Title

HIGH-ENERGY PARTICLE DATA. Volume II RANGE-ENERGY AND dE/dx PLOTS OF CHARGED PARTICLES IN MATTER

Author

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University of California

Ernest O. Lawrence Radiation Laboratory

HIGH-ENERGY PARTICLE DATA
Volume III
KINEMATICS OF PARTICLES AS A FUNCTION OF MOMENTUM

Berkeley, California

UCRL-2426
Vol. III (1963 Edition)
UC-34 Physics
TID-4500 (30th Ed.)

UNIVERSITY OF CALIFORNIA

Lawrence Radiation Laboratory
Berkeley, California

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October 1, 1964

ERRATA

TO: All recipients of UCRL-2426, Vol. III (1963 Edition)

FROM: Technical Information Division

Subject: UCRL-2426, Vol. III (1963 Edition) "High-Energy Particle Data, Volume III, Kinematics of Particles as a Function of Momentum," W. Peter Trower, July 1, 1963.

Please make the following corrections on subject report.

Page iii — Line 4 from bottom "Protons" should read Photons.

Page 2 — Eq. (a) should read

$$w = [M_1^2 + M_2^2 + 2M_2(M_1^2 + P_1^2)^{1/2}]^{1/2}$$

and Eq. (d) should read

$$p = (w_3^2 - M_3^2)^{1/2} = (w_4^2 - M_4^2)^{1/2}.$$

Page 3 — Line 11 "The term $(\gamma\cos\theta_3)\cdots$" should read The term $(\gamma p)\cdots$

Page 23 — Curves labeled "$(\gamma-1)\times 10^{-5}$" and "$(\gamma-1)\times 10^{-6}$" should be labeled $(1-\beta)\times 10^{-5}$ and $(1-\beta)\times 10^{-6}$ respectively.

Pages 69-73 — In the heading, change "Protons" to Photons.

Page 70 — Change abscissa scale to 10, 20, 30, 40, 50, 60, 70, 80, 90, 100.

Pages 74, 75, 79, 80, 84, 85, 89, 93, 97, and 101 — To upper left-hand corners, add

ηM_p and w in MeV
$1\ b = 10^{-24}\ cm^2$

Pages 76, 77, 81, 82, 86, 90, 94, 98, and 102 - To upper left-hand corners, add

ηM_p and w in MeV
1 mb = 10^{-27} cm^2

Pages 78, 83, 88, 92, 96, 100, and 104 - To upper left-hand corners, add

ηM_p and w in MeV
1 μb = 10^{-30} cm^2

Page 79 Curve labeled "$(\pi\lambda\!\!\!^{-2})$ mb" should read $(\pi\lambda\!\!\!^{-2} \times 1000)$b.

Page 82 Curve labeled "$(\pi\lambda\!\!\!^{-2} \times 10)$mb" should read $(\pi\lambda\!\!\!^{-2} \times 10)\mu$b.

Page 90 Curve labeled "$\beta \times 10^{-2}$" should read $\beta \times 10^{-1}$.

Page 106 Equation (b) at top of page should read

$$R: \quad T_1 = T\left(\frac{m_2 + T/2}{m_2 + T/2 + m_1 + T/2}\right) = T\left(\frac{m_2 + T/2}{m_1 + m_2 + T}\right)$$

Page 107 Line 15 "$\theta = 0_2 + \theta_3$" should read $\theta = \theta_2 + \theta_3$

and Eq. (b) should read

$$P_1 \sin\theta_2 = P_3 \sin\theta \leq (M_1/M_2)P_{c.m.}$$

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HIGH-ENERGY PARTICLE DATA

Volume III

KINEMATICS OF PARTICLES AS A FUNCTION OF MOMENTUM

W. Peter Trower

July 1, 1963

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HIGH-ENERGY PARTICLE DATA
Volume III
KINEMATICS OF PARTICLES AS A FUNCTION OF MOMENTUM

Contents

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Volume III
KINEMATICS OF PARTICLES AS A FUNCTION OF MOMENTUM

W. Peter Trower

Lawrence Radiation Laboratory
University of California
Berkeley, California

July 1, 1963

INTRODUCTION

The first section of this volume displays the kinematic quantities T, $(\gamma-1)$, $H\rho$, β or $(1-\beta)$, and $P\beta$, as functions of particle momentum for the most common elementary particles (the notation is given below). The second part presents the dynamic quantities $\pi\lambda^2$, ηM_p, β or $(1-\beta)$, η, $(\gamma-1)$, and w, as functions of incident-particle momentum for the most common elementary particles interacting with a proton. Σ^-, Σ^0, Ξ^0, and n were not plotted, being so close in mass to one of the particles that has been included that they cannot be distinguished graphically.

Now a word about notation. In this publication, mass, energy, and momentum are expressed in terms of energy: M is an abbreviation for mc^2, and momentum P is an abbreviation for Pc. The total energy (W) equals the kinetic energy (T) plus a rest energy (M). Capital letters refer to quantities relative to the laboratory (LAB) system, and lower-case letters refer to quantities relative to the center-of-mass (c.m.) system. The Greek letters β, γ, and η are used for the transformation quantities that relate motion of the c.m. system to the LAB system. The numerical subscripts designate the particle to which a quantity refers:

LAB	Name	c.m.
T	Kinetic energy	t
P	Momentum	p
W	Total energy	w
Θ	Angle	θ

We are concerned only with reactions of the form $M_1 + M_2 \rightarrow M_3 + M_4$, and of these possible cases we discuss only that one in which M_2 is at rest and bombarded by M_1 with a momentum P_1. The final state of the reaction is represented by M_3 and M_4.

ELLIPSE-PLOTTING METHOD FOR SOLUTION OF THE RELATIVISTIC TWO-BODY PROBLEM

A useful graphic solution for the relativistic two-body problem is known as the ellipse-plotting method.[1] One may wish to know, for a given P_1 in the LAB system, what the momenta P_3 and P_4 are that correspond to a θ_3 or θ_4 (where $\theta_3 = \pi - \theta_4$). We first perform the following calculations, recalling that the quantities described above in the Introduction, can all be obtained from the graphs constituting the bulk of this volume:

(a) Find the total energy in both systems from

$$w = \left[M_1^2 + M_2^2 + 2M_2^2\left(P_1^2 + M_1^2\right)^{1/2}\right]^{1/2},$$

where w is the invariant that is plotted in the dynamics-of-collision plots, and

$$W = \left[P_1^2 + M_1^2\right]^{1/2} + M_2 .$$

(b) Find the transformation quantities from

$$\gamma = \frac{W}{w}, \quad \beta = P_1/W, \quad \text{and} \quad \eta = \beta\gamma = \frac{P_1}{w} .$$

(c) Find the total energies of particles 3 and 4, in the c. m. system, from

$$w_3 = \frac{w^2 + M_3^2 - M_4^2}{2w}; \quad w_4 = w - w_3 .$$

(d) Find the momentum, in the c. m. system, from

$$p = p_3 = p_4, \quad p = \left(w_3^2 - M_3^2\right)^{1/2} = \left(w_4^2 - M_4^2\right)^{1/2} .$$

(e) Finally, compute ηw_3, ηw_4, and γp .

---

[1] J. Blaton, On a Geometrical Interpretation of Energy and Momentum Conservation in Atomic Collisions and Disintegration Processes, Kgl. Danske Videnskab. Selskab., Mat. -fys. Medd. 24, No. 20 (1950); and M. L. Stevenson, Reaction Dynamics for Scanners, Lawrence Radiation Laboratory, University of California, Berkeley, Alvarez Group Memo 300 (unpublished).

The construction of the ellipse follows directly from the Lorentz transformation of the c.m. to the LAB quantities:

$$\begin{pmatrix} \gamma & 0 & 0 & \eta \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ \eta & 0 & 0 & \gamma \end{pmatrix} \begin{pmatrix} p\cos\theta \\ p\sin\theta \\ 0 \\ w \end{pmatrix} = \begin{pmatrix} \gamma p\cos\theta + \eta w \\ p\sin\theta \\ 0 \\ \eta p\cos\theta + \gamma w \end{pmatrix} = \begin{pmatrix} P\cos\Theta \\ P\sin\Theta \\ 0 \\ W \end{pmatrix}.$$

Taking the quantities associated with particle 3 for our construction, we can write

$$P_x = P_3\cos\Theta_3 = \gamma p\cos\theta_3 + \eta w_3,$$
$$P_y = P_3\sin\Theta_3 = p\sin\theta_3,$$

which are simply the parametric equations of an ellipse. The term $(\gamma\cos\theta_3)$ is a measure of the eccentricity, the degree to which the circle in the c.m. system has been dilated because of momentum P_1. The quantity ηw_3 represents the translation of the center of the coordinate origin in going from the LAB to the c.m. system owing to momentum P_1. When $\theta_3 = 0$, P_x is the semimajor axis, and when $\theta_3 = \pi/2$, P_y is the semiminor axis.

Having calculated all the necessary quantities, we construct a plot by performing the following geometrical operations (see Fig. 1):

(a) Construct a Cartesian base (y ordinate and x abscissa).

(b) Draw a circle of radius p with its center at the origin.

(c) Mark off an arc on the circle corresponding to the desired θ_3.

(d) Through this point on the circle, draw a line parallel to the x axis. We then have steps (c) and (d) represented in Fig. 1 by two dashed lines.

(e) Multiply the value for x at y by γ, and plot the new point $(\gamma x, y)$.

(f) Repeat (c) through (e), picking enough points to generate an ellipse.

(g) On the x axis, locate a point (3) at a distance ηw_3 to the left of the y axis, and a point (4) at a distance ηw_4 to the right of the y axis. The distance between points (3) and (4) equals the LAB momentum of the bombarding particle P_1, and is the vector sum of $\vec{P}_3$ and $\vec{P}_4$.

(h) Draw verticals through points (3) and (4), and locate points (3') and (4') at distance M_3 and M_4 above (3) and (4) respectively.

(i) From points (3) and (4) draw momentum vectors $\vec{P}_3$ and $\vec{P}_4$ to points on the ellipse corresponding to the c.m. angles desired.

(j) Measure LAB angles Θ_3 and Θ_4 with a protractor.

(k) If desired, $\vec{P}_3$ and $\vec{P}_4$ can be measured, and their values converted to T_3 and T_4 respectively, by reference to the graphs. Or alternatively, T_3 and T_4 can be ascertained by construction by applying Pythagoras' theorem as follows:

Place one tip of a pair of dividers on point (3), and extend the other tip a distance P_3 on the ellipse. Pivot the dividers about point (3) until the other tip touches the major axis of the ellipse, at point (a). Then pivot the dividers about point (a), and open them so that the other tip is on point (3'). The dividers now show a separation

$$W_3 = \left[P_3{}^2 + M_3{}^2\right]^{1/2}.$$

Next, pivot the dividers about point (3') to where the other tip is vertically below points (3') and (3), at point (b). Point (b) then lies at a distance T_3 below point (3), since $T_3 = W_3 - M_3$. Now an energy scale plotted downward from point (3) permits a direct reading of T_3. This procedure, once mastered, is simple to perform. It is demonstrated in Fig. 2.

It is interesting to note that in the extreme relativistic limit a particle going backward in the c.m. system (i.e., $\theta_3 = \pi$) approaches a constant momentum in the LAB system; namely,

$$(M_3{}^2 - M_2{}^2) / 2M_2 .$$

Fig. 1

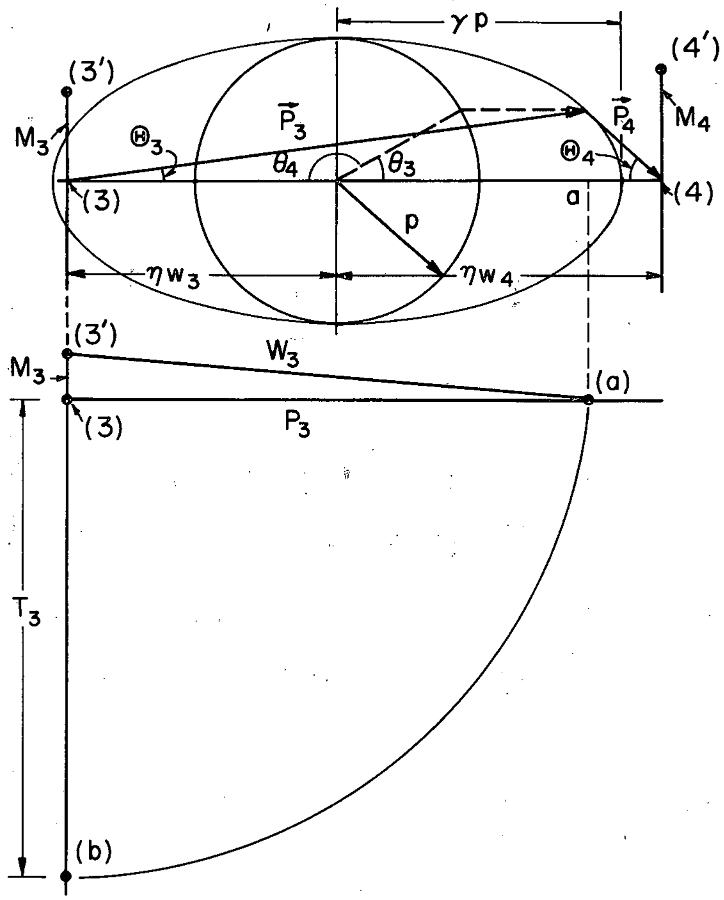

Fig. 2

MU-29966

Figs. 1-2. Typical diagram for a reaction of the type $\pi^- + p \rightarrow K^+ + \Sigma^-$. M_1 is the bombarding particle with kinetic energy T_1. M_3 and M_4 are the rest masses of the resulting particles.

ACKNOWLEDGMENTS

The author is indebted to Dr. Arthur H. Rosenfeld, Dr. M. Lynn Stevenson, and Dr. Robert D. Tripp for their comments and suggestions during the preparation of this collection of graphs. The actual presentation was made possible through the patient art work of Mr. Robert Stevens and the tireless editing of Mr. Howard Rogers.

This work was done under the auspices of the U. S. Atomic Energy Commission.

OTHER VOLUMES IN THE UCRL-2426 SERIES

Other volumes available in the UCRL-2426 series are:

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Vol. II (1963 Revision), Range and Energy (and Momentum) Plots of Passage of Particles through Matter, October 1963.

UCRL-2426 Vol. III
(1963 Edition)

KINEMATICS OF ELEMENTARY PARTICLES

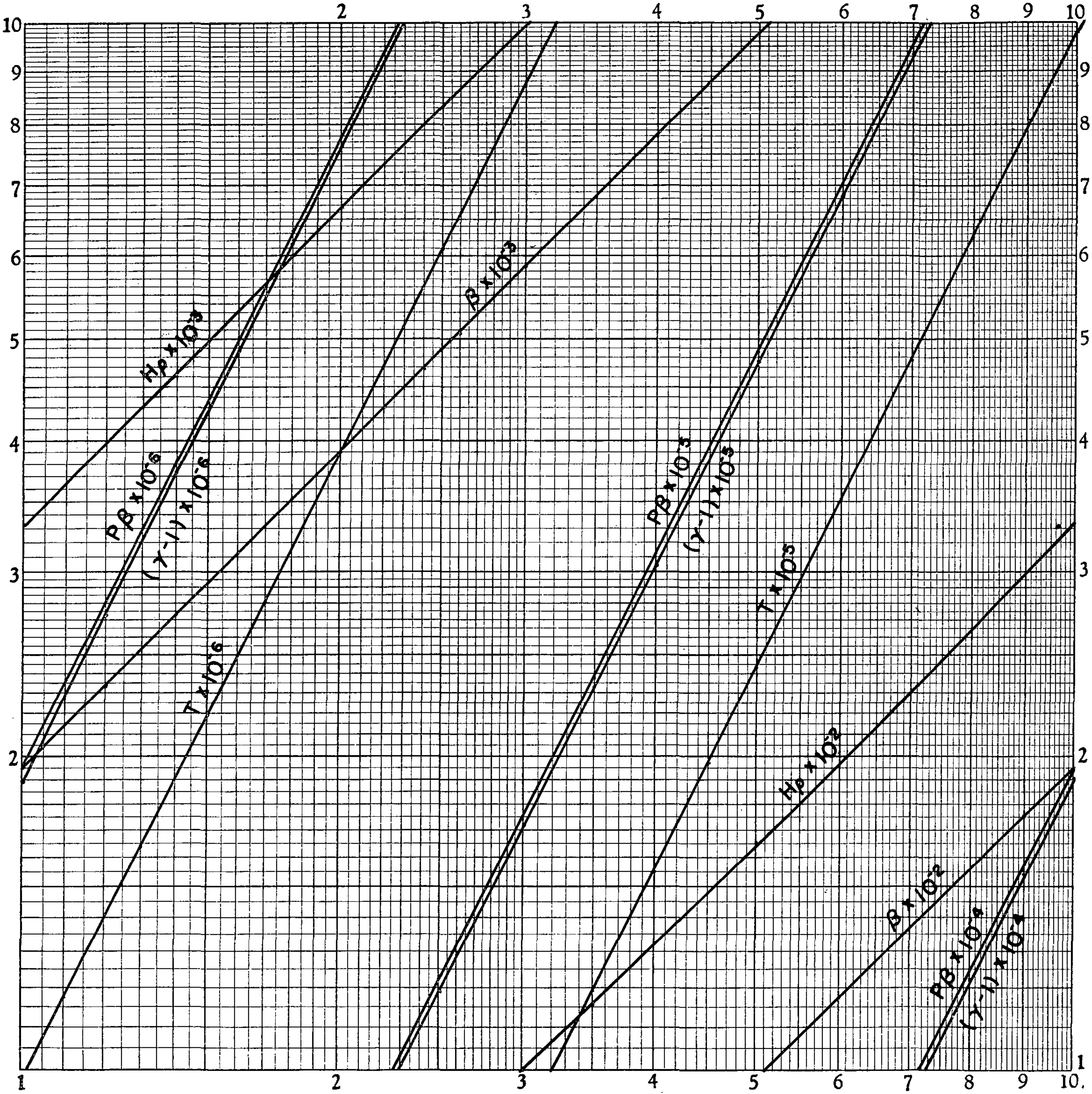
ELECTRONS

1 keV/c to 10 keV/c

β, (γ-1), T, Pβ, Hρ

T in MeV
Hρ in kgauss-cm
Pβ in MeV/c

M_e = 0.510976 MeV
= 1 m

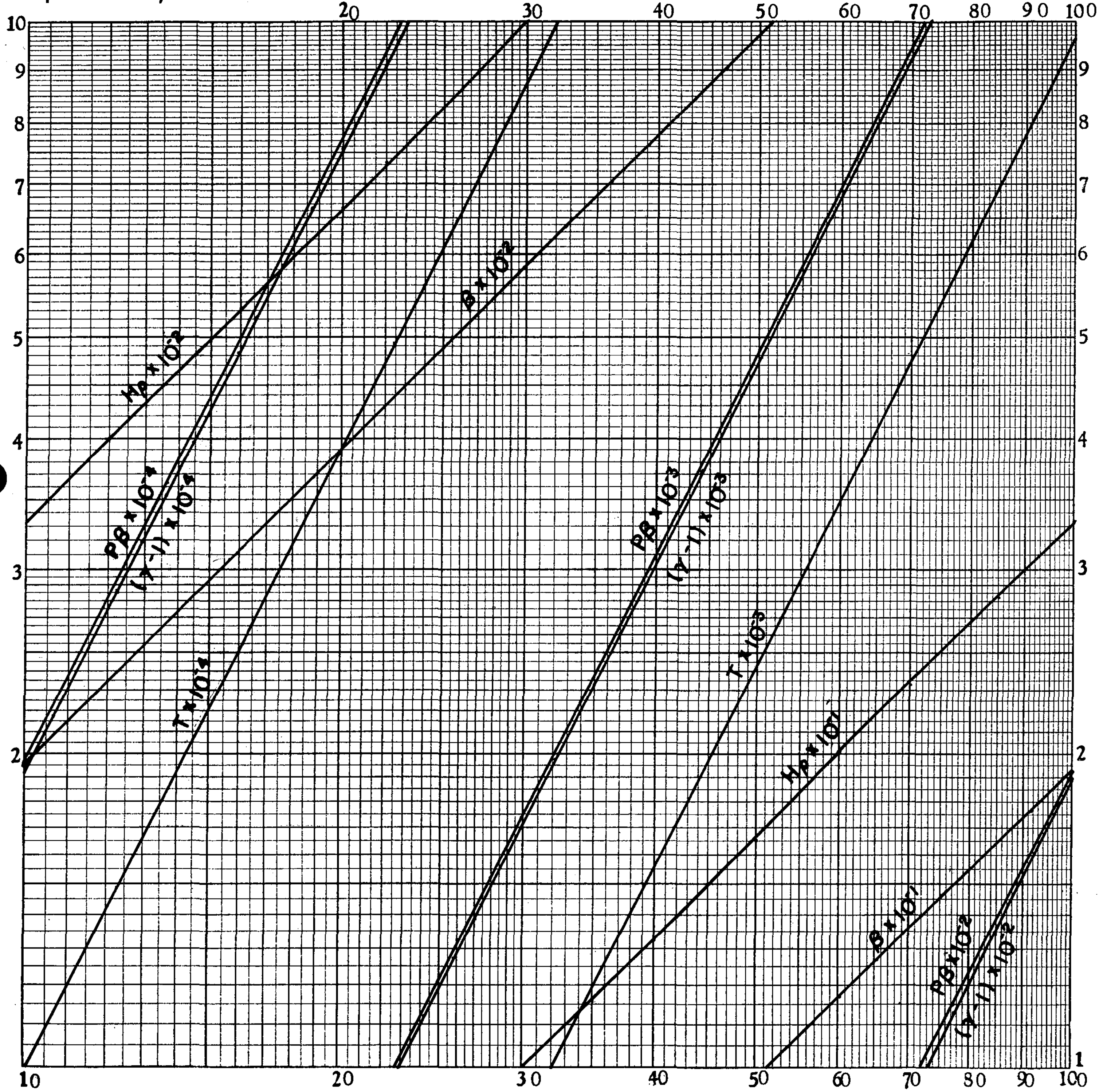


ELECTRONS

T in MeV
Hρ in kgauss-cm
Pβ in MeV/c

10 keV/c to 100 keV/c
β, (γ-1), T, Pβ, Hρ

M_e = 0.510976 MeV
= 1 m



P (keV/c)

ELECTRONS

100 keV/c to 1 MeV/c

β, $(1-\beta)$, $(\gamma-1)$, T, $P\beta$, $H\rho$

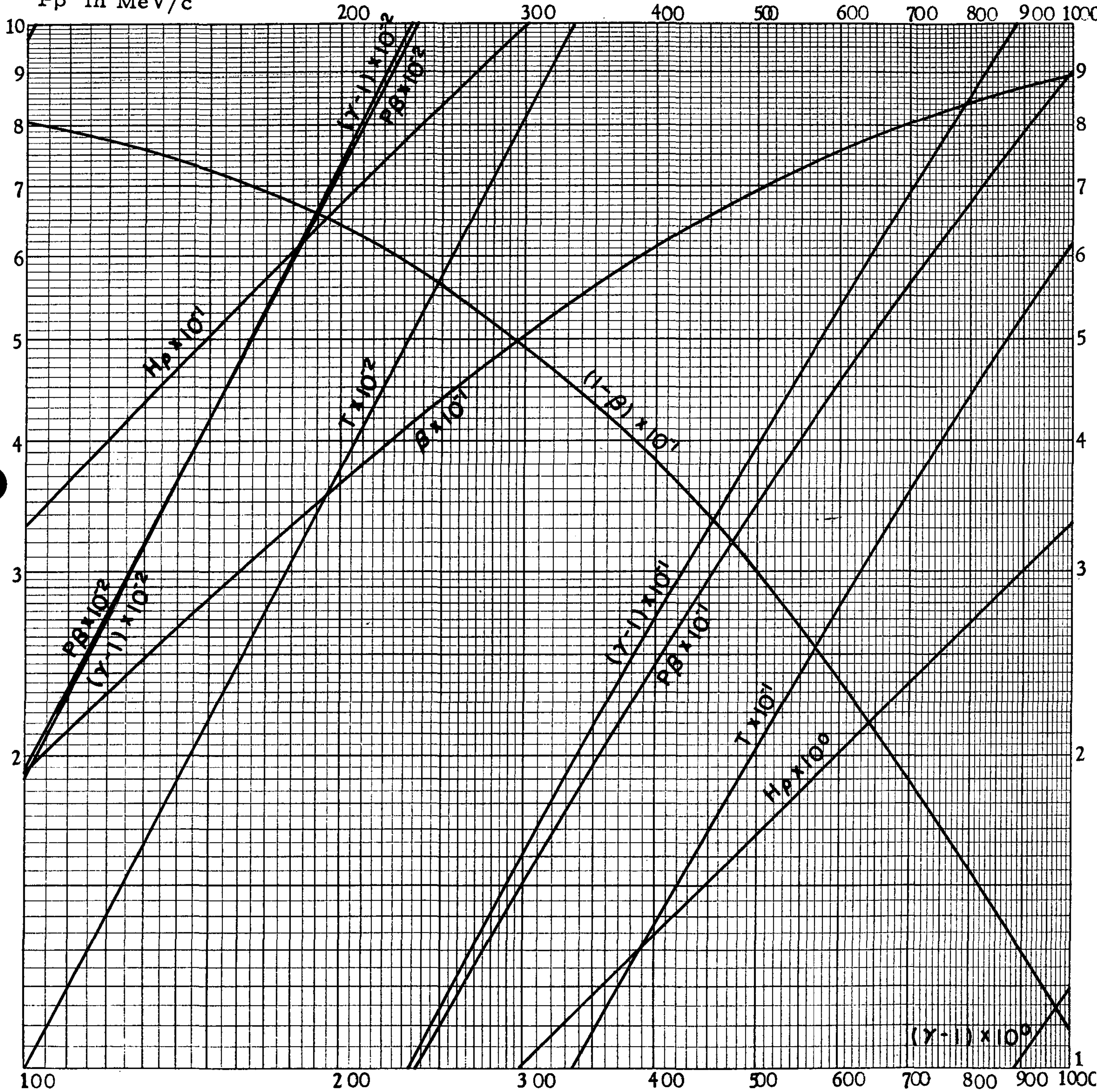
T in MeV

$H\rho$ in kgauss-cm

$P\beta$ in MeV/c

M_e = 0.510976 MeV

= 1 m



P (keV/c)

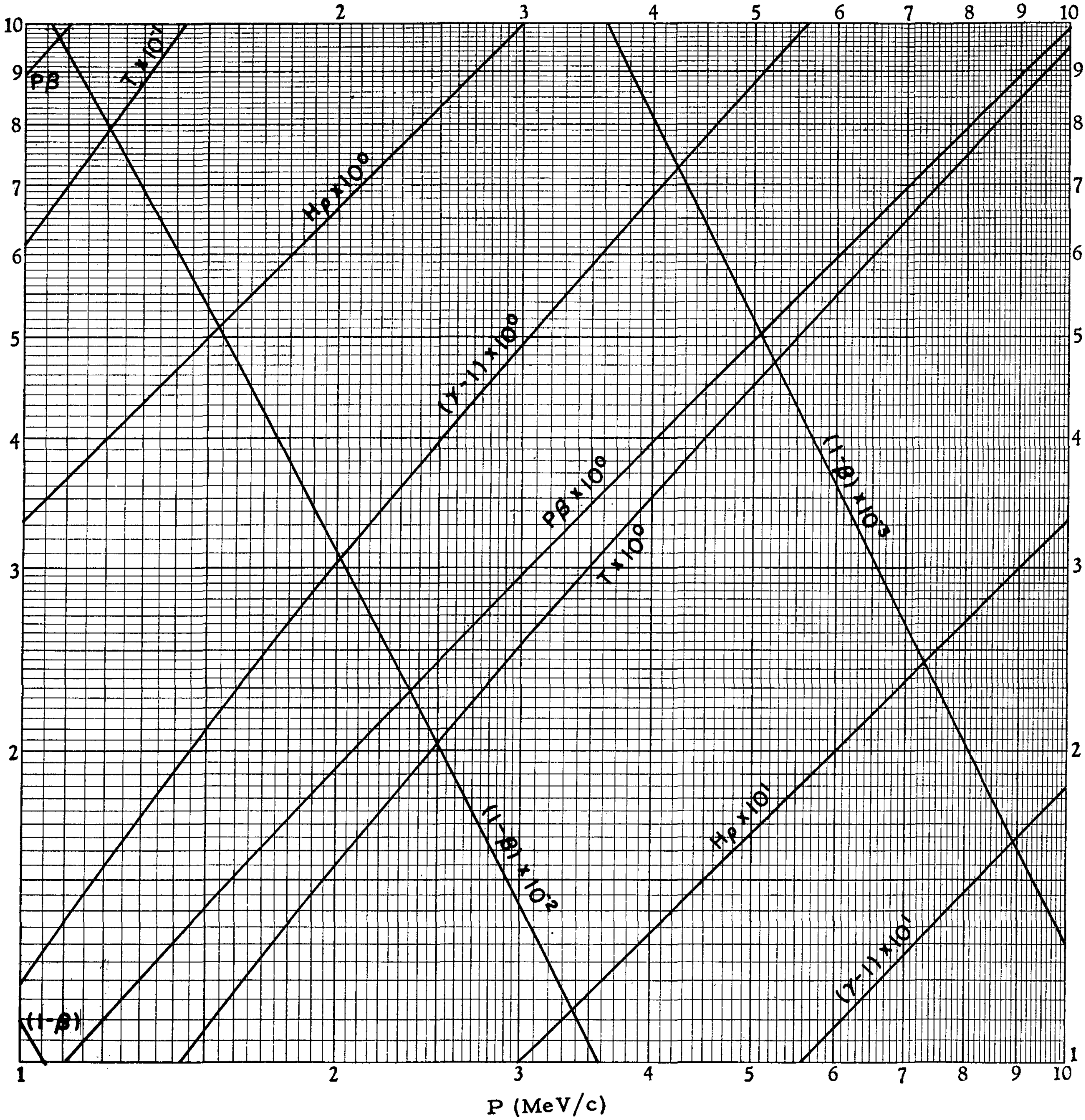
ELECTRONS

T in MeV
Hρ in kgauss-cm
Pβ in MeV/c

1 MeV/c to 10 MeV/c

(1 - β), (γ - 1), T, Pβ, Hρ

M_e = 0.510976 MeV
= 1 m



0 0 1 0 1 2 0 4 7 1 0

T in MeV

Hρ in kgauss-cm

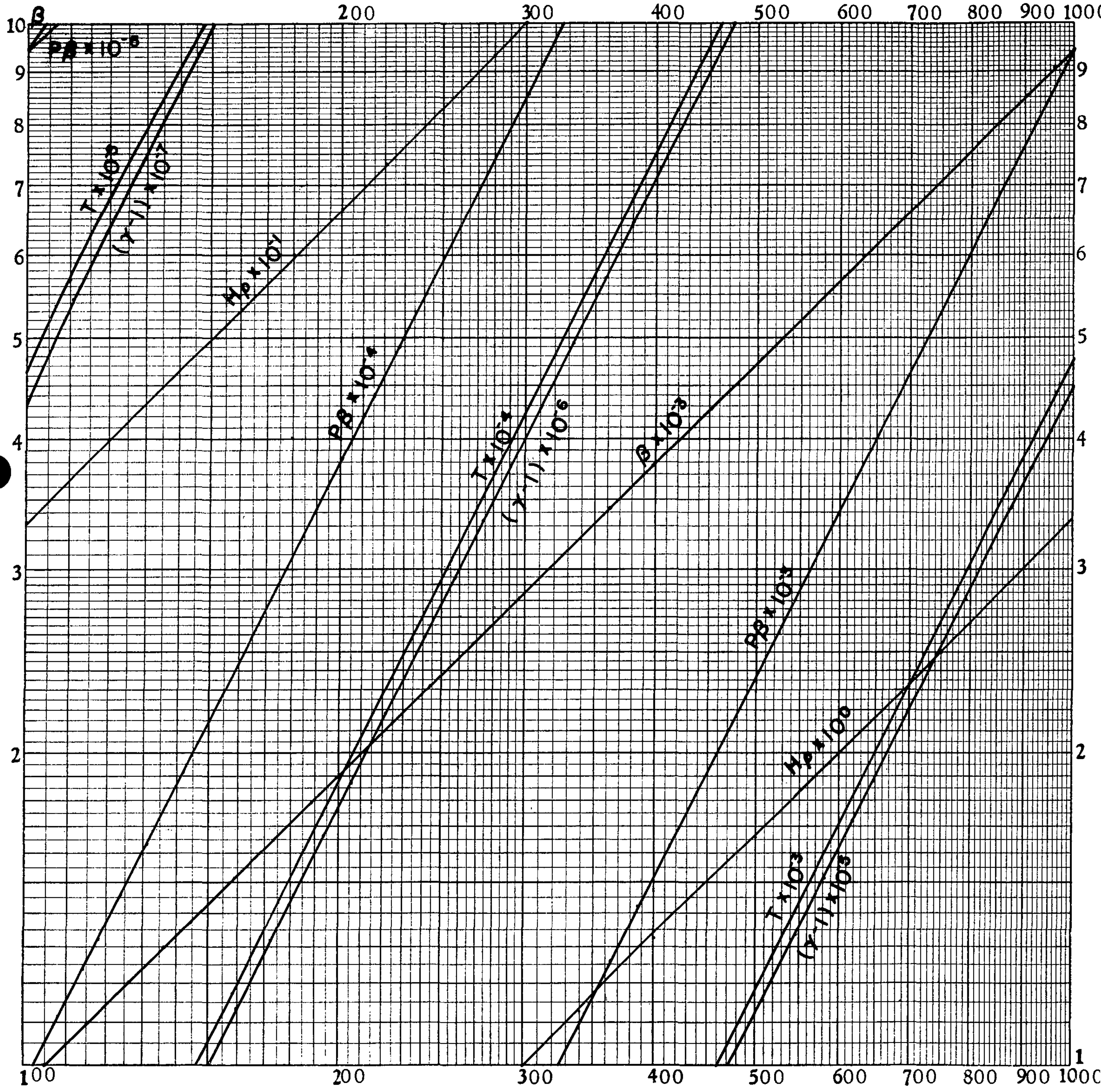
Pβ in MeV/c

100 keV/c to 1 MeV/c

β, (γ - 1), T, Pβ, Hρ

$M_{\mu^{\pm}}$ = 105.655 MeV

= 206.77 m



P (keV/c)

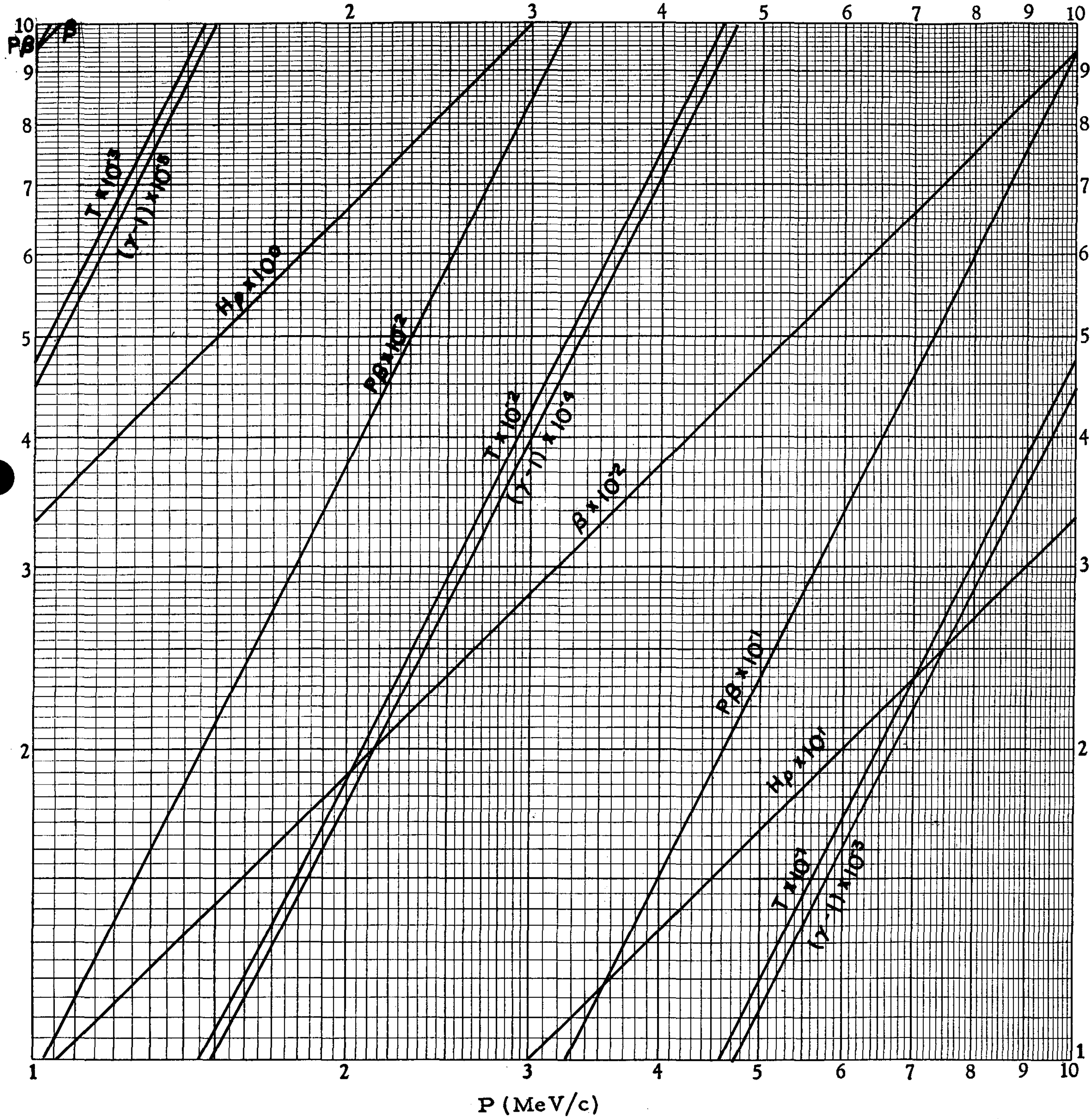
$\mu^{\pm}$ LEPTONS

1 MeV/c to 10 MeV/c

β, (γ - 1), T, Pβ, Hρ

T in MeV
Hρ in kgauss-cm
Pβ in MeV/c

$M_{\mu^{\pm}}$ = 105.655 MeV
= 206.77 m



T in MeV

Hρ in kgauss-cm

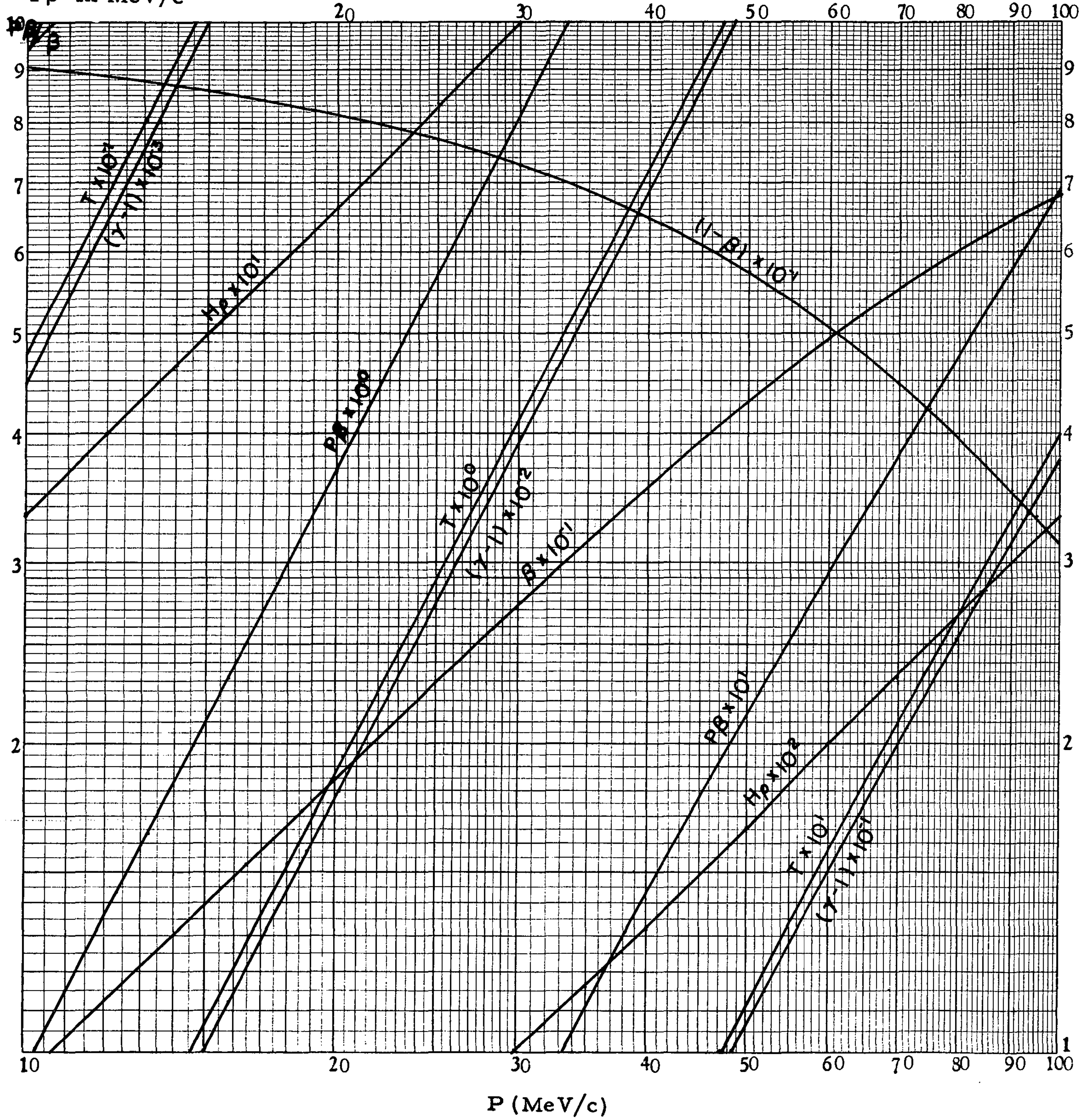
Pβ in MeV/c

10 MeV/c to 100 MeV/c

β, (1 - β), (γ - 1), T, Pβ, Hρ

$M_{\mu^\pm}$ = 105.655 MeV

= 206.77 m



$\mu^\pm$ LEPTONS

100 MeV/c to 1 BeV/c

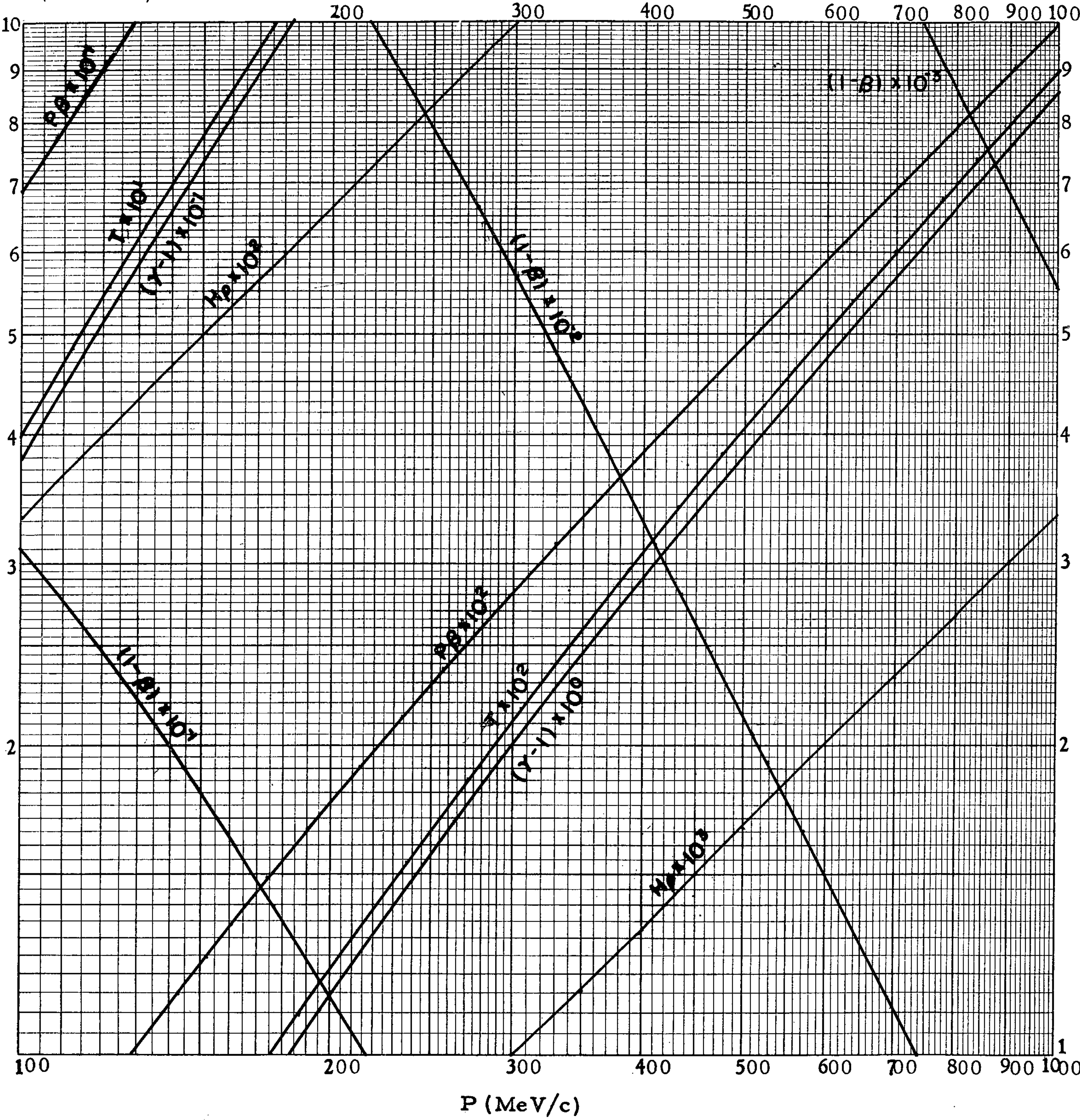
(1 - β), (γ - 1), T, Pβ, Hρ

T in MeV

Hρ in kgauss-cm

Pβ in MeV/c

$M_{\mu^\pm}$ = 105.655 MeV
= 206.77 m



$\mu^{\pm}$ LEPTONS

T in MeV

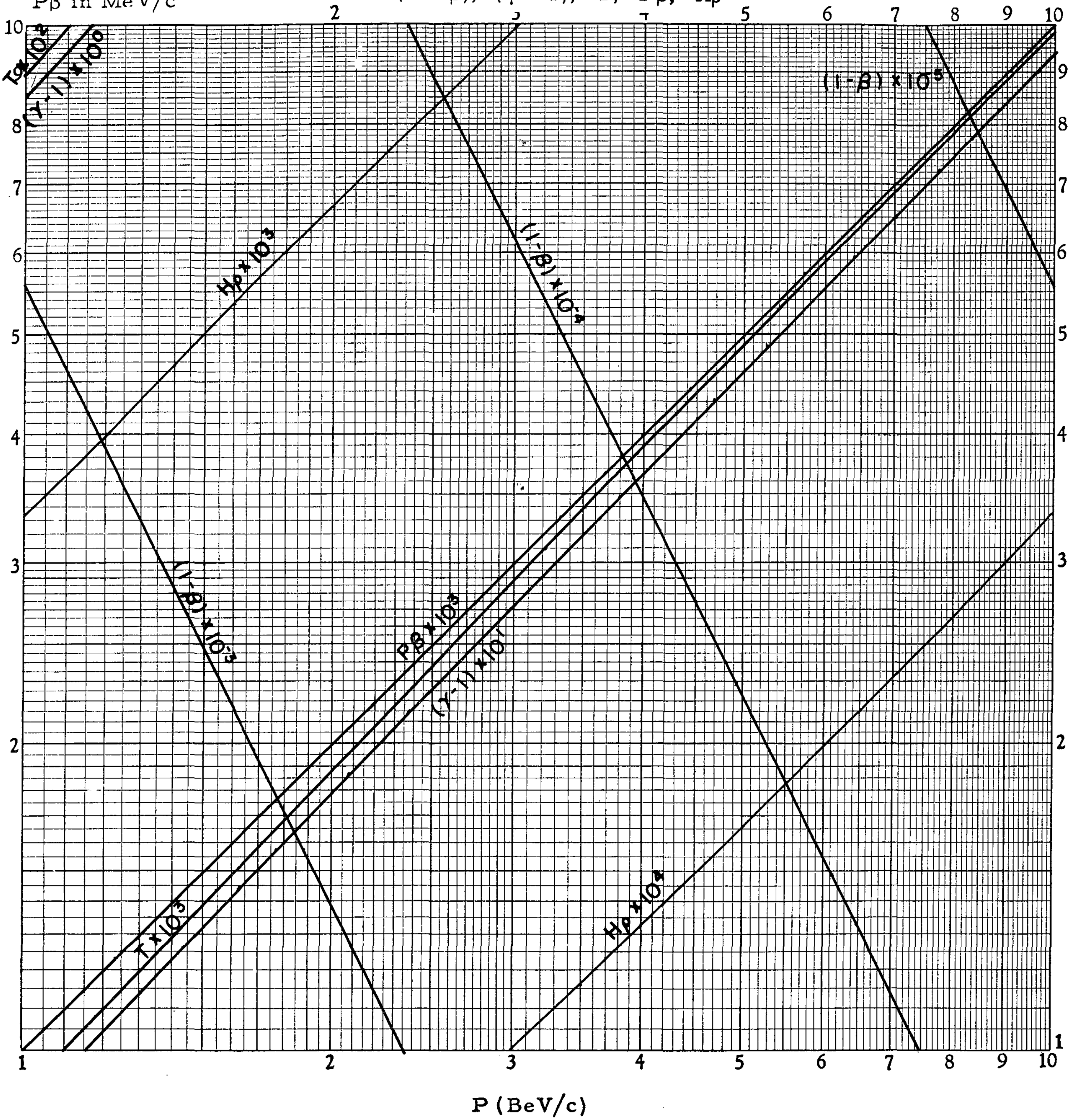
Hρ in kgauss-cm

Pβ in MeV/c

1 BeV/c to 10 BeV/c

(1 - β), (γ - 1), T, Pβ, Hρ

$M_{\mu^{\pm}}$ = 105.655 MeV
= 206.77 m



$\mu^{\pm}$ LEPTONS

T in MeV

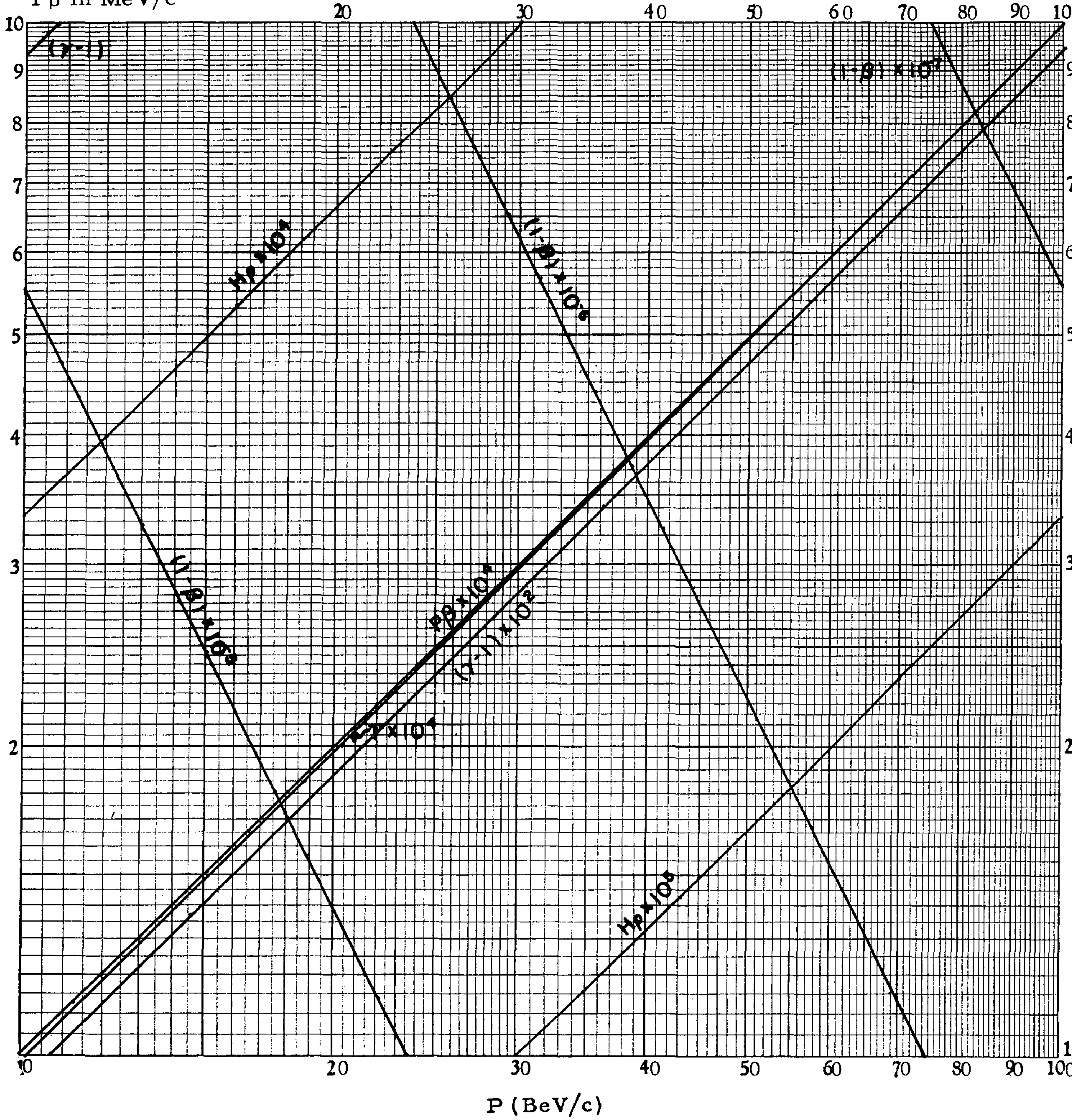
Hρ in kgauss-cm

Pβ in MeV/c

10 BeV/c to 100 BeV/c

(1 - β), (γ - 1), T, Pβ, Hρ

$M_{\mu^{\pm}}$ = 105.655 MeV
= 206.77 m



π^0 MESONS

100 keV/c to 1 MeV/c

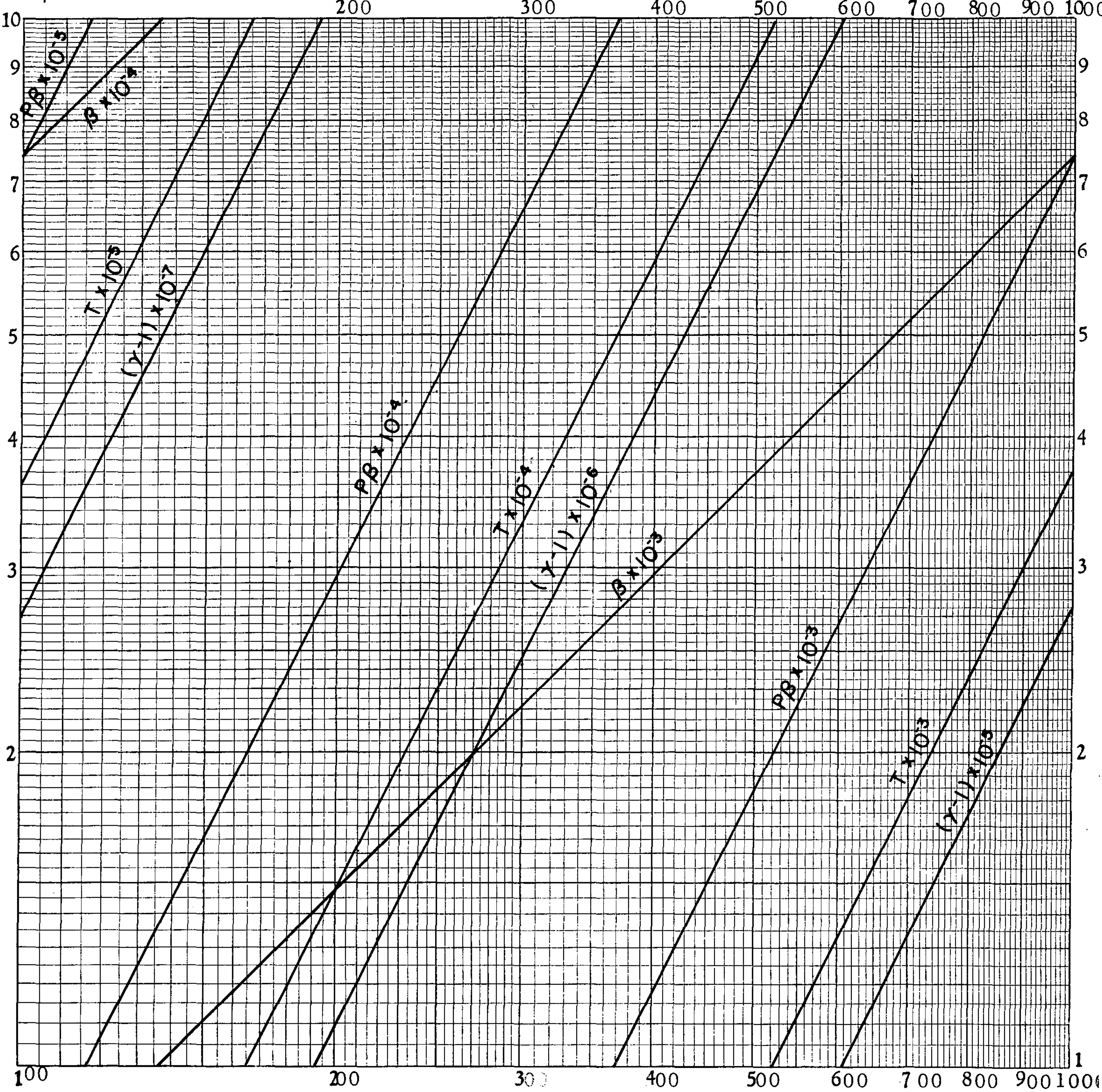
β, $(\gamma - 1)$, T, Pβ

M_{π^0} = 135.00 MeV

= 264.42 m

T in MeV

Pβ in MeV/c



P (keV/c)

