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Characterization of Toroidal Alfvén Eigenmode and Fishbone Instabilities in Mega-Amp
Spherical Tokamak/-Upgrade

A dissertation submitted in partial satisfaction
of the requirements for the degree
Doctor of Philosophy in Physics

by

Henry Hingyin Wong

2025

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ABSTRACT OF THE DISSERTATION

Characterization of Toroidal Alfvén Eigenmode and Fishbone Instabilities in Mega-Amp
Spherical Tokamak/-Upgrade

by

Henry Hingyin Wong

Doctor of Philosophy in Physics

University of California, Los Angeles, 2025

Professor Troy A. Carter, Co-Chair

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This dissertation investigates low-frequency, neutral beam injection (NBI)-driven instabilities—Toroidal Alfvén Eigenmodes (TAEs) and fishbone (FB) instabilities—in the spherical tokamaks MAST and MAST-U, with a focus on their effects on fast-ion confinement and bulk plasma behavior. Using a combination of diagnostics, including beam emission spectroscopy (BES) and soft X-ray measurements, detailed experimental characterizations of these instabilities are performed to support validation of theoretical and simulation models and to inform reduced fast-ion transport calculations.

First, global radial mode structure measurement of TAEs in the MAST discharge 26887 at 180 ms are made with soft X-ray and inverted to obtain the emissivity fluctuation spatial structure using a novel technique. The structure shows characteristics consistent with theory for $(n,m,m+1) = (1,1,2)$ TAEs. The global linear gyrokinetic simulations of TAEs in MAST using the Gyrokinetic Toroidal Code (GTC) are verified and validated against the measurements. The results demonstrate good agreement in mode frequencies and radial structures,

providing one of the first successful validations of gyrokinetic models in spherical tokamaks. Discrepancies observed in the poloidal harmonics motivate further experimental efforts to measure fast-ion anisotropy and mode structure in greater detail.

Second, this work explores the impact of FB instabilities on core confinement in MAST and MAST-U, revealing evidence of substantial magnetic axis shifts and thermal species temperature drops, despite the absence of conventional sawtooth activity. Novel core poloidal mode structures measurements are obtained first time in a spherical tokamak using a 2D BES system. The measurements reveal that not all FB-like modes have the expected $(n,m) = (1,1)$ dominant mode numbers and structures in the core. More importantly, noticeable thermal species temperature drops occur after the appearances of these modes. This study highlights the importance to improve understanding and predictive capabilities for these modes to mitigate their associated thermal species temperature drops, and hence the direct impacts on the Lawson criterion.

Finally, reduced-model transport calculations using ORBIT-Kick are performed for both TAEs and FBs. By incorporating measured mode amplitudes and frequency chirping, ORBIT-Kick captures the enhanced fast-ion transport induced by rapidly evolving instabilities. These simulations highlight how differences in mode structure and orbit location determine whether transport leads to losses or internal redistribution. The calculated transport signatures are positioned for future validation against diagnostic measurements such as neutron emission and FILD signals.

Collectively, the results contribute to improved predictive capability for fast-ion behavior in the presence of MHD activity, and provide a framework for integrating diagnostic measurements into transport modeling for spherical tokamaks.

The dissertation of Henry Hingyin Wong is approved.

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*To my mother, my brother, White-White-the-Poodle,
and the mighty God.*

TABLE OF CONTENTS

1	Introduction	1
1.1	Nuclear fusion reaction	1
1.2	Basic principles of controlled nuclear fusion	2
1.3	Configurations of Tokamaks	4
1.3.1	Non-inductive heating mechanisms	6
1.4	Comparison of Conventional and Spherical Tokamaks	6
1.4.1	Mega-Amp Spherical Tokamak (MAST)	7
1.4.2	Mega-Amp Spherical Tokamak-Upgrade (MAST-U)	8
1.5	A brief introduction to instabilities and their associated fast ion transport in Tokamaks	9
1.5.1	Alfvén instabilities	10
1.5.2	MHD instabilities	11
1.6	Overview of the Dissertation	12
1.6.1	Outline of the dissertation	14
2	Fast ion resonant condition and orbit topology	16
3	MHD frameworks for TAEs and FBs	20
3.1	Drift-kinetic model	20
3.2	Single-fluid ideal MHD model	22
3.3	Ideal MHD descriptions for TAE	23
3.3.1	Limitations of the ideal MHD model for TAEs in STs	25

3.4	MHD-based models for Internal kink, infernal, and Fishbone instabilities . . .	27
3.4.1	Internal kink mode	27
3.4.2	Infernal mode	30
3.4.3	Fishbone instability	32
3.4.4	Limitations of these models for FBs in STs	35
3.5	Summary of this chapter	36
4	Data analysis methodology	37
4.1	Short-time Fourier transform (STFT) and its inverse	37
4.1.1	Time-frequency resolution trade-off in STFT	39
4.2	Magnetic fluctuations and toroidal mode numbers inferred from the outboard Mirnov coil array	40
4.3	Time- and frequency-dependent band-pass filtering	42
4.4	Linear regression model for fluctuations	43
4.5	Mode spatial structure diagnostics	46
4.5.1	Tangential Soft X-Ray camera array diagnostic	46
4.5.2	Beam Emission Spectroscopy diagnostic	50
4.6	Summary of this chapter	52
5	Measurements of TAE in MAST for Verification and Validation of a Linear Global Gyrokinetic Simulation	53
5.1	MAST test case: experimental profiles and TAE measurements	55
5.2	GTC gyrokinetic formulation and its reduction to ideal MHD	66
5.3	Maxwellian and Anisotropic distributions in GTC	72
5.4	Continuum and NOVA simulations of the MAST TAE test case	74

5.5	Antenna-excited TAEs in GTC	76
5.6	TAEs excited by Maxwellian and slowing-down fast ion distributions in GTC	83
5.7	Summary of this chapter	94
6	Reduced fast ion transport calculations for TAEs MAST	99
6.1	GTC vs. NOVA mode structures as inputs for fast ion transport calculations	100
6.2	Effects of mode amplitudes on TAE associated fast ion transport	104
6.3	Effects of chirping on TAE associated fast ion transport	106
6.4	Summary of this chapter	108
7	Fishbone instabilities and their impacts on core confinement in MAST and MAST-U	116
7.1	Fishbone measurements in MAST	118
7.2	Fishbone measurements in MAST-U	123
7.2.1	Fast ion transport associated measurements	128
7.2.2	Thermal ion transport associated measurements	131
7.2.3	2D BES measurements on the FB-like bursts	132
7.2.4	Discussions on the FB-associated thermal and fast ion transport in MAST-U	140
7.3	Summary of this chapter	150
8	Reduced fast ion transport calculations for FBs in MAST-U	152
8.1	Constructing the poloidal harmonics of a MAST-U FB for transport calculations	153
8.2	Effects of mode amplitudes on FB associated fast ion transport	155
8.3	Effects of chirping on FB associated fast ion transport	165

8.4	Summary of this chapter	167
9	Conclusions and future efforts	168
9.1	Measurements of TAE in MAST for Verification and Validation of a Linear Global Gyrokinetic Simulation	168
9.2	Fishbone instabilities and their impacts on core confinement in MAST and MAST-U	170
9.3	Reduced fast ion transport calculations for TAEs MAST and FBs in MAST-U	171

LIST OF FIGURES

1.1	(a) Schematic of the tokamak configuration (from [65]) ;(b) Tokamak poloidal cross-section and the schematic of the tokamak coordinate system	5
1.2	Illustration of the spherical tokamak aspect ratio compared to the conventional tokamak aspect ratio from [92]	8
2.1	Example of the poloidal projections of the guiding-centre trajectories of JET fast deuterium orbits, from [56]	18
2.2	Example of the orbit topology boundaries for different types of MAST fast deuterium orbits in the phase space at a fixed energy.	19
3.1	(a) Dispersion relation of two waves with toroidal mode number $n=4$ and poloidal harmonics of $m=4$ and $m=5$ in a plasma with a q profile that increases monotonically with radius. Frequencies are plotted as positive for both signs of $k_{ }$. The frequency gap increases with radius because r/R increases, which increases the modulation of N . The two waves are counter-propagating in the frequency gap. (b) Dispersion relation with (solid) and without (dashed) toroidal coupling of the waves. The $m=6$ and $m=7$ waves are also shown and intersect at larger radii. The figures are based on [34] and from [47].	26
3.2	Comparison of a hypothetical $q(r) = 0.8 + r^2$ with $r_s = 0.45$ and the corresponded internal kink radial mode structures with $\xi_0 = 1.5$ and three different $\gamma\tau_A$. . .	30
3.3	Comparison of a hypothetical $q(r)$ with $r_0 = 0.5$, a hypothetical pressure profile with $r_p = 0.3$, and the corresponded internal mode radial structure.	33
3.4	Time evolution of $\dot{B}_\theta$ of two fishbones from [72], with an artistic depiction of the shape of the signal as a "fishbone".	36

4.1	$n \in [0, 8]$ in MAST-U discharge 47001 from 100 to 300ms, 0 to 160kHz.	40
4.2	(a) Time-frequency varying band pass filter of a δB_R signal from discharge 47128 in MAST-U; (b) filtered signal before (green) and after linear regression (red).	46
4.3	The set of B-spline polynomials P_j used to invert δI_{SXR} in MAST	50
5.1	MAST discharge 26887 raw signals of (a) one of the outboard Mirnov coils that measure δB_z and; (b) tangential SXR camera with impact parameters (p) from 28.82 to 130.52cm, from 0.16 to 0.21 seconds	56
5.2	(a) MAST discharge 26887 δB_z spectrogram with half-overlapping 0.1ms windows. Only the $n = 1$ components are displayed. The targeted TAE is circled in pink. $(n,m) = 1,1$ TAE frequency and core plasma rotation frequency are labeled in black lines; core (at $R = 1.0\text{m}$) electron (b) density and (c) temperature from Thomson scattering; (d) NBI power; (e) total neutron emission from fission chamber, and low pass filtered (DC) neutron emission showing the slow evolving part of the total neutron emission.	57
5.3	Spectrograms (with half-overlapping 0.1ms windows) of 13 tangential SXR camera channels with impact parameters (p) from 28.82 to 130.52cm in MAST discharge 26887 from 150 to 250ms.	59
5.4	Cross coherences (using 0.6ms and 30kHz coherence averaging windows) of STFT of measurements from 13 tangential SXR camera channels with impact parameters (p) from 28.82 to 130.52cm and STFT of δB_z in MAST discharge 26887 from 150 to 250ms.	60
5.5	(a) filtered line-integrated (b) and inverted SXR emissivity fluctuations associated with the targeted TAE in MAST pulse 26887 in the major radius domain	62
5.6	(a) Inverted SXR emissivity fluctuations and (b) $-q-$ profile associated with the targeted TAE in MAST pulse 26887	63

5.7	(a) 2D and (b) mid-plane radial fast ion density profiles from NUBEAM/TRANSP for MAST pulse 26887, $t = 180\text{ms}$	65
5.8	(a) Radial density and (b) temperature profiles from NUBEAM/TRANSP from thermal ions, electrons and fast ions used in GTC simulations.	67
5.9	Fast ion distribution from NUBEAM calculated for MAST pulse 26887, $t = 180\text{ms}$. (a) λ vs. R near injection energy; (b) λ vs. E at $(R,Z) = 1.08\text{m}, 0.01\text{m}$; (c) the analytic fast ion slowing-down distribution used in a linear GTC run, mimicking the injection λ and three birth energies (labeled in dash lines) apparent in the distribution shown in (b)	68
5.10	Mode structure FWHM from measurement (dashed line), NOVA (green line) and GTC (red line) together with Alfvén continuum in the range of TAE gap generated by GTC and NOVA calculated for MAST pulse 26887, $t = 180\text{ms}$. . .	77
5.11	Radial displacement of TAE at $\approx 89\text{kHz}$ from NOVA calculated for MAST pulse 26887, $t = 180\text{ms}$	78
5.12	The saturated amplitudes versus antenna frequencies for antenna excited TAEs (case I in Table.5.1)	83
5.13	The waveform (solid curves) of $(n,m) = (1,1)$ near $-q = 1.5$ of the antenna excited modes, with their $e^{i\omega_E t} e^{\gamma_{DT} t}$ fits (dashed curves) for the period after the antenna was turned off	84
5.14	2D (R,Z) snapshots from GTC runs that excite $n = 1$ TAE with (a1) 70kHz antenna in single-fluid MHD plasma without ϕ_{eff} ; (b1) 72kHz antenna in double-fluid MHD plasma with ϕ_{eff} ; (c1) 70kHz antenna with gyro-kinetic thermal ions and ϕ_{eff} ; (d1) Maxwellian fast ion profile with $1 * (\beta_{FI,NUBEAM})_{max}$, (e1) Maxwellian fast ion profile with $2 * (\beta_{FI,NUBEAM})_{max}$, (f1) analytic fast ion slowing-down distribution with injection $\lambda = 0.65$. (a2) - (f2) show the corresponding poloidal harmonics $m = [0,4]$	85

5.15	(a) $n_{e0}(\rho)$ from TRANSP, its profile fitted to a Gaussian function as $n_{e0,fit} = n_{e0,max} e^{-\frac{(\rho-\mu)^2}{2\sigma^2}}$, and the fitting uncertainty; (b) $-\nabla_\rho n_{e0,fit} = \frac{(\rho-\mu)}{\sigma^2} n_{e0,fit}$ and the propagated fitting uncertainties	88
5.16	Poloidal harmonics $m = [1,2]$ of $(-\frac{m}{\rho} \nabla_\rho n_{e0,fit})(\delta\phi)$ from GTC runs that excite $n = 1$ TAE with (a) Maxwellian fast ion profile with $1 * (\beta_{FI,NUBEAM})_{max}$; (b) analytic fast ion slowing-down distribution with injection $\lambda = 0.65$, with dashed curves represent the propagated $n_{e0,fit}$ fitting uncertainties	89
5.17	(a) Growth rates and (b) eigen-frequencies (in the plasma frame) of $n = 1$, $m = 1$ TAEs excited by analytic Maxwellian fast ion β_{FI} of 1, 1.5, 2 and 2.5 times of $(\beta_{FI,NUBEAM})_{max}$ (by scaling n_{FI}), sampled near $-q = 1.5$ surface	90
5.18	2D (R,Z) $\delta B_r/B$ structures of (a) $m = 1$ and (b) $m = 2$ from NOVA; (c) $m = 1$ and (d) $m = 2$ from linear GTC with the mode excited by analytic Maxwellian fast ion β_{FI} of $3 * (\beta_{FI,NUBEAM})_{max}$ (by scaling n_{FI})	91
6.1	Kick probability matrix ($p(\lambda, P_\zeta, \mathcal{E})$) from result of ORBIT-Kick that uses NOVA structure, showing ΔP_ζ of resonant fast ions and the resonance bands for $(n, p, l) = (1, 1, 2)$ and $(1, \leq -1, 1)$ in phase space representations for $\mathcal{E}$ at (a) 16.7 keV, (b) 26.0 keV, (c) 35.3 keV, and (d) 44.7 keV	102
6.2	Histogram of $\Delta \mathcal{E}$ and its standard deviation ($SD_{\Delta \mathcal{E}}$) (in keV) of fast ions in ORBIT-Kick calculation using NOVA structure at 88kHz (case 1 in table 6.1) as an input.	104
6.3	Kick probability matrix ($p(\lambda, P_\zeta, \mathcal{E} = 35.3\text{keV})$) from result of ORBIT-Kick that uses (a) NOVA structure at 88kHz, (b) Maxwellian GTC structure at 73kHz, and (c) NOVA structure at 73kHz, showing ΔP_ζ of resonant fast ions and the resonance bands for $(n, p, l) = (1, 1, 2)$ and $(1, \leq -1, 1)$ in phase space representations.	105

6.4	2D ($\lambda = \mu B_0/E$ and $\mathcal{E}$) histograms for fast ions with their energies modified greater than $3 * SD_{\Delta\mathcal{E}}$ in ORBIT-Kick calculations using (a) NOVA structure at 88kHz (case 1 in table 6.1), (b) GTC structure at 72kHz (case 2), and (c) NOVA structure at 73kHz (case 3).	105
6.5	mode chirping (a) frequency and (b) amplitude profiles that mimic the measurements used for ORBIT-Kick TAE cases 6 and 7 (table 6.1)	109
6.6	Kick probability matrix ($p(\lambda, P_\zeta, \mathcal{E} = 26.0\text{keV})$) from result of ORBIT-Kick that uses (a) evolving NOVA structure with a fixed frequency at 88kHz, and (b) evolving NOVA structure with a frequency chirping down from 88kHz to 68kHz (as shown in Fig.6.5), showing ΔP_ζ of resonant fast ions and the resonance bands for $(n, p, l) = (1, 1, 2)$ and $(1, \leq -1, 1)$ in phase space representations.	110
6.7	2D ($\lambda = \mu B_0/E$ and $\mathcal{E}$) histograms for fast ions with their energies modified greater than $3 * SD_{\Delta\mathcal{E}}$ in ORBIT-Kick calculations using (a) evolving NOVA structure with a fixed frequency at 88kHz (case 6 in table 6.1), and (b) evolving NOVA structure with a frequency chirping down from 88kHz to 68kHz (case 7).	111
6.8	Kinetic Poincaré plot of fast ions in case 7 (table 6.1), with the orbit that jumps during the ORBIT-Kick simulation highlighted in red	112
6.9	The outermost point of orbit for particles with initial (a) $\psi = 0.8$, (b) $\psi = 0.91$, and energy of particles with initial (c) $\psi = 0.8$, (d) $\psi = 0.91$ in case 7 (table 6.1)	113
7.1	MAST discharge 29976 raw signals of (a) one of the outboard Mirnov coils that measure δB_z and; (b) tangential SXR camera with impact parameters (p) from 28.82 to 130.52cm, from 0.18 to 0.25 seconds, covering the period with appearances of fishbone-like bursts	121

7.2	MAST discharge 26887 raw signals of (a) one of the outboard Mirnov coils that measure δB_z and; (b) tangential SXR camera with impact parameters (p) from 28.82 to 130.52cm, from 0.21 to 0.30 seconds, covering the period with appearances of fishbone-like bursts	122
7.3	(a) the spectrogram of δB_z of shot 29976. Only $n = 1$ is shown. The black line represents the core plasma rotation frequency; (b) Mid-plane q profile 29976 at 201ms.	122
7.4	(a) the spectrogram of δB_z of shot 26887. Only $n = 1$ is shown. The black line represents the core plasma rotation frequency; (b) Mid-plane q profile of 26887 at 262ms.	123
7.5	(a) spectrogram of δB_z of shots 29976; (b) amplitude of $\delta\epsilon_{FB}$ from SXR after band pass filter, linear regression and emissivity inversion major radius versus time with the magnetic axis location from EFIT labeled in a turquoise line; (c) temporal phase variation of band pass filtered and inverted SXR versus band pass filtered δB_z at $R \approx 92\text{cm}$ for FB bursts	124
7.6	(a) spectrogram of δB_z of shots 26887; (b) amplitude of $\delta\epsilon_{FB}$ from SXR after band pass filter, linear regression and emissivity inversion major radius versus time with the magnetic axis location from EFIT labeled in a turquoise line; (c) temporal phase variation of band pass filtered and inverted SXR versus band pass filtered δB_z at $R \approx 89\text{cm}$ for FB bursts	125
7.7	(a) real parts of the FB associated $\delta B_z, \delta B_R, \delta B_T$; (b) angle between real parts of $\vec{\delta B}$ components; (c) ratio of δB_z and δB_T amplitude; (d) the passive FIDA signal from a channel with the chord impact parameter at $R = 1.20\text{m}$ in the discharge 29976.	126
7.8	MAST-U discharge 47128 raw signals of (a) one of the outboard Mirnov coils that measure δB_R and; (b) eight mid-plane located BES APD cameras	129

7.9	MAST-U discharge 47132 raw signals of (a) one of the outboard Mirnov coils that measure δB_R and; (b) eight mid-plane located BES APD cameras	130
7.10	(a) spectrogram of δB_R with labels for chirps A, B, C and D; (b) Field pitch versus time inferred from MSE with contour lines labeling pitches of -0.04 to 0.04; (c) magnetic axis locations versus time inferred from MSE and from the EFIT++; (d) neutron emission from fission chamber; (e) Thermal ion rotation speed near $R = 1.0\text{m}$ from CXRS, for discharge 47128 from 250ms to 500ms, with the labels for times of occurrences of various instabilities under the time axis.	133
7.11	(a) spectrogram of δB_R with labels for chirps A, B, C and D; (b) Field pitch versus time inferred from MSE with contour lines labeling pitches of -0.04 to 0.04; (c) magnetic axis locations versus time inferred from MSE and from the EFIT++; (d) neutron emission from fission chamber; (e) Thermal ion rotation speed near $R = 1.0\text{m}$ from CXRS, for discharge 47132 from 250ms to 500ms, with the labels for times of occurrences of various instabilities under the time axis.	134
7.12	MAST-U discharge 47128. (a) Toroidal number identifications for modes from 0 to 60kHz using toroidally separated Mirnov coils (b) Plasma current; (c) On-axis neutral beam injection power; (d) thermal electron densities inferred from the average of ten Thmoson scattering channels near $R = 1.0\text{m}$ and $R = 1.2\text{m}$; (e) thermal electron temperatures inferred from the average of ten Thmoson scattering channels near $R = 1.0\text{m}$ and $R = 1.2\text{m}$; (f) thermal ion densities inferred from the average of four CXRS channels near $R = 1.0\text{m}$ and $R = 1.2\text{m}$; (g) on-axis $ q_0 $ for discharge 47128 from 250ms to 500ms inferred from the MSE, with the labels for times of occurrences of various instabilities under the time axis.	135

7.13	MAST-U discharge 47132. (a) Toroidal number identifications for modes from 0 to 60kHz using toroidally separated Mirnov coils (b) Plasma current; (c) On-axis neutral beam injection power; (d) thermal electron densities inferred from the average of ten Thomson scattering channels near $R = 1.0\text{m}$ and $R = 1.2\text{m}$; (e) thermal electron temperatures inferred from the average of ten Thomson scattering channels near $R = 1.0\text{m}$ and $R = 1.2\text{m}$; (f) thermal ion densities inferred from the average of four CXRS channels near $R = 1.0\text{m}$ and $R = 1.2\text{m}$; (g) on-axis $ q_0 $ for discharge 47128 from 250ms to 500ms inferred from the MSE, with the labels for times of occurrences of various instabilities under the time axis.	136
7.14	The thermal ion profiles inferred from CXRS in discharge 47128 at (a) 350ms and 360ms, during and after chirp D (the last burst in the series of double FB-like bursts), and (b) before the another FB-like and during MHD-like bursts	137
7.15	Spectrograms of 4 mid-plane BES channels in MAST-U discharge 47128 from 250 to 360ms.	141
7.16	Cross coherences of STFT of measurements from 4 mid-plane BES channels and STFT of δB_R in MAST-U discharge 47128 from 250 to 360ms.	142
7.17	(a) spectrogram of δB_R of shots 47128; (b) amplitude of $\delta I/I_0$ from BES associated with chirps A and B after band pass filter and regression in major radius versus time with the magnetic axis location from EFIT++ labeled in a turquoise curve; (c) averaged temporal phase shift of two band pass filtered BES channels above and below the mid-plane at $R \approx 98\text{cm}$ associated with chirps A and B. . .	143

7.18	(a) spectrogram of δB_R of shots 47128; amplitude of $\delta I/I_0$ from BES associated with (b) chirp C only and (c) chirp D only after band pass filter and regression in major radius versus time with the magnetic axis location from EFIT++ labeled in a turquoise curve; (d) averaged temporal phase shift of two band pass filtered BES channels above and below the mid-plane at $R \approx 98\text{cm}$ associated with chirps C and D.	144
7.19	the (R,Z) snapshots of chirp B associated $real(\delta I/I_0)$ without linear regression at (a) $t = 325.925\text{ms}$ and (b) 325.975ms . $\rho = 0.1$ and 0.2 surfaces are labeled in solid and dashed curves, respectively	145
7.20	the (R,Z) snapshots of chirp C associated $real(\delta I/I_0)$ without linear regression at (a) $t = 348.010\text{ms}$ and (b) 348.025ms . $\rho = 0.1$ and 0.2 surfaces are labeled in solid and dashed curves, respectively	145
7.21	the (R,Z) snapshots of chirp C associated $real(\delta I/I_0)$ without linear regression at (a) $t = 348.325\text{ms}$, (b) 348.350ms and (c) 348.375ms . $\rho = 0.1$ and 0.2 surfaces are labeled in solid and dashed curves, respectively	146
7.22	the (R,Z) snapshots of chirp B associated $real(\delta I/I_0)$ without linear regression at (a) $t = 349.300\text{ms}$ and (b) 349.350ms . $\rho = 0.1$ and 0.2 surfaces are labeled in solid and dashed curves, respectively	146
7.23	(a) the normalized chirp B associated $abs(\delta I/I_0)$ (solid line) and $abs(\delta B_R)$ (dashed line) without linear regression; (b) the real part of chirp B associated $\delta I/I_0$ without linear regression, showing the waveform of chirp B at (R,Z) = (96cm, -6.15cm) , (R,Z) = (96cm, 8.16cm) and (R,Z) = (94cm, 3.92cm).	147

7.24	the absolute values of chirps C and D associated δB_R (solid lines) and δB_R (dashed lines) without linear regression. The chirp C associated δB_R and δB_R are normalized by the maximum value of chirp D associated δB_R and δB_R , respectively; (b) the real part of chirp C associated $\delta I/I_0$ without linear regression, showing the waveform of chirp C at at (R,Z) = (96cm, -6.15cm) , (R,Z) = (96cm, 8.16cm) and (R,Z) = (94cm, 3.92cm); (c) the real part chirp D associated $\delta I/I_0$ without linear regression, showing the waveform of chirp D at at (R,Z) = (96cm, -6.15cm) , (R,Z) = (96cm, 8.16cm) and (R,Z) = (94cm, 3.92cm).	148
8.1	The radial profiles of $abs(\delta I/I_o)$ from BES associated with (a) chirp D in 47128 at t = 325.9ms with the viewing window centered at R = 95cm; (b) first of the double FB-like burst in 47132 at t = 326.2ms with the viewing window centered at R = 115cm.	156
8.2	The radial profiles of $abs(\delta I/I_o)$ from BES associated with (a) chirp C in 47128 at t = 347.5ms with the viewing window centered at R = 95cm; (b) first of the double FB-like burst in 47132 at t = 348ms with the viewing window centered at R = 115cm.	157
8.3	(a) 2D and fast ion density profiles and (b) the radial gradient of the product of fast ion density (n_{FI}) and temperature (T_{FI}) from NUBEAM/TRANSP for MAST-U discharge 47128	158
8.4	The fast ion distribution near the (a) magnetic axis and (b) $R = 1.3m$ and mid-plane from NUBEAM/TRANSP for MAST-U discharge 47128	158
8.5	Tests case for constraining the FB mode structure inputs. (a1) - (d1): the input poloidal harmonics (in radial plasma displacements) for ORBIT-Kick and (a2) - (d2) to be the resulting kick probability matrix ($p(\lambda, P_\zeta, \mathcal{E} = 54.0\text{keV})$)	159

8.6	kick probability matrix ($p(\lambda, P_\zeta, \mathcal{E} = 54.0\text{keV})$) from result of ORBIT-Kick that uses structure (shown in Fig.8.5(d1)), showing ΔP_ζ of resonant fast ions and the resonance bands for $(n, p, l) = (1, 1, 2)$ at 25.4 and 33 kHz in phase space representations	160
8.7	$\delta n_e/n_{e0}$ measurements from BES compared to different $\delta n_e/n_{e0}$ from two A_{scale} settings in ORBIT-Kick for chirp B in MAST-U discharge 47128.	161
8.8	(a) fixed and chirping frequencies for different cases, with a comparison with the plasma rotation frequency at $R = 1.0\text{m}$ and (b) amplitude profiles that mimic the measurements used for ORBIT-Kick FB cases	162
8.9	kick probability matrices ($p(\lambda, P_\zeta, \mathcal{E} = 54.0\text{keV})$) from results of ORBIT-Kick that use the structure (shown in Fig.8.5(d1)) with (a) $A_{scale} = 5E - 3$ and fixed frequency (case 3 in Table.8.1); (b) $A_{scale} = 5E - 3$ and chirping frequency (case 6 in Table.8.1); (c) $A_{scale} = 4E - 4$ and fixed frequency (case 1 in Table.8.1); (d) $A_{scale} = 4E - 4$ and chirping frequency (case 4 in Table.8.1), showing ΔP_ζ of resonant fast ions and the resonance bands for $(n, p, l) = (1, 1, 2)$ at 27 and 33 kHz in phase space representations	163
8.10	2D ($\lambda = \mu B_0/E$ and $\mathcal{E}$) histograms for fast ions with their energies modified greater than $3 * SD_{\Delta \mathcal{E}}$ in ORBIT-Kick calculations using the structure (shown in Fig.8.5(d1)) with (a) $A_{scale} = 5E - 3$ and fixed frequency (case 3 in Table.8.1); (b) $A_{scale} = 5E - 3$ and chirping frequency (case 6 in Table.8.1); (c) $A_{scale} = 4E - 4$ and fixed frequency (case 1 in Table.8.1); (d) $A_{scale} = 4E - 4$ and chirping frequency (case 4 in Table.8.1).	164
8.11	Kinetic Poincaré plot at $\lambda = \mu B_0/E = 0.4$ of fast ions for cases of (a) fixed frequency at 33 kHz (case 3 in Table.8.1); (b) chirping frequency (case 6 in Table.8.1). Orbits are labeled by different colors.	166

LIST OF TABLES

1.1	Main parameters for the JET, ITER, MAST and MAST-U tokamaks	9
5.1	Various simulation cases to measure the mode eigen-frequencies and total growth/damping rates with different combinations of damping and driving mechanisms of the $n = 1$ TAE located near the $q = 1.5$ surface	98
6.1	Inputs and settings for ORBIT-Kick runs for MAST TAE cases	113
6.2	Results of ORBIT-Kick runs for MAST TAE cases	114
8.1	Inputs and settings for ORBIT-Kick runs for MAST-U FB cases	166
8.2	Results of ORBIT-Kick runs for MAST-U FB cases	167

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Wong, H.H., Huang, H., Liu, P., Yu, Y., Wei, X., Brochard, G., Fil, N., Lin, Z., Podesta, M., Bonfiglioli, P.J. and McClements, K.G., 2024. Linear gyrokinetic simulations of toroidal Alfvén eigenmodes in the Mega-Amp Spherical Tokamak. *Physics of Plasmas*, 31(11). [106]

Rivero-Rodríguez, J.F., McClements, K.G., Fitzgerald, M., Sharapov, S.E., Cecconello, M., Crocker, N.A., Dolby, I., Dreval, M., Fil, N., Galdón-Quiroga, J. and García-Muñoz, M., 2024. Overview of fast particle experiments in the first MAST Upgrade experimental campaigns. *Nuclear Fusion*, 64(8), p.086025. [82]

Harrison, J.R., Aboutaleb, A., Ahmed, S., Aljunid, M., Allan, S.Y., Anand, H., Andrew, Y., Appel, L.C., Ash, A., Ashton, J. and Bachmann, O., 2024. Overview of physics results

from MAST upgrade towards core-pedestal-exhaust integration. Nuclear Fusion, 64(11), p.112017. [45]

CHAPTER 1

Introduction

1.1 Nuclear fusion reaction

Nuclear fusion is the process where two or more atomic nuclei combine to form heavier nuclei. The discovery of nuclear fusion emerged from investigations into the mechanisms that sustain the brightness of stars, as chemical processes alone cannot account for the energy output required to maintain their radiance over billions of years. In 1920, the atomic nucleus mass defect was discovered [7], revealing that the mass of a nucleus is less than the combined mass of its constituent nucleons. This missing mass is converted into nuclear binding energy via the mass-energy equivalence principle, which stabilizes nucleons against electrostatic repulsion. This insight suggested that nuclear energy could be released either through the splitting of heavy nuclei (nuclear fission) or the fusion of lighter nuclei into heavier ones, a process termed nuclear fusion. These findings led to the hypothesis that nuclear fusion powers the energy output of stars [27].

Nuclear fusion occurs naturally in stars but is rare elsewhere due to the significant Coulomb repulsion between positively charged nuclei. Fusion in stars is facilitated not only by their extreme heat and pressure but also by the abundance of reactant nuclei. The quantum tunneling effect allows nuclei to overcome the Coulomb barrier, although with very low probability. However, the vast number of nuclei present in stars ensures a sufficient amount of reactions to occur to ignite the fusion reaction. Outside stellar environments, achieving fusion requires extremely high temperatures to provide nuclei with enough kinetic energy to

overcome the Coulomb barrier. Consequently, confinement methods are essential to maintain the high temperatures needed to sustain fusion reactions in artificial settings.

1.2 Basic principles of controlled nuclear fusion

Despite the need for confinement methods, artificial controlled nuclear fusion holds significant potential for energy production. The concept of magnetically controlling fusion reactions to harness energy emerged in the 1950s. Soviet scientists Andrei Sakharov and Igor Tamm proposed the tokamak design in 1950, while Lyman Spitzer in the United States introduced the stellarator concept in 1951 [8]. These approaches are categorized under Magnetic Confinement Fusion (MCF). Alternatively, Inertial Confinement Fusion (ICF), which involves igniting reactions by compressing the target, was also proposed during the same period. All these approaches require heating the fuel to extremely high temperatures and/or pressures through various methods.

The fusion reaction between deuterium (2_1D) and tritium (3_1T) is commonly selected for controlled fusion due to its relatively low ignition temperature compared to other fusion reactions. In principle, nuclei with nucleon numbers below that of iron-56 can release energy through fusion. However, many such reactions are impractical for controlled fusion. Most leading fusion concepts therefore focus on the and reaction:

$${}^2_1D + {}^3_1T \rightarrow {}^4_2He [3.56 \text{ MeV}] + n [14.03 \text{ MeV}], \quad (1.1)$$

where 4_2He is a helium nucleus (or alpha particle), and n is a neutron. The energy values in brackets represent the kinetic energy of each particle, yielding a total energy release of 17.6 MeV per D-T fusion reaction.

While D-T fusion has a high theoretical energy yield, achieving a net positive energy output in artificial settings is challenging due to various power loss mechanisms. The fusion

amplification factor Q is defined as:

$$Q = \frac{P_{\text{fusion}}}{P_{\text{heating}}}, \quad (1.2)$$

where P_{fusion} is the fusion power output and P_{heating} is the auxiliary heating power input. A value of greater than 1 is required to sustain the fusion reaction. Although D-T fusion appears promising due to its high energy output, a significant portion (14.03 MeV out of 17.6 MeV) of the released energy is carried away by neutrons, which cannot be confined electromagnetically. Only the alpha particles contribute to the self-heating process. Therefore, the combined alpha particle power and auxiliary heating power must exceed the power losses :

$$P_{\text{loss}} < P_{\alpha} + P_{\text{heating}} = P_{\alpha} \left(1 + \frac{17.6}{3.56Q} \right) \approx P_{\alpha} \left(1 + \frac{5}{Q} \right). \quad (1.3)$$

The power losses during confinement are characterized as:

$$P_{\text{loss}} = \frac{3nk_B T}{\tau_E}, \quad (1.4)$$

where T is the temperature, n is the particle density, τ_e is the confinement time, and k_B is the Boltzmann constant. Balancing the power yields the Lawson criterion [32]:

$$n\tau_E = \frac{60k_B T}{\langle \sigma v \rangle E_{\text{fusion}} (1 + 5/Q)}, \quad (1.5)$$

where $\langle \sigma v \rangle$ is the reaction's cross-section, and E_{fusion} is the energy released per fusion reaction.

The Lawson criterion defines the minimum conditions required to ignite a fusion reaction. For example, ICF typically achieves a high density with a very short confinement time by compressing the target using powerful lasers [107]. Although ICF has the advantage of high density, its compression mechanisms often lack the repeatability and efficiency needed for practical energy production. This thesis focuses on MCF methods, which aims to confine plasma at a lower density over a longer timescale. For instance, achieving a temperature of approximately 14 keV in MCF devices with magnetic fields of 1-5 T is required to meet the conditions [107].

1.3 Configurations of Tokamaks

Tokamaks are a type of Magnetic Confinement Fusion (MCF) device known for their exceptional plasma confinement performance. These devices use externally applied magnetic fields to confine hot plasma. Two sets of coil systems generate toroidal and poloidal magnetic fields, both of which are essential for plasma confinement, as illustrated in Fig.1.1 [65]. The central solenoid plays a key role by inducing a toroidal electric field inside the tokamak through variations in its current. This electric field drives a toroidal plasma current due to the plasma's finite resistivity. The resulting toroidal current generates a poloidal magnetic field, which is critical for stabilizing the plasma in the tokamak. The poloidal magnetic field imparts helicity to the total magnetic field, with its strength varying over the minor radius, measured from the point where the magnetic field is purely toroidal.

The helicity of the magnetic field can be quantified using the safety factor :

$$q = \frac{r \cdot B_t}{R \cdot B_p}, \quad (1.6)$$

where R is the major radius measured from the center of the tokamak, and B_t and B_p are the toroidal and poloidal components of the magnetic field, respectively. The safety factor is so named because higher values of reduce the susceptibility of the plasma to macroscopic magnetohydrodynamic (MHD) instabilities.

To describe the plasma in tokamaks, field-aligned coordinate systems are used instead of Cartesian coordinates. The dynamics of plasma species are often expressed in terms of the magnetic flux at each location, which can be calculated using the Grad-Shafranov equations [40, 84]. Detailed derivations can be found in White (2006) [103]. Charged particles in the plasma typically travel along magnetic flux surfaces unless cross-field transport occurs. The separatrix, or the last closed flux surface (LCFS), defines the boundary of the confined plasma. Beyond the LCFS, flux surfaces are no longer closed, and particles are vented out

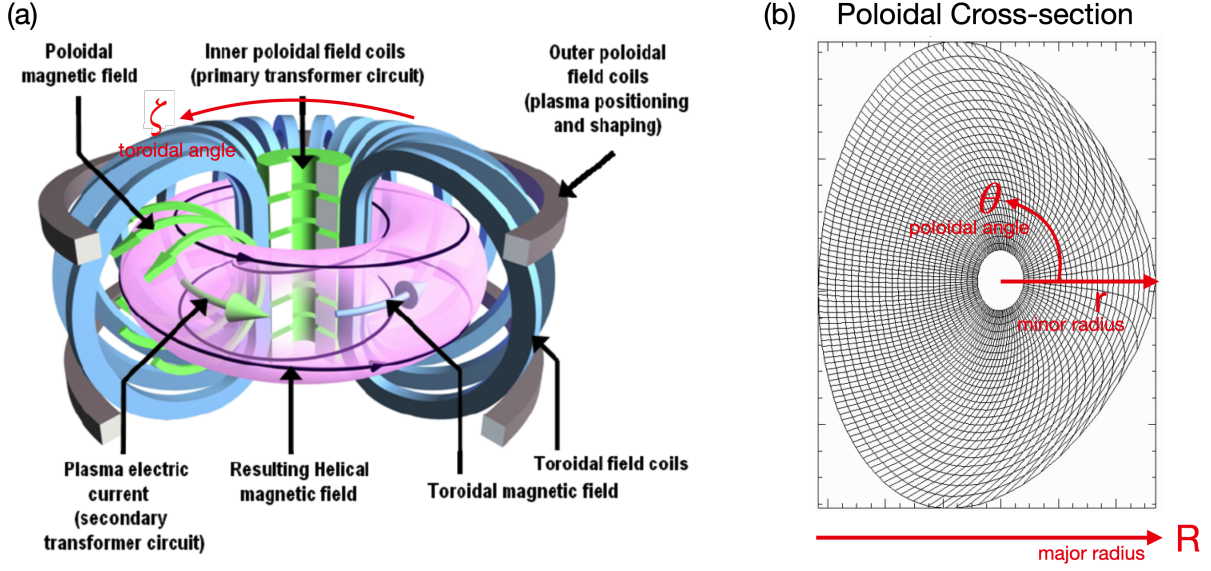


Figure 1.1: (a) Schematic of the tokamak configuration (from [65]) ;(b) Tokamak poloidal cross-section and the schematic of the tokamak coordinate system

through the divertor. Flux surfaces are often labeled by the normalized magnetic flux value:

$$\rho_t(r) = \sqrt{\frac{\psi_t(r)}{\pi B_{\phi 0}}}, \quad (1.7)$$

where ρ_t is the toroidal magnetic flux, and $B_{\phi 0}$ is the equilibrium toroidal magnetic field. ρ_t can be normalized to a range of $\in [0, 1]$ by dividing its value at the LCFS ($\rho_t(a)$), such that $\rho = \rho_t/\rho_t(a)$. This label of the normalized flux surface is used throughout this dissertation.

Additional theoretical descriptions of flux surface shapes, such as triangularity and elongation, are discussed in White (2006) [103]. However, these parameters are not explored in detail here, as the cases presented in this dissertation involve similar plasma shaping. Multi-device or multi-shape configuration comparisons are beyond the scope of this work.

1.3.1 Non-inductive heating mechanisms

In addition to generating the poloidal magnetic field for plasma stability, the toroidal current also contributes to plasma heating. However, the efficiency of inductive heating decreases over time as the plasma resistivity diminishes with increasing plasma temperature. Consequently, inductive heating becomes less effective as the plasma is heated during the course of a tokamak discharge. To address these limitations, non-inductive heating methods are employed.

Tokamaks typically utilize two primary non-inductive heating techniques. The first technique involves cyclotron resonances to heat ions or electrons in the plasma. Electromagnetic waves, known as Ion Cyclotron Resonance Heating (ICRH) at frequencies around 50 MHz, and Electron Cyclotron Resonance Heating (ECRH) at approximately 150 GHz, resonate with ions and electrons, respectively. Although these methods sometimes provide effective heating, they are not the focus of this dissertation.

The second technique, which will be discussed in detail in later chapters, is Neutral Beam Injection (NBI). This method involves injecting a beam of neutral energetic particles into the plasma, where they collide with plasma species and transfer energy. The neutral particles launched by the NBI penetrate deeply into the plasma core and are subsequently ionized. Plasma species are not always heated uniformly by these methods. As a result, fast (energetic) particles with significantly higher temperatures than the bulk plasma species are often generated in specific regions of the tokamak plasma. For instance, NBI-induced fast ions may be deposited at locations determined by factors such as beam power and bulk plasma density.

1.4 Comparison of Conventional and Spherical Tokamaks

Large-scale tokamaks can achieve unprecedented fusion performance but are associated with high construction and operational costs. For example, the International Thermonuclear

Experimental Reactor (ITER), currently under construction in southern France, promises significant improvement in the Q value compared to its predecessor, the Joint European Torus (JET) tokamak. This improvement is achievable by increasing ITER's main parameters, such as plasma volume, plasma current, and magnetic field amplitude, by factors of 2 to 6 compared to JET. Although achieving a high Q is desirable and remarkable for the field of fusion research, the cost of constructing and operating such large devices remains a significant barrier to the commercialization of fusion power. Hence, reducing the size of fusion tokamaks is a desirable objective.

One way to make a tokamak smaller is by reducing the aspect ratio ($A = R/a$, where R is the major radius and a is the minor radius at the LCFS). The concept of the spherical tokamak arises when the aspect ratio approaches 1, compared to the typical value of 2 to 3 in conventional tokamaks. As a result, the plasma in spherical tokamaks resembles a "cored apple" rather than the "donut-like" shape seen in conventional tokamaks, as demonstrated in Fig.1.2. With the device size significantly reduced, the challenges in constructing the vacuum chamber to house the plasma, as well as other coil and heating systems, are greatly alleviated. Additionally, plasma stability and energy confinement performance have been observed to improve in spherical tokamaks, as demonstrated in the systematic study of plasma in the low-aspect-ratio National Spherical Torus Experiment (NSTX) [58].

1.4.1 Mega-Amp Spherical Tokamak (MAST)

This dissertation focuses on the Mega Ampere Spherical Tokamak (MAST) and its upgrade (MAST-U). MAST [23] operated at the Culham Centre for Fusion Energy (CCFE), Oxfordshire, UK, from 1999 to 2013. The objectives of MAST were to explore the performance of sustained plasma at a spherical tokamak aspect ratio, develop a physics database for spherical tokamaks, and advance the understanding of conventional tokamaks by providing data for aspect ratio scaling studies. Key parameters of MAST are summarized in Table 1.1. Notably, the minor radius of MAST was 65% of JET's, but MAST's major radius was less

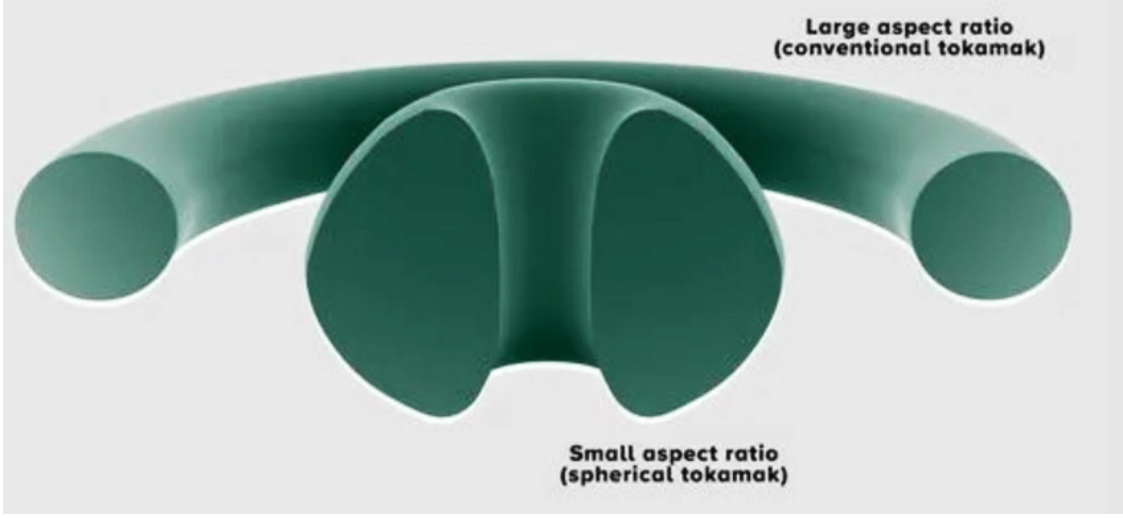


Figure 1.2: Illustration of the spherical tokamak aspect ratio compared to the conventional tokamak aspect ratio from [92]

than 30% of JET's, highlighting its compact design compared to conventional tokamaks.

MAST employed two on-axis Neutral Beam Injectors (NBIs) for auxiliary heating. These two beams targeted the plasma core, injecting from different toroidal locations, referred to as south-south (SS) and south-west (SW). Both beams were tangential to the device's mid-plane, aiming close to the magnetic axis where B_p is zero. As a result, the fast-ion distribution induced by the beams was similar. The primary purpose of having two beams was to increase the overall heating power without heavily influencing plasma parameters based on which beam was used. The injection angles of both beams were fixed and could not be tilted. In experiments requiring fast ions to be deposited away from the magnetic axis, the plasma was shifted vertically, either upward or downward, from the device's mid-plane.

1.4.2 Mega-Amp Spherical Tokamak-Upgrade (MAST-U)

MAST-U is the successor experiment to MAST. One of the most notable features of MAST-U is its new Super-X divertor, which significantly reduces heat loads on the divertor walls.

.	JET	ITER	MAST	MAST-U
Major radius (m)	3	6.2	0.85	0.85
Minor radius (m)	1	2	0.65	0.65
Plasma Volume (m^3)	125	830	8	8
Plasma current (MA)	6	15	1.3	2
Magnetic field	3.4	5.3	0.52	0.75
Pulse length (s)	10	400	0.6	5

Table 1.1: Main parameters for the JET, ITER, MAST and MAST-U tokamaks

This feature provides valuable insights for the design of future tokamak-based fusion power plants, although it is not the primary focus of this dissertation. The upgraded device retains the same major and minor radii as MAST but includes enhancements to key parameters such as NBI power, plasma current, magnetic field strength, and pulse duration. A comparison of these parameters between MAST and MAST-U is provided in Table 1.1. MAST-U began operations in 2020 and continues to conduct experiments to this day.

MAST-U features a slightly modified NBI setup compared to MAST, offering greater experimental flexibility. While the SS beam maintains the same injection angle and toroidal location as in MAST, the SW beam in MAST-U is vertically displaced by approximately 10 cm. This modification allows for a much broader range of fast-ion distribution and density control than was previously possible [82] [45].

1.5 A brief introduction to instabilities and their associated fast ion transport in Tokamaks

While tokamaks offer significant advantages, certain challenges arise when considering their potential as candidates for fusion energy production. These challenges can hinder the success of tokamak-based plants if not thoroughly studied and addressed before implementation.

One of the major challenges is the instabilities driven by fast-ions. The associated energetic particle (EP) transport and losses caused by these instabilities can degrade fusion performance and significantly reduce the lifespan of plasma-facing components in tokamaks. Below is a brief overview of some of these fast ion-driven instabilities, providing context before this dissertation focuses on specific cases.

These instabilities can interact, sometimes non-linearly, with energetic particles, causing their transport and loss. Power transfer between EPs (e.g., fast ions) and plasma modes occurs when the average of the dot product of the particle velocity vectors and the electric field vector of the mode is nonzero. An important part of this power transfer is facilitated in curved magnetic fields due to the drift velocity and the transverse electric field. The energy exchange is typically analyzed over several orbital cycles, as the mode growth rates are much smaller than the mode frequencies. This transfer is most effective under resonance conditions, where the sum of integer multiples of the EP toroidal and poloidal orbit frequencies equals the mode frequency.

1.5.1 Alfvén instabilities

Shear Alfvén waves driven by energetic particles are commonly observed in both laboratory and natural plasmas, such as in the ionospheric plasma of the upper atmosphere. Shear Alfvén waves have frequencies well below the $\vec{k} c_S$, where $\vec{k}$ is the wavenumber vector of the mode with toroidal ($\approx n/R$) and poloidal ($\approx m/a$) components, and $c_S = \sqrt{(2\gamma_e T_e + \gamma_i T_i)/(m_i)}$ is the deuterium/electron plasma's speed of sound, with γ_e and γ_i to be adiabatic indices for electrons and ions, respectively. These waves are a branch of Alfvén waves and represent transverse, low-frequency electromagnetic waves propagating along the magnetic field direction. In cylindrical plasma devices, shear Alfvén waves in the continuum spectrum are rarely destabilized because they are subject to strong damping due to phase mixing with other Alfvén waves, causing the energy to leak away along field lines [59] [93]. However, in toroidal plasmas, the toroidicity breaks poloidal symmetry, creat-

ing coupling between the different cylindrical modes, leading to the formation of gaps in the Alfvén continuum. These gaps allow weakly damped discrete eigenmodes to emerge at specific frequencies, locations, and mode numbers [114].

The destabilization of these Alfvén eigenmodes (AEs) is driven by two main factors: the existence of continuum gaps and the free energy present in the EP distribution function. In principle, various types of AEs can be distinguished based on their specific characteristics. For instance, reversed-shear AEs (RSAEs) are localized at the minimum safety factor (q_{min}) value, while beta-induced AEs (BAEs) arise due to plasma acoustic compressibility effects. Global AEs (GAEs) occur because of a minimum in the Alfvén continuum, and toroidicity-induced AEs (TAEs) are induced by the toroidal geometry of the plasma. However, due to the aspect ratio and the typically weakly shear safety factor profiles of the plasma in spherical tokamaks, the actual Alfvén eigenmodes can sometimes be linear superpositions of these local theory Alfvénic modes. These modes had been found in high beta plasmas in spherical tokamaks, such as in the National Spherical Torus Experiment [24]. Modes with TAE-like characteristics are studied in this project, hence TAEs are discussed in detail in this dissertation.

1.5.2 MHD instabilities

In addition to shear Alfvén instabilities, various magnetohydrodynamic (MHD) instabilities can also contribute to EP transport and losses in tokamaks. Similar to AEs, toroidicity facilitates the coupling of modes, leading to the existence of some ballooning MHD instabilities. However, these instabilities are unrelated to the Alfvén continuum and are instead driven by plasma current or pressure gradients, which can arise in both ohmic- and NBI-heated scenarios.

Examples of MHD instabilities include kink modes, which are driven by current and pressure gradients in high- β plasmas. Tearing modes result from resistive instabilities, while sawtooth oscillations involve periodic redistributions of plasma energy in the core. Besides

their associated thermal species transport and plasma disruptions, these instabilities can also lead to fast ion transport and losses, degrading fusion performance and potentially damaging plasma-facing components [109][82][75].

MHD instabilities exist in different physical regimes and require various models for accurate characterization. Some instabilities are sensitive to the plasma’s resistivity and require resistive MHD models. Others, like ideal kink modes, can often be described by ideal MHD models. However, while the MHD model plays a role, certain instabilities, such as Fishbone (FB) modes, are strongly influenced by EP dynamics. Characteristics of these instabilities are determined jointly by the plasma fluid and fast-ion populations, and cannot treat fast-ion properties as small perturbations in MHD limits. Hence non-perturbative theory are required to describe these instabilities. This brings challenges to study them to mitigate their associated fast ion transport and losses. It is important to note that this dissertation primarily focuses on FB instabilities and does not cover all the mentioned MHD modes in detail.

1.6 Overview of the Dissertation

This dissertation presents results on the measurements and simulations of TAEs and FB instabilities in MAST and MAST-U, using multiple diagnostic tools. The primary goal is to enhance understanding of these low-frequency, NBI-induced instabilities to mitigate their impacts on fusion performance and the potential damage caused by fast ion transport to wall components in future tokamak-based fusion devices.

Measurements from various diagnostics are utilized to investigate the characteristics of these instabilities. These data support plasma phenomenology studies, verify and validate simulations, and serve as inputs for reduced-model-based calculations. The spectral characteristics of TAEs and FBs in MAST and MAST-U are primarily captured by the outboard Mirnov coil array, which offers high sensitivity and the ability to measure magnetic fluctua-

tions at frequencies up to the megahertz range. Spatial structures, representing the spatial patterns of fluctuations associated with these instabilities, are measured by tangential soft X-ray cameras in MAST. Beam Emission Spectroscopy (BES) was used to measure mode spatial structures during those campaigns, not only because the tangential soft X-ray cameras were not operational during the early physics campaigns in MAST-U, but the BES diagnostic offers full coverage of all poloidal angles in the core, which is useful for inferring poloidal characteristics of modes and it obtains first-of-its-kind measurements in a spherical tokamak. Additional diagnostics, such as Thomson scattering, charge exchange recombination spectroscopy, neutron emission fission chambers, and motional Stark effect diagnostics, are employed to measure various plasma equilibrium parameters.

A linear gyrokinetic (GK) model is verified and validated using measurements of TAEs in MAST. While ideal MHD models are commonly used to describe TAEs in conventional and spherical tokamaks (STs), they exclude fast ions and the kinetic effects of thermal ions, and hence cannot predict some of the TAE characteristics such as the growth rate of the mode that are significantly influenced by the fast ions. Especially, these effects are more pronounced due to the relatively high fast-ion-to-thermal-ion beta ratio in STs. Therefore, a kinetic description of both the thermal plasma and beam fast ions is critical for assessing the stability of these NBI-induced modes. Global gyrokinetic codes, such as the Gyrokinetic Toroidal Code (GTC) [66], provide promising capabilities for cross-scale integrated simulations of fast-particle-driven modes [67]. However, unique features of STs, such as the steep field gradient on the high-field side and the high fast-ion-to-thermal-ion beta ratio, have not been extensively tested in GK simulations. This dissertation addresses this gap by validating the first global GK simulations of TAEs in MAST.

The dissertation also investigates FBs in MAST and MAST-U, focusing on their phenomenology. Historically, FBs have received less attention regarding their impact on bulk plasma and flux surfaces, with most studies emphasizing their role in fast-ion transport. However, observations of FB-like instabilities in MAST and MAST-U challenge this limited

perspective. Experimental investigations in MAST-U plasmas, where the minimum safety factor (q_{min}) approaches unity, reveal significant differences in FB behavior. While many FBs in MAST and MAST-U exhibit characteristics similar to those observed in other tokamaks, measurements from Mirnov coil arrays, soft X-ray emission (in MAST), and BES (in MAST-U) reveal substantial distortions in spatial structures during plasma discharges for some FB-like instabilities. These events are associated with pronounced fast ion transport and marked shifts in the magnetic axis, along with crashes in thermal electron and ion temperature profiles. Such phenomena indicate rapid and significant rearrangements of flux surfaces, highlighting deviations from traditional FB models and previous observations.

Beyond validating physics models or simulations, measurements of TAEs and FBs in MAST and MAST-U can also be used as inputs for reduced fast-ion transport model calculations in various ways. For instance, the radial mode structures and poloidal harmonics obtained from both ideal MHD and GK simulations of TAEs in MAST are used as inputs for ORBIT-Kick [77] [78], a reduced energetic particle transport model. However, linear models cannot predict saturated mode amplitudes or chirping (rapid frequency variations). ORBIT-Kick accommodates these measured amplitudes and chirping frequencies, integrating them with simulation results for fast-ion transport calculations. For FBs, where self-consistent simulations are less common than for TAEs, radial mode structure measurements guide the construction of analytical mode structures as inputs for ORBIT-Kick. This approach facilitates studies of FB-associated fast-ion transport in the absence of fully self-consistent simulations, reducing computational complexity and resource demands.

1.6.1 Outline of the dissertation

This dissertation is divided into seven chapters.

Chapter 1 (this chapter) is the introductory chapter.

Chapter 2 introduces fast ion resonances and orbit topology.

Chapter 3 introduces MHD models for TAEs, FBs, and discusses the limitations of these models

Chapter 4 introduces the data analysis methodology used throughout the dissertation.

Chapter 5 presents the results and analysis on the work TAE in MAST, as well as using them for verification and validation of a global gyrokinetic simulation.

Chapter 6 presents the results on guiding center based reduced fast ion transport calculations for TAEs MAST.

Chapter 7 presents the results and analysis on fishbone instabilities and their impacts on core confinement in MAST and MAST-U

Chapter 8 presents the results on guiding center based reduced fast ion transport calculations for FBs in MAST-U.

Chapter 9 provides conclusions and discusses the possible future efforts for the projects presented in this dissertation.

CHAPTER 2

Fast ion resonant condition and orbit topology

Low-frequency modes, such as TAEs and FBs, can induce fast-ion transport, displacing them from their original orbits by mode-particle resonances. For a resonance to occur, a particle must return periodically to experience the same perturbation. This requirement leads to a discrete poloidal transit time [104]:

$$(n\omega_\zeta - \omega)T = 2\pi p ; \omega_\theta T = 2\pi l \quad (2.1)$$

where ω is the mode frequency, ω_ζ and ω_θ are the toroidal and poloidal frequencies, respectively, T is the poloidal transit time interval, and p and l are integers. Resonance occurs when

$$p\omega_\theta - nl\omega_\zeta + l\omega = 0 \quad (2.2)$$

It is important to note that the resonance condition is defined by the integers (n, p, l) , but the poloidal resonance periodicity is not strictly determined by the poloidal mode harmonic m due to drift motion effects [104]. Additionally, the shape of the fast ion orbits are related to the quantity $-p$, for example, $p \leq -1$ corresponds to banana-like orbits.

Fast ion orbits are described by the conservation of constants of motion. Unlike thermal species, energetic particles such as fast ions traverse different magnetic surfaces due to their large velocities. Hence their orbits follow the conservation of constants of motion (if collisions

are neglected), which define the topological boundaries of different orbit types instead of magnetic flux surfaces. Perturbations can modify the constants of motion of fast ions, leading to cross-field transport.

The guiding center approximation is commonly used to calculate the orbits of fast ions. While full-orbit calculations based on the Lorentz force can, in principle, provide exact solutions, they are computationally expensive and prone to numerical noise. For low-frequency perturbations, it is sufficient to use the guiding center approximation, which averages out the fast cyclotron motion to simplify orbit calculations [102]. The Hamiltonian in this approximation is given by :

$$H = \frac{1}{2}mv_{\parallel}^2 + \mu B(\psi, \theta) + \Phi(\psi, \theta, \zeta), \quad (2.3)$$

where Φ is the electrostatic potential. The equations of motion and momenta are derived from the Hamiltonian as:

$$\dot{\theta} = \frac{\partial H}{\partial P_{\theta}}, \quad \dot{P}_{\theta} = -\frac{\partial H}{\partial \theta} \quad (2.4)$$

$$\dot{\zeta} = \frac{\partial H}{\partial P_{\zeta}}, \quad \dot{P}_{\zeta} = -\frac{\partial H}{\partial \zeta}. \quad (2.5)$$

Since fast ions conserve their constants of motion in equilibrium, we have $\mu = mv_{\perp}^2/(2B)$. Using this and the Hamiltonian, the canonical toroidal angular momentum is defined as:

$$P_{\zeta} = \frac{v_{\parallel}}{B}g - \psi_p, \quad (2.6)$$

where ψ_p is the poloidal flux variable, and $g = RB_t$. This is used to define different types of orbits. Boundaries can be calculated to separate the regions of different types of orbits in the phase space ($\mu B_0/E$ and P_{ζ}/Ψ_W), as shown in an example in Fig.2.2.

The conservation of motion can be defined in terms of the canonical angular momentum, energy $\mathcal{E}$, toroidal mode number n , and mode frequency $\omega = 2\pi f$:

$$\omega P_{\zeta} - n\mathcal{E} = \text{constant}. \quad (2.7)$$

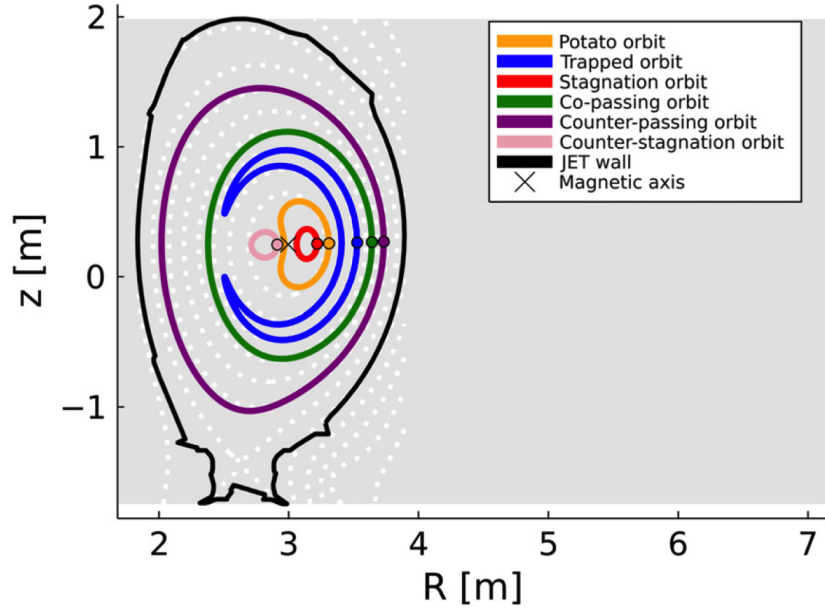


Figure 2.1: Example of the poloidal projections of the guiding-centre trajectories of JET fast deuterium orbits, from [56]

This has been shown in the reference [102]. The variations in $\mathcal{E}$ and P_ζ due to a single mode are related to its n and ω by:

$$\frac{\Delta P_\zeta}{\Delta \mathcal{E}} = \frac{n}{\omega}, \quad (2.8)$$

which constrains the possible trajectories of the particles. The perturbations in P_ζ and $\mathcal{E}$ cause transport effects that alter the trajectories and orbit types of fast ions and, in some cases, lead to their loss from the plasma.

