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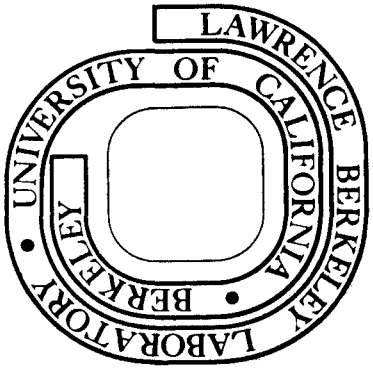

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# Proposed Experiment to Realize and Detect the Three Dimensional Wigner Lattice

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## ABSTRACT

This paper suggests and discusses possible experimental means of realizing and detecting the three dimensional Wigner electron lattice. It is concluded that very pure germanium, doped with antimony, offers the possibility of obtaining extrinsic electron densities low enough to form the Wigner lattice. It is proposed that Brillouin scattering of photons by the LA and TA modes of vibration of the lattice offers a method of experimental detection. This is because the Brillouin scattering spectrum of the electron lattice is anticipated to include structure not found in the spectrum of the electron gas of the same density.

## I. Introduction

The Wigner lattice [1] is a body-centered cubic lattice of electrons whose formation in a solid is predicted to occur at a sufficiently low value of the electron density. With the present availability of very pure semiconductors with variable extrinsic electron densities, it is interesting to consider the possibility of experimentally realizing and detecting the three-dimensional [2] Wigner lattice. The aim of this paper is the suggestion and discussion of possible experiments for this purpose. To the best of the author's knowledge, this is the first paper discussing the possible experimental realization and detection of the three-dimensional Wigner lattice.

Considering an assembly of electrons in the presence of a uniform background of fixed positive charge, Wigner [1] argued that, as their potential energy became large compared to their kinetic energy, the electrons would become localized in space. This would make the kinetic energy ineffective in opposing this spatial localization and lead to the formation of a lattice of electrons. If n is the electron density in a solid of dielectric constant $\varepsilon$, then the potential energy V of interaction of the electrons is roughly

$$V \simeq (-e)^2/\varepsilon r_o$$

where $r_o$ is the average distance between electrons. Making the Wigner-Seitz approximation, $(4\pi r_o^3/3) = (1/n)$, the potential energy may be

expressed as

$$V = (4\pi/3)^{1/3}(e^2/\varepsilon)n^{1/3}.$$

The average kinetic energy T of an electron may be taken as equal to (3/5) of the Fermi energy $E_F$, so

$$T \cong \frac{3}{5}E_F = (3\hbar^2/10m)(3\pi^2 n)^{2/3},$$

where m is the free electron mass. Then the ratio

$$(V/T) = (4\pi/3)^{1/3}[\,10/3(3\pi^2)^{2/3}\,](me^2/\hbar^2\varepsilon)n^{-1/3},$$

and (V/T) will become large at low values of the electron density n. Defining an effective Bohr radius [3] $a_o^*$ by the relation

$$a_o^* \equiv (\hbar^2\varepsilon/m^*e^2),$$

where we are now considering electrons of effective mass $m^*$, one obtains

$$(V/T) = C(r_o/a_o^*) \equiv Cr_s \qquad (1)$$

where the constant $C = (10/9)(16/3\pi^2)^{1/3} = 0.905$. The expression (1) shows that (V/T) increases as the dimensionless parameter $r_s$ increases,

leading to the crystallization of the electron gas at sufficiently large values of $r_s$, say larger than a critical value [4] of $r_s = r_c$.

The ground state energy of the electron gas and lattice has been the subject of a number of theoretical calculations [4-8]. The values of $r_c$ obtained are generally between 14 and 20, leading to the expectation that the Wigner lattice will be stable at electron densities corresponding to values of $r_s$ greater than about 15. Large values of $r_s$ are favored by the low values of the electron density which can be achieved in extrinsic semiconductors at low temperatures. It is thus natural to consider such semiconductors as potential "hosts" in which to examine the possibility of achieving and detecting the Wigner electron lattice.