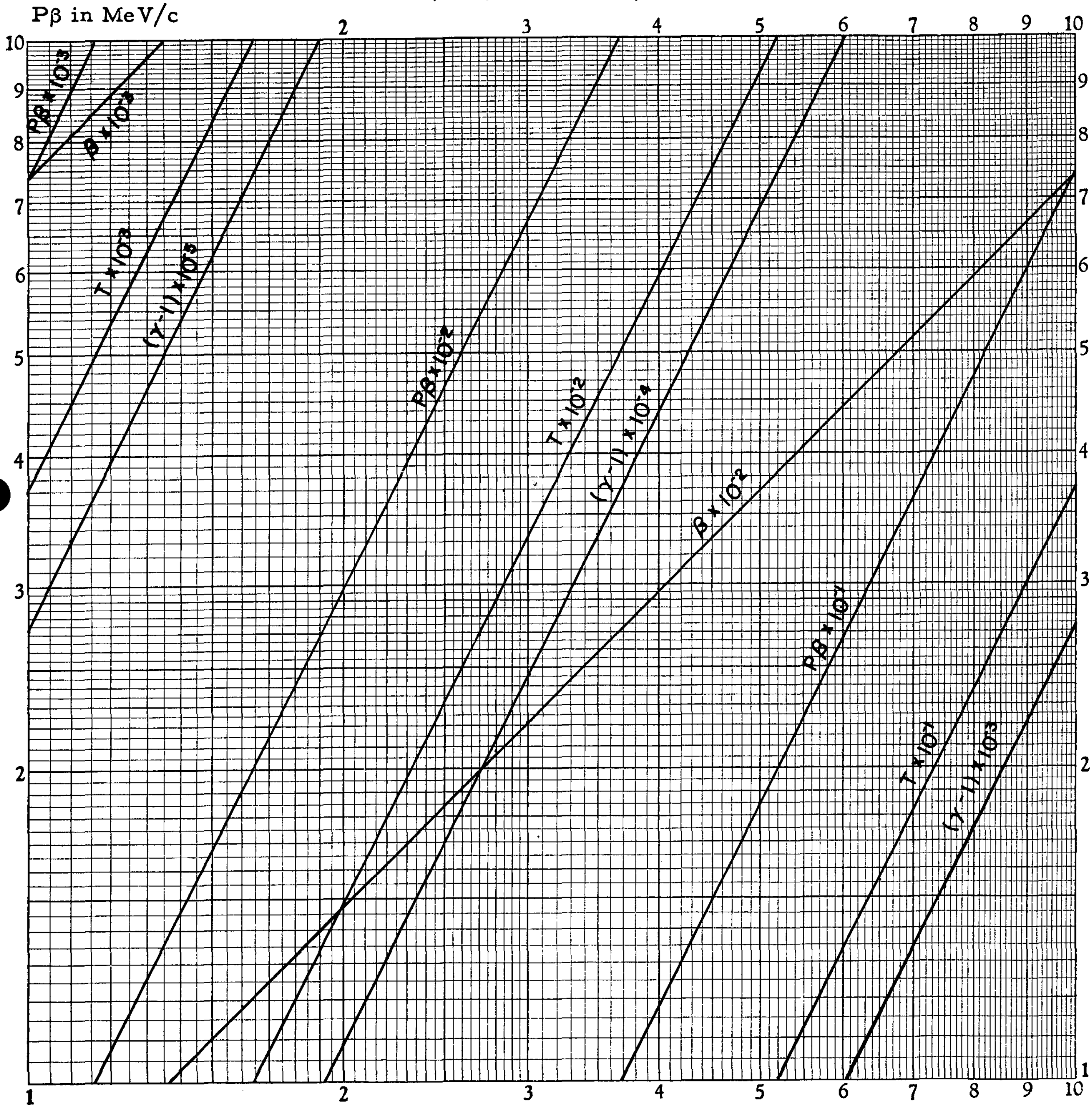
1 MeV/c to 10 MeV/c

β, $(\gamma - 1)$, T, $P\beta$

M_{π^0} = 135.00 MeV = 264.42 m

T in MeV

$P\beta$ in MeV/c



P (MeV/c)

π^0 MESONS

10 MeV/c to 100 MeV/c

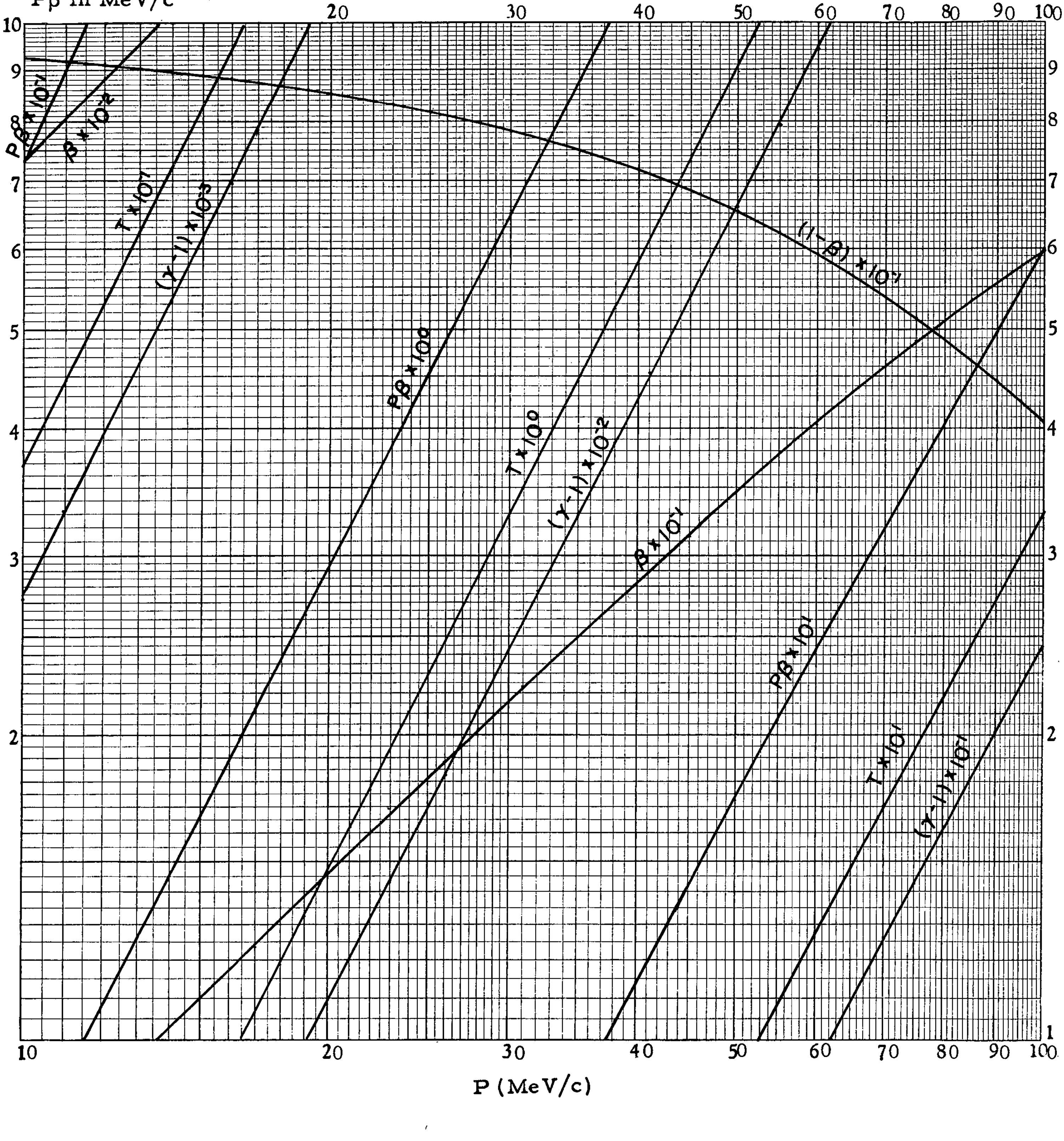
β, (1 - β), (γ - 1), T, Pβ

M_{π^0} = 135.00 MeV

= 264.42 m

T in MeV

Pβ in MeV/c



P (MeV/c)

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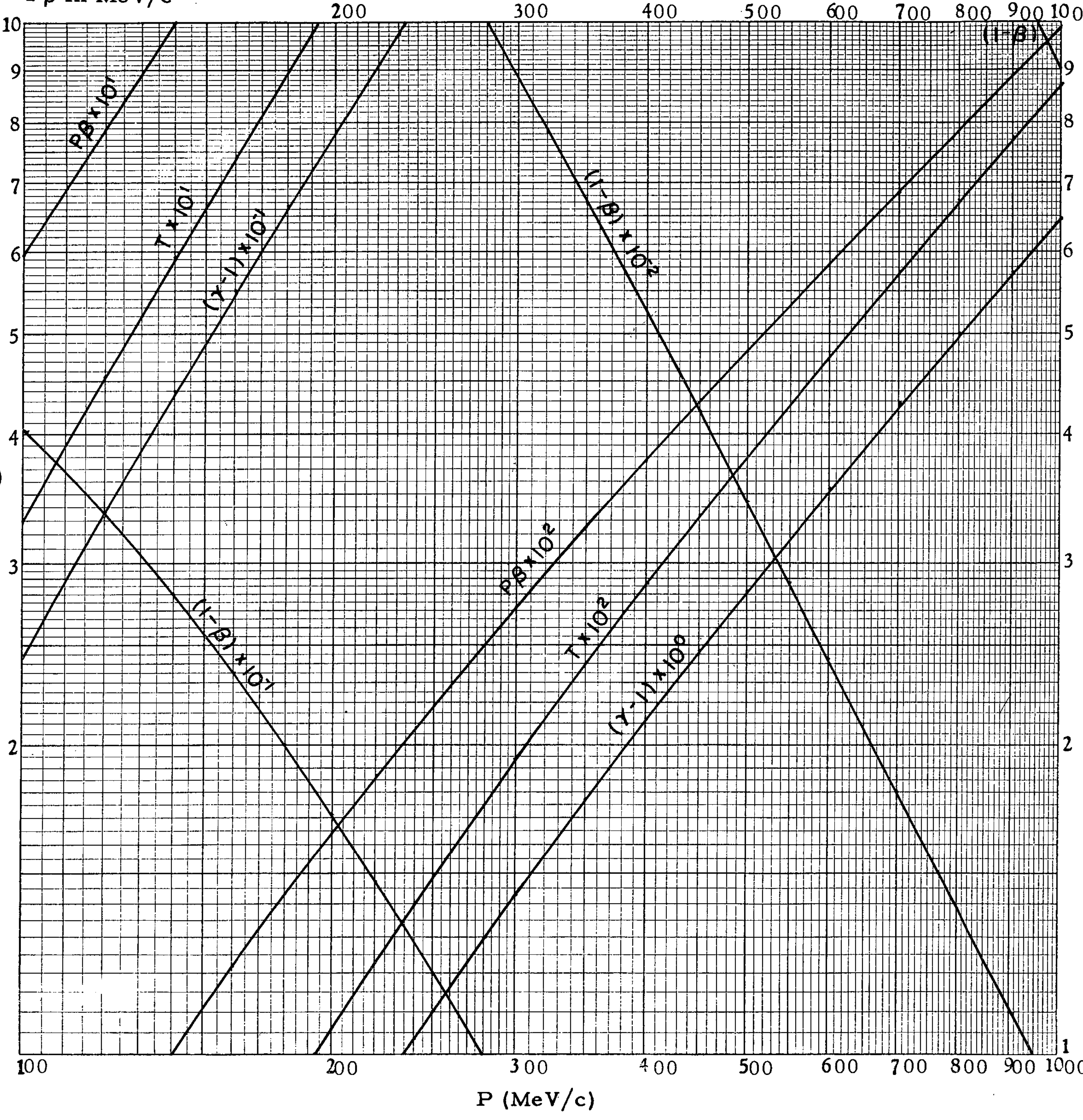
π^0 MESONS

100 MeV/c to 1 BeV/c

(1 - β), (γ - 1), T, Pβ

M_{π^0} = 135.00 MeV
= 264.42 m



π^0 MESONS

1 BeV/c to 10 BeV/c

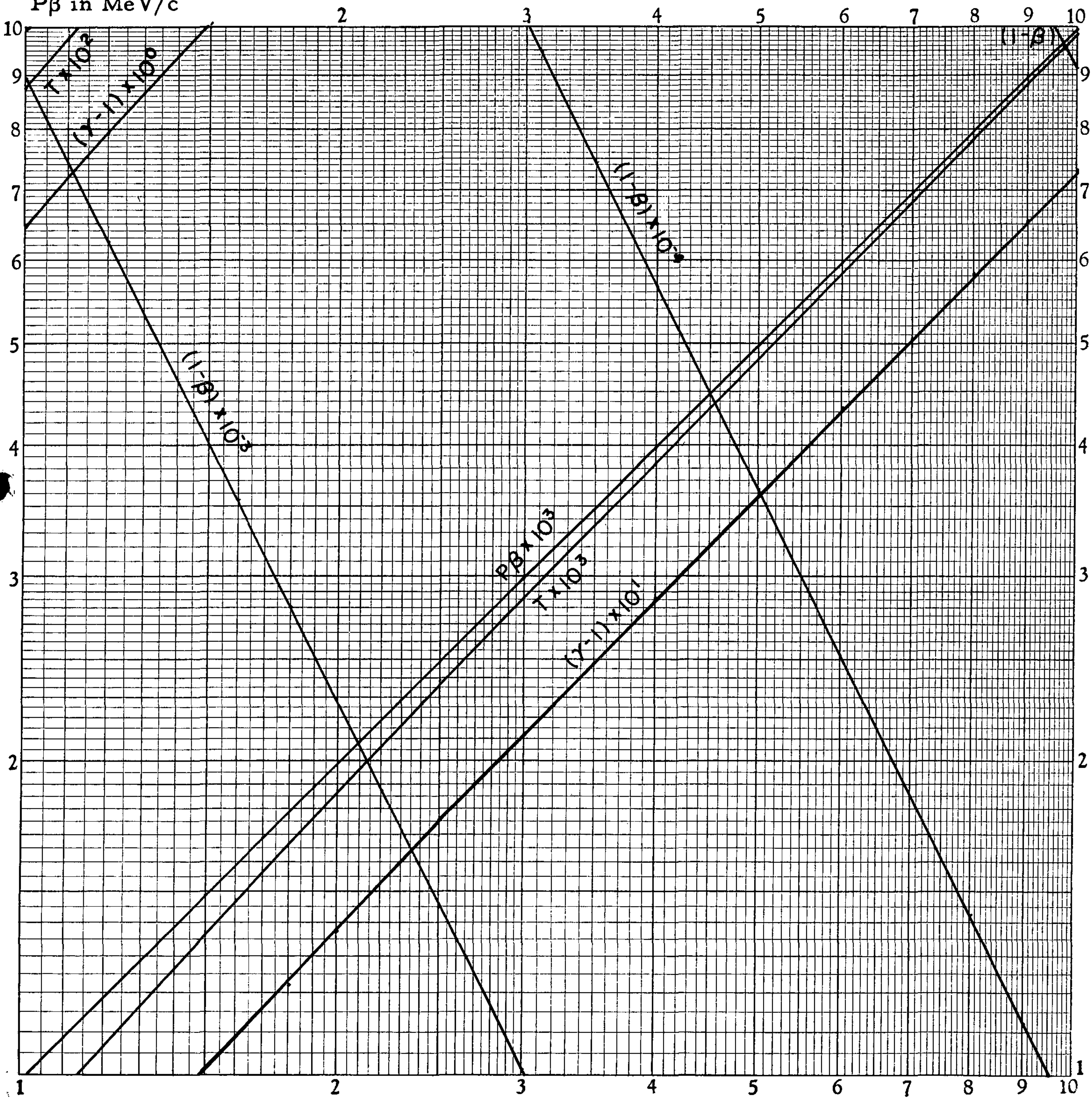
M_{π^0} = 135.00 MeV

= 264.42 m

T in MeV

Pβ in MeV/c

β, (1 - β), (γ - 1), T, Pβ



P (BeV/c)

π^0 MESONS

10 BeV/c to 100 BeV/c

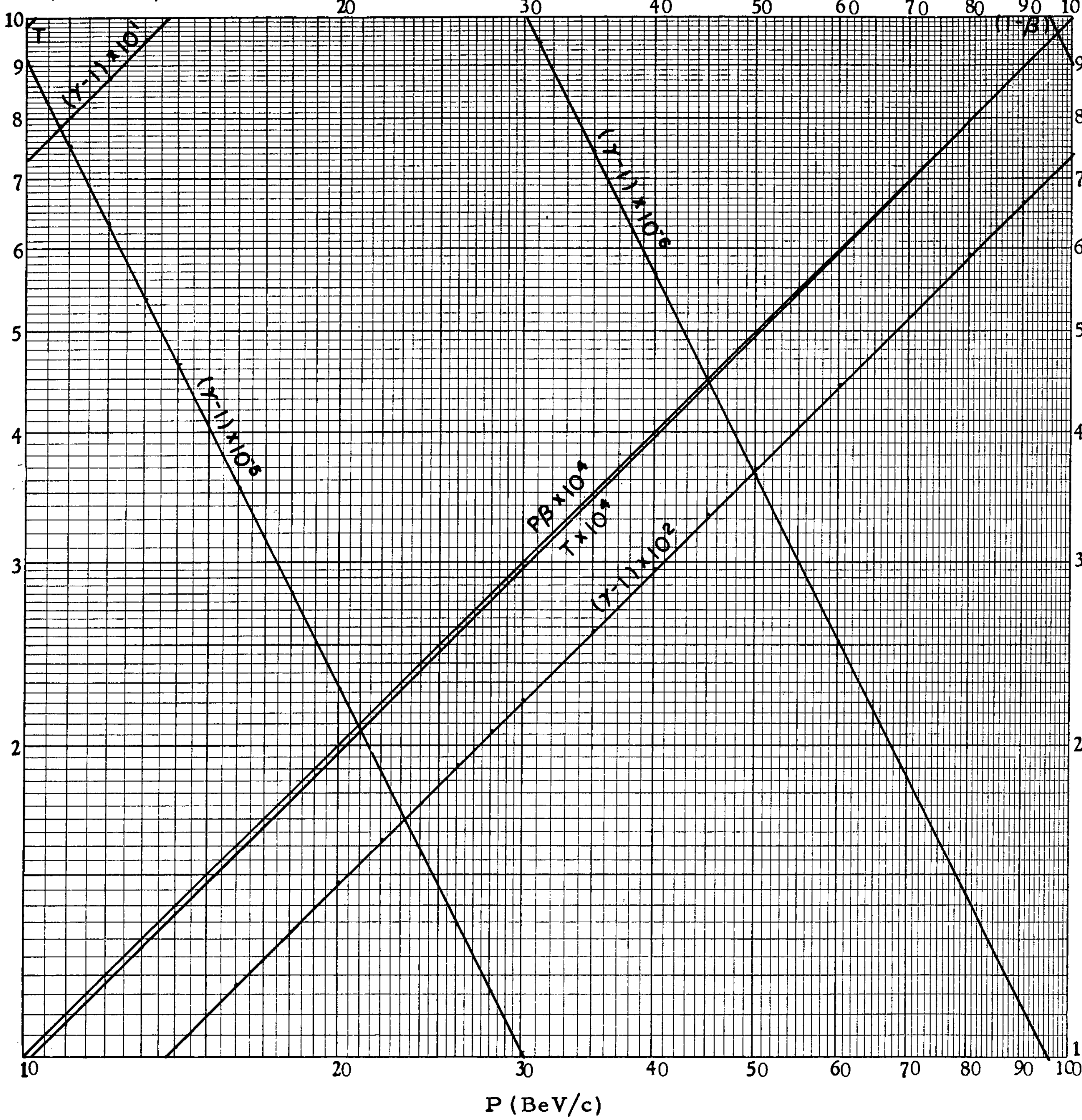
β, $(1-\beta)$, $(\gamma-1)$, T, $P\beta$

T in MeV

$P\beta$ in MeV/c

$M_{\pi 0}$ = 135.00 MeV

= 264.42 m



$\pi^{\pm}$ MESONS

T in MeV

Hρ in kgauss-cm

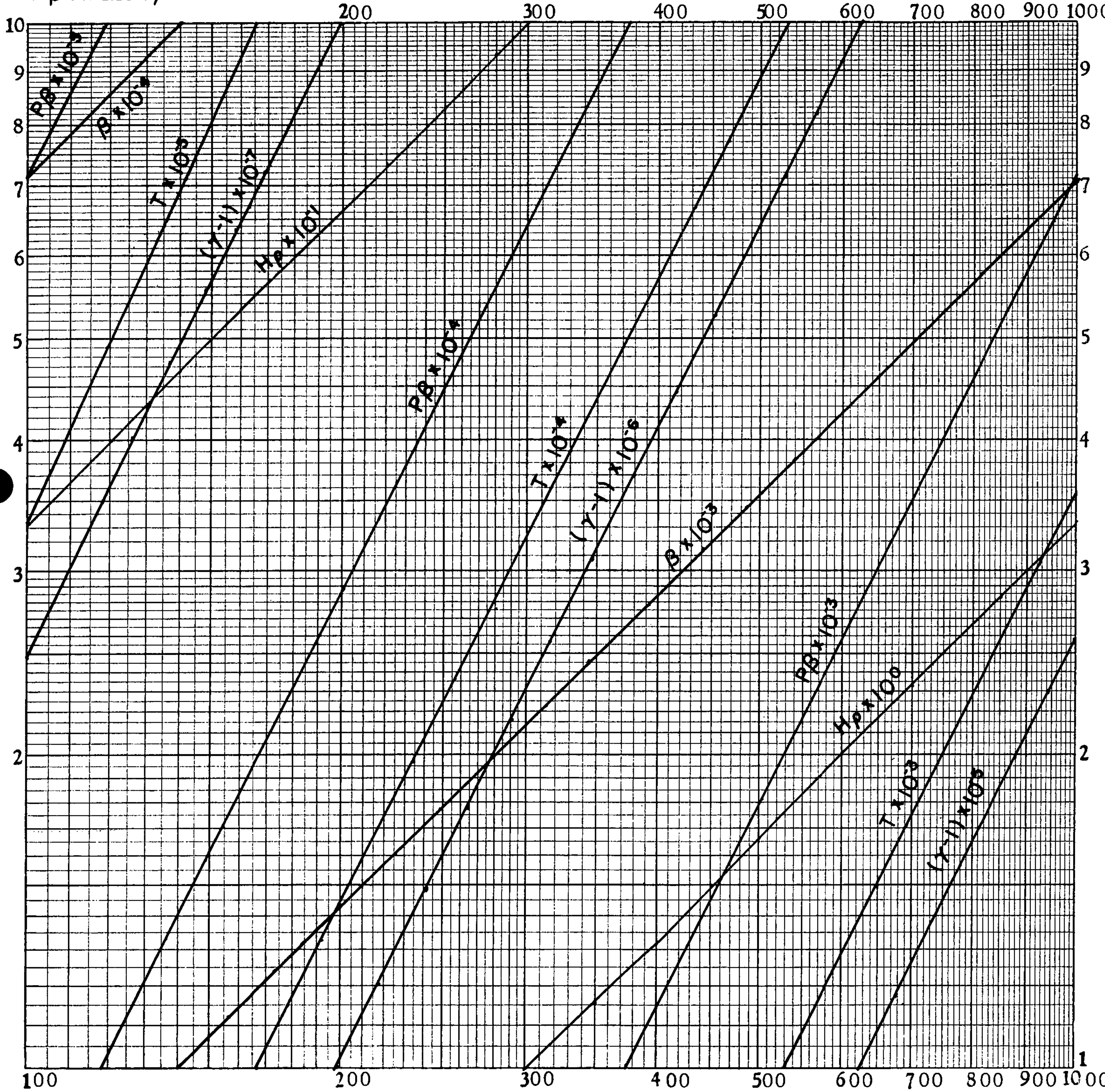
Pβ in MeV/c

100 keV/c to 1 MeV/c

β, (γ - 1), T, Pβ, Hρ

$M_{\pi^{\pm}}$ = 139.59 MeV

= 273.18 m



P (keV/c)

T in MeV

Hρ in kgauss-cm

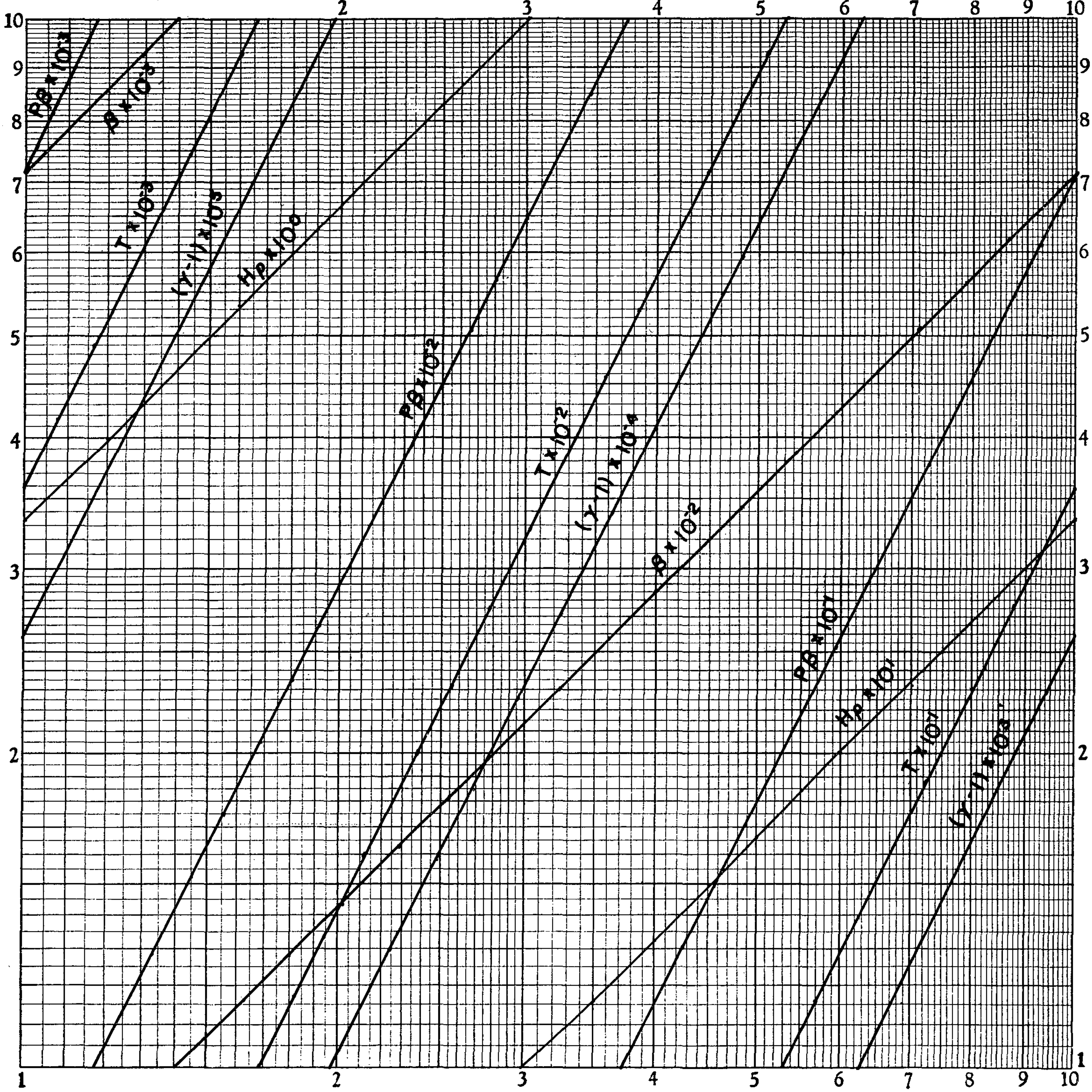
Pβ in MeV/c

$\pi^{\pm}$ MESONS

1 MeV/c to 10 MeV/c

β, (γ - 1), T, Pβ, Hρ

$M_{\pi^{\pm}}$ = 139. 59 MeV
= 273. 18 m



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$\pi^{\pm}$ MESONS

10 MeV/c to 100 MeV/c

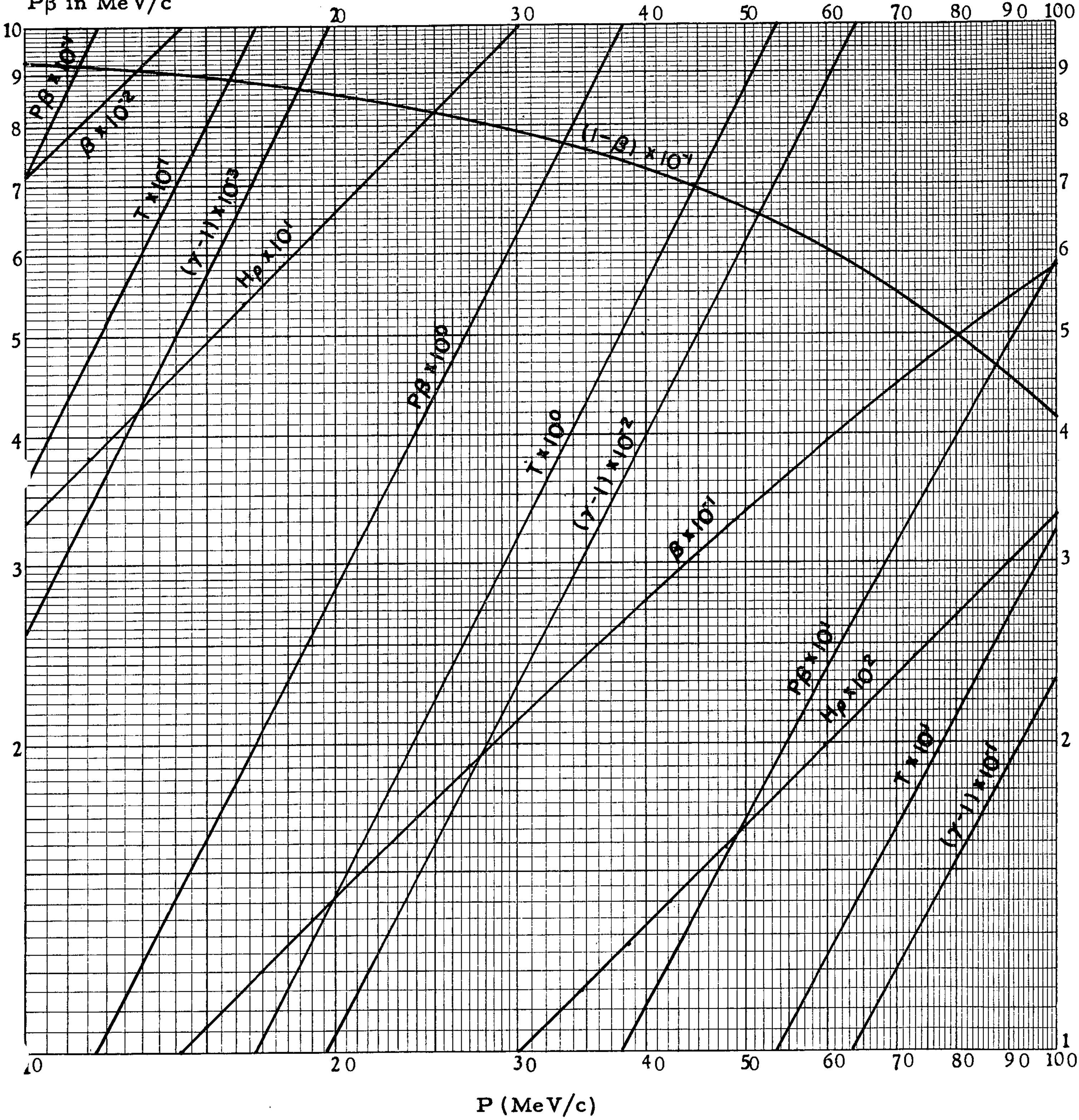
β, $(1 - \beta)$, $(\gamma - 1)$, T, Pβ, Hρ

T in MeV

Hρ in kgauss-cm

Pβ in MeV/c

$M_{\pi^{\pm}}$ = 139.59 MeV
= 273.18 m



$\pi^{\pm}$ MESONS

100 MeV/c to 1 BeV/c

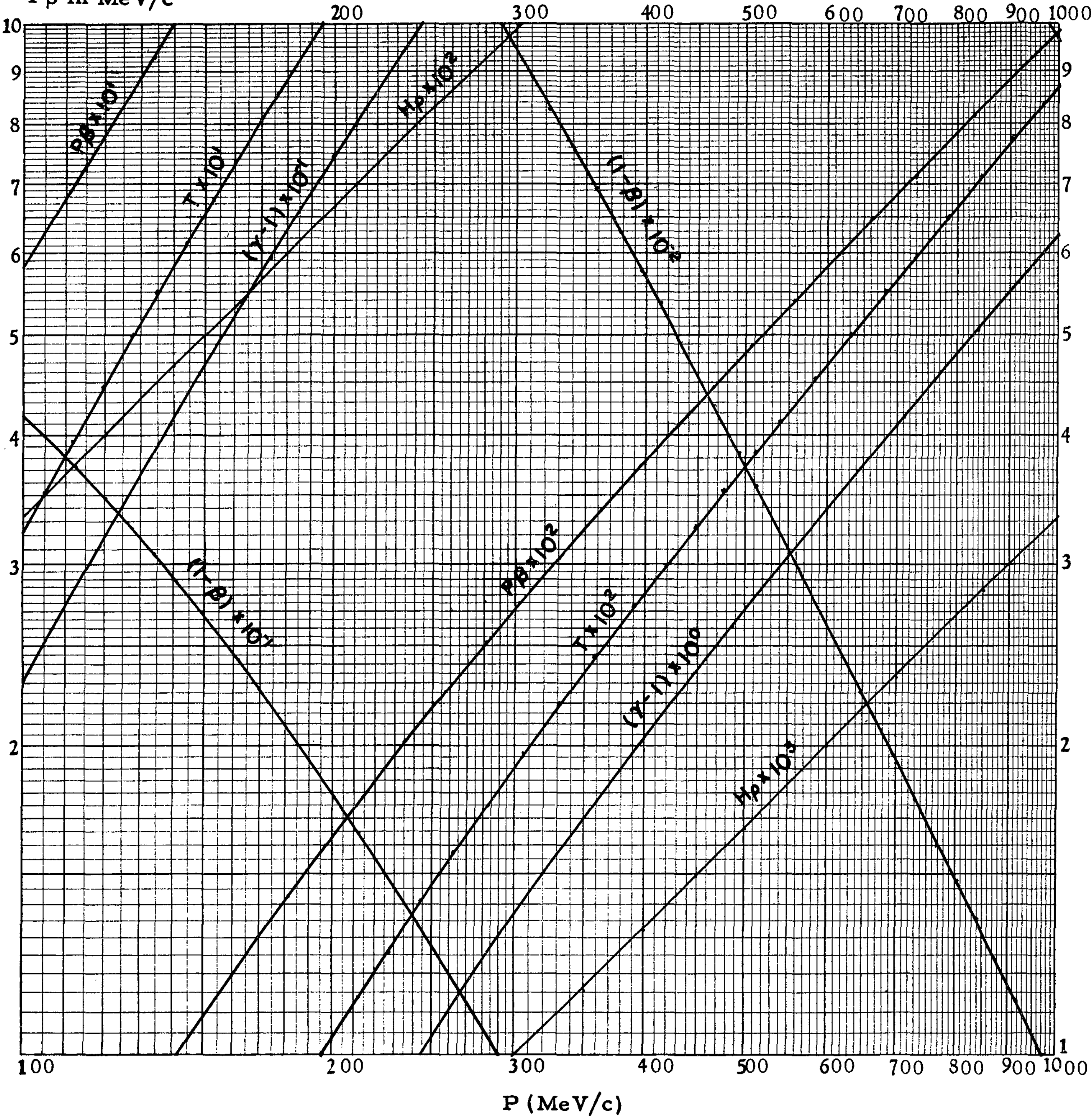
$(1 - \beta)$, $(\gamma - 1)$, T, $P\beta$, $H\rho$

T in MeV

$H\rho$ in kgauss-cm

$P\beta$ in MeV/c

$M_{\pi^{\pm}}$ = 139.59 MeV
= 273.18 m



$\pi^{\pm}$ MESONS

1 BeV/c to 10 BeV/c

(1 - β), (γ - 1), T, Pβ, Hρ

T in MeV

Hρ in kgauss-cm

Pβ in MeV/c

$M_{\pi^{\pm}}$ = 139.59 MeV
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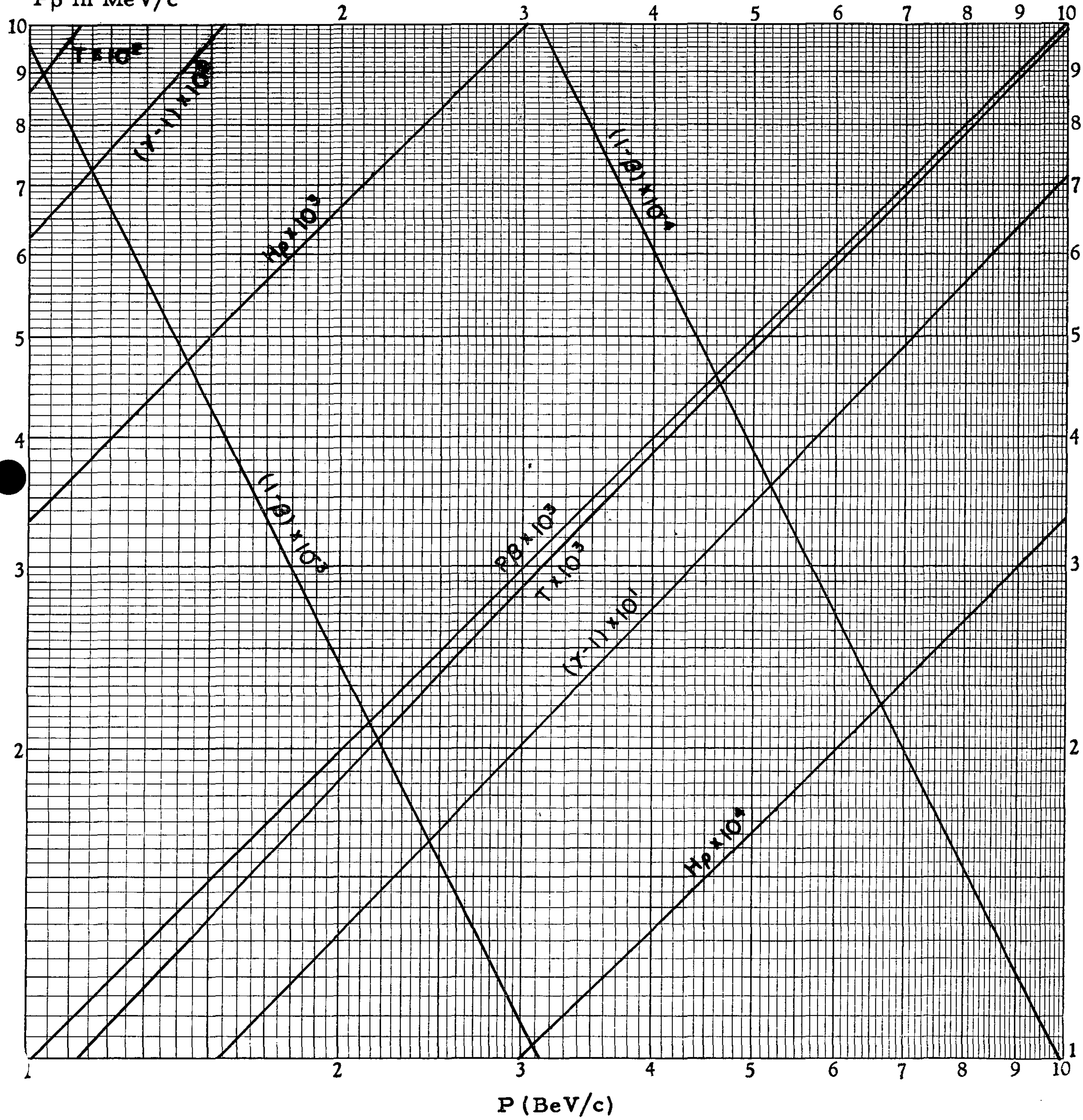


$\pi^{\pm}$ MESONS

T in MeV

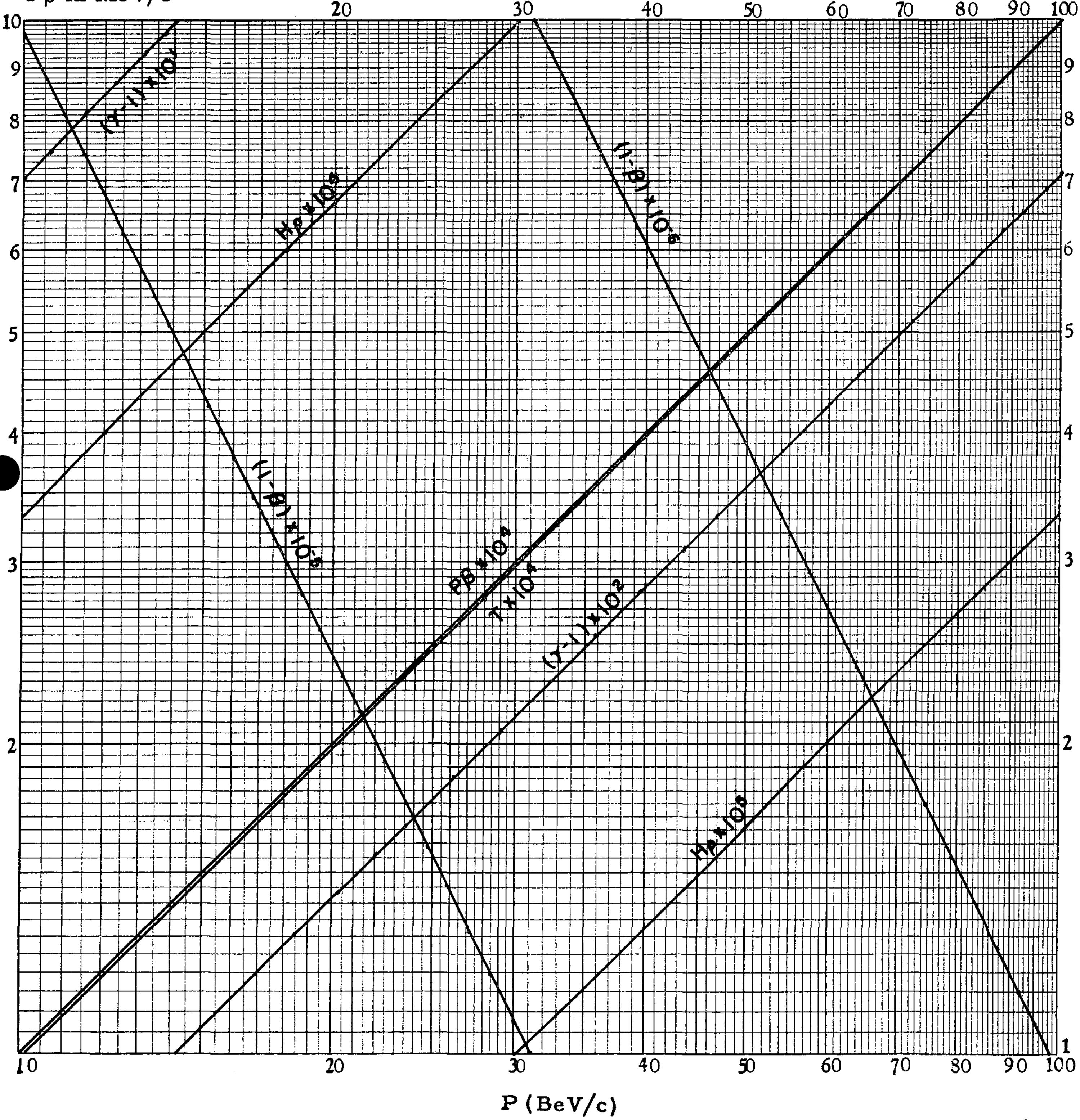
Hρ in kgauss-cm

Pβ in MeV/c

10 BeV/c to 100 BeV/c

(1 - β), (γ - 1), T, Pβ, Hρ

$M_{\pi\pm}$ = 139.59 MeV
= 273.18 m



0 0 1 0 1 2 0 4 7 2 8

K$^{\pm}$ MESONS

T in MeV

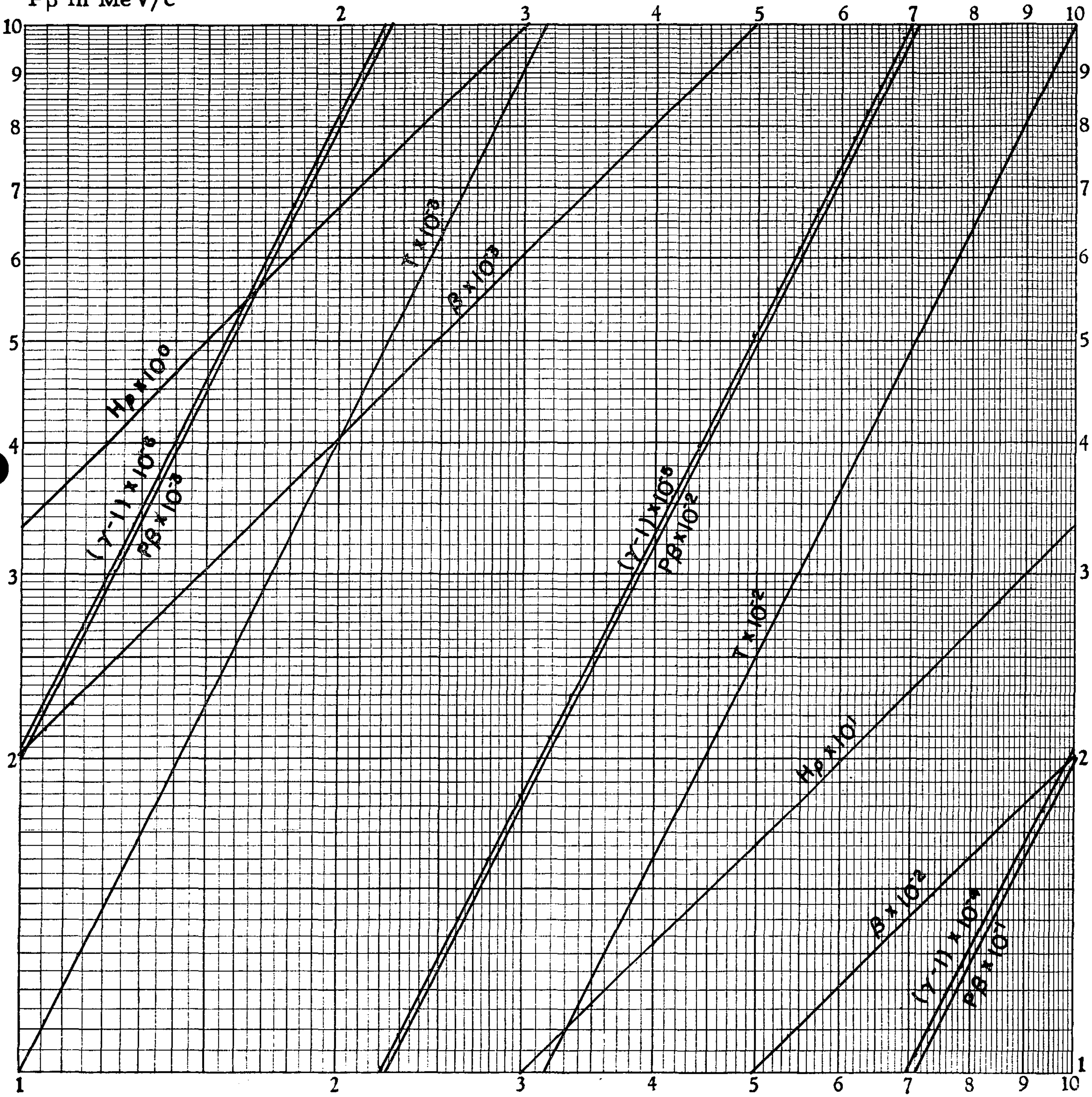
Hρ in kgauss-cm

Pβ in MeV/c

1 MeV/c to 10 MeV/c

β, (γ - 1), T, Pβ, Hρ

$M_{K^{\pm}}$ = 493.9 MeV
= 966.58 m



P (MeV/c)

