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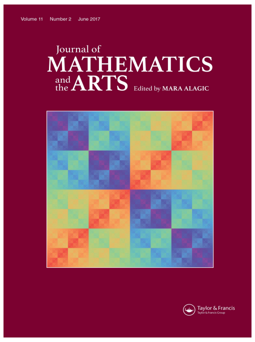

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Quantifying patterns in art and nature

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ABSTRACT

Many different types of artworks mimic the properties of natural fractal patterns – in particular, statistical self-similarity at different scales. Here, we describe examples of abstract art created by us and well-known artists such as Ruth Asawa and Sam Francis that evoke the repetition and variability of biological forms. We review the 'drip' paintings of Jackson Pollock that display statistical self-similarity at varying scales, and discuss studies that measured the fractal dimension of Pollock's drip paintings. The contemporary environmental artist Edward Burtynsky who captures aerial photographs of man-created and man-altered landscapes that resemble natural patterns is also discussed. We measure fractal dimension and a second shape parameter – fractional concavity – for borders in three of Burtynsky's photographs of man-made landscapes and of biological tissues that resemble his compositions. This specifies the complexity of patterns in Burtynsky's photographs of diverse man-impacted landscapes and underscores their similarity to fractal patterns found in nature.

Graphical Abstract: Log Booms # 1. Photograph © Edward Burtynsky, courtesy Robert Koch Gallery, San Francisco / Nicholas Metivier Gallery, Toronto.

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1. Introduction

Many different types of artworks mimic the statistical properties of natural fractal patterns – in particular, self-similarity at different scales. In a paper titled 'Entropy: Order and Disorder in Math, Science, Nature and Art', Neeman and Marshahak described the psychological impact and effect of repetition and variability in art. They stated, 'When we are introduced with an irregularity, a piece of information that is out of place, we try to make sense of it, and when we succeed we have a feeling of tension reduction, which gives us pleasure' (Neeman, & Maharshak, 2006).

Here, we present examples of abstract art created by us and other artists that are inspired by the statistical self-similarity of natural fractal patterns. We review the 'drip' paintings of Jackson Pollock that also mimic these properties of natural patterns, and describe studies that measured fractal dimension of Pollock's drip paintings from different years of his career. The work of Edward Burtynsky, a contemporary environmental artist that captures artistic photographs of man-made and man-altered landscapes with motifs repeated at varying scales is discussed. We then calculate fractal dimension of three of Burtynsky's photographs of man-impacted landscapes and of biological cells and tissues that resemble his compositions.

2. Our artwork based on natural patterns in biological cells

As a first example of art based on natural patterns, we discuss our own abstract watercolour paintings inspired by microscopic images of cellular and molecular forms. The first painting, Array of Cells, is based on an image of neural ectoderm (prospective spinal cord) tissue from a frog embryo (Figure 1A) (Elul et al., 1997). The painting depicts a collection of differently coloured cells with similar yet variable morphologies interdigitating with one another to elongate the spinal cord tissue (Figure 1A). Our second painting, Fractal Bubble Chain, resembles an electron micrograph of a folded DNA chain (Figure 1B) (Wasserman et al., 1985). In the painting, the smaller folds of the chain are disordered while on a larger scale the chain displays more directionality (Figure 1B). These folds and twists of the chain modulate the accessibility of different enzymes that replicate and transcribe the DNA.

In addition to our watercolour paintings, we have developed computational visualizations that mimic the temporal patterns in changes of cellular morphologies. These animations were developed using the Processing programming language (www.processing.org), based on observations of time lapse microscopic sequences of cells captured in living embryonic tissues (Patel et al., 2017). To best simulate the fractal patterns in the dynamic cell contours, we incorporated both deterministic (repetitive) and random (variable) number parameters into each of the programmes. In our first simulation of neural cell motility, we included deterministic parameters to specify the number of nodes and direction of cell movement, along with a random number generator to mimic the rotation of the nodes (Figure 2A). Similarly, in an animation of neuronal branching, a specific parameter for branch number was used together with a random number generator for branching angle (Figure 2B).

