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Journal
The Astrophysical Journal, 320(2)

ISSN
0004-637X

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Publication Date
1987-09-01

DOI
10.1086/165560

Peer reviewed

THE ASTROPHYSICAL JOURNAL, 320:439–447, 1987 September 15

THE QUARK-HADRON PHASE TRANSITION AND PRIMORDIAL NUCLEOSYNTHESIS

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Received 1987 January 20; accepted 1987 March 13

ABSTRACT

The dynamics and statistical mechanics of the quark-hadron phase transition are explored using the bag model and the known spectrum of hadronic states. We compute the maximum amplitude for isothermal baryon number density fluctuations to emerge from this phase transition and their effects on primordial nucleosynthesis, as a function of the coexistence temperature (or bag constant) and the fractional volume of the universe which will remain in quark-gluon plasma when the release of latent heat no longer compensates the cooling due to expansion. For values of the bag constant, $B < (260 \text{ MeV})^4$, it is possible to find a pressure, temperature, and baryon-chemical-potential equilibrium between the quark-gluon phase and the hadron phase. We calculate the difference in baryon number concentration between these phases. This difference in baryon number concentration may lead to isothermal baryon density fluctuations. For $B > (260 \text{ MeV})^4$ phase equilibrium is not possible for temperatures below 300 MeV. Thus any differences in baryon concentration are small. In the extreme but interesting case of a Hagedorn limiting hadronic temperature and large bag constant we even find that the QCD vacuum energy can produce a mini-inflationary epoch. We discuss computations of the primordial nucleosynthesis yields corresponding to our estimates of the maximum baryon density fluctuations in a universe with $\Omega = 1$ in baryons. We find that ^{4}He, ^{3}He, and deuterium are within observed constraints for a large part of the parameter space. However, ^{7}Li is overproduced. Within the parameter space for an $\Omega = 1$ universe we find only a very small abundance of elements heavier than mass 11 (total mass fraction $X \leq 10^{-8}$).

Subject headings: cosmology — elementary particles — nucleosynthesis

I. INTRODUCTION

There has been a great deal of interest recently (Applegate, Hogan, and Scherrer 1987, hereafter AHS; Sale and Mathews 1986) in the possibility that the quark-hadron phase transition may have been a production site for primordial isothermal baryon number fluctuations. Such fluctuations could substantially change the predicted abundances from big bang nucleosynthesis and influence ideas on the formation and nature of the dark matter component. For example, it may be possible to have $\Omega = 1$ in baryons (Sale and Mathews 1986; AHS) and yet still have acceptable big bang nucleosynthesis.

In the scenario discussed by Witten (1984), and Applegate and Hogan (1985), the phase transition from the quark-gluon plasma (i.e., unconfined color charges) to the hadronic (i.e., confined) phase should take place through the following process (assuming that the phase transition is first order as suggested by lattice QCD with fermions, cf. Kogut 1986). As the universe expands the temperature drops through T_c, the critical temperature, where phase separation and coexistence is possible. Supercooling occurs until the probability to nucleate bubbles of the hadronic phase is high. The initially nucleated bubbles of hadron phase release latent heat from the QCD vacuum energy and reheat the universe to T_c, where further nucleation of the confined phase is inhibited. The confined and unconfined phases now coexist in pressure and baryon-chemical-potential equilibrium. As the universe expands the temperature is kept at T_c by the growth of the confined phase at the expense of the unconfined phase. This constant temperature evolution may continue until all of the universe has been converted to confined phase (cf. Kajantie and Kurki-Suonio 1986).

However, the universe may not remain at T_c until all of the quark-gluon plasma has been converted into hadrons. At some point the release of latent heat may not be rapid enough to compensate the cooling due to expansion (Witten 1984). This would lead to decoupled regions of quark-gluon plasma which nucleate at a later time and thus could produce nearly isothermal baryon number density fluctuations. Witten (1984) also pointed out that it may be possible for the shrinking bubbles of quark-gluon plasma to cool and stabilize, leading to quark nugget formation. Whether of not such nuggets are absolutely stable will not be discussed here, but Alcock and Farhi (1985) have shown that all nuggets smaller than 10^{52} baryons will evaporate on a time scale of milliseconds also leaving behind local, nearly isothermal, baryon number density enhancements. For our purposes we do not require formation of Witten nuggets, but only the more general scenario of decoupled regions of quark-gluon plasma before the phase transition has run to completion.

We have employed the bag model to give the bulk volume energy of the quark-gluon plasma in a manner to be described in § II. The uncertainties inherent in bag models fall into two main categories: the treatment of surface effects in nucleons and other hadrons; and the value of the bag constant itself. The first uncertainty is relatively unimportant for our purposes because we are interested in macroscopic regions of quark-gluon plasma where bulk, or volume, energy dominates any surface terms. The bag constant, which gives this volume energy, is uncertain itself, especially at the high temperatures inherent in the cosmological phase transition we consider (cf. the review of bag models in Thomas 1984). For this reason we consider a range of bag constants, B. In our model, the bag constant helps to determine the coexistence temperature, T_c which, as we will see, sets the amplitude of the isothermal baryon number fluctuations. This will be shown with a very simple statistical argument.

We show that for $B < (260\text{ MeV})^4$ the above picture of constant-temperature coexistence, coalescence, and baryon number fluctuation can occur. However, if $B > (260\text{ MeV})^4$ no coexistence is possible (below $kT \sim 300$ MeV) because the pressure from the known baryonic resonances always exceeds the pressure in the quark-gluon plasma. Therefore, if bubbles of hadronic phase are nucleated they will grow explosively into the quark-gluon plasma, and the liberation of latent heat will not be sufficient to achieve coexistence of phases and phase separation. In the most extreme version of this scenario, the QCD vacuum energy will come to dominate the pressure of the universe and a de Sitter exponential expansion can ensue for a short time. This mini-inflation results in only a 15% increase in the scale factor. The important point is that the nucleation properties of the hadronic phase are much different; i.e., the nucleation rate is very rapid and is not arrested by release of latent heat. A high nucleation rate suppresses the large-scale separation of phases needed to form isothermal baryon number fluctuations. Even if coexistence of phases occurs at higher temperatures, the difference in baryon number concentration between the phases is small and, again, isothermal baryon number fluctuations will be negligible.

