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Superconducting Nanobridge SQUID Magnetometers for Spin Sensing

by

Natania Antler

A dissertation submitted in partial satisfaction of the
requirements for the degree of
Doctor of Philosophy

in

Physics

in the

Graduate Division

of the

University of California, Berkeley

Committee in charge:

Professor Irfan Siddiqi, Chair
Professor John Clarke
Professor Sayeef Salahuddin

Fall 2014

Superconducting Nanobridge SQUID Magnetometers for Spin Sensing

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Abstract

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Doctor of Philosophy in Physics

University of California, Berkeley

Professor Irfan Siddiqi, Chair

As the cutting edge of science and technology pushes towards smaller length scales, sensing technologies with nanoscale precision become increasingly important. In this thesis I will discuss the optimization and application of a 3D nanobridge SQUID magnetometer for studying solid state spin systems, in particular for sensing impurity spins in diamond. Solid state spins have proposed applications in memory and computation for both classical and quantum computing. Isolated spins typically have longer coherence times, making them attractive qubit candidates, but necessitating the development of very sensitive detectors for readout.

This 3D nanobridge SQUID combines the exquisite spatial sensitivity of a traditional nanoSQUID with a large non-linearity on par with a tunnel junction SQUID. This allows us to build a highly sensitive magnetometer which can act as both an efficient flux transducer as well as a nearly quantum limited lumped Josephson Parametric Amplifier. We show that the device has a minimum flux noise of $17 \pm 0.9 \text{ n}\Phi_0/\text{Hz}^{1/2}$ with only a factor of ~ 2.5 increase in flux noise up to 61 mT. A second generation device with a smaller capacitor achieves field tolerance up to 75 mT. The maximal bandwidth values range from 25-40 MHz in the parametric amplification regime to 70 MHz in the linear regime. This combination of large bandwidth, low flux noise, large flux coupling and field tolerance make this sensor a promising candidate for near-single-spin dynamics measurements.

In the last part of this thesis we begin to demonstrate the utility of a nanobridge SQUID magnetometer for characterizing spin systems in the solid state. We use the magnetometer to measure the decay characteristics of P1 centers in diamond. We find that the spin-lattice relaxation time varies with temperature, with an order of magnitude decrease in the decay time between 25 mK and 370 mK.

To David Reber, my ever-patient parents, and my teachers.

In memory of David Morgan-Grenville.

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Chapter 1

Introduction

In quantum computing, spin-based computation and memory are attractive, albeit at times quixotic, goals. In the difficulty of using spins for quantum computing therein also lies the strength: spins are well isolated and thus very difficult to control or readout, yet their isolation leads to much longer coherent lifetimes [1]. In other words, spins are very difficult and slow to access, but under certain circumstances they retain quantum information longer than most other engineered quantum systems.

There has been much recent interest in developing hybrid quantum computation systems. Such systems aim to marry the best aspects of proposed qubit implementations while avoiding their shortcomings. For example, there have been several proposals to couple superconducting qubits with solid-state spins [2–4]. In these proposals, the spins act as a memory lending their long lifetimes to the system, while the superconducting qubits lend their speed, addressability and ease of fabrication. One recently realized implementation of these schemes demonstrated coupling between an ensemble of nitrogen-vacancy (NV) centers in diamond and superconducting qubits via a resonator [5, 6].

Of course, if such a system is to succeed, one must have a very good handle on the physics governing both quantum systems. Superconducting qubits are well understood, and the body of knowledge for these qubits is growing steadily [7]. However, the sheer complexity and variety of electron spin-donor systems in the solid state means that there is much still to learn about these systems, especially at dilution fridge temperatures for applications to computing. A magnetometer that operates with high sensitivity at these temperatures that could be used to study these spin-systems would be useful from this perspective as well as a broader basic science perspective.

Our candidate for such a device is a novel nanobridge Superconducting Quantum Interference Device (SQUID) magnetometer, which combines the superior flux coupling ability of a nanoSQUID design, with the low noise and large bandwidth characteristics of a lumped element Josephson parametric amplifier (LJPA) [8–11]. In this thesis I will show work towards characterization and application of this nanobridge SQUID magnetometer. To characterize the device we took detailed measurements of its sensitivity under a variety of bias conditions and in the magnetic fields needed for practical magnetometry measurements.

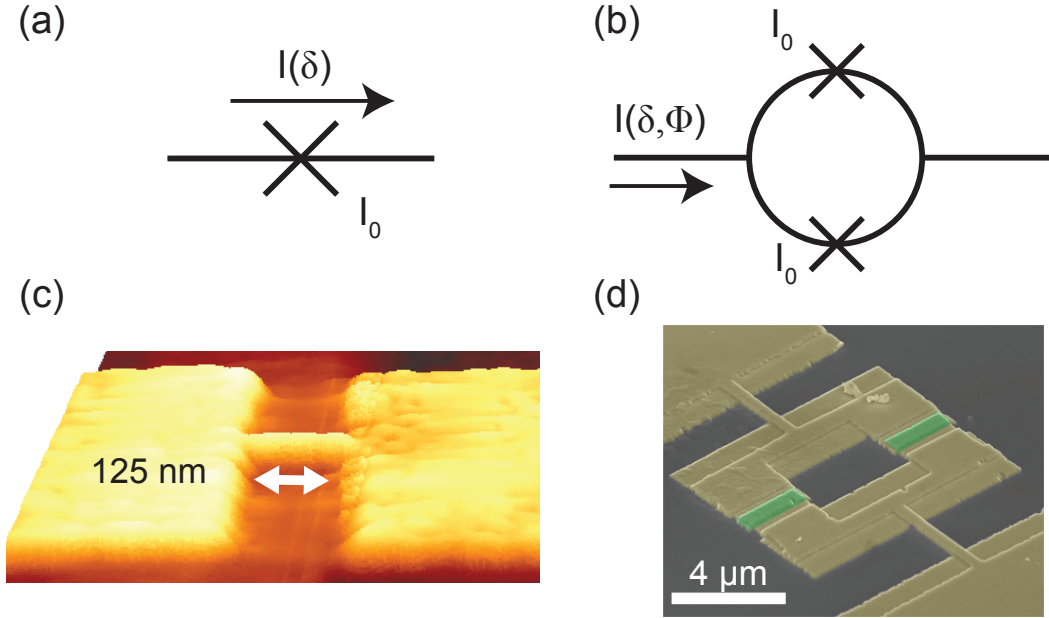


Figure 1.1: (a) and (b) depict the circuit symbols for Josephson junctions and SQUIDs respectively. In (c) we show an AFM image of a 2D nanobridge, or “weak link” Josephson junction and (d) is a SEM image of tunnel junction SQUID.

Our first application of the magnetometer was to perform detailed measurements of the spin-lattice relaxation times of P1 centers in diamond. These defects consist of a substitutional nitrogen atom in the carbon lattice [12] and are particularly relevant to solid state quantum information processing [13–17]. The properties of P1 centers in diamond have been under investigation since they were first discovered in 1959 [12, 18], but studies of the relaxation decay of these centers at ultra-low temperatures have only recently begun [19, 20], making this a great system on which to test-drive our device.

1.1 SQUID and Josephson Junction Introduction

The key component of our nanobridge SQUID magnetometer is of course the SQUID, or Superconducting QUantum Interference Device. A SQUID consists of a loop of superconducting material interrupted by one or two so-called Josephson junctions. These junctions typically consist of an insulating material inserted into the superconductor in a sandwich-like construction. This superconductor-insulator-superconductor (SIS) junction is typically called a “tunnel” junction and is the most common Josephson junction. One can also construct a junction out of a constriction or “weak-link” in the superconducting material, as

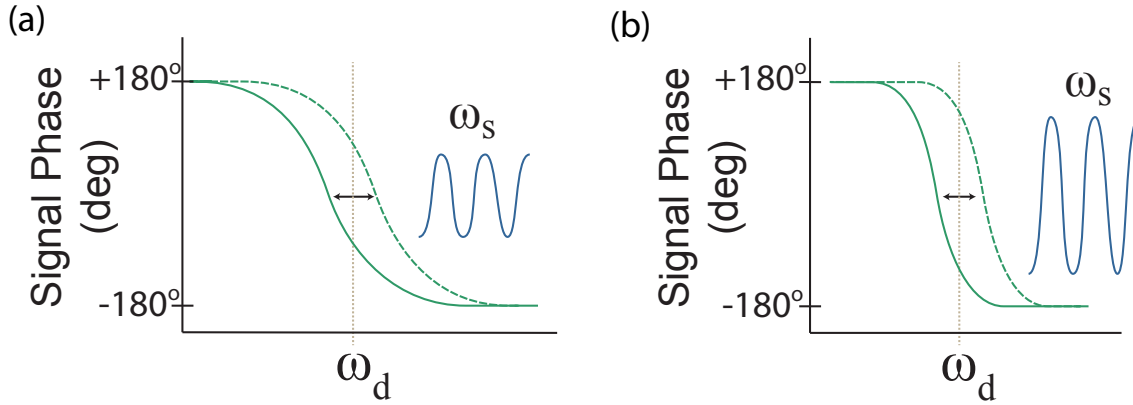


Figure 1.2: In (a) we show an illustration of the phase response of a linear resonator. Given an input flux signal at ω_s , the resonant frequency will modulate back and forth causing a periodic change in the phase response. If we focus our measurement at a single frequency, ω_d , we will observe the phase modulation at ω_s . In this regime the magnetometer responds in a linear fashion, simply transducing flux into a voltage signal. In (b) we illustrate the steepening of the curve as the power of ω_d increases, with a corresponding increase in the amplitude of the phase modulation. In this regime the magnetometer performs both flux transduction and parametric amplification.

shown in Figure 1.1c. The junctions in the SQUID design used for the magnetometer are “variable thickness” or 3D nanobridge junctions. These junctions have the same footprint as the “weak link” or 2D junction shown in Figure 1.1c, but the banks on either side of the bridge are 3 to 5 times thicker (or taller) than the bridge.

Whatever their construction, Josephson junctions and SQUIDs are crucial parts of superconducting circuits. These elements add nonlinearity, allowing us to build amplifiers or qubits. A single Josephson junction will add a current-drive-dependent nonlinearity to a circuit, but a SQUID will add magnetic flux dependence. This allows construction of circuits for an entirely different application: magnetometry.

1.2 Nanobridge SQUID Magnetometer

A SQUID acts as a flux-dependent inductor, as described in Chapter 2. If we put the SQUID in parallel with a capacitor, we will obtain a resonant LC circuit. This circuit will then have a flux-dependent resonant frequency. We can measure this resonant frequency by probing on resonance and measuring the phase of the reflected signal. Changes in flux through the SQUID loop will then register as changes in the reflected phase, as illustrated in Figure 1.2. If we increase the power of the probe tone, that is begin to drive the circuit harder, we will access the nonlinearity of the Josephson junctions and enable parametric amplification of the

flux signal tone. In this way the SQUID magnetometer acts as both a flux transducer and an amplifier. More details on the physics of how this device operates are given in Chapter 2.

We use a 3D nanobridge SQUID in particular because the weak link geometry allows efficient electromagnetic coupling between the SQUID loop and magnetic system under study (see Section 2.1.6). The three-dimensional, or variable thickness geometry allows for enhanced nonlinearity and hence better sensitivity as compared to planar nanoSQUIDs [21–24].

1.3 Solid State Spins for Hybrid Quantum Computing

In order to choose a spin system for use in hybrid quantum computing, with an eye towards the requirements for fault tolerant quantum computation [25], one must consider a number of important system properties: coherence times, addressability, and ease of positioning or fabrication. At least as far as hybrid spin-superconducting qubit systems go, the last consideration has biased the field towards solid state spin systems (with a notable exception [26]), as these often allow for controlled fabrication and positioning.

Addressability means that the spins must be easy to control and readout. Solid state spins have the advantage of fixed physical location, so they can be more easily addressed in that respect. In addition it is possible to choose systems with convenient microwave transitions for coupling to microwave resonators or superconducting systems. Additionally, some solid state spin systems have convenient optical transitions that allow an additional avenue for control and coupling [27–30], and in some cases might allow the mapping of qubit quantum states onto photons for quantum communication applications, eg. quantum repeaters [31].

The first criteria, coherence time, is of course crucial in any quantum computing system. This is particularly crucial for spin-based memories, which have been promoted on the basis of the generally long coherence times of spins in contrast to many other proposed qubit implementations. There is generally a balance between ease of readout and coherence times. Typically, the denser the spin ensemble, the easier it is to read out. Unfortunately, denser spins also mean stronger spin-spin interactions and faster decoherence.

Nitrogen-vacancy (NV) centers in diamond have been a very popular candidate of late for many possible applications, including magnetometry, thermometry, qubits and quantum memories [29]. They have the advantage that they have easily experimentally accessible microwave and optical transitions that allow for optically induced polarization in the ground state. This broad addressability makes them very attractive for use in hybrid systems. This defect consists of a nitrogen substitutional impurity in the carbon lattice next to a vacancy.

Defects in silicon-carbide are another class of spin systems which have drawn much recent interest [27, 28]. These are attractive due to their optical and microwave transitions, as well as the ease of fabrication of circuitry on the silicon-carbide substrate. Bismuth and phosphorus in silicon are yet two other solid state electron donor systems that have drawn recent interest [32–35]. These consist of phosphorous or bismuth Group V atoms, which act as donors in the silicon. Both have easily accessible microwave transitions, and allow for easy fabrication of companion circuits on chip and well controlled spin-positioning. Additional

spin systems of recent interest include ruby [36] and rare earth ions, including Er^{3+} doped Y_2SiO_5 [30]. The latter has the advantage of convenient microwave transitions in addition to optical transitions in a common low-loss telecommunications band.

P1 centers in diamond are a spin-1/2 system which has been well studied since its discovery in 1959 [12]. Of all of these options for spin systems, we chose to study P1 centers in diamond as they are well enough studied that we can easily obtain samples and identify the spin transitions. At the same time there still remain open questions about the physics of these centers' spin dynamics at dilution fridge temperatures, a regime in which our device is well suited for operation.

1.4 Thesis Overview

In the next chapter I will discuss the theory of the nanobridge SQUID magnetometer, as well as give a brief introduction to the P1 and NV centers in diamond. In the third chapter I discuss the experimental setup and device fabrication details. In the fourth chapter I will discuss work done to fully characterize the device with detailed measurements of its sensitivity, or flux noise performance, under a variety of bias conditions and in the magnetic fields needed for practical magnetometry measurements. In the last chapter I will describe an experiment where we used this nanobridge SQUID magnetometer to study in detail the spin-lattice relaxation times of P1 centers in diamond. In the appendices I discuss the design and operation of the Chase Fridge where most of these measurements were completed. In addition I discuss additional work to study NV centers in diamond using three and two dimensional cavities, and then lastly I go through in more detail some of the peculiarities of the magnetometer which we discovered while attempting to use it to study a variety of different systems. This section will be especially useful for someone planning to build on this work.

1.5 Summary of Key Results

The key results presented in this thesis relate to the characterization and application of our nanobridge SQUID magnetometer. We find that the optimized device has a state-of-the-art flux noise and bandwidth combination with $17 \text{ n}\Phi_0/\text{Hz}^{1/2}$ and 25 MHz of bandwidth. In addition we find that the newest magnetometer design is tolerant of in-plane magnetic fields up to 75 mT. After characterizing the magnetometer we show that we are able to measure an ensemble of P1 centers in diamond as a function of magnetic field, pulse power and temperature. We find that the shape of the decay curve is dependent on pulse power and polarizing field. In addition we find that the relaxation decay time constant is weakly temperature dependent, with a more pronounced temperature dependence at higher magnetic fields.

Chapter 2

Magnetometer Theory and Introduction to Spins

In this chapter I will go over the basic theory of operation of the magnetometer and also give a brief introduction to the spin systems we study in this thesis. More details on the experimental realization of the magnetometer can be found next in Chapter [3](#).

2.1 Magnetometer Theory

In this section I will show how one of the fundamental building blocks of a superconducting circuit—Josephson junctions—can be used to create a highly sensitive and fast detector of magnetic fields. In particular, I will show how three-dimensional (3D) nanobridge type Josephson junctions will be crucial for the design of a magnetometer with high spatial sensitivity.

2.1.1 Josephson Relations

Josephson junctions are typically a central part of the superconducting circuits that are designed for quantum information applications. These junctions add circuit nonlinearity, allowing us to build amplifiers and qubits. In this section I will give some motivation for how this nonlinearity arises.

The simplest and most common form of Josephson junction is a tunnel junction which consists of a superconductor–insulator–superconductor sandwich. The two superconductors form the electrodes of the junction. The Josephson relations govern how these junctions work, and were first put forth in a seminal paper by B. Josephson [\[37\]](#). The first equation, known as the dc Josephson relation, is given by,

$$I = I_0 \sin \delta \tag{2.1}$$

where I is the supercurrent across the junction, I_0 the junction critical current, and δ is the gauge invariant phase difference across the junction.

The fundamentally surprising thing about this relation is that it means that current can flow across the junction without the presence of a voltage difference; the only requirement is a phase difference, δ . The gauge invariant phase difference between the superconducting condensates of the electrodes is given by:

$$\delta = \phi_1 - \phi_2 + \frac{2\pi}{\Phi_0} \int_1^2 \mathbf{A} \cdot d\mathbf{s} = \Delta\phi + \frac{2\pi}{\Phi_0} \int_1^2 \mathbf{A} \cdot d\mathbf{s} \quad (2.2)$$

Where $\Phi_0 \equiv h/(2e)$ is a constant known as a flux quantum. $\mathbf{A}$ is the vector potential of any magnetic field present. The superconducting phase in each electrode, ϕ_i , can be thought of as the phase of the superconductor wavefunction, or order parameter, $\Psi_i(\vec{r}, t) = \Psi_{0,i}(\vec{r}, t)e^{i\phi(\vec{r}, t)}$, where $|\Psi_i|^2 = n_i$. In this formalism, n is proportional to the density of charge carriers: Cooper pairs. The phase describes the center of mass motion of the Cooper pairs. Since a 2π change in wavefunction phase of either superconducting electrode should have no physical effect, the relationship between superconducting current across the junction and the phase difference (or current phase relation, CPR) should also be 2π periodic. Any periodic function can be expressed as a Fourier series, and it turns out that the simplest Fourier series, a sine, is what applies in the case of the CPR for a tunnel Josephson junction [38–40].

The second relation, known as the ac Josephson effect, is given by:

$$\frac{d\delta}{dt} = \frac{2eV}{\hbar} = \frac{2\pi V}{\Phi_0} \quad (2.3)$$

where V is the voltage difference between the sandwiching superconductors.

2.1.2 Junction Voltage State and Switched Readout

If the current applied to the junction exceeds the critical current, I_0 , the junction will enter the voltage state with $V = I_0 R_N$, where R_N is the normal state resistance of the junction. Equation 2.3 implies that the finite voltage present in the voltage state will lead to a rapid increase in δ . One can visualize this effect with the so-called “tilted washboard potential” which can be expressed as [39]:

$$U_J \equiv \frac{\Phi_0}{2\pi} (I_0(1 - \cos\delta) - I\delta). \quad (2.4)$$

As shown in Figure 2.1, if $I < I_0$ the superconducting phase will be confined to oscillate around a local minimum, but as I increases the potential tilts until $I > I_0$ and no local minima remain and the phase “escapes.”

A traditional nanoSQUID readout technique is to switch a SQUID into the voltage state via a fixed-rate current ramp. The experimenter measures the time it takes to trigger a voltage pulse, and that yields the switching current I_{SW} ¹. This switching measurement can

¹ I_0 and I_{SW} would be identical in an ideal SQUID, but thermal, electrical and even quantum noise will cause the SQUID to switch at a current lower than I_0 .

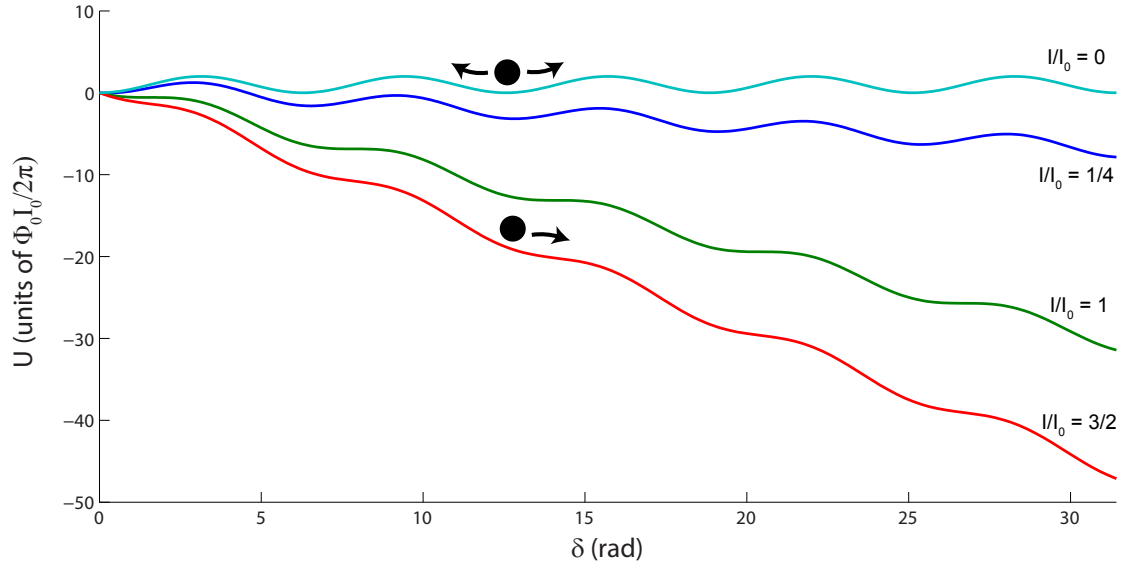


Figure 2.1: Tilted washboard potential for various values of I/I_c , as marked at right. The phase is trapped oscillating back and forth in a single well for $I < I_c$, but as I exceeds the critical current the phase will escape and begin to rapidly increase.

be typically performed with rates up to 10 kHz [21, 41]. Monitoring the switching current will yield information about the flux through the SQUID loop.

There are two main disadvantages of this readout technique. The first is that it is bandwidth limited. These measurements are relatively slow due to the time it takes for the SQUID to reach its equilibrium temperature after it returns to the superconducting state [42]. It is possible to increase the bandwidth by resistively shunting the SQUID and operating at a current bias above the critical current, but this leads to dissipation and heating at the chip that can produce severe backaction on the quantum system under study [41]. One can avoid this problem by operating with an unshunted SQUID, but there still may be backaction associated with the switch to the voltage state.

In order to avoid these problems we operate the magnetometer only in the superconducting regime, where it acts as a flux dependent nonlinear inductor.

2.1.3 Junction Inductance

As mentioned in the previous section, our goal is to operate the magnetometer only in the superconducting regime, where the SQUID acts as a flux-dependent nonlinear inductor. To see how this arises, first we show how the inductance of a *single* Josephson junction is dependent on the superconducting phase.

If we take the time derivative of Equation 2.1 we obtain:

$$\frac{dI}{dt} = I_0 \cos \delta \frac{d\delta}{dt} \quad (2.5)$$

Substituting in Equation 2.3 for $\frac{d\delta}{dt}$, we obtain:

$$\frac{dI}{dt} = I_0 \cos \delta \frac{2\pi V}{\Phi_0} \quad (2.6)$$

Rearranging Equation 2.6 to solve for V we find:

$$V = \frac{\Phi_0}{2\pi I_0 \cos \delta} \frac{dI}{dt} = \frac{L_0}{\cos \delta} \frac{dI}{dt} \quad (2.7)$$

This expression makes it clear that the tunnel junction forms a superconducting phase-dependent inductance:

$$L_J(\delta) = \frac{L_0}{\cos \delta}. \quad (2.8)$$

For a junction with a more general (non-sinusoidal) current phase relationship, which will be discussed in the next section, the inductance can be defined as:

$$L_J = \frac{\varphi_0}{\partial I(\delta)/\partial \delta} \quad (2.9)$$

where $\varphi_0 = \Phi_0/(2\pi)$.

2.1.4 Current Phase Relationships

The current phase relationship (CPR) is very important for determining the physics of a junction, as we saw in the previous section. As stated, the CPR for a tunnel junction is a simple sinusoid. It is possible to think of other types of junctions that will have different CPRs. For example, a constriction in the superconducting material will lead to restrictions on the phase that can produce tunnel-junction-like behavior. All CPRs, however, must adhere to a few simple physically motivated rules, nicely enumerated in Ref. 43 and reproduced below:

1. A 2π change in the superconducting phase in either junction electrode has no physical effect, so it should not have an effect on the CPR: $I(\delta) = I(\delta + 2\pi)$.
2. Changing the sign of the supercurrent flow should change the sign of the phase difference: $I(\delta) = -I(-\delta)$ in the most common cases.²

²This is not the case for superconductors with broken time reversal symmetry, eg. superconductor-ferromagnet-superconductor junctions.

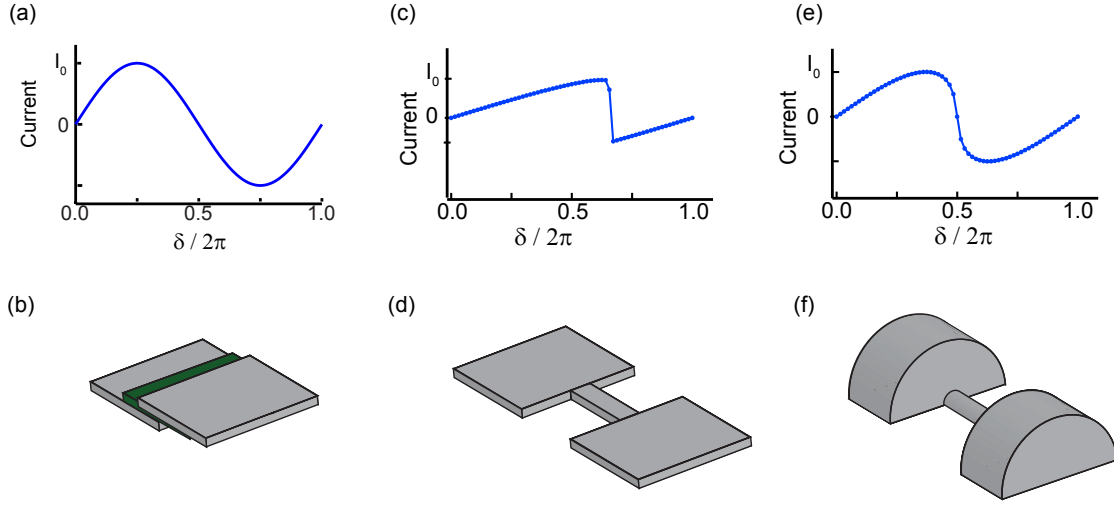


Figure 2.2: In (a) we show the current-phase relationship (CPR) for a tunnel junction and in (b) we show a cartoon of the geometry. Similarly in (c) and (d) we show the CPR and geometry of a 2D nanobridge. Note that this CPR is multivalued and the average of the second branch (suppressed for clarity) and the first branch yields $I(\pi) = 0$. Finally in (e) and (f) we show the CPR and geometry of a 3D nanobridge. The 2D and 3D nanobridge CPRs were calculated numerically, and more details on the calculations can be found in Reference 23.

3. A gradient in the superconducting phase is required for a DC supercurrent³, so if there is no phase change across the junction, ie. $\delta = 0$, there should be no current: $I(2\pi n) = 0$, with $n = 0, \pm 1, \pm 2, \dots$
4. Items 1 and 2 also imply that $I(\pi n) = 0$ for $n = 0, \pm 1, \pm 2, \dots$. Taking for example $\delta = -\pi$, (1) implies $I(-\pi) = I(\pi)$ and (2) implies $I(-\pi) = -I(\pi)$, meaning $I(-\pi) = -I(-\pi) = 0$.

These rules imply that the CPR can be written as a Fourier series, or a sum of sines and cosines, where the cosines vanish in the cases where (2) holds.

One particular junction geometry consisting of a constriction in the superconductor, which is also referred to as a “weak-link”, or as a micro- or nano-bridge, suggests an attractive direction for research for a number of reasons. The first is that these junctions are particularly well-suited for magnetometry applications as will be discussed in Section 2.1.6. These junctions also allow for higher critical currents per area, allowing for higher pump

³The velocity of a single Cooper pair can be expressed as $v_p = \frac{1}{m_p} (\hbar \nabla \phi - q_p \mathbf{A})$, so the current will be proportional to $\nabla \phi$.

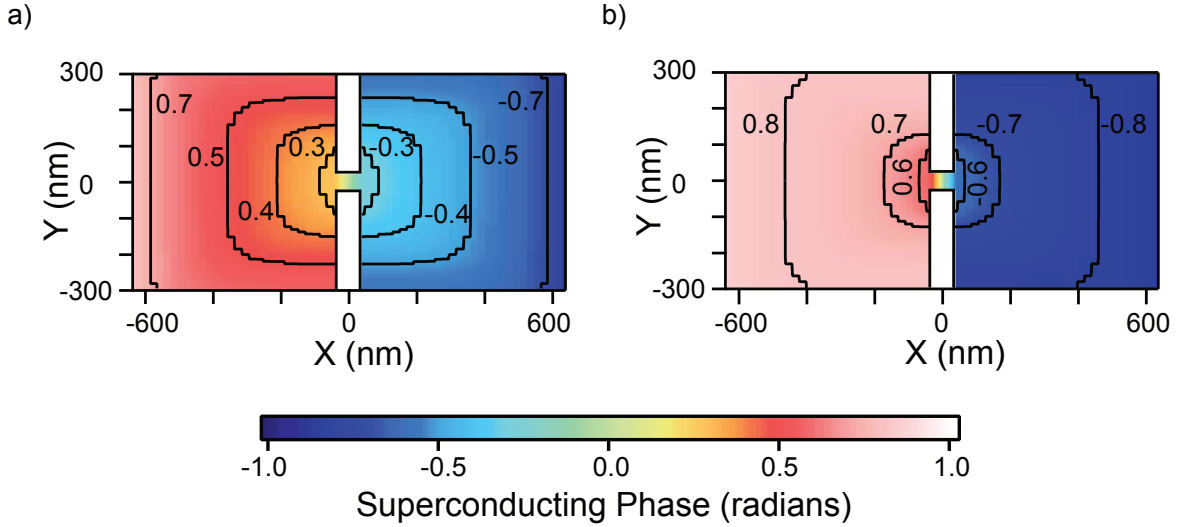


Figure 2.3: A plot of the numerical solutions to the Usadel equations for the nanobridge geometries shown in Figure 2.2(d) and (f). In this figure we show a plot of the superconducting phase distribution with position for a 2D nanobridge in (a) and a 3D nanobridge in (b). The superconducting phase is well confined within the 3D nanobridge, but spreads outward from the 2D nanobridge. More details can be found in Reference 23.

powers and lower noise when these nanobridges are put in the magnetometer circuit, as discussed in Section 2.1.7. Finally these junctions can be used to study the physics of quasiparticles in superconductors, which is beyond the scope of this thesis but is discussed in detail in References 44 and 45. Much research was done by Levenson-Falk and Vijay, et al., to determine the properties of these nanobridge junctions [23, 24, 46]. They studied two particular nanobridge geometries, shown in Figure 2.2, and determined that nanobridges with high bank to bridge thickness ratios (three-dimensional nanobridges) have a much stronger nonlinearity than single-thickness (two-dimensional) nanobridges.

To solve for these CPRs in these various nanobridge geometries Vijay et al. [23] numerically solved the Usadel equations which govern the superconducting phase behavior. They showed that a three-dimensional (3D) nanobridge, as shown in Figure 2.2 has a nearly sinusoidal CPR. A two-dimensional (2D) nanobridge, also shown in Figure 2.2, has a much more linear CPR. Note that this CPR is multivalued, and the average of first and second branch yield $I(\pi) = 0$. The weak nonlinearity of the 2D junction means that it is less useful in the context of a dispersive magnetometer. As we will see in Sections 2.1.5 and 2.1.7, weak nonlinearity will lead to reduced modulation in the flux-tuning curve and therefore reduced sensitivity. In addition, it turns out that the threshold for the onset of chaotic behavior in a 2D junction is much lower than for a 3D one, meaning that it is difficult to bias a 2D magnetometer in the parametric amplification regime, and impossible for some sample parameters [22–24, 46].

As shown in Figure 2.3, the behavior of the superconducting phase is very different between the 2D and 3D cases; it is tightly confined within the 3D nanobridge, whereas the phase gradient spills into the electrodes in the 2D case. A rough physical explanation for this is that the total current must be constant through the electrodes and bridge, meaning that the smaller cross-sectional area of the bridge will result in a higher current density. The Cooper pair density n is fixed through the superconductor, so in order to increase the current density the phase gradient must be higher within the bridge to compensate.⁴ The amount of compensation depends on the aspect ratio between bridge and electrode cross sections, so the larger aspect ratio in the 3D case creates a steeper phase gradient. However, this intuition only holds within a few coherence lengths of the superconducting material, so increasing the 3D geometry aspect ratio beyond a certain limit does nothing.

As long as the bridge length is within a factor of a few coherence lengths and the width roughly equivalent to the coherence length, $\xi(T = 0) \approx 30$ nm for Al, then the phase change across the junction remains well confined within the bridge [23]. This confinement implies increased nonlinearity in the current phase relationship (CPR), which we will see in Section 2.1.5 will allow increased modulation depth of the critical current with flux and therefore greater sensitivity [22–24]. The use of aluminum rather than niobium, which has a shorter ξ , allows straightforward fabrication with electron beam lithography. More details on the specifics of these calculations are given in References 44 and 23.

2.1.5 SQUID Flux Tuning and the SQUID Equations

In the following I will first derive how the critical current of a tunnel junction SQUID tunes with magnetic field, and then describe how this can be extended to a SQUID with junctions of arbitrary CPR. Various versions of this derivation can be found in References 38, 39 and 40 among others.

We first consider a SQUID loop, as shown in Figure 2.4a where the SQUID electrodes are assumed to be of greater width than the penetration depth. If we take a line-integral path at the far interior of the SQUID loop, as shown with dashed line, where the magnetic field penetrates the superconductor and there is no supercurrent flow, we find that $v_p = 0 = \frac{1}{m_p} (\hbar \nabla \phi - q_p \mathbf{A})$, so $\mathbf{A} = (\Phi_0/2\pi) \nabla \phi$ within the SQUID electrodes. Since $\mathbf{B} = \nabla \times \mathbf{A}$, the line integral of $\mathbf{A}$ around the path will be equal to the enclosed flux Φ . So we can write an expression for the flux as:

$$\Phi = \oint \mathbf{A} \cdot d\mathbf{s} = (\Phi_0/2\pi) \int_{\text{electrodes}} \nabla \phi \cdot d\mathbf{s} + \int_{\text{junctions}} \mathbf{A} \cdot d\mathbf{s} \quad (2.10)$$

Using Equation 2.2, the expression for the gauge-invariant phase difference across each junction, and substituting in for $\int_{\text{junctions}} \mathbf{A} \cdot d\mathbf{s}$, we obtain:

$$\Phi = \frac{\Phi_0}{2\pi} \left(\left[\int_{\text{electrodes}} \nabla \phi \cdot d\mathbf{s} + \Delta\phi_2 - \Delta\phi_1 \right] + \delta_1 - \delta_2 \right). \quad (2.11)$$

⁴Note again that the Cooper pair velocity is proportional to the phase gradient.