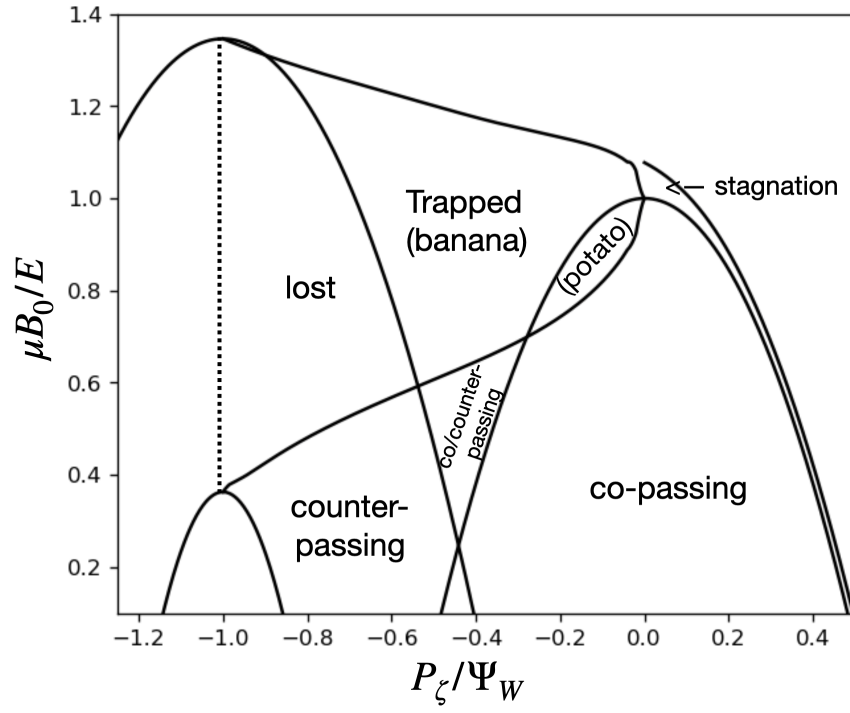


Figure 2.2: Example of the orbit topology boundaries for different types of MAST fast deuterium orbits in the phase space at a fixed energy.

CHAPTER 3

MHD frameworks for TAEs and FBs

The origins of TAEs and FB instabilities in tokamaks were briefly introduced in Chapter 1. In this chapter, the frameworks that describe some of the characteristics of these instabilities are presented. It begins with an introduction to the drift-kinetic model, which is the framework for understanding the interaction of energetic particles with MHD modes. Following this, the reduction of the drift-kinetic model to the ideal MHD framework is presented. The chapter then discusses the details on how TAEs, internal kink, and infernal modes emerge in the ideal MHD limit. The chapter describes internal kink or infernal modes are destabilized by beam induced fast ions and become FBs. These descriptions are in MHD and conventional limits, hence they might not be accurate for high fast ion beta spherical tokamak. The shortcomings of these models are discussed in this chapter as well.

3.1 Drift-kinetic model

The drift-kinetic model provides a framework for describing the across-field transport of particles, momentum, and heat in tokamaks. The time evolution of the distribution function of a collisional plasma is governed by the Fokker-Planck equation:

$$(\partial_t + \dot{\mathbf{X}} \cdot \nabla + (Z_i e / m_i)(\mathbf{E} + \mathbf{v} \times \mathbf{B}) \cdot \partial_{v_{\parallel}})[f_i(\mathbf{X}, v, t)] = C(f_i(\mathbf{X}, v, t)) \quad (3.1)$$

Here, i is the label for plasma's species, $f_i(\mathbf{X}, v, t)$ is the 6+1 dimensional distribution function for i species, $\mathbf{X}$ represents location's coordinates, v denotes the velocity, Z_i is the effective charge number for the i species, $\mathbf{E}$ is the electric field, $\mathbf{B}$ is the magnetic field, and

$C()$ is the collision operator.

The Fokker-Planck equation is derived by combining Maxwell's equations and the Boltzmann equation into the Vlasov equation, then incorporating the Landau kinetic equation to account for collisions in plasma. While this formulation is comprehensive, it includes variables and dimensions unnecessary for describing low-frequency instabilities in tokamak plasmas. To simplify, the equation can be reduced for such scenarios. The time and length scales for these instabilities are within the limits of $t > 2\pi/\Omega_i$ and $L > v_\perp/\Omega_i$, respectively, where $\Omega_i = ZeB/m_i$ is the cyclotron frequency for the i species, and $v_\perp$ is the velocity component perpendicular to the field. Under these constraints, the Fokker-Planck equation can be reduced to the drift-kinetic equation:

$$\partial_t f_i + v_\parallel \mathbf{b} \cdot \nabla f_i + (v_E + v_D) \cdot \nabla f_i + \nabla \cdot (v_p f_i) + (Z_i e/m_i)(v_\parallel E_\parallel + v_D \cdot \mathbf{E}) \partial_\parallel f_i = C(f_i) \quad (3.2)$$

Here, $v_\parallel$ is the velocity component parallel to the field, v_E is the $E \times B$ drift, v_D is the gradient and curvature drift, v_p is the polarization drift, and $\mathbf{b} = \mathbf{B}/B_0$ where B_0 is the equilibrium magnetic field magnitude at the magnetic axis. This equation is sufficient to describe the interaction of energetic particles with low-frequency, long-wavelength MHD modes.

Further simplifications can be achieved by linearizing the drift-kinetic equation (Eq. 3.2). This approach reduces the complexity of the problem and allows for analytically solvable results. Linear MHD frameworks assume that instabilities are not strong enough to significantly alter the equilibrium distribution on time scales shorter than the instability's frequency. Therefore, the distribution function f_i can be approximated as the sum of an equilibrium part f_{i0} and a small sinusoidal perturbation δf :

$$f_i \approx f_{i0} + \delta f, \quad (3.3)$$

when $\delta f \ll f_{i0}$. The toroidal and poloidal periodicities of tokamak plasmas allow the

Fourier decomposition of δf into a discrete spectrum:

$$\delta f \propto \sum_{m,n} e^{i(n\zeta - m\theta - \omega t)}, \quad (3.4)$$

where ζ and θ are toroidal and poloidal angle in Boozer coordinates, respectively, and (n,m) are representing the toroidal and poloidal mode numbers. From this point onward, Einstein summation notation is adopted for equations containing summations of m and n , omitting $\sum_{m,n}$ for simplicity.

3.2 Single-fluid ideal MHD model

The Vlasov equation (Eq. 3.1 without collisionality) and Maxwell's equations can be combined to derive the single-fluid ideal magnetohydrodynamic (MHD) model under specific limits. Several key assumptions are required for the validity of the ideal MHD model.

First, the length scale of the system must be much larger than all length scales characterizing the microscopic dynamics of the particles in the plasma, i.e., $L_{MHD} \gg \lambda_L$, where L_{MHD} is the MHD length scale and r_L is the gyroradius (or Larmor radius) of the particles. Additionally, L_{MHD} must be much larger than the Debye length, $\lambda_D = \sqrt{2\epsilon_0 k_B T_i / (n_i e^2)}$, ensuring that particles do not experience the Coulomb interactions of other particles directly. This condition allows the quasi-neutrality approximation in MHD. Similarly, the time scale of the system must be much larger than the characteristic time scales of the microscopic particle dynamics for the ideal MHD model to remain applicable.

The ideal MHD formalism assumes a thermal bulk plasma consisting of two species—electrons and ions—treated as a single fluid. Since the electron mass is negligible compared to the ion mass, the fluid's mass density and velocity are determined solely by the ions, i.e., $\rho_f = \rho_i$ and $\mathbf{v} = \mathbf{v}_i$. However, the scalar pressure of the fluid includes contributions from both species, $p = p_i + p_e$. Under these assumptions, the fundamental continuity equations [33] are given

by:

$$\partial_t \rho + \nabla \cdot (\rho \mathbf{v}) = 0 \quad (3.5)$$

$$\rho(\partial_t \mathbf{v} + (\mathbf{v} \cdot \nabla) \mathbf{v}) = \mathbf{J} \times \mathbf{B} - \nabla p \quad (3.6)$$

$$d_t \left(\frac{p}{\rho_f^\gamma} \right) = 0 \quad (3.7)$$

Here, $\gamma = 5/3$ is the specific heat ratio, $\mathbf{J}$ is the current density, and $d_t = \partial_t + \mathbf{v} \cdot \nabla$ represents the material derivative.

The single-fluid assumption also modifies Ohm's law and consequently affects Maxwell's equations. In the ideal MHD regime, resistive effects are negligible compared to other terms. Thus, Ohm's law simplifies to:

$$\mathbf{E} = -\mathbf{v} \times \mathbf{B} \quad (3.8)$$

This leads to the Maxwell's equations in the ideal MHD limit:

$$\nabla \cdot \mathbf{B} = 0 \quad (3.9)$$

$$\nabla \times \mathbf{B} = \mu_0 \mathbf{J} \quad (3.10)$$

$$\partial_t \mathbf{B} + \nabla \times \mathbf{E} = 0 \quad (3.11)$$

These equations collectively describe the dynamics of single-fluid ideal MHD [33], given the limit conditions are satisfied.

3.3 Ideal MHD descriptions for TAE

Some of the characteristics of TAEs can be derived in the ideal MHD limits and the tokamak boundary conditions of the drift-kinetic model. As previously noted in chapter 1, TAEs in

the ideal MHD regime exist within an incompressible plasma with low β , where the plasma pressure is small compared to the magnetic pressure. The frequency of a shear Alfvén wave is defined as $\omega = k_{\parallel}(r)c_A$, where $k_{\parallel}$ is the parallel wave number, the Alfvén velocity $c_A = B/\sqrt{\mu_0\rho}$ with B is the magnetic flux density, μ_0 to be the permeability of the vacuum, and ρ to be the plasma's mass density. Using a Fourier transform, $\mathbf{b} \cdot \nabla$ can be expressed as $ik_{\parallel}$, where the periodic boundary conditions of a tokamak relate $k_{\parallel}$ to the toroidal and poloidal mode numbers (n and m):

$$\mathbf{b} \cdot \nabla = \frac{b_{\theta}}{r} \partial_{\theta} + \frac{b_{\zeta}}{R} \partial_{\zeta} = -\frac{imb_{\theta}}{r} + \frac{inb_{\zeta}}{R} = ik_{\parallel} \quad (3.12)$$

. Here, b_{θ} and b_{ζ} are the poloidal and toroidal components of $\mathbf{b}$, respectively. Although ST is the main focus of this dissertation, it is convenient to use conventional aspect ratio limits of $b_{\zeta} \approx 1$ and $b_{\theta} \approx r/(Rq)$, where q is the safety factor. This yields the following expression for the parallel wave number for specific n, m and q:

$$k_{\parallel,m}(r) = \frac{n - m/q(r)}{R} \quad (3.13)$$

.

Several conditions allow further simplification to the drift-kinetic equation (Eq. 3.2). TAEs are shear Alfvén modes, implying no compressional components $B_{\parallel}$ and thus $\mathbf{A}_{\perp} = 0$. The parallel ($E_{\parallel}$) and perpendicular ($E_{\perp}$) components of the electric field can be written as:

$$E_{\parallel} = -\nabla_{\parallel}\phi - \partial_t A_{\parallel} ; E_{\perp} = -\nabla_{\perp}\phi \quad (3.14)$$

. However, $E_{\parallel}$ can be eliminated in the ideal MHD limit under the quasi-neutrality and single-fluid assumptions. Then the drift-kinetic equation can be linearized for small amplitude modes. This results in the Hain-Lüst equation for modes with poloidal mode number m:

$$H_m = \partial_r(r^3(\frac{\omega^2}{c_A^2} - k_{\parallel,m}^2)\partial_r E_m) - (m^2 - 1)r(\frac{\omega^2}{c_A^2} - k_{\parallel,m}^2)E_m + (\frac{\omega^2}{c_A^2})'r^2 E_m = 0 \quad (3.15)$$

. This equation, derived by retaining only the leading a/R terms from the Taylor series expansion of Eq. 3.2 and summing over all species, describes a continuous eigenvalue spectrum $\omega^2 = k_{\parallel,m}^2 c_A^2$, as shown in Fig.3.1(a). However, because $k_{\parallel,m}$ and c_A vary radially, the

eigenvalues form a continuous spectrum across the plasma minor radius. At resonant layers ($r = r_0$) where $\omega = k_{||,m}(r_0)c_A(r_0)$, waves are absorbed, leading to damping.

While the Hain-Lüst equation explains AEs in cylindrical boundary conditions, it neglects the coupling of adjacent poloidal harmonics (E_{m+1} and E_m), critical for describing TAEs in tokamaks. The poloidal coupling terms containing (E_{m+1} in the m order, and E_m in the $m + 1$ order) can be put back to the Hain-Lüst equation to describe TAEs:

$$H_m + \left(\frac{3a}{2R}\partial_r\left(\frac{\omega^2 r^4}{c_A^2 a}\right)\partial_r\right)E_{m+1} = 0 ; H_{m+1} + \left(\frac{3a}{2R}\partial_r\left(\frac{\omega^2 r^4}{c_A^2 a}\right)\partial_r\right)E_m = 0 \quad (3.16)$$

. As a result, the new spectra resemble those from the cylindrical spectra, except close to their crossing points where the coupling creates spectral gaps near crossing points ($k_{||,m} = -k_{||,m+1}$), as shown in Fig.3.1(b). Within these gaps, the continuum damping is weakened, allowing modes such as TAEs to exist. The location and frequency of TAEs in an ideal MHD description with conventional aspect ratio is given by:

$$q(r) = \frac{m + 1/2}{n} ; \omega \approx \frac{c_A}{2qR} \quad (3.17)$$

. Energetic particles, such as fast ions generated by NBI, can excite TAEs at these locations due to the weakly damped environment in the continuum gaps.

3.3.1 Limitations of the ideal MHD model for TAEs in STs

The ideal MHD description for TAEs has limitations. In conventional tokamaks, the width of the continuum gap is often narrow, limiting the number of mode combinations that can exist. In high- β , high NBI power spherical tokamaks such as MAST and MAST-U, the gap widths are significantly larger, enabling multiple mode combinations beyond the typical m and $m + 1$ pairs. Furthermore, ideal MHD excludes collisional and kinetic effects, which influence the dispersion relation and TAE characteristics. The exclusion of collisionality and low temperature assumptions cause inaccurate expression for the TAE dispersion relation, as seen in Eq.3.13 compared to the expression presented in [36].

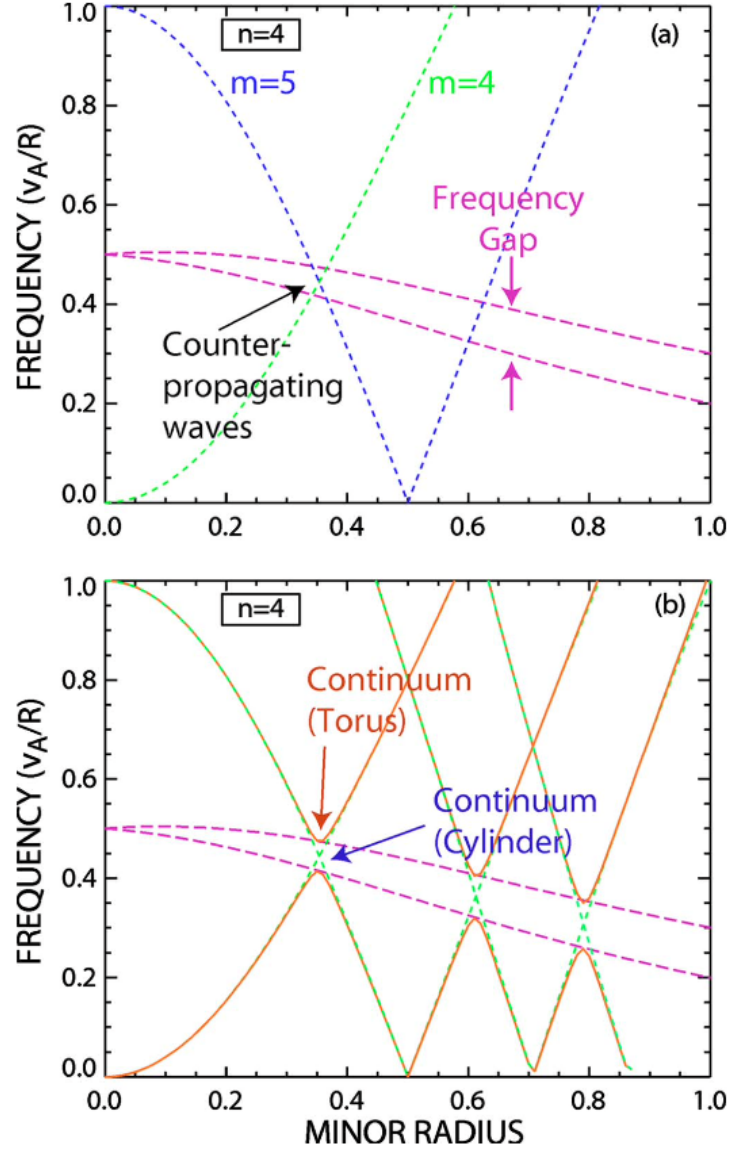


Figure 3.1: (a) Dispersion relation of two waves with toroidal mode number $n=4$ and poloidal harmonics of $m=4$ and $m=5$ in a plasma with a q profile that increases monotonically with radius. Frequencies are plotted as positive for both signs of $k_{\parallel}$. The frequency gap increases with radius because r/R increases, which increases the modulation of N . The two waves are counter-propagating in the frequency gap. (b) Dispersion relation with (solid) and without (dashed) toroidal coupling of the waves. The $m=6$ and $m=7$ waves are also shown and intersect at larger radii. The figures are based on [34] and from [47].

In addition, the ideal MHD description for TAEs only predicts gap locations for ranges of n and m , but it cannot predict the actual mode frequency and mode numbers of fast-ion-driven TAEs that will be unstable in experiments without the considerations of fast ion properties and the mode-fast ion interaction. For example, changes in beam injection geometry alter the fast ion birth pitch-energy distribution, which are found to have effects on TAE mode characteristics in other tokamaks (for example, in DIII-D [94] [96]). These factors underscore the need for developing and testing of sophisticated models to better capture the behavior of TAEs.

3.4 MHD-based models for Internal kink, infernal, and Fishbone instabilities

Kink modes might have similar mode number as TAEs and also exhibit low frequency, but they are driven by fundamentally different mechanisms. This section provides a brief overview of internal kink modes, which occur when the minimum safety factor (q_{min}) falls below unity, infernal modes that arise when $q_{min} > 1$, and the excitation of Fishbone (FB) instabilities from these modes.

3.4.1 Internal kink mode

The internal kink mode is one of many MHD instabilities observed in tokamaks. It is characterized by internal plasma perturbations that do not deform the plasma boundary, giving rise to its name. A fundamental requirement for the internal kink mode is the presence of a $q = 1$ surface, where resonance occurs at $q = m/n$. This implies that an $n = m = 1$ kink mode resonates at $q = 1$, with its mode spectrum dominated by the $m = 1$ harmonic.

The structure and dispersion relation of the $n = m = 1$ internal kink mode can be derived from the linearized ideal MHD equations. Assuming small-amplitude perturbations that evolve slowly relative to the equilibrium state, any field quantity (denoted as Ξ) can be

decomposed into equilibrium and perturbed components as $\Xi(\mathbf{X}, t) = \Xi_0(\mathbf{X}) + \Xi_1(\mathbf{X})e^{-i\omega t}$, similar to Eq.3.4. The perturbation displaces the plasma from equilibrium, described by the MHD displacement vector $\xi(\mathbf{X}, t)$, where the plasma velocity is given by $\dot{\xi}(\mathbf{X}, t) = \mathbf{v}(\mathbf{X}, t)$. Using this formulation, the MHD equations can be rewritten as [33]:

$$p_1 = -\xi \cdot \nabla p - \gamma p \nabla \cdot \xi \quad (3.18)$$

$$\mathbf{B}_1 = \nabla \times (\xi \times \mathbf{B}) \quad (3.19)$$

$$-\omega^2 \rho_f \xi = \mathbf{F}(\xi), \quad (3.20)$$

where $\mathbf{F}(\xi)$ is defined as:

$$\mathbf{F}(\xi) = \frac{1}{\mu_0} ((\nabla \times \mathbf{B}_1) \times \mathbf{B} + (\nabla \times \mathbf{B}) \times \mathbf{B}_1) + \nabla(\xi \cdot \nabla p + \gamma p \nabla \cdot \xi) \quad (3.21)$$

. As in the case for TAE discussed in section 3.3, ξ serves as the eigenvector of the system, with ω^2 as the eigenvalue, which can be determined once ξ is defined.

The energy principle [33] provides a method to determine the global mode structure of ξ analytically. The system's total energy consists of equilibrium and perturbed contributions:

$$\mathcal{E}(\xi, \xi^*) = K(\xi, \xi^*) - \delta W(\xi, \xi^*) = 0, \quad (3.22)$$

where K and δW are obtained by integrating ξ , or the product of ξ and $\mathbf{F}(\xi, \xi^*)$ over the plasma volume. The detailed expression of K is given in [33], while the eigenvalue ω^2 depends on δW :

$$\omega^2 = \frac{\delta W}{(1/2) \int_V dV |\xi|^2} \quad (3.23)$$

. This implies the frequency becomes imaginary when $\delta W \leq 0$, , meaning that the mode grows exponentially and hence the plasma becomes unstable. The terms contributing to δW can provide insight into stabilizing and destabilizing effects:

$$\delta W = \frac{1}{2} \int_V dV \left(\underbrace{\frac{|\tilde{B}^2|}{\mu_0}}_1 + \underbrace{\Gamma p_0 |\nabla \cdot \xi|^2}_2 + \underbrace{(\xi \cdot \nabla p_0) \nabla \cdot \xi}_3 - \underbrace{\xi \cdot J_0 \times \nabla \times (\xi \times \mathbf{B}_0)}_4 \right), \quad (3.24)$$

where Γ is a field line compressional coefficient. Here, terms 1 and 2 represent the field line bending and compression effects, respectively, as they are always positive. In contrast, terms 3 and 4, representing the pressure and current drives respectively, can be negative and become destabilizing effects.

A nearly cylindrical plasma approximation is used to obtain the radial structure and growth rate of the $n = m = 1$ internal kink mode. While the application of the formalism on the toroidal curvature has been done in [18], the cylindrical assumption is more widely used for its simplicity. The plasma is assumed to be in a cylinder of length $l = 2\pi R_0$. The radial structure of the displacement is periodic ($\xi(\mathbf{X}) \propto e^{i(n\zeta - m\theta)}$) in ζ and θ because of the boundary conditions. Further assumptions include straight-field-line coordinates, a strictly positive safety factor ($|q| > 0$), and incompressibility.

The radial mode structure of $n = m = 1$ can be obtained by minimized energy method. The detailed derivations of this method are beyond the focus of this dissertation. Those who are interested can find the derivations in [33]. The descriptions of each step, however, are physically meaningful and should be stated here. To summarize, the displacement vector $\xi(\mathbf{X})$ needs to be written in terms of ξ_r and $\xi_{||}$ representing the radial and parallel components, respectively. The δW is minimized against $\xi_{||}$ due to the incompressibility of the plasma, satisfying the following:

$$\nabla \cdot \xi = 0 \quad (3.25)$$

. The final step involves putting back ξ to δW to solve for frequency and the dispersion relation of the mode.

The radial eigenfunction for the $m = 1$ harmonic ($\xi_{r,m=1}$) found by minimizing Eq.3.24 under the condition in Eq.3.25 is:

$$\xi_{r,m=1} = \xi_0 \left(\frac{1}{2} - \frac{1}{\pi} \arctan \left(\frac{nsx}{\gamma\tau_A r_s} \right) \right), \quad (3.26)$$

where $s = \frac{r}{q} \frac{dq}{dr}$ is the magnetic shear in terms of radial gradient of q , $n = 1$ is the toroidal mode number, $\gamma\tau_A$ is the mode growth rate, $x = r - r_s$, and r_s is the minor radius of the $q =$

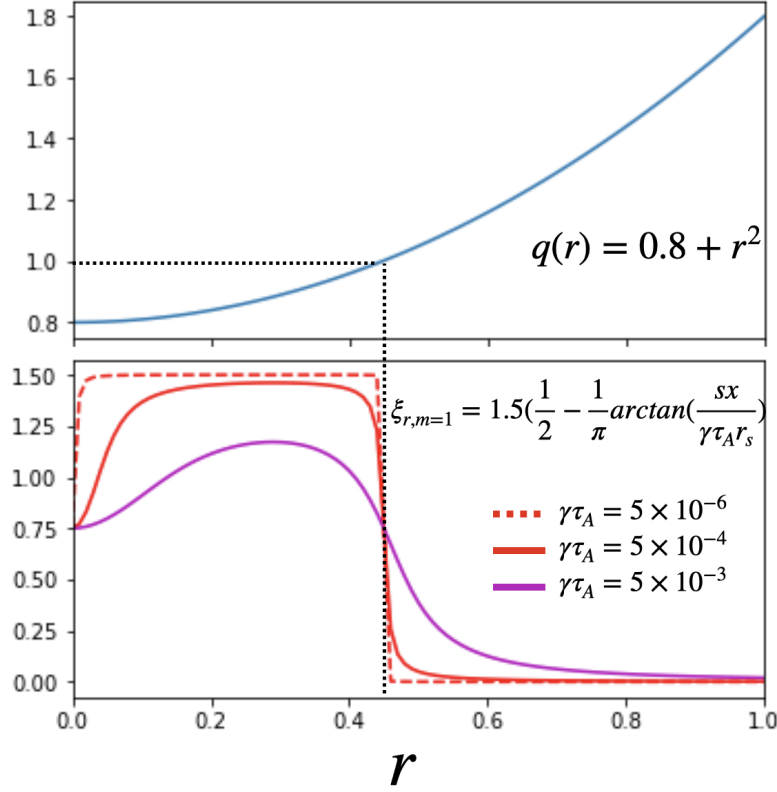


Figure 3.2: Comparison of a hypothetical $q(r) = 0.8 + r^2$ with $r_s = 0.45$ and the corresponded internal kink radial mode structures with $\xi_0 = 1.5$ and three different $\gamma\tau_A$

1 surface. This expression describes an internal kink mode structure resembling a smoothed Heaviside step function, peaking near the magnetic axis, reaching its minimum near the last closed flux surface (LCFS), and exhibiting the steepest gradient at the $q = 1$ surface. The smoothness of the radial mode structure depends on the growth rate of the mode, as demonstrated in Fig.3.2 in a case of a hypothetical q profile ($q(r) = 0.8 + r^2$) with $r_s = 0.45$ and several different $\gamma\tau_A$.

3.4.2 Infernal mode

Although internal kink modes and infernal modes share some similarities, their underlying mechanisms and assumptions differ. Unlike internal kink modes, infernal modes do not

require the presence of a $q = 1$ surface or resonance at $q = m/n$. Specifically, the $n = m = 1$ infernal mode is also referred to as the quasi-interchange mode. In plasmas with a flat q -profile, the field line bending stabilization effect (term 1 in Eq. 3.24) is weakened, allowing global modes to be destabilized by strong pressure gradients in low-shear ($dq/dr \approx 0$) regions. As a result, an $n = m = 1$ infernal (quasi-interchange) mode can exist at locations where q is slightly above unity and pressure gradients are steep.

Although ideal MHD assumptions still hold, the stability analysis of the infernal mode differs from that of the internal kink mode. Instead of following the energy principle approach, infernal mode stability is determined by solving the MHD eigenmode equation, as detailed in [41]. For this analysis, the safety factor q profile is divided into two regions: a low-shear region ($r < r_0$): $dq/dr = 0$, meaning that q is nearly constant and approaches (but does not reach) $q = 1$; a high-shear region ($r > r_0$): The q -profile steepens.

In the high-shear region, the displacement amplitude of higher harmonics ($m > 1$) is small, approximately $(\gamma/\omega_A)\xi_{m=1}$, where γ is the mode growth rate and ω_A is the Alfvén frequency. Consequently, the radial displacement of the dominant $m = 1$ mode in this region is negligible. While $m > 1$ harmonics can have finite amplitudes, their derivation is outside the scope of this work, as the $m = 1$ component remains dominant.

In the low-shear region, the $m = 1$ mode follows the dispersion relation:

$$\frac{1}{r} \frac{d}{dr} \left[r^3 \mathcal{Q} \frac{d\xi_{r,m}}{dr} \right] + \left[(1-m)^2 \mathcal{Q} + \frac{r\alpha}{R_0} \left(\frac{n^2}{m^2} - 1 \right) - \frac{\alpha^2}{2} \right] \xi_{r,m} + \frac{\alpha}{2} \left(\frac{r^m L_+}{+m} \right) = 0, \quad (3.27)$$

where

$$\frac{L_+}{1+m} = \frac{1+m}{r_0^{2+2m}} \left(\frac{2+m+\hat{c}}{m-\hat{c}} \right) \int_0^{r_0} r^{1+m} \alpha \xi_{r,m} dr, \quad (3.28)$$

$$\hat{c} = \left(\frac{r d\xi_{r,m+1}/dr}{\xi_{r,m}} \right) \Big|_{r_0+\epsilon}, \quad (3.29)$$

$$\alpha = - \left(\frac{2R_0 q^2}{B_0^2} \right) \frac{dp_0}{dr}, \quad (3.30)$$

is the ballooning parameter, and

$$\mathcal{Q} = k_{\parallel}^2 + \frac{\gamma^2}{\omega_A^2}. \quad (3.31)$$

. The key feature of Eq. 3.27 is that the radial displacement $\xi_{r,m}$ explicitly depends on the pressure gradient dp_0/dr via Eq. 3.30.

Solving Eq. 3.27 analytically is challenging, and even for simple pressure profiles, the solutions are often complex. However, to understand the qualitative behavior of infernal modes, a simplified model can be used. A "staircase-like" pressure profile is considered in this simple case, where the pressure gradient is concentrated at a single radial location r_p , as shown in Fig.3.3. Under this assumption, the radial displacement of the $m = 1$ infernal mode takes the form:

$$\xi_{r,m=1} = \begin{cases} 0, & r < r_p, \\ \xi_0 \frac{(r_p/r_0)^2}{(r/r_0)^2} = \xi_0 \frac{r_p^2}{r^2}, & r_p < r < r_{\text{LCFS}}. \end{cases} \quad (3.32)$$

. Here, ξ_0 is a constant, and r_p is the location of the strongest pressure gradient. Although this assumption does not perfectly represent realistic pressure profiles, Eq. 3.32 provides insight into how the infernal mode responds to pressure gradients. This expression is frequently used to approximate the mode's growth rate based on the dispersion relation (Eq. 3.27), making it a useful tool for qualitative analysis.

3.4.3 Fishbone instability

Fishbone (FB) instabilities were first observed in the Poloidal Divertor eXperiment (PDX) in 1983 [72]. The instability was named after the characteristic fishbone-like pattern seen in the temporal evolution of mode-associated fluctuations, as shown in Fig.3.4. However, this name does not reflect the underlying driving mechanism. FBs arise when internal kink or infernal modes are destabilized by beam-driven fast ions, primarily due to the fast ion pressure gradient. It is worth noting that FBs are not the only instabilities related to internal kink

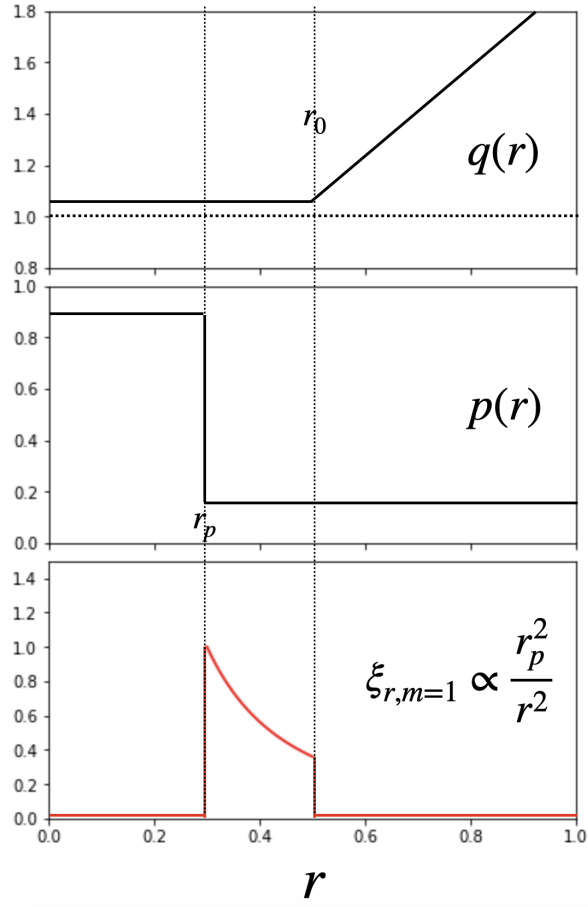


Figure 3.3: Comparison of a hypothetical $q(r)$ with $r_0 = 0.5$, a hypothetical pressure profile with $r_p = 0.3$, and the corresponded infernal mode radial structure.

or infernal modes. They can also be destabilized into other types of instabilities by various mechanisms.

The mechanism by which fast ions destabilize an internal kink mode, leading to FB instability, was first proposed by Chen in 1984 [20]. The presence of fast ions modifies the perturbation energy as:

$$\delta W_{\text{tot}} = \delta W_{\text{MHD}} + \delta W_k, \quad (3.33)$$

where δW_{MHD} is the same as in the absence of fast ions (Eq. 3.24), and δW_k represents the contribution from kinetic species:

$$\delta W_k = \int_V dV \xi^* \cdot \delta \mathbf{P}_h, \quad (3.34)$$

, where $\delta \mathbf{P}_h$ is the perturbed fast particle pressure tensor, as defined in [4]. Following the same energy principle approach as in the kink mode case, the FB dispersion relation is obtained by minimizing δW_{tot} with respect to $\xi_{\parallel}$, assuming plasma incompressibility (Eq. 3.25). Unlike Eq. 3.24, the term δW_k contains only a destabilizing pressure-driven contribution, implying that the presence of fast ions enhances kink mode instability.

Fast ions can also destabilize infernal modes, leading to fishbone instabilities in the $q_{\text{min}} > 1$ regime. These so-called infernal-like FBs were first observed in NSTX, and their excitation mechanism was proposed by Kolesnichenko, Marchenko, and White in 2006 [60].

The initial steps for deriving infernal-like FB characteristics follow the ideal MHD infernal mode analysis but include the additional δW_k term (Eq. 3.34). Consequently, the dispersion relation of the infernal-like FB differs from that of the standard infernal mode (Eq. 3.27). The detailed derivation is provided in [60], where the mode displacement for the $n = m = 1$ infernal-like FB is expressed as:

$$\xi_{r,m=1,n=1} = \frac{2\epsilon^2 e \hat{\beta}_p (\nu + 1) (r^{2\nu} - 1)}{[(2\nu + 1)^2 - 1] l_{\alpha}(\omega, r) + 4\nu(\nu + 1) \left[\left(\frac{1}{q} - 1 \right)^2 - (\omega/\omega_A)^2 \right]}, \quad (3.35)$$

where most terms and variables are defined in [60]. The key components include the $\hat{\beta}_p \propto \int_0^r r^2 (dp_c/dr) dr$, representing the species' pressure-to-magnetic-field ratio, and $r^{2\nu} - 1$, where ν characterizes an analytic pressure profile:

$$p_c \propto 1 - \left(\frac{r}{r_{\text{LCFS}}} \right)^{2\nu}. \quad (3.36)$$

. Here, ν increases with stronger radial pressure gradients. Consequently, the FB radial displacement ξ_r generally exhibits higher amplitude with increasing bulk and fast ion β , peaking near the location of the steepest pressure gradients. This shows the structure of infernal-like FBs are primarily determined by the beam-induced fast ion pressure gradients.

3.4.4 Limitations of these models for FBs in STs

The FB models discussed above, whether for internal-kink-like or infernal-like FBs, may not fully capture the characteristics of FBs in spherical tokamaks (STs). The derivation of analytic solutions for both internal kink and infernal modes typically begins with the assumption of a cylindrical plasma, with some studies [18] incorporating toroidicity corrections. While these approaches may provide marginally accuracy for conventional aspect ratio tokamaks, they become less reliable for describing internal kink and infernal modes in STs, where the geometry and equilibrium profiles differ significantly.

Additionally, many hybrid kinetic-MHD models assume $\delta W_k \ll \delta W_{\text{MHD}}$ for simplification. However, in STs such as MAST/-U and NSTX/-U, the fast ion beta to bulk ion beta ratio (β_f/β_i) is considerably higher than in conventional tokamaks. In addition, the wave-particle interaction with the fast ions, as well as the alteration of fast ion profiles due to FB associated fast ion transport are not considered in this model, similar to the case of the TAE model. These factors challenge the applicability of existing FB models and suggest that alternative modeling approaches may be necessary for accurately describing FB behavior in STs.

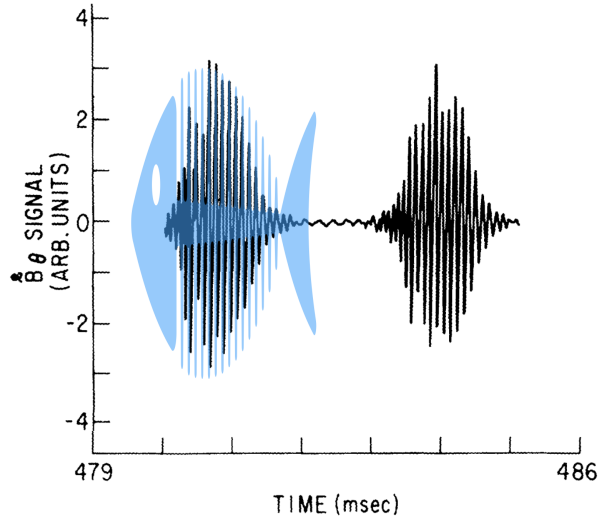


Figure 3.4: Time evolution of $\hat{B}_\theta$ of two fishbones from [72], with an artistic depiction of the shape of the signal as a "fishbone".

3.5 Summary of this chapter

This chapter outlined the descriptions of TAEs and FBs in the MHD picture. These models have limitations in describing both instabilities in spherical tokamaks due to their assumptions, including the conventional aspect ratio approximation, low or zero fast ion beta, and the neglecting of nonlinear effects. While these models provide a foundation for the discussions in later chapters, they also highlight the need for more advanced modeling approaches to accurately capture TAEs and FBs in STs.

CHAPTER 4

Data analysis methodology

High-quality measurements of various characteristics of TAEs and FBs are essential for advancing the understanding of these instabilities. MAST and MAST-U are equipped with a suite of diagnostics that capture both equilibrium and fluctuating quantities associated with low-frequency modes, making them well-suited for studying TAEs and FBs.

However, extracting and isolating the relevant physical quantities from these measurements requires several data analysis techniques. This chapter outlines the methodology used to determine the toroidal mode number (n) of each instability — an important parameter for characterizing these modes. Next, the chapter discusses the Short-time Fourier transform (STFT)-based time-frequency mode decomposition method - a useful technique that is rare in plasma physics research. Following is the discussion on the implementation of a linear regression model to remove non-global contributions from the signals. Finally, the chapter presents an overview of mode spatial diagnostics and the procedures for applying the analysis techniques to the data, which include a novel pseudo-matrix-inverse based technique to invert line integrated soft x-ray signal using localized B-spline polynomial functions.

4.1 Short-time Fourier transform (STFT) and its inverse

The Short-Time Fourier Transform (STFT) [3] forms the foundation of the spectral analysis performed in this dissertation. It is particularly well-suited for analyzing signals whose frequency content evolves over time. By applying the Fourier Transform to short, overlap-

ping segments of a signal, the STFT provides a time-frequency representation that reveals both spectral and temporal features. This section introduces the discrete STFT and its inverse, and highlights the role of the Hanning window in ensuring accurate signal reconstruction—especially after applying time-frequency domain masking.

In the discrete-time STFT, the signal is divided into overlapping frames, each multiplied by a window function to mitigate spectral leakage. The Fourier transform is then applied to each windowed segment. The STFT of a signal $x[t]$, denoted $\hat{x}(T, f)$, is defined as:

$$\hat{x}(T, f) = \sum_{t=0}^{N-1} x[t + TH] w[t] e^{-i(2\pi f)t/N} \quad (4.1)$$

where $w[t]$ is the window function of length N , H is the hop size between successive frames, T is the frame index, and f is the discrete frequency index with resolution determined by the FFT size N . A spectrogram can be obtained by computing the squared magnitude, $|\hat{x}(T, f)|^2$, which is commonly used for visual inspection of the signal's time-frequency power spectrum.

The choice of window function and frame overlap are critical to enabling accurate signal reconstruction via the inverse STFT. This dissertation employs the Hanning window, defined as:

$$w[t] = 0.5 \left(1 - \cos \left(\frac{2\pi t}{T-1} \right) \right), \quad 0 \leq t < T \quad (4.2)$$

The Hanning window smoothly tapers to zero at the boundaries, reducing spectral leakage and providing a good balance between temporal localization and frequency resolution [110]. It also satisfies the Constant Overlap-Add (COLA) condition when used with 50% overlap (i.e., $H = N/2$) [57], which is essential for accurate signal reconstruction.

The original signal can be reconstructed from its STFT representation via the inverse STFT. The discrete inverse STFT, $x_{\text{inverse}}[t]$, is given by:

$$x_{\text{inverse}}[t] = \frac{\sum_T w[t - TH] \tilde{\hat{x}}_T[t - TH]}{\sum_T w^2[t - TH]} \quad (4.3)$$

where $\tilde{x}_T[t]$ is the inverse Fourier transform of each time frame T , defined as:

$$\tilde{x}_T[t] = \frac{1}{N} \sum_{f=0}^{N-1} \hat{x}(T, f) e^{i(2\pi f)t/N} \quad (4.4)$$

It is important to note that the amplitude attenuation introduced by the Hanning window during the forward STFT is effectively compensated by the energy accumulation from overlapping frames in the inverse STFT. As a result, a simplified numerical approximation is often sufficient:

$$x_{\text{inverse}}[t] \approx \sum_T \tilde{x}_T[t - TH] \quad (4.5)$$

This approximation reduces computational complexity while still achieving accurate signal reconstruction in practice.

4.1.1 Time-frequency resolution trade-off in STFT

A well-known limitation of the STFT is its inability to provide arbitrarily fine resolution in both time and frequency simultaneously. This is a consequence of the Gabor limit, which states that the product of the time and frequency resolutions must satisfy $\Delta T \cdot \Delta f \geq 1/4\pi$.

For example, consider a diagnostic system sampling at 2 MHz for 0.1 seconds (i.e., 200,000 data points). If a frequency resolution of $\Delta f = 5$ kHz is desired, then record length must be $1/5\text{kHz} = 0.2$ ms, which is $0.2 \text{ ms} * 2 \text{ MHz} = 400$ samples. With a 50% overlap, this results in 200 time frames, and the effective temporal resolution (frame spacing) is $\Delta T = 0.1\text{ms}$. However, if higher frequency resolution is needed, for example, $\Delta f = 1$ kHz, then the window size must increase fivefold to 2000 samples. This reduces the temporal resolution by a factor of five, yielding $\Delta t = 0.5\text{ms}$. Thus, STFT parameters must be chosen to balance temporal and frequency resolution depending on the nature of the signal. For example, a chirping mode with a 80 kHz frequency shift over a 1–2 ms interval requires finer temporal resolution to capture rapid frequency evolution. In contrast, a long-lived mode lasting 10–20 ms with slowly evolving frequency would benefit more from higher frequency resolution. Therefore,

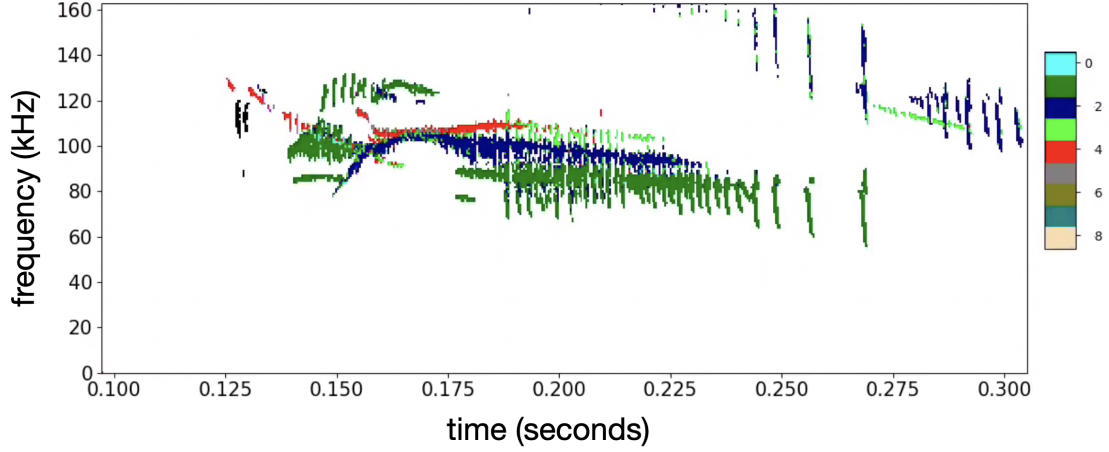


Figure 4.1: $n \in [0, 8]$ in MAST-U discharge 47001 from 100 to 300ms, 0 to 160kHz.

the STFT configuration must be carefully tailored to suit the physical characteristics of the signal under investigation.

4.2 Magnetic fluctuations and toroidal mode numbers inferred from the outboard Mirnov coil array

The toroidally extended outboard Mirnov coil array (OMAHA) [5] can be used to infer the magnetic fluctuations and toroidal mode number (n) associated with low frequency global modes in MAST and MAST-U. Core-localized, large-scale, low-frequency instabilities are typically readily detected by the OMAHA system owing to its high signal-to-noise ratio and temporal sampling rate [52], even though the system is located outboard away from the instabilities. The OMAHA system in MAST was composed by six, five, and two coils that measured the vertical (δB_z), toroidal (δB_ϕ) and radial (δB_r) components of the magnetic fluctuations, respectively. Importantly, modes in a tokamak tend to have toroidal sinusoidal structure since the plasma equilibrium is toroidal symmetric to a high degree. Hence, it is rare to find modes with different toroidal mode numbers occur with the same frequency at

the same time. Hence it enables n mode number to be determined effectively from a set of toroidally separated Mirnov coils measuring the same component (e.g. δB_z). The inferred n is essential for classifying the modes, for example, to distinguish whether a mode is a TAE, a FB, or other type fast ion- or MHD-induced instability according to its n and frequency. The same analysis procedure applies to the data in MAST-U as well but with a different coil location configuration. In addition, Mirnov signals in MAST-U are band-pass filtered to eliminate grounding noise at about 0.5kHz, which were not an issue in MAST. This does not affect the targeted instabilities reported in this dissertation that typically have much higher frequencies than the grounding noise.

The n are identified for each time and frequency using a Singular Value Decomposition (SVD)-based method pseudo-inverse [52]. The procedure starts by applying the STFT to the OMAHA signals to infer the time, frequency and the mode amplitude of each instability. Assuming a single mode with a fixed frequency and n mode number traveling in the tokamak, then phase differences appear in signals picked up by the toroidally separated coils at a certain time and frequency: $e^{in\phi_x}$, where ϕ_x represents the toroidal locations of x coils. There would be $N \times M$ of these phase difference combinations, where M is the number of distinct n modes and N is the number of toroidally separated coils. With these assumptions and settings, the signals from the coils can be written as:

$$\mathbf{F} = \gamma \cdot \alpha \quad (4.6)$$

$$\mathbf{F} = \begin{pmatrix} F_1 \\ F_2 \\ \vdots \\ F_N \end{pmatrix}, \gamma = \begin{pmatrix} e^{in_1\phi_1} & \dots & e^{in_M\phi_1} \\ e^{in_1\phi_2} & \dots & e^{in_M\phi_2} \\ \vdots & & \vdots \\ e^{in_1\phi_N} & \dots & e^{in_M\phi_N} \end{pmatrix}, \alpha = \begin{pmatrix} \alpha_1 \\ \alpha_2 \\ \vdots \\ \alpha_M \end{pmatrix}, \quad (4.7)$$

for each frequency and time of the STFT. The γ matrix has to be decomposed into:

$$\gamma = \mathbf{U} \cdot \text{diag}(w_i) \cdot V^H \quad (4.8)$$

. Here, matrices $\mathbf{U}$, $\mathbf{V}$ and weights w_i are solved by the SVD method, with the superscript H stands for Hermitian transpose. The goal here is to find a combination of α for a single dominant n with the smallest residual:

$$r = |\gamma \cdot \alpha - \mathbf{F}|/|\mathbf{F}| \quad (4.9)$$

. This can be done by representing the α vector as:

$$\alpha = \mathbf{V} \cdot \text{diag}(1/w_i) \cdot U^H \cdot \mathbf{F} \quad (4.10)$$

. These procedures yield the dominant toroidal mode number for the instabilities in each plasma discharge in MAST and MAST-U. For example, the $n \in [0, 8]$ harmonics are identified in MAST-U discharge 47001, as shown in Fig.4.1. They can be compared with other measurements or expressions, such as the core rotation and the conventional aspect ratio ideal MHD TAE frequencies to determine their types of instabilities. The range of n needs to be carefully chosen as the identification method will produce incorrect results when the actual value is excluded in the selected range of n in γ . It is also worth noting the accuracy of this SVD-based identification decreases for large n , as it is limited by the number of the coils. Yet the setup in MAST and MAST-U are sufficient for the modes with low frequencies and low n numbers.

4.3 Time- and frequency-dependent band-pass filtering

The unique characteristics of TAEs and FBs in MAST and MAST-U necessitate unconventional band-pass filtering methods for analysis. Many observed TAEs and FBs interact with fast ions, causing their frequencies to change rapidly (chirping). Additionally, some of these chirps occur in close proximity in both time and frequency domains, making conventional band-pass filtering—typically based on a rectangular time-frequency range—ineffective for isolating individual chirps in certain cases. An alternative approach involves extracting mode characteristics, such as amplitude and spatial structure, directly from the short-time Fourier

transform (STFT) at a specific time and frequency. However, due to rapid chirping, contributions from multiple frequencies may be present within a single burst. To address this, a time-frequency-varying band-pass filter is required to accommodate these rapid frequency shifts. This STFT-based decomposition method has been used in other branches of science, engineering [113], and commercially available signal or audio processing tools [1].

The band-pass filtering process begins by isolating the region around the targeted chirp. A masking array $\mathbf{A}_{mask}$ is constructed with the same frequency (f) and time (T) dimensions as the STFT, where each element $A_{T,f}$ is defined as:

$$A_{T,f} = \begin{cases} 1, & \text{for the desired } f \text{ and } T \\ 0, & \text{elsewhere} \end{cases} \quad (4.11)$$

. This matrix is then applied to mask the STFT of the signal through element-wise (Hadamard) multiplication:

$$\hat{X}(T, f)_{filtered} = \mathbf{A}_{mask} \odot \hat{X}(T, f) \quad (4.12)$$

. This operation sets all elements of $\hat{X}(T, f)$ outside the masking range to zero. The inverse short-time Fourier transform (ISTFT), denoted as $(\)^{-1}$, is then applied to $\hat{X}(T, f)_{filtered}$ to obtain the filtered signal in the time domain. Notably, $\hat{X}(T, f)_{filtered}^{-1}$ remains a complex signal when negative frequency contributions are discarded during the construction of $\mathbf{A}_{mask}$. This preserves the phase information of mode-associated fluctuations, which is useful for various analyses discussed in later sections and chapters. Although different window functions can be chosen for each T in an STFT, a half-overlap Hann window is used exclusively in this study to reduce artifacts in masked STFT reconstructions [62].

4.4 Linear regression model for fluctuations

Despite its high sensitivity and signal-to-noise ratio, relying solely on OMAHA is insufficient for capturing certain aspects, such as the spatial structures of the modes inside the bulk

plasma. Mode structure diagnostics like BES [29] and SXR [54] measurements often have lower signal-to-noise ratios compared to OMAHA. The following methods were employed to filter the signals to isolate fluctuations associated with the global modes from noise and turbulence.

OMAHA captures magnetic field fluctuations ($\delta\vec{B}$) at the edge, while BES captures the spatial structure of local electron density fluctuations, and weighted sums of line-integrated δn_e , electron temperature fluctuations (δT_e), and effective charge fluctuations (δZ_{eff}) are captured by tangential SXR. These low-frequency fluctuations are assumed to have predominantly transverse polarization and result from magnetic field line displacement as an initial approach [95]. Since both BES and SXR measurements may also capture noise or fluctuations not associated with the displacement parameter (ξ) (e.g., turbulence), the measured $\delta\vec{B}$ can be used as the independent variable in a linear regression model for both BES and SXR signals [55]:

$$\delta x(t) = L_x(\delta\vec{B}(t)) + \text{noise}, \quad (4.13)$$

where $\delta x(t)$ represents the fluctuation of either BES or SXR signals extracted by short-time Fourier transform (STFT: $\hat{X}(T, f)$, T : time window of Fourier transform); L_x is an unknown linear function, linking $\delta x(t)$ and $\delta\vec{B}(t)$. The fluctuations selected for this study, when observed in the STFT of SXR and BES signals, must appear as time-evolving, narrow-band spectral peaks adequately coherent with a time-evolving, narrow-band spectral peak in the STFT of $\delta\vec{B}$. The cross-coherence between δx and $\delta\vec{B}$ is defined as:

$$C_{xB} = \frac{|\langle S_{xB} \rangle|}{\sqrt{\langle S_{xx} \rangle \langle S_{BB} \rangle}}, \quad (4.14)$$

where $\langle S_{xB} \rangle$ is the boxcar convolution of multiple time and frequency points of the cross-spectrum of either SXR or BES signals with $\delta\vec{B}$. The values of C_{xB} range from 0 to 1. Particularly, the C_{xB} can be used to determine the uncertainties of signal x under the assumption of a linear response with the addition of noise contribution used in the model in

Eq 4.13. The relative noise level can be defined as:

$$U_x = \sqrt{1 - (C_{xB})^2} \quad (4.15)$$

In practice, the STFT of the signals are defined as the spectrum of the signals used to compute S_{xx} , S_{xB} , and S_{BB} in this study. Hence the $C_{xB}(f, T)$ and $U_x(f, T)$ are computed to determine the coherence and noise level of the signals for each convolution window in the STFT domains. This would ensure the selected activities in this study have strong correlations between the spectral signatures of the edge and core measurements.

To isolate the contributions of FB chirps to SXR or BES signals and δB_z signals, a band-pass filter technique and linear regression, as shown in Fig. 4.2, were applied. The technique described in section 4.3 is applied to discard the negative frequency component of the signals, yielding analytic signals that preserve complex components for extracting phases and other physical quantities in this study.

BES and SXR signals may also capture non-global features such as turbulence and noise from the diagnostics' electronics. Linear regression is applied to the band-pass filtered SXR signals to isolate the global part by eliminating the uncorrelated portion of the signals:

$$\delta x(t)_{mode} = \frac{LP(\widehat{\delta x}(T, f)_{filtered}^{-1} * (\widehat{\delta B_z}(T, f)_{filtered}^{-1})^\dagger)}{LP(\widehat{\delta B_z}(T, f)_{filtered}^{-1} * (\widehat{\delta B_z}(T, f)_{filtered}^{-1})^\dagger)^{1/2}}, \quad (4.16)$$

where δx_{mode} represents the amplitude of either BES or line-integrated SXR signal relative to $\delta \vec{B}(t)$ after the linear regression process, $\hat{X}(T, f)_{filtered}^{-1}$ is the inverse STFT of a band-pass filtered STFT of a signal, dagger sign ($X^\dagger$) represents complex conjugate of any complex signal X , and $LP(\cdot)$ denotes a low-pass filter. This process is justifiable if we assume that SXR signals of certain chirps are linear functions of δB_z .

The measured SXR signal fluctuations are line-integrated, and further inversion to local emissivity, with the techniques to be shown in section 4.5.1, is needed for subsequent studies.

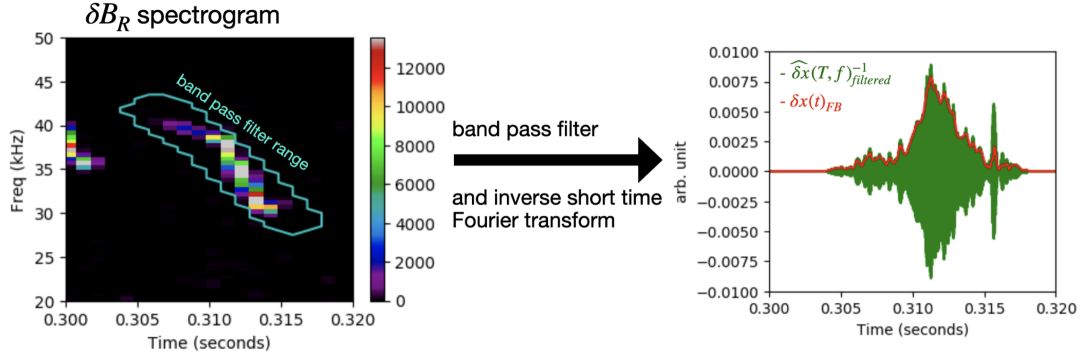


Figure 4.2: (a) Time-frequency varying band pass filter of a δB_R signal from discharge 47128 in MAST-U; (b) filtered signal before (green) and after linear regression (red).

4.5 Mode spatial structure diagnostics

The tangential SXR and BES diagnostics presented in this section measure the spatial structures of low frequency instabilities in MAST and/or MAST-U. A linear regression technique shown in the previous section is applied to the band-pass filtered SXR and BES signals to isolate the contribution of the global mode before further analysis to extract the spatial structures.

4.5.1 Tangential Soft X-Ray camera array diagnostic

The tangential SXR system comprises 18 lines-of-sight with collimated views observing the plasma tangentially in the horizontal plane, but only 8 to 10 of these cameras are used for analysis due to some of the views are of regions with little or no fluctuating or mode-related signal, such as the central column. The SXR cameras capture line-integrated soft X-ray emissions along lines of sight lying on the midplane, with the array of sight lines diverging from the aperture.

The emissivity of SXR is related to the electron density and temperature [54]:

$$\epsilon(R) = \epsilon_c n_e^2 \sqrt{T_e} Z_{eff} e^{-\frac{E_{photon}}{T_e}} \quad (4.17)$$

where ϵ_c is a constant, E_{photon} is the lowest energy of soft x-ray photon that can pass through the beryllium foil in front of each SXR camera, which is $\approx 1keV$ for the SXR system in MAST and MAST-U [71], and $Z_{eff,0}$ is the equilibrium effective ion charge state, which is influenced by the densities of impurities. $Z_{eff,0}$ is usually less than 1.5 in the core region [51], meaning there are only low concentrations of impurities in the core. On the other hands, the mode frequency (below 100 kHz) is much lower to the ion cyclotron frequency (in the MHz range) and hence the mode wavelengths are large compared to the gyroradii of all the ion species. As a result, the $E \times B$ flows affect all ion species the same, maintaining the proportions of impurity and main ion densities. Therefore, the contribution of the fluctuation of effective charge ($\delta Z_{eff}/Z_{eff,0}$) to the SXR fluctuation is expected to be small, and negligible in the core.

Assuming $\delta f \ll f_0$, where $f = f_0 + \delta f$ and δf represents the fast fluctuating part, we can obtain $\delta\epsilon(R)$ using the first-order Taylor expansion of $\epsilon(R)$:

$$\frac{\delta\epsilon}{\epsilon_0} = 2 \frac{\delta n_e}{n_{e0}} + \left(\frac{1}{2} + \frac{E_{photon}}{T_{e0}} \right) \frac{\delta T_e}{T_{e0}} + \frac{\delta Z_{eff}}{Z_{eff,0}} \quad (4.18)$$

Expressing $\delta n_e/n_{e0}$ and $\delta T_e/T_{e0}$ in terms of plasma displacement ξ using the linearized electron continuity and energy equations gives:

$$\frac{\delta n_e}{n_{e0}} = -\nabla \cdot \xi - \xi \cdot \frac{\nabla n_{e0}}{n_{e0}} \quad \text{and} \quad \frac{\delta T_e}{T_{e0}} = -(\gamma - 1) \nabla \cdot \xi - \xi \cdot \frac{\nabla T_{e0}}{T_{e0}}, \quad (4.19)$$

where γ is the adiabatic index, or ratio of specific heats, for the electron fluid. For low-frequency modes, it is typically assumed that parallel transport process are fast enough, so that T_e is constant along field lines, leading to $\gamma = 1$. Considering the compressional component of the mode to be small when $nc_S/R \ll \omega$ and $mc_S/a \ll \omega$, we can assume $\nabla \cdot \xi \approx 0$. This leads to $\delta f/f_0 \approx -\xi \cdot \frac{\nabla f_0}{f_0}$ for both $\delta n_e/n_{e0}$ and $\delta T_e/T_{e0}$. This shows that the SXR emissivity fluctuation is closely related to radial plasma displacement.