II. QUARK-HADRON PHASE TRANSITION DYNAMICS

Using the bag model, the thermodynamic potential, Ω, for the quark-gluon plasma in the limit of vanishing quark masses is

$$\Omega_{qg} = \frac{-N_c N_f V}{6(\hbar c)^3}\left[\frac{7\pi^2}{30}(kT)^4 + \mu_q{}^2(kT)^2 + \frac{1}{2\pi^2}\mu_q{}^4\right] - \frac{\pi^2}{45(\hbar c)^3} N_g V(kT)^4 + BV , \tag{1}$$

where N_c is the number of colors (3), N_f is the number of relativistic quark flavors (2 corresponding to u and d quarks at lower T, and 3 at higher T where the s quark is also important), k is the Boltzmann constant, and B is the bag constant. The quark chemical potential is $\mu_q = \mu_b/3$, where μ_b is the baryon chemical potential. The system volume is V. The expression in equation (1) includes both quarks and antiquarks and is exact for any μ_q and T so long as the quarks are relativistic. The quantity $N_g = 8$ is the number of gluons. The thermodynamic variables corresponding to Ω are then

$$P = -\left(\frac{\partial\Omega}{\partial V}\right)_{T,\mu} = -\frac{\Omega}{V} \tag{2a}$$

$$n = -\frac{1}{V}\left(\frac{\partial\Omega}{\partial\mu}\right)_{V,T} \tag{2b}$$

$$S = -\left(\frac{\partial\Omega}{\partial T}\right)_{V,\mu} \tag{2c}$$

$$E = -PV + ST + \mu n V , \tag{2d}$$

where P, n, S, E are respectively pressure, baryon number density, entropy, and energy. We note that the pressure from the bag (QCD vacuum) is negative. Therefore, the larger the bag constant the smaller the size of bound color singlets. We discuss here the pressure and energy density for the strongly interacting component only, i.e., quarks, gluons, baryons, and mesons. Of course, the contributions to the total pressure and energy density from the background photons, leptons, and antileptons are the same in both phases. These relativistic particles contribute a pressure

$$P = 2.04 \times 10^{-7}\text{ MeV fm}^{-3}(kT/\text{MeV})^4 \tag{2e}$$

to each phase (including photons, three flavors of neutrino, and relativistic electrons and muons).

For bound hadrons in the confined phase we compute Ω separately for baryons and mesons. For small μ ($\mu = 0$ for mesons) we follow the notation of Fowler and Hoyle (1964) and derive for mesons

$$\Omega = -\frac{gV(kT)^4}{\pi^2(\hbar c)^3}\sum_{n=1}^{\infty}\left[\frac{1}{n^4}\bar{K}_2\left(n\frac{mc^2}{kT}\right)\right] \tag{3a}$$

from which the mesonic pressure is

$$P_\pi = \frac{3(kT)^4}{\pi^2(\hbar c)^3}\sum_{n=1}^{\infty}\left[\frac{1}{n^4}\bar{K}_2\left(n\frac{mc^2}{kT}\right)\right], \tag{3b}$$

where $\bar{K}_2$ is related to the modified Bessel function of the second order,

$$K_2(x) = (2/x^2)\bar{K}_2(x) , \tag{3c}$$

and we note that

$$\lim_{x\to 0}\bar{K}_2(x) = 1 .$$

The dominant source of pressure in the hadronic phase is contributed by the most relativistic particle, the pion. However, in what follows we sum the pressure contribution from all known hadronic states. Masses, spins, and isospins of these particles are taken from the particle data group summary (Aguilar-Benitez *et al.* 1986).

For baryons we obtain for small μ ($|\mu| < mc^2$)

$$\Omega = -\frac{2g(kT)^4V}{\pi^2(\hbar c)^3}\sum_{n=1}^{\infty}\frac{(-1)^{n+1}}{n^4}\cosh\left(\frac{n\mu}{kT}\right)\bar{K}_2\left(n\frac{mc^2}{kT}\right) \tag{4a}$$

$$n_b = \frac{2g(kT)^3}{\pi^2(\hbar c)^3}\sum_{n=1}^{\infty}\frac{(-1)^{n+1}}{n^3}\sinh\left(\frac{n\mu}{kT}\right)\bar{K}_2\left(n\frac{mc^2}{kT}\right), \tag{4b}$$

where the notation is as above, with $g = (2J + 1)(2I + 1)$, J the spin, I the isospin, and n_b the baryon number density. Note that since $\mu/kT \sim 10^{-8}$ the small μ limit is justified.

In addition to summing the contribution to Ω in equations (3a) and (4a) over the 128 known hadronic resonances, we have performed another calculation in which we integrate the thermodynamic potential over an exponentially increasing (Hagedorn 1973) spectrum of hadron masses. The pressure-temperature curves (for a fixed baryon chemical potential, $\mu \ll mc^2$) for the quark-gluon and hadronic phases are shown in Figure 1*a*, *b*, *c*.

Figure 1*a* gives the pressure-temperature curves for a low bag constant corresponding to the MIT bag model, $B = (145\text{ MeV})^4$. The hadronic curve is calculated using the spectrum of known hadronic states. Equilibrium phase coexistence is possible where the two curves cross.