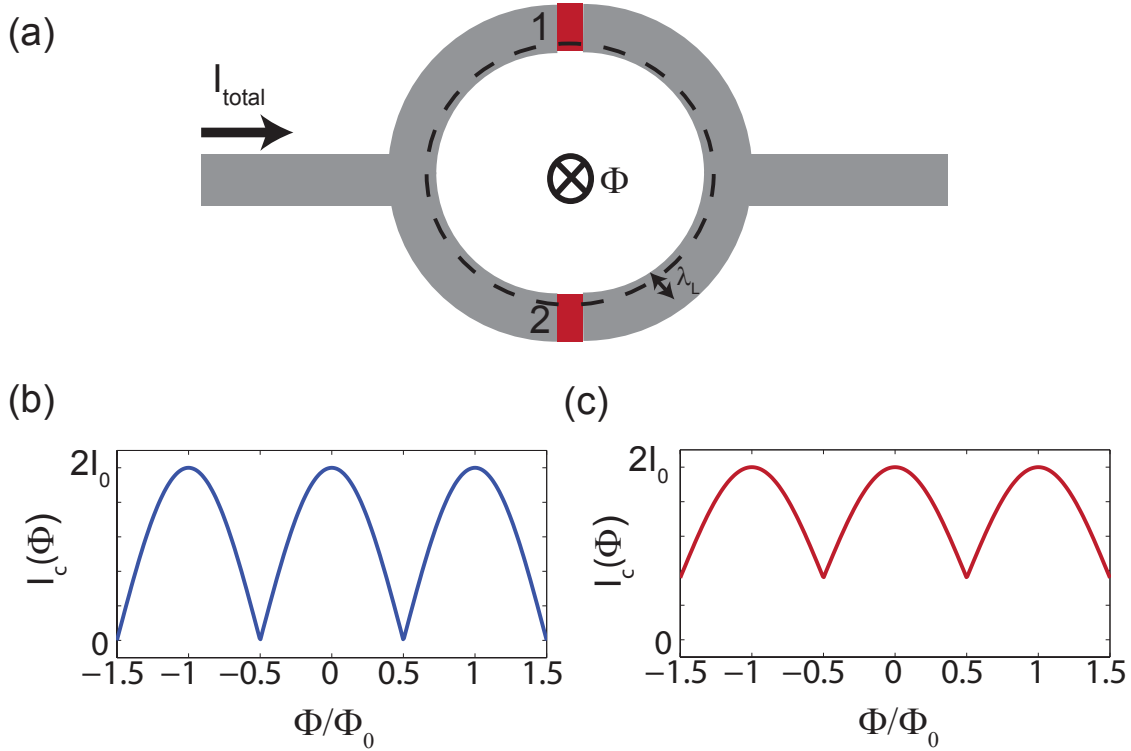


Figure 2.4: The dashed circle on the SQUID illustrated in (a) denotes the path used for integration in Equation 2.10. In (b) we plot the flux modulation for a tunnel junction SQUID as given in Equation 2.15, and in (c) we plot an example flux tuning curve for a SQUID with decreased modulation depth.

The term in brackets is just equal to a multiple of 2π since it is just the integral of the superconducting phase around the loop, and ϕ must be single-valued. Substituting in $2\pi n$ for the term in brackets and rearranging, we can write:

$$2\pi \left(\frac{\Phi}{\Phi_0} - n \right) = \delta_1 - \delta_2 \text{ with } n = 0, \pm 1, \pm 2 \dots \quad (2.12)$$

Since this equation does not fully define δ_i , we can rewrite this in terms of two equations with some arbitrary phase δ_0 :

$$\begin{aligned} \delta_1 &= \delta_0 + \pi \frac{\Phi}{\Phi_0} + \pi n \\ \delta_2 &= \delta_0 - \pi \frac{\Phi}{\Phi_0} - \pi n \end{aligned} \quad (2.13)$$

Now, if we sum the current through the two SQUID junctions, assuming they have identical

critical currents, I_0 , we find:

$$\begin{aligned} I_{total} &= I_0 \left(\sin \left(\delta_0 + \pi \frac{\Phi}{\Phi_0} + \pi n \right) + \sin \left(\delta_0 - \pi \frac{\Phi}{\Phi_0} - \pi n \right) \right) \\ &= \pm 2I_0 \sin \delta_0 \cos \frac{\pi \Phi}{\Phi_0} \end{aligned} \quad (2.14)$$

In the last line we have used the sum and difference formulas twice and taken $\cos(\pi n) = \pm 1$. We don't know what δ_0 is equal to, but that doesn't matter if all we are interested in is the maximum current through the SQUID, or its critical current I_c . For maximum current, we take $\sin \delta_0 = \pm 1$, and can write:

$$I_c(\Phi) = 2I_0 \left| \cos \frac{\pi \Phi}{\Phi_0} \right| \quad (2.15)$$

thus confirming that the SQUID critical current will tune as a function of flux through the SQUID loop. This expression is plotted in Figure 2.4b. To find the inductance of the SQUID, we model it as a junction with a flux dependent CPR, and take the derivative of Equation 2.14 with respect to δ_0 , and then plug that derivative into the denominator of Equation 2.9 and evaluate at $\delta_0 = 0$ yielding:

$$L_{SQUID}(\Phi) = \frac{\varphi_0}{I_c(\Phi)} \quad (2.16)$$

The preceding equations assume that the geometric and kinetic loop inductance of the SQUID, L , is vanishingly small, or $\beta_L = \frac{2LI_0}{\Phi_0} \ll 1$. In the case of finite β_L or a non-sinusoidal CPR, one must solve the full SQUID equations instead. These are given by:

$$2\pi n = \delta_1 - \delta_2 + \beta_{L1} \frac{I_1(\delta_1)}{I_{0,1}} - \beta_{L2} \frac{I_2(\delta_2)}{I_{0,2}} + \Phi_a/\varphi_0 \quad (2.17)$$

$$\phi_t = \left(\delta_1 + \beta_{L1} \frac{I_1(\delta_1)}{I_{0,1}} + \delta_2 + \beta_{L2} \frac{I_2(\delta_2)}{I_{0,2}} \right) / 2 \quad (2.18)$$

where Φ_a is applied flux and ϕ_t is the total superconducting phase difference across the entire SQUID and is analogous to δ_0 in the above derivation. $\beta_{Li} = I_0 L_i / \varphi_0$ can be thought of as the ratio between the junction inductance and the linear inductance of the SQUID arm containing that junction, and a large value means that there will be large induced screening currents in the arm. Equation 2.17 is simply an extension of Equation 2.12 where we have separated Φ into its applied and induced flux components. Equation 2.18 is an average of the phases across each SQUID arm. These two equations form a system which can be solved for δ_i in terms of Φ and ϕ_t . If we then take the sum of the junction currents, we find for the total current:

$$I_s(\phi_t, \Phi) = I_1(\delta_1(\phi_t, \Phi)) + I_2(\delta_2(\phi_t, \Phi)) \quad (2.19)$$

To find the critical current, $I_c(\Phi)$ we maximize $I_s(\phi_t, \Phi)$ with respect to ϕ_t , and to find the inductance, $L_s(\Phi)$, we take $L_s(\Phi) = \varphi / (\partial I_s(\phi_t, \Phi) / \partial \phi_t)$.

Due to the complicated CPR of a nanobridge SQUID, the inductance and critical current are not simply related, and are best calculated numerically or measured experimentally [23, 24]. However, one can get a rough idea of what to expect with some further reasoning. For a tunnel junction SQUID with $\beta_L = 0$, Equation 2.17 yields $\delta_1 - \delta_2 = \pi$ at half a flux quantum of applied flux. Using Equation 2.19 and 2.1 we find that $I = I_0(\sin \delta_1 + \sin(\delta_1 - \pi)) = 0$, an exact cancellation of the critical current. However, if β_L is not exactly zero *or* the junction CPR is not purely sinusoidal and has some linear element, this cancellation will be imperfect. In these cases the modulation of the critical current from $2I_0$ to zero will be suppressed. We define modulation depth as $(I_{c,max} - I_{c,min})/I_{c,max}$. An example of $\sim 60\%$ modulation suppression is shown in Figure 2.4c. For an ideal diffusive weak-link, as derived by Kulik and Omel'yanchuk in what is known as KO-1 theory [47], the maximum modulation depth is 81%, as compared to 100% for a tunnel junction and 25% for a long wire with linear CPR [24]. The weak nonlinearity of 2D nanobridges in SQUIDs leads to drastically suppressed modulation as well as an almost triangularly (compared to cosinusoidally) shaped modulation curve [22, 24]. 3D nanobridges also have some modulation suppression, but approach ideal weak-link behavior as the bridge length decreases. See References 24 and 44 for experimental measurements of these flux modulation curves for 2D and 3D nanobridge SQUIDs with varying bridge lengths.

2.1.6 Nanobridge flux coupling

Apart from a higher critical current, one of the other key advantages for a nanobridge SQUID lies in its use for magnetometry. Nanobridges allow much greater flux coupling for a single spin as compared to a tunnel junction [48, 49]. As shown in Figure 2.5, due to the geometry and field exclusion due to the Meisner effect [38], the spin is effectively farther away from the SQUID loop when next to a tunnel junction and so will couple flux less efficiently into the loop.

We calculate the flux coupled into an ideal nanobridge SQUID loop, approximated as a thin wire loop, with dimensions $1 \times 1 \mu\text{m}^2$. We find that for a spin about 80 nm away from the edge of the SQUID loop we will have $\sim 20 \text{ n}\Phi_0$ of flux. A plot of this flux coupling, as calculated numerically, is shown in Figure 2.6.

2.1.7 SQUIDs in Circuits: Flux Transduction and Parametric Amplification

In the previous sections we have established that a nanobridge SQUID will act as an very sensitive flux-dependent inductor. The next question is how can we put this SQUID into a circuit in order to reliably transduce a flux signal into a signal that we can easily measure (ie. a voltage signal). It turns out that if we shunt the SQUID with a capacitor, we will find that with the appropriate parameters we can obtain both flux transduction *and* amplification. How does this occur? The capacitively shunted SQUID acts as a parallel LC circuit that has

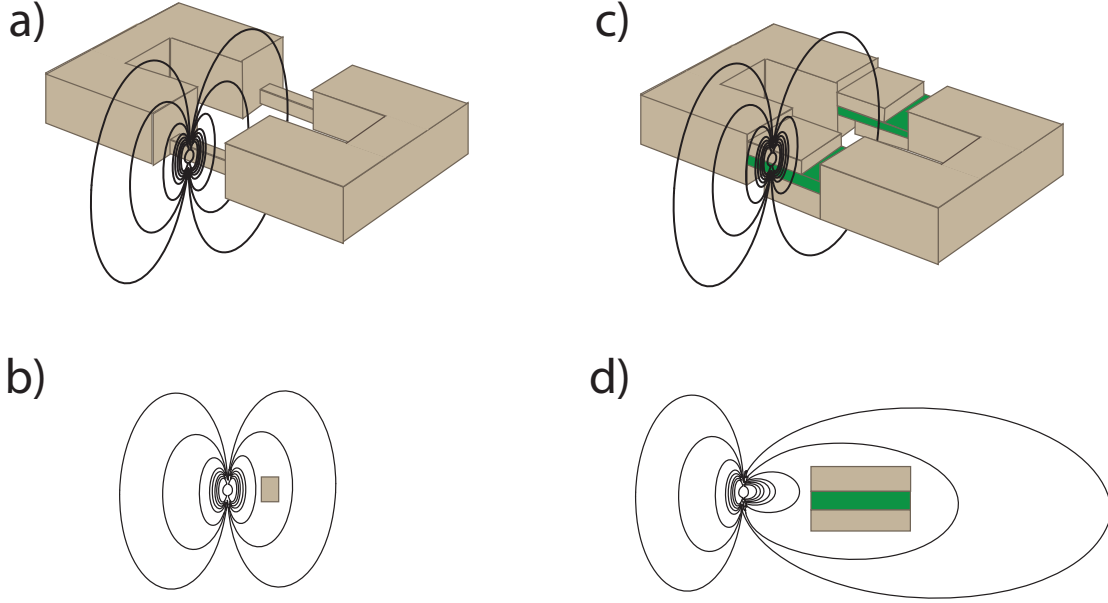


Figure 2.5: (a) An illustration (not to scale) of the nanobridge SQUID and flux coupling of a single spin into the SQUID loop. (b) A view of (a) from the side. (c) An illustration (not to scale) of a tunnel junction SQUID with a single spin, and a side view in (d) showing the field exclusion and a spin that is effectively farther away from the SQUID loop center.

a flux dependent resonant frequency:

$$\omega(\Phi) = \frac{1}{\sqrt{L(\Phi)C}} \quad (2.20)$$

A low frequency flux signal, $\Delta\Phi(\omega_s)$, that threads the SQUID loop will cause a periodic change in the inductance, thus modulating the resonant frequency. We can track this modulation by measuring the phase of a microwave probe tone reflected off the resonator, using for instance a vector network analyzer (VNA). The phase will undergo a 360° phase shift as the probe is tuned through resonance. If we bias our probe tone at a single frequency near the resonance center, we will see its phase periodically modulated at ω_s . In the frequency domain this phase modulation will appear as sidebands on the probe tone, ω_d , at the locations $\omega_d \pm \omega_s$, as shown in Figure 2.7b. If we increase the strength of our probe tone so that it now begins to strongly pump the resonator, we will access the nonlinearity of the SQUID, and the phase response curve will steepen, as shown in simplified cartoon form in Figure 2.7c. In this regime the flux signal is both transduced and amplified, as shown in Figure 2.7d.

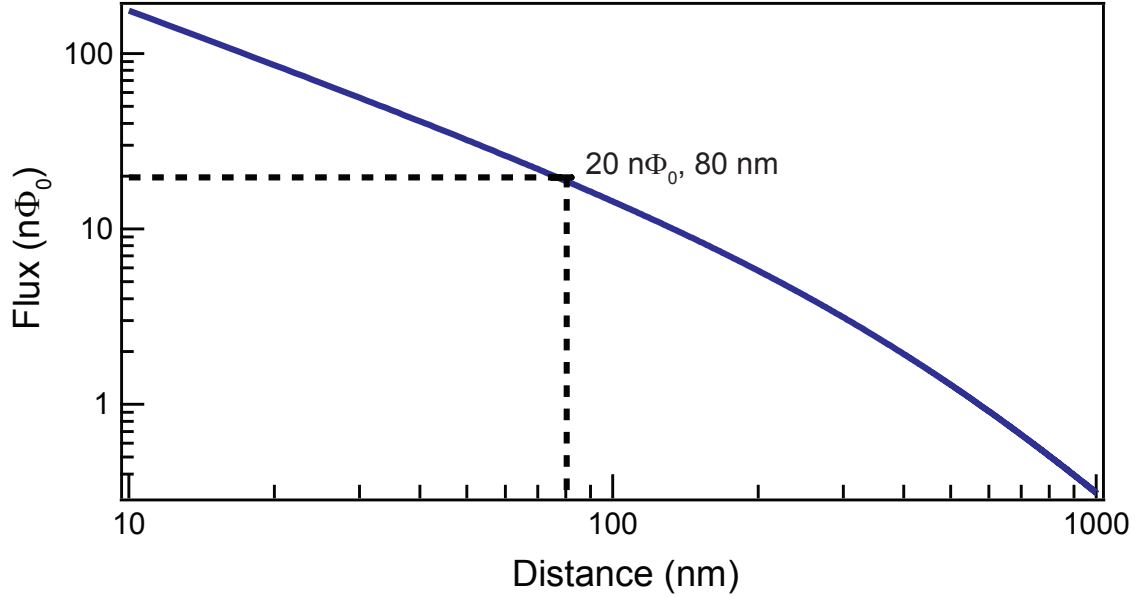


Figure 2.6: A plot of the magnetic flux from a single electron spin through an $1 \times 1 \mu\text{m}^2$ loop of infinitesimal thickness, for varying distances of the dipole from the SQUID loop edge.

A more complete illustration of the behavior of this nonlinear resonator with drive power is shown in Figure 2.8. The resonance frequency is fixed for a large range of low probe tone powers, this is the linear regime. This regime is useful for magnetometry of large magnetic signals, as were investigated in Chapter 5. As the probe tone power increases and drives the resonator harder, at a certain point the resonance frequency will begin to decrease and the phase response will sharpen, or become nonlinear. This is the parametric amplification regime, or “paramp” regime, which is used for both magnetometry and qubit readout [8–11, 44, 50]. Above the critical point, P_c , the resonator phase response will bifurcate. This regime is used for latched readout of qubits and is the basis for the Josephson Bifurcation Amplifier (JBA) [51].

As we have established, this magnetometer can be thought of as a two-stage device, a flux transducer and a parametric amplifier. The total transduced signal for a unit flux signal, normalized by the gain G , is given by [44]:

$$T \sim \frac{\partial V}{\partial \Phi} \frac{\sqrt{\frac{\sin^2 \theta_t}{G} + G \cos^2 \theta_t}}{\sqrt{G}} \quad (2.21)$$

At high gains this expression reduces to:

$$T \sim \frac{\partial V}{\partial \Phi} \cos \theta_t. \quad (2.22)$$

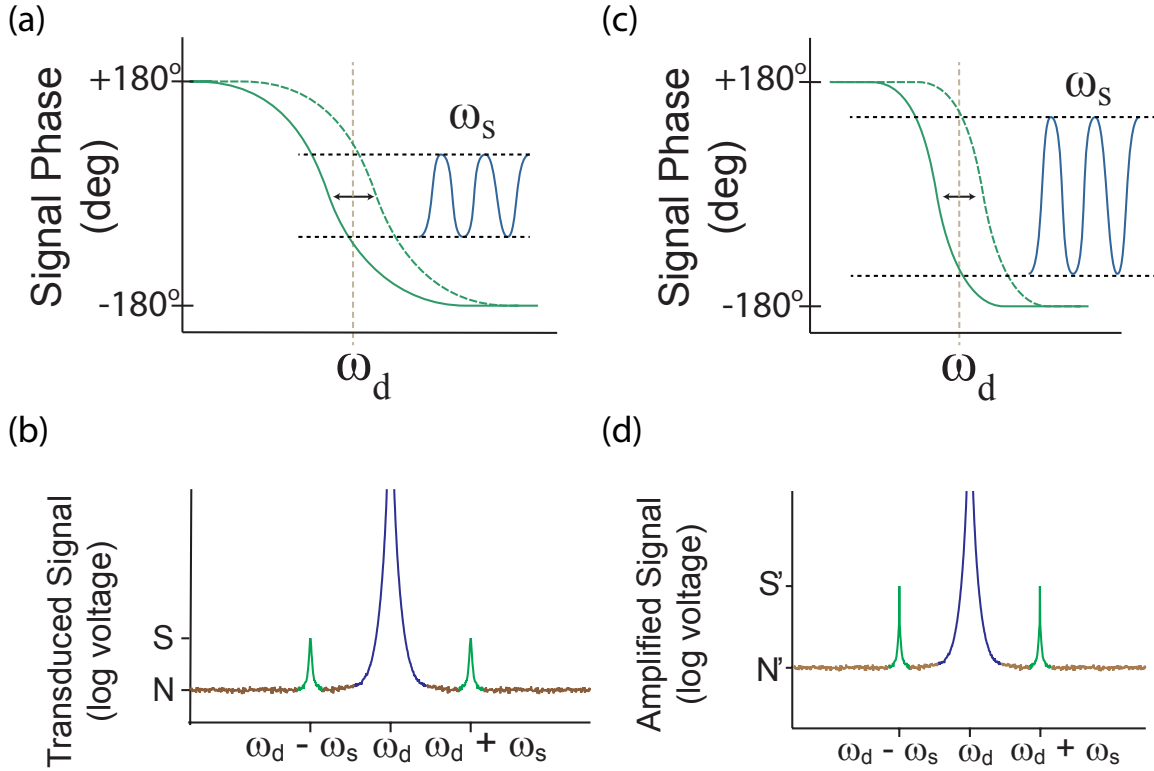


Figure 2.7: In (a) we give a cartoon of the phase response of the magnetometer operating in the linear regime. In (b) we show the pump signal and transduced sidebands at $\omega_d \pm \omega_s$. With a stronger drive tone the magnetometer enters the parametric amplification (paramp) regime and the phase response curve steepens, as illustrated in (c). In (d) we show the transduced and now amplified sidebands, again at $\omega_d \pm \omega_s$.

The term $\frac{\partial V}{\partial \Phi}$ is the *transduction factor* which depends on the strength of the pump or input drive signal, V_{in} , and the slope of the flux tuning curve at the applied flux, $\frac{d\omega(\Phi)}{d\Phi}|_{\Phi_a}$, where Φ_a is the magnitude of the applied flux. Reduced modulation in the flux-tuning curve, which is most prominent in the case of a 2D nanobridge SQUID, will lead to reduced transduction and by extension reduced sensitivity. Note that G and θ_t are also dependent on the pump strength.

The angle θ_t is an intrinsic parameter of the nonlinear oscillator (as described by the Duffing model) and represents the phase difference between the pump tone (or the amplified quadrature) and the up-converted signal. In the linear regime when $G \approx 1$ the θ_t dependence drops out and the transduction is purely dependent on the transduction factor, $\frac{\partial V}{\partial \Phi}$, as expected. In the paramp regime, when G is maximized, θ_t approaches 60° , and the transduc-

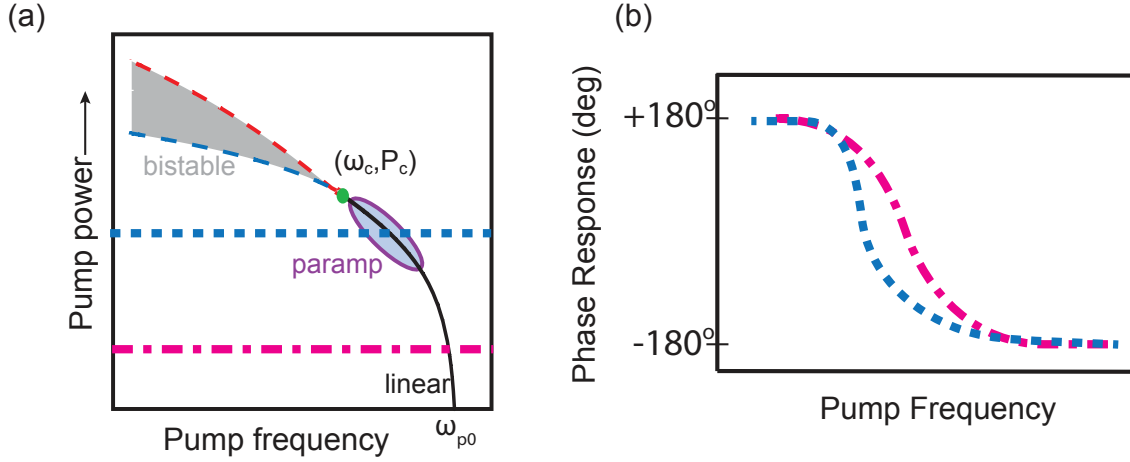


Figure 2.8: Shown in (a) is a cartoon of the location of the resonator frequency as a function of drive power and probe frequency. In (b) we draw a cartoon of the phase response of the resonator in the linear and paramp regimes, at the powers indicated by the dashed lines in (a). An experimental version of this plot is given in Chapter 4.1.

tion is halved [10, 11, 44, 52]. As we will show experimentally in Section 4.2 [11], this means that a single-stage magnetometer (no following paramp) will not have optimal sensitivity at the maximal gain point. Rather the lowest flux noise will be obtained with a drive power that lies between the maximal transduction and maximal gain points.

2.1.8 Dynamical Equations

It is possible to analytically derive the behavior of a tunnel junction magnetometer, due to its simple CPR. Nanobridge junction and SQUID behavior can be numerically calculated from the same starting point. The calculation for the tunnel junction magnetometer and related calculations for the LJPA and JBA are derived in several theses and papers [8, 10, 50, 51] so I will give only a very brief sketch of the results. The derivation begins by writing down a differential equation based on the resistively- and capacitively-shunted junction (RCSJ) model [39, 53, 54], which is broadly applicable even to nanobridge junctions. If we were to model a junction with a general CPR, we could model it as a parallel RLC circuit, where R could represent internal loss in the junction, a physical shunt resistor, or most frequently in our case the impedance of the microwave environment. C represents the combination of a intrinsic junction capacitance and a physical shunt capacitor. We assume that the physical inductance of the circuit is small, such that the circuit inductance is dominated by the junction. Using Kirchhoff's current law we can write an “equation of motion” for the junction:

$$C\dot{V} + \frac{V}{R} + I(\delta) = I_{drive} \quad (2.23)$$

Using Equation 2.3 we can rewrite this equation in terms of the superconducting phase difference:

$$\varphi_0 C \ddot{\delta} + \frac{\varphi_0}{R} \dot{\delta} + I(\delta) = I_{drive} \quad (2.24)$$

Where we have defined $\varphi_0 \equiv \Phi_0/(2\pi)$. From this point we must insert an expression for the current phase relationship, $I(\delta)$. In the case of a tunnel junction, this is simply Equation 2.1. In the case of a nanobridge junction, one can solve this equation numerically [23, 44, 46].

In the following we will sketch the analytical results that can be obtained for a tunnel junction magnetometer. These are helpful for getting a basic idea of how the effective two stage model for the magnetometer, as an upconverting transducer and amplifier, can be justified mathematically. For the tunnel junction SQUID magnetometer, we write down a version of Equation 2.24 where we model the SQUID as a single junction with a flux dependent critical current, $I_c(\Phi_a)$, with $\omega_{p0}^2 = I_c(\Phi_a)/(\varphi_0 C)$. The sinusoidal CPR is truncated after the first nonlinear term:

$$\frac{\partial^2 \delta}{\partial t^2} + 2\Gamma \frac{\partial \delta}{\partial t} + \omega_{p0}^2(\Phi) \left(\delta - \frac{\delta^3}{6} \right) = \frac{2\pi}{\Phi_0 C} I_d \cos(\omega_d t). \quad (2.25)$$

Here $\Gamma = (2Z_0 C)^{-1}$, where Z_0 is the impedance of the directly coupled transmission line. We assume here that the loop inductance of the SQUID is very small, $\beta_L \ll 1$. As well we assume that there is an applied flux consisting of a DC bias component and a single low frequency signal, $\Phi_a = \Phi_b + \Delta\Phi \cos(\omega_s t)$, with $\Delta\Phi \ll \Phi_0$. Plugging an ansatz of the form $\delta(t) = \delta_0 \cos(\omega_d t - \theta) + \epsilon(t)$ into Equation 2.25, where the first term is the steady state solution with $\Delta\Phi = 0$ and the second is a small perturbation of the junction phase due to $\Delta\Phi$, we can obtain after several approximations:

$$\begin{aligned} & \frac{d^2 \epsilon}{dt^2} + 2\Gamma \frac{d\epsilon}{dt} + \epsilon \omega_{p0}^2 \left(1 - \frac{\delta_0^2}{4} \right) \left[1 - \frac{\delta_0^2}{4 - \delta_0^2} \cos(2\omega_d t - 2\theta) \right] \\ &= \frac{\Delta\Phi}{Z_0 C \Phi_0} \frac{\partial V_{rf}}{\partial \Phi} [\cos(\omega_d t + \omega_s t - \theta) + \cos(\omega_d t - \omega_s t - \theta)]. \end{aligned} \quad (2.26)$$

Upon examination, the left side of this equation is that of a parametrically driven harmonic oscillator. Examination of the right hand side shows that the flux signal has been parametrically upconverted, through interaction with the drive tone, and becomes an effective double sideband rf signal input into the parametric amplifier. The sideband signal can be expressed as a single quadrature signal with components at $\omega_d \pm \omega_s$ and a phase θ relative to the drive tone. This angle θ is analogous to the angle θ_t that we presented for the general case. The size of these sidebands will be proportional to the strength of the flux signal $\Delta\Phi$ and the transduction factor. In the case of the tunnel junction magnetometer we can write down a specific equation for the transduction factor:

$$\frac{\partial V_{rf}}{\partial \Phi} = \pi \frac{2I_0 Z_0}{4\Phi_0} \sin\left(\frac{\pi\Phi_b}{\Phi_0}\right) \left(\delta_0 - \frac{\delta_0^3}{8} \right) \cdot \left(\frac{-\Phi_0}{2} < \Phi_b < \frac{\Phi_0}{2} \right) \quad (2.27)$$

As expected for the tunnel junction magnetometer, the transduction increases as we bias the DC flux towards the bottom of the flux tuning curve, or where $\Phi_b = \Phi_0/2$. The magnitude of the final output signal, after amplification of the transduced double sideband signal, will be given by $\Delta\Phi\eta\frac{\partial V_{rf}}{\partial\Phi}\sqrt{G}$, where η accounts for the effect of the angle θ given the nature of phase sensitive parametric amplification, in analogy to Equation 2.21:

$$\eta \sim \gamma \frac{\sqrt{\frac{\sin^2\theta}{G} + G\cos^2\theta}}{\sqrt{G}} \quad (2.28)$$

where $\gamma = 1$ or 2 is a numerical factor that depends on the mode of operation and accounts for the correlation between sidebands.

2.1.9 Flux Noise

The flux noise, $S_\phi^{1/2}$ is an important measure of the sensitivity of our magnetometer. We define the voltage signal-to-noise ratio as the ratio between the magnitude of our input signal, $\Delta\Phi$ (units of Φ_0) and the product of the flux noise $S_\phi^{1/2}$ (units of $\Phi_0/\sqrt{\text{Hz}}$) and the square root of the measurement bandwidth B (units of $\sqrt{\text{Hz}}$):

$$V_{SNR} = \frac{\Delta\Phi}{S_\phi^{1/2}\sqrt{\gamma B}}. \quad (2.29)$$

Here the factor γ accounts for correlation between the noise at the transduced signal sideband frequencies and depends on the mode of operation of the magnetometer and the technique we use to measure the signal-to-noise ratio. This will be explained further when we describe the characterization experiments in Sections 4.2 and 4.3. As one can see from Equation 2.29, a smaller flux noise will lead to a better signal-to-noise ratio.

While much of this thesis will be devoted to characterizing the *nanobridge* SQUID magnetometer by measuring its flux noise, it is still instructive to show the results of an analytic calculation of the flux noise for a *tunnel junction* SQUID magnetometer. These results follow References 10 and 50 closely. We begin by writing an expression for the single-sided spectral density of flux noise in analogy to the Johnson-Nyquist noise formula [55, 56]:

$$\begin{aligned} S_{\Phi_{eff}}^{1/2}(\Phi_0 H z^{-1/2}) &= \frac{\sqrt{2(k_B T_{sys} + \hbar\omega_d/2)Z_0}}{\eta \frac{\partial V_{rf}}{\partial\Phi} \Phi_0} \\ &= \frac{4}{\pi} \frac{\sqrt{2(k_B T_{sys} + \hbar\omega_d/2)Z_0}}{2I_0 Z_0 \eta \sin\left(\frac{\pi\Phi_b}{\Phi_0}\right) \left(\delta_0 - \frac{\delta_0^3}{8}\right)} \left[\frac{\Phi_0}{\sqrt{\text{Hz}}} \right]. \end{aligned} \quad (2.30)$$

Here T_{sys} is the added noise of the amplification chain, and we have referred the output voltage noise in the numerator to an input flux noise via the placement of the transduction coefficient $\eta\partial V_{rf}/\partial\Phi$ in the denominator. In the second line we have substituted in Equation 2.27 for the transduction factor.

For weak drives, where the resonator response is linear and $G = 1$, the system noise temperature is dominated by the HEMT and is much larger than the quantum limit. We can also express the junction oscillation amplitude in terms of the voltage drive, $\delta_0 - \delta_0^3/8 \approx \delta_0 \approx V_d Q / (Z_0 2I_0)$. We take $\eta = 1$, and using these approximations we find that in the limit of $G \sim 1$ the flux noise reduces to:

$$S_{\Phi_{eff}}^{1/2}(\Phi_0 H z^{-1/2}) \approx \frac{4}{\pi} \frac{\sqrt{2k_B T_{HEMT} Z_0}}{\sin\left(\frac{\pi\Phi_b}{\Phi_0}\right) V_d Q} \left[\frac{\Phi_0}{\sqrt{\text{Hz}}} \right]. \quad (2.31)$$

In this regime, higher sensitivity can be obtained by increasing resonator Q and the drive power. However, too much drive power and the resonator becomes nonlinear, invalidating this approximation.

For strong drives, the resonator becomes nonlinear and we have $G \gg 1$. In the high gain limit, the noise temperature is dominated by quantum fluctuations, as we have overwhelmed the noise of the following HEMT amplifier, so we can write $T_{sys} \approx \hbar\omega_d / (2k_B)$. We can also approximate $\delta_0 - \delta_0^3/8 \approx \delta_c = 4/(3^{1/4}\sqrt{Q})$ when $Q \gtrsim 10$ and $\delta_c \leq 1$. Since $\theta \rightarrow 60^\circ$ at high gain, we can take $\eta \approx 2 \cos \theta \approx 1$:

$$S_{\Phi_{eff}}^{1/2}(\Phi_0 H z^{-1/2}) \approx \frac{(2\sqrt{3})^{1/2}}{\pi} \frac{\sqrt{\hbar\omega_d}}{2I_0 \sin\left(\frac{\pi\Phi_b}{\Phi_0}\right)} \sqrt{\frac{Q}{Z_0}} \left[\frac{\Phi_0}{\sqrt{\text{Hz}}} \right]. \quad (2.32)$$

This equation suggests, counterintuitively, that in the paramp regime, a low Q circuit is desirable for the best sensitivity. The Q dependence comes from $\delta_c \propto Q^{-1/2}$. Additionally, lower Q leads to a higher bandwidth for a given gain, making it doubly desirable.

2.2 Introduction to Superconducting Circuits and Spins

In the following sections I will discuss some of the basic background for the diamond impurities that are studied in Chapter 5 and Appendix B. We will be referring to only two types of the very large number of diamond impurity centers. We will discuss P1 centers, a single nitrogen substitutional impurity, and NV centers which have a nitrogen substitution adjacent to a vacancy in the carbon lattice. Both of these defects have effective spin-1/2 or 1 electron spins that can be excited at microwave frequencies, and in principle read out with our magnetometer. A cartoon of these two centers is shown in Figure 2.9. In addition I will discuss the basics of two superconducting circuits that can also be used to read out these spins, and how these readout techniques differ from the magnetometer measurement.

2.2.1 Diamond Types and Methods of Fabrication

There are many types of naturally occurring diamonds, categorized by type and concentration of impurities. While many of the first experiments on diamond were conducted on mined

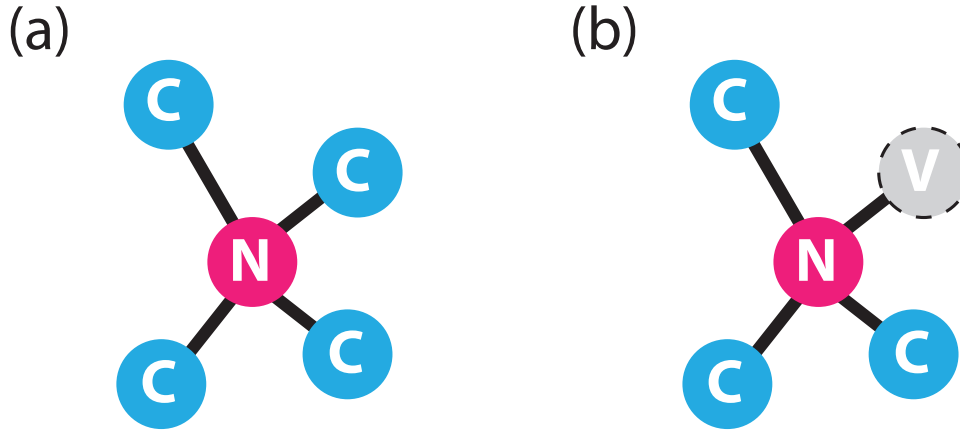


Figure 2.9: Here we show cartoons of the P1 and NV center defect structures. P1 centers, shown in (a) consist of a single nitrogen substitutional impurity. NV centers have an additional vacancy adjacent to the substitutional nitrogen, as shown in (b).

stones, typical modern samples are synthetically grown. There are two types of methods for creating artificial diamonds. Diamonds with a high nitrogen concentration (~ 100 ppm or more) are denoted type Ib and are yellow in color. These diamonds are typically made using the high-pressure high-temperature (HPHT) technique. A cleaner way of manufacturing diamonds is with chemical vapor deposition (CVD). This method allows for mid-range (~ 1 ppm) and very low (~ 1 ppb) concentrations of nitrogen, which can be carefully controlled.

