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Black Hole Thermodynamics

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Abstract

The discovery in the early 1970s that black holes radiate as black bodies has radically affected our understanding of general relativity, and offered us some early hints about the nature of quantum gravity. In this chapter I will review the discovery of black hole thermodynamics and summarize the many independent ways of obtaining the thermodynamic and (perhaps) statistical mechanical properties of black holes. I will then describe some of the remaining puzzles, including the nature of the quantum microstates, the problem of universality, and the information loss paradox.

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1 Introduction

The surprising discovery that black holes behave as thermodynamic objects has radically affected our understanding of general relativity and its relationship to quantum field theory. In the early 1970s, Bekenstein [1, 2] and Hawking [3, 4] showed that black holes radiate as black bodies, with characteristic temperatures and entropies

$$kT_{\text{H}} = \frac{\hbar\kappa}{2\pi}, \qquad S_{\text{BH}} = \frac{A_{\text{hor}}}{4\hbar G}, \tag{1.1}$$

where κ is the surface gravity and A_{hor} is the area of the horizon. These quantities appear to be inherently quantum gravitational, in the sense that they depend on both Planck's constant $\hbar$ and Newton's constant G. The resulting black body radiation, Hawking radiation, has not yet been directly observed: the temperature of an astrophysical black hole is on the order of a microkelvin, far lower than the cosmic microwave background temperature. But the Hawking temperature and the Bekenstein-Hawking entropy have been derived in so many independent ways, in different settings and with different assumptions, that it seems extraordinarily unlikely that they are not real.

In ordinary thermodynamic systems, thermal properties reflect the statistical mechanics of underlying microstates. The temperature of a cup of tea is a measure of the average energy of its molecules; its entropy is a measure of the number of possible microscopic arrangements of those molecules. It seems natural to suppose the same to be true for black holes, in which case the temperature and entropy (1.1) might tell us something about the underlying quantum gravitational states. This idea has been used as a consistency check for a number of proposed models of quantum gravity, and has suggested important new directions for research.

In one key aspect, though, black hole entropy is atypical. For an ordinary nongravitational system, entropy is extensive, scaling as volume. Black hole entropy, on the contrary, is "holographic," scaling as area. If the Bekenstein-Hawking entropy really counts black hole microstates, this holographic scaling suggests that a black hole has far fewer degrees of freedom than we might expect. But it has also been argued that a black hole provides an upper bound on the number of degrees of freedom in a given volume: if one tries to pack too many degrees of freedom into a region of space, they may inevitably collapse to form a black hole. This has led to the conjecture that all of Nature may be fundamentally holographic, and that our usual counting of local degrees of freedom vastly overestimates their number.

Black hole thermodynamics leads to a number of other interesting puzzles as well. One is the "problem of universality." If the Bekenstein-Hawing entropy S_{BH} counts the black hole degrees of freedom in an underlying quantum theory of gravity, one might expect that the result would depend on that theory. Oddly, though, the expression (1.1) can be obtained from a number of very different approaches to quantum gravity, from string theory to loop quantum gravity to induced gravity, in which the microscopic states appear to be quite different. This suggests the existence of an underlying structure shared by all of these approaches, but although we have some promising ideas, we do not know what that structure is.

A second puzzle is the "information loss paradox." Consider a black hole initially formed by the collapse of matter in a pure quantum state, which then evaporates completely into Hawking radiation. If the radiation is genuinely thermal, this would represent a transition from a pure state to a mixed state, a process that violates unitarity of evolution and is forbidden in ordinary quantum mechanics. If, on the other hand, the Hawking radiation is secretly a pure state, this would appear to require correlations between "early" and "late" Hawking particles that have never been in causal contact. The problem was recently sharpened by Almheiri *et al.* [5], who argue that one must sacrifice some cherished principle: the equivalence principle, low energy effective field theory, or the nonexistence of high-entropy "remnants" at the end of black hole evaporation. These issues are currently areas of active research, and the matter remains in flux.

2 Prehistory: black hole mechanics and Wheeler's cup of tea

The prehistory of black hole thermodynamics might be traced back to Hawking's 1972 proof that the area of an event horizon can never decrease [6]. This property is reminiscent of the second law of thermodynamics, and the correspondence was strengthened with the discovery of analogs of other laws of thermodynamics. This work culminated in the publication by Bardeen, Carter, and Hawking [7] of the "four laws of black hole mechanics": for a stationary asymptotically flat black hole in four dimensions, uniquely characterized by a mass M, an angular momentum J, and a charge Q,

0. The surface gravity κ is constant over the event horizon.
1. For two stationary black holes differing only by small variations in the parameters M, J, and Q,

$$\delta M = \frac{\kappa}{8\pi G}\delta A_{\rm hor} + \Omega_H \delta J + \Phi_H \delta Q, \tag{2.1}$$

   where Ω_H is the angular velocity and Φ_H is the electric potential at the horizon.
2. The area of the event horizon of a black hole never decreases,

$$\delta A_{\rm hor} \geq 0. \tag{2.2}$$

3. It is impossible by any procedure to reduce the surface gravity κ to zero in a finite number of steps.

These laws are numbered in parallel with the four usual laws of thermodynamics, and they are clearly formally analogous, with κ playing the role of temperature and $A_{\rm hor}$ of entropy. As in ordinary thermodynamics, there are a number of formulations of the third law, not all strictly equivalent; for a summary of the current status, see Wall [8]. While the four laws were originally formulated for four-dimensional "electrovac" spacetimes, they can be extended to more dimensions, more charges and angular momenta, and to other "black" objects such as black strings, rings, and branes. The first law, in particular, holds for arbitrary isolated horizons [9], and for much more general gravitational actions, for which the entropy can be understood as a Noether charge [10].

But despite the formal analogy with the laws of thermodynamics, Bardeen, Carter, and Hawking argued that κ cannot be a true temperature and $A_{\rm hor}$ cannot be a true entropy. The defining characteristic of temperature, after all, is that heat flows from hot to cold. But if one places a classical black hole in contact with a heat reservoir of any temperature, energy will flow into the black hole, but never out. A classical black hole thus has a temperature of absolute zero.

A second thread in the development of black hole thermodynamics began in 1972, when John Wheeler asked his student, Jacob Bekenstein, what would happen if he dropped his cup of tea into a passing black hole [11]. The initial state would be a cup of tea (with a nonzero entropy) plus a back hole; the final state would be no tea, but a slightly larger black hole. Where did the entropy go?

Bekenstein's ultimate answer was that the black hole must have an entropy proportional to its area. Using a series of thought experiments [1, 2], he argued further that the entropy ought to be of the form

$$S = \eta \frac{A_{\text{hor}}}{\hbar G} \tag{2.3}$$

where η is a constant of order one. For example, a spherical particle with a proper radius equal to its Compton wavelength will increase the entropy (2.3) by an amount of order one bit [2], a result that is, remarkably, independent of the spin and charge of the black hole. A small harmonic oscillator dropped into a black hole will increase (2.3) by an amount at least as great as its entropy [2], as will a beam of black body radiation [1]. Bekenstein's "generalized second law of thermodynamics" — the claim that the total of ordinary entropy of matter plus black hole entropy S_{BH} never decreases — has now been tested, and confirmed, over a very wide range of settings [8, 12, 13].

3 Hawking radiation

Despite the intriguing parallels between area and entropy, the fact that a classical black hole had temperature $T = 0$ seemed to present an insurmountable obstacle to identifying the two. Zel'dovich had suggested in 1970 that quantum effects might be relevant, causing spinning black holes to radiate in particular modes [14], but the arguments were not worked out in detail. In 1974, however, Hawking [3, 4] used newly developed techniques for treating quantum field theory in curved spacetime [16] to show that *all* black holes radiate, with a black body temperature T_{H}. With this identification of temperature, the first law of black hole mechanics (2.1) determines the entropy, and Bekenstein's expression (2.3) is confirmed, with $\eta = 1/4$.*

There are several ways to describe Hawking's results. Perhaps the most intuitive is to say that quantum mechanics allows particles to tunnel out through the event horizon; but while Hawking himself used this as a heuristic picture [4], the full mathematical description of such a tunneling process was not worked out until much later [17]. Hawking's calculation was based, rather, on a key feature of quantum field theory in curved spacetime: the fact that the vacuum is not unique, but depends on a choice of time. In particular, Hawking showed that the Minkowski vacuum of a past observer watching the collapse of a star differed from the vacuum of a future observer looking at the resulting black hole.

3.1 Quantum field theory in curved spacetime

To be more explicit, consider a free massless scalar field φ. In Minkowski space, the standard approach to quantization [18] is to first expand φ in its Fourier modes,

$$\varphi = \sum_{\mathbf{k}} \left(a_{\mathbf{k}} f_{\mathbf{k}}(t, \mathbf{x}) + a^\dagger_{\mathbf{k}} f^*_{\mathbf{k}}(t, \mathbf{x}) \right) \quad \text{with } f_{\mathbf{k}} = c_{\mathbf{k}} e^{i\mathbf{k}\cdot\mathbf{x} - i\omega_{\mathbf{k}} t},\ \omega_{\mathbf{k}} = |\mathbf{k}|. \tag{3.1}$$

The $f_{\mathbf{k}}$ are positive frequency, positive energy modes, which may be completely characterized by the conditions

$$\Box f_{\mathbf{k}}(t, \mathbf{x}) = 0, \quad \partial_t f_{\mathbf{k}}(t, \mathbf{x}) = -i\omega_{\mathbf{k}} f_{\mathbf{k}}(t, \mathbf{x}) \qquad (\text{with } \omega_{\mathbf{k}} > 0). \tag{3.2}$$

*Strictly speaking, the first law determines black hole entropy only up to an additive constant. But Pretoruis *et al.* have designed an (idealized) process that *reversibly* forms a black hole with entropy S_{BH}, strongly hinting that no additive constant appears [15].