Figure 1*b* shows the pressure-temperature curves for a high bag constant corresponding to $B = (278\text{ MeV})^4$. Phase equilibrium is not possible over the temperature range plotted. Figure 1*b* also shows the P-T curve if the hadronic phase is characterized by a Hagedorn spectrum of masses. For this

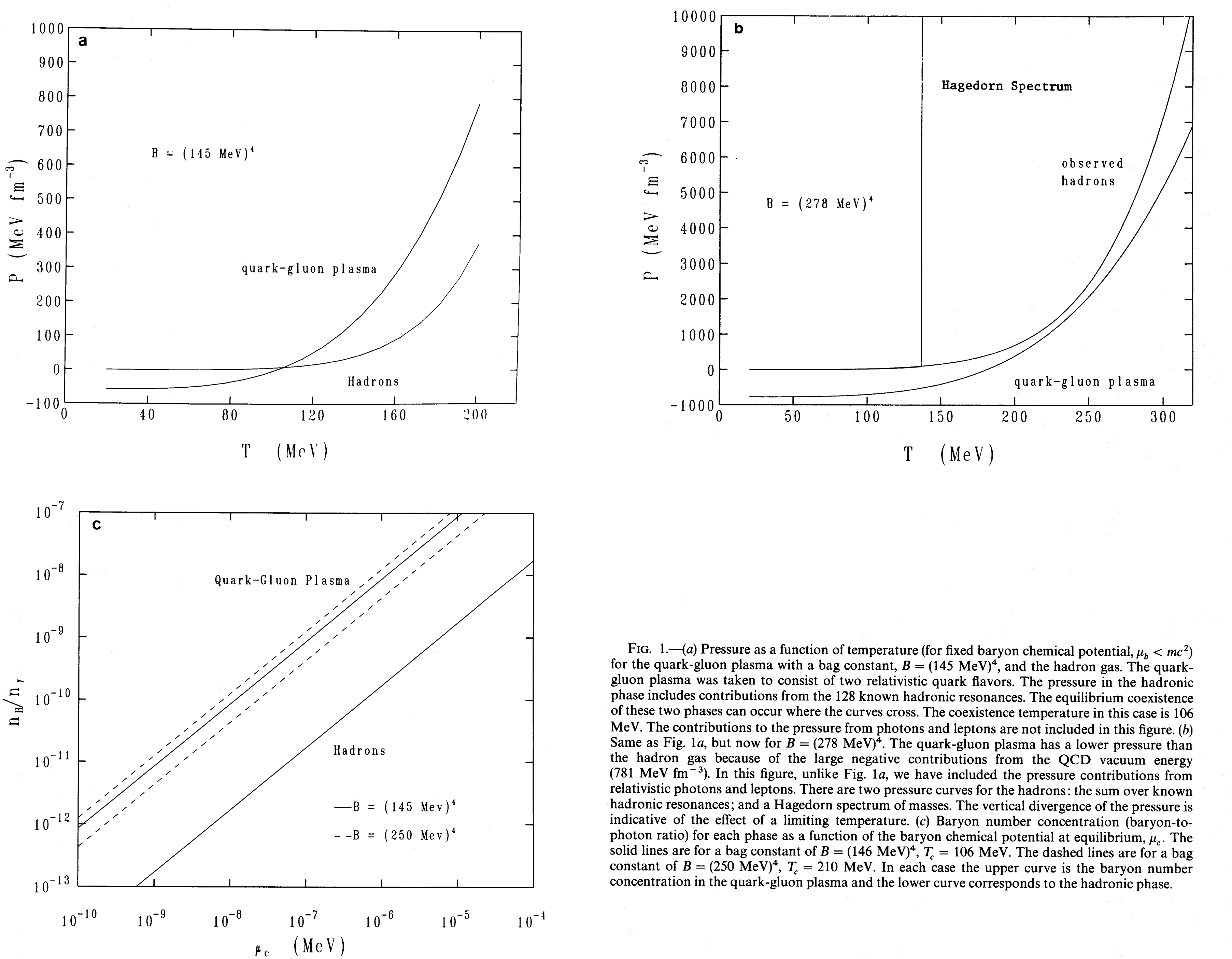

FIG. 1.—(*a*) Pressure as a function of temperature (for fixed baryon chemical potential, $\mu_b < mc^2$) for the quark-gluon plasma with a bag constant, $B = (145\ \mathrm{MeV})^4$, and the hadron gas. The quark-gluon plasma was taken to consist of two relativistic quark flavors. The pressure in the hadronic phase includes contributions from the 128 known hadronic resonances. The equilibrium coexistence of these two phases can occur where the curves cross. The coexistence temperature in this case is 106 MeV. The contributions to the pressure from photons and leptons are not included in this figure. (*b*) Same as Fig. 1*a*, but now for $B = (278\ \mathrm{MeV})^4$. The quark-gluon plasma has a lower pressure than the hadron gas because of the large negative contributions from the QCD vacuum energy (781 MeV fm^{-3}). In this figure, unlike Fig. 1*a*, we have included the pressure contributions from relativistic photons and leptons. There are two pressure curves for the hadrons: the sum over known hadronic resonances; and a Hagedorn spectrum of masses. The vertical divergence of the pressure is indicative of the effect of a limiting temperature. (*c*) Baryon number concentration (baryon-to-photon ratio) for each phase as a function of the baryon chemical potential at equilibrium, μ_c. The solid lines are for a bag constant of $B = (146\ \mathrm{MeV})^4$, $T_c = 106$ MeV. The dashed lines are for a bag constant of $B = (250\ \mathrm{MeV})^4$, $T_c = 210$ MeV. In each case the upper curve is the baryon number concentration in the quark-gluon plasma and the lower curve corresponds to the hadronic phase.

extreme calculation we used the Hagedorn (1973) mass spectrum with a limiting temperature of $T_{\rm Hag} = 140$ MeV. It is known that finite hadron size effects will truncate the Hagedorn spectrum at some point so that the limiting temperature effect is never achieved (cf. Suhonen [1982] and references therein). Olive (1981) has considered this effect and has shown that the actual hadronic *P-T* curve should lie between the two extremes plotted in Figure 1*b*. Nevertheless, the physical behavior is clear. For large values of the bag constant, there is a large negative contribution to the pressure in the quark-gluon phase such that the quark-gluon pressure is always lower than the hadronic pressure. This is true for temperatures lower than ~ 300 MeV, but for higher temperatures, QCD corrections may become large enough to cause the curves to cross (Suhonen 1982). This dependence of the phase coexistence thermodynamics on the bag constant is important for the nucleation properties as well as for the amplitude of the emergent baryon density fluctuations.