## II. Possible Formation of the Lattice

The semiconductor presently obtainable in the purest form is germanium, in which net carrier concentrations of $10^9$ to $10^{11}$ $cm^{-3}$ may be obtained[9]. The existence of germanium of such high purity suggests that extrinsic electron concentrations low enough to produce the Wigner lattice may be achievable. Using an effective mass value[10] $m^* = 0.12m$ for electrons and a dielectric constant $\varepsilon = 16$, equation (1) may be used to calculate values of $r_s$ as a function of the electron density n in germanium. If a typical theoretical value of $r_c = 15$ is chosen, it is found that $r_s$ is larger than 15 if the electron density n is less than $8.7 \times 10^{14}$ $cm^{-3}$. Considering antimony as a donor in germanium (with an ionization energy $E_d = 0.0096$ eV[11]), one may calculate [12] the electron concentration n as a function of temperature for a given value [13] of the donor atom concentration $N_d$. If $N_d = 10^{16}$ atoms of antimony per $cm^3$, the electron density n is less than $3.5 \times 10^{14}$ $cm^{-3}$ for temperatures below 16K. Such conditions are readily achievable in the very pure germanium available today. It thus appears that values of $r_s$ larger than the calculated values of the critical value $r_c$ may be obtained for the extrinsic electron gas in antimony doped germanium [14].

We next calculate some parameters of the electron gas at a low density. Using a representative value of the electron density $n = 10^{14}$ $cm^{-3}$, corresponding to $r_s = 19$ in germanium, the lattice constant a of the bcc electron lattice is given by $n = (2/a^3)$, or $a = 2.7 \times 10^{-5}$ cm. It is well known [15] that the electron gas in a semiconductor of simple band structure [16] has a plasma oscillation

mode of frequency $\omega_p$, where $\omega_p^2 = (4\pi ne^2/m^*\varepsilon)$. For germanium with $n = 10^{14}$ electrons per $cm^3$, $m^* = 0.12m$, and $\varepsilon = 16$, a value of $\omega_p = 4.1 \times 10^{11}$ $sec^{-1}$ is obtained, corresponding to a $\Gamma(k=0)$ plasmon energy $\hbar\omega = 0.269 \times 10^{-3}$ eV = 0.27 meV.

## III. Possible Light Scattering Experiment to Detect the Lattice

Assuming that the Wigner lattice does form under the conditions suggested, how might it be detected experimentally? It is well-known [17] that plasmons in the conduction electron gas in semiconductors have been detected by light (Brillouin) scattering experiments. In such experiments, an incident photon of frequency $\omega_o$ is inelastically scattered with the creation of a plasmon of frequency $\omega_p$ and a scattered photon of frequency $\omega_s$. The result is a frequency shift $\Delta\omega \equiv (\omega_o - \omega_s) = \omega_p$ for pure plasmon scattering without plasmon-phonon interactions [17]. For example, such Brillouin scattering has been observed [18] in n-type GaAs. In this case, the observed plasma frequency $\omega_p \simeq 100\ cm^{-1}$, corresponding to a plasmon energy $\hbar\omega_p = 1.2$ meV, at an electron concentration of about $1.16 \times 10^{17}\ cm^{-3}$.

If the conduction electron gas in a semiconductor does crystallize into an electron lattice, it would be expected that transverse acoustic [19] vibrations would be possible in addition to a longitudinal acoustic branch. This expectation is borne out in the calculation by Clark [20] of the vibrational spectrum of an electron lattice in the presence of a uniform background of positive charge [21]. Clark calculates the frequencies of both longitudinal (LA) and transverse (TA) acoustic modes as a function of wave vector for several directions of propagation in the bcc electron lattice. The LA mode is unusual in that its frequency is not zero at the zone center; this is due to the long range of the Coulomb potential. (This behavior of the vibrational spectrum is the same as that observed for plasma oscillations in the electron gas [22].) The electron lattice thus has vibrational modes

whose phonon aspects may be thought of approximately as longitudinal and transverse "plasmons". The former are the analog of longitudinal plasma oscillations in the electron gas, but the latter should be something new.

Clark's result is that the frequencies $\omega_\ell$ and $\omega_t$ of the LA and TA modes are given by

$$\omega_\ell = (1/\sqrt{2\pi})\Lambda_\ell\omega_p \quad ; \quad \omega_t = (1/\sqrt{2\pi})\Lambda_t\omega_p ,$$

where $\omega_p^2 = (4\pi ne^2/m^*\varepsilon)$ is the plasma frequency of the electron gas of density n, etc. The quantities $\Lambda_\ell$ and $\Lambda_t$ are reduced frequencies whose calculated values [20] at various special points [23] of the Brillouin Zone are given in Table I. While the frequency of the zone center LA mode is the same as the plasma frequency of the electron gas of the same density, the zone edge (points H and P) LA and TA modes of the electron lattice are calculated to have frequencies of about 0.6 $\omega_p$.