SXR cameras measure line-integrated emission along their lines of sight, resulting in the following raw signals:

$$I_i = \int_{L_i} \epsilon(R) dL_i \quad (4.20)$$

and similarly for the fluctuations:

$$\delta I_{i,n} = \int_{L_i} \delta \epsilon_n(R) e^{in\zeta_i} dL_i \quad (4.21)$$

where n is the toroidal mode number inferred from Mirnov coil array measurements, ζ_i is the toroidal angle at different points along each line-of-sight i , and δI_i are the signals from SXR lines-of-sight after bandpass filtering and linear regression shown in section 2. Since the tangential SXR camera lines of sight lie in the mid-plane only, there is no dependence on poloidal wavenumber. The subscript n under δI means that the band-pass filtered SXR signals represent a specific toroidal number only.

Inversions of δI_i to $\delta \epsilon(R)$ are performed using a least-square fit method. The $\delta \epsilon(R)$ is assumed to be a linear combination of B-spline polynomials ($P_j(R)$):

$$\delta \epsilon_n(R) = \sum_j c_j P_j(R), \quad (4.22)$$

where c_j are the coefficients to be solved for. Line integrations along the lines of sight of the SXR cameras are applied to the model and organized into a system of equations, which can be rewritten as a matrix equation:

$$\delta \mathbf{I}_n = \mathbf{X} \cdot \mathbf{C} \quad (4.23)$$

$$\delta \mathbf{I}_n = \begin{pmatrix} \delta I_{1,n} \\ \delta I_{2,n} \\ . \\ . \\ \delta I_{I,n} \end{pmatrix}, \mathbf{X} = \begin{pmatrix} X_{1j} & \dots & X_{1J} \\ X_{2j} & \dots & X_{2J} \\ . & & . \\ . & & . \\ X_{Ij} & \dots & X_{IJ} \end{pmatrix} \mathbf{C} = \begin{pmatrix} c_1 \\ c_2 \\ . \\ . \\ c_J \end{pmatrix}, \quad (4.24)$$

, with I SXR channels and J B-spline polynomials. The matrix $\mathbf{X}$ has elements $X_{ij} = \frac{1}{L_i} \int_{L_i} P_j e^{in\zeta_i} dL_i$, where L_i is the length of each chord from the camera aperture to the edge of the vacuum vessel, and vector $\mathbf{C}$ has c_j as its elements. Solving for $\mathbf{C}$ involves $\mathbf{C} = \mathbf{X}^{-1} \cdot \delta\mathbf{I}_{\text{SXR}}$, where $\mathbf{X}^{-1}$ is the inverted $\mathbf{X}$ matrix. For the case when $I \geq J$, the square matrix can be exactly inverted without residue errors. However, the $\mathbf{X}^{-1}$ has to be obtained using the SVD-based pseudo-inverse for $I < J$ cases. The SXR emissivity can now be reconstructed using the solved c_j in vector $\mathbf{C}$ and the model $\delta\epsilon(R) = \sum_j^\beta c_j P_j(R)$. These procedures enable the inference of SXR emissivity. It is worth noting that the residue error increases when the matrix gets more rectangular. For $I \ll J$, using a matrix with a high condition number in the pseudo-inverse process increases the residue error [10]. Hence there is a trade-off between minimizing the error versus obtaining fine details in the radial structure from the inversion using a large amount of small scale P_j . On the other hands, it is possible to have the data fitted to the noise when $I \gg J$. A set of B-spline polynomials shown in Fig.4.3 have been chosen for the MAST SXR signal inversion. Future work can focus on optimizing the number of j using concepts such as the Akaike information criterion [2].

The noises of the integrated SXR emission measurements propagate to the uncertainties of coefficients c_j during the matrix multiplication in the inversion process. The noises of the SXR signals $U_{\delta I_{\text{SXR}}}$ can be defined by Eq. 4.15 under the assumption of linear relations between SXR and edge $\delta\vec{B}$ measurements. The uncertainty of each c_j is propagated from δI_{SXR} , defined as:

$$U_{c_j} = \sqrt{\sum_{i=1}^N (X_{ij}^{-1})^2 \cdot \sigma_{\delta I_{\text{SXR},i}}^2}, \quad (4.25)$$

under the assumption that there is no uncertainty in the inversion of the basis matrix $\mathbf{X}^{-1}$. It is worth noting that the residue error from the pseudo-inverse process cannot be too high or this assumption would breakdown, highlighting the importance of not choosing the large amount of basis functions for inversion.

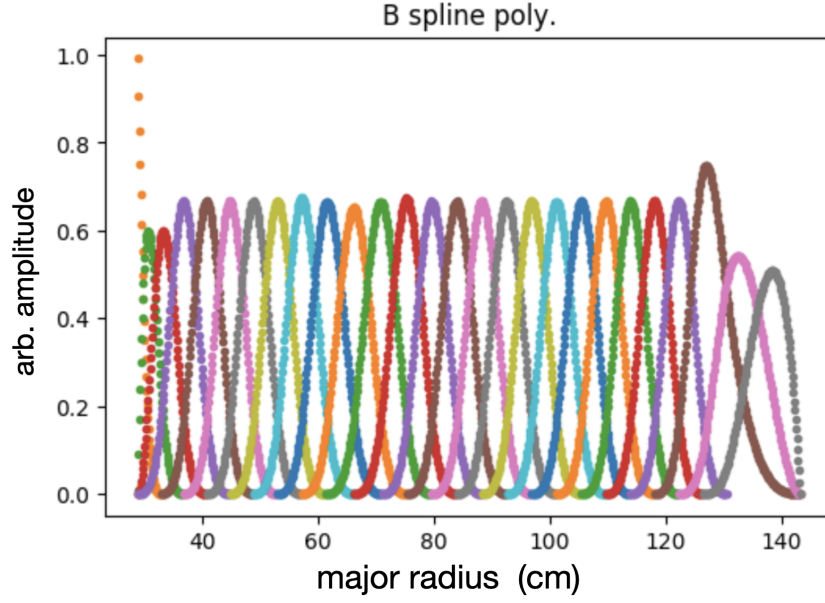


Figure 4.3: The set of B-spline polynomials P_j used to invert δI_{SXR} in MAST

4.5.2 Beam Emission Spectroscopy diagnostic

The 2D Beam Emission Spectroscopy (BES) diagnostic measures the emission fluctuations resulting from the collisional excitation of injected beam neutrals by plasma ions and electrons. BES systems in conventional and spherical tokamaks are primarily used to observe turbulent associated density fluctuations. They are less commonly used to observe global modes such as TAE and FB. The system captures the Doppler shifted deuterium beam Balmer-alpha (D_α) emission [49] with wavelength of about 660nm from the energetic deuterium atoms in the range of 50 to 100 keV [29].

The BES diagnostic can measure electron density fluctuation when it is used in appropriate temporal and spatial regimes. The fluctuation of beam D_α emission ($\delta I/I_0$) has the following expression [30]:

$$\frac{\delta I}{I_0} \approx \frac{\delta n}{n} \frac{1}{1 + n\langle\sigma v\rangle_3/A_{30}}, \quad (4.26)$$

where $\delta n/n_0$ is the fractional plasma density fluctuation, A_{30} is the spontaneous radiative

decay rate of the excited level 3 to the ground state, and $\langle\sigma v\rangle_3$ is collisional depopulation rate coefficient for level 3 (including ionization and collisional de-excitation). This expression shows that the BES response is attenuated relative to the underlying density fluctuation by the factor $(1 + R)^{-1}$, with

$$R \equiv \frac{n\langle\sigma v\rangle_3}{A_{30}}. \quad (4.27)$$

The critical rate coefficient $\langle\sigma v\rangle_{\text{crit}}$, defined as:

$$\langle\sigma v\rangle_{\text{crit}} \equiv \frac{A_{30}}{n_e}, \quad (4.28)$$

can be used to estimate the attenuation ratio. For example, using values relevant to the MAST and MAST-U experiments, such as deuterium neutral beam with energy $E_{\text{beam}} = 60 \text{ keV}$ injected into a deuterium plasma with density $n_e = 3 \times 10^{19} \text{ m}^{-3}$ and temperatures $T_e = T_i = 2 \text{ keV}$, the spontaneous emission rate is $A_{30} \approx 4.4 \times 10^7 \text{ s}^{-1}$. This yields a critical depopulation rate coefficient:

$$\langle\sigma v\rangle_{\text{crit}} = \frac{A_{30}}{n_e} \approx \frac{4.4 \times 10^7}{3 \times 10^{19}} \approx 1.5 \times 10^{-12} \text{ m}^3/\text{s}. \quad (4.29)$$

From collisional-radiative modeling (for example, ADAS or Yacora databases [91] [108]), the effective collisional depopulation coefficient at $T = 2 \text{ keV}$ is estimated to be in the range of $\langle\sigma v\rangle_j \sim (3\text{--}6) \times 10^{-13} \text{ m}^3/\text{s}$. Hence, the ratio is

$$R = \frac{n\langle\sigma v\rangle_j}{A_{30}} = \frac{\langle\sigma v\rangle_j}{\langle\sigma v\rangle_{\text{crit}}} \sim 0.2\text{--}0.4. \quad (4.30)$$

The attenuation factor $(1 + R)^{-1}$ is therefore 0.7 to 0.8, indicating that the measured BES emission fluctuations will be about 20–30% smaller than the underlying density fluctuations under these plasma conditions in MAST and MAST-U.

The 2D BES system in MAST-U consists of an 8 by 8 avalanche photodiode (APD) array and a series of lens, motors, mirrors and filters in MAST-U. An optical filter selects the D_α light emission from the beam neutrals whilst reducing the light emission from background D_α and other impurity line emission. The APD array used by the BES system have an

≈ 2 cm separation, sampling a region $\approx 14 \times 16$ cm (R by z) in the plasma core, which covers approximately one-third of the minor radius. The viewing location can be moved radially between shots using a series of stepper motors that control an in-vessel mirror, external focusing lens, and the rotation of the APD array.

4.6 Summary of this chapter

This chapter introduced the OMAHA diagnostic settings and the analysis techniques used to determine the toroidal mode number of low-frequency instabilities in MAST and MAST-U. Additionally, the application of linear regression methods to remove non-global contributions from the signals was discussed. The chapter also covered the use of the tangential soft X-ray camera array and beam emission spectroscopy diagnostics, along with the procedures for inferring spatial profiles of mode-associated fluctuations. These techniques serve as essential tools for the analyses presented in the following chapters.

CHAPTER 5

Measurements of TAE in MAST for Verification and Validation of a Linear Global Gyrokinetic Simulation

The development of simulations capable of accurately modeling these instabilities and their associated fast ion transport is essential for the success of future fusion tokamaks. Linear MHD models, as described in Chapter 3.2, are often used to simulate modes excited in tokamaks by fast ions, including those arising from NBI [39]. However, to capture kinetic effects of thermal plasma at the microscopic scale that could affect the instability-induced fast ion transport, it is important to use a kinetic description for both the thermal plasma and the fast ions to assess the stability of TAEs accurately.

Global gyrokinetic (GK) codes, such as the Gyrokinetic Toroidal Code (GTC), offer promising capabilities for cross-scale integrated simulations of fast particle-driven modes [67]. However, these codes must be validated through comparisons with other simulation models and experimental measurements. GTC has been benchmarked against ideal and resistive MHD by suppressing kinetic effects of thermal plasmas, as well as other GK simulations by keeping kinetic effects of thermal plasmas, for toroidicity-induced Alfvén eigenmodes (TAEs) and reversed shear Alfvén eigenmodes (RSAEs) in conventional tokamaks like DIII-D [87][26][25]. However, investigations into a TAE in a spherical tokamak (ST) using global GK simulations remain notably rare and unreported compared to studies using ideal MHD models. In addition to the difference in aspect ratio, the ratio of fast ion beta (β_{FI}) to thermal ion beta (β_i) is typically much higher in some STs, such as the Mega-Amp Spherical Tokamak (MAST) [23], compared to conventional tokamaks. Global GK simulations have

not yet been verified in this β_{FI} regime in the STs.

This project aims to bridge this gap by comparing the results of linear GTC and ideal MHD simulations of a TAE in MAST to identify the non-perturbative and kinetic effects of thermal plasma. New experiments dedicated to TAEs are planned for MAST-U (the upgraded version of MAST) [82] and provide opportunities for model validations. For example, there is an ongoing effort to investigate the damping channels of $n = 3$ TAEs in MAST-U using ORB5 (another GK model) [22], but without the use of fast ion anisotropy. However, MAST presents a simpler NBI configuration that is more straightforward for modeling fast ion anisotropy in GTC, which is the first time it is being used in a GK simulation of a fast ion-driven AE in a ST. In addition, global radial mode structure measurements from tangential soft X-ray cameras were accessible in MAST, but they were unavailable during the early physics campaigns in MAST-U. These factors make the targeted TAE case in MAST more suitable for the model validation compared to the cases from MAST-U.

The remainder of this chapter is organized as follows. The experimental parameters and measurements of the targeted TAE case are first presented in Section 5.1. The underlying formulation of GTC is then compared with hybrid GK-MHD and ideal MHD models in the long wavelength limit (Section 5.2). The Maxwellian distributions used to describe both thermal and fast ion species, and Anisotropic distributions used to describe fast ion species in GTC, are introduced in Section 5.3. The frequency and spatial mode structure of the TAE computed using the ideal MHD code NOVA [21] (Section 5.4) are compared with those of the modes excited by an antenna in GTC, along with calculations of damping rates due to continuum damping, radiative damping, ion Landau damping, and finite Larmor radius (FLR) effects (Section 5.5; cases I - IV in Table 5.1). Additionally, TAEs are driven by Maxwellian and anisotropic slowing-down fast ion distributions in linear GTC simulations to examine the effects of these different types of drive on mode structure, frequency, and growth rate (Section 5.6; cases V - VI in Table 5.1).

5.1 MAST test case: experimental profiles and TAE measurements

MAST plasma discharge 26887 was selected for analysis. This device had a major radius $R_0 \approx 0.9$ m and a minor radius $a \approx 0.6$ m. Notably, the a/R_0 ratio for MAST (≈ 0.66) is considerably larger than that of many conventional tokamak cases (e.g., $a/R_0 \approx 0.333$ for JET [80]), which are typically used to validate and benchmark GTC simulations. The plasma current is approximately 800 kA and the toroidal field on the magnetic axis is around 0.43 T.

The mode frequency and radial structure of a TAE-like burst at $t \approx 180$ ms with toroidal mode number $n = 1$ in discharge 26887 are inferred from various diagnostics. Core-localized, large-scale, low-frequency instabilities are typically readily detected by a toroidally extended Mirnov coil array (OMAHA) [5] (with the raw signal from one of the coils shown in Fig 5.1(a)), owing to its high signal-to-noise ratio. The toroidal mode numbers (n) were identified for each time and frequency using a SVD-based method [52] described in Chapter.4.2. The selected $n = 1$ burst begins at a frequency of approximately 90 kHz and then chirps down to around 65 kHz (in the lab frame) in approximately 2 ms, as shown in Fig.5.2 (a). The core plasma rotation frequency inferred from charge exchange recombination spectroscopy (CXRS) [105] is about 10 kHz, meaning the mode frequency is about 10 kHz lower from the measurement in the plasma frame. The electron density and temperature are measured using a Thomson scattering diagnostic, showing increases in both core electron density and temperature (taken at $R = 1.0$ m) over the period of the appearance of TAEs in Fig.5.2 (b) and (c). Plasma heating is provided by an on-axis mid-plane NBI, the power increasing from 1.0 MW to approximately 1.6 MW at 170 ms, as shown in Fig.5.2 (d). The neutron emission inferred from fission chamber measurement increases slowly over time after 180 ms (shown in Fig.5.2 (e)) in the presence of a steady NBI power input.

The safety factor (q) profile and the TAE radial structure are also inferred or reproduced

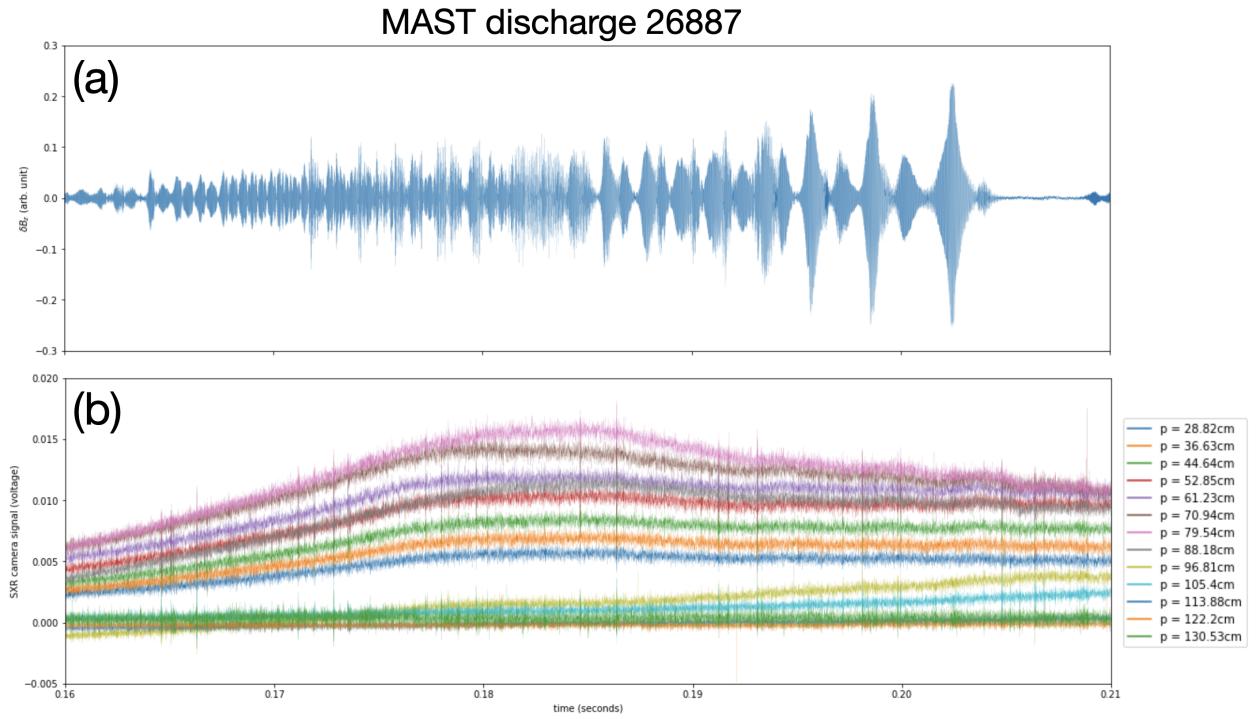


Figure 5.1: MAST discharge 26887 raw signals of (a) one of the outboard Mirnov coils that measure δB_z and; (b) tangential SXR camera with impact parameters (p) from 28.82 to 130.52cm, from 0.16 to 0.21 seconds

from various diagnostics and calculations. The q profile and magnetic configuration are reproduced from the MSE constrained EFIT++ reconstruction [63][64][6], with minimum safety factor $|q_{min}| = 1.23$ at the flux surface of $\rho = \sqrt{\psi_{pol}} \approx 0.45$ and central safety factor $|q_0| = 1.4$ at $t \approx 180\text{ms}$ (shown in Fig.5.6 (b)), which is the time of the selected TAE-like burst.

The radial structure of that burst is inferred from tangential Soft X-ray (SXR) cameras [54] (with the raw signals from the SXR channels used shown in Fig 5.1(b)). The beam emission spectroscopy diagnostic that measures electron density fluctuations ($\delta n_e/n_{e0}$) was not yet available when the experiment was conducted, and electron cyclotron emission (ECE), which is commonly used to measure electrons temperature fluctuations ($\delta T_e/T_{e0}$) in conventional aspect ratio tokamaks, is not available in MAST because of the cyclotron frequency is much lower than the plasma frequency due to the low magnetic field strength. Hence the SXR camera array provided the most viable option for measuring the global spatial structure

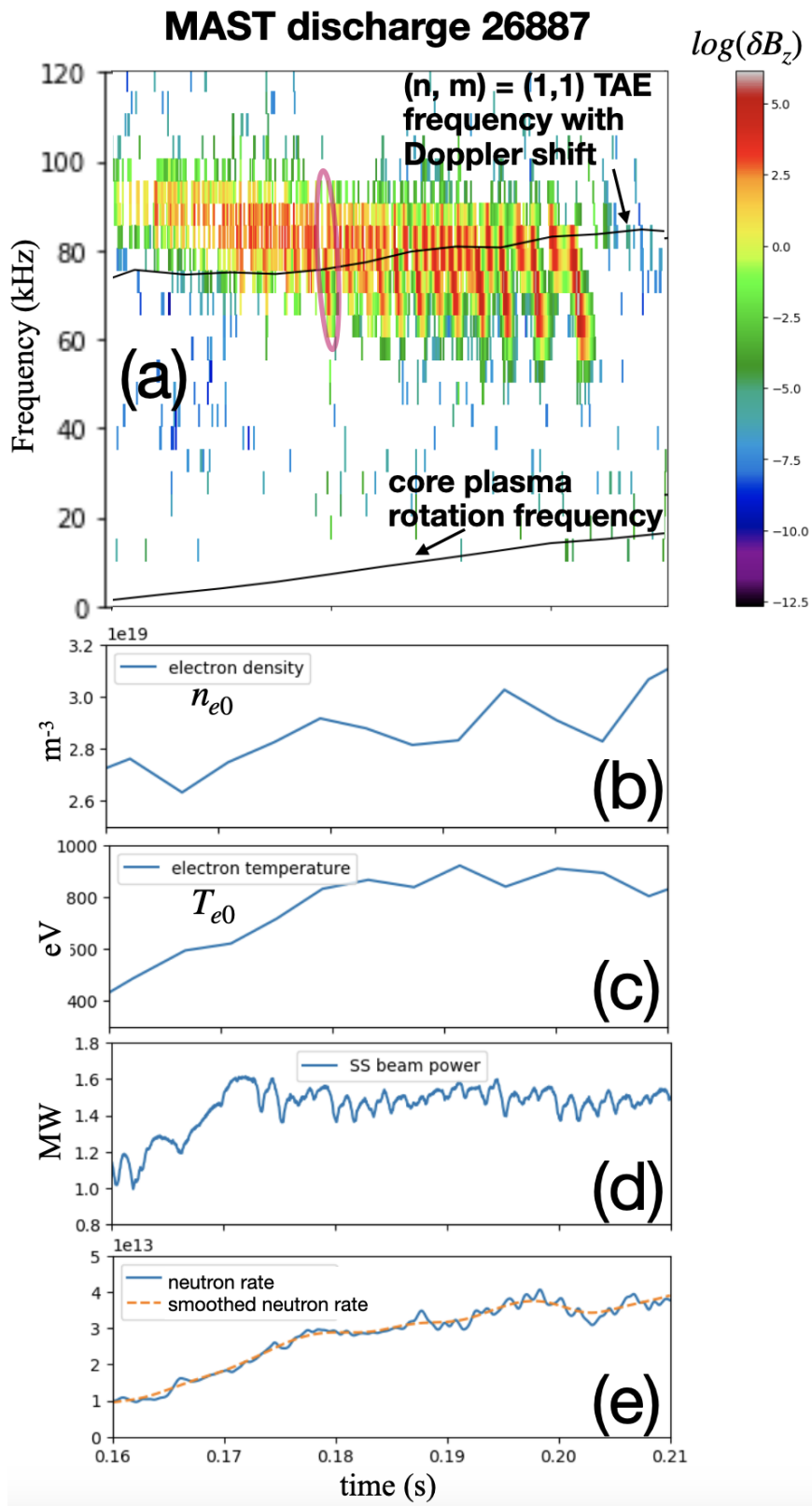


Figure 5.2: (a) MAST discharge 26887 δB_z spectrogram with half-overlapping 0.1ms windows. Only the $n = 1$ components are displayed. The targeted TAE is circled in pink. $(n, m) = 1, 1$ TAE frequency and core plasma rotation frequency are labeled in black lines; core (at $R = 1.0\text{m}$) electron (b) density and (c) temperature from Thomson scattering; (d) NBI power; (e) total neutron emission from fission chamber, and low pass filtered (DC) neutron emission showing the slow evolving part of the total neutron emission.

of TAEs in this pulse. The signal-to-noise ratios of these SXR signals are noticeably lower compared to the edge located Mirnov coil signal. This can be seen by comparing the Mirnov coil spectrogram in Fig. 5.2 and SXR spectrograms in Fig. 5.3. The cross coherence analysis (using 0.6ms and 30kHz coherence averaging windows) explained by Eq. 4.14 in Ch.4.4 can be useful in this situation to distinguish TAE-related signals and noises in SXR measurements. The Fig. 5.4 demonstrates one of the reasons why the TAE-like chirp at 180ms is selected. Although $C_{\delta I_{SXR}, \delta \hat{B}_z} = 0.8$ gives out the relative noise level of 60%, calculated using Eq.4.15 described in Chapter 4.4, the noise levels associated with the targeted chirp are still lower compared to some of the neighboring activities for signals from core SXR channels. It is worth noting that, the signals after linear regression described in Eq. 4.16 would have the uncertainty much lower than the relative noise level, because the signals are averaged over some numbers (N) of time and frequency windows. Hence it will have a $1/\sqrt{N}$ factor that lowers the overall uncertainty level.

The SXR line-integrated signals are first band-pass filtered to isolate the frequency and time domain of the targeted chirp. Linear regressions discussed in Chapter 4.4 are then applied to the fluctuations of the SXR emissivities to eliminate non-global contributions [55], as shown in Fig. 5.5(a). Lastly, the filtered signals are inverted to obtain the local emissivities in the major radius domain [71] using the method discussed in Chapter 4.5.1. The inverted SXR emissivity fluctuation is represented as a linear combination of $\delta n_e/n_{e0}$, $\delta T_e/T_{e0}$ and $\delta Z_{eff}/Z_{eff,0}$, as shown in Eq.4.18. The SXR emissivity fluctuation is closely related to radial plasma displacement, as discussed in Chapter 4.5.1. This quantity is commonly used for expressing TAE mode structure in various models (e.g. NOVA) because it fully characterizes incompressible modes in ideal MHD. It is important to note that the uncertainty levels of SXR channels prohibit the reliable inferring of the exact mode amplitude of the targeted TAE, but it is still sufficient to the peaking location of the mode in the major radius domain. The radial profile of the inferred TAE-associated emissivity fluctuation ($\delta \epsilon/\epsilon_0$, as in eq.4.18) peaks near $R = 125\text{cm}$, as shown in Fig. 5.5(b), which is at $\rho \approx 0.65$

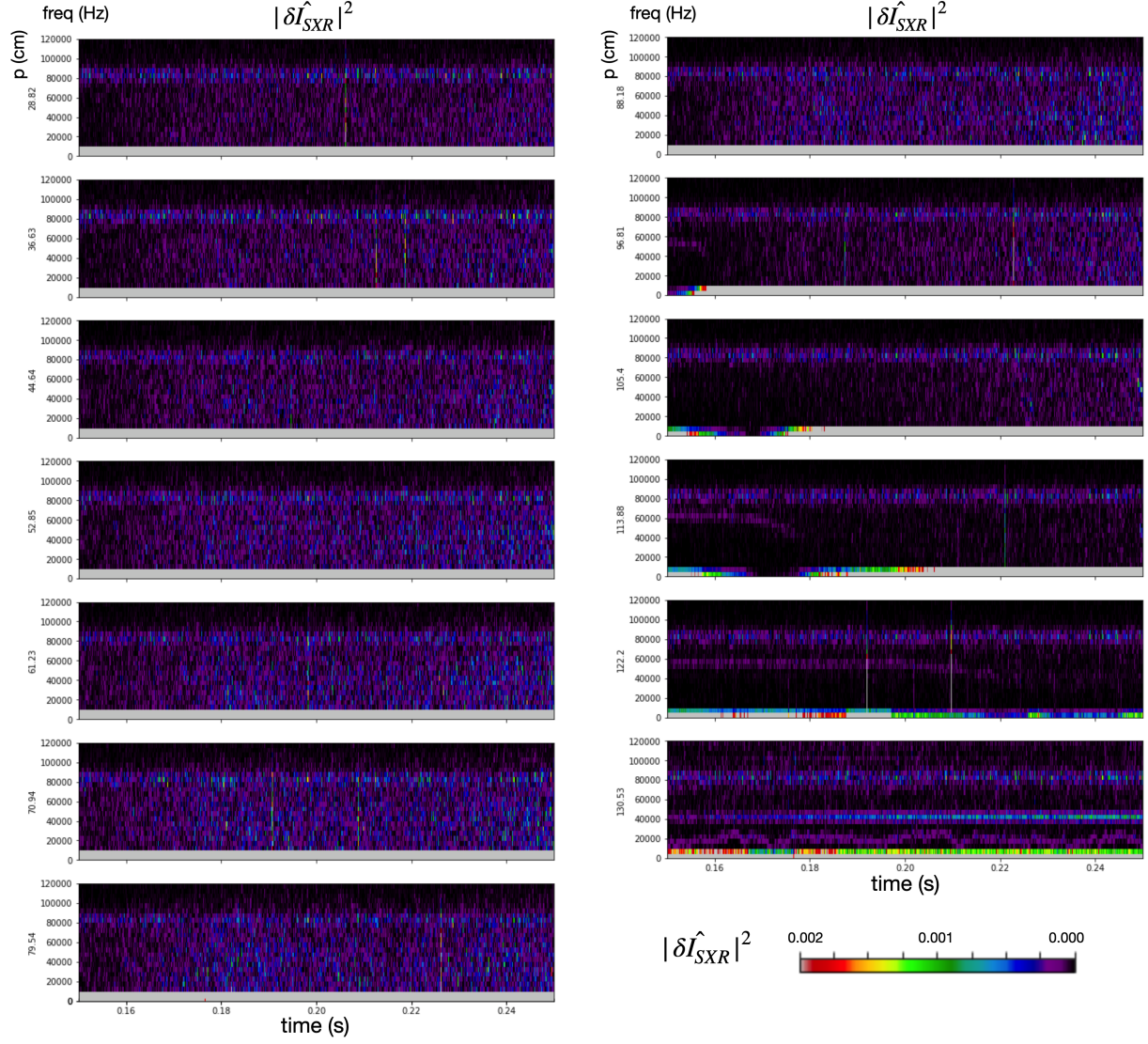


Figure 5.3: Spectrograms (with half-overlapping 0.1ms windows) of 13 tangential SXR camera channels with impact parameters (p) from 28.82 to 130.52cm in MAST discharge 26887 from 150 to 250ms.

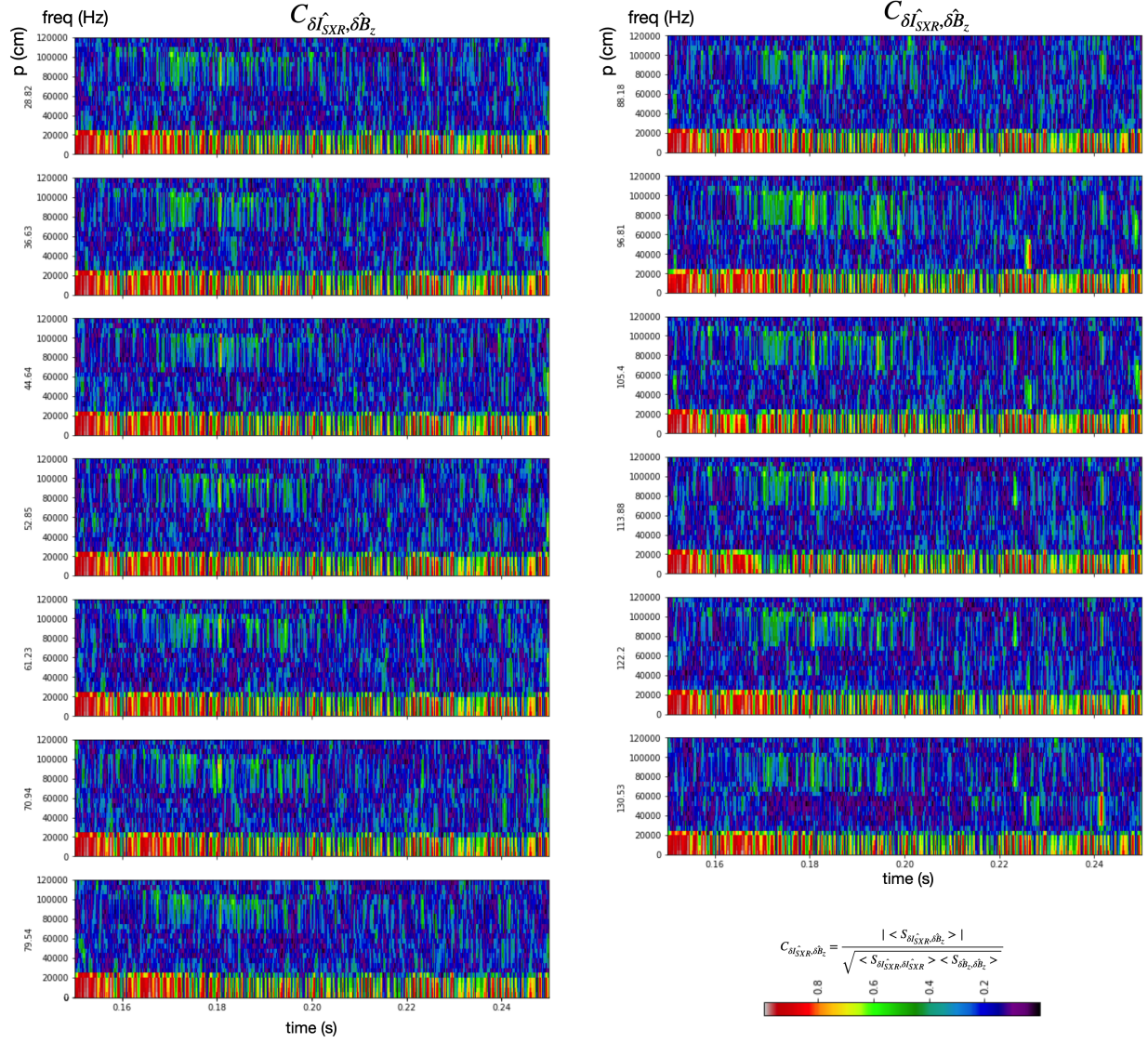


Figure 5.4: Cross coherences (using 0.6ms and 30kHz coherence averaging windows) of STFT of measurements from 13 tangential SXR camera channels with impact parameters (p) from 28.82 to 130.52cm and STFT of δB_z in MAST discharge 26887 from 150 to 250ms.

in the flux coordinate, where $|q| \approx 1.5$ on the outboard (low field) side, as shown in Fig.5.6 (a). In the MHD model, the $n = 1$, $m = [1, 2]$ TAE gap is located near $q = 1.5$ with the frequency of about 70KHz (in the plasma frame) in the equilibrium of the targeted discharge. Measured radial mode structure and mode frequency (with the consideration of rotation Doppler shift) are located inside that $n = 1$, $m = [1, 2]$ TAE gap, suggesting the targeted TAE would have $m = [1, 2]$ of dominant poloidal harmonics in the MHD picture. It is worth noting that the emissivity fluctuation is weaker on the inboard (high field) side than on the outboard side after the consideration of uncertainties, indicating that the measured mode has a ballooning-like structure, which is common for TAEs with frequencies near the bottom of the $n = 1$ TAE continuum gap [70].

The selected MAST discharge is studied through GK simulations using the GTC code, as well as ideal MHD simulations using the eigenvalue NOVA code [21], along with the single-fluid ideal MHD feature integrated within GTC. Temperature and density profiles for thermal electrons and ions used for the simulations (shown in Fig.5.8) are obtained from interpretive TRANSP [14], while the magnetic configuration utilized in the simulations is obtained from MSE and pressure constrained EFIT++, as mentioned previously. TRANSP samples density and temperature radial profiles of thermal ions and electron from experimental data (Thomson scattering and charge exchange recombination spectroscopy), and maps them on the flux surface positions obtained from the EFIT++ equilibrium reconstruction. The symmetrization of the radial profiles from the low and high field side is done by taking averages of data points on each flux surface. Fast ion profiles are modeled using the NUBEAM [76] module of TRANSP without the assumption on activate anomalous fast ion diffusion. The two-dimensional (R,Z) fast ion density profile lacks radial and poloidal symmetry. Hence the GTC runs that have the modes excited by fast ions use NUBEAM results on the outboard (low field) side and at $\rho > 0.3$ only as the reference for fast ion density and temperature profile inputs to provide sufficient fast ion gradients for driving TAE-like modes in cases that have Maxwellian and anisotropic initial fast ion distributions. Notably,

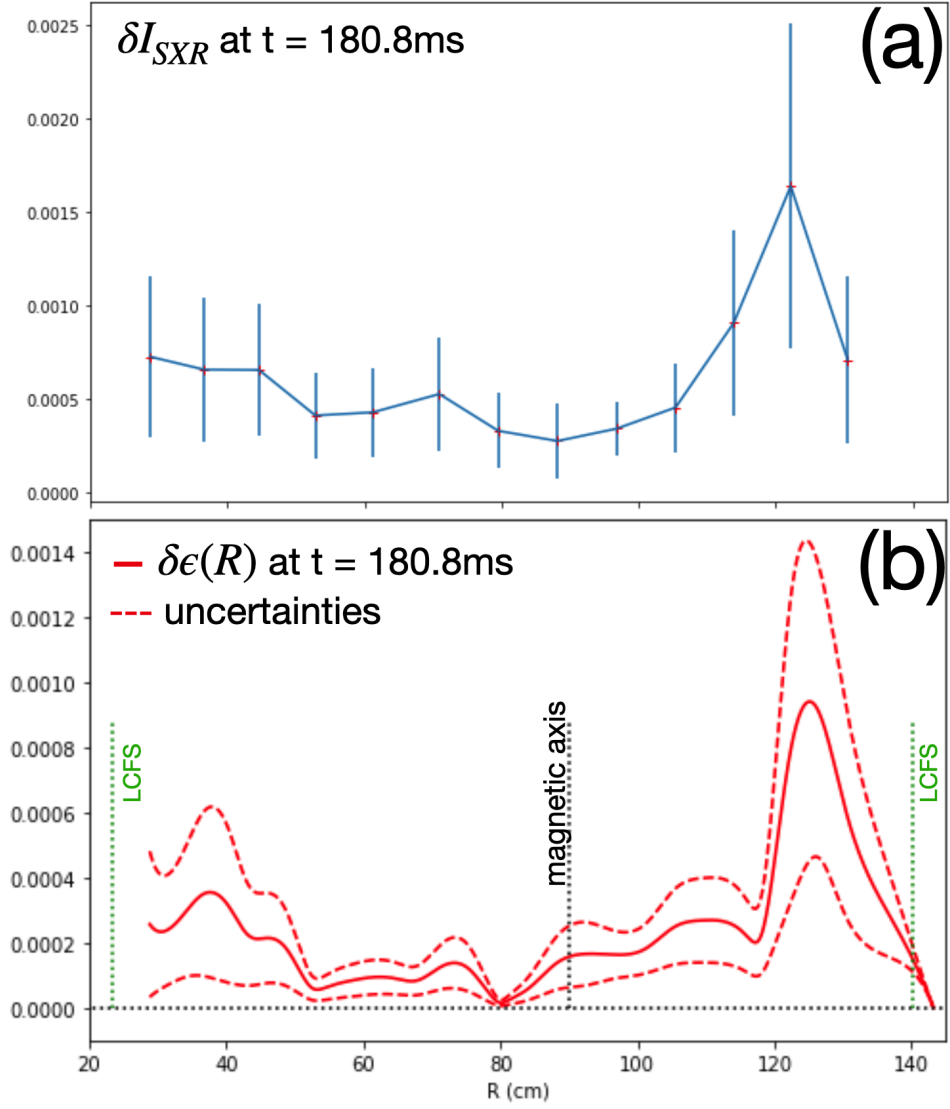


Figure 5.5: (a) filtered line-integrated (b) and inverted SXR emissivity fluctuations associated with the targeted TAE in MAST pulse 26887 in the major radius domain

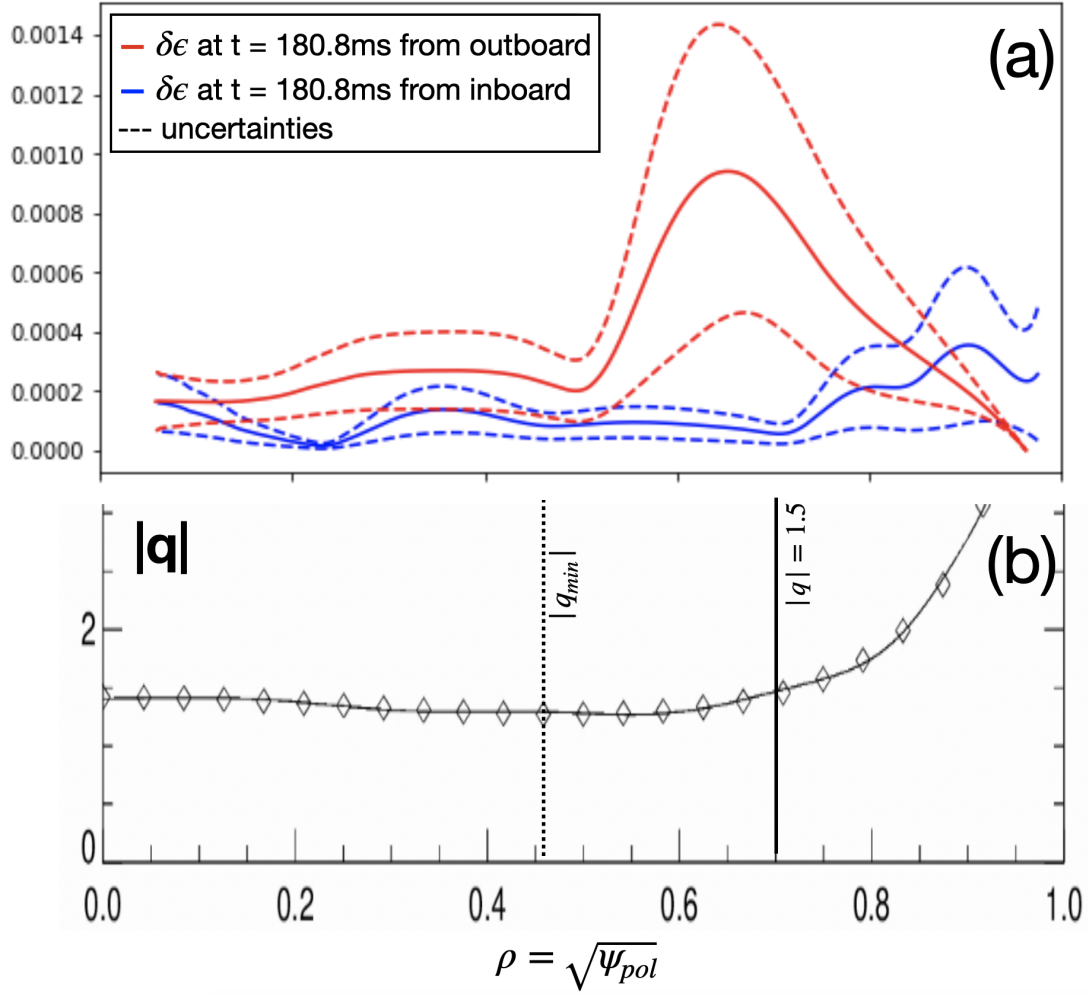


Figure 5.6: (a) Inverted SXR emissivity fluctuations and (b) — q — profile associated with the targeted TAE in MAST pulse 26887

the fast ion temperature radial profile obtained by taking the average of fast ion energy at each radial position suggests that the fast ions are significantly hotter (more than 20 times) and roughly one-fifth as dense as the thermal ions in the core. Hence the maximum fast ion beta $((\beta)_{max} \propto (n * T)_{max})$ is about 5 times higher than the maximum thermal beta in the core. This regime was rarely investigated using global GK simulations previously.

The two-dimensional (R,Z) fast ion density distribution peaks in the outboard region and lacks poloidal symmetry. Consequently, the fast ion pressure gradient is highest in the outboard region, with the peak outboard fast ion pressure gradient occurring near $\rho_{out} = 0.5$, as shown in Fig.5.7(a). However, GTC can only load the fast ions based on poloidally symmetric density profiles, and using the poloidally averaged NUBEAM fast ion density profile leads to an insufficient initial pressure gradient to drive the TAE, especially considering modes with ballooning structure are more sensitive to the outboard pressure than the inboard region. Therefore, an analytic profile with the maximum gradient peaking at $\rho = 0.6$ (as shown in Fig.5.8(a)), which is close to $q = 1.5$ and similar to the outboard fast ion density profile shown in Fig.5.7(b), is used in GTC simulations for the purpose of loading the fast ion distributions. After loaded to the simulations, the full orbit fast ion profiles will eventually become poloidally asymmetric as the fast ions move through their orbits.

Besides radial and 2D profiles, the NUBEAM calculation also provides slowing-down fast ion distributions in pitch and energy. Initially, fast ions generated by NBI concentrate near injection energy and pitch, undergoing slowing down, diffusion, and scattering through collisions and diffusion with the electrons and ions in a thermal background plasma [86][85][89][74]. The beam injects a mix of ions at the full (60keV), 1/2 and 1/3 energy levels of the nominal beam voltage, with most of the ions being at the full energy. The resulting slowing-down distribution retains most fast ions near their birth pitch at three energy levels. NUBEAM accounts for the geometries of NBI and the tokamak, determining the birth pitch angle of fast ions at different radii. Although fast ions are born within a relatively short segment of the beam path through the plasma, this segment is a significant fraction of

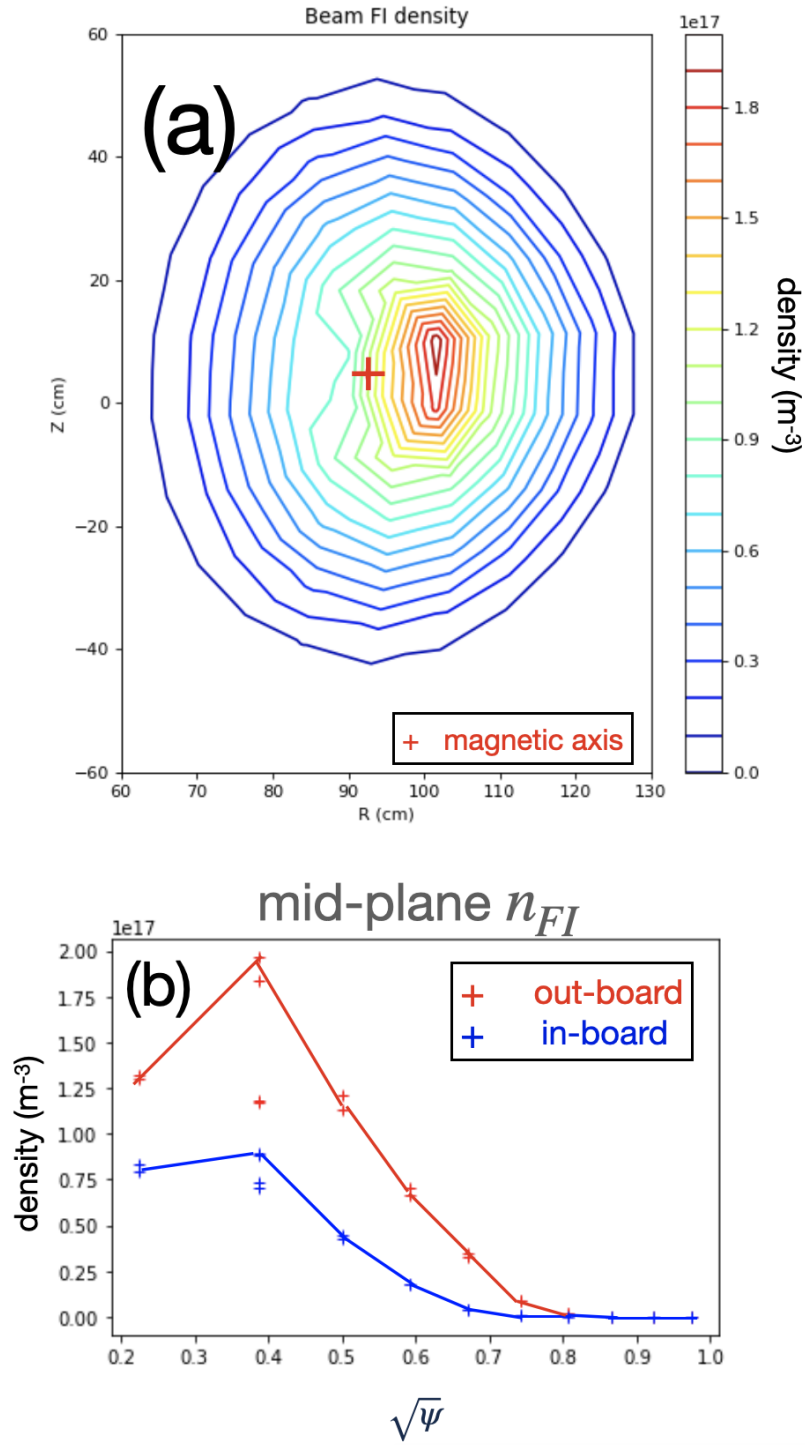


Figure 5.7: (a) 2D and (b) mid-plane radial fast ion density profiles from NUBEAM/TRANSP for MAST pulse 26887, $t = 180\text{ms}$

the minor radius in compact spherical tokamak pulse like the one in MAST presented here. Consequently, the birth pitch changes rapidly, from $\mu B_0/E \approx 0.6$ at $R \approx 1.1$ m near the location of peak fast ion density, to $\mu B_0/E \approx 0.15$ at $R \approx 1.3$ m near the $q = 1.5$ surface, as shown in Fig.5.9.

All GTC simulations, including single-, two-fluid ideal MHD and GK simulations employing various methods to excite the mode, have nearly identical simulation parameters (except for slight modifications to the boundary settings) to ensure consistency. The TAE simulations using antenna excitation have the inner boundary set to $\rho = 0.1$, and simulations that include fast ion distributions have the inner boundary set to about $\rho = 0.2$. The region beyond $\rho = 0.9$ up to the last closed flux surface (LCFS) is also excluded in all simulations. Radial linear decays near the inner and outer boundaries of the simulation perimeter are applied. These boundary settings have been carefully selected from a series of GTC test runs with the lowest levels of numerical instabilities among the test runs to mitigate their effects on accurately measuring the mode characteristics. The GTC simulations exclusively incorporate $n = 1$ and $m \in [0, 7]$ modes, which should sufficiently account for the dominant harmonics of the low frequency $n = 1$ TAE.

5.2 GTC gyrokinetic formulation and its reduction to ideal MHD

The Gyrokinetic Toroidal Code (GTC) [66] employs a gyrokinetic particle-in-cell (PIC) method to handle both thermal and fast ions, but can also be operated using reduced models including two- and a single-fluid models of thermal species by suppressing kinetic effects of thermal plasmas. The gyrokinetic equation (Equation 2 in [53]) is used to describe toroidal plasmas in the inhomogeneous magnetic field [15] with the gyrocenter position, magnetic moment μ , and parallel velocity $v_{\parallel}$ as independent variables. The distribution functions of different species can be decomposed into equilibrium (f_{0i}) and perturbed (δf_i) parts. Although this work solely focuses on linear perturbative simulations, GTC is capable of

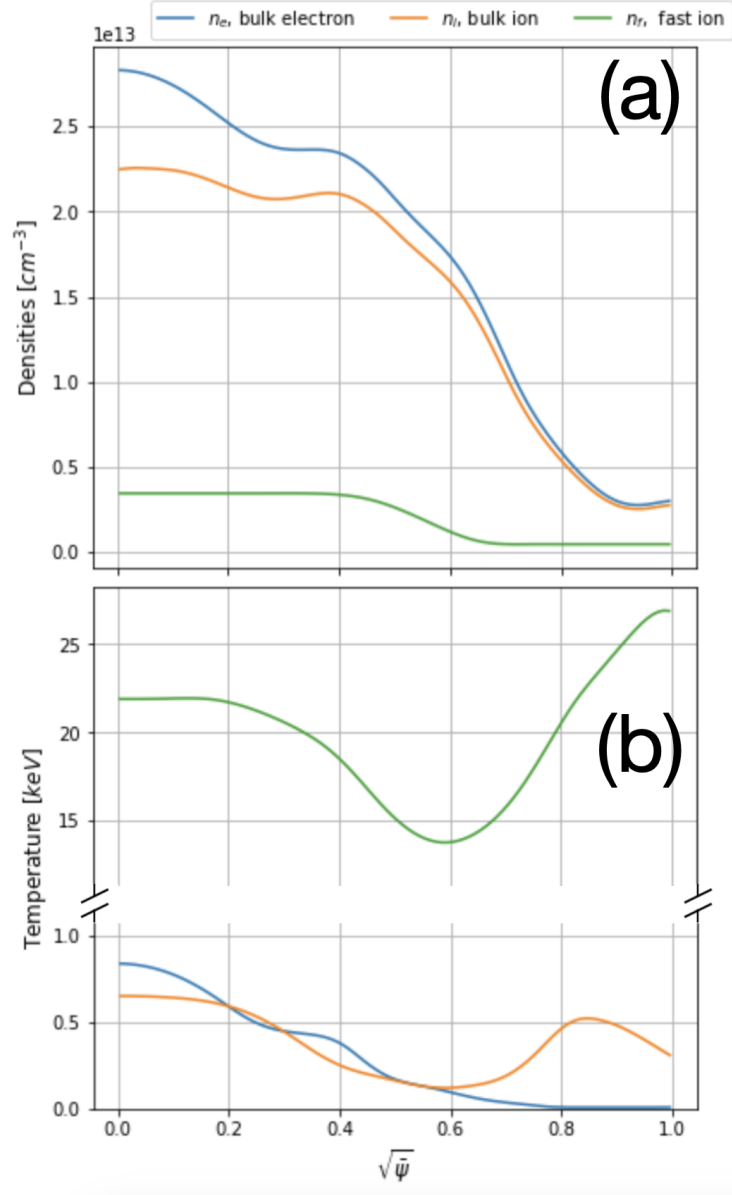
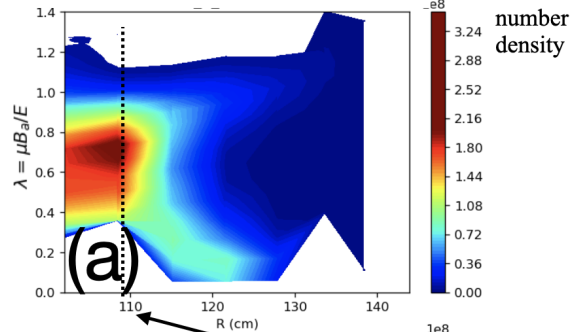
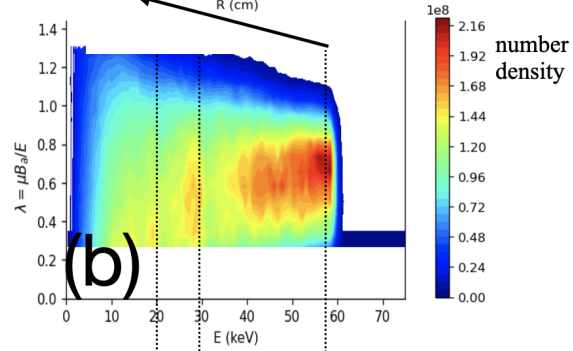


Figure 5.8: (a) Radial density and (b) temperature profiles from NUBEAM/TRANSP from thermal ions, electrons and fast ions used in GTC simulations.

NUBEAM λ vs. R
near injection E
of 56keV



NUBEAM λ vs. E
at R = 1.08m,
mid-plane



analytic λ vs. E
fast ion
distribution used
in GTC

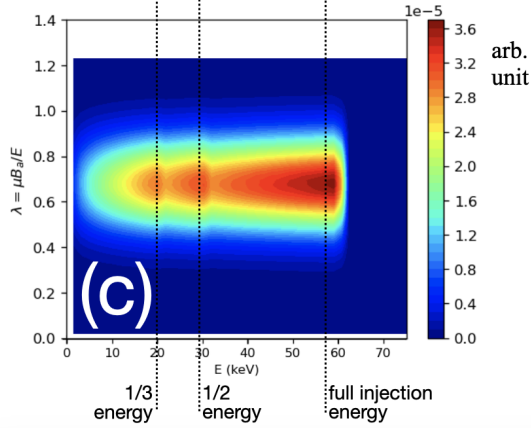


Figure 5.9: Fast ion distribution from NUBEAM calculated for MAST pulse 26887, $t = 180\text{ms}$. (a) λ vs. R near injection energy; (b) λ vs. E at $(R,Z) = 1.08\text{m}, 0.01\text{m}$; (c) the analytic fast ion slowing-down distribution used in a linear GTC run, mimicking the injection λ and three birth energies (labeled in dash lines) apparent in the distribution shown in (b)

handling non-linear evolution of full- f and δf distributions. The gyrokinetic equation is expressed as follows, with the parallel perturbed magnetic field $\delta B_{||}$ neglected for low- β plasma [25]:

$$(\partial_t + \dot{\mathbf{X}} \cdot \nabla + v_{||} \partial_{v_{||}})[f_0(\mathbf{X}, \mu, v_{||}) + \delta f(\mathbf{X}, \mu, v_{||}, t)] = 0. \quad (5.1)$$

Here, $\dot{\mathbf{X}}$, $v_{||}$, and μ are defined in [25], in terms of equilibrium magnetic field (B_0), perturbed magnetic field (δB), electrostatic potential (ϕ), and the parallel vector potential ($A_{||}$). With gyro-averaging applied to δB , ϕ , and $A_{||}$ in the formulation, assumptions of zero equilibrium electric field ($\phi_0 = 0$) and time-independent equilibrium magnetic field ($\partial_t A_{||0} = 0$) are made. Consequently, only the perturbed parts with zonal ($\phi_{00}, A_{||00}$) and nonzonal ($\delta\phi, \delta A_{||}$) components are necessary to describe the potentials $\phi = \phi_{00} + \delta\phi$ and $A_{||} = A_{||00} + \delta A_{||}$.

In the fluid-kinetic electron hybrid model, electron distribution function is expanded into an adiabatic part (calculated by a fluid equation) and non-adiabatic part (calculated by the PIC method). The fluid continuity equation describes the electrons as a fluid with using the velocity from the massless momentum equation, and the non-adiabatic part of the electron distribution function defines the electron kinetic effects using a realistic electron mass. The equilibrium distributions of thermal electrons and ions are described as Maxwellian distributions, while either Maxwellian or analytic slowing-down distributions can be used to define the initial states of the full orbit fast ions. The zonal and non-zonal perturbed electrostatic potentials (ϕ_{00} and $\delta\phi$, respectively) are obtained by solving the gyrokinetic Poisson's equation [25]. The parallel electric field is defined as

$$\delta E_{||} = -\mathbf{b}_0 \cdot \nabla \phi_{eff}, \quad (5.2)$$

where $\mathbf{b}_0 = \mathbf{B}_0/|B_0|$, and the effective potential ϕ_{eff} contributes to the inductive potential $\phi_{ind} = \phi_{eff} - \delta\phi$. Subsequently, the perturbed parallel vector potential can be calculated by:

$$\partial_t \delta A_{||} = \mathbf{b}_0 \cdot \nabla \phi_{ind} \quad (5.3)$$

The inclusion of ϕ_{eff} results in a model similar to an extended hybrid GK-MHD model discussed in [115] and [100] but with the addition of pressure-driven terms. This model can be

further reduced to ideal MHD by setting $\delta E_{\parallel} = 0$, resulting in the exact formulation shown in Equation (53) in [25], which includes terms that describe the bending of field lines responsible for shear Alfvén modes, current-driven modes such as the kink mode, and pressure-driven instabilities. A notable distinction between the extended hybrid GK-MHD model and the ideal MHD model lies in the inclusion of $\delta E_{\parallel}$. A parallel electric field can lead to particle acceleration, alter the shear wave dispersion relation [36], and introduce additional damping compared to the ideal MHD model or the conventional hybrid GK-MHD model without parallel electric fields.

GTC offers different run modes by a systematic reduction of the gyrokinetic equation to explore various growth and damping mechanisms by treating the ϕ_{ind} and the gradient of fast ion distributions differently. The gyrokinetic Poisson equation and Ampère's law can be combined and reduced in the long wavelength ($k_{\perp}\rho_i < 1$) limit, taking $\partial_t \rightarrow -i\omega$ for a mode with single frequency:

$$\frac{\omega^2}{v_A^2} \nabla_{\perp}^2 \delta\phi + B_0 \cdot \nabla \left[\frac{\nabla_{\perp}^2 (b_0 \cdot \nabla \phi_{ind})}{B_0} \right] + i\omega \frac{4\pi}{c^2} \sum_{\alpha=i,e} (-i\omega Z_{\alpha} \delta n_{\alpha} + \nabla \cdot \delta J_{\parallel\alpha}) = 0 \quad (5.4)$$

. Here, $\delta J_{\parallel\alpha}$ and δn_{α} represent current and density perturbations, respectively, resulting from the integral of velocity and/or δf in gyro-center coordinate, with Z_{α} being the effective charges of ions and electrons.

The ion gyrokinetic equation can be further reduced in the long wavelength limit to describe δn_{α} and $\delta J_{\parallel\alpha}$ [25]. As a result, the equation can be simplified into the linear model for low-frequency modes:

$$\begin{aligned} 0 = & \frac{\omega(\omega - \omega_* P)}{v_A^2} \nabla_{\perp}^2 \delta\phi + \underbrace{B_0 \cdot \nabla \left[\frac{\nabla_{\perp}^2 (b_0 \cdot \nabla \phi_{ind})}{B_0} \right]}_{(1)} - \underbrace{\nabla (b_0 \cdot \nabla \phi_{ind}) \times b_0 \cdot \nabla \frac{b_0 \cdot \nabla \times B_0}{B_0}}_{(2)} \\ & \underbrace{-i\omega \frac{4\pi}{c} \left[\nabla \times b_0 \cdot \nabla \left(\frac{\delta P_{\parallel}}{B_0} \right) + b_0 \times \nabla B_0 \cdot \left(\nabla \frac{\delta P_{\perp}}{B_0^2} \right) + \frac{\nabla \times b_0 \cdot \nabla B_0}{B_0^2} \delta P_{\perp} \right]}_{(3)}. \end{aligned} \quad (5.5)$$

Here, $\delta P_\perp$ and $\delta P_\parallel$ represent perpendicular and parallel pressure, respectively, and ω_* is the ion diamagnetic frequency. The term (1) describes the bending of field lines responsible for shear Alfvén modes, term (2) is responsible for current-driven modes such as the kink mode, and term (3) pertains to pressure-driven instabilities. This equation is similar to the ideal MHD model shown in [25] but keeping a non-zero ϕ_{eff} . It resembles a hybrid MHD-GK model shown in [115] and [100] but with the addition of pressure-driven terms.

