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# Intermediated Surge Pricing

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## Abstract

I study a market in which a profit-maximizing intermediary facilitates trade between buyers and sellers. The intermediary sets prices for buyers and sellers, and keeps the difference as her fee. Optimal prices increase with demand and, under plausible conditions, the optimal percent fee decreases with demand. However, if the intermediary keeps a constant percent fee regardless of demand, as is the case for some intermediaries, the price paid by buyers during high (low) demand increases (decreases) even further; that is, surge pricing is amplified.

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# 1 Introduction

Intermediaries abound in markets, facilitating trade between buyers and sellers. The services they provide include reduction in search costs, information exchange, access to inventory, and diversification of risk. While the digital economy has diminished the need for intermediaries in some areas it has also created new markets intermediated by middlemen.

Some of these new intermediated markets are for goods or services for immediate delivery. Uber, the online transportation company, provides a motivating example. The firm is an intermediary between car drivers and passengers. Its superior matching technology reduces search costs on both sides of the market. Uber sets a price (per mile) that passengers pay and takes a fixed percent fee from each transaction it mediates. Uber's software system allows it to monitor local demand and supply conditions in real time. While Uber responds to sharp increases in demand by raising price, it keeps the same percent fee, usually 20% of revenue, at each demand level. Thus, the payment that a car driver receives is a fixed percent of the payment made by the passenger. Conceivably, if Uber were to reduce its percent fee during high demand, it may increase profits by enticing more drivers to enter the supply pool. Further, the flexibility afforded by not having the payment by a passenger and the payment to a driver in lock step may moderate price increases during periods of high demand.

These questions are explored in a simple market model with a monopolist intermediary who sets prices for buyers and sellers. The focus is on how optimal prices vary with changes in demand, especially when there is some inflexibility in the prices set by the intermediary.

Consider a market with an intermediary who facilitates trade between a large number of buyers and sellers. Search costs for buyers and sellers are larger than the gains to trade; hence without the intermediary there is (essentially) no trade. This is the case for several markets that have experienced extraordinary growth after the advent of the internet and digital communication enabled intermediaries to reduce search costs. In the model, each buyer's value and each seller's cost for the good are

private information. Consequently, the intermediary does not have the ability to price discriminate between buyers or between sellers. The intermediary sets a price for all buyers and a price for all sellers to maximize her profit. As there are a large number of buyers and sellers, it is optimal for each to act as a price-taker. The intermediary's profit is the difference between the buyer price and the seller price multiplied by the number of units traded.

The technological advances that have reduced search costs and enabled better matches between buyers and sellers have also made it easier for the intermediary to monitor demand and supply and adjust prices accordingly. Changes in optimal prices with changes in market conditions are a focus of this paper. Of particular interest is the revenue share<sup>1</sup> of the intermediary at optimal prices and changes in this optimal share with changes in demand and supply. To capture changes in market conditions in a tractable manner, in the model there is a continuum of buyers and a continuum of sellers rather than a large finite number of each.<sup>2</sup> An increase in the mass of buyers corresponds to an increase in demand relative to supply.

It is shown that there exists a unique set of optimal prices at which the intermediary's profit is maximized. As the mass of buyers increases, the optimal price charged to buyers and paid to sellers increase – also known as *surge pricing*. However, the optimal share (i.e., percent fee) of the intermediary may increase or decrease.

The intermediary's role consists of two interlinked parts: it acts as a monopolist in its interactions with buyers and as a monopsonist in its interactions with sellers. The marginal cost of the intermediary monopolist is determined by the intermediary monopsonist, while the marginal revenue of the intermediary monopsonist is determined by the intermediary monopolist. The change in the optimal percent fee of the intermediary due to an increase in the mass of buyers depends on the elasticities of demand and supply through their effect on the gross mark-up of the intermediary monopolist and on the gross mark-down of the intermediary monopsonist. As demand increases, the optimal percent fee of the intermediary decreases if and only if the product of the gross mark-up and gross mark-down decreases.

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<sup>1</sup>The percent of buyer price that is taken by the intermediary as fees.

<sup>2</sup>Myerson and Satterthwaite [12] obtain the optimal mechanism for a profit-maximizing intermediary with one buyer and one seller.

Consequently, a constant percent fee is rarely optimal for the intermediary. For a sufficiently large increase in buyer mass, the optimal percent fee of the intermediary decreases. During periods of very high demand, reducing the intermediary's percent fee draws more sellers in, thereby increasing the number of trades and making it optimal for the intermediary to capture a smaller fraction of this larger pie. However, firms such as Uber and Lyft extract the same fraction, charging a constant percent fee of 20% of buyer price, regardless of the level of demand. This inflexibility in pricing results in a match between demand and supply that is less than optimal for the intermediary.

Apart from being sub-optimal, a constant percent fee may also amplify the surge in buyer prices when there is a sharp increase in demand. Recall that in the absence of a constant percent fee constraint, it is optimal for the intermediary to reduce its percent fee when the demand increase is large enough. In this scenario, under a constant percent fee constraint the intermediary will charge a fee that is smaller than optimal during low demand and greater than optimal during high demand. This in turn increases the surge in buyer prices. Specifically, a constant percent fee increases the buyer price during high demand and decreases the buyer price during low demand, compared to optimal prices when the intermediary's fee is flexible. Under these conditions, a constant percent fee also diminishes the surge in seller prices which leads to fewer sellers during high demand than is optimal for the intermediary.

There may be a legal rationale for Uber's constant percent fee. Currently, the firm is a defendant in lawsuits that question its standing as an intermediary. The plaintiffs are sellers (Uber car drivers) who claim that they should be considered employees of Uber and are entitled to various benefits under labor regulations. An important consideration in the lawsuit is who controls the prices and fees.<sup>3</sup> Plausibly, if Uber were to exercise greater flexibility in its pricing, such as fine tune its percent fee to changes in demand, then Uber's legal claim that it acts as an intermediary may be weakened. This imposes a constraint on its pricing decisions. Thus, while a constant percent fee may strengthen Uber's position in pending litigation, it is costly for two reasons. Not only does a constant percent fee reduce profits, it also exacerbates surge

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<sup>3</sup>See Douglas O'Connor et al. v. Uber Technologies et al., 2015 [8]. Similar lawsuits have been filed against other intermediaries including Lyft, Postmates, Instacart, GrubHub, and Shyp.

pricing leading to a loss in buyer goodwill.<sup>4</sup>

The question addressed in this paper – the impact of some rigidity in pricing on surge pricing – has not been asked in the literature. Several papers, notably Armstrong [1], Caillaud and Jullien [7], and Rochet and Tirole [14, 15], analyze two-sided markets, where two types of agents interact on a platform provided by a third party. Indirect network externalities play a significant role in such markets. In particular, costs on one side of the market cannot be easily passed through to the other side. Consequently, the terms of trade depend not only on the total amount that the two parties to a transaction pay but also on the division of the payments.

An older literature investigates equilibrium and efficiency in markets with intermediaries. Rubinstein and Wolinsky [16] obtain a steady-state equilibrium in a market with buyers, sellers, and intermediaries, highlighting the relationship between trading rules and the endogenous terms of trade. Stahl [18], Spulber [17], and Yanelle [19] explore conditions under which competition between intermediaries leads to (in)efficient outcomes. The welfare-improving role of a monopolist intermediary in a market with search costs is examined in Yavas [20]. Biglaiser [4] shows that a fully-informed intermediary can overcome adverse selection between a buyer and a seller in a lemons market, while Glode and Opp [9] show that one or more moderately-informed intermediaries can also achieve efficiency in such a market.

Recent work in operations management investigates static and dynamic contracts in intermediated markets. See Banerjee, Johari, and Riquelme [2], Cachon, Daniels, and Lobel [6], and Bai et. al [3].

The paper is organized as follows. The model is presented in the next section. In Section 3, optimal prices are derived and comparative statics for the optimal fee for the intermediary with respect to changes in demand are investigated. Pricing under a constant-fee constraint is examined in Section 4. Section 5 concludes. All proofs are in an appendix.

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<sup>4</sup>See Grubb [10], for instance.

## 2 The model

There are (potential) gains to trade between buyers and sellers. Each buyer has utility for one unit of a homogenous object and each seller has one unit of the object to sell. I assume that there is a continuum of buyers and a continuum of sellers, with each buyer and seller being infinitesimal. There is a mass  $\mu$  of buyers and a unit mass of sellers.<sup>5</sup> Changes in demand are captured by changes in  $\mu$ . Each buyer's valuation  $v$  has cumulative distribution function  $F_b$  with strictly positive and continuous density function  $f_b$  on support  $[0, 1]$ . Each seller's cost  $c$  has cumulative distribution function  $F_s$  with strictly positive and continuous density function  $f_s$  on support  $[0, 1]$ .