Similarly, the $f^*_{\mathbf{k}}$ are negative frequency, negative energy modes. The theory has a natural scalar product,

$$(\varphi_1, \varphi_2) = -i \int_\Sigma n^a (\varphi_1 \overleftrightarrow{\partial}_a \varphi_2^*) \sqrt{g_\Sigma}\, d^3x, \tag{3.3}$$

where Σ is an arbitrary Cauchy surface and n^a is its unit normal. The modes (3.2) are then orthonormal, provided that we choose the normalization constants

$$c_{\mathbf{k}} = \frac{1}{\sqrt{(2\pi)^3 2\omega}}. \tag{3.4}$$

To quantize the theory, we now interpret the coefficients of the $f_{\mathbf{k}}$ as annihilation operators, and the coefficients of the $f^*_{\mathbf{k}}$ as creation operators. These obey the standard commutation relations, and can be used to construct the usual Fock space of free particle states. In particular, the vacuum is the state annihilated by the $a_{\mathbf{k}}$,

$$a_{\mathbf{k}}|0\rangle = 0. \tag{3.5}$$

In a more general spacetime, or even a noninertial frame in Minkowski space, the standard Fourier modes no longer exist. Suppose, though, that we can still choose a preferred time coordinate t. We can then find a complete set of orthonormal modes satisfying (3.2), perform an expansion of the form (3.1) with respect to these modes, and once again define creation and annihilation operators and a vacuum state.

What is new, though, is that there may now be more than one preferred time. Given two choices, say t and t', two expansions exist:

$$\varphi = \sum_i \left(a_i f_i + a_i^\dagger f_i^* \right) = \sum_i \left(a'_i f'_i + a'^\dagger_i f'^*_i \right). \tag{3.6}$$

Furthermore, since the $\{f_i, f_i^*\}$ are a complete set of functions, we can write

$$f'_j = \sum_i \left(\alpha_{ji} f_i + \beta_{ji} f_i^* \right) \tag{3.7}$$

for some coefficients α_{ji} and β_{ji}. This relation is known as a Bogoliubov transformation, and the coefficients α_{ji} and β_{ji} are Bogoliubov coefficients [19]. The coefficients may be read off from (3.7) by using the inner product (3.3); in particular,

$$\beta_{ji} = -(f_i^*, f'_j). \tag{3.8}$$

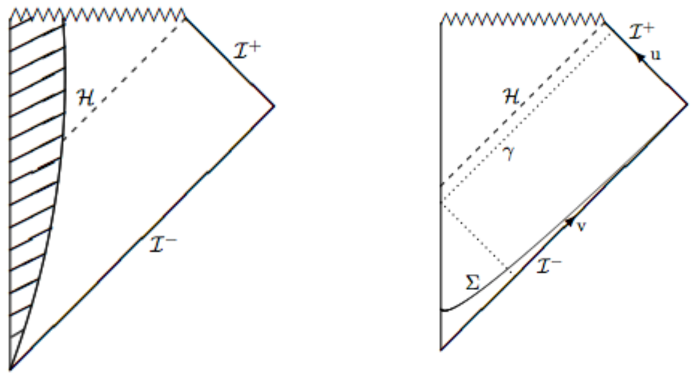
We now have two vacua, a state $|0\rangle$ annihilated by the a_i and a state $|0'\rangle$ annihilated by the a'_i, and two number operators, $N_i = a_i^\dagger a_i$ and $N'_i = a'^\dagger_i a'_i$. Using (3.7) and the orthonormality of the mode functions, it may be easily checked that

$$\langle 0'|N_i|0'\rangle = \sum_j |\beta_{ji}|^2. \tag{3.9}$$

Thus if the coefficients β_{ji} are not all zero, the "primed" vacuum will have a nonvanishing "unprimed" particle content.

3.2 Hawking's calculation

In his seminal work on black hole radiation and evaporation [3, 4], Hawking computed the Bogoliubov coefficients between an initial vacuum outside a collapsing star and a final vacuum after the formation of a black hole. He showed that a "primed" observer in the distant future would see a thermal distribution of particles at the Hawking temperature (1.1). While Hawking's full calculation is too technical to include in this review — see, for example, Traschen's very nice paper [20] for details — it is possible to sketch out the basic argument.



(a) Carter-Penrose diagram for a star collapsing to form a black hole

(b) Coordinates, ray-tracing, and mode functions

Figure 1: The set-up for Hawking's calculation.

Hawking's starting point is the collapse of a star to form a black hole, as shown in the Carter-Penrose diagram of figure 1(a). The shaded area is the collapsing star; the horizon $\mathcal{H}$ forms at some time after the collapse has begun. In the distant past, the spacetime far from the star is nearly flat, and we can define a standard Minkowski vacuum at past null infinity $\mathcal{I}^-$. In the distant future, the black hole has settled down to a (nearly) stationary configuration, and we can define a vacuum at future null infinity $\mathcal{I}^+$. From the previous discussion, we need to find the orthonormal modes at $\mathcal{I}^\pm$, propagate them to a common Cauchy surface — Σ in figure1(b) — and take the inner product (3.8) to determine the Bogoliubov coefficients.

For the first step, let us choose null coordinates u ("retarded time") and v ("advanced time") in the exterior of the black hole, chosen so that u is an affine parameter along $\mathcal{I}^+$ and v is an affine parameter along $\mathcal{I}^-$, as shown in figure 1(b). It is easy to check that the affine condition implies that near $\mathcal{I}^\pm$, the metric takes the form $ds^2 = dudv +$ angular terms. As a result, the Klein-Gordon equation greatly simplifies, and the asymptotic modes become

$$\begin{aligned} f_{\omega\ell m} &\sim \frac{1}{r} Y_{\ell m}(\theta,\phi) e^{-i\omega v} \qquad \text{near } \mathcal{I}^-, \\ f'_{\omega\ell m} &\sim \frac{1}{r} Y_{\ell m}(\theta,\phi) e^{-i\omega u} \qquad \text{near } \mathcal{I}^+. \end{aligned} \tag{3.10}$$

To calculate the inner product (3.8), Hawking evolved the mode $f'_{\omega\ell m}$ from $\mathcal{I}^+$ backwards to $\mathcal{I}^-$. In principle, this requires solving the Klein-Gordon equation with boundary conditions at $\mathcal{I}^+$. For the relevant frequencies, though, the geometric optics approximation, in which massless particles move along null geodesics, is good enough. The dotted line labeled γ in figure 1(b) shows a null geodesic propagating backwards from $\mathcal{I}^+$, "reflecting" off $r = 0$, and continuing to $\mathcal{I}^-$. (One may visualize this as an ingoing spherical wave shrinking to a point at the origin, passing through itself, and expanding outward.) This geodesic maps a point u on $\mathcal{I}^+$ to a point $p(u)$ on $\mathcal{I}^-$, and the mode functions $f'_{\omega\ell m}$ in turn map to functions of the form

$$\frac{1}{r}Y_{\ell m}(\theta,\phi)e^{-i\omega p^{-1}(v)}. \tag{3.11}$$

The meat of Hawking's calculation is the determination of this function $p(u)$, which he finds by tracking the position of the geodesic γ relative to the horizon $\mathcal{H}$. The result is that

$$p(u) \sim A - Be^{-\kappa u}, \tag{3.12}$$

where κ is the surface gravity and A and B are constants. The exponential dependence on u reflects the extreme blue shift of waves passing near the horizon: two successive crests at $\mathcal{I}^+$, labeled by u and $u+\delta u$, will be mapped to a separation $p'(u)\delta u \sim \kappa Be^{-\kappa u}\delta u$ at $\mathcal{I}^-$. Viewed in the opposite direction, the function $p(u)$ describes the "peeling" of null geodesics away from the horizon [21].

The computation of the Bogoliubov coefficients is now straightforward. The inner product (3.8) on a Cauchy surface Σ near $\mathcal{I}^-$ now involves integrals of the form

$$\int dv\, e^{i\omega' v}e^{-i\frac{\omega}{\kappa}\ln v}.$$

These yield gamma functions of complex arguments, whose absolute squares give the exponential behavior of a thermal distribution with the Hawking temperature.

I have left out many steps, of course. In particular, future null infinity $\mathcal{I}^+$ is not a Cauchy surface, so the modes $f'_{\omega\ell m}$ in (3.10) are not a complete set; they must be supplemented by modes on the horizon $\mathcal{H}$. These account for backscattering into the black hole. This scattering is energy-dependent, and distorts the black body spectrum, yielding instead a "grey body spectrum"

$$\langle N\rangle = \frac{\Gamma_\omega}{e^{\frac{2\pi\omega}{\kappa}} - 1} \tag{3.13}$$

where the reflection coefficient Γ_ω can be calculated explicitly [22, 23]. Generalizations to particles other than massless scalars are straightforward, and give a consistent thermodynamic picture.