The above hybrid model can be reduced to ideal MHD by setting $\delta E_\parallel = 0$. Applying the normal mode assumption yields a one-dimensional wave vector $i\vec{k} = \nabla$. Consequently, ϕ_{ind} reduces to $\delta\phi$, and $b_0 \cdot \nabla$ becomes $ik_\parallel$, resulting in the ideal MHD model:

$$0 = \frac{\omega(\omega - \omega_* P)}{v_A^2} \nabla_\perp^2 \delta\phi + \underbrace{iB_0 \cdot \nabla \left[\frac{\nabla_\perp^2 (k_\parallel \delta\phi)}{B_0} \right]}_{(1)} - \underbrace{i \nabla (k_\parallel \delta\phi) \times b_0 \cdot \nabla \frac{b_0 \cdot \nabla \times B_0}{B_0}}_{(2)} \quad (5.6)$$

$$\underbrace{-i\omega \frac{4\pi}{c} \left[\nabla \times b_0 \cdot \nabla \left(\frac{\delta P_\parallel}{B_0} \right) + b_0 \times \nabla B_0 \cdot \left(\nabla \frac{\delta P_\perp}{B_0^2} \right) + \frac{\nabla \times b_0 \cdot \nabla B_0}{B_0^2} \delta P_\perp \right]}_{(3)} \quad (5.7)$$

A notable distinction between equations (5.5) and (5.7) lies in the wave vectors in terms (1) and (2), which describe shear Alfvén modes and current-driven modes, respectively. In the hybrid model, energy is "shared" between $k_\parallel$ and other directions, unlike in the ideal model, where it is solely attributed to $k_\parallel$. Additionally, differences in term (3) stem from the dependence on ϕ_{eff} in $\delta P_\perp$ and $\delta P_\parallel$. GTC assumes their lowest order terms $\delta P_\perp^{(0)}$ and $\delta P_\parallel^{(0)}$ to be dependent on the lowest order component of the effective potential ($\phi_{eff}^{(0)}$), which is influenced by the poloidal and toroidal curvatures of the density profile [53]. These terms, previously overlooked in cases of low-frequency shear modes in conventional tokamaks, may become more significant in the stronger curvatures found in STs. Such modifications, due to the presence of ϕ_{eff} , would impact the mode frequency, as well as the growth and damping rates. GTC offers different run modes to explore these regimes. For instance, it can test

both models by toggling ϕ_{eff} on or off, examining the ideal MHD (described in Chapter.3.2) and kinetic regimes for low-frequency modes.

The $\delta A_{||}$ and $\delta\phi$ perturbations defined previously are used in the gyrokinetic particle weight ($w_i = \delta f_i / f_i$) equation that handles the evolution of perturbed distribution functions, rewritten as the following:

$$\begin{aligned} \frac{dw_i}{dt} = (1 - w_i) & \left[-(v_{||} \frac{\delta \mathbf{B}}{B_0} + v_E) \cdot \frac{\nabla f_{0i}}{f_{0i}}|_{\mu} + \left(\mu \frac{\delta \mathbf{B}}{B_0} \cdot \nabla B_0 \right. \right. \\ & \left. \left. + eZ_i \frac{\mathbf{B}^*}{B_0} \cdot \nabla \delta\phi + eZ_i \partial_t \delta A_{||} \right) \left(\frac{1}{m_i f_{0i}} \frac{\partial f_{0i}}{\partial v_{||}} \right) \right], \end{aligned} \quad (5.8)$$

where i is the species, v_E is the $E \times B$ drift velocity, $\mathbf{B}$ is the total magnetic field, $\mathbf{B}^*$ is related to $\mathbf{B}$ (defined in [53]), and eZ_i is the effective charge of the species. Some of the quantities in the weight equation will be handled differently to test various physics regimes. For instance, the code provides the option of turning finite $\delta E_{||}$ on and off by treating $\partial_t \delta A_{||}$ differently in the two- and single-fluid ideal MHD run modes, respectively, to test continuum and radiative damping mechanisms separately. Ion Landau damping can also be added on top of these damping mechanisms when the thermal ions are in the full- or gyro-kinetic regimes.

5.3 Maxwellian and Anisotropic distributions in GTC

GTC incorporates thermal ion temperature and density gradients, which contribute to the TAE instability drives, into Maxwellian distributions in the simulations that consider gyrokinetic thermal ions. The Maxwellian distribution is defined as follows:

$$f_{0a} = \frac{n_{0a}}{(2\pi T_{0a}/m_i)^{3/2}} \exp \left(-\frac{2\mu B_0 + m_a v_{||}^2}{2T_{0a}} \right), \quad (5.9)$$

where the subscript a represents the species, and n_{0a} , T_{0a} , and m_a represent the species' density, temperature, and mass, respectively, with B_0 being the equilibrium magnetic field.

The gyrokinetic weight equation (Equation 5.8) shows that the mode structure depends on the gradients of the distributions ($\nabla f_{0a}/f_{0a}$ and $(1/f_{0a})(\partial f_{0a}/\partial v_{||})$) defined as:

$$\begin{aligned} \frac{\nabla f_{0a}}{f_{0a}}|_{\mu} = & \left(\frac{1}{n_{0a}} \frac{\partial n_{0a}}{\partial \psi} + \left(-\frac{3}{2T_{0a}} + \frac{2\mu B_0 + m_a v_{||}^2}{2T_{0a}^2} \right) \frac{\partial T_{0a}}{\partial \psi} \right) \nabla \psi \\ & - \frac{\mu}{T_{0a}} \nabla B_0 \end{aligned} \quad (5.10)$$

and

$$\frac{1}{f_{0a}} \frac{\partial f_{0a}}{\partial v_{||}} = -\frac{m_a v_{||}}{T_{0a}}, \quad (5.11)$$

respectively.

Fast ions can also be included in GTC to excite modes. These fast ions can be initially loaded in a Maxwellian distribution, similar to the treatment of thermal ions (Equation 5.9). The fast ion profiles will eventually become poloidally asymmetric as the fast ions move through their orbits. Alternatively, the fast ions can also be initially loaded in an analytic fast ion slowing-down distribution [74], defined as:

$$f_{0f} = \frac{n_{0f} H(v_0 - v)}{v^3 + v_c^3} \exp \left(- \left(\frac{\lambda - \lambda_0}{\Delta \lambda} \right)^2 \right) \quad (5.12)$$

Here f is the label for the species, which in this case is just for the fast ions from a single NBI injection while the thermal species are still represented by the Maxwellian distribution (Equation 5.9). $v = \sqrt{v_{||}^2 + v_{\perp}^2}$ where $v_{||}$ is the parallel velocity of the particles, $v_c = v_{c,0} \sqrt{T_f/m_f}$ is the critical velocity (where $v_{c,0}$ is a dimensionless scale factor, and T_f is the fast ions' temperature), $v_0 = v_{0,0} \sqrt{T_f/m_f}$ is the birth velocity (where $v_{0,0}$ is a dimensionless scale factor), H is the Heaviside step function, $\lambda = \mu B_0/E = (B_0/B)(v_{\perp}^2/v^2)$ is related to the pitch of the particle with respect to the on-axis magnetic field B_0 , E is the fast ion's kinetic energy, λ_0 is a constant associated with the birth pitch, and $\Delta \lambda$ is the

width of the pitch angle distribution. The gradients of the analytic slowing-down fast ion distribution ($\nabla f_{0f}/f_{0f}$ and $(1/f_{0a})(\partial f_{0a}/\partial v_{||})$) are given by:

$$\begin{aligned} \frac{\nabla f_{0f}}{f_{0f}}|_{\mu} &= \left(\frac{1}{n_{0f}} \frac{\partial n_{0f}}{\partial \psi} - \frac{3v_c^3}{2(v^3 + v_c^3)T_f} \frac{\partial T_{0f}}{\partial \psi} \right) \nabla \psi \\ &+ \left(\frac{2\lambda^2}{B_a \Delta \lambda^2} (\lambda - \lambda_0) - \frac{1}{m_f} \frac{3\mu v}{v^3 + v_c^3} \right) \nabla B_0 \end{aligned} \quad (5.13)$$

and

$$\frac{1}{f_{0a}} \frac{\partial f_{0a}}{\partial v_{||}} = \frac{2m_a v_{||} \lambda^2}{\mu B_0 \Delta \lambda^2} (\lambda - \lambda_0) - \frac{3v v_{||}}{v^3 + v_c^3}, \quad (5.14)$$

respectively. These gradients determine the mode structure when an anisotropic fast ion distribution is considered.

5.4 Continuum and NOVA simulations of the MAST TAE test case

The Alfvén continuum is computed using the continuum modules bundled with NOVA and GTC, as shown in Fig.5.10. Unlike the NOVA code, the GTC code in δf run mode itself does not calculate the continuum directly. Instead it uses a bundled ALCON module [?] to solve ideal MHD Alfvén continua in tokamaks with the provided inputs for GTC. Comparing these continua is crucial to ensure similarity in inputs used by NOVA and GTC, in view of the differences in the treatment of magnetic configurations and equilibrium conditions [21]. Additionally, the continuum provides insights into the TAE mode frequency and radial structure.

Generally, the continua from NOVA and single-fluid GTC align well with each other, exhibiting an $n = 1$ TAE gap ranging from about 80 to 130 kHz at $\rho = 0.2$, and widening to a range from 60 to 200 kHz at $\rho = 0.5$. The TAE gap in the targeted MAST case is notably

broader than in conventional tokamaks due to the low aspect ratio [70]. Discrepancies arise near the edge ($\rho > 0.8$) in the part of the continuum associated with higher m harmonics, probably due to the different approaches employed by the two codes in handling equilibrium conditions. For example, maxima of the lower GTC continuum and minimum of the upper GTC continuum near $\rho = 0.65$ are both shifted down compared to those of the NOVA continuum, possibly due to differences on the treatments on the input profiles, such as the magnetic shear profiles used by the two continuum modules. The discrepancies here can in principle result in differences in mode characteristics (such as eigen-frequencies) calculated by GTC and NOVA. Discrepancies in GTC and NOVA continua also exist near $\rho = 0.85$. However, the measured mode extent of the targeted TAE stays within $\rho < 0.85$. Hence the discrepancy near $\rho = 0.85$ is unlikely to significantly impact the low-mode-number ($m \in [0, 2]$) TAEs found in the simulations.

The NOVA code handles the magnetic configuration and plasma thermal profiles slightly differently from GTC. The code samples the region either above or below the mid-plane and assumes up-down symmetry in the magnetic configuration. The actual magnetic configuration, inferred from MSE-constrained EFIT++, indicates a slight up-shift of the magnetic axis by less than 3 cm at $t \approx 180$ ms. Consequently, the lower region is selected to avoid issues from the axis location. The NOVA simulations encompass $n = 1$ and $m \in [0, 19]$ modes. Like GTC, solutions from NOVA are sometimes contaminated by non-TAE-like continuum intersection when a mode touches the continuum, resulting in strong continuum damping. However, unlike GTC, which is an initial value code, the eigenvalue code NOVA yields numerous possible solutions. Only solutions exhibiting minimal continuum interactions are analyzed. Note that the mode spatial structures and poloidal harmonics from NOVA are represented by the plasma radial displacement (ξ) while GTC solutions are expressed in terms of the electrostatic potential perturbation ($\delta\phi$). The two can be linked through their definitions of fields [101]:

NOVA:

$$\delta B = \nabla \times (\xi \times B_0), \delta E = -\partial_t \xi \times B_0 \quad (5.15)$$

GTC:

$$\delta B = \nabla \times (\delta A_{\parallel} b_0), \delta E = -\nabla \delta \phi - \partial_t \delta A_{\parallel} b_0 \quad (5.16)$$

Ideal MHD simulations are conducted using the NOVA code to establish a baseline for the other simulations reported in this chapter. NOVA simulations on TAEs in NSTX have been validated and reported numerous times [38][31], yielding solutions that agree with the frequency and radial mode structure at the linear stage (without or at the beginning of frequency chirping) of the mode. Using NOVA, a cluster of $n = 1$ TAE solutions in MAST near 90 kHz (in the lab frame) is identified, consistent with the frequency measured by OMAHA before chirping occurred in MAST shot 26887 at approximately 181 ms. The radial mode structures and poloidal harmonics are similar among the cluster of solutions, with $m = 1, 2$ dominating and peaking near the $q = 1.5$ surface near $\rho = 0.6$, shown in Fig.5.11. This agrees well with analytic theory in the large aspect ratio limit, which indicates that TAE peaks at the $q = \frac{m+1/2}{n}$ surface [34].

5.5 Antenna-excited TAEs in GTC

Eigenmodes excited by antennas are used to determine mode frequencies and damping rates in single- and two-fluid ideal MHD and linear GK regimes in GTC. In GTC, the antenna inserts an additional synthetic potential (ϕ_{ant}) into the inductive potential: $\phi_{ind} = \phi_{eff} - \delta\phi + \phi_{ant}$, with ϕ_{ant} represented as a standing sinusoidal signal [112]:

$$\phi_{ant} = \phi_{ant,0}(\zeta, \theta, x) \cos(\omega_{ant} t), \quad (5.17)$$

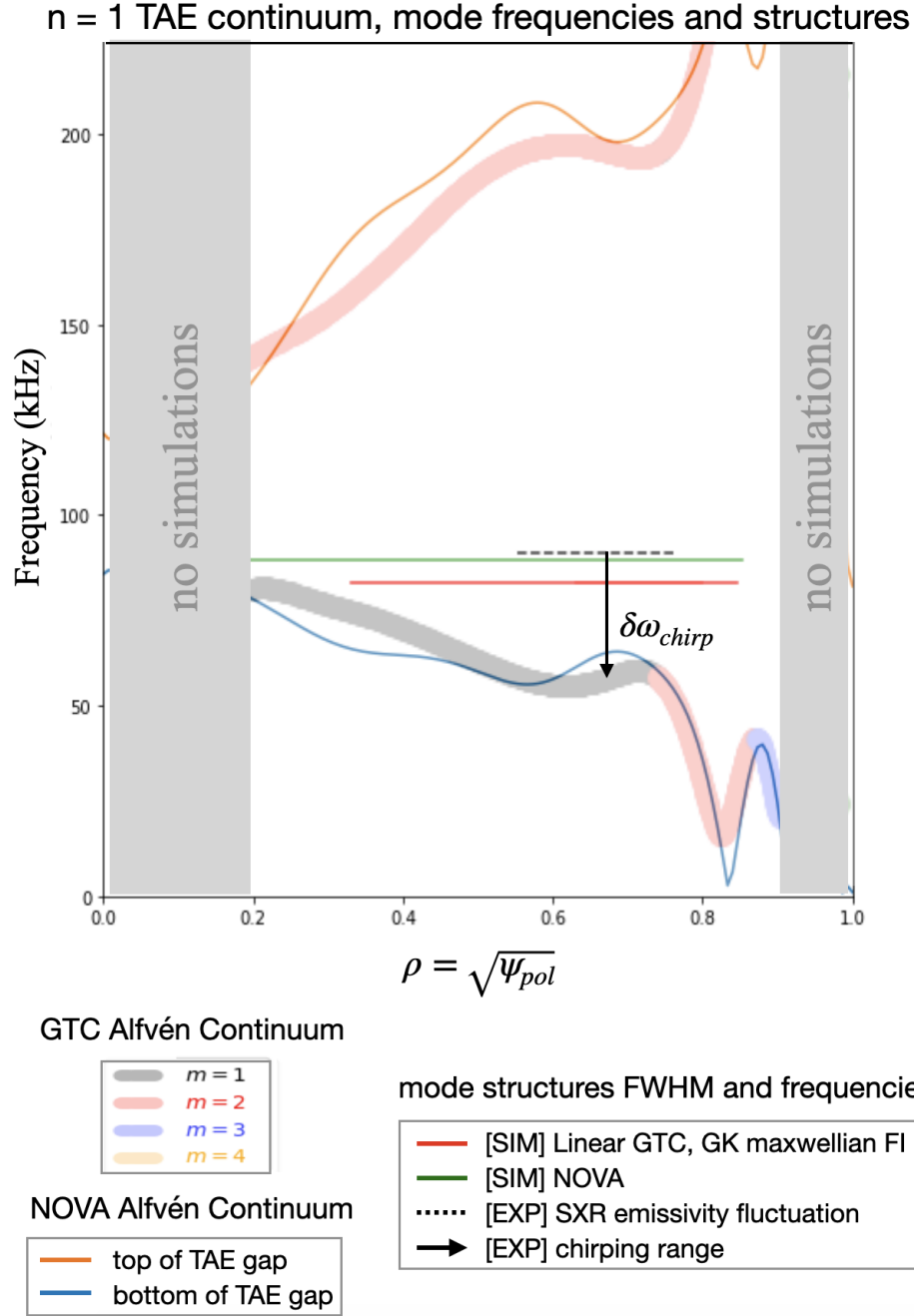


Figure 5.10: Mode structure FWHM from measurement (dashed line), NOVA (green line) and GTC (red line) together with Alfvén continuum in the range of TAE gap generated by GTC and NOVA calculated for MAST pulse 26887, $t = 180\text{ms}$.

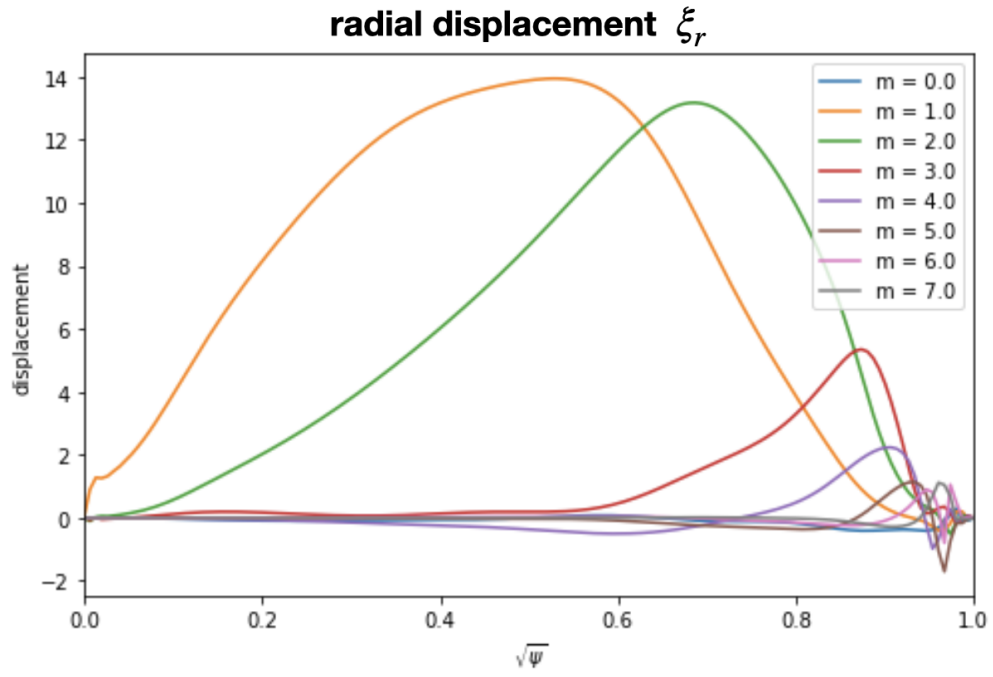


Figure 5.11: Radial displacement of TAE at $\approx 89\text{kHz}$ from NOVA calculated for MAST pulse 26887, $t = 180\text{ms}$.

where ζ and θ are toroidal and poloidal angles, respectively, and the antenna spatial profile $\phi_{ant,0}(\zeta, \theta, x)$ in the absence of fast ions is defined as follows:

$$\phi_{ant,0} = \begin{cases} C_{amp} \left(\frac{\exp\left(\frac{-2(x-x_{ctr})^2}{\sigma_w^2}\right) - \exp\left(\frac{-0.5x_{bd}^2}{\sigma_w^2}\right)}{(1 - \exp\left(\frac{-0.5x_{bd}^2}{\sigma_w^2}\right))} \right) \cos(m\theta - n\zeta), & \text{if } |x - x_{ctr}| < x_{bd} \\ 0 & \text{otherwise} \end{cases}, \quad (5.18)$$

where $x \in [0, 100]$ is the radial grid in GTC simulations, $C_{amp} = 10^{-4}$ is the antenna amplitude, $x_{ctr} = 70$ is the radial location of peak antenna amplitude in the GTC grid set near the $q = 1.5$ surface, x_{bd} is the radial size of the antenna in the GTC grid, and the width $\sigma_w = x_{bd}/2$. x_{bd} is limited to 20 to ensure that the antenna radial profile does not touch simulation boundaries. The experimental modes were not antenna driven. Hence the purpose of exciting the TAE with an antenna is to accurately measure the mode frequencies and damping rates without influences from numerical instabilities near boundaries, instead of aiming to recover the global radial mode structure computed by NOVA or measured by SXR. Hence the radial extent of the antenna envelope ($x_{bound} = 20$) is limited to region around $q = 1.5$ only to ensure that no modes or instabilities other than the $n = 1, m = 1$ TAE are excited by the antenna.

The excited mode will follow one of two scenarios. The modes are observed to grow linearly up to the amplitude where the plasma damping balances with the antenna driving when the antenna frequencies match the TAE eigen-frequency. Otherwise, the mode intensity grows with oscillations due to the frequency mismatch and saturates at a lower amplitude. The relationship between saturated intensity, antenna frequency, eigen-frequency, and damping rate follows the driven resonant cavity theory [111]:

$$A^2 \propto \frac{1}{(\omega_E^2 + \gamma_D^2 - \omega_{ant}^2)^2 + 4\gamma_D^2\omega_{ant}^2}, \quad (5.19)$$

where ω_E is the eigen-frequency, γ_D is the damping rate, and A^2 is the normalized saturated intensity. This relation indicates that the mode would exhibit the strongest A^2 when $\omega_E =$

ω_{ant} for a fixed γ_D . TAEs exist in the continuum gap, hence a resonant cavity model is used instead of a regular driven-damped mode model. A scan of antenna frequencies with the same settings of other parameters, such as the antenna spatial profiles, gives out a saturated intensity profile related to the antenna frequencies, as shown in Fig. 5.12. The eigen-frequency of the TAE is near the frequency with the peak normalized saturated intensity.

Another set of simulations is performed to investigate TAE properties in the GTC single-fluid model. The mode eigen-frequency, damping rate due to continuum damping and the ratio of poloidal harmonics are determined using the antenna module in single-fluid run modes in GTC. After measuring the frequency with the peak normalized saturated intensity, a new calculation is performed to first excite the mode at that frequency, and then to measure to decay of the mode begins after the antenna is turned off, where γ_D and the actual ω_E can be measured by fitting the decaying oscillator function: $A(t) \propto e^{i\omega_E t} e^{\gamma_D t}$. A TAE with $n = 1$, $m = 1, 2$ is expected peak near the $q = 1.5$ surface. Therefore, $\phi_{ant,0}$ is positioned near $|q| = 1.5$ with mode numbers $n = 1$ and $m = 1$ in the antenna excitation of TAE. The single-fluid run mode has $\phi_{eff} = 0$ and hence $\delta E_{||} = 0$. The antenna is kept on until the amplitude of the excited mode is saturated, then the antenna is turned off to measure an eigen-frequency of 70.1 kHz, suggesting a TAE eigen-frequency close to 80 kHz after considering the plasma rotation frequency of about 10 kHz in this case. This frequency is about 8kHz (or 10 %) lower than the value predicted by NOVA and the initial frequency of the TAE. But it is interesting to note that the frequency from GTC matches closely the expected large aspect ratio ideal MHD (n,m) = 1,1 TAE frequency ($v_A/2qR$, where v_A is the Alfvén speed). Similar to the continuum calculations shown in Fig.5.10, the discrepancy in the predicted mode frequencies from both codes would be due to the difference in NOVA and GTC predicted frequency mostly by differences input profiles, not the differences in physics. The damping rate is measured to be $\gamma_D = -(2\pi) * 3342 \text{ s}^{-1}$ and $\gamma_D/\omega = -4.77\%$. The antenna-excited $m = 1$ harmonic closely resembles the results from ideal MHD NOVA, as shown in Fig.5.14 (a1)(a2), but the $m = 2$ harmonic has a much lower contribution to the

overall mode structure in GTC compared to NOVA. Non-perturbative effects or differences in the treatments for equilibrium could contribute to the discrepancies in the mode frequency and the ratio of poloidal harmonic. Contributions from reverse shear Alfvén eigenmodes (RSAEs) might also cause these discrepancies. RSAEs generally have eigen-frequencies in the range below the TAE frequency with only a single poloidal harmonic peaking near the q_{min} in a reversed shear ($|q_0| > |q_{min}|$) plasma. However, RSAEs could be in an intermediate regime with both characteristics of TAEs and RSAEs, which have two coupled poloidal harmonics and in an intermediate frequency [11]. The equilibrium used in GTC simulations is slightly reversed shear with q_{min} location ($\rho \approx 0.45$) not too far away from the $q = 1.5$ surface at $\rho \approx 0.7$, as shown in Fig.5.6(b). Hence it is possible to have small contributions of $m = 1$ RSAE excited alongside $m = 1$, $m = 2$ TAE in GTC, and alter the overall mode eigen-frequency and the ratio of poloidal harmonics from a conventional TAE.

The mode exhibits a similar eigen-frequency and greater damping rate when radiative damping is included in GTC. The mode is excited with the same antenna setting but in the two-fluid model with finite $\delta E_{||}$. The eigen-frequency of the mode at 72 kHz (before considering the rotation-induced Doppler shift) is similar to that found in the single-fluid MHD simulation. The relative amplitude of $m = 1$ and $m = 2$ $\delta\phi$ near the $|q| = 1.5$ surface is similar to that found in the single-fluid MHD simulation. However, $\delta\phi$ is weaker at q_{min} in the two-fluid simulation (Fig.5.14 (b1)(b2)), suggesting that a weaker coupling between the TAE and the RSAE in producing the global eigenmode structure when radiative damping caused by the finite $\delta E_{||}$ is taken into account. The mode damping rate measured at $|q| = 1.5$ after the antenna is turned off is greater ($\gamma_{tot}/\omega = -4.77\%$ versus -5.97% , as shown in Table 5.1), also as a result of the $\delta E_{||}$ introduced by radiative damping.

Antenna-excited TAEs saturate at much lower amplitude and exhibit significantly greater damping rate when kinetic thermal ions are included in GTC. The TAE is excited by the antenna in the same way as before but now with full-kinetic thermal ions, with finite $\delta E_{||}$ and without gyro-averaging. Thus, continuum, radiative, and ion Landau damping mechanisms

are all included in this run mode. The eigen-frequency is found to be slightly lower than in the previous cases, 69.3 kHz before Doppler shift. Its mode structure and poloidal harmonics (Fig.5.14 (c1)(c2)) are similar to the two-fluid MHD case near $|q| = 1.5$. However, the damping rate at $|q| = 1.5$ after the antenna is turned off is more significant at $\gamma_D/\omega = -11.0\%$, most likely due to the addition of ion Landau damping similar to that found in RSAE cases in DIII-D [?]. The saturated amplitude is much lower compared to previous ideal MHD cases, as shown in Fig.5.13. This is likely due to the more significant damping rate and agree with the resonant cavity mode model (Equation 5.19). The simulation can also include gyrokinetic thermal ions with gyro-averaging, which introduces the FLR effect. However, the saturated amplitude, eigen-frequency and damping rate in the gyro-averaging case are nearly the same as the full-kinetic case, suggesting that ion Landau damping is responsible for most of the additional damping rather than the FLR effect in gyro-kinetic simulation. The difference in damping rates is also reflected in the ratios between $\delta E_{\parallel}$ (defined in Eq. 5.2) and the electrostatic parallel electric fields $E_{\parallel}^{ES} = -\mathbf{b}_0 \cdot \nabla \delta \phi$ [68]. The mode is Alfvénic in the ideal MHD description and experiencing no Landau nor radiative damping effects when $\delta E_{\parallel}/E_{\parallel}^{ES} = 0$, as in the single-fluid MHD case shown in Table 5.1 (case I). The $\delta E_{\parallel}/E_{\parallel}^{ES}$ in the region near the peak of the modes ($0.5 < \rho < 0.8$) in cases II - IV are still much smaller than 1. This indicates that the modes in these cases are still predominantly Alfvénic, but the damping effects are more significant compared to case I and not negligible. Besides the differences in the net damping rates and the $\delta E_{\parallel}/E_{\parallel}^{ES}$, the ion Landau damping might also affect the spatial structures of the modes. The ion Landau effect is the strongest when the ion thermal velocity approaches the wave parallel phase velocity $\omega_E/k_{\parallel}$ where $k_{\parallel} = (n - m/q)/R$ (in the large aspect ratio limit) is the parallel wave vector. The T_i is at its maximum near the core (as shown in Fig.5.8(b)), suggesting that $v_{phase,ion}$ is likely to be closest to $\omega_E/k_{\parallel}$ in the region and hence that ion Landau effect is the strongest near the core. As a result, the mode has a much weaker $\delta \phi$ in the core region ($\rho < 0.5$) in the simulation that includes gyro-kinetic thermal ions compared to the $\delta \phi$ from single-fluid

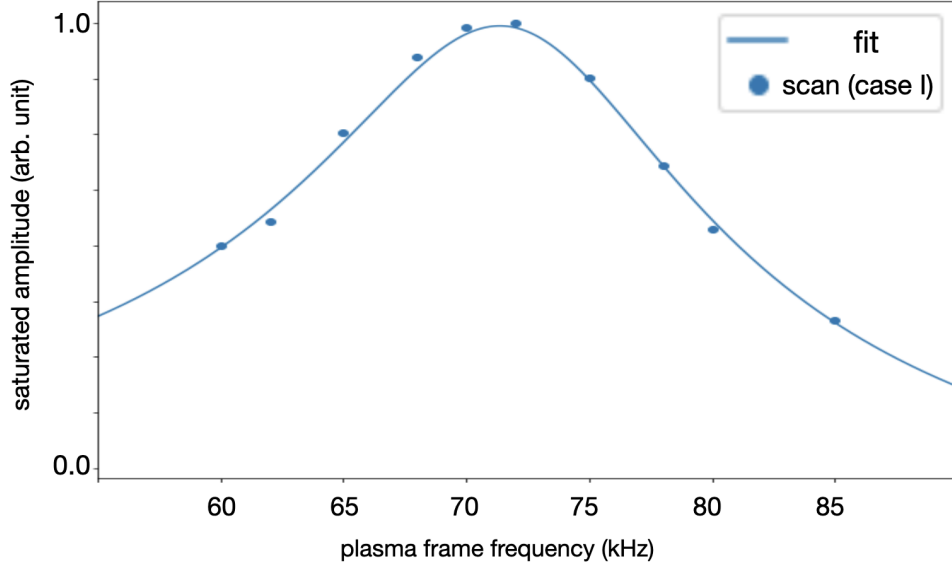


Figure 5.12: The saturated amplitudes versus antenna frequencies for antenna excited TAEs (case I in Table.5.1)

simulation (Figures 5.14(c1)(c2) vs. (a1)(a2), respectively).

5.6 TAEs excited by Maxwellian and slowing-down fast ion distributions in GTC

TAEs can also be excited by Maxwellian fast ion distributions in GTC as described in Equation 5.9. Analytic fast ion density profiles are employed to excite TAEs in the linear GK regime in GTC and reveal the instability threshold of the mode. The experimental fast ion distributions induced by NBI are computed by NUBEAM [76]. Spatial profiles of fast ion temperature and density are considered, while their pitch and energy distributions are neglected for modes excited by Maxwellian distributions. Analytic fast ion density profiles with scales of 1, 1.5, 2, and 2.5 times the maximum fast ion density computed with NUBEAM ($(n_{FI,NUBEAM})_{max}$) are used in linear GTC runs to determine the instability threshold of the targeted TAE. The increase of the fast ion density scale factor subsequently increases

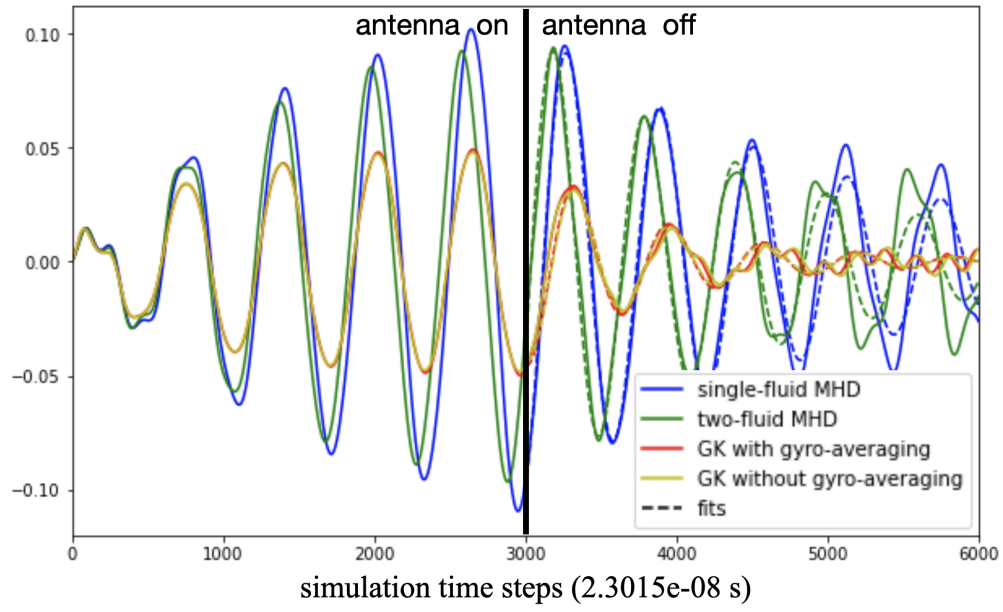


Figure 5.13: The waveform (solid curves) of $(n,m) = (1,1)$ near $—q— = 1.5$ of the antenna excited modes, with their $e^{i\omega_E t} e^{\gamma_D t}$ fits (dashed curves) for the period after the antenna was turned off

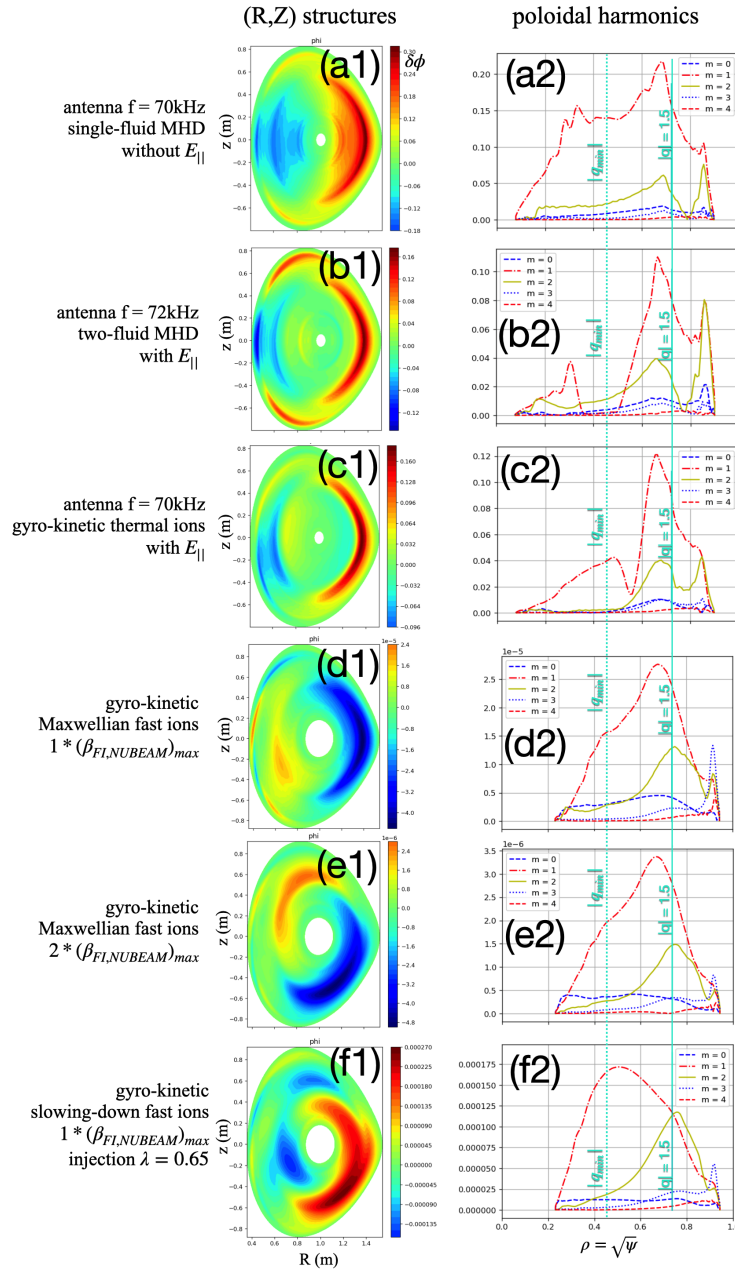
$\delta\phi$


Figure 5.14: 2D (R,Z) snapshots from GTC runs that excite $n = 1$ TAE with (a1) 70kHz antenna in single-fluid MHD plasma without ϕ_{eff} ; (b1) 72kHz antenna in double-fluid MHD plasma with ϕ_{eff} ; (c1) 70kHz antenna with gyro-kinetic thermal ions and ϕ_{eff} ; (d1) Maxwellian fast ion profile with $1 * (\beta_{FI,NUBEAM})_{max}$, (e1) Maxwellian fast ion profile with $2 * (\beta_{FI,NUBEAM})_{max}$, (f1) analytic fast ion slowing-down distribution with injection $\lambda = 0.65$. (a2) - (f2) show the corresponding poloidal harmonics $m = [0,4]$

the β_{FI} and keep T_{FI} unchanged in the scan. The TAE is marginally unstable when it is excited with $(\beta_{FI,sim})_{max} = 1 * (\beta_{FI,NUBEAM})_{max}$. Simulations with β_{FI} slightly below $1 * (\beta_{FI,NUBEAM})_{max}$ are dominated by noise and numerical instabilities, indicating that the mode is stable, and hence no accurate mode structures, frequencies, or growth rates can be extracted from these solutions. This suggests that the β_{FI} threshold for an unstable TAE is slightly below the maximum fast ion pressure reproduced by NUBEAM. The ratio of the net growth rate ($\gamma_{tot} = \gamma_L + \gamma_D$) to the mode frequency (ω) for modes excited by 1, 1.5, 2, and 2.5 times $(\beta_{FI,NUBEAM})_{max}$ are 3.63%, 7.56%, 9.52%, and 11.77%, respectively, as shown in Fig.5.17 (a). The γ_{tot}/ω appears to be nearly linearly proportional to β_{FI} , which is similar to the approximation for low-n TAEs in large aspect ratio discussed in the Equation 10 of [34], despite having a significantly greater β_{FI}/β_i . The ratio of $\delta E_{||}$ and $E_{||}^{ES}$ in these fast-ion-excited modes are also measured in the same way mentioned previously in Section 5.5, and shown in Table 5.1. However, since the fast-ion-excited modes are continuously driven by the fast ions, the measured $\delta E_{||}/E_{||}^{ES}$ values reflect both damping and driven effects. Hence these values cannot be directly compared with those found in the antenna-excited cases.

The frequencies of modes excited by the analytic fast ion profile with different pressure scales are similar. Previous benchmarks of GTC have demonstrated non-perturbative kinetic contributions from fast ions [112], resulting in differences in mode frequency compared to eigen-frequencies found in antenna scans without $\delta E_{||}$. The frequencies of modes excited by a fast ion profile with the pressure at 1 times $(\beta_{FI,NUBEAM})_{max}$ peak near 72 kHz before Doppler shift. The frequencies shift downward in cases when the $(\beta_{FI,NUBEAM})_{max}$ is scaled up, as shown in Fig.5.17 (b). The shift could be the result of the significantly increased fast ion density in the cases with higher fast ion pressure scales that alter the equilibrium when quasi-neutrality is considered, in addition to kinetic effects. The spatial structure of the modes excited by profiles with these scales is similar when they are in the exponential growth phase. The $\delta\phi$ peaks near $|q| = 1.5$ (Fig.5.14 (d1)(e1)) with $m = 1$ dominant poloidal harmonic (Fig.5.14 (d2)(e2)). The contributions from $m = 2$ are more pronounced

compared to the antenna-excited modes. However, the ratio of $m = 2$ to $m = 1$ is still not as much as those in ideal MHD NOVA, even when both are converted to the same physical quantity $\delta B_r/B$ for comparisons, as shown in Fig.5.18. The ratio of $m = 2$ to $m = 1$ ($(\delta B_{r,m=2})_{max}/(\delta B_{r,m=1})_{max}$) in the GTC solution is lower than the NOVA solution (≈ 0.68 versus ≈ 0.92 , respectively). This suggests that there are still contributions from $m = 1$ RSAE to the overall mode structure and frequency, similar to the antenna-excited cases.

The radial TAE structure excited by a fast ion distribution in GTC is compared to inverted SXR measurements after an preliminary attempt on converting $\delta\phi$ to δn_e . The conversion done by multiplying $\delta\phi$ from GTC by the radial gradient of electron density, poloidal mode number (m) and $1/r$, as the following:

$$\delta n_e \approx -\frac{m}{r} \frac{\delta\phi}{b_0} \nabla_r n_{e0}, \quad (5.20)$$

where b_0 is the magnetic field strength on axis. The $\nabla_r n_{e0}/r$ term might alter mode peaking location and radial extent when it is represented in terms of electron density fluctuations. Hence this conversion is needed before direct comparisons between GTC results and SXR measurements. $\nabla_r n_{e0}(r)$ can be overly sensitive to jumps in $n_{e0}(r)$ from either TRANSP results or Thomson measurements. Hence n_{e0} is first smoothed out by fitted to a Gaussian function as $n_{e0,fit} = n_{e0,max} e^{-\frac{(\rho-\mu)^2}{2\sigma^2}}$ (as shown in Fig.5.15(a)) before finding $\nabla_r n_{e0,fit}(r)$ (as shown in Fig.5.15(b)). The use of Gaussian ensures smoothness of $n_{e0,fit}$, as well as having a fixed expression of $-\nabla_\rho n_{e0,fit} = \frac{(\rho-\mu)}{\sigma^2} n_{e0,fit}$ in terms of $n_{e0,fit}$ and fitting parameters. This makes estimating the propagation of fitting uncertainties to δn_e straightforward. It is worth noting that the fitting uncertainties are large at some radial locations. This methodology can be refined in by future researchers to improve the results.

The mode peaking location of $(-\frac{m}{\rho} \nabla_\rho n_{e0,fit})(\delta\phi)$ of a TAE excited by Maxwellian fast ion distributions in GTC is similar to the measured structure inferred from SXR measurements. b_0 and the minor radius a are constants, hence $(-\frac{m}{\rho} \nabla_\rho n_{e0,fit})(\delta\phi)$ (where $\rho \approx r/a$ is sufficient

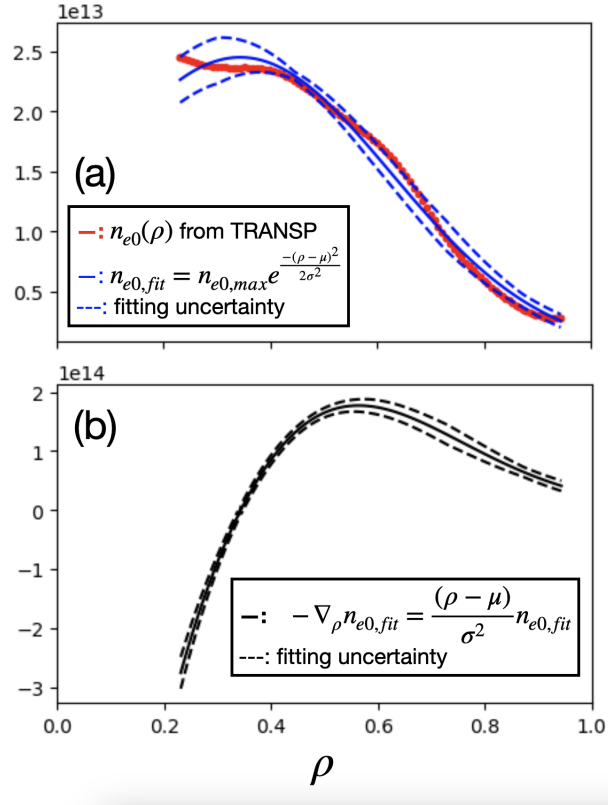


Figure 5.15: (a) $n_{e0}(\rho)$ from TRANSP, its profile fitted to a Gaussian function as $n_{e0,fit} = n_{e0,max} e^{-\frac{(\rho-\mu)^2}{2\sigma^2}}$, and the fitting uncertainty; (b) $-\nabla_{\rho} n_{e0,fit} = \frac{(\rho-\mu)}{\sigma^2} n_{e0,fit}$ and the propagated fitting uncertainties

to compare the radial profiles of δn_e in Eq.5.20 with the SXR measurements. Both SXR measurements and Maxwellian fast ion excited GTC exhibit a peak near $q = 1.5$ surface, as shown in Fig.5.6(a) vs. Fig.5.16(a), respectively. However, the radial extent of the mode from GTC is slightly wider than the measured structure. This could be the result of poor fitted $n_{e0,fit}$ at the region $\rho < 0.4$ and future researchers can improve the results by using better fitting model for $n_{e0,fit}$.

Apart from Maxwellian distributions, analytic fast ion slowing-down distributions can be utilized to excite the TAE in GTC as described in Equation 5.12. Although the model does not accommodate the rapidly changing birth pitch across different major radii in spherical

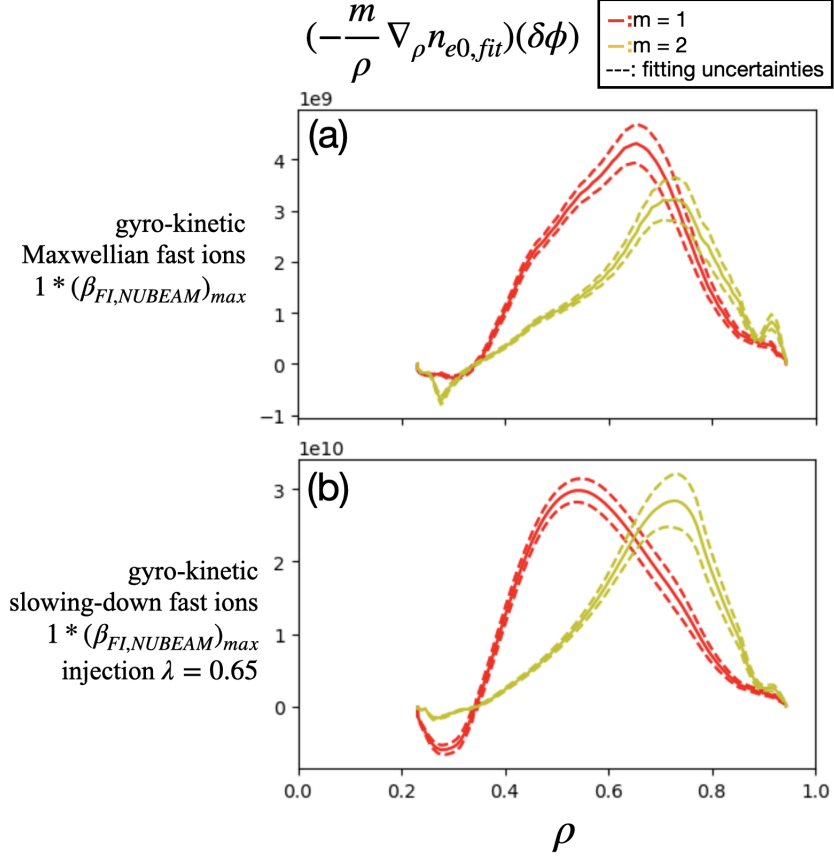


Figure 5.16: Poloidal harmonics $m = [1,2]$ of $(-\frac{m}{\rho} \nabla_{\rho} n_{e0,fit})(\delta\phi)$ from GTC runs that excite $n = 1$ TAE with (a) Maxwellian fast ion profile with $1 * (\beta_{FI,NUBEAM})_{max}$; (b) analytic fast ion slowing-down distribution with injection $\lambda = 0.65$, with dashed curves represent the propagated $n_{e0,fit}$ fitting uncertainties

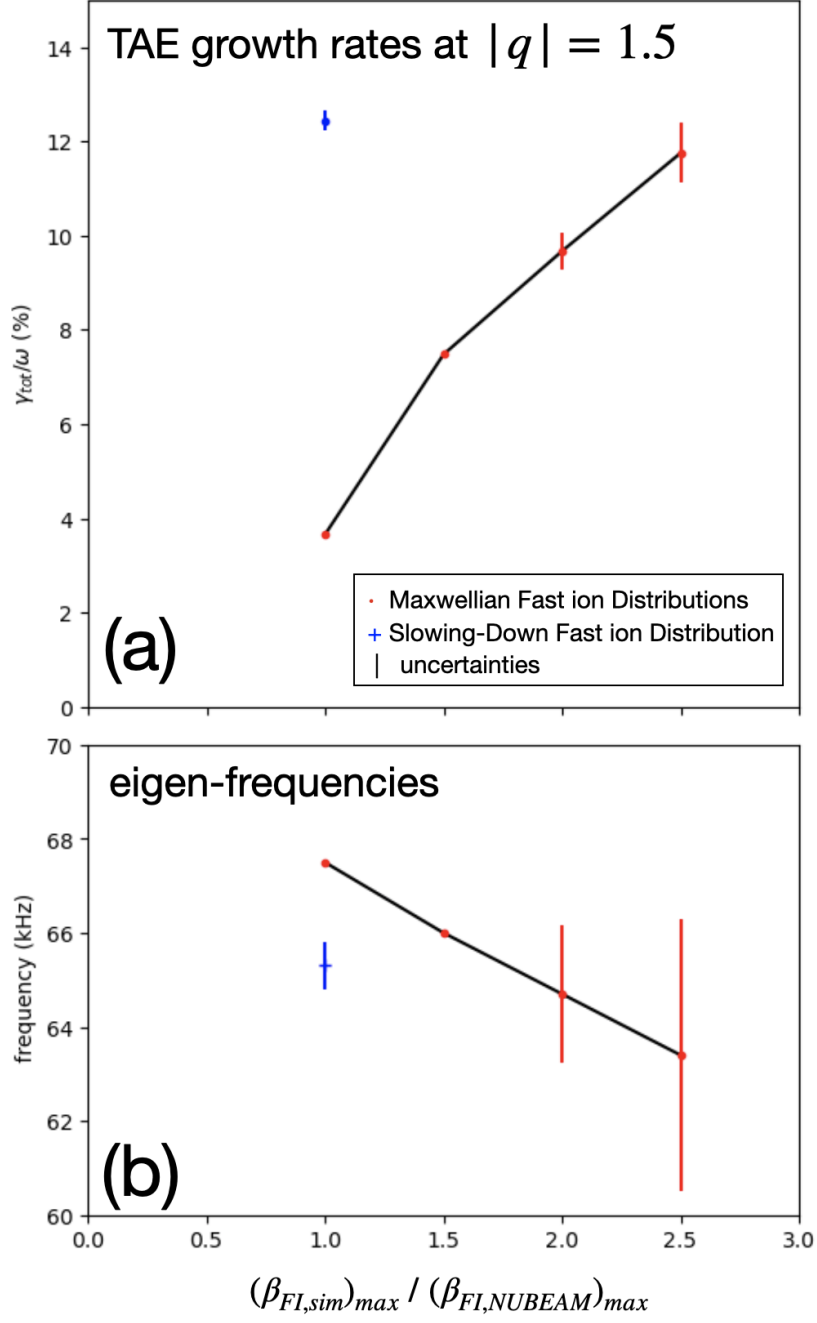


Figure 5.17: (a) Growth rates and (b) eigen-frequencies (in the plasma frame) of $n = 1$, $m = 1$ TAEs excited by analytic Maxwellian fast ion β_{FI} of 1, 1.5, 2 and 2.5 times of $(\beta_{FI,NUBEAM})_{max}$ (by scaling n_{FI}), sampled near $|q| = 1.5$ surface

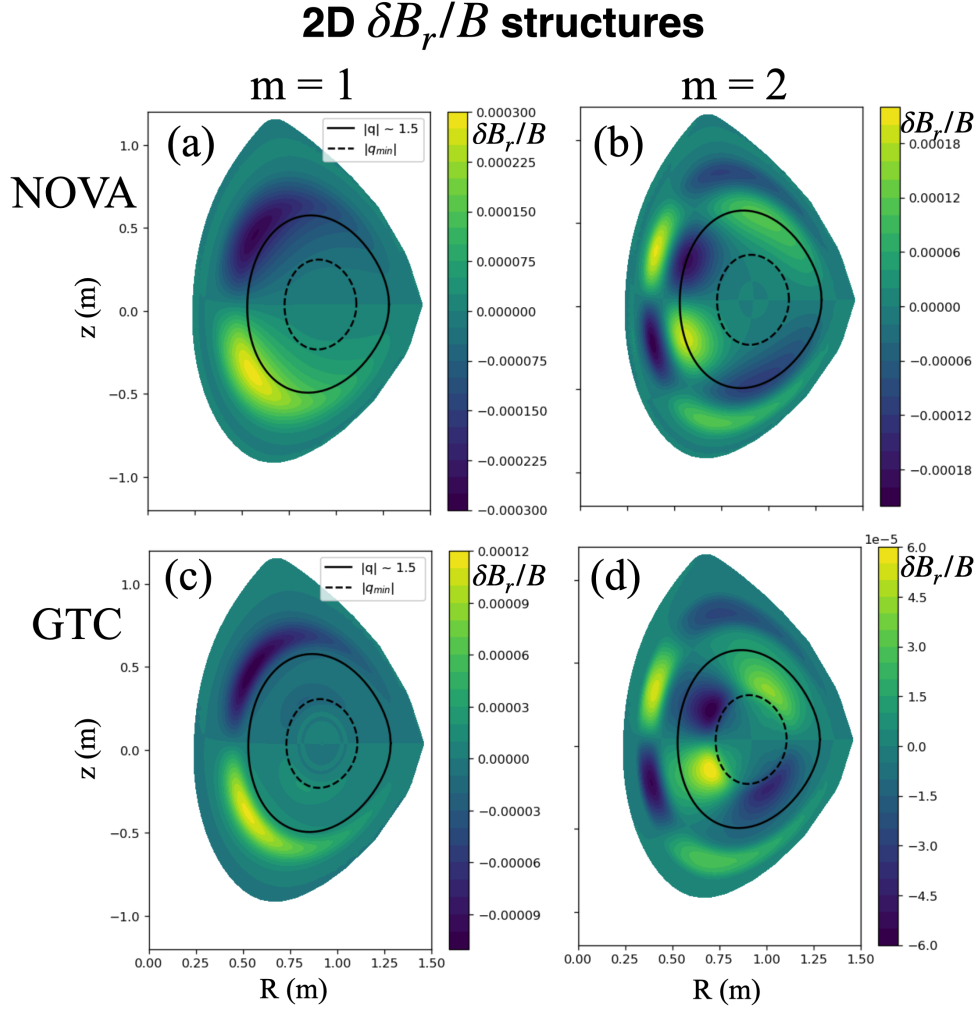


Figure 5.18: 2D (R,Z) $\delta B_r/B$ structures of (a) $m = 1$ and (b) $m = 2$ from NOVA; (c) $m = 1$ and (d) $m = 2$ from linear GTC with the mode excited by analytic Maxwellian fast ion β_{FI} of $3 * (\beta_{FI,NUBEAM})_{max}$ (by scaling n_{FI})

tokamaks as shown in Fig.5.9(a) in Section 5.1, it is sufficient to represent most of the beam-induced fast ions as they are generated near $R \approx 1.1$ m with birth pitch variable concentrated near $\lambda = 0.65$. Hence, analytical slowing-down distributions that fit the fast ion pitch-energy distributions there (Figures 5.9(b)(c)) could realistically represent the trapped particles interacting with the TAE.

The TAE driven by the analytic slowing-down fast ion distribution exhibits a higher growth rate and slightly different mode spatial structure compared to modes excited by Maxwellian distributions, probably due to the anisotropic distribution of fast ions [17]. The radial fast ion density and temperature (for determining v_c and v_0) profiles are taken from the Maxwellian fast ion case with $(\beta_{FI,sim})_{max} = 1 * (\beta_{FI,NUBEAM})_{max}$ for consistency. The frequencies of modes from the two cases remain similar to those of the mode excited solely by the Maxwellian distribution, around 70 kHz before a Doppler shift correction. However, the growth rate of the mode excited by the slowing-down distribution $(n, m) = (1, 1)$ is significantly higher ($\gamma_{tot}/\omega = 10.75\%$) than the mode excited solely by the Maxwellian distribution using the same fast ion pressure profiles. This indicates that the mode becomes more unstable with the use of the slowing-down distribution, probably due to the anisotropic fast ion distributions peak at higher λ , providing more fast ions to be in shear Alfvén resonance. The effects of fast ion anisotropy on the growth rate had also been observed in the stability study of TAEs in ITER scenarios when the pitch distribution is considered in the kinetic-MHD code NOVA-K [39]. The difference of TAE growth rates between fast ion Maxwellian and slowing down distributions excited modes with the same n_{FI} in GTC, however, is greater than those found using NOVA/-K. The discrepancy could be the result of additional kinetic effects included in GTC compared to NOVA/-K, and also the differences in fast ion beta and aspect ratio in MAST case compared to the ITER case discussed in [39].

The use of an analytic slowing-down distribution in GTC also alters the radial structure of the mode compared to the case with Maxwellian distribution excitation. The peak of the $m = 1$ component in $\delta\phi$ shifts to $\rho = 0.5$ close to $|q_{min}|$ (Fig.5.14 (f2)) from $\rho = 0.65$ near

$|q| = 1.5$ in cases excited solely by the Maxwellian distributions. In the slowing-down case, the ratio of the $m = 1$ to $m = 2$ component in $\delta\phi$ is comparable at $|q| = 1.5$, but $m = 1$ is dominant near the $|q_{min}|$. Additionally, a spiral feature emerges in the 2D (R, Z) structure of $\delta\phi$ for the mode excited by the $\lambda = 0.65$ slowing-down distribution (Fig.5.14 (f1)), absent in the mode excited by the Maxwellian distribution alone. This spiral suggests a stronger k_r , implying more significant radial energy transfer than the previous case. These characteristics suggest the use of an anisotropic fast ion distribution causes the mode to exhibit distinctive features of both TAE at $q = 1.5$ and RSAE at q_{min} . This could be due to the differences in $\nabla f_{0f}/f_{0f}|_\mu$ (Equation 5.13) and $(1/f_{0a})(\partial f_{0a}/\partial v_{||})$ (Equation 5.14) peaking locations when the pitch of fast ions is considered, and hence altering the fast ion pressure gradients. This can make both $m = 1$ RSAE and $m = 1,2$ TAE more distinguishable at their associated q surfaces.

The radial extent of a TAE excited by an anisotropic fast ion distribution in GTC is much wider than the measured TAE structure. Similar to the case excited by Maxwellian fast ion distribution, the $\delta\phi$ from the anisotropic case is also converted to $(-\frac{m}{\rho}\nabla_\rho n_{e0,fit})(\delta\phi)$ for comparison with SXR measurements. The $m = 1$ poloidal harmonic still peaks much closer to the core in the anisotropic case (shown in Fig.5.16(b)) compared to the Maxwellian case after the conversion (shown in Fig.5.16(a)). This causes the mode excited by anisotropic fast ion distribution to be wider than both Maxwellian case and the SXR measurement (shown in Fig.5.6(a)), even neglecting the region $\rho < 0.4$ that has poor fitted $n_{e0,fit}$. Surprisingly, the less realistic Maxwellian fast ion distribution seems to produce a result more similar to the measurements compared to the anisotropic fast ion distribution. Besides improving $n_{e0,fit}$, future researcher can investigate whether the anisotropic fast ion distribution used in GTC simulations (Fig.5.9(c)) is the source of the discrepancy and require improvements as well.

5.7 Summary of this chapter

Global radial mode structure measurement of TAEs in the MAST discharge 26887 at 180 ms are made with soft X-ray and inverted to obtain the emissivity fluctuation spatial structure using a novel technique. The structure shows characteristics consistent with theory for $(n,m,m+1) = (1,1,2)$ TAEs. It is worth noting that the dominant mode numbers and mode frequency of a NBI-excited TAE depend on the beam geometry. In this particular MAST case, the $(n,m,m+1) = (1,1,2)$ gap location is the closest one to the beam induced fast ion deposition location near the core, and hence the TAE with this set of mode number is excited. Dominant mode numbers of the TAEs might be different if beam does not have on-axis injection geometry. In addition, although not included in the MHD picture of TAE, a different beam injection geometry can alter the mode frequency by changing the poloidal transit times for fast ions, which affect mode-fast ion resonance (Eq. 2.2). Future researchers can study the effects of geometry to TAE's characteristics and stability in MAST-U or similar devices (e.g. NSTX-U) using neutral beams with different injection geometries.

The radial mode structure measurements are also compared with linear gyrokinetic simulations using various GTC run modes, as summarized in Table 5.1. These simulations demonstrate GTC's capability to incorporate various mechanisms that significantly affect the TAE, which are absent in MHD models. Antenna-driven TAEs in single-fluid (without $\delta E_{||}$) and two-fluid (with $\delta E_{||}$) plasma in GTC are subject to similar damping rates of about 5-6%. This indicates that radiative damping introduced by the two-fluid model does not significantly increase the damping beyond the continuum level. However, the damping rate increases significantly to 11% when the thermal ions are in the gyrokinetic regime, which includes the ion Landau effect. On the other hand, the damping rate only changes slightly when FLR effects are included, introduced by the gyro-averaging of the thermal ions. This suggests that the ion Landau effect makes the most significant contribution to the mode damping rate, emphasizing the need for an accurate treatment of kinetic effects (absent in

ideal MHD codes) for precise TAE stability predictions. The ratios of $\delta E_{\parallel}$ and $E_{\parallel}^{ES}$ in these antenna-excited modes also reflect the difference in the damping mechanisms, as they are more significant compared to the two-fluid case.