0 0 1 0 1 2 0 4 7 2 9

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T in MeV

Hρ in kgauss-cm

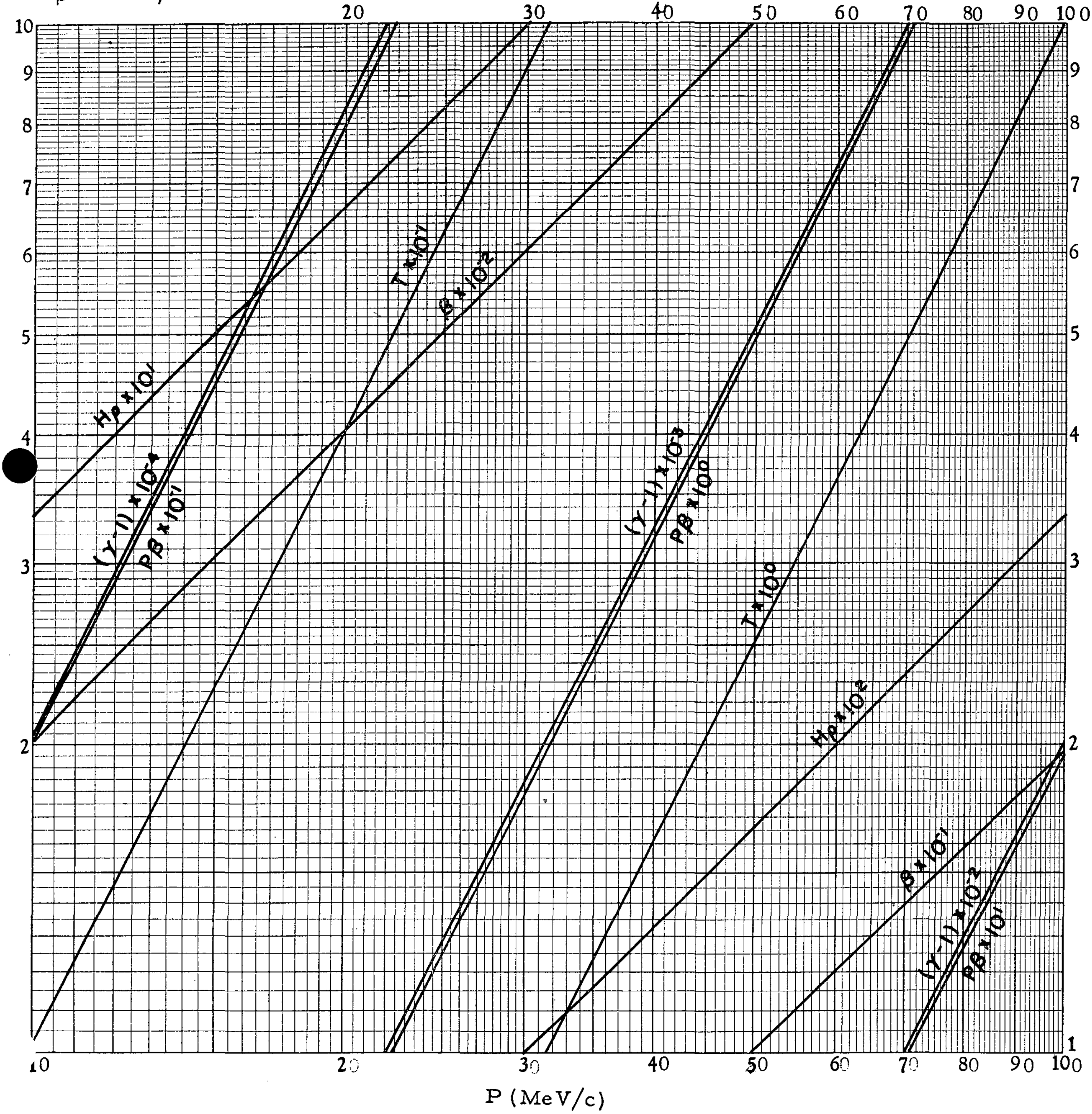
Pβ in MeV/c

K$^{\pm}$ MESONS

10 MeV/c to 100 MeV/c

β, (γ - 1), T, Pβ, Hρ

$M_{K^{\pm}}$ = 493.9 MeV
= 966.58 m



T in MeV

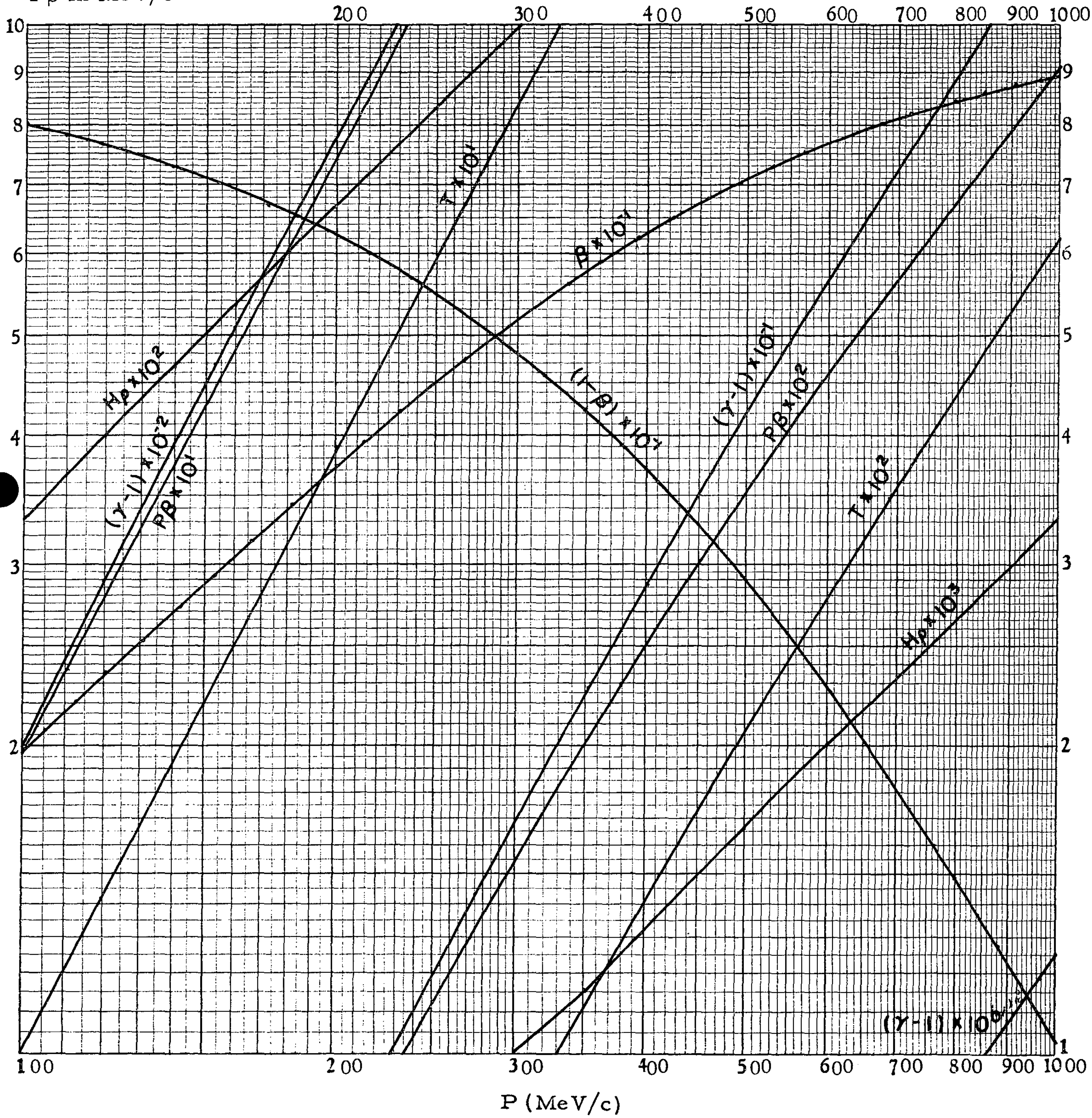
Hρ in kgauss-cm

Pβ in MeV/c

100 MeV/c to 1 BeV/c

β, (1 - β), (γ - 1), Pβ, Hρ

$M_{K^{\pm}}$ = 493.9 MeV
= 966.58 m



$K^{\pm}$ MESONS

1 BeV/c to 10 BeV/c

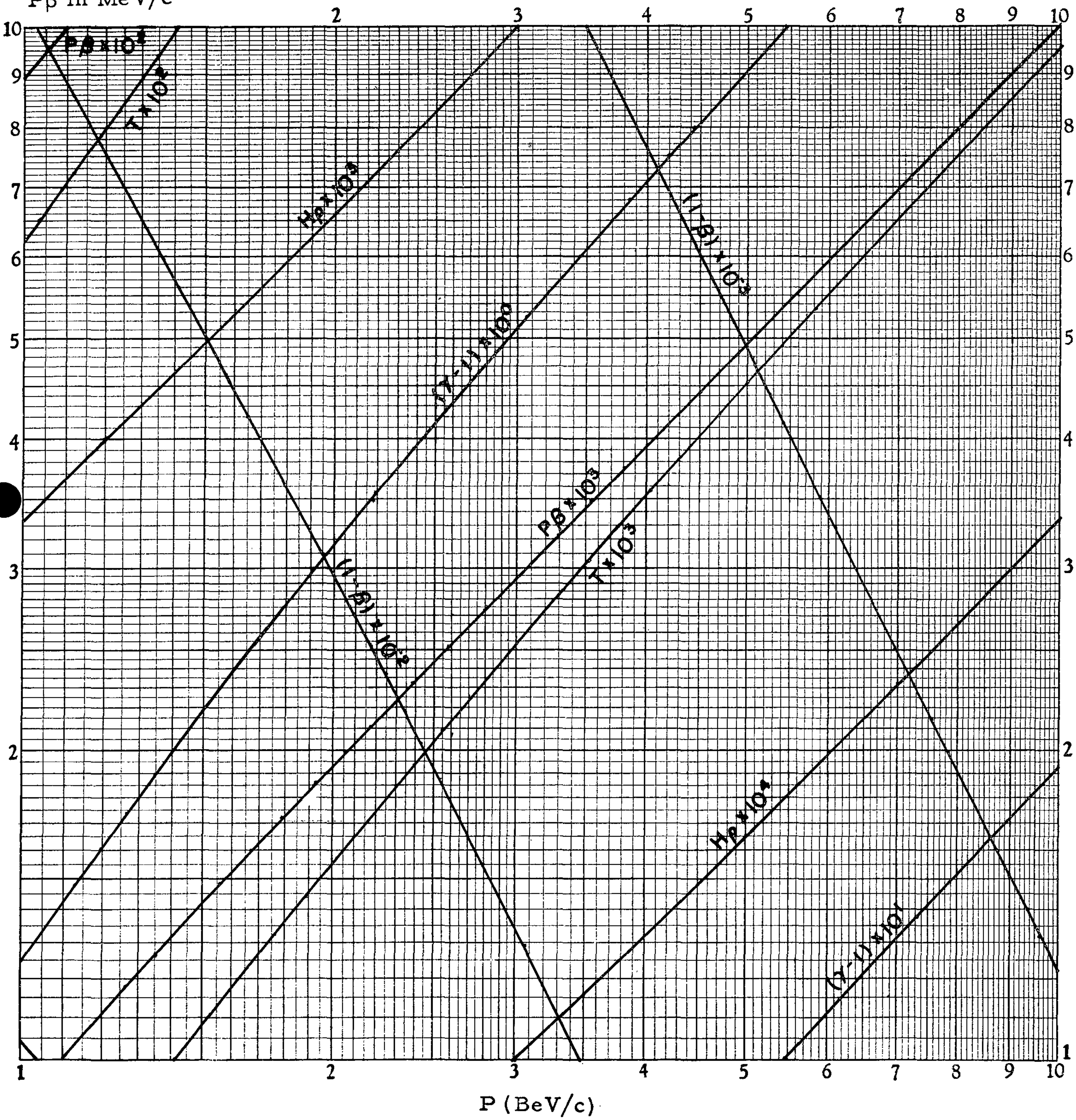
(1 - β), (γ - 1), Pβ, Hρ

T in MeV

Hρ in kgauss-cm

Pβ in MeV/c

$M_{K^{\pm}}$ = 493.9 MeV
= 966.58 m



K± MESONS

T in MeV

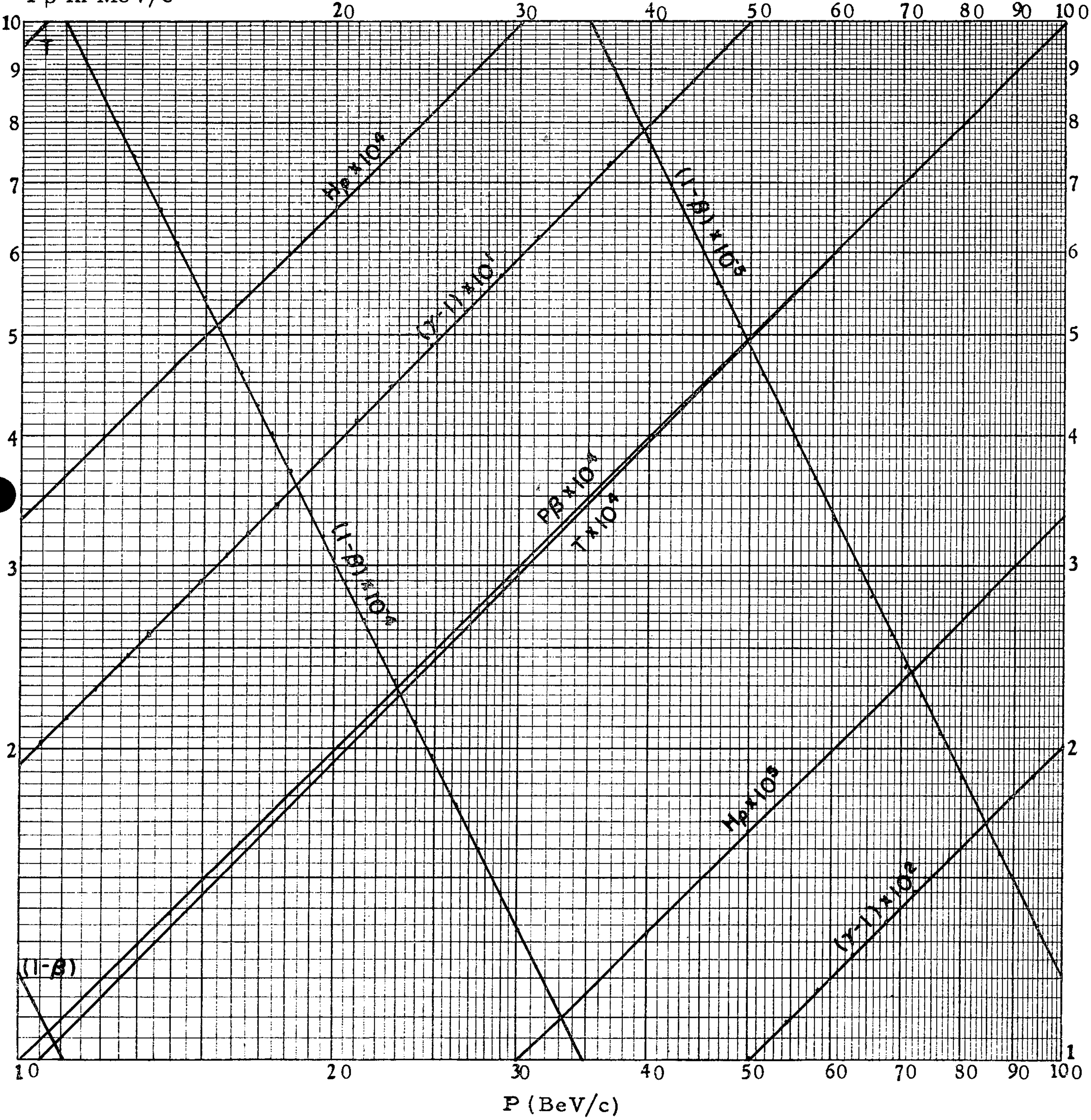
Hρ in kgauss-cm

Pβ in MeV/c

10 BeV/c to 100 BeV/c

(1 - β), (γ - 1), T, Pβ, Hρ

$M_{K^{\pm}}$ = 493.9 MeV
= 966.58 m



K^0 MESONS

T in MeV

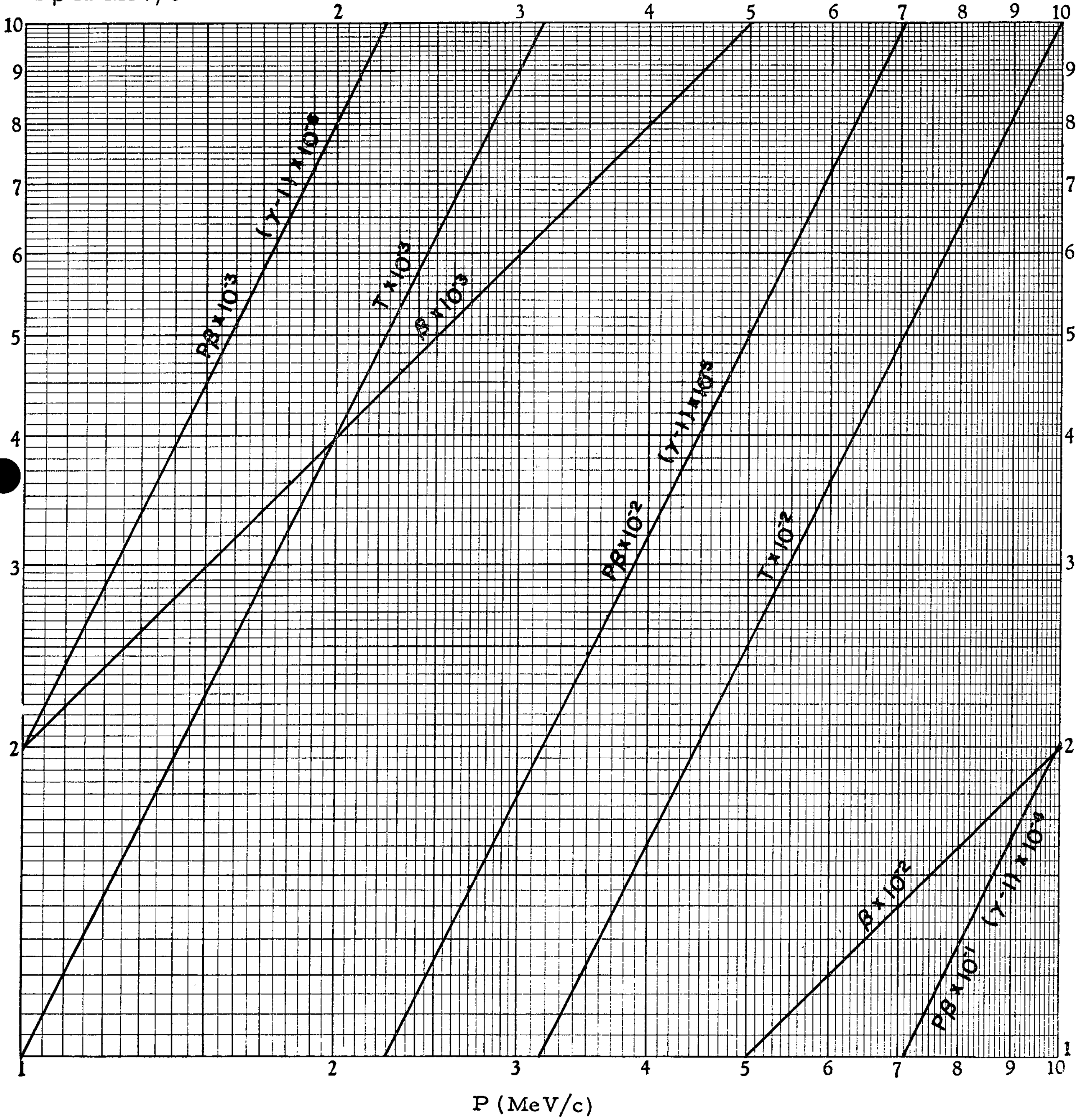
Pβ in MeV/c

1 MeV/c to 10 MeV/c

β, (γ - 1), T, Pβ

M_{K^0} = 497.8 MeV

= 974.42 m

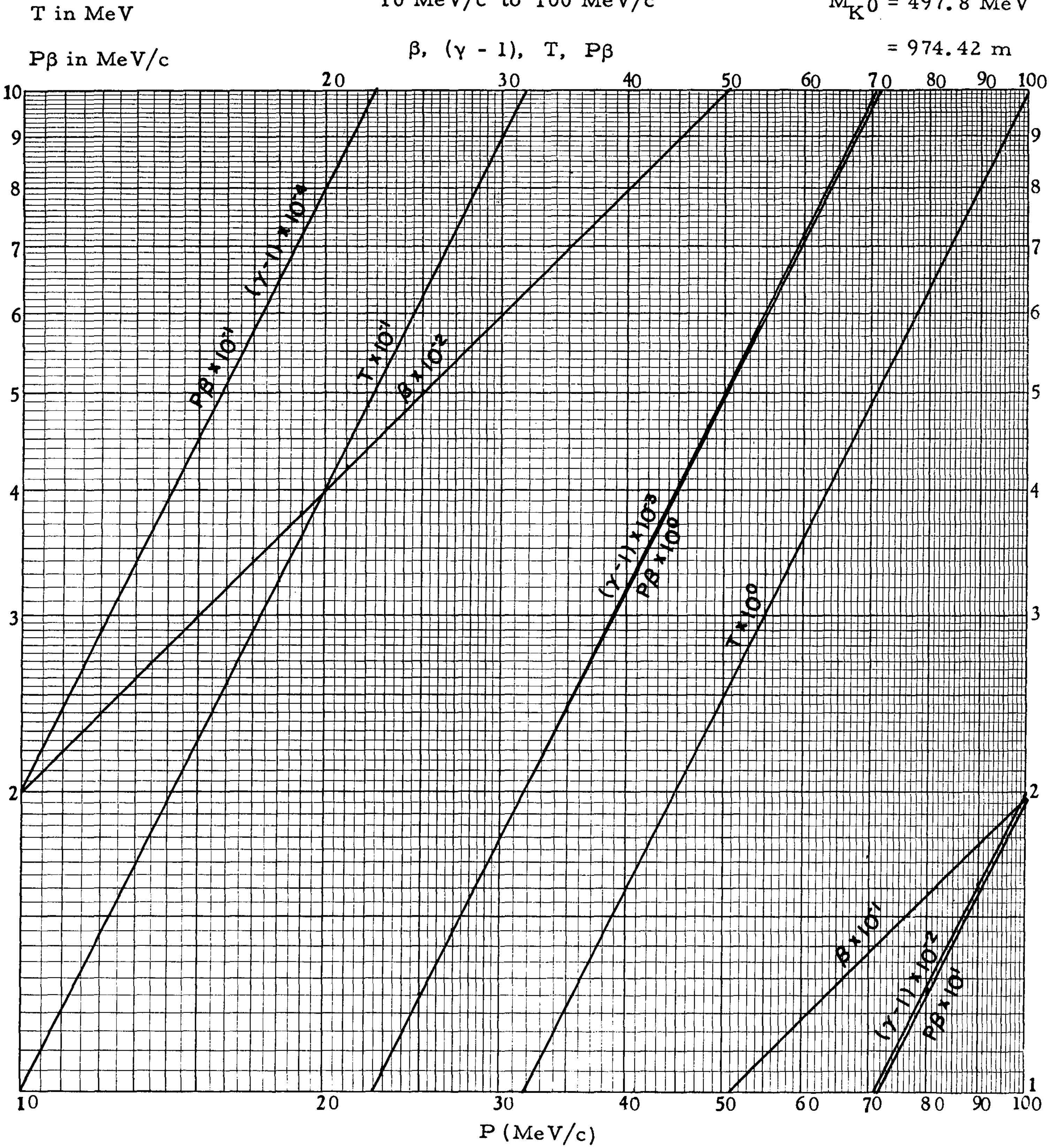


K^0 MESONS

10 MeV/c to 100 MeV/c

β, $(\gamma - 1)$, T, Pβ

M_{K^0} = 497.8 MeV
= 974.42 m



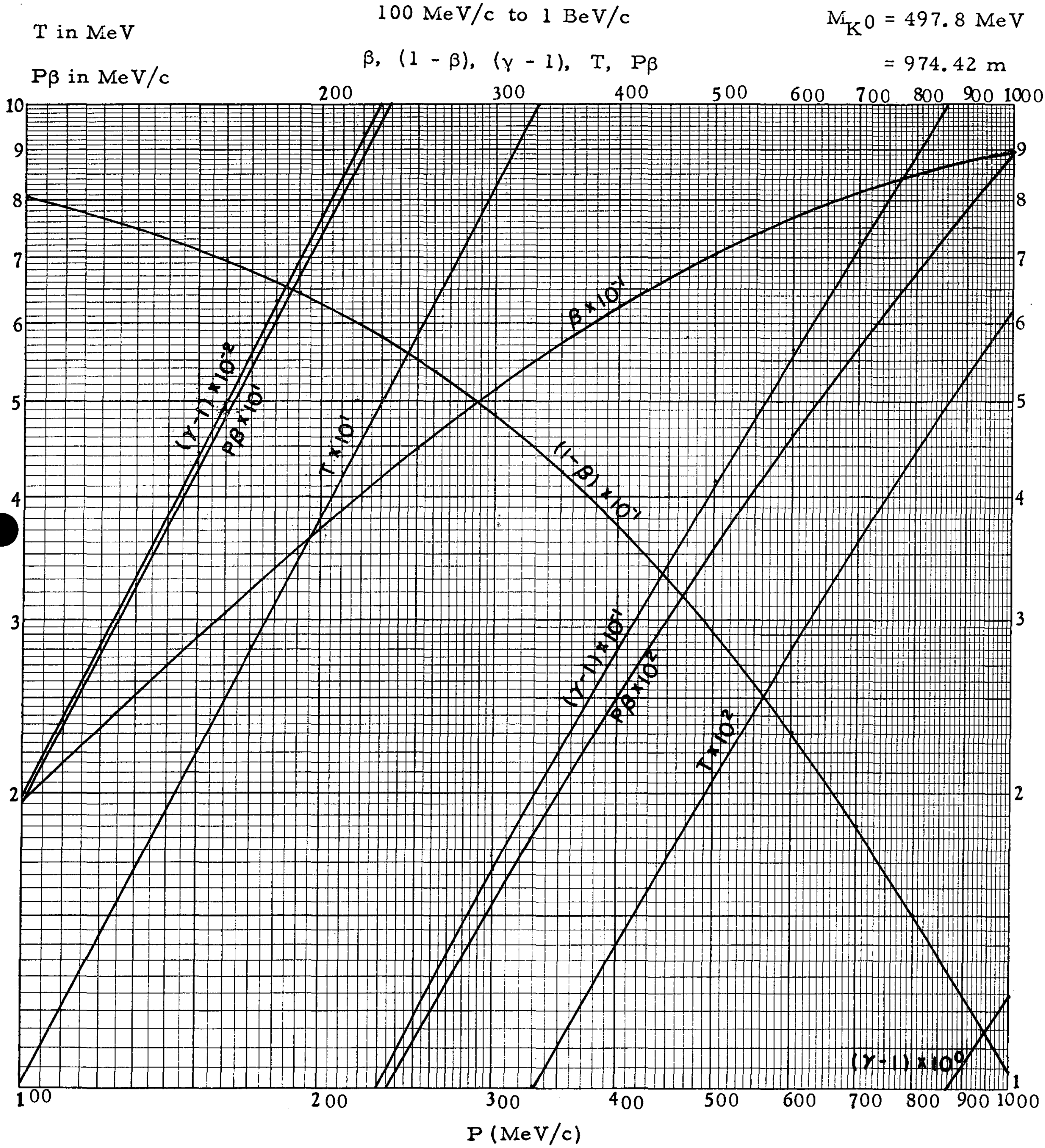
0 0 1 0 1 2 0 4 7 3 5

K^0 MESONS

100 MeV/c to 1 BeV/c

β, $(1-\beta)$, $(\gamma-1)$, T, $P\beta$

M_{K^0} = 497.8 MeV
= 974.42 m



0 0 1 0 1 2 0 4 7 3 6

K^0 MESONS

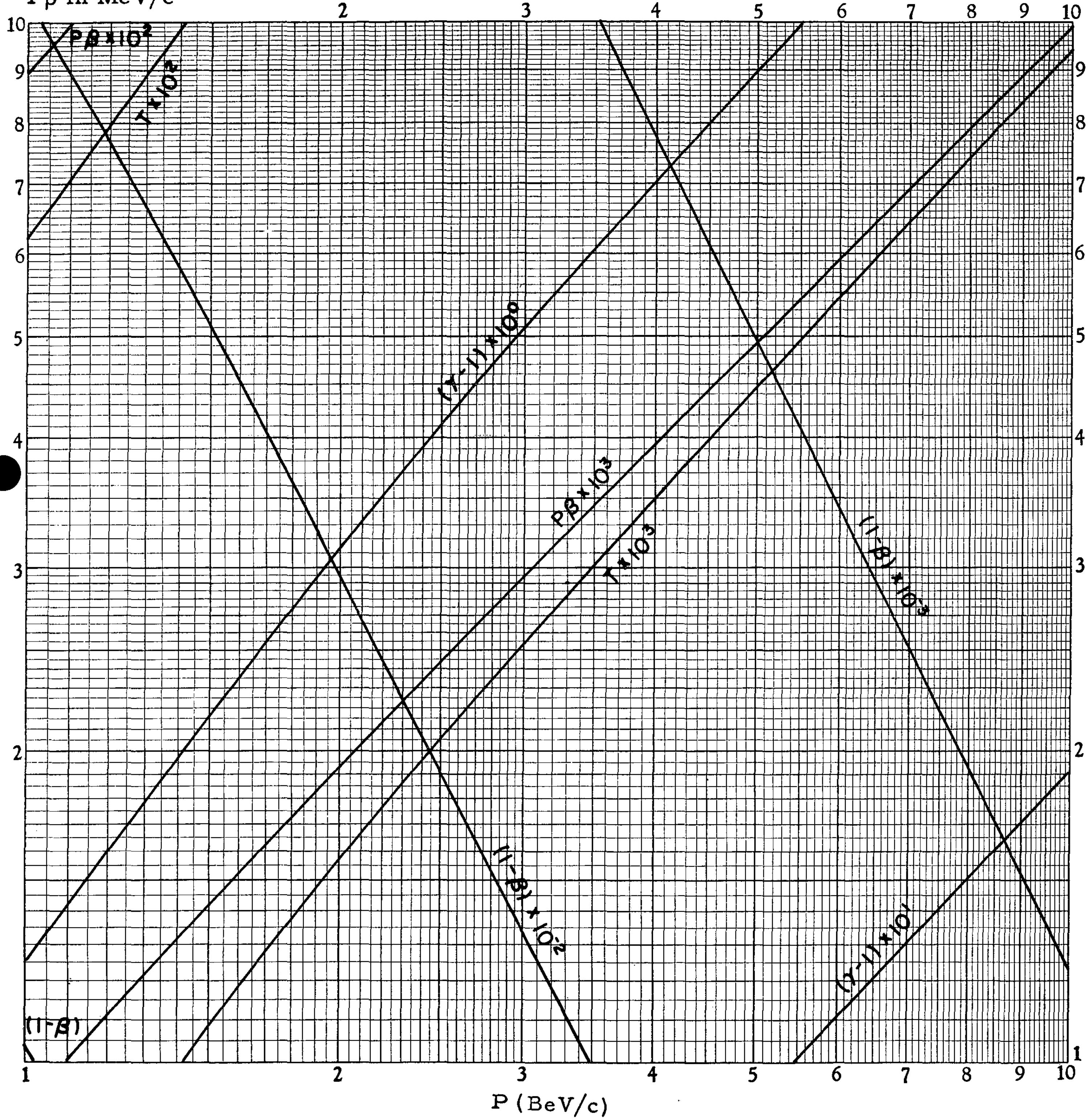
1 BeV/c to 10 BeV/c

β, $(1 - \beta)$, $(\gamma - 1)$, T, $P\beta$

M_{K^0} = 497.8 MeV = 974.42 m

T in MeV

$P\beta$ in MeV/c

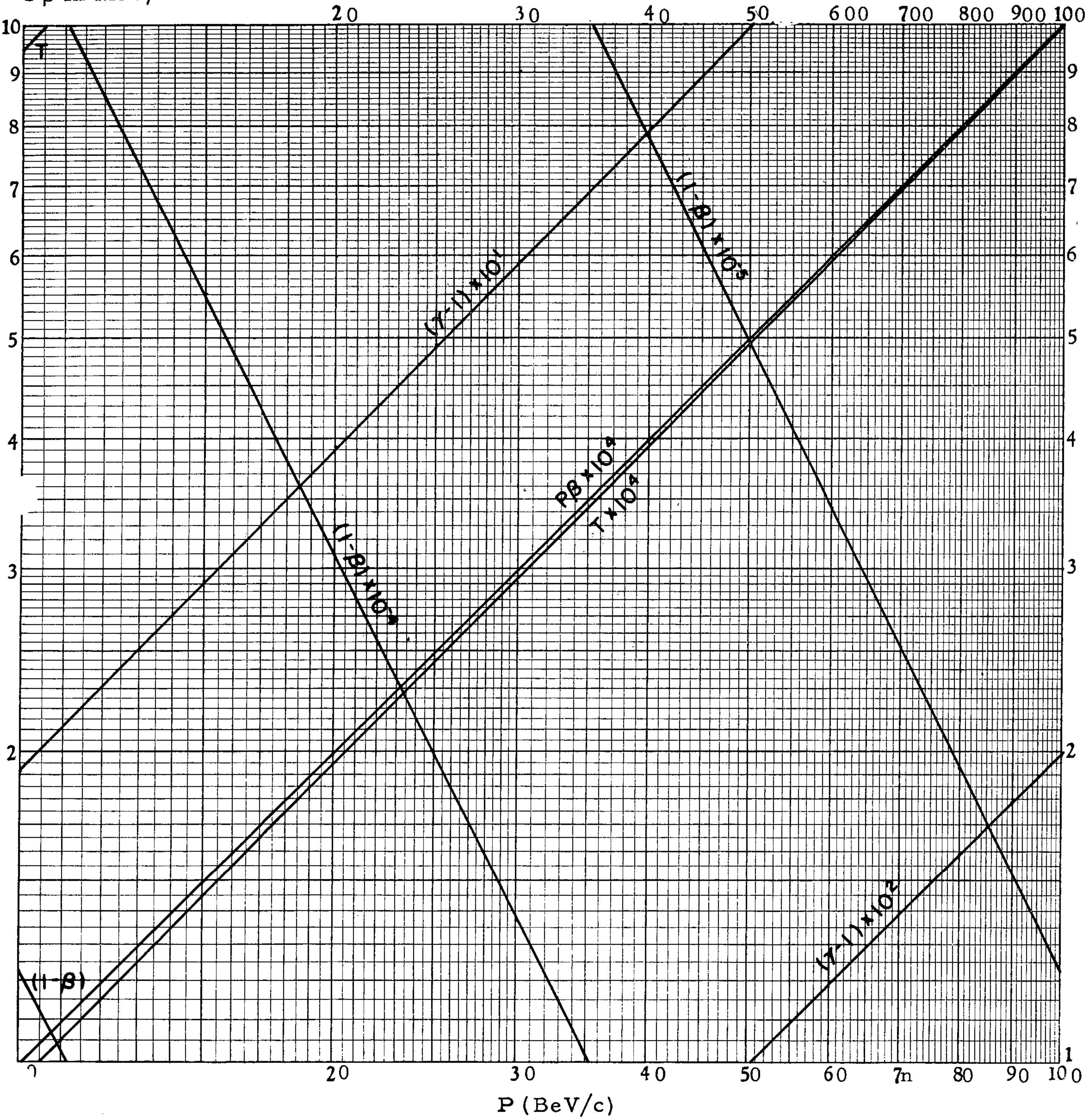


10 BeV/c to 100 BeV/c

(1 - β), (γ - 1), T, Pβ

M_{K^0} = 497.8 MeV
= 974.42 m

T in MeV
Pβ in MeV/c



PROTONS

T in MeV

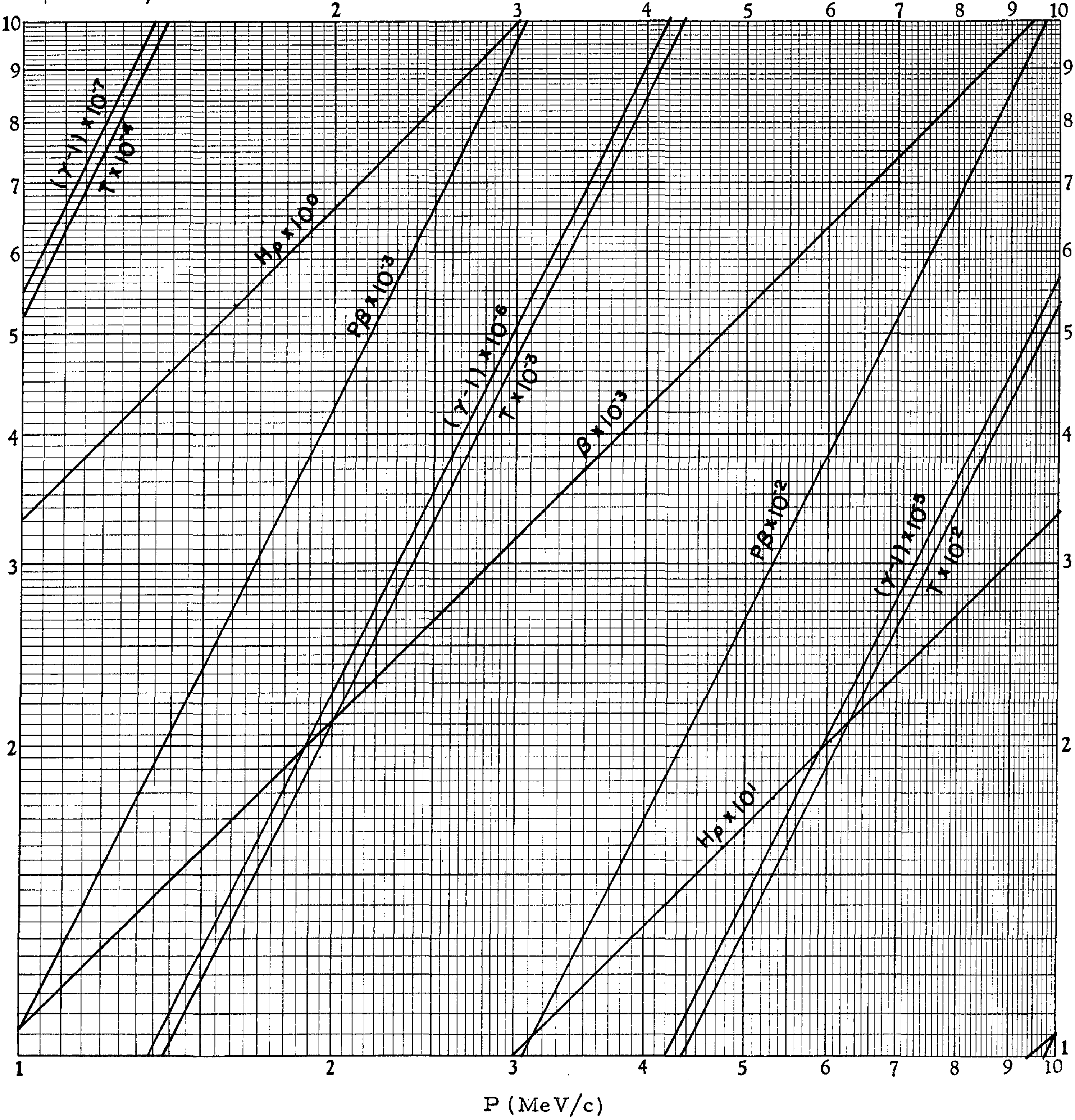
Hρ in kgauss-cm

Pβ in MeV/c

1 MeV/c to 10 MeV/c

β, (γ - 1), T, Pβ, Hρ

M_p = 938.213 MeV
= 1836.12 m



PROTONS

T in MeV

Hρ in kgauss-cm

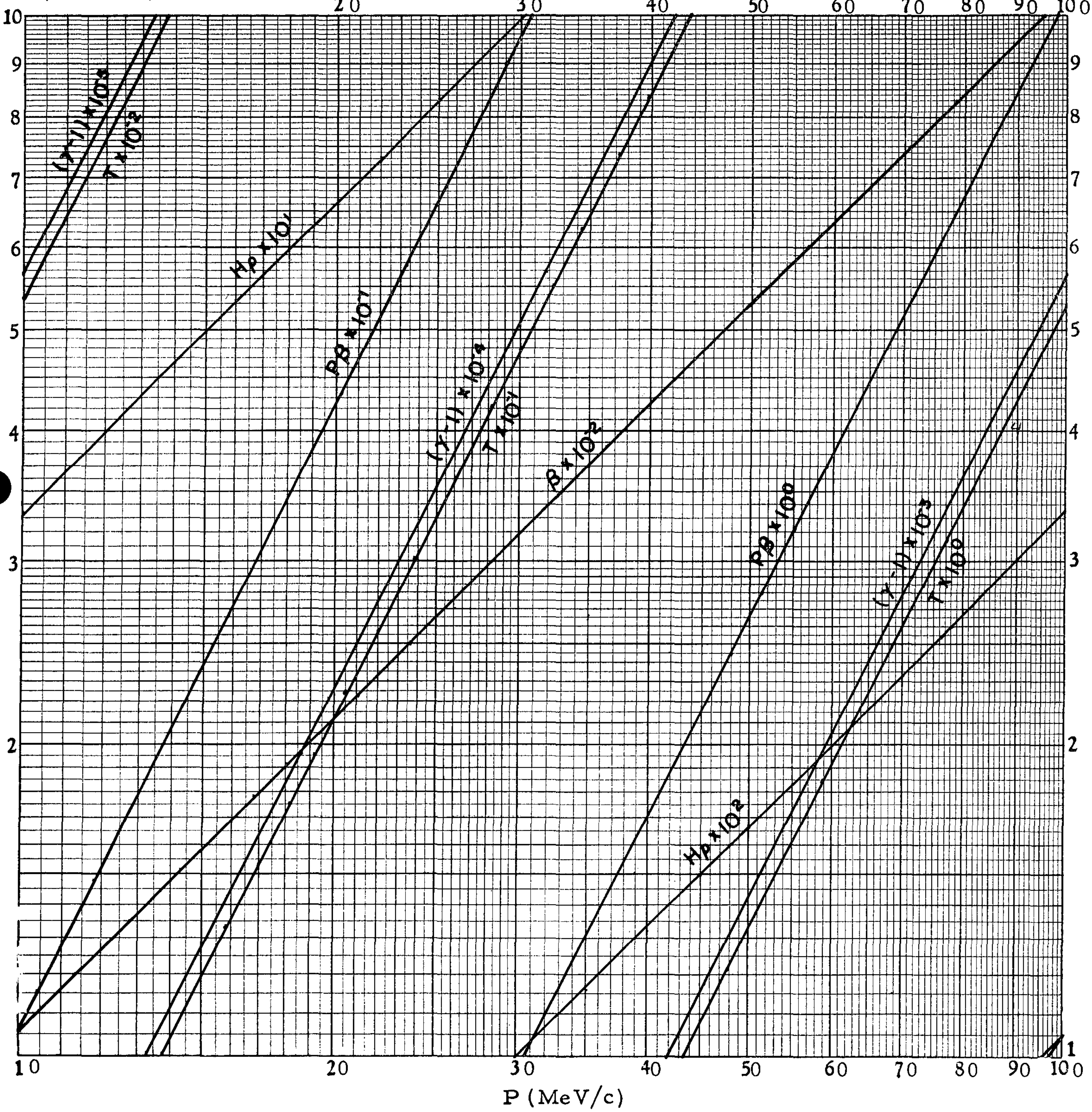
Pβ in MeV/c

10 MeV/c to 100 MeV/c

β, (1 - β), (γ - 1), Pβ, Hρ

M_p = 938.213 MeV

= 1836.12 m



PROTONS

T in MeV

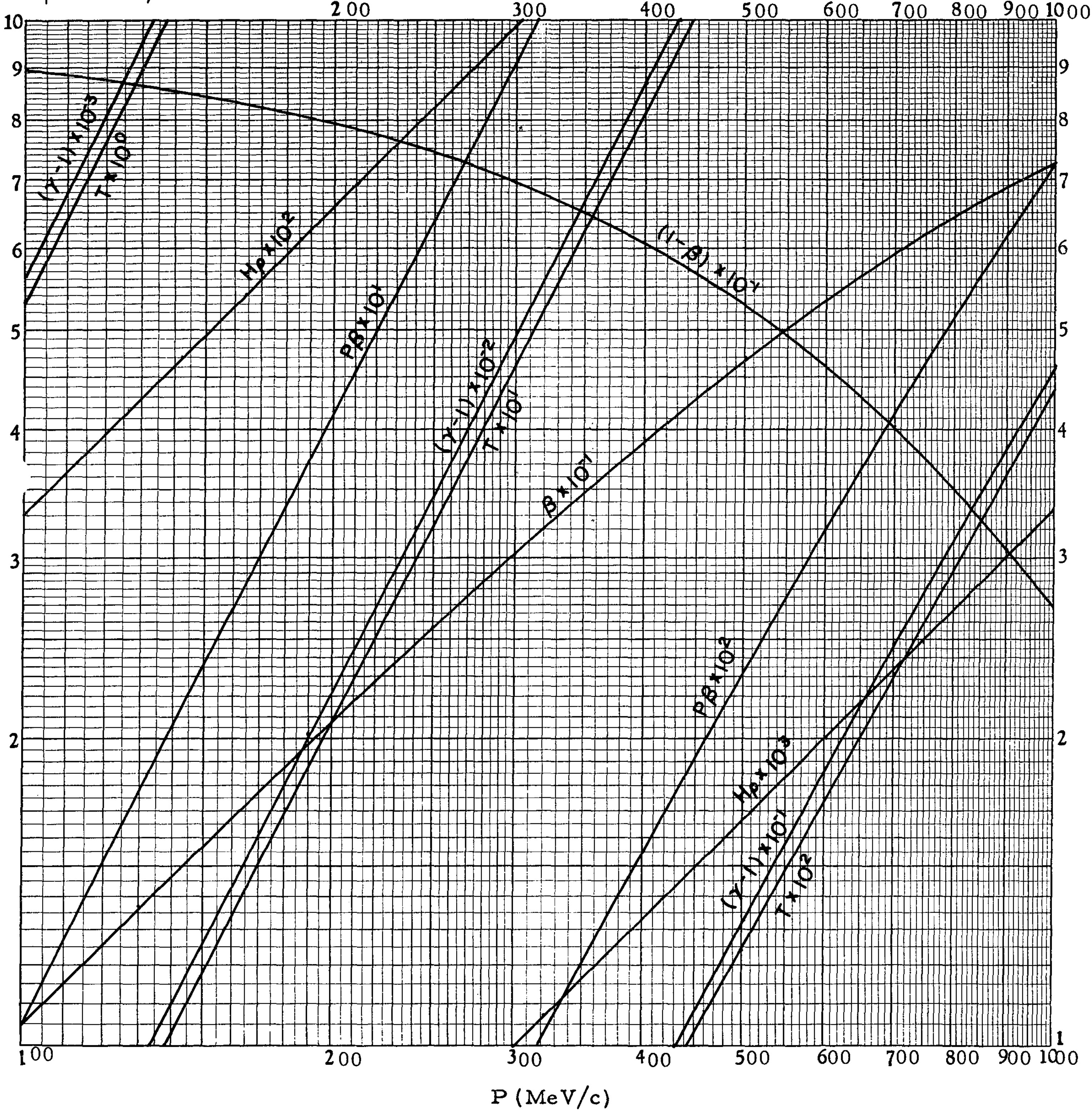
Hρ in kgauss-cm

Pβ in MeV/c

100 MeV/c to 1 BeV/c

β, (1 - β), (γ - 1), T, Pβ, Hρ

M_p = 938.213 MeV
= 1836.12 m



PROTONS

T in MeV

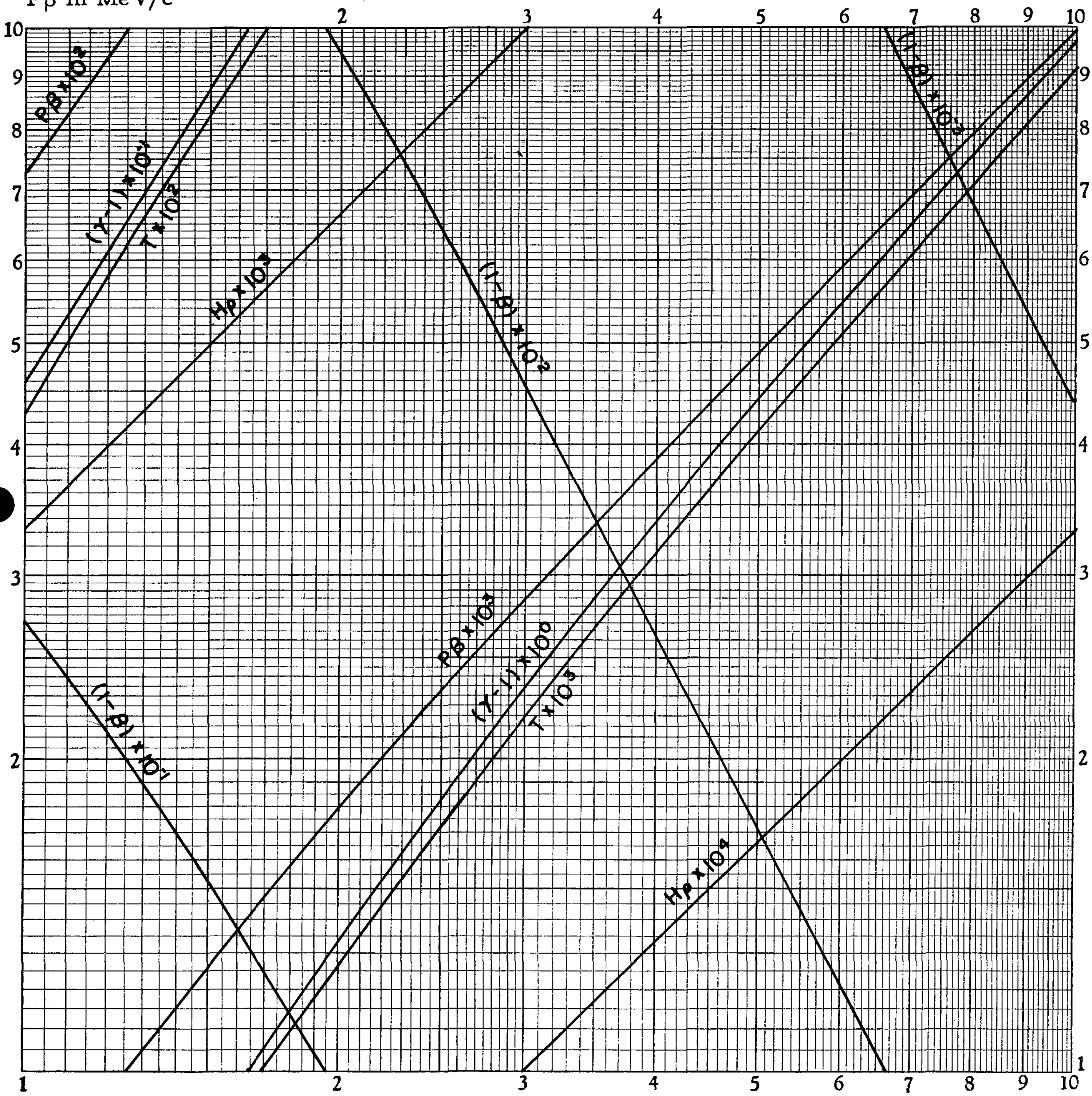
Hρ in kgauss-cm

Pβ in MeV/c

1 BeV/c to 10 BeV/c

β, (1 - β), (γ - 1), T, Pβ, Hρ

M_p = 938.213 MeV
= 1836.12 m



P (BeV/c)