In the price-theoretic interpretation of mechanism design due to Bulow and Roberts [5], the demand curve is  $q = \mu(1 - F_b(p_b))$  and the supply curve is  $q = F_s(p_s)$ , where  $p_b$  is the buyer price and  $p_s$  is the seller price.

The search costs for buyers and sellers are assumed to be prohibitively high. An intermediary with superior matching technology enables trade between the two sides. The intermediary in our model is a matchmaker who does not trade but simply matches buyers with sellers. She is not a market maker who buys and holds inventory. This role as a matchmaker but not a market maker is appropriate in markets for immediate delivery of perishable goods.

The intermediary knows  $\mu$ ,  $F_b$ , and  $F_s$ . However, a buyer's value or a seller's cost are not known to the intermediary. Thus, perfect price discrimination is not an option for the intermediary. To simplify the notation, the intermediary's fixed cost and marginal costs per transaction are assumed to be zero.

An assumption that is maintained throughout is that the distributions  $F_b$  and  $F_s$  are *regular* in the sense of Myerson [11]. That is, the virtual utility of buyers,  $v - \frac{1-F_b(v)}{f_b(v)}$ , is strictly increasing in  $v$  and the virtual cost of sellers,  $c + \frac{F_s(c)}{f_s(c)}$ , is strictly increasing in  $c$ .<sup>6</sup>

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<sup>5</sup>The assumption of a unit mass of sellers is without loss of generality as  $\mu$  may be viewed as the mass of buyers per unit mass of sellers.

<sup>6</sup>Most of the results in the paper can be proved if the virtual utility and virtual cost functions are weakly increasing. However, the proofs are simpler when these functions are assumed to be strictly increasing.

From earlier work, it follows that the intermediary cannot increase her profits by using a randomized selling mechanism. In a general environment, Riley and Zeckhauser [13] showed that a seller's best strategy is to charge a take-it-or-leave-it price to buyers. Myerson and Satterthwaite [12] showed that it is optimal for a profit-maximizing intermediary between one buyer and one seller to charge take-it-or-leave-it prices. These papers imply that in the model with a continuum of buyers and sellers considered here, we may restrict attention to deterministic selling mechanisms, i.e., to prices.

**Proposition A:** (MYERSON AND SATTERTHWAITE [12], RILEY AND ZECKHAUSER [13])  
*The optimal strategy for the intermediary is to announce a take-it-or-leave-it price for buyers and a take-it-or-leave-it price for sellers.*

Hence, in order to maximize profits, the intermediary selects a price  $p_b$  for buyers and a price  $p_s$  for sellers. Any buyer with value greater than  $p_b$  will purchase a unit and any seller with cost less than  $p_s$  will sell a unit, through the intermediary. Optimal prices are derived in the next section.

### 3 Optimal intermediation

At price  $p_b = v$ , buyers demand  $\mu(1 - F_b(v))$  units and at price  $p_s = c$ , sellers are willing to supply  $F_s(c)$  units. Thus,  $q = \min\{\mu(1 - F_b(v)), F_s(c)\}$  is the amount traded. If  $\mu(1 - F_b(v)) < F_s(c)$  then the intermediary can lower the seller price  $p_s$  slightly below  $c$  and still trade  $q$  units. Similarly, if  $\mu(1 - F_b(v)) > F_s(c)$  then the intermediary can raise the buyer price  $p_b$  slightly above  $v$  and still trade  $q$  units. Hence, intermediary profit-maximization implies that optimal prices  $p_b = v$  and  $p_s = c$  are such that demand equals supply:

$$q = \mu[1 - F_b(v)] = F_s(c) \tag{1}$$

Thus,  $v = F_b^{-1}(1 - \frac{q}{\mu})$ ,  $c = F_s^{-1}(q)$  and the intermediary's profit as a function of  $q$  is

$$\begin{aligned} \Pi_I(q) &= q[v - c] \\ &= q[F_b^{-1}(1 - \frac{q}{\mu}) - F_s^{-1}(q)] \end{aligned}$$



$$\begin{aligned}
\Rightarrow \quad \frac{d\Pi_I}{dq} &= F_b^{-1}\left(1 - \frac{q}{\mu}\right) - F_s^{-1}(q) + q \frac{dF_b^{-1}\left(1 - \frac{q}{\mu}\right)}{dq} - q \frac{dF_s^{-1}(q)}{dq} \\
&= F_b^{-1}\left(1 - \frac{q}{\mu}\right) - F_s^{-1}(q) - \frac{q}{\mu f_b(F_b^{-1}(1 - \frac{q}{\mu}))} - \frac{q}{f_s(F_s^{-1}(q))} \\
&= v - c - \frac{1 - F_b(v)}{f_b(v)} - \frac{F_s(c)}{f_s(c)} \tag{2}
\end{aligned}$$

The first-order condition,  $\frac{d\Pi_I}{dq} = 0$ , implies that at optimal prices  $p_b = v$  and  $p_s = c$ ,

$$v - \frac{1 - F_b(v)}{f_b(v)} = c + \frac{F_s(c)}{f_s(c)} \tag{3}$$

Thus, a necessary condition for optimality is that the virtual utility of the marginal buyer equals the virtual cost of the marginal seller. Following Bulow and Roberts [5], this necessary condition may be interpreted as stating that marginal revenue equals marginal cost.

Note that (3) is independent of  $\mu$ . There are many solutions to (3), one for each  $\mu$ . As shown below, the demand equals supply condition (1) pins down a unique solution.

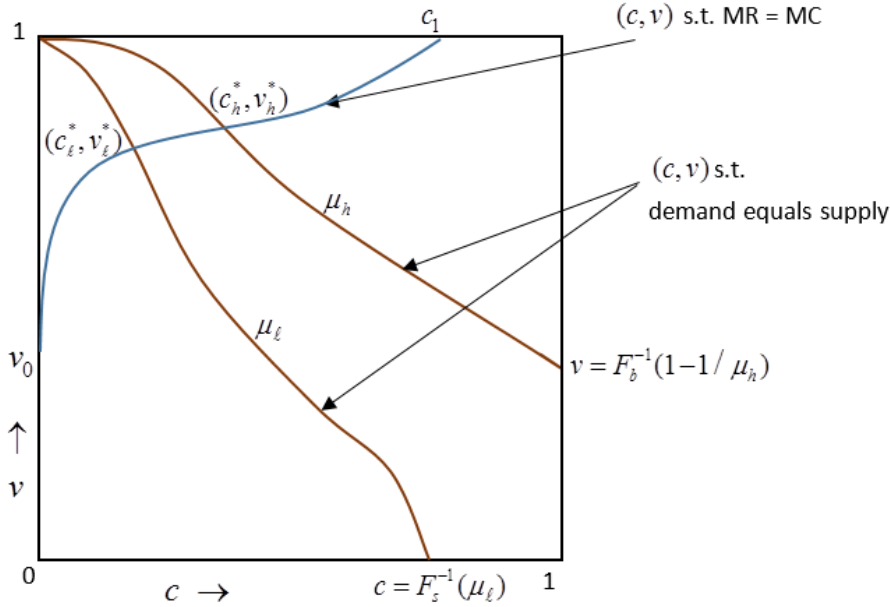


Figure 1

As the densities  $f_s$  and  $f_b$  are strictly positive and continuous on their support the locus of points  $(c, v)$  satisfying (3) is a continuous curve. This is the blue curve, labeled

‘MR=MC,’ in Figure 1. This curve is a positively-sloped function in the regular case. To see this, start at any point  $(c, v)$  on the ‘MR=MC’ curve. If the cost is increased from  $c$  to  $c + \Delta c$ , then the right-hand side of (3) increases by regularity. To restore equality in (3) the buyer’s value must be increased from  $v$ , once again by regularity. Hence, there is a  $\Delta v > 0$  such that  $(c + \Delta c, v + \Delta v)$  is on the ‘MR=MC’ curve. The points  $v_0 > 0$  and  $c_1 < 1$  are obtained from  $v_0 - \frac{1-F_b(v_0)}{f_b(v_0)} = 0$  and  $c_1 + \frac{F_s(c_1)}{f_s(c_1)} = 1$ , respectively. Buyers with  $v < v_0$  and sellers with  $c > c_1$  do not trade. Note also that  $MR = v - \frac{1-F_b(v)}{f_b(v)} > c + \frac{F_s(c)}{f_s(c)} = MC$  above the ‘MR=MC’ curve and  $MR < MC$  below this curve.