Table I. Vibrational Frequencies of the bcc Electron Lattice.

| Special Point | Γ(000) | H(100) | P(111) |
|---|---|---|---|
| $\Lambda_\ell$ | 2.50 | 1.45 | 1.45 |
| $\Lambda_t$ | 0.0 | 1.45 | 1.45 |
| $(\omega_\ell/\omega_p)$ | 1.00 | 0.58 | 0.58 |
| $(\omega_t/\omega_p)$ | 0.0 | 0.58 | 0.58 |

One may contrast the expected features of the Brillouin scattering [24] spectrum of the electron lattice with those observed in Brillouin scattering by plasmons in the elctron gas. In the latter case, the central feature of the spectrum is [18] a peak at the photon frequency shift $\Delta\omega = \omega_p$ due to scattering by the plasmons which are the LA vibrational mode of the electron gas. For the electron crystal, one would also expect to observe Brillouin scattering at a photon frequency shift $\Delta\omega = \omega_p$ due to the zone center LA vibrational mode whose calculated frequency is $\omega_p$. However, it is <u>also</u> to be expected that the additional vibrational modes possible due to the formation of the electron lattice would result in new and additional features in the Brillouin scattering spectrum, and that these features would not be present in the equivalent spectrum for the electron gas. Of special interest are the transverse vibrations of the electron lattice; these would be expected to couple strongly to incident photons. However, for these TA modes, the zone-center frequency is zero, so it is of particular interest and importance that, for the electron lattice, one would expect Brillouin scattering by LA and TA modes whose wave vectors are significantly different from zero.

The reason for this is the fact that the lattice constant a of the electron lattice is very large compared to the values of the lattice constant found in most crystals. This results in the electron lattice having a much smaller Brillouin Zone than those usually encountered. For an electron density $n = 10^{14} cm^{-3}$, the lattice constant a is

approximately $2.7 \times 10^{-5}$cm, so the Brillouin Zone boundary ($\pi/a$) is at a wave vector about equal to $10^{5}$cm$^{-1}$, a value about three orders of magnitude smaller than those found for the zone boundary of a typical crystal. The conservation of wave vector in scattering by lattice phonons requires that

$$\underline{k}_o = \underline{k}_s + \underline{q}$$

for phonon creation, where $\underline{k}_o$, $\underline{k}_s$, and $\underline{q}$ are the wave vectors of the incident photon, the scattered photon, and the emitted phonon, respectively. Since, on assuming little dispersion of the refractive index of the medium, $|\underline{k}_o|$ is approximately equal to $|\underline{k}_s|$, conservation of wave vector leads to the result [24] that

$$q = |\underline{q}| = 2k_o \sin\theta, \qquad (2)$$

where $k_o = |\underline{k}_o|$ and $\theta$ is the angle between $\underline{k}_o$ and $\underline{k}_s$. The physical importance of equation (2) for the present discussion is the fact that $2k_o$ is the approximate magnitude of the <u>maximum</u> phonon wave vector allowed in Brillouin scattering. Since, for usual crystal lattices, ($\pi/a$) is much larger than the wave vector $k_o$ of the incident photon, Brillouin scattering involves only phonons with small values of $q \simeq 0$ near the zone center. However, for the electron lattice, ($\pi/a$) is roughly equal to $k_o$ for visible and near infrared photons, so it is to be

expected that LA and TA vibrational modes with wave vectors $\underline{q}$ of magnitude equal to the zone edge values would take part in the Brillouin scattering process.

It appears reasonable, then, to conclude that the Brillouin scattering spectrum (i.e., intensity as a function of photon frequency shift $\Delta\omega$ for a given scattering direction $\theta$) of the electron lattice should show new features in addition to scattering by the LA ($k = 0$) mode at $\Delta\omega = \omega_p$ found for the electron gas. Since both the LA and TA vibrations with wave vectors covering, approximately, the whole Brillouin Zone would be expected to take part, spectral structure at values of $\Delta\omega$ smaller than $\omega_p$ is to be anticipated. The relative intensities will depend, among other things, on the occupation numbers of the various phonon modes of the electron lattice and, hence, on the temperature. It is also to be expected that $\omega_\ell$ and $\omega_t$ would, through their proportionality to $\omega_p$, be directly proportional to the square root of the electron density n. This behavior, which has been observed [25] for Brillouin scattering by plasmons in the electron gas, would serve to identify observed scattering peaks as due to scattering by vibrations of the electron lattice. In any event, it appears, based on the ideas presented above, reasonable to expect significant qualitative differences between the Brillouin scattering spectrum due to plasmons in the electron gas and the features anticipated for the spectrum of the electron lattice.