T in MeV

Hρ in kgauss-cm

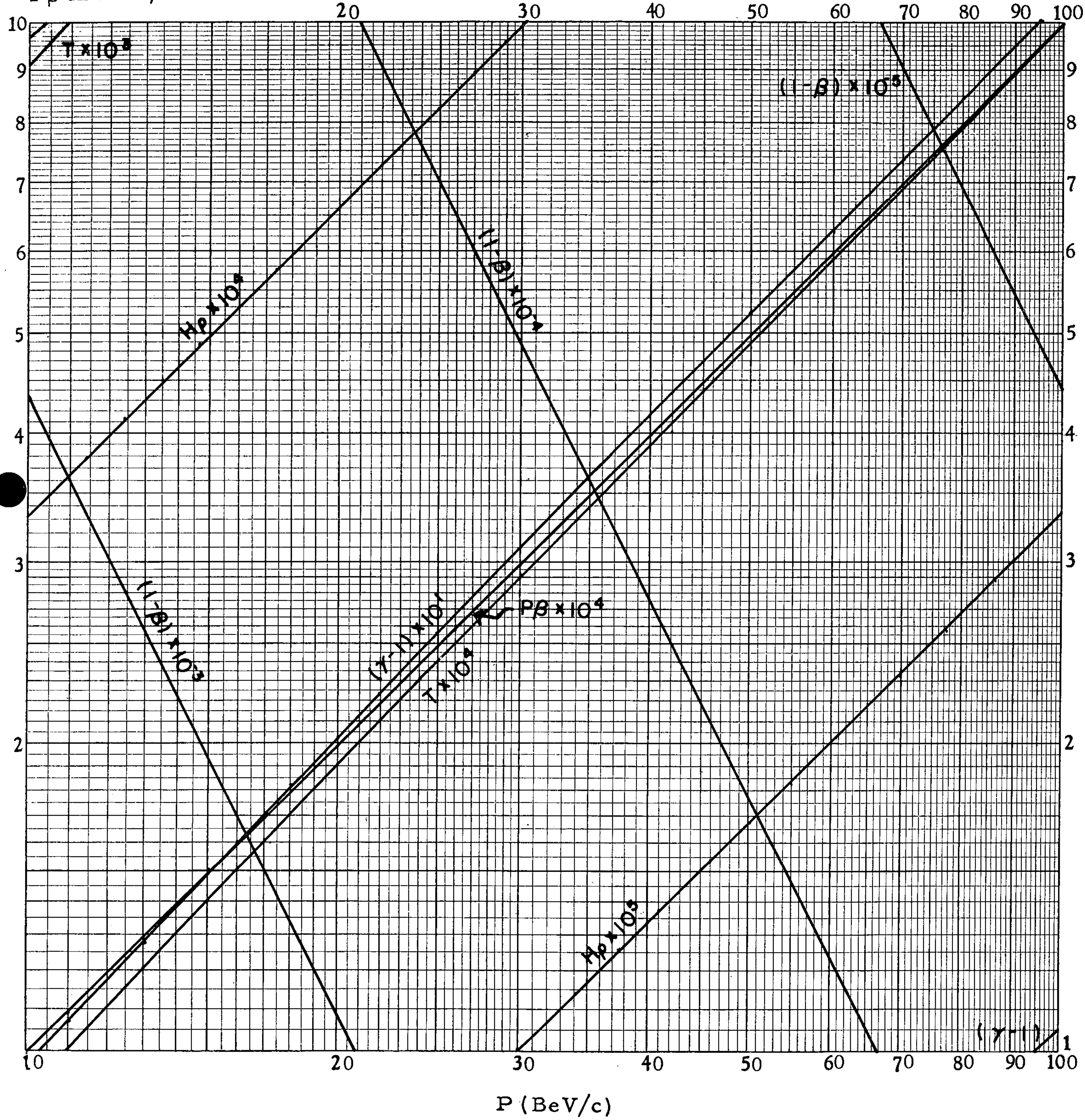
Pβ in MeV/c

PROTONS

10 BeV/c to 100 BeV/c

(1 - β), (γ - 1), T, Pβ, Hρ

M_p = 938.213 MeV
= 1836.12 m



Λ HYPERONS

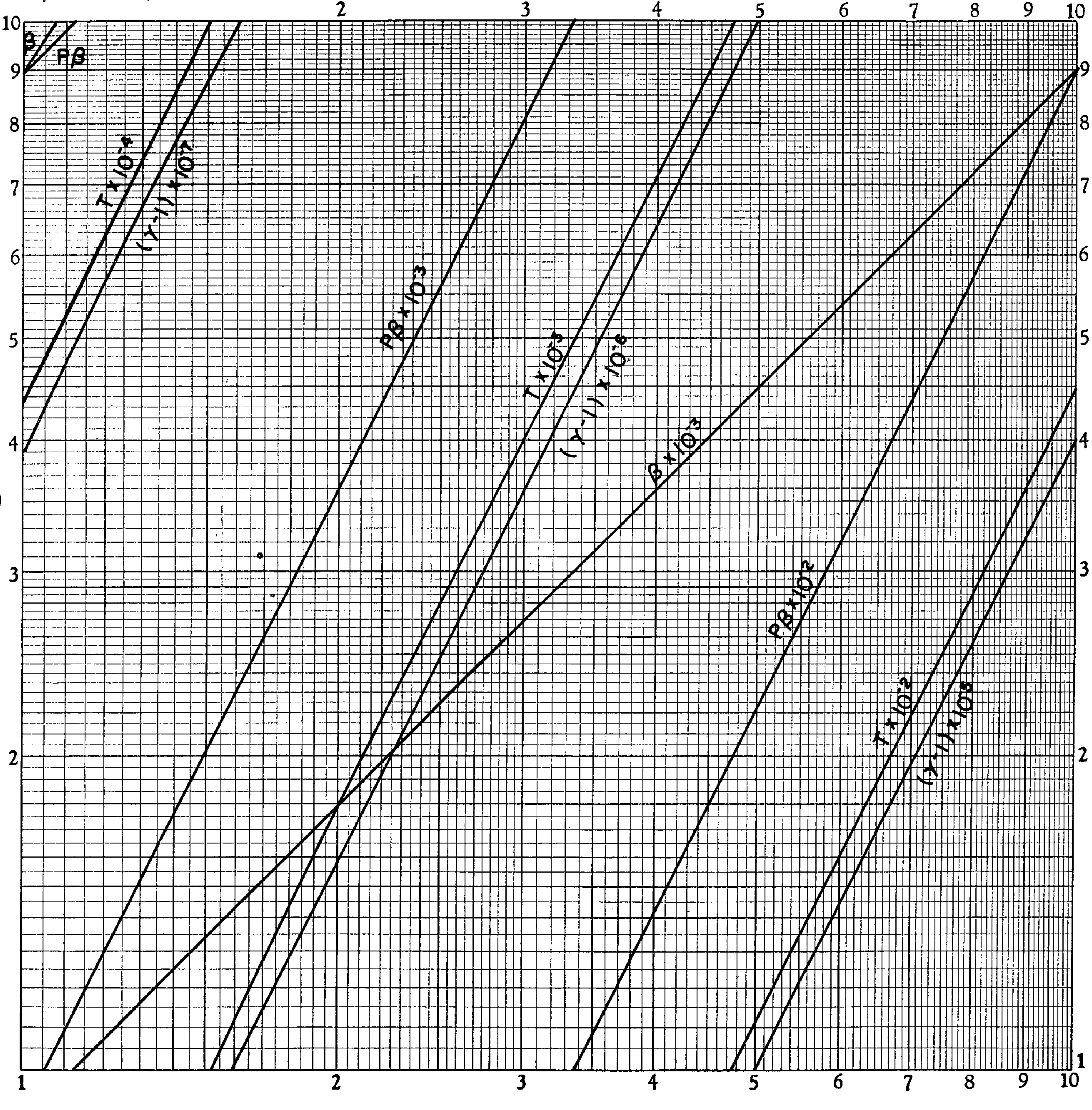
T in MeV

Pβ in MeV/c

1 MeV/c to 10 MeV/c

β, (γ - 1), T, Pβ, Hρ

M_Λ = 1115.36 MeV
= 2182.80 m



Λ HYPERONS

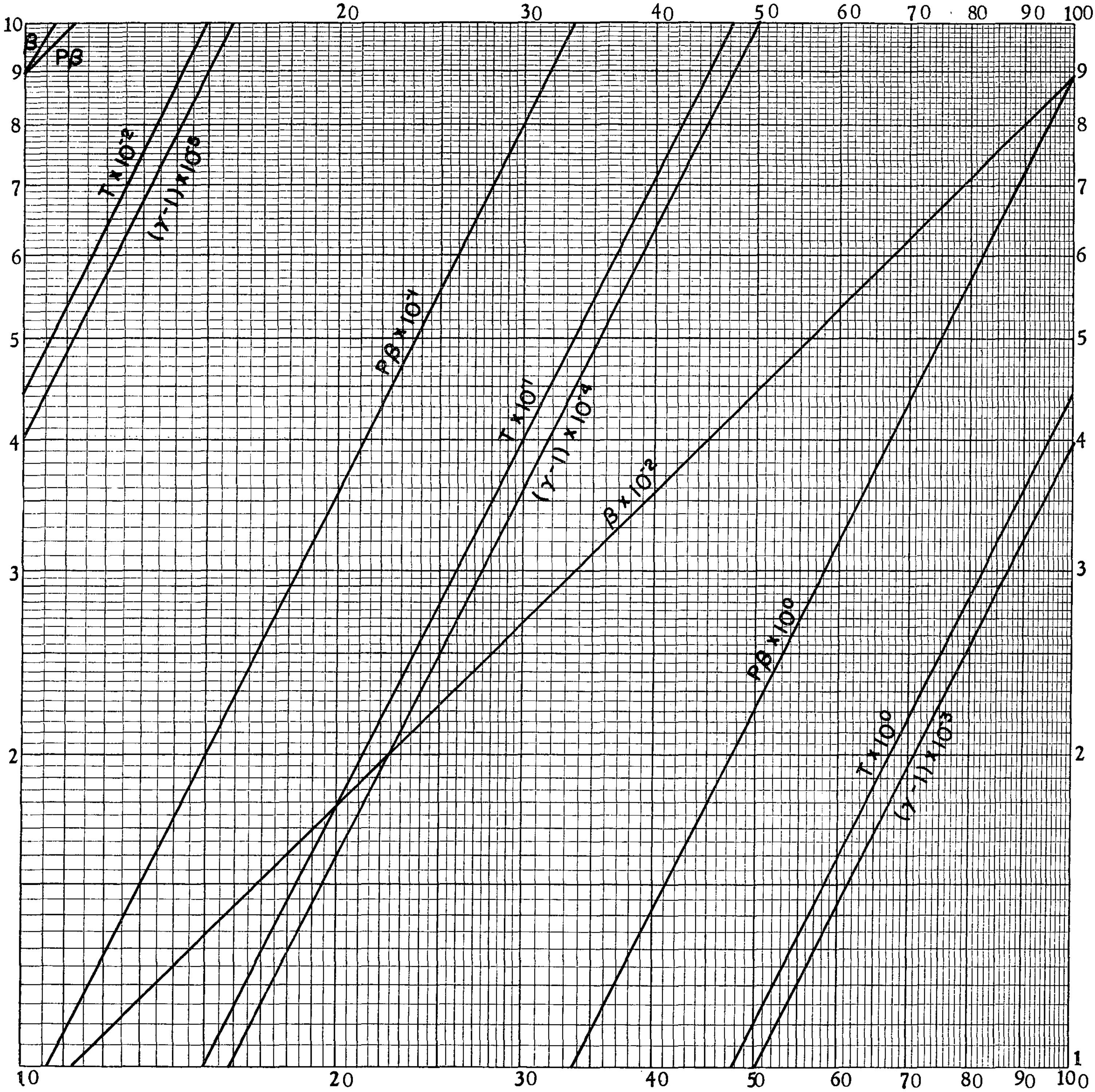
10 MeV/c to 100 MeV/c

β, (γ - 1), T, Pβ

T in MeV

Pβ in MeV/c

M_Λ = 1115.36 MeV
= 2182.80 m



0 0 1 0 1 2 0 4 7 4 5

Λ HYPERONS

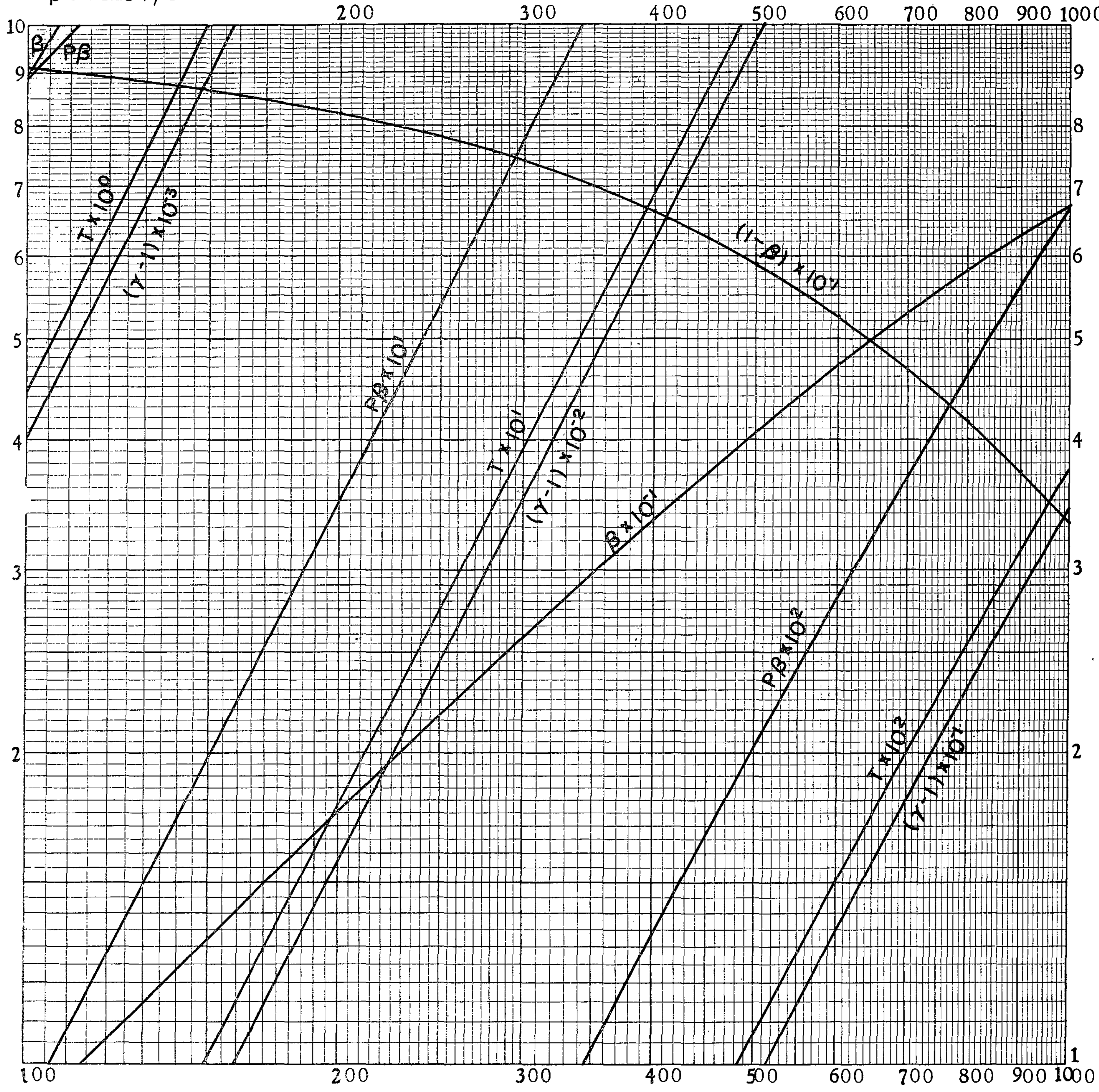
100 MeV/c to 1 BeV/c

β, (1 - β), (γ - 1), T, Pβ

T in MeV

Pβ in MeV/c

M_Λ = 1115.36 MeV
= 2182.80 m



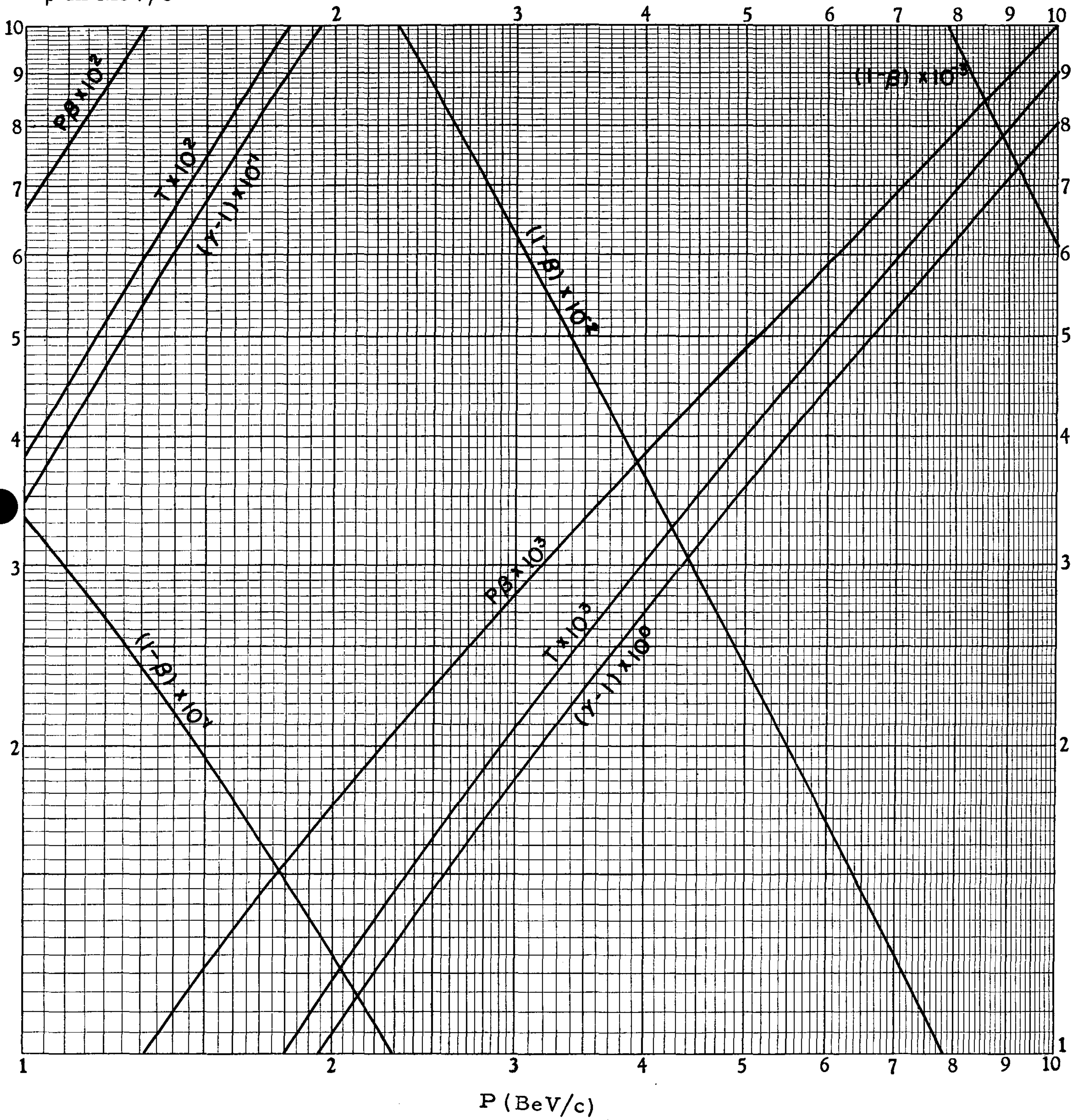
T in MeV

Pβ in MeV/c

1 BeV/c to 10 BeV/c

(1 - β), (γ - 1), T, Pβ

M_Λ = 1115.36 MeV
= 2182.80 m



0 0 1 0 1 2 0 4 7 4 7

Λ HYPERONS

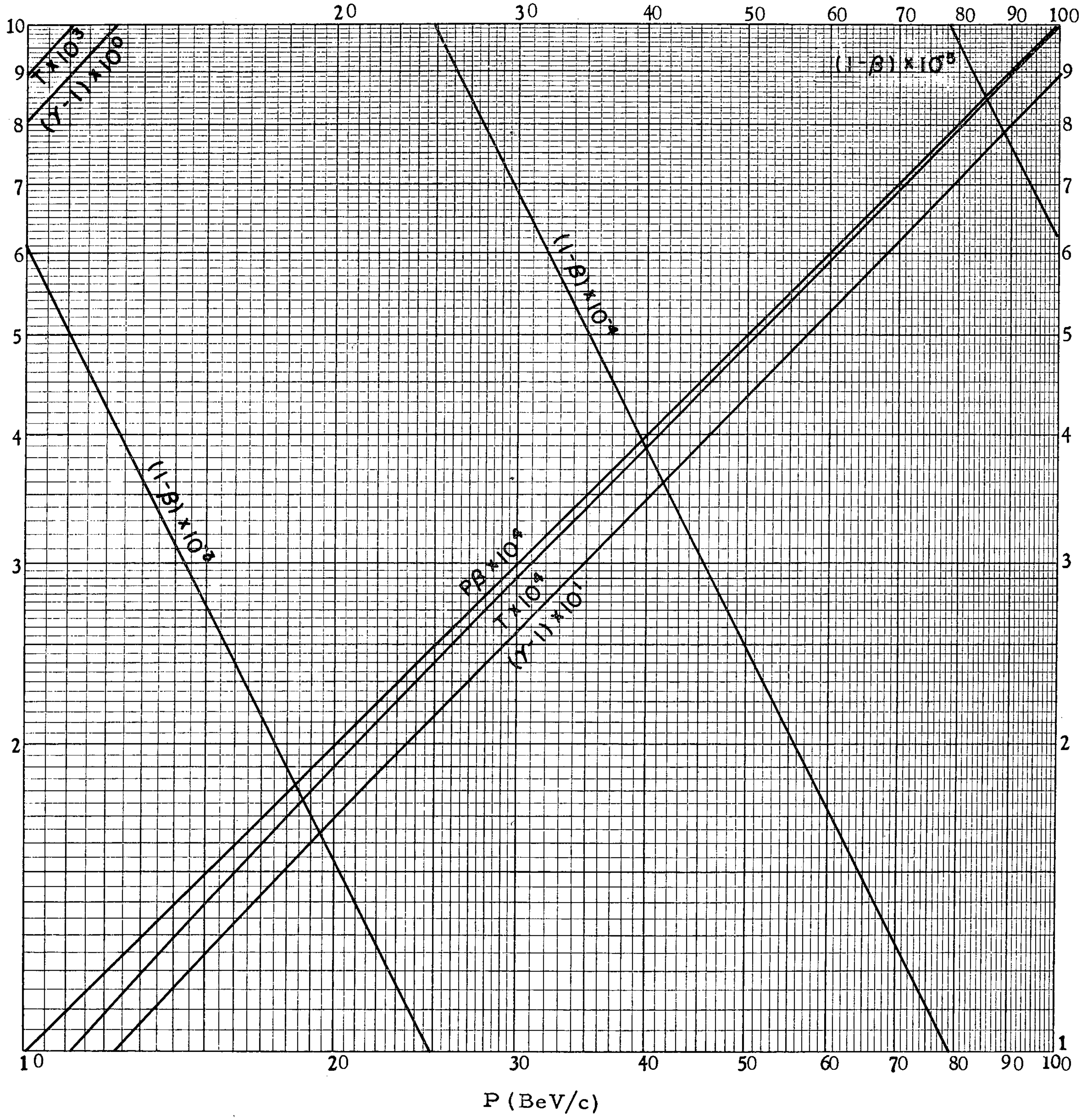
10 BeV/c to 100 BeV/c

(1 - β), (γ - 1), T, Pβ

T in MeV

Pβ in MeV/c

M_Λ = 1115.36 MeV
= 2182.80 m



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Σ^+ HYPERONS

T in MeV

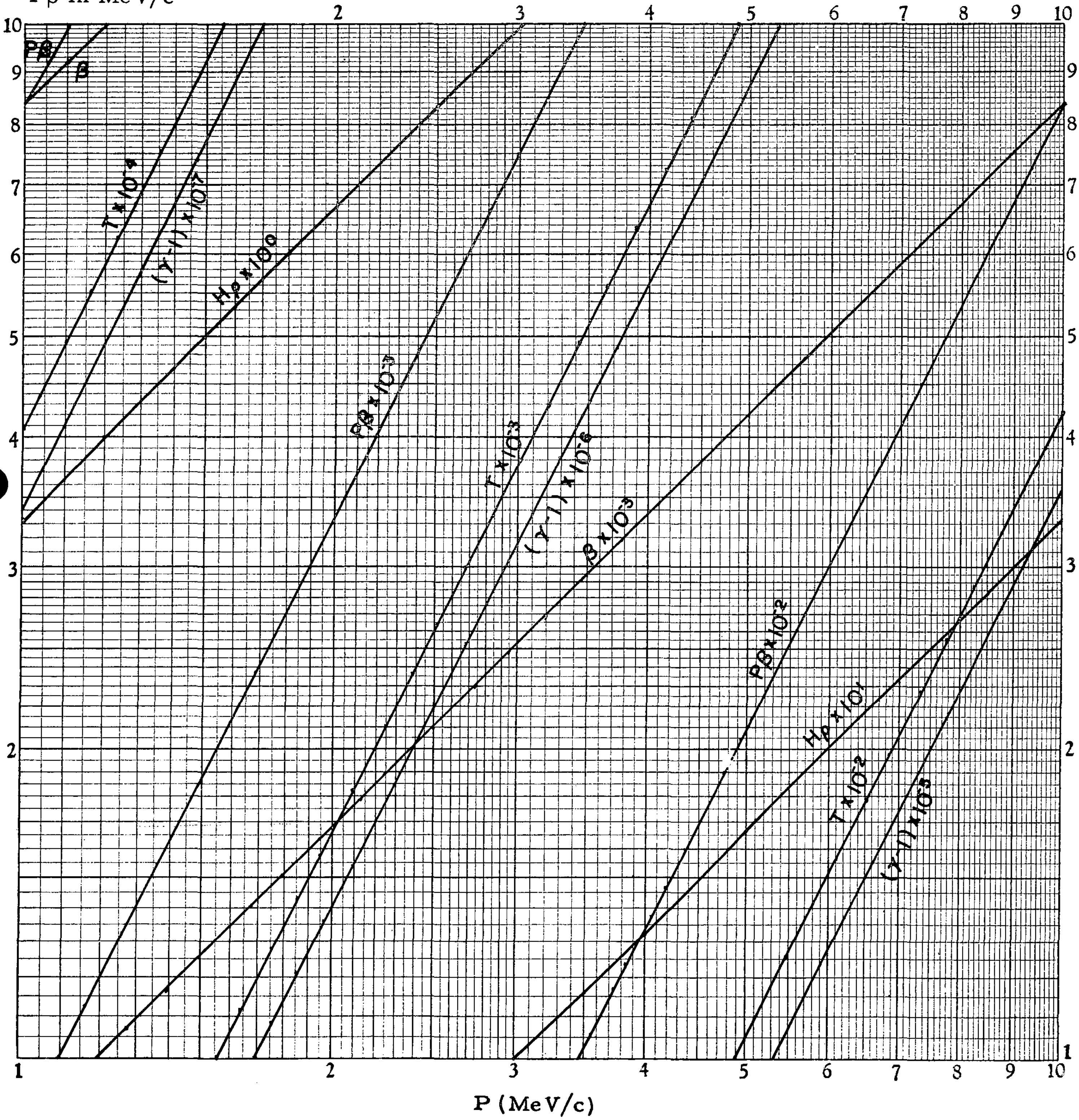
Hρ in kgauss-cm

Pβ in MeV/c

1 MeV/c to 10 MeV/c

β, (γ - 1), T, Pβ, Hρ

M_{Σ^+} = 1189.40 MeV
= 2321.83 m



Σ^+ HYPERONS

10 MeV/c to 100 MeV/c

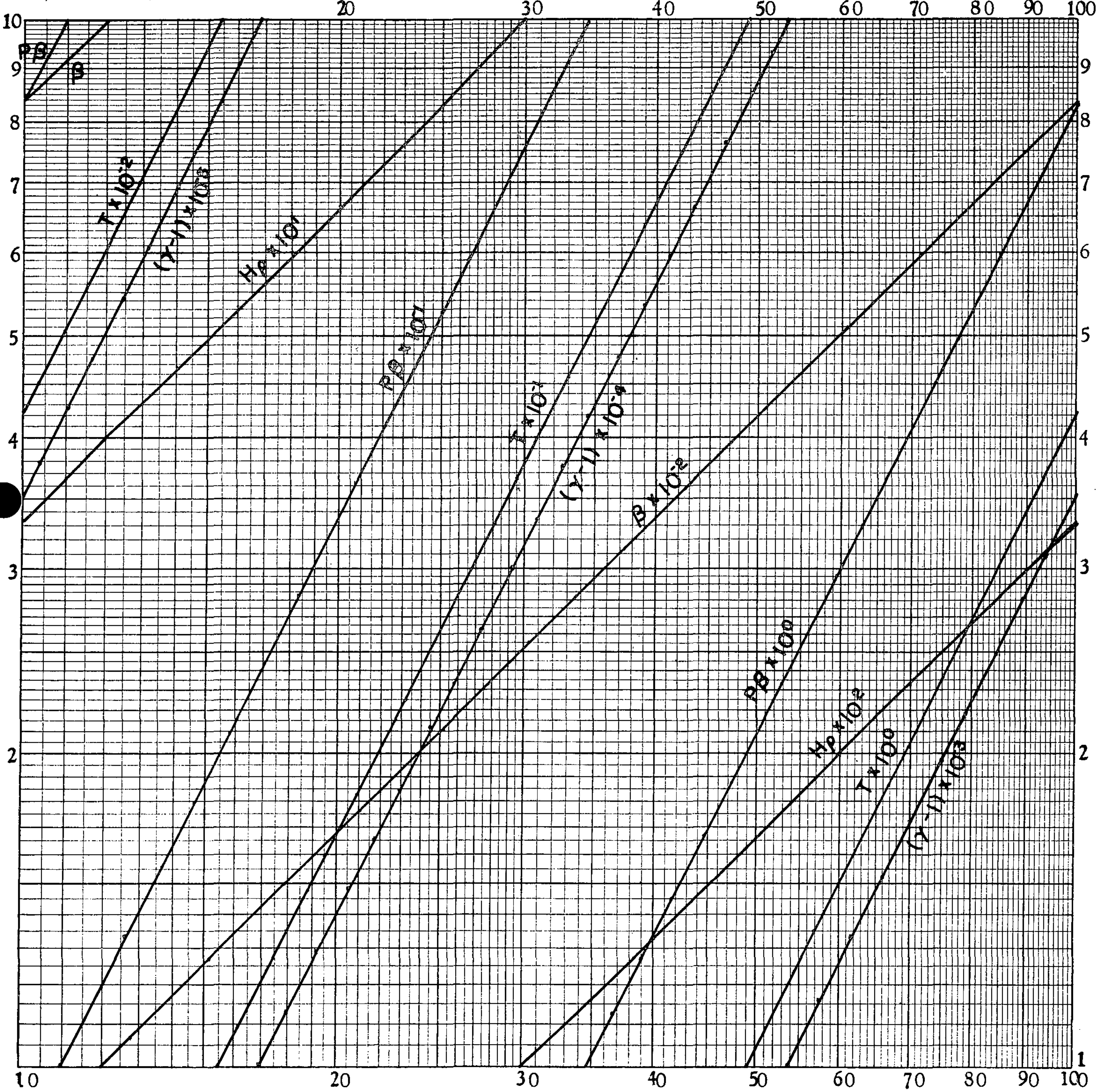
β, $(\gamma - 1)$, T, Pβ, Hρ

T in MeV

Hρ in kgauss-cm

Pβ in MeV/c

M_{Σ^+} = 1189.40 MeV
= 2321.83 m



P (MeV/c)

T in MeV

Hρ in kgauss-cm

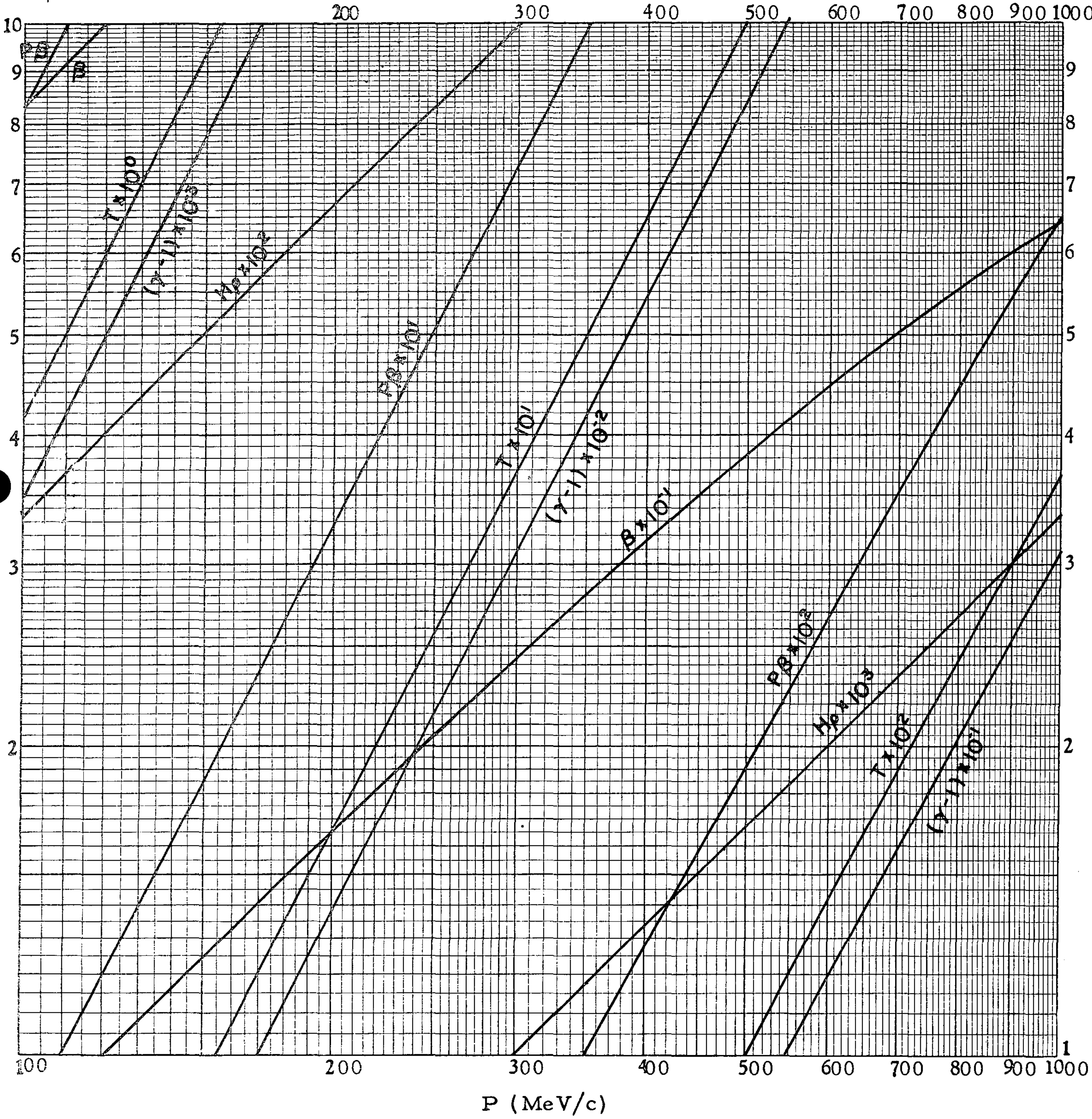
Pβ in MeV/c

Σ^+ HYPERONS

100 MeV/c to 1 BeV/c

β, (γ - 1), T, Pβ, Hρ

M_{Σ^+} = 1189.40 MeV
= 2321.83 m



Σ^+ HYPERONS

1 BeV/c to 10 BeV/c

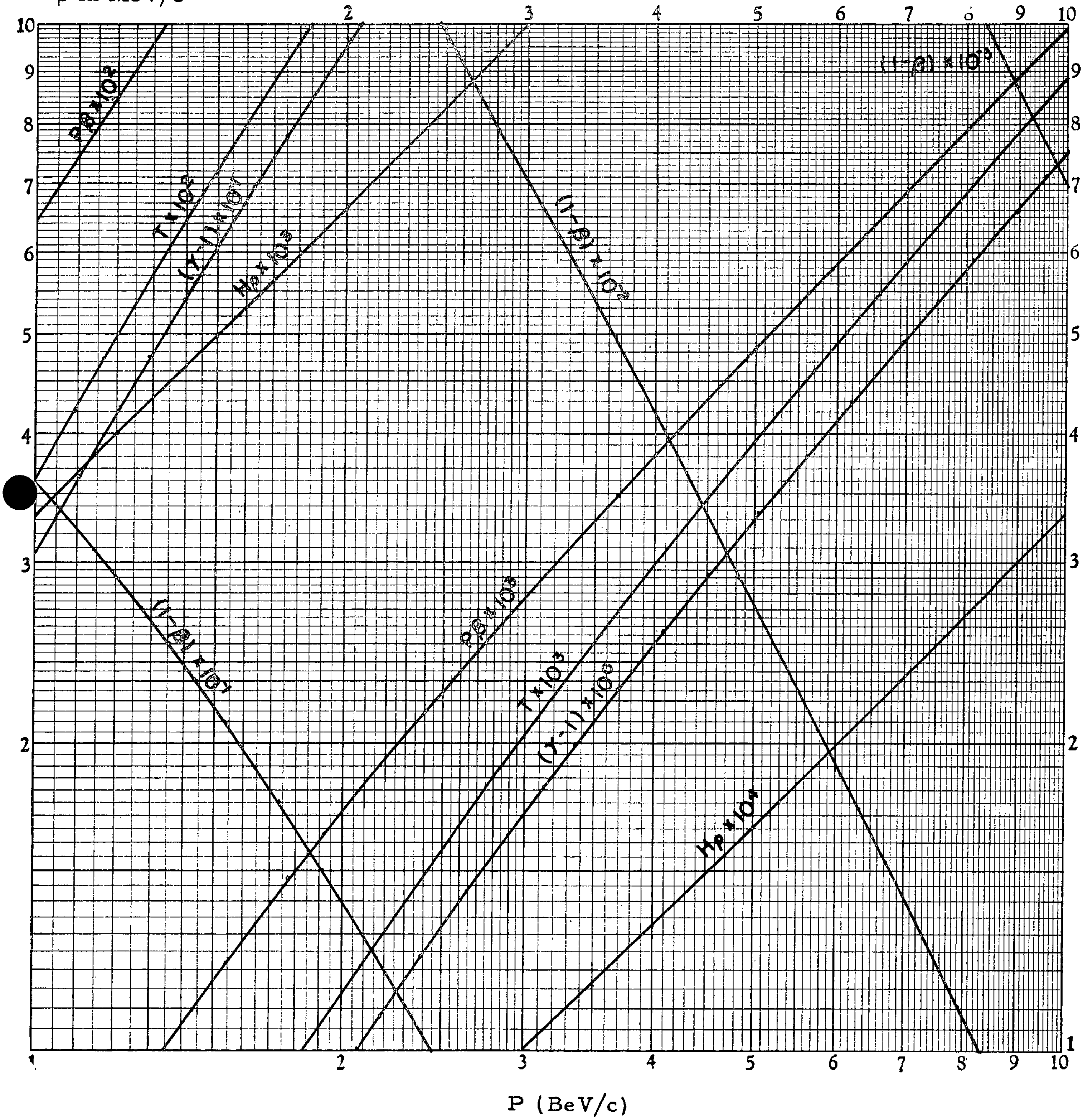
$(1-\beta)$, $(\gamma-1)$, T, $P\beta$, $H\rho$

