Title

THE CONVECTIVE HEATING & COOLING OF A BODY

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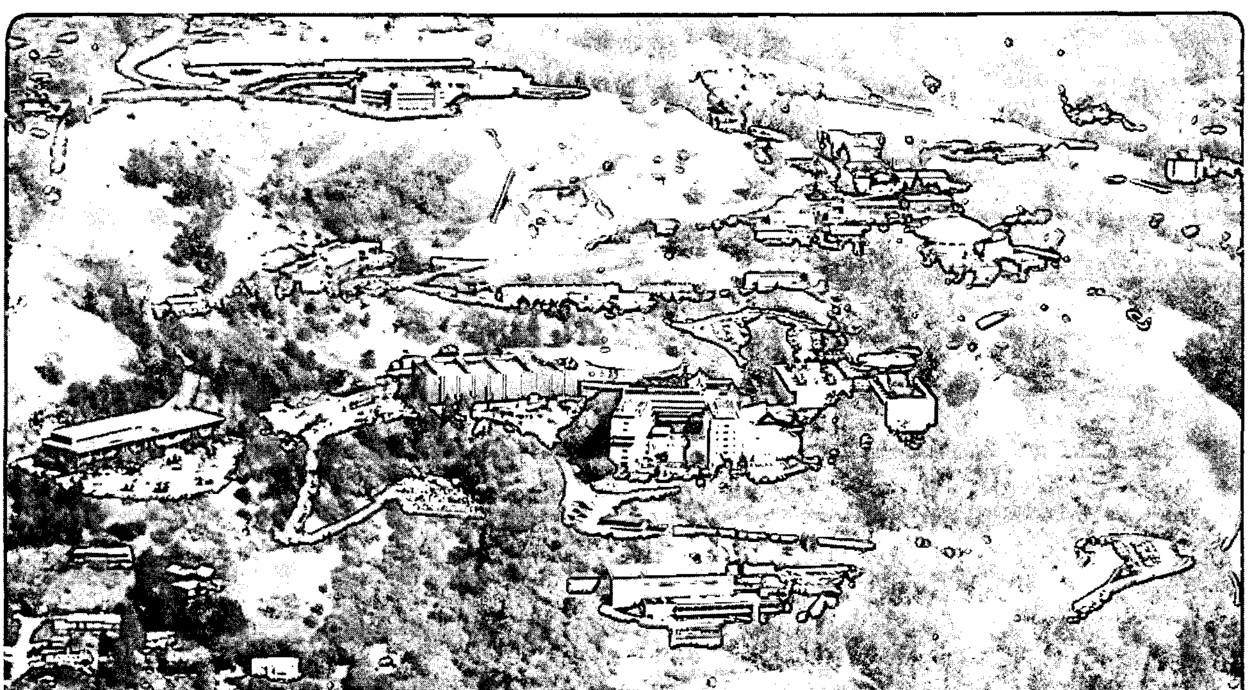

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ENGINEERING NOTE

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ANALYSIS

TITLE: THE CONVECTIVE HEATING & COOLING OF A BODY

SUMMARY & INTRODUCTION

A Revision 11/18/81

Equations are developed which allow estimates of the time for the convective heating or cooling of a body. Three cases are considered, constant mass flow of coolant and constant heat capacities, variable solid* heat capacity ($C_S = CT$) and both variable heat capacity of the solid and variable coolant mass flow (constant pressure drop). In all cases heat exchanger effectiveness is assumed constant thru the process. The variable solid heat capacity gives rise to a non symmetry in the heating and cooling times. For equal pressure drops, cooling times will be substantially less than the heating times between the same temperature limits. The expressions obtained are evaluated for several sets of temperature extremes for both heating and cooling and the results applied to an example problem. The solid temperature is assumed uniform in this development. However the results obtained should be useful for making first cut estimates of the temperature response of a distributed system where the solid temperature may vary over the length of the gas or liquid path.

*The term solid really has no significance here. It could just as well pertain to a liquid with an immersed heat exchanger.