The two negatively-sloped brown curves, labeled demand equals supply, represent the points  $(c, v)$  that satisfy (1) for buyer mass  $\mu_\ell$  and  $\mu_h$ , respectively. In the figure,  $\mu_\ell < 1 < \mu_h$ . The intersection of the (blue) ‘MR=MC’ curve with a (brown) demand equals supply curve corresponding to buyer mass  $\mu_k$ ,  $k = \ell, h$  yields the optimal  $(c_k^*, v_k^*)$ . This is proved next.

**Proposition 1** *Assume that  $F_b$  and  $F_s$  are regular and the mass of buyers is  $\mu$ . There exists a unique pair of optimal prices  $(c^*(\mu), v^*(\mu))$  at which the intermediary’s profit is maximized. Moreover,  $\frac{dc^*(\mu)}{d\mu} > 0$  and  $\frac{dv^*(\mu)}{d\mu} > 0$ .*

The following example illustrates the equilibrium.

EXAMPLE 1: If  $F_b$  and  $F_s$  are uniformly distributed on  $[0, 1]$  then (1) and (3) are

$$\begin{aligned} \mu(1 - v^*) &= c^*, & 2v^* - 1 &= 2c^* \\ \implies v^* &= 1 - \frac{1}{2(1 + \mu)}, & c^* &= \frac{\mu}{2(1 + \mu)}, & \Pi_I &= \frac{\mu}{4(1 + \mu)} \end{aligned}$$

The distributions in this example are regular. Proposition 1 implies that optimal prices increase with  $\mu$ . This can be verified directly as,

$$\frac{dv^*}{d\mu} = \frac{1}{2(1 + \mu)^2} > 0, \quad \frac{dc^*}{d\mu} = \frac{1}{2(1 + \mu)^2} > 0$$

The optimal percent fee of the intermediary (expressed as a fraction of the buyer price),  $\alpha^*$  is

$$\alpha^* \equiv \frac{v^* - c^*}{v^*} = \frac{1 + \mu}{1 + 2\mu}$$

Note that for any  $\mu$ ,  $v^* - c^* = 0.5$ . Thus, as  $v^*$  increases with  $\mu$ ,  $\alpha^*$  decreases

$$\frac{d\alpha^*}{d\mu} = -\frac{1}{(1+2\mu)^2} < 0$$

□

In the example, as buyer mass  $\mu$  increases, the intermediary's optimal percent fee,  $\alpha^*$ , decreases. However, in general  $\alpha^*$  may increase or decrease with  $\mu$ , depending on the elasticities of the demand and supply curves.

The intermediary is both a monopolist and a monopsonist. It acts as a monopolist in its interactions with buyers, with its marginal cost determined by the equilibrium in the sellers' market. The intermediary also acts as a monopsonist in its interactions with sellers, with its marginal revenue from a unit of input determined by the equilibrium in the buyers' market. With this in mind, define price elasticities of demand and supply

$$\begin{aligned}\eta_b(v) &= \frac{v}{q} \frac{dq}{dv} = -v \frac{f_b(v)}{1 - F_b(v)} \\ \eta_s(c) &= \frac{c}{q} \frac{dq}{dc} = c \frac{f_s(c)}{F_s(c)}\end{aligned}$$

where we use  $q = \mu(1 - F_b(v))$  and  $q = F_s(c)$ .

Consider the intermediary in its role as a monopolist with constant marginal cost  $c$ . From price theory we know that the price  $v$  charged by this monopolist satisfies

$$\frac{v}{c} = \frac{1}{1 + \frac{1}{\eta_b(v)}}$$

Call  $\frac{1}{1 + \frac{1}{\eta_b(v)}}$  the *gross mark-up* for our intermediary monopolist. Next, consider the intermediary in its role as a monopsonist with constant marginal revenue product from a unit of input equal to  $v$ . It will set price for the input at  $c$  such that

$$\frac{v}{c} = 1 + \frac{1}{\eta_s(c)}$$

Call  $1 + \frac{1}{\eta_s(c)}$  the *gross mark-down* for our intermediary monopsonist.

The product of the gross mark-up and the gross mark-down determines whether the intermediary's optimal percent fee increases or decreases with  $\mu$ .

**Proposition 2** *The intermediary's optimal percent fee,  $\alpha^*(\mu) = \frac{v^*(\mu) - c^*(\mu)}{v^*(\mu)}$  decreases with  $\mu$  if and only if the product of the gross mark-up of the intermediary monopolist and the gross mark-down of the intermediary monopsonist*

$$\frac{1 + \frac{1}{\eta_s(c^*)}}{1 + \frac{1}{\eta_b(v^*)}} \quad (4)$$

*decreases with  $\mu$ .*

By Proposition 1, optimal prices  $c^*(\mu)$  and  $v^*(\mu)$  increase as  $\mu$  increases. It is reasonable that  $|\eta_b(v)|$  increases (i.e., demand becomes more elastic) as  $v$  increases;<sup>7</sup> therefore the denominator of the expression in (4) increases. If, in addition,  $\eta_s(c)$  either increases or does not decrease too fast as  $c$  increases, then  $\alpha^*(\mu)$  decreases with  $\mu$ .

Another necessary and sufficient condition for  $\frac{d\alpha^*(\mu)}{d\mu} < 0$  is that  $\frac{c^*}{v^*} \frac{dv^*}{dc^*}$ , the elasticity of  $v^*$  with respect to  $c^*$  along the locus of points satisfying equation (3), is less than 1. This is established next.

**Proposition 3** *The intermediary's optimal percent fee,  $\alpha^*(\mu) = \frac{v^* - c^*}{v^*}$  decreases with  $\mu$  if and only if*

$$\frac{dv^*}{dc^*} < \frac{v^*}{c^*}$$

*In particular, if  $\frac{dv^*}{d\mu} \leq \frac{dc^*}{d\mu}$  then  $\frac{d\alpha^*}{d\mu} < 0$ .*

The necessary and sufficient condition in Proposition 3 admits a geometric interpretation. This condition states that, at a specific value of  $\mu$ , the slope of the 'MR=MC' curve in Figure 1 (the locus of points satisfying equation 3) is less than the slope of the straight line from the origin to the point on the 'MR=MC' curve.

While Propositions 2 and 3 are stated for local changes in buyer mass  $\mu$ , they are readily adapted to large changes in  $\mu$ . Let  $\mu_h > \mu_\ell$ . Then  $\alpha^*(\mu_\ell) > \alpha^*(\mu_h)$  if and only if

$$\frac{1 + \frac{1}{\eta_s(c^*(\mu_\ell))}}{1 + \frac{1}{\eta_b(v^*(\mu_\ell))}} > \frac{1 + \frac{1}{\eta_s(c^*(\mu_h))}}{1 + \frac{1}{\eta_b(v^*(\mu_h))}}$$

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<sup>7</sup>This is certainly true if the hazard rate  $\frac{f_b(v)}{1-F_b(v)}$  of  $F_b$  increases with  $v$ , i.e.,  $1 - F_b(v)$  is log concave.

if and only if

$$\frac{v^*(\mu_h) - v^*(\mu_\ell)}{c^*(\mu_h) - c^*(\mu_\ell)} \leq \frac{v^*(\mu_\ell)}{c^*(\mu_\ell)}$$

The proof is omitted.

As shown next, when starting from a low buyer mass, the optimal percent fee for the intermediary decreases for a sufficiently large increase in buyer mass. This is regardless of the elasticities of supply and demand.

**Proposition 4** *There exist  $\underline{\mu}$  and  $\bar{\mu}$ ,  $\underline{\mu} \leq \bar{\mu}$ , such that for any  $\mu_\ell < \underline{\mu}$  and any  $\mu_h \geq \bar{\mu}$ , we have  $\alpha^*(\mu_h) < \alpha^*(\mu_\ell)$ .*

The intuition behind Proposition 4 is as follows. Observe that the intermediary will never trade with a buyer with negative marginal revenue,  $v < v_0$ , or a seller with marginal cost greater than 1,  $c > c_1$ . If  $\mu$  is much smaller than 1, then the optimal buyer price is  $v_0 + \epsilon$  where  $\epsilon$  is small, positive. As  $\mu \ll 1$ , demand from buyers with value greater than  $v_0 + \epsilon$  can be satisfied by a small fraction of the unit mass of sellers – those with costs less than  $\epsilon'$ , where again  $\epsilon'$  is small and positive. Thus, the intermediary's fee is  $\frac{v_0 + \epsilon - \epsilon'}{v_0}$ , which is close to 1 as  $\epsilon$  and  $\epsilon'$  become arbitrarily small as  $\mu$  approaches 0. If instead,  $\mu \gg 1$ , then the optimal buyer price is  $1 - \hat{\epsilon}$  and the optimal seller price is  $c_1 - \hat{\epsilon}'$ . The intermediary's fee in this case is  $\frac{1 - \hat{\epsilon} - c_1 + \hat{\epsilon}'}{1 - \hat{\epsilon}}$  which is substantially less than 1 as  $c_1 \in (0, 1)$  is a fixed number while  $\hat{\epsilon}$  and  $\hat{\epsilon}'$  become arbitrarily small as  $\mu$  increases. The continuity of  $\alpha^*(\mu)$  with respect to  $\mu$  implies the proposition.