T in MeV

$H\rho$ in kgauss-cm

$P\beta$ in MeV/c

M_{Σ^+} = 1189.40 MeV
= 2321.83 m



Σ^+ HYPERONS

T in MeV

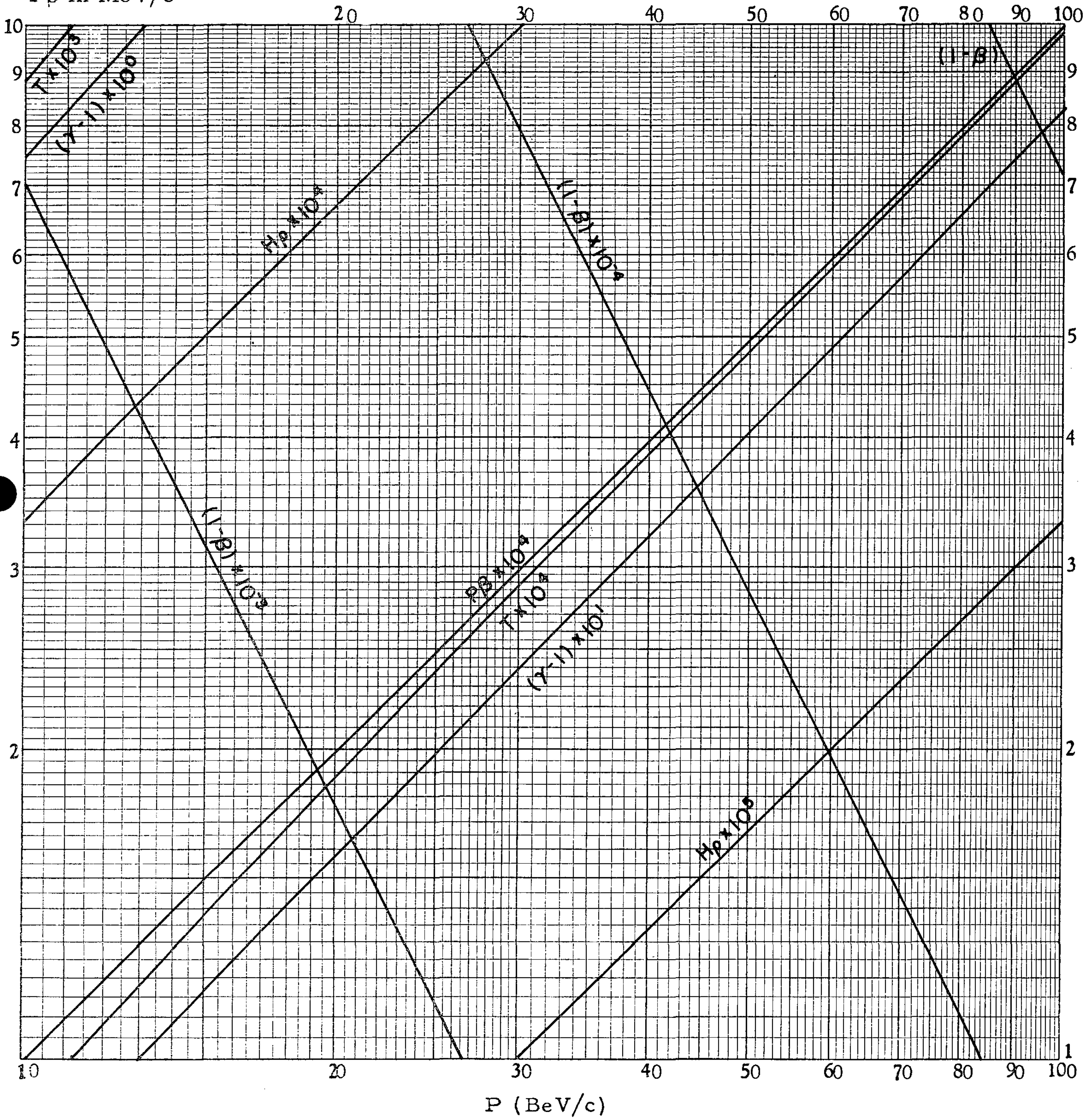
Hρ in kgauss-cm

Pβ in MeV/c

10 BeV/c to 100 BeV/c

β, (1 - β), (γ - 1), T, Pβ, Hρ

M_{Σ^+} = 1189.40 MeV
= 2321.83 m



0 0 1 0 1 2 0 4 7 5 3

Ξ^- HYPERONS

1 MeV/c to 10 MeV/c

β, (γ - 1), T, Pβ, Hρ

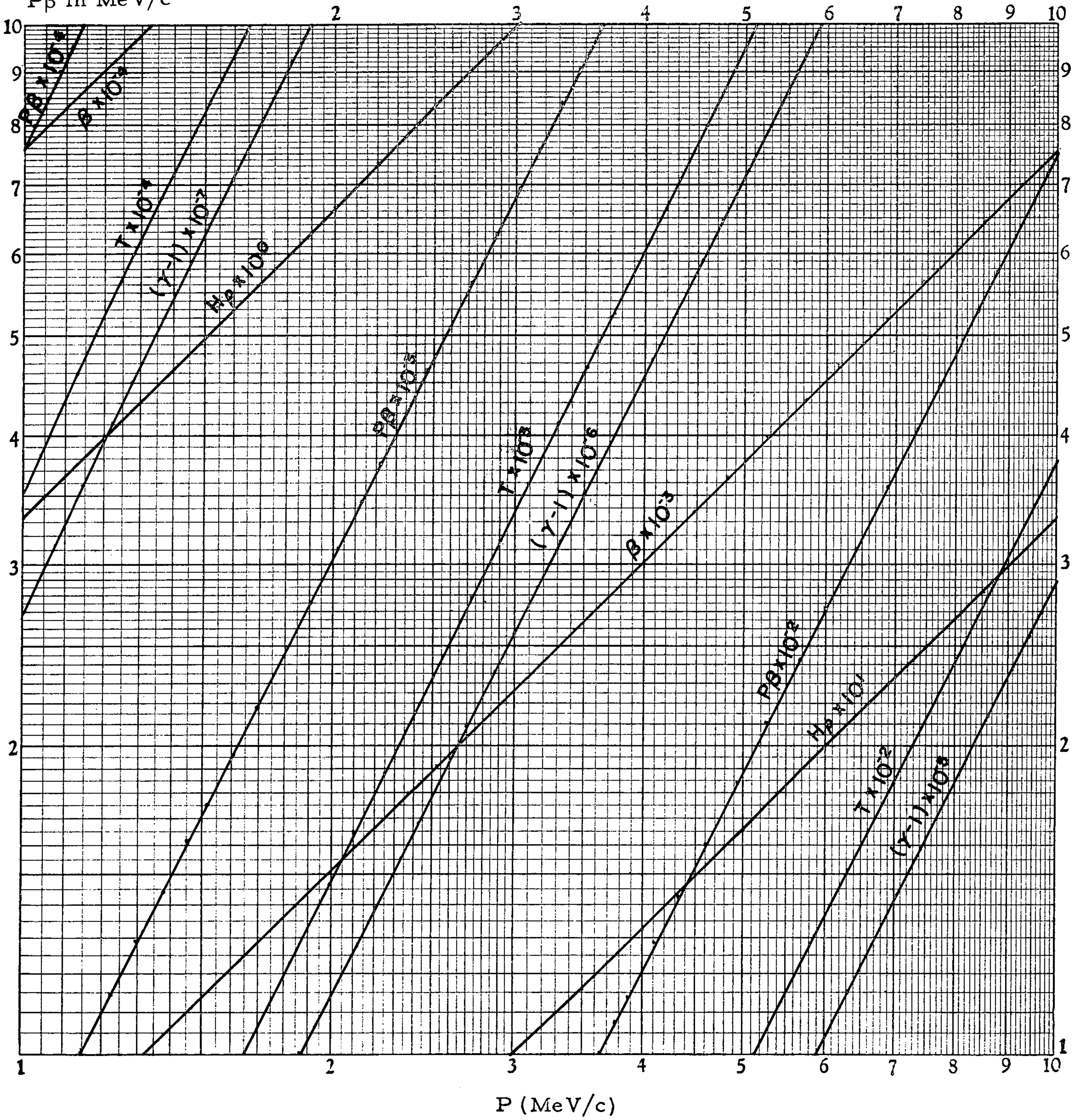
T in MeV

Hρ in kgauss-cm

Pβ in MeV/c

M_{Ξ^-} = 1318.4 MeV

= 2580.16 m

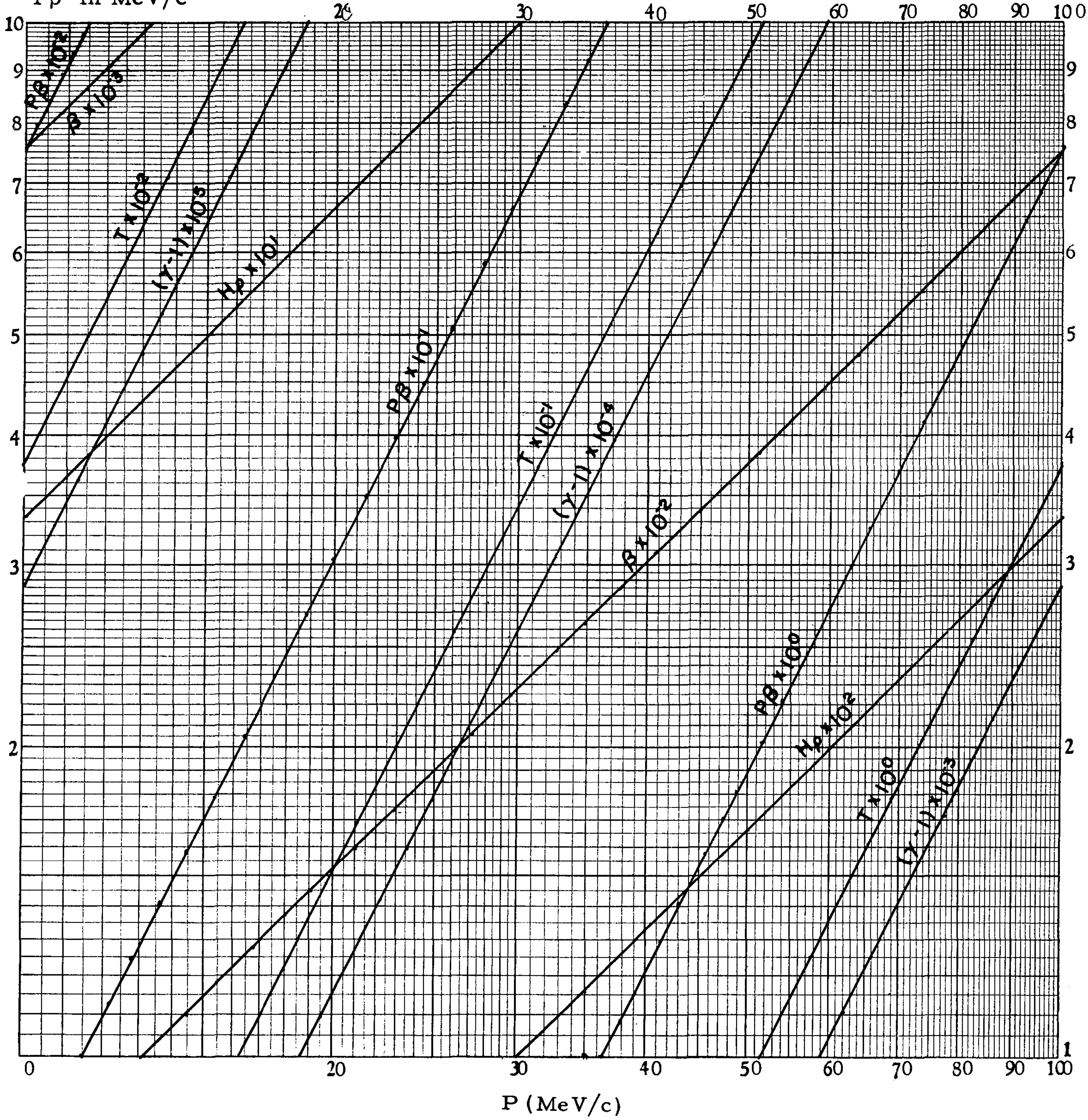


T in MeV
Hρ in kgauss-cm
Pβ in MeV/c

10 MeV/c to 100 MeV/c

β, (γ - 1), T, Pβ, Hρ

M_{Ξ^-} = 1318.4 MeV
= 2580.16 m



Ξ⁻ HYPERONS

100 MeV/c to 1 BeV/c

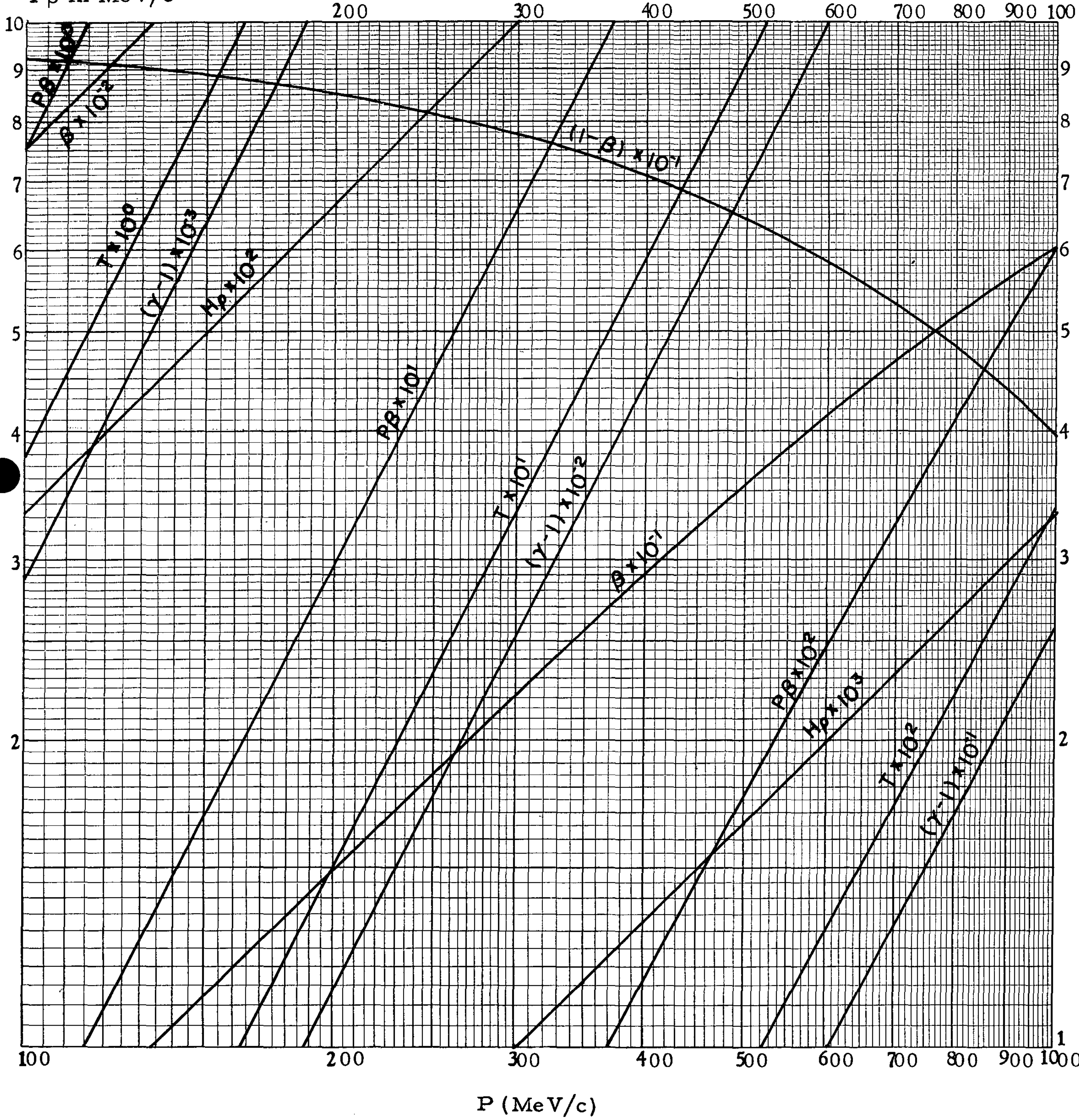
β, (1 - β), (γ - 1), T, Pβ, Hρ

T in MeV

Hρ in kgauss-cm

Pβ in MeV/c

M_{Ξ^-} = 1318.4 MeV
= 2580.16 m



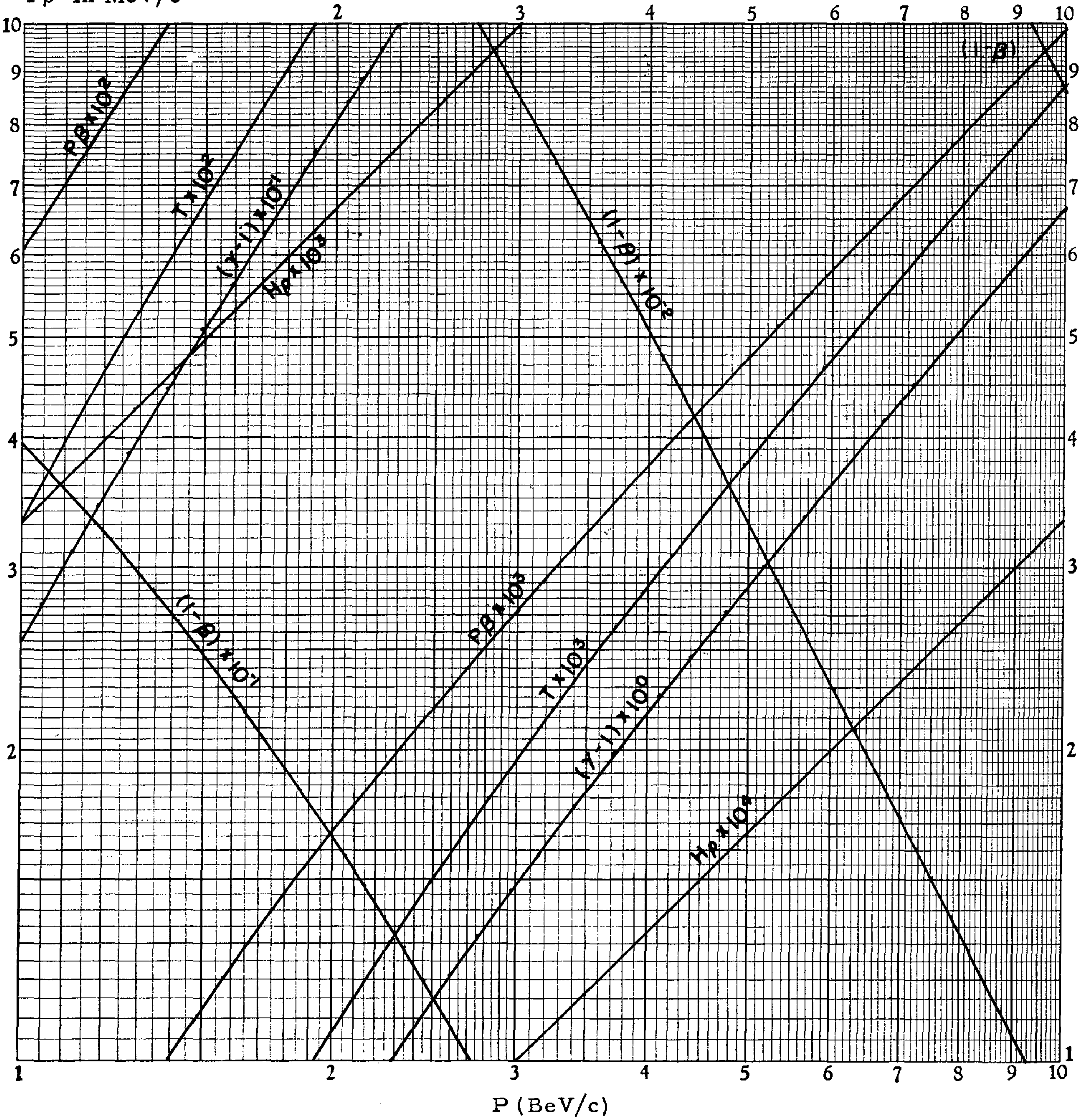
Ξ^- HYPERONS

T in MeV
Hρ in kgauss-cm
Pβ in MeV/c

1 BeV/c to 10 BeV/c

(1 - β), (γ - 1), T, Pβ, Hρ

M_{Ξ^-} = 1318.4 MeV
= 2580.16 m



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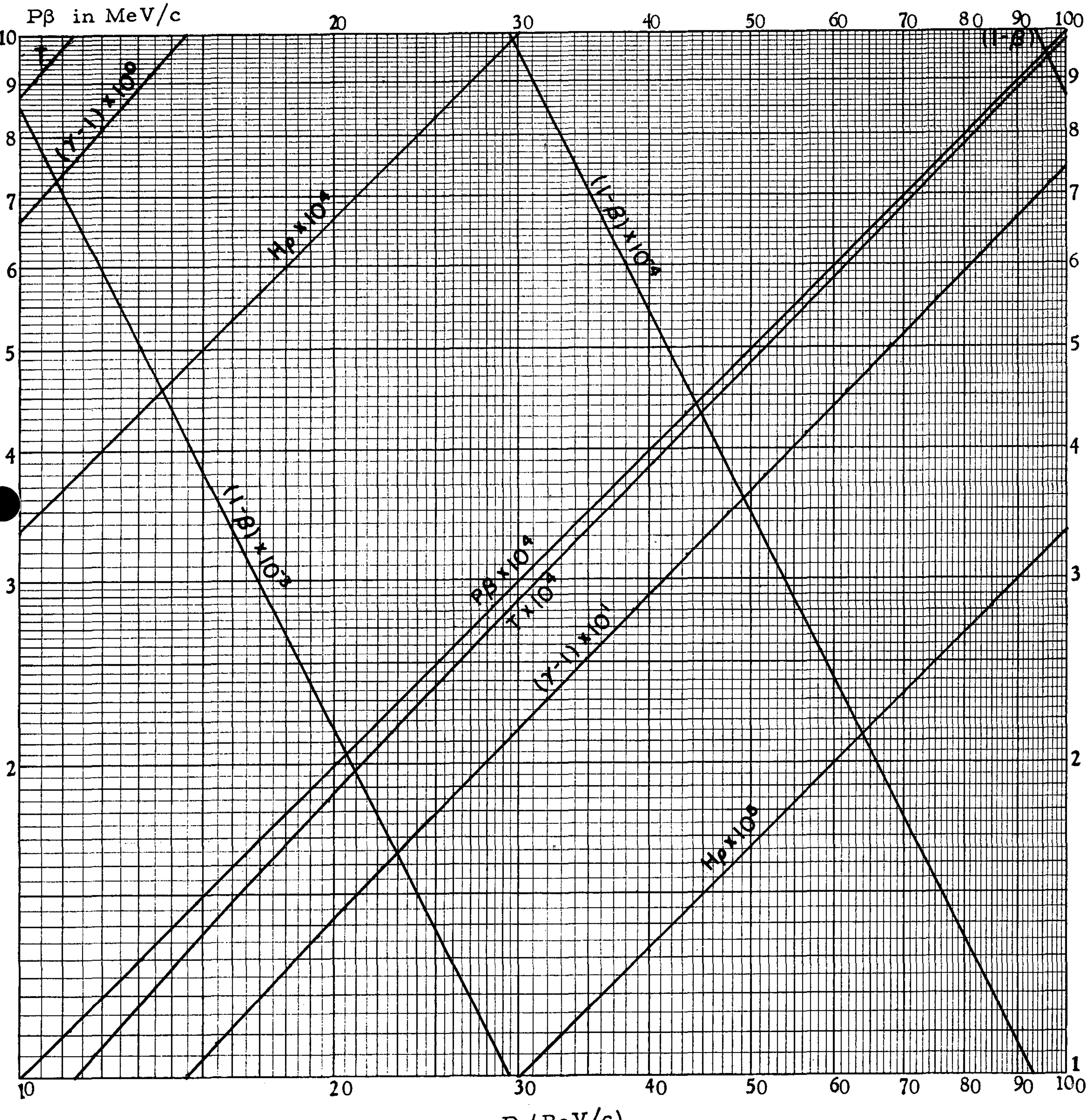
Ξ^- HYPERONS

T in MeV
Hρ in kgauss-cm
Pβ in MeV/c

10 BeV/c to 100 BeV/c
(1 - β), (γ - 1), T, Pβ, Hρ

M_{Ξ^-} = 1318.4 MeV
= 2580.16 m



T in MeV

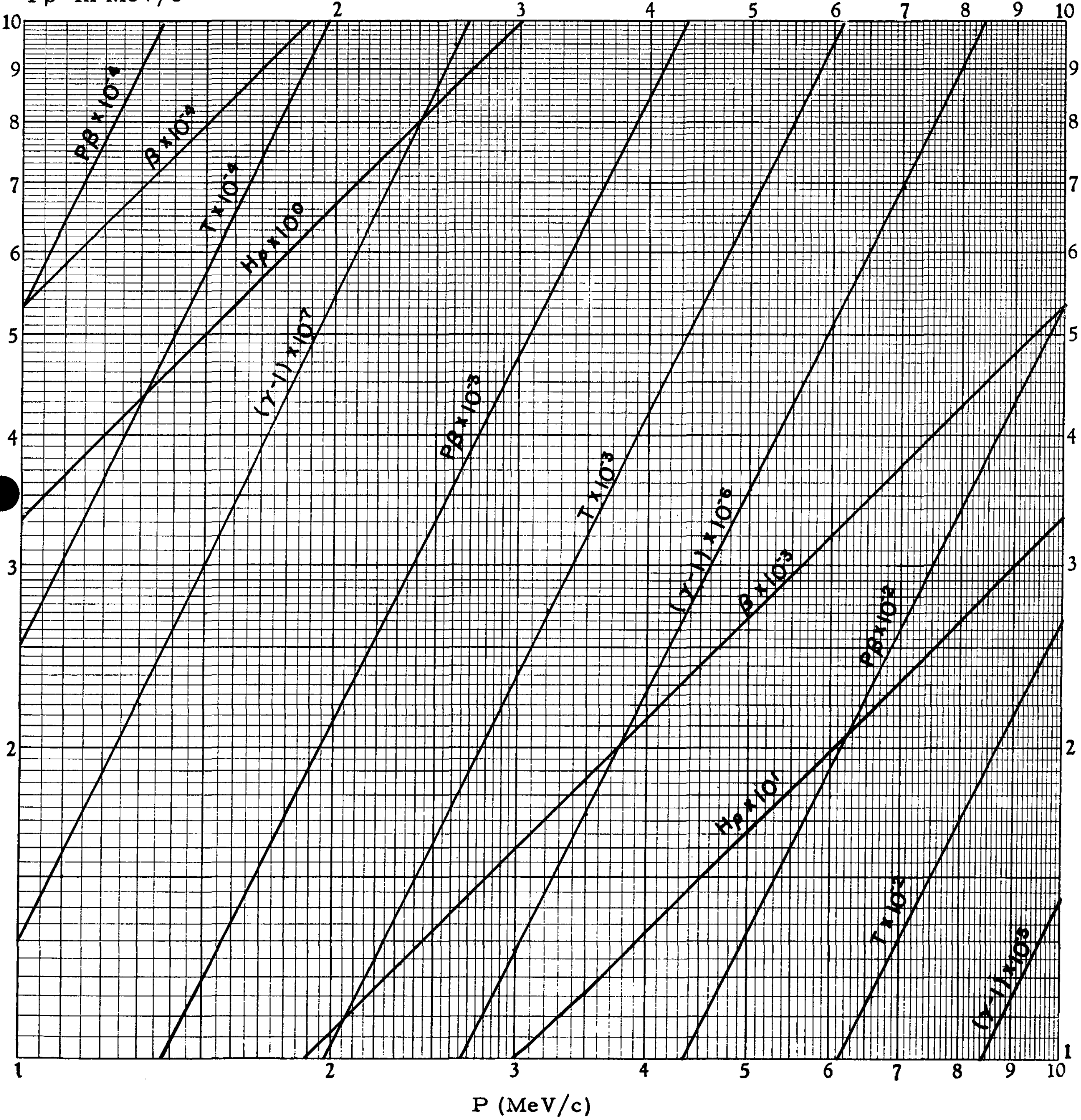
Hρ in kgauss-cm

Pβ in MeV/c

1 MeV/c to 10 MeV/c

β, (γ - 1), T, Pβ, Hρ

M_d = 1875.49 MeV
= 3670.40 m



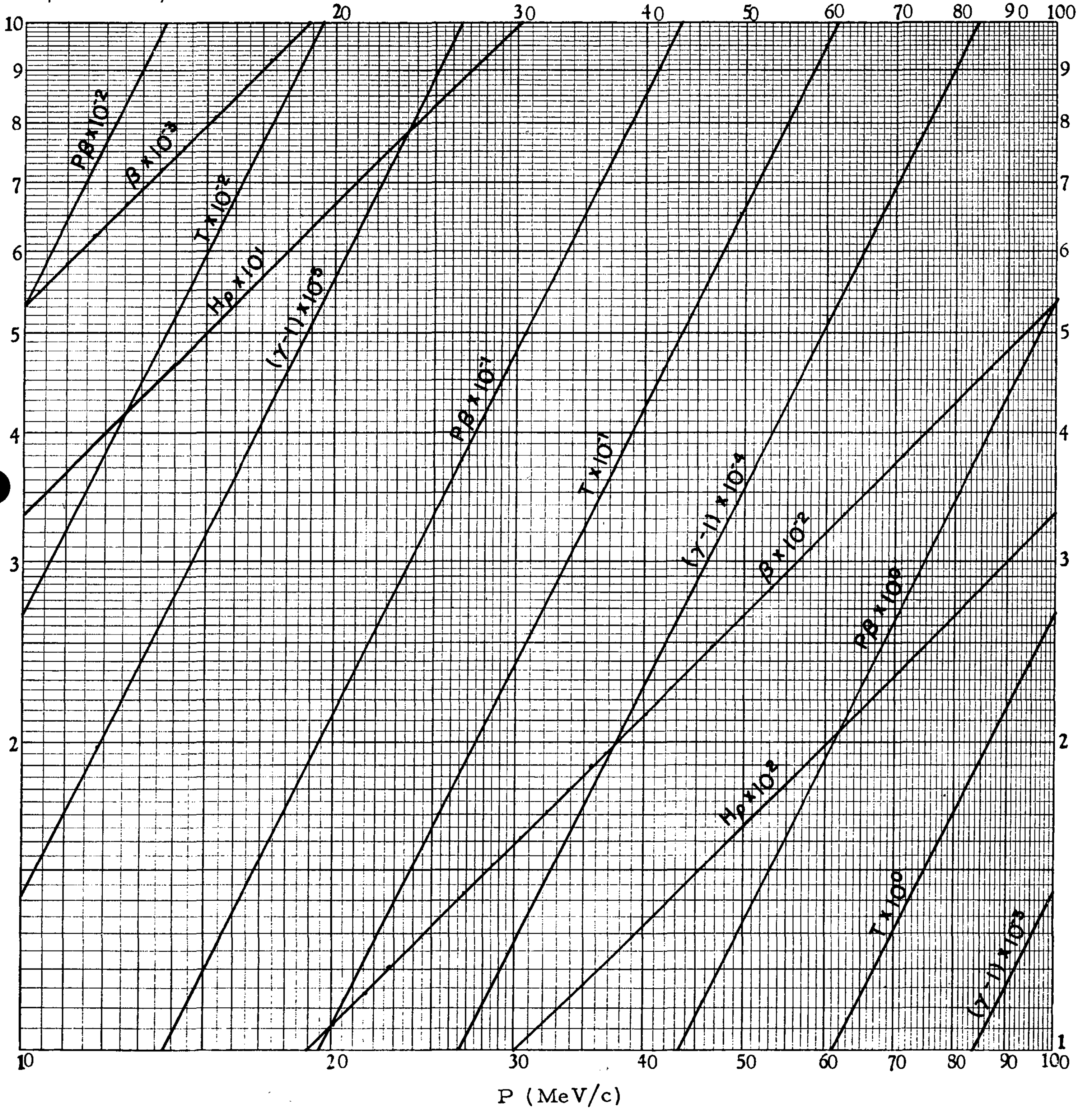
DEUTERONS

T in MeV
Hρ in kgauss-cm
Pβ in MeV/c

10 MeV/c to 100 MeV/c

β, (γ - 1), T, Pβ, Hρ

M_d = 1875.49 MeV
= 3670.40 m



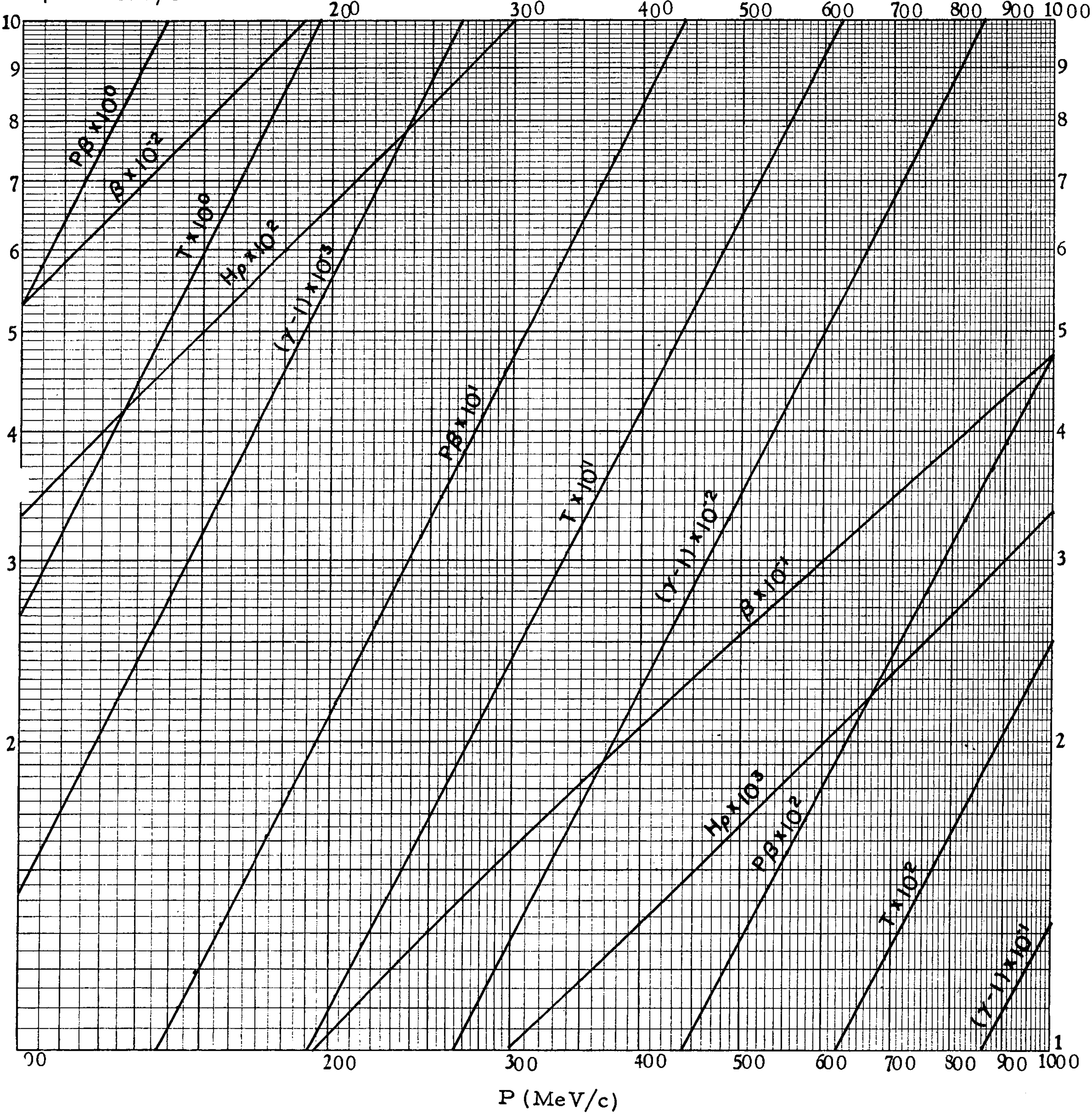
DEUTERONS

T in MeV
Hρ in kgauss-cm
Pβ in MeV/c

100 MeV/c to 1 BeV/c

β, (γ - 1), T, Pβ, Hρ

M_d = 1875.49 MeV
= 3670.40 m

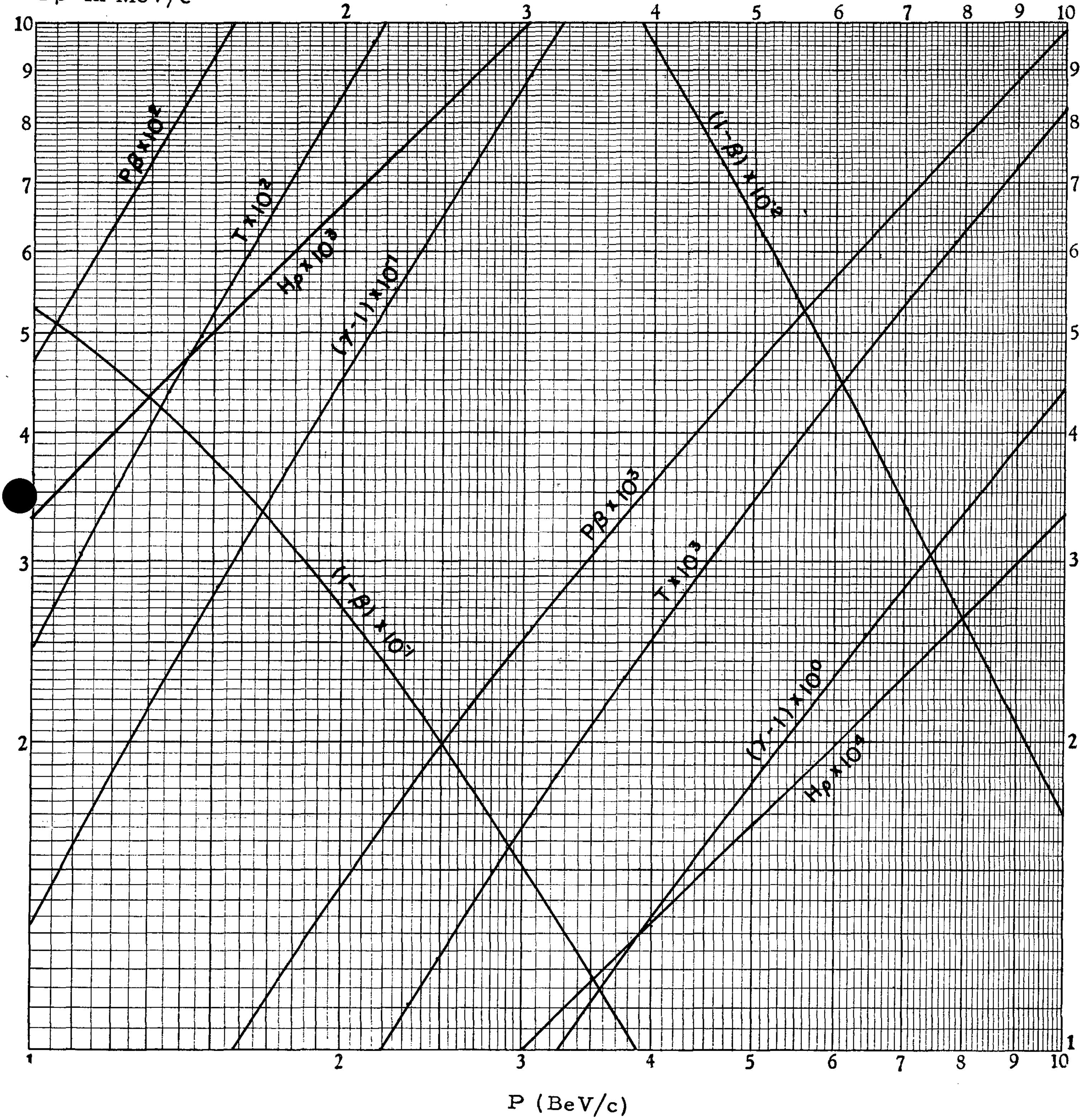


DEUTERONS

T in MeV
Hρ in kgauss-cm
Pβ in MeV/c

(1 -β), (γ - 1), T, Pβ, Hρ
1 BeV/c to 10 BeV/c

M_d = 1875.49 MeV
= 3670.40 m



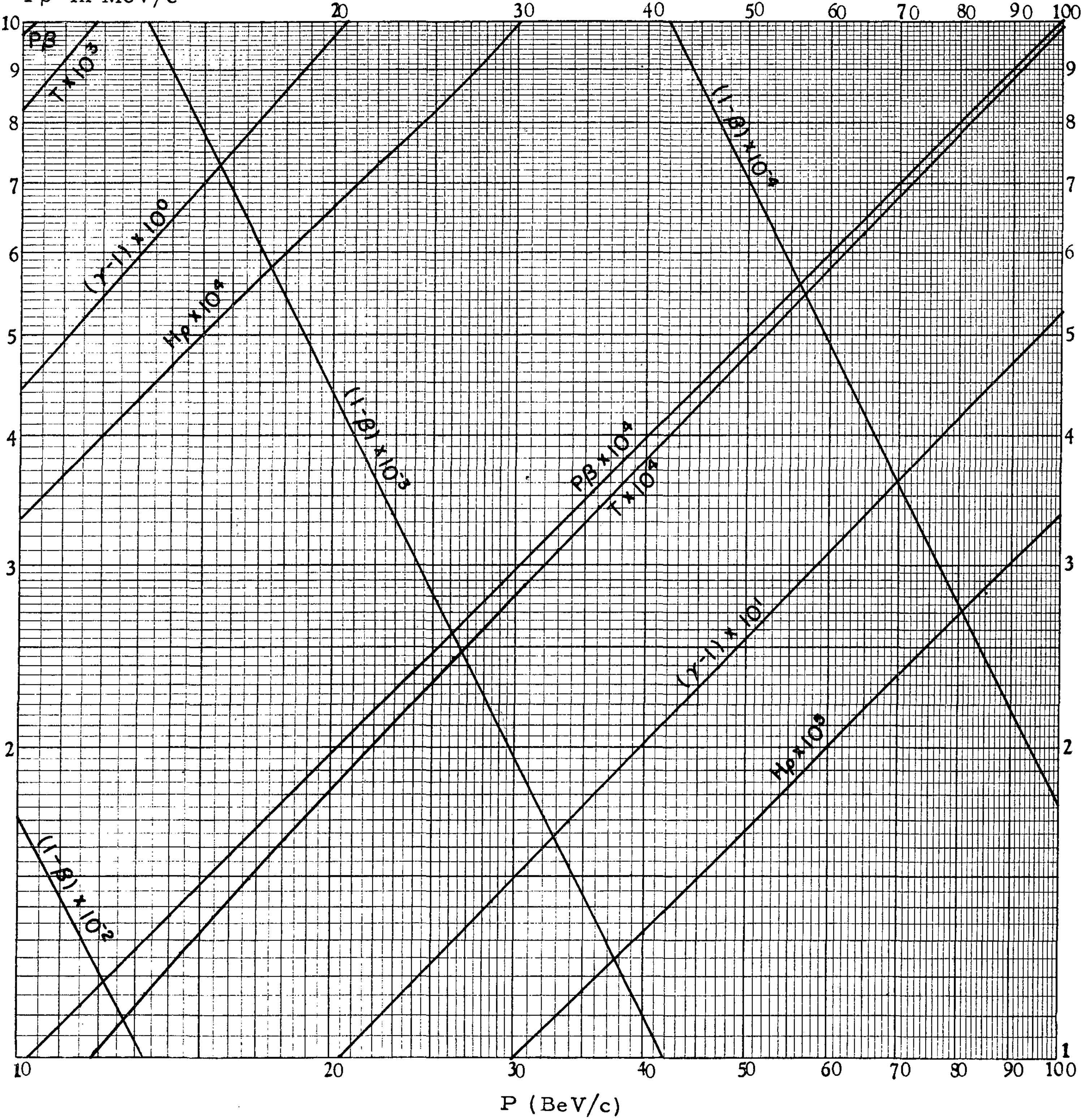
DEUTERONS

T in MeV
Hρ in kgauss-cm
Pβ in MeV/c

10 BeV/c to 100 BeV/c

(1 -β), (γ - 1), T, Pβ, Hρ

M_d = 1875.49 MeV
= 3670.40 m

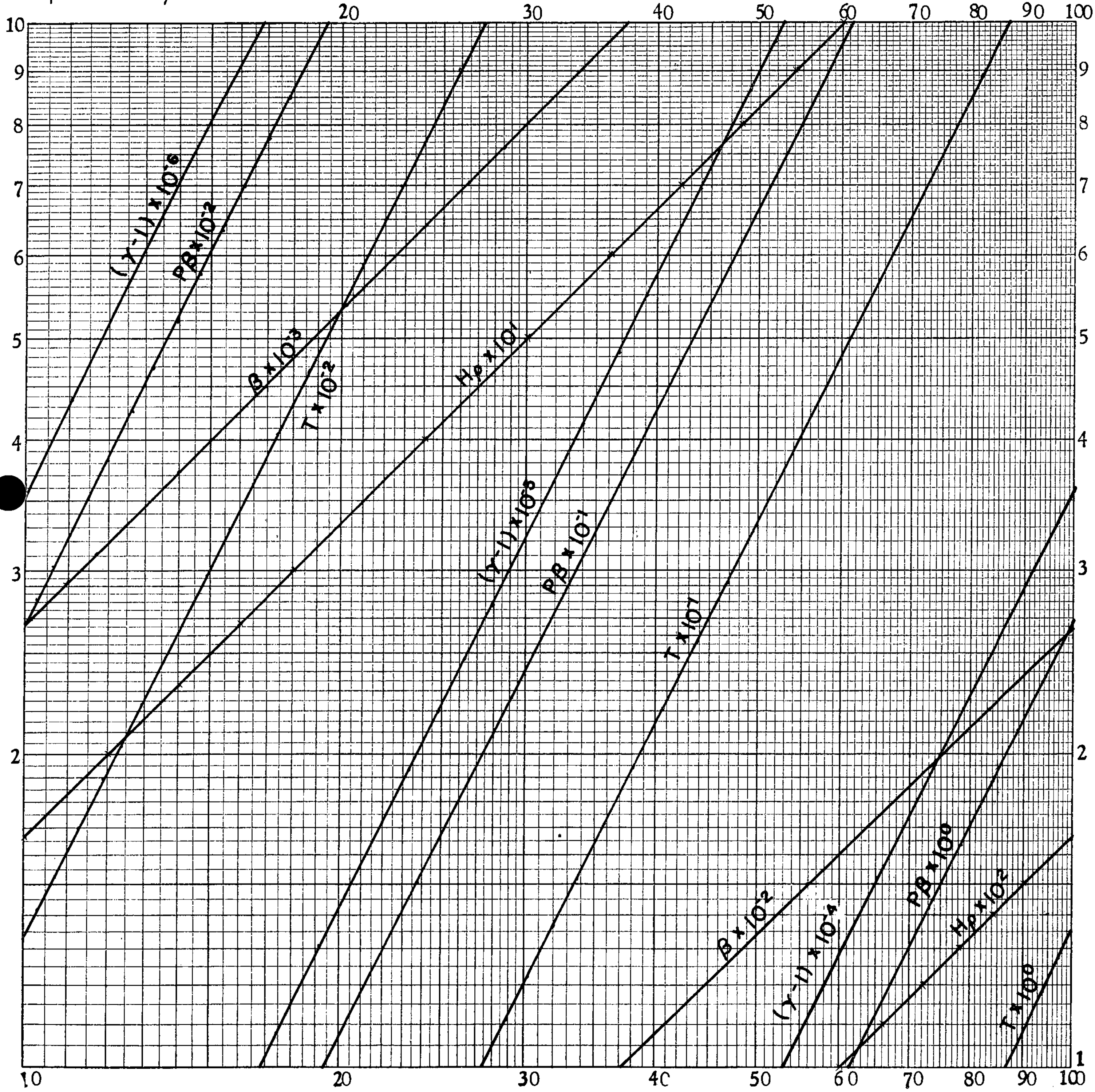


ALPHA PARTICLES

T in MeV
Hρ in kgauss-cm
Pβ in MeV/c

10 MeV/c to 100 MeV/c
β, (γ - 1), T, Pβ, Hρ

M_a = 3727.23 MeV
= 7294.47 m



P (MeV/c)

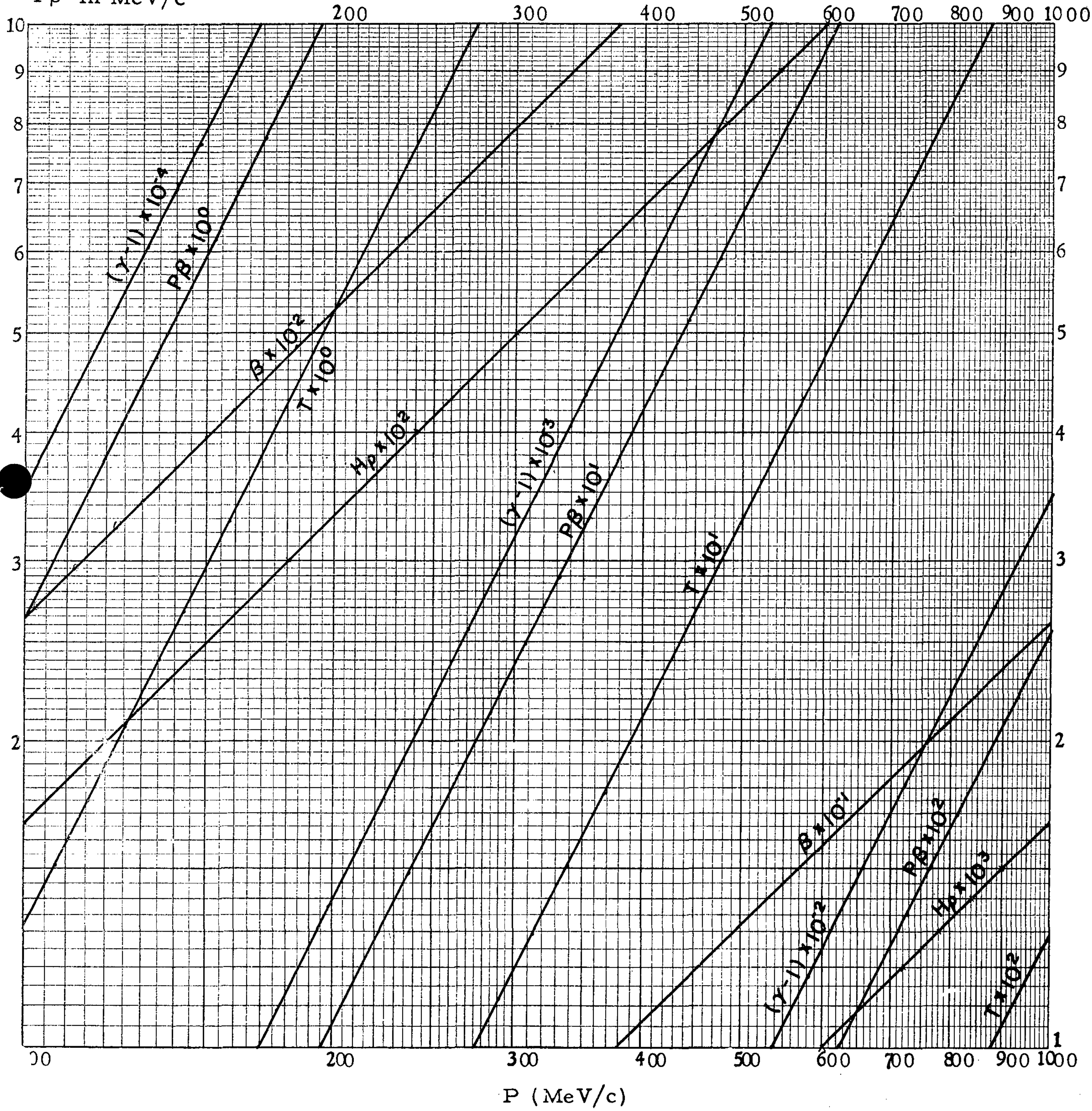
ALPHA PARTICLES

100 MeV/c to 1 BeV/c

β, (γ - 1), T, Pβ, Hρ

T in MeV
Hρ in kgauss-cm
Pβ in MeV/c

M_α = 3727.23 MeV
= 7294.47 m



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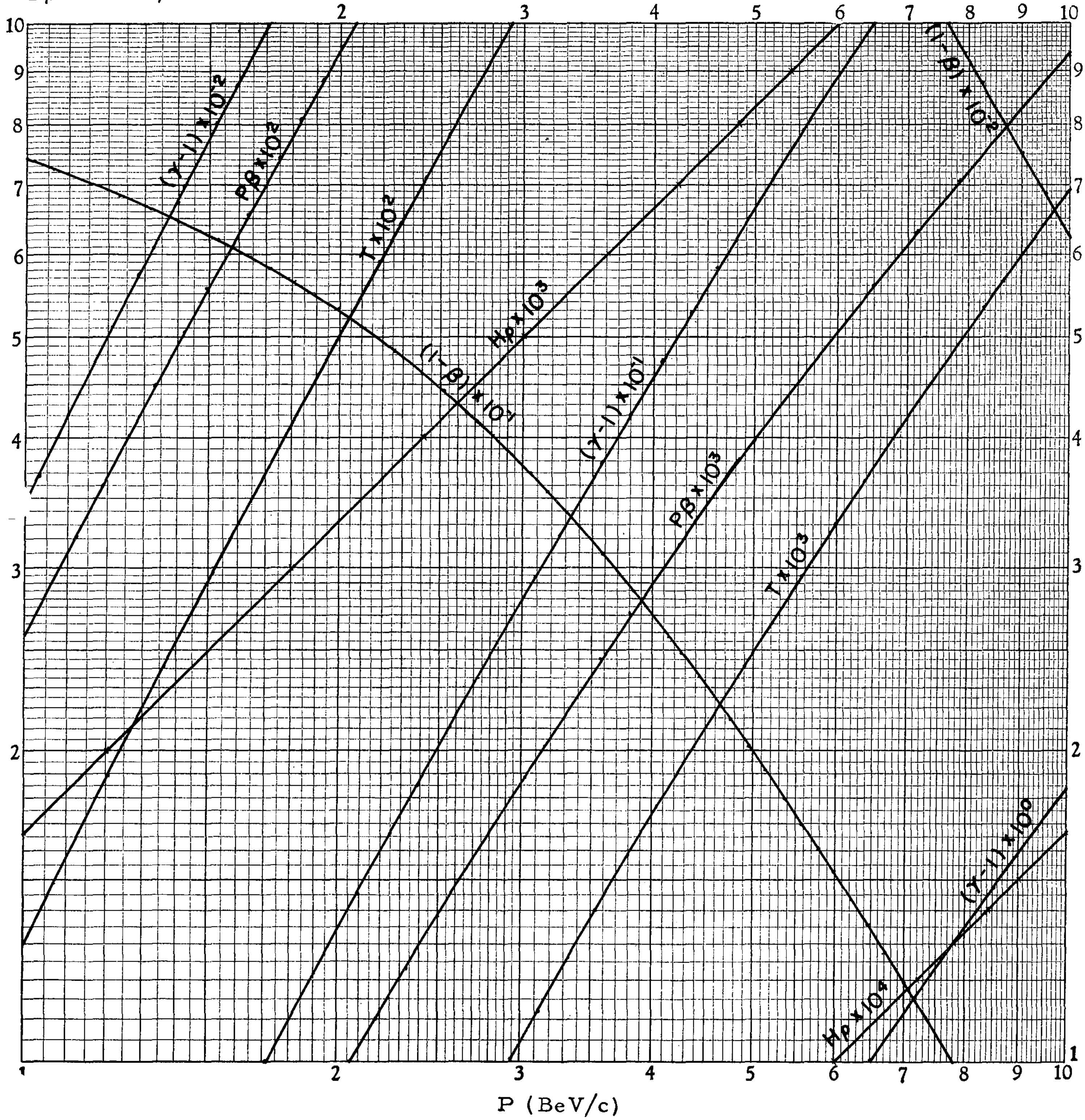
T in MeV
Hρ in kgauss-cm
Pβ in MeV/c

1 BeV/c to 10 BeV/c

(1 -β), (γ - 1), T, Pβ, Hρ

M_a = 3727.23 MeV
= 7294.47 m



0 0 1 0 1 2 0 4 7 6 6

T in MeV

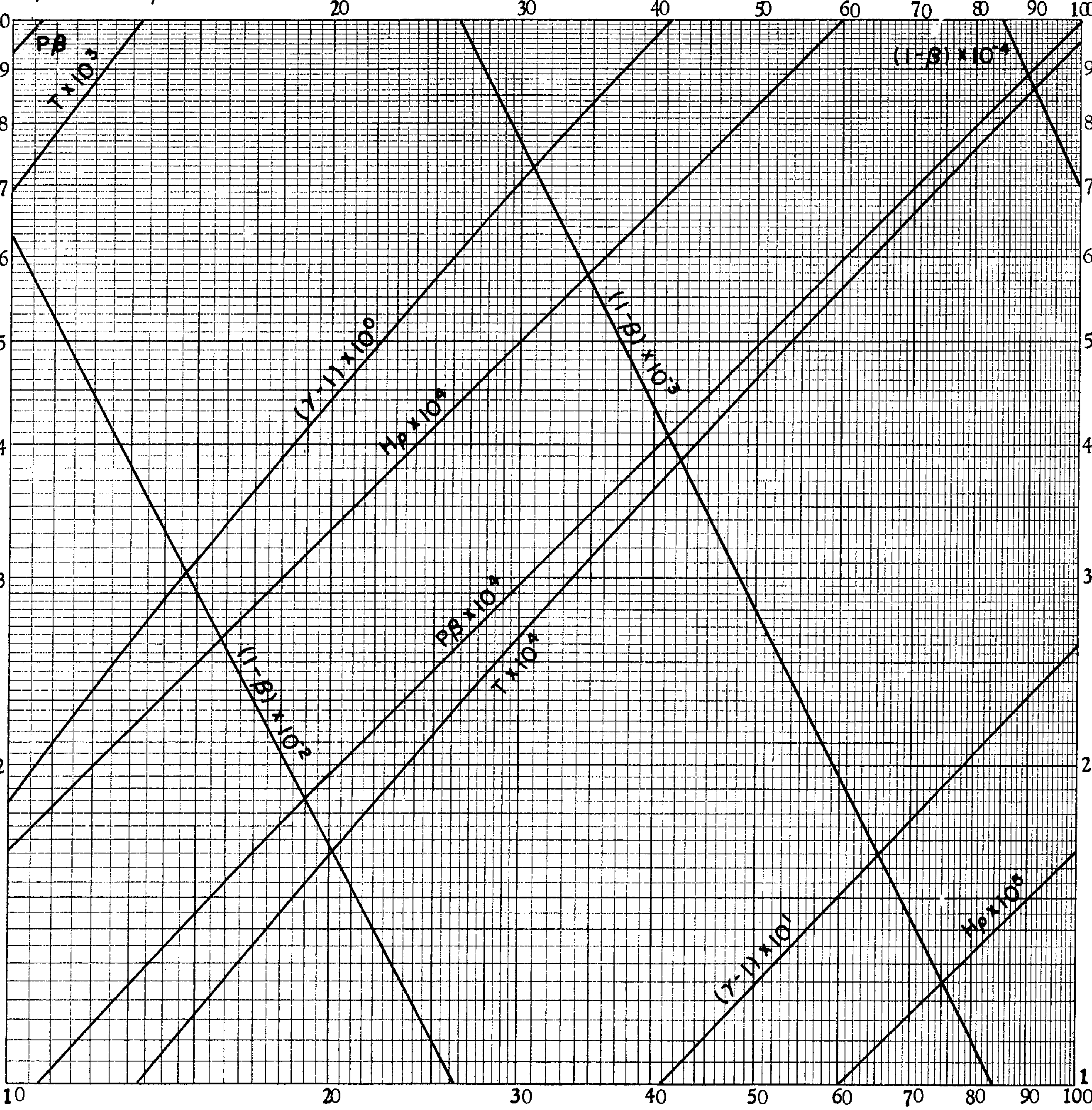
Hρ in kgauss-cm

Pβ in MeV/c

10 BeV/c to 100 BeV/c

(1 -β), (γ - 1), T, Pβ, Hρ

M_a = 3727.23 MeV = 7294.47 m



P (BeV/c)