ANALYSIS

Consider a body at temperature T_s which exchanges heat with a gas stream which approaches at temperature T_{g1} and which leaves at T_{g2}. An energy balance for the time interval $d\theta$ yields,

$$\dot{m}_g C_{pg}(T_{g1}-T_{g2})\,d\theta = m_s C_s\, dT_s \qquad (1)$$

To account for imperfect heat transfer we define a heat exchanger effectiveness ϵ

$$\epsilon = \frac{T_{g1}-T_{g2}}{T_{g1}-T_s} \qquad (2)$$

Combining these expressions, separating variables and integrating from 0 to θ and T_{s1} to T_{s2} yields

$$\frac{\epsilon\, \dot{m}_g C_{pg}}{m_s C_s}\int_0^\theta d\theta = \int_{T_{s1}}^{T_{s2}} \frac{dT_s}{T_{g1}-T_s} \qquad (3)$$

or

$$\frac{\dot{m}_g C_{pg}\,\theta}{m_s C_s} = \ln\left(\frac{T_{g1}-T_{s1}}{T_{g1}-T_{s2}}\right) \qquad (4)$$

If the solid heat capacity is allowed to vary with temperature according to $C_s = C T_s$, the result is

$$\frac{\epsilon\, \dot{m}_g C_{pg}\,\theta}{m_s C} = T_{s1}-T_{s2}+T_{g1}\ln\frac{T_{g1}-T_{s1}}{T_{g1}-T_{s2}} \qquad (5)$$

Note that for the same end temperatures, equation (4) will predict identical heating and cooling times whereas there is no such symmetry in equation (5). The reason for this is that as the end point of the cooling or heating period is approached the temperature difference between the gas stream and solid is decreasing and therefore the amount of refrigeration or heating available in the gas stream is likewise decreasing. This, however, is compensated in the cooling case by the diminished heat capacity whereas in the heating case the solid heat capacity is approaching a maximum. Cooling times will, therefore, be less than heating times for equal mass flows and temperature differences.

If now, rather than assuming a constant mass flow a constant pressure drop across the load is assumed the mass flow will vary inversly as the square root of the gas temperature, ie,

$$\dot{m}_g = \frac{K}{\sqrt{\bar{T}_g}} \qquad (6)$$

where $\bar{T}_g$ is the mean value of the gas temperature and for simplicity we use the arithmetic average

$$\bar{T}_g = \frac{T_{g1} + T_{g2}}{2} = \left(1 - \frac{\epsilon}{2}\right) T_{g_1} + \frac{\epsilon}{2} T_S \qquad (7)$$

The form of (6) does not lead to an easy integration. Therefore we approximate the mass flow with

$$\dot{m}_g = A + \frac{B}{\bar{T}_g} \qquad (6a)$$

which really is not too bad over most of the temperature range. Using (2), (6a), (7) and $C_S = CT_S$ in (1) and integrating from T_{S_1} to T_{S2} and 0 to θ we get

$$\int_{T_{S_1}}^{T_{S2}} \frac{T_S \, dT_S}{\left(A + \frac{B}{\bar{T}_g}\right)(T_{g_1} - T_S)} = \frac{C_{pg}\,\epsilon}{m_S C} \int_0^{\theta} d\theta \qquad (8)$$

Which after some algebra yields

$$\frac{A C_{pg} \epsilon \theta}{m_s C} = F(T_{S1}, T_{S2}, T_{g1}) \tag{9}$$

where

$$F(T_{S1},T_{S2},T_{g1}) = T_{S1} - T_{S2} + \left[\frac{1}{\epsilon}\frac{B}{A} - \frac{T_{g1}}{2}\right] \ln\left\{\frac{\frac{\epsilon}{2}T_{S2}^2 + \left[(1-\epsilon)T_{g1} + \frac{B}{A}\right]T_{S2} + \left(1-\frac{\epsilon}{2}\right)T_{g1}^2 + \frac{B}{A}T_{g1}}{\frac{\epsilon}{2}T_{S1}^2\left[(1-\epsilon)T_{g1} + \frac{B}{A}\right]T_{S1} + \left(1-\frac{\epsilon}{2}\right)T_{g1}^2 + \frac{B}{A}T_{g1}}\right\}$$

$$+ \frac{\frac{T_{g1}^2}{2} + \left(\frac{1}{\epsilon} - \frac{1}{2}\right)\frac{B}{A}T_{g1} + \frac{1}{\epsilon}\frac{B^2}{A^2}}{T_{g1} + \frac{B}{A}} \ln\left\{\frac{\epsilon T_{S2} + (2-\epsilon)T_{g1} + 2\frac{B}{A}}{\epsilon T_{S1} + (2-\epsilon)T_{g1} + 2\frac{B}{A}} \times \frac{T_{g1} - T_{S1}}{T_{g1} - T_{S2}}\right\}$$