4 Back-of-the-envelope estimates

The description I have given of Hawking's derivation of black hole radiation is not, for most people, terribly intuitive. It is useful to understand some back-of-the-envelope estimates, which can give a better feel for the physics. The following examples are taken from a set of my earlier lectures [24], slightly updated.

4.1 Entropy

We start with an estimate of black hole entropy, using the generalized second law [25]. Consider a cubic box of gas with volume ℓ^3, mass m, and temperature T, dropped into a Schwarzschild black hole of mass

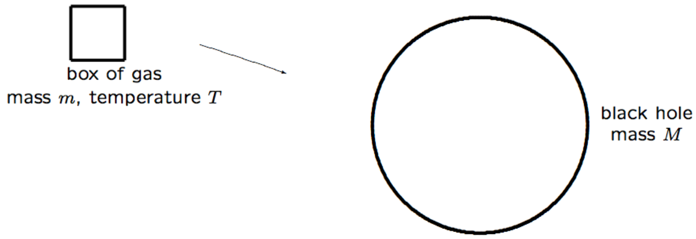


Figure 2: A thought experiment for Bekenstein-Hawking entropy

M (and therefore having a horizon area of $A = 16\pi G^2 M^2$). Let us assume that the box is at least as large as the thermal wavelength of the gas, $\ell \sim \hbar/T$. Then the descent of the gas into the black hole will lead to a decrease of entropy

$$\Delta S_{\text{gas}} \sim -\frac{m}{T} \sim -\frac{m\ell}{\hbar}. \tag{4.1}$$

The box of gas will coalesce with the black hole when its proper distance ρ from the horizon is of order ℓ. For a Schwarzschild black hole, this proper distance is

$$\rho = \int_{2GM}^{2GM+\delta r} \frac{dr}{\sqrt{1-2GM/r}} \sim \sqrt{GM\delta r},$$

so $\rho \sim \ell$ when $\delta r \sim \ell^2/GM$. The gas initially has mass m, but its energy as seen from infinity is red-shifted as the box falls toward the black hole; when the box reaches $r = 2GM + \delta r$, the black hole will gain a mass

$$\Delta M \sim m\sqrt{1 - \frac{2GM}{2GM+\delta r}} \sim \frac{m\ell}{GM}.$$

The change in the horizon area is thus

$$\Delta A \sim G^2 M \Delta M \sim Gm\ell. \tag{4.2}$$

Comparing (4.1) and (4.2), we see that the black hole must gain an entropy

$$\Delta S_{\text{BH}} \sim -\Delta S_{\text{gas}} \sim \frac{\Delta A}{\hbar G}. \tag{4.3}$$

4.2 Temperature

We next estimate the Hawking temperature, at a similar level of hand-waving [26]. As in section 3, we consider inequivalent vacua, but following a heuristic approach suggested by Hawking [4], we now compare vacua inside and outside the horizon.

As usual in quantum field theory, let us view the vacuum as a sea of virtual pairs of particles, each pair consisting of a particle with positive energy E relative to an observer at infinity and a particle with

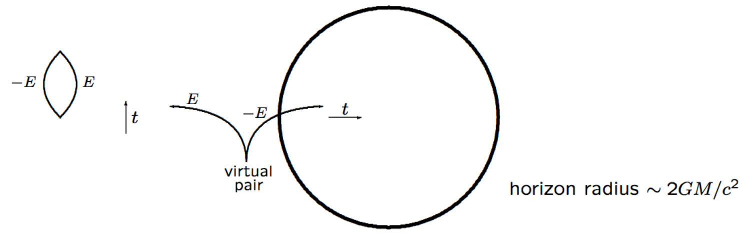


Figure 3: A thought experiment for Hawking temperature

negative energy $-E$. Normally, by the uncertainty principle, a negative energy particle can exist only for a time $t \sim \hbar/|E|$. Recall, however, that the distinction between "positive" and "negative" energy can depend on the choice of time. For a Schwarzschild black hole, with a metric

$$ds^2 = \left(1 - \frac{2M}{r}\right) dt^2 - \left(1 - \frac{2M}{r}\right)^{-1} dr^2 - r^2 d\Omega^2, \tag{4.4}$$

the coordinate t is a time coordinate in the exterior, but a spatial coordinate in the interior, where the coefficient of dt^2 changes sign.† Similarly, the coordinate r is a spatial coordinate in the exterior, but a time coordinate in the interior, where "forward in time" means "towards the center of the black hole." A particle with negative energy relative to an exterior observer may thus have positive energy relative to an interior observer, and if it crosses the horizon quickly enough, this will allow us to evade the uncertainty principle.

Now consider a virtual pair of massless particles momentarily at rest at a coordinate distance δr from the horizon. As in the preceding section, the proper distance — and therefore the proper time for the negative energy partner to reach the horizon — will be

$$\tau \sim \sqrt{GM\delta r}.$$

Setting this equal to the lifetime $\hbar/E$ of the pair, we find that

$$|E| \sim \frac{\hbar}{\sqrt{GM\delta r}}.$$

which should match the energy of the escaping positive-energy partner. This is the energy at $2GM + \delta r$, though. As in the preceding section, the energy at infinity will be red-shifted, becoming

$$E_\infty \sim \frac{\hbar}{\sqrt{GM\delta r}} \sqrt{1 - \frac{2GM}{2GM + \delta r}} \sim \frac{\hbar}{GM}, \tag{4.5}$$

independent of the initial position δr. We thus expect a black hole to radiate with a characteristic temperature $kT \sim \hbar/GM$, matching the Hawking temperature (1.1).

†Strictly speaking, the coordinates labeled r and t outside the horizon are not the same as those inside, since the two coordinate patches do not overlap. The argument can be rephrased in terms of proper times of infalling observers, though, in a way that avoids this issue.

5 The many derivations of black hole thermodynamics

Hawking's derivation of the black hole temperature seems to depend only on simple properties of quantum field theory in curved spacetime. Still, one may worry that these properties have been pushed beyond their realm of validity. In particular, it is evident from section 2 that the calculation of the Bogoliubov coefficients requires that we follow outgoing modes backwards in time to a region in which they are very highly blue-shifted, with energies far above the Planck scale [27–29].

This "trans-Planckian problem" has been addressed in a number of ways: by imposing high-energy cut-offs [28], by restricting fields to discrete lattices [30], by altering dispersion relations to break the connection between high frequencies and high energies [31, 32], and by considering analog systems such as Unruh's "dumb holes," fluid flows that generate sonic event horizons [27]. It turns out to be very hard to "break" Hawking's results: standard thermal Hawking radiation, with the usual Hawking temperature, appears even in models that never involve Planck scale physics [33].

Perhaps the strongest evidence for the reality of black hole thermodynamics, though, comes from the number of independent derivations, each relying on different assumptions and approximations. In this section I will briefly summarize some of these results.

5.1 Other settings

Hawking's original derivation involved a particular physical setting, a star collapsing to form a black hole. This determined the choice of vacuum state, and one could worry that the result might be too narrow. It has been subsequently shown, though, that one can generalize the derivation considerably, for instance by replacing the initial Minkowski vacuum in the distant past with the vacuum for a freely falling observer near the horizon [34]. Indeed, there is now fairly strong evidence that only a few fundamental features are needed to obtain thermal radiation: a vacuum near the horizon as seen by a freely falling observer, vacuum fluctuations that start in the ground state, and subsequent adiabatic evolution [32, 35]. In particular, the Einstein field equations are not required [36].

We may also ask which ingredients in the derivation of section 2 were essential to the result [21]. Consider any setting in which, as in figure 1(b), one can trace null geodesics (such as γ in the figure) from part of $\mathcal{I}^+$ to part of $\mathcal{I}^-$, defining a map $p(u)$. One can then *define* a time-dependent "surface gravity" $\kappa(u)$ by

$$\kappa(u) = -\frac{p''(u)}{p'(u)}. \tag{5.1}$$

Now pick a point u_* on $\mathcal{I}^+$. Barceló *et al.* show [21] that an observer near u_* will observe nearly thermal Hawking radiation, with a time-dependent temperature $\hbar\kappa(u_*)/2\pi$, provided an adiabatic condition

$$\kappa'(u_*) \ll \kappa(u_*)^2 \tag{5.2}$$

holds. In particular, it is not even necessary for an event horizon to form; the exponential "peeling" (3.12) is sufficient.

One can also analyze an eternal black hole at equilibrium with its Hawking radiation [37], obtaining consistent results. Moreover, the computation may be extended beyond the number operator (3.9) to find an expression for the full final state in terms of the initial vacuum; it is exactly thermal, at least in the approximation that one ignores back-reaction of Hawking radiation on the black hole [38, 39].

5.2 Unruh radiation

By the principle of equivalence, the gravitational field near a black hole horizon should be locally equivalent to uniform acceleration in a flat spacetime. Hawking radiation is not entirely local, so one might not expect an exact equivalence, but one should be able to test Hawking's results by comparing the vacua of an inertial observer and a uniformly accelerating observer in Minkowski space. This was first done by Unruh [34]. The Bogoliubov coefficients can be derived fairly easily, and the conclusion is that the accelerated observer with a constant proper acceleration a sees a thermal flux of particles with a temperature

$$T_U = \frac{\hbar a}{2\pi}, \tag{5.3}$$

in almost exact analogy with the Hawking temperature (1.1).