A calculation of the Fermi-Thomas screening length $\lambda_{FT} \equiv (6\pi ne^2/\varepsilon E_F)^{-1/2}$, where $E_F$ is the Fermi energy, for an electron

density $n = 10^{14}\text{cm}^{-3}$ in germanium yields a value of $\lambda_{FT} = 200\text{Å}$. Further, one would expect to use an incident photon frequency $\omega_o$ for scattering experiments such that

$$\omega_p < \omega_o < (E_G/\hbar)$$

where $E_G$ is the energy gap of the semiconductor in question. For the germanium described above $\omega_p = 4.1 \times 10^{11}\text{sec}^{-1}$ and $(E_G/\hbar) = 1.14 \times 10^{15}\text{sec}^{-1}$. The corresponding wavelengths $\lambda_p = (2\pi c/\omega_p) = 0.463$ cm and $\lambda_G = 2\pi c/(E_G/\hbar) = 1.654 \times 10^{-4}$cm. The incident photon wavelength $\lambda_o$ used will then be such that it is less than $\lambda_p$ and greater than $\lambda_G$, so it will then be true that $\lambda_o \gg \lambda_{FT}$. This means that the electron gas at this density should exhibit collective (i.e., plasmon) behavior, rather than single particle behavior, for an incident wavelength $\lambda_o$ in the Brillouin scattering experiments suggested above.

It is therefore reasonable to suggest that an experimental study of Brillouin scattering in very pure germanium (doped with antimony) would be a reasonable method of searching for the Wigner lattice. A series of such samples with varying values of the donor atom density $N_d$ could be studied at appropriate temperatures, yielding a series of Brillouin scattering spectra at varying values of the electron density n in a given temperature. A search for the spectral features described above would then probe the presence or absence of the electron lattice.

## IV. Stability of the Lattice

In order to investigate the potential stability of the Wigner lattice in the temperature range in question, the magnitude and sign of the cohesive energy may be calculated. The cohesive energy $\Delta E$, per electron, of the lattice is the difference between the energy $E_g$ of the electron gas per electron and the energy $E_s$ of the crystal per electron:

$$\Delta E \equiv (E_g - E_s).$$

A positive value of $\Delta E$ will imply the stability of the lattice relative to the gas for the value of the electron density n being considered. The energy $E_s$ at T = 0 K, when the vibrations of the lattice have the average phonon occupation number $\langle n \rangle = 0$, is given [4] by

$$E_s = [\ (-3e^2/2r_o) + (1/2)\sqrt{3}\ \hbar\omega_p\ ],$$

and the energy $E_g$ may be taken as $(3/5)\ E_F$, where $E_F$ is the Fermi energy at the electron density n. The expression for $E_s$ may be modified to

$$E_s = [\ (-3e^2/2r_o) + (\langle n \rangle + 1/2)\sqrt{3}\ \hbar\omega_p]$$

to include the case of values of $\langle n \rangle$ not equal to zero.

If we consider T = 16K, $k_BT = 1.38 \times 10^{-3}$eV, and, for $n = 10^{14}$ $cm^{-3}$, $\hbar\omega_p = 2.69 \times 10^{-4}$eV, so $k_BT > \hbar\omega_p$, leading to the result that

$$\langle n \rangle \cong (k_BT/\hbar\omega_p) \cong 5.$$

Using $\langle n \rangle = 5$ and $r_o = (3/4\pi)^{1/3} n^{-1/3} = 1.34 \times 10^{-5}$ cm, one obtains $E_s = -2.16 \times 10^{-14}$ ergs per electron, $E_g = 0.624 \times 10^{-16}$ ergs per electron, and $\Delta E$ is found to be equal to $2.17 \times 10^{-14}$ ergs per electron or $1.36 \times 10^{-2}$ eV per electron. The cohesive energy is thus positive and its value may be compared to the cohesive energy of solid neon (melting point 24K) of 0.02 eV per atom given in Reference 11, page 99. We see also that $(\Delta E/\hbar\omega_p) \tilde{=} 50$ and $(\Delta E/k_B T) \tilde{=} 10$, showing that the cohesive energy per electron of the Wigner lattice is large, in this temperature range, compared to both $k_B T$ and the largest quantum $\hbar\omega_p$ of the vibrational spectrum of the lattice. It is therefore reasonable to conclude that the electron lattice is, at least in principle, stable in this temperature range.