TAEs excited by Maxwellian fast ion distributions with various fast ion pressures in GTC demonstrate the sensitivity of TAE growth rates to the fast ion pressure gradient. A scan of analytic fast ion pressure profiles with densities ranging from 1 to 2.5 times the level computed by NUBEAM was conducted. The spatial structures of these TAEs are similar to each other and to NOVA ideal MHD eigenmode calculations, with comparable proportions of $m = 1$ and $m = 2$ components in $\delta\phi$. The TAEs excited in the fast ion density scan have growth rates ranging from 3.9 to 16.1%. This indicates that the experimental fast ion density computed by NUBEAM is sufficient to drive the mode and suggests that the stability threshold lies between 0.5 and 1 times the NUBEAM fast ion density.

Fast ion anisotropy significantly affects the stability of TAEs. Besides fast ion Maxwellian distributions, GTC can also be used to model the excitation of TAEs with analytic slowing-down fast ion distributions. The growth rate increases significantly to about 11% when an analytic slowing-down distribution that mimics the NUBEAM fast ion pitch/energy distribution near the peak fast ion density ($R \approx 1.1\text{m}$) with the same analytic fast ion density profile used in the Maxwellian case is used to excite the mode. This demonstrates that mode stability is sensitive to the pitch and energy distribution of the beam-induced fast ions. The $m = 1$ and $m = 2$ components at $q = 1.5$ are similar to the previous case excited solely by the Maxwellian distributions, but $m = 1$ $\delta\phi$ extends further to the core near the q_{min} location. A more noticeable spiral 2D structure, implying a stronger k_r compared to the mode excited by the Maxwellian distribution, appears in the mode excited by the slowing-down distribution. This suggests that additional contributions from an RSAE are present due to the differences in $\nabla f_{0f}/f_{0f}$ peaking locations arising from the replacement of a Maxwellian distribution with an anisotropic slowing-down distribution, and hence altering the fast ion pressure gradients. As with the kinetic effects mentioned earlier, fast ions are absent in ideal

MHD codes, which limits their ability to assess the impact of fast ion distributions on TAE stability. This emphasizes the importance of kinetic codes like GTC, which can account for the effects of fast ions.

Overall, the GTC simulation results for TAE radial mode structure and frequency are in good agreement with the measurements. Despite different representations of thermal and fast ions, the eigen-frequencies of TAEs excited in various GTC run modes fall within a narrow range of 69 to 72 kHz. The laboratory frame frequencies are close to 80 kHz after accounting for the ≈ 10 kHz plasma rotation Doppler shift. This is about 9% lower than the frequency computed by NOVA and the measured initial frequency of the targeted chirping TAE. Interestingly, the frequencies from GTC are closer to the TAE frequency at its peak mode amplitude, as well as to the ideal MHD value for the $(n, m) = (1, 1)$ TAE frequency, once the rotation Doppler shift is considered. The discrepancy could be attributed to differences in the input profiles and equilibria for NOVA and GTC, given that there is a noticeable downshift in the GTC continuum compared to the one from NOVA. This could explain the downshift in frequencies from various GTC run modes compared to NOVA result. The radial mode structures from the GTC simulations that include fast ions also align with experimental results inferred from tangential SXR. The $\delta\phi$ from these simulations all peak around $\rho = 0.6$ to 0.8 , close to the expected mode location for ideal MHD $n = 1$ and $m = 1, 2$ TAEs, as well as the measured SXR $\delta\epsilon$. Differences in the radial mode extent in GK fast ion simulations are found, possibly due to the treatment of fast ion distributions (whether they are Maxwellian or anisotropic) in these simulations. Surprisingly, the less realistic Maxwellian fast ion distribution seems to produce a result with better agreements with the measurements compared to the anisotropic fast ion distribution. Future researchers can investigate whether the anisotropic fast ion distribution used in GTC simulations (Fig.5.9(c)) is the source of the discrepancy and require improvements as well. Future experimental efforts could also focus on directly inferring fast ion anisotropy using tomography techniques [69] and on measuring poloidal mode structures and harmonics with

poloidally viewing diagnostics. These efforts could provide valuable references for detailed comparisons between Maxwellian and anisotropic fast ion models used in the GTC in the future. Future efforts can also be done to assess whether the anisotropic fast ion distribution used in GTC is sufficient to model TAEs excited by different beam geometry.

case	description	damping/driving mechanisms	eigen-frequency (kHz) without Doppler shift	γ_{tot}/ω (%)	$(\delta E_{ }/E_{ }^{ES})_{mean}$ ($0.5 < \rho < 0.8$)
I	single-fluid MHD	antenna, continuum	70.1	-4.77	0
II	two-fluid (with $\delta E_{ }$)	case I + radiative	72.0	-5.97	0.10401
III	drift-kinetic thermal ions (without gyro-averaging)	case II + ion Landau	69.2	-11.0	0.098139
IV	gyro-kinetic thermal ions (with gyro-averaging)	case III + FLR	69.3	-10.8	0.10299
V	gyro-kinetic thermal + fast ions (Maxwellian distribution)	case IV without antenna + with fast ion gradient drive	69-72 (Fig.5.17(b))	3.6 - 11.8 (Fig.5.17(a))	0.034561
VI	gyro-kinetic thermal + fast ions (slowing down distribution)	case IV without antenna + with fast ion gradient drive	66 (Fig.5.17(b))	10.8 (Fig.5.17(a))	0.026967

Table 5.1: Various simulation cases to measure the mode eigen-frequencies and total growth/damping rates with different combinations of damping and driving mechanisms of the $n = 1$ TAE located near the $q = 1.5$ surface

CHAPTER 6

Reduced fast ion transport calculations for TAEs MAST

Fast ion transport associated with TAEs is estimated using a guiding-center-based reduced model. Simulations of fast-ion-driven instabilities in tokamaks are essential for deepening and broadening our understanding of these instabilities, as well as for assisting in the planning and control of future plasma discharges. First-principles, initial-value simulations, such as the linear GTC described in Chapter 5, aim to capture the fundamental driving mechanisms and mode dynamics. However, these simulations are highly complex and require significant computational resources. Although they offer high fidelity, the inclusion of multiple physics mechanisms governing fast-ion-driven instabilities can introduce numerical challenges, necessitating extensive verification, validation, and case development efforts. In practice, the computational demands and complexity of first-principles simulations can become impractical for analyzing specific, well-defined problems—especially when prior knowledge of the instabilities exists from measurements or other calculations. Reduced models are developed to mitigate this challenge by leveraging such prior information as inputs, significantly simplifying calculations and reducing computational costs.

The ORBIT-Kick model [77] is used to calculate fast ion transport and losses induced by plasma instabilities. The model accounts for variations in energy (E) and canonical toroidal angular momentum (P_ζ) due to resonant wave-particle interactions, governed by the constant-of-motion relationship ($\omega P_\zeta - nE = \text{constant}$) given in Eq. 2.7 (Chapter 2). To apply this model, fast ions must be characterized in terms of their orbit topology [103]. Initial

fast ion phase-space distributions from NUBEAM are used to set the starting conditions for the calculations. Importantly, the model does not assume whether the transport is diffusive or convective; instead, the resulting radial transport emerges naturally from the phase-space dynamics of fast ions. The model requires three primary inputs: the mode’s spatial structure, amplitude (A_{mode}), and frequency, which are used to characterize the ”kicks” (Eq. 2.8). However, these inputs can originate from experimental data, analytical models, or other numerical simulations, making the ORBIT-Kick model highly flexible. This adaptability enables the study of fast ion transport under different conditions across multiple sources.

This chapter presents a project that explores the use of different input sources for ORBIT-Kick transport calculations related to TAE-induced transport. The first part focuses on comparing ORBIT-Kick calculations based on solutions from NOVA and linear GTC for the TAE case in MAST, as discussed in Chapter 5. By utilizing ORBIT-Kick, it is possible to compute the severity of fast ion transport, the number of lost particles, and resonance conditions that linear simulations alone cannot address. Additionally, measured chirping profiles—unaddressable by linear simulations—can be incorporated into ORBIT-Kick to assess the effects of frequency chirping on fast ion transport and losses. These results demonstrate that combining experimental measurements with reduced modeling provides a viable approach for studying fast ion transport, offering a valuable tool for planning plasma discharges and mitigating TAE-induced fast ion transport in fusion devices.

6.1 GTC vs. NOVA mode structures as inputs for fast ion transport calculations

Radial structures and frequencies from various simulations can serve as inputs for ORBIT-Kick transport model calculations. The ORBIT-Kick model does not impose restrictions on whether the mode spatial structure, amplitude (A_{mode}), and frequency originate from experimental data, analytical models, or numerical simulations. This flexibility allows for

a comparative study of fast ion transport associated with the TAE discussed in Chapter 5 using solutions from both NOVA and linear GTC.

To establish a fundamental understanding of resonance conditions for different orbit types, ORBIT-Kick is initially applied using the TAE radial structure from NOVA, with fast ions uniformly distributed in phase space (E, P_ϕ, μ) , without incorporating NUBEAM inputs. Trapped particles exhibit the most significant kicks in P_ζ around $\mathcal{E} \approx 26$ keV, as demonstrated by the kick probability matrix, showing the ΔP_ζ distribution for particles initially loaded in different (E, P_ϕ, μ) in Fig. 6.1. This result corresponds to the $m = 1, 2$ TAE from NOVA in a 2 ms ORBIT-Kick calculation, with a maximum mode perturbation of $\delta B_r/B = 3.65 \times 10^{-4}$ and 300,000 fast particles. The analysis reveals strong interactions among trapped fast ions in the energy range $15 \text{ keV} < E < 45 \text{ keV}$, with some contribution from passing particles as well.

ORBIT-Kick also computes poloidal and toroidal frequencies to infer resonance conditions for different orbit types [104]. It calculates ω_ζ and ω_θ throughout phase space, allowing for the identification of the integers p and l corresponding to resonant trapped and passing particles. The resonant passing particles are concentrated near $(n, p, l) = (1, 1, 2)$, while resonant trapped particles exhibit $l = 1$ and values of p scattered at -1 or lower, as shown in Fig. 6.1. This suggests that there are more fast ions following banana-like than potato-like orbits, with variations in orbit shape and width.

ORBIT-Kick calculations using spatial structure inputs from GTC yield results similar to those obtained with NOVA inputs. In these simulations, the TAE spatial structure and frequency from both GTC and NOVA are employed as inputs in ORBIT-Kick runs and compared. To ensure a more accurate representation of fast ion interactions with the wave, the fast ion distribution from NUBEAM is used in all cases. While some particles undergo resonance and exhibit variations in $\mathcal{E}$, ρ , and P_ζ , most remain unchanged during ORBIT-Kick runs. To assess the intensity of mode-induced kicks and identify resonance conditions, the standard deviations of these values— $SD_{\Delta\mathcal{E}}$ (illustrated in Fig. 6.2), and $SD_{\Delta P_\zeta}$ —as

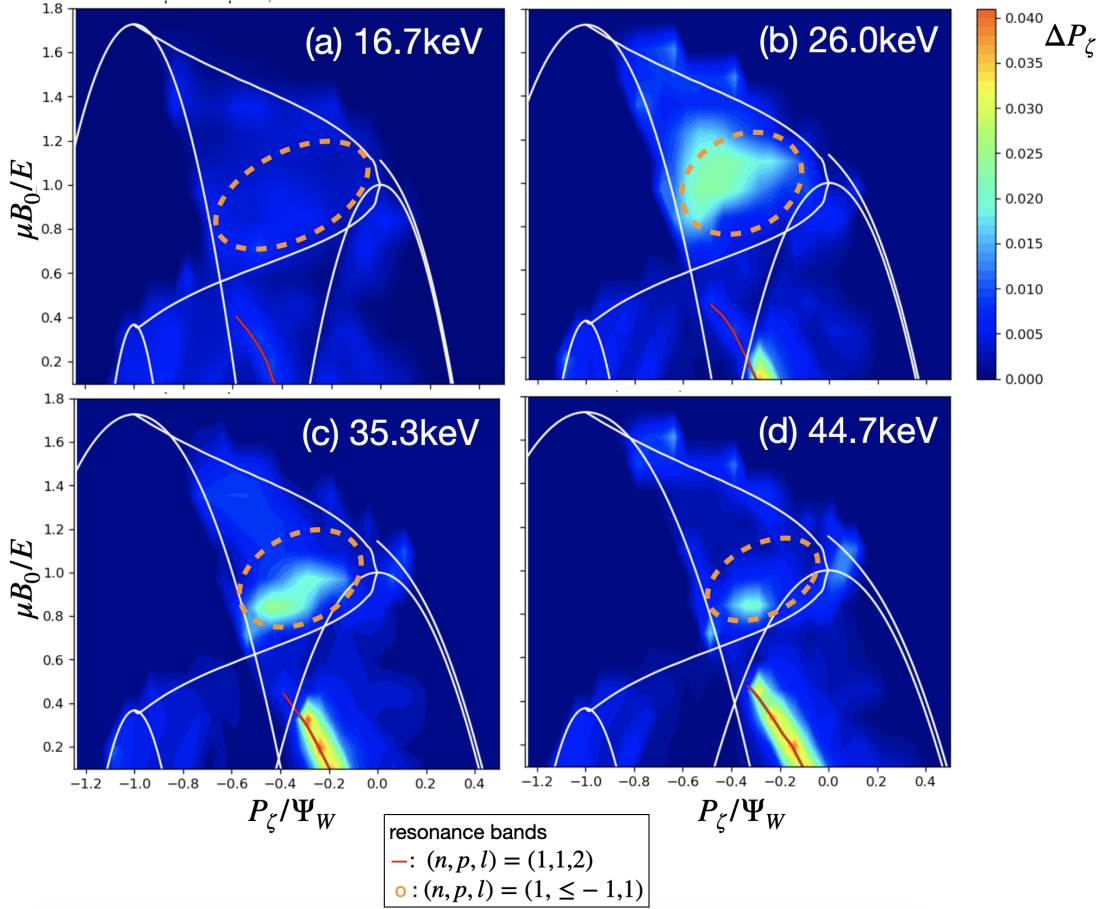


Figure 6.1: Kick probability matrix ($p(\lambda, P_\zeta, \mathcal{E})$) from result of ORBIT-Kick that uses NOVA structure, showing ΔP_ζ of resonant fast ions and the resonance bands for $(n, p, l) = (1, 1, 2)$ and $(1, \leq -1, 1)$ in phase space representations for $\mathcal{E}$ at (a) 16.7 keV, (b) 26.0 keV, (c) 35.3 keV, and (d) 44.7 keV

well as the number of fast ion losses (defined as particles transported to $\rho > 1$), serve as key metrics. When using similar mode amplitudes (Table 6.1) in ORBIT-Kick runs, the standard deviations from calculations with TAE structures from both GTC (Fig 5.14(d)), NOVA (Fig 5.11), using the NUBEAM FI distribution (Fig.5.9(a)(b)) are of the same order of magnitude (cases 1 and 2 in Table 6.1 6.2).

The primary difference between the GTC and NOVA cases lies in the resonance conditions. In both cases (cases 1 and 2 in Table 6.1 6.2), similar resonances are observed for the $(n, p, l) = (1, 1, 2)$ particle (with the resonance band in $(\mathcal{E}, \lambda)$ space at $P_\zeta/\Psi_W = -0.34$, labeled in red in Fig. 6.4), the $(n, p, l) = (1, -1, 1)$ trapped particle (labeled in orange), and the lost trapped particle (labeled in blue at $P_\zeta/\Psi_W = -0.84$). However, a notable difference arises for trapped particle resonance bands associated with integer values of $p \ll -1$. In the run using the GTC solution at a mode frequency of 73 kHz (circled in an orange dashed line in Fig. 6.4 (b)), these resonance bands shift toward the zero point in $(\mathcal{E}, \lambda)$ space compared to the NOVA case at 88 kHz (Fig. 6.4 (a)). The differences can also be seen in the kick probability matrix, showing the ΔP_ζ distribution for particles initially loaded in different (E, P_ϕ, μ) in Fig. 6.3 for cases 1, 2 and 3 in Table 6.1. Fewer trapped particles experience kicks when the mode frequencies are reduced to 73 kHz, shown in Fig. 6.3(b)(c) for cases 2 and 3 in Table 6.1. On the other hand, the resonance bands for passing particles shift toward higher μ . This implies that the resonance conditions for both trapped and passing particles are altered by lowering the mode frequencies.

However, similar shifts in resonance conditions and fast ion transport intensity are observed in ORBIT-Kick runs using modified NOVA solutions with the same spatial mode structure but a reduced frequency of 73 kHz (case 3 in Table 6.1, Table 6.2, and Fig. 6.4 (c)). This suggests that small differences in spatial mode structure between GTC and NOVA do not substantially affect the intensity of fast ion transport or total loss counts. Instead, differences in frequency primarily modify the resonance conditions for the $p \ll -1$ banana-like trapped orbits. The ratios of lost co-passing and lost trapped particles (Table 6.2) differ

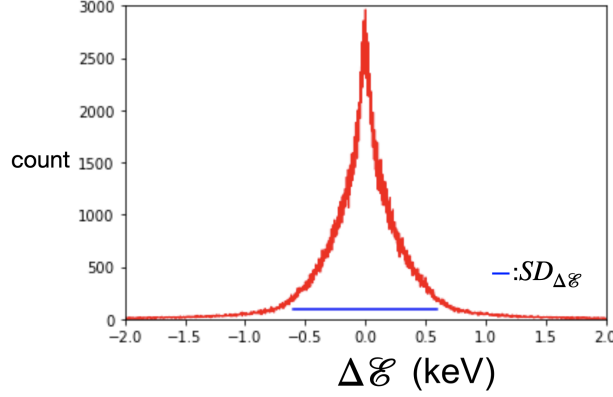


Figure 6.2: Histogram of $\Delta\mathcal{E}$ and its standard standard deviation ($SD_{\Delta\mathcal{E}}$) (in keV) of fast ions in ORBIT-Kick calculation using NOVA structure at 88kHz (case 1 in table 6.1) as an input.

slightly between the 88 kHz and 73 kHz cases (≈ 0.26 vs. ≈ 0.21 for cases 1 and 3 in Table 6.2, respectively, with histogram uncertainties of about 4 to 5 %), indicating that the resonance condition shift leads to a slight reduction in co-passing particle losses.

6.2 Effects of mode amplitudes on TAE associated fast ion transport

Fast ion transport is found to depend on the mode amplitude in ORBIT-Kick calculations. While using spatial structures from GTC or NOVA as inputs does not substantially alter fast ion transport calculations, as shown previously, the structure from NOVA is utilized in a scan of mode amplitude (cases 3, 4 and 5 in 6.1). However, the frequency is shifted to 73 kHz to mimic GTC solutions and align with the frequency exhibiting the peak mode amplitude from the experiment. The case 3 with $\delta B_r/B$ in the order of 10^{-4} serves as the baseline and is compared with case 4 with $\delta B_r/B \sim 10^{-3}$ and case 5 with $\delta B_r/B \sim 10^{-2}$. Generally, the intensity of transport (case 4 and 5 shown in Table 6.1 and 6.2) increases with $\delta B_r/B$, as loss count $SD_{\Delta E}$ and $SD_{\Delta P_\zeta}$ increase roughly in the same order of magnitude

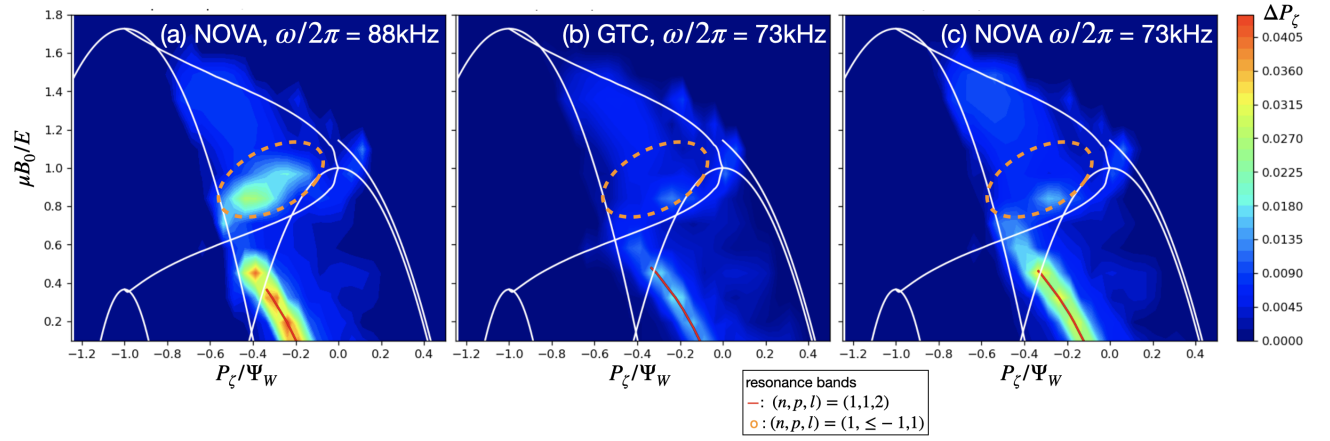


Figure 6.3: Kick probability matrix ($p(\lambda, P_\zeta, \mathcal{E} = 35.3\text{keV})$) from result of ORBIT-Kick that uses (a) NOVA structure at 88kHz, (b) Maxwellian GTC structure at 73kHz, and (c) NOVA structure at 73kHz, showing ΔP_ζ of resonant fast ions and the resonance bands for $(n, p, l) = (1, 1, 2)$ and $(1, \leq -1, 1)$ in phase space representations.

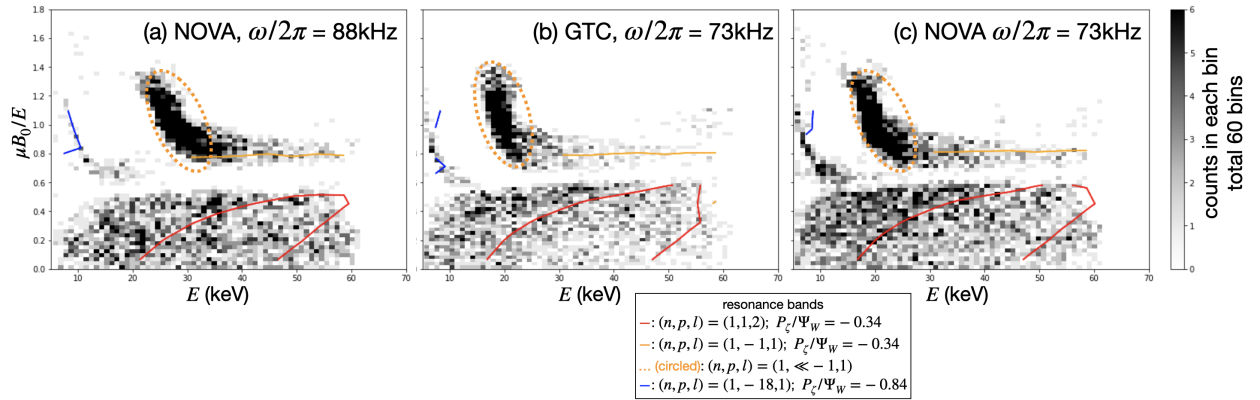


Figure 6.4: 2D ($\lambda = \mu B_0/E$ and $\mathcal{E}$) histograms for fast ions with their energies modified greater than $3 * SD_{\Delta \mathcal{E}}$ in ORBIT-Kick calculations using (a) NOVA structure at 88kHz (case 1 in table 6.1), (b) GTC structure at 72kHz (case 2), and (c) NOVA structure at 73kHz (case 3).

when $\delta B_r/B$ increases from $\sim 10^{-4}$ to $\sim 10^{-3}$. However, this is not the case when $\delta B_r/B$ increases from $\sim 10^{-3}$ to $\sim 10^{-2}$. It is important to note that actual mode amplitudes are typically not as strong as our $\sim 10^{-2}$ test case for modes with small perturbations like TAEs. Moreover, besides the overall increases in losses for both co-passing and trapped particles, the lost co-passing to lost trapped particles ratio jumps from about 1/5 to nearly 1 when $\delta B_r/B$ increases from $\sim 10^{-4}$ to $\sim 10^{-2}$. Most of the particles interacting with the TAEs are usually trapped particles. The jump in lost ratio indicates the resonance conditions have differed from the baseline cases. Hence other cases to be reported in the rest of the section use $\delta B_r/B$ well under 10^{-2} .

6.3 Effects of chirping on TAE associated fast ion transport

Incorporating evolving mode amplitude and frequency into ORBIT-Kick transport calculations allows for mimicking the amplitudes and frequencies of chirping modes. The evolving mode amplitude cycle has been previously employed in ORBIT-Kick to compute sawtooth instability-associated transport [79]. Additionally, decreasing mode frequency can be simultaneously applied in the mode evolving cycle to simulate chirping TAEs. A_{mode} and ω are modified, and new resonance conditions are considered in each time step in ORBIT-Kick when both evolving cycles are used. It is important to note that ORBIT-Kick currently does not account for the effects of $\partial_t \omega$ on the resonance conditions. Hence, further study is needed in the future to verify this assumption before applying ORBIT-Kick to all energetic particle mode-like phenomena.

The chirping of mode frequency induces more significant variations in fast ion energy and momentum, as well as modifying the resonance conditions compared to simulations with a fixed frequency in the mode evolving cycle in ORBIT-Kick calculations. Linear simulations, whether in ideal MHD or GK regimes, cannot provide solutions with evolving growth rates and frequencies. Hence, the time-evolving mode amplitude and frequency profiles (shown

in Fig.6.5) used in the ORBIT-Kick runs are fitted to experimental measurements. In a case with NOVA structure and both evolving mode amplitude and frequency that fit the measurements, fast ion energy and momentum variations ($\Delta\mathcal{E}$ and ΔP_ζ , respectively) are roughly twice as large as in another case with evolving mode amplitude but a fixed frequency of 88 kHz (shown cases 6 vs. 7 in Table 6.2, respectively). This demonstrates the presence of frequency chirping induces more fast ion transport compared to simulations with a fixed mode frequency. While the kick probability matrices ($p(\lambda, P_\zeta, \mathcal{E})$) of these two cases (shown in Fig.6.6) look similar to those from the baseline cases discussed in Section 6.1, the $(n, p, l) = (1, -1, 1)$ trapped particle resonance band disappears in the $(\mathcal{E}, \lambda)$ representation of the resonance map (shown in Fig.6.7 (b)) compared to other cases. Instead, the $p \ll -1$ bands remain and responsible for most of the kicks and losses to the trapped particles in the chirping case. This suggests the change of mode frequency greatly affects the trapped orbits.

Alterations of the resonance condition due to frequency chirping might cause fast ions to jump into different orbits, thereby inducing more significant transport and losses. The fast ion kinetic Poincaré plot [102][104], showing the orbits of fast ions in (θ, ψ) space, is a useful tool included in ORBIT-Kick for describing the behavior of fast ions. The particles could be in passing- or potato-like orbits, circling the magnetic axis. Alternatively, they can be in banana-like orbits, staying on one side of the axis and appearing as islands in the Poincaré plot. The Poincaré plot for case 7 sampled at $\lambda = 0.6$ reveals unusual behavior of some fast ions near the edge of plasma. For example, some fast ions near the edge of the plasma initially exhibit passing- or potato-like orbits but then transition into orbiting around an island and finally shift into other passing- or potato-like orbits during the frequency chirping cycle, highlighted in red in Fig.6.8.

ORBIT-Kick can also track individual particles in high time resolution to examine the behavior of their orbits. For instance, a fast ion with an initial position at $\psi = 0.8$ and $\lambda = 0.6$ initially circles the axis in a passing- or potato-like orbit, reaching $\psi \approx 0.3$ at the innermost point before returning to $\psi = 0.8$ to complete one cycle. The presence of a mode perturbs

the orbit, causing precessions and sometimes even losses when the orbit is perturbed beyond $\psi = 1$. The outermost point of the orbit changes over time. When the mode profile used in case 6 induces perturbations to the orbit of a fast ion starting at $\psi = 0.8$ and $\lambda = 0.6$, the orbit oscillates for less than $\psi = 0.05$. However, this is not the case when frequency chirping is imposed in ORBIT-Kick. There is a more significant jump in the outermost point of the orbit than in the previous case (Fig.6.9 (a)), even using the same mode amplitude profile. The final orbit has the outermost point at $\psi \approx 0.9$, meaning the fast ion is being transported to a different orbit. Conversely, for another fast ion with an initial position at $\psi = 0.91$ and $\lambda = 0.6$, the particle is lost (moved $\psi > 1$) without frequency chirping. Nevertheless, it is transported back to $\psi \approx 0.8$ in the presence of frequency chirping (Fig.6.9 (b)). These examples highlight the increased transport intensity when frequency chirping is present.

The impact of frequency chirping of the TAE is particularly pronounced for particles initiated near the last closed flux surface (LCFS), which are more susceptible to loss. Any perturbations caused by frequency chirping can easily eject these particles beyond the LCFS. This effect is reflected in the shift in the lost co-passing to lost trapped particle ratio, which increases from 1/5 in Case 1 (baseline) to 0.42 in the chirping case.

6.4 Summary of this chapter

This chapter presents the ORBIT-Kick calculations performed to study fast ion transport in NBI-driven TAEs in MAST discharge 26887 using linear GTC and NOVA results reveal that fast ion transport depends on mode amplitude, frequency chirping, and resonance conditions. These studies demonstrate the potential applications of the ORBIT-Kick model for improving our understanding of TAEs, their associated fast ion transport, and for guiding future experiments. GTC simulations provide multiple inputs for ORBIT-Kick, and initial comparisons show that using TAE spatial structures from GTC versus NOVA leads to minimal differences in fast ion resonance conditions and transport intensities. This similarity is

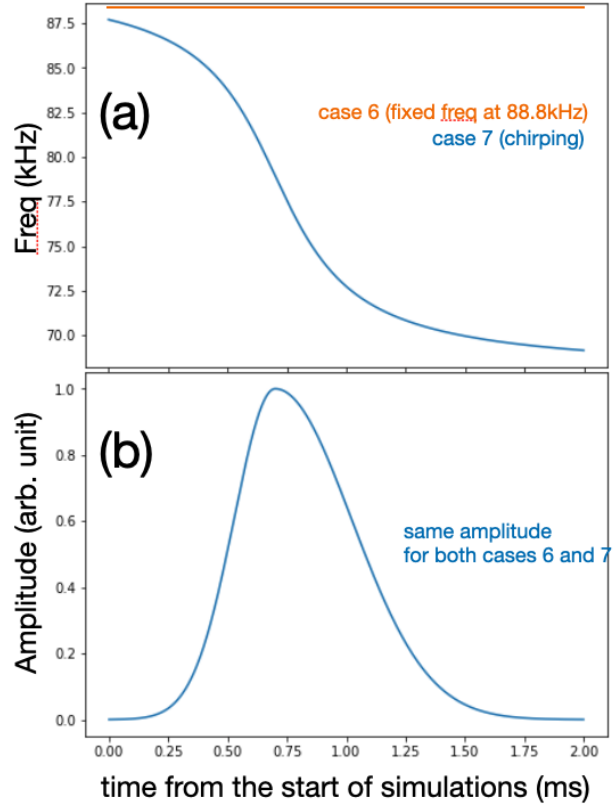


Figure 6.5: mode chirping (a) frequency and (b) amplitude profiles that mimic the measurements used for ORBIT-Kick TAE cases 6 and 7 (table 6.1)

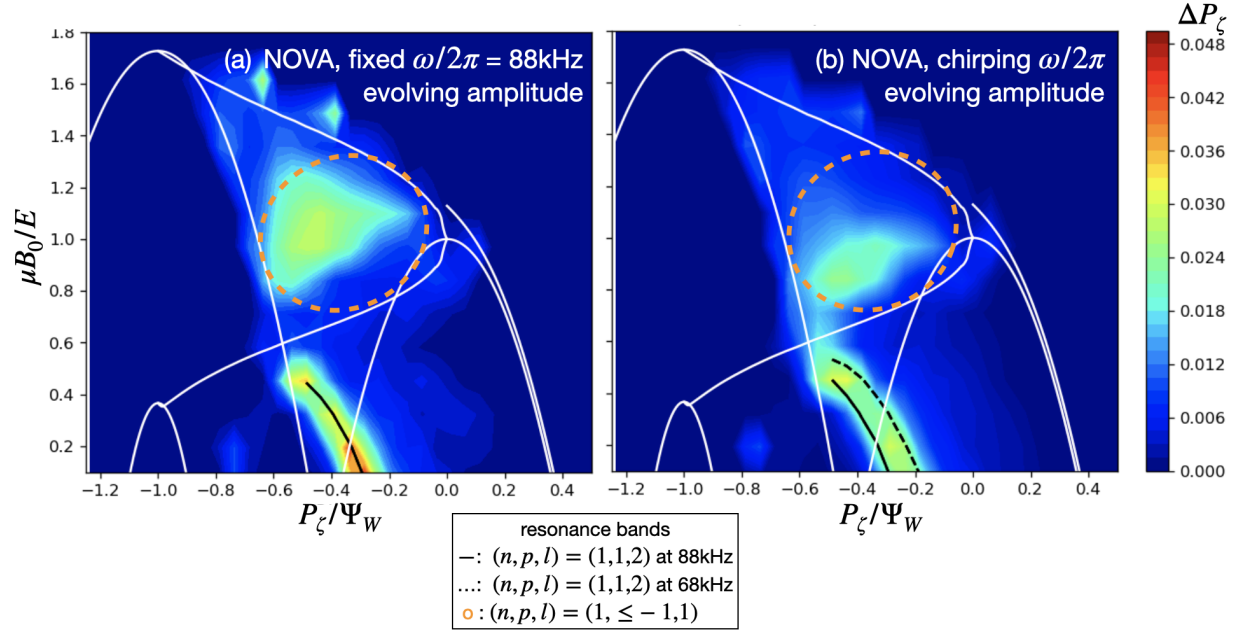


Figure 6.6: Kick probability matrix ($p(\lambda, P_\zeta, \mathcal{E} = 26.0\text{keV})$) from result of ORBIT-Kick that uses (a) evolving NOVA structure with a fixed frequency at 88kHz, and (b) evolving NOVA structure with a frequency chirping down from 88kHz to 68kHz (as shown in Fig.6.5), showing ΔP_ζ of resonant fast ions and the resonance bands for $(n, p, l) = (1, 1, 2)$ and $(1, \leq -1, 1)$ in phase space representations.

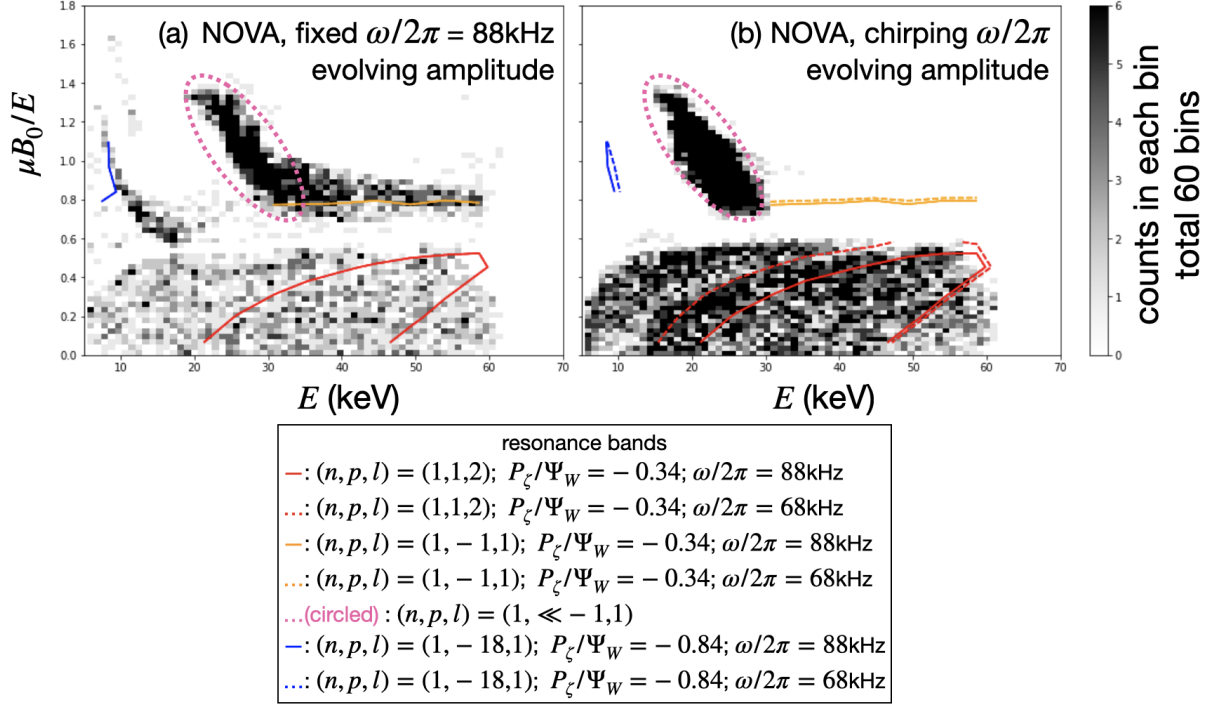


Figure 6.7: 2D ($\lambda = \mu B_0/E$ and $\mathcal{E}$) histograms for fast ions with their energies modified greater than $3 * SD_{\Delta\mathcal{E}}$ in ORBIT-Kick calculations using (a) evolving NOVA structure with a fixed frequency at 88kHz (case 6 in table 6.1), and (b) evolving NOVA structure with a frequency chirping down from 88kHz to 68kHz (case 7).

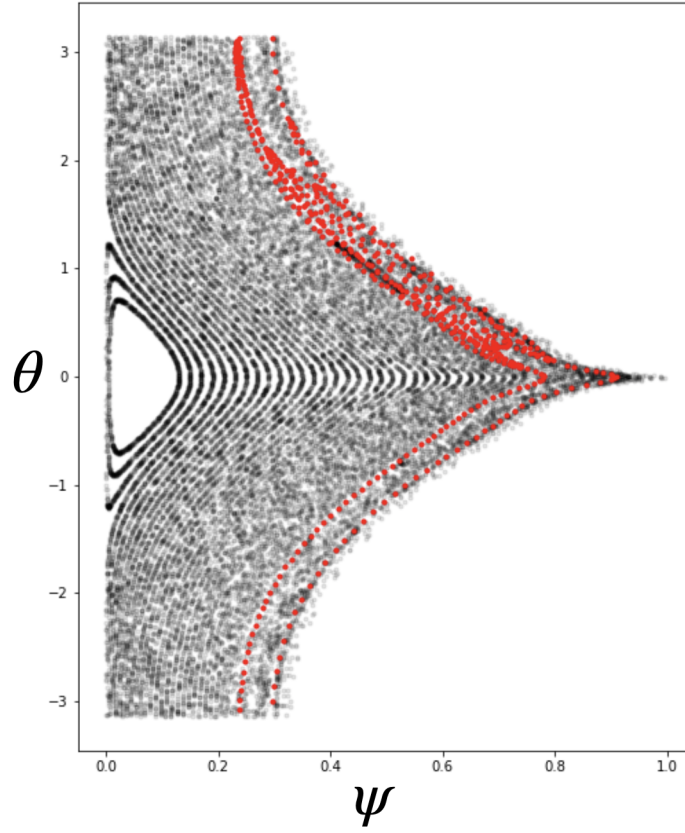


Figure 6.8: Kinetic Poincaré plot of fast ions in case 7 (table 6.1), with the orbit that jumps during the ORBIT-Kick simulation highlighted in red

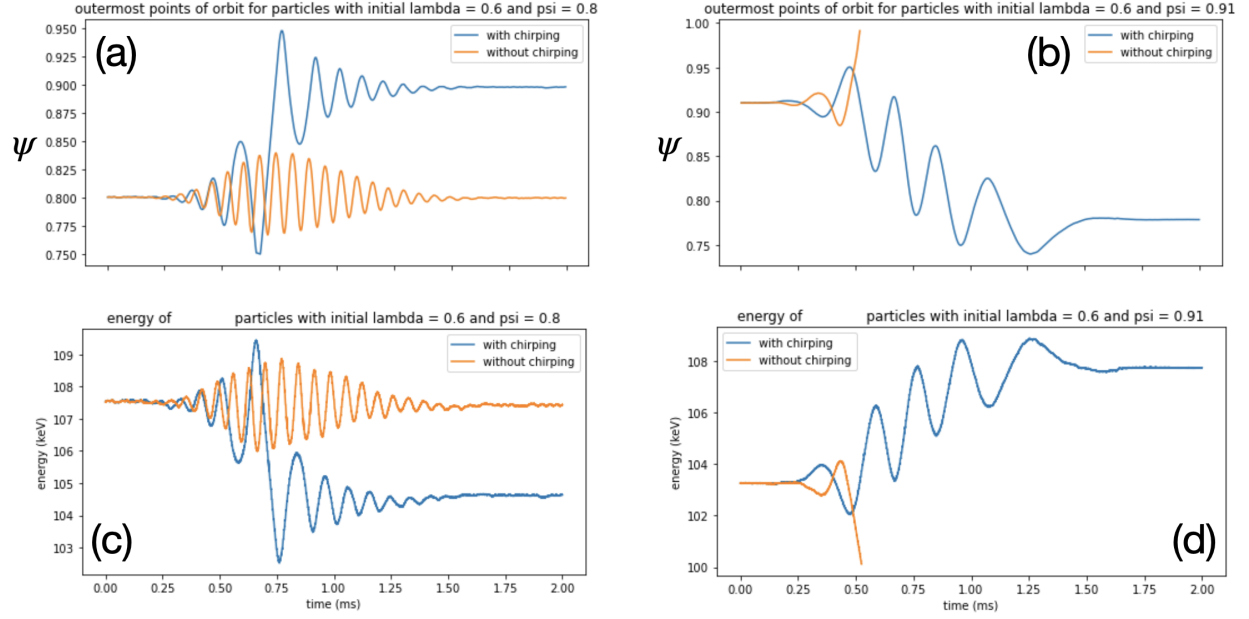


Figure 6.9: The outermost point of orbit for particles with initial (a) $\psi = 0.8$, (b) $\psi = 0.91$, and energy of particles with initial (c) $\psi = 0.8$, (d) $\psi = 0.91$ in case 7 (table 6.1)

Inputs quantities or settings	case 1	case 2	case 3	case 4	case 5	case 6	case 7
mode	NOVA	GTC	NOVA	NOVA	NOVA	NOVA	NOVA
structure		(fig.5.14(c))					
A_{scale}	5E-04	5E-04	5E-04	5E-03	5E-02	Evolving (Fig.6.5)	Evolving (Fig.6.5)
$(\delta B_r/B)_{\text{max}}$	3.65E-04	3.78E-04	3.65E-04	3.65E-03	3.65E-02	7.30E-04	7.30E-04
frequency (kHz)	88	72	73	73	73	88	Chirping (Fig.6.5)

Table 6.1: Inputs and settings for ORBIT-Kick runs for MAST TAE cases

results	case 1	case 2	case 3	case 4	case 5	case 6	case 7
$SD_{\Delta E}$ (keV)	0.6015	0.3423	0.4832	3.5088	17.8137	0.6403	1.3287
$SD_{\Delta P}$	0.0134	0.00919	0.0129	0.0932	0.364	0.0143	0.0338
loss (per 300000 particles)	1992	1406	2006	20692	74427	2107	2570
lost trapped/total trapped (%)	1.01%	0.727%	1.07%	10.3%	32.2%	0.961%	1.18%
lost co-passing/total co-passing (%)	0.343%	0.199%	0.295%	4.45%	21.6%	0.526%	0.645%
lost co-passing/lost trapped	0.260	0.210	0.212	0.349	0.968	0.418	0.419

Table 6.2: Results of ORBIT-Kick runs for MAST TAE cases

expected, given the comparable spatial structures of $m = 1$ modes from both codes. While mode frequency shifts from GTC modify the energy of resonant trapped particles, they do not significantly affect cross-field fast ion transport and losses in ORBIT-Kick calculations, reinforcing the model's reliability, particularly for large-scale TAE scenarios. A scan of mode amplitudes in ORBIT-Kick indicates that fast ion transport and losses increase proportionally with amplitude until reaching a physically unrealistic scale. Additionally, the ratio of lost co-passing to lost trapped particles rises with increasing mode amplitude. Frequency chirping further increases fast ion transport, while also increasing the ratio of lost co-passing to lost trapped particles.

CHAPTER 7

Fishbone instabilities and their impacts on core confinement in MAST and MAST-U

Fishbones (FBs) have traditionally received less attention with regard to their impact on bulk plasma and flux surfaces compared to their role in fast-ion transport. However, observations of FBs-like instabilities in MAST-U challenge this perspective. Since its discovery in 1983, considerable efforts have been made to describe the characteristics of the FB and its effects on EPs, as discussed in Chapter 3.4.3. However, the models typically used to describe FBs in conventional tokamaks do not always apply to those found in spherical tokamaks (STs). STs often feature flat central safety factor ($q(r)$) profiles in high plasma poloidal beta (β_p), and low magnetic field strength scenarios with strong neutral beam injections. In some cases, the minimum safety factor ($|q|_{min}$) value is slightly above 1, resulting in a core region devoid of any $q = 1$ surface. Such FBs in infernal kink regime (infernal-like FBs) were described as quasi-interchange fishbones by Kolesnichenko et al. [60]. In this scenario, where particles cannot interact with the $q = 1$ surface, the existence of infernal-like FBs with frequency at $\omega \sim \omega_{di}$ is unlikely. Consequently, the FBs found in such scenarios are likely to be a type of energetic particle mode (EPM), with their characteristics determined by resonant EPs (predominantly fast ions).

Measurements of various physical quantities associated with infernal-like FBs in the Mega Amp Spherical Tokamak (MAST), reported in Section 7.1, raise questions and have provided the motivation for new experiments dedicated to FBs in the upgrade of this device now operating, MAST-U. The MAST measurements utilize an outboard array of Mirnov coils to

measure poloidal, radial, and toroidal components of outboard magnetic fluctuations (δB_z , δB_R , δB_T), enabling identification of the toroidal mode number (n). Additionally, line-integrated soft x-ray (SXR) emissions from an array of tangential views are employed to measure the spatial structure of the FB in the plasma. Techniques developed for eliminating non-global contributions to signals and converting them into analytic signals, along with methods to invert line-integrated SXR measurements to local SXR emissivity fluctuations ($\delta\epsilon$), are demonstrated in sections 4.4 and 4.5.1. The observed temporal phase variations between near-edge δB_z and near-core $\delta\epsilon$ associated with the FBs (discussed in section 7.1) reveal the possible distortion of modes during the chirping period.

Dedicated experiments (reported in Section 7.2) have been performed for FBs in MAST-U (the upgraded version of MAST [23]), yielding intriguing insights into the transient modification of plasma equilibrium during FBs. The severity of crashes in bulk ion temperatures are comparable to those observed during sawtooth events in the same discharge. While transient changes to thermal profiles have been reported in other tokamaks [42], the observations go beyond alterations in fast ion profiles, extending to modifications in the magnetic field. In a specific discharge, notable movements of the magnetic axis and crashes in bulk ion temperature occurred following the appearance of two FB-like chirps in close temporal proximity in the frequency-time domain, as detailed in Sections 7.2.1 and 7.2.2. The 2D beam emission spectroscopy (BES) [28][29] with radially adjustable viewing locations was used to measure the local density fluctuations (δn_e) associated with FBs around the magnetic axis in the experiment. Similar to the MAST case, temporal phase variations between measurements from near-axis BES channels and Mirnov coils occur during the FB chirping period, as shown in section 7.2.3. These drops in thermal species temperatures and distortions are more pronounced during or after the appearance of two FB-bursts in close proximity in time-frequency space. The FB waveforms and 2D mode structures inferred from the 2D BES array show that mode-mode interactions of a (1,1) FB-like mode and another mode likely cause the measured drop of thermal species temperatures, also showed in section 7.2.3 and discussed

in section 7.2.4.

7.1 Fishbone measurements in MAST

Both $q_{\min} < 1$ classic FBs and $q_{\min} > 1$ infernal-like FBs are observed in many MAST discharges. These FBs have frequencies well below the ideal MHD $(n,m) = 1,1$ Toroidicity-driven Alfvén Eigenmode (TAE) frequency $(v_A/2qR)$ and near the plasma rotation frequency at $R = 1.0$ m, such as the FBs in shot 29976 at 201 ms and 26887 at 262 ms, both circled in pink in Figures 7.3(a) and 7.4(a) in the spectrograms of δB_r measured by one of the Mirnov coil (with their raw signals shown in Fig 7.1(a) and 7.2(a), respectively). Toroidal mode numbers were identified for each time and frequency using an SVD-based method [52]. Plasma equilibria of both cases were reconstructed with MSE-constrained EFIT++ [63][64][6], shown in Figures 7.3(b) and 7.4(b). The q profiles of these two cases are quite different. In shot 29976 at 201 ms, $q > 1$ everywhere except near the magnetic axis, where q is just slightly above 1. In contrast, shot 26887 at 262 ms has $q < 1$ over most of the minor radius. The minimum in q at $R \approx 120$ cm is away from the magnetic axis at $R \approx 95$ cm, indicating a reversed shear feature. The FB in 29976, with $q_{\min}$ slightly above 1, resembles an infernal-like FB, while the FB in 26887 has a q profile consistent with a classic FB.

The SXR emissivities inferred from the inversion of signals demonstrate the time evolution of the mode structures of FBs from shots 29976 and 26887 (with the raw signals from the SXR channels used shown in Fig 7.1(b) and 7.2(b), respectively). The emissivities $\delta\epsilon_{FB}$ associated with targeted FBs in 29976 and 26887 (shown in Figures 7.5(b) and 7.6(b), respectively) peak near the magnetic axis, suggesting both are on-axis FBs tied to $(n,m) = (1,1)$ modes regardless of whether their $q_{\min}$ is above or below 1. The $\delta\epsilon_{FB}$ associated with the targeted FB in 26887 peaks roughly at the time of peak δB_z (shown in 7.6(a)), with no noticeable changes in peak location throughout the burst’s growth and decay. However, the $\delta\epsilon_{FB}$ associated with the targeted FB in 29976 peaks before the peak of δB_z (shown in 7.5(a)),

indicating a shift in mode structure over time. This has been documented in [71], where it was suggested that the FB started as an energetic particle mode (EPM) but the shift in peak location indicated the mode could no longer be an EPM by the end of the burst. The absence of a $q = 1$ surface could lead to weaker growth of the kink instability compared to cases with the $q = 1$ surface, and FB-like bursts in the $q_{min} > 1$ regime might require strong mode-EP interaction to be excited. Hence $q_{min} > 1$ infernal-like FBs are more likely to be EPM-like at the early stage of the bursts compared to conventional internal-like FBs.

The method described in section 4.5.2 retains the imaginary part of the signals after inverse STFT and phase differences between two diagnostics: $\Delta\theta(\delta\epsilon_{FB}, \delta B_z) = \arctan\left(\frac{Im(\delta\epsilon_{FB})}{Re(\delta\epsilon_{FB})}\right)$, in contrast with the analysis method of [71]. The real and imaginary parts of the time-dependent inverted SXR structures differ from each other, implying phase shifts of the mode at some locations. For example, the phases in both cases near the magnetic axis shifted by about $\pi/2$ over the period of chirping in about 2 ms, as shown in Figures 7.5(c) and 7.6(c). This phase analysis offers an alternative understanding of FB spatial structure evolution, in which the apparent “shift” in axis location observed in the real part, reported in [71], is due to an evolving difference in the phase between the radially separated $\delta\epsilon_{FB}$ and δB_z fluctuations.

The toroidal mode structure of the measured FBs was considered in the SXR signal inversion discussed in section 4.5.2. Therefore, toroidal separation in the measurement locations of the $\delta\epsilon$ and δB_z fluctuations should not produce phase variation between inverted SXR and B_z fluctuations over time. The radial separations of the diagnostics can produce a phase difference if the wave propagates radially. However, since the radial separation is static, a change in the induced phase differences over time could suggest a changing radial wave number. Radial propagation might naturally be expected if the sources and sinks of wave energy are radially separated (see e.g. [61]), and an evolving radial wave number due to the distortion of the propagation medium would be unsurprising since the sources and sinks of the FB energy must necessarily evolve over the wave’s lifetime as it grows and decays.

The FB mode distortions appear to be associated with strong fast-ion redistribution during the chirping period. FIDASIM [35][88] computations will be needed to interpret how fast ion spatial and phase space distributions are modified by transport in future work. However, passive FIDA measurements alone are sufficient to demonstrate significant fast ion transport if noticeable spikes in the signals occur. The passive FIDA [73] signals from various locations in pulse 29976 during the FB chirp are observed to peak between 201.0 and 201.5 ms (Fig.7.7(d)). This passive FIDA emission results from fast ions undergoing charge exchange with neutrals at the edge, and the sudden peaking is interpreted as a surge in the edge density of fast ions, a phenomenon documented in other tokamaks [12][43]. The growth and peaking of the passive FIDA signals are delayed compared to those of δB_z (Fig.7.7(a)). The amplitude of δB_z starts growing before 200.0 ms, peaks at 201.0 ms, and then decays. The passive FIDA did not grow significantly until after 200.5 ms. This delay is most likely because fast ion transport depends on mode amplitude. Peak fast ion transport likely occurs at the time of peak FB amplitude, but the passive FIDA signals peak later. The delay in the FIDA signal could be due to the passive FIDA time resolution. The phase between SXR emission fluctuations and δB_z at 91 cm changes most rapidly between 201.0 ms and 201.5 ms. As noted previously, the outboard Mirnov system in MAST has a set of three coils that measured B_z , B_R , B_T at the same toroidal location. Neither the angle between δB_z and δB_R (Fig.7.7(c)), nor their relative amplitude, change significantly, suggesting no change in the direction of the poloidal magnetic field at the coil location (and perhaps no vertical shift in the plasma). However, the amplitudes of δB_z increased significantly relative to δB_T between 200.2 and 201.2 ms (Fig.7.7(d)), indicating a change in the magnetic field pitch at the coil location. The changing structure of $\delta \vec{B}$ and the fast-ion redistribution both potentially contribute to the measured temporal phase variations, as an FB can be viewed as a wave propagating in the combined medium of the fast ions and thermal plasma.

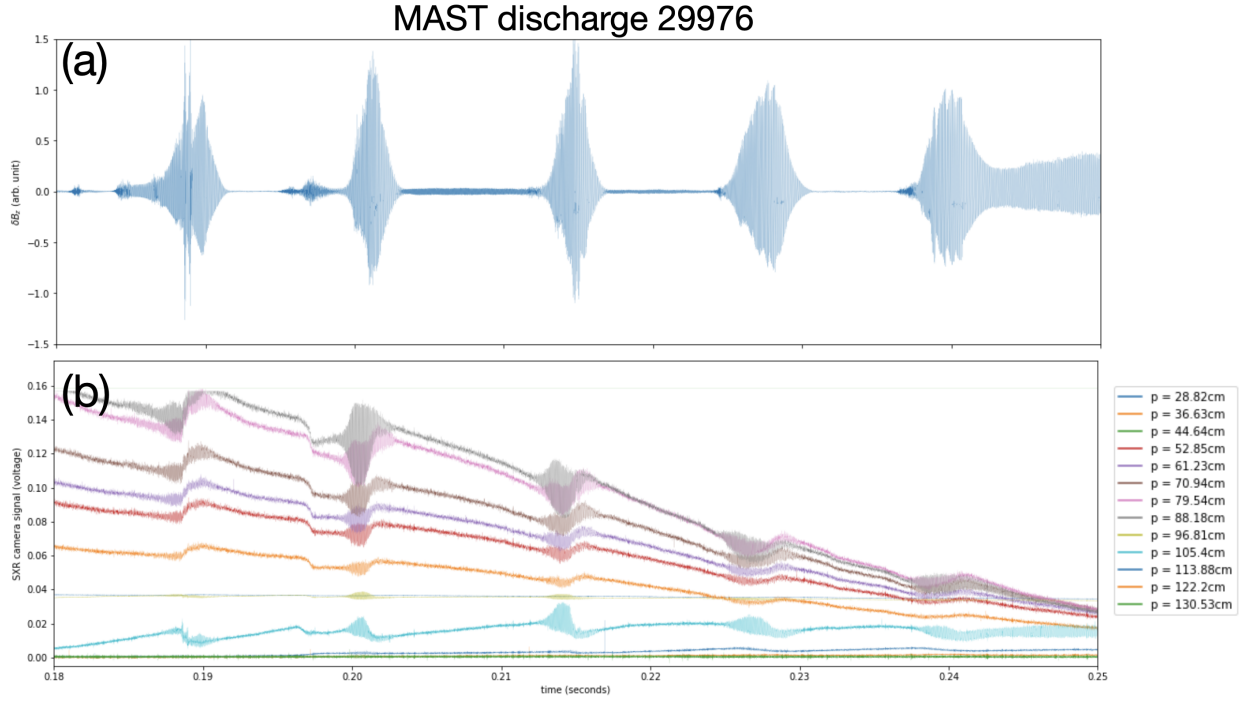


Figure 7.1: MAST discharge 29976 raw signals of (a) one of the outboard Mirnov coils that measure δB_z and; (b) tangential SXR camera with impact parameters (p) from 28.82 to 130.52cm, from 0.18 to 0.25 seconds, covering the period with appearances of fishbone-like bursts

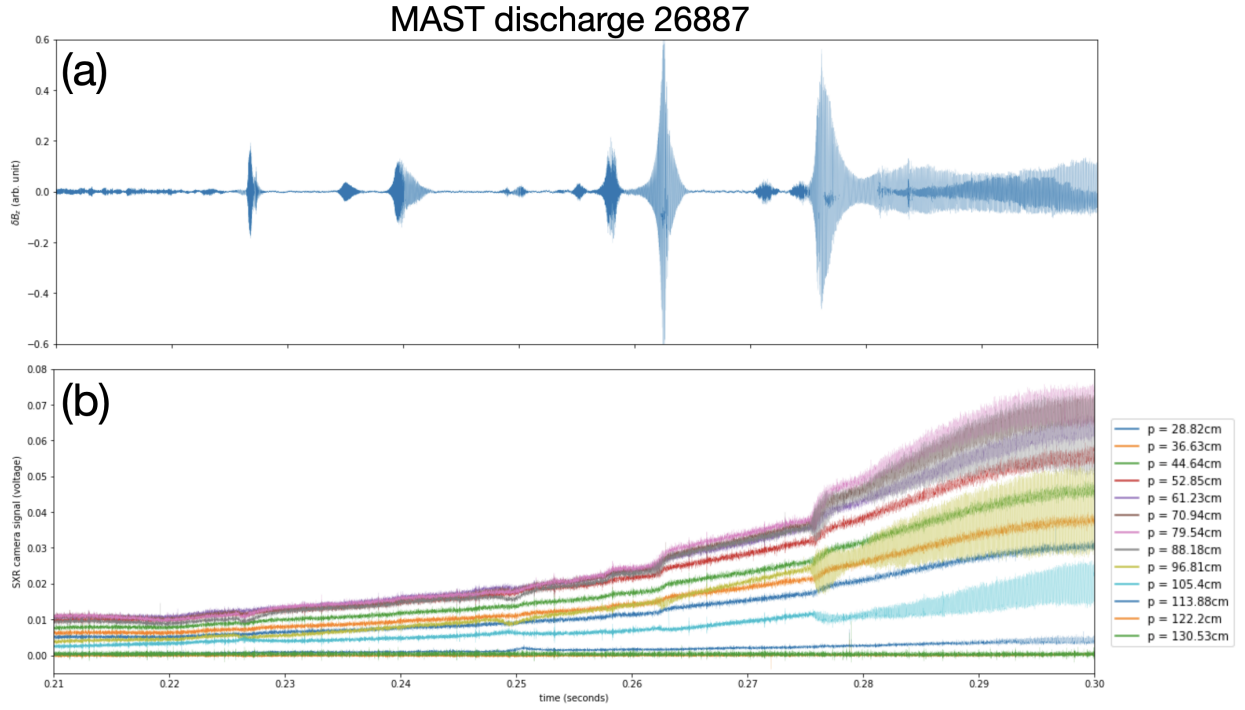


Figure 7.2: MAST discharge 26887 raw signals of (a) one of the outboard Mirnov coils that measure δB_z and; (b) tangential SXR camera with impact parameters (p) from 28.82 to 130.52cm, from 0.21 to 0.30 seconds, covering the period with appearances of fishbone-like bursts

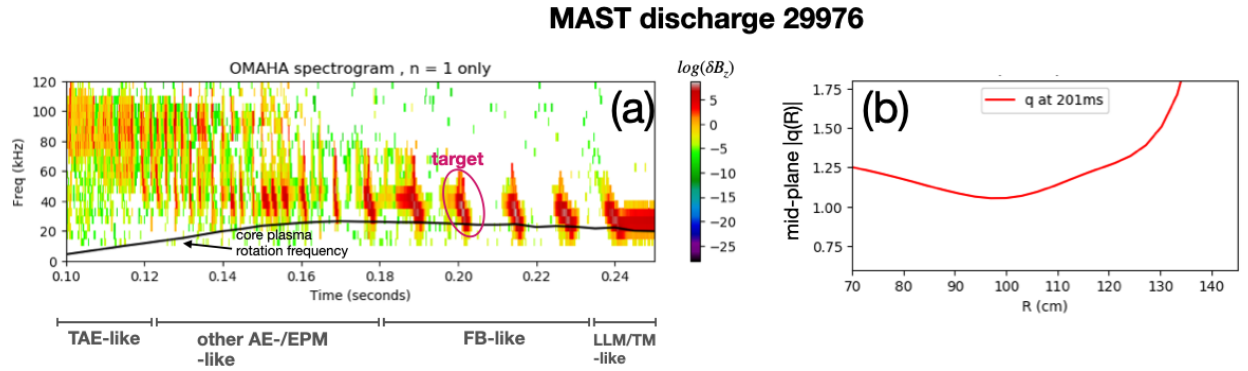


Figure 7.3: (a) the spectrogram of δB_z of shot 29976. Only $n = 1$ is shown. The black line represents the core plasma rotation frequency; (b) Mid-plane q profile 29976 at 201ms.