3. Artwork created by others that captures natural fractal patterns

Beyond our own work, there are several examples of well-known abstract artists whose art was inspired by repetition and variability in natural patterns. One is Ruth Asawa, a

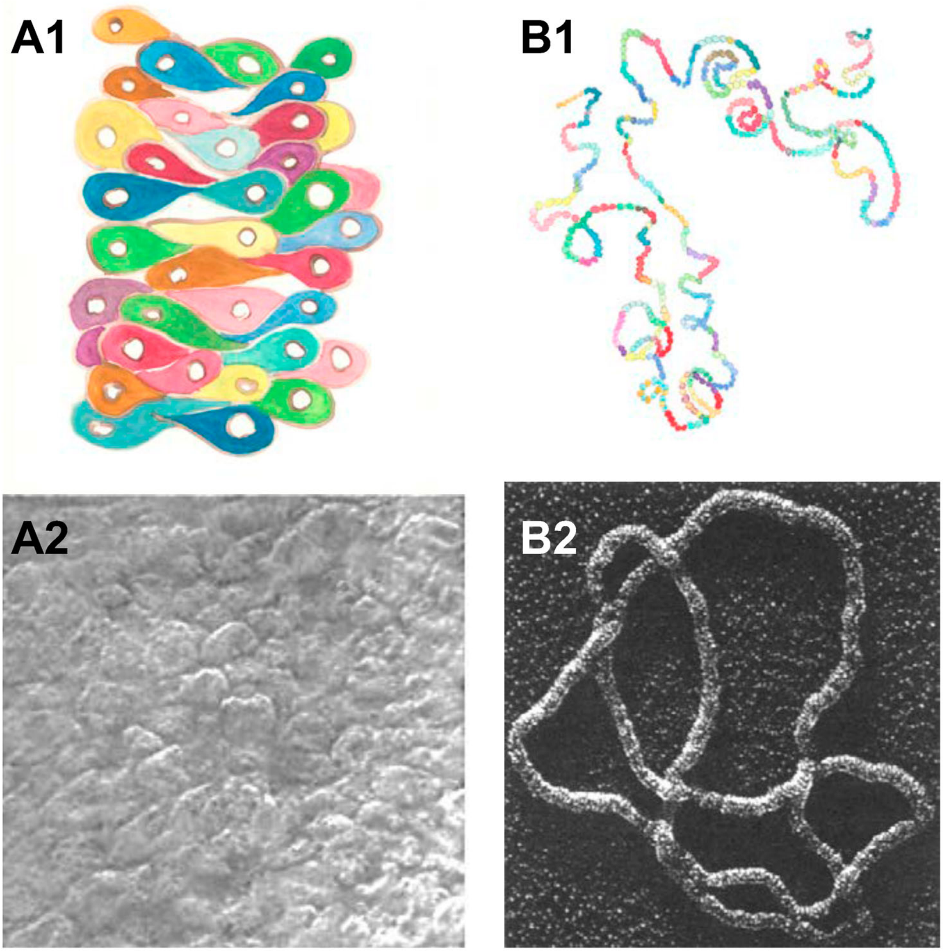

Figure 1. Watercolour paintings by Tamira Elul: (A1) Array of Cells; (B1) Fractal Bubble Chain. Microscopic Images: (A2) Explant of neural ectoderm cells from frog embryo; Micrograph of neural ectoderm cells: also see Elul et al. (1997); (B2) Knotted DNA chain-Electron micrograph of DNA chain, reproduced from (Wasserman et al., 1985).