Coincidentally, at almost the same time as Unruh's work, Bisognano and Wichmann gave an independent, more difficult but mathematically rigorous proof of the Unruh effect in quantum field theory [40]. Their results are quite general; while they require flat spacetime, the quantum field theory need not be a free theory.

5.3 Particle detectors

Hawking's derivation and similar approaches to black hole thermodynamics depend heavily on the standard quantum field theoretical definitions of vacuum states, Fock space, and particle number. But since one of the main conclusions is that states in curved spacetime are rather different from those we are used to in Minkowski space — for instance, the vacuum is no longer unique — we might worry that we are overinterpreting the formalism. True physical observables are notoriously difficult to find in quantum gravity, so this is not a trivial concern.

To investigate this question, Unruh [34] and DeWitt [41] developed simple models of particle detectors in black hole backgrounds. They showed that such detectors do, in fact, see thermal radiation at the Hawking temperature. Similarly, Yu and Zhou [42] have shown that a two-level atom outside a black hole will spontaneously excite as if it were in a thermal bath at the Hawking temperature.

5.4 Tunneling

Hawking radiation is a quantum process, and we may try to apply our intuition about quantum mechanics to seek a deeper understanding. As noted in section 3, an obvious guess is that while escape from a black hole is classically forbidden, quantum particles might tunnel out. For instance, instead of visualizing virtual pairs forming outside the horizon as in section 2, we might reverse the picture and imagine pairs forming inside the horizon, with one member then tunneling out.

While the idea of a tunneling description can be traced to Damour and Ruffini [43] and Srinivasan and Padmanabhan [44], its modern incarnation owes itself to a key insight of Parikh and Wilczek [17], who realized that rather than thinking of a particle as tunneling through the horizon, one could think of the horizon as tunneling past the particle. Consider an initial Schwarzschild black hole of mass M that emits a particle in an s-wave state, represented as a spherical shell of mass $\omega \ll M$. In the WKB approximation, the tunneling rate is

$$\Gamma = e^{-2\,Im\,I/\hbar}, \tag{5.4}$$

where I is the action for the outgoing shell, moving in a background metric of a black hole as it crosses the horizon from r_{in} to r_{out}. The exponent, in turn, can be written as

$$Im\, I = Im \int_{r_{in}}^{r_{out}} p_r dr = Im \int_{r_{in}}^{r_{out}} \int_0^{p_r} dp'_r\, dr = Im \int_M^{M-\omega} \int_{r_{in}}^{r_{out}} \frac{dr}{\dot{r}} dH, \tag{5.5}$$

where I have used Hamilton's equations of motion to write $dp_r = dH/\dot{r}$ and noted that the horizon moves inward from GM to $G(M-\omega)$ as the particle is emitted.

To evaluate this integral, Parikh and Wilczek use the Painlevé-Gullstrand form of the black hole metric,

$$ds^2 = \left(1 - \frac{2G(M-\omega)}{r}\right) dt^2 - 2\sqrt{\frac{2G(M-\omega)}{r}} dt\, dr - dr^2 - r^2 d\Omega^2, \tag{5.6}$$

which avoids coordinate singularities at the horizon. Outgoing radial null geodesics then satisfy

$$\dot{r} = 1 - \sqrt{\frac{2G(M-\omega)}{r}}$$

and the integral (5.5) can be evaluated by a standard contour deformation, yielding a tunneling rate

$$\Gamma = e^{-8\pi\omega G\left(M - \frac{\omega}{2}\right)/\hbar} = e^{\Delta S_{\mathrm{BH}}} \tag{5.7}$$

where ΔS_{BH} is the change in the Bekenstein-Hawking entropy (1.1) due to the emission of the shell. By the first law of thermodynamics, $\Delta S_{\mathrm{BH}} = -\hbar\omega/T_{\mathrm{H}}$, and we recover the expected emission rate for thermal Hawking radiation.

The tunneling approach was initially developed for the simplest black holes, but has by now been vastly extended, to apply to virtually every known black hole solution, including even nonstationary configurations [45]. The method has also been generalized; in particular, one can replace the assumption that outgoing particles follow null geodesics by the more general approximation that the action I satisfies a Hamilton-Jacobi equation [46].

5.5 Hawking radiation from anomalies

From early on, it was clear that black hole thermodynamics should be visible in the behavior of the stress-energy tensor near a horizon [41,47]. The computation of the expectation value $\langle T_{ab}\rangle$ is a complicated and technical subject, beyond the scope of this review; see, for instance, the book by Birrell and Davies [18] for a pedagogical introduction. The results, however, are consistent with Hawking's discovery. In particular, the heuristic approach of section 2 is supported: an ingoing flux of negative energy near the horizon balances the outgoing flux of positive energy at infinity. Energy is conserved, and as in the tunneling derivation of section 4, the back-reaction of Hawking radiation reduces the mass of the black hole.

In general, the computation of the expectation value $\langle T_{ab}\rangle$ requires a mixture of analytic approximations and numerical methods. In one special case, though, the situation becomes drastically simpler. Consider a conformally invariant field (for instance, a massless scalar) in two spacetime dimensions. Classically, conformal invariance forces the trace $T^a{}_a$ of the stress-energy tensor to vanish, and its quantum correction, the "trace anomaly," is completely determined as

$$\langle T^a{}_a\rangle = \frac{c}{24\pi} R,$$

where c, the central charge, is fully determined by the properties of the field in a flat background [48]. Christensen and Fulling [49] realized in 1977 that in such a case, the full stress-energy tensor is completely determined from $\langle T^a{}_a \rangle$ by conservation laws and boundary conditions. The result exactly matched expectations, with fluxes of positive energy at infinity, describing thermal radiation, balanced by fluxes of negative energy through the horizon.

Christensen and Fulling's original analysis relied crucially on the restriction to two dimensions. More recently, though Wilczek and collaborators [50–52] have shown how to generalize the argument to a much wider setting. The central idea is that one can impose an effective description of a vacuum state near the horizon, the equivalent of the freely falling observer's vacuum, by excluding matter fields that correspond to outgoing ("horizon-skimming") modes. This restriction makes the field theory chiral, though, and a chiral theory contains a new anomaly, a diffeomorphism anomaly. By analyzing the resulting theory one partial wave at a time, one can again use this anomaly to reconstruct the stress-energy tensor, recovering the characteristic thermal radiation. This method is actually closely related to that of Christensen and Fulling — in two dimensions, one can always trade a conformal anomaly for a diffeomorphism anomaly by adding appropriate counterterms — but the newer approach seems more suitable for treating separate partial waves, and thus for application to more than two dimensions.

In a remarkable piece of work, Iso, Morita, and Umetsu [53] and Bonora, Cvitan, Pallua, and Smolić [54, 55] have extended this approach to obtain a much more detailed picture of Hawking radiation. They show that if one considers not just the stress-energy tensor, but higher spin currents as well, one can recover the full thermal spectrum. Hawking radiation can thus be obtained, with only fairly minimal assumptions, from the symmetries of the near-horizon region of a black hole.

5.6 Periodic Greens functions

In ordinary quantum field theory, thermal Greens functions have a hidden periodicity. Consider, for instance, the Greens function of a scalar field φ in a thermal ensemble at temperature $T = 1/\beta$:

$$\begin{aligned} G_\beta(x,0;x',t) &= Tr\left(e^{-\beta H}\varphi(x,0)\varphi(x',t)\right) = Tr\left(e^{-\beta H}\varphi(x,0)e^{-\beta H}e^{\beta H}\varphi(x',t)\right) \\ &= Tr\left(\varphi(x,0)e^{-\beta H}e^{\beta H}\varphi(x',t)e^{-\beta H}\right) = Tr\left(\varphi(x,0)e^{-\beta H}\varphi(x',t+i\beta)\right) \\ &= G_\beta(x',t+i\beta;x,0) \end{aligned} \tag{5.8}$$

where I have used cyclicity of the trace and the fact that the Hamiltonian generates time translations, so $e^{\beta H}\varphi(x',t)e^{-\beta H} = \varphi(x',t+i\beta)$. In particular, a thermal Greens function that is symmetric in its arguments must be periodic in imaginary time, with period $i\beta$. Conversely, in axiomatic quantum field theory such periodicity, formalized as the KMS condition [56–58], serves to define a thermal state.

For a uniformly accelerating observer in Minkowski space, it was shown in 1976 that the Greens function has just such a periodicity [40], with a temperature equal to the Unruh temperature (5.3). By the arguments of section 2, one should expect an analogous result for a stationary observer near a black hole horizon. This is indeed the case [59, 60], and β is just the inverse of the Hawking temperature (1.1). Moreover, the response of a particle detector of the sort discussed in section 3 is determined by a closely related Greens function, and the periodicity ensures quite generally that such a detector will see a thermal bath at temperature T_{H}.

5.7 Gravitational partition function

One may ask whether the periodicity properties of the Greens functions may be extended to the gravitational field itself. For ordinary quantum mechanical systems, it is well known that a thermal partition function may be obtained from the usual path integral by analytically continuing to periodic imaginary time with period β [61]. The meaning of such a continuation is not so clear in general relativity, though: there is usually no preferred time coordinate to make imaginary, and there is no simple relationship between solutions of the field equations with "real time" (Lorentzian signature) and "imaginary time" (Riemannian signature).