Some types of natural diamond may have nitrogen vacancy centers already present, but extra steps are required to create these centers in artificial diamond. For very high concentrations (~ 10 ppm) of NVs, Acosta et al. irradiated ~ 100 ppm nitrogen HPHT diamonds with 3 MeV electrons with varying doses to create vacancies in the lattice. High temperature annealing causes the vacancies to diffuse towards the nitrogens which act as a trap for the vacancies, creating nitrogen vacancy (NV) centers. Using this technique, Acosta et al. were able to create high densities of bulk NVs with a maximal conversion rate of nitrogens to and NV^- centers of $\sim 16\%$ [57]. Another method of creating NV centers is by implantation. Additional nitrogen is implanted in unmasked spots on an electronic grade (CVD, 1 ppb nitrogen diamond) diamond. The diamond is irradiated and annealed, allowing creation of precisely located single NV centers [58, 59].

2.2.2 Introduction to the NV and P1 Centers

In general, defects in diamond can be modeled as effective molecules formed by the dangling bonds and available electrons of the defect atom(s). These defects have highly localized wavefunctions (ie. within a few lattice constants) which are subject to strong hyperfine interactions with the nearest nuclear spins, but other spin-spin interactions enter more weakly

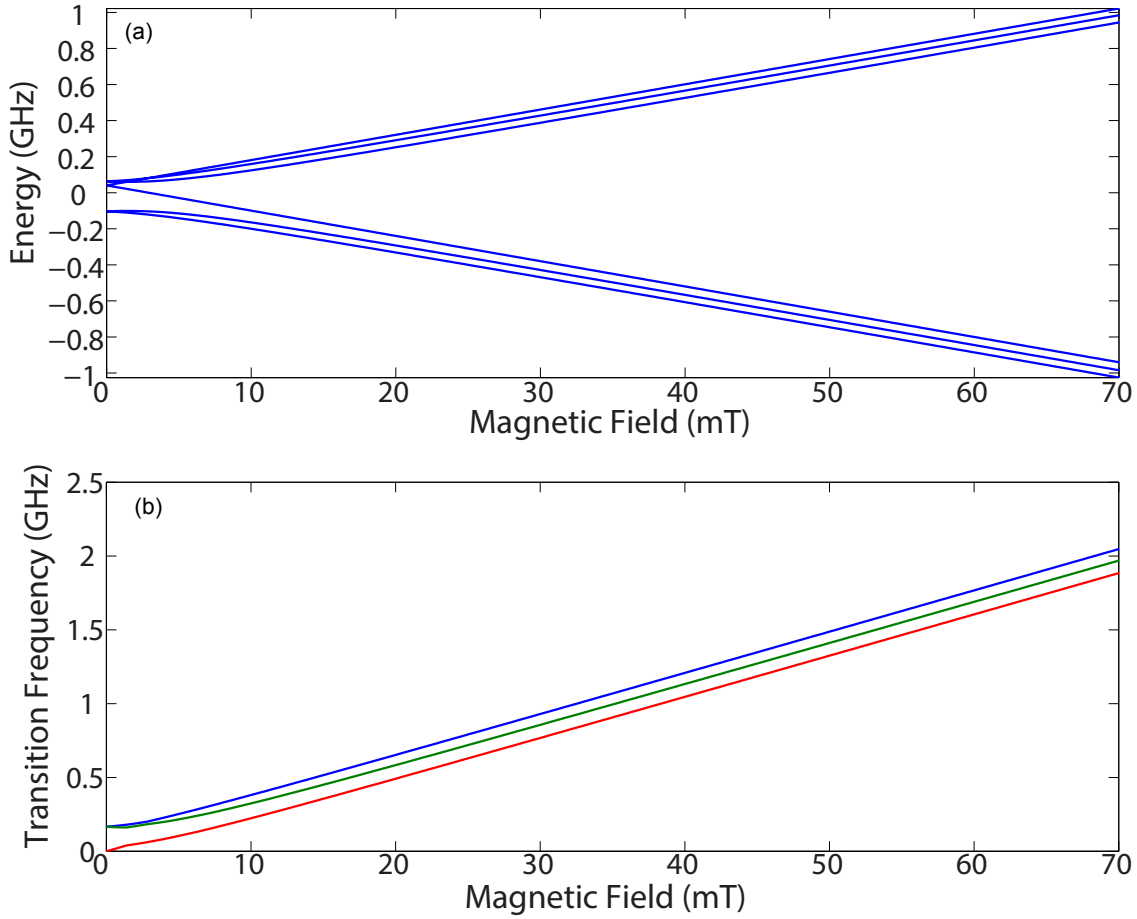


Figure 2.10: In (a) we plot the energy levels of the P1 defect as a function of magnetic field along the [100] diamond axis. In (b) we plot the allowed transitions, again with field. All lines were calculated with EASYSPIN [60].

as dipolar coupling terms. In this way P1 and NV centers can be modeled as effective single electron spins with a fixed spin of 1 or 1/2.

The P1 Hamiltonian is relatively straightforward, as the P1 centers are spin-1/2 defects with some hyperfine splitting due to the ^{14}N nucleus with $I = 1$ [61]:

$$\mathcal{H}_{P1}/h = \frac{\mu_{P1}}{h} \vec{S} \cdot \vec{B} + A_{\perp} (S_x I_x + S_y I_y) + A_{\parallel} S_z I_z - \frac{\gamma_N}{2\pi} \vec{B} \cdot \vec{I} + P(I_z^2 - \frac{I^2}{3}). \quad (2.33)$$

Here P is the quadrupole coupling with $P = -3.97$ MHz, and the hyperfine splittings are $A_{\perp} = 113.984$ MHz and $A_{\parallel} = 81.344$ MHz [61]. $\mu_{P1}/h = 28.0$ MHz/mT and γ_N is the gyromagnetic ratio for nitrogen. The P1 energy level and microwave transition diagrams are shown in Figure 2.10. As one can see from the plots, fields up to 100 mT allow easy

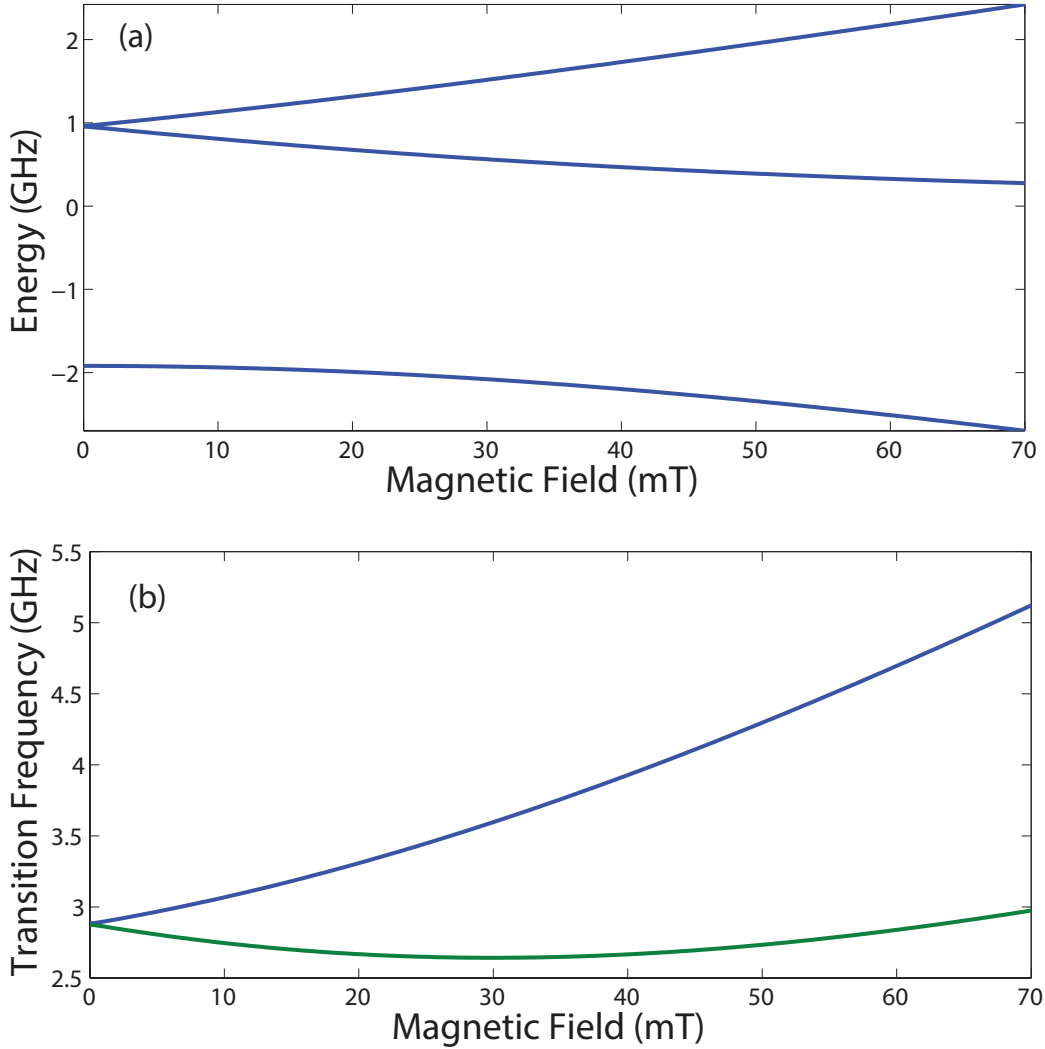


Figure 2.11: In (a) we plot the energy levels of the ground state manifold of the NV⁻ defect with the magnetic field aligned along the [100] axis of the diamond ($\sim 55^\circ$ from the defect symmetry axis). In (b) we plot the allowed spin-flip transitions. All lines were calculated with EASYSPIN [60].

addressability of the spins using microwave circuitry. Note that the P1 center undergoes a Jahn-Teller-like distortion in the lattice such that one of the four N-C bonds lengthens. This distortion breaks the defect's four-fold degeneracy, and creates a C_{3v} symmetry [62]. Tunneling between the four possible orientations of the defect, accompanied by a spin flip, was proposed as a possible channel for energy loss at low temperatures (~ 6 K) [20].

The NV⁻ defect has a triplet ground state, 3A_2 with a zero-field splitting of $D/2\pi \sim 2.88$

GHz. The higher energy $m_s = \pm 1$ states can be Zeeman split, as shown in Figure 2.11a. There is a bright optical transition between 3A_2 and the excited state triplet 3E , which can be pumped with 532 nm green laser light. The excitation decays back to the ground state manifold either with a spin-state conserving 638-800 nm light emission or via a dark-state manifold of singlet states, denoted 1A_1 and 1E . The latter decay path polarizes the spins into the $m_s = 0$ ground state. A simplified Hamiltonian for the ground state which neglects hyperfine interactions is given by [63]:

$$\mathcal{H}_{NV} = \hbar D S_z^2 + \hbar E (S_X^2 - S_Y^2) + g_{NV} \mu_B \vec{S} \cdot \vec{B} \quad (2.34)$$

where $E/2\pi$ can vary between ~ 100 kHz for dilute NV centers in pure diamond [64] to order of ~ 10 MHz for highly irradiated type 1b samples [63]. The full Hamiltonian, including hyperfine terms, can be found in Reference [29]. In Figure 2.11 we plot the ground state energy levels, and allowed transitions for a magnetic field along the $[100]$ axis, which is at $\sim 55^\circ$ to the defect symmetry axis which lies along the N-V axis. There are four possible orientations for each NV defect within the crystal structure. The $[100]$ axis makes the same angle with each possible orientation of the defect, but other magnetic field directions will create further splitting according to the effective magnetic field strength for each of the four populations.

2.2.3 Spin Coherence Times

One of the key spin system properties that will be measured using the magnetometer in the last chapter of this thesis is the spin-lattice relaxation, or T_1 time. There are typically two time constants of import for a spin system: T_1 and T_2 . The T_1 time is the characteristic time for irreversible energy loss of a spin to its environment, and T_2 is the characteristic time for decoherence of the spin's phase. These time constants can be added ad hoc into the classical equations for the precession of a magnetic dipole in magnetic fields, yielding the semi-classical Bloch equations [65]. The full quantum mechanical description of relaxation processes requires density matrix formalism, and is described generally by Bloch-Wangness-Redfield Theory [66]. T_1 and T_2 roughly represent how long a system will remain in a prepared quantum state, so the quest for long T_1 and T_2 times for qubits is a key goal in quantum computing research.

To intuitively grasp the significance of these time constants it is easiest to introduce the Bloch sphere. We will use this sphere, as shown in Figure 2.12, to describe the ensemble average of many two-level systems. The arrow in red represents the average magnetization vector for the ensemble and the poles of the sphere represent the states $|0\rangle$ and $|1\rangle$. This classic picture was first applied to interpret magnetic resonance experiments performed on nuclear spin ensembles. In a typical experiment the spins are placed in a large static magnetic field, B_0 , and precess about that field at the Larmor frequency, given by γB , where $\gamma_N/(2\pi) = 42.6$ MHz/T for a proton and $\gamma_e/(2\pi) = 28$ MHz/mT for an electron. At this point it is helpful to make the rotating wave approximation and switch into a frame rotating at

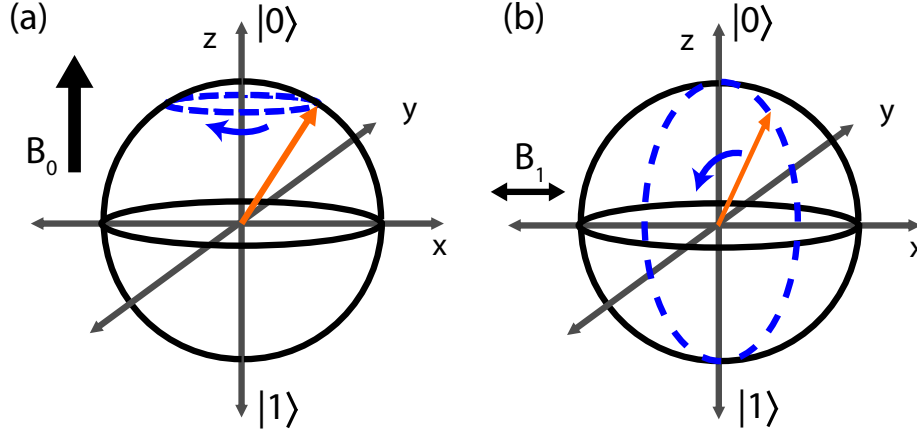


Figure 2.12: In (a) we illustrate the precession of the magnetization about B_0 at the Larmor frequency, and in (b) we illustrate Rabi oscillations in the rotating frame created by a resonant field B_1 along the x-axis.

the Larmor frequency such that the ensemble vector no longer precesses and remains fixed at some small angle to the polar axis. An alternating magnetic field, B_1 , at the Larmor frequency⁵ applied perpendicular to the large static field will cause the spin ensemble to oscillate periodically between the $|0\rangle$ and $|1\rangle$ states in a so-called Rabi oscillation. In the rotating frame this appears as a precession about the B_1 axis (for simplicity we assume the field is along the x-axis in Figure 2.12.) The essence of the magnetic resonance condition is that as long as the frequency of B_1 is very close to the $|0\rangle \rightarrow |1\rangle$ transition frequency, the magnitude of B_1 can be much smaller than B_0 and still create these oscillations. With B_1 exactly on resonance with the transition the Rabi oscillation frequency will be directly proportional to the strength of B_1 .

The effects of T_1 and T_2 can be most easily be seen in practice when we consider a pulsed B_1 drive. If we turn on B_1 in a short pulse that is timed such that the ensemble can only oscillate from $|0\rangle$ to $|1\rangle$ but not return, we call this a π -pulse. Half of this time is a $\pi/2$ -pulse, and will leave the magnetization in the x-y plane of the Bloch sphere. After a π -pulse that leaves ensemble in $|1\rangle$, the time constant of the exponential decay back towards $|0\rangle$ is defined as T_1 , or the longitudinal decay time. If the ensemble is prepared in $|0\rangle$ and then subjected to a $\pi/2$ -pulse, the magnetization vector will end up in the x-y plane of the Bloch sphere. After the characteristic time T_2 , or transverse relaxation time, the magnetization vector will have dephased. We represent this as a spreading or smearing of the magnetization vector in the x-y plane such that it no longer has a well defined azimuthal angle. T_2 decoherence in an ensemble can arise from many sources, with dipolar interactions between neighboring spins typically dominant. Some other sources of T_2 decoherence, among many, include diffusion of

⁵More generally the excitation must be at the transition frequency between $|0\rangle$ and $|1\rangle$, which may not be the Larmor frequency if there are additional splittings such as hyperfine or zero-field terms involved.

the excitation through the sample and inhomogeneities in the magnetic field. Both T_1 and T_2 effects also apply in the case of a single spin, but the theoretical treatment requires more careful language and a quantum mechanical description (eg. References 66–68).

In spin ensembles T_1 times are typically much longer than T_2 times, usually by an order of magnitude or more. The relaxation times are highly dependent on the specific physics of the system under study, including the spin density, temperature and environment. T_1 times of electron spins in solids typically range from milliseconds to several or thousands of seconds, with the latter values typically occurring in sparser and colder samples [19, 20, 34]. T_2 times can range from nanoseconds to seconds [34], again depending strongly on the spins' surroundings and temperature. This sensitive dependence is why these relaxation times are a valuable probe for spin system physics.

The overall picture can get quite complicated, but electron spins in solids typically lose their energy via phonons, or lattice excitations [69]. The phonon loss channels include direct processes where one phonon and a spin flip combine and Raman processes where two phonons and a spin inelastically scatter. Other decay modes include Orbach processes which involve a higher energy electronic excitation, cross-relaxation between two distinct spin populations, and spin-orbit phonon-induced tunneling which involves tunneling between Jahn-Teller distortions [20]. Below about 4 K it is generally suspected that phonon processes tend to freeze out and spin diffusion dominates in the T_1 decay, however only limited data is available at these temperatures and therefore one goal of our research is to explore this temperature regime.

2.2.4 Superconducting Resonators and Flux Qubits for Spin Readout

There have been several proposals for coupling diamond impurities to superconducting circuits, as well as a flurry of recent experimental work towards realizing this goal [2–6, 30, 36, 63, 70–73]. The two experiments that first experimentally realized coupling between a solid state spin and superconducting qubit involved an NV ensemble in a bulk NV diamond sample coupled either directly to a flux qubit [71] or coupled to a transmon qubit via a resonator [6].

It should be noted that the nature of the coupling between device and spin differs between these implementations and our magnetometer measurements. The former is a resonant coupling which can be modeled with coupled electromagnetic modes, while the latter is simply a dispersive⁶ measurement of flux through the SQUID loop. Both thrusts of research have advantages and disadvantages. Resonator-spin coupling can be modeled simply using quantum mechanics and is an entirely coherent coupling in certain limits [36]. Coupling strength is a limiting factor, however, with single-shot state-of-the-art sensitivities in the range of 10^6 spins/Hz^{1/2} [74, 75]. The magnetometer measurement, on the other hand, has

⁶In this context we use dispersive to mean that the magnetometer's resonant frequency is far detuned (several GHz) from the spin transition.

near-single spin sensitivity in principle, and allows for a straightforward and low-backaction readout.

Chapter 3

Design, Fabrication, and Experimental Setup

In this section I will describe the magnetometer and microwave launch design, sketch out the 3D nanobridge fabrication process and then discuss the general experimental setup including 3-axis magnet and fridge wiring.

3.1 Device Design and Optimization

As shown in Figure 3.1, the magnetometer consists of a 3D nanobridge SQUID shunted in parallel with a parallel-plate capacitor, with about 7 pF of capacitance. The capacitor pads are made of aluminum, with a virtual ground plane made of niobium. The first generation of devices (pictured in Figure 3.1) used ~ 300 nm thick silicon nitride (SiN) as the capacitor dielectric, and had the ground plane below the capacitor pads. The capacitor pads were approximately $100\text{ }\mu\text{m}$ on a side. This style of devices was characterized in Refs 11, 76. In the fall of 2012, Dr. Shay Hacoen-Gourgy developed a process to put the ground plane on top of the capacitor plates, using an aluminum oxide (AlO_x) dielectric layer instead. This more easily controlled process allowed us to deposit a much thinner layer of dielectric (~ 16 nm), thus allowing us to make a thinner capacitor, with $88\text{ }\mu\text{m}$ lateral dimensions. These thinner and slightly smaller area capacitors increased the in-plane field tolerance of the device by $\sim 10\text{-}15$ mT, as discussed in Section 4.3. All magnetometers have a fast flux (FF) line that is a shorted coplanar waveguide which supports excitations from DC to GHz frequencies. This line is used to put a known flux signal through the SQUID for characterization purposes, as well as for exciting spins.

We show in Figure 3.2 AFM images of a 3D nanobridge SQUID and a 2D nanobridge. In the magnetometer the SQUID nanobridge is typically 100 nm long, $15\text{-}20\text{ nm}$ tall and 30 nm wide. The SQUID loop is typically $\sim 2 \times 1.5\text{ }\mu\text{m}^2$ with a $70\text{-}80\text{ nm}$ pad thickness. With these thicknesses the junction banks are about five times thicker than the nanobridges, yielding a true 3D construction. Given these bridge dimensions, the nanobridge SQUID critical current

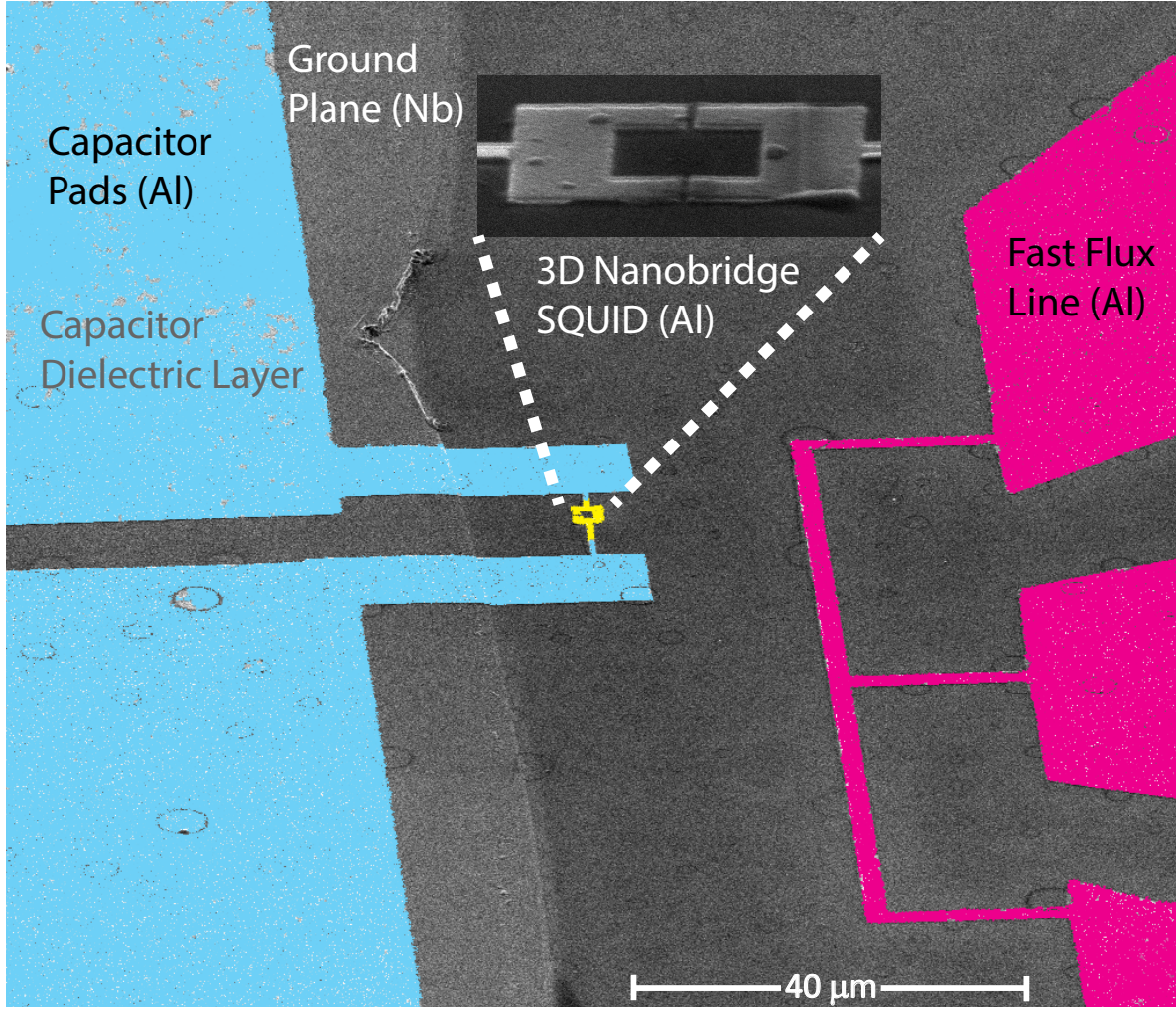


Figure 3.1: This is a false-color SEM image of the magnetometer. At center is the nanobridge SQUID highlighted in yellow, and expanded with a close up image. A clearer close up AFM image of a nanobridge SQUID is given in Figure 3.2. At left in blue is the parallel plate capacitor. The capacitor pads are Al, with a AlOx or SiN dielectric layer (not shown). The ground plane is made of Nb. At right, highlighted in pink, is the fast flux line made of Al. This is used to put a known flux signal through the SQUID for sensitivity characterization, or for exciting spins.

is approximately $I_c \approx 20 \mu A$.

Note that increasing the SQUID critical current is desirable up to a certain point, as this maximizes the critical drive power¹ and thus maximizes the transduction [51]. However, increasing critical current will decrease the junction inductance, and at a certain point the

¹The critical drive power is the maximal drive power before the resonator bifurcates.

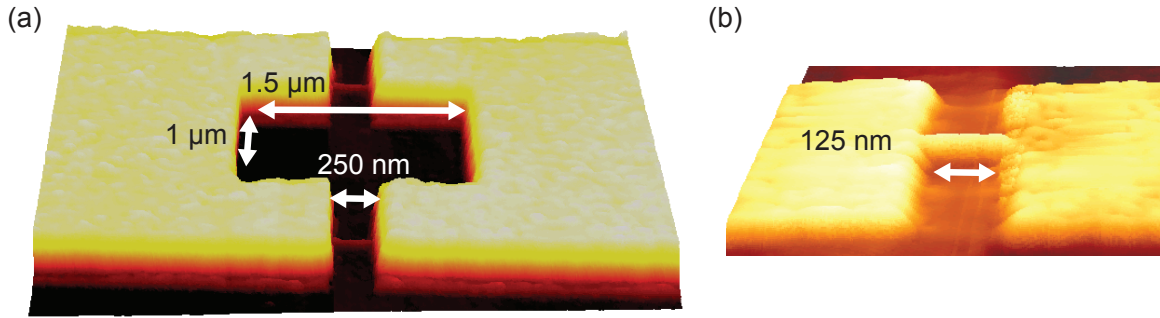


Figure 3.2: (a) An AFM image of a 3D nanobridge SQUID. Note that the bridge length is typically 100 nm, rather than the 250 nm shown in the image. The bridge cross-section is typically 15-20 nm tall by 30 nm wide. The $\sim 2 \times 1.5 \mu\text{m}^2$ SQUID loop is typical, with ~ 70 -80 nm of thickness. (b) An AFM image of a 2D nanobridge, for comparison.

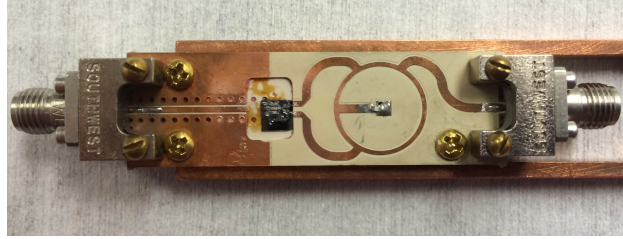


Figure 3.3: A picture of the microwave launch. The launch consists of a 180° rat-race hybrid at right that directly couples to the magnetometer, and a CPW line which is wirebonded to the FF line at left.

participation ratio² will become too small and the flux tuning of the device will be reduced. Reduced flux tuning, as discussed in Section 2.1.7, will decrease the efficiency of the flux-to-voltage transduction and thereby reduce sensitivity. This will also decrease the available nonlinearity of the junction, making it increasingly difficult to access the paramp regime [44].

3.2 Microwave Launch

In Figure 3.3 we show a picture of the the microwave launch. Constructed on Rogerscorp TMM6 using our in-house LPKF PCB milling machine, this launch was designed to easily fit within a 1" magnet bore. The magnetometer circuit is differentially driven via the 180° rat-race hybrid [77] which is optimized for 6 GHz operation and works well in the 4-8 GHz band. The hybrid directly couples the resonator to the 50Ω environment, producing the

²Participation ratio in this context is the ratio between the junction inductance and the linear inductance of the rest of the circuit.

target $Q \sim 30$. The fast flux line is wirebonded to a coplanar wave guide (CPW) line, which supports input signals from DC to several GHz. Both the hybrid and CPW lines are connected to coaxial cables with Southwest SMA end-launch microwave connectors (Part # 292-04A-5).³

3.3 3D Nanobridge Fabrication Process

The 3D nanobridge fabrication process was developed by Eli Levenson-Falk, and I refer the reader to his Ph.D. thesis appendix for the full recipe [44]. In the following I will give a brief sketch of the fabrication process.

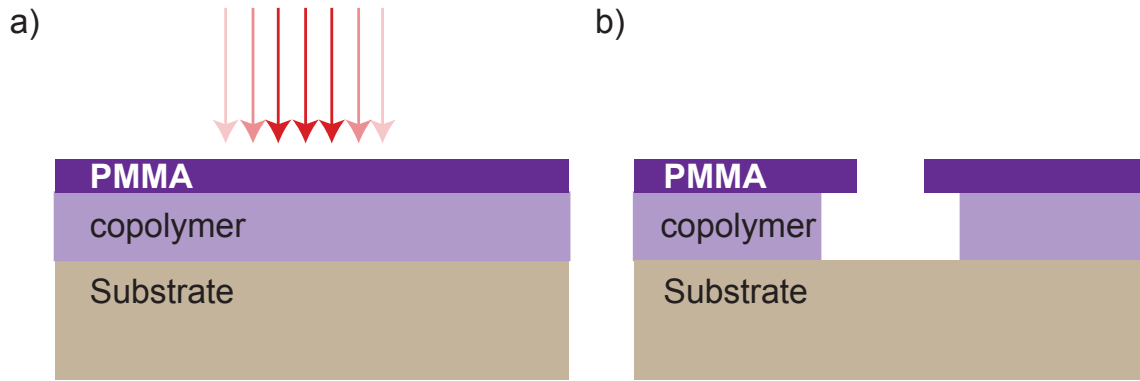


Figure 3.4: E-beam exposure with a high dose at the center and low doses on the sides (as shown in (a)) will produce an undercut in the PMMA/copolymer resist stack after development due to the greater sensitivity of the copolymer (MMA based) to low doses (shown in (b)). This undercut will be crucial for the evaporation and liftoff processes.

As shown in Figure 3.4, we spin a bilayer of MMA based copolymer and PMMA resist on the chip, and then expose with an electron beam writer. The center of the features is exposed with the highest dose, with lower doses on the sides of the features to produce an undercut in the resist after it is developed. The MMA is more sensitive to the ebeam exposure, so the low dose will expose it, while leaving the PMMA intact.

After the e-beam write step, the resist is developed in a precisely chilled methyl isobutyl ketone (MIBK) and isopropyl alcohol bath, in order to maximize resolution and dose consistency. After the developing, the chip is placed in an e-beam evaporator. Aluminum is first evaporated onto the chip head on, and then the sample stage is tilted to 35° for a second evaporation step. As shown in Figure 3.5, the first evaporation step deposits metal for the

³In the future, for a more compact mount within a magnet, one could consider using the low-profile version of these connectors: Part # 292-04A-6. Note that neither of the part numbers listed are for the non-magnetic version of these connectors, which does exist.

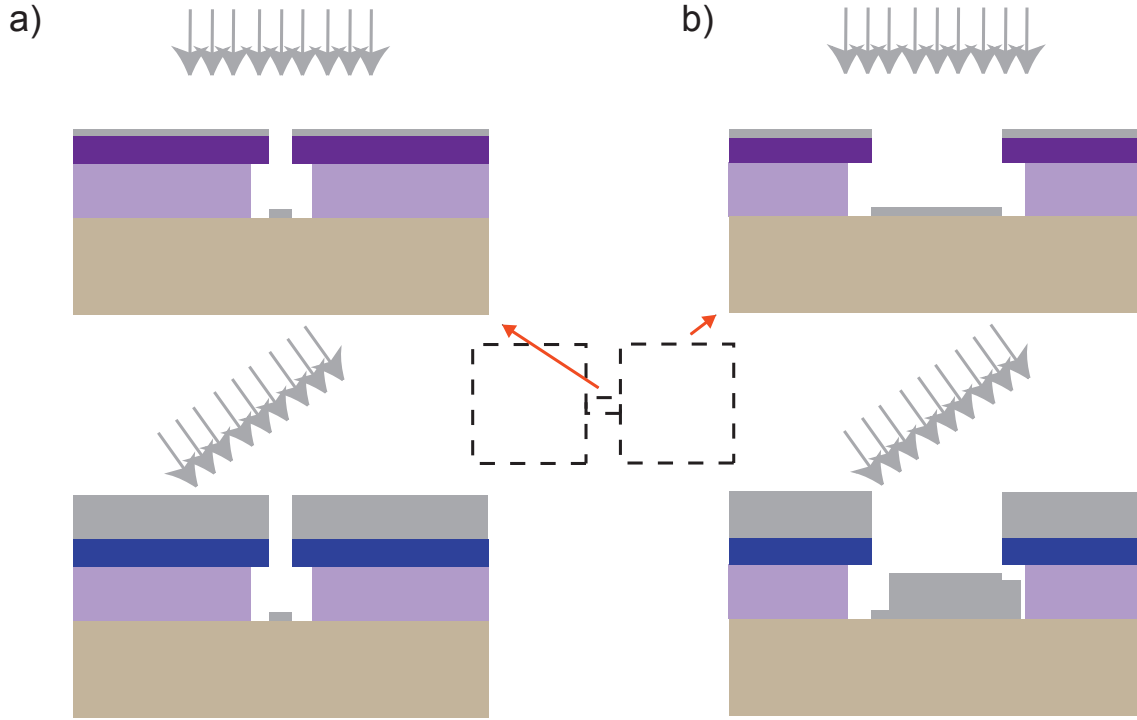


Figure 3.5: Here we present a diagram of the aluminum double-angle evaporation process. In (a) we show a cross-section of the location on the developed wafer and resist stack where the nanobridge will be fabricated. In (b) we show a cross-section of the junction bank. The wider opening for the banks allows metal to accumulate during both steps (top and bottom illustrations). The narrower opening for the bridge only allows metal to accumulate during the first step (top left), while metal during the second step is blocked (bottom left). This double-angle evaporation process leads to the desired thin nanobridge relative to the thick junction banks.

bridge and bottom part of the 3D banks. The second step continues to build up metal in the 3D bank region (Fig. 3.5b) but the incident aluminum is blocked from entering the narrow opening above the bridge (Fig. 3.5a). In the end the bridges end up with about 15-20 nm of thickness, and the banks about 70 nm.

After evaporation the chip is placed in an acetone bath in order to dissolve the resist and “liftoff” the excess aluminum that built up on the resist covered areas. The undercut in the resist is important for ensuring that the liftoff process goes smoothly [44].