MAST discharge 26887

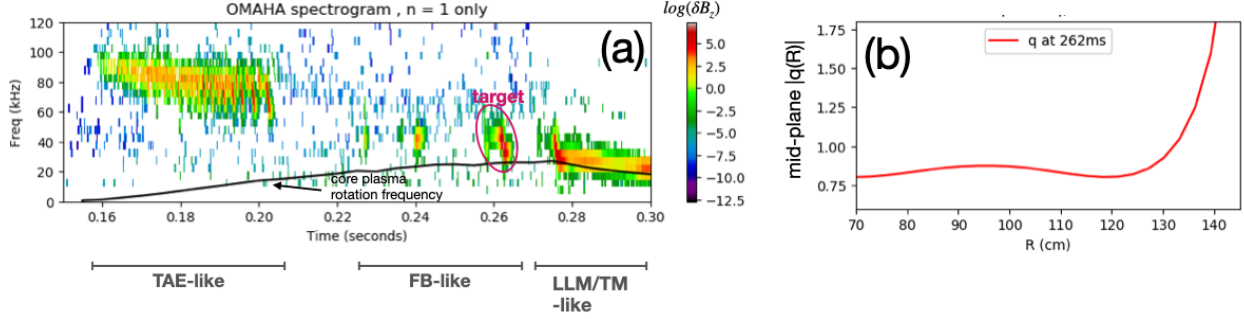


Figure 7.4: (a) the spectrogram of δB_z of shot 26887. Only $n = 1$ is shown. The black line represents the core plasma rotation frequency; (b) Mid-plane q profile of 26887 at 262ms.

7.2 Fishbone measurements in MAST-U

Dedicated experiments were conducted in MAST-U to measure characteristics of various low-frequency modes, including FBs [44][45][82]. The discharges from this experiment aim to excite FBs before the appearance and domination of other low-frequency instabilities, such as long-lived modes (LLMs), tearing modes (TMs), and sawtooth oscillations which appear as $|q|_{min}$ decreases. Discharges 47128 and 47132 were selected for analysis due to their similarities (with the raw signals from one of the Mirnov coils and mid-plane beam emission spectroscopy (BES) APD cameras shown in Fig. 7.8 and 7.9, respectively). It is worth noting that the MAST-U OMAHA signals shown here are high-pass-filtered as mentioned in Chapter 4.2, but the same does not apply to the BES signals. Hence a 0.5 kHz signal appears in BES but does not show up in the Mirnov signals. Key parameters, including the spectral characteristics of other modes found in δB_R spectrograms (Figures 7.10(a) and 7.11(a)), plasma current (Figures 7.12 (b) and 7.13(b)) and core electron density (Figures 7.12(d) and 7.13(d)), are comparable around the time of FB-like and other MHD activities. One primary difference is the heating power, which is slightly higher in the nearly steady state at about 1.5 MW in discharge 47132 (Fig.7.13(c)) compared to about 1.3 MW in 47128 (Fig.7.12(c)). This higher heating power in 47132 results in marginally higher core ion temperatures and neutron emission (Fig.7.11(e)) compared to 47128 (Fig.7.11(e)) and a slightly elevated $|q_0|$ inferred from the motional Stark effect (MSE) diagnostic [63] [64] [81] by about 0.2 (Figures

MAST discharge 29976

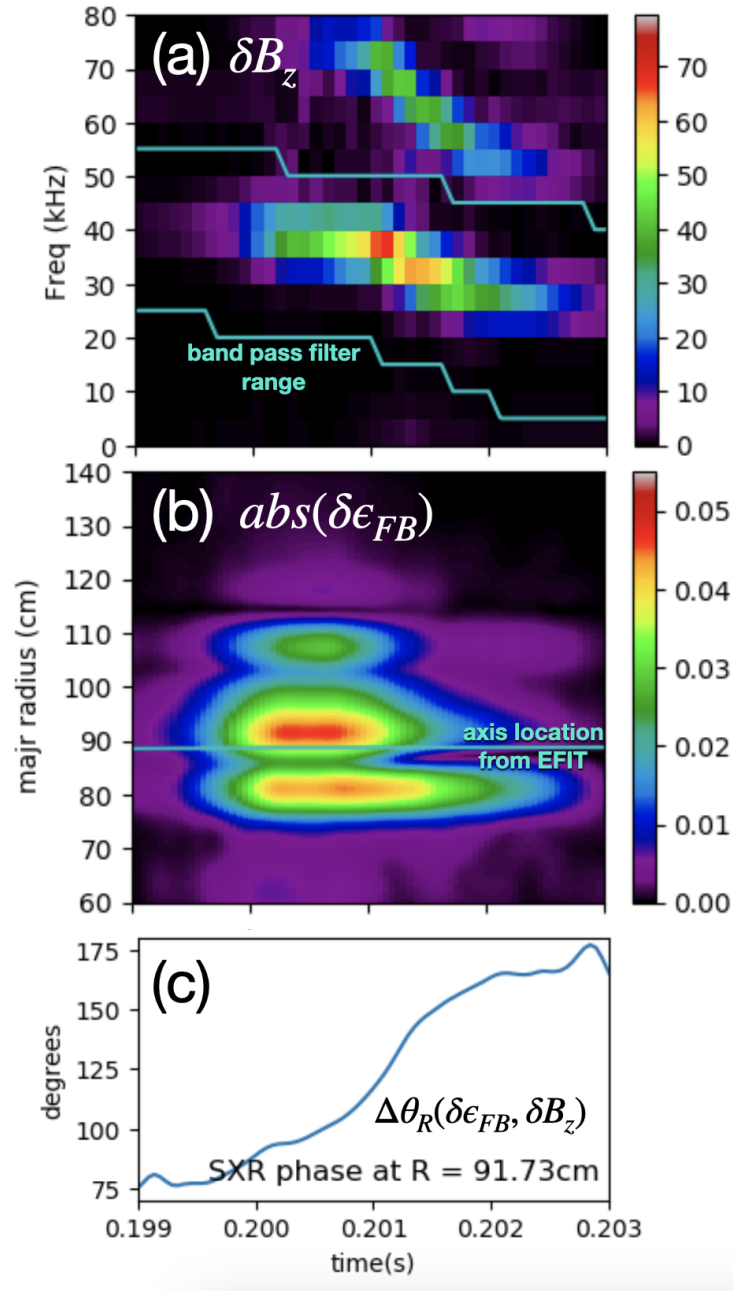


Figure 7.5: (a) spectrogram of δB_z of shots 29976; (b) amplitude of $\delta \epsilon_{FB}$ from SXR after band pass filter, linear regression and emissivity inversion major radius versus time with the magnetic axis location from EFIT labeled in a turquoise line; (c) temporal phase variation of band pass filtered and inverted SXR versus band pass filtered δB_z at $R \approx 92\text{cm}$ for FB bursts

MAST discharge 26887

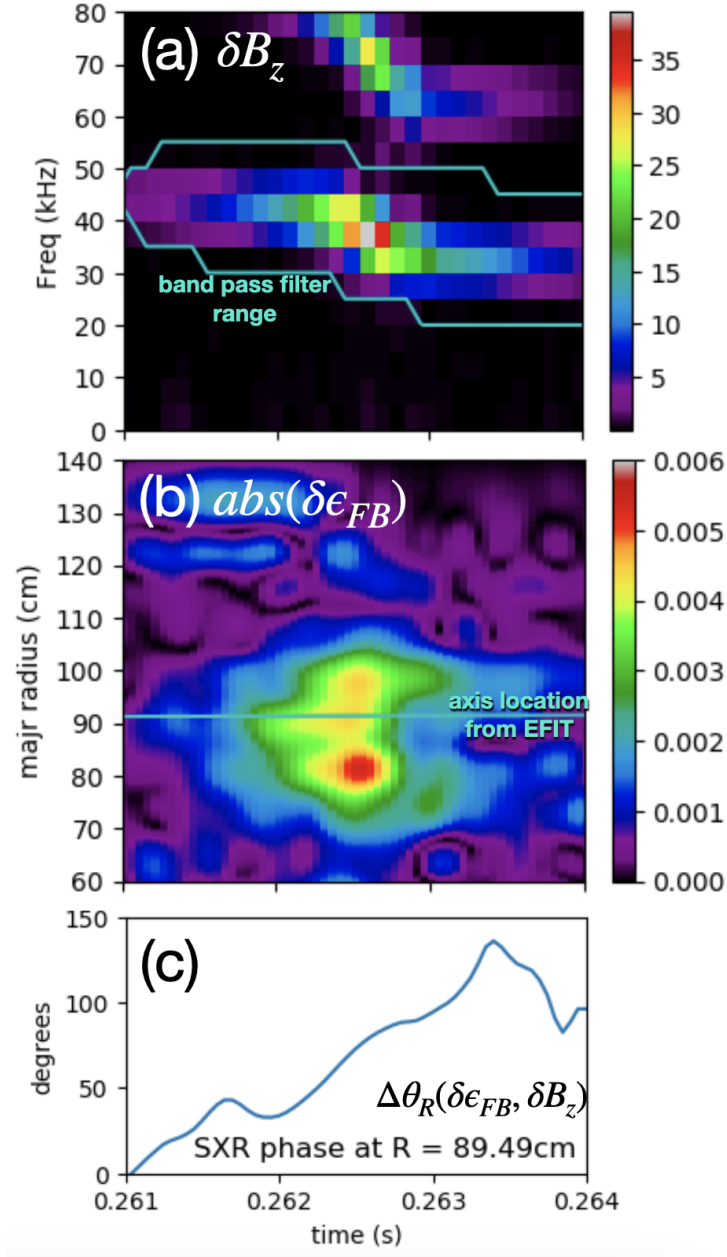


Figure 7.6: (a) spectrogram of δB_z of shots 26887; (b) amplitude of $\delta\epsilon_{FB}$ from SXR after band pass filter, linear regression and emissivity inversion major radius versus time with the magnetic axis location from EFIT labeled in a turquoise line; (c) temporal phase variation of band pass filtered and inverted SXR versus band pass filtered δB_z at $R \approx 89\text{cm}$ for FB bursts

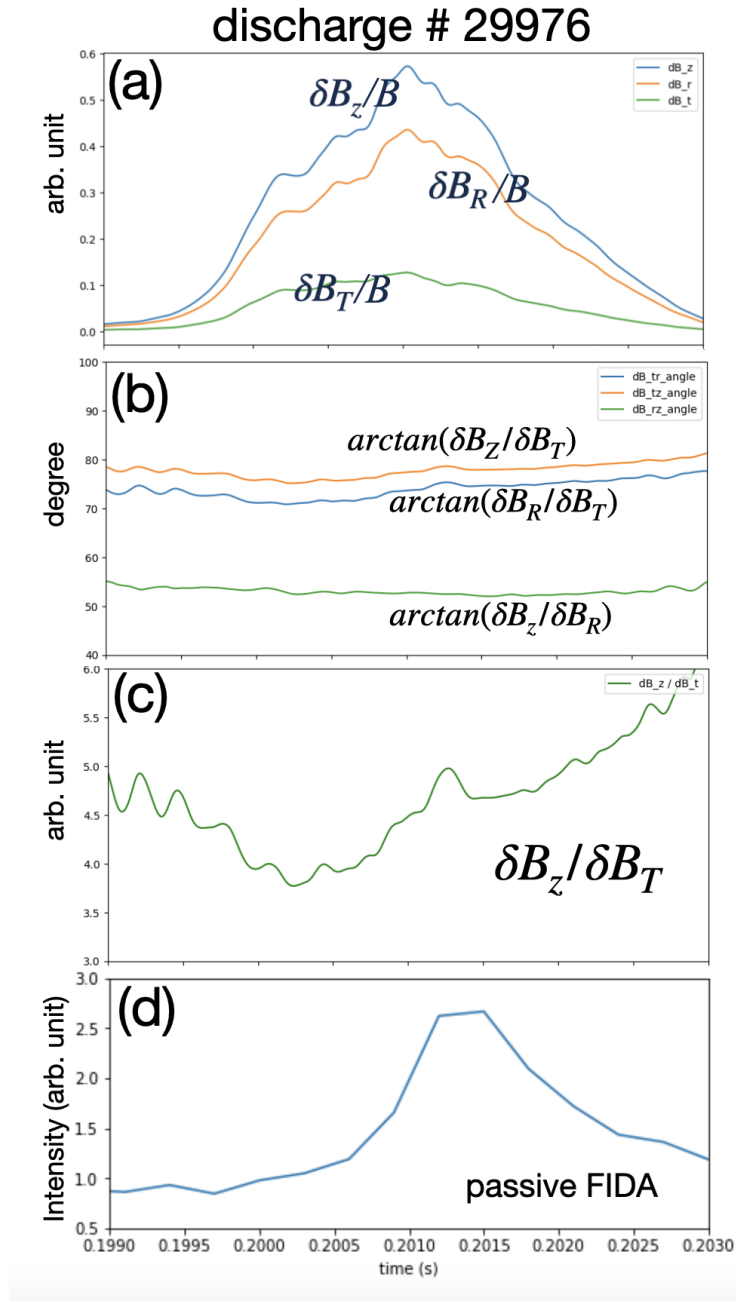


Figure 7.7: (a) real parts of the FB associated $\delta B_z, \delta B_R, \delta B_T$; (b) angle between real parts of $\vec{\delta B}$ components; (c) ratio of δB_z and δB_T amplitude; (d) the passive FIDA signal from a channel with the chord impact parameter at $R = 1.20\text{m}$ in the discharge 29976.

7.13(g) and 7.12(g)). This makes the $|q_0|$ remains well above 1 in discharge 47132 for a longer period of time as a consequence, while in 47128, $|q_0|$ drops below 1 after the disappearance of FB-like chirps.

Despite the differences in the heating power and $|q_0|$, the spectral signatures of various beam-driven low frequency instabilities are similar in discharges 47128 and 47132. Bursts are identified based on comparisons of their mode frequencies and toroidal mode numbers (n) with other instabilities in the same discharge. These FB-like bursts usually have frequencies well below the expected large aspect ratio ideal MHD (n, m) = 1,1 Toroidicity-driven Alfvén Eigenmode (TAE) [70] [9] frequency ($v_A/2qR$, where m is the poloidal mode number) and are closer to the plasma rotation frequency at $R = 1.0$ m. The n were identified for each time and frequency using an SVD-based method [52]. The bursts to be reported here are classified as FB-like only if they have $n = 1$ with the mode frequencies much lower than $v_A/2qR$ and closer to the core plasma rotation frequency. In both discharges, FB-like bursts initially appear at about 40 kHz, with their frequencies decreasing as $|q_0|$ decreases. Each chirp lasts about 3–5 ms (Figures 7.12(b) and 7.13(b)). All FB-like bursts in 47128 and 47132 occur when $|q_0| > 1$, even accounting for uncertainties in MSE measurements (Figures 7.12(g) and 7.13(g)), indicating that they are in the infernal-like ($|q|_{min} > 1$) regime.

This study focuses on the FB-like bursts in discharge 47128. Although the beam-driven low frequency instabilities are similar in both discharges, the BES diagnostic in discharge 47128 was configured to capture 2D structures near the axis, enabling detailed characterization of FB-like modes. Data from discharge 47132 demonstrate the repeatability and similarity of the equilibria, as well as fast and thermal ion transport, between the two discharges. However, these observations are not discussed in detail, as the analysis focuses on four FB-like bursts (denoted as chirps A, B, C, and D) in discharge 47128. The FB-like chirps are first about 15ms apart of each other, such as the bursts at 310ms (chirp A) and at 325ms (chirp B) in discharge 47128. Earlier chirps were excluded because their $|q_0|$ values are much higher than 1, and their frequencies are elevated relative to the core plasma rota-

tion frequency, suggesting potential contributions from Alfvén eigenmodes, similar to those observed in NSTX-U simulations [67]. The later chirps C and D appear in close proximity in time-frequency space (the "double FB-like" regime labeled in Figures 7.10 and 7.12). The burst following the appearance of double FBs initially resembles a FB but then quickly transition to MHD- or LLM-like bursts for 50-70 ms. This type of synergy in NSTX plasma is well-documented [37] [99] [109], where $(n, m) = (1, 1)$ modes trigger nonlinear interactions with $(1, 2)$ modes at the $q = 2$ surface, leading to pressure and rotation profile flattening and reduced neutron emission. Similar interactions and their effects on bulk confinement have also been observed in other MAST-U discharges [19] [44] [82]. For this reason, the FB-like burst following chirp D is excluded from this study to avoid misidentification of the mode due to mixed FB and MHD characteristics.

7.2.1 Fast ion transport associated measurements

Measurements from MSE, charge exchange recombination spectroscopy (CXRS) [105], and neutron emission via fission chambers indicate significant fast-ion transport associated with various bursts in discharge 47128. The MSE array measures magnetic field pitch angles at different locations in the plasma mid-plane accessible by one of the neutral beams. The pitch angle shows rapid changes during or after many bursts in 47128 (Fig.7.10(b)). During each burst, the magnetic axis's radial position initially moves outward and then rapidly inward, as inferred from EFIT++ equilibrium reconstruction constrained by MSE measurements. Fig.7.10(c) presents an alternative determination of the magnetic axis location directly inferred from MSE pitch measurements without relying on equilibrium reconstruction. Given that the pitch is zero at the magnetic axis by definition, the radial position R of the magnetic axis is inferred by interpolating pitch vs R to find where pitch equals zero. Changes in plasma pressure can alter Shafranov shifts [83], causing rapid movements of the magnetic axis. Therefore, quick redistributions of fast ions and/or bulk thermal plasma species can lead to rapid magnetic axis movements observed in discharge 47128. Drops in neutron emis-

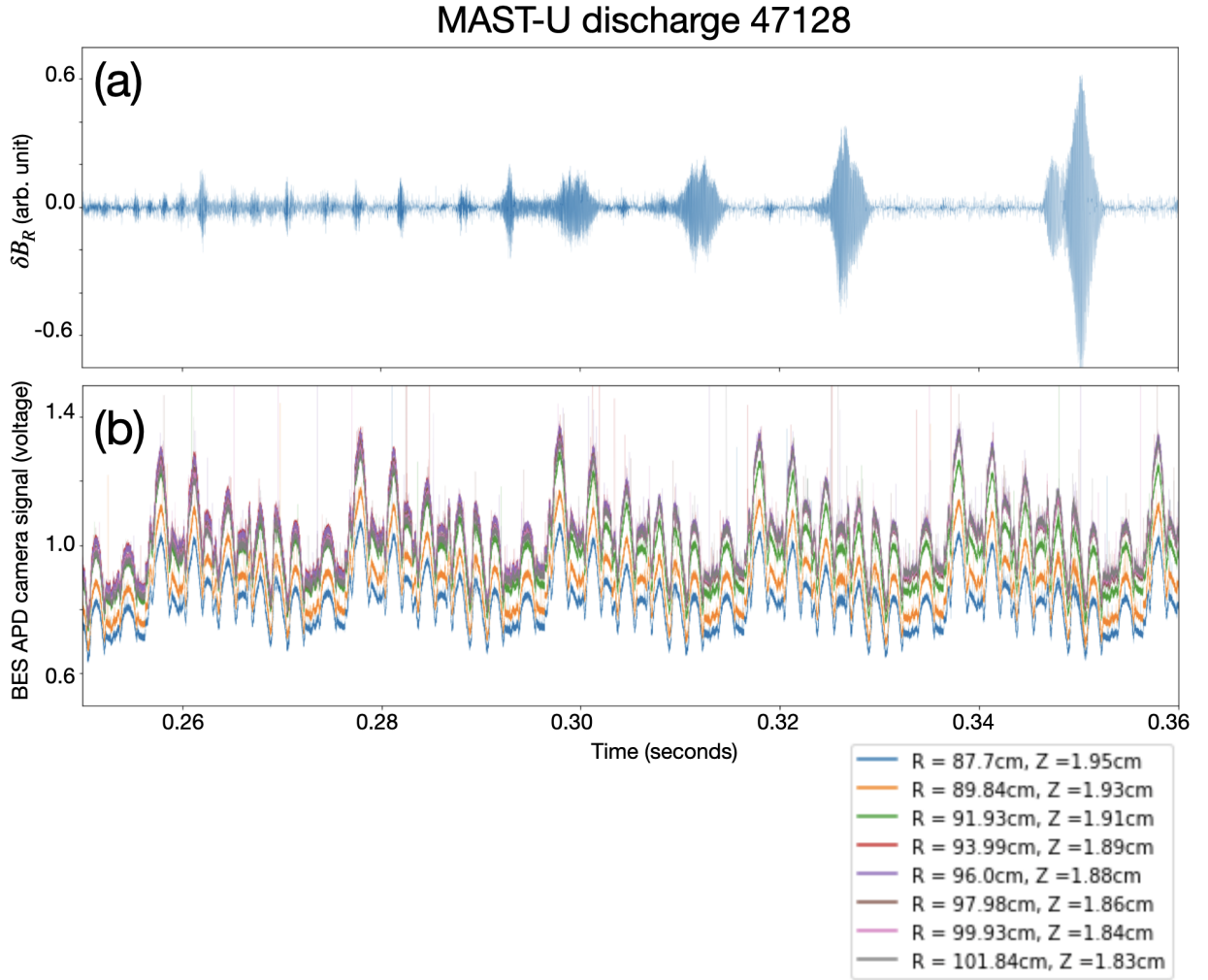


Figure 7.8: MAST-U discharge 47128 raw signals of (a) one of the outboard Mirnov coils that measure δB_R and; (b) eight mid-plane located BES APD cameras

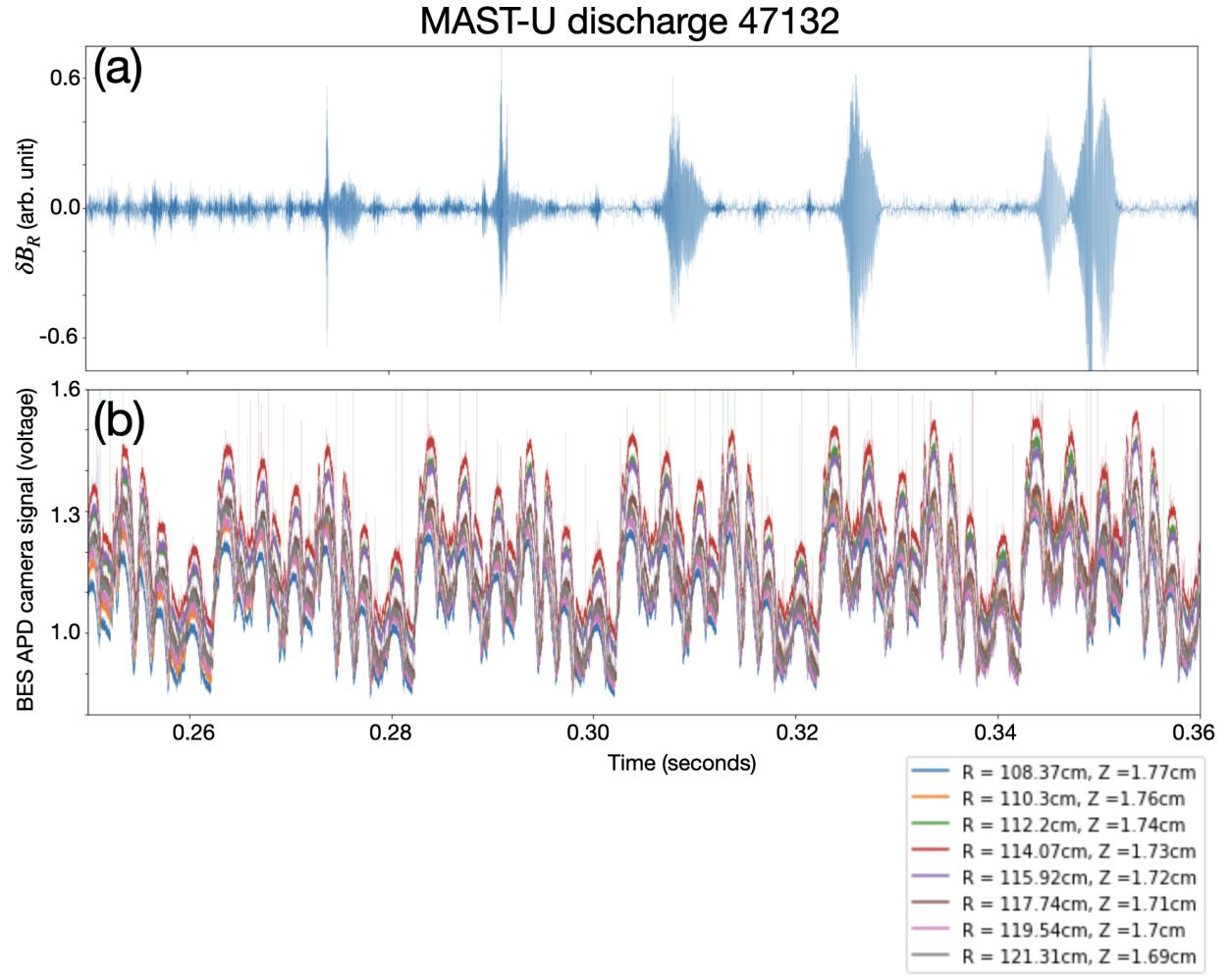


Figure 7.9: MAST-U discharge 47132 raw signals of (a) one of the outboard Mirnov coils that measure δB_R and; (b) eight mid-plane located BES APD cameras

sion rates were also observed during or after bursts in shot 47128 (Fig.7.10(d)), indicating reductions in fusion reactions likely due to fast ion transport or losses and/or reductions in bulk plasma confinement. CXRS measurements of carbon impurity ion rotation reveal significant deceleration near the core after many bursts (Figures 7.10(e)). Since neutral beam injection predominantly drives plasma rotation in MAST-U, sudden decreases in rotation may indicate substantial losses of beam-sourced fast ions in the core.

Comparisons of analysis of profiles inferred from various diagnostics highlights the $|q_0|$ dependency of frequencies and fast ion transport during single-chirp FB-like bursts in discharge 47128. For instance, chirp B exhibits a lower $|q_0|$ compared to chirp A. Although the axis movements during chirps A and B are comparable, the neutron rate drop is slightly more pronounced after chirp B than after chirp A, possibly due to differences in ion densities during the appearances of chirps A and B. On the other hand, while the magnetic axis movements are comparable, the FB chirps with lower $|q_0|$ and lower frequency tend to induce more significant drops in neutron rates. This aligns with previous FB simulation results, where $(n, m) = (1, 1)$ modes tend to be more unstable (and consequently higher growth rates and saturated mode amplitudes) at lower $|q_0|$ [67].

7.2.2 Thermal ion transport associated measurements

Surprisingly, notable thermal transport are associated not only with MHD and sawtooth-like bursts but also with double FBs in discharge 47128. CXRS also measures thermal ion temperature profiles in addition to ion rotation. The CXRS sampling rate is sufficient to detect profile changes occurring at intervals of 1 ms or longer. The CXRS captures noticeable core thermal ion temperature drops during double FB-like bursts (chirps C and D) prior to interplay between a FB and another MHD activity, although less pronounced (approximately 8-9%) and no flattening of the thermal ion temperature profiles as shown in Fig.7.14(b) compared to Fig.7.14(a). However, it is unlikely that these bursts are associated with other MHD bursts, considering their spectral characteristics and the relatively high

$|q|_{min}$, compared to the MHD activities that appear later in discharge 47128.

Further investigation into these modes is necessary due to their impact on both fast and thermal ion confinement. Detailed studies of their mode structure and poloidal mode numbers are required to determine their characteristics and the mechanisms triggering the transport associated with double FB-like bursts in these discharges.

7.2.3 2D BES measurements on the FB-like bursts

The 2D BES system has been upgraded to provide radial and poloidal mode structure measurements for FBs in MAST-U. A scan of BES viewing locations was conducted in MAST-U with the upgraded diagnostic [29]. The radial structures inferred from BES in discharge 47128 are particularly noteworthy, as the viewing window (centered at $R = 0.95$ m) covered the magnetic axis. The signal-to-noise ratios of these BES signals are not as high as the the edge located Mirnov coil signal, which can be seen by comparing the Mirnov coil spectrogram in Fig. 7.10 and BES spectrograms in Fig. 7.15. The cross coherence analysis explained by Eq. 4.14 in Ch.4.4 can be useful FB-related signals from noise or other local non-global fluctuations in BES measurements. The selected bursts must have have $C_{\delta I_{SX R}, \delta \hat{B}_z}$ above 0.9, which gives uncertainties below 40%, as shown in Fig. 7.16.

The mid-plane radial structures are determined by averaging the fluctuations from two channels located at each radius and approximately 1 cm above and below the mid-plane. Many bursts in discharge 47128, such as chirps A, B and D, show mid-plane radial BES fluctuations peaking at two radial extremities of the viewing window, with minimum fluctuations near the axis locations inferred from MSE (Fig.7.17(b) for chirps A and B, and Fig.7.18(c) for chirp D). Chirp C in 47128, however, exhibit a rather different and unusual radial mode structure. The chirp C associated BES measurement has only one peak located in the high-field (in-board) side without a minimum located in the location of the magnetic axis.

discharge # 47128

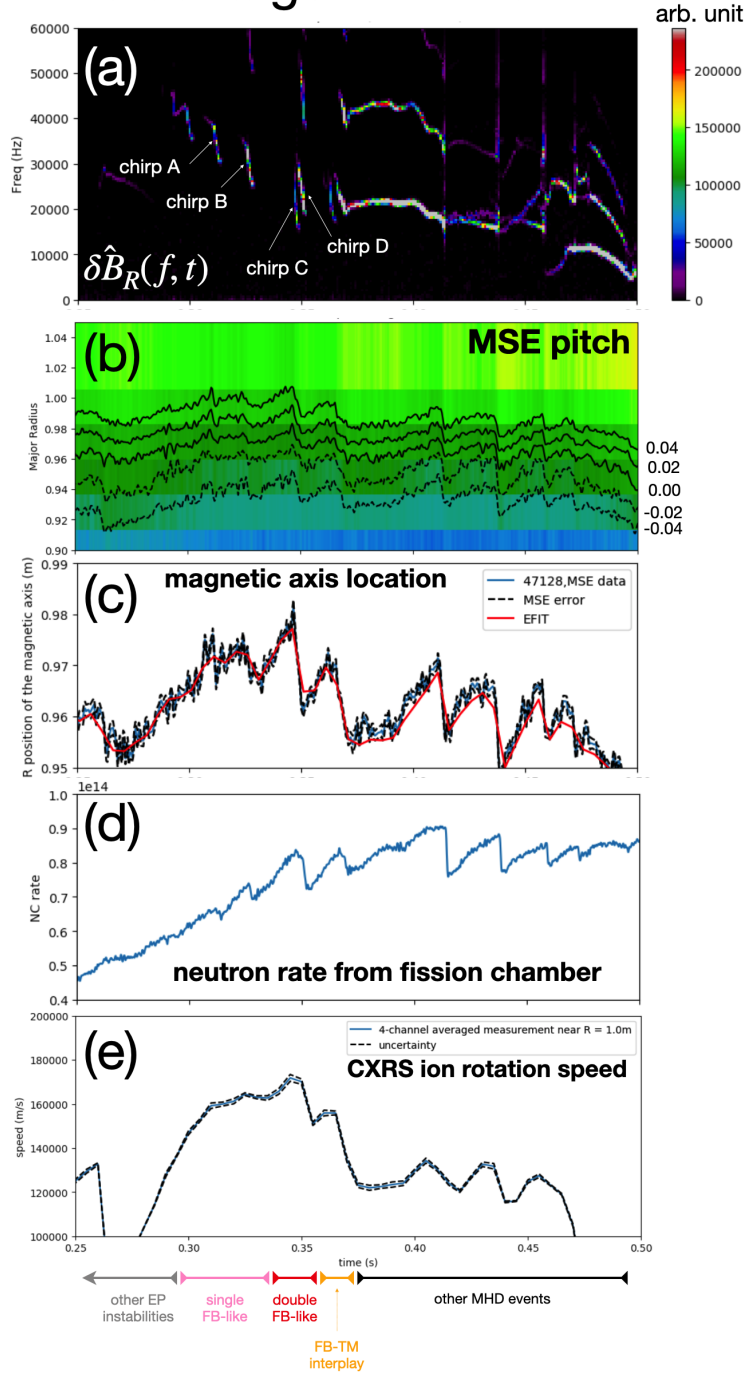


Figure 7.10: (a) spectrogram of δB_R with labels for chirps A, B, C and D; (b) Field pitch versus time inferred from MSE with contour lines labeling pitches of -0.04 to 0.04; (c) magnetic axis locations versus time inferred from MSE and from the EFIT++; (d) neutron emission from fission chamber; (e) Thermal ion rotation speed near $R = 1.0\text{m}$ from CXRS, for discharge 47128 from 250ms to 500ms, with the labels for times of occurrences of various instabilities under the time axis.

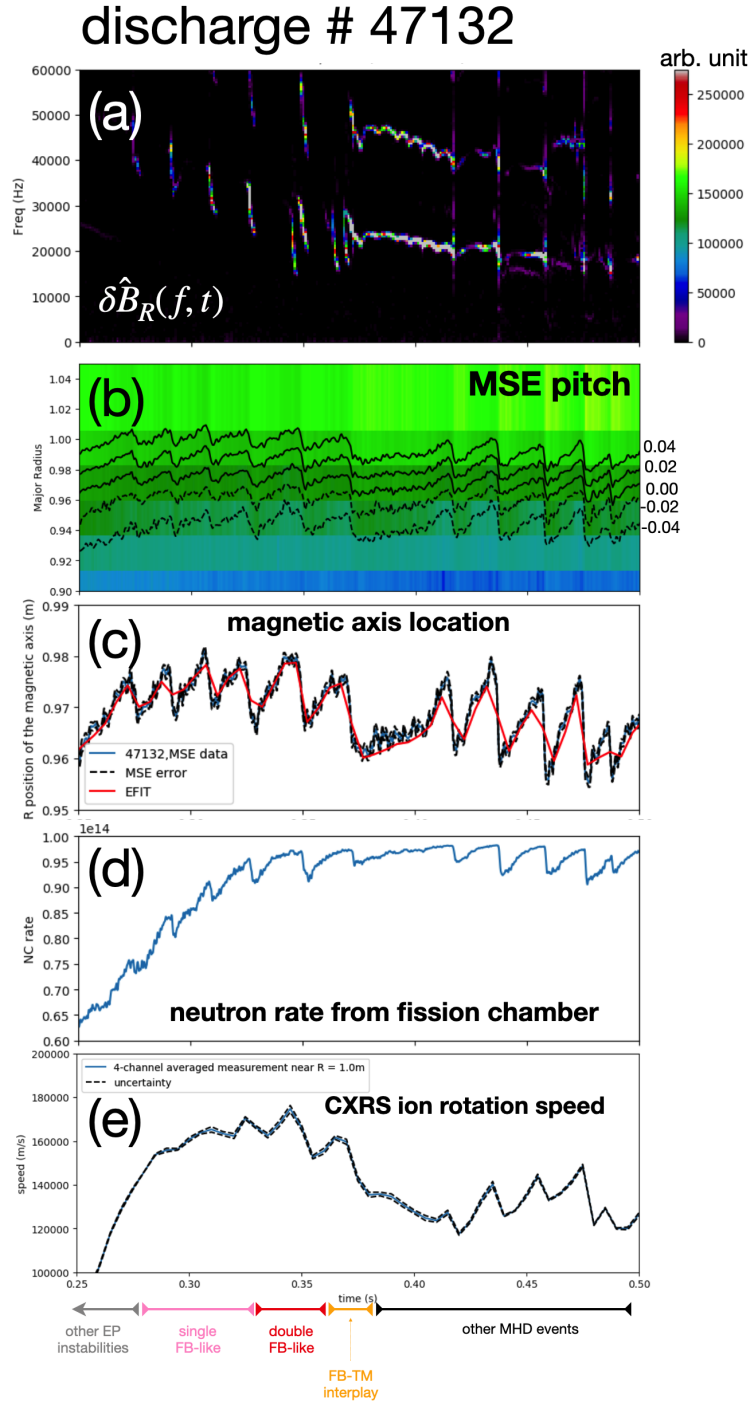


Figure 7.11: (a) spectrogram of δB_R with labels for chirps A, B, C and D; (b) Field pitch versus time inferred from MSE with contour lines labeling pitches of -0.04 to 0.04; (c) magnetic axis locations versus time inferred from MSE and from the EFIT++; (d) neutron emission from fission chamber; (e) Thermal ion rotation speed near $R = 1.0m$ from CXRS, for discharge 47132 from 250ms to 500ms, with the labels for times of occurrences of various instabilities under the time axis.

discharge # 47128

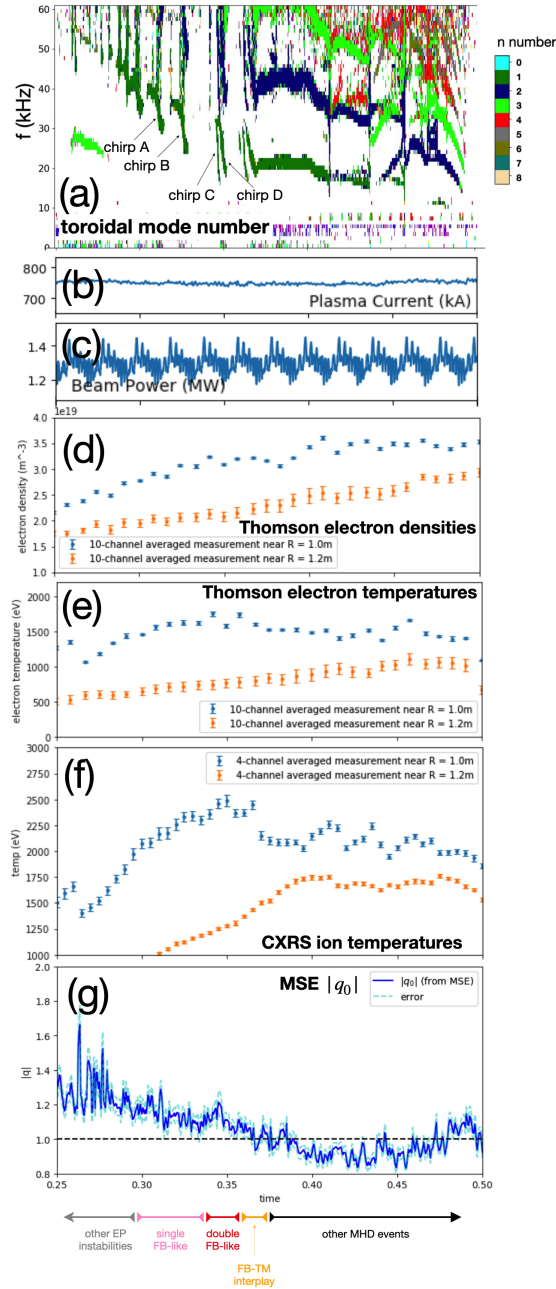


Figure 7.12: MAST-U discharge 47128. (a) Toroidal number identifications for modes from 0 to 60kHz using toroidally separated Mirnov coils (b) Plasma current; (c) On-axis neutral beam injection power; (d) thermal electron densities inferred from the average of ten Thomson scattering channels near $R = 1.0$ m and $R = 1.2$ m; (e) thermal electron temperatures inferred from the average of ten Thomson scattering channels near $R = 1.0$ m and $R = 1.2$ m; (f) thermal ion densities inferred from the average of four CXRS channels near $R = 1.0$ m and $R = 1.2$ m; (g) on-axis $|q_0|$ for discharge 47128 from 250ms to 500ms inferred from the MSE, with the labels for times of occurrences of various instabilities under the time axis.

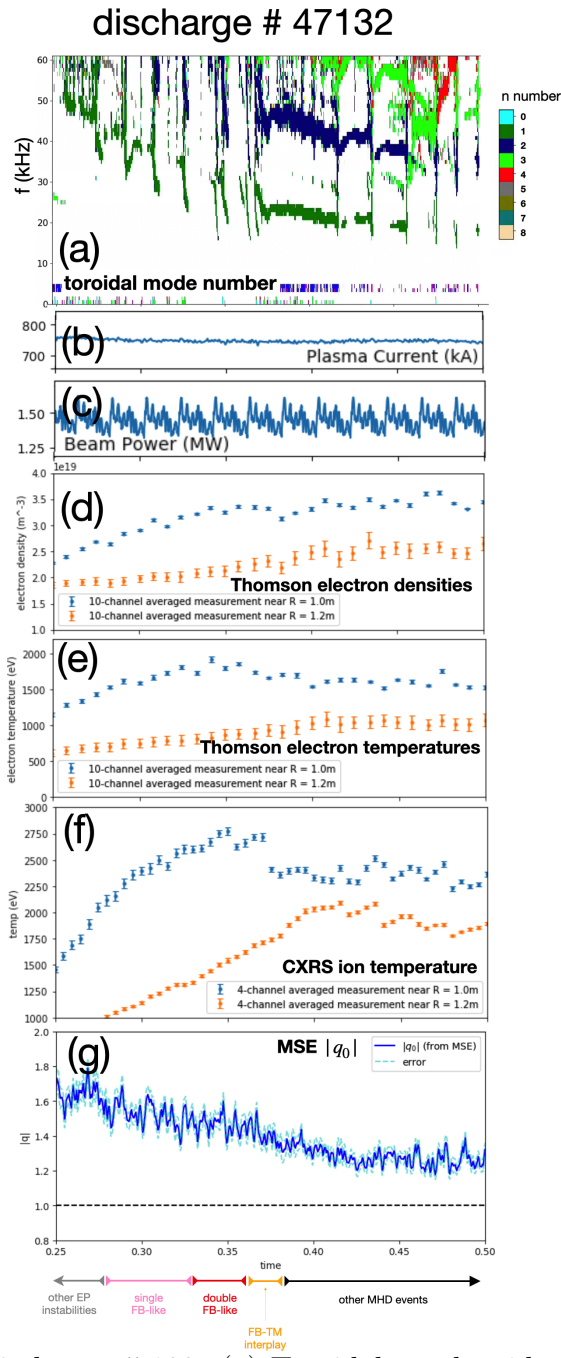


Figure 7.13: MAST-U discharge 47132. (a) Toroidal number identifications for modes from 0 to 60kHz using toroidally separated Mirnov coils (b) Plasma current; (c) On-axis neutral beam injection power; (d) thermal electron densities inferred from the average of ten Thomson scattering channels near $R = 1.0\text{m}$ and $R = 1.2\text{m}$; (e) thermal electron temperatures inferred from the average of ten Thomson scattering channels near $R = 1.0\text{m}$ and $R = 1.2\text{m}$; (f) thermal ion densities inferred from the average of four CXRS channels near $R = 1.0\text{m}$ and $R = 1.2\text{m}$; (g) on-axis $|q_0|$ for discharge 47128 from 250ms to 500ms inferred from the MSE, with the labels for times of occurrences of various instabilities under the time axis.

discharge # 47128, CXRS ion temperature

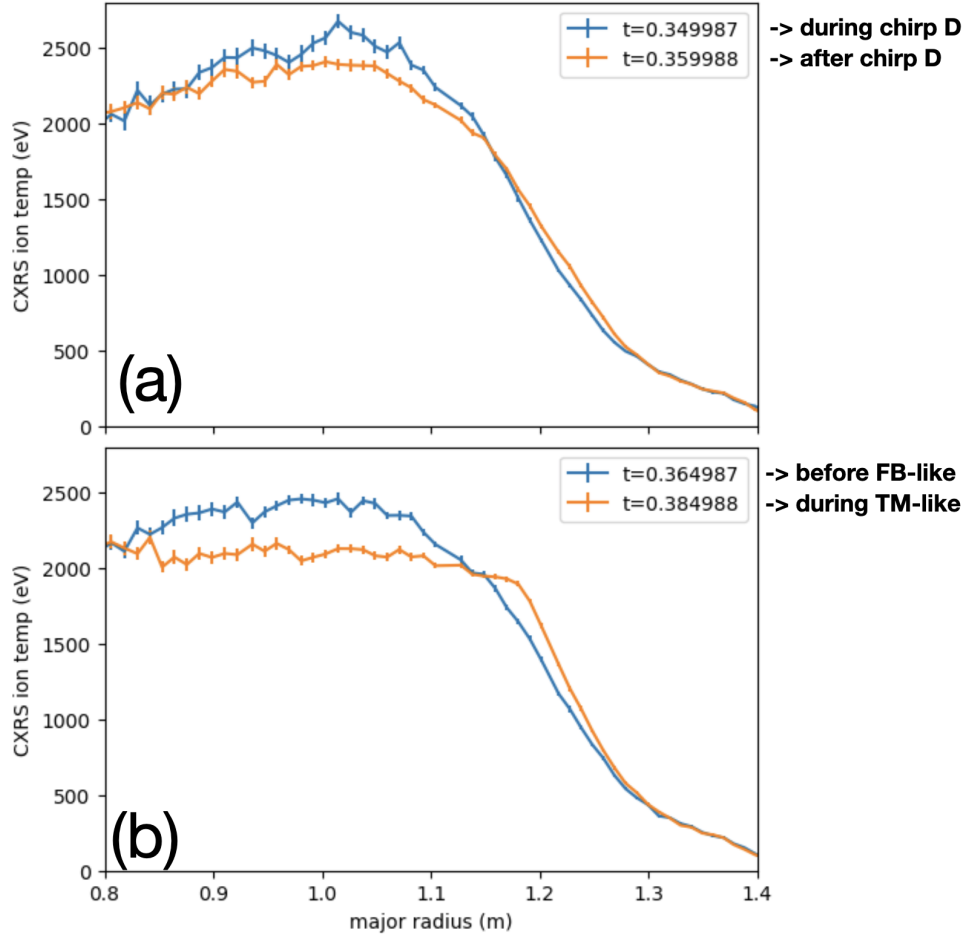


Figure 7.14: The thermal ion profiles inferred from CXRS in discharge 47128 at (a) 350ms and 360ms, during and after chirp D (the last burst in the series of double FB-like bursts), and (b) before the another FB-like and during MHD-like bursts

The evolution of phases between near-axis BES and the edge-localized Mirnov coil measurements can be used to determine the effects of shifts in magnetic axis and distortion of magnetic fields to the complex structures of the FB-like bursts. Accurate BES versus Mirnov phase determination across different chirps requires sufficient statistical accuracy. Sampling phase variations when δI_{FB} amplitudes are small can lead to false impressions of significant temporal phase variations. The full-width half-maxima (FWHM) of each peak in the low-pass filtered δI_{FB} from a channel near the mid-plane at approximately 98 cm is used to determine the time range for sampling phase variations. The mid-plane phases are determined by averaging the phases from two channels located at $R \approx 98$ cm and approximately 1 cm above and below the mid-plane. Using this method, the temporal phase of chirp A at 310 ms varies from $\sim 210^\circ$ to $\sim 120^\circ$, and chirp B at 326 ms varies from $\sim 205^\circ$ to $\sim 130^\circ$ (Fig.7.17(c)). Chirp C shows similar temporal phase variation to chirps A and B, while chirp D shows slightly larger temporal phase variation compared to other targeted chirps in discharge 47128 (Fig.7.24(d)). The phase relationship between δI_{FB} and δB_R is governed by the physics of mode stability and propagation. The thermal and fast ion transport shown in sections 7.2.1 and 7.2.2 alter the medium the modes are traveling in, and can cause distortions of modes when they are observed in the lab frame. Notably, chirp C has an average phase about 180° different from chirp D and other chirps, which all have similar average phases. This suggests that the chirp C may be a fundamentally different mode compared to other FB-like modes in discharge 47128.

The comparisons between the mode amplitudes measured by core localized BES and edge Mirnov coil also reveal the differences between chirps C and D. The peak chirp C associated BES ($real(\delta \hat{I}/I_0)_{filtered}^{-1}$) amplitude are about 60% compared to chirp D. In contrast, the peak chirp C associated Mirnov coil signal is only about one-third of the peak chirp D Mirnov coil signal amplitude (Fig.7.24(a)). The discrepancy of chirp C associated signals inferred from core localized and edge measurements suggests the mode is core-localized with a much smaller spatial extent compared to other FB-like burst, or localized in the in-board

(high-field) side. This eliminates the possibility of chirp C being an off-axis mode, such as the off-axis FBs found in other tokamaks (e.g. in DIII-D [50]) that locate near the edge of the out-board side and could have strong interactions with the wall.

The 2D BES measurements suggest that not all bursts that exhibit FB-like spectral characteristics share similar mode structures. The dominant poloidal mode numbers (m) for chirps C and D in discharge 47128 are determined by the 2D BES. The 2D viewing window of BES in this shot covered regions immediately above and below the magnetic axis. Thus, the dominant m numbers for the targeted chirps in the core can be identified if the their poloidal structures exhibit features typically found in modes with such m . For example, snapshots of (R, Z) of the real part of the band-pass filtered (without linear regression) BES signals ($real(\delta\hat{I}/I_0)_{filtered}^{-1}$) associated with chirp B show an obvious $(n,m) = (1,1)$ structure near the axis, where where a positive and negative nodes separated for about 180 degrees of poloidal angles and rotate poloidally over time, as shown in Fig.7.19. The (R, Z) snapshots associated with chirp D at 349ms also exhibit similar features (Fig.7.22). These measurements suggest both chirps B and D are likely to be (1,1)-dominant at the times of their peak amplitude. However, the (R, Z) snapshots associated with chirp C reveal rather unusual features for a FB-like burst. There is no distinctive $m = 1$ structure within the BES viewing window. Instead, the BES captures the tails of two nodes with opposite parities associated with chirp C at first at 348.01ms, as shown in Fig.7.20. 0.315ms later at 348.325ms, the core structure of chirp C evolves to have a single node near $\rho = 0.15$ only, and it only oscillates over time but without any poloidal rotation, as shown in Fig.7.21. This suggests a dominant $m = 0$ structure associated with chirp C in the region near the axis ($\rho < 0.1$) at 348.325ms, which is rare for a mode that exhibit FB-like spectral features.

The waveform measurements from poloidally separated BES channels show that the dominant poloidal mode number of chirp D in the core is affected by chirp C. The evolution of m dominance for FB-like chirps can be estimated by comparing waveform measurements from three poloidally separated BES channels located near $\rho = 0.1$ over a much longer

period of time compared to the (R,Z) snapshots shown previously. For instance, the chirp B associated $real(\delta\hat{I}/I_0)_{filtered}^{-1}$ from two channels located above and below the axis are about 180 degrees out of phase, while the signal near the mid-plane in the high field side has the phase in between (Fig.7.23). This persists for the majority of the time (about 1ms) over the period of the chirp B, indicating it likely to be a (1,1)-dominant mode. In contrast, chirp C associated signals from all of these three locations are almost in phase from 347.5 ms to 348.5 ms (Fig.7.24(c)). The waveform of these chirp C associated measurements, alongside with the 2D and radial structures mentioned previously, suggest the chirp C associated near-axis $m = 0$ structure persists throughout most of the time of its appearance. On the other hands, a transition of m dominance occurs for chirp D. The chirp D associated $real(\delta\hat{I}/I_0)_{filtered}^{-1}$ from two channels located above and below the axis are partially out of phase in the beginning of the appearance of chirp D when its mode amplitude is comparable to chirp C. These signals transitions to be similar to chirp B. This suggests chirp D has the mixed odd-even m at first, and later becomes (1,1)-dominant when chirp C disappears. This suggests the occurrence of a mode-mode interaction, altering the mode characteristics of one the modes that appears at the same time with different frequencies.

7.2.4 Discussions on the FB-associated thermal and fast ion transport in MAST-U

The mode amplitudes of FB-like bursts can explain the severity of fast ion transport but cannot fully account for the observed drops in central thermal ion temperatures during and after double FB-like bursts. The EP convective resonant transport model suggests that the EP transport and losses caused by FBs scale linearly to their mode amplitudes [90], as the $E \times B$ drifts that drive the EPs are linearly proportional to the electric field (E) [48]. The drops in neutron emission in discharge 47128 (Fig.7.10(d)) are proportional to the amplitudes of FBs (Fig.7.8(a)). Chirp D has the highest mode amplitude among other FB-like bursts in the same discharge, and the additional contribution from chirp C result in significant fast

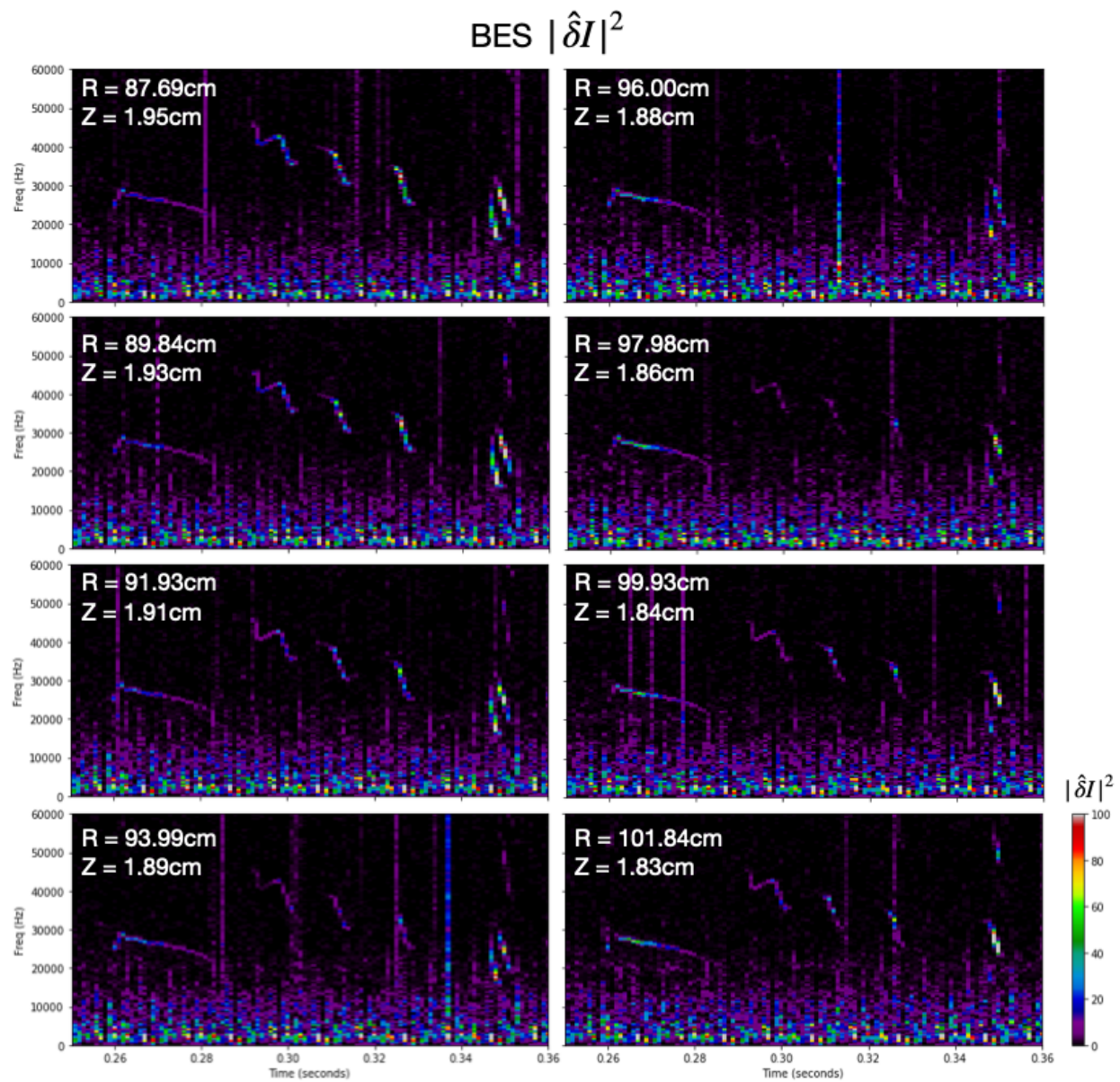


Figure 7.15: Spectrograms of 4 mid-plane BES channels channels in MAST-U discharge 47128 from 250 to 360ms.

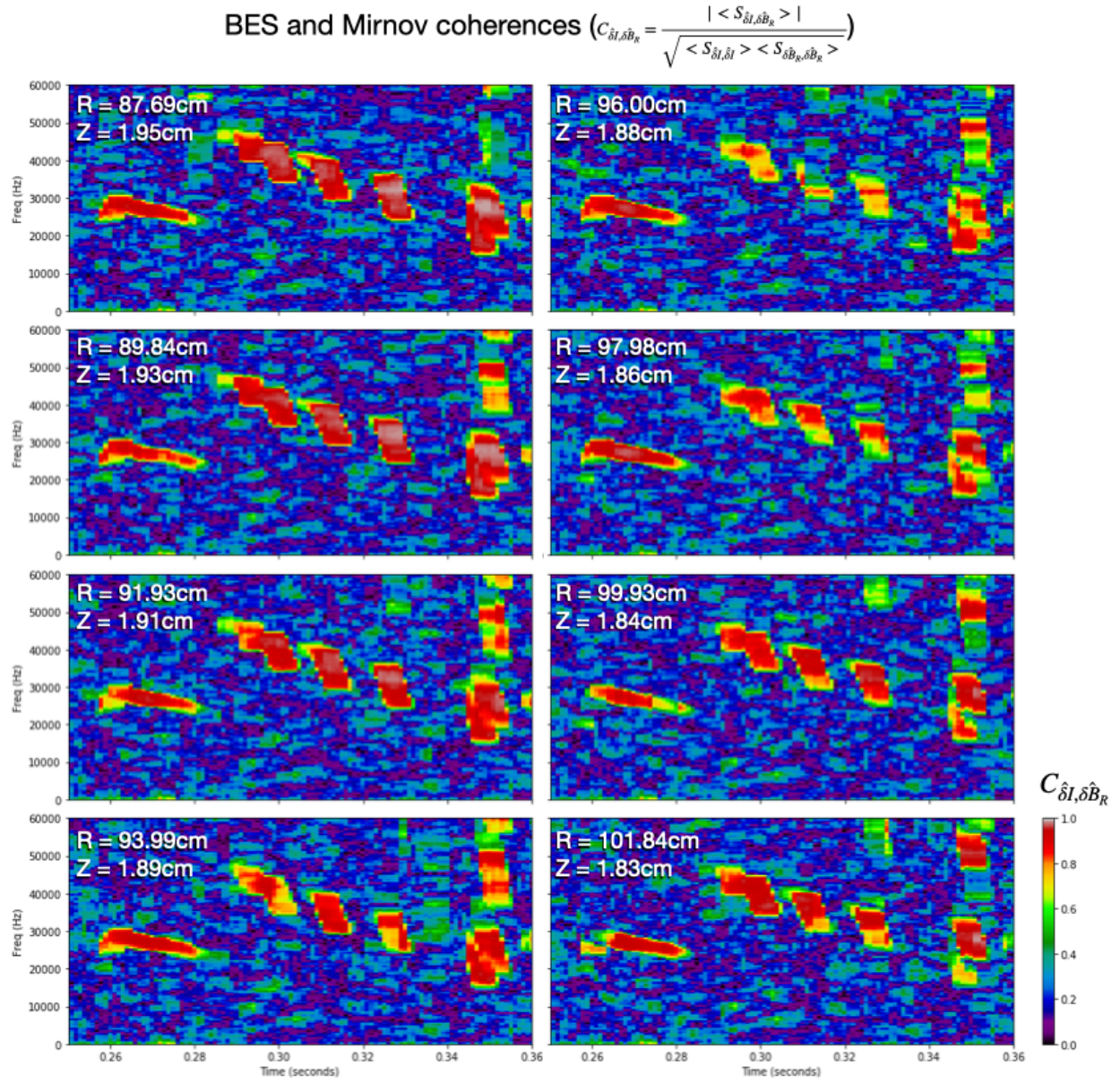


Figure 7.16: Cross coherences of STFT of measurements from 4 mid-plane BES channels and STFT of δB_R in MAST-U discharge 47128 from 250 to 360ms.

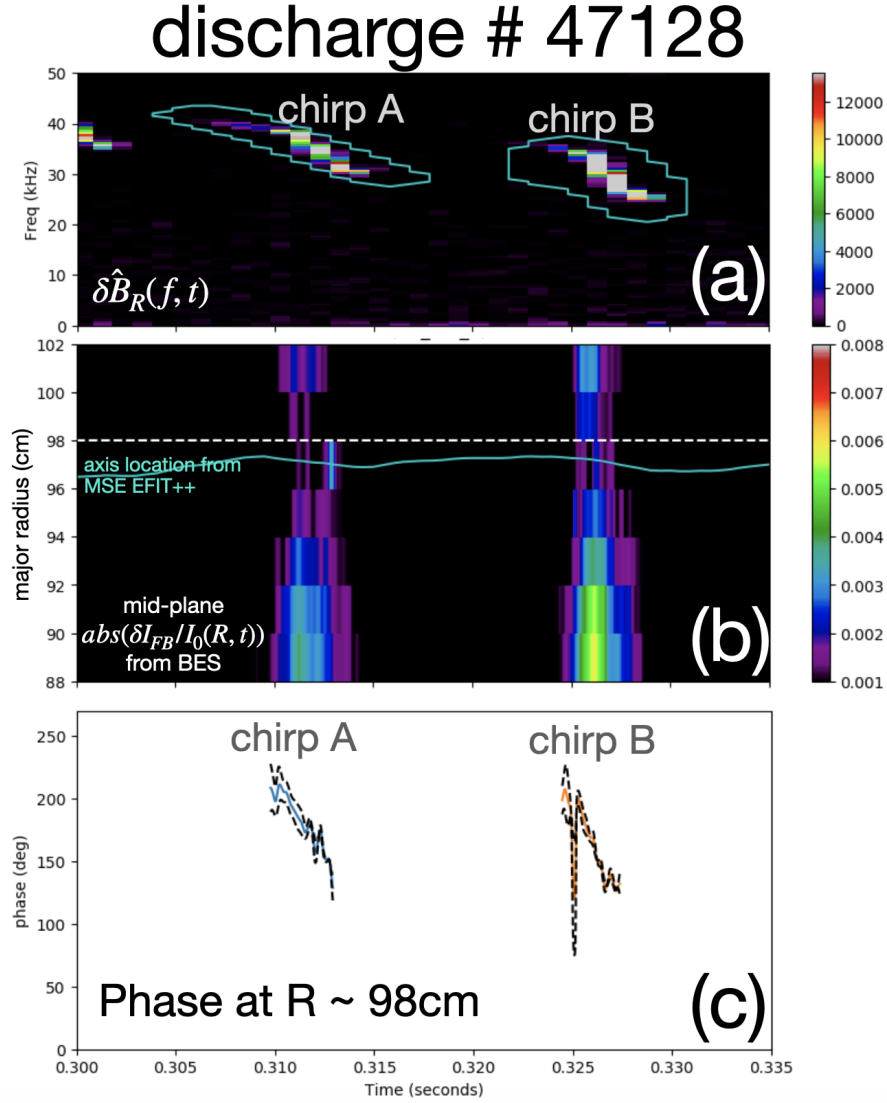


Figure 7.17: (a) spectrogram of δB_R of shots 47128; (b) amplitude of $\delta I/I_0$ from BES associated with chirps A and B after band pass filter and regression in major radius versus time with the magnetic axis location from EFIT++ labeled in a turquoise curve; (c) averaged temporal phase shift of two band pass filtered BES channels above and below the mid-plane at $R \approx 98\text{cm}$ associated with chirps A and B.