In one setting, though, there is a natural choice. If a spacetime is stationary — that is, if it admits a timelike Killing vector — the Killing vector defines a preferred time coordinate. In particular, for a nonextremal stationary black hole, the generic metric near enough to the horizon takes the form

$$ds^2 = 2\kappa(r-r_+)dt^2 - \frac{1}{2\kappa(r-r_+)}dr^2 - r_+^2 d\Omega^2,$$

where r_+ is the location of the horizon and κ is again the surface gravity. Setting $t = i\tau$ and replacing r by the proper distance to the horizon,

$$\rho = \frac{1}{\kappa}\sqrt{2\kappa(r-r_+)},$$

we obtain the "Euclidean black hole" metric

$$ds^2 = d\rho^2 + \kappa^2\rho^2 d\tau^2 + r_+^2 d\Omega^2. \tag{5.9}$$

We can immediately recognize the first two terms of (5.9) as the metric of a flat plane in polar coordinates. The horizon $r = r_+$ has collapsed to a point $\rho = 0$: the "Euclidean section" includes only the black hole exterior. But the ρ–τ plane is flat only if $\kappa\tau$ has a period 2π; otherwise, the origin is a conical singularity, and the metric will not extremize the action at that point. This periodicity, in turn, requires that τ have a period $2\pi/\kappa = 1/T_{\text{H}}$, exactly as in the preceding section.

We now see that the periodicity of Greens functions did not depend on details of quantum fields, but arose directly from the geometry. But we can do more: as Gibbons and Hawking argued [63], we can use this result to obtain a saddle point approximation to the gravitational path integral.

By analogy with ordinary quantum field theory, the Euclidean path integral for the gravitational partition function can be formally written as

$$Z[\beta] = \int [dg] e^{I_{\text{Euc}}} \tag{5.10}$$

where I_{Euc} is the "Euclidean" Einstein-Hilbert action — that is, the Einstein-Hilbert action for metrics of Riemannian signature — and the integral is over all metrics that are, in some sense, periodic in "Euclidean" time with period β. This is, of course, an ill-defined expression: general relativity is nonrenormalizable, so even if we knew exactly what "periodic" meant here, we would not know how to make sense of the functional integral. Still, though, there ought to be some sense in which (5.10) has a meaning in effective field theory [62], and even if the full expression is ill-defined, a saddle point approximation could give a physically reasonable result.

Naively, the saddle point contributions (5.9) gives a vanishing contribution to I_{Euc}: the "bulk" Einstein-Hilbert action

$$\frac{1}{16\pi G}\int d^4x\,\sqrt{|g|}R$$

is zero for any classical solution of the vacuum field equations. The key observation of Gibbons and Hawking [63] was that on a manifold with boundary, the "bulk" action must be supplemented by a boundary term, without which the action will typically have no true extrema [64]. The boundary was originally taken to be at infinity, but subsequent work has shown that it may also be placed at the horizon [65–67]. The resulting boundary term may be evaluated in a number of ways; for instance, if one dimensionally reduces the action to the ρ–τ plane, the problem becomes purely topological [65]. The final result is that

$$I_{\text{Euc}} = \frac{A_{\text{hor}}}{4\hbar G} - \beta(M + \Omega J + \Phi Q). \tag{5.11}$$

The saddle point contribution $e^{I_{\text{Euc}}}$ to the partition function (5.10) may then be recognized as the grand canonical partition function for a system with inverse temperature $\beta = 2\pi/\kappa$ and entropy $S_{\text{BH}} = A_{\text{hor}}/4\hbar G$, just as expected.

The same analysis applies to much more general stationary geometries [68]. Just as above, Killing horizons in the Lorentzian configurations translate into zeros of the Killing vector in the Riemannian continuation, and boundary terms resembling (5.11) again appear. Recently Neiman has argued that even in Lorentzian signature, the usual Gibbons-Hawking boundary term acquires an imaginary piece whenever the boundary involves "signature flips," points at which the signature of the *boundary* changes from spacelike to timelike [69]. For black hole spacetimes, the resulting boundary term looks very much like the one in the Euclidean action.

Alternatively, one may try to evade the ambiguities in "imaginary time" by starting with the Hamiltonian formulation of general relativity [66]. Perhaps unsurprisingly, the same boundary term (5.11) appears. In canonical quantization, this boundary term gives rise to a new term in the Wheeler-DeWitt equation, from which one can again recover the Bekenstein-Hawking entropy [70].

5.8 Pair production of black holes

As Heisenberg and Euler noticed in 1936 [71], the quantum vacuum is unstable in the presence of a strong electric field. Virtual pairs of charged particles can be "pulled out of the vacuum" by the field, becoming physical particle-antiparticle pairs. The pair production rate per unit volume for a fermion of mass m and charge e in a constant field $\mathcal{E}$ was worked out in detail by Schwinger [72], and takes the form

$$W \sim \frac{\alpha^2\mathcal{E}^2}{\pi^2}e^{-\pi m^2/|e\mathcal{E}|}.$$

For $\mathcal{N}$ identical species of fermions, this expression would be multiplied by $\mathcal{N}$.

Now consider pair production of charged black holes in an electric field. Without a full quantum theory of gravity we cannot compute an exact production rate, but as in the preceding section, we can use the Euclidean path integral to obtain a semiclassical approximation. If the Bekenstein entropy is a statistical mechanical entropy, we would expect a black hole to have $\exp\{S_{\text{BH}}\}$ microscopic states, with a corresponding enhancement of the pair production rate by $\mathcal{N} = \exp\{S_{\text{BH}}\}$.

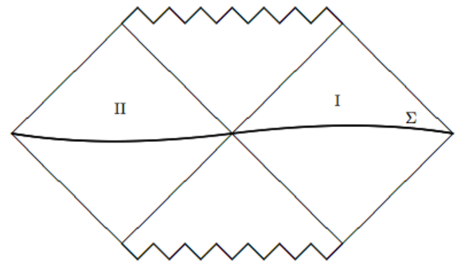


Figure 4: A Carter-Penrose diagram for an eternal black hole

This is exactly what is found. The first computation, by Garfinkle, Giddings, and Strominger [73], considered magnetically charged black holes, and compared the pair production rate to the rate for magnetic monopoles of the same mass. Subsequent work extended these results to a much wider variety of black holes [74–76]. In every known case, pair production is enhanced just as one would expect if the Bekenstein-Hawking entropy represented an actual counting of states.

5.9 Quantum field theory and the eternal black hole

Hawking's derivation of black hole thermodynamics was based on a collapsing star, with a Minkowski-like vacuum in the distant past. But we can also write down solutions of the Einstein field equations that represent eternal black holes.

Recall that the maximally extended eternal black hole spacetime has a Carter-Penrose diagram with four regions, as shown in figure 4. Now consider a quantum state Ψ defined on a Cauchy surface Σ passing through the black hole bifurcation sphere, as shown in the figure. No information can pass from region II to region I, so even if Ψ is a pure state, an observer in region I will see only a density matrix, obtained by tracing over region II. This at least opens up the possibility that physics for an observer in region I will be thermal.

Indeed, for a free quantum field there is at most one quantum state, the Hartle-Hawking vacuum state, that is regular everywhere on the horizon [77, 78]. For a scalar field, a direct computation shows that the density matrix obtained by tracing over region II is thermal, with a temperature $T_{\rm H}$ [79]. For more general fields, the same can be shown by means of fairly sophisticated quantum field theoretical arguments [77, 78] or by an analysis of the path integral [80]. Thus even if matter is secretly in a pure state, the eternal black hole is effectively also a thermal system.

5.10 Quantized gravity and classical matter

Although they differ in detail, the derivations of Hawking radiation so far have all analyzed quantum matter in a classical black hole background. This is a reasonable approximation, except perhaps for the "trans-Planckian problem" discussed above. In one special case, though, the direction can be reversed: Hawking radiation can be derived from classical matter in the presence of a quantum black hole.

This special case is that of the BTZ black hole [81], a black hole in (2+1)-dimensional asymptotically anti-de Sitter space. This is a rather peculiar solution: the vacuum Einstein field equations in 2+1 dimensions imply that spacetime has constant negative curvature, and, indeed, the BTZ black hole can be described as a region of anti-de Sitter space with certain identifications [82]. Nevertheless, it is a genuine black hole, the final state of collapsing matter, with a normal event horizon and (in the rotating case) a normal inner Cauchy horizon [83].

The BTZ black hole presents a puzzle for any statistical mechanical explanation of black hole entropy. As a corollary to the constant curvature required by the field equations, (2+1)-dimensional gravity contains no propagating degrees of freedom that could serve as candidates for microscopic states. As first suggested by Carlip [84] and confirmed in detail by Strominger [86] and, independently, Birmingham, Sachs, and Sen [87], the natural candidate for the microscopic degrees of freedom are "edge states," additional degrees of freedom at infinity that appear because the boundary conditions break diffeomorphism invariance.

I will discuss these states further in section 9. For now, the key feature is that they allow an "effective" description of the quantum BTZ black hole in terms of a particular two-dimensional conformal field theory at infinity [88]. Emparan and Sachs have shown how to couple this quantum theory to classical matter [89], and to use the coupling to calculate transition amplitudes between quantum black hole states induced by interactions with this "background" matter. Detailed balance arguments then allow them to determine the emission rate of the matter fields, obtaining the correct Hawking radiation spectrum, even including the correct greybody factors.