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DYNAMICS OF COLLISIONS WITH A PROTON TARGET

PROTONS: $\gamma + p$

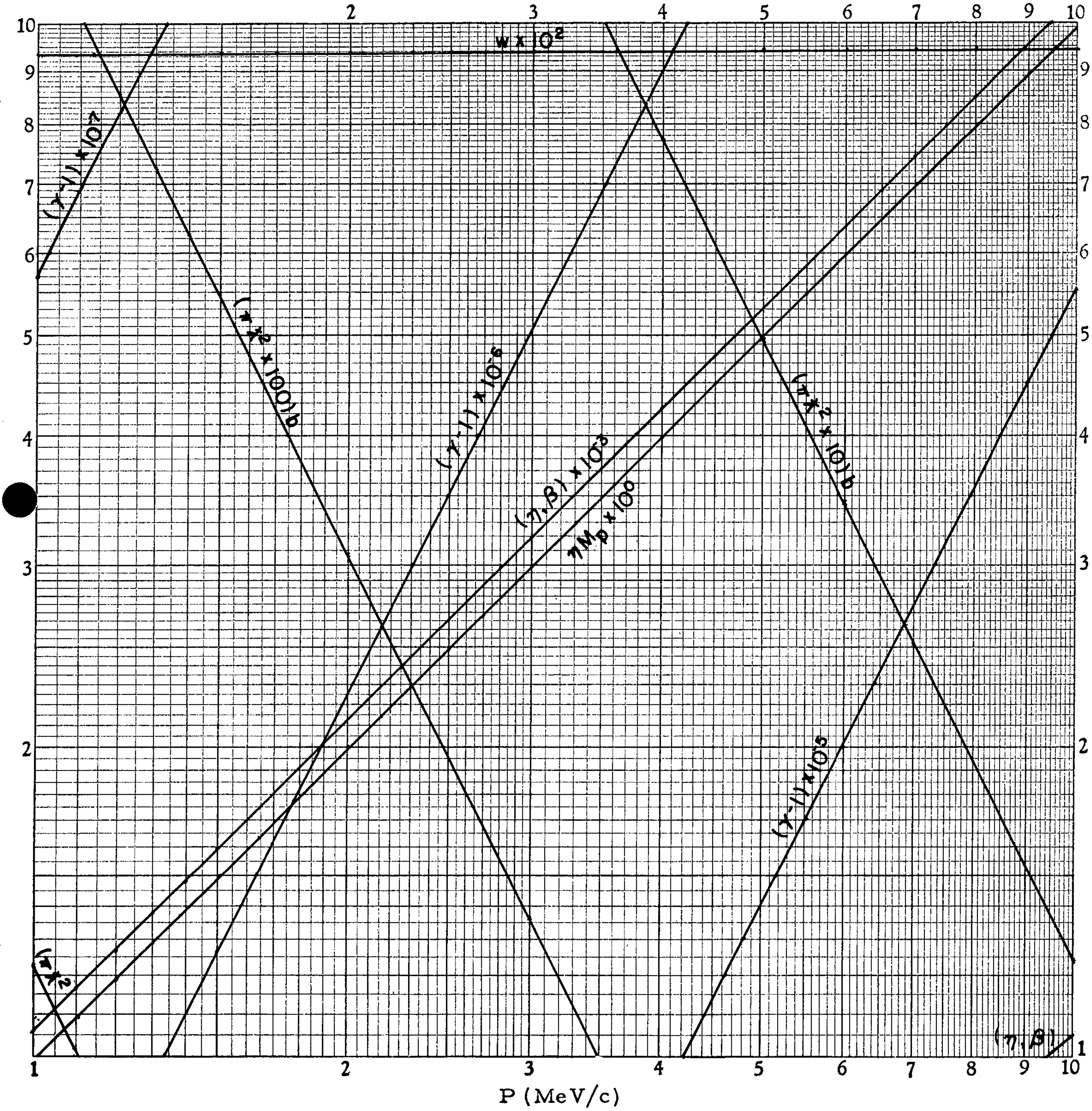
ηM_p in MeV
w in MeV
b = 10^{-24} cm^2

1 MeV/c to 10 MeV/c

β, $(\gamma - 1)$, η, ηM_p, w, $\pi \lambda\!\!\!^{-2}$

M_γ = 0 MeV

M_p = 938.213 MeV



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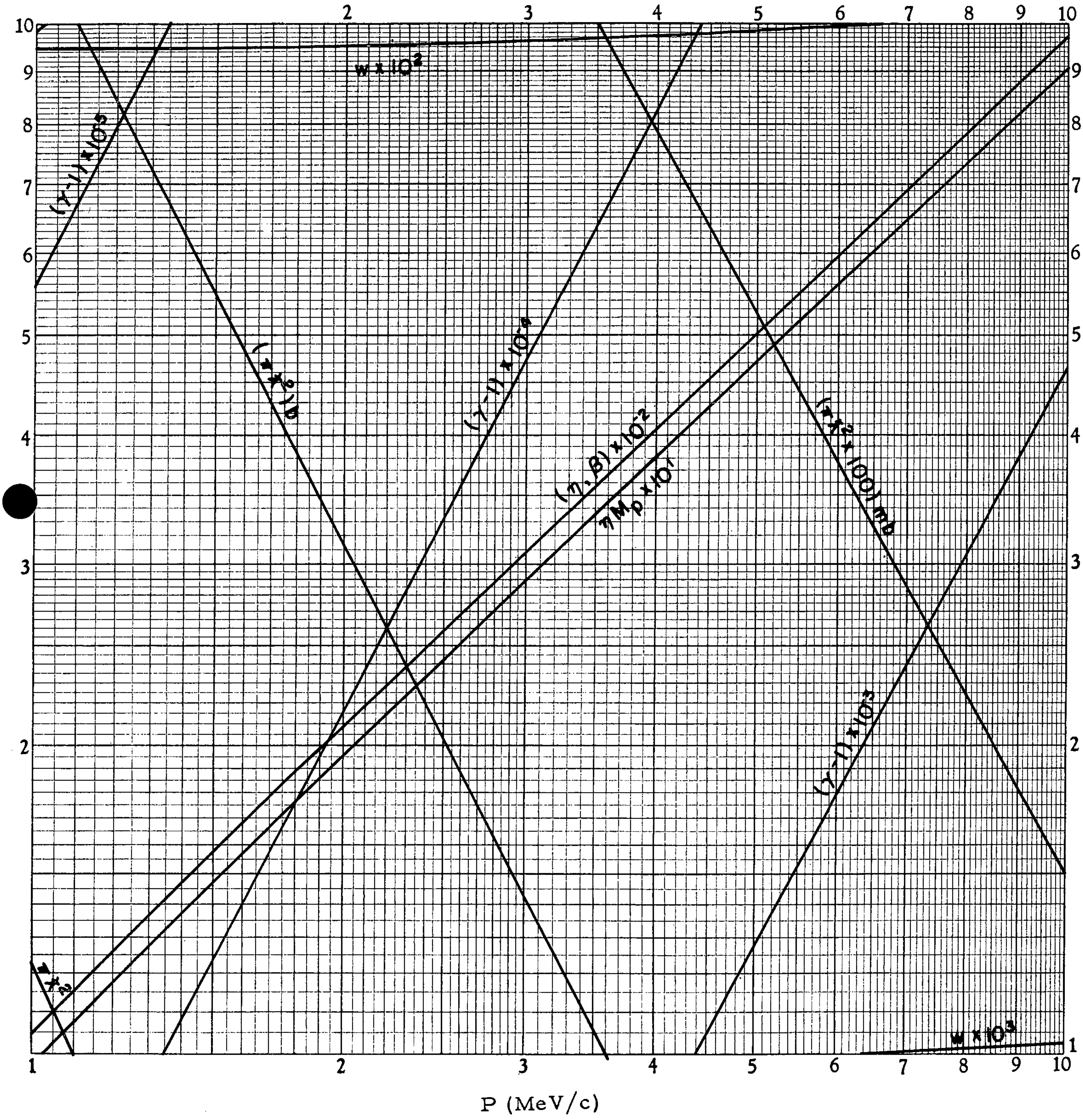
PROTONS: $\gamma + p$

ηM_p in MeV
w in MeV
$b = 10^{-24} cm^2$

10 MeV/c to 100 MeV/c
β, $(\gamma - 1)$, η, ηM_p, w, $\pi \lambda\!\!\!^{-2}$

M_γ = 0 MeV
M_p = 938.213 MeV



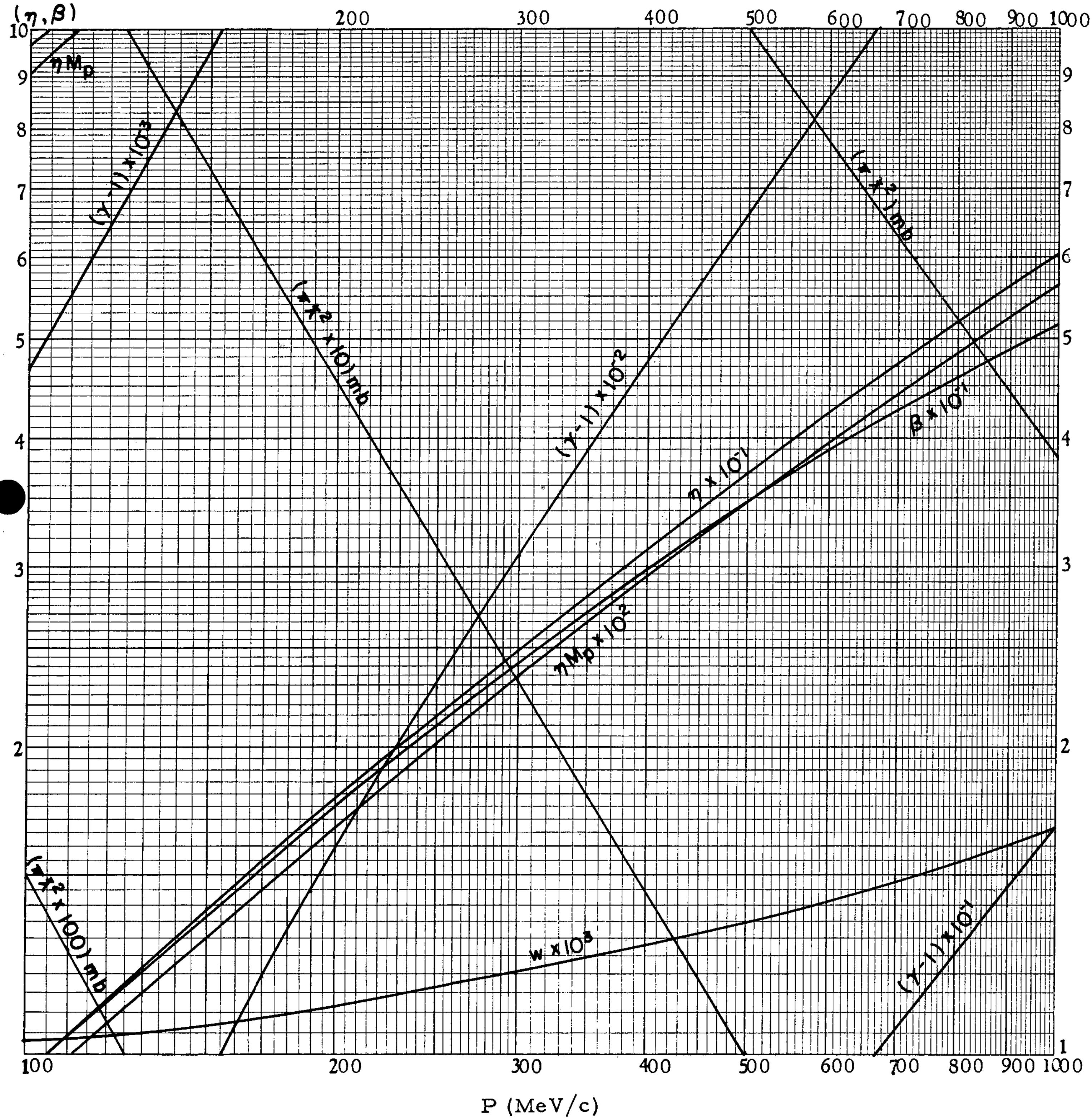
PROTONS: $\gamma + p$

100 MeV/c to 1 BeV/c

β, $(\gamma - 1)$, η, ηM_p, w

ηM_p in MeV
w in MeV
mb = 10^{-27} cm^2

M_γ = 0 MeV
M_p = 938.213 MeV



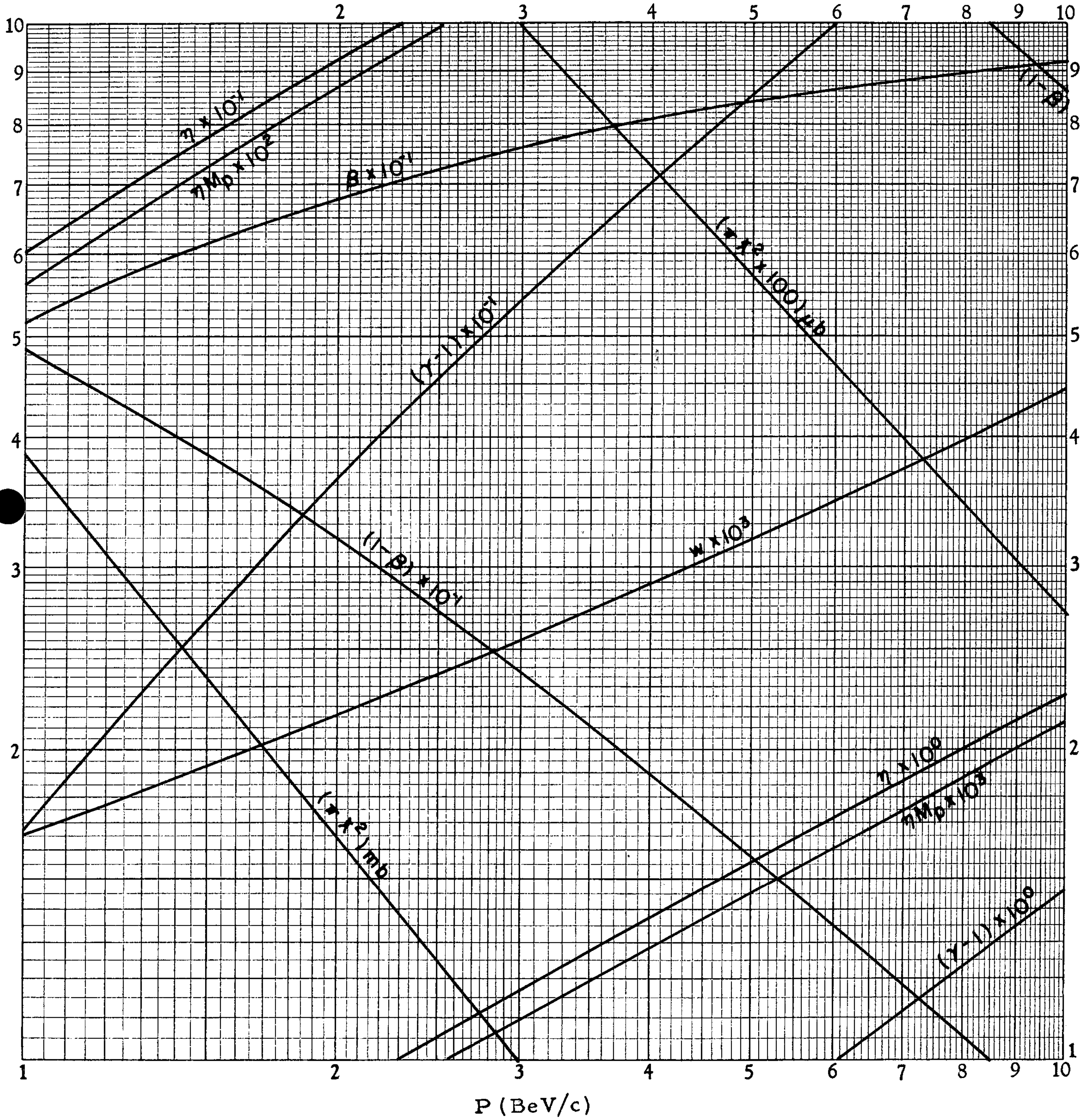
ηM_p in MeV
w in MeV
mb = 10^{-27} cm^2

1 BeV/c to 10 BeV/c

β, $(\gamma - 1)$, η, ηM_p, w

M_γ = 0 MeV

M_p = 938.213 MeV



PROTONS: $\gamma + p$

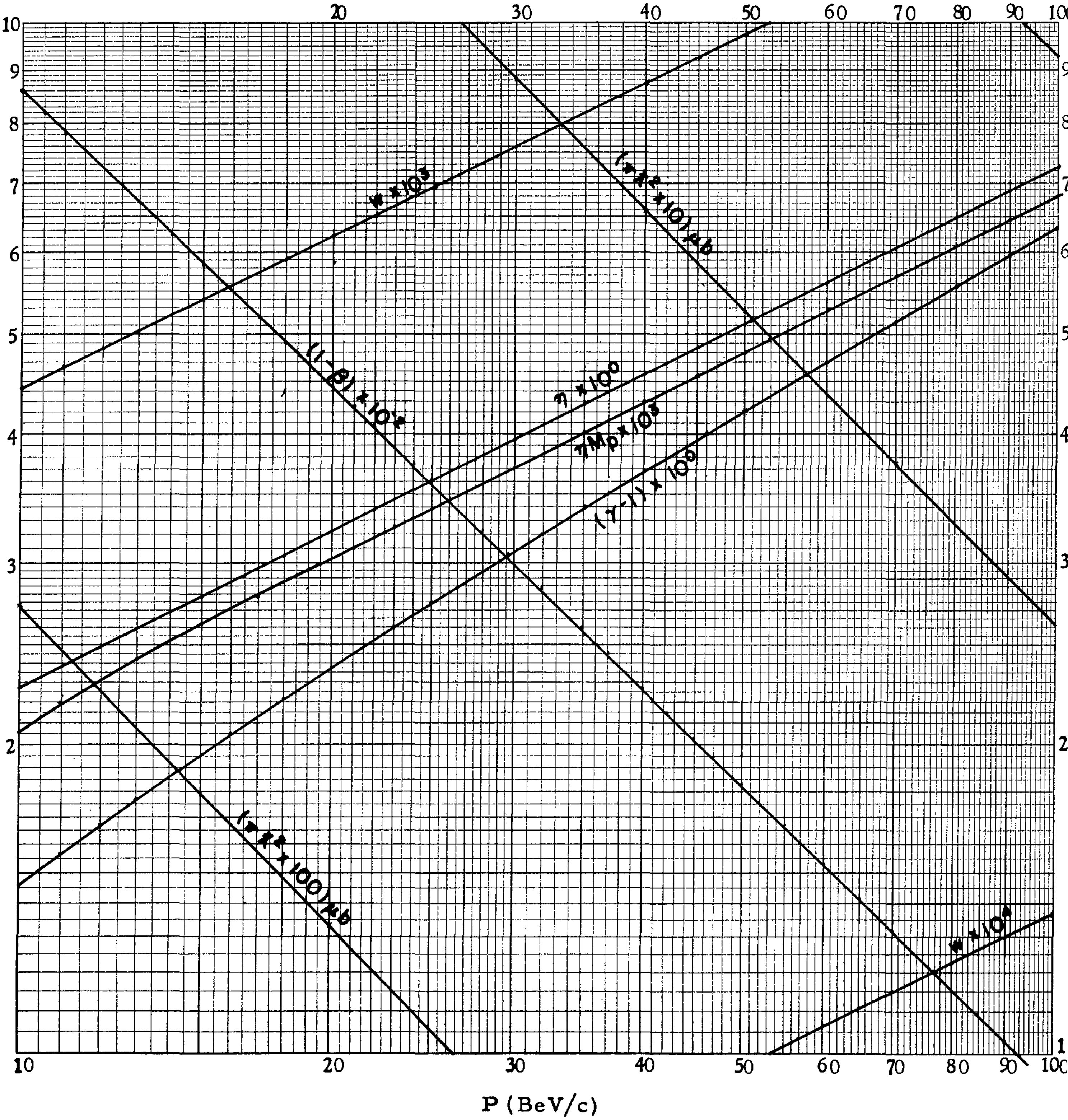
ηM_p in MeV
w in MeV
$\mu b = 10^{-30}\ cm^2$

10 BeV/c to 100 BeV/c

$(1 - \beta)$, $(\gamma - 1)$, η, ηM_p, w

M_γ = 0 MeV

M_p = 938.213 MeV

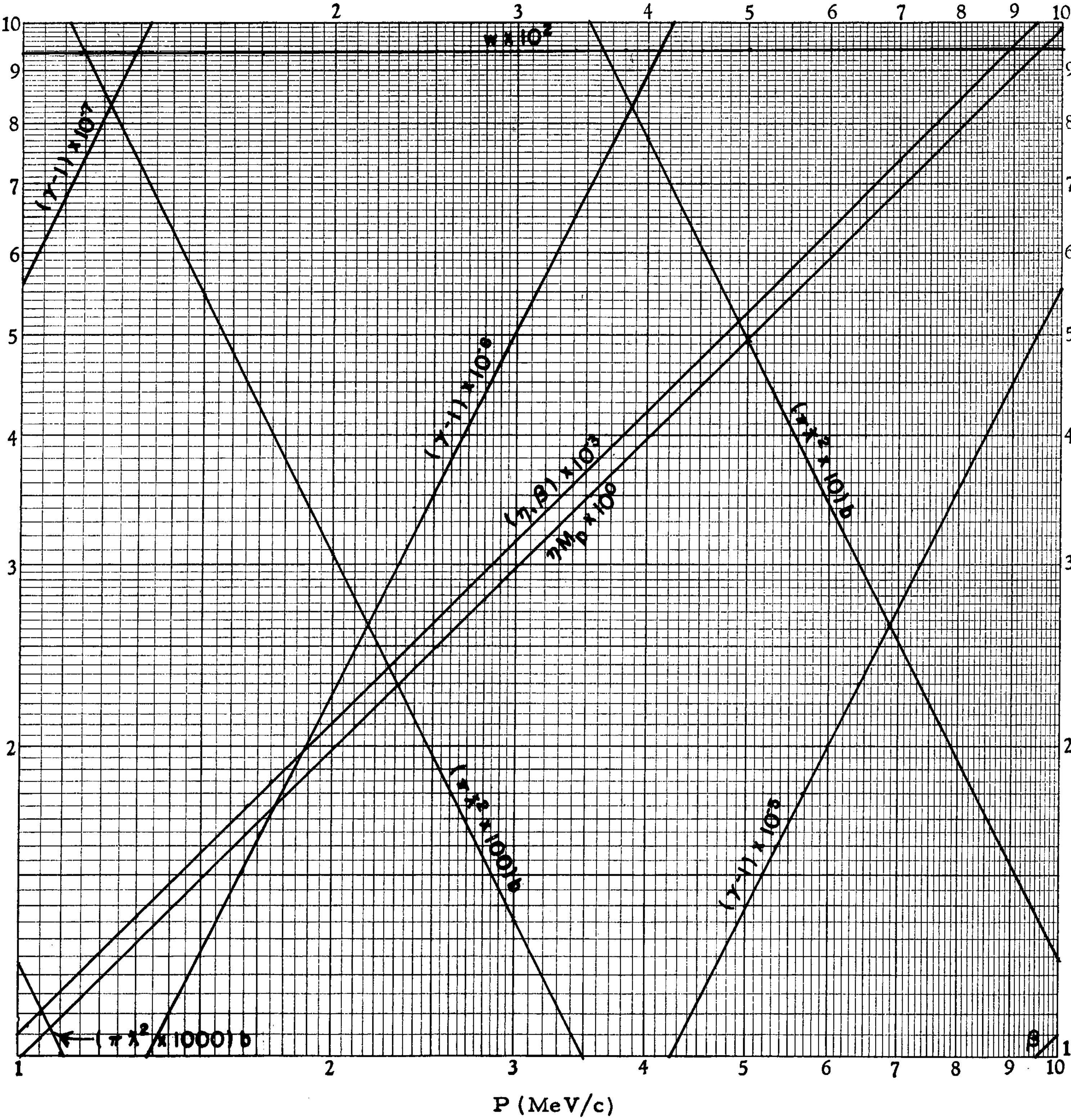


ELECTRONS: $e^{\pm} + p$

1 MeV/c to 10 MeV/c

β, $(\gamma - 1)$, η, ηM_p, w

$M_{e\pm} = 0.510976$ MeV
$M_p = 938.213$ MeV

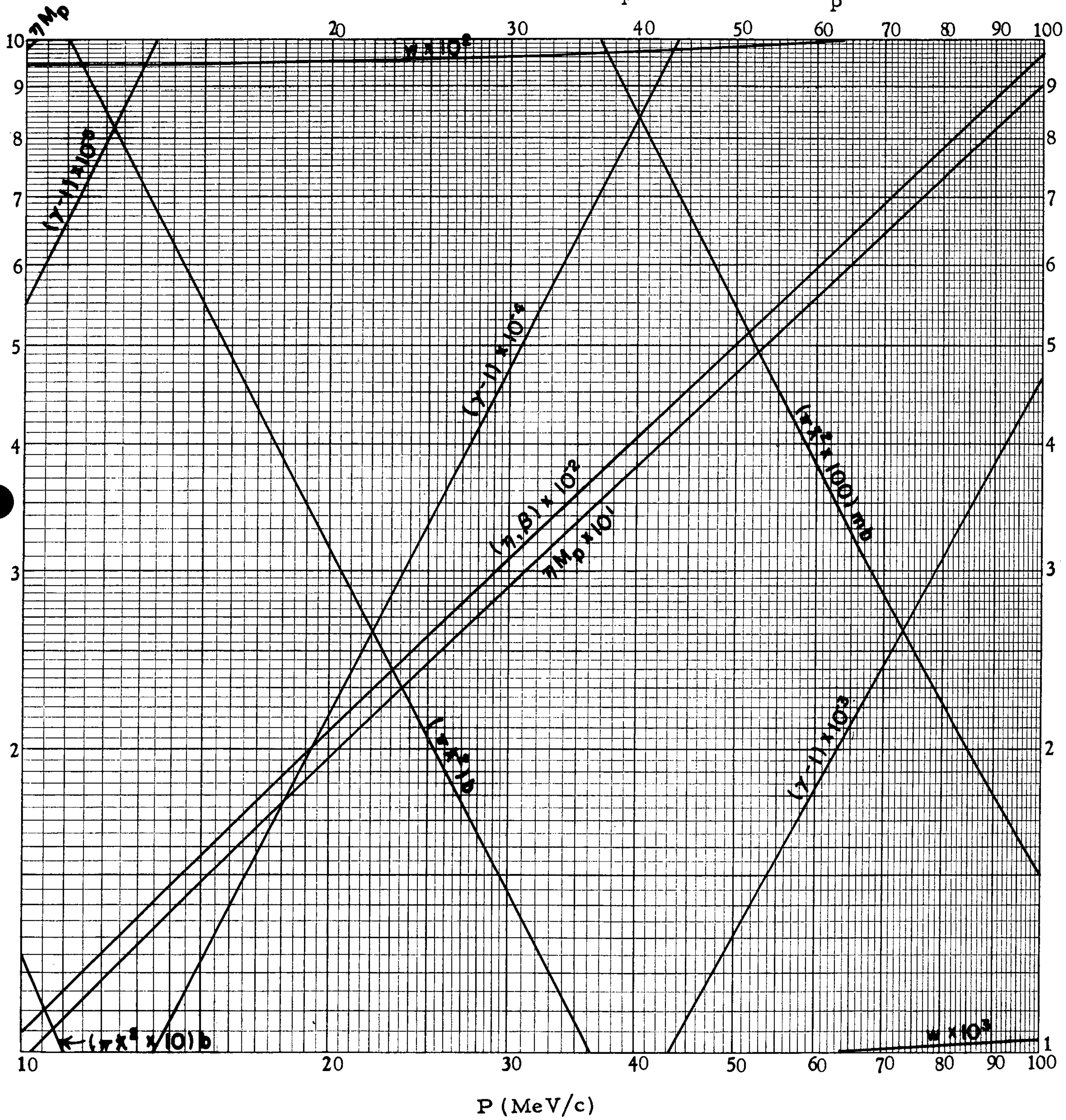


ELECTRONS: $e^{\pm} + p$

10 MeV/c to 100 MeV/c

β, $(\gamma - 1)$, η, ηM_p, w

$M_{e^{\pm}} = 0.510976$ MeV
$M_p = 938.213$ MeV



0 0 1 0 1 2 0 4 7 7 5

ELECTRONS: $e^{\pm} + p$

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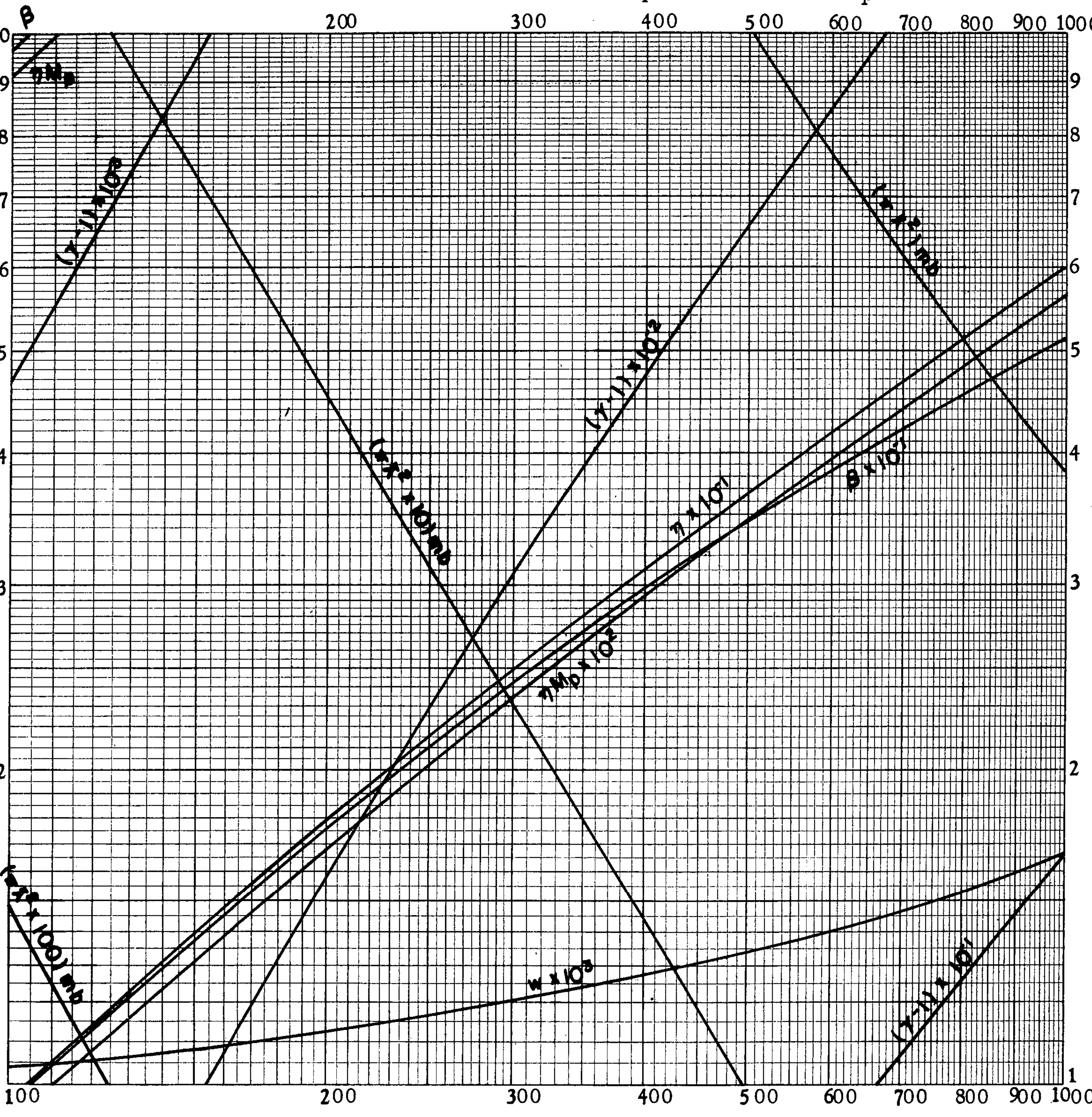
100 MeV/c to 1 BeV/c

β, $(\gamma - 1)$, η, ηM_p, w

$M_{e\pm} = 0.510976$ MeV

$M_p = 938.213$ MeV



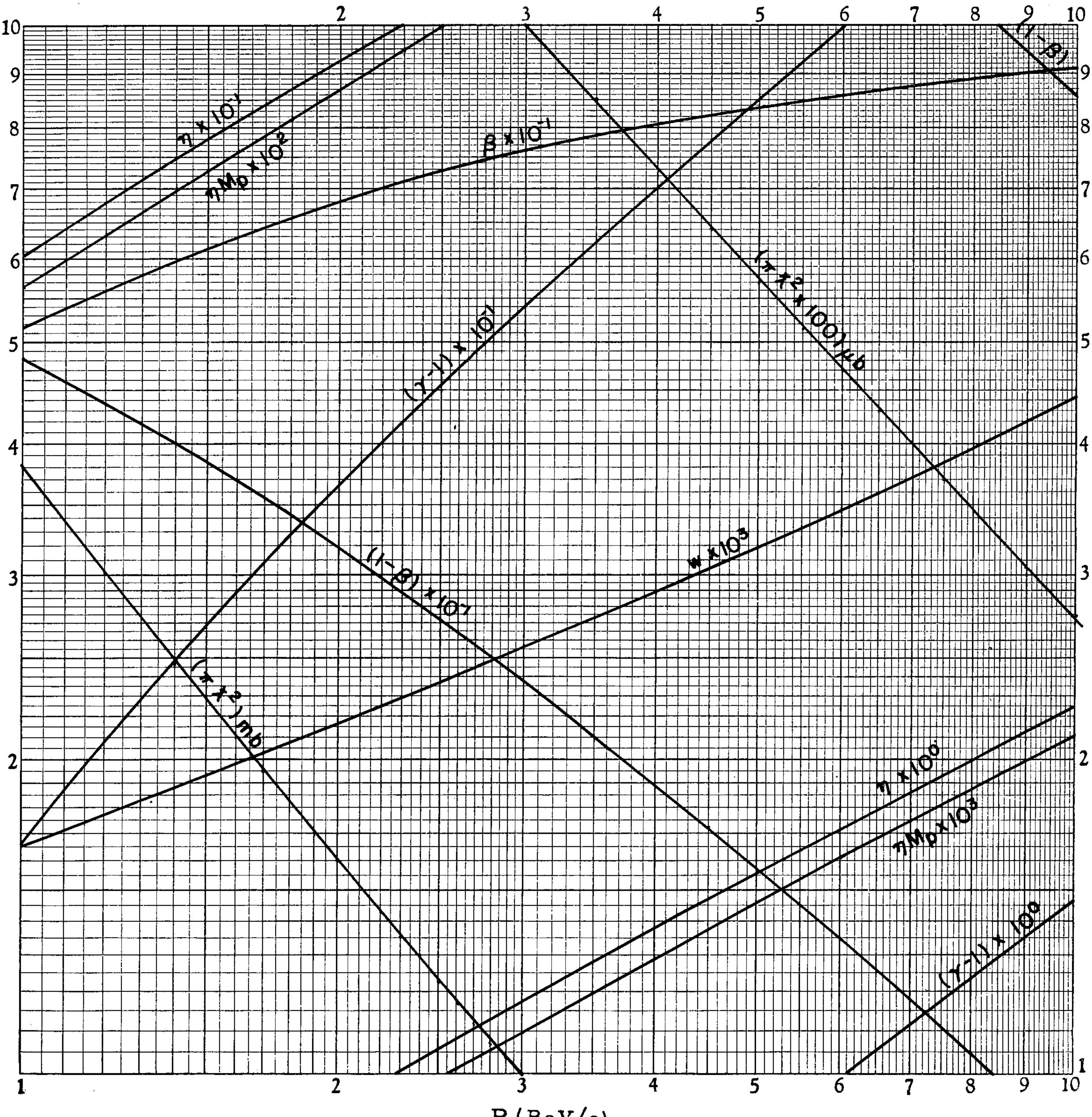
P (MeV/c)

ELECTRONS: $e^{\pm} + p$

1 BeV/c to 10 BeV/c

$M_{e^{\pm}}$ = 0.510976 MeV
M_p = 938.213 MeV

β, $(1 - \beta)$, $(\gamma - 1)$, η, ηM_p, w



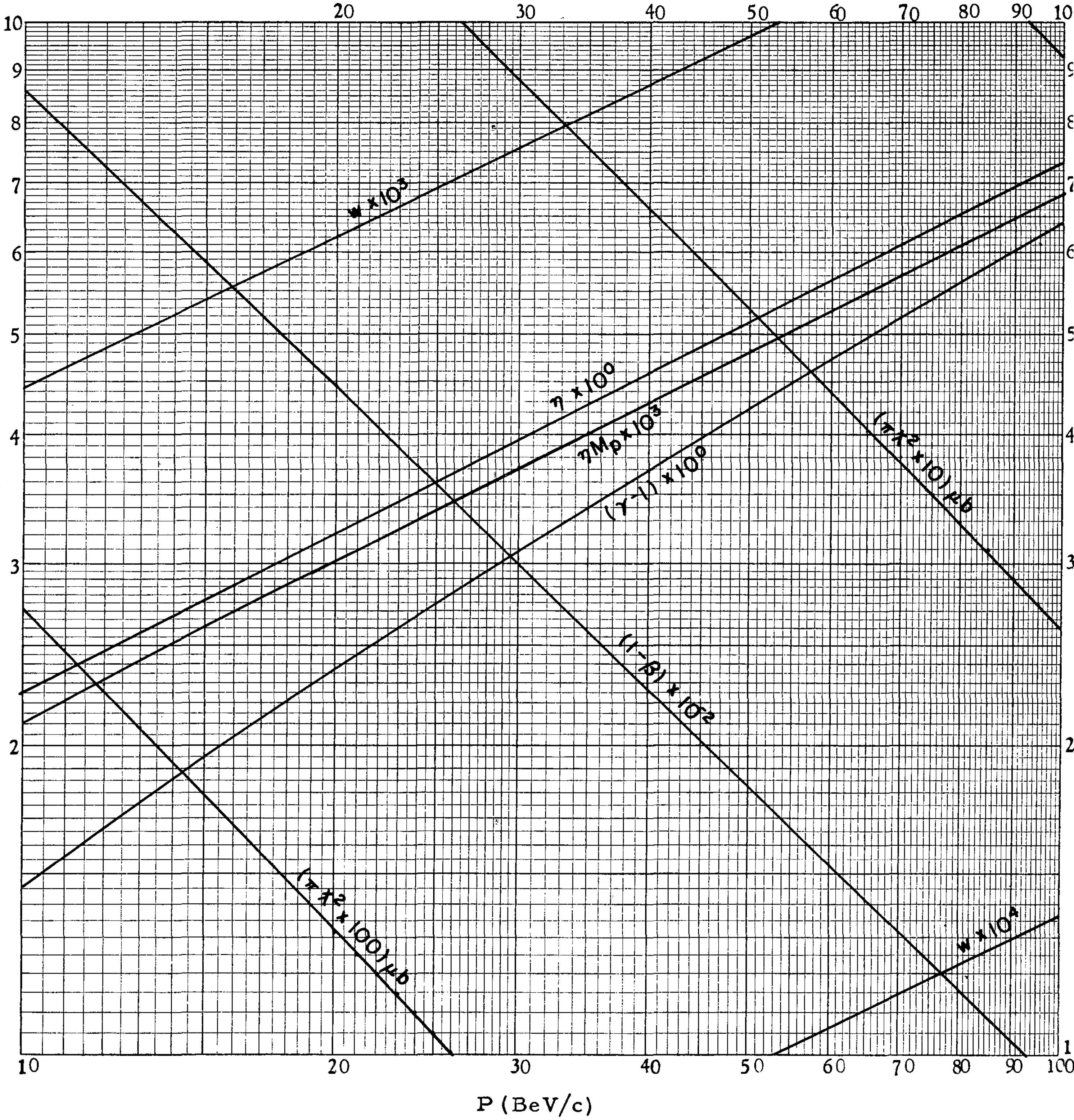
0 0 1 0 1 2 0 4 / 7 7

ELECTRONS: $e^{\pm} + p$

10 BeV/c to 100 BeV/c

$(1 - \beta)$, $(\gamma - 1)$, η, ηM_p, w

$M_{e^{\pm}} = 0.510976$ MeV
$M_p = 938.213$ MeV



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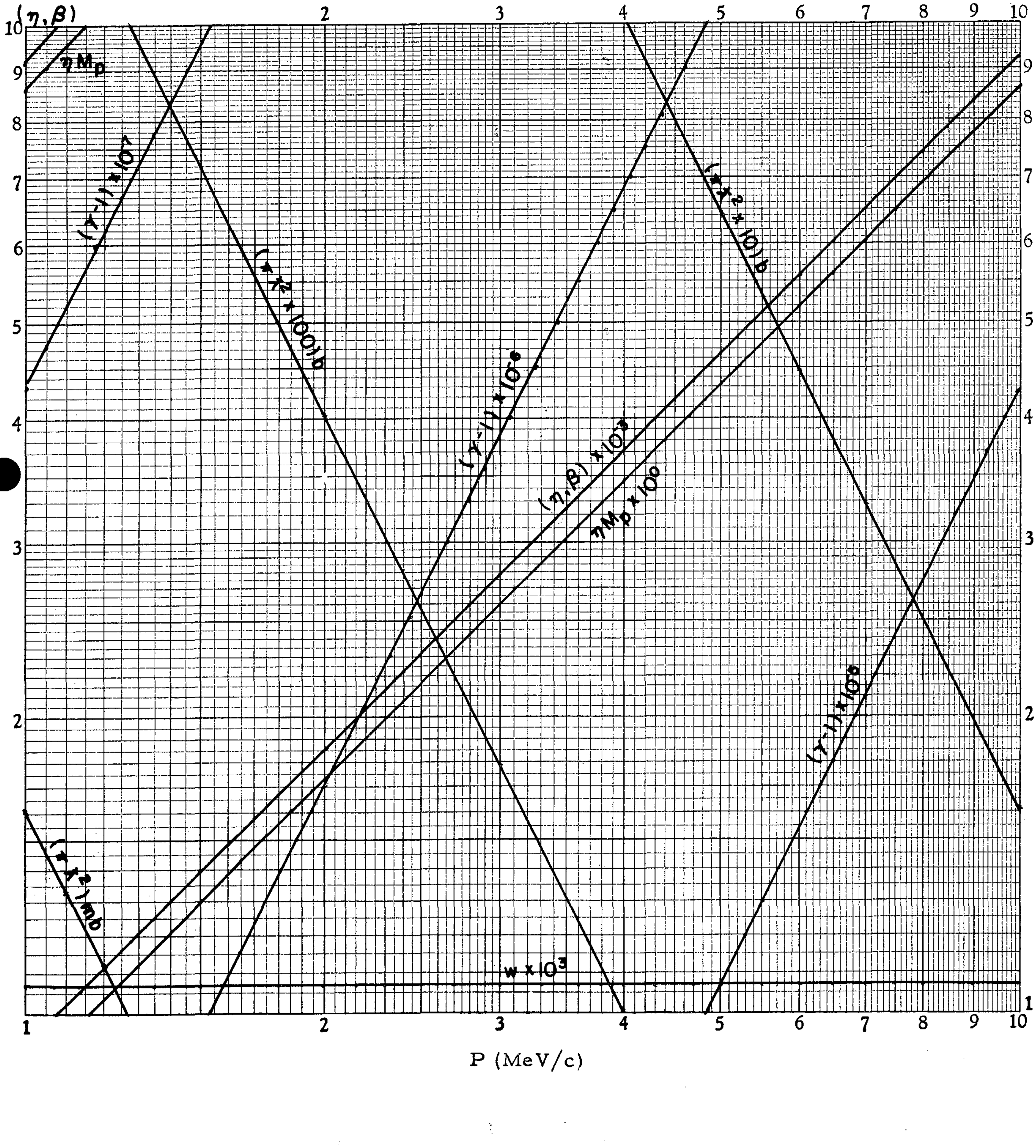
π MESONS: $\pi^{\pm} + p$