That  $\underline{\mu} = \bar{\mu}$  is possible in Proposition 4 follows from Example 1 where it was directly established that  $\frac{d\alpha^*(\mu)}{d\mu} < 0$  for each  $\mu$ . In this example, the locus of points satisfying (3) is  $v^* = c^* + 0.5$ . Thus,  $\frac{dv^*}{dc^*} = 1 < \frac{v^*}{c^*}$ , verifying the necessary and sufficient condition of Proposition 3. Alternatively, note that in Example 1,

$$\begin{aligned} \eta_b(v) &= -\frac{v}{1-v} & \eta_s(c) &= 1 \\ \implies \frac{1 + \frac{1}{\eta_s(c^*)}}{1 + \frac{1}{\eta_b(v^*)}} &= \frac{2}{1 - \frac{1-v^*}{v^*}} = \frac{2v^*}{2v^* - 1} \end{aligned}$$

which decreases with  $v^*$  and hence with  $\mu$ . This verifies the necessary and sufficient condition of Proposition 2.

## COMPARISON WITH A BENCHMARK MONOPOLIST

It is useful to compare pricing by the intermediary with pricing by a monopolist who acquires the productive assets and services of all the sellers who contract with the intermediary. I make this comparison under the assumption that the monopolist's total cost of providing goods remain unchanged.<sup>8</sup>

It is easy to show that the marginal cost of the monopolist at  $q = F_s(c)$  units is  $c$ , which is less than  $c + \frac{F_s(c)}{f_s(c)}$ , the marginal cost at  $q$  units for the intermediary. The marginal revenue of the monopolist is the same as that of the intermediary. Consequently, the benchmark monopolist's optimal buyer price is lower and the quantity higher than the corresponding optimal values for the intermediary. Monopoly prices,  $(c_m, v_m)$ , are less inefficient than intermediary prices,  $(c^*, v^*)$ , as can be seen from

$$v_m - c_m = \frac{1 - F_b(v_m)}{f_b(v_m)} < \frac{1 - F_b(v_m)}{f_b(v_m)} + \frac{F_s(c_m)}{f_s(c_m)}$$

Thus, while  $(c^*, v^*)$  is on the 'MR=MC' curve in Figure 1,  $(c_m, v_m)$  is below this curve. Moreover, it can be shown that  $v^* > v_m$  and  $c_* < c_m$ . Consequently, as  $v^* - c^* > v_m - c_m > 0$ , the monopoly outcome is more efficient than the intermediary outcome.

Observe that if the hazard rate of  $F_b$  is increasing in  $v$  then  $\frac{1-F_b(v)}{f_b(v)}$  decreases with  $v$ . Therefore,  $v_m - c_m$  decreases as  $\mu$  increases and so does the monopolist's percent fee  $\alpha_m = \frac{v_m - c_m}{v_m}$ .

## 4 Surge pricing under constrained intermediation

As mentioned earlier, intermediaries such as Uber and Lyft keep a constant percent of the buyer price regardless of demand conditions. I show below that, under reasonable conditions, if an intermediary operates under the constraint that its percent fee is

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<sup>8</sup>From the resistance of some intermediaries to classify sellers as employees rather than as independent contractors it appears that this may not be a tenable assumption. That is, total costs may be higher if the intermediary acquired the assets and services of the sellers. Nevertheless, it is instructive to consider this benchmark monopolist with the same cost of providing service as the sellers collectively.

constant then as demand increases the magnitude of surge in buyer price is amplified and the magnitude of surge in seller price is diminished, compared to unconstrained optimal prices.

It is sufficient to consider two possible levels of demand, high or low. The mass of buyers is  $\mu_h$  during high demand and  $\mu_\ell$  during low demand, with  $\mu_h > \mu_\ell$ . The fraction of time that demand is high is  $r$ . Alternatively,  $r$  may be viewed as the probability that demand is high at any given moment. The constraint is that the intermediary keeps the same fraction  $\alpha$  of the buyer price regardless of the level of demand.<sup>9</sup> Thus, rather than choose any prices  $v_h, c_h$  during high demand and any prices  $v_\ell, c_\ell$  during low demand, the intermediary is constrained to choose  $v_h, v_\ell$  and  $\alpha$  and set  $c_h = (1 - \alpha)v_h, c_\ell = (1 - \alpha)v_\ell$ . That is, prices must satisfy

$$\frac{1}{1 - \alpha} = \frac{v_h}{c_h} = \frac{v_\ell}{c_\ell} \quad (5)$$

for some  $\alpha \in (0, 1)$ . I refer to (5) as the constant-fee constraint.

Let  $(\hat{c}_\ell, \hat{v}_\ell)$  and  $(\hat{c}_h, \hat{v}_h)$  be the optimal prices at  $\mu_\ell$  and at  $\mu_h$  respectively under the constant-fee constraint. Let

$$\hat{\alpha} = \frac{\hat{v}_\ell - \hat{c}_\ell}{\hat{v}_\ell} = \frac{\hat{v}_h - \hat{c}_h}{\hat{v}_h}$$

be the optimal fee of the intermediary under this constraint.

As already mentioned, the object being sold is perishable and cannot be stored. Thus, optimality implies that demand must equal supply under each of the two demand scenarios. That is, (1) is satisfied at  $\mu_\ell$  and at  $\mu_h$ . Therefore,  $(\hat{c}_k, \hat{v}_k)$  lies on the demand equals supply curve for  $\mu_k, k = \ell, h$  in Figure 2. A consequence is that

$$\hat{c}_h > \hat{c}_\ell \quad \text{and} \quad \hat{v}_h > \hat{v}_\ell \quad (6)$$

To see this, note that the constant-fee constraint states that the straight line through  $(\hat{c}_\ell, \hat{v}_\ell)$  and  $(\hat{c}_h, \hat{v}_h)$  passes through the origin and has slope  $\frac{1}{1 - \hat{\alpha}} > 0$ . As the demand equals supply curve for  $\mu_h$  lies above the demand equals supply curve for  $\mu_\ell$  (see proof of Proposition 1), (6) follows.

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<sup>9</sup>Equivalently, the intermediary adds a constant mark-up of  $\frac{\alpha}{1 - \alpha}\%$  to the seller price to obtain the buyer price.

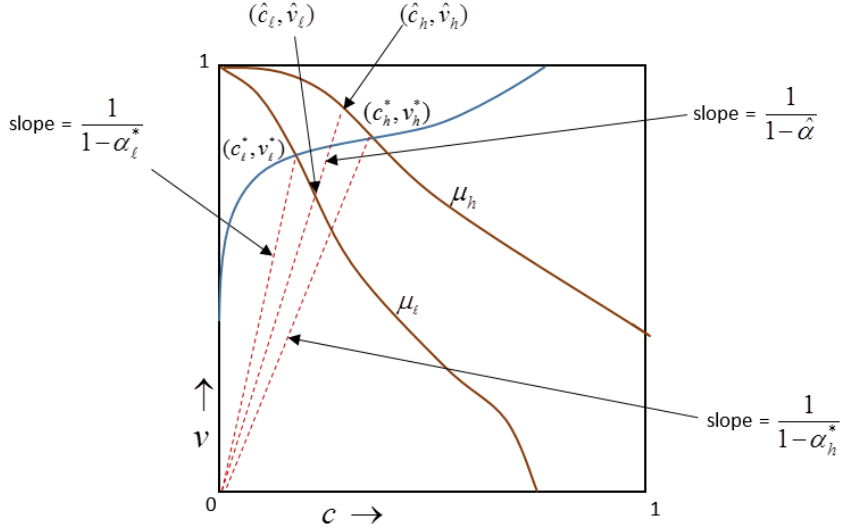


Figure 2

Let  $v_k^* = v^*(\mu_k)$ ,  $c_k^* = c_k^*(\mu_k)$ , and  $\alpha_k^* = \alpha_k^*(\mu_k)$  be the (unconstrained) optimal prices and percent fee at buyer mass  $\mu_k$ ,  $k = \ell, h$ . The next proposition says that the constrained optimal fee for the intermediary is in between the unconstrained optimal fees at  $\mu_\ell$  and  $\mu_h$ .