To determine the flow constants A & B we enforce the following conditions

$$\text{at } T_g = T_g^* \qquad \frac{K}{\sqrt{T_g^*}} = A + \frac{B}{T_g^*} \tag{10}$$

$$\text{and } \int_{T_{g0}}^{T_{g3}} \frac{K}{\sqrt{T_g}}\, dT_g = \int_{T_{g0}}^{T_{g3}} \left(A + \frac{B}{T_g}\right) dT_g \tag{11}$$

we find

$$\frac{B}{A} = \frac{2(T_g^*)^{1/2}\left(T_{g_3}^{1/2} - T_{g_0}^{1/2}\right) - \left(T_{g3} - T_{g0}\right)}{\ln\left(\frac{T_{g3}}{T_{g0}}\right) - 2\left(\frac{T_{g3}^{1/2} - T_{g0}^{1/2}}{T_g^{*1/2}}\right)} \tag{12}$$

So we see that $\frac{B}{A}$ is a function only of temperature and

$$\frac{A}{K} = T_g^{*-1/2} - \frac{2\left(T_{g3}^{1/2} - T_{g0}^{1/2}\right) - (T_g^*)^{-1/2}(T_{g3} - T_{g0})}{T_g^*\left[\ln\left(\frac{T_{g3}}{T_{g0}}\right) - \frac{T_{g3} - T_{g0}}{T_g^*}\right]} \tag{13}$$

For $Tg^* = 200K$, $Tg_0 = 4K$, $Tg_3 = 290K$ we get

$$\frac{B}{A} = 64.45\ K$$

$$\frac{A}{K} = 0.0536\ K^{-1/2}$$

For the case of constant flow eq (9) reduces to eq (5) on the substitution of $B=0$

The function $F(Ts_1, Ts_2, Tg_1)$ is plotted vs the final solid temperature, Ts_2, for the case of cooling from 290K with 4.5, 20 & 80K gas in Fig 1, cooling from 30K with 4.5K gas in Fig 2 and heating from 4K with 300K gas in Fig 3.

The flow constants A & K may be evaluated as follows. For a compressible, constant area flow with friction (See for instance, Venard, Elementary Fluid Mechanics 3rd Ed pg 209)

$$P_1^2 - P_2 = \frac{\dot{m}g^2 R T_g}{S^2}\left[2 \ln \frac{u_2}{u_1} + \frac{fL}{d_H}\right] \qquad (14)$$

where R is the gas constant, S the flow area, u the flow velocity. If $fL/d_H \gg 2\ln(u_2/u_1)$ as is generally the case we get

$$\dot{m}g = \sqrt{\frac{(P_1^2 - P_2^2)S^2}{(fL/d_H)RT_g}} \qquad (14a)$$

(which is identical to the result obtained by evaluating the density at the arithmetic average pressure in the incompressible approximation.)