Japanese-American San Francisco based sculptor, educator and arts activist, who worked from the 1940s to 1980s (D'Souza et al., 2019). Her hanging woven wire sculptures, now held in museum collections all over the world, contain self-similar forms at varying scales and evoke biological forms (Figure 3A). Underscoring the influence of natural patterns on her work, Asawa stated:

> These forms come from observing plants, the spiral shell of a snail, seeing light through insect wings, watching spiders repair their webs in the early morning, and seeing the sun through the droplets of water suspended from the tips of pine needles while watering my garden.

A second post-World War II artist that captured characteristics of natural patterns in their art is Sam Francis (Selz, 1975). Francis originally studied biology and psychology at the University of California, Berkeley, but after World War II he returned to Berkeley to obtain degrees in art and art history. In the 1950s, Francis moved to Paris where he began

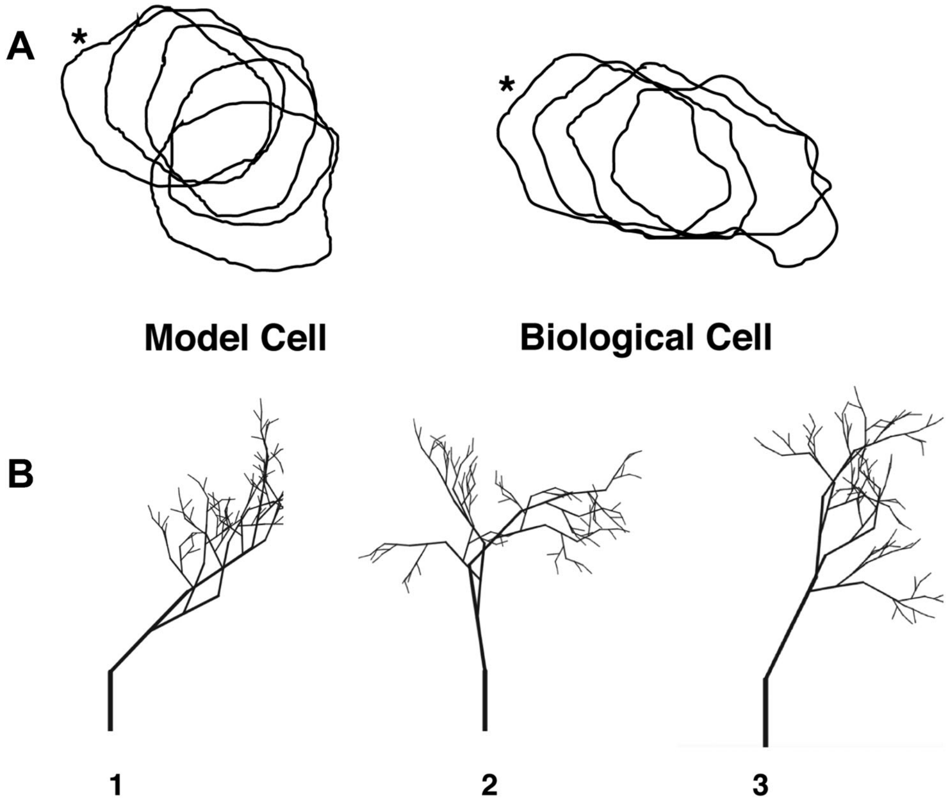

Figure 2. (A) Border of model cell over sequential iterations (left) and of actual cell over time (right). (B) Sequential iterations of animation of neuronal arbour branching. Reproduced from (Patel et al., 2017).

to create large paintings containing lyrical biomorphic forms at varying scales (Figure 3B). In a previous paper we measured two shape parameters – aspect ratio (length/width) and circularity index ($4^{*}\pi^{*}$ area / (perimeter)^2) – for forms in three of Francis' paintings and microscopic biological cells that resembled his paintings (Lakhani et al., 2016). This paper mathematically specified the morphological similarity between Francis' art and biological forms, which may underlie part of the appeal of his painted forms.