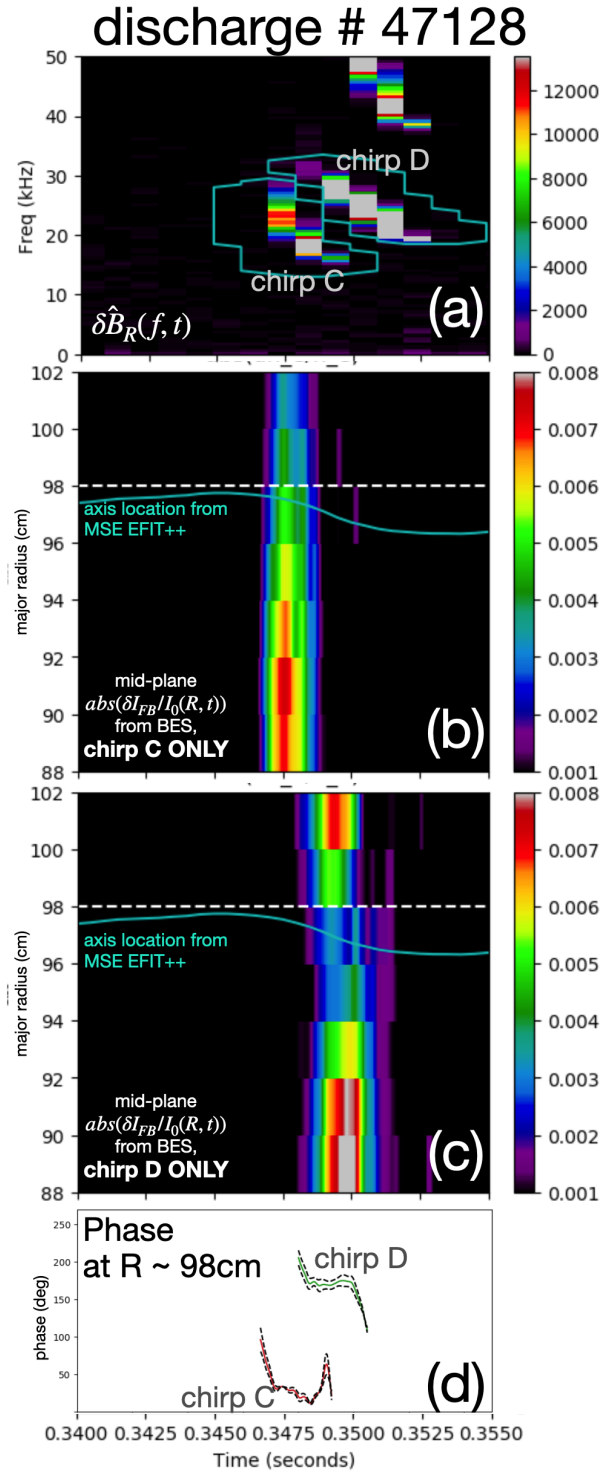


Figure 7.18: (a) spectrogram of δB_R of shots 47128; amplitude of $\delta I/I_0$ from BES associated with (b) chirp C only and (c) chirp D only after band pass filter and regression in major radius versus time with the magnetic axis location from EFIT++ labeled in a turquoise curve; (d) averaged temporal phase shift of two band pass filtered BES channels above and below the mid-plane at $R \approx 98\text{cm}$ associated with chirps C and D.

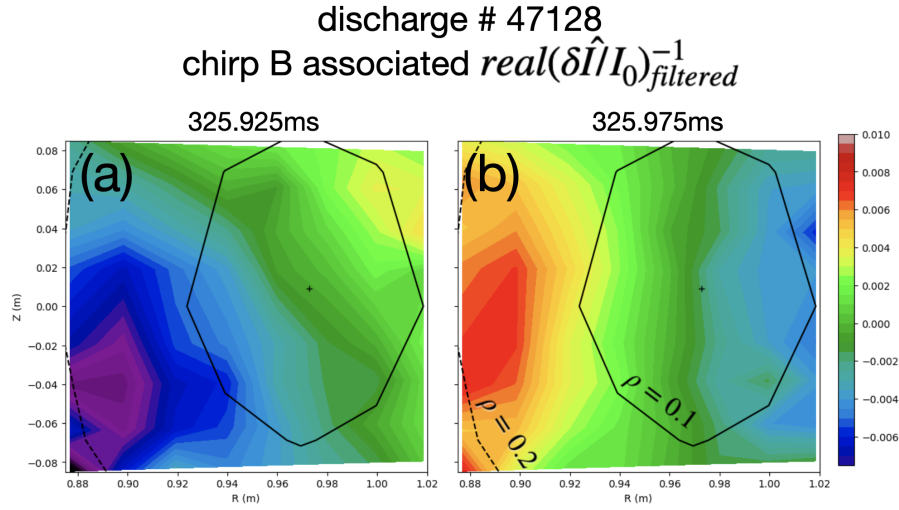


Figure 7.19: the (R,Z) snapshots of chirp B associated $real(\delta I/I_0)$ without linear regression at (a) $t = 325.925\text{ms}$ and (b) 325.975ms . $\rho = 0.1$ and 0.2 surfaces are labeled in solid and dashed curves, respectively

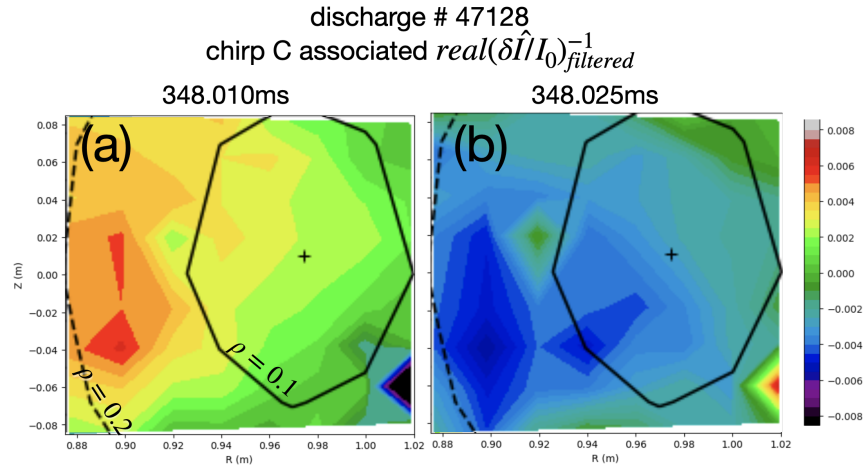


Figure 7.20: the (R,Z) snapshots of chirp C associated $real(\delta I/I_0)$ without linear regression at (a) $t = 348.010\text{ms}$ and (b) 348.025ms . $\rho = 0.1$ and 0.2 surfaces are labeled in solid and dashed curves, respectively

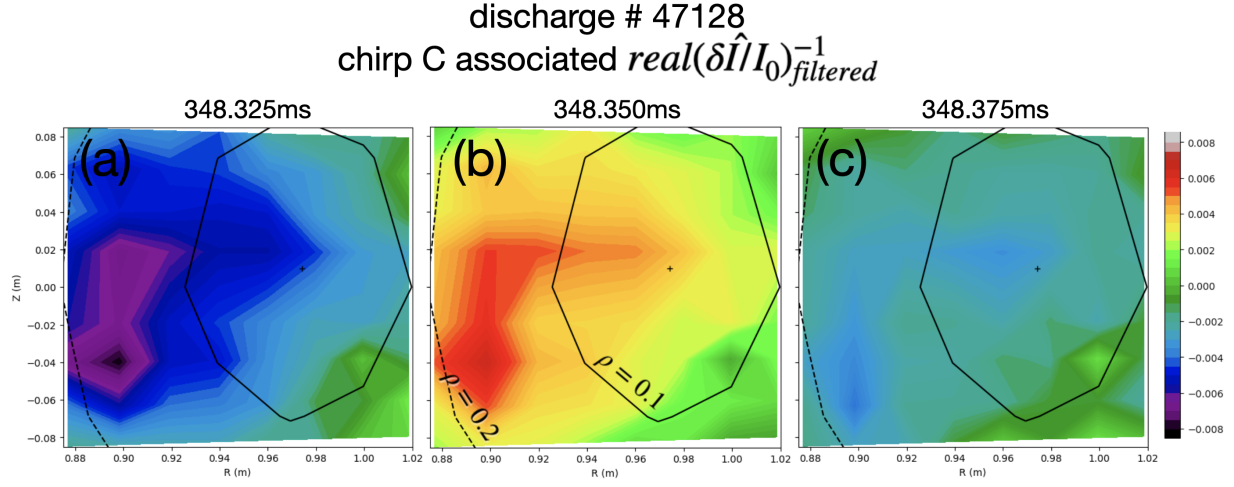


Figure 7.21: the (R,Z) snapshots of chirp C associated $real(\delta I/I_0)$ without linear regression at (a) $t = 348.325\text{ms}$, (b) 348.350ms and (c) 348.375ms . $\rho = 0.1$ and 0.2 surfaces are labeled in solid and dashed curves, respectively

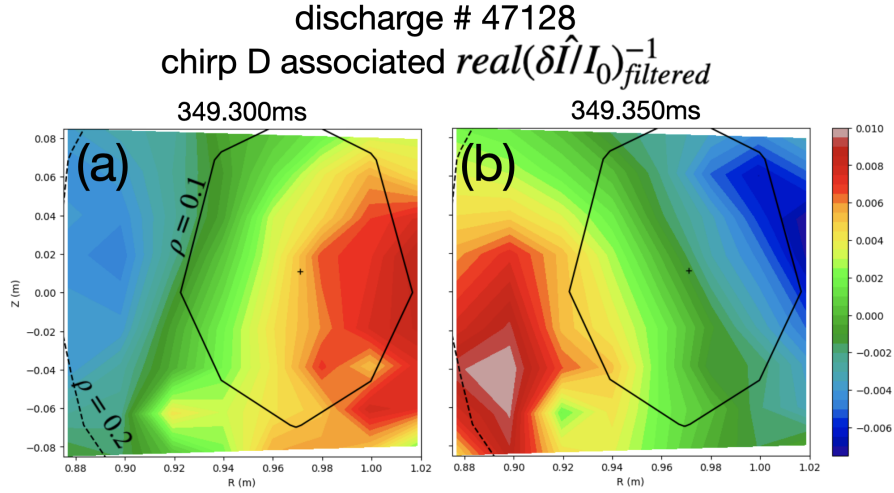


Figure 7.22: the (R,Z) snapshots of chirp B associated $real(\delta I/I_0)$ without linear regression at (a) $t = 349.300\text{ms}$ and (b) 349.350ms . $\rho = 0.1$ and 0.2 surfaces are labeled in solid and dashed curves, respectively

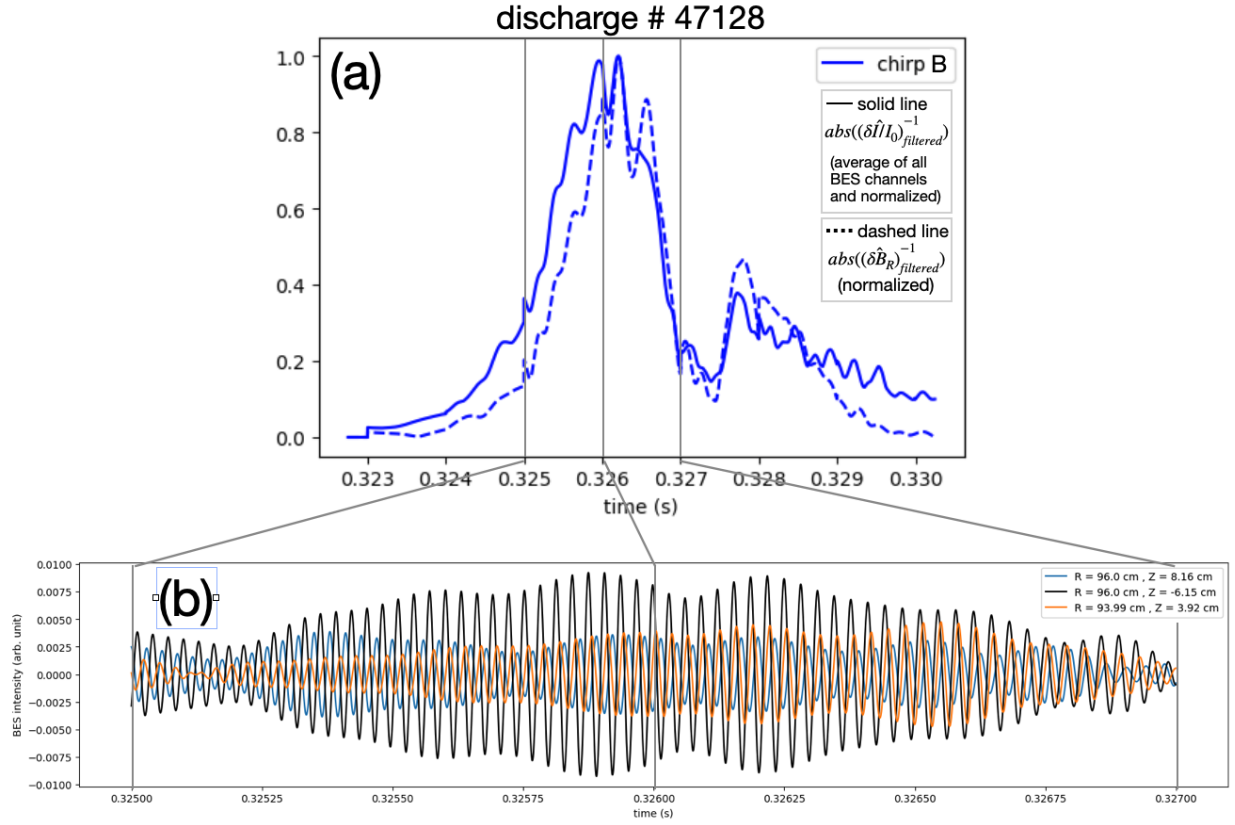


Figure 7.23: (a) the normalized chirp B associated $abs(\delta I/I_0)$ (solid line) and $abs(\delta B_R)$ (dashed line) without linear regression; (b) the real part of chirp B associated $\delta I/I_0$ without linear regression, showing the waveform of chirp B at $(R,Z) = (96\text{cm}, -6.15\text{cm})$, $(R,Z) = (96\text{cm}, 8.16\text{cm})$ and $(R,Z) = (94\text{cm}, 3.92\text{cm})$.

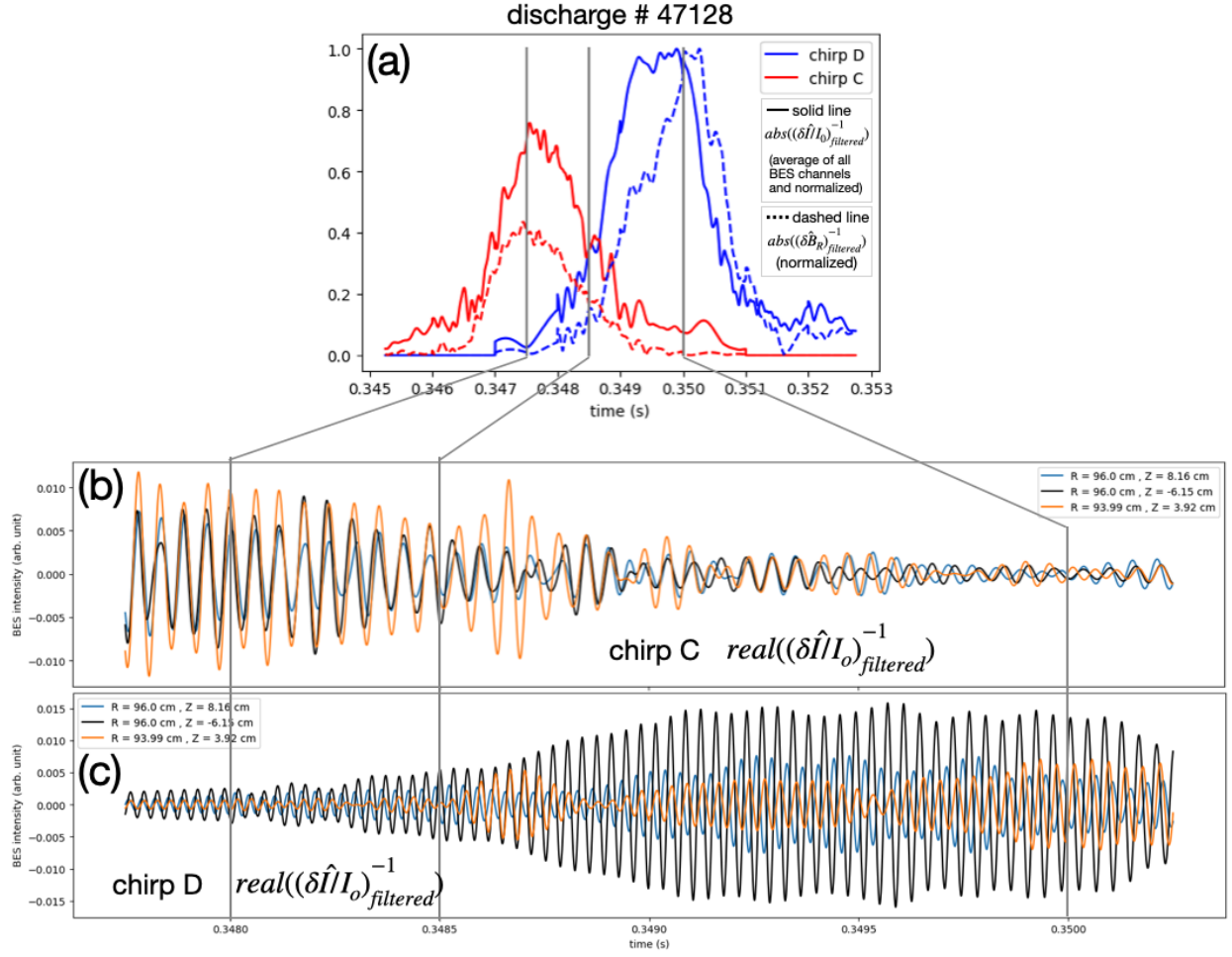


Figure 7.24: the absolute values of chirps C and D associated δB_R (solid lines) and δB_R (dashed lines) without linear regression. The chirp C associated δB_R and δB_R are normalized by the maximum value of chirp D associated δB_R and δB_R , respectively; (b) the real part of chirp C associated $\delta I/I_0$ without linear regression, showing the waveform of chirp C at at (R,Z) = (96cm, -6.15cm) , (R,Z) = (96cm, 8.16cm) and (R,Z) = (94cm, 3.92cm); (c) the real part chirp D associated $\delta I/I_0$ without linear regression, showing the waveform of chirp D at at (R,Z) = (96cm, -6.15cm) , (R,Z) = (96cm, 8.16cm) and (R,Z) = (94cm, 3.92cm).

ion transport consistent with the EP convective resonant transport model. Another form of interaction between the double bursts is also possible where the first burst moves fast ions to a part of phase space that is strongly affected by the second burst, leading to greater transport of fast ions. The proportionality between mode amplitudes and neutron emission drop levels also appears in FB-like bursts in discharge 47132, as shown in Fig. 7.11(d) and 7.9(a).

However, unlike the drops in neutron emissions, the reduction of central thermal ion temperatures after some of the FB-like bursts do not seem to be linearly proportional to the mode amplitudes. Beam-induced fast ions provide the heating for the plasma in the targeted discharges. Therefore, a significant reduction in core fast ions could, in principle, remove the primary heat source, resulting in drops in the central thermal ion temperature. If this explanation held true, the thermal ion temperature drops would be proportional to the reductions in neutron emissions (and hence the mode amplitudes), and also depend on the time difference between peaks of bursts and temperature drop measurements. Nevertheless, the central thermal ion temperatures do not consistently drop proportionally, and they only show significant decreases during or after the double FB-like bursts in both 47128 and 47132 (figures 7.12 and 7.13). In addition, the thermal confinement times are typically greater than 10ms, which is much longer than the time differences between observed drops versus peaks of bursts. This observation suggests that strong mode amplitudes and associated fast ion transport alone cannot fully explain the temperature drops.

The observed drops in thermal ion temperature after double FB-like bursts are possibly related to the presence of the core-localized $m = 0$ mode. The stability of FBs generally correlates with the $|q|_{min}$, as demonstrated in 3D kinetic-MHD simulations of FBs in NSTX [67] (a spherical tokamak similar to MAST-U). The $|q|_{min}$ during the single burst (e.g. chirp B in 47128) before the appearance of the double bursts are comparable. However, the drops in central thermal ion temperatures are much more pronounced with the appearance of double bursts compared to single bursts. It is possible that the simultaneous presence of two

core-localized modes at slightly different frequencies increases the stochasticity of the fields and flux surfaces near the axis, leading to more significant transport of core thermal ions and shifts of the magnetic axis, as observed in discharge 47128.

7.3 Summary of this chapter

The FB-like bursts cause fast and thermal ion transport in both MAST-U discharges 47128 and 47132. The occurrences of flux surface rearrangements and bulk plasma temperature crashes measured by various diagnostics show that drops in thermal species temperatures are comparable to sawtooth crashes cannot be totally avoided even with $|q|_{min} > 1$. Drops in plasma rotation speeds and neutron emissions are likely the results of core fast ion transport. The levels of drops of these fast ion associated measurements are roughly linearly proportional to the mode amplitudes of the FB-like bursts, which can be explained by the convective EP transport model [48]. However, this model cannot explain the drops in thermal species temperatures. Changes in magnetic equilibrium inferred from MSE-constrained EFIT++ and drops in core thermal ion temperatures (at levels comparable to those caused by MHD bursts) suggest the occurrences of thermal transport due to FB-like bursts. However, the spectral characteristics of these bursts and the $|q|_{min}$ values suggest that the drops are unlikely to be associated with MHD bursts. The drop in core thermal ion temperature and the distortions of the modes become more pronounced when two bursts occur in close proximity in time-frequency space (labeled "double FB-like" in plots), such as chirps C and D in 47128. The $|q|_{min}$ during the presence of the double FB-like bursts in 47128 are comparable to the $|q|_{min}$ during the prior single FB-like burst. However, the core thermal ion temperature drop could only be observed during and after the presence of double bursts.

2D BES poloidal mode structure measurements, which are for the first time obtained in a spherical tokamak, suggest some of the FB-like modes do not have expected poloidal structures for pressure driven infernal-like FBs. The measurements show that chirp C is

likely to have a $m = 0$ dominated structure near the axis, while chirp D is more likely to be a $(n, m) = (1, 1)$ dominated mode similar to other typical FBs. Additional analytic and empirical efforts are needed in the future to describe the burst with the measured core $m = 0$ structure.

The bulk plasma temperature crashes raise concerns for future fusion devices. While the MAST-U experiments aimed to avoid sawtooth crashes, drops in thermal species temperatures associated with FB-like bursts instead of sawtooth still occurred even with $|q|_{min}$ well above unity. The drops in thermal species temperatures impact the Lawson criterion and hence hinder the fusion performance directly. Future fusion devices may operate under conditions with higher fields and lower $|q|_{min}$, where FBs are expected to play a significant role. For instance, scenarios involving FBs are under consideration for future fusion devices [16] [46]. However, thermal confinement in these scenarios could still be compromised by FB-induced sawtooth-like crashes, similar to those observed in MAST-U, even in the absence of conventional sawtooth oscillations. Therefore, further studies are critical to improving the understanding of FB-like bursts and developing strategies to mitigate their impacts on performances of future fusion devices.

CHAPTER 8

Reduced fast ion transport calculations for FBs in MAST-U

Besides the TAE in MAST, measurements of FBs in MAST-U can also be utilized as inputs for ORBIT-Kick to analyze FB-induced fast ion transport. Unlike TAEs, the mode structures and frequencies of FBs in MAST-U cannot be fully resolved using MHD codes. Although there are efforts on determining these properties using kinetic simulations, they are beyond the scope of this dissertation. Instead, measurements of mode amplitude, two-dimensional spatial structure, and frequency chirping, as described in Chapter 7.2.3, can guide the construction of analytic mode structures for ORBIT-Kick transport calculations. This approach enables FB-associated transport calculations even in the absence of self-consistent FB simulations. The severity of fast ion transport, loss counts, and resonance conditions associated with an FB in MAST-U discharge 47128 are computed using measured data as inputs. These calculations provide critical insights into FB-driven fast ion transport in STs, contributing to a better understanding of fast ion behavior in future fusion devices.

The ORBIT-Kick model can be used to calculate fast ion transport and losses associated with FBs in MAST-U, but it requires experimental measurements as guidance for constructing analytic inputs. Unlike TAEs, kinetic simulations for FBs in STs are rare and remain at the frontier of this research field. The development and the verification and validation of high-fidelity simulations are beyond the scope of this dissertation. Consequently, no outputs from such simulations are available for use as inputs in ORBIT-Kick calculations.

Fortunately, many of the measurements on FBs in MAST-U, discussed in Chapter 7.2, can

serve as a reference for setting input profiles and parameters in ORBIT-Kick calculations. For example, the amplitude and frequency evolutions measured by OMAHA coils can be incorporated into the calculations, similar to the TAE case discussed in Section 6.3. Most of the additional effort is focused on constructing the poloidal harmonics and constraining the mode amplitude of an FB using 2D measurements from the BES diagnostics, as global mode structure measurements were not available in MAST-U during the experiment.

8.1 Constructing the poloidal harmonics of a MAST-U FB for transport calculations

The poloidal harmonics of an FB in MAST-U discharge 47128 are constructed as inputs for ORBIT-Kick calculations using BES measurements, the infernal mode model, and NUBEAM fast ion distribution calculations. The FB-like instability "chirp B" in discharge 47128 (Fig. 7.10, Fig.7.12) is selected for analysis because it is milder than the "double-chirps" C and D, yet exhibits a stronger mode amplitude and a greater neutron emission drop compared to chirp A, making it a valuable case for study.

An appropriate radial structure of the $m = 1$ harmonic is found under the guidance from the BES measurement and the FI pressure gradient. Fig.8.1 indicates that the electron density fluctuation associated with the mode ($\delta n_e/n_{e0}$) increases from the magnetic axis to $\rho = 0.2$ and remains low near the $|q|_{min}$ location at $\rho = 0.6$. Additionally, the 2D $\delta n_e/n_{e0}$ structure from BES suggests that the poloidal harmonic $m = 1$ dominates the region from the magnetic axis to $\rho = 0.2$. However, mode structure measurements are unavailable for the region between $\rho = 0.2$ and $\rho = 0.5$. The $|q|_{min}$ value during chirp B is approximately 1.15, which remains above unity even when considering uncertainties in the q-profile inferred from MSE. This suggests that chirp B could be a fast-ion pressure-gradient-driven infernal mode. Thus, in addition to serving as an input for ORBIT-Kick, the fast ion distribution from NUBEAM (Fig. 8.3) is used to constrain the mode structure between $\rho = 0.2$ and 0.5.

The radial gradient of the fast ion pressure ($\nabla(n_{FI}T_{FI})$) peaks near $\rho = 0.2$, indicating that the radial mode structure of chirp B should decrease gradually, rather than sharply, from $\rho = 0.2$ to 0.5—consistent with the "toy model" (Fig. 3.3) discussed in Chapter 3.4.2 for fast-ion-gradient-driven infernal modes. The $m = 2$ and $m = 3$ harmonics are assumed to be kink-related and associated with the $|q| = 2$ and $|q| = 3$ surfaces, respectively. Their radial structures and relative amplitudes (compared to $m = 1$) are determined using analytic kink solutions previously implemented in ORBIT-Kick for sawtooth transport calculations [79]. The resulting poloidal harmonic input for ORBIT-Kick is shown in Fig. 8.5(d1).

A series of ORBIT-Kick test cases using hypothetical $m = 1$ modes peaking at different radial locations reveals the relationship between input mode structures and the orbit types of resonant fast ions. The kick matrix ($p(\lambda, P_\zeta, \mathcal{E} = 54.0\text{keV})$) shown in Fig. 8.5(d2), obtained from an ORBIT-Kick calculation with a Maxwellian fast ion distribution, indicates that co-passing, trapped banana, and stagnation particles interact with the mode. However, distinguishing resonances associated with the known $m = 1$ structure (from BES) or assumed mode structures for $\rho > 0.2$ from Fig. 8.5(d2) is challenging. To investigate this further, test runs using hypothetical narrow $m = 1$ modes peaking at $\rho = 0.2, 0.4$, and 0.6 (Fig. 8.5(a1)-(c1)), with a fixed frequency of 35 kHz and a fixed mode amplitude ($A_{scale} = 5E - 4$), reveal that modes localized at $\rho \leq 0.4$ primarily interact with co-passing and stagnation particles (Fig. 8.5(a2)-(b2)), while modes at $\rho \geq 0.6$ predominantly interact with trapped banana particles (Fig. 8.5(c2)). However, BES measurements of the actual mode structure show low amplitude at $\rho = 0.6$, suggesting that interactions with trapped particles may arise from the tail of the $m = 1$ harmonic, as well as from the potential influence of $m = 2$ and $m = 3$ harmonics. Nevertheless, the predominant resonant fast ions appear to be in co-passing and stagnation orbits.

The resonance integers p and l , along with mode frequencies corresponding to resonant particles, are identified using the poloidal and toroidal bounce frequencies (ω_ζ and ω_θ) calculated by ORBIT-Kick in a test run with the poloidal harmonics from Fig. 8.5(d1) at 33

kHz and $A_{scale} = 2E - 3$. The resonant trapped particles are scattered across bands with $p \ll -1$, indicating banana orbits with a large amount of elliptic points. The resonant passing particles form two distinct bands at $\lambda < 0.4$ and $0.4 < \lambda < 0.6$, both with mode numbers $(n, p, l) = (1, 1, 2)$. Notably, the resonant passing particles at $0.4 < \lambda < 0.6$ align with the measured initial mode frequency of 33 kHz, while those at $\lambda < 0.4$ correspond to a band near the plasma rotational frequency (at $R = 1.0$ m) of 25.4 kHz, as shown in Fig. 8.6. This suggests that, in addition to the supplied poloidal harmonics at 33 kHz, a co-moving resonance with a near-zero frequency in the plasma frame is triggered and plays a role in kicking co-passing particles. Although most of the NBI-induced fast ions populate near the magnetic axis with a λ of 0.5 as shown in Fig.8.4 (a), some fast ions are induced by the beam away from the axis with $\lambda \leq 0.2$ as shown in Fig.8.4 (b). These fast ions are susceptible to the kicks generated by the co-moving resonance.

8.2 Effects of mode amplitudes on FB associated fast ion transport

In addition to determining the dominant poloidal harmonic in the core, BES measurements from discharge 47128 can also be used to constrain the mode amplitude settings in ORBIT-Kick. The electron density fluctuation associated with the FB in this discharge ($\delta n_e/n_{e0}$) is measured up to $\rho = 0.2$. The $\delta n_e/n_{e0}$ associated with the input mode structure in ORBIT-Kick is controlled by the A_{scale} parameter. The kinetic Poincaré simulations are used to track the perturbations of thermal electron trajectories due to the input mode structure in ORBIT-Kick. The associated $\delta n_e/n_{e0}$ can then be estimated with reference to the initial n_{e0} profile from TRANSP. This methodology follows that used in [13] [13] [79] [98], but $\delta n_e/n_{e0}$ is estimated instead of $\delta T_e/T_{e0}$ shown in the references. The resulting $\delta n_e/n_{e0}$ from the setting of $A_{scale} = 4E - 4$ lies near the center of the distribution of measured $\delta n_e/n_{e0}$ values (Fig. 8.7). To account for possible variations in the peak $\delta n_e/n_{e0}$, a case with $A_{scale} = 5E - 3$ that fits the upper uncertainties of $\delta n_e/n_{e0}$ is also considered. This

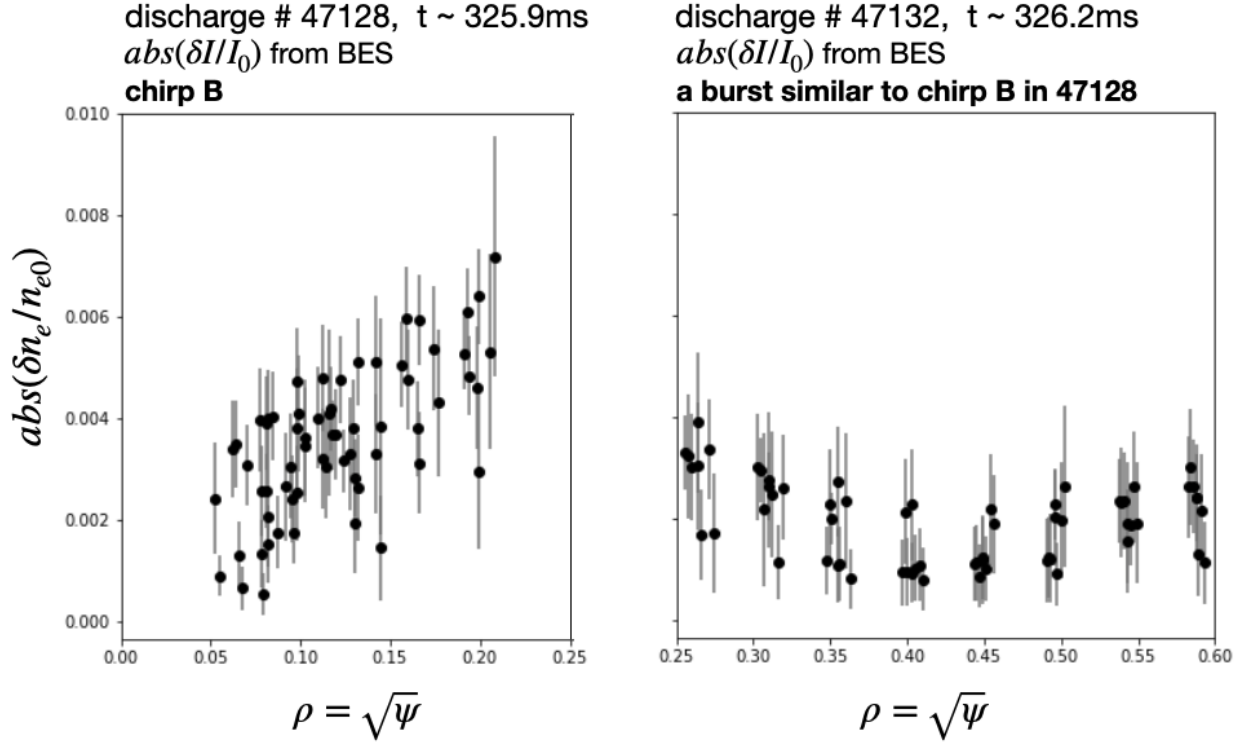


Figure 8.1: The radial profiles of $abs(\delta I/I_0)$ from BES associated with (a) chirp D in 47128 at $t = 325.9\text{ms}$ with the viewing window centered at $R = 95\text{cm}$; (b) first of the double FB-like burst in 47132 at $t = 326.2\text{ms}$ with the viewing window centered at $R = 115\text{cm}$.

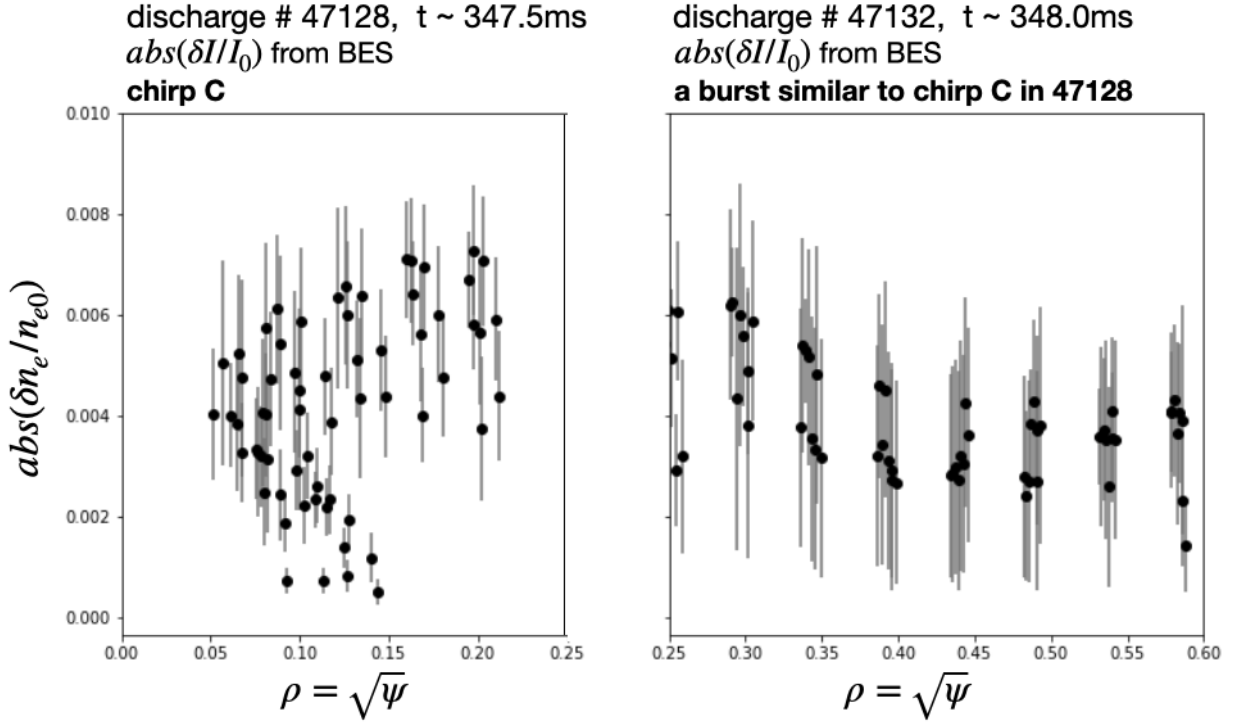


Figure 8.2: The radial profiles of $abs(\delta I/I_o)$ from BES associated with (a) chirp C in 47128 at $t = 347.5\text{ms}$ with the viewing window centered at $R = 95\text{cm}$; (b) first of the double FB-like burst in 47132 at $t = 348\text{ms}$ with the viewing window centered at $R = 115\text{cm}$.

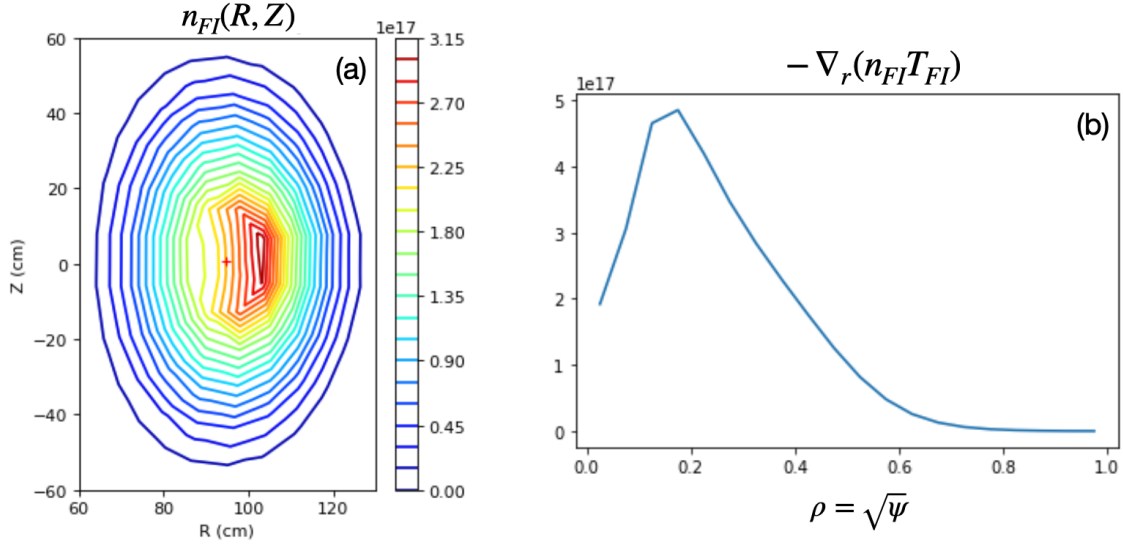


Figure 8.3: (a) 2D and fast ion density profiles and (b) the radial gradient of the product of fast ion density (n_{FI}) and temperature (T_{FI}) from NUBEAM/TRANSP for MAST-U discharge 47128

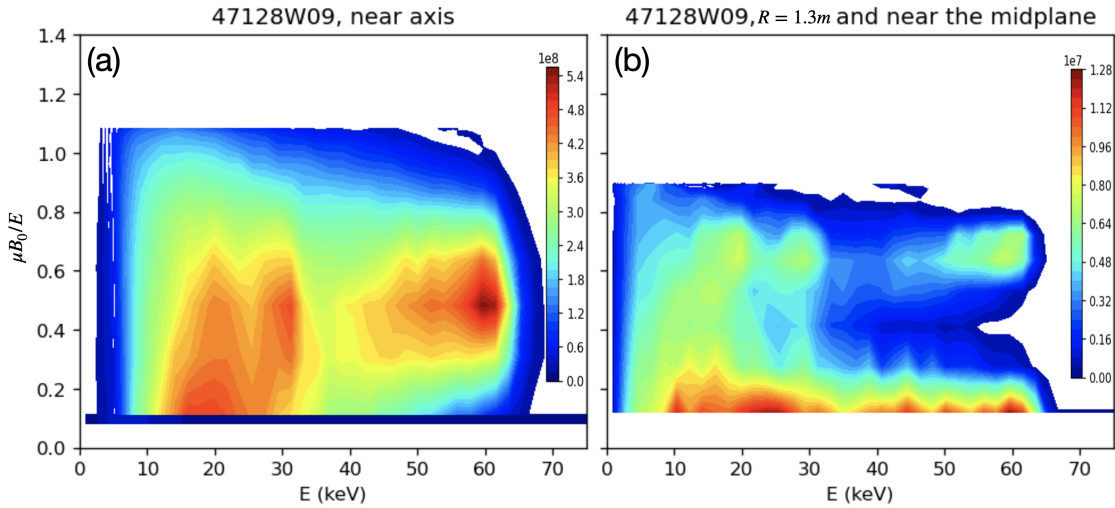


Figure 8.4: The fast ion distribution near the (a) magnetic axis and (b) $R = 1.3m$ and mid-plane from NUBEAM/TRANSP for MAST-U discharge 47128

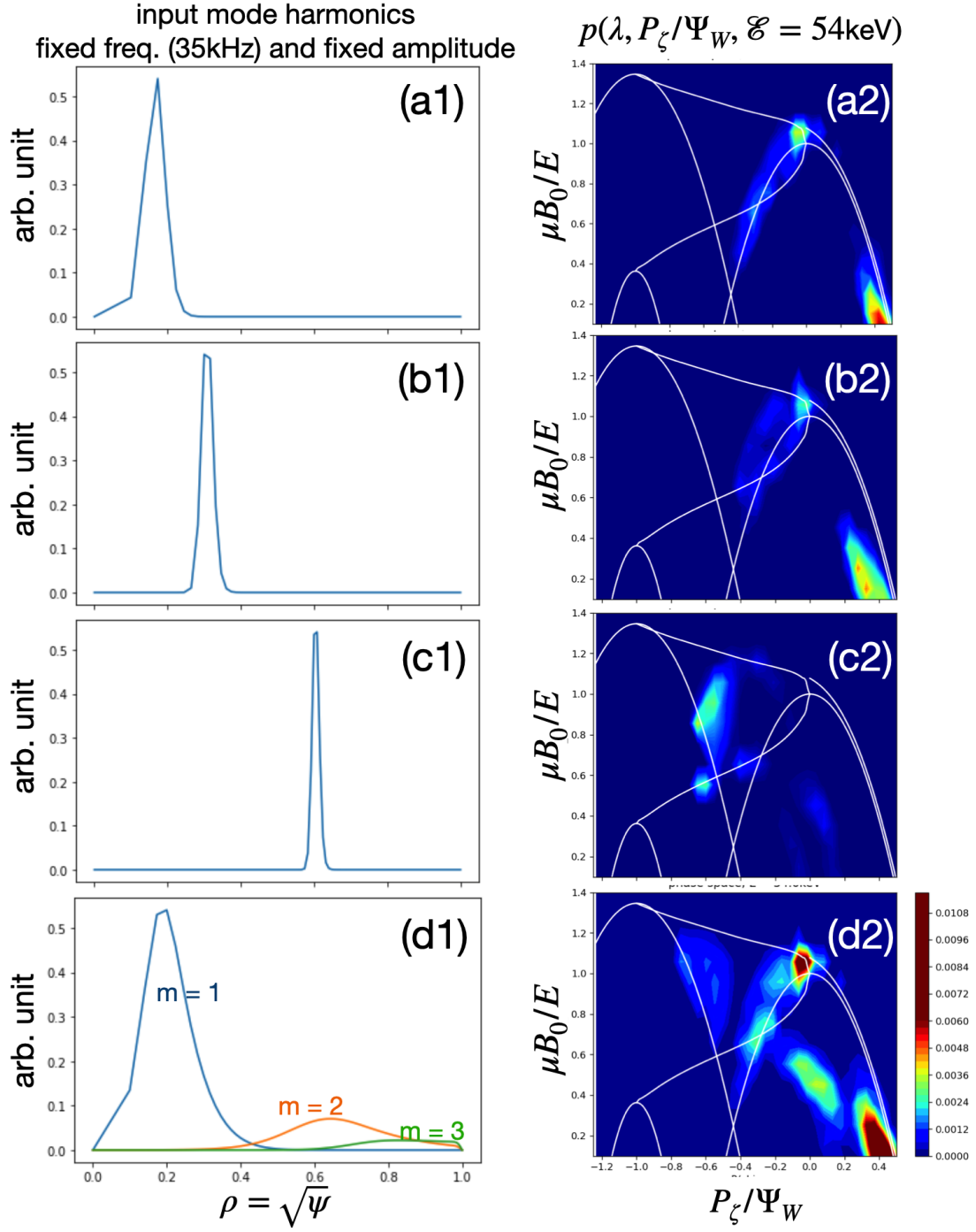


Figure 8.5: Tests case for constraining the FB mode structure inputs. (a1) - (d1): the input poloidal harmonics (in radial plasma displacements) for ORBIT-Kick and (a2) - (d2) to be the resulting kick probability matrix ($p(\lambda, P_\zeta, \mathcal{E} = 54.0\text{keV})$)

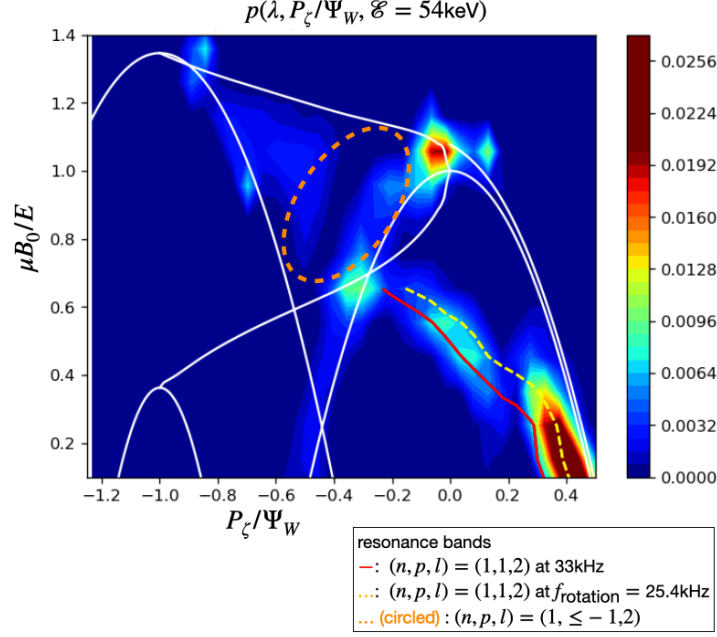


Figure 8.6: kick probability matrix ($p(\lambda, P_\zeta, \mathcal{E} = 54.0\text{keV})$) from result of ORBIT-Kick that uses structure (shown in Fig.8.5(d1)), showing ΔP_ζ of resonant fast ions and the resonance bands for $(n, p, l) = (1, 1, 2)$ at 25.4 and 33 kHz in phase space representations

amplitude setting provides an upper bound for $\delta n_e/n_{e0}$ near $\rho = 0.2$ and allows uncertainties in ORBIT-Kick calculations to be assessed when compared with measurements from other diagnostics in future efforts. It is worth noting that the $\delta n_e/n_{e0}$ only increases less than a factor of 2 with the increase of a factor of more than 10 in A_{scale} . This can be due to uncertainties induced by insufficient radial bins and/or number of thermal electrons used in the ORBIT-Kick Poincaré calculation. Future efforts can be done to refine the input settings for more accurate estimations of $\delta n_e/n_{e0}$.

Similar to the case of TAEs in MAST, fast ion transport associated with an FB in MAST-U is found to depend on mode amplitude in ORBIT-Kick calculations. Each ORBIT-Kick run is 6ms long with an evolving mode amplitude versus time profile shown in Fig.8.8(b), mimicking the measured profile in Fig.7.23 (a). Although the different A_{scale} values used in these runs are relatively close—primarily to improve statistical significance—some differences in the intensity of fast ion transport are still observed. In general, increasing the

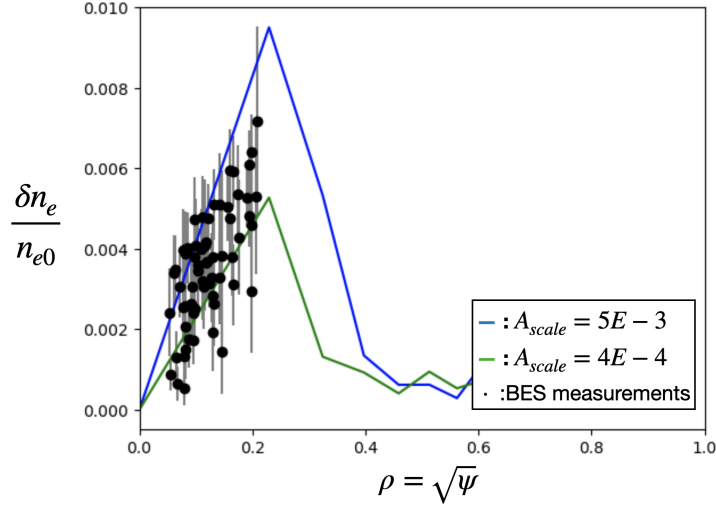


Figure 8.7: $\delta n_e/n_{e0}$ measurements from BES compared to different $\delta n_e/n_{e0}$ from two A_{scale} settings in ORBIT-Kick for chirp B in MAST-U discharge 47128.

mode amplitude does not change the resonances at which particles interact with the mode. However, it results in more kicks for both trapped and co-passing particles (Fig. 8.9(a) and (c)). As expected, the overall number of lost particles increases with a higher A_{scale} .

The most notable effect of increasing A_{scale} is a slight decrease in the ratio of lost co-passing to lost trapped particles (cases 1, 2 and 3 shown in Table.8.2), suggesting that trapped orbits are more susceptible to stronger kicks. This can also be seen in the energy versus λ representation of the kick matrices, where the increased amount of particles in the trapped orbits experience resonances in a higher A_{scale} case compared to a lower A_{scale} case (shown in Fig.8.10 (a) and (c)). This behavior contrasts with the TAE cases discussed in Chapter 6, where co-passing particles were more affected. This difference is likely due to variations in frequency and mode localization between TAEs and FBs in MAST/MAST-U, which influence trapped and co-passing particle orbits in different ways.

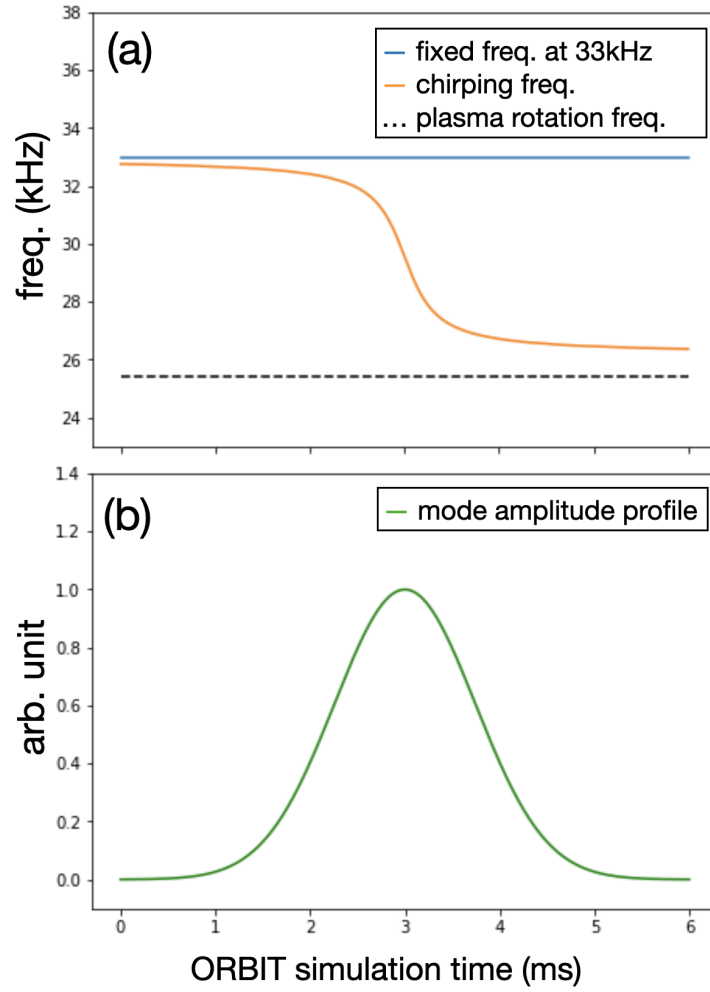


Figure 8.8: (a) fixed and chirping frequencies for different cases, with a comparison with the plasma rotation frequency at $R = 1.0\text{m}$ and (b) amplitude profiles that mimic the measurements used for ORBIT-Kick FB cases

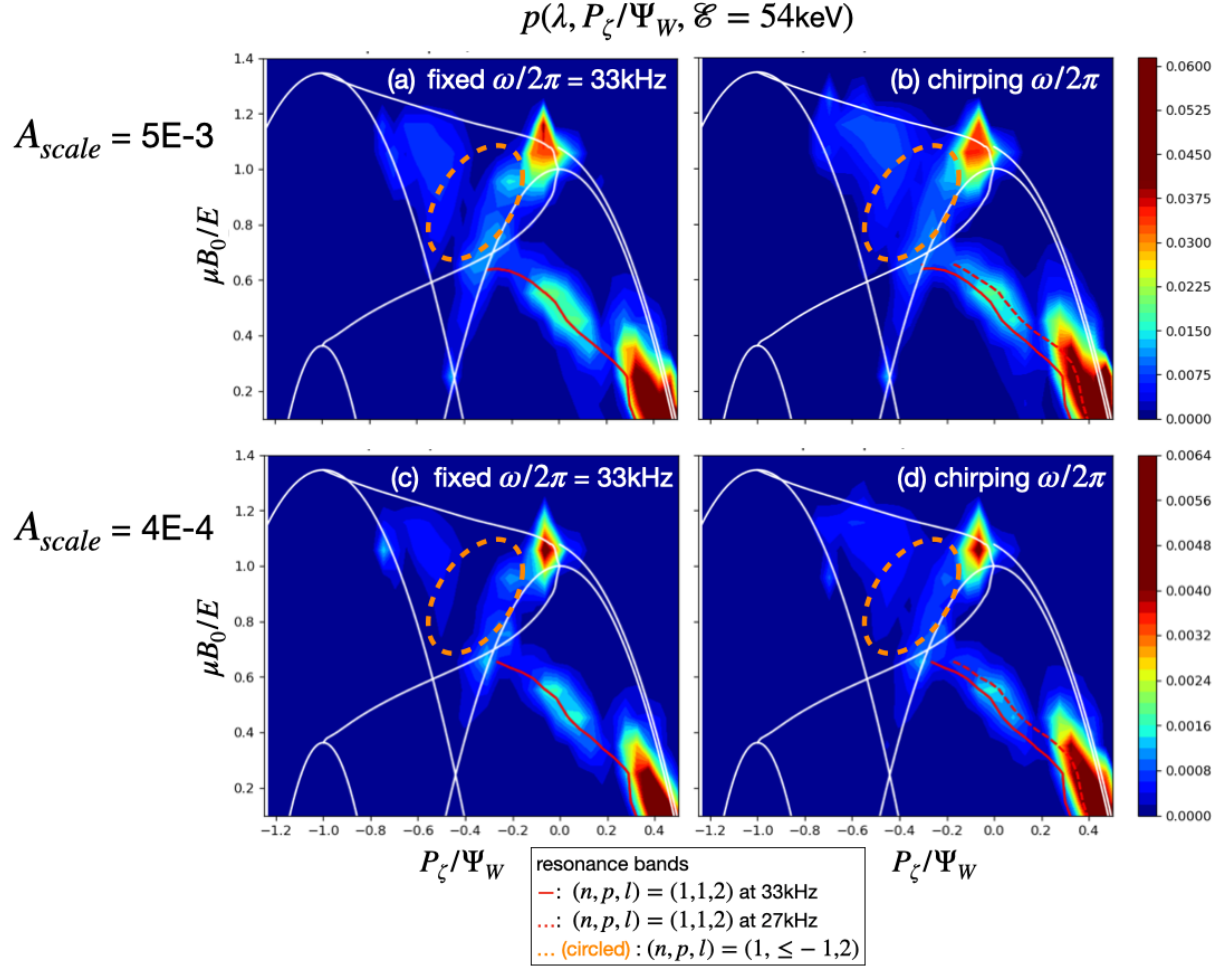


Figure 8.9: kick probability matrices ($p(\lambda, P_\zeta, \mathcal{E} = 54.0\text{keV})$) from results of ORBIT-Kick that use the structure (shown in Fig.8.5(d1)) with (a) $A_{scale} = 5E - 3$ and fixed frequency (case 3 in Table.8.1); (b) $A_{scale} = 5E - 3$ and chirping frequency (case 6 in Table.8.1); (c) $A_{scale} = 4E - 4$ and fixed frequency (case 1 in Table.8.1); (d) $A_{scale} = 4E - 4$ and chirping frequency (case 4 in Table.8.1), showing ΔP_ζ of resonant fast ions and the resonance bands for $(n, p, l) = (1, 1, 2)$ at 27 and 33 kHz in phase space representations

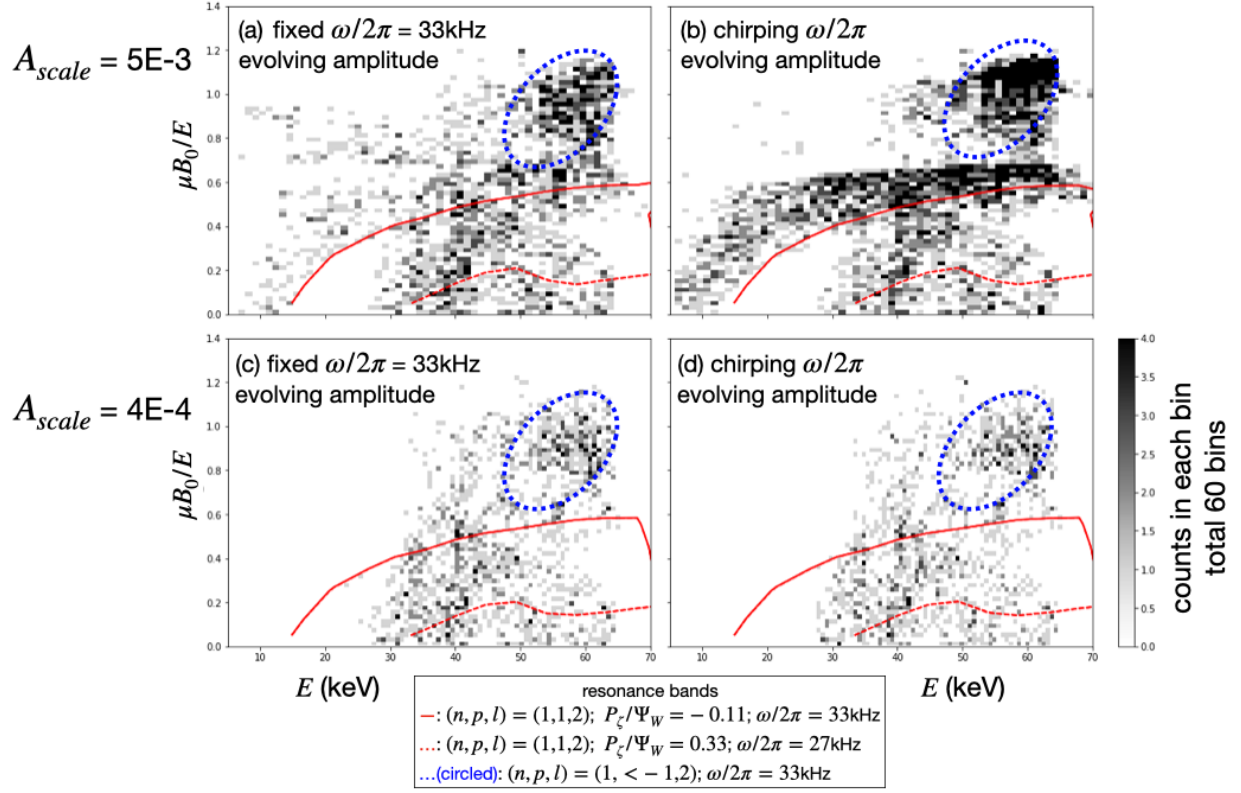


Figure 8.10: 2D ($\lambda = \mu B_0 / E$ and E) histograms for fast ions with their energies modified greater than $3 * SD_{\Delta E}$ in ORBIT-Kick calculations using the structure (shown in Fig.8.5(d1)) with (a) $A_{scale} = 5E - 3$ and fixed frequency (case 3 in Table.8.1); (b) $A_{scale} = 5E - 3$ and chirping frequency (case 6 in Table.8.1); (c) $A_{scale} = 4E - 4$ and fixed frequency (case 1 in Table.8.1); (d) $A_{scale} = 4E - 4$ and chirping frequency (case 4 in Table.8.1).