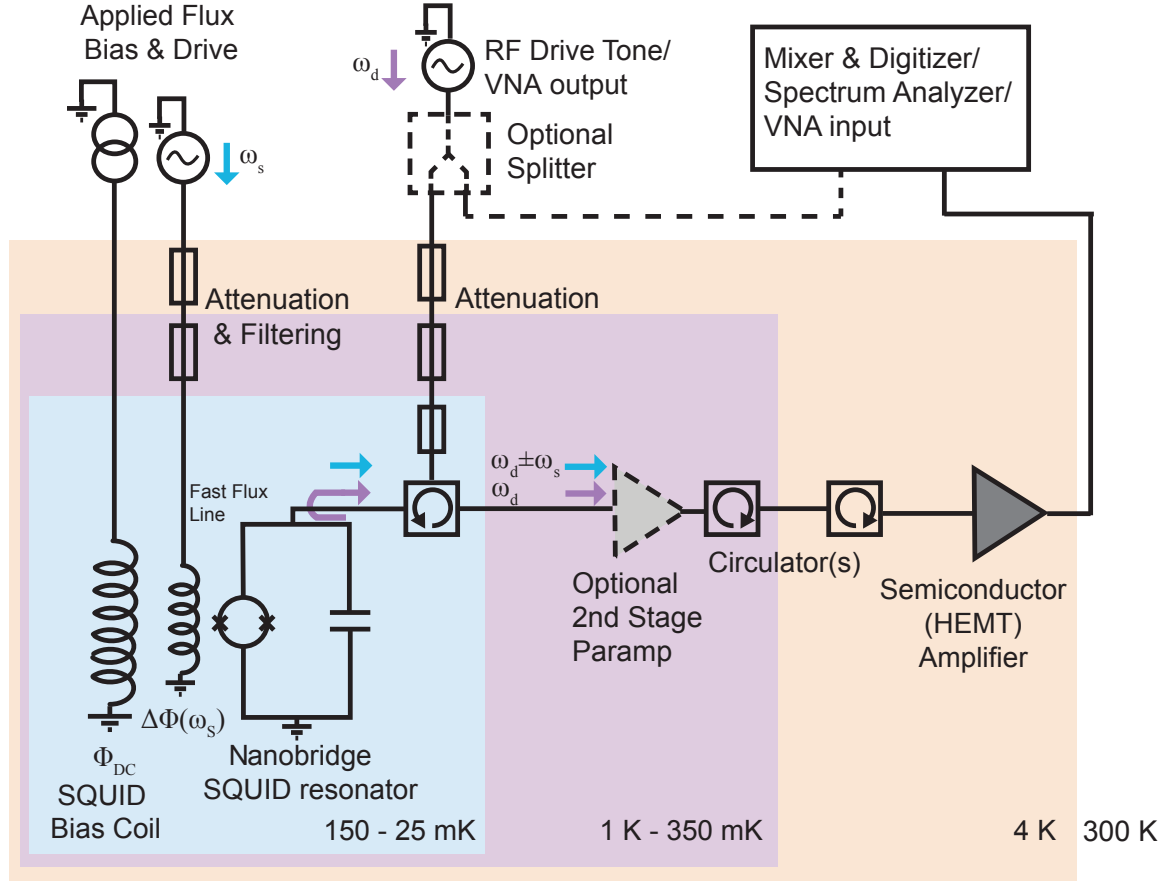


Figure 3.6: Here we show a general setup for the fridge wiring. An input microwave line is attenuated at several temperatures before reaching the base stage. The input tone enters the magnetometer circuit via a circulator, and then exits through the output line via an optional paramp, more circulators (acting as isolators) and a HEMT amplifier. The output tone is then measured either via mixer and digitizer, vector network analyzer (VNA) or spectrum analyzer. An additional attenuated microwave input line is connected to the on-chip fast flux line. Lastly, a DC bias field is applied to the SQUID via a superconducting magnet. The solenoid producing in-plane field for magnetometry and field tolerance measurements is not shown.

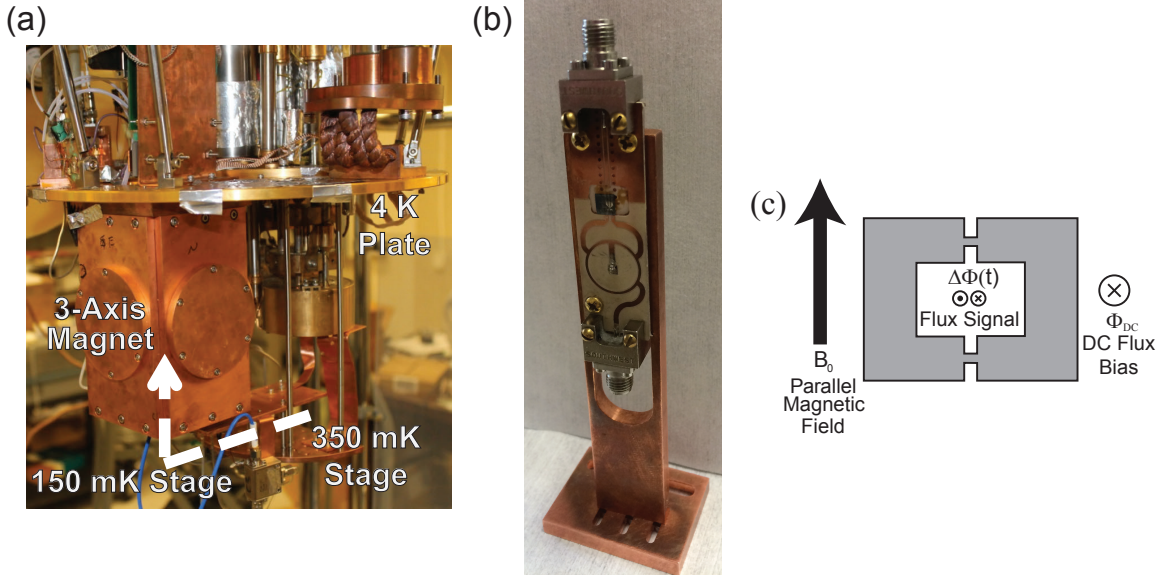


Figure 3.7: (a) A picture of the 3-axis magnet, 150 mK plate and sample stage within the magnet. (b) A picture of the sample holder that is placed inside the magnet. (c) An illustration of the magnetic field directions with respect to the SQUID. The main solenoid field, B_0 , is in-plane. A perpendicular field provides the DC flux bias on the SQUID, Φ_{DC} , and also shims the main coil. The varying flux signal, $\Delta\Phi(t)$ is provided by either the fast flux line or the spin system under measurement.

3.4 3-Axis Magnet, Sample Stage and Fridge Wiring

In Figure 3.6 we show typical wiring for these magnetometry experiments. All measurements are done in dilution refrigerators with base temperatures between 25 and 150 mK. More specific setups are given in the following chapters where several experiments are discussed. In general, the magnetometer requires a heavily attenuated microwave input line, a microwave output line with low-noise amplifier at 4 K, an additional more lightly attenuated microwave input line for the fast flux line, and a DC wiring pair for the SQUID bias coil. Additional wiring is necessary if a large solenoid coil for in-plane field is required. Note that the circulators shown in Figure 3.6 are non-reciprocal microwave components that allow microwave signal propagation in the direction indicated by the arrow with only ~ 0.5 dB of loss, and have ~ 20 dB of attenuation for counter-propagating signals. When the third port is terminated (as shown in the 1 K and 4 K circulators in Figure 3.6) the circulators act as isolators. The low noise amplifier at 4 K is a high electron mobility transistor (HEMT) amplifier, which has been optimized for low noise operation at cryogenic temperatures.

Figure 3.7 shows pictures of the 3-axis magnet and sample stage and an illustration of the magnetic field directions at the SQUID. See Appendix A for more details and pictures of

the 3-axis magnet. The main solenoid coil produces an in-plane field which can be used to produce Zeeman splitting. The Helmholtz pairs are used to shim the main solenoid as well as provide a perpendicular field for a DC flux bias on the SQUID, Φ_{DC} as shown in Figure 3.7c. We denote time dependent flux through the SQUID loop as $\Delta\Phi(t)$, and this is produced by either a known flux signal from the fast flux line (used for flux noise measurements) or by the spins under study. The sample holder shown in Figure 3.7b is designed for easy installation and removal within the magnet bore. The direction of installation inside the magnet is indicated by the dashed white arrow in Figure 3.7a.

Chapter 4

Magnetometer Characterization

In this chapter we will discuss the detailed experiments that were performed to characterize this nanobridge SQUID magnetometer. Note that these results followed closely on the heels of experiments to characterize a prototype tunnel-junction magnetometer which have been covered elsewhere [10, 50]. In Section 4.1 we discuss some of the basic characterization experiments that we perform on almost every device. In Section 4.2 we fully characterize the flux noise, transduction and amplification behavior of the nanobridge SQUID magnetometer and compare with the current state of the field. The bulk of the results presented in this section were published in Reference 11, and the research was led by Dr. R. Vijay and Dr. Eli Levenson-Falk.

The presence of thick banks in the 3D nanobridge SQUID raises the question of whether these structures will operate in large parallel magnetic field, due to the relatively low critical field of aluminum in bulk. In Section 4.3 we demonstrate that the 3D nanobridge SQUID operates reliably in magnetic fields up to 75 mT, which is enough field tolerance to study many interesting spin systems. The majority of these results were published in Reference 76.

4.1 Device Characterization: First Steps

In this section I will present a number of first order characterization measurements that we perform on nearly all devices. These types of measurements can be performed on devices that are optimized for use as magnetometers or as flux-tunable paramps. This section should be a useful introduction for those just beginning to learn how to use paramps and magnetometers in measurements.

4.1.1 Flux Tuning

In Figure 4.1 we show the results of a measurement to illustrate the tuning of our magnetometer as a function of perpendicular field through the SQUID loop. The experimental technique for gathering this measurement is very simple: fix the amount of perpendicular

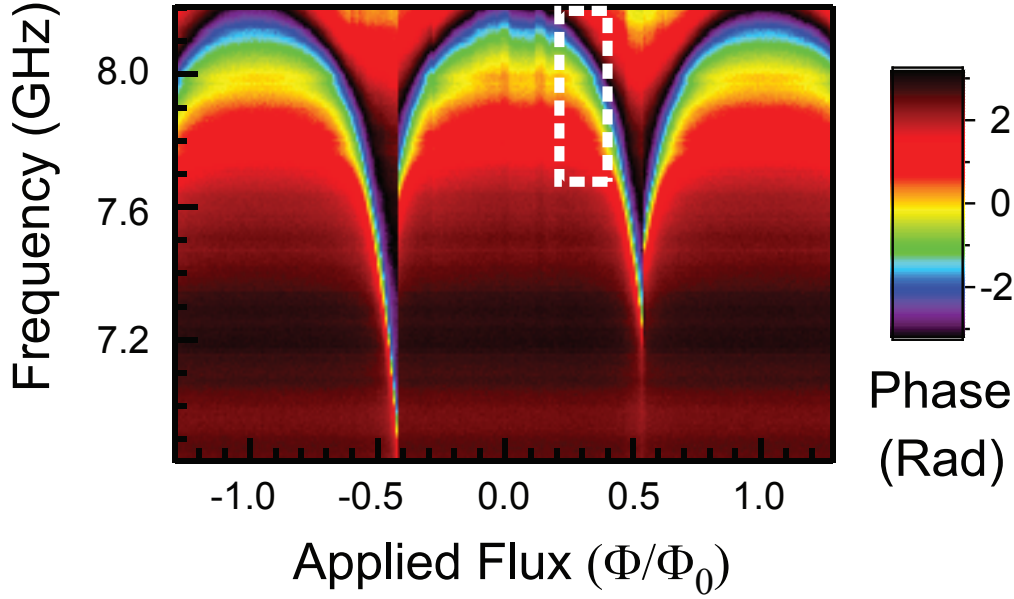


Figure 4.1: A plot of resonance frequency versus flux bias. Reflected phase is encoded in color, with 0 radians indicating the resonance center. The dashed white box denotes the typical DC flux bias for the measurements performed in Sections 4.2 and 4.3.

field through the SQUID loop with a DC flux bias magnet coil, take a trace of the resonator phase response with a vector network analyzer (VNA), step the flux bias by a small amount, take another phase response trace, repeat.

This plot is the experimental version of the cartoon illustration in Figure 2.4c. One subtlety not shown in Figure 2.4c is the hysteretic nature of the 3D nanobridge flux tuning. The inductance of the 3D nanobridge SQUID is multivalued, leading to hysteretic jumps between solution branches as the flux is tuned. This occurs because as the nanobridge SQUID is tuned near $\Phi_0/2$ the SQUID potential develops two wells with different inductances, and will hysteretically jump between those wells, thus dramatically shifting the inductance [44].¹ In Figure 4.1 the flux was tuned from negative to positive values. As one can see at $\sim \pm 0.5\Phi_0$ the resonance frequency tunes down dramatically as the flux is tuned and then abruptly jumps branches to a higher frequency. A flux sweep in the other direction would yield a plot that is an approximate reflection of Figure 4.1 about $\Phi/\Phi_0 = 0$.

From performing this type of measurement one can find a conversion between current on the DC bias coil and flux through the SQUID loop. For the 3-Axis magnet setup shown in Figure 3.7, tuning over a flux quantum usually requires between 20-30 mA of current on the Helmholtz coil. For more typical paramp setups with smaller and closer DC bias coils, the amount of current to tune over a full flux quantum is typically only a few milliamps or less.

¹See Section 4.4 of Reference 44 for more details

4.1.2 Resonator Phase Response with Pump Power

It is often helpful to experimentally map out the linear, paramp and bifurcation regimes of the magnetometer. These regimes were explained in detail in Section 2.1.7. In Figure 4.2a we show an experimental plot of the resonator phase response as a function of drive power and frequency. The numbered white-dashed lines correspond to the phase traces shown in Figure 4.2b. In the region of pump power directly above the critical point, labeled P_c , the resonator bifurcates. Below the critical point is the parametric (paramp) amplification regime, and below that the linear regime.

To take this plot we use the power sweep functionality on the VNA. At each drive frequency, we take one trace with the power ramped up and one with it ramped down. In this way we keep the device from experiencing large jumps in the applied drive power, and thus are able to avoid instability. The bifurcation behavior that is visible in the striped pattern above the marked critical point would not be visible without this zig-zag pattern of power ramps.

4.1.3 Paramp Bias and Gain Profile

In order to find a good paramp point for the magnetometer we first tune to a flux bias where we have the maximum sensitivity or the lowest flux noise. This is typically near $\Phi_0/4$ for a nanobridge SQUID magnetometer. This point reflects a balance between the additional transduction that is available when the resonator is tuned down towards $\pi/2$ and the increased limits on the drive power as the nanobridge SQUID is tuned down. As we discussed in Section 2.1.7, the transduction factor $\frac{\partial V}{\partial \Phi}$ increases with the slope of $\frac{df_{res}}{d\Phi}$ and the drive power. The slope of the tuning curve, $\frac{df_{res}}{d\Phi}$, increases as we tune towards $\Phi_0/2$. However, at the same time, the critical current of the SQUID drops and thus the non-linearity increases [24]. This means that it takes less drive power to bifurcate the resonator, meaning that we are limited to increasingly lower drive powers as we tune the SQUID down if we want to remain in the paramp regime. Experimentally we find that these two competing factors balance to an optimally low flux noise point near $\Phi_0/4$ [11].

To find a paramp point we begin by measuring the resonator phase response through resonance with a small vector network analyzer (VNA) probe power (typically ~ -50 dBm at the VNA output). The VNA trace should look similar to trace 3 in Figure 4.2. We increase the VNA input tone power until the trace begins to steepen and tune down in frequency (trace 1 in Figure 4.2). We find and note the frequency at the center of the steep part of the resonance curve (~ 7.49 GHz in Figure 4.2), reduce the VNA probe power, and set the paramp pump generator to the noted frequency. Typically the paramp pump tone is coupled into the magnetometer input line in addition to the VNA input tone via a directional coupler. We center the VNA readout with a span of about ~ 100 MHz² around the noted frequency, and switch from a phase to magnitude measurement. We gradually increase the paramp

²This range is of course dependent on the device bandwidth.

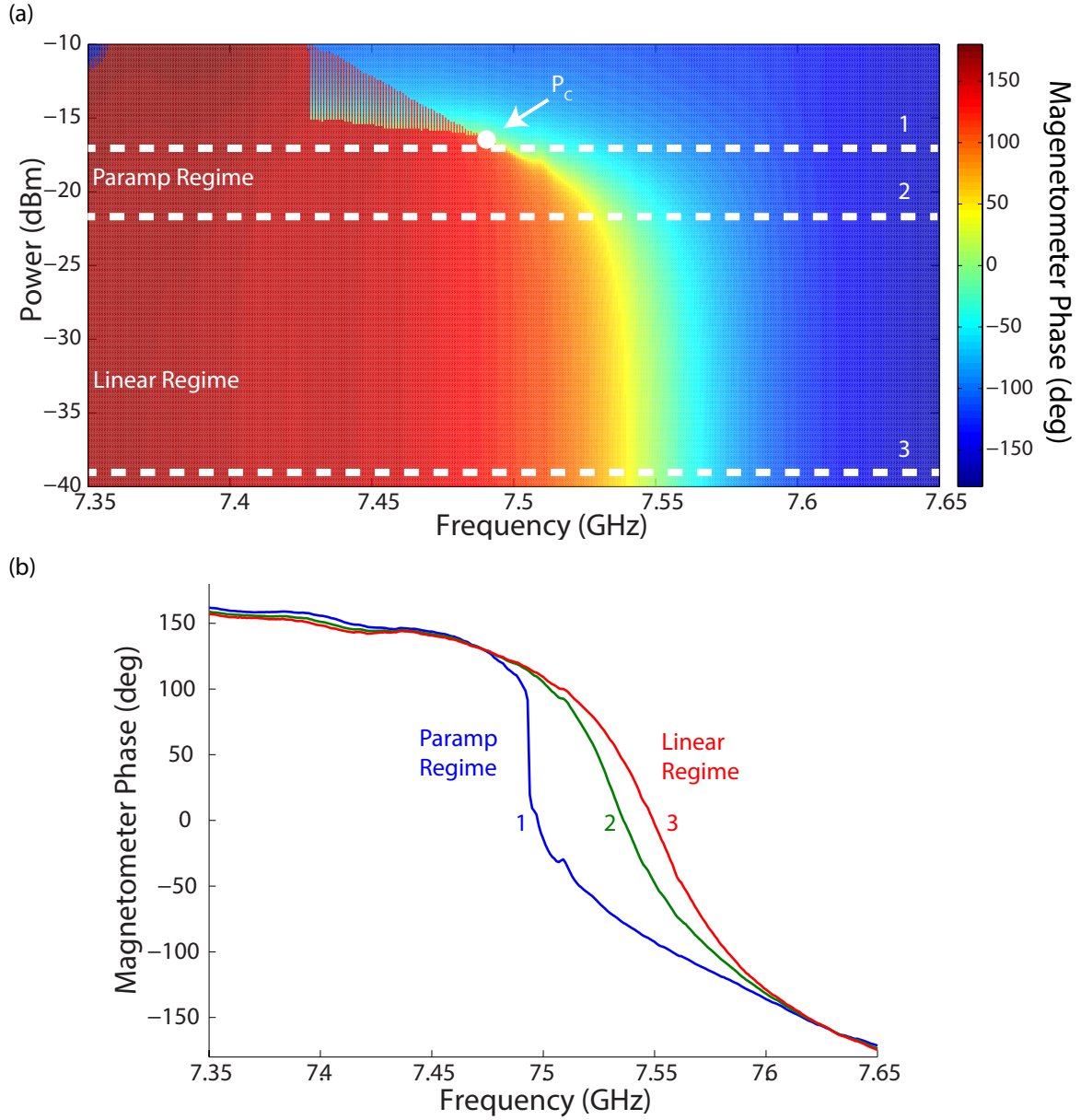


Figure 4.2: In (a) we show an experimental plot of the resonator phase response as a function of drive power and frequency. The numbered and dashed-white lines correspond to the numbered phase response curves shown in (b). Note that the given pump power is at the VNA output. Subtract ~ 50 dB of attenuation in the fridge plus cable attenuation for the power at the resonator. Also, note that the wiggle at ~ 7.51 GHz is likely due to impedance variations in the microwave line.

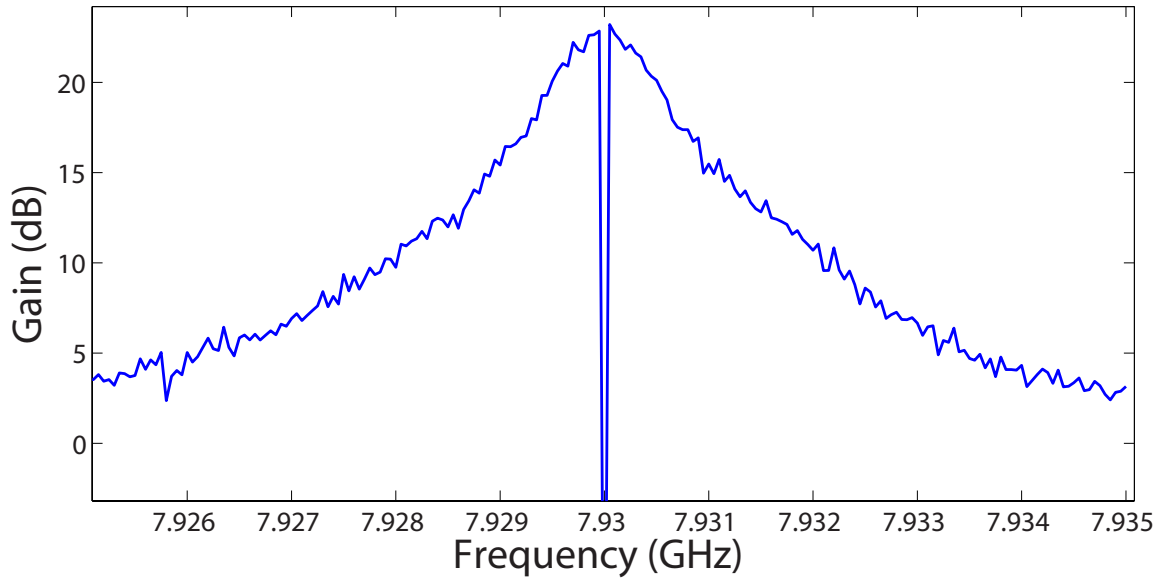


Figure 4.3: Here we show a typical gain profile for a nanobridge SQUID magnetometer. This bias point had roughly ~ 24 dB of gain. The discontinuity at center is due to leakage from the pump tone into the VNA.

pump tone power³ until we begin to see a gain profile emerge. If the gain is too much, one can go to a higher frequency and lower pump power, and vice versa for too little gain. In all cases we want to bias the paramp pump so we get close to the maximum gain. For a nanobridge SQUID the lowest flux noise is at a pump power just below the maximum gain point (see Section 4.2).

If desired, one can turn the pump off, take a memory trace, and then divide new measurements by the memory trace in order to show the normalized gain amount, as well as minimize gain profile distortions due to impedance variations. A sample (normalized) gain profile is shown in Figure 4.3.

It should be emphasized that this is a non-dissipative device. As a consequence, under ideal conditions in the absence of gain there is no variation with frequency of the reflected signal magnitude and instead we measure the reflected phase. In practice, however, there is usually some loss in the device that creates a small peak or dip on resonance in the magnitude measurement. These losses are more pronounced at larger in-plane fields. In addition, reflections due to impedance variations can appear as losses and “wiggles” in the frequency response. The measurement is especially vulnerable to impedance variations in the coaxial line between the magnetometer and first circulator, so this cable should be kept in good repair and as short in length as possible.

³We often split the paramp pump tone so that it can be used at the LO on a mixer. In this case we put a variable attenuator after the splitter so we can control the pump power input to the magnetometer without changing the LO input power.

4.1.4 Fast Flux Line Calibration

In order to characterize the magnetometer it is very important to be able to produce a flux signal of known magnitude at the SQUID. We obtain this by calibrating the strength of the magnetic field produced by our on-chip fast flux line with a series of measurements. First we bias the SQUID to some finite flux (which flux doesn't matter, as long as it is kept consistent throughout the measurement). Then we put in a fixed amplitude voltage excitation on the fast flux line and measure the amplitude of the phase response of the magnetometer. This measurement gives us the conversion between fast flux line input (Volts) and magnetometer phase response (degrees). We change the coil bias by a small amount, and again measure the magnetometer phase response. This gives a conversion between coil bias change (in mA) and magnetometer response (degrees). Lastly, from a flux tuning curve (as shown in Figure 4.1) we find how much current change in the coil will yield a flux quantum. Given these quantities we can find out what percentage of a Φ_0 a fast flux line input voltage will produce.

In equation form:

$$\text{Conversion} \left[\frac{\text{Volts}}{\Phi_0} \right] = \frac{\text{FF input}}{\text{Phase Response}} \left[\frac{\text{Volts}}{\text{Degrees}} \right] \times \frac{\text{Phase Response}}{\delta \text{ Coil}} \left[\frac{\text{Degrees}}{\text{mA}} \right] \times \frac{\Delta \text{ Coil}}{\text{Flux Quantum}} \left[\frac{\text{mA}}{\Phi_0} \right] \quad (4.1)$$

A larger conversion factor generally means that the fast flux line is farther away from the SQUID loop. Note that it can be possible to do this conversion without using the DC bias coil, if the fast flux line is able to take enough current to tune the SQUID through a flux quantum. This generally only occurs with FF lines that are close to the SQUID and which are specially engineered for high critical currents, typically with the use of very thick superconducting traces.

4.2 Characterization of Flux Noise, Transduction and Amplification

4.2.1 Experimental Setup

In this section we present data to characterize the flux noise, transduction and amplification of the 3D nanobridge SQUID magnetometer. In Figure 4.4 we show a simplified schematic of the experimental setup (omitting attenuators and filters). This setup is very similar to the general setup shown in Figure 3.6. These measurements were taken in a standard Oxford Triton dilution refrigerator that reached a base temperature of 25 mK. In this setup a lumped-element Josephson parametric amplifier (LJPA) was optionally switched into the amplification chain using a microwave transfer switch. Room temperature amplifiers are not shown in the figure. As we described in Section 2.1.7, a pump tone, ω_d is sent into the

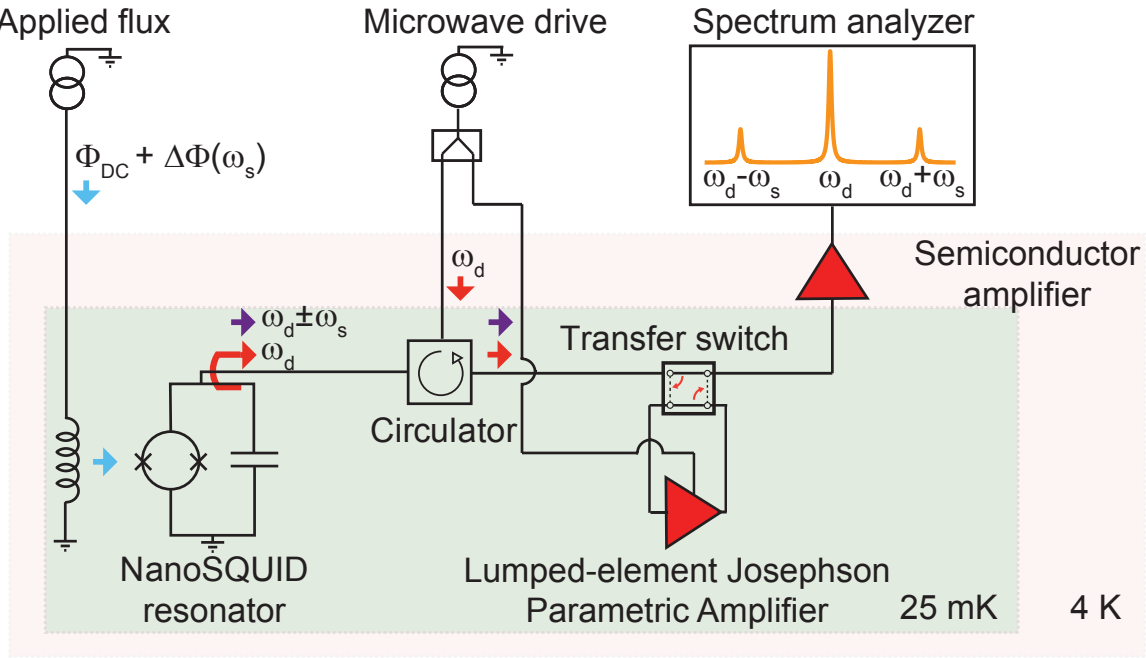


Figure 4.4: Here we show the setup for the experiment discussed in Section 4.2. A strong microwave tone from a generator is split and used to drive both the magnetometer and an optional LJPA. A phase shifter (not shown) is located before the LJPA so that the phase of the amplified quadrature can be controlled relative to the magnetometer output. The flux noise is measured by recording the signal to noise ratio at the sidebands located at $\omega_d \pm \omega_s$ using a spectrum analyzer. All of these measurements were done in a Oxford Triton dilution refrigerator.

microwave input line, goes through a circulator and reflects off the nanoSQUID resonator. The input flux tone $\Delta\Phi(\omega_s)$ is upconverted and transduced to sidebands on this drive tone at $\omega_d \pm \omega_s$, which are transmitted out through the circulators and the amplification chain. The reflected drive tone and sidebands are then measured with the spectrum analyzer. We characterize the magnetometer while it operates in both the linear and param regime, with and without the following LJPA switched into the amplification chain.

4.2.2 Flux Noise Measurement

In order to measure flux noise with a spectrum analyzer we put in a known flux signal, $\Delta\Phi(\omega_s)$, and measure the voltage signal-to-noise ratio, V_{SNR} , at one of the sidebands. We can rearrange Equation 2.29 yielding an equation for the flux noise at ω_s :

$$S_\phi^{1/2}(\omega_s) = \frac{\Delta\Phi}{V_{SNR}\sqrt{\gamma B}}. \quad (4.2)$$

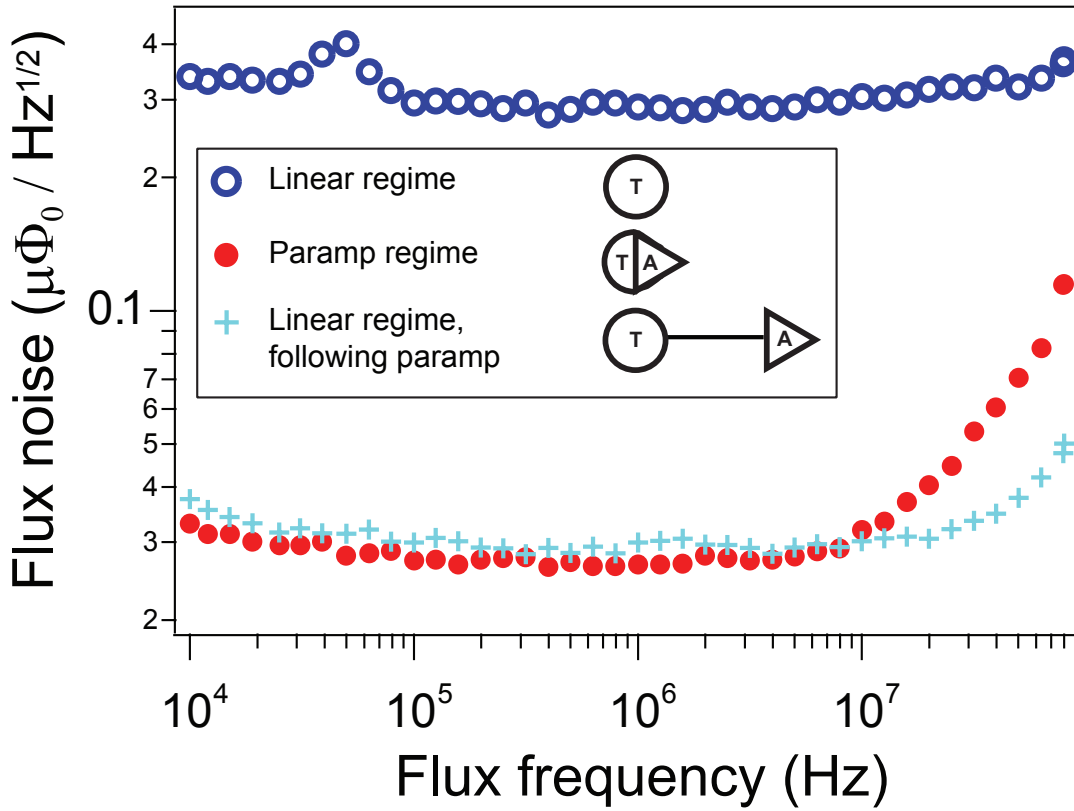


Figure 4.5: In this figure we present the flux noise of the nanobridge SQUID magnetometer as operated in three regimes. As shown in the legend, the open blue circles represent linear regime operation, and the closed red circles represent paramp regime operation. The light blue crosses depict two-stage operation with the magnetometer operating in the linear regime with a following LJPA.

For spectrum analyzer measurements of the flux noise we use the resolution bandwidth for the quantity B . If the magnetometer is being operated in the linear regime, the noise is uncorrelated between the two sidebands, but the sidebands contain the same information, meaning that our voltage signal to noise ratio is effectively $\sqrt{2}$ times larger than the measured quantity V_{SNR} . This reduces the flux noise by a factor of $\sqrt{2}$, which is accounted for by setting the parameter $\gamma = 2$ in Equation 4.2. In the paramp regime, however, the noise at the sidebands is correlated, so the effective SNR is the same as the measured SNR and we set $\gamma = 1$. With this technique, in order to get the entire flux noise spectrum one must step the flux signal frequency, ω_s , and remeasure V_{SNR} at each point.

The resulting flux noise measurements for various modes of operation are shown in Figure 4.5. The top line (open blue circles) shows flux noise for the magnetometer in the linear regime. The trace with red closed circles shows flux noise for the magnetometer in the

paramp regime. The light-blue crosses show two-stage operation with the magnetometer operated in the linear regime with a following LJPA switched into the circuit. One can see that for the single-stage measurement the bandwidth is reduced in the paramp regime, from ~ 100 MHz to ~ 20 MHz, as expected for an amplifier with a constant gain bandwidth product [10]. About ~ 60 MHz of this bandwidth is recovered with two stage-operation, while maintaining the same low flux noise of $30 \text{ n}\Phi_0/\text{Hz}^{1/2}$ [11].

We note that the factor of ~ 5 improvement in flux noise as compared to the tunnel junction prototype [10] is attributed to the ~ 5 increase in critical current of the nanobridge SQUID. The critical drive amplitude depends linearly on the junction critical current [51], so maximum drive amplitude, and therefore maximal transduction, increases proportionally.

Also, we note that the reason for the dramatic change in flux noise between the linear regime and the paramp regime can be explained by system noise temperature. We can get some idea of what to expect from the results of the tunnel junction magnetometer flux noise calculations [10, 50] which were briefly summarized in Section 2.1.9. In the linear regime the noise is dominated by the that of the HEMT following amplifier. The following amplification chain was measured to have a noise temperature of about 8 K during this measurement. In the paramp regime when the power gain is greater than ~ 20 dB, the amplified quantum noise overwhelms the noise of the following amplifiers which limits the system noise to near the quantum limit [10, 11, 44, 50].

4.2.3 Transduction and Gain Measurements

As discussed in Section 2.1.7, the magnetometer can be thought of as a two stage device that both transduces and amplifies incoming signals. However, separating these two effects, which occur in a single device with only one port, is not necessarily straightforward. An added complication, as discussed in Section 2.1.7, is the presence of relative angle θ_t between the amplified quadrature of the paramp and the transduced signal.

As shown in Figure 4.6a and b, we can model the upconverted signal as a combination of deamplified and amplified quadrature signals.⁴ As shown in Figure 4.6a, at small gain and drive amplitudes θ_t is small, and we measure a signal which is a combination of amplified and deamplified components. As the drive increases, the size of the transduced and upconverted signal (ghosted green arrow) will increase since $\partial V/\partial \Phi$ depends on drive power. Unfortunately, θ_t and G also increase with drive, so an increasingly large component of the upconverted signal will be deamplified (ghosted blue arrow) and will not figure in our measured signal (green arrow). In the paramp regime (shown in Figure 4.6b) we measure the direct projection of this upconverted signal onto the amplified quadrature, and θ_t approaches 60° , which halves the strength of the transduced signal. Since sensitivity depends on a trade-off between low noise (high gain) and high transduction we will expect that the lowest flux noise will lie somewhere in between the maximum gain and maximum transduction points.

⁴The amplified and deamplified quadrature plane is a feature of phase-sensitive parametric amplification [8, 9, 51]. As one might expect, a signal which lies along the deamplified quadrature will be reduced in power by a factor of $1/G$, while a signal along the amplified quadrature will increased by a factor of G .