Higher bag constants are favored in the chiral bag models (cf. Thomas 1984). We present results for $B = (278\ \text{MeV})^4$ because it is representative of the behavior we are trying to illustrate and because this is the value of the bag constant used by many recent papers (cf. Suhonen 1982 and Kajantie and Kurki-Suonio 1986). Brown and Bethe (1986) favor a lower bag constant, $B = (220\ \text{MeV})^4$, which will give results similar to the low bag constant scenario. It should be kept in mind that finite temperature corrections to the bag constant are important and uncertain. In keeping with the exploratory spirit of this work, we treat the bag constant as a parameter which characterizes the coexistence temperature.

The critical bag constant where the division between the high bag constant and low bag constant scenarios occur is found to be $B \sim (260\ \text{MeV})^4$. This is an interesting value because it falls between the low values which fit the hadronic spectrum best, as in the MIT bag model with $B = (145\ \text{MeV})^4$, and the high values of B, which are found in the pionic or chiral bag models (cf. the review of bag models in Thomas 1984). Furthermore, the critical bag constant may not be too far from the value of the bag constant necessary to reproduce the phase transition temperature derived from lattice QCD (Fukugita and Ukawa 1986).

We also note here a curious phenomenon which could occur if the Hagedorn mass spectrum is not too far from the truth. As the universe filled with quark-gluon plasma cools the positive pressure contribution from photons, leptons, quarks, and gluons eventually could be dominated by the negative pressure from the QCD vacuum energy. A short de Sitter phase would then ensue in which the scale factor of the universe increases by about 15%. The ratio of the scale factor at the beginning of the de Sitter phase ($R_{\rm cross}$, where the total pressure is zero) to the scale factor at the end of the inflation ($R_{\rm Hag}$, where the temperature has cooled below the Hagedorn limiting temperature) is

$$R_{\rm Hag}/R_{\rm cross} = T_{\rm cross}/T_{\rm Hag} \sim \exp\left[(t_{\rm Hag} - t_{\rm cross})/\tau\right] , \qquad (5)$$

where $T_{\rm cross}$ and $T_{\rm Hag}$ are the respective temperatures corresponding to $R_{\rm cross}$ and $R_{\rm Hag}$. For the case shown in Figure 1*b*, $t_{\rm Hag} - t_{\rm cross} \sim 4\ \mu$s is only a small fraction of the expansion time scale $\tau \sim 20\ \mu$s. This mini-inflationary epoch is thus very short-lived and could not contribute to the large-scale structure of the universe. On the other hand, it could drastically affect the nucleation properties during the quark-hadron phase transition.

III. BARYON NUMBER DENSITY

The critical parameter for isothermal baryon density fluctuations is the difference in the net baryon number concentration of the two phases while they coexist. The larger this difference, the larger will be the baryon density fluctuations when, and if, quark-gluon plasma decoupling takes place. The net baryon concentration in the unconfined (quark) phase can be computed from equations (1) and (2b):

$$n_b{}^q \approx \tfrac{2}{9}(kT/\hbar c)^3(\mu_b/kT) , \qquad (6a)$$

where we have neglected terms of higher order than $(\mu_b/kT) \sim 10^{-8}$. The number of quarks is comparable to the number of photons, but the net baryon number is, of course, much smaller since it depends on the difference between the number of quarks and antiquarks. By contrast, the net baryon number in a gas of neutrons and protons is only

$$n_b{}^h \approx (8/\pi)^{1/2}(kT/\hbar c)^3(\mu_b/kT)(mc^2/kT)^{3/2} \exp\left[-mc^2/kT\right] , \qquad (6b)$$

where, $mc^2 \approx 938$ MeV is the nucleon rest mass. Even though the temperature is much less than the nucleon mass, the number of nucleons and antinucleons is comparable, $(n_{\bar{N}}/n_N) \approx \exp(-2\mu_b/kT) \approx 1$, but the net baryon number in equation (6c) is much smaller than in the quark phase. If we include only the contribution from neutrons and protons in the hadronic phase then, for illustration, the ratio of the baryon density in the two phases can be written analytically,

$$\frac{n_b{}^q}{n_b{}^h} \approx \left(\frac{\pi^3}{2}\right)^{1/2} \frac{e^z}{z^{3/2}} \approx \begin{matrix} 1042 \text{ at } kT \approx 106 \text{ MeV} \\ 25 \text{ at } kT \approx 240 \text{ MeV} \end{matrix} , \qquad (6c)$$

where $z = mc^2/kT$. The large disparity in baryon concentration in the two phases is clearly due to the much greater statistical weight for putting baryon number in nearly massless relativistic quarks, as opposed to the nonrelativistic baryons. The inclusion of the other known baryonic resonances, however, has the effect of lowering the ratio in equation (6c) at a given coexistence temperature. The ratio of baryon to photon density for the two phases (including all known hadron resonances in the hadronic phase) as a function of baryon chemical potential is shown in Figure 1*c* for two different values of the critical temperature (or bag constant).