Therefore

$$A = 0.0536K = 0.0536\ \dot{m}g\sqrt{T_g} \qquad (15)$$

FIG 1 TEMPERATURE FUNCTION FOR COOLING FROM AN INITIAL TEMPERATURE OF 290K

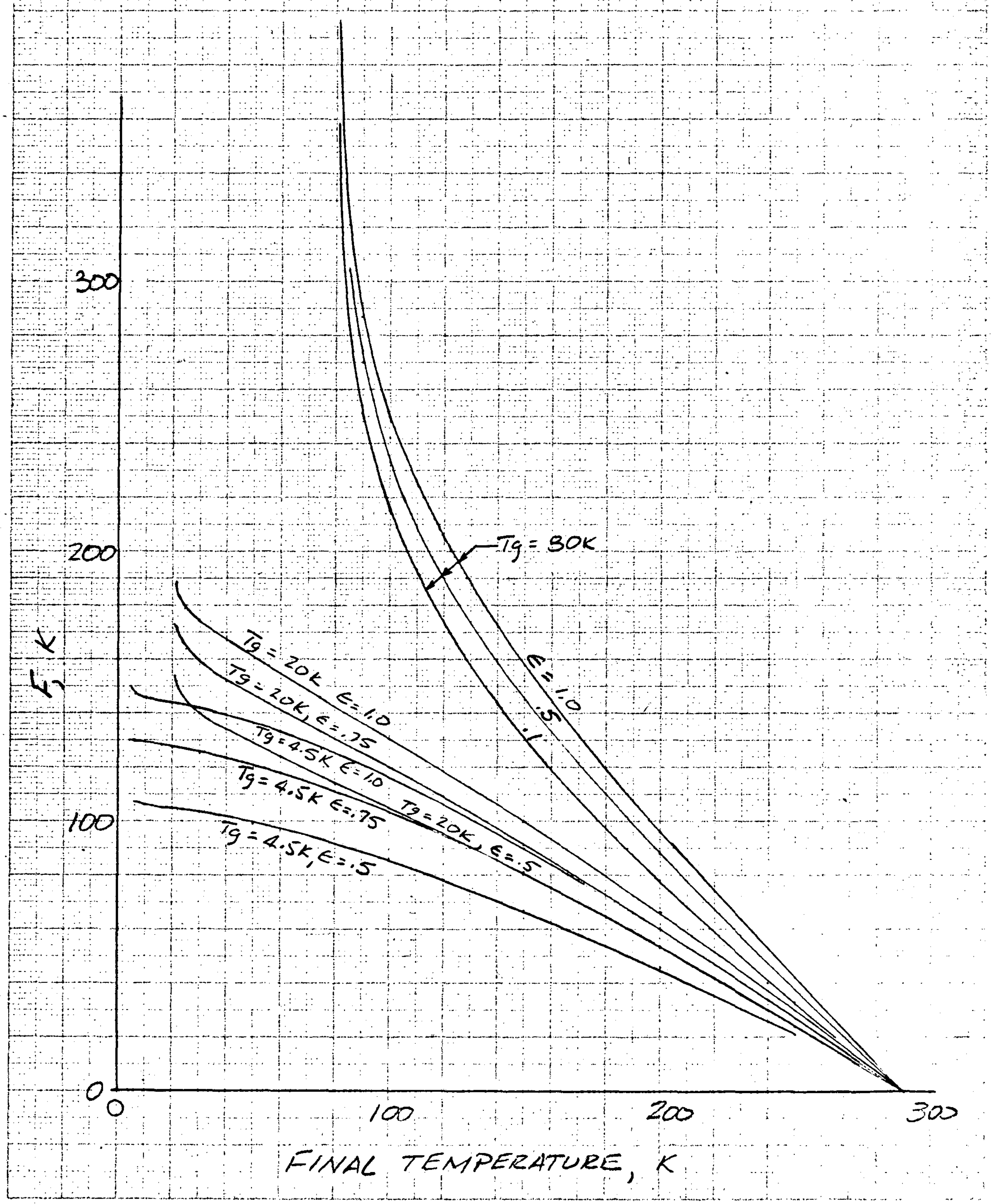
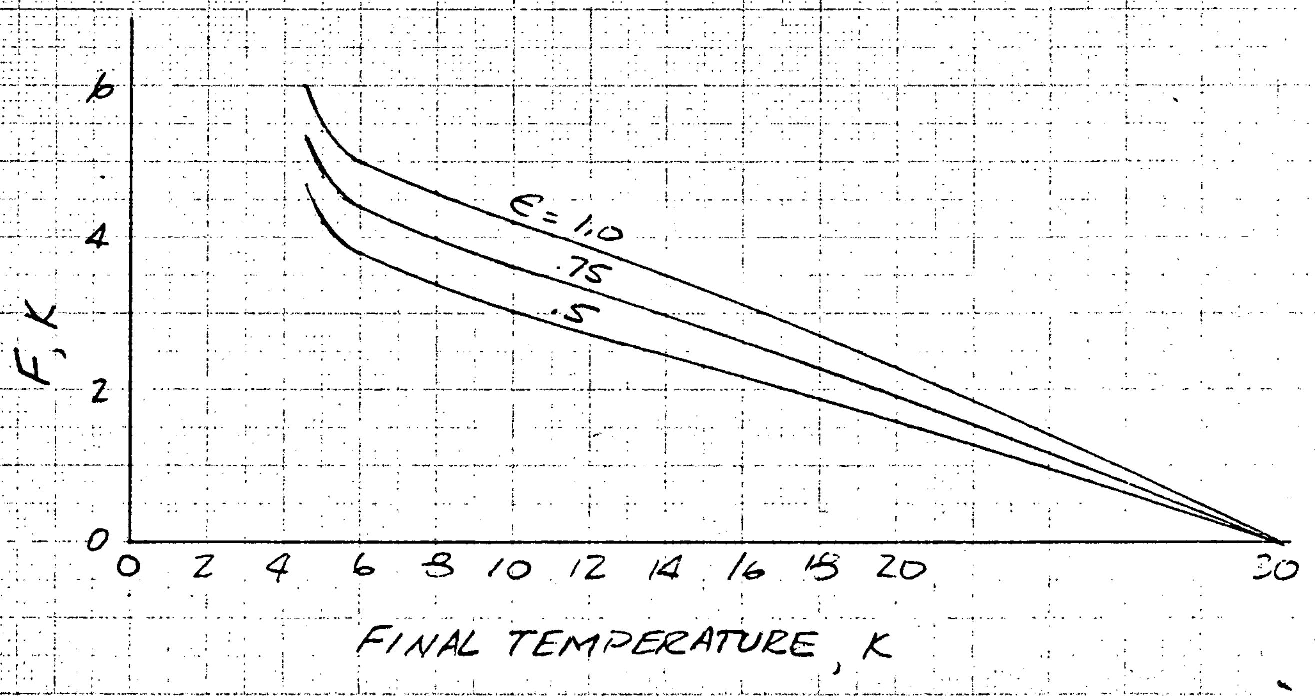