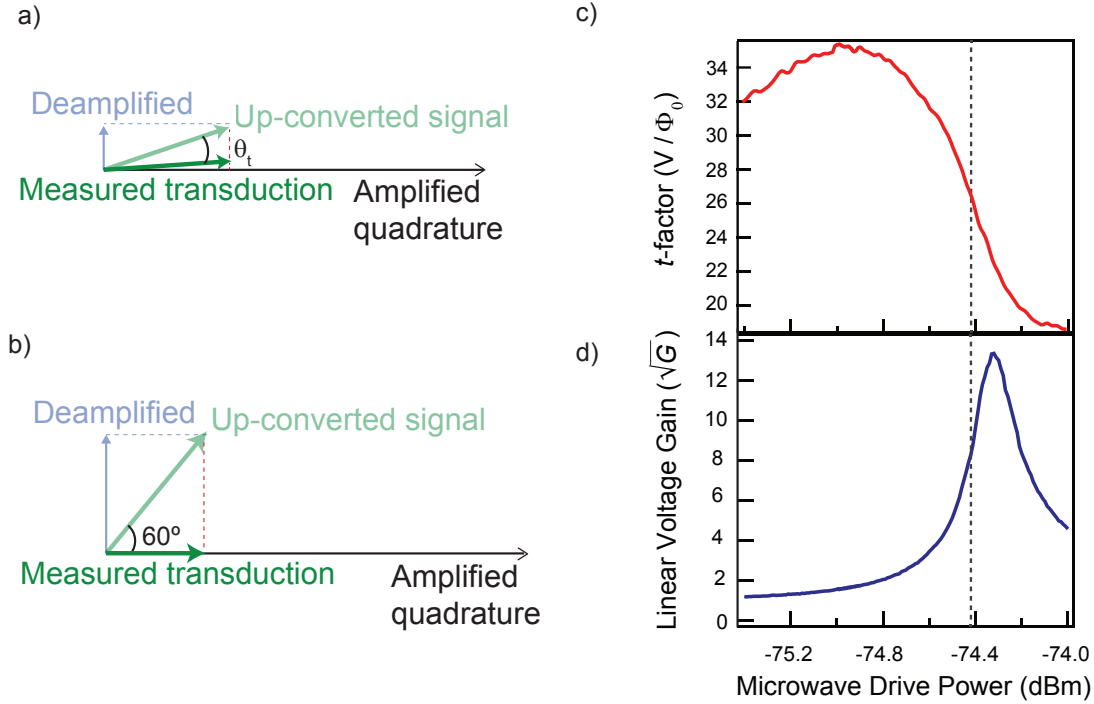


Figure 4.6: In (a) and (b) we represent the transduced and upconverted signal (ghosted-green arrow), which will be an effective input to the amplification stage of the magnetometer, as the sum of amplified and deamplified components. At low gain, as shown in (a), θ_t will be small and the measured signal will be a combination of amplified and deamplified components. As shown in (b) at high gain $\theta_t \rightarrow 60^\circ$ and the measured signal is the projection of the input signal onto the amplified quadrature. In (c) we plot the experimentally measured t-factor and in (d) we plot the measured voltage gain both as a function of drive power. The dashed gray line denotes the drive power with the lowest measured flux noise.

In the following measurement we quantify the effect of θ_t on the magnetometer operation. First we bias the magnetometer in the paramp regime at a flux and frequency point which allows a maximal gain of 20 dB (voltage gain of 10) at the optimal drive amplitude. Then we begin at a small drive power where the magnetometer is in the linear regime, and then slowly step the drive amplitude up in power. At each point we measure the paramp gain by measuring the gain on a weak microwave tone input near the drive frequency. The measured voltage gain, $\sqrt{G}$, is plotted in Figure 4.6d.

We additionally measure the transduction at each drive power point by inputting a small known-amplitude flux signal on the FF line, $\Delta\Phi(\omega_s)$, where $\omega_s = 1$ MHz in this measurement. We measure the amplitude of the upconverted sidebands and divide by the measured paramp voltage gain $\sqrt{G}$ and the flux signal strength $\Delta\Phi$ (units of Φ_0). The resulting measure of the transduction, which we call the ‘t-factor’, is given by $t = TG_{sys}$. In this expression, T

is given by Equation 2.21 and depends on θ_t and G . G_{sys} is the total gain of the following amplification chain, which does not include any paramp gain, and is approximately 10^5 in this system.

The t-factor is plotted in Figure 4.6a. At low pump powers the transduction initially increases with drive power, as we expect in the linear regime. As drive power increases the transduction begins to drop quickly, eventually reaching a factor of two reduction, as expected from Equation 2.21 with $\theta_t \rightarrow 60^\circ$ as the paramp reaches maximal gain. The drive power with minimal measured flux noise is indicated by the dashed gray line in Figure 4.6. As expected, this lies between the points of maximal transduction and maximal gain.

In one last experiment, the magnetometer was driven at the maximal transduction point as shown in Figure 4.6a, and then followed with the switched in LJPA which was biased such that the two devices gave a combined gain of 20 dB. This allowed independent control over the transduction and amplification, as the phase difference between the transduced signal and the independent LJPA drive tone could now be minimized. Operation in this two-stage fashion allowed a minimal flux noise of $23 \text{ n}\Phi_0\text{Hz}^{-1/2}$ with a bandwidth of 20 MHz, which was limited by the combined gain-bandwidth product of the two devices. This measurement did not quite recover the factor of 2 decrease in t-factor due to θ_t . It is suspected that the estimated loss of 2-3 dB in microwave cables and circulators between the magnetometer and following LJPA accounts for a decrease in SNR that would explain this discrepancy [11, 44].

4.2.4 Benchmarking

In order to put these results in context, we show in Figure 4.7 a plot of widely reported results of flux noise and bandwidth for a variety of different SQUIDs. The green symbols depict measurements where the SQUID was read out in a dissipative fashion, either with switched readout or a shunt resistor. The blue symbols denote non-dissipative readout. Note that the axis for flux noise is oriented such that low flux noise is at the right hand side of the plot. The largest bandwidth and low flux noise combinations can be found in the top-right corner of the plot, where the results reported in this section and References 11 and 44 are placed.

To calculate magnetic *field* noise from the measured *flux* noise we simply divide by the SQUID loop area. One might think this would allow us to increase the field sensitivity to arbitrarily large values by increasing the SQUID loop size or adding flux coupling loops. This has indeed been done to a certain extent in many experiments [39, 86, 87]. Additional SQUID loop area added to our device, however, would increase the linear inductance and thus suppress the detector nonlinearity, making operation in the low noise parametric amplification regime increasingly difficult. For this reason, we find that flux noise is a more appropriate sensitivity metric for our magnetometer.

Converting the quoted flux noise to field noise can still be useful, however, for the sake of comparison to other technologies. Assuming a SQUID loop size of $(1.5 \text{ }\mu\text{m})^2$ and a flux noise of $\sim 30 \text{ n}\Phi_0/\text{Hz}^{1/2}$, we obtain a field sensitivity of $\sim 20 \text{ pT}/\text{Hz}^{1/2}$. Other SQUID magnetometers which were optimized for field rather than flux noise, with $\sim 1 \text{ cm}^2$ pickup

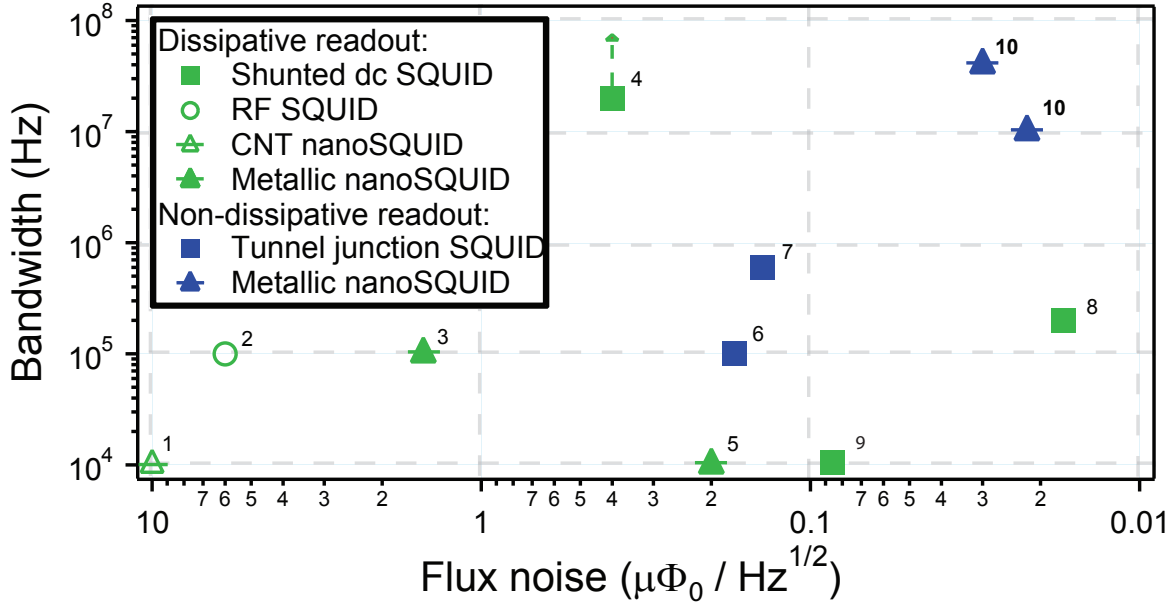


Figure 4.7: To put our results in context we show widely circulated results for flux noise and bandwidth for a variety of SQUIDs. Note that low flux noise values are towards the right of the plot. Green symbols denote SQUIDs read out in a dissipative fashion, either with a resistive shunt or switching readout. Blue symbols denote non-dissipative readout of tunnel junction SQUIDs and nanoSQUIDs. The numbers at each point refer to the following references: (1) Cleuziou et al. [78]; (2) Rogalla and Heiden [79]; (3) Troeman et al. [80]; (4) Drung et al. [81]; (5) Hao et al. [82]; (6) Mates et al. [83]; (7) Hatridge et al. [10]; (8) Van Harlingen et al. [84]; (9) Awschalom et al. [85]; (10) Levenson-Falk et al. [11].

coils areas, achieved sensitivities of 0.9-1.4 fT/Hz^{1/2} with ~250 kHz - 6 MHz of bandwidth [86, 87]. To my knowledge the only technology rivaling SQUIDs for field sensitivity is atomic magnetometry with sensitivities of 0.54 fT/Hz^{1/2}, although this has only on order of ~100 Hz of bandwidth. More recently there has been much interest in NV center magnetometers, which can operate at room temperature. A recent demonstration achieved 2.5 nT/Hz^{1/2} with 250 kHz of bandwidth and a projected improvement to 70 pT/Hz^{1/2} [88]. These comparisons, in addition to considerations of probe volume, dynamic range and other metrics, illustrate that no one magnetometer emerges as clearly superior, but rather each technology has its own range of applications.

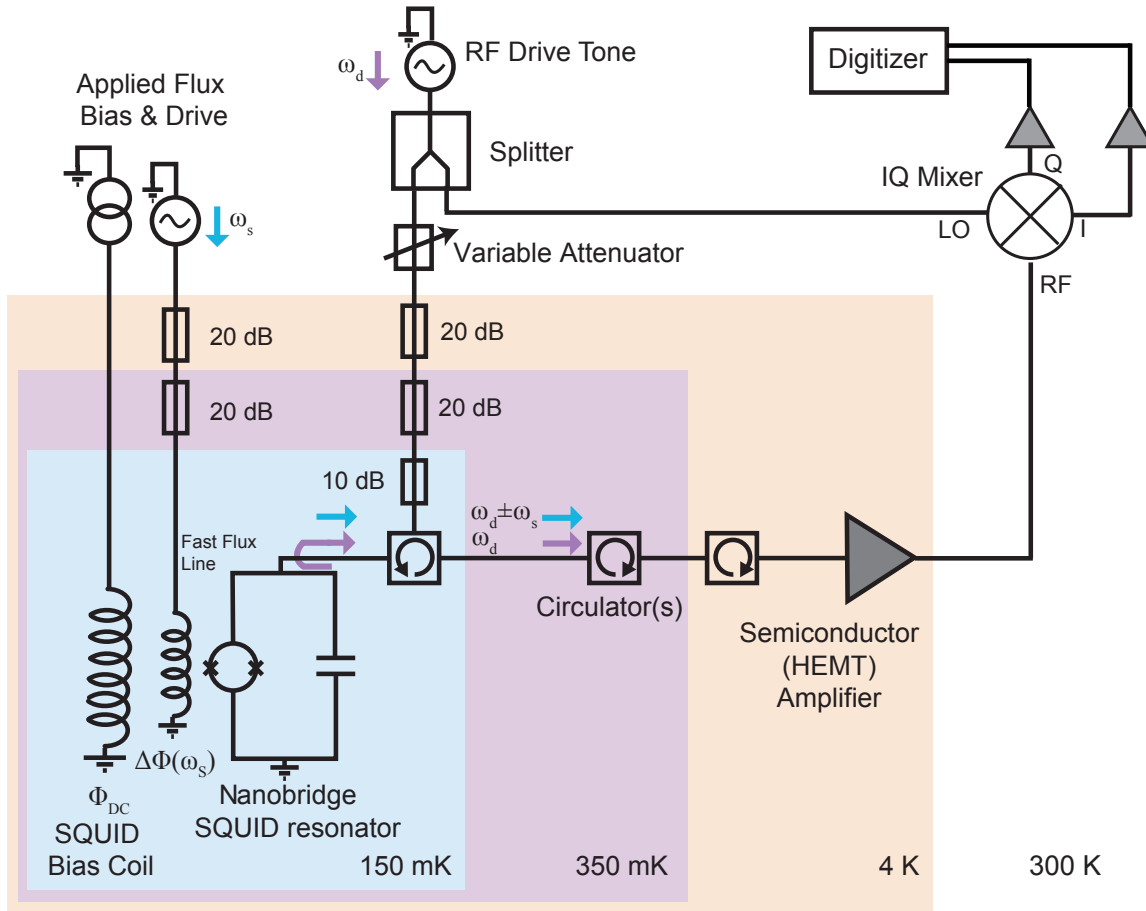


Figure 4.8: Schematic of the magnetometer and measurement circuit. A RF drive tone, ω_d is sent into the device. The tone reflects off the resonator with sidebands at $\omega_d \pm \omega_s$ where ω_s is the flux signal frequency.

4.3 Characterization of Magnetic Field Tolerance

4.3.1 Experimental Setup

As discussed in Chapter 3, the magnetometer consists of an aluminum nanobridge SQUID [11, 24, 46] shunted by a parallel plate capacitor formed by two aluminum pads and a niobium ground plane. Given the 3D geometry of these nanobridge SQUIDs, and the low critical field of bulk aluminum, the question arose as to how well these devices would operate in magnetic fields typically required for performing magnetometry experiments. In this section we will demonstrate the successful operation of the magnetometer in moderate applied in-plane magnetic fields, that are large enough for a host of applications. We study magnetometers made with SiN dielectric capacitors as well as a second generation device with much thinner

and slightly smaller footprint AlO_x capacitors. Planar 2D nanoSQUID (with 15 nm thick banks and bridges) structures were produced for comparison with the first generation design but with only a single metallization step at normal incidence. All devices had an on-chip fast flux line to inject calibrated flux signals from dc up to GHz frequencies.

As discussed in previous sections, the capacitively shunted nanobridge SQUID forms a nonlinear resonant circuit. The SQUID acts as a flux dependent nonlinear inductor, with inductance $L_S(\Phi)$. A varying flux signal coupled into the SQUID loop causes a change in the inductance and thus a change in the resonant frequency of the circuit, $\omega_0 = 1/\sqrt{L_S(\Phi)C}$, where C is the shunt capacitance. When the device is pumped with a microwave tone near its resonance, a flux signal modulates the phase of the reflected microwave pump signal. Thus, if the circuit is pumped at ω_d , the reflected signal will exhibit sidebands at $\omega_d \pm \omega_s$ in the frequency domain where ω_s is the flux signal frequency. [10, 11] A schematic of the device and measurement circuit is shown in Figure 4.8.

This set of measurements was performed in a cryogen-free sorption-pumped dilution refrigerator at 150 mK within a custom 3-axis magnet (see Appendix A). The 4 K stage of the refrigerator is cooled with a mechanical pulse tube (PT) cooler and holds the 3-axis magnet, which consists of a large solenoid surrounded by two orthogonal Helmholtz pairs. A picture of the 3-Axis magnet and sample stages, as well as an illustration of the SQUID as arranged in the magnetic field is shown in Fig 3.7. We use the Helmholtz coils to produce a static flux bias and also to shim the main solenoid. This shimming ensures the static flux bias is kept constant as the in-plane field is ramped up. We bias near $\Phi_{DC} = \Phi_0/4$, as depicted by the white-dashed rectangle in Figure 4.1.

Note that as the parallel magnetic field increases the resonance frequency of the magnetometer decreases, shifting the entire flux tuning curve down in frequency as the superconductivity is suppressed. In order to stay at the same flux bias point it is not sufficient (or often even possible) to keep the resonance tuned at the same frequency. Instead, as the parallel field increases one must track the frequency of the highest point on the flux tuning curve ($\Phi = 0$) and then tune the SQUID down from that point by a set amount using the Helmholtz coils to provide the perpendicular field.

4.3.2 Flux Noise Measurement with Magnetic Field

In Figure 4.9 we show sample flux noise spectra. These traces were acquired using homodyne detection of the reflected output (as shown in Figure 4.8). The magnetometer pump tone is split and used to pump the device and also downconvert the output signal, yielding a spectrum from dc to the Nyquist frequency associated with our digitizer sampling rate. A known low frequency flux signal is also sent into the device via the fast flux line and used to calibrate the voltage spectrum into flux noise units. This calibration signal is the peak at 500 kHz visible in all of the spectra. To perform this calibration we use Equation 2.29 as

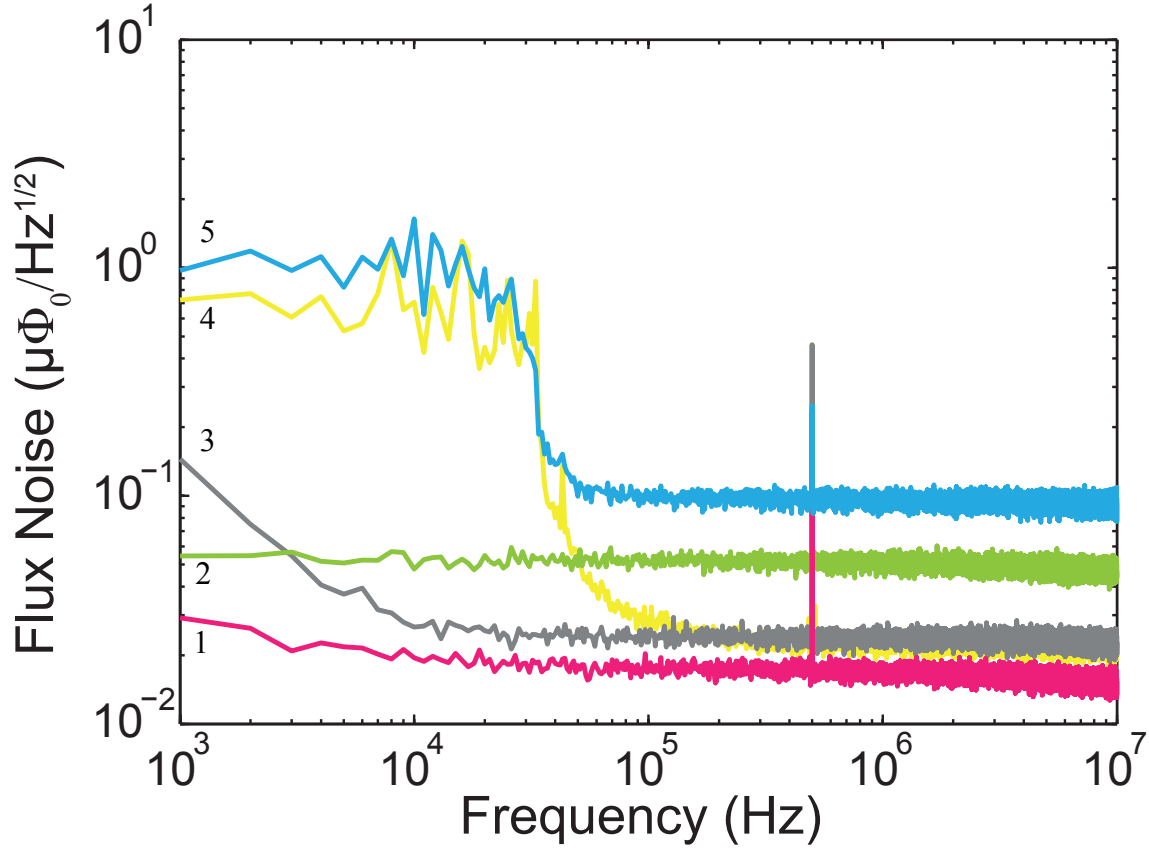


Figure 4.9: Flux noise spectra taken at various in-plane magnetic field values. Each spectrum is numbered at the left from 1 to 5. Spectra 1 and 2 were taken at 0 mT parallel field with the pulse tube (PT) cooler on. The magnetometer was in the paramp regime for spectrum 1 and the linear regime for 2. Spectra 3 and 4 were taken in the paramp regime with 33 mT of parallel field. Spectrum 3 was taken with the PT off to illustrate that the noise below 100 kHz in a large magnetic field is dominated by the PT. Spectrum 5 was taken in the linear regime at 41 mT with the PT on. Overall, the maximal values of bandwidth varied from 40 MHz in the paramp regime to 70 MHz in the linear regime.

a guide⁵ and divide the voltage noise spectrum by the measured voltage of the flux signal, yielding a dimensionless signal-to-noise spectrum. We then multiply by the previously known amplitude of the flux noise signal in units of Φ_0 ⁶ and divide by the square root of the resolution bandwidth of the Fourier transform⁷.

⁵With this measurement we set $\gamma = 1$ for all magnetometer modes of operation, as the homodyne measurement mixes the sidebands together.

⁶We get this from the conversion factor that was discussed in Section 4.1.4.

⁷This is just t_{meas}^{-1} , the inverse of the total measurement trace time.

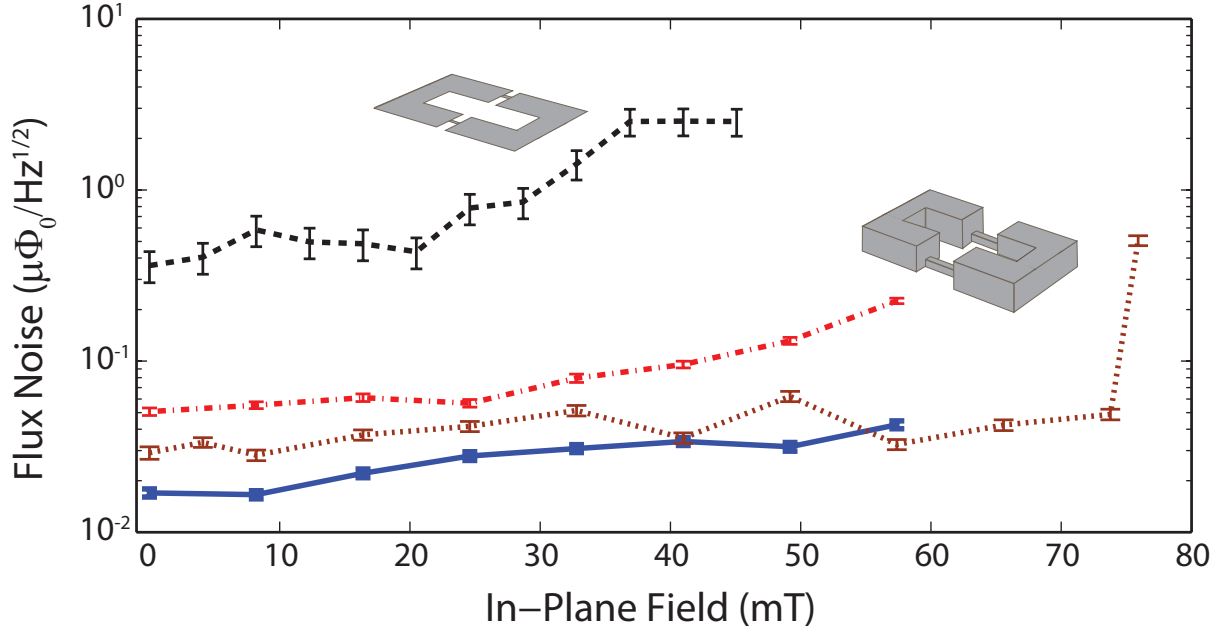


Figure 4.10: Flux noise at 1 MHz of both 3D and 2D magnetometers as a function of in-plane field. The blue line (solid) at bottom is the flux noise of the 3D nanobridge device biased in the paramp regime. Flux noise ranged from $17.0 \pm 0.9 \text{ n}\Phi_0/\text{Hz}^{1/2}$ at zero field to $42 \pm 2 \text{ n}\Phi_0/\text{Hz}^{1/2}$ at 61 mT. The red line (dot-dashed) in the middle is data for the same 3D device in the linear regime. The flux noise ranged from $51 \pm 3 \text{ n}\Phi_0/\text{Hz}^{1/2}$ at zero field to $225 \pm 8 \text{ n}\Phi_0/\text{Hz}^{1/2}$ at 61 mT. The black line (dashed) at top is data taken for the 2D magnetometer in the linear regime. The flux noise ranges from $0.36 \pm 0.07 \mu\Phi_0/\text{Hz}^{1/2}$ to $2.5 \pm 0.4 \mu\Phi_0/\text{Hz}^{1/2}$ at 45 mT. The brown (dotted) line shows data from a 2nd generation magnetometer with a thinner and smaller footprint AlO_x capacitor. It has a field tolerance of ~ 75 mT.

Figure 4.9 shows spectra taken with and without parallel fields in both the linear and paramp regimes. The spectra at 33 mT were taken with and without the pulse tube (PT) cooler to illustrate that the noise below 100 kHz in large parallel fields is dominated by the acoustic (and possibly electrical) noise of the pulse tube. Data are not shown below 1 kHz as the flux noise is limited by system (primarily PT) noise rather than intrinsic $1/f$ noise in this range.

The flux noise at 1 MHz for both 3D and 2D magnetometer devices is plotted versus magnetic field in Figure 4.10. The bottom two traces show flux noise for the 3D nanobridge device run in the paramp (bottom, blue solid) and linear regime (middle, red dot-dashed). The brown dotted trace shows data from a second generation magnetometer with a thinner and smaller footprint AlO_x capacitor. This device had a $\sim 23\%$ increase in field tolerance over the first generation devices. Although the flux noise was slightly higher, this device had

not been fully optimized for low flux noise performance. The top trace (black dashed) is flux noise of the 2D device in the linear regime. The 2D device has larger error bars due to greater uncertainty in the calibration stemming from an irregular flux versus phase tuning curve. This 2D device could not be operated in the paramp regime [23, 24]. The minimum flux noise measured on the 3D nanobridge device was $17.0 \pm 0.9 \text{ n}\Phi_0/\text{Hz}^{1/2}$. We define the device bandwidth as the frequency beyond the white noise floor (flat region of the spectra, cf. Fig. 4.9) where sensitivity degrades by a factor of $\sqrt{2}$. Maximal values of the bandwidth for the quoted flux noise values range from 25-40 MHz in the paramp regime to 70 MHz in the linear regime.

At fields below 60 mT the slowly increasing flux noise with in-plane field is likely due to a combination of factors, including microwave losses, instability of bias points, and a decreased flux-to-voltage signal transduction. The latter is a result of a decreased slope in the flux tuning curve. Suppression of the superconductivity as in-plane field increases leads to decreased resonant frequency at $\Phi_{DC} = 0$. At fields higher than 60 mT, microwave losses in the 3D nanobridge magnetometer become pronounced with a large absorption peak.

There is no discernible difference in field tolerance between the 2D and 3D SQUID devices. Thus we believe that we may be limited by superconductivity suppression in the 200 nm thick niobium ground planes rather than in the aluminum SQUIDs themselves. If suppression of superconductivity begins at the edges of the device ground plane, and if the capacitor pads of the device extend all the way to these edges, we would expect to see a detrimental effect on the capacitor properties as in-plane field increased. These devices would exhibit greater loss and a lower field tolerance than devices with pads smaller relative to the ground plane. Such behavior was observed for several devices. In addition, the supposition that field tolerance is limited by the capacitor and ground plane is further bolstered the 23% larger field tolerance of the second generation devices that have much thinner and slightly smaller footprint capacitors.

Chapter 5

P1 Center Readout with the Magnetometer

As discussed in previous chapters, we have developed a novel nanobridge superconducting quantum interference device (SQUID) magnetometer that should allow us to continuously measure spin systems using a non-dissipative, high bandwidth and low flux noise readout [10, 11, 76]. Given recent experiments and proposals to couple solid state spin systems to superconducting devices [2–6, 30, 36, 63, 70–73], as discussed in Section 2.2.4, it could be useful to develop our magnetometer to independently assess and characterize these solid state spin systems at dilution fridge temperatures for use in hybrid devices. We have chosen P1 centers in diamond, substitutional nitrogen atoms in the carbon lattice [12] as discussed in Section 2.2, as a magnetic system for this magnetometer to study as a proof of principle. P1 centers are particularly relevant to solid state quantum information processing as they are an important source of decoherence of nitrogen-vacancy (NV) centers [13, 14], and have been proposed for use as ancilla qubits [15] or as part of an NV-N magnetometry scheme [16, 17]. The properties of P1 centers in diamond have been under investigation since they were first discovered in 1959 [12, 18], but studies of the relaxation decay of these defects at ultra-low temperatures have only recently begun [19, 20].

In the paper by Reynhardt, et al. [20], the T_1 decay time of P1 centers Type 1b diamond were measured using an ESR spectrometer over temperature range of 300 to 6 K. The authors proposed that the dominant source of relaxation at these low temperatures was spin-orbit phonon-induced tunneling. This process is characterized by phonon-induced tunneling between the four orientational states of the defect, where the 4-fold degeneracy is lifted by a Jahn-Teller-like distortion in the C-N bonds [20, 62]. A recent study by Ranjan, et al., in which the P1s were coupled to a superconducting resonator, found temperature independence between 250 mK and 1 K [19] at 86 mT suggesting spin diffusion as a potential limiting mechanism for T_1 .

In this work we hope to further explore parameter space by studying in detail the T_1 decay of P1 centers in a type 1b high pressure high temperature (HPHT) diamond sample as a function of magnetic field, temperature and excitation power at dilution fridge temper-

atures. The bulk of these measurements were performed using our novel nanobridge SQUID magnetometer, making this the first dispersive SQUID measurement of the magnetic flux for a spin system of this type. ESR-type measurements of these systems typically require a resonant coupling at either a fixed field or frequency – whereas we are able to directly measure the spin signal at any magnetic field up to the tolerance of our detector at 70 mT of in-plane field [76]. For comparison, we also measure the T_1 decay with temperature of the same diamond sample using a $\lambda/4$ superconducting Nb coplanar waveguide (CPW) resonator.

5.1 Experimental Setup

Our magnetometer operates dispersively and acts as an upconverting flux transducer on the P1 spin signal [10, 11]. A schematic of the device is shown in Figure 5.1. The nanobridge SQUID [23, 24, 46] acts as a inductor in the circuit with a variable inductance that changes with flux through the SQUID loop. The SQUID is shunted with a parallel plate capacitor which forms an LC circuit with a resonant frequency that is flux-dependent. As shown in Figure 5.1, the input drive tone, ω_d , is reflected off the magnetometer circuit, travels through a circulator and then to our HEMT amplifier. The flux sensitive changes in resonant frequency appear as phase modulations on ω_d . The device can achieve exquisite sensitivity when operated in the nonlinear parametric amplification regime, but in this work we operate in the linear regime. The measurement is performed by probing the magnetometer phase response at a constant frequency on resonance and at low power (-105 dBm at the magnetometer) using a vector network analyzer. We typically use measurement bandwidths of 1 to 5 Hz and take traces between one and five hours long due to the long P1 relaxation times.

We place an Element Six Type 1b HPHT diamond (~ 100 ppm Nitrogen, [100] cut) with a layer of N vacuum grease on top of a nanobridge magnetometer as shown in Figure 5.2. We align the diamond such that the magnetic field is along [100] and in-plane with the magnetometer, as marked in Figure 5.2. The magnetic field is ramped from zero to values ranging between 15 to 64 mT. The maximum field tolerance for the magnetometer is ~ 70 mT [76]. We can excite the spins using a fast flux line which consists of a shorted coplanar-waveguide (CPW) co-fabricated on the silicon chip with the magnetometer circuit. The three large CPW traces are visible under the diamond in Figure 5.2, and the full SEM image of the device is given in Chapter 3.

Measurements were performed in two separate cryogen-free dilution refrigerators with varying capabilities. The first data sets were taken in a fridge that could operate stably only at 150 mK and 370 mK. The same magnetometer and diamond sample were then measured in a second fridge with temperature stability between 25 mK and 290 mK.

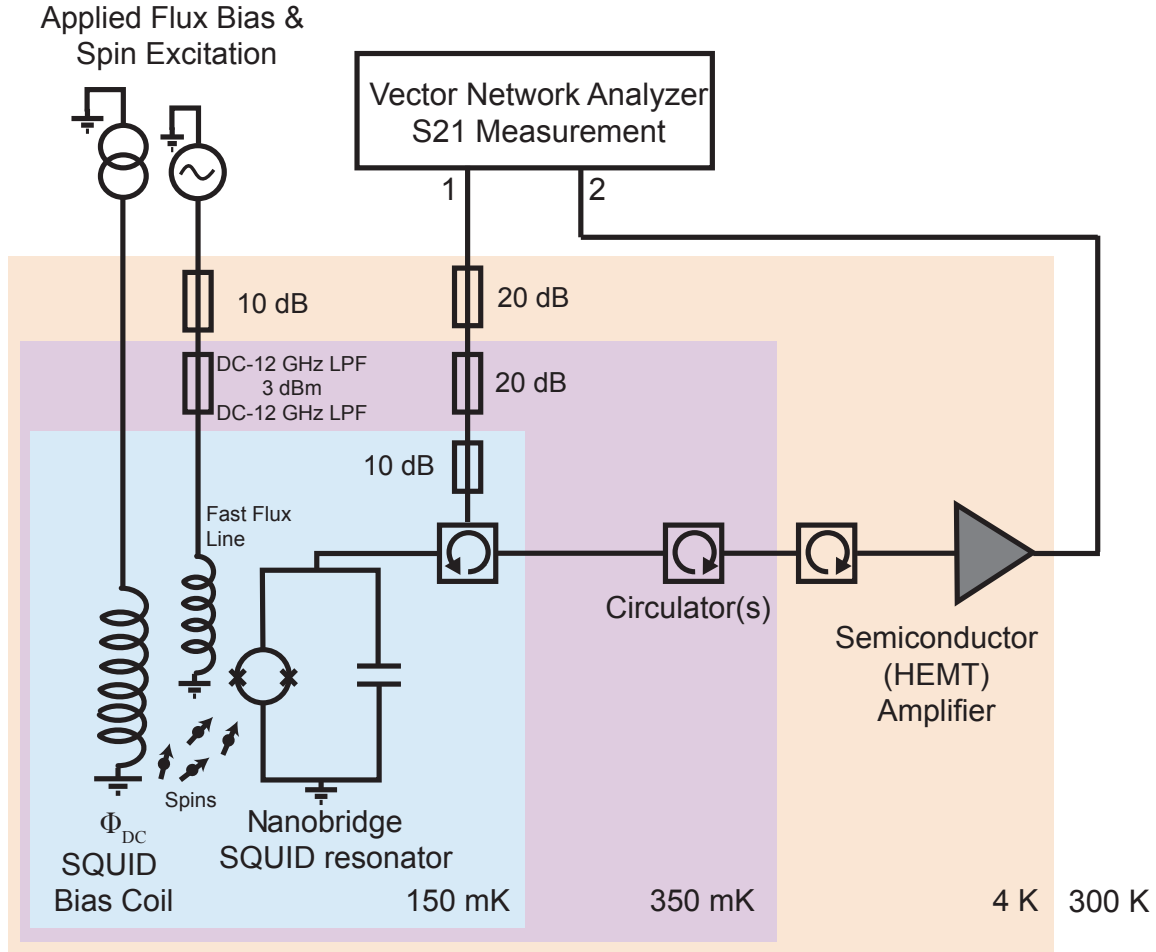


Figure 5.1: In this figure we show a schematic of the magnetometer and measurement circuit for the P1 T_1 decay measurements. These measurements were done in a 3-axis magnet in a sorption-pumped dilution refrigerator (see Appendix A). Decay traces were taken by monitoring the magnetometer phase with a weak probe tone from the vector network analyzer (VNA).