Equation (6c) and Figure 1*c* make it clear that the difference in the baryon number concentration between the two phases goes down as the coexistence temperature increases (i.e., value of B increases). This effect is enhanced by the inclusion of more hadronic resonances. When regions of quark-gluon plasma decouple and cool, this difference in baryon number concentrations translates into baryon density fluctuations which are essentially isothermal (Witten 1984).

We emphasize the importance of the value of the bag constant in determining the coexistence temperature. Previous studies have shown that, for the most part, the dynamics and large-scale structure of the universe going through the quark-hadron phase transition are relatively insensitive to the value of the bag constant (Suhonen 1982; Lodenquai and Dixit 1983; Dixit and Suhonen 1983). We point out that since the bag constant determines the coexistence temperature, it will strongly influence the baryon number concentration difference and hence, primordial nucleosynthesis as discussed below. Values of the bag constant greater than $(260\ \text{MeV})^4$ imply that any coexistence must be at a temperature greater than 300 MeV where the difference in baryon concentration is small.

As far as which value of the coexistence temperature (or bag constant) is best to use, there is still considerable uncertainty. Lattice QCD with fermions on an $8^3 \times 4$ lattice would place the phase transition temperature at about 200–230 MeV (Fukugita and Ukawa 1986). Calculations on a $10^3 \times 6$ lattice however place the transition temperature at ~ 160 MeV (Kogut 1986). These results indicate the trend. However, the lattice sizes are probably still too small for a reliable extraction of the continuum physics. The bag constant is also uncertain due to finite temperature corrections which will reduce the value of B (McLerran and Svetitsky 1981). Thus, the coexistence temperature is probably close to the value obtained with lower values for the bag constant which produce significant baryon number concentration differences between the phases, but we emphasize the uncertainty in this parameter.

IV. NUCLEATION

In the standard case, where the bag constant is less than $(260\ \mathrm{MeV})^4$, the universe will supercool until the classical nucleation rate becomes large. This initial nucleation releases latent heat and reheats the universe back to the coexistence temperature. The latent heat is carried by neutrinos and by relativistic shocks generated by the expanding hadonic bubbles (Kajantie and Kurki-Suonio 1986). Once the universe is heated back to the coexistence temperature, further nucleation is inhibited due to the steep temperature dependence of the nucleation rate.

If, however, the bag constant is large ($B > [260\ \mathrm{MeV}]^4$), and we insist that the nucleation of the hadronic phase occur for $kT < 300$ MeV, then the nucleation will take place out of equilibrium. In this case, the nucleation rate will be much higher than in the standard separation of phases scenario. A higher nucleation rate implies a smaller initial volume for each bubble of hadronic phase, many more bubbles per unit volume, and hence, less efficient separation of phases. This is an additional effect which will tend to diminish the amplitude of baryon density fluctuations if the bag constant is large.

It is very difficult to make an accurate prediction of the properties of the nucleation. However, it is useful to make some estimate of the number of nucleation sites per unit volume in order to model the effects of nucleon diffusion from the regions of the high density. This diffusion will have an important effect on the primordial nucleosynthesis, as we shall see. In order to have an indication of the nucleation rate, we follow Landau and Lifshitz (1969) and Kajantie and Kurki-Suonio (1986) to estimate the nucleation rate, $p(T)$

$$p(T) = C T_c^4 \exp\left[-D T_c^2/(T_c - T)^2\right] , \qquad (7a)$$

where C and D are parameters which depend on microscopic details of the phase transition, and T_c is the coexistence temperature. If we define the supercooling parameter to be $\eta = (T_c - T_f)/T_c$, where T_f is the lowest temperature reached before reheating, then the number of nucleated sites per unit volume, N_n, will be

$$N_n(ct)^3 \approx \frac{D^3}{8\pi v_s^3 \eta^9} , \qquad (7b)$$

where ct is the horizon scale and v_s is the velocity of the shocks which are assumed to be responsible for the reheating (Kajantie and Kurki-Suonio 1986). The amount of super cooling depends relatively weakly on the phase transition parameters from equation (7a)

$$\eta \approx \frac{D^{1/2}}{[195 + \ln C - 4 \ln D - 4 \ln (T_c/\mathrm{MeV}) + 12 \ln \eta]^{1/2}} \qquad (7c)$$

so that to a fair approximation, the number density of nucleated sites can be written

$$N_n(ct)^3 \approx 10^9 D^{-1.5} . \qquad (7d)$$

For $D \sim 1$, then $\eta \sim 8 \times 10^{-2}$ and $N_n(ct)^3 \sim 10^9$. For a horizon scale of 10 km, we find that the nucleated hadron bubbles are on the order of a meter apart, in agreement with Kajantie and Kurki-Suonio (1986). We assume that there is a duality between the centers of nucleated hadron bubbles at the beginning of the phase transition and the centers of regions of quark-gluon plasma at the end of the phase transition.

We note the relatively weak dependence on the phase transition parameters, C and D. For several orders of magnitude variation in D we estimate that the average separation between nucleation sites should be ~ 1 to 10 m at the phase transition which would correspond to 10^3 to 10^4 m during nucleosynthesis. Thus, we find that the separation between baryon density fluctuations should be within the neutron diffusion length (~ 30 km) yet larger than the proton diffusion length (~ 300 m) computed by AHS at the time of nucleosynthesis.