FIG 2 TEMPERATURE FUNCTION FOR COOLING FROM 30K WITH $T_g = 4.5K$



for helium

$$\frac{A}{S} = 11.91\,P_2\sqrt{\left(\frac{P_1^2}{P_2^2}-1\right)\Big/\frac{fL}{d_H}} \quad g/s\,cm^2 \quad (P \text{ in atm}) \qquad (16)$$

The use of Figures 1 2 & 3 along with eq (9) & (16) are demonstrated for a constant pressure drop case in the following example.

EXAMPLE

Determine the cooling time from 290 to 6.5K using 4.5K gas and the heating time from 4 to 280K using 300K gas for a 5 m long, 7500 kg cold iron magnet. Eight, $\frac{1}{2}$ inch diameter coolant passages are provided thru the iron laminations. In each case gas is supplied at 1.3 atm and exhausted to compressor suction at 1.07 atm.

SOLUTION

The iron laminations are assumed to be aligned to ±.002 inch. Therefore, the coolant passages can be treated as a rough pipe with relative roughness

$$\frac{e}{d} = \frac{(2)(.002)}{0.5} = 0.008$$

which results in a friction factor of 0.035 and which is very nearly independent of Reynolds number. Further, we will ignore the velocity head loss at the exit, Then

$$\frac{fL}{d} = \frac{(0.035)(5)(3.28)(12)}{0.5} = 13.8$$

From (16)

$$\frac{A}{S} = (11.91)(1.07)\sqrt{\left(\frac{1.3^2}{1.07^2}-1\right)/13.8} = 2.37$$