5.2 Spectroscopy

We can perform spectroscopy by stepped sweeping of the frequency of a series of short microwave pulses on the fast flux line. At the frequencies resonant with the spins we see a step in the magnetometer response, with roughly 10 MHz of linewidth. A sample spectroscopy trace, taken at 27 mT, is shown in the inset of Figure 5.3. These traces generally take about 30 seconds to complete. A full spectroscopy plot of field versus resonant excitation frequency is shown in Figure 5.3. The three dashed theory lines were calculated with EASYSPIN [60]

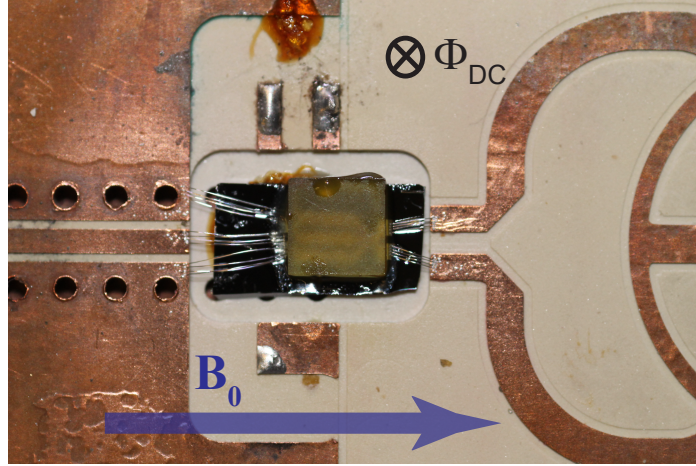


Figure 5.2: A photo of the microwave launch with the magnetometer chip and yellow diamond placed on top. The direction of polarizing field, B_0 , is denoted with an arrow. The direction of DC flux bias, Φ_{DC} , is perpendicular to the photo plane, as shown.

using the hyperfine transition values given in Ref. 61. We scaled the current applied to the magnetic field coil (units of mA) to magnetic field values (units of mT) using a single scaling constant for all data points, 11.05 mA/mT. This is within 10% of the current-to-field conversion ratio measured using a Hall sensor in a separate cooldown.

5.3 T_1 Decay Measurements

In order to perform the decay time constant measurements, we must first allow the spins to equilibrate in the polarizing magnetic field, waiting between 1.5 and 10 hours, depending on the magnetic field and temperature. After this, we prepare the spins with a near-saturating resonant microwave pulse on the fast flux line. A typical spin response curve after an excitation pulse is shown in Figure 5.4 and the inset shows a timing diagram of the experimental sequence. The curve fit to the magnetometer phase, highlighted in red, is given by the equation,

$$\phi(t) = Ae^{-(t/T_1)^n} + C \quad (5.1)$$

where $0 < n < 1$. Note that Ranjan et al. [19] used a biexponential to fit their data, given by:

$$\phi(t) = Ae^{t/T_{1a}} + Be^{t/T_{1b}} + C. \quad (5.2)$$

Ranjan, et al. suggested that this curve shape might be due to two or more spin populations with different densities varying by a factor of ~ 8 [19, 89, 90]. We found that a stretched exponential fit our data better, and had the added benefit of allowing us to easily track the

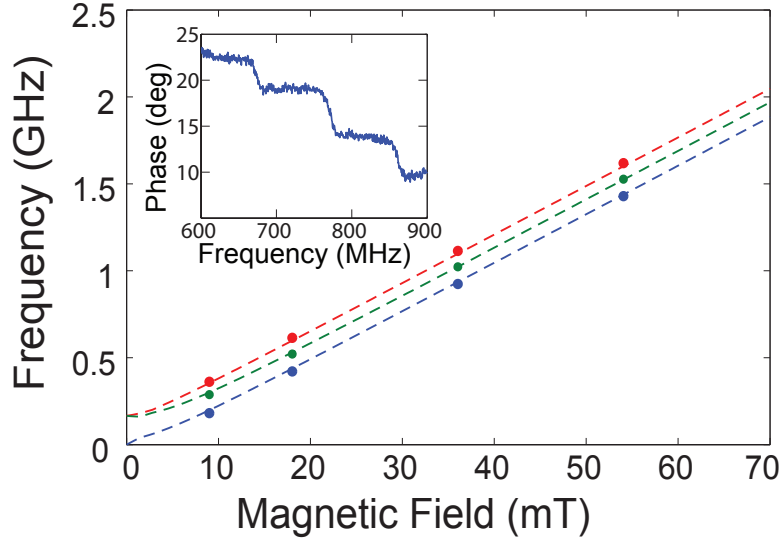


Figure 5.3: Spectroscopy of spin transition frequencies with magnetic field. Dots indicate experimental data. The three dashed lines were calculated with EASYSPIN [60] using the hyperfine transition values given in Ref. 61 for a ^{14}N nucleus with spin 1 and magnetic field along the [100] diamond axis. The magnet current values for the experimental data points were converted to magnetic field units using a global constant which was scaled to fit the overall slope of the calculated spectrum. Inset: A sample spectroscopy trace, taken at 27 mT, which shows the magnetometer phase response as the excitation pulse frequency is swept through the spin resonances.

decay curve shape by plotting the parameter n . Stretched exponentials have been used to fit spin-ensemble relaxation in many NMR-type experiments [91–95].

The resonant pulse time was varied in inverse proportion to the pulse power in order to conserve total excitation energy at each field point. However, the input pulse energy was decreased as the magnetic field increased in order to keep the spin response constant. Due to increased spin polarization with field, a constant pulse input energy would have yielded a spin response that exceeded the dynamic range of the magnetometer. Typical pulse powers at the sample range from -30 to -50 dBm, with pulse times ranging from 100 ms to 1000 s depending on the pulse power and the polarizing field. Note that all quoted pulse powers are calibrated from room temperature measurements of the attenuation on the microwave line that feeds the on-chip fast flux line. Absolute pulse power at the chip while cold was not calibrated, but will be slightly higher due to decreased cable attenuation at cryogenic temperatures.

The spin signal is on order of $10\text{ m}\Phi_0$, corresponding to $\sim 10\text{ }\mu\text{T}$ of field for a $(1.5\text{ }\mu\text{m})^2$ SQUID loop. All data that are shown were taken with pulses resonant with the central

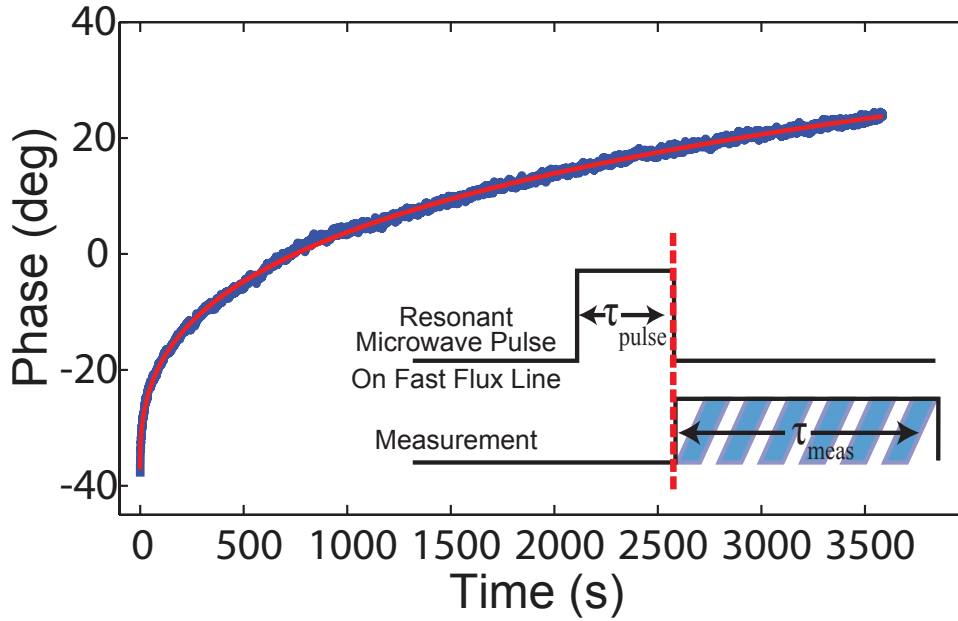


Figure 5.4: A typical time trace of the spin response after a resonant microwave pulse. This trace was taken after a 100 ms and -30 dBm pulse with a 36 mT polarizing magnetic field. This trace was fit with Equation 5.1, and the result is shown in red, with $T_1' = 6.6 \times 10^4$ s and $n = 0.346$. Inset: A timing diagram for the resonant microwave pulse sequence. Typical pulse powers at the sample range from -30 to -50 dBm, with pulse times ranging from 100 ms to 1000 s depending on the pulse power and the polarizing field. Pulse time and power are adjusted in inverse proportion to keep the input RF energy, and corresponding spin response magnitude, constant.

hyperfine line. For simplicity, pulses resonant with the upper and lower hyperfine transitions are not shown, as these did not yield significantly different results.

The first observed trend is that the curve shape is power dependent. In Figure 5.5 we plot three decay traces at three different powers on a log scale, and one can clearly see their differing shapes. We fit these three traces to the stretched exponential function given in Equation 5.1. This function provides a good fit for the trace from $t \gtrsim 10$ s to the end of the trace at 3600 s. Only the first 1000 seconds are plotted in order to clearly show the initial curve shape.

In Figure 5.6 we display just the first 100 seconds of the decay trace on a log-log plot to illustrate the difficulty of fitting the initial decay. From this plot it would appear that a power law model given by,

$$\phi(t) = \left(\frac{t}{T_1'} \right)^n \quad (5.3)$$

could be the best fitting function, but it turns out that this model fits poorly over the

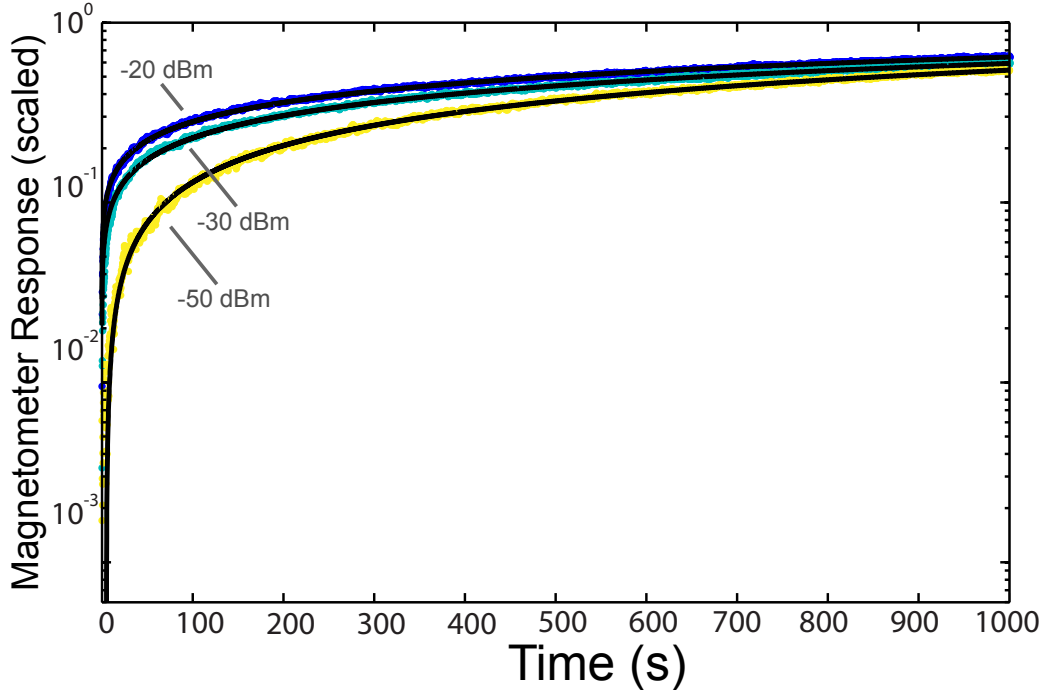


Figure 5.5: Here we show the effect of excitation pulse power on the shape of the magnetometer response. The plot shows the magnetometer signal after excitation pulses at three different powers at a constant magnetic field of 54 mT and temperature of 150 mK. The highest power pulse, about -30 dBm at the sample, produces a much steeper curve than the -50 dBm pulse. Note that the magnetometer signal has been scaled between 0 and 1. Each trace is fit to a stretched exponential function, Equation 5.1, yielding good fits for $t \gtrsim 10$ s.

full length of the trace. In the absence of a theoretical model to fit these decay traces, we decided to use the stretched exponential fit function, Equation 5.1. The stretched exponential provides a much better fit than the power law function over the entire trace length, and it does a better job than the biexponential function of tracking the initial decay.

Figure 5.7 illustrates the correspondence between the fit parameter n and the excitation pulse power, showing quantitatively how the pulse power effects the curve shape. At low powers the decay approaches that of a pure exponential, ie. $n \rightarrow 1$, but becomes strongly stretched ($n \rightarrow 0$) at higher powers. We show more data in Figure 5.8, plotting now both n and T_1' with power at two field and temperature points. Qualitatively, these plots suggest that the power dependence of n may become weaker with increased magnetic field.

Upon further examination of Figure 5.8a and 5.8b it appears that the T_1' is temperature dependent, with shorter T_1' at increased temperature. The separation of the T_1' values, which is clear at 54 mT, becomes less apparent as the magnetic field decreases. In Figure 5.9 we plot $1/T_1'$ in 3D as a function of both field and pulse power, to show that the separation of T_1' values between the two temperatures is established over many data points.

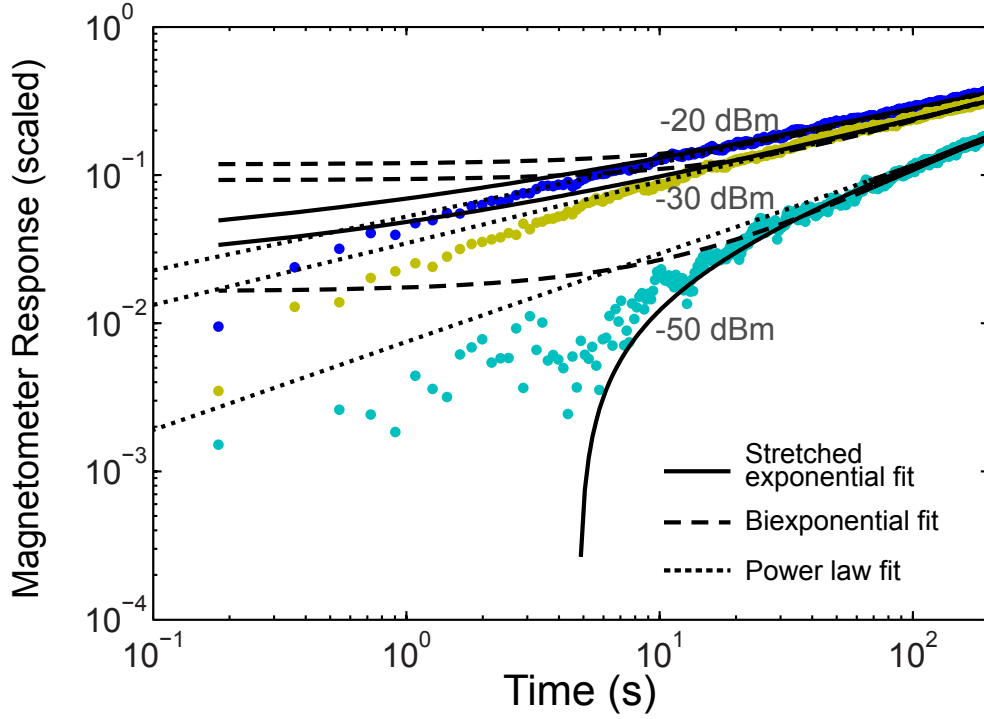


Figure 5.6: Here we show a log-log plot of the first 100 seconds of the spin decay, to illustrate the difficulty of fitting the initial spin response. We show fits to biexponential (cf. Equation 5.2, stretched exponential (cf. Equation 5.1) and power law (cf. Equation 5.3) decays, which all have difficulty tracking the initial response. From the first 100 seconds it would appear that a power law model is most appropriate, but this produces a much poorer fit over the length of the entire trace.

After taking the data plotted in the previous figures we decided that it would be instructive to obtain data over a wider temperature range. Figure 5.10 shows scaled decay traces for three different temperatures, showing the effect of a varying T_1' value for fixed n . In order to make a fair comparison between T_1' times as we vary temperature, we bin the T_1' data points by n value. For values of $n < 0.5$ the T_1 times have a very large spread even between points taken under the same conditions, leading us to believe that our fits are unreliable when the curve shape has such a steep decay. For this reason we take only data between $0.5 < n < 0.7$ for the plot of T_1' versus temperature shown in Figure 5.11. Data with $0.5 < n < 0.6$ are plotted with open blue circles and data with $0.6 < n < 0.7$ are plotted with closed green diamonds. This figure illustrates the order of magnitude change in T_1' for a single diamond sample over the range of 25 mK to 370 mK.

Note that each point in Figure 5.11 is a stretched exponential fit to a single data trace except for the open blue circle with errorbars at 370 mK. This point is an average of four data points, with the errorbars produced from a 95% confidence interval using the Student

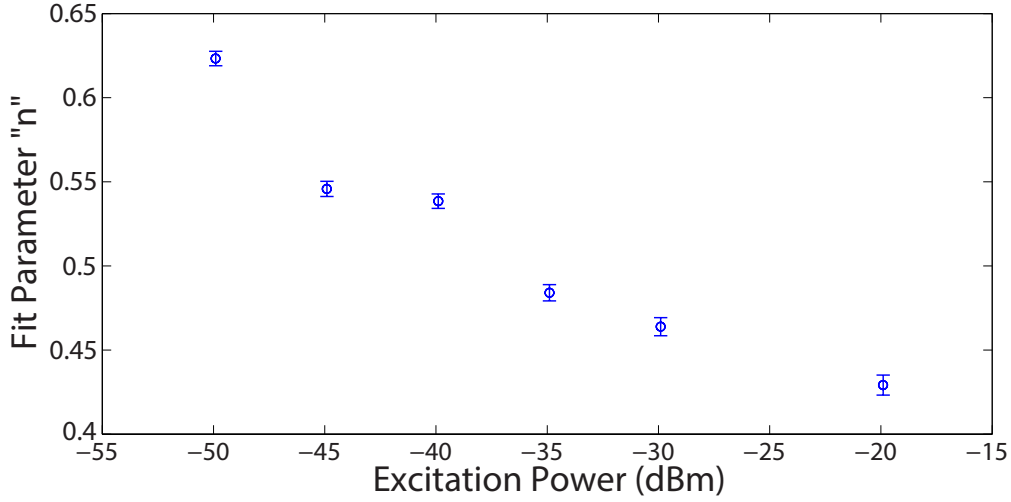


Figure 5.7: In this figure we quantitatively show the relationship between excitation pulse power and the curve shape, by tracking the fit parameter “ n ”. These measurements were taken with constant field and temperature of 54 mT and 150 mK respectively. The errorbars are derived from 95% confidence intervals from the fit. We find from this plot that the curve shape, parameterized by “ n ”, tracks the pulse power for the range of $n = 0.6$ to 0.4 . At low powers the decay approaches that of a pure exponential, ie. $n \rightarrow 1$, but becomes strongly stretched ($n \rightarrow 0$) at higher powers.

T_1 's distribution. Since the traces taken at 370 mK had the lowest SNR due to lower spin polarization, this gives an estimated upper bound for the errorbars on the lower temperature data. The relative scarcity of data points at these lower temperatures is due to the impracticality of taking many traces with these very long T_1' times. Also note that the errors resulting from the fit to each trace yield errorbars smaller than the symbol size. The data points at 150 mK and 370 mK were taken at 54 mT in the first fridge setup, while the other temperature points were taken in the second fridge setup at 64 mT. We also have limited magnetometer measurements for a second diamond sample which was measured with both a magnetometer and superconducting resonator. These results will be discussed in the following section.

5.4 Coplanar Waveguide Measurements of T_1 Decays

For comparison, we also show data taken with a $\lambda/4$ coplanar waveguide (CPW) resonator measured in reflection, with an additional fast flux line (shorted CPW) on chip. The weakly coupled avoided crossing between the resonator and the P1 spins, shown in Figure 5.12, allows us to resonantly probe the spins. First we tune the spins into resonance with the resonator using a static magnetic field in-plane with the resonator. The spin frequency is indicated by the white dashed line in Figure 5.12. Then we resonantly pulse the spins on either the

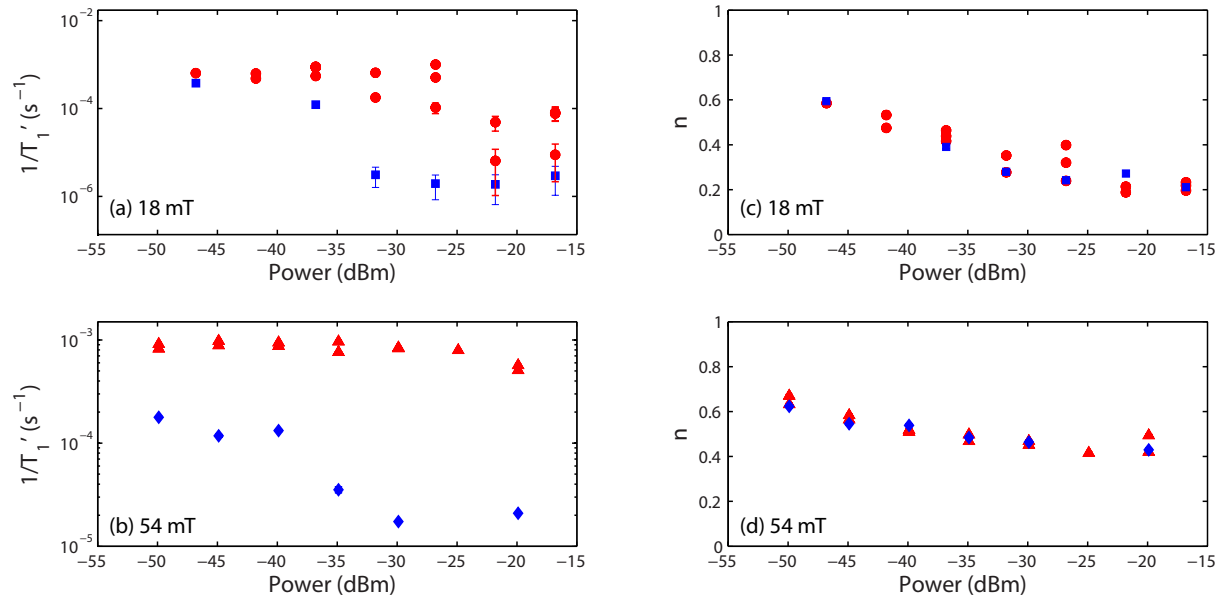


Figure 5.8: This figure shows the results of fits of the spin response to Equation 5.1 after a resonant pulse excitation. All pulses were resonant with the central P1 line. 95% confidence interval error bars from the fits are shown only when they are larger than the symbol size. Both (a) and (b) plot the fit parameter $1/T_1'$ as a function of resonant pulse power at the sample. In (a) data was taken at 18 mT, with blue squares denoting 150 mK fridge operation and red circles, 370 mK operation. In (b) data was taken at 54 mT, with blue diamonds denoting 150 mK and red triangles, 370 mK. (c) Results for the fit parameter n are shown at 150 mK, again as a function of resonant pulse power at the sample. Blue diamonds again represent 150 mK data, and red triangles 370 mK. (d) Results for n at 54 mT, with blue squares at 150 mK and red circles at 370 mK. This data was taken on diamond sample D11.

fast flux line or the through the resonator and then measure the response by monitoring the magnitude and phase of the reflected -50 dBm resonant probe tone using a vector network analyzer. A plot of fits to the stretched exponential (Equation 5.1) of the resulting spin decays is shown in Figure 5.13. The errorbars are given by the standard deviation of fits to multiple decay traces. With this resonator we are able to probe over a wider temperature range than the magnetometer, although with lower signal to noise ratio. We again observe a weak temperature dependence, with T_1' slightly decreasing with increasing temperature.

In order to compare our resonator data with previously published work [19], we also plot data taken with the spin excitation applied to the resonator instead of the fast flux line at 370 mK and 2 K. The resulting time constants are shorter, but still appear to correlate with the temperature. If this time constant shortening effect is real (after all we only have two data points) it may be an interesting effect to study in the future.

The decay times measured with the magnetometer were significantly longer than those

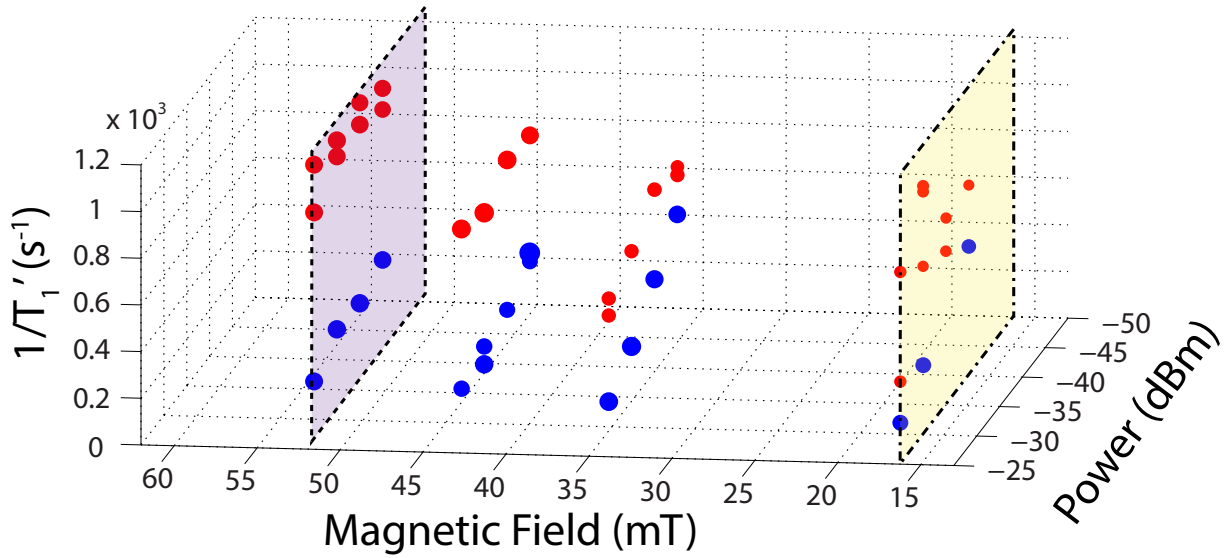


Figure 5.9: Here we plot the fit parameter $1/T'_1$ in Equation 5.1 as a function of both magnetic field and pulse power. We plot data for pulses with less than -30 dBm of power, at 150 mK (blue dots) and 370 mK (red dots). The data crosssection highlighted at 18 mT with the yellow rectangle (dot-dashed border) is shown in Figure 5.8a, and the data highlighted at 54 mT with purple rectangle (dashed border) is plotted in Figure 5.8b.

measured with the resonator. We measured $1/T'_1 \sim 2 \times 10^{-4} \text{ s}^{-1}$ in contrast to the $\sim 1 \times 10^{-3} \text{ s}^{-1}$ values measured with the resonator on the same sample at the same temperature. The reason for this discrepancy is still unknown. Given the measured spin-cavity coupling, $\sim 2.5 \text{ MHz}$ and cavity linewidth, $\sim 2 \text{ MHz}$, the Purcell effect yields a $\sim 1 \text{ MHz}$ decay rate, meaning that this does not account for the effect. This discrepancy could be an interesting avenue for further study.

On the other hand, the rate of change with temperature between the magnetometer and the resonator data does appear to be about the same, albeit over different temperature ranges. A linear fit to the resonator data yields a slope of $\sim 1 \times 10^{-3} \text{ s}^{-1}/\text{K}$ between 150 mK and 2 K, which is within a factor of a few of the slope from the magnetometer data from the same sample of $\sim 3 \times 10^{-3} \text{ s}^{-1}/\text{K}$ between 25 mK and 370 mK. While a linear fit is likely a crude approximation, this is still valuable as a rough sanity check.

5.5 Comparison to Previous Work

With these measurements we hope to add another piece to the puzzle of what governs the relaxation dynamics of P1 centers at dilution fridge temperatures. In Table 5.1 we

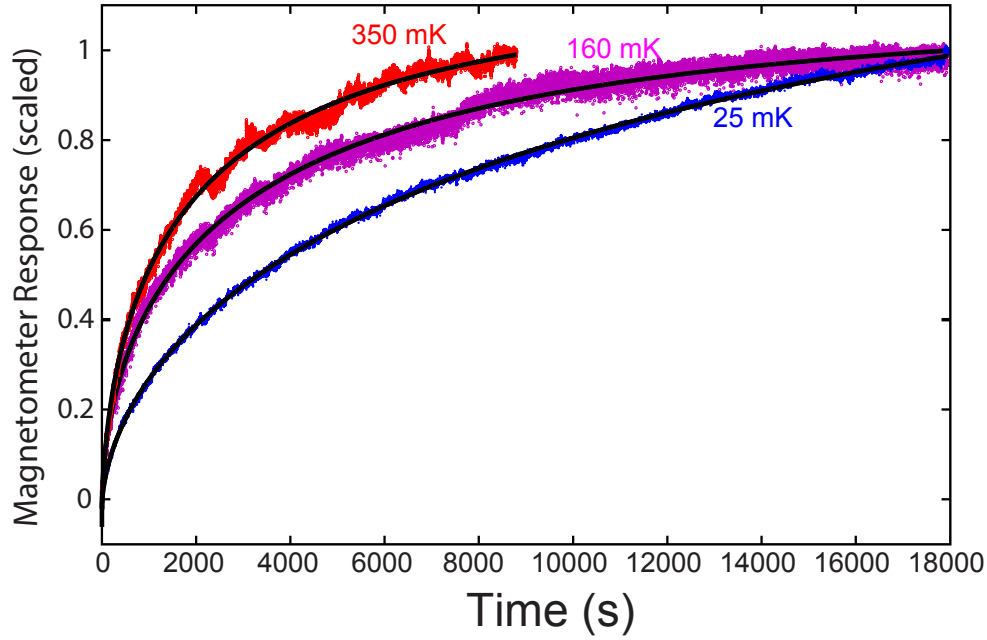


Figure 5.10: In this figure we show three decay curves taken at different temperatures, in order to illustrate the main trend we observe: the decay time constant becomes shorter at higher temperatures, as one can see from the decay traces. These curves were chosen such that their fit parameter “ n ” lies in a narrow range between $n = 0.55$ to 0.58 , in order to ensure a fair comparison. All curves in this plot were taken at a magnetic field of 64 mT with -63 dBm of power at the sample.

show a summary of results from previous work and compare to our results. As mentioned earlier, higher temperature and field ESR measurements by Reynhardt, et al. [20] showed temperature dependence of T_1 above 6 K. This temperature dependence was attributed to spin-orbit phonon-induced tunneling between Jahn-Teller-like distortions of the C-N bonds [20, 62]. Measurements from Takahashi et al were performed on a sample containing both P1 and NV centers down to 40 K and are in reasonable agreement. Reynhardt et al. proposed a model for the phonon-induced temperature dependence of the spin-lattice decay with the form:

$$\frac{1}{T_1} = AT + BT^5. \quad (5.4)$$

The coefficients for the Reynhardt, et al. sample (type 1b HPHT with 95 ppm concentration of P1s) were measured to be $A = 5 \times 10^{-3} \text{ K}^{-1}\text{s}^{-1}$ and $B = 1.1 \times 10^{-10} \text{ K}^{-5}\text{s}^{-1}$ for the central hyperfine line [20]. If we use these values for the coefficients and solve for lower temperatures we obtain $T_1 = 8 \times 10^3 \text{ s}$ at 25 mK and $T_1 = 5.5 \times 10^2 \text{ s}$ at 370 mK. The predicted values are shorter than our magnetometer results by roughly a factor of two. Since

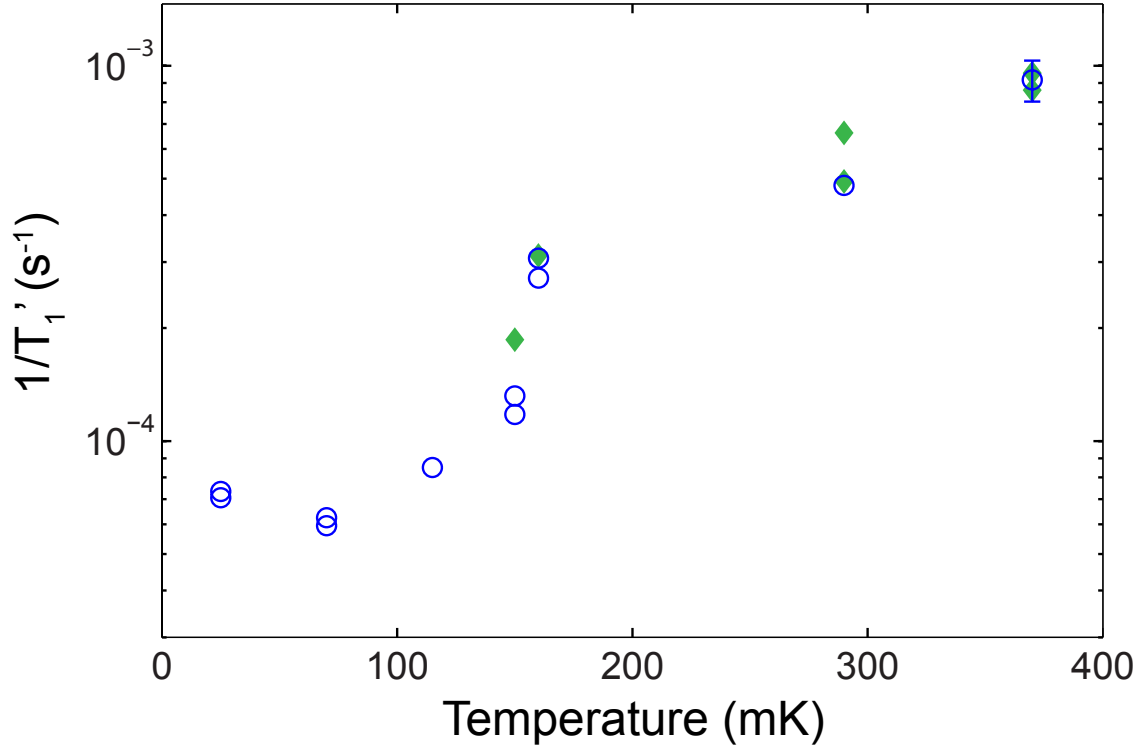


Figure 5.11: Here we show the relationship between the decay time constant T'_1 and temperature. In this plot we bin the data according to curve shape, with $n = 0.5$ to 0.6 represented by open blue circles and $n = 0.6$ to 0.7 with closed green diamonds. We observe an increase in $1/T'_1$ by an order of magnitude between 25 mK and 370 mK. Note that each point is a stretched exponential fit to a single data trace except for the open blue circle with errorbars at 370 mK. This point is an average of four data points, with the errorbars produced from a 95% confidence interval. Since the traces taken at 370 mK had the lowest SNR due to lower spin polarization, this gives an estimated upper bound for the errorbars on the lower temperature data. The relative scarcity of data points at these lower temperatures is due to the impracticality of taking many traces with these very long T'_1 times. Also note that the errors resulting from the fit to each trace yield errorbars smaller than the symbol size. The data points at 150 mK and 370 mK were taken at 54 mT, while the other temperature points were taken in a different fridge at 64 mT.