8.3 Effects of chirping on FB associated fast ion transport

Similar to the case of TAEs in MAST, the measured frequency chirping of FBs can be used as an input for ORBIT-Kick transport calculations. The time-evolving frequency profile used in these runs (Fig. 8.3(a)) is fitted to the measured chirping profile from Fig. 7.17(a), where the frequency decreases from 33 kHz to 27 kHz over approximately 2 ms.

The main effect of frequency chirping is an increased spread of particle distribution compared to fixed-frequency cases at different amplitudes, as shown in Fig. 8.10(b) and (d). Notably, the final chirped frequency is very close to the core plasma rotational frequency, potentially leading to stronger resonances in the $(n, p, l) = (1, 1, 2)$ band at 25.4 kHz, as previously shown in Fig. 8.6. While the total number of lost particles increases slightly with chirping, the ratio of lost co-passing to lost trapped particles decreases compared to fixed-frequency cases at the same mode amplitudes.

As observed in MAST TAE runs comparing fixed versus chirping frequencies, the most significant differences in MAST-U FB runs lie in the standard deviations of $\Delta\mathcal{E}$ and ΔP . Specifically, in the $A_{scale} = 5E - 3$ case, $SD_{\Delta\mathcal{E}}$ and $SD_{\Delta P}$ increase by approximately 24% and 72%, respectively, as shown in Table 8.2. This effect is evident in the Poincaré plots comparing fixed and chirping frequency runs: in the fixed-frequency ORBIT-Kick runs, orbits and islands near the peak of the $m = 1$ harmonic remain well-defined, whereas they become more disrupted in the chirping-frequency run (Fig. 8.11(a) vs. (b)).

These results suggest that frequency chirping can make fast ion orbits stochastic, leading to increased $SD_{\Delta\mathcal{E}}$ and $SD_{\Delta P}$. However, since the mode is far from the plasma edge, the resulting stochastic orbits do not guarantee the increase of overall particle losses beyond the last closed flux surface (LCFS), unless the fast ion orbit islands induced by the mode is large enough. The width of these islands are determined by both the radial extent and amplitude of the mode. This highlights the importance to constrain inputs using measurements for accurate estimations on fast ion transport and losses.

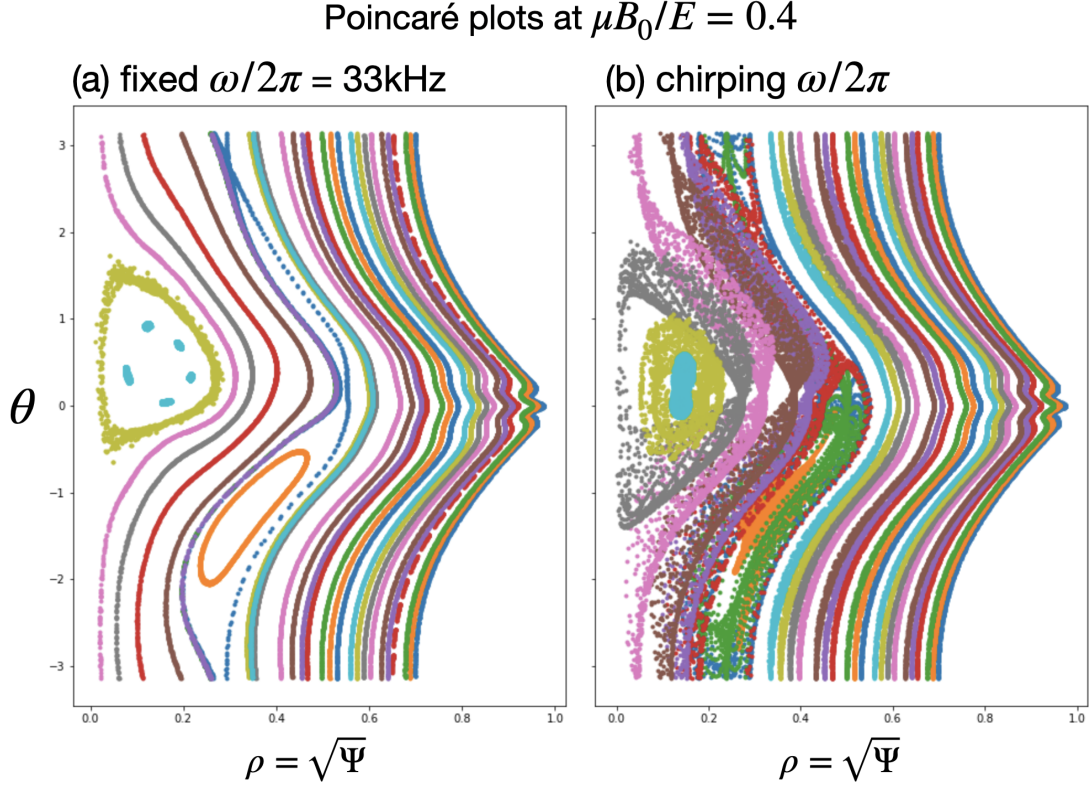


Figure 8.11: Kinetic Poincaré plot at $\lambda = \mu B_0/E = 0.4$ of fast ions for cases of (a) fixed frequency at 33 kHz (case 3 in Table.8.1); (b) chirping frequency (case 6 in Table.8.1). Orbits are labeled by different colors.

Inputs quantities or settings	case 1	case 2	case 3	case 4	case 5	case 6
A_{scale}	4E-04	2E-03	5E-03	4E-04	2E-03	5E-03
$(\delta n_e/n_{e0})_{\text{max}}$	0.00527	0.00768	0.00950	0.00527	0.00768	0.00950
frequency (kHz)	33	33	33	chirping (Fig.8.8(a))	chirping (Fig.8.8(a))	chirping (Fig.8.8(a))

Table 8.1: Inputs and settings for ORBIT-Kick runs for MAST-U FB cases

results	case 1	case 2	case 3	case 4	case 5	case 6
$SD_{\Delta E}$ (keV)	0.334	0.359	0.4266	0.342	0.4039	0.5283
$SD_{\Delta P}$	0.00202	0.00699	0.01382	0.00455	0.01334	0.02376
loss (per 300000 particles)	1557	1659	2032	1563	1773	2437
lost trapped/total trapped (%)	0.763%	0.814%	1.03%	0.766%	0.887%	1.28%
lost co-passing/total co-passing (%)	0.416%	0.444%	0.509%	0.420%	0.456%	0.567%
lost co-passing/lost trapped	0.516	0.516	0.469	0.519	0.487	0.420

Table 8.2: Results of ORBIT-Kick runs for MAST-U FB cases

8.4 Summary of this chapter

This chapter presents the ORBIT-Kick calculations performed to study fast ion transport in NBI-driven FBs in MAST-U using various input sources. Unlike TAEs, no reliable simulations are available to resolve the poloidal harmonics of FBs in spherical tokamaks at the time of this study. Instead, BES measurements from MAST-U are used to analytically reconstruct radial structures and constrain the amplitudes of FB poloidal harmonics for low m numbers. ORBIT-Kick calculations based on these analytical poloidal harmonics show that fast ion transport and losses scale with mode amplitude, similar to the TAE case. However, in contrast to TAEs, the ratio of lost co-passing to lost trapped particles decreases with increasing mode amplitude. This difference is likely due to variations in resonance conditions between these two types of modes, which have distinct structures and frequencies. Frequency chirping of the FB does not significantly increase particle loss but has a more pronounced effect on particle energy and momentum compared to fixed-frequency cases. This is because the disturbed orbits remain far from the plasma edge, preventing particles from being lost beyond the LCFS.

CHAPTER 9

Conclusions and future efforts

This dissertation explores the measurement of TAEs and FB instabilities in MAST and MAST-U using multiple diagnostic tools to enhance our understanding of these phenomena. These measurements serve various purposes, including the verification and validation of first-principles simulations, phenomenological studies, and inputs for reduced fast-ion transport models. The overarching goal is to mitigate the impact of these instabilities on fusion performance and minimize potential damage to wall components in future tokamak-based fusion devices. Beyond these contributions, the work presented here lays the foundation for future research aimed at further advancing the understanding of low-frequency NBI-driven instabilities.

9.1 Measurements of TAE in MAST for Verification and Validation of a Linear Global Gyrokinetic Simulation

Global radial mode structure measurement of TAEs in the MAST discharge 26887 at 180 ms are made with soft X-ray and inverted to obtain the emissivity fluctuation spatial structure using a novel technique. The structure shows characteristics consistent with theory for $(n,m,m+1) = (1,1,2)$ TAEs in this particular MAST case. However, these characteristics might be altered if the beam injection geometry is different in other cases. Future researchers can study the effects of geometry to TAE's characteristics and stability in MAST-U or similar devices (e.g. NSTX-U) using neutral beams with different injection geometries.

A linear gyrokinetic (GK) model is verified and validated using measurements of TAEs in MAST. Global gyrokinetic codes, such as the Gyrokinetic Toroidal Code (GTC) [66], offer promising capabilities for cross-scale integrated simulations of fast-particle-driven modes [67]. However, key characteristics of spherical tokamaks (STs)—such as the steep field gradient on the high-field side and the high fast-ion-to-thermal-ion beta ratio—have not been extensively tested in GK simulations. This dissertation addresses this gap by validating the first global GK simulations of TAEs in MAST.

Overall, the GTC simulation results for TAE radial mode structure and frequency show good agreement with experimental measurements. Despite different representations of thermal and fast ions, the eigenfrequencies of TAEs excited in various GTC run modes fall within a narrow range of 69 to 72 kHz. After accounting for the ≈ 10 kHz plasma rotation Doppler shift, the laboratory frame frequencies are close to 80 kHz—approximately 9% lower than the frequency computed by NOVA and the measured initial frequency of the targeted chirping TAE. Interestingly, the frequencies from GTC align more closely with the TAE frequency at its peak mode amplitude, as well as with the ideal MHD prediction for the $(n, m) = (1, 1)$ TAE frequency when the Doppler shift is considered. This discrepancy may stem from the treatment of input profiles, given the similar differences observed between NOVA and GTC ideal MHD simulations.

The radial mode structures from GTC simulations that include fast ions also align well with experimental results inferred from tangential soft X-ray (SXR) measurements. The simulated electrostatic potential perturbation ($\delta\phi$) peaks around $\rho = 0.6$ to 0.8 , matching the expected mode location for ideal MHD $n = 1$, $m = 1, 2$ TAEs and the measured SXR perturbation ($\delta\epsilon$). Discrepancies in radial mode structures in GK fast-ion simulations arise primarily when considering poloidal structures and harmonics, likely due to the treatment of fast-ion distributions (whether modeled as Maxwellian or anisotropic). However, since neither poloidal structures nor harmonics are measured or inferred from the available diagnostics in this dissertation, the differences in these aspects do not affect the overall agreement

between simulations and experiments.

Future experimental efforts could further validate and refine the GTC model. The linear GTC predicts the growth and damping rates of TAEs in MAST, yet the absence of data capturing the linear stage of TAEs in the targeted case prevents direct comparisons of these quantities. New experiments aimed at observing TAEs during the transition from a linear, non-chirping stage to a chirping stage are necessary to compare with linear GTC predictions. Additionally, experimental efforts should focus on directly inferring fast-ion anisotropy through tomography techniques [69] and measuring poloidal mode structures and harmonics using poloidally viewing diagnostics. These studies would provide valuable references for detailed comparisons between Maxwellian and anisotropic fast-ion models in GTC simulations.

Beyond new experimental efforts, extending GTC simulations of the targeted TAE in MAST into the nonlinear regime is crucial for resolving frequency chirping and nonlinear damping effects. The rapid frequency and amplitude variations of TAEs significantly influence fast ion transport phenomena that linear simulations cannot capture. The targeted TAE case is particularly suitable for comparison with δf nonlinear simulations in GTC, as its equilibrium conditions remain relatively stable despite TAE activity. Investigating this case in the nonlinear regime would offer deeper insights into the frequency evolution and nonlinear damping mechanisms of TAEs in MAST.

9.2 Fishbone instabilities and their impacts on core confinement in MAST and MAST-U

This dissertation also investigates fishbones (FBs) in MAST and MAST-U. In MAST, phases between FB associated fluctuations measured by two radial separated diagnostics are observed to be varying over time, suggesting either distortions on the equilibrium or FB mode structures. This observation motivated new experiments in MAST-U with additional and

detailed measurements, including a first-of-its-kind poloidal mode structure measurements obtained using the 2D BES system. The BES measurements reveal that not all FB-like bursts with similar spectral characteristics have the expected core structures and dominant m numbers for pressure-driven infernal-like FBs.

The impacts of FB-like instabilities on bulk plasma and flux surfaces are also observed in MAST-U. Experimental studies in MAST-U plasmas, where the minimum safety factor (q_{min}) approaches unity, reveal substantial distortions in spatial structures during plasma discharges for certain FB-like instabilities. These events are associated with pronounced fast-ion transport, significant shifts in the magnetic axis, and sudden crashes in thermal species temperatures. Such observations indicate rapid and large-scale rearrangements of flux surfaces, suggesting deviations from conventional FB models and previously documented behaviors. The observed bulk plasma temperature drops raise concerns for future fusion devices as they hinder the fusion performances directly by impacting the Lawson criterion. In the MAST-U experiments discussed in this dissertation, sawtooth crashes were intentionally avoided, yet sawtooth-like drops in thermal species temperatures linked to FBs still occurred, despite q_{min} being well above unity. Future fusion devices operating at higher fields—and thus lower q_{min} may still experience degraded thermal confinement due to FB induced sawtooth-like crashes, even in the absence of conventional sawtooth instabilities. Therefore, further research is essential to improve understanding and predictive capabilities for these modes in next-generation fusion devices.

9.3 Reduced fast ion transport calculations for TAEs MAST and FBs in MAST-U

Beyond validating physics models and simulations, measurements of TAEs in MAST and FBs in MAST-U can also serve as inputs for reduced fast-ion transport model calculations. As previously mentioned, linear models cannot predict frequency chirping. To address this,

ORBIT-Kick incorporates measured amplitudes and chirping frequencies, integrating them with simulation results to calculate fast-ion transport. For FBs, where self-consistent simulations are less common than for TAEs radial mode structure measurements from the BES diagnostic are used to reconstruct analytical poloidal harmonics, which are then used as inputs for ORBIT-Kick. This approach enables studies of FB associated fast ion transport without requiring fully self-consistent simulations, significantly reducing computational complexity and resource demands.

Frequency chirping is found to intensify fast-ion transport in both TAE and FB cases. In both cases of TAEs and FBs, orbit structures and island formations are disrupted by the changing mode frequency. However, differences in mode structures and frequencies between these instabilities affect how fast ions are transported. For example, in cases where the disrupted orbits remain far from the plasma edge, such as those affected by FBs in MAST-U fast ions may not be lost beyond the LCFS and hence significantly contribute to particle losses.

Future efforts should focus on comparing ORBIT-Kick calculations for TAEs and FBs with fast-ion-associated measurements from various diagnostics. For instance, fast-ion transport or losses can influence neutron emission, as both processes reduce heating efficiency and hinder fusion reaction rates. The kick-matrix computed by ORBIT-Kick can be used as an input for TRANSP to predict changes in neutron flux due to instabilities. These predictions can then be compared with neutron rate measurements from the fission chamber or neutron camera diagnostics [19]. Additionally, ORBIT-Kick’s calculated ratios of lost co-passing and lost trapped particles can be validated against Fast Ion Loss Detector (FILD) measurements in MAST-U [97]. Such comparisons would help assess the accuracy of the ORBIT-Kick model and enhance its predictive capabilities for future fusion experiments.

Bibliography

- [1] Adobe Inc. *Selecting audio & audio ranges in Adobe Audition*, 2021. URL <https://helpx.adobe.com/audition/using/selecting-audio.html>. HelpX documentation; describes the Marquee, Lasso, and Paintbrush tools in spectral display.
- [2] Hirotogu Akaike. *Information Theory and an Extension of the Maximum Likelihood Principle*, page 199–213. Springer New York, 1998. ISBN 9781461216940. doi: 10.1007/978-1-4612-1694-015. URL <http://dx.doi.org/10.1007/978-1-4612-1694-015>.
- [3] J. Allen. Short term spectral analysis, synthesis, and modification by discrete fourier transform. *IEEE Transactions on Acoustics, Speech, and Signal Processing*, 25(3): 235–238, June 1977. ISSN 0096-3518. doi: 10.1109/tassp.1977.1162950. URL <http://dx.doi.org/10.1109/TASSP.1977.1162950>.
- [4] T. M. Antonsen and Y. C. Lee. Electrostatic modification of variational principles for anisotropic plasmas. *The Physics of Fluids*, 25(1):132–142, January 1982. ISSN 0031-9171. doi: 10.1063/1.863612. URL <http://dx.doi.org/10.1063/1.863612>.
- [5] L. C. Appel and M. J. Hole. Calibration of the high-frequency magnetic fluctuation diagnostic in plasma devices. *Review of Scientific Instruments*, 76(9), September 2005. ISSN 1089-7623. doi: 10.1063/1.2009107. URL <http://dx.doi.org/10.1063/1.2009107>.
- [6] L.C. Appel and I. Lupelli. Equilibrium reconstruction in an iron core tokamak using a deterministic magnetisation model. *Computer Physics Communications*, 223:1–17, February 2018. ISSN 0010-4655. doi: 10.1016/j.cpc.2017.09.016. URL <http://dx.doi.org/10.1016/j.cpc.2017.09.016>.
- [7] F. W. ASTON. The constitution of the elements. *Nature*, 105(2627):

- 8–8, March 1920. ISSN 1476-4687. doi: 10.1038/105008a0. URL <http://dx.doi.org/10.1038/105008a0>.
- [8] Matteo Barbarino. A brief history of nuclear fusion. *Nature Physics*, 16(9): 890–893, June 2020. ISSN 1745-2481. doi: 10.1038/s41567-020-0940-7. URL <http://dx.doi.org/10.1038/s41567-020-0940-7>.
- [9] S.H Batha, F.M Levinton, D.A Spong, R.V Budny, Z Chang, D.S Darrow, E.D Fredrickson, G.Y Fu, E Mazzucato, R Nazikian, and S.J Zweben. Experimental study of toroidicity induced alfvén eigenmode (tae) stability at high $q(0)$. *Nuclear Fusion*, 35(12):1463–1468, December 1995. ISSN 0029-5515. doi: 10.1088/0029-5515/35/12/i06. URL <http://dx.doi.org/10.1088/0029-5515/35/12/I06>.
- [10] David A. Belsley, Edwin Kuh, and Roy E. Welsch. *Regression Diagnostics: Identifying Influential Data and Sources of Collinearity*. Wiley, June 1980. ISBN 9780471725152. doi: 10.1002/0471725153. URL <http://dx.doi.org/10.1002/0471725153>.
- [11] J. Bland and S.E. Sharapov. Core-localized toroidal alfvén eigenmodes in spherical tokamaks. *Fusion Engineering and Design*, 190:113694, May 2023. ISSN 0920-3796. doi: 10.1016/j.fusengdes.2023.113694. URL <http://dx.doi.org/10.1016/j.fusengdes.2023.113694>.
- [12] Nathan G. Bolte, William W. Heidbrink, David Pace, Michael Van Zee-land, and Xi Chen. Measurement and simulation of passive fast-ion d-alpha emission from the diii-d tokamak. *Nuclear Fusion*, 56(11):112023, September 2016. ISSN 1741-4326. doi: 10.1088/0029-5515/56/11/112023. URL <http://dx.doi.org/10.1088/0029-5515/56/11/112023>.
- [13] P.J. Bonfigli, M. Podestà, M. Vallar, N.N. Gorelenkov, V. Kiptily, R.B. White, C. Giroud, S. Brezinsek, and JET Contributors. Numerical studies on saturated kink and sawtooth induced fast ion transport in jet iter-like plasmas. *Nuclear Fusion*, 62

- (11):112002, September 2022. ISSN 1741-4326. doi: 10.1088/1741-4326/ac888c. URL <http://dx.doi.org/10.1088/1741-4326/ac888c>.
- [14] Joshua Breslau, Marina Gorelenkova, Francesca Poli, Jai Sachdev, Alexei Pankin, Gopan Perumpilly, Xingqiu Yuan, and Laszlo Glant. Transp, 2018. URL <https://www.osti.gov/doecode/biblio/12542>.
- [15] A. J. Brizard and T. S. Hahm. Foundations of nonlinear gyrokinetic theory. *Reviews of Modern Physics*, 79(2):421–468, April 2007. ISSN 1539-0756. doi: 10.1103/revmodphys.79.421. URL <http://dx.doi.org/10.1103/RevModPhys.79.421>.
- [16] G. Brochard, R. Dumont, H. Lütjens, and X. Garbet. Linear stability of the iter 15 ma scenario against the alpha fishbone. *Nuclear Fusion*, 60(8):086002, July 2020. ISSN 1741-4326. doi: 10.1088/1741-4326/ab9255. URL <http://dx.doi.org/10.1088/1741-4326/ab9255>.
- [17] G. Brochard, C. Liu, X. Wei, W. Heidbrink, Z. Lin, N. Gorelenkov, C. Chrystal, X. Du, J. Bao, A.R. Polevoi, M. Schneider, S.H. Kim, S.D. Pinches, P. Liu, J.H. Nicolau, and H. Lütjens. Saturation of fishbone instability by self-generated zonal flows in tokamak plasmas. *Physical Review Letters*, 132(7), February 2024. ISSN 1079-7114. doi: 10.1103/physrevlett.132.075101. URL <http://dx.doi.org/10.1103/PhysRevLett.132.075101>.
- [18] M. N. Bussac, R. Pellat, D. Edery, and J. L. Soule. Internal kink modes in toroidal plasmas with circular cross sections. *Physical Review Letters*, 35(24):1638–1641, December 1975. ISSN 0031-9007. doi: 10.1103/physrevlett.35.1638. URL <http://dx.doi.org/10.1103/physrevlett.35.1638>.
- [19] M Cecconello, I J Dolby, A Sperduti, J Rivero-Rodriguez, G Ericsson, I Fitzgerald, S Y Allan, J Voller, B Honey, B A Nizar, and S D Elmore. First observations of confined fast ions in mast upgrade with an upgraded neutron camera. *Plasma Physics and*

- Controlled Fusion*, 65(3):035013, February 2023. ISSN 1361-6587. doi: 10.1088/1361-6587/acb50c. URL <http://dx.doi.org/10.1088/1361-6587/acb50c>.
- [20] Liu Chen, R. B. White, and M. N. Rosenbluth. Excitation of internal kink modes by trapped energetic beam ions. *Physical Review Letters*, 52(13):1122–1125, March 1984. doi: 10.1103/physrevlett.52.1122. URL <https://doi.org/10.1103/physrevlett.52.1122>.
- [21] C. Z. Cheng and M. S. Chance. Low- n shear alfvén spectra in axisymmetric toroidal plasmas. *The Physics of Fluids*, 29(11):3695–3701, November 1986. ISSN 0031-9171. doi: 10.1063/1.865801. URL <http://dx.doi.org/10.1063/1.865801>.
- [22] N. Chulu Chinn, F. Palermol, B. McMillan, C. Roach, M. Fitzgerald, K. McClements, S. Blackmore, M. Dreval, A. Mishchenko, and T. Hayward-Schneider. Gyrokinetic investigation of tae damping channels with comparison to theory and application to mast-u. —, (to be submitted), 2024.
- [23] M Cox. The mega amp spherical tokamak. *Fusion Engineering and Design*, 46(2–4):397–404, November 1999. ISSN 0920-3796. doi: 10.1016/S0920-3796(99)00031-9. URL [http://dx.doi.org/10.1016/S0920-3796\(99\)00031-9](http://dx.doi.org/10.1016/S0920-3796(99)00031-9).
- [24] N. A. Crocker, E. D. Fredrickson, N. N. Gorelenkov, G. J. Kramer, D. S. Darrow, W. W. Heidbrink, S. Kubota, F. M. Levinton, H. Yuh, J. E. Menard, B. P. LeBlanc, and R. E. Bell. Alfvén cascade modes at high β in the national spherical torus experiment. *Physics of Plasmas*, 15(10), October 2008. ISSN 1089-7674. doi: 10.1063/1.2993182. URL <http://dx.doi.org/10.1063/1.2993182>.
- [25] W. Deng, Z. Lin, and I. Holod. Gyrokinetic simulation model for kinetic magnetohydrodynamic processes in magnetized plasmas. *Nuclear Fusion*, 52(2):023005, January 2012. ISSN 1741-4326. doi: 10.1088/0029-5515/52/2/023005. URL <http://dx.doi.org/10.1088/0029-5515/52/2/023005>.

- [26] W. Deng, Z. Lin, I. Holod, Z. Wang, Y. Xiao, and H. Zhang. Linear properties of reversed shear alfvén eigenmodes in the diii-d tokamak. *Nuclear Fusion*, 52(4): 043006, March 2012. ISSN 1741-4326. doi: 10.1088/0029-5515/52/4/043006. URL <http://dx.doi.org/10.1088/0029-5515/52/4/043006>.
- [27] A. S. EDDINGTON. The internal constitution of the stars. *Nature*, 106 (2653):14–20, September 1920. ISSN 1476-4687. doi: 10.1038/106014a0. URL <http://dx.doi.org/10.1038/106014a0>.
- [28] A. R. Field, D. Dunai, N. J. Conway, S. Zoletnik, and J. Sárközi. Beam emission spectroscopy for density turbulence measurements on the mast spherical tokamak. *Review of Scientific Instruments*, 80(7), July 2009. ISSN 1089-7623. doi: 10.1063/1.3170034. URL <http://dx.doi.org/10.1063/1.3170034>.
- [29] A. R. Field, D. Dunai, R. Gaffka, Y.-c. Ghim, I. Kiss, B. Mészáros, T. Krizsanóczy, S. Shibaev, and S. Zoletnik. Beam emission spectroscopy turbulence imaging system for the mast spherical tokamak. *Review of Scientific Instruments*, 83(1), January 2012. ISSN 1089-7623. doi: 10.1063/1.3669756. URL <http://dx.doi.org/10.1063/1.3669756>.
- [30] R. J. Fonck, P. A. Duperrex, and S. F. Paul. Plasma fluctuation measurements in tokamaks using beam-plasma interactions. *Review of Scientific Instruments*, 61 (11):3487–3495, November 1990. ISSN 1089-7623. doi: 10.1063/1.1141556. URL <http://dx.doi.org/10.1063/1.1141556>.
- [31] E.D. Fredrickson, N.A. Crocker, D.S. Darrow, N.N. Gorelenkov, G.J. Kramer, S. Kubota, M. Podesta, R.B. White, A. Bortolon, S.P. Gerhardt, R.E. Bell, A. Di-allo, B. LeBlanc, F.M. Levinton, and H. Yuh. Fast-ion energy loss during tae avalanches in the national spherical torus experiment. *Nuclear Fusion*, 53(1):013006, December 2012. ISSN 1741-4326. doi: 10.1088/0029-5515/53/1/013006. URL <http://dx.doi.org/10.1088/0029-5515/53/1/013006>.

- [32] Jeffrey P. Freidberg. *Plasma Physics and Fusion Energy*, page 60–84. Cambridge University Press, February 2007. doi: 10.1017/cbo9780511755705.006. URL <http://dx.doi.org/10.1017/CB09780511755705.006>.
- [33] Jeffrey P. Freidberg. *Ideal MHD*. Cambridge University Press, June 2014. ISBN 9780511795046. doi: 10.1017/cbo9780511795046. URL <http://dx.doi.org/10.1017/CB09780511795046>.
- [34] G. Y. Fu and J. W. Van Dam. Excitation of the toroidicity-induced shear alfvén eigenmode by fusion alpha particles in an ignited tokamak. *Physics of Fluids B: Plasma Physics*, 1(10):1949–1952, October 1989. ISSN 0899-8221. doi: 10.1063/1.859057. URL <http://dx.doi.org/10.1063/1.859057>.
- [35] Benedikt Geiger, Luke Stagner, William W Heidbrink, Ralph Dux, Rainer Fischer, Yutaka Fujiwara, Alvin Garcia, Asger Schou Jacobsen, Anton Jansen vanVuuren, Alexander N Karpushov, Deyong Liu, Philip Adrian Schneider, Igor Sfiligoi, Peter Zsolt Poloskei, and Markus Weiland. Progress in modelling fast-ion d-alpha spectra and neutral particle analyzer fluxes using fidasim. *Plasma Physics and Controlled Fusion*, 2020. URL <http://iopscience.iop.org/10.1088/1361-6587/aba8d7>.
- [36] W. Gekelman, S. Vincena, B. Van Compernelle, G. J. Morales, J. E. Maggs, P. Pribyl, and T. A. Carter. The many faces of shear alfvén waves. *Physics of Plasmas*, 18(5):055501, May 2011. ISSN 1089-7674. doi: 10.1063/1.3592210. URL <http://dx.doi.org/10.1063/1.3592210>.
- [37] S.P. Gerhardt, D.P. Brennan, R. Buttery, R.J. La Haye, S. Sabbagh, E. Strait, M. Bell, R. Bell, E. Fredrickson, D. Gates, B. LeBlanc, J. Menard, D. Stutman, K. Tritz, and H. Yuh. Relationship between onset thresholds, trigger types and rotation shear for the $m/n = 2/1$ neoclassical tearing mode in a high- spherical torus. *Nuclear Fusion*, 49(3):032003, March 2009. ISSN 1741-4326. doi: 10.1088/0029-5515/49/3/032003. URL <http://dx.doi.org/10.1088/0029-5515/49/3/032003>.

- [38] N. N. Gorelenkov, E. Belova, H. L. Berk, C. Z. Cheng, E. Fredrickson, W. W. Heidbrink, S. Kaye, and G. J. Kramer. Beam ion driven instabilities in the national spherical tokamak experiment. *Physics of Plasmas*, 11(5):2586–2593, May 2004. ISSN 1089-7674. doi: 10.1063/1.1689667. URL <http://dx.doi.org/10.1063/1.1689667>.
- [39] N.N Gorelenkov, H.L Berk, and R.V Budny. Beam anisotropy effect on alfvén eigenmode stability in iter-like plasmas. *Nuclear Fusion*, 45(4):226–237, March 2005. ISSN 1741-4326. doi: 10.1088/0029-5515/45/4/002. URL <http://dx.doi.org/10.1088/0029-5515/45/4/002>.
- [40] H Grad and H Rubin. Hydromagnetic equilibria and force-free fields. Technical report, New York Univ., New York. Inst. of Mathematical Sciences, 10 1958. URL <https://www.osti.gov/biblio/4315300>.
- [41] J P Graves, M Coste-Sarguet, and C Wahlberg. Pressure driven long wavelength mhd instabilities in an axisymmetric toroidal resistive plasma. *Plasma Physics and Controlled Fusion*, 64(1):014001, December 2021. ISSN 1361-6587. doi: 10.1088/1361-6587/ac3496. URL <http://dx.doi.org/10.1088/1361-6587/ac3496>.
- [42] S Günter, A Gude, J Hobirk, M Maraschek, S Saarelma, S Schade, R.C Wolf, and ASDEX Upgrade Team. Mhd phenomena in advanced scenarios on asdex upgrade and the influence of localized electron heating and current drive. *Nuclear Fusion*, 41(9): 1283–1290, September 2001. ISSN 0029-5515. doi: 10.1088/0029-5515/41/9/317. URL <http://dx.doi.org/10.1088/0029-5515/41/9/317>.
- [43] G Z Hao, W W Heidbrink, D Liu, M Podesta, L Stagner, R E Bell, A Bortolon, and F Scotti. Measurement of the passive fast-ion d-alpha emission on the nstx-u tokamak. *Plasma Physics and Controlled Fusion*, 60(2): 025026, January 2018. ISSN 1361-6587. doi: 10.1088/1361-6587/aa9fde. URL <http://dx.doi.org/10.1088/1361-6587/aa9fde>.

- [44] J. Harrison, et al., et al., et al., et al., et al., et al., et al., et al., et al., et al., et al., and et al. 2023 29th iaea fusion energy conference.
- [45] J.R. Harrison, A. Aboutaleb, S. Ahmed, M. Aljunid, S.Y. Allan, H. Anand, Y. Andrew, L.C. Appel, A. Ash, J. Ashton, O. Bachmann, M. Barnes, B. Barrett, D. Bayer, D. Beckett, J. Bennett, J. Berkery, M. Bernert, W. Boeglin, C. Bowman, J. Bradley, D. Brida, P.K. Browning, D. Brunetti, P. Bryant, J. Bryant, J. Buchanan, N. Bulmer, A. Carruthers, M. Cecconello, Z.P. Chen, J. Clark, C. Cowley, M. Coy, N. Crocker, G. Cunningham, I. Cziegler, T. Da Assuncao, Y. Damizia, P. Davies, I.E. Day, G.L. Derks, S. Dixon, R. Doyle, M. Dreval, M. Dunne, B.P. Duval, T. Eagles, J. Edmond, H. El-Haroun, S.D. Elmore, Y. Enters, M. Faitsch, F. Federici, N. Fedorczak, F. Felici, A.R. Field, M. Fitzgerald, I. Fitzgerald, R. Fitzpatrick, L. Frassinetti, W. Fuller, D. Gahle, J. Galdon-Quiroga, L. Garzotti, S. Gee, T. Gheorghiu, S. Gibson, K.J. Gibson, C. Giroud, D. Greenhouse, V.H. Hall-Chen, C.J. Ham, R. Harrison, S.S. Henderson, C. Hickling, B. Hnat, L. Howlett, J. Hughes, R. Hussain, K. Imada, P. Jacquet, P. Jepson, B. Kandan, I. Katramados, Y.O. Kazakov, D. King, R. King, A. Kirk, M. Knolker, M. Kochan, L. Kogan, B. Kool, M. Kotschenreuther, M. Lees, A.W. Leonard, G. Liddiard, B. Lipschultz, Y.Q. Liu, B.A. Lomanowski, N. Lonigro, J. Lore, J. Lovell, S. Mahajan, F. Maiden, C. Man-Friel, F. Mansfield, S. Marsden, R. Martin, S. Mazzi, R. McAdams, G. McArdle, K.G. McClements, J. McClenaghan, D. McConville, K. McKay, C. McKnight, P. McKnight, A. McLean, B.F. McMillan, A. McShee, J. Measures, N. Mehay, C.A. Michael, F. Militello, D. Morbey, S. Mordijck, D. Moulton, O. Myatra, A.O. Nelson, M. Nicassio, M.G. O'Mullane, H.J.C. Oliver, P. Ollus, T. Osborne, N. Osborne, E. Parr, B. Parry, B.S. Patel, D. Payne, C. Paz-Soldan, A. Phelps, L. Piron, C. Piron, G. Prechel, M. Price, B. Pritchard, R. Proudfoot, H. Reimerdes, T. Rhodes, P. Richardson, J. Riquezes, J.F. Rivero-Rodriguez, C.M. Roach, M. Robson, K. Ronald, E. Rose, P. Ryan, D. Ryan, S. Saarelma, S. Sabbagh, R. Sarwar, P. Saunders, O. Sauter, R. Scannell,

- T. Schuett, R. Seath, R. Sharma, P. Shi, B. Sieglin, M. Simmonds, J. Smith, A. Smith, V. A. Soukhanovskii, D. Speirs, G. Staebler, R. Stephen, P. Stevenson, J. Stobbs, M. Stott, C. Stroud, C. Tame, C. Theiler, N. Thomas-Davies, A.J. Thornton, M. Tobin, M. Vallar, R.G.L. Vann, L. Velarde, K. Verhaegh, E. Viezzer, C. Vincent, G. Voss, M. Warr, W. Wehner, S. Wiesen, T.A. Wijkamp, D. Wilkins, T. Williams, T. Wilson, H.R. Wilson, H. Wong, M. Wood, and V. Zamkovska. Overview of physics results from mast upgrade towards core-pedestal-exhaust integration. *Nuclear Fusion*, 64(11):112017, August 2024. ISSN 1741-4326. doi: 10.1088/1741-4326/ad6011. URL <http://dx.doi.org/10.1088/1741-4326/ad6011>.
- [46] Hongda He, Yueqiang Liu, Guangzhou Hao, Jinxia Zhu, Yong Shen, and Guoyao Zheng. Resistive wall mode and fishbone mode in iter steady state scenario: roles of fusion-born alphas and plasma flow. *Nuclear Fusion*, 64(9):096023, August 2024. ISSN 1741-4326. doi: 10.1088/1741-4326/ad63b7. URL <http://dx.doi.org/10.1088/1741-4326/ad63b7>.
- [47] W. W. Heidbrink. Basic physics of alfvén instabilities driven by energetic particles in toroidally confined plasmas. *Physics of Plasmas*, 15(5), February 2008. ISSN 1089-7674. doi: 10.1063/1.2838239. URL <http://dx.doi.org/10.1063/1.2838239>.
- [48] W. W. Heidbrink and R. B. White. Mechanisms of energetic-particle transport in magnetically confined plasmas. *Physics of Plasmas*, 27(3), March 2020. ISSN 1089-7674. doi: 10.1063/1.5136237. URL <http://dx.doi.org/10.1063/1.5136237>.
- [49] W W Heidbrink, K H Burrell, Y Luo, N A Pablant, and E Ruskov. Hydrogenic fast-ion diagnostic using balmer-alpha light. *Plasma Physics and Controlled Fusion*, 46(12):1855–1875, November 2004. ISSN 1361-6587. doi: 10.1088/0741-3335/46/12/005. URL <http://dx.doi.org/10.1088/0741-3335/46/12/005>.
- [50] W W Heidbrink, M E Austin, R K Fisher, M García-Muñoz, G Matsunaga,

- G R McKee, R A Moyer, C M Muscatello, M Okabayashi, D C Pace, K Shinohara, W M Solomon, E J Strait, M A Van Zeeland, and Y B Zhu. Characterization of off-axis fishbones. *Plasma Physics and Controlled Fusion*, 53(8):085028, July 2011. ISSN 1361-6587. doi: 10.1088/0741-3335/53/8/085028. URL <http://dx.doi.org/10.1088/0741-3335/53/8/085028>.
- [51] S. Henderson. Impurity transport studies on mast. *PhD Thesis*, 2014.
- [52] M J Hole and L C Appel. A modulation model for mode splitting of magnetic perturbations in the mega ampere spherical tokamak. *Plasma Physics and Controlled Fusion*, 51(4):045002, January 2009. doi: 10.1088/0741-3335/51/4/045002. URL <https://doi.org/10.1088/0741-3335/51/4/045002>.
- [53] I. Holod, W. L. Zhang, Y. Xiao, and Z. Lin. Electromagnetic formulation of global gyrokinetic particle simulation in toroidal geometry. *Physics of Plasmas*, 16(12):122307, December 2009. ISSN 1089-7674. doi: 10.1063/1.3273070. URL <http://dx.doi.org/10.1063/1.3273070>.
- [54] I. H. Hutchinson. *Principles of Plasma Diagnostics*. Cambridge University Press, July 2002. ISBN 9780511613630. doi: 10.1017/cbo9780511613630. URL <http://dx.doi.org/10.1017/CB09780511613630>.
- [55] Naofumi Iwama, Hiromasa Yoshida, Hitoshi Takimoto, Yun Shen, Shuichi Takamura, and Takashige Tsukishima. Phillips–tikhonov regularization of plasma image reconstruction with the generalized cross validation. *Applied Physics Letters*, 54(6):502–504, February 1989. doi: 10.1063/1.100912. URL <https://doi.org/10.1063/1.100912>.
- [56] H. Järleblad, L. Stagner, M. Salewski, J. Eriksson, M. Nocente, J. Rasmussen, Ž. Štancar, Ye.O. Kazakov, B. Simmendefeldt, and JET Contributors. Fast-ion orbit sensitivity of neutron and gamma-ray diagnostics for one-step fusion reactions. *Nu-*

- clear Fusion*, 62(11):112005, September 2022. ISSN 1741-4326. doi: 10.1088/1741-4326/ac63d3. URL <http://dx.doi.org/10.1088/1741-4326/ac63d3>.
- [57] Douglas L. Jones and Thomas W. Parks. Time-frequency window leakage in the short-time fourier transform. *Circuits, Systems, and Signal Processing*, 6(3): 263–286, September 1987. ISSN 1531-5878. doi: 10.1007/bf01599994. URL <http://dx.doi.org/10.1007/BF01599994>.
- [58] S.M Kaye, M.G Bell, R.E Bell, E.D Fredrickson, B.P LeBlanc, K.C Lee, S Lynch, and S.A Sabbagh. Energy confinement scaling in the low aspect ratio national spherical torus experiment (nstx). *Nuclear Fusion*, 46(10):848–857, August 2006. ISSN 1741-4326. doi: 10.1088/0029-5515/46/10/002. URL <http://dx.doi.org/10.1088/0029-5515/46/10/002>.
- [59] C. E. Kieras and J. A. Tataronis. The shear alfvén continuous spectrum of axisymmetric toroidal equilibria in the large aspect ratio limit. *Journal of Plasma Physics*, 28(3):395–414, December 1982. ISSN 1469-7807. doi: 10.1017/s0022377800000386. URL <http://dx.doi.org/10.1017/S0022377800000386>.
- [60] Ya. I. Kolesnichenko, V. V. Lutsenko, V. S. Marchenko, and R. B. White. Stabilization of the quasi-interchange mode in tokamaks by circulating energetic ions. *Physics of Plasmas*, 14(1), January 2007. doi: 10.1063/1.2434791. URL <https://doi.org/10.1063/1.2434791>.
- [61] G.J. Kramer, B.J. Tobias, A. Turnbull, and E.M. Bass. Enhanced radial energy transport induced by radially curved alfvén eigenmode wavefronts. *Nuclear Fusion*, 59(9):094001, July 2019. doi: 10.1088/1741-4326/ab2ee3. URL <https://doi.org/10.1088/1741-4326/ab2ee3>.
- [62] Tsubasa Kusano, Yoshiki Masuyama, Kohei Yatabe, and Yasuhiro Oikawa. De-

- signing nearly tight window for improving time-frequency masking, 2018. URL <https://arxiv.org/abs/1811.08783>.
- [63] L.L. Lao, H. St. John, R.D. Stambaugh, A.G. Kellman, and W. Pfeiffer. Reconstruction of current profile parameters and plasma shapes in tokamaks. *Nuclear Fusion*, 25(11): 1611–1622, November 1985. ISSN 1741-4326. doi: 10.1088/0029-5515/25/11/007. URL <http://dx.doi.org/10.1088/0029-5515/25/11/007>.
- [64] L.L. Lao, J.R. Ferron, R.J. Groebner, W. Howl, H. St. John, E.J. Strait, and T.S. Taylor. Equilibrium analysis of current profiles in tokamaks. *Nuclear Fusion*, 30(6): 1035–1049, June 1990. ISSN 1741-4326. doi: 10.1088/0029-5515/30/6/006. URL <http://dx.doi.org/10.1088/0029-5515/30/6/006>.
- [65] S. Li, H. Jiang, Z. Ren, and C. Xu. Optimal tracking for a divergent-type parabolic pde system in current profile control. *Abstract and Applied Analysis*, 2014:1–8, 2014. ISSN 1687-0409. doi: 10.1155/2014/940965. URL <http://dx.doi.org/10.1155/2014/940965>.
- [66] Z. Lin, T. S. Hahm, W. W. Lee, W. M. Tang, and R. B. White. Turbulent transport reduction by zonal flows: Massively parallel simulations. *Science*, 281(5384): 1835–1837, September 1998. ISSN 1095-9203. doi: 10.1126/science.281.5384.1835. URL <http://dx.doi.org/10.1126/science.281.5384.1835>.
- [67] P. Liu, X. Wei, Z. Lin, G. Brochard, G.J. Choi, W.W. Heidbrink, J.H. Nicolau, and G.R. McKee. Regulation of alfvén eigenmodes by microturbulence in fusion plasmas. *Physical Review Letters*, 128(18), May 2022. ISSN 1079-7114. doi: 10.1103/physrevlett.128.185001. URL <http://dx.doi.org/10.1103/PhysRevLett.128.185001>.
- [68] Yaqi Liu, Zhihong Lin, Huasen Zhang, and Wenlu Zhang. Excitation of low frequency alfvén eigenmodes in toroidal plasmas. *Nuclear Fusion*, 57(11):

- 114001, July 2017. ISSN 1741-4326. doi: 10.1088/1741-4326/aa7d14. URL <http://dx.doi.org/10.1088/1741-4326/aa7d14>.
- [69] B. Madsen, M. Salewski, J. Huang, A. S. Jacobsen, O. Jones, and K. G. McClements. Velocity-space tomography using prior information at mast. *Review of Scientific Instruments*, 89(10), oct 2018. ISSN 1089-7623. doi: 10.1063/1.5035498. URL <http://dx.doi.org/10.1063/1.5035498>.
- [70] K G McClements, M P Gryaznevich, S E Sharapov, R J Akers, L C Appel, G F Counsell, C M Roach, and R Majeski. Physics of energetic particle-driven instabilities in the start spherical tokamak. *Plasma Physics and Controlled Fusion*, 41(5): 661–678, January 1999. ISSN 1361-6587. doi: 10.1088/0741-3335/41/5/307. URL <http://dx.doi.org/10.1088/0741-3335/41/5/307>.
- [71] K G McClements, J Young, L Garzotti, OM Jones, and C A Michael. Abel inversion of soft x-ray fluctuations associated with fast particle-driven fishbone instabilities in MAST plasmas. *Plasma Research Express*, 3(3):034001, September 2021. doi: 10.1088/2516-1067/ac256a. URL <https://doi.org/10.1088/2516-1067/ac256a>.
- [72] K. McGuire, R. Goldston, M. Bell, M. Bitter, K. Bol, K. Brau, D. Buchenauer, T. Crowley, S. Davis, F. Dylla, H. Eubank, H. Fishman, R. Fonck, B. Grek, R. Grimm, R. Hawryluk, H. Hsuan, R. Hulse, R. Izzo, R. Kaita, S. Kaye, H. Kugel, D. Johnson, J. Manickam, D. Manos, D. Mansfield, E. Mazzucato, R. McCann, D. McCune, D. Monticello, R. Motley, D. Mueller, K. Oasa, M. Okabayashi, K. Owens, W. Park, M. Reusch, N. Sauthoff, G. Schmidt, S. Sesnic, J. Strachan, C. Surko, R. Slusher, H. Takahashi, F. Tenney, P. Thomas, H. Towner, J. Valley, and R. White. Study of high-beta magnetohydrodynamic modes and fast-ion losses in pdx. *Physical Review Letters*, 50(12):891–895, March 1983. ISSN 0031-9007. doi: 10.1103/physrevlett.50.891. URL <http://dx.doi.org/10.1103/PhysRevLett.50.891>.

- [73] C A Michael, N Conway, B Crowley, O Jones, W W Heidbrink, S Pinches, E Braeken, R Akers, C Challis, M Turnyanskiy, A Patel, D Muir, R Gaffka, and S Bailey. Dual view fida measurements on mast. *Plasma Physics and Controlled Fusion*, 55(9): 095007, July 2013. ISSN 1361-6587. doi: 10.1088/0741-3335/55/9/095007. URL <http://dx.doi.org/10.1088/0741-3335/55/9/095007>.
- [74] Dmitry Moseev and Mirko Salewski. Bi-maxwellian, slowing-down, and ring velocity distributions of fast ions in magnetized plasmas. *Physics of Plasmas*, 26(2), February 2019. ISSN 1089-7674. doi: 10.1063/1.5085429. URL <http://dx.doi.org/10.1063/1.5085429>.
- [75] C M Muscatello, W W Heidbrink, Ya I Kolesnichenko, V V Lutsenko, M A Van Zeeland, and Yu V Yakovenko. Velocity-space studies of fast-ion transport at a sawtooth crash in neutral-beam heated plasmas. *Plasma Physics and Controlled Fusion*, 54(2): 025006, January 2012. ISSN 1361-6587. doi: 10.1088/0741-3335/54/2/025006. URL <http://dx.doi.org/10.1088/0741-3335/54/2/025006>.
- [76] Alexei Pankin, Douglas McCune, Robert Andre, Glenn Bateman, and Arnold Kritz. The tokamak monte carlo fast ion module nubeam in the national transport code collaboration library. *Computer Physics Communications*, 159(3): 157–184, June 2004. ISSN 0010-4655. doi: 10.1016/j.cpc.2003.11.002. URL <http://dx.doi.org/10.1016/j.cpc.2003.11.002>.
- [77] M Podestà, M Gorelenkova, and R B White. A reduced fast ion transport model for the tokamak transport code transp. *Plasma Physics and Controlled Fusion*, 56(5):055003, April 2014. ISSN 1361-6587. doi: 10.1088/0741-3335/56/5/055003. URL <http://dx.doi.org/10.1088/0741-3335/56/5/055003>.
- [78] M Podestà, M Gorelenkova, N N Gorelenkov, and R B White. Computation of alfven eigenmode stability and saturation through a reduced fast ion transport model

- in the transp tokamak transport code. *Plasma Physics and Controlled Fusion*, 59(9):095008, July 2017. ISSN 1361-6587. doi: 10.1088/1361-6587/aa7977. URL <http://dx.doi.org/10.1088/1361-6587/aa7977>.
- [79] M Podestà, M Gorelenkova, N N Gorelenkov, R B White, P J Bonfiglioli, F M Poli, A Teplukhina, J Yang, M Cecconello, and M Vallar. Development of a reduced model for energetic particle transport by sawteeth in tokamaks. *Plasma Physics and Controlled Fusion*, 64(2):025002, December 2021. ISSN 1361-6587. doi: 10.1088/1361-6587/ac37fe. URL <http://dx.doi.org/10.1088/1361-6587/ac37fe>.
- [80] P. H. Rebut and B. E. Keen. The jet experiment: Evolution, present status, and prospects. *Fusion Technology*, 11(1):13–42, January 1987. ISSN 0748-1896. doi: 10.13182/fst87-a24999. URL <http://dx.doi.org/10.13182/FST87-A24999>.
- [81] B.W Rice. q profile measurements with the motional stark effect diagnostic in the diii-d tokamak. *Fusion Engineering and Design*, 34–35:135–142, March 1997. ISSN 0920-3796. doi: 10.1016/S0920-3796(96)00684-9. URL [http://dx.doi.org/10.1016/S0920-3796\(96\)00684-9](http://dx.doi.org/10.1016/S0920-3796(96)00684-9).
- [82] J.F. Rivero-Rodríguez, K.G. McClements, M. Fitzgerald, S.E. Sharapov, M. Cecconello, N.A. Crocker, I. Dolby, M. Dreval, N. Fil, J. Galdón-Quiroga, M. García-Muñoz, S. Blackmore, W. Heidbrink, S. Henderson, A. Jackson, A. Kappatou, D. Keeling, D. Liu, Y.Q. Liu, C. Michael, H.J.C. Oliver, P. Ollus, E. Parr, G. Prechel, T. Rhodes, D. Ryan, P. Shi, M. Vallar, L. Velarde, T. Williams, and H. Wong. Overview of fast particle experiments in the first mast upgrade experimental campaigns. *Nuclear Fusion*, 64(8):086025, June 2024. ISSN 1741-4326. doi: 10.1088/1741-4326/ad56a2. URL <http://dx.doi.org/10.1088/1741-4326/ad56a2>.
- [83] V. D. Shafranov. Equilibrium of a toroidal pinch in a magnetic field. *Soviet Atomic Energy*, 13(6):1149–1158, November 1963. ISSN 1573-8205. doi: 10.1007/bf01312317. URL <http://dx.doi.org/10.1007/BF01312317>.

- [84] V.D. Shafranov. Plasma equilibrium in a magnetic field. *Reviews of Plasma Physics*, 2:103, 1966.
- [85] D V Sivukhin. Coulomb collisions in a fully ionized plasma. *Rev. Plasma Phys. (USSR)(Engl. Transl.)*, Vol: 4, 1 1966. URL <https://www.osti.gov/biblio/4541439>.
- [86] L. Spitzer. Physics of fully ionized gases. *Interscience, New York*, 1956.
- [87] D. A. Spong, E. M. Bass, W. Deng, W. W. Heidbrink, Z. Lin, B. Tobias, M. A. Van Zeeland, M. E. Austin, C. W. Domier, and N. C. Luhmann. Verification and validation of linear gyrokinetic simulation of alfvén eigenmodes in the diii-d tokamak. *Physics of Plasmas*, 19(8), August 2012. ISSN 1089-7674. doi: 10.1063/1.4747505. URL <http://dx.doi.org/10.1063/1.4747505>.
- [88] L. Stagner, B. Geiger, and W.W. Heidbrink. FIDASIM: A Neutral Beam and Fast-ion Diagnostic Modeling Suite. URL <https://doi.org/10.5281/zenodo.1341369>.
- [89] T H Stix. Heating of toroidal plasmas by neutral injection. *Plasma Physics*, 14 (4):367–384, April 1972. ISSN 0032-1028. doi: 10.1088/0032-1028/14/4/002. URL <http://dx.doi.org/10.1088/0032-1028/14/4/002>.
- [90] J.D. Strachan, B. Grek, W. Heidbrink, D. Johnson, S.M. Kaye, H.W. Kugel, B. Le Blanc, and K. McGuire. Studies of energetic ion confinement during fishbone events in pdx. *Nuclear Fusion*, 25(8):863–880, August 1985. ISSN 1741-4326. doi: 10.1088/0029-5515/25/8/001. URL <http://dx.doi.org/10.1088/0029-5515/25/8/001>.
- [91] H.P. Summers, W.J. Dickson, M.G. O’Mullane, N.R. Badnell, A.D. Whiteford, D.H. Brooks, J. Lang, S.D. Loch, and D.C. Griffin. Ionisation state, excited populations and emission of impurities in dynamic finite-density plasmas. i: The generalised collisional–radiative model for light elements. *Accepted at arXiv*, 2005.

- [92] Sundaresan Sridhar. Study of tokamak plasma disruption and runaway electrons in a metallic environment. 2020. doi: 10.13140/RG.2.2.22495.38562. URL <http://rgdoi.net/10.13140/RG.2.2.22495.38562>.
- [93] J. A. Tataronis. Energy absorption in the continuous spectrum of ideal mhd. *Journal of Plasma Physics*, 13(1):87–105, February 1975. ISSN 1469-7807. doi: 10.1017/s0022377800025897. URL <http://dx.doi.org/10.1017/S0022377800025897>.
- [94] B. Tobias, E.M. Bass, I.G.J. Classen, C.W. Domier, B.A. Grierson, W.W. Heidbrink, N.C. Luhmann, R. Nazikian, H.K. Park, D.A. Spong, and M.A. Van Zeeland. Alfvén eigenmode structure during off-axis neutral beam injection. *Nuclear Fusion*, 52(10): 103009, August 2012. ISSN 1741-4326. doi: 10.1088/0029-5515/52/10/103009. URL <http://dx.doi.org/10.1088/0029-5515/52/10/103009>.
- [95] M. A. VanZeeland, G. J. Kramer, M. E. Austin, R. L. Boivin, W. W. Heidbrink, M. A. Makowski, G. R. McKee, R. Nazikian, W. M. Solomon, and G. Wang. Radial structure of alfvén eigenmodes in the DIII-d tokamak through electron-cyclotron-emission measurements. *Physical Review Letters*, 97(13), September 2006. doi: 10.1103/physrevlett.97.135001. URL <https://doi.org/10.1103/physrevlett.97.135001>.
- [96] J Varela, D A Spong, L Garcia, Y Ghai, D Zarzoso, D del Castillo-Negrete, H Betar, J Ortiz, D C Pace, M A Van Zeeland, X Du, R Sanchez, V Tribaldos, and J M Reynolds-Barredo. Effect of the neutral beam injector operational regime on the alfvén eigenmode saturation phase in diii-d plasma. *Plasma Physics and Controlled Fusion*, 65(12):125004, October 2023. ISSN 1361-6587. doi: 10.1088/1361-6587/ad05d4. URL <http://dx.doi.org/10.1088/1361-6587/ad05d4>.
- [97] Lina Velarde, J F Rivero-Rodríguez, J Galdón-Quiroga, T Williams, J Rueda-Rueda, P Cano-Megías, R Chacartegui, M García-Muñoz, S Blackmore, K G McClements,

- L Sanchís, and E Viezzer. Velocity-space analysis of fast-ion losses measured in mast-u using a high-speed camera in the fld detector. *Plasma Physics and Controlled Fusion*, 67(1):015024, December 2024. ISSN 1361-6587. doi: 10.1088/1361-6587/ad9be4. URL <http://dx.doi.org/10.1088/1361-6587/ad9be4>.
- [98] C. Perez von Thun, A. Perona, T. Johnson, S.E. Sharapov, M. Reich, V.G. Kiptily, M. Cecconello, A. Salmi, V. Ya Goloborod’ko, S.D. Pinches, M. García-Muñoz, D. Darrow, M. Brix, and I. Voitsekhovitch. Mev-range fast ion losses induced by fishbones on jet. *Nuclear Fusion*, 50(8):084009, July 2010. ISSN 1741-4326. doi: 10.1088/0029-5515/50/8/084009. URL <http://dx.doi.org/10.1088/0029-5515/50/8/084009>.
- [99] Feng Wang, G. Y. Fu, J. A. Breslau, Kevin Tritz, and J. Y. Liu. Simulation of non-resonant internal kink mode with toroidal rotation in the national spherical torus experiment. *Physics of Plasmas*, 20(7), July 2013. ISSN 1089-7674. doi: 10.1063/1.4816026. URL <http://dx.doi.org/10.1063/1.4816026>.
- [100] X. Wang, S. Briguglio, L. Chen, C. Di Troia, G. Fogaccia, G. Vlad, and F. Zonca. An extended hybrid magnetohydrodynamics gyrokinetic model for numerical simulation of shear alfvén waves in burning plasmas. *Physics of Plasmas*, 18(5), May 2011. ISSN 1089-7674. doi: 10.1063/1.3587080. URL <http://dx.doi.org/10.1063/1.3587080>.
- [101] Zhixuan Wang, Zhihong Lin, Wenjun Deng, Ihor Holod, W. W. Heidbrink, Y. Xiao, H. Zhang, W. Zhang, and M. Van Zeeland. Properties of toroidal alfvén eigenmode in diii-d plasma. *Physics of Plasmas*, 22(2), February 2015. ISSN 1089-7674. doi: 10.1063/1.4908274. URL <http://dx.doi.org/10.1063/1.4908274>.
- [102] R B White, N Gorelenkov, W W Heidbrink, and M A Van Zeeland. Particle distribution modification by low amplitude modes. *Plasma Physics and Controlled Fusion*, 52(4): 045012, March 2010. ISSN 1361-6587. doi: 10.1088/0741-3335/52/4/045012. URL <http://dx.doi.org/10.1088/0741-3335/52/4/045012>.

- [103] R.B. White. *The Theory of Toroidally Confined Plasmas*. Imperial College Press, 2006.
- [104] Roscoe White and Andreas Bierwage. Particle resonances in toroidal fusion devices. *Physics of Plasmas*, 28(3), March 2021. ISSN 1089-7674. doi: 10.1063/5.0040975. URL <http://dx.doi.org/10.1063/5.0040975>.
- [105] Marco Wisse. *Charge-Exchange Spectroscopy in the MAST Tokamak*. PhD thesis, Imperial College London, 10 2007.
- [106] HH Wong, S. Blackmore, C Michael, K.G. McClements, Crocker N.A., M. Fitzgerald, L Garzotti, R. Scannell, S. Thomas, D. Dunai, A.R. Field, A.R. Jackson, M. Podesta, J.F. Rivero-Rodriguez, M. Cecconello, A. Sladkomedova, and T.A. Carter. Fishbone instabilities and the impacts on core confinement in mast and mast-u. *Nuclear Fusion*, (submitted and awaiting for peer review), 2024.
- [107] Samuel E. Wurzel and Scott C. Hsu. Progress toward fusion energy breakeven and gain as measured against the lawson criterion. *Physics of Plasmas*, 29(6), June 2022. ISSN 1089-7674. doi: 10.1063/5.0083990. URL <http://dx.doi.org/10.1063/5.0083990>.
- [108] D. Wunderlich, M. Giacomini, R. Ritz, and U. Fantz. Yacora on the web: Online collisional radiative models for plasmas containing h, h2 or he. *Journal of Quantitative Spectroscopy & Radiative Transfer*, 2020. doi: 10.1016/j.jqsrt.2019.106695.
- [109] J Yang, M Podestà, and E D Fredrickson. Synergy of coupled kink and tearing modes in fast ion transport. *Plasma Physics and Controlled Fusion*, 63(4): 045003, February 2021. ISSN 1361-6587. doi: 10.1088/1361-6587/abd9e4. URL <http://dx.doi.org/10.1088/1361-6587/abd9e4>.
- [110] Qingbo Yin, Liran Shen, Mingyu Lu, Xiangyang Wang, and Zhi Liu. Selection of optimal window length using stft for quantitative snr analysis of lfm signal. *Journal of Systems Engineering and Electronics*, 24(1):26–35, February 2013. ISSN 1004-4132. doi: 10.1109/jsee.2013.000004. URL <http://dx.doi.org/10.1109/JSEE.2013.000004>.

- [111] H. S. Zhang, Z. Lin, I. Holod, X. Wang, Y. Xiao, and W. L. Zhang. Gyrokinetic particle simulation of beta-induced alfvén eigenmode. *Physics of Plasmas*, 17(11), November 2010. ISSN 1089-7674. doi: 10.1063/1.3498761. URL <http://dx.doi.org/10.1063/1.3498761>.
- [112] Wenlu Zhang, Ihor Holod, Zhihong Lin, and Yong Xiao. Global gyrokinetic particle simulation of toroidal alfvén eigenmodes excited by antenna and fast ions. *Physics of Plasmas*, 19(2), February 2012. ISSN 1089-7674. doi: 10.1063/1.3685703. URL <http://dx.doi.org/10.1063/1.3685703>.
- [113] Wei Zhou, Wei-Jian Li, and Wei-Xin Ren. Stft-based time-frequency mode decomposition: A fast and robust method for multicomponent signal analysis, 2025. URL <https://arxiv.org/abs/2507.11919>.
- [114] Fulvio Zonca and Liu Chen. Resonant damping of toroidicity-induced shear-alfvén eigenmodes in tokamaks. *Physical Review Letters*, 68(5):592–595, February 1992. ISSN 0031-9007. doi: 10.1103/physrevlett.68.592. URL <http://dx.doi.org/10.1103/PhysRevLett.68.592>.
- [115] Fulvio Zonca and Liu Chen. Resonant and non-resonant particle dynamics in alfvén mode excitations. *Plasma Physics and Controlled Fusion*, 48(5): 537–556, March 2006. ISSN 1361-6587. doi: 10.1088/0741-3335/48/5/004. URL <http://dx.doi.org/10.1088/0741-3335/48/5/004>.