4. Measurement of fractal dimension in Jackson Pollock's drip paintings

Jackson Pollock is yet another well-known abstract expressionist artist whose paintings mimic the properties of natural fractal patterns. In several earlier studies, Taylor and associates quantitatively assessed the patterns in Pollock's drip paintings by measuring fractal dimension, a parameter that specifies the rate of change of detail with scale in a self-similar pattern (Brosch et al., 1992; Edmonds et al., 2011; Feldman, 2012; Kim et al., 2011; Mandelbrot, 1982; Nonnenmacher et al., 1994; Smith et al., 1996; Takeda et al., 1992; Taylor et al., 1999). Fractal dimension ranges between 1 and 2, with patterns with less (more) fine structure having lower (higher) fractal dimensions. These researchers showed that the fractal dimension in Pollock's drip paintings increased from a relatively low value of 1.1 to a significantly higher value of 1.8 during the years he was creating these drip paintings

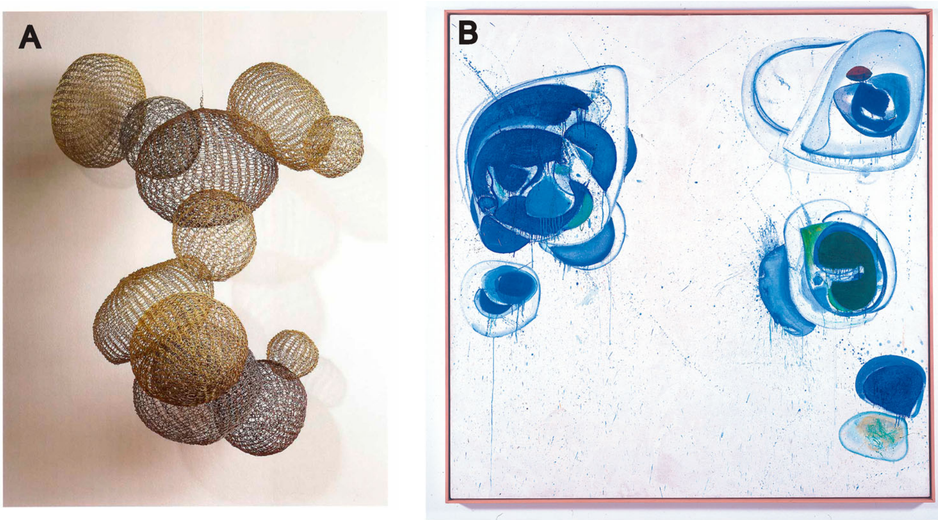

Figure 3. (A) Ruth Asawa sculpture, Untitled (Hanging Asymmetrical Twelve Interlocking Bubbles), ca. 1957, (304.8 × 30.5 × 30.5 cm). © Estate of Ruth Asawa. (B) Sam Francis painting, Blue Balls VII, ca. 1962, (206.4 × 203.4 cm) © Sam Francis Foundation, California/Artists Rights Society (ARS), NY.

Figure 4. Jackson Pollock's paintings: (A) Freeform, ca. 1946 (129.2 × 76.5 cm). (B) One: Number 31,1950, ca.1950 (269.5 × 530.8 cm). © Pollock-Krasner Foundation/ Artists Rights Society (ARS), NY.

(Taylor et al., 1999). These changes in fractal dimension specified how the statistically self-similar patterns in Pollock's drip paintings became more complex over the course of his career (Feldman, 2012; Street et al., 2016; Taylor et al., 1999). Foe example, compare Pollock's painting *Freeform*, painted in 1946, which has less fine structure corresponding to a lower fractal dimension (Figure 4A), to his painting *One: Number 31*, painted in 1950, which is more complex in structure and reflects a higher fractal dimension (Figure 4B).