5.11 Other approaches

I have tried to give an overview of the most common approaches to black hole thermodynamics, but I have necessarily omitted a few. For example:

- Instead of considering only the region outside the horizon, one may choose a time-slicing that crosses the horizon and continues to the singularity. A black hole interior is not static — the Killing vector that gives time translation invariance in the exterior becomes spacelike in the interior — and the Hamiltonian in horizon-crossing coordinates is therefore time-dependent. Nevertheless, one can perform a Heisenberg picture quantization to obtain time-dependent states which, although pure, behave "thermally" in the sense that they excite radiation detectors and have a net flux of radiation at infinity that matches Hawking's results [90].

- One may repeat the computation of Unruh radiation in the functional Schrödinger picture, in which the quantum state is a wave functional of the field configuration. The results are again what would be expected [91]: the ground states for the stationary and accelerated observers are different, and the difference appears as a thermal distribution of "Minkowski" particles in the accelerated observer's state.

- York has proposed a model in which fluctuations of the event horizon lead to a "quantum ergosphere" through which particles can tunnel [92]. The picture is too complicated to compute an exact Hawking temperature, but by restricting the horizon fluctuations to those corresponding to the lowest quasinormal mode, he obtains a value for the temperature that is accurate to to within about 2%.

The derivations in this section are not entirely independent. For instance, the response function of a particle detector (section 3) depends on a Greens function, and it is the periodicity of this function (section 6) that determines the thermal response. This periodicity, in turn, may be traced the periodicity of the black hole metric in imaginary time (section 7). The tunneling approach (section 4) and the original analysis by Hawking both lead to relations of the form

$$P(\text{emis}) = e^{-\beta E} P(\text{absor})$$

between emission and absorption probabilities, and both results come from the behavior of complex functions near the horizon. Still, the derivations are sufficiently independent, with different enough assumptions and approximations, that together they provide very powerful evidence for the reality of Hawking radiation.

6 Thermodynamic properties of black holes

There is more to thermodynamics than temperature and entropy. If black holes are thermal objects, we should be able to apply the rest of thermodynamic theory, from Carnot cycles to phase changes, to systems containing black holes. In this section, after a short discussion of black hole evaporation, I will review four selected aspects: heat capacity, phase transitions, thermodynamic volume, and an argument that black hole thermodynamics prohibits certain violations of Lorentz invariance.

6.1 Black hole evaporation

We have seen that a black hole radiates as a grey body, at a temperature T_{H}. By the Stefan-Boltzmann law, it will therefore lose mass at a rate

$$\frac{dM}{dt} = -\varepsilon\sigma T_{\mathrm{H}}^4 A_{\mathrm{hor}} \tag{6.1}$$

where ε is some averaged emissivity, summed over the species of particles that can be radiated. For the Schwarzschild black hole, $T_{\mathrm{H}} \sim 1/M$ and $A_{\mathrm{hor}} \sim M^2$, so the lifetime goes as M^3. To determine the exact coefficients one must treat the greybody factors carefully [22]; the result is that

$$\tau \sim 10^{71}(M/M_{\odot})^3$$

where $M_{\odot}$ is the mass of the Sun. For charged or rotating black holes, evaporation is more complicated [93]: black holes can "discharge" via Schwinger pair production, but the Hawking temperature decreases with increasing charge and angular momentum.

6.2 Heat capacity

A Schwarzschild black hole has a temperature $T = \hbar/8\pi M$. As an isolated black hole evaporates, it radiates energy, its mass decreases, and its temperature consequently *increases*. That is, a Schwarzschild black hole has a negative heat capacity,

$$C = T\frac{\partial S}{\partial T} = -\frac{1}{8\pi T^2}. \tag{6.2}$$

For a rotating black hole, the situation is more complicated: as Davies has shown [94], the heat capacity at fixed angular velocity is always negative, but the heat capacity at fixed angular momentum is negative for low J, diverges as a critical value, and then becomes positive, a behavior indicative of a phase transition.

While negative heat capacity is unusual, it is not a peculiar feature of black holes, but holds even for Newtonian self-gravitating systems. As a satellite in low Earth orbit loses energy to atmospheric friction, its orbit drops to a lower altitude, *increasing* its kinetic energy. For gravitationally bound stellar systems, Lynden-Bell has analyzed this behavior, which he calls it the "gravothermal catastrophe," in detail [95]. In Newtonian gravity, the runaway behavior is a consequence of the purely attractive, long range nature of the gravitational interaction, and it stops when gravity is no longer the principle interaction governing the system. For a relativistic black hole, on the other hand, there seems to be no competing interaction, at least until the evaporating black hole has shrunk to the Planck scale.

There are two standard (theoretical) ways to stabilize a black hole. The first is to place it in a reflecting cavity, allowing it to come into equilibrium with its Hawking radiation. For a spherical cavity of area $A = 4\pi r_B^2$, the heat capacity for a Schwarzschild black hole becomes [96]

$$C_A = 8\pi M^2 \left(1 - \frac{2M}{r_B}\right)\left(\frac{3M}{r_B} - 1\right)^{-1}, \tag{6.3}$$

which is positive for a small enough cavity, $r_B < 3M$. The second is to consider a black hole in asymptotically anti-de Sitter space [97], in which the geometry serves as a sort of "cavity." For a given temperature, one finds two possible anti-de Sitter black hole configurations, a "small" black hole with negative heat capacity and a "large" black hole with positive heat capacity.

6.3 Phase transitions

As already indicated, black holes can have a complicated phase structure, including phase transitions of various types. The most famous of these is the Hawking-Page transition for a black hole in asymptotically anti-de Sitter space [97], a first order phase transition between thermal radiation at low temperatures and a black hole at higher temperatures. In the AdS/CFT correspondence of string theory, this transition has an interesting "dual" description as a confinement-deconfinement transition [98].

We have come to understand that this kind of behavior is not at all exceptional. As one example among many [99], consider a black hole with charge Q in asymptotically flat space, enclosed in a cavity whose walls are maintained at a fixed temperature $1/\beta$. One finds a phase diagram in the Q–β plane containing a line of first-order phase transitions, terminating at a critical point which is the location of a second-order phase transition with calculable critical exponents.

6.4 Thermodynamic volume

The first law of black hole mechanics (2.1) closely resembles the standard form of the first law of thermodynamics, but it is missing a term, the pressure term $-pdV$. There is, in fact, a candidate for "pressure": the cosmological constant Λ has the correct dimension, and acts as a pressure in cosmological settings. But Λ is not normally considered a state function; it is a coupling constant that is fixed by the theory, not by the state of the thermodynamic system.

There are, however, situations in which Λ is a dynamical field, forced to be constant only by virtue of its field equations [100]. In that case, it may make sense to treat it as a state function [101], and to generalize the first law to the form [102, 103]

$$\delta M = T\delta S + \Omega\delta J + \Phi\delta Q + V_{therm}\delta p, \tag{6.4}$$

where $p = -\Lambda/8\pi G$. Note that in this formulation, the mass M should be interpreted as enthalpy (that is, $U + pV$, where U is the internal energy) [103].

The "thermodynamic volume" V_{therm} is defined by (6.4), and exists even in the limit $\Lambda \to 0$. Its physical meaning is not very well understood, though. For a Schwarzschild black hole, it is easy to check that

$$V_{therm} = \frac{4}{3}\pi {r_+}^3, \tag{6.5}$$

where r_+ is the location of the horizon in Schwarzschild coordinates. This relation generalizes to higher dimensional static black holes, including those with charge: the thermodynamic volume is the volume of a *flat* round ball whose surface area is equal to the area of the event horizon. But the generalization to rotating black holes is complicated, and the corresponding thermodynamic volume does not yet have a simple physical interpretation [104].

The inclusion of a volume term in the first law (6.4) enriches the phase structure of the theory, with a number of interesting consequences [105, 106]. For example, the equation of state for a charged, rotating anti-de Sitter black hole becomes very close to that of a van der Waals gas; rotating anti-de Sitter black holes in more than six dimensions exhibit reentrant phase transitions; and anti-de Sitter black holes in six dimensions with multiple spins have a triple point.

6.5 Lorentz violation and perpetual motion machines

Lorentz invariance guarantees that there is a single "maximum speed" c, the same for all particles. I have implicitly used this assumption throughout this review, choosing units $c = 1$. Suppose, though, that Lorentz invariance is broken, in such a way that two species of particles — say A and B — have different maximum speeds $c_B > c_A$. Then Dubovsky and Sibiryakov have shown [107] that a black hole can be used to build a "perpetual motion machine of the second kind," a device that transfers heat from a cold reservoir to a hot reservoir without any net use of energy. In some Lorentz-violating theories, there is even a classical mechanism, analogous to the Penrose process, for "mining" a black hole [108].

To understand the argument, first note that the event horizon for the more slowly moving species A will lie outside the event horizon for the faster species B. This is physically intuitive, but also follows directly if we restore the factors of c in the definition of the Schwarzschild event horizon, $r_+ = 2GM/c^2$. Correspondingly, the Hawking temperature $T_{A,H}$ of species A should be lower than the Hawking temperature $T_{B,H}$ of species B.