**Proposition 5** *Under the constant-fee constraint, (5), the optimal fee for the intermediary,  $\hat{\alpha}$ , satisfies*

$$\min\{\alpha_\ell^*, \alpha_h^*\} \leq \hat{\alpha} \leq \max\{\alpha_\ell^*, \alpha_h^*\}, \quad (7)$$

*with strict inequalities if  $\alpha_\ell^* \neq \alpha_h^*$ . Further, optimal prices  $(\hat{c}_\ell, \hat{v}_\ell)$ ,  $(\hat{c}_h, \hat{v}_h)$  and an optimal fee  $\hat{\alpha} = \frac{\hat{v}_k - \hat{c}_k}{\hat{v}_k}$  exist.*

The necessity of (7) can be seen from Figure 2. As  $\alpha_\ell^* > \alpha_h^*$  in the figure, we have  $\frac{1}{1-\alpha_\ell^*} > \frac{1}{1-\alpha_h^*}$ . That is, the line from the origin to  $(c_\ell^*, v_\ell^*)$  is steeper than the line from the origin to  $(c_h^*, v_h^*)$ . According to Proposition 5,  $\alpha_\ell^* > \hat{\alpha} > \alpha_h^*$ . Hence, as depicted in Figure 2, the line from the origin to  $(\hat{c}_\ell, \hat{v}_\ell)$  and  $(\hat{c}_h, \hat{v}_h)$ <sup>10</sup> is less steep [steeper] than the line from the origin to  $(c_\ell^*, v_\ell^*)$  [( $c_h^*, v_h^*$ )]. If, say  $\hat{\alpha} > \alpha_\ell^*$  ( $> \alpha_h^*$ ), then the line from the origin to the purported constrained optimal prices would be steeper than the line from the origin to  $(c_\ell^*, v_\ell^*)$ ; the constrained optimal prices would lie on the portions of the demand equals supply curves for  $\mu_\ell$  and for  $\mu_h$  that lie

<sup>10</sup>Both points lie on the same line due to the constant-fee constraint.



above the ‘MR=MC’ curve. In this region, marginal revenue exceeds marginal cost. Decreasing  $\hat{\alpha}$  by lowering buyer prices and raising seller prices, while staying on the demand equals supply curves, would increase the intermediary’s profit at  $\mu_\ell$  and at  $\mu_h$ . Thus,  $\hat{\alpha} > \max\{\alpha_\ell^*, \alpha_h^*\}$  cannot be optimal. A symmetric argument establishes that  $\hat{\alpha} < \min\{\alpha_\ell^*, \alpha_h^*\}$  is not optimal.

The preceding analysis shows that if  $\alpha_\ell^* > \alpha_h^*$  then  $(\hat{c}_h, \hat{v}_h)$  is to the northwest of  $(c_h^*, v_h^*)$  and  $(\hat{c}_\ell, \hat{v}_\ell)$  is to the southeast of  $(c_h^*, v_h^*)$ . This leads to the result that the constant-fee constraint amplifies surge pricing for buyers while diminishing surge pricing for sellers.

**Proposition 6** *If  $\mu_\ell < \mu_h$  is such that  $\alpha_\ell^* > \alpha_h^*$ , then as the mass of buyers increases from  $\mu_\ell$  to  $\mu_h$  the surge in buyer prices is greater and the surge in seller prices is smaller under the constant-fee constraint than at unconstrained optimal prices. That is,*

$$\begin{aligned} \hat{v}_h &> v_h^* > v_\ell^* > \hat{v}_\ell \\ \hat{c}_h^* &> \hat{c}_h > \hat{c}_\ell > c_\ell^* \end{aligned}$$

From Proposition 4 we know that there exist  $\underline{\mu} \leq \bar{\mu}$  such that  $\alpha_\ell^* > \alpha_h^*$  for any  $\mu_\ell < \underline{\mu}$  and any  $\mu_h \geq \bar{\mu}$ . Together with Proposition 6 we have

**Corollary 1** *There exist  $\underline{\mu}$  and  $\bar{\mu}$ ,  $\underline{\mu} \leq \bar{\mu}$ , such that for any  $\mu_\ell < \underline{\mu}$  and any  $\mu_h \geq \bar{\mu}$ , as the mass of buyers increases from  $\mu_\ell$  to  $\mu_h$  the surge in buyer prices is greater and the surge in seller prices is smaller under the constant-fee constraint than at unconstrained optimal prices.*

Similarly, from Propositions 2 and 6 we have

**Corollary 2** *If the product of the gross mark-up of the intermediary monopolist and the gross mark-down of the intermediary monopsonist decreases as the mass of buyers increases from  $\mu_\ell$  to  $\mu_h$  the surge in buyer prices is greater and the surge in seller prices is smaller under the constant-fee constraint than at unconstrained optimal prices.*

The impact of a constant fee on efficiency is mixed. Under the constant-fee constraint, efficiency deteriorates (improves) in the demand condition with the lower

(higher) unconstrained percent fee. To see this, observe that at the efficient outcome the intermediary's profit is zero: the efficient outcome with buyer mass  $\mu_k$ ,  $k = \ell, h$  is  $v_k^e = c_k^e$  obtained by the intersection in Figure 2 of the demand equals supply curve for  $\mu_k$  with the positive-sloped diagonal (not shown in the figure). The unconstrained optimal prices are inefficient as  $v_k^* > c_k^*$ , i.e.,  $(c_k^*, v_k^*)$  is above the diagonal. Any movement from  $(c_k^*, v_k^*)$  towards (away from) the diagonal along the demand equals supply curve increases (decreases) the gains from trade. In Figure 2,  $\alpha_h^* < \alpha_\ell^*$ , and thus  $\alpha_h^* < \hat{\alpha} < \alpha_\ell^*$  by Proposition 5. Consequently, the constrained optimal prices  $(\hat{c}_h, \hat{v}_h)$  are further away from the diagonal than  $(c_h^*, v_h^*)$  and the constant-fee constraint decreases efficiency in the high demand setting. Similarly,  $\alpha_h^* < \alpha_\ell^*$  implies that efficiency increases in the low demand setting under the constant-fee constraint.

A necessary condition for optimality under the constant-fee constraint is presented below.

**Lemma 1** *Under the constant-fee constraint, (5), the optimal prices  $(\hat{c}_\ell, \hat{v}_\ell)$  at  $\mu_\ell$  and  $(\hat{c}_h, \hat{v}_h)$  at  $\mu_h$  satisfy*

$$\begin{aligned} \left[ \hat{v}_h - \frac{1 - F_b(\hat{v}_h)}{f_b(\hat{v}_h)} \right] - \left[ \hat{c}_h + \frac{F_s(\hat{c}_h)}{f_s(\hat{c}_h)} \right] &= \frac{\lambda \hat{c}_\ell}{r} \frac{1 - F_b(\hat{v}_h)}{f_b(\hat{v}_h)} + \frac{\lambda \hat{v}_\ell}{r} \frac{F_s(\hat{c}_h)}{f_s(\hat{c}_h)} \\ &= \frac{\lambda(1 - \hat{\alpha})\hat{v}_\ell\hat{v}_h}{r} \left[ \frac{1}{|\eta_b(\hat{v}_h)|} + \frac{1}{\eta_s(\hat{c}_h)} \right] \end{aligned} \quad (8)$$

$$\begin{aligned} \left[ \hat{v}_\ell - \frac{1 - F_b(\hat{v}_\ell)}{f_b(\hat{v}_\ell)} \right] - \left[ \hat{c}_\ell + \frac{F_s(\hat{c}_\ell)}{f_s(\hat{c}_\ell)} \right] &= -\frac{\lambda \hat{c}_h}{1 - r} \frac{1 - F_b(\hat{v}_\ell)}{f_b(\hat{v}_\ell)} - \frac{\lambda \hat{v}_h}{1 - r} \frac{F_s(\hat{c}_\ell)}{f_s(\hat{c}_\ell)} \\ &= -\frac{\lambda(1 - \hat{\alpha})\hat{v}_\ell\hat{v}_h}{1 - r} \left[ \frac{1}{|\eta_b(\hat{v}_\ell)|} + \frac{1}{\eta_s(\hat{c}_\ell)} \right] \end{aligned} \quad (9)$$

where  $\lambda$  is a Lagrangian multiplier.