5. Edward Burtynsky's photographs of man-impacted landscapes

A contemporary exemplar of an artist that emulates natural patterns in their art is Edward Burtynsky. Burtynsky is an iconic Canadian environmental photographer who captures aerial photographs of man-created and man-altered landscapes that contain repeating

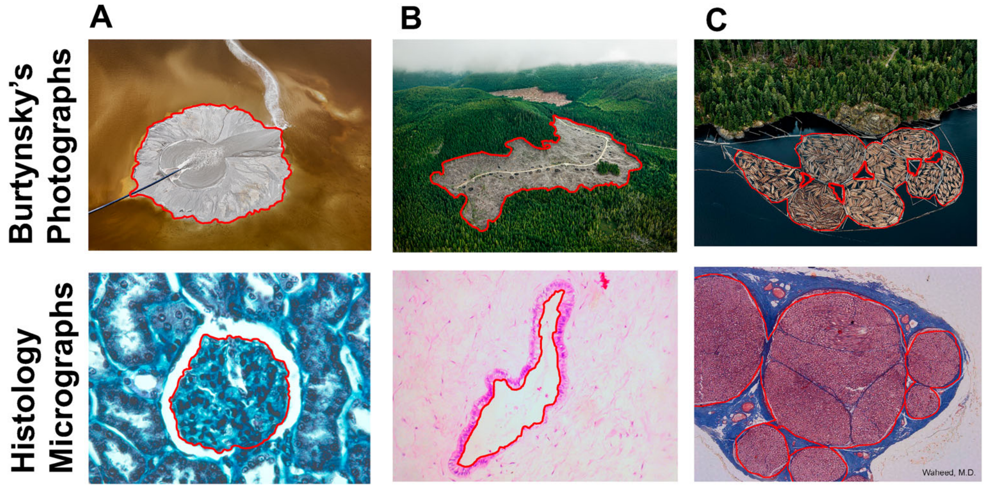

Figure 5. (A) Phosphor Tailings # 5 (top), renal corpuscle (bottom); (B) Clear Cut # 4 (top), mesenteric duct (bottom); (C) Log Booms # 1 (top), peripheral nerve (bottom). Red outlines in all images added by TE to indicate border that was measured. Photographs © Edward Burtynsky, courtesy Robert Koch Gallery, San Francisco /Nicholas Metivier Gallery, Toronto. Renal corpuscle, Terrence Miller, Ph.D.; Mesenteric duct, Ed Uthman, M.D.; Peripheral nerve, Rehan Waheed. M.D.

motifs at varying scales (Bishop, 2017; Katchadourian, 2016). Here, we measured fractal dimension and a second shape parameter – fractional concavity – for the borders of the dominant elements in three of Burtynsky's photographs of man-made landscapes, all from his recent Anthropocene project (Figure 5). To determine how Burtynsky's photographs of these man-impacted landscapes relate to natural patterns, we also compare their shape parameters to those of three histology micrographs of biological tissues that visually resemble the photographs (Figure 5).

6. Measurement of fractal dimension and fractional concavity

For each Burtynsky photograph, the dominant element was selected based on its large size and central location within the image, as well as its resemblance to a biological tissue in a histology micrograph (Figure 5). To calculate fractal dimension for borders in Burtynsky's photographs and the micrographs, we used the box counting method (Feldman, 2012; Nonnenmacher et al., 1994; Smith et al., 1996; Wahl). The border of the selected forms and tissues were outlined (red lines, Figure 5), and the border outlines were superposed on grids (tables) of different box sizes (r) in Powerpoint (Figure 6A). The boxes that contained any portion of the border outline were coloured and counted (N) (Figure 6A). Using Excel, we plotted log N versus log (1/r) on an x-y scatter plot, and fit the data points with a linear regression line. The slope of this regression line was the fractal dimension. To determine fractional concavity, the border outlines were imported into Image J software (NIH). In Image J, we used the freehand tool to measure the total border length as well as the lengths of all the visually identified concave segments of the border (Figure 6B). In Excel, the lengths of all the concave segments were summed and this number was divided by the total border length. This ratio was the fractional concavity.