V. BARYON DENSITY FLUCTUATIONS AND PRIMORDIAL NUCLEOSYNTHESIS

We have seen in the previous sections that isothermal baryon number density fluctuations are a possible consequence of the phase transition from quark-gluon plasma to hadronic matter. If this is true, then these fluctuations can have an interesting effect on primordial nucleosynthesis. There are two influences which must be considered. One is the fact that more than one baryon-to-photon ratio must be averaged to produce the final nucleosynthesis yield. Such an averaging by itself tends to alter significantly the yields from primordial nucleosynthesis (Sale and Mathews 1986). The second intriguing effect was recently suggested by AHS. It is that the neutron and proton components of these density fluctuations will diffuse differently after the weak reactions fall out of equilibrium. Essentially, the neutron mean free path is much larger (by more than an order of magnitude) since the neutron interacts only weakly with the background plasma via nucleon scattering or via the small neutron dipole moment. The diffusion of protons, on the other hand, is slowed by the more dominant proton-electron scattering.

The influence of these two effects must be studied by following the evolution of the proton and neutron fluids, the nucleosynthetic reaction rates, and the universal expansion. To model these processes we use the following scenario which should correspond to the maximum possible effect on primordial nucleosynthesis.

For reasons of theoretical prejudice, we consider a universe with total average $\Omega_b = 1$ during the present baryon-dominated epoch. This average present Ω is determined by the contributions from the high baryon density and low baryon density regions produced by the quark-hadron phase transition

$$\Omega = f_v \Omega_q + (1 - f_v)\Omega_h = 1 , \qquad (8)$$

where f_v is the volume fraction of the universe in the high baryon density regions which were originally in the form of quark-gluon plasma when these regions began to decouple from the expansion and cool. Note that our definition of f_v differs from that of AHS. In that work f_v refers to the volume fraction in the low-density neutron-rich regions. The quantity Ω_q is a measure of the baryon density in the formerly quark-gluon plasma regions, and similarly Ω_h a measure of the hadronic baryon density before neutron diffusion. From Figure 1*c*, and equation (6c), $\Omega_h \sim 0.02\Omega_q$ for $T_c = 106$ MeV (bag constant of [145 MeV]^4), if we ignore baryon diffusion before the freezeout of the weak interaction rates.

To include the effects of nucleon diffusion in this picture, we make two simplifying assumptions: (1) the nucleation rate is sufficiently great that the mean separation between fluctuations is significantly less than a neutron diffusion length at the time of nucleosynthesis; (2) the proton diffusion can be ignored. These assumptions are a fair approximation to the results of our nucleation rate studies discussed above and to the diffusion rates of AHS, where it was shown that the diffusion length for baryons is small until the time when the weak interactions can be neglected. Our approximation to their result is to ignore baryon diffusion altogether until the temperature falls below $T \approx 1.3 \times 10^9$ K as discussed below. The differential diffusion process occurs in the ensuing $\sim$100 s before nucleosynthesis, during which time the nucleons are locked into their identities (except for the slow neutron decay, which we include).

To implement these assumptions we have utilized the big bang nucleosynthesis code of Wagoner (1973) with three neutrino flavors, a neutron half-life of 10.6 minutes, and a number of nuclear reactions updated from the original version. We begin the nucleosynthesis with a baryon density corresponding to Ω_q or Ω_h as a function of f_v from equation (8).

The weak interaction drops out of equilibrium at a temperature of $T \sim 1$ MeV. That is, the forward and reverse rates for electron (positron) and neutrino (antineutrino) capture reactions are not equal for temperatures lower than this. However, the rates of these reactions are still appreciable down to temperatures of $T \sim 0.11$ MeV (1.3×10^9 K), where the final neutron-to-proton ratio is set and then subsequently changes only through neutron decay (cf. the discussion on p. 550 of Weinberg [1972]). Nucleosynthesis begins about 100 s later when the temperature has dropped to $T \approx 0.9 \times 10^9$ K.

As nucleosynthesis begins, the universe consists of two components: one a high baryon density proton-rich region (1), with

$$\Omega^{(1)} = X_n + \Omega_q(1 - X_n)\,, \quad (9)$$

$$X_n{}^{(1)} = X_n/\Omega^{(1)}\,. \quad (10)$$

(where X_n is the neutron mass fraction before diffusion at $T = 1.3 \times 10^9$ K) and a low baryon density neutron-rich region (2),

$$\Omega^{(2)} = X_n + \Omega_h(1 - X_n) \quad (11)$$

$$X_n{}^{(2)} = X_n/\Omega^{(2)}\,. \quad (12)$$

The final averaged mass fraction for each nuclide is then determined from

$$\bar{X}_i = f_v X_i{}^{(1)}\Omega^{(1)} + (1 - f_v)X_i{}^{(2)}\Omega^{(2)}\,. \quad (13)$$

In Figures 2*a*–*c* we show the nucleosynthesis yields in each of the two phases and final averaged abundances as a function of the high-density region volume factor, f_v. Figure 2*d* shows the relative baryon densities, $\Omega^{(1)}$ and $\Omega^{(2)}$, after neutron diffusion. For a broad range of models ($0.1 \leq f_v \leq 0.9$) most of the averaged nucleosynthesis yields are consistent with observation, except for an overabundance of ^{7}Li. This overabundance may not be too significant due to the uncertainties in the ^{7}Li abundance, the nuclear reaction rates, and stellar destruction of ^{7}Li (Boesgaard and Steigman 1985). Nevertheless, this overabundance appears to be an unavoidable consequence of these models due to the large contribution from the ^{3}He(α, γ)^{7}Be reaction in the high baryon density regions, and the ^{3}H(α, γ)^{7}Li reaction in the low baryon density regions. This seems to be a general problem with nucleosynthesis in inhomogeneous cosmologies, unless the high baryon density regions selectively collapse to form dark matter (Sale and Mathews 1986). This may, therefore, be a significant argument against such inhomogeneities being present during nucleosynthesis. Other authors have considered the effect on big bang nucleosynthesis of isothermal inhomogeneities (Barrow and Morgan 1983; Epstein and Petrosian 1975; Gisler, Harrison, and Rees 1974; Harrison 1968; Sale and Mathews 1986; Wagoner 1973; Yang *et al.* 1984; and Zel'dovich 1975), but none has taken into account the differential diffusion of neutrons and protons pointed out by AHS.