As noted earlier, if  $\alpha_\ell^* > \alpha_h^*$  then  $\text{MR} > \text{MC}$  at  $(\hat{c}_h, \hat{v}_h)$  and  $\text{MR} < \text{MC}$  at  $(\hat{c}_\ell, \hat{v}_\ell)$ . That is,

$$\hat{v}_h - \frac{1 - F_b(\hat{v}_h)}{f_b(\hat{v}_h)} > \hat{c}_h + \frac{F_s(\hat{c}_h)}{f_s(\hat{c}_h)} \quad \text{and} \quad \hat{v}_\ell - \frac{1 - F_b(\hat{v}_\ell)}{f_b(\hat{v}_\ell)} < \hat{c}_\ell + \frac{F_s(\hat{c}_\ell)}{f_s(\hat{c}_\ell)}$$

Thus, the Lagrange multiplier  $\lambda$  in (8) and (9) is positive when  $\alpha_\ell^* > \alpha_h^*$  and negative if  $\alpha_\ell^* < \alpha_h^*$ . In either event, the divergence between the marginal revenue and marginal

cost at  $(\hat{c}_k, \hat{v}_k)$  increases as (i) the absolute value of the elasticity of demand at  $\hat{v}_k$  decreases or (ii) the elasticity of supply at  $\hat{c}_k$  decreases or (ii) the fraction of time that buyer mass is  $\mu_k$  decreases.

In the example below, constrained optimal prices are easily computed with the necessary conditions in Lemma 1.

**EXAMPLE 2:** Consider Example 1 under the constant-fee constraint. The buyer mass is  $\mu_\ell = 1$  half the time and  $\mu_h = 2$  half the time, i.e.,  $r = 0.5$ . As  $F_b$  and  $F_s$  are uniformly distributed on  $[0,1]$ ,  $\eta_b(v) = -\frac{v}{1-v}$  and  $\eta_s(c) = 1$ . Substituting in the first-order conditions above, we obtain

$$\begin{aligned} 2\hat{\alpha}\hat{v}_h - 1 &= 2\lambda(1 - \hat{\alpha})\hat{v}_\ell \\ 1 - 2\hat{\alpha}\hat{v}_\ell &= 2\lambda(1 - \hat{\alpha})\hat{v}_h \end{aligned}$$

Moreover, the demand equals supply condition implies that

$$\hat{v}_k = \frac{\mu_k}{1 - \hat{\alpha} + \mu_k}, \quad k = \ell, h$$

Substituting  $\hat{v}_k$  in the first-order conditions we have two equations in two unknowns:  $\hat{\alpha}$  and  $\lambda$ . For  $\mu_\ell = 1$ ,  $\mu_h = 2$ , the constrained optimal solution is

$$\hat{\alpha} = 0.6316, \quad (\hat{c}_\ell, \hat{v}_\ell) = (0.2692, 0.7308), \quad (\hat{c}_h, \hat{v}_h) = (0.3111, 0.8444)$$

The unconstrained optimal solutions at high and low demand are

$$\alpha_\ell^* = 0.6667, \quad (c_\ell^*, v_\ell^*) = (0.25, 0.75), \quad \alpha_h^* = 0.6, \quad (c_h^*, v_h^*) = (0.3333, 0.8333)$$

Note the amplification of the surge in buyer prices and the reduction of the surge in seller prices under the constant-fee constraint, as established in Proposition 6.

As the quantity sold equals the seller price in this uniform distribution example, it is easy to see from  $\hat{c}_h < c_h^*$  that the quantity of trades completed during high demand is less than optimal. Similarly, compared to the unconstrained optimal solution, too many trades are completed during low demand. A direct computation shows that change (increase) in efficiency (at constrained optimal prices in comparison to unconstrained optimal prices) during low demand is

$$\Delta E_\ell = \frac{1}{2}(\hat{c}_\ell - c_\ell^*)(\hat{v}_\ell - \hat{c}_\ell + v_\ell^* - c_\ell^*) = +0.00923$$

while the change (decrease) in efficiency during high demand is

$$\Delta E_h = \frac{1}{2}(\hat{c}_h - c_h^*)(\hat{v}_h - \hat{c}_h + v_h^* - c_h^*) = -0.01146$$

As  $r = 0.5$ , the net effect of constrained pricing is a reduction in efficiency.  $\square$

## 5 Concluding Remarks

Optimal prices set by an intermediary are obtained and related to the intermediary's behavior as a monopsonist and as a monopolist in its interactions with sellers and buyers, respectively. The model permits a simple analysis of the impact of changes in relative demand and supply. Conditions under which the intermediary's optimal share of the pie decreases with demand are derived. It is shown that charging a constant fee reduces intermediary profits and, surprisingly, may also magnify the surge in buyer prices and attenuate the surge in seller prices during high demand periods.

In the model, the intermediary does not hold inventory, which is also the case in several markets. Relaxing this assumption would reduce price swings, both in the optimal prices and in the constrained prices, but would probably not change the conclusion that a constant percent fee for the intermediary increases the surge in buyer prices.

That the intermediary has market power in its interactions with buyers and sellers has implications for anti-trust policy. One's initial intuition might be that if an intermediary hires its sellers, an issue in pending litigation, then its marginal costs would decrease, thereby reducing the distortions of an intermediary monopsonist. The comparison with a benchmark monopolist in Section 3 supports this view. However, this argument assumes that the total cost and the value proposition to buyers would remain unchanged if sellers were to become employees. This assumption may not be tenable. First, many car drivers sell their services to both Uber and Lyft, switching between the two depending on which intermediary provides a closer next passenger pick-up. This practice reduces idle time and increases the efficiency of the sellers; it would likely be curtailed if each seller were to become an employee of one of the two firms. Second, the intermediary's marginal cost might decrease only if it bought the

productive assets of the sellers along with their services; this would reduce the scale of operations of the intermediary due to capital constraints which in turn may reduce the value of the service to buyers. These considerations are also germane to anti-trust policy.

## 6 Appendix: Proofs

**Proof of Proposition 1:** First, it is shown that the necessary conditions for optimality, (1) and (3), are satisfied at exactly one set of prices.

If  $(c, v)$ ,  $(c', v')$ , satisfy (1) then  $v < v'$  if and only if  $c > c'$ . That is, a demand equals supply curve in Figure 1 has strictly negative slope. To see this, note that  $v < v'$  implies  $F_b(v) < F_b(v')$ , as  $f_b$  is strictly positive on its support. Thus,  $F_s(c) = \mu(1 - F_b(v)) > \mu(1 - F_b(v')) = F_s(c')$ . Hence,  $c > c'$ . Reversing this argument we have  $c > c'$  implies  $v < v'$ .

It was argued earlier that the locus of points  $(c, v)$  that satisfy (3) has strictly positive slope. Thus, as the locus of points that satisfy (1) has strictly negative slope, there is a unique  $(c^*(\mu), v^*(\mu))$  that satisfies the two necessary conditions for optimality: the demand equals supply condition (1) and the marginal revenue equals marginal cost condition (3).

As the intermediary's profit function is maximized on a compact domain  $\{0 \leq c \leq 1, 0 \leq v \leq 1, v \geq c\}$ , a maximum exists. Further, this maximum must occur in the interior of the domain because the intermediary profit is zero at any point on the boundary of the domain while it is strictly positive at any point in the interior. As the two necessary conditions are satisfied at this interior maximum, this maximum must occur at the unique  $(c^*(\mu), v^*(\mu))$  that satisfies (1) and (3).