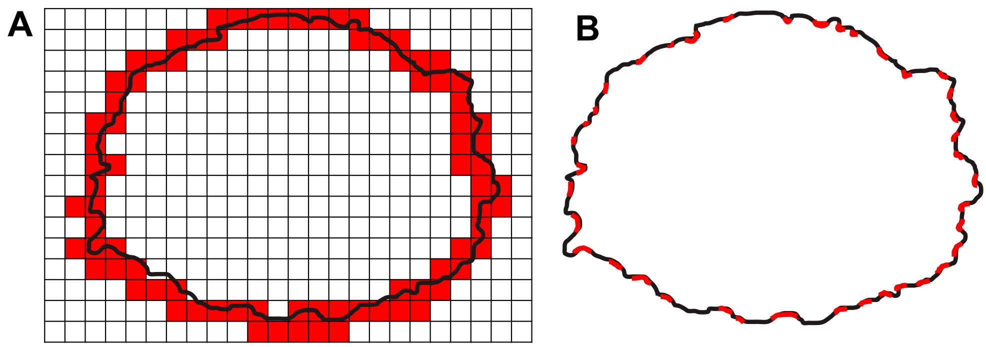

Figure 6. (A) Border of Phosphor Tailings # 5 on one grid used to calculate fractal dimension. Boxes filled with red contain a portion of the border. (B) Border of Phosphor Tailings # 5 with concave segments highlighted in red, used for calculating fractional concavity.

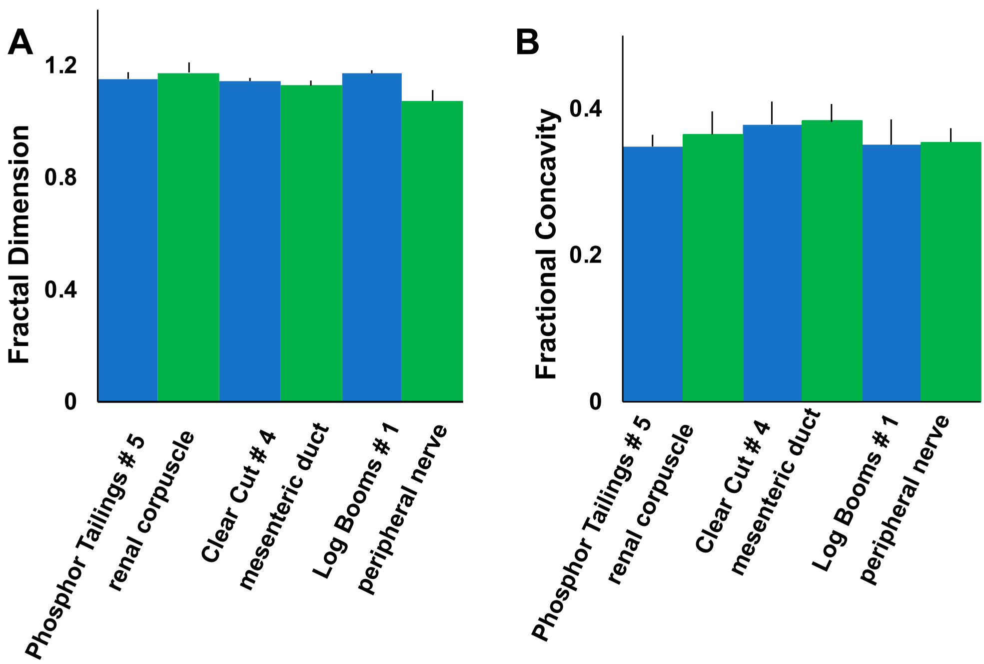

Figure 7. Mean fractal dimension (A) and fractional concavity (B) for borders in Burtynsky's photographs (blue) and in histology micrographs (green). Error bars – SEM.

For each image, fractal dimension and fractional concavity were determined three times and averaged (Figure 7). Student's t-test (two-tailed) was used to compare mean values of these parameters between corresponding photographs and micrographs.