1 MeV/c to 10 MeV/c

β, (γ - 1), η, ηM_p, w

$M_{\pi^{\pm}} = 139.59$ MeV
$M_p = 938.213$ MeV

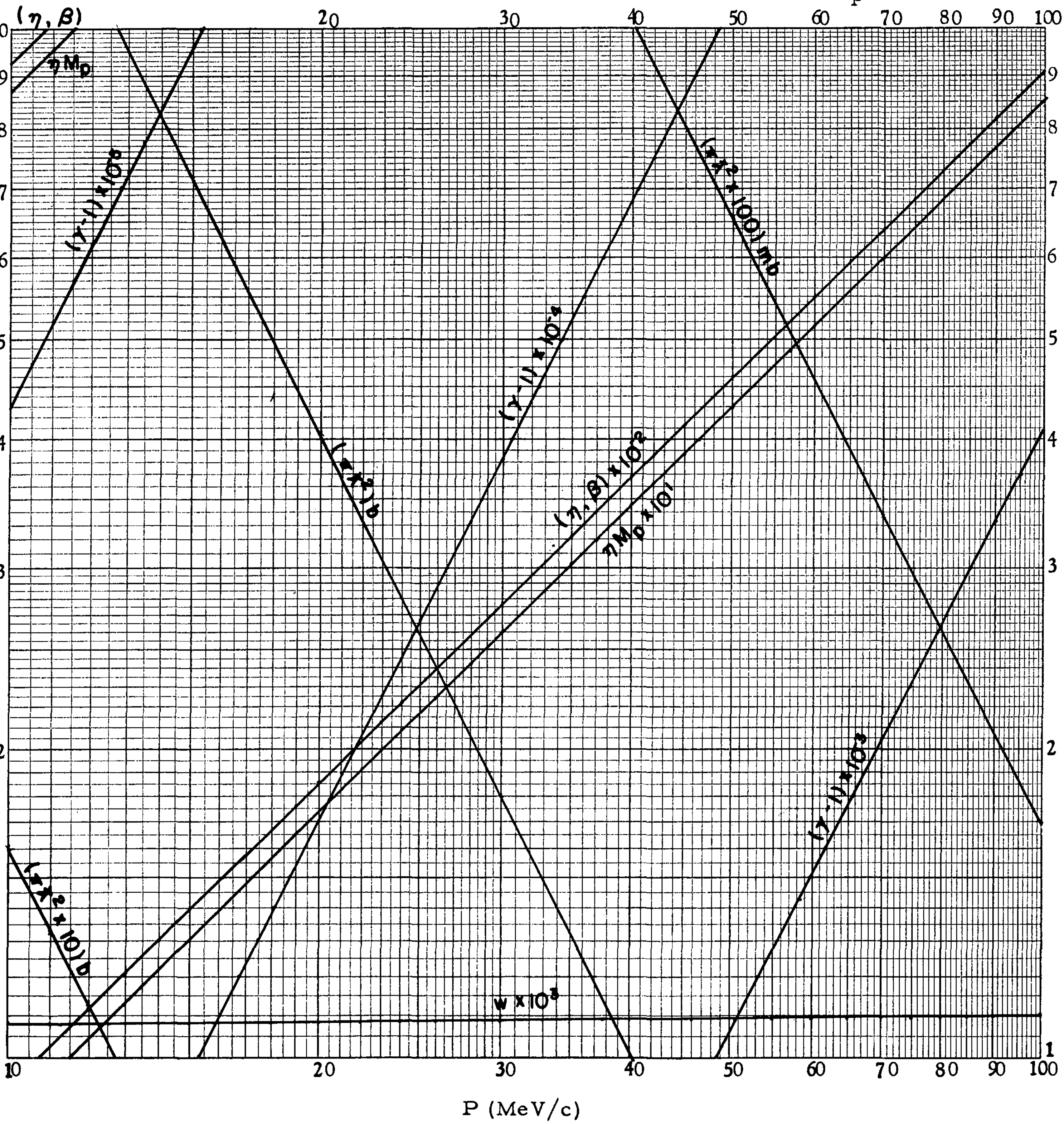


π MESONS: $\pi^{\pm} + p$

10 MeV/c to 100 MeV/c

β, $(\gamma - 1)$, η, ηM_p, w

$M_{\pi^{\pm}} = 139.59$ MeV
$M_p = 938.213$ MeV



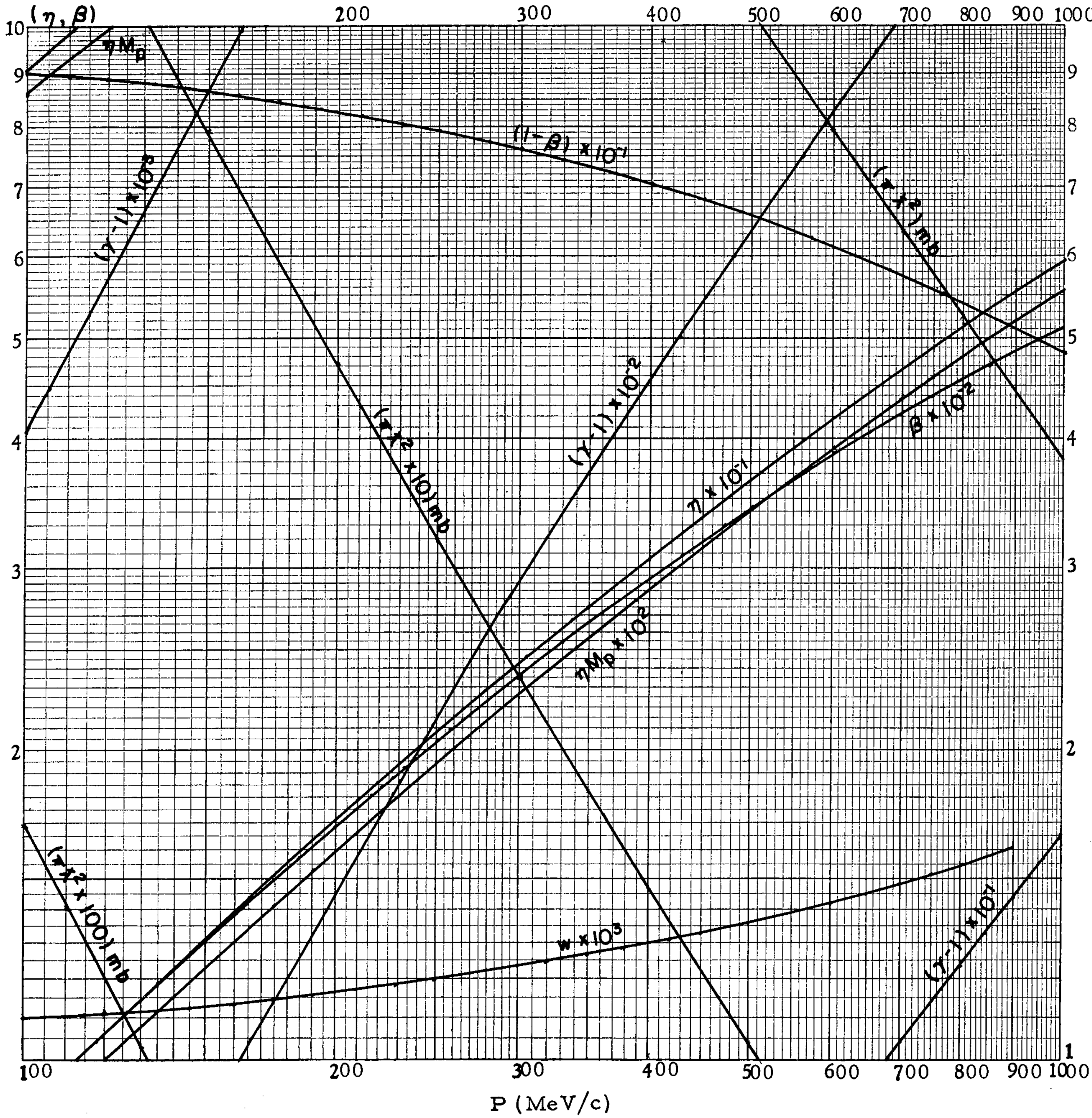
0 0 1 0 1 2 0 4 7 8 0

100 MeV/c to 1 BeV/c

β, $(\gamma - 1)$, η, ηM_p, w

$M_{\pi^{\pm}} = 139.59$ MeV

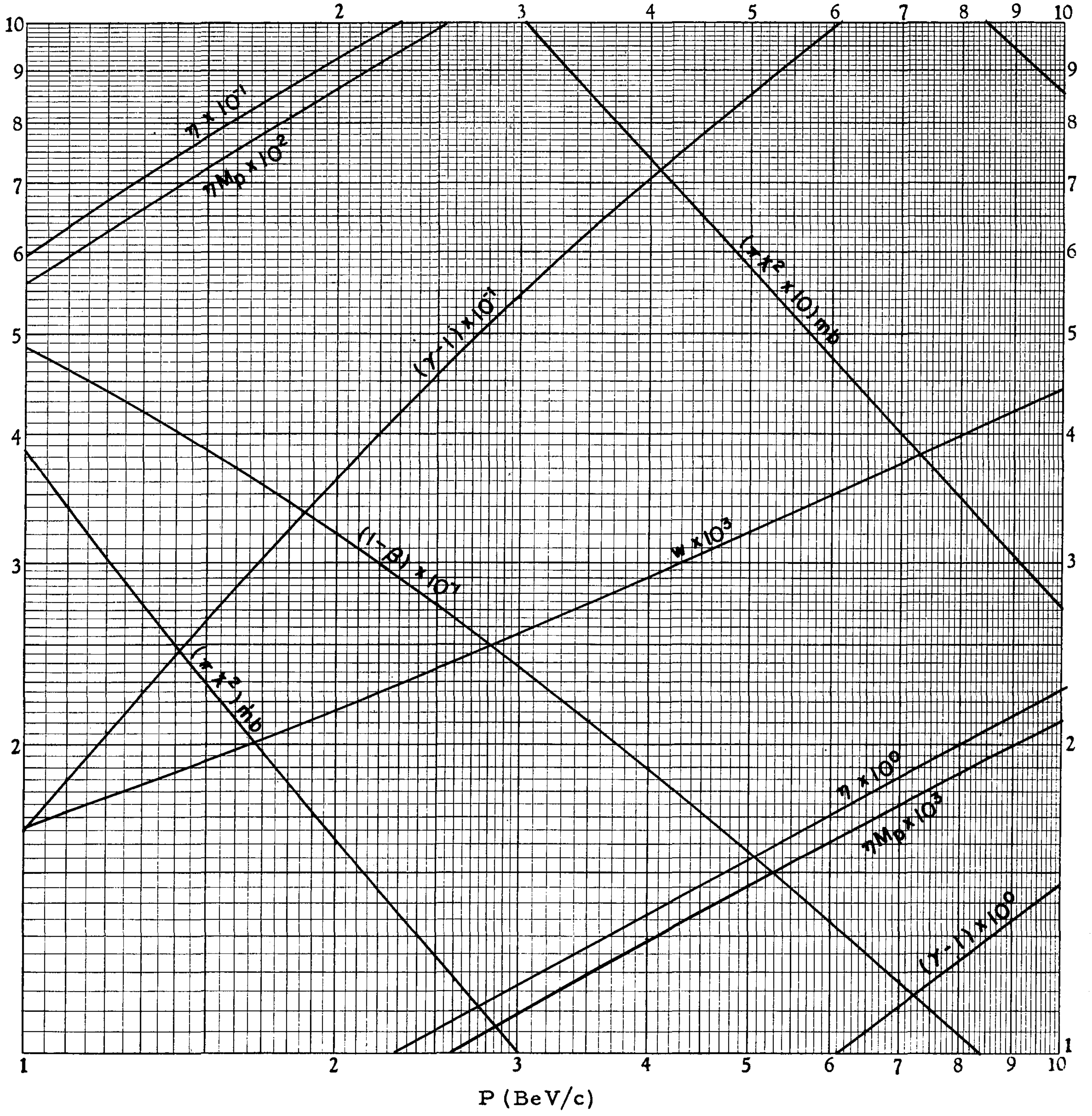
$M_p = 938.213$ MeV



1 BeV/c to 10 BeV/c

(1 - β), (γ - 1), η, ηM_p, w

$M_{\pi^{\pm}}$ = 139.59 MeV
M_p = 938.213 MeV



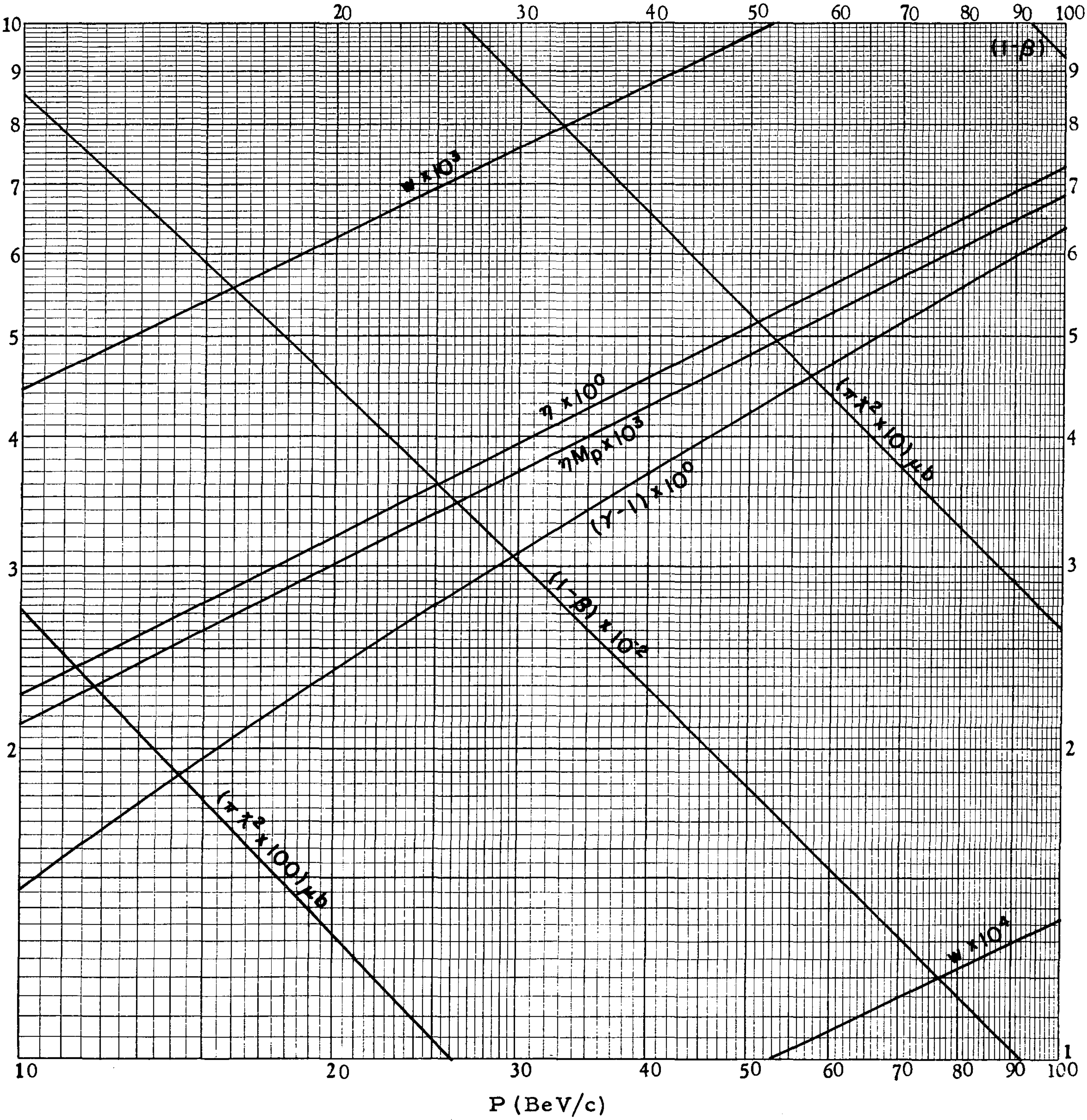
0 0 1 0 1 2 0 4 7 8 2

10 BeV/c to 100 BeV/c

$(1 - \beta)$, $(\gamma - 1)$, η, ηM_p, w

$M_{\pi^{\pm}} = 139.59$ MeV

$M_p = 938.213$ MeV

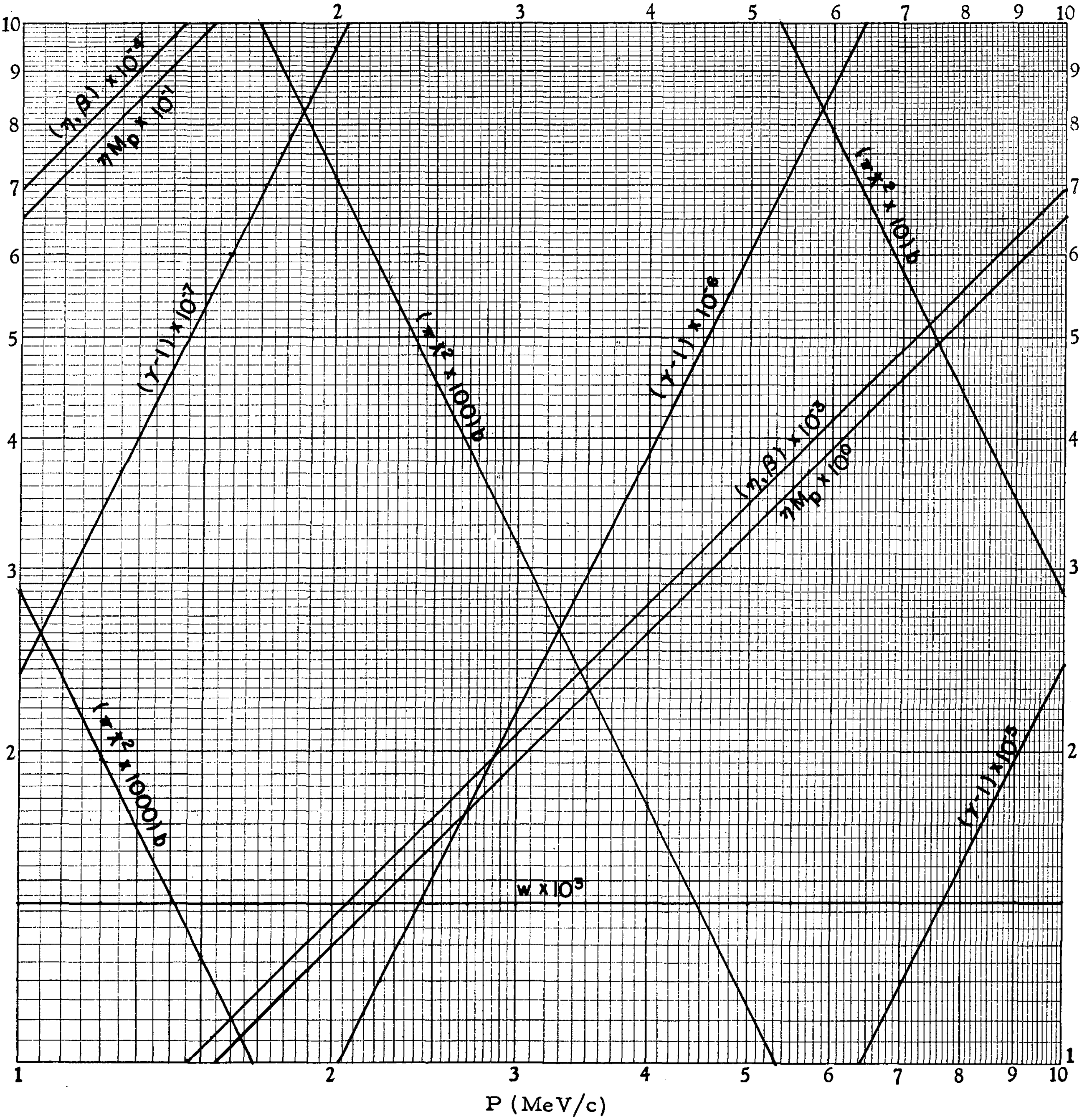


1 MeV/c to 10 MeV/c

$M_{K^{\pm}} = 493.9$ MeV

β, $(\gamma - 1)$, η, ηM_p, w

$M_p = 938.213$ MeV



0 0 1 0 1 2 0 4 7 8 4

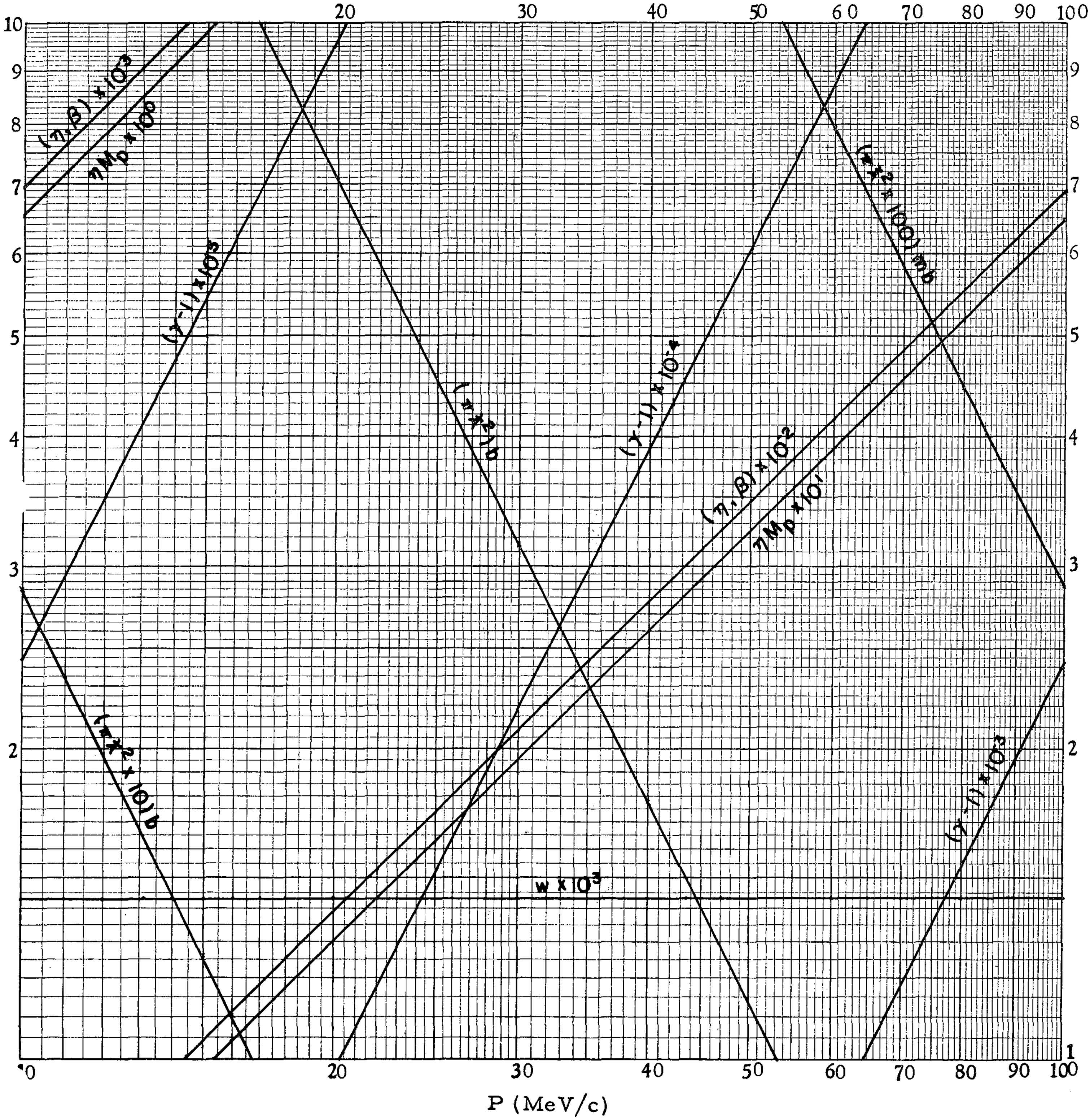
K MESONS: $K^{\pm} + p$

10 MeV/c to 100 MeV/c

β, $(\gamma - 1)$, η, ηM_p, w

$M_{K^{\pm}} = 493.9$ MeV

$M_p = 938.213$ MeV



0 0 1 0 1 2 0 4 7 8 5

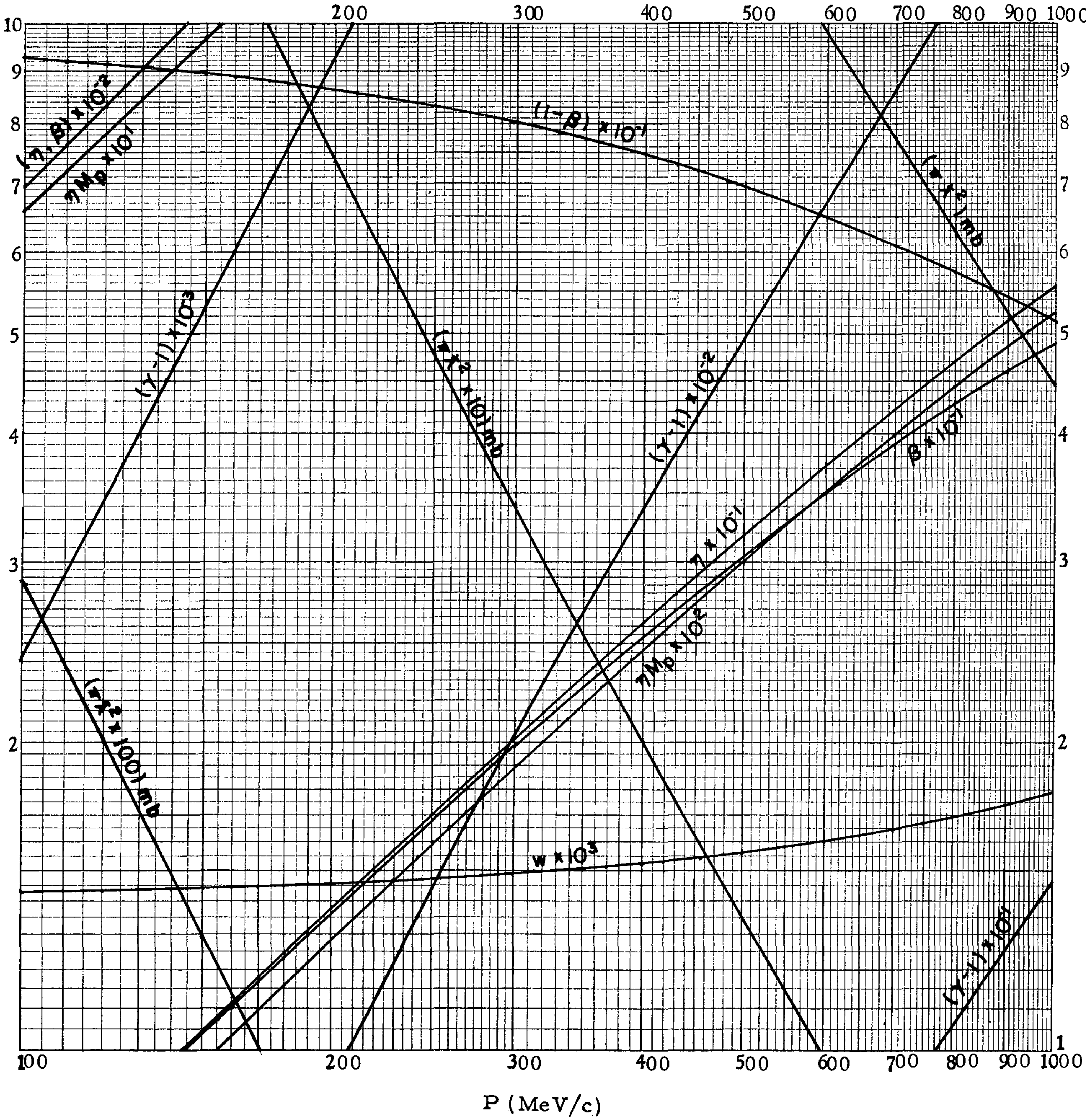
K MESONS: $K^{\pm} + p$

100 MeV/c to 1 BeV/c

$M_{K^\pm} = 493.9$ MeV

β, $(1-\beta)$, $(\gamma-1)$, η, ηM_p, w

$M_p = 938.213$ MeV

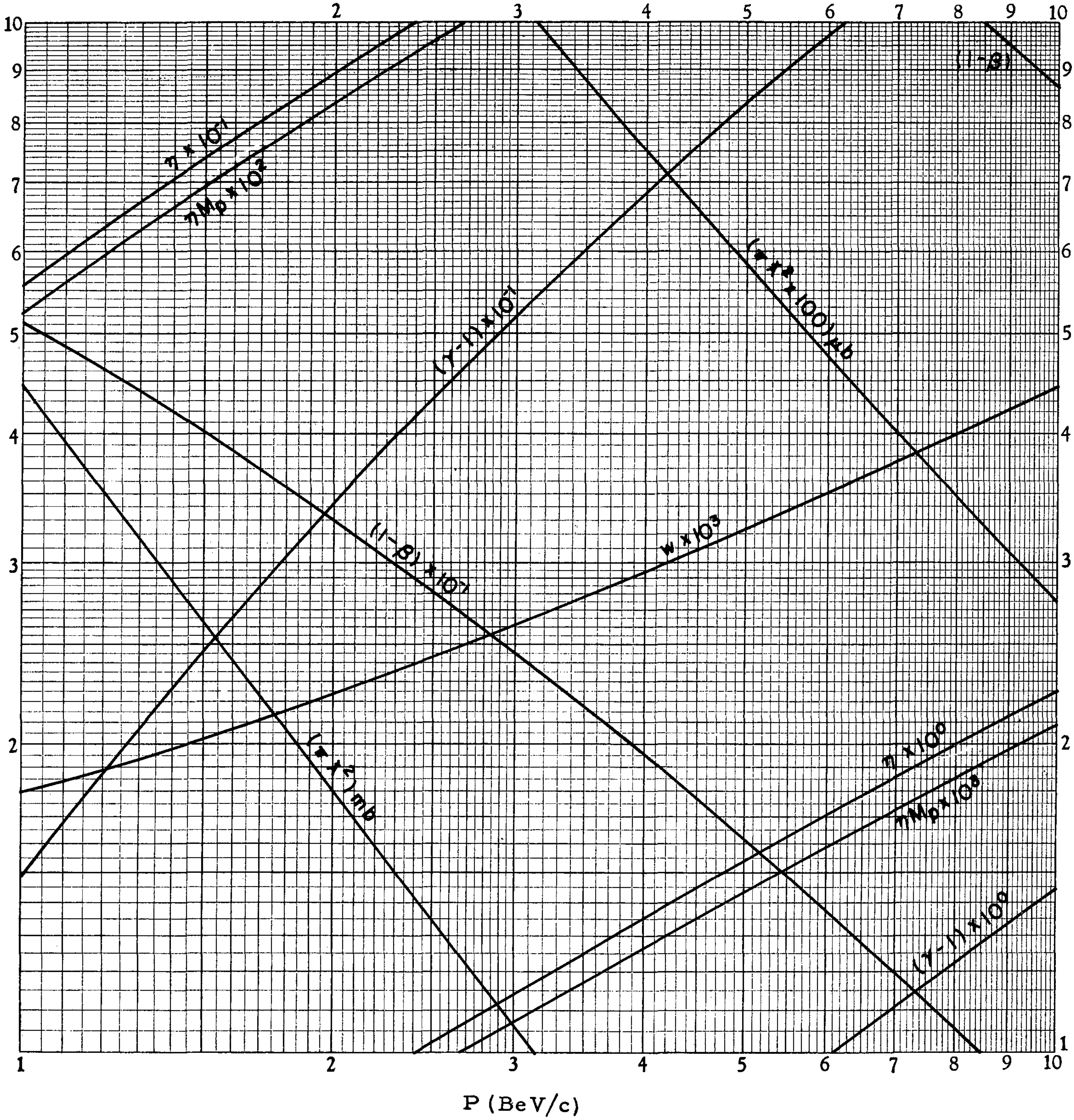


ηM_p in MeV
w in MeV
$mb = 10^{-27} cm^2$

1 BeV/c to 10 BeV/c

$(1 - \beta)$, $(\gamma - 1)$, η, ηM_p, w, $\pi \lambda^2$

$M_{K^{\pm}} = 493.9$ MeV
$M_p = 938.213$ MeV



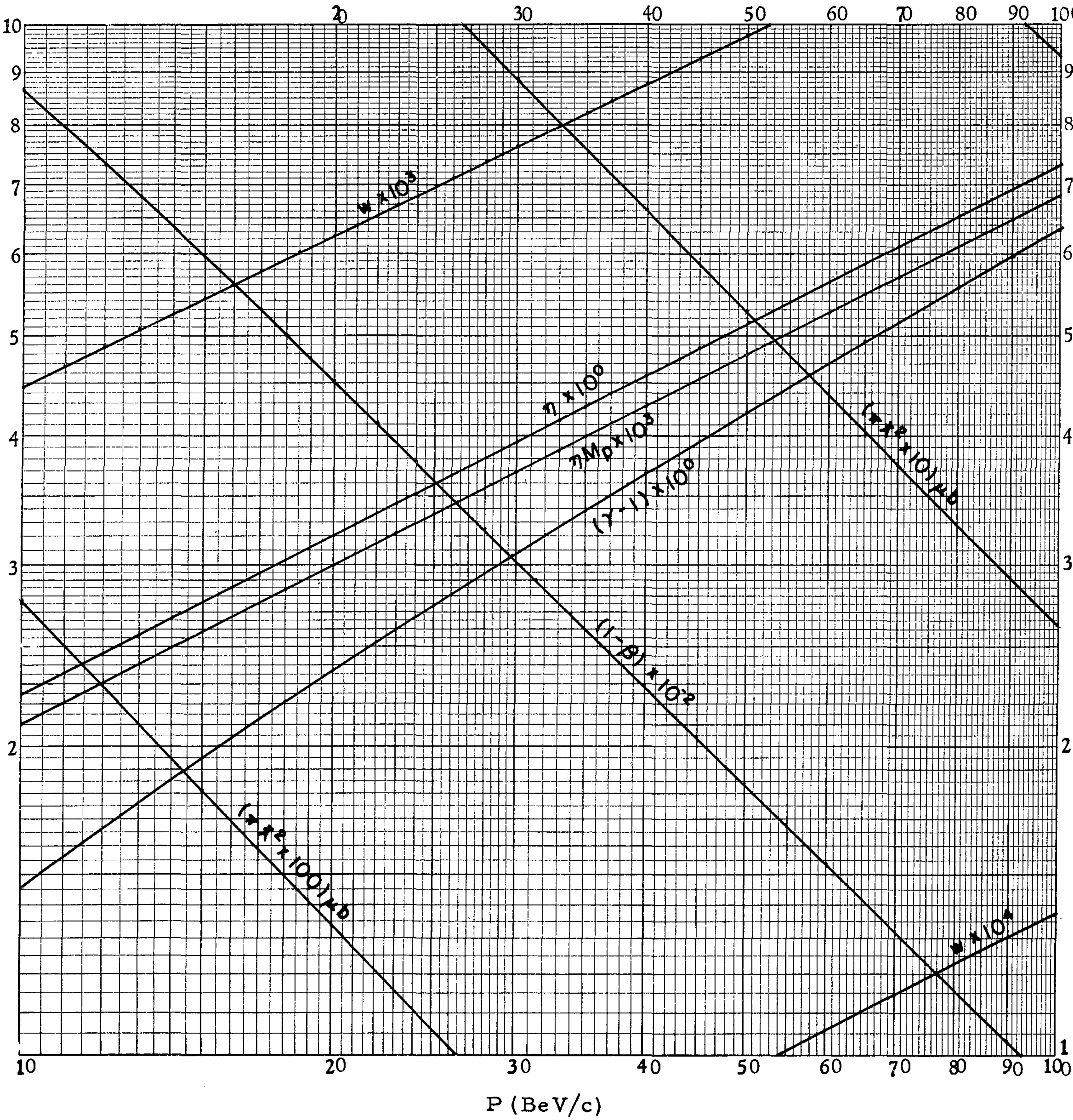
0 0 1 0 1 2 0 4 7 8 7

10 BeV/c to 100 BeV/c

$(1 - \beta)$, $(\gamma - 1)$, η, ηM_p, w

$M_{K^{\pm}} = 493.9$ MeV

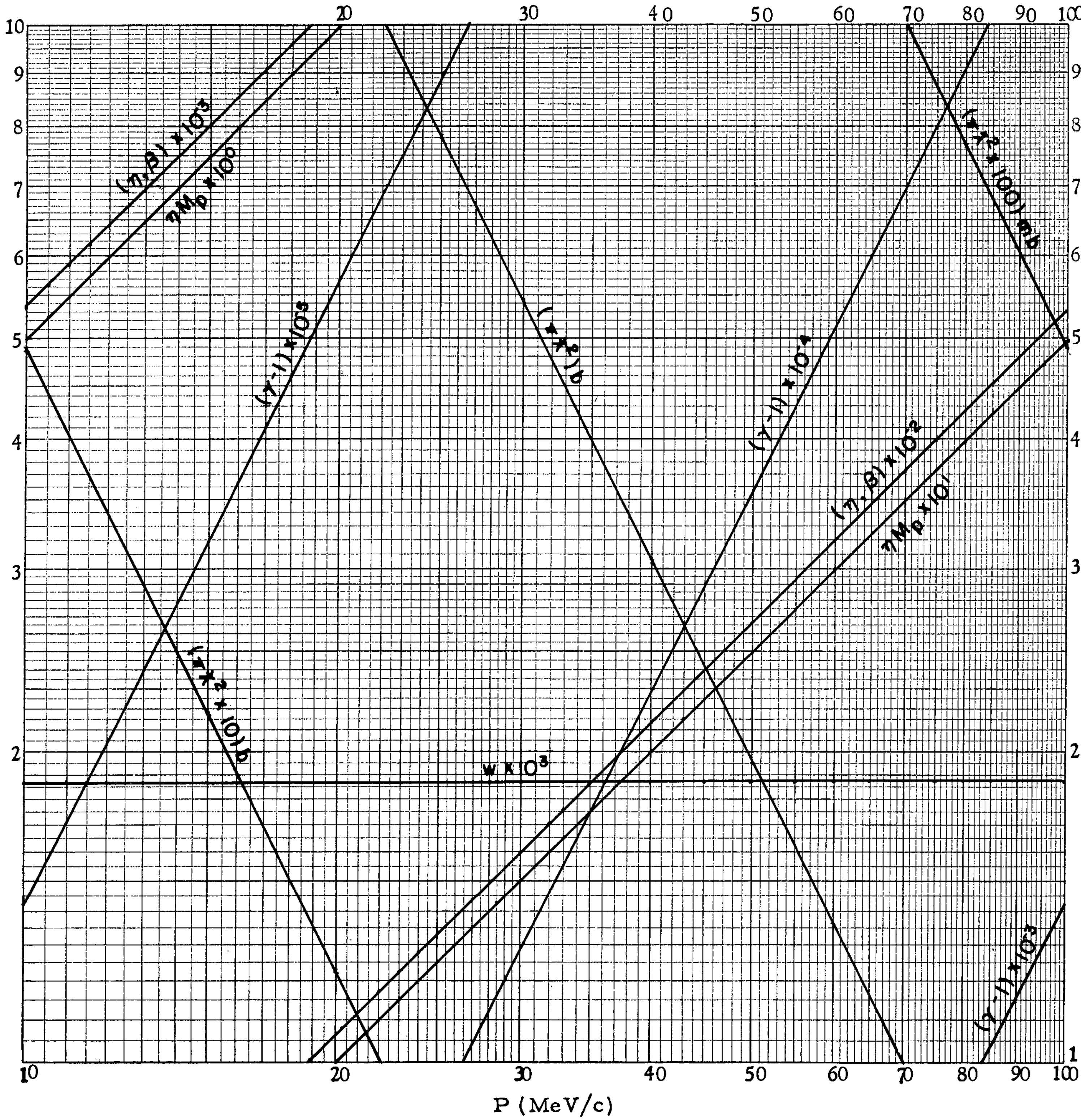
$M_p = 938.213$ MeV



10 MeV/c to 100 MeV/c

β, $(\gamma - 1)$, η, ηM_p, w

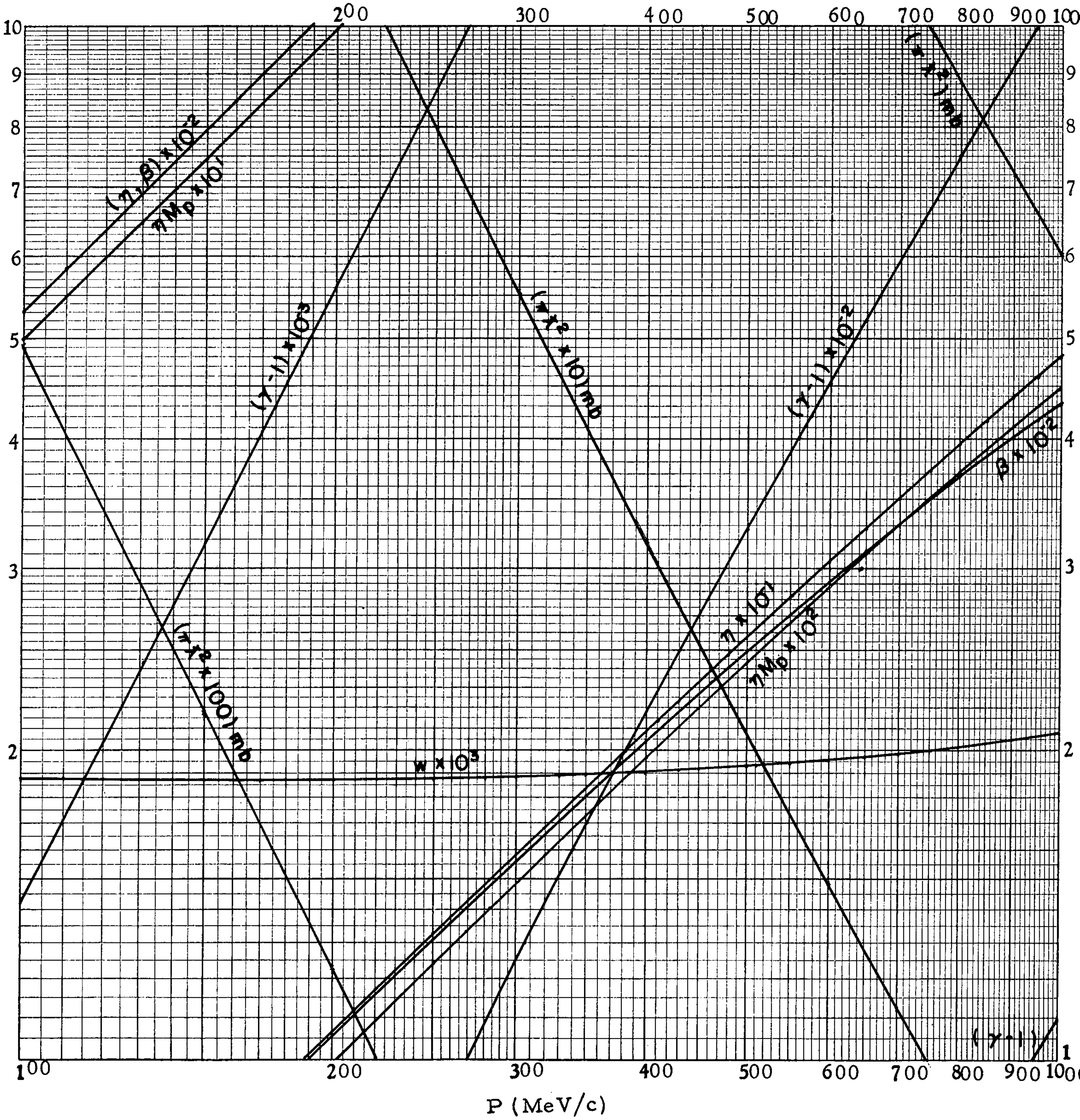
M_p = 938.213 MeV



100 MeV/c to 1 BeV/c

β, (γ - 1), η, ηM_p, w

M_p = 938.213 MeV

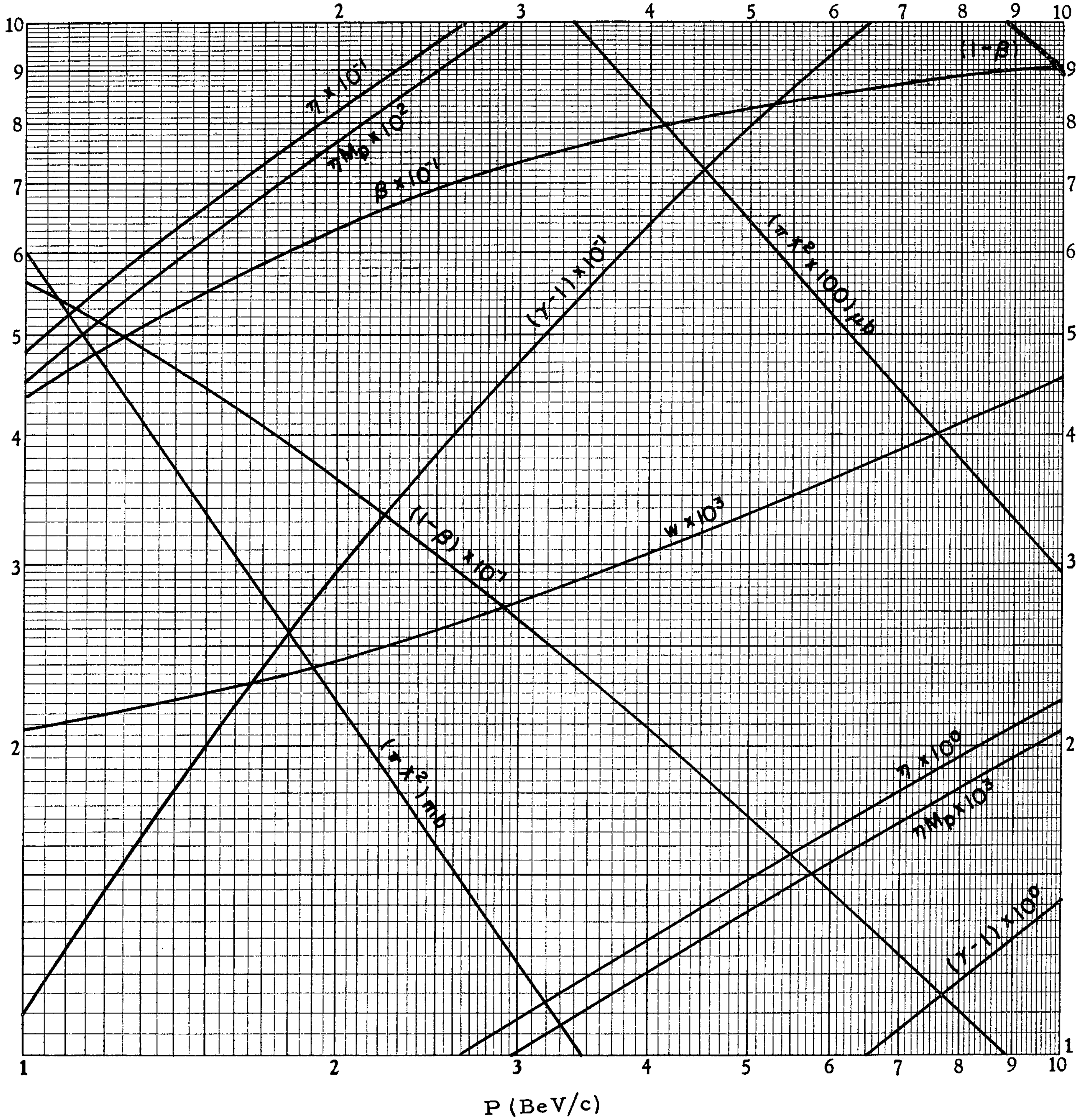


ηM_p in MeV
w in MeV
mb = 10^{-27} cm^2

1 BeV/c to 10 BeV/c

β, $(1 - \beta)$, $(\gamma - 1)$, η, ηM_p, w, $\pi \lambda\!\!\!^{-2}$

M_p = 938.213 MeV

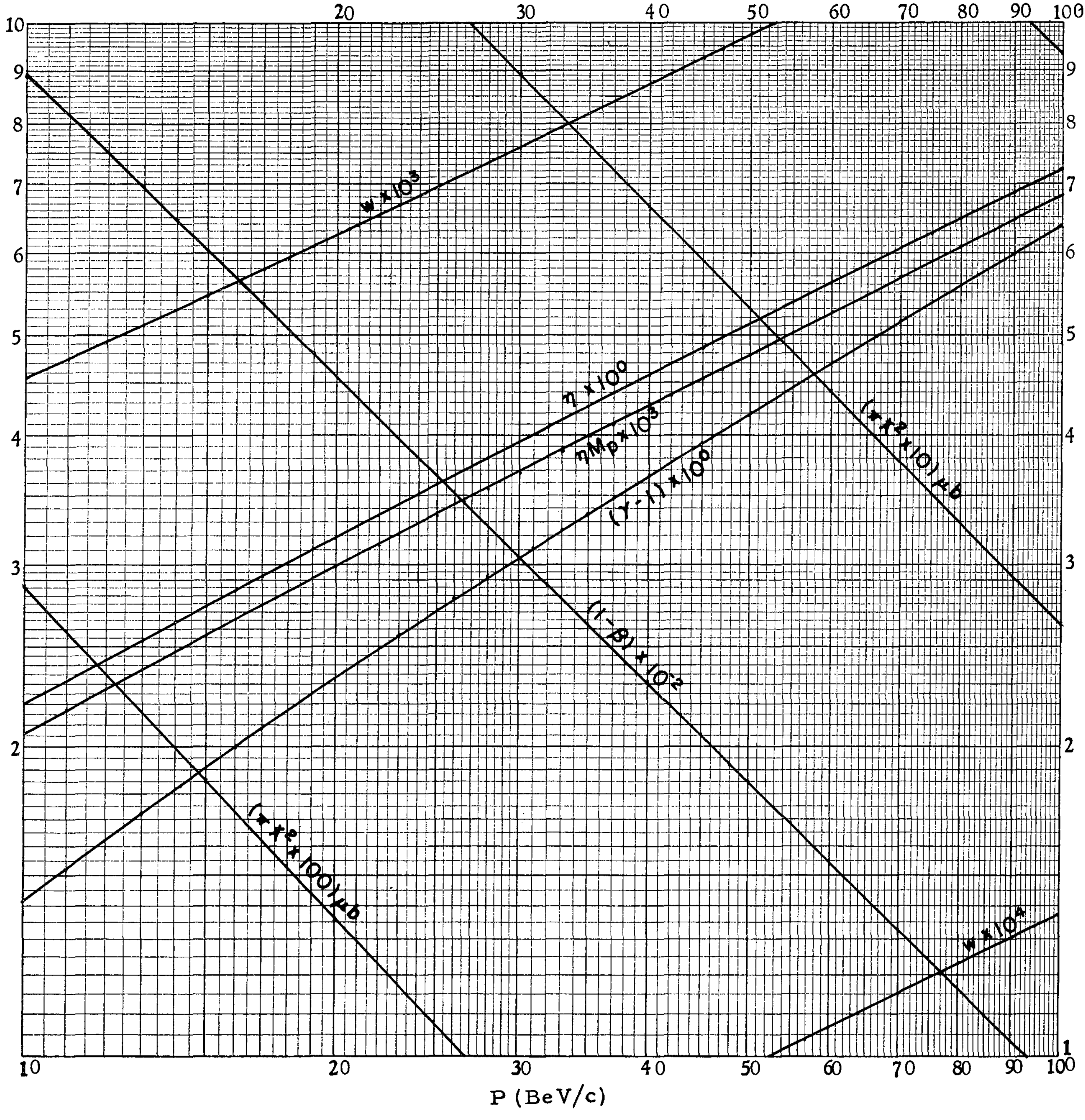


0 0 1 0 1 2 0 4 7 . 9 1

10 BeV/c to 100 BeV/c

(1 - β), (γ - 1), η, ηM$_p$, w

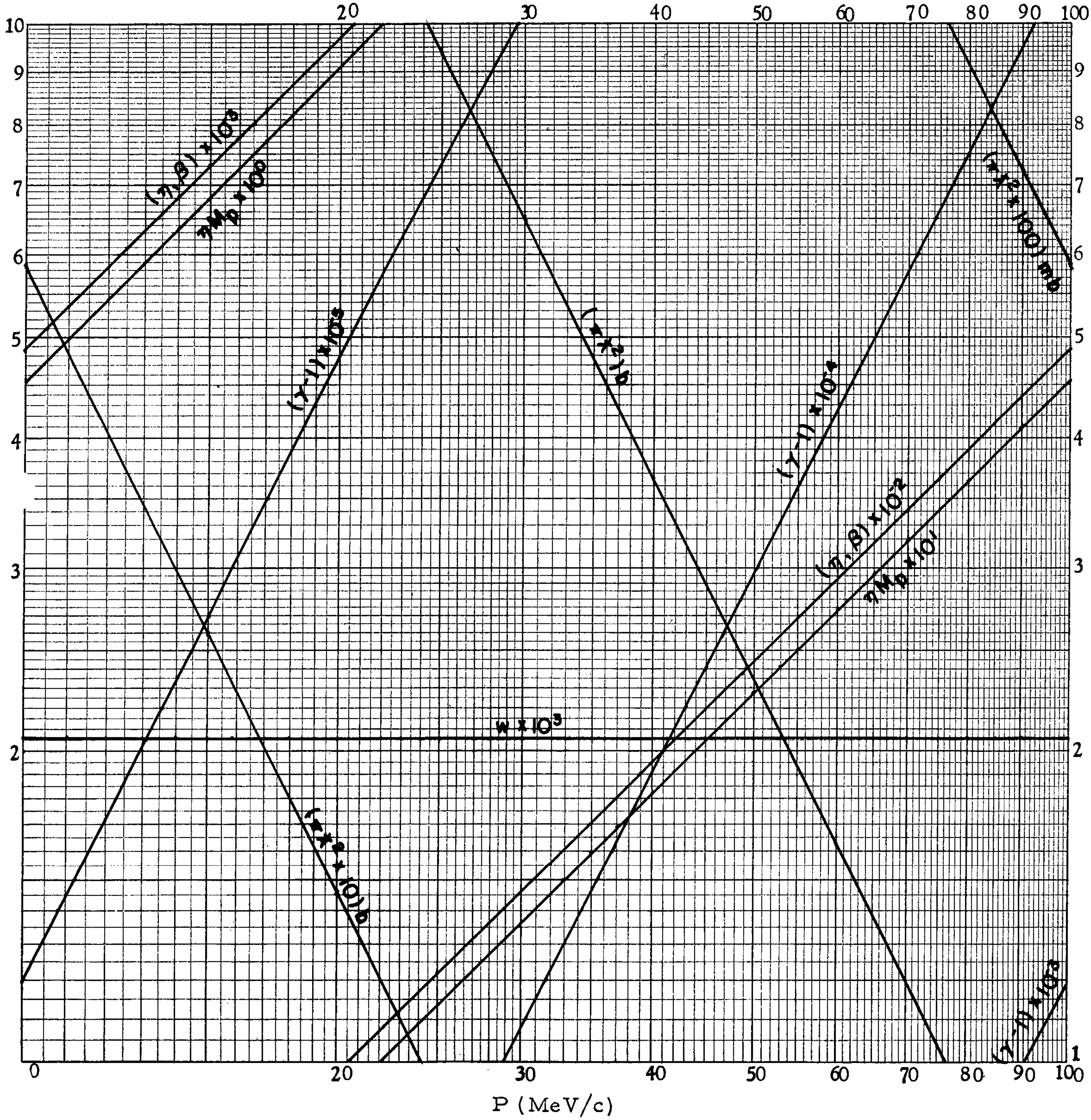
M_p = 938.213 MeV



10 MeV/c to 100 MeV/c

β, (γ - 1), η, ηM$_p$, w

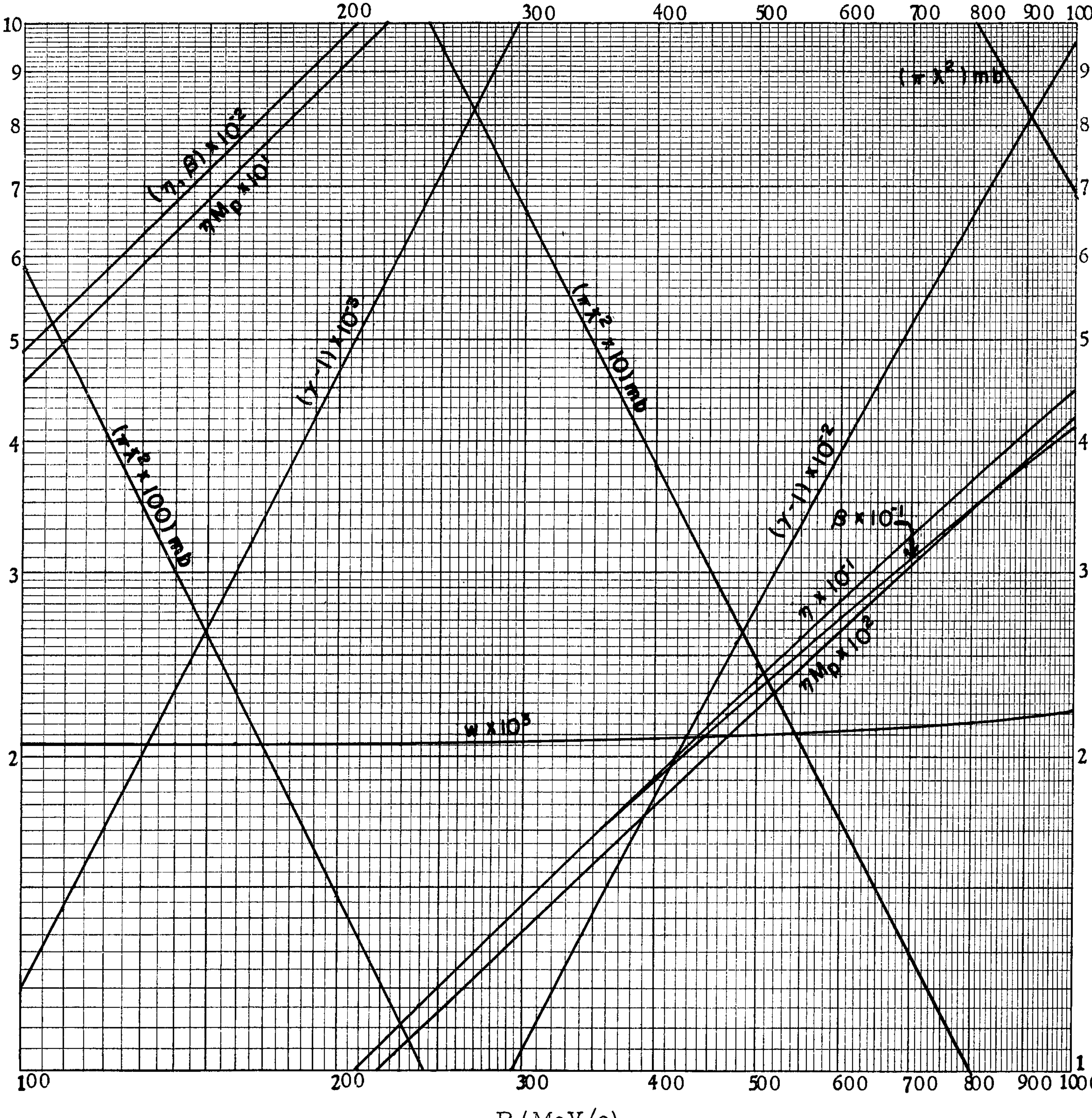
$M_\Lambda = 1115.36$ MeV
$M_p = 938.213$ MeV