Dubovsky and Sibiryakov's "construction manual" is then the following. As heat reservoirs, construct a shell A at temperature $T_{A,shell}$ that interacts only with species A, and a second shell B at temperature $T_{B,shell}$ that interacts only with species B. Arrange the temperatures so that

$$T_{B,H} > T_{B,shell} > T_{A,shell} > T_{A,H}. \tag{6.6}$$

This will result in a net energy flux from shell A into the black hole, and from the black hole out to shell B. It is not hard to check that one can adjust the shell temperatures so that the flux into the black hole balances the flux out. The net result, then, is a flux of energy from the "cold" shell A to the "hot" shell B, violating the second law of thermodynamics.

Arguments like this should be treated cautiously. Historically, the generalized second law of thermodynamics has shown remarkably resilience, often involving subtle and unexpected effects. There may be natural Lorentz-violating theories in which all matter shares a single universal horizon, for instance, or theories in which the black hole interior carries hidden entropy. But the apparent paradox shows, at least, that simple thermodynamic arguments about black holes may place surprising restrictions on nongravitational physics.

7 Approaches to black hole statistical mechanics

In ordinary thermodynamic systems, thermodynamic properties reflect the statistical mechanics of underlying microscopic states. Entropy, in particular, is a measure of the number of accessible states. One might hope for the same to be true for black holes. Then, since the Bekenstein-Hawking entropy depends on both Newton's constant G and Planck's constant $\hbar$, it is plausible that black hole thermodynamics is telling us something about quantum gravitational states.

Black hole thermodynamics may therefore present us with a rare opportunity. There is very little that we actually know about quantum gravity, so any insight is likely to be valuable. Like many opportunities, though, this one is accompanied by a serious difficulty: in the absence of a quantum theory of gravity, how do we start to understand the quantum states of a black hole?

Fortunately, while we do not yet have a complete quantum theory of gravity, we have a number of research programs, at various levels of maturity, attempting to develop such a theory. Some of these are advanced enough to be able to make predictions about at least certain classes of black holes. Indeed, as I shall discuss in section 9, the real problem may be not that we have no microscopic explanation of black hole entropy, but that we have too many.

7.1 "Phenomenology"

From the earliest days of black hole thermodynamics, there was an obvious simple "phenomenological" model for Bekenstein's area law [2]: one had to merely assume that black hole horizon area was quantized, with discrete "plaquettes" of area on the order of the Planck area. The simplest choice — partially justified by the observation that the horizon area is an adiabatic invariant [109] — would a discrete, evenly spaced area spectrum.

Suppose, for instance, that the Bekenstein-Hawking entropy were *exactly* the logarithm of the number of states. This number is, of course, an integer, so successive values of the area would have to differ by an amount

$$\Delta A = 4\hbar G \ln k \tag{7.1}$$

for some integer k. As Hod first observed [110], one could then appeal to the Bohr correspondence principle to determine the area spacing. While black holes have no stable excitations, they do have damped "ringing modes," called quasinormal modes, with complex frequencies [111]. The most strongly damped quasinormal modes of the Schwarzschild black hole have frequencies

$$Re\,\omega \sim \ln 3/8\pi GM. \tag{7.2}$$

The Bohr correspondence principle would then suggest that these frequencies should correspond to transitions between adjacent area eigenstates, with energies

$$\Delta M = \hbar\omega \ \Rightarrow\ \Delta A/32\pi G^2 M = \hbar \ln 3/8\pi GM, \tag{7.3}$$

which exactly matches (7.1) with $k = 3$.

Arguments of this sort are suggestive, but must also be treated with care. For instance, within the same framework, Maggiore [112] has argued — based in part on an analogy with a damped harmonic oscillator — that relevant frequency one should use for the Bohr correspondence principle is not the real part $Re\,\omega$ of the quasinormal frequency, but the modulus $|\omega|$. For highly damped modes,

$$|\omega| \sim \frac{1}{4GM}\left(n + \frac{1}{2}\right), \tag{7.4}$$

and the Bohr correspondence principle then gives

$$\Delta A = 8\pi\hbar G. \tag{7.5}$$

A wide range of simple models have been explored — Medved's review [113] gives a partial list — and they do not agree about the area spectrum. The lesson seems to be, not surprisingly, that thought experiments and analogies can only go so far; one needs a genuine quantum theory of gravity to obtain a real answer.

In the remainder of this section, I will briefly summarize some of the more specific approaches to black hole statistical mechanics from the point of view of particular models of quantum gravity.

7.2 Entanglement entropy

Consider a space that is divided into two disjoint pieces A and B. Suppose a quantum system on that space is characterized by a pure state $|\Psi\rangle$, with a corresponding density matrix $\rho_{AB} = |\Psi\rangle\langle\Psi|$. The von Neumann entropy of such a pure state,

$$S_{vN} = Tr\,\rho\ln\rho \tag{7.6}$$

is, of course, zero.

Now, however, imagine an observer who is restricted to the region A, and cannot observe anything in B. All observations of such an observer can be described by a reduced density matrix, $\rho_A = Tr_B\,\rho_{AB}$, whose entropy will in general be nonzero. For a local theory with a finite-dimensional Hilbert space, for instance, we can write the state $|\Psi\rangle$ in a Schmidt decomposition,

$$|\Psi\rangle = \sum c_i|\phi_i\rangle|\psi_i\rangle, \tag{7.7}$$

where $\{|\phi_i\rangle\}$ and $\{|\psi_i\rangle\}$ are orthonormal bases of states in regions A and B. Then

$$\rho_A = \sum |c_i|^2|\phi_i\rangle\langle\phi_i| \tag{7.8}$$

and the von Neumann entropy of region A is simply

$$S_A = Tr\,\rho_A\ln\rho_A = \sum |c_i|^2\ln|c_i|^2, \tag{7.9}$$

the Shannon entropy for the probabilities $p_i = |c_i|^2$.

This "entanglement entropy" is now commonplace in quantum theory, but it was first introduced by Sorkin in the context of black hole thermodynamics [114]. The idea is that an observer outside the horizon ("region A") knows nothing about the state inside the horizon ("region B"), and therefore sees a mixed state with nonvanishing entropy. This picture arises naturally from the eternal black hole of section 5.9; the density matrix that gives the entropy (7.9) is the same as the thermal density matrix that gives the Hawking spectrum. For many states. including the vacuum state, the entanglement entropy for a black hole is proportional to the horizon area [114–117], since the main contribution comes from correlations among degrees of freedom very close to the horizon, for which the "bulk" state is irrelevant.

The *coefficient* of this area term, on the other hand, diverges, for essentially the same reason: the more closely degrees of freedom are localized to the horizon, the higher their energies. The simplest cut-off, a "brick wall" approximately one Planck length in proper distance from the horizon [118], gives a coefficient for the Bekenstein-Hawking entropy of the right order of magnitude. In general, though, the exact value depends sensitively on the number and type of entangled fields — that is, on the particle content of the Universe.

A possible solution to this "species problem" comes from the observation by Susskind and Uglum that the same modes responsible for the entanglement entropy also renormalize Newton's constant, which also appears in the Bekenstein-Hawking entropy formula [119]. It is plausible that these contributions cancel. In particular, Cooperman and Luty [120] have recently argued that for a certain class of states defined by a path integral, the leading renormalized term in the entanglement entropy is given by the corresponding renormalized Bekenstein-Hawking formula, independent of the particle content of the theory.

Inspired by the AdS/CFT correspondence of string theory, Ryu and Takayanagi have proposed an alternative "holographic" approach to defining and regulating entanglement entropy [121, 122]. In the description I have given above, the regions A and B lie on a $(d-1)$-dimensional spacelike slice of a d-dimensional spacetime. Ryu and Takayanagi propose embedding this spacetime as the asymptotic boundary of $(d+1)$-dimensional anti-de Sitter space. The boundary between regions A and B can then be extended into the "bulk" anti-de Sitter space as a d-dimensional minimal surface; Ryu and Takayanagi propose that the entanglement entropy should be proportional to the area of this surface. While this approach is obviously inspired by black hole thermodynamics, can be proven without any assumptions about black holes, at least for the static case [123]. The holographic entanglement entropy can then be used to determine the entropy of a black hole, and one again obtains the Bekenstein-Hawking expression, with no divergences or species problem [124].

7.3 String theory

The leading research program in quantum gravity today is string theory. I will not attempt to summarize the theory here, but will instead focus on three rather different approaches to black hole statistical mechanics within string theory.

7.3.1 Weakly coupled strings and branes

The first successful calculation of black hole entropy in string theory, by Strominger and Vafa [125], looked at a class of multiply charged, extremal (that is, maximally charged), supersymmetric (BPS) black holes in five dimensions. As classical objects, such black holes are completely determined by their charges: the mass, horizon area, and all other macroscopic quantities can be expressed as functions of the charges. As string theoretical objects, on the other hand, they are strongly bound states of the fundamental constituents of string theory, strings and D-branes. Strominger and Vafa suggested "turning down" the strength of the gravitational interaction, until a black hole became a weakly coupled system of strings and D-branes. These constituents would still have the prescribed charges, but at weak coupling one could also count the number of states with such charges. The result, translated back into the black hole parameters, exactly matched the Bekenstein-Hawking entropy.