## V. Other Possible Experiments

There are, of course, other possible experiments to consider for the detection of the electron lattice. Among these is a study of the specific heat $C_v$ as a function of electron density. It appears, however, that the Debye temperature [26] for the electron lattice is about 3000K, while that of the host germanium is 374K. The lattice specific heat of the germanium would thus be a factor of about $10^3$ larger than that of the electron crystal, making experimental detection of the latter difficult. One might consider observing the vanishing of the term in $\gamma T$ in the specific heat [27] as the electron gas crystallizes. Howver, a calculation (by the author) of $\gamma$ for an electron gas of density $n = 10^{14} cm^{-3}$ leads to a value of $\gamma = 9 \times 10^{-2}$ erg-deg$^{-2}$. The electron gas specific heat $\gamma T$ is equal to 1.5 erg-deg$^{-1}$ at T = 16K, compared to a lattice specific heat of $1.1 \times 10^5$ erg-deg for germanium, both values being for a volume of 1 cm$^3$. It appears, therefore, that experimentally detecting the electron lattice using specific heat measurements would probably be difficult. Another type of possible experiment might involve the scattering [28] of phonons by the conduction electrons in a solid. One might anticipate that the ordered electron lattice would scatter phonons less than does the electron gas, and that this difference might be probed with thermal conductivity measurements, or perhaps with an experiment involving the scattering by electrons of monochromatic phonons generated by the methods used by Dynes and Narayanamurti [29].

## VI. Conclusion

In summary, it is proposed that conditions under which the three dimensional Wigner electron lattice is predicted to form may be achieved in extrinsic germanium at low temperatures. Of several possible experiments which might detect this lattice, it appears that the most feasible is the study of Brillouin scattering of photons by the LA and TA modes of vibration of the electron lattice.

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10. This value of m* is an average over the longitudinal and transverse masses $m_\ell = 1.64m$ and $m_t = 0.082m$ [see, for example, R. Dalven, Infrared Phys. 6, 129 (1969)] using the relation $m^* = 3m_t m_\ell/(2m_\ell + m_t)$.
11. C. Kittel, "Introduction to Solid State Physics", Fourth Edition, (John Wiley, 1971), page 374, Table 3.

12. C. Kittel, Reference 11, page 376, equation (16). The requisite condition that $E_d \gg k_B T$ is satisfied for the cases considered in this paper.

13. The decrease of $E_d$ with increasing donor concentration $N_d$ for values of $N_d$ larger than about $10^{15} cm^{-3}$ [see, for example, S. Wang, "Solid State Electronics", (McGraw-Hill, 1966), page 157] has been neglected for simplicity.

14. A second possible semiconductor for this purpose is GaAs. Even though GaAs has a simpler conduction band structure than does germanium, the present technology of GaAs is not as advanced as is that of germanium, and it is doubtful that GaAs of purity sufficient for these purposes is available.

15. P. M. Platzman and P. A. Wolff, Reference 3, page 14 and 85.

16. By a "simple semiconductor" is meant a semiconductor with a single, isotropic conduction band minimum. Examples are GaAs (see Note 14) and InSb. Germanium, with its many-valley conduction band, is not member of this class, but the complications thereby introduced (see Reference 3, pages 99ff) are ignored in this paper for simplicity.

17. For a review, see P. M. Platzman and P. A. Wolff, Reference 3, Section 34, pages 85ff.

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19. The bcc lattice has one lattice point per primitive unit cell.

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25. See Reference 3, page 86.

26. Calculated by the author, using $r_s = 19$, from the relation given by Carr on page 1444 of his paper in Reference 6. This value of the Debye temperature of the electron lattice must, however, be regarded with caution since it corresponds to the very high value of $2 \times 10^9$ cm/sec for the velocity of sound.

27. C. Kittel, Reference 11, page 252.

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