100 MeV/c to 1 BeV/c

β, (γ - 1), η, ηM_p, w

M_Λ = 1115.36 MeV
M_p = 938.213 MeV

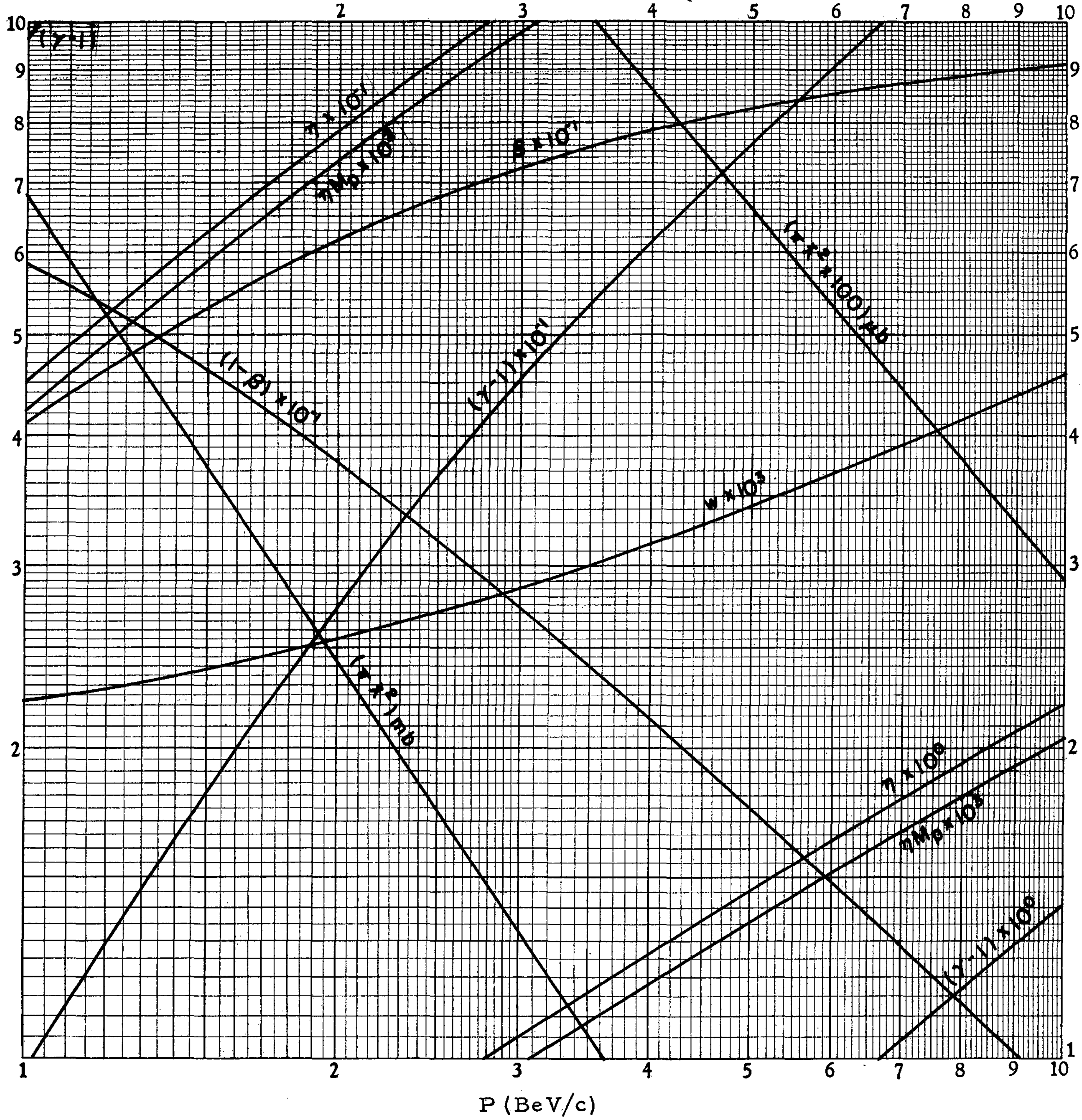


ηM_p in MeV
w in MeV
mb = 10^{-27} cm^2

1 BeV/c to 10 BeV/c

β, (1 - β), (γ - 1), η, ηM_p, w, $\pi \lambda^2$

M_Λ = 1115.36 MeV
M_p = 938.213 MeV

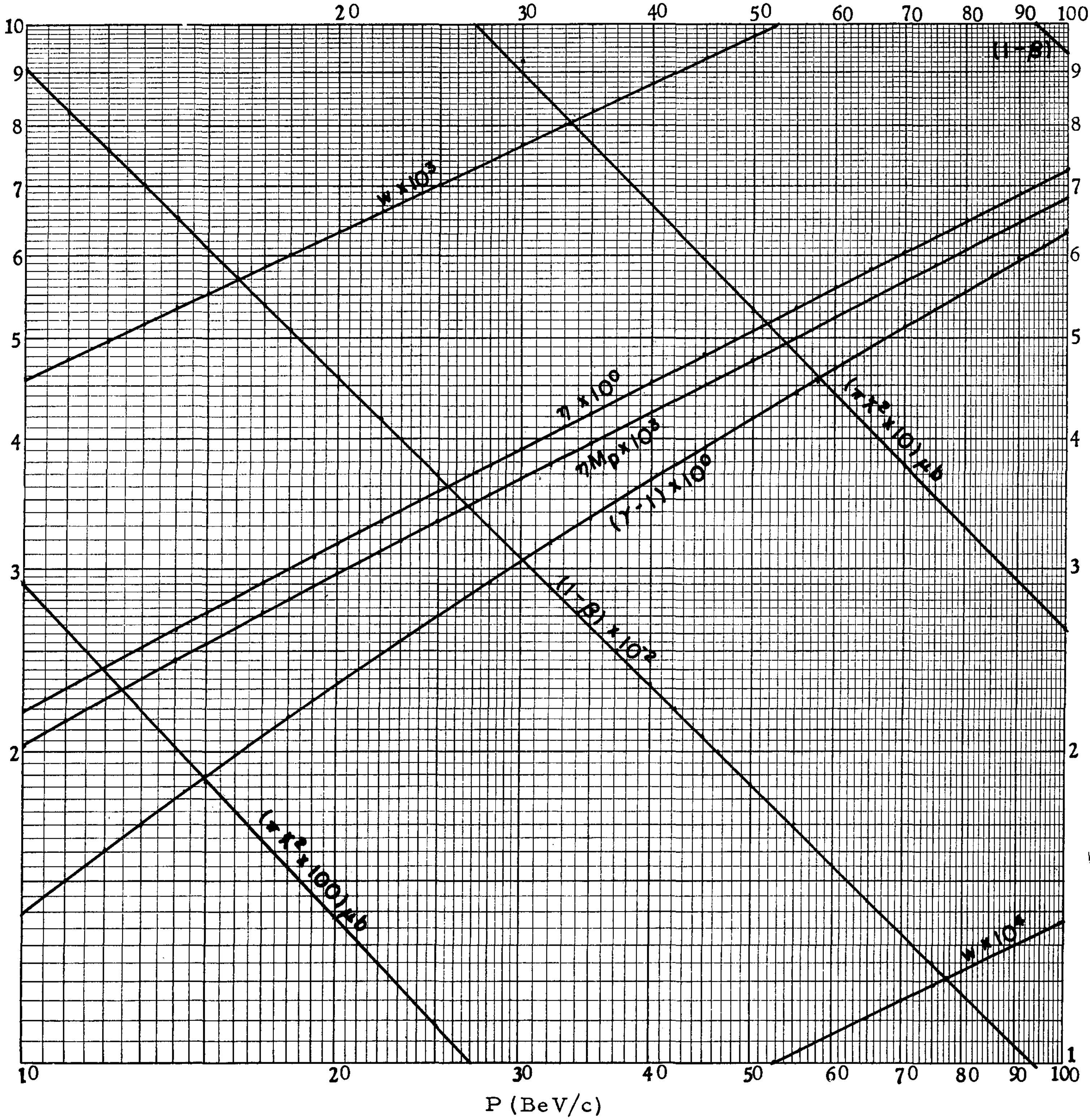


0 0 1 0 1 2 0 4 7 9 5

10 BeV/c to 100 BeV/c

M_Λ = 1115.36 MeV
M_p = 938.213 MeV

β, (1 - β), (γ - 1), η, ηM_p, w

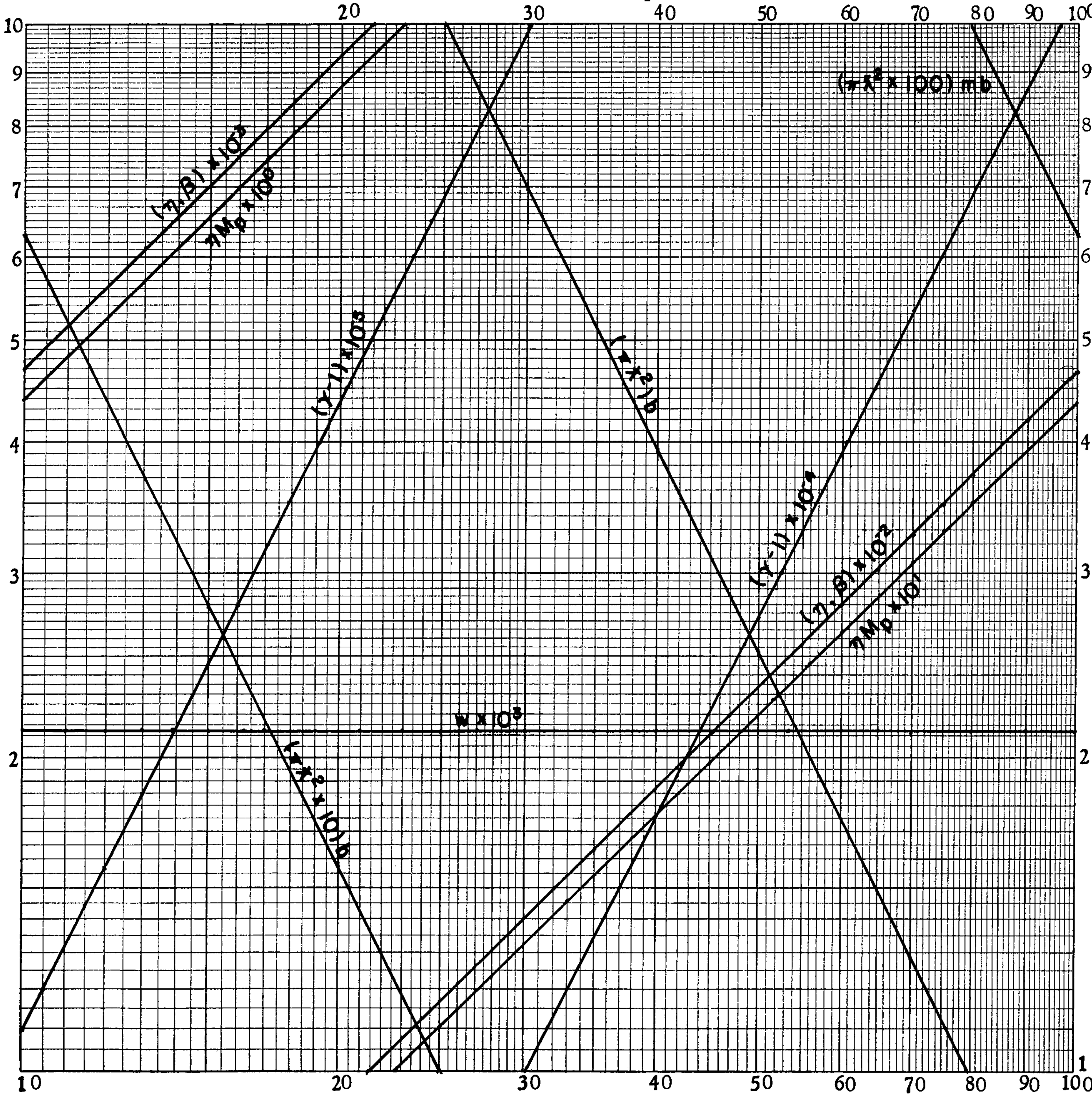


Σ HYPERONS: $\Sigma^+ + p$

10 MeV/c to 100 MeV/c

β, $(\gamma - 1)$, η, ηM_p, w

$M_{\Sigma^+} = 1189.40$ MeV
$M_p = 938.213$ MeV



P (MeV/c)

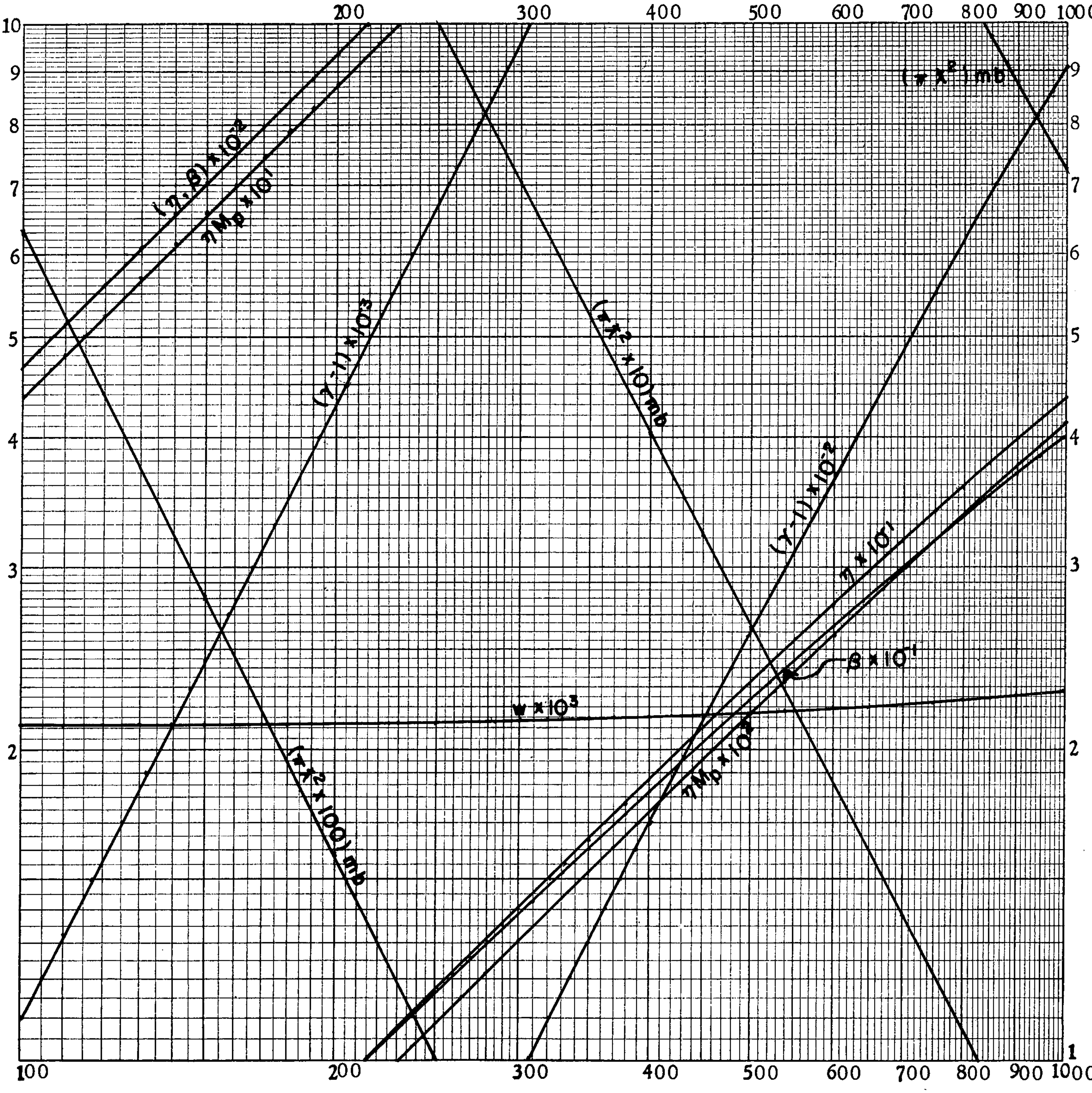
Σ HYPERONS: $\Sigma^+ + p$

100 MeV/c to 1 BeV/c

β, $(\gamma - 1)$, η, ηM_p, w

$M_{\Sigma^+} = 1189.40$ MeV

$M_p = 938.213$ MeV

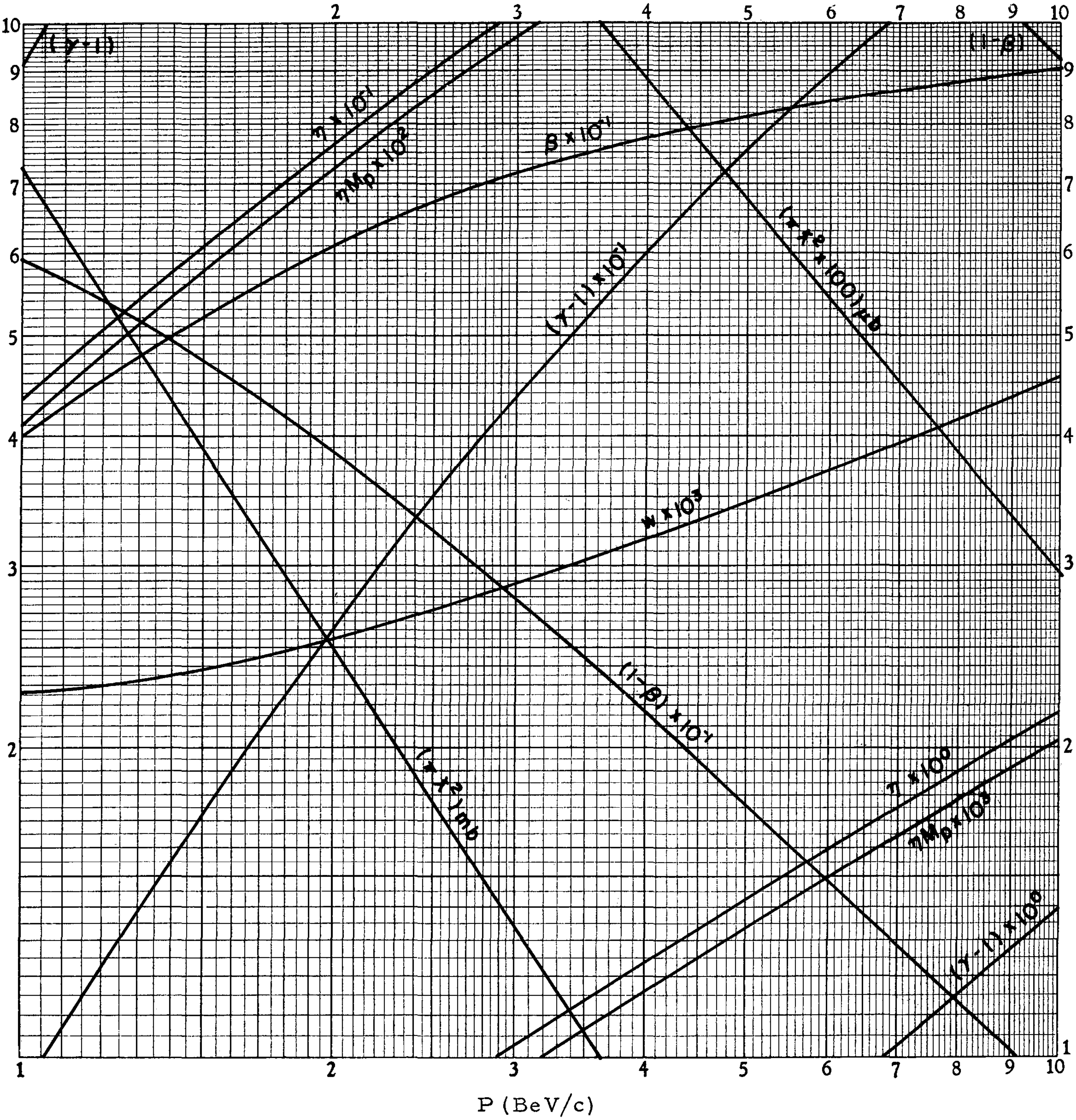


ηM_p in MeV
w in MeV
mb = 10^{-27} cm^2

1 BeV/c to 10 BeV/c

β, $(1-\beta)$, $(\gamma-1)$, η, ηM_p, w, $\pi \lambda^2$

M_{Σ^+} = 1189.40 MeV
M_p = 938.213 MeV



Σ HYPERONS: $\Sigma^+ + p$

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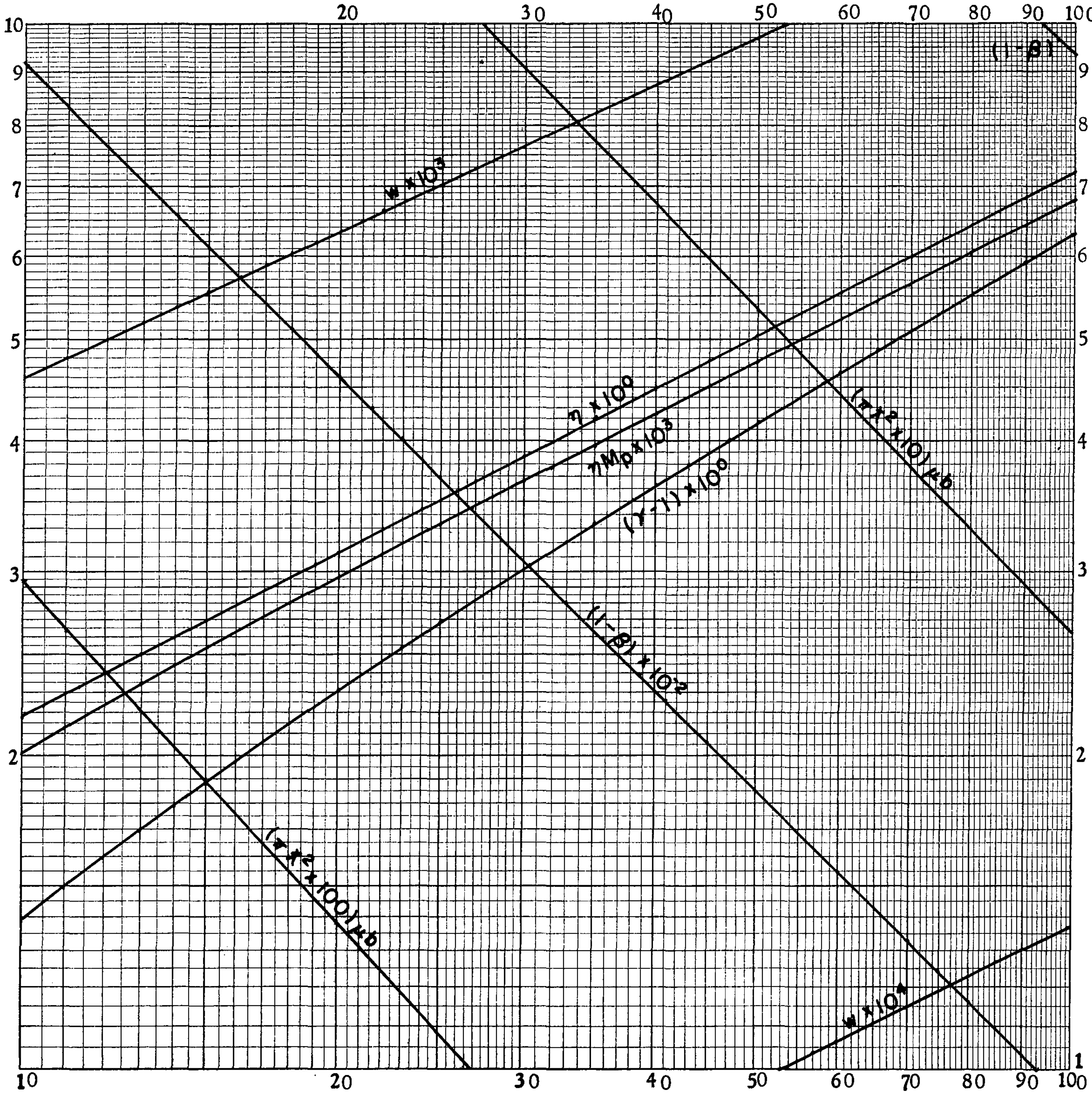
10 BeV/c to 100 BeV/c

$(1 - \beta)$, $(\gamma - 1)$, η, ηM_p, w

M_{Σ^+} = 1189.40 MeV
M_p = 938.213 MeV



P (BeV/c)

0 0 1 0 1 2 0 4 8 0 0

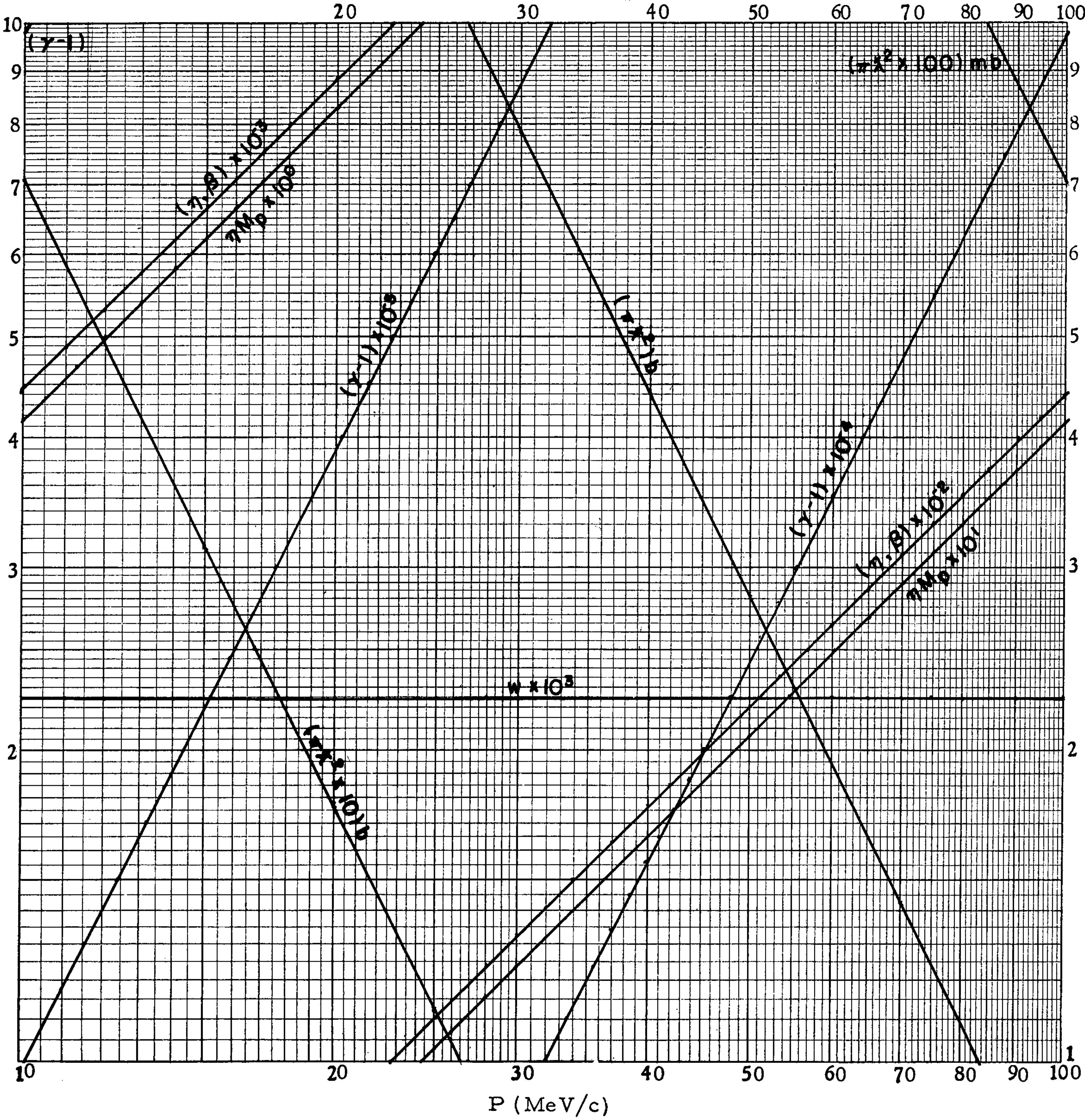
Ξ HYPERONS: $\Xi^- + p$

10 MeV/c to 100 MeV/c

$M_{\Xi^-} = 1318.4$ MeV

β, (γ - 1), η, ηM_p, w

$M_p = 938.213$ MeV



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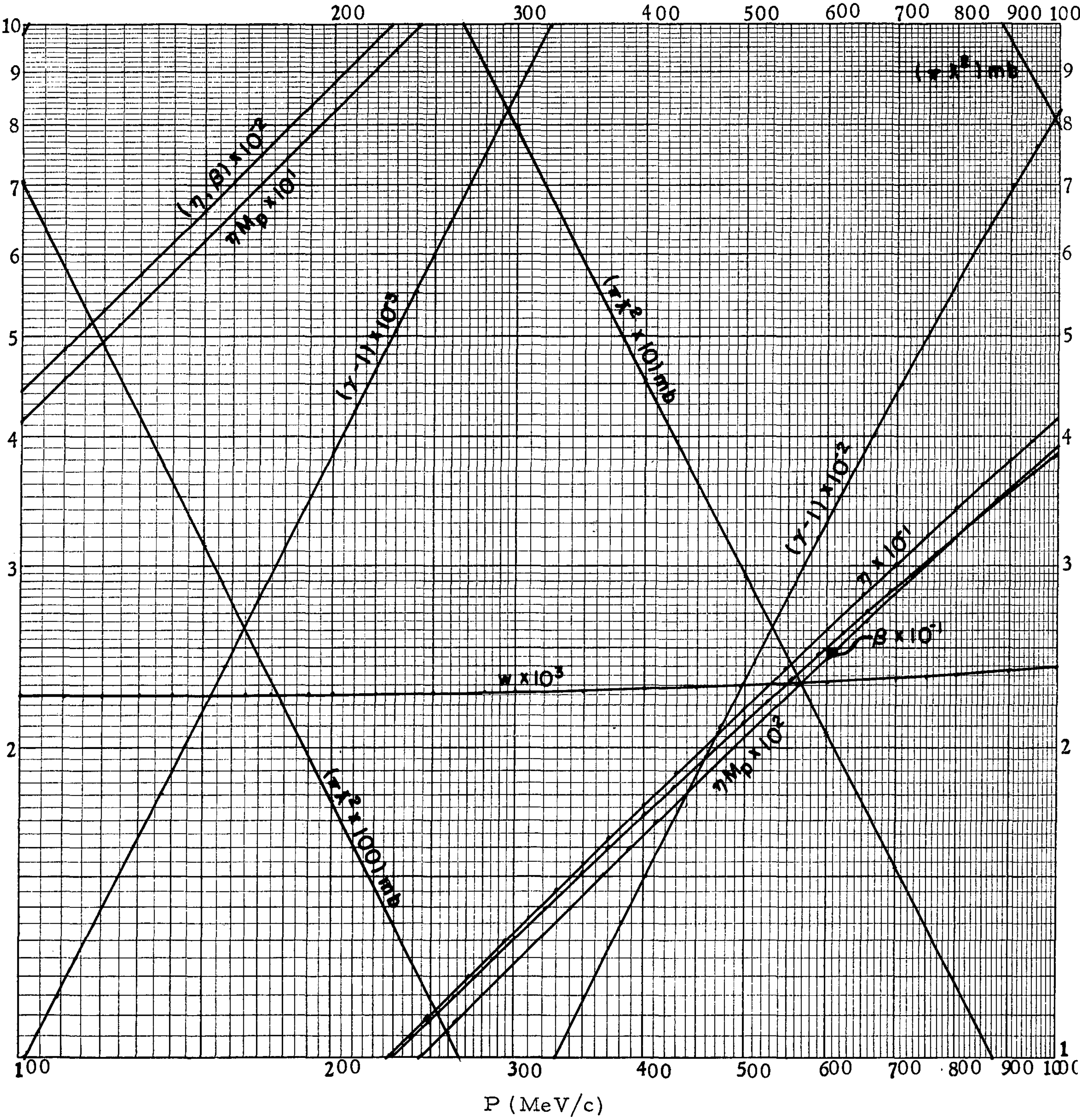
Ξ HYPERONS: $\Xi^- + p$

100 MeV/c to 1 BeV/c

β, $(\gamma - 1)$, η, ηM_p, w

$M_{\Xi^-} = 1318.4$ MeV

$M_p = 938.213$ MeV



0 0 1 0 1 2 0 4 8 0 2

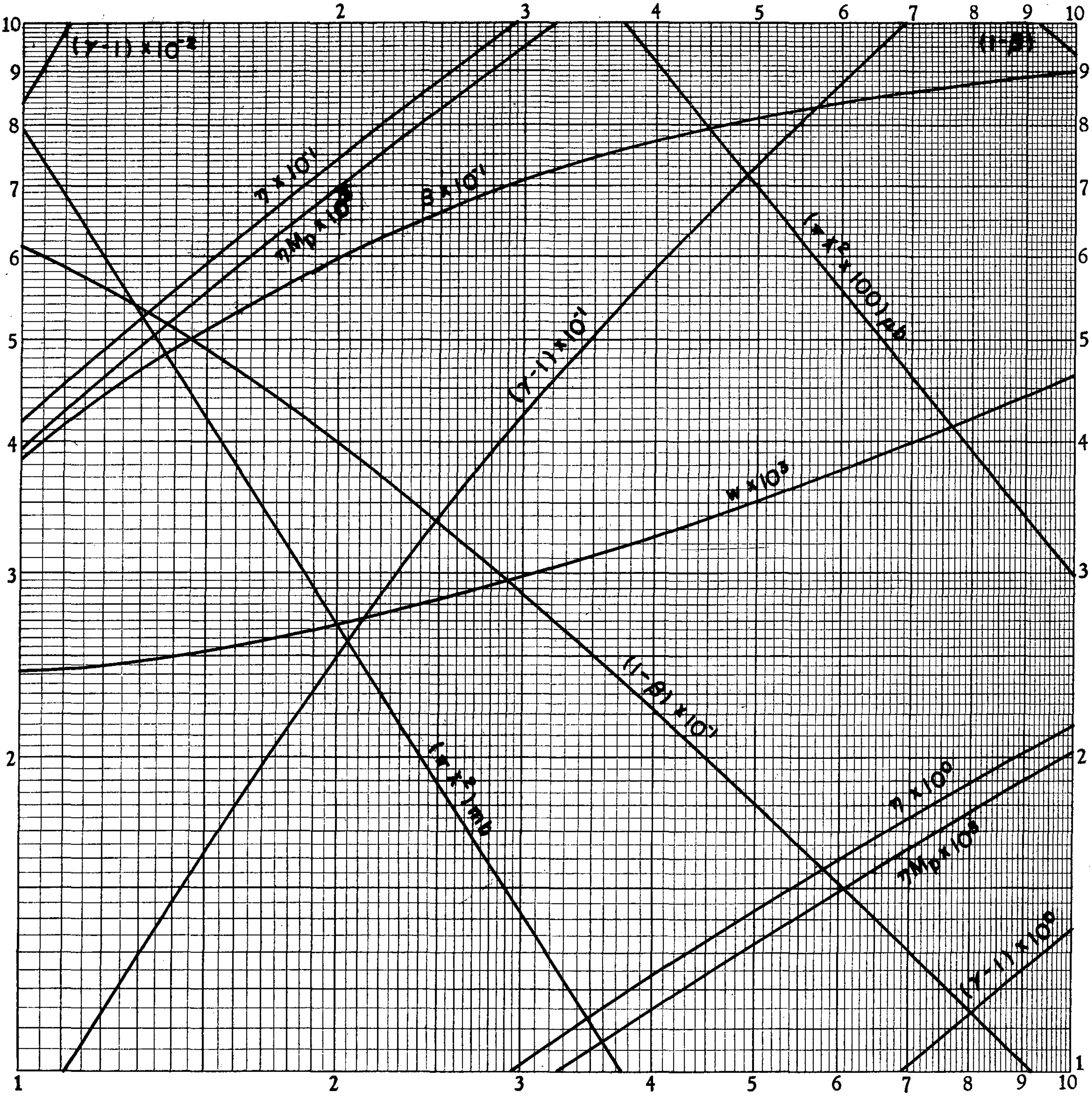
Ξ HYPERONS: $\Xi^- + p$

ηM_p in MeV
w in MeV
mb = 10^{-27} cm^2

1 BeV/c to 10 BeV/c

β, $(1 - \beta)$, $(\gamma - 1)$, η, ηM_p, w, $\pi \lambda^2$

M_{Ξ^-} = 1318.4 MeV
M_p = 938.213 MeV



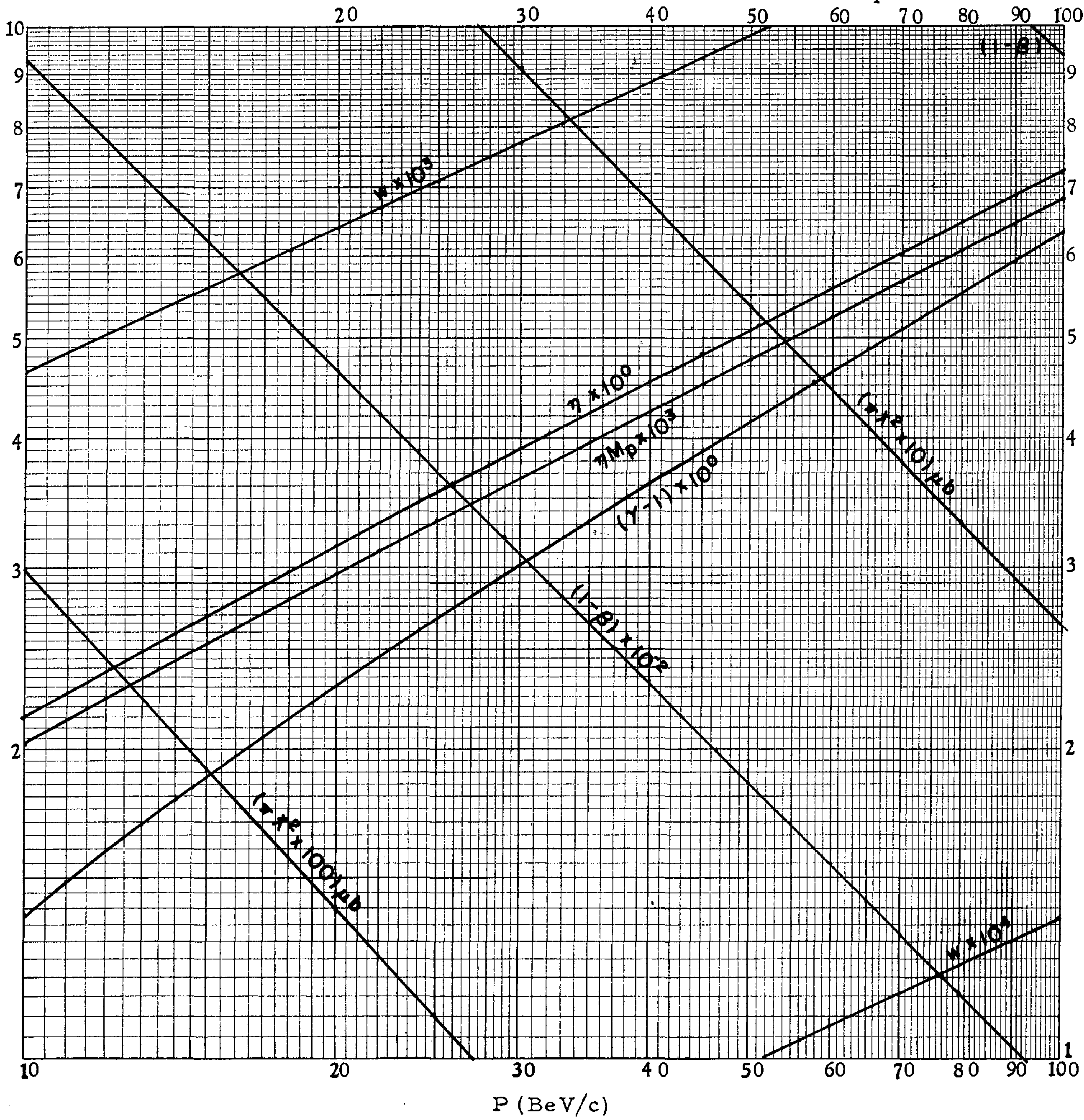
P (BeV/c)

10 BeV/c to 100 BeV/c

$(1 - \beta)$, $(\gamma - 1)$, η, ηM_p, w

$M_{\Xi^-} = 1318.4$ MeV

$M_p = 938.213$ MeV



MNEMONIC DEVICE FOR RELATIVISTIC KINEMATIC FORMULAS[2]

Frank S. Crawford, Jr.

In order to obtain slide-rule accuracy for kinetic energies, one should use formulae expressed in terms of kinetic energies, because if one uses the total energy, the rest mass often uses up the first few significant figures.

The following mnemonic device enables one to write down the exact relativistic (R) formula, if one remembers the nonrelativistic (NR) one:

(a) Write the correct NR formula.

(b) To the rest energy of each moving particle add one-half the total kinetic energy in the center-of-mass (c.m.) system.

Example 1:

A particle of rest mass m_1 and LAB system kinetic energy T_1^0 is incident on a stationary particle of rest mass m_2. Letting $c = 1$, what is the total kinetic energy T in the c.m. system?

(a) NR: $$T = T_1^0 \left(\frac{m_2}{m_2 + m_1}\right);$$

(b) R: $$T = T_1^0 \left(\frac{m_2}{m_2 + m_1 + (T/2)}\right)$$

To solve this quadratic expression numerically for T, it is easier and faster to consider this as a recursion formula, and use a slide rule, rather than to rewrite and solve by radicals. For example, a 600-MeV proton on a proton gives

$$T_{(n+1)} = 600 \left(\frac{938}{938 + 938 + T_{(n)}/2}\right) = (0),\ \underset{\uparrow\ \text{NR}}{300},\ 278,\ \underset{\uparrow\ \text{R}}{279},\ 279, \cdots \text{MeV}.$$

Example 2:

Two particles of rest mass m_1 and m_2 share the kinetic energy $T = T_1 + T_2$ in their c.m. system. How do they divide-up T?

2. From Am. Jour. Phys. 26, 376 (1958).

(a) NR: $T_1 = T \left(\frac{m_2}{m_2 + m_1}\right)$;

(b) R: $T_1 = T \left(\frac{m_2 + T/2}{m_2 + t/2 + m_1 + T/2}\right) = T \left(\frac{m_2 + T/2}{m_1 + m_2 + T}\right)$

For instance, what is the kinetic energy of the μ meson (rest mass 106 MeV) in the decay of a π meson (rest mass 140 MeV) $\pi \rightarrow \mu + \nu$?

$$T_\mu = 34 \frac{(0 + 34/2)}{(0 + 106 + 34)} = 4.13 \text{ MeV.}$$

Example 3:

A single particle m_1 moves relative to the LAB system origin, where an infinite mass is located (so that the laboratory and c.m. systems are equivalent). Express the particle's kinetic energy in terms of its momentum and rest energy.

(a) NR: $T = \frac{p^2}{2m}$;

(b) R: $T = \frac{p^2}{2(m+(T/2)}$.

This example exposes the underlying root of the "mnemonic," in the exact relativistic formulae. We added an infinite mass at the origin so that we could use the "mnemonic" without modifying the phrase "in the c.m. system." One could perhaps say that the infinite mass provides the inertial frame in which special relativity is true.

SOME SIMPLE RULES OF RELATIVISTIC KINEMATICS

Frank T. Solmitz

I. Energy-Angular Distributions

In many problems in particle physics one has to transform an energy-angular distribution from one frame of reference to another (say, from the LAB system to the c.m. systems of a reaction). This transformation is given simply by

$$\frac{1}{P}\frac{d^2n}{dW\,d\Omega} = \frac{1}{P'}\frac{d^2n}{dW'\,d\Omega'},$$

where P, W, and Ω are the momentum, total energy, and solid angle in one frame, and the corresponding primed quantities refer to the other frame. In other words, $(P\,dW\,d\Omega)$ is Lorentz-invariant.

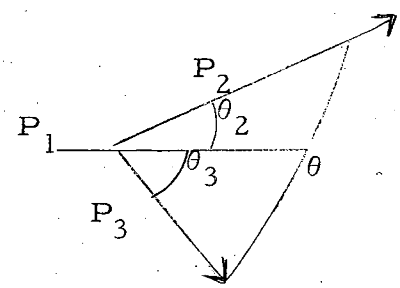
II. Two-Body Decays

Consider particle 1 decaying into particles 2 and 3, with respective masses and momenta M_1, M_2, M_3, P_1, P_2, P_3; let the angle between the directions of particles 2 and 1 be θ_2, and that between 3 and 1 be θ_3, and let $\theta \equiv \theta_2 + \theta_3$. Given two of the momenta and one angle, or both angles and one momentum, one can identify the type of decay process with the help of kinematics tables and graphs.[3] However, one can often rule out certain possible identifications, even if only two quantities are measured, by the use of simple inequalities:

$$P_2 \sin\theta_2 = P_3 \sin\theta_3 \leq P_{c.m.}, \qquad \text{(a)}$$

$$P_1 \sin\theta_2 = P_3 \sin\theta \leq (M_1/M_2)\, P_{c.m.}, \qquad \text{(b)}$$

$$P_1 \sin\theta_3 = P_2 \sin\theta \leq (M_1/M_3)\, P_{c.m.}, \qquad \text{(c)}$$



Here $P_{c.m.}$ denotes the momentum of particle 2 (or 3) in the rest frame of particle 1:

$$P_{c.m.} = \frac{1}{2M_1}\left[(M_1 + M_2 + M_3)(M_1 - M_2 - M_3)(M_1 + M_2 - M_3)(M_1 - M_2 + M_3)\right]^{1/2}.$$

[3] See, for example, Howard S. White, Identification Curves for Heavy-Meson and Hyperon Decays, University of California Radiation Laboratory Report UCRL-3514, September (unpublished).

Equation (a) is a consequence of the invariance of the momentum components transverse to the direction of flight of particle 1. Similarly, Eq. (b) follows from consideration of the momentum components transverse to the direction of flight of particle 2, and from noting that the momentum of particle 1 (or 3), in the rest frame of particle 2, is $(M_1/M_2)\, P_{c.m.}$.

SOLID ANGLE SUBTENDED BY A FINITE RECTANGULAR COUNTER

Frank S. Crawford, Jr.

A geometry problem that arises in particle detection is the calculation of the solid angle Ω subtended by a "finite" detector at a source of particles. For a rectangular detector and a point source, a simple formula can be obtained for the integrated solid angle. First consider the special case in which the point source P is located a distance c perpendicularly above a corner of a rectangle of length a and width b (see Fig. 3a). Then we have

$$\tan \Omega = \frac{ab}{r^2_{eff}},$$

where ab = area of rectangle,

$r_{eff} = \sqrt{cd}$ = geometric mean of smallest and largest distances from P to the rectangle,

$d = (c^2 + a^2 + b^2)^{1/2}$.

Thus the finite solid-angle formula is obtained from that of an infinitesimal detector by replacing r^2 by r^2_{eff}, and Ω by $\tan \Omega$.

The above holds only for the special case in which the perpendicular from P to the plane of the detector intersects one corner of the detector. We can now use this result to obtain the solid angle subtended by a rectangle oriented arbitrarily with respect to P. Let σ be the intersection with the plane of the rectangle of the perpendicular from P to the plane of the rectangle. If σ lies inside the rectangle (Fig. 3b), it implies the four sub-rectangles A, B, C, and D; we simply apply the formula to them and add the results. If σ lies outside the rectangle (Fig. 3c), then we apply the formula to the four rectangles A + B + C + D, B + C, C + D, and C, and find the desired quantity from A = (A + B + C + D) - (C + D) - (B + C) + C.

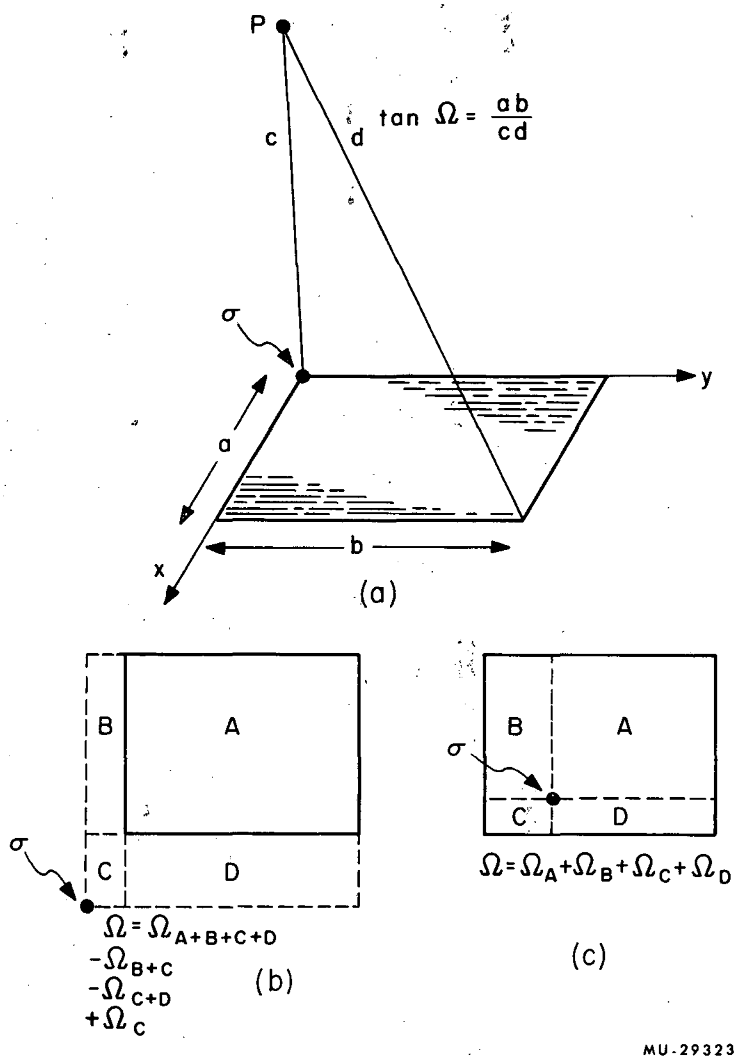


Fig. 3. Geometrical solution for the solid angle subtended by a finite rectangular counter.