As the locus of points  $(c, v)$  that satisfy (3) has strictly positive slope, either (i)  $\frac{dc^*(\mu)}{d\mu} > 0$  and  $\frac{dv^*(\mu)}{d\mu} > 0$  or (ii)  $\frac{dc^*(\mu)}{d\mu} < 0$  and  $\frac{dv^*(\mu)}{d\mu} < 0$ . To rule out (ii), it is enough to show that the locus of points satisfying (1) at  $\mu + \Delta\mu$  is above the locus of points satisfying (1) at  $\mu$ , where  $\Delta\mu > 0$ . If  $\mu(1 - F_b(v)) = F_s(c)$  then  $(\mu + \Delta\mu)(1 - F_b(v)) > F_s(c)$ . hence there exists a  $\Delta v > 0$  such that  $(\mu + \Delta\mu)(1 - F_b(v + \Delta v)) = F_s(c)$ . Hence, if  $(c, v)$  satisfies (1) at  $\mu$  then there exists  $\Delta v > 0$  such that  $(c, v + \Delta v)$  satisfies (1) at  $\mu + \Delta\mu$ .  $\square$

**Proof of Proposition 2:** Equation (3) may be written as

$$v^* \left[ 1 - \frac{1}{v^*} \frac{1 - F_b(v^*)}{f_b(v^*)} \right] = c^* \left[ 1 + \frac{1}{c^*} \frac{F_s(c^*)}{f_s(c^*)} \right]$$

$$\begin{aligned} \Leftrightarrow \quad & v^* \left[ 1 + \frac{1}{\eta_b(v^*)} \right] = c^* \left[ 1 + \frac{1}{\eta_s(c^*)} \right] \\ \Leftrightarrow \quad & \frac{1}{1 - \alpha^*(\mu)} = \frac{v^*}{c^*} = \frac{1 + \frac{1}{\eta_s(c^*)}}{1 + \frac{1}{\eta_b(v^*)}} \end{aligned}$$

Observe that  $\alpha^*(\mu)$  decreases with  $\mu$  if and only if  $\frac{1}{1 - \alpha^*(\mu)}$  decreases with  $\mu$ .  $\square$

**Proof of Proposition 3:** We have

$$\begin{aligned} & \frac{d\alpha^*}{d\mu} = \frac{d}{d\mu} \left[ \frac{v^* - c^*}{v^*} \right] < 0 \\ \Leftrightarrow \quad & v^* \left[ \frac{dv^*}{d\mu} - \frac{dc^*}{d\mu} \right] - \frac{dv^*}{d\mu} [v^* - c^*] = c^* \frac{dv^*}{d\mu} - v^* \frac{dc^*}{d\mu} < 0 \\ \Leftrightarrow \quad & \frac{c^*}{v^*} \frac{dv^*/d\mu}{dc^*/d\mu} < 1 \\ \Leftrightarrow \quad & \frac{c^*}{v^*} \frac{dv^*}{dc^*} < 1 \\ \Leftrightarrow \quad & \frac{dv^*}{dc^*} < \frac{v^*}{c^*} \end{aligned}$$

As  $c^* < v^*$ , this inequality is satisfied if  $\frac{dv^*}{d\mu} \leq \frac{dc^*}{d\mu}$ .  $\square$

**Proof of Proposition 4:** As  $\mu \rightarrow 0$ , we have  $v^*(\mu) \rightarrow v_0 > 0$ ,  $c^*(\mu) \rightarrow 0$  and thus,  $\alpha^*(\mu) \rightarrow 1$ . Next, as  $\mu \rightarrow \infty$ ,  $v^*(\mu) \rightarrow 1$ ,  $c^*(\mu) \rightarrow c_1$  and thus,  $\alpha^*(\mu) \rightarrow 1 - c_1$ .

Thus  $1 = \alpha^*(0) > 1 - c_1 = \alpha^*(\infty)$ . By continuity of the  $(c^*, v^*)$  curve, there exist  $\underline{\mu} \leq \bar{\mu}$  for any  $\mu_\ell < \underline{\mu}$  and any  $\mu_h \geq \bar{\mu}$ , we have  $\alpha^*(\mu_h) < \alpha^*(\mu_\ell)$ .<sup>11</sup>  $\square$

**Proof of Proposition 5:** Because the good cannot be stored, at any optimal solution the demand equals supply constraint must be met during both the high demand and the low demand periods; the argument is the same as in the unconstrained case. Hence, if quantity  $q_k$  is sold at buyer mass  $\mu_k$  then the buyer price is  $v_k = F_b^{-1}(1 - \frac{q_k}{\mu_k})$  and the seller price is  $c_k = F_s^{-1}(q_k)$ ,  $k = \ell, h$ . The intermediary's expected profit under the constant-fee constraint is

$$\Pi_I(q_h, q_\ell) = r q_h \left[ F_b^{-1} \left( 1 - \frac{q_h}{\mu_h} \right) - F_s^{-1}(q_h) \right] + (1 - r) q_\ell \left[ F_b^{-1} \left( 1 - \frac{q_\ell}{\mu_\ell} \right) - F_s^{-1}(q_\ell) \right]$$

<sup>11</sup>Note that there are many selections of  $\underline{\mu} \leq \bar{\mu}$  for which the lemma holds.

s.t.

$$F_s^{-1}(q_\ell)F_b^{-1}\left(1 - \frac{q_h}{\mu_h}\right) = F_s^{-1}(q_h)F_b^{-1}\left(1 - \frac{q_\ell}{\mu_\ell}\right)$$

The profit may be written as the integral of its derivative obtained in (2)

$$\begin{aligned} & r \int_{v_h}^1 \int_0^{c_h} \left( v - \frac{1 - F_b(v)}{f_b(v)} - \left[ c + \frac{F_s(c)}{f_s(c)} \right] \right) f_s(c) f_b(v) dc dv \\ & + (1 - r) \int_{v_\ell}^1 \int_0^{c_\ell} \left( v - \frac{1 - F_b(v)}{f_b(v)} - \left[ c + \frac{F_s(c)}{f_s(c)} \right] \right) f_s(c) f_b(v) dc dv \end{aligned}$$

subject to the constant-fee constraint.

The argument below is followed in Figure 2. Note that if  $\hat{\alpha}$  is the optimal fee, then the optimal prices  $(\hat{c}_k, \hat{v}_k)$  are at the intersection of a straight line through the origin with slope  $\frac{1}{1-\hat{\alpha}}$  and the demand equals supply curve for  $\mu_k$ .

If  $\hat{\alpha} < \min\{\alpha_\ell^*, \alpha_h^*\}$  then, in Figure 2, the straight line through the origin with slope  $\frac{1}{1-\hat{\alpha}}$  is less steep than each of the two straight lines from the origin to  $(c_k^*, v_k^*)$ ,  $k = \ell, h$ .<sup>12</sup> Each  $(\hat{c}_k, \hat{v}_k)$  lies below the ‘MR=MC’ curve, the region where  $\text{MR} = v - \frac{1-F_b(v)}{f_b(v)} < c + \frac{F_s(c)}{f_s(c)} = \text{MC}$ . Thus, selling a little less by decreasing  $c_k$  and increasing  $v_k$  slightly, while maintaining the constant-fee constraint and the two demand equals supply conditions, will increase profit in each of the two states  $\ell$  and  $h$  and thereby increase  $\Pi_I(q_h, q_\ell)$ . This contradicts the optimality of  $\hat{\alpha}$ .

Similarly, if  $\hat{\alpha} > \min\{\alpha_\ell^*, \alpha_h^*\}$  then each  $(\hat{c}_k, \hat{v}_k)$  is above the ‘MR=MC’ curve, the region where  $\text{MR} > \text{MC}$ . Selling a little more will increase profit in each the two states.

Thus, (7) must hold.

Suppose that  $\alpha_\ell^* \neq \alpha_h^*$  and that it is optimal to set  $\hat{\alpha} = \alpha_\ell^*$ . Therefore,  $(\hat{c}_\ell, \hat{v}_\ell) = (c_\ell^*, v_\ell^*)$ . Thus, at low demand the prices are unconstrained optimal but not at high demand (as  $\alpha_\ell^* \neq \alpha_h^*$ ). Consequently, marginal revenue equals marginal cost at low demand but not at high demand. Because marginal revenue and marginal cost are continuous functions, the intermediary’s profits are greater if the constant percent fee is set at  $\hat{\alpha} + \epsilon$  rather than at  $\hat{\alpha}$ , where  $|\epsilon|$  is small with  $\epsilon > 0$  if  $\alpha_h^* > \alpha_\ell^*$  and  $\epsilon < 0$  if  $\alpha_h^* < \alpha_\ell^*$ . This contradicts the assumption that  $\hat{\alpha}$  is optimal. An identical argument

<sup>12</sup>These lines have slope  $\frac{1}{1-\alpha_k^*}$ ,  $k = \ell, h$ . Further,  $\frac{1}{1-\alpha_k^*} > \frac{1}{1-\hat{\alpha}}$  if and only if  $\alpha_k^* > \hat{\alpha}$ .



implies that  $\hat{\alpha} \neq \alpha_h^*$ . Thus, each of the two inequalities in (7) must be strict when  $\alpha_\ell^* \neq \alpha_h^*$ .