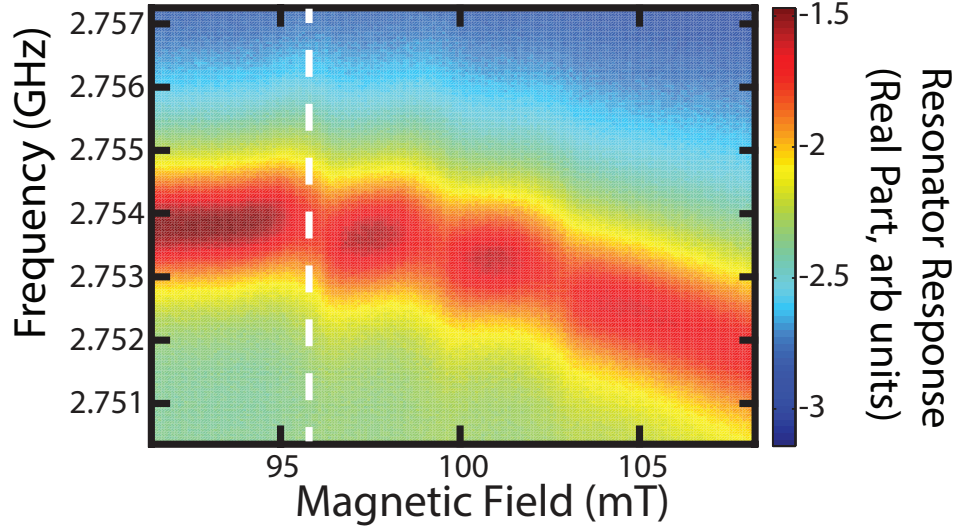


Figure 5.12: Here we plot the avoided crossing of the resonator with the three P1 hyperfine lines, with the color representing the real part of the resonator response. The magnet current values were scaled to units of milliTesla with the same conversion coefficient as the magnetometer measurements. The white dashed line indicates the magnetic field bias point.

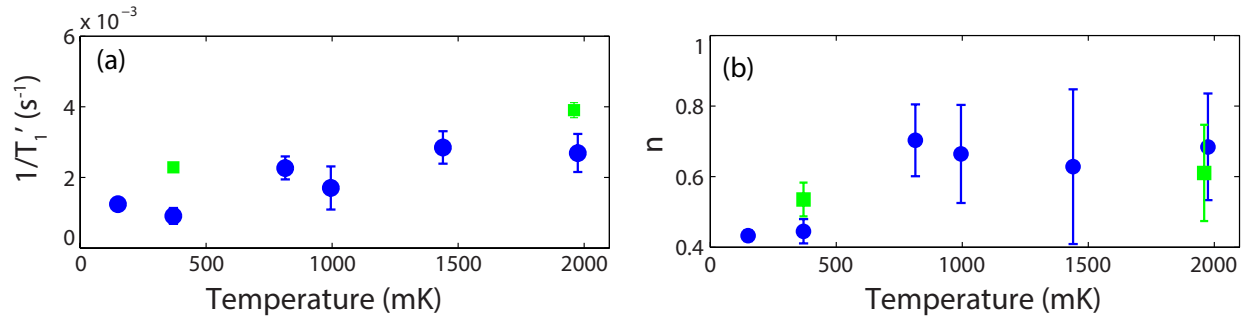


Figure 5.13: In this figure we plot the result of the $\lambda/4$ CPW Nb resonator measurements. In (a) we plot the fit parameter $1/T'_1$, and in (b) we plot n , both as a function of temperature. Blue circles denote fast flux line spin excitation, whereas green squares denote spin excitation via a strong pulse on the resonator. Each data point is the mean of fits to multiple measurement traces, with errorbars given by the standard deviation of the resulting fit values. Magnetometer measurements on this same sample (not shown) yielded $1/T'_1 = (1.8 \pm 0.8) \times 10^{-4} s^{-1}$ with $n = 0.73 \pm 0.03$ at 150 mK, and $1/T'_1 = (5 \pm 1) \times 10^{-4} s^{-1}$ with $n = 0.77 \pm 0.02$ at 370 mK.

Source	Technique	T_1 Time (s)	Magnetic Field (mT)	Temperature Dependent?
Reynhard et al. [20]	Pulsed ESR	2.5 @ 6 K	343	Yes over 6 - 300 K
Takahashi et al. [96]	Pulsed ESR	8 @ 40 K	8600	Yes from 40 - 300K
Ranjan et al. [19]	Supercond. Resonator	20 and 160	86 and 263	No over 0.25 - 1 K
This Work	SQUID Magnetometer	1×10^4 @ 25 mK, 1×10^3 @ 370 mK	64 and 54	Yes over 25 - 370 mK
This Work	Supercond. Resonator	1×10^3 @ 150 mK, 3×10^3 @ 2 K	95	Yes, weakly over 150 - 2 K

Table 5.1: Here we present a summary of previous work to measure T_1 decays in P1 centers, along with our own results. Note that the Takahashi et al. [96] experiment was performed on a sample containing both NVs and P1 centers. As well, note that the Ranjan et al. results for T_1 list two time constants as they used a biexponential fit. Our results T_1 results are for a stretched exponential fit to Equation 5.1.

we would expect the values of these coefficients to vary slightly from sample to sample [96], agreement within a factor of a few is a reasonable result. Note that the model given in Equation 5.4 is still a poor fit to both the resonator and magnetometer data, but since this data is over a relatively narrow temperature range and some variance, we cannot entirely rule out this model from the data alone.

We can compare our result to that found by Ranjan, et al. with their superconducting resonator experiments: no temperature dependence of the T_1 time between 250 mK and 1 K [19]. With this result, they attributed the energy loss to spin-diffusion out of the resonator volume. From our data we can conclude that whatever model is proposed for the spin-lattice relaxation it must still account for temperature dependence at these low temperatures and moderate magnetic fields.

Takahashi et al. showed in Reference 96 that a simple model of spin-flip limited diffusion was a good fit to their temperature dependent T_2 data. The model is in essence that spin diffusion rate is proportional to the number of pairs with opposite spin. Using this model the decay rate is given by

$$\frac{1}{T_2} \equiv CP_{m_s=-1/2}P_{m_s=1/2} + \Gamma_{res} = \frac{c}{(1 + e^{T_{Ze}/T})(1 + e^{-T_{Ze}/T})} + \Gamma_{res} \quad (5.5)$$

where Γ_{res} is the residual relaxation rate, T_{Ze} is the temperature corresponding to the Zeeman energy, $\mu B/k_B$, and C is a temperature independent parameter. Unfortunately,

our data does not fit to this simple model. However, we suspect that more sophisticated theoretical investigations along these lines may prove fruitful.

In conclusion, we have shown strong temperature dependence of T_1' , with an order of magnitude increase of T_1' time between 370 and 25 mK. With this data we hope to aid future experimental and theoretical investigations to determine the mechanism for spin-lattice decay of P1 centers at low temperatures and moderate magnetic fields.

Chapter 6

Conclusions and Future Work

In this section I will summarize the conclusions of the work presented in this thesis, and give a sketch of future experiments with the magnetometer.

6.1 Summary and Conclusions

In this thesis we have demonstrated the desirable characteristics and successful operation of our 3D nanobridge SQUID magnetometer. We have shown that the device has a minimum flux noise of $17 \pm 0.9 \text{ n}\Phi_0/\text{Hz}^{1/2}$ with only a factor of ~ 2.5 increase in flux noise in a parallel magnetic field up to 61 mT. A second generation device with a thinner and slightly smaller footprint capacitor has a field tolerance up to 75 mT. The maximal bandwidth values range from 25-40 MHz in the paramp regime to 70 MHz in the linear regime. This combination of large bandwidth, low flux noise, large flux coupling and field tolerance make this sensor a promising candidate for near-single-spin dynamics measurements.

In the previous chapter we began to demonstrate the utility of a nanobridge SQUID magnetometer for characterizing spin systems in the solid state. We successfully used the magnetometer to study the decay characteristics of P1 centers in diamond. We found that the decay time constant was temperature dependent, with an order of magnitude increase in the decay time from 370 mK to 25 mK with a moderate 54-64 mT of magnetic field. With this new data we hope to add a further information towards determining the mechanism behind spin-lattice relaxation decays of P1 centers at dilution fridge temperatures.

6.2 Future Work

Further work with this magnetometer will certainly include further studies of diamond. In addition to P1 centers in diamond, we also plan to measure NV centers in a diamond with a [111] cut. Unfortunately, the magnetometer is insensitive to NV spins in the more common [100] cut diamond, as the summation of the flux due to the four NV orientations cancels out

geometrically. Luckily we will have access to a newly irradiated and annealed [111] diamond with a high concentration of NVs¹ in the near future.

In addition to diamond impurities there are several other interesting spin systems that could be studied with this magnetometer. Silicon carbide is a spin system which has generated much interest recently [27, 28], and could be measured in bulk with our flip chip technique or perhaps with a small patch implanted on chip with the magnetometer. We have attempted to measure Bismuth donors implanted in ^{28}Si in the past, but were unable to do so for a variety of reasons (see Appendix C). However, it should be possible to measure these donors in bulk, and may be possible in a carefully shielded setup to see them in smaller concentrations. These spins have low-decoherence clock transition point that is within the field tolerance of the magnetometer and would provide an interesting feature to study. [32, 35].

¹These are very rare!

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Appendix A

Chase Fridge

The green fridge in 115 LeConte has many names. It goes by Kinda Cold, Yu-Cold, or Chase Fridge. The latter is perhaps the most common name, as the dilution unit – the heart of the fridge – was made by Simon Chase in England. In my first three years in grad school, I worked along with three undergraduates, Sun Yu-Dong, Anirudh Narla and Ravi Naik, to assemble this fridge and the accompanying 3-axis magnet.

In the following appendix I will attempt to give a brief summary of how the fridge works and its various components. Along the way, I will give some introduction to basic fridge design, construction and operation principles with the aim of introducing a new student to some of the considerations we face when working with this equipment. The full fridge documentation, wiring information, Chase unit manual, fridge software and the OneNote notebook can be found in the QNL server under `\hardware\Kinda Cold`. Additional information is available via hardcopy in the physical fridge notebook and blue envelope labeled Kinda-Cold that should be stored near the fridge.

A.1 Methods of Cooling: Theory

Each stage of the Kinda-Cold is cooled in a slightly different manner. In this section I will briefly describe the theory for each of these cooling techniques. Most of this section closely follows the highly recommended *Matter and Methods at Low Temperatures* by F. Pobell [97], to which I refer the reader for more detail.

A.1.1 Pulse Tube Refrigeration

Pulse tube refrigeration operates under a similar principles to regular mechanical refrigeration, and allows cooling down to about 2 K. This limit is due to the fact that the thermal expansion coefficient of the working gas, usually ^4He , goes to zero at this temperature. Pulse tube refrigeration uses a thermodynamic cycle that operates on a working gas taking energy from a cold bath and transferring it to a hot one, with a certain efficiency. A simple pulse

tube configuration is a linear volume which consists of a compressor at one end, a regenerator (heat reservoir made of high heat capacity, often magnetic, material) connected to the pulse tube cold end, a length of pulse tube, and then the pulse tube hot end which is connected through an orifice or flow impedance to a reservoir volume. The long length of pulse tube serves to insulate the two ends of the pulse tube from each other, so gas particles never traverse the entire length of the tube before the flow reverses. During the compression phase of the cycle, the heat of compression at the compressor is dissipated by a water cooled heat exchanger. The regenerator takes heat from the gas flowing from the compressor to the cold end of the pulse tube. Gas from the hot end of the pulse tube flows through the orifice into reservoir volume and gives off heat. During the expansion phase the gas reverses direction. Gas flowing from the cold end of the pulse tube to the regenerator removes heat from the pulse tube cold end, creating the desired cooling effect. Further down the chain, gas flowing from the pulse tube through the regenerator towards the compressor takes up heat from the regenerator which is then dissipated in the water cooled heat exchanger.

The Cryomech PT415 in the Kinda-Cold is a two-stage pulse tube cooler, which is more complicated than the description given above. Two stage coolers typically have two PTs in parallel and are run with a Gifford-McMahon style cycle. For more details, I refer the reader to brief descriptions in Refs. [97, 98] with more detail and references given in the review article by Waele [99].

A.1.2 Evaporation

The pre-cooler cold head in the chase unit is cooled via evaporative cooling, performed sequentially on volumes of ^4He and ^3He gas, enabling cooling down to 300 mK. *Matter and Methods at Low Temperatures* by F. Pobell [97] gives a good description of the math describing this evaporation, which I will follow in this section.

The basic principle behind evaporative cooling is that energetic atoms or molecules will tend to enter the vapour phase above a liquid. This first order phase change will cost some energy, a latent heat, that will cause the overall energy, and thus temperature, of the liquid to drop. If you pump on the liquid, up to a certain limit this will increase the rate of evaporation, and thus the rate of cooling.

The cooling power can be expressed as:

$$\dot{Q} = \dot{n} (H_{liq} - H_{vap}) = \dot{n}L \quad (\text{A.1})$$

Where H_{liq} and H_{vap} denote the enthalpy of an atom associated with the liquid and gas phases, and their difference is the latent heat, L . $\dot{n}$ is the rate of particles/time that are moving from the liquid to gas phase. If a pump with a constant volume pumping speed is used, the rate $\dot{n}$ will be proportional to the vapor pressure. Pobell derives the vapor pressure to first approximation, assuming an ideal gas and latent heat constant with temperature:

$$P_{vap}(T) \propto e^{-L/RT} \quad (\text{A.2})$$

Where R is the ideal gas constant. So, we can write:

$$\dot{Q} \propto LP_{vap}(T) \propto e^{-1/T} \quad (\text{A.3})$$

which tells us that the cooling power of a pumped liquid will decrease with temperature. Eventually the cooling power will balance with external heat input, stabilizing temperature. Of course, this process ends when the liquid is entirely pumped away.

A.1.3 Dilution

Dilution allows cooling down to 5 mK in highly optimized fridges. This cooling technique is in a certain sense analogous to evaporative cooling, in that it also relies on a phase transition. In this case, the two phases have to do with the mixing between ^4He and ^3He isotopes. The low energy state of a mixture of ^3He and ^4He below 867 mK at saturated vapor pressure is a two-phase state consisting of a concentrated ^3He liquid and a dilute mixture of ^3He in a ^4He rich liquid. With gravity (which is necessary for the dilution process to occur) the pure ^3He liquid will float on top of the dilute mixture. This ground state is not intuitive – under classical reasoning we would expect completely separated ^4He and ^3He liquids for a zero entropy state – however the quantum nature of the liquids at these temperatures is what leads to the finite solubility of ^3He in the ^4He liquid. The cooling process works by forcing ^3He atoms down into the dilute mixture, and this requires a “heat of mixing” which leads to overall cooling of the mixture. The dilution cooling power goes like $\dot{Q} \propto T^2$ rather than the $\dot{Q} \propto e^{-1/T}$ for evaporative cooling. This weaker temperature dependence is due to the finite solubility of ^3He in the ^4He rich liquid even down to really low temperatures. Essentially one never “runs out” of ^3He atoms for dilution – unlike what eventually happens when a liquid evaporates away. I refer the reader to Ref. [97] for diagrams of this process in action and more details.

A.2 Fridge Parts

In this section I will give a brief overview of the fridge components that are necessary for its operation. Labeled pictures of the fridge in its initial and final configuration are shown in Figures A.1 and A.2.

A.2.1 Fridge Plates, Pulse Tube Cooler and Outer Vacuum Can

The fridge has four plates that are fixed at four different temperatures (40 K, 4 K, 350 mK and 150 mK) during steady-state operation, as shown in Figure A.1.¹ These temperature

¹The top plate which runs near 40 K is often referred to as the 77 K stage in much of the fridge documentation. This is for historical reasons because in wet fridges (fridges that take liquid cryogenics) the analogous plate is maintained at liquid nitrogen temperatures. Also, you may find that in the fridge software and documentation the 350 mK and 150 mK plates may be referred to as 300 mK and 70 mK stages. This is because the cold heads of the Chase unit will reach these temperatures when they are not thermally loaded.

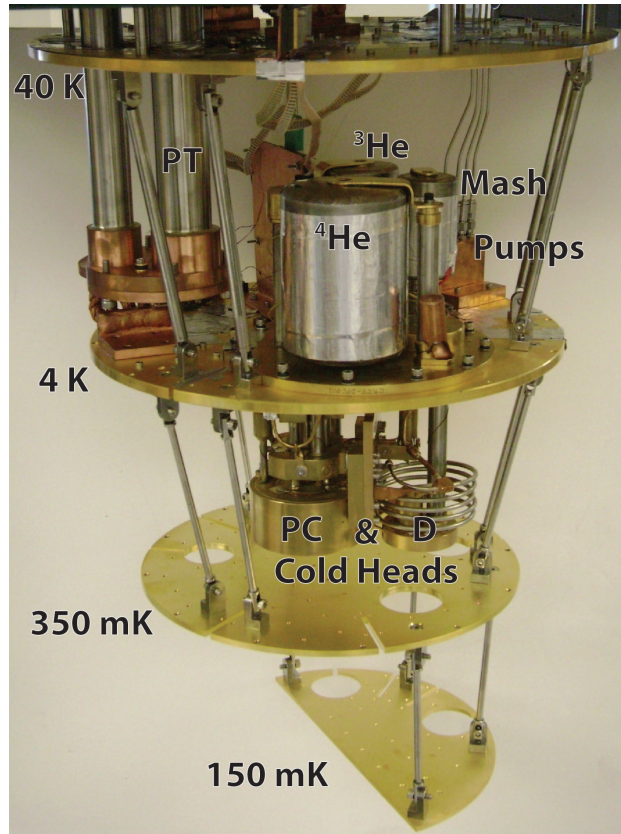


Figure A.1: Labeled picture of the fridge, in its first configuration with large plates on the 350 and 150 mK stages. Cooldown times with these large plates were very long, due to their large heat mass, and they were later abandoned in favor of at least two other designs which ultimately led to the current configuration.

gradients are maintained by excellent thermal isolation between the stages. The stainless steel structural supports located between each stage are very thin-walled tubing and all DC and microwave wiring was done with care to minimize thermal conductivity while still providing adequate electrical conductivity and acceptable loss characteristics (see Section A.4 on wiring for further information).

The 40 K and 4 K plates are cooled using a mechanical pulse tube (PT) cooler, Cryomech PT410-RM. This operates in a similar fashion to a regular household refrigerator, except that the working gas is helium, and the compressor runs at 11 kW. The compressor requires a steady source of cooling water, and shares a pump box with the Oxicrod, located currently in 119 LeConte. The PT cold heads are well anchored to the 40 K and 4 K plates using a copper plates bolted to the head and stage with welded copper braids in between. This configuration is used so that any strain between the cold head and the plate produced during the cooldown due to differing coefficients of thermal contraction can be mitigated by the connecting braids.

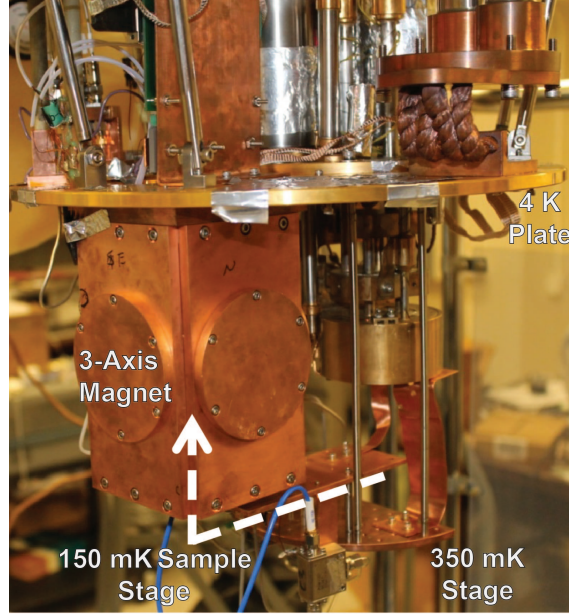


Figure A.2: Current configuration of the fridge, showing the 3-axis magnet, and current 350 and 150 mK plates.

In order to avoid condensing vapor on the fridge plates and PT heads, we must perform all cooldowns in vacuum. The outer can (green) on the fridge is a vacuum vessel with O-ring.² This can is much heavier than the other fridge cans as it has a thicker wall. Note that much care must be taken when bolting and unbolting the can – as due to its heavy weight many of the threaded inserts have become unusable. Before beginning operation of the PT, one should pump down the can to 5×10^{-2} mbar of pressure. If this takes more than 15 to 20 minutes, you might consider checking for leaks.

A.2.2 Chase Unit

The heart of this fridge consists of Simon Chase’s first-of-its-kind He7 and Dilutor Module System. A picture of the unit is shown in Figure A.3. The unit consists of a pre-cooler (PC) cold head and dilutor cold head. The PC cold head is connected to separate ^4He and ^3He chambers with charcoal sorption pumps. The dilutor cold head has a 50:50 mix (also known as “mash” in England) of ^4He and ^3He gas, and a matching sorption pump. The pumps are heated and cooled by way of heater resistors and active gas-gap heat switches³. More details

²This can is made out of a single cast aluminum tube—in contrast to the rolled and welded cans on the other fridges. Irfan says that tubing of this size is commercially available because it is used for missile tubes!

³Gas gap heat switches are small tubes filled with gas (in this situation ^3He or ^4He) that have an absorber. Passive heat switches will conduct heat from end to end of the tube until the absorber cools enough to absorb the gas. Active heat switches have an absorber that can be heated at will by the user, using voltage

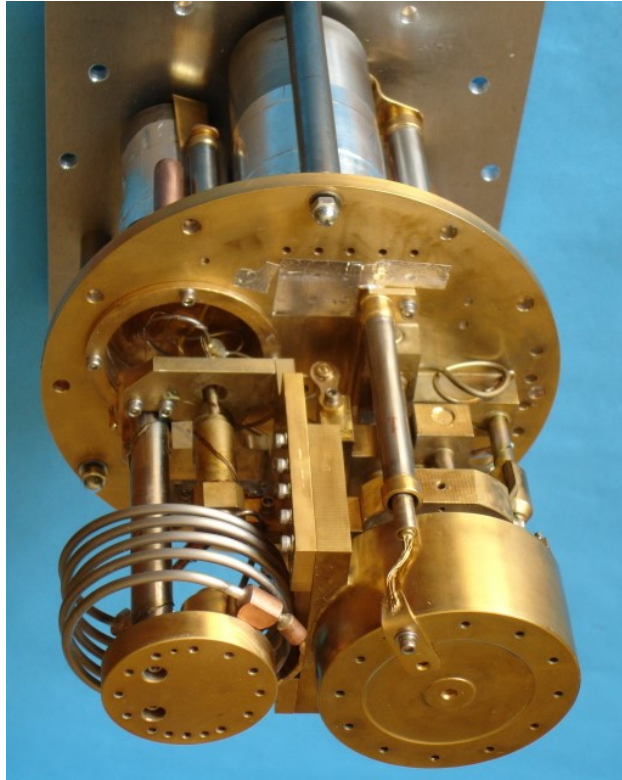


Figure A.3: The chase unit before installation. Picture credit: Simon Chase.

about the fridge can be found in the Chase Unit Manual, but I will give a brief summary of its operation next. If you are learning how to operate the Chase fridge, I recommend reading this section first, and then tackling the manual. The manual will be clearer after you are familiar with the basic fridge operation. Note that the fridge automation software (written by me and Anirudh Narla) takes care of most of this operation automatically – but when tweaking or fixing bugs it is of course helpful to understand the basics of the fridge operation.

A.3 Fridge Operation

A.3.1 Initial Cooldown (300 K–4 K)

After installing the two radiation shields and outer vacuum can, and pumping out the fridge to less than 5×10^{-2} mbar, we can start the pulse tube cooler, via a button on the compressor. A plot of the initial cooldown is shown in Figure A.4. After about 12 hours the 4 K and 77

across a heater resistor. Within the design temperature range these switches allow user controlled heat conduction. [100]

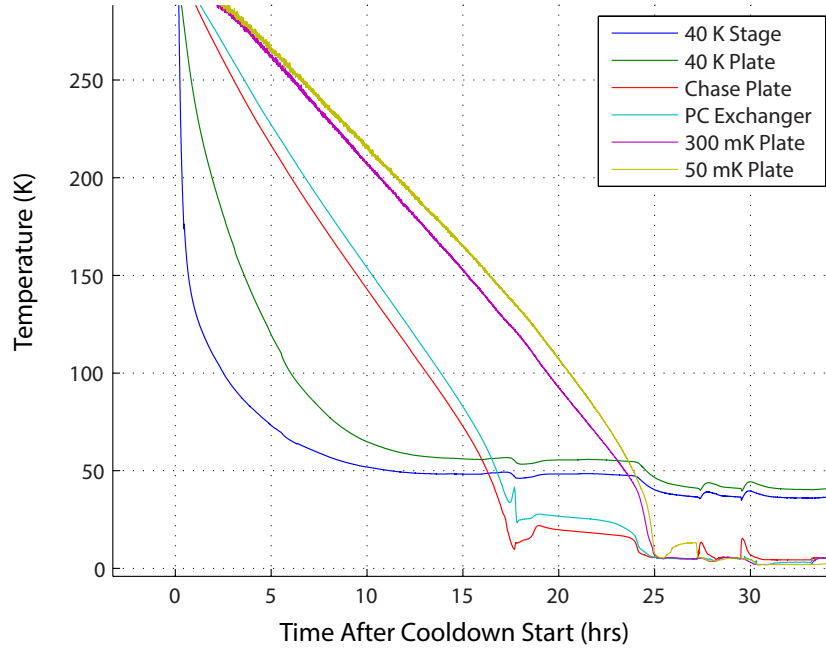


Figure A.4: Temperatures as the fridge cools down from room temperature to near 4 K.

K plates will have cooled to close to their final temperatures. However, the 350 mK and 150 mK base stages will still be warm, due to fact that they are very well isolated from the other stages. Note that there is a passive gas-gap heat switch between the bottom of the PC cold head and the 4 K plate that is meant to help with the initial cooldown, but this has only small thermal conduction. To enhance the conduction we heat all the pumps⁴ to release the adsorbed gas which will increase the thermal link between the cold heads (connected to bottom plates) and the 4K plate. With this technique, after about 24 hours from the cooldown start the 350 and 150 mK plates will have cooled below 5 K.

A.3.2 Cycle to Base (4 K–150 mK)

The basic idea of the fridge cycle is as follows:

1. Heat the mash pump to 21 K. At this temperature the pump will be weakly pumping on the mash, allowing circulation in the dilutor circuit.
2. Heat the ^4He and ^3He pumps to release their adsorbed gas
3. Wait some time for the released ^4He gas to liquefy as it is cooled by the 4K PT stage.

⁴Before heating these pumps, we have to wait for the pump heat switches to cool below about 15 K, to ensure that they do not provide a direct link between the pump heaters and the 4 K plate.

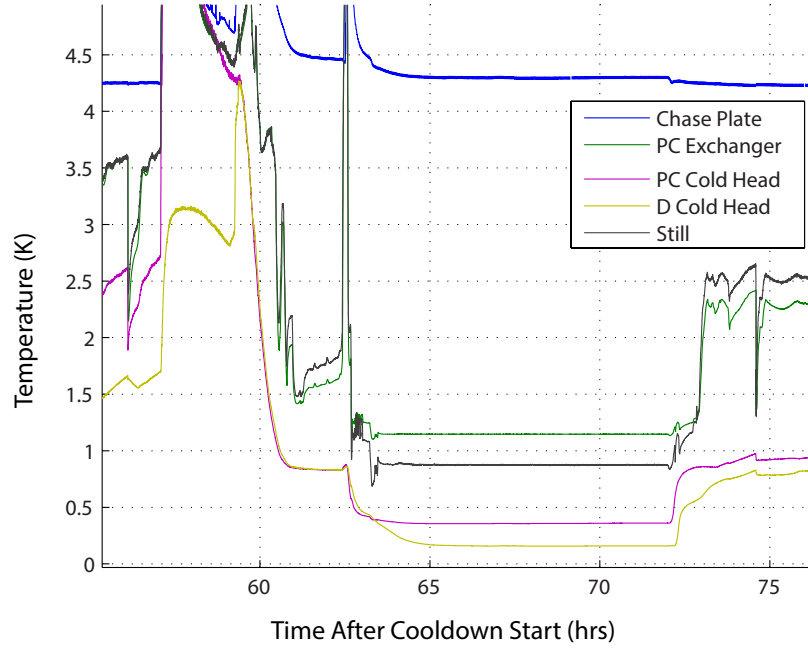


Figure A.5: Temperatures as the fridge cycles from 4 K to base for the cold heads, 4 K stage, and pre-cooler exchanger.

4. Pump on the ^4He liquid, thus cooling the PC cold head by evaporative cooling.
5. Wait for the ^4He to expire (all gas is adsorbed). During this time the PC cold-head is cooled to 800 mK which is cold enough to liquefy the ^3He gas.
6. After the ^4He expires, pump on the ^3He , which will bring the PC cold head to 350 mK.
7. During this time the PC Cold Head and Dilutor cold heads are linked with a gas-gap heat switch. Step 5 will cool the dilutor cold head to a temperature which is cold enough for dilution to begin occurring (see Section A.1.3 for more information on this process) around 400 mK. After the plates reach about 450 mK, we turn off the link between the cold heads (named the dilutor cold head switch) and this allows the dilutor cold head to cool below the PC cold head and reach base.

As mentioned in Step 2, we must heat both the ^4He and ^3He pumps to begin the cycle. The current fridge configuration requires that we heat the ^4He to 70 K and the ^3He pump to 45 K and hold those for 70 minutes. The appropriate times and temperatures will vary depending on the heat loads on the plates and the temperature of the 4 K plate. If this plate is running above 4.4 K, reliable operation of the fridge becomes much more difficult. A hotter 4 K plate and/or more heat load on the 350 and 150 mK plates, will necessitate longer

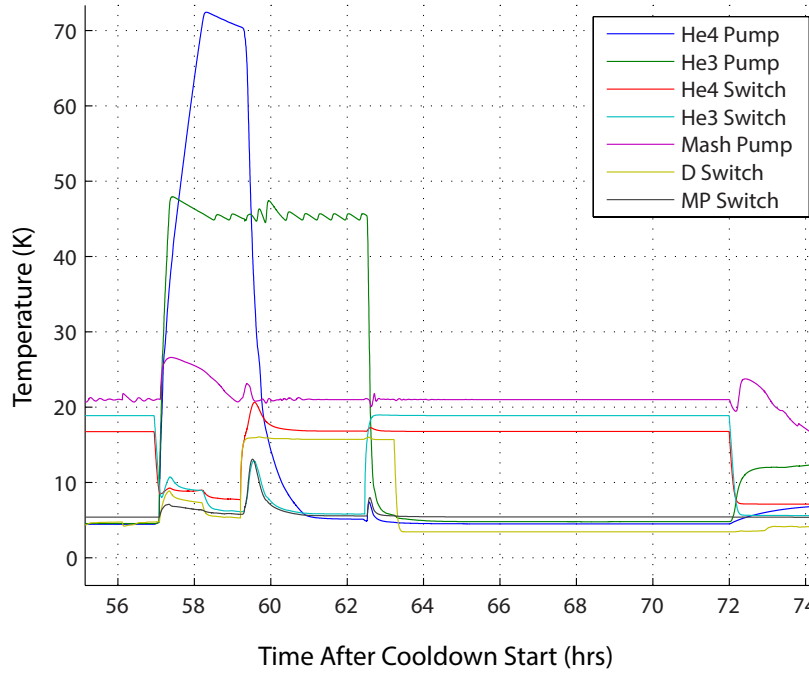


Figure A.6: Temperatures as the fridge cycles from 4 K to base for the pumps and heat switches. D denotes “dilutor” for the dilutor heat switch.

heating times and higher heating temperatures in order to liquefy enough ^4He gas. We must liquefy enough ^4He gas so that the fridge can stay at 800 mK for long enough to liquefy the required amount of ^3He for a good hold time. Since the dilution mix is continually cycling, the amount of time the fridge stays at base (typically 8-9 hours) is actually determined by how long the 350 mK stage is maintained at its base temperature by pumping on liquid ^3He . Note that the waiting for ^4He expiry step is crucial to get right. Under normal operation the fridge software detects expiry very reliably, but occasionally this algorithm will fail, usually due to some irregularity in the cooldown or user error. The fridge software detects the expiry by looking for a spike in the PC exchanger temperature⁵ as well as the start of an increase in the cold head temperatures. If a false positive for expiry is detected for some reason, and not all the ^4He is pumped away, a superfluid ^4He leak may form which will thermally connect the cold head and 4 K plate, and cause the fridge to fail to cool properly. Alternatively, if the expiry goes undetected, the software will wait forever at the waiting for expiry step and the fridge cycle will fail.

⁵This exchanger intertwines the tubes connecting the ^4He and ^3He pumps with the PC cold head, facilitating thermal connection between the gases.

A.3.3 Running in ^3He Mode (4 K – 370 mK)

The Chase unit can also be run ^3He mode. This allows for a slightly longer hold time (typically 15 hours) at 370 mK. The procedure for operating in this mode is as follows:

1. Heat ^4He and ^3He pumps with the mash pump cold (times and temperatures for this step will be shorter and lower than in Step 1 for a regular cycle, typically 30 min at 55 K on the ^4He Pump). To keep the mash pump cold we turn on the mash pump heat switch.
2. Turn off heat to the ^4He pump and turn on the pump heat switch – thus pumping on ^4He by cooling the charcoal sorption pump.
3. Wait for expiry of ^4He . In this mode, the PC-exchanger continually warms up for several hours and then spikes. The software does not detect expiry very well in this mode, so it is usually best to turn off the automation and run the fridge manually after this step.
4. Pump on ^3He as was done with ^4He in step 2.
5. If the sample is connected to the 150 mK plate, keep the dilutor cold head switch on. Otherwise, if the sample is on the 350 mK plate, you can turn off this heat switch and let the dilutor cold head drift up in temperature.

A.3.4 Warming Up

To warm up the fridge follow these steps:

1. Turn off all heaters at heat switches on the chase unit, by hitting “Reset All Channels” on the fridge control panel. I recommend hitting this button twice, and double checking that all the currents showing on the ADC are 0 mA (which actually reads as 3-4 mA on the ADC).
2. Make sure all magnet coil currents are off.
3. Turn off the pulse tube by hitting the “off” button on the compressor.
4. Double check that the pulse tube is off!
5. Open the valve before the pressure release valve all the way, while keeping the second (outer) valve closed. For a full 1-2 day warmup, you can keep the outermost valve closed, and safely let the fridge warm up. The pressure relief valve will relieve any potentially dangerous excess pressure.

6. For 12-15 hour warmup, slowly vent the can to nitrogen, by drawing vapor off of a dewar of LN2. Vent until the can pressure rises to 1-2 mbar. For the fastest warm up (7-12 hours), ensure that the can pressure goes to 2 mbar and begins to rise after the top plates warm past 77 K. Note that the valve and LN2 tube may ice up in this case.
7. Point a fan at the OVC to help aid the warmup, and remove any sensitive equipment sitting below the OVC in case condensation forms on the outside.
8. Ensure that all (non-RuOx) thermometers read above the room dewpoint temperature before venting to air and opening the can.

A.4 Wiring

Wiring a dilution fridge is a bit of an art form. One must balance adequate electrical conductivity and acceptable loss characteristics while still minimizing thermal conductivity. This balance is quantified by the Wiedemann-Franz law which states that the thermal conductivity and electrical conductivity are related by the product of a constant and the temperature [97, 101, 102]:

$$\kappa/\sigma = L_0 T \quad (\text{A.4})$$

Where $L_0 = 2.4453 \times 10^{-8} \text{ W } \Omega \text{ K}^{-2}$. This law is just an approximation, and most typically holds true at low ($\sim 4 \text{ K}$) and high temperatures ($\sim 300 \text{ K}$) but not in between.