and

$$A = (2.37)(8)\left(\frac{\pi}{4}\right)(1.27)^2 = 24.0 \; g/s$$

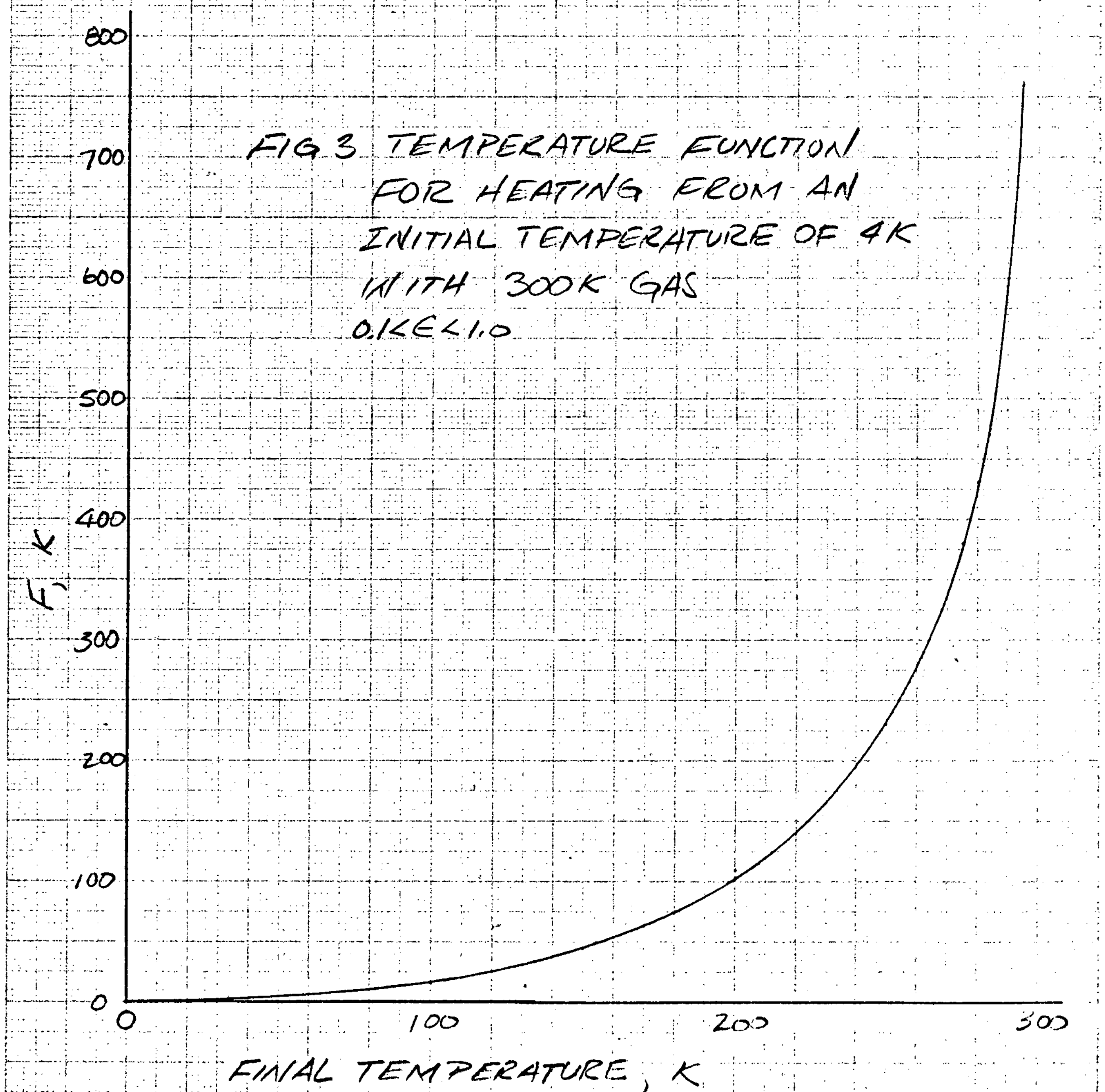
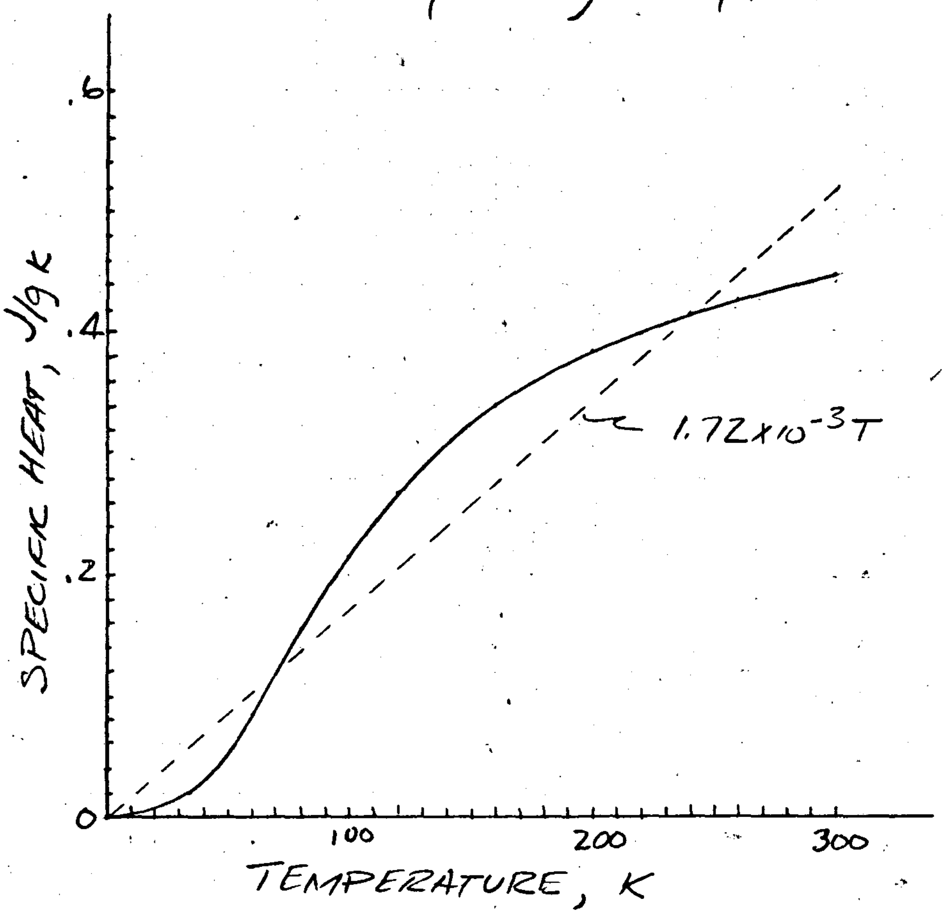