These results were quickly extended to a large number of extremal and near-extremal black holes, and, through dualities, to some particular nonextremal black holes as well [126, 127]. A straightforward extension of the argument gives Hawking radiation, coming from decay of excited states of the constituent strings and D-branes, complete with the correct greybody factors [128]. One might worry about the consistency of the procedure: the process of turning down the gravitational coupling need not preserve the number of states. For supersymmetric black holes, though, the number of states is protected by nonrenormalization theorems. For nearly supersymmetric black holes, the procedure continues to work well, perhaps even better than one might expect; far from extremality, it becomes harder to obtain the right factor of 1/4 in the entropy.

There is one peculiar feature of these calculations, which I will return to in section 9. While the final result is an expression for entropy in terms of horizon area, the connection is indirect. On the strong coupling side, one may start with the horizon area, but one must reexpress it in terms of charges. On the weak coupling side, one may start with the number of states, but one must again reexpress it in terms of charges. It is only when the two processes are compared that one obtains the Bekenstein-Hawking entropy. This means that each new type of black hole requires a new calculation; some crucial aspect of universality is missing.

7.3.2 Fuzzballs

Mathur has proposed running the analysis of the preceding section backwards [129]. Suppose one starts on the weak coupling side, with a particular collection of strings and D-branes whose charges match those of a desired black hole. One may then turn the gravitational coupling up, and try to see what geometry appears at strong coupling. The result is typically *not* a black hole, but rather a "fuzzball," a configuration with no horizon and no singularity, but with a geometry that looks very much like that of a black hole outside a would-be horizon [130, 131]. This phenomenon seems to require the extra dimensions available in string theory, but it provides an intriguing new picture of black holes. For example, Hawking radiation can appear as a result of a classical instability in a fuzzball geometry [132].

In a few special cases, one can count the number of such classical "fuzzball" geometries and reproduce the Bekenstein-Hawking entropy. In general, though, it is likely that many of the states relevant for determining the Bekenstein-Hawking entropy will not have classical geometric descriptions.

7.3.3 The AdS/CFT correspondence

A third approach approach to black holes in string theory exploits Maldacena's celebrated AdS/CFT correspondence [133,134]. This extremely well-supported conjecture states that string theory in a d-dimensional asymptotically anti-de Sitter spacetime is dual to a conformal field theory in a flat $(d-1)$-dimensional space that can, in a sense, be viewed as the boundary of the AdS spacetime. For asymptotically anti-de Sitter black holes, this means that one can, in principle, compute the entropy by counting states in a (nongravitational) dual conformal field theory.

The most straightforward application of this duality occurs for the (2+1)-dimensional BTZ black hole discussed in section 5.10. Here, the dual description is given by a two-dimensional conformal field theory. As I shall describe in more detail in section 9, two-dimensional conformal field theories have an exceptionally large symmetry group that controls many of their properties. In particular, the density of states is determined by a single parameter c, the central charge, which characterizes the conformal anomaly. As we saw in section 5.5, anomalies can provide very useful tools; here, they are powerful enough to determine the density of states.

For asymptotically anti-de Sitter gravity in 2+1 dimensions, the central charge is dominated by a classical contribution, which was discovered some time ago by Brown and Henneaux [85]. Strominger [86] and Birmingham, Sachs, and Sen [87] independently realized that this result could be used to compute the BTZ entropy, precisely reproducing the Bekenstein-Hawking expression. As noted in section 5.10, the same methods can be used to compute Hawking radiation for the BTZ black hole.

While this result appears to be rather specialized, it has an important extension. Many higher dimensional near-extremal black holes, including black holes that are not themselves asymptotically anti-de

Sitter, have a near-horizon geometry of the form $BTZ \times \textit{trivial}$, where the "trivial" part merely renormalizes constants in the calculation of entropy. As a result, the BTZ results can be used to obtain the entropy of a large class of string theoretical black holes, including most of the black holes whose states could be counted in the weak coupling approach of section 7.3.1 [135].

7.4 Loop quantum gravity

The second major research program in quantum gravity is loop quantum gravity, or "quantum geometry." I will again focus only on those elements relevant to black hole thermodynamics.

The key features relevant to this question are these:

1. A basis for the kinematical states of the theory is given by spin networks, graphs with edges labeled by spins and vertices labeled by SU(2) intertwiners.
2. Given any surface Σ, each spin network is an eigenstate of the area operator $\hat{A}_\Sigma$, with eigenvalue

$$A_\Sigma = 8\pi\gamma G \sum_j \sqrt{j(j+1)} \tag{7.10}$$

   where the sum is over the spins j of edges of the spin network that cross Σ. Note that this area is quantized, but in a rather complicated way, with a spectrum that can be shown to have an elaborate substructure [137, 138].
3. Operators such as the area and volume depend on a parameter γ, the Barbero-Immirzi parameter. The significance of this parameter is quite poorly understood; theories with different values of γ are probably inequivalent, but it has been suggested that γ may not appear in properly renormalized observables [139] or in a somewhat different approach to quantization [140].
4. The physical states are obtained from the spin networks by imposing the Hamiltonian constraint, a procedure that is not yet completely under control.

7.4.1 Microcanonical approach

Given this structure, a natural first step is to choose the surface Σ to be a black hole horizon and count the number of spin network states that give a prescribed area [141, 142]. This is a bit tricky, since the true event horizon is defined by the global properties of the spacetime. On the other hand, we know that Hawking radiation depends only on local properties, so a local characterization should be adequate. The relevant definition is probably that of an "isolated horizon," a null surface with vanishing expansion that obeys the laws of black hole mechanics [9, 136].

As we shall see in section 9, the specification of such a horizon is a sort of boundary condition, which requires the introduction of boundary terms in the Einstein-Hilbert action. For loop quantum gravity, these boundary terms induce a three-dimensional Chern-Simons action on the horizon. The careful version of the state-counting [143, 144] can then be translated into a counting of Chern-Simons states, a well-understood procedure [145], although with slightly subtle combinatorics. The ultimate result is a microcanonical black hole entropy [146, 147]

$$S = \frac{\gamma_M}{\gamma}\frac{A_{\text{hor}}}{4\hbar G}, \tag{7.11}$$

where γ is the Barbero-Immirzi parameter and

$$\gamma_M \approx .23753 \tag{7.12}$$

is a constant determined as the solution of a particular combinatoric problem. If one chooses $\gamma = \gamma_M$, one recovers the standard Bekenstein-Hawking entropy.

The significance of this peculiar value γ_M is not understood, and it may reflect an inadequacy in the quantization procedure or the definition of the area operator [140]. Note, though, that the Barbero-Immirzi parameter appears only in the combination $G\gamma$, where G is the "bare" Newton's constant, *i.e.*, the parameter appearing in the action. The relevant constant in the Bekenstein-Hawking entropy, however, is the renormalized value [139], and since it is not known how Newton's constant is renormalized in loop quantum gravity, the "physical" value of γ remains uncertain. Ideally, one could address this question by computing the attraction between two test masses in the Newtonian limit, but this seemingly straightforward problem is not yet solved — loop quantum gravity is defined at the Planck scale, and the extrapolation to physically realistic distances remains extremely difficult.

In any case, though, once γ is fixed for one type of black hole — the Schwarzschild solution, for instance — the loop quantum gravity computations give the correct entropy for a wide variety of others, including charged black holes, rotating black holes, black holes with dilaton couplings, black holes with higher genus horizons, and black holes with arbitrarily distorted horizons [148, 149]. In particular, there is no need to restrict oneself to near-extremal black holes.

As an interesting variation within this framework [150], one may again look at the horizon area (7.10), but instead of counting states of the boundary Chern-Simons theory one may count the number of ways the boundary spins can be joined to a single interior vertex. This means, in essence, that one is completely course-graining the interior state of the black hole, much as one does in an entanglement entropy computation; there are, in fact, interesting relationships to entanglement entropy. The end result is again an entropy proportional to the horizon area, but now requiring a different value of the Barbero-Immirzi parameter to match the Bekenstein-Hawking result.

7.4.2 Other ensembles

The microcanonical approach described above involves a hidden assumption: that all spin networks that give the same horizon area occur with equal probability. This is a standard assumption in ordinary thermodynamics, but gravitating systems are not quite ordinary. Moreover, the microcanonical approach uses only properties of the "kinematical Hilbert space," that is, the space of states before the Hamiltonian constraint has been imposed. But we know in other contexts that although Hawking radiation is basically kinematical, the Bekenstein-Hawking entropy depends on the dynamics [36], including the Hamiltonian constraint.

In the past several years, interesting progress has been made in moving away from thesse assumptions. In particular,

– The quantum Hamiltonian that generates translations along the horizon has been identified [151], and yields a local temperature and energy, as measured by an idealized particle detector, that agree with semiclassical expectations [152]. This is not yet an enumeration of microscopic states, but it is a version of the thermodynamic computation of section 5.3 in a fully quantum gravitational setting.

– One can define a sort of grand canonical ensemble, in which the number of punctures — that is, of edges of the spin network that cross the horizon — appears with a corresponding chemical potential [153].

The Barbero-Immirzi parameter still appears, but in a different form: the entropy becomes

$$S = \frac{A_{\rm hor}}{4\hbar G} + N\sigma(\gamma) \tag{7.13}$$

where $\sigma(\gamma)$ is a Lagrange multiplier that vanishes when γ takes the value (7.12). It has been argued that if one considers additional couplings to matter, and makes the "holographic" assumption that the matter entropy near the horizon scales with the area, then the Lagrange multiplier term vanishes in the classical limit, giving back the