We also note that the mass fraction for heavy nuclei ($A > 11$) in the neutron-rich regions is not large enough to contribute significantly to the heavy-element abundances as speculated in AHS. The reasons for this difference are due to the constraint (eq. [8]) that Ω be unity, and our calculated ratio of baryon densities in the two phases. These two constraints imply that $\Omega^{(2)} < 0.3$ (or baryon-to-photon ratio, $\eta < 10^{-9}$) for $f_v > 0.1$ (see Fig. 2*d*), which corresponds to baryon densities too low for significant heavy-element formation even in neutron-rich regions. This low heavy-element abundance will probably be made even lower (Fowler and Malaney 1986) by new reaction rates for the production of $A = 11$. Nevertheless, Applegate (1986) speculates that such neutron-rich regions could be a site for primordial *r*-process nucleosynthesis.

VI. SUMMARY AND CONCLUSION

We have made a simple model for the quark-hadron phase transition based on the bag model for the quark-gluon plasma, the known spectrum of hadronic states, and elementary statistical mechanics. We believe that these simple ingredients provide an adequate parameterization of the phase transition allowing us to mock up the essential features needed to gauge the maximum size of the isothermal baryon number density fluctuations. The most important parameters are (1) the bag constant, which determines the coexistence temperature and therefore the difference in baryon number concentration; and (2) the fraction of the volume of the universe which is in high baryon density regions left over from the decoupling of quark-gluon plasma.

We have investigated primordial nucleosynthesis in the parameter space discussed above. For high coexistence temperatures, characteristic of a large bag constant, we find that the isothermal baryon number fluctuations are small and we recover standard big bang nucleosynthesis. We find for coexistence temperatures not too much greater than that derived from the standard MIT bag model, $T_c = 106$ MeV, $B = (145\ \mathrm{MeV})^4$, we are able to confirm the hypothesis that isothermal baryon inhomogeneities (from the quark-hadron

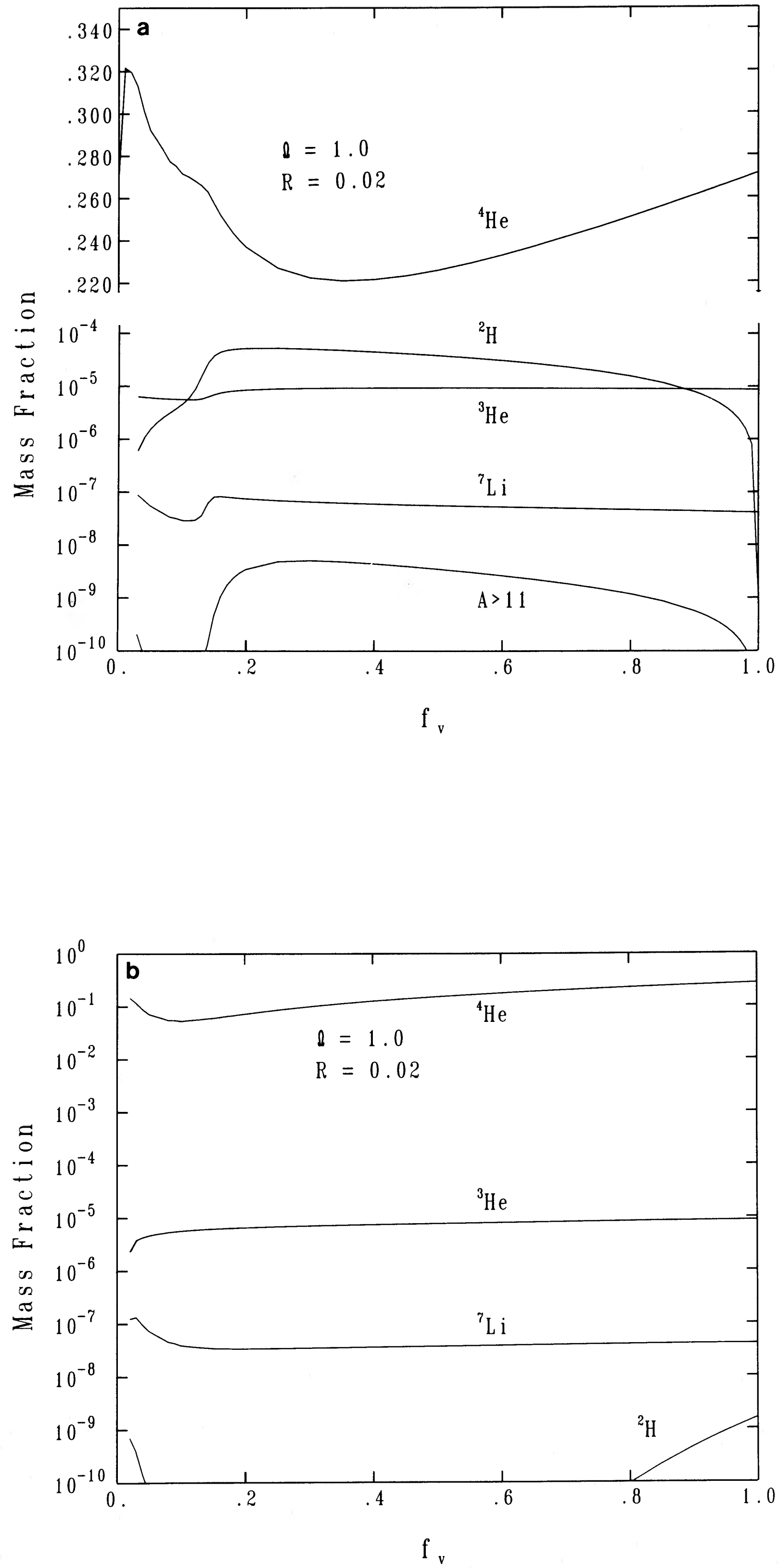

FIG. 2.—(*a*) Averaged nucleosynthesis yields as a function of the fraction of the volume of the universe (after freezeout of the weak reactions) in high baryon density regions after the quark-hadron phase transition. (*b*) Nucleosynthesis yields from the high baryon density proton-rich regions. (*c*) Nucleosynthesis yields from the low baryon density neutron-rich regions. (*d*) Relative baryon densities in the proton-rich and neutron-rich regions after neutron diffusion. The ratio of the baryon densities in the two regions before neutron diffusion is given by *R*. This ratio after neutron diffusion can be derived from Fig. 2*d*.