FIG 3 TEMPERATURE FUNCTION FOR HEATING FROM AN INITIAL TEMPERATURE OF 4K WITH 300K GAS

$0.1 < \epsilon < 1.0$

The iron heat capacity is plotted below vs temperature



The heat capacity curve is approximated by the dashed line which is determined so that the two curves have equal areas (enthalpy) in the interval 0-300 K. Therefore $C = 1.72 \times 10^{-3}$ J/g K²

From Fig 1

$\epsilon = 0.5$ $F_{cooling} = 106K$

$\epsilon = 1.0$ $F_{cooling} = 148 K$

From Fig 3

$0.1 < \epsilon < 1.0$ $F_{Heating} = 430K$

From eq (9) we find for cooling with a heat exchanger effectiveness of 0.5

$$\theta_{cooling} = \frac{(106)(7500 \times 10^{3})(1.72 \times 10^{-3})}{(24.0)(5.2)(.5)} = 21900S = 6.1\ hr.$$

Or for $\epsilon' = 1.$

$$\theta_{cooling} = \frac{(143)(7500\times10^{-3})(1.72\times10^{-3})}{(24)(5.2)(1.)} = 15300\ s = 4.3\ hr$$

For the heating case, since $F_{heating}$ is nearly independent of ϵ, the heating time is inversely proportional to ϵ, so from eq (9)

$$\epsilon\,\theta_{heating} = \frac{(430)(7500\times10^{-3})(1.72\times10^{-3})}{(24.0)(5.2)} = 44{,}450.\ S = 12.4\ hr$$

so that

$\epsilon = 0.5$, $\theta_{heating} = 24.8$ hr

$\epsilon = 1.0$ $\theta_{heating} = 12.4$ hr

For the same pressure drop the heating time is seen to be much greater than the cooling time. However, much higher flows are generally available with warm gas than for the cold gas supplied by a refrigerator. The pressures chosen for this example are typical of that available downstream of the JT valve on a refrigerator. If the warm up time is critical pressures up to typically 18 atm are available from the refrigerator compressor. For example assume that P_1 is 3 atm in the heating case above, then A becomes 91.0 g/s and the heating times become 6.5 and 3.3 hr for $\epsilon = 0.5$ or 1.0 respectively.
Another example in the use of this method is given in M5817

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