For DC wiring we typically we use Maganin wire, which has a high resistivity, for temperature sensors, and thin copper wires where necessary for fridge heaters, HEMT bias, and other high current applications. All microwave wiring is done in coax with stainless steel inner and outer conductor UT-085-SS-SS cable down to the 4K stage. Input lines below 4K are also done with SS-SS coax. Signal loss is a concern on the output, so the lines are done with superconducting Nb cable⁶ between 150 mK and 350 mK and with Cu-Ni coax cable between 350 mK and 4 K. Note that superconductors will conduct heat only via quasiparticles which appear in proportion to $e^{(-\Delta E/k_B T)}$, where $\Delta E = 1.76 k_B T_c$ for most elemental superconductors [38, 97, 103]. Since 150 mK is well below the Nb gap, the superconductor will have very little thermal conductivity. We can afford more thermal load on the 350 mK stage, so Cu-Ni cable is a good compromise due to its intermediate thermal and electrical conductivity. At 4 K the signal is amplified by 40 dB with the HEMT, so loss from the stainless steel coax on the output above 4 K is not a problem.

The magnet leads for the 3-Axis magnet will be discussed in detail in Section A.6, but I will note here that these are high-Tc superconducting leads. Due to the high currents that must be used to run the magnet coils, the small gauge wire that would be necessary to safely run these currents would be too much heat load on the 4K plate so we have to use a low

⁶Part number: SC-219/50-Nb-Nb assembly 50ohm semi-rigid coaxial cable assembly, cable diameter 2.19mm, niobium center and outer conductors, PTFE dielectric, outer jacket stripped 3mm on each end and plated at minimum of 1.0" (approx. 26mm). The overall cable length is set by the application.

thermal conductivity but high electrical conductivity superconductor that works between 4 K and 40 K.

Note that all the thermometry and fridge heater wiring enters the fridge through at 72 pin hermetic connector. It is then broken out on a room temperature board, followed by a 40 K stage breakout board, and then a 4 K board. From the 4K board, two loom⁷ cables go to the Chase unit. These cables contain all the thermometry and heater wiring for the Chase unit, and their integrity is crucial for the operation of the fridge.⁸ Additional wiring in the fridge includes two hermetic reichenbach connectors which carry the HEMT bias wiring and 4 additional copper DC wires that are currently unused. These have been used in the past for Hall sensor or small magnet coils. Lastly, note that there is a heater on the 150 mK plate that can be used for stage temperature control. This heater plugs into the 4 K breakout board, and has a BNC output on the Anirudh box, which will be discussed in the next section.

A.5 Fridge Control and Temperature Readout

The fridge heaters and thermometry are all routed through a 72-pin cable that goes to a box built by Anirudh Narla. Thermometry readout wires are broken out inside the box, and then re-routed again within an external box that is placed behind the Narla box. This small box routes the wires to the Lakeshore 370 Resistance Bridge and two Lakeshore 218 instruments. The Lakeshore 370 reads out Cernox and RuOx thermometers.⁹ The Lakeshore 218s read out the diode thermometry.

Heater channels are controlled via a digital to analog converter (DAC) inside the Narla box. An analog to digital converter (ADC) reads the output from the DAC in order to ensure that the DAC output is set correctly. Relays that can switch the output to each channel on and off provide an extra layer of security against errant current output by the DAC. All thermometry and heater channels can be controlled with a Labview program on the fridge computer, entitled with the somewhat baroque name: “Yu-Cold Control Panel With Automation improved further changed.vi” This program has automation that takes care of the initial cooldown and cycle from 4 K. Unless the fridge heat loads change from a change in wiring or a touch, the automation works reliably for cooling to 150 mK.

⁷Cryogenic loom cable typically consists of 12 twisted pair wires. Typically these are wired to 25 pin Lakeshore connectors. The loom can be made of copper, NbTi, Manganin, BeCu or Constantan alloys. The choice of wire material will depend on the electrical and thermal conductivity needs. Cables which contain heater lines are typically made of copper, while thermometry-only cables will be Manganin.

⁸Note that cable between the 4 K breakout board and the chase unit (the pre-cooler wiring one) is failing at the connector on the 4 K breakout board side. We have jury-rigged a solution by soldering wires directly to the 4 K board, but note that this cable may continue to fail in the future, and may need to be replaced. Instructions for making new cables were compiled by Ravi Naik and are available on the Antelope under \hardware\MicroD Cable Making.

⁹The Kinda-Cold cernox thermometers operate over a range from 300 K down to 2 K. The RuOx thermometers operate most reliably in the 10 K to 10 mK range.

Coil Name	Max Current (A)	Conversion (mA/mT)
Z Coil	3.5	12.2
X Coil	1+	250
Y Coil	1+	166

Table A.1: Information on the 3-Axis magnet. The conversion factors were measured with a cryogenic Lakeshore Hall sensor (located in a second blue envelope near the fridge documentation.) Note that field calibration with P1 centers in diamond yields a 11 mA/mT conversion factor for the Z-coil, which is within 10% of the values measured here. Note that the hall sensor and diamond were on different sample mounts, and were likely at slightly different positions within the field.

Note that this software is unfortunately wedded to the Windows XP operating system. This is because there are no official drivers for the DAC that will work for Windows 7+, as unfortunately the DAC is no longer supported. It is possible that there are other unofficial drivers somewhere, or that the DAC could be replaced with a newer National Instruments DAC, but at the moment this is an important constraint on the system.

A.6 3-Axis Magnet and Leads

The Chase fridge contains a home-built 3-Axis magnet on the 4 K plate, which is very useful for performing magnetometry experiments. The main solenoid coil (named the Z-coil) has a 1" bore, and can take up to about 3.5 A of current, corresponding to about 280 mT. The main solenoid is surrounded by two Helmholtz pairs (X and Y coils) which can be used to shim the coil or provide a stronger bias. A picture of the main solenoid coil before installation of the Helmholtz coils is shown in Figure A.11a. A picture with the current setup including all the Helmholtz coils is shown in Figure A.2. All coils were wound with NbTi wire with a copper matrix.¹⁰ The Helmholtz coils have about 5–7% of the strength of the main solenoid for the same input current. The conversion constants for current versus field are shown in Table A.1. As well, analytically calculated plots of the main solenoid field, Helmholtz pair field, Z-coil inhomogeneity, and fringing fields are shown in Figures A.7, A.8, A.9 and A.10. The code for these calculations can be found on the Antelope\Data\Natania\Magnet Coil Inhomogeneity. These calculations give some sense of what went into the design of the magnet.

The high-Tc superconducting leads were made by HTS-110 and distributed by GMW, with part number CS010030 and length 170 mm. These leads have a maximum operating current of 150 A, more than enough for our needs. Before buying the leads we attempted to build these HTC leads ourselves, and the results are shown in Figure A.11b. We constructed

¹⁰Part number: 54S43 with an insulated diameter of 0.33 mm, and purchased from Supercon, Inc.

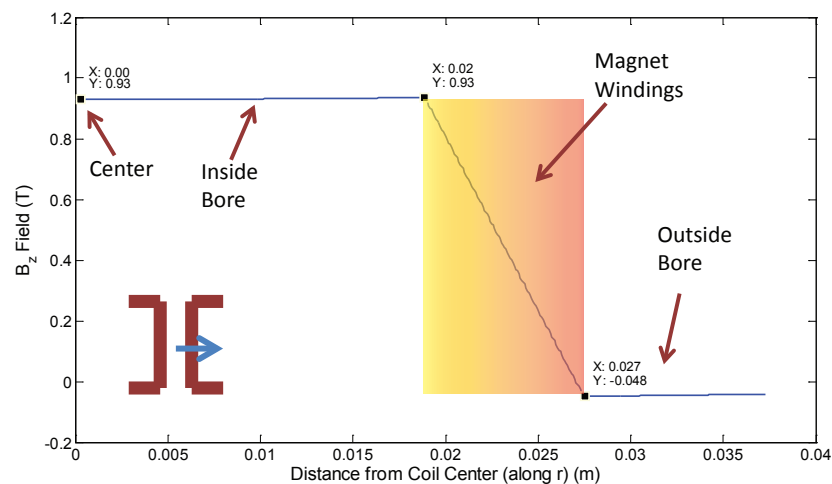


Figure A.7: A calculation to show the field of the main solenoid coil with 10 A of current. This calculation ended up overestimating the measured field by about 13%.

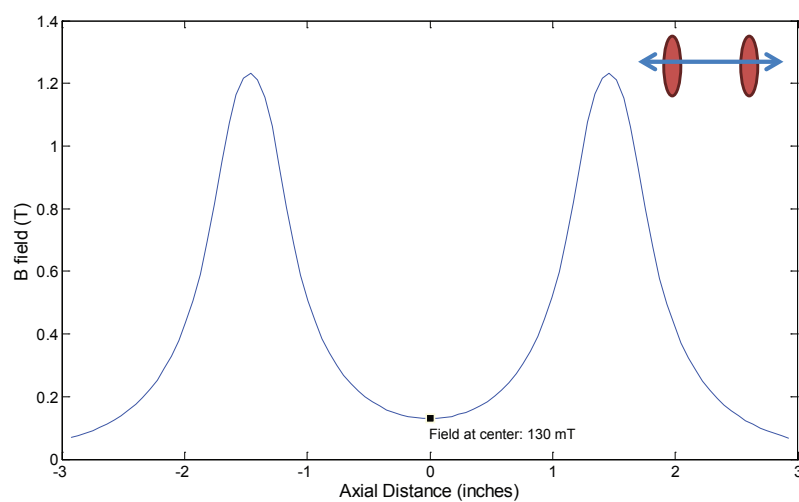


Figure A.8: A calculation to show the field of larger Helmholtz coil with 10 A of current. This calculation ended up overestimating the field by about 50%.

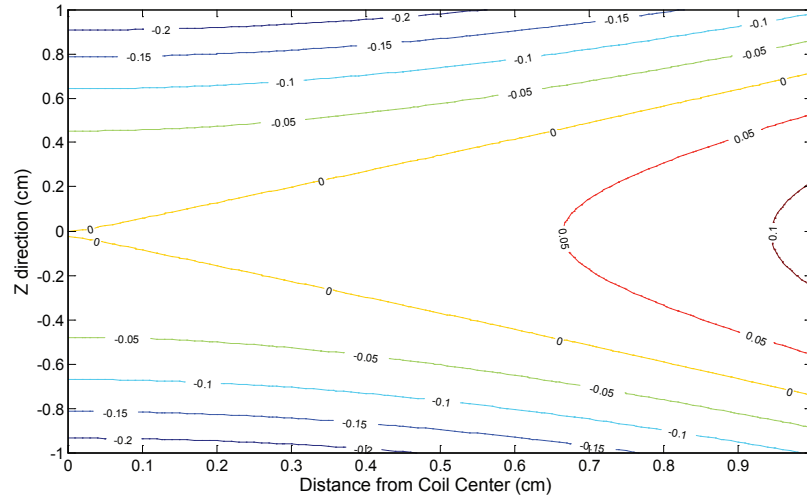


Figure A.9: A plot of the main solenoid inhomogeneity referenced to the center of the coil. This was calculated analytically, but has not been experimentally verified.

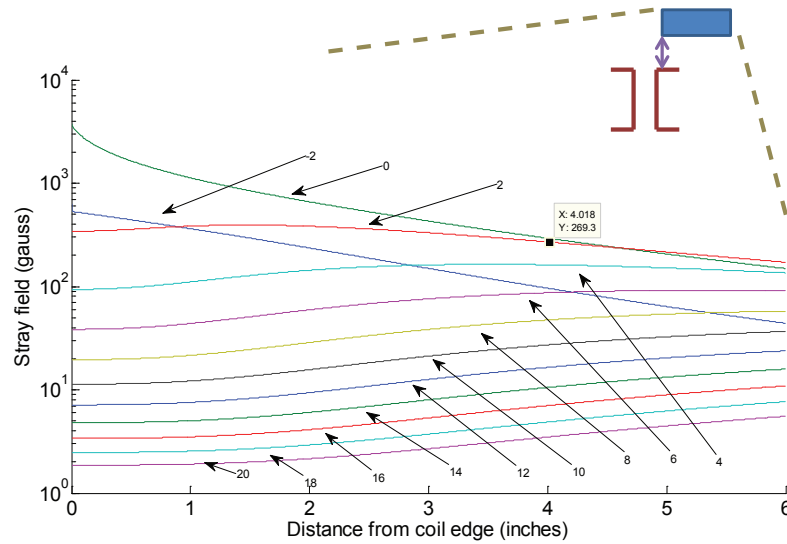


Figure A.10: A theoretical plot of the main solenoid fringing fields in Gauss as function of distance from the coil edge, for 10 A of current through the coil. Height above the coil (in inches) is noted for each curve. Again, this was calculated analytically and has not been experimentally verified.

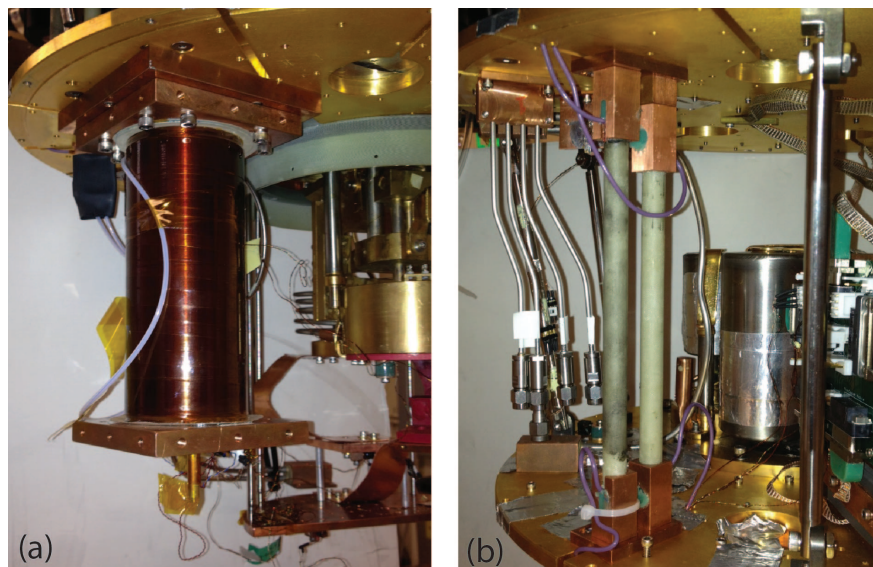


Figure A.11: (a) A picture of the main solenoid coil before the addition of the Helmholtz pairs. (b) A picture of the home-built HTC magnet leads. As described in the text, these produced just a bit too much heat load, so were replaced with commercial leads.

the leads using YBCO tape inside G10 tubing filled with Stycast. The ends were carefully heatsunk with copper blocks as shown in the picture. This tack almost worked, but the leads produced just a little too much heat load on the 4 K plate for the Chase Unit to operate reliably. The purchased leads do still add to the heat load on the 4 K plate, but not enough to affect operation. These were carefully heatsunk with copper standoffs as well, in order to make sure that superconductor is well thermalized.

A.7 Failure Modes: a.k.a. what NOT to do

The Kinda-Cold has remarkably few catastrophic failure modes for a dilution refrigerator. This is one reason why it is ideal for operation by beginning students. Here, however, is a list of things NOT to do with the fridge:

- Do NOT hit the Chase unit with any force. Do not walk into it accidentally and take steps to make sure no one else does. Go near it only very carefully with wrenches or screw drivers or ball drivers. If you puncture any of the structure, very high pressure and expensive gas will escape, and the unit will unusable. Furthermore, an explosion might result which could injure you or someone nearby.
- Do NOT vent the outer vacuum can while the pulse tube is running, and only very carefully (to nitrogen or helium only) after it has been turned off for warmup (see the

warmup section for more details). Leakage of air into the can will result in ice/liquid condensing on the pulse tube and HEMT, possibly rendering them inoperable.

- Do NOT run current to the heaters on the Chase unit while the unit is warming up or at room temperature. Double check the ADC readout to confirm these heaters are off before warmup. Currents in excess of 1 mA may burn out these delicate resistors when the unit is warm.
- Do NOT run current to the superconducting coils while the unit is warming up or at room temperature.
- Be cautious of pump carts that are used during the cool down. Do NOT move a turbo pump while it is running, as this could cause the pump blades to crash, and conservation of momentum could result in a very dangerous pump cart missile.

These items are mostly common sense, but good to keep in mind when working on and around the fridge.

A.8 Fridge Incarnations, Limitations and Future Work

This fridge went through a number of iterations before we were able to come to a configuration that works reliably, with a reasonable cool down time. In the course of the fridge construction we experimented with many different configurations, which I will describe below, by way of context for what we eventually ended up with. The main constraints of this fridge are that the base stage only has about 1 μ Watt of cooling power, a loaded base temperature of 150 mK, and a hold time of 8-9 hours before it must be recycled from 4 K (a process which takes 7 to 8 hours). For comparison, the commercial Oxford fridges have two orders of magnitude larger cooling power at base temperature which is 20-30 mK, and are run continuously without need to recycle.

The limited hold time and base temperature of the Kinda Cold limit the type of experiments that can be performed in it. Qubit measurements are not possible, but the fridge is well suited to magnetometry and amplifier characterization. The low cooling power limits the amount of wiring that can be added to the base stage, which means that the fridge is likely limited to a single sample per cooldown. Despite its limitations, however, this fridge is extremely reliable, with few catastrophic failure modes, making it ideal for beginning students. One great advantage of its reliability and user-serviceability is that it is not prone to long down-times while waiting for repair.

As shown in Figure A.1, the original fridge design had large 150 and 350 mK plates, in a similar fashion to the commercial Oxford fridges. Unfortunately, because the fridge lacked a pre-cool circuit, this configuration took 36+ hours to cool to 4 K due to the large heat mass of the plates above 77 K and limited thermal conduction between stages. To solve this

problem, and because this fridge was initially envisioned as a fast-cycle fridge, we floated designs for mechanical or large gas gap heat switches, but settled on a liquid nitrogen and helium precool circuit. We installed two versions of this pre-cool, with the last one containing heat-exchangers based on the design from the Oxford fridges. Unfortunately, the thin-walled stainless steel tubing required for the pre-cool circuit still caused too much heat load on the base stage, and we had to abandon the project.

Fortunately, by then we had decided to use the fridge for magnetometry experiments, and install a 3-axis magnet. Our design allowed for much smaller plates at 350 and 150 mK¹¹, which had proportionally smaller heat mass, and allowed for a faster initial cooldown (currently about 25 hours to 4 K). After designing and installing the magnet, 350 mK and 150 mK stages, the fridge was in its final configuration.

Future work on the fridge, other than wiring changes for a different experimental setup, will most likely include a software upgrade. As described in the fridge control section, the LabView software for the fridge could be rewritten for longer term operation.

¹¹We settled on very thin-walled and long stainless steel tubing for the plate supports. Note that we tried aluminum supports between the 150 mK and 350 mK stage, as we hoped these would go superconducting, but unfortunately these had too much heat load.

Appendix B

Measuring NVs in Diamond with 2D and 3D Cavities

In this section I will briefly describe the work we did to measure the NV centers in diamond using 2D and 3D cavities. The former was done with an eye towards coupling the NV centers to a qubit via a superconducting resonator, and I will briefly describe work towards this goal in Section B.1. Kubo et al. in Reference [6] were able to couple the NV centers to a qubit before we were able to succeed in this goal, so instead we decided to pursue another avenue of research: investigating the source of the loss that we observed when placing the diamonds on top of the 2D cavities. We found that the diamonds greatly lowered the 2D cavity quality factor (Q) when placed on top of the sample, and were not sure why this occurred. In these measurements we used a spring clip to hold the diamond in place, ruling out any loss that might come from using vacuum grease to glue the sample down (see Figure B.1). We aimed to study this loss effect by placing a variety of diamond samples into 3D cavities with fundamental modes near the NV zero-field-splitting and measuring how the the 3D cavity Q-factor was affected. This work is described in Section B.2.

B.1 2D Cavity Measurements

In this section I show data from measurements where we extracted the coupling between the NV spin ensemble and resonator as well as the spin linewidth. We measured two diamond samples that were characterized by Acosta et al. in Reference [57] and one other sample that had similar parameters. We used a fixed-frequency coplanar waveguide superconducting Nb resonator at ~ 3.2 GHz and a ‘flip-chip’ technique to place the diamond sample. We placed the diamond sample on top of the resonator, as shown in Figure B.1, and held it in place with a spring clip.

In order to observe coupling between the spins and the resonator we tuned the magnetic field until the NVs came into resonance. At this point the spin ensemble and the resonator can be modeled as two coupled harmonic oscillators and we observe an avoided crossing [36].

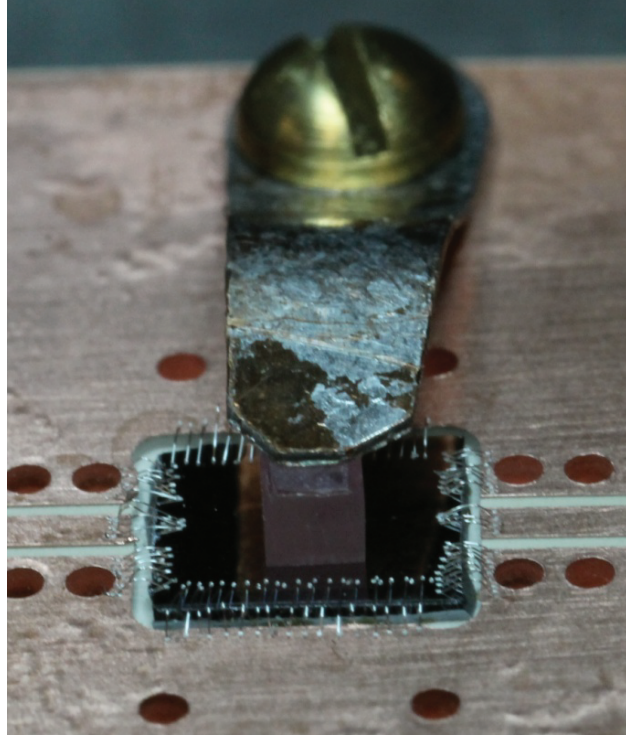


Figure B.1: Here we show a photo of the 2D resonator with the sample flip-chipped on top and held in place with a spring clip. In this photo the sample is actually ruby, but the arrangement was identical for a NV diamond.

Due to the ~ 2.8 GHz zero field splitting of the NV centers, this avoided crossing could be achieved within about 16 mT of magnetic field along the $[100]$ diamond axis. Such an avoided crossing is shown in Figure B.2.

Sample Name	Concentration (NVs/cm ³)	Spin Linewidth (MHz)	Coupling (MHz)
S6	1.5×10^{18}	5.4	2.8
S8	5.5×10^{16}	5.6	0.136
B2	$\sim 1.5 \times 10^{18}$	8.3	2.3

Table B.1: Here is a table showing the results of fits to Equation B.1. S6 and S8 were samples characterized in Reference [57], and B2 was made using an identical process to S6.

In order to extract the linewidth and coupling values we fit the real-part of the resonator S12 measurement at each magnetic field point in order to extract the resonator half-linewidth,

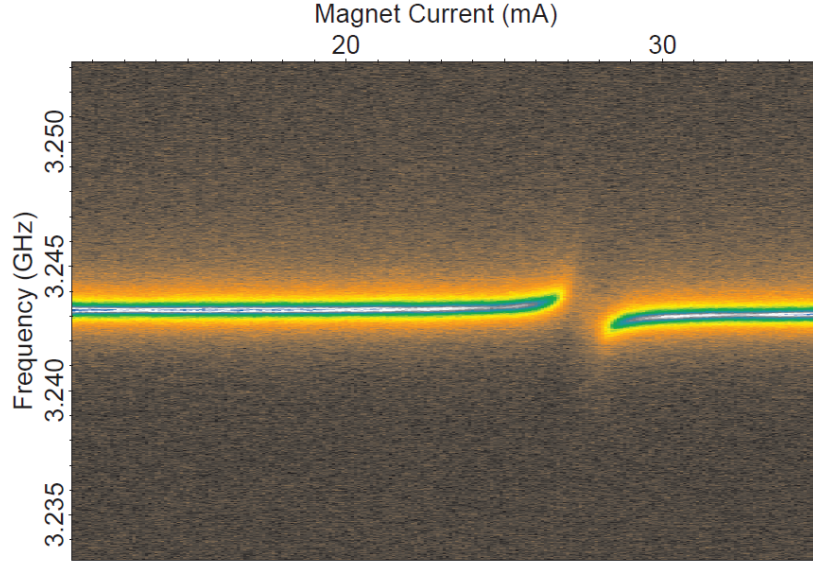


Figure B.2: In this plot we show data for the real part of a S12 measurement of a fixed frequency superconducting resonator with a NV diamond on top (Sample S6 [57]) as the magnetic field is varied. As the magnetic field tunes the spins into resonance with the resonator we see an avoided-crossing. To extract the spin linewidth and spin-cavity coupling from this data we fit the resonator profile at each magnetic field point to a Lorentzian in order to extract the cavity half-linewidth, and then fit the resulting data for κ as a function of magnetic field (Δ) to Equation B.1. For this data the resulting fit yielded $g_c = 2.8$ MHz and $\gamma_s = 5.4$ MHz.

κ . We then fit the resulting data to another Lorentzian curve, given by:

$$\kappa = \kappa_c + g_c^2 \gamma_s / (\Delta^2 + \gamma_s^2) \quad (\text{B.1})$$

where g_c is the cavity-spin ensemble coupling, κ_c is the intrinsic cavity half linewidth far away from the spins, γ_s is the spin linewidth, and Δ is the magnetic field detuning [104]. The resulting coupling and spin-linewidth values are given in Table B.1.

B.2 3D Cavity Measurements

In this section I will describe some of the preliminary measurements we did to try to figure out why we observed that several diamond samples when placed on top of the 2D resonator lowered the cavity quality factor (Q). We placed the diamonds into the center of specially machined 3D aluminum cavities, as shown in Figure B.3, with fundamental modes of ~ 3 GHz. This located the diamonds at an antinode of the electric field, thus maximizing any contribution they might have to the loss. We measured the 3D cavity Q-factor with and

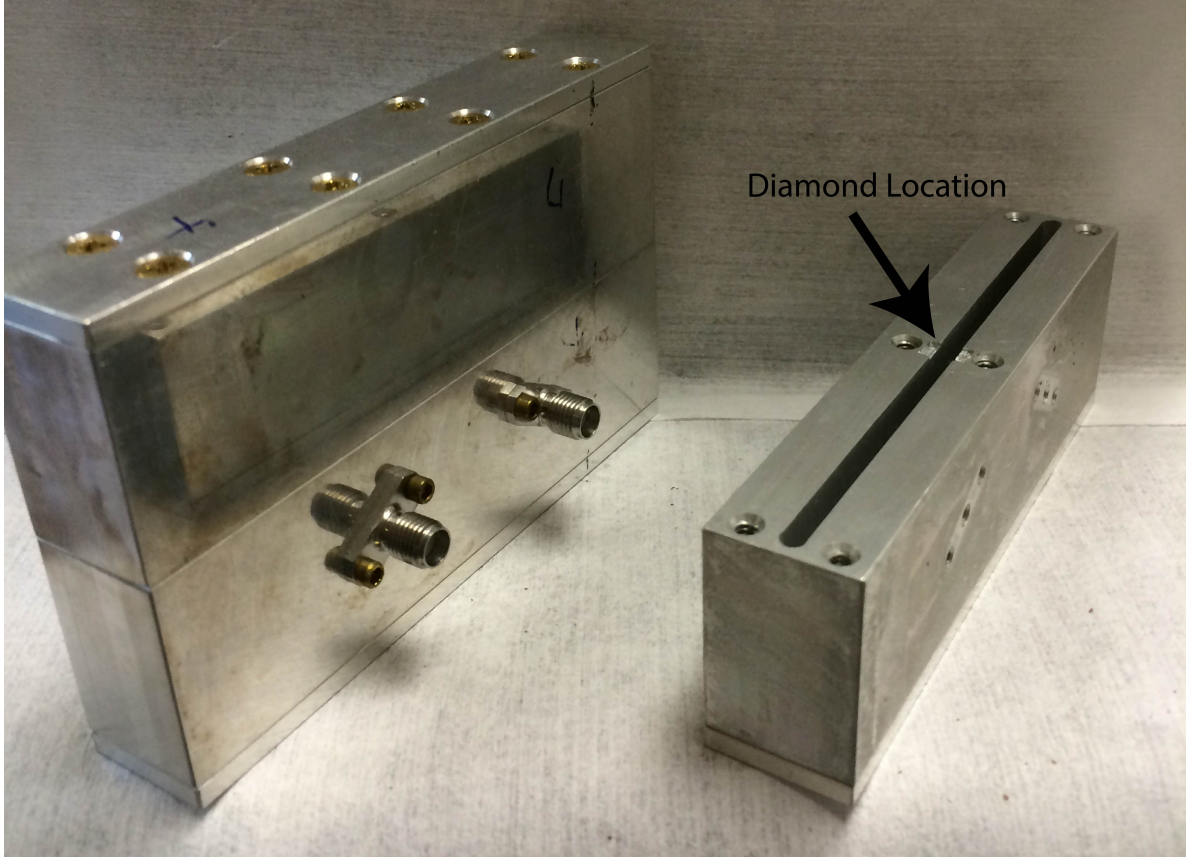


Figure B.3: Here is a photo of one of the specially machined 3D aluminum cavities that we used to measure the loss at ~ 3 GHz of diamonds with various nitrogen concentrations and in several stages of the NV production process. The diamond is placed in the milled out spot indicated by the arrow.

without the diamonds at both room temperature and 30 mK. We piranha cleaned all samples before performing these measurements, although there was some time delay (1-4 hours) before the samples were placed in vacuum and cooled.

We found that the Q of the cavities ($Q \sim 1 \times 10^6$ when empty and at 30 mK) were limited to 300-1000 when diamonds with ppm concentrations of nitrogen were placed at the electric field antinode in the cavity and the cavity was cooled to 30 mK. This result held true for one sample that had just been irradiated (CVD E6¹), two that had been both irradiated and annealed to produce NVs (CVD E6), two that were just ppm nitrogen concentration without irradiation or anneal (CVD E6), and one Type 1b E6 HPHT diamond. We found, however, that E6 PPB diamonds (1 with small N implant spot and 1 plain) did not significantly decrease the cavity Q when cooled.

¹E6 is short for Element Six which is a synthetic diamond supply company.

When the ppm nitrogen diamonds were placed at the cavity magnetic field antinode they did not affect cavity Q at room temperature (whereas these diamonds at the e-field antinode did affect the Q at room temp). We did not have the opportunity to perform this measurement at cryogenic temperatures. We concluded from these measurements that the losses observed in the 2D cavities could be mitigated by making sure that the diamonds did not cover the capacitors in the CPW line² where the electric field is concentrated.

Long after this work was completed, in a private communication with Yuimaru Kubo, who has done additional unpublished work on the subject, he suggested that the loss can be mitigated by a careful acid clean. He additionally suggested that the polish on the diamond sides is also very important. In these measurements the ppb samples were all polished on both sides of the diamond. Unfortunately, I did not keep good records for polish on all of the ppm samples, but it is highly likely that they were all only polished on one side. This surface roughness may account entirely for the additional losses that we observed with the ppm nitrogen samples in the 3D cavities rather than any dependence of the loss on the bulk nitrogen concentration.

²The resonator capacitors were small finger capacitors in the central CPW trace.

Appendix C

Magnetometer Experimental Details: Pitfalls, Limitations and Other Useful Tidbits

This section contains a summary of the work Dr. Shay Hacoen-Gourgy and I did from Sep. 2012-Aug. 2013 and describes some of the challenges we encountered during this time. In August 2013 we first successfully measured P1 centers in an NV-containing diamond, and began the work described in Chapter 5. Before that, we tried many different magnetometer, measurement sequence, and sample variations in order to try to measure a signal from a small spin ensemble. We encountered many limitations in our experimental setup and with the magnetometer itself which made these measurements trickier. I have listed some of these difficulties and subtleties encountered below in the hopes of making life easier for anyone who wishes to continue this project.

C.1 Effects of the Fast Flux Line on the SQUID

Likely the biggest issue with the magnetometer is the effect of the fast flux line on the magnetometer. We found that strong RF pulses suppress the critical current, lowering the resonant frequency and drastically changing the optimal bias parameters. After the pulse the magnetometer would eventually return to its original bias point, but this would take tens to hundreds of microseconds, likely due to trapped quasiparticles. Unfortunately, this recovery timescale is long compared to T_2 times in most solid state systems with high electron spin concentrations, (eg. P1 centers in diamond saturate to $\sim 250 \mu s$ in a crystal with 10^{19} to 10^{20} cm^{-3} of nitrogen density at 1.2 K [96]) making coherent measurement difficult.

C.2 Gradiometer

In order to solve this RF sensitivity problem we made a spatial gradiometer which would be insensitive to RF pulses but sensitive to local magnetic flux changes from spin signals. Unfortunately, insensitivity to global field changes means that this gradiometer cannot be flux biased with an external magnetic field coil. Instead, an additional DC line is required on chip to create a magnetic bias field that is asymmetric with respect to the gradiometer. Dr. Shay Hacoen-Gourgy successfully fabricated several double-loop gradiometers, although without the separate DC line. Instead he used high-critical current fast flux lines which were aligned such that they could be used for DC flux biasing. This allowed us to measure the flux noise of one device ($\sim 100 \text{ n}\Phi_0\text{Hz}^{-1/2}$) but did not solve the RF pulse sensitivity issue. We did not pursue the next step to build a functioning gradiometer with an DC bias line as the experiments described in Chapter 5 were successful with a magnetometer of regular geometry.

C.3 Limits on Spin Signal Frequency and Magnitude

The spin signal frequency should fall within the white noise floor frequency range for the best sensitivity. This starts at 10-100 kHz and ends at the bandwidth edge of 10-100 MHz. Unfortunately, in order to obtain Rabi oscillations in this frequency range a very strong drive is required, which will suppress the SQUID critical current. This makes measurement of continuous Rabi oscillations difficult. In addition if the signal is very small and has a short transverse coherence time, we will only be able to acquire signal for a short period of time. The flux noise is proportional to the inverse square root of the acquisition time so short times will have high flux noise. One should be able to average to mitigate this problem, but unfortunately if the SNR of a signal trace is $\ll 1$, then the SNR does not grow with $\sqrt{n_{\text{avg}}}$ but rather $SNR \sim 1 + \frac{1}{2} \frac{S^2}{N^2} n_{\text{avg}}$, meaning that there is only very small improvement from averaging.

C.4 Experimental Setup Issues: Pulse Tube Vibrations and Ground Noise

In addition to $1/f$ noise limiting the sensitivity at low frequencies, an additional consideration is vibrational noise. This is most prominent at high magnetic fields in setups where the magnet and sample stage are not bolted together (as in the Chase fridge) but can still be an issue even in more mechanically stable setups. The pulse tube (PT) coolers in our lab pulse at 1.4 Hz. This signal, its harmonics and additional acoustic frequencies (most prominent around 10 kHz) are apparent in all flux noise spectra. It is possible that electrical noise from the PT plays a role as well. Vibrations from a fridge turbo pump will also appear in flux noise spectra. One proposed solution to mitigate the PT vibrations and electrical noise is

to buy a driver that drives the pulse tube with a sine-wave, rather than the default pulsed drive. These drivers will void the Cryomech warranty, but also allow PT frequency tuning which can optimize cooling power. We did not try this solution, but it is a something to consider if continuing experiments in this vein.

As well, we discovered that ground noise in the setup became particularly apparent after many averages. While trying to pick out a small spin signal with many averages, we encountered tens to hundreds of spurious peaks in the Fourier spectrum. Some of the many peaks could be traced to our measurement setup or HEMT saturation, some to the pulse tube and pump vibrations and some appeared to be due to 60 Hz harmonics and other ground noise sources. The Chase fridge was relatively quiet in terms of ground noise, but it could only stay cold for 8 hours, limiting our averaging time for very small signals. The ideal setup for these sorts of measurements would be a wet fridge with minimal mechanical vibrations, and a very quiet and properly wired electrical ground and measurement chain.