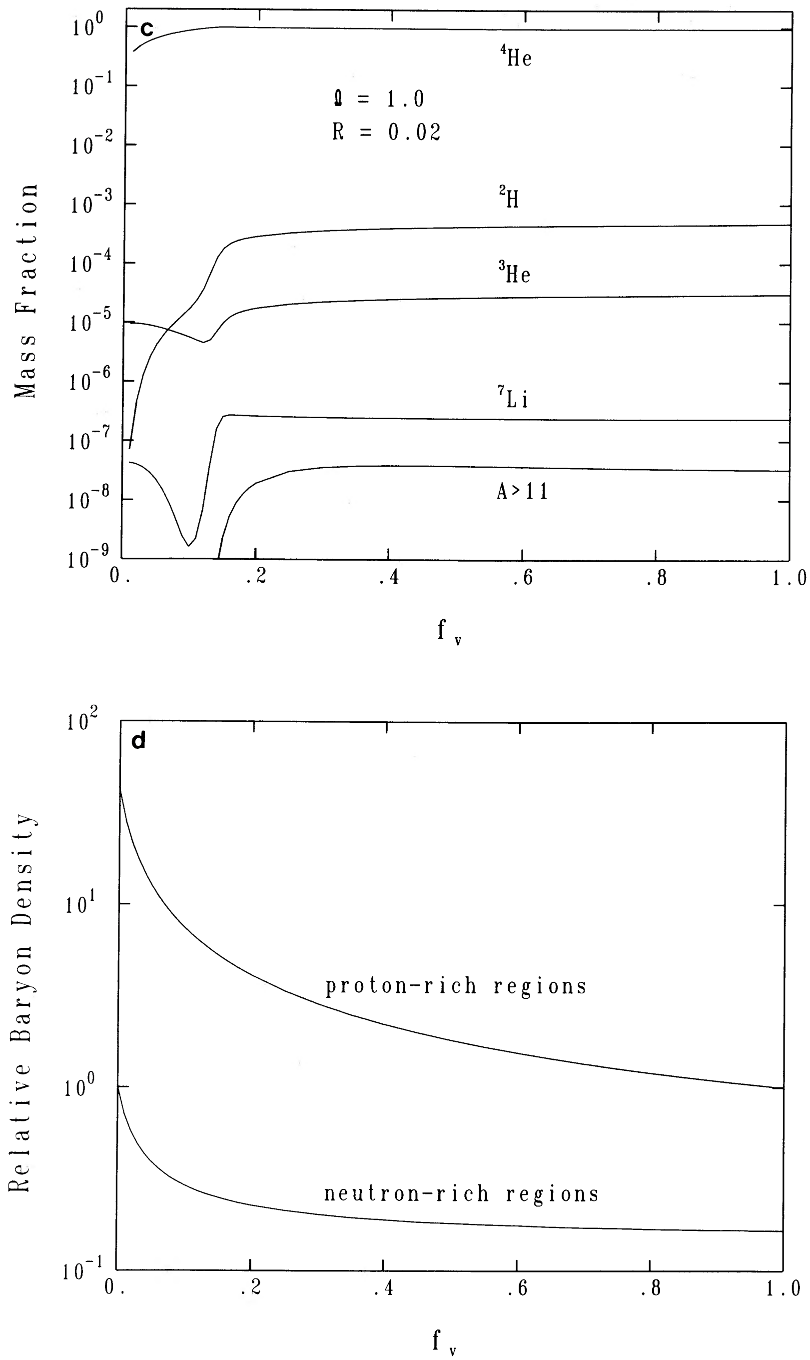

FIG. 2—*Continued*

phase transition) lead to predictions of primordial nucleosynthesis consistent with most light-element abundances even with $\Omega_b = 1$, but with an overproduction of ^{7}Li. However, over the entire parameter space we find that only an insignificant mass fraction in heavy elements with $A > 11$ can be produced in a universe with $\Omega_b = 1$.

In addition, we have pointed out that the inclusion of the known spectrum of hadronic masses implies a critical bag constant, $B = (260\ \mathrm{MeV})^4$, above which the separation of phase scenario described above cannot be realized at temperatures below 300 MeV, since the pressure in the quark-gluon plasma is always less than that in the hadron phase. In the extreme case of a Hagedorn limiting temperature a mini-inflation period can ensue in which the hadron-phase nucleation rate is high and the baryon number fluctuations are small so that standard big bang nucleosynthesis is recovered. It is significant that these two very different scenarios for the quark-hadron transition depend on a quantity, the bag constant, which is at the center of research and debate in nuclear physics.

The authors acknowledge useful discussions with G. F. Bertsch, M.-Y. Chu, W. A. Fowler, C. J. Hogan, A. Kerman, R. Perry, K. E. Sale, D. N. Schramm, and S. E. Woosley. Work performed under the auspices of the US Department of Energy by Lawrence Livermore National Laboratory under contract no. W-7405-ENG-48.

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