7. Results

Using the box counting method, we calculated a mean fractal dimension of 1.15 (SE = 0.0234) for Burtynsky's photograph Phosphor Tailings # 5 and of 1.17 (SE = 0.037) for the microscopic image of the renal corpuscle (Figure 7A; $p > 0.05$). The fractal dimension for the border in Burtynsky's photograph Clear Cut # 4 was 1.14 (SE = 0.011) whereas that for the border in the mesenteric duct micrograph was 1.13 (SE = 0.018) (Figure 7A; $p > 0.05$). Finally, applying the box counting technique to the borders in Edward Burtynsky's photograph Log Booms # 1 and the histology image of peripheral nerve, we determined average fractal dimensions of 1.17 (SE = 0.011) and 1.07 (SE = 0.038), respectively (Figure 7A; $p > 0.05$).

The fractional concavity for the border in Burtynsky's photograph Phosphor Tailings # 5 was 0.35 (SE = 0.027), whereas that for the border in the micrograph of the renal corpuscle was 0.37 (SE = 0.054) (Figure 7B; $p > 0.05$). For the photograph Clear Cut # 4 we determined a slightly greater fractional concavity of 0.39 (SE = 0.054), while the fractional concavity for the microscopic image of the mesenteric duct border was similarly 0.38 (SE = 0.042) (Figure 7B; $p > 0.05$). Finally for the borders in Burtynsky's photograph Log Booms #1 and in the micrograph of peripheral nerve we measured fractional concavities of 0.35 (SE = 0.035) and 0.35 (SE = 0.019) (Figure 7B; $p > 0.05$).

8. Discussion

Our analysis shows that the borders in three of Burtynsky's photographs of man-created or man-altered landscapes exhibit a small range of fractal dimensions between approximately 1.1–1.2, which are similar to the biological tissues that visually resemble the photographs. The fractional concavity measurements showed that Burtynsky's photographs also exhibited a correlated small range of fractional concavities from 0.35 to 0.4, similar to the biological look alike images. These results mathematically specify the scaling and

Figure 8. *Teasel Grey Fault Line*. Hand built porcelain sculpture by Nuala O'Donovan. © nualaodonovan.com 2020. Photograph by Sylvain Deleu.

complexity of three of Burtynsky's photographic compositions of diverse man-impacted landscapes and underscore their similarity to micrographs of biological tissues.

The similarity of scaling in patterns in Burtynsky's photographs of man-created and man-altered landscapes to organic tissues may explain part of the artistic appeal of Burtynsky's compositions. Studies have shown that humans find the statistical self-similarity of both natural and artistic fractal patterns aesthetically appealing and stress reducing (Taylor, 2017). Several reports demonstrated that humans specifically prefer natural and artistic fractal patterns with low to intermediate fractal dimensions (around 1.3) (Street et al., 2016; Taylor et al., 2011; Viengkham & Spehar, 2018). This may be because, although patterns in the natural world span a large range of fractal dimension, patterns with intermediate fractal dimensions are particularly prevalent (Street et al., 2016; Taylor et al., 2011; Viengkham & Spehar, 2018). One direction for future study could be to explore whether Burtynsky's photographs that are close to this putatively aesthetically optimal fractal dimension, such as Log Booms #1, are preferred relative to other photographs of his that have much greater fractal dimension. Another future research question could be to determine whether the surrounding landscapes of the chosen fractal elements (such as the trees and water in Log Booms # 1) also contribute to the aesthetic appeal of Burtynsky's photographs.

9. Summary

Scale-independent statistical self-similarity is a fundamental property of fractal patterns in nature, and is also mimicked in many different types of art. The contemporary Irish sculptor Nuala O'Donovan, who constructs intricate biomorphic sculptures based on natural fractals, summarized the significance of such patterns, stating 'It is the imperfections in the patterns ... that are the evidence of a life force' (Figure 8) (Dhruba, 2013).

Disclosure statement

No potential conflict of interest was reported by the author(s).

Data availability statement

Data are stored in the Box Repository at Touro University California.

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