To prove that an optimal fee and prices exist first note that for each  $\alpha$  there exist a unique set of prices  $(c_k(\alpha), v_k(\alpha))$  that satisfies the demand equals supply constraint:

$$\mu_k(1 - F_b(v_k)) = F_s(c_k) = F_s((1 - \alpha)v_k)$$

As  $v_k$  is increased from 0, the left-hand side decreases from  $\mu_k$  and the right-hand side increases from 0, with equality at a unique point  $(c_k(\alpha), v_k(\alpha))$  where  $c_k(\alpha) = (1 - \alpha)v_k(\alpha)$ . Hence, the profit function may be written as a function of  $\alpha$ :

$$\begin{aligned} \Pi_I(\alpha) &= r \int_{v_h(\alpha)}^1 \int_0^{c_h(\alpha)} \left( v - \frac{1 - F_b(v)}{f_b(v)} - \left[ c + \frac{F_s(c)}{f_s(c)} \right] \right) f_s(c) f_b(v) dc dv \\ &\quad + (1 - r) \int_{v_\ell(\alpha)}^1 \int_0^{c_\ell(\alpha)} \left( v - \frac{1 - F_b(v)}{f_b(v)} - \left[ c + \frac{F_s(c)}{f_s(c)} \right] \right) f_s(c) f_b(v) dc dv \end{aligned}$$

where the constant-fee constraint is satisfied as  $c_k(\alpha) = (1 - \alpha)v_k(\alpha)$ . The domain for continuous function  $\Pi_I(\alpha)$  is a compact set  $[0, 1]$ .<sup>13</sup> Hence, there exists an  $\hat{\alpha}$  at which  $\Pi_I(\alpha)$  is maximized.  $\square$

**Proof of Proposition 6:** As  $\alpha_\ell^* > \alpha_h^*$ , Proposition 5 implies that  $\hat{\alpha}$  satisfies  $\alpha_h^* \leq \hat{\alpha} \leq \alpha_\ell^*$ . Thus, the constrained optimal prices satisfy

$$\frac{1}{1 - \alpha_h^*} = \frac{v_h^*}{c_h^*} \leq \frac{\hat{v}_h}{\hat{c}_h} = \frac{1}{1 - \hat{\alpha}} = \frac{\hat{v}_\ell}{\hat{c}_\ell} \leq \frac{v_\ell^*}{c_\ell^*} = \frac{1}{1 - \alpha_\ell^*}$$

That is,  $(\hat{c}_\ell, \hat{v}_\ell)$  is below the ‘MR=MC’ curve (and on the demand equals supply line for  $\mu_\ell$ ) and, similarly,  $(\hat{c}_h, \hat{v}_h)$  is above ‘MR=MC’ curve; this can be seen in Figure 2 where  $\alpha_\ell^* > \alpha_h^*$ . As the demand equals supply curve is negatively sloped (see proof of Proposition 1), it follows immediately that  $\hat{v}_h > v_h^*$  and  $v_\ell^* > \hat{v}_\ell$ . That  $v_h^* > v_\ell^*$  follows from Proposition 1.

A symmetric argument implies that  $c_h^* > \hat{c}_h$  and  $\hat{c}_\ell > c_\ell^*$ . That  $\hat{c}_h > \hat{c}_\ell$  follows from (6).  $\square$

<sup>13</sup>In fact, the search for an optimal  $\alpha$  may be restricted to the compact set  $[\min\{\alpha_\ell^*, \alpha_h^*\}, \max\{\alpha_\ell^*, \alpha_h^*\}]$ .

**Proof of Lemma 1:** As  $v_* = F_b^{-1}(1 - \frac{q_*}{\mu_*})$  and  $c_* = F_s^{-1}(q_*)$ , the intermediary's expected profit under the constant-fee constraint is

$$\Pi_I(q_h, q_\ell) = r q_h \left[ F_b^{-1}\left(1 - \frac{q_h}{\mu_h}\right) - F_s^{-1}(q_h) \right] + (1 - r) q_\ell \left[ F_b^{-1}\left(1 - \frac{q_\ell}{\mu_\ell}\right) - F_s^{-1}(q_\ell) \right]$$

s.t.

$$F_s^{-1}(q_\ell) F_b^{-1}\left(1 - \frac{q_h}{\mu_h}\right) = F_s^{-1}(q_h) F_b^{-1}\left(1 - \frac{q_\ell}{\mu_\ell}\right)$$

The Lagrangian for the intermediary profit-maximization problem is

$$\begin{aligned} \mathbb{L}_I(q_h, q_\ell, \lambda) &= r q_h \left[ F_b^{-1}\left(1 - \frac{q_h}{\mu_h}\right) - F_s^{-1}(q_h) \right] + (1 - r) q_\ell \left[ F_b^{-1}\left(1 - \frac{q_\ell}{\mu_\ell}\right) - F_s^{-1}(q_\ell) \right] \\ &\quad + \lambda \left[ F_s^{-1}(q_\ell) F_b^{-1}\left(1 - \frac{q_h}{\mu_h}\right) - F_s^{-1}(q_h) F_b^{-1}\left(1 - \frac{q_\ell}{\mu_\ell}\right) \right] \end{aligned}$$

$$\begin{aligned} \implies \frac{\partial \mathbb{L}_I}{\partial q_h} &= r \left[ F_b^{-1}\left(1 - \frac{q_h}{\mu_h}\right) - F_s^{-1}(q_h) \right] + r q_h \frac{dF_b^{-1}\left(1 - \frac{q_h}{\mu_h}\right)}{dq_h} - r q_h \frac{dF_s^{-1}(q_h)}{dq_h} \\ &\quad + \lambda \left[ F_s^{-1}(q_\ell) \frac{dF_b^{-1}\left(1 - \frac{q_h}{\mu_h}\right)}{dq_h} - F_b^{-1}\left(1 - \frac{q_\ell}{\mu_\ell}\right) \frac{dF_s^{-1}(q_h)}{dq_h} \right] \\ &= r \left[ F_b^{-1}\left(1 - \frac{q_h}{\mu_h}\right) - F_s^{-1}(q_h) \right] - r \frac{q_h}{\mu f_b\left(F_b^{-1}\left(1 - \frac{q_h}{\mu_h}\right)\right)} - r \frac{q_h}{f_s\left(F_s^{-1}(q_h)\right)} \\ &\quad - \lambda F_s^{-1}(q_\ell) \frac{q_h}{\mu f_b\left(F_b^{-1}\left(1 - \frac{q_h}{\mu_h}\right)\right)} - \lambda F_b^{-1}\left(1 - \frac{q_\ell}{\mu_\ell}\right) \frac{q_h}{f_s\left(F_s^{-1}(q_h)\right)} \\ &= r(v_h - c_h) - (r + \lambda c_\ell) \frac{1 - F_b(v_h)}{f_b(v_h)} - (r + \lambda v_\ell) \frac{F_s(c_h)}{f_s(c_h)} \end{aligned}$$

As  $\frac{\partial \mathbb{L}_I}{\partial q_h} = 0$  at optimal prices  $(\hat{c}_\ell, \hat{v}_\ell)$  and  $(\hat{c}_h, \hat{v}_h)$ , we have

$$\hat{v}_h - \frac{1 - F_b(\hat{v}_h)}{f_b(\hat{v}_h)} - \frac{\lambda \hat{c}_\ell}{r} \frac{1 - F_b(\hat{v}_h)}{f_b(\hat{v}_h)} = \hat{c}_h + \frac{F_s(\hat{c}_h)}{f_s(\hat{c}_h)} + \frac{\lambda \hat{v}_\ell}{r} \frac{F_s(\hat{c}_h)}{f_s(\hat{c}_h)} \quad (10)$$

Similarly,  $\frac{\partial \mathbb{L}_I}{\partial q_\ell} = 0$  at optimal prices  $(\hat{c}_\ell, \hat{v}_\ell)$  and  $(\hat{c}_h, \hat{v}_h)$  implies

$$\implies \hat{v}_\ell - \frac{1 - F_b(\hat{v}_\ell)}{f_b(\hat{v}_\ell)} + \frac{\lambda \hat{c}_h}{1 - r} \frac{1 - F_b(\hat{v}_\ell)}{f_b(\hat{v}_\ell)} = \hat{c}_\ell + \frac{F_s(\hat{c}_\ell)}{f_s(\hat{c}_\ell)} - \frac{\lambda \hat{v}_h}{1 - r} \frac{F_s(\hat{c}_\ell)}{f_s(\hat{c}_\ell)} \quad (11)$$

Equations (10) and (11) imply (8) and (9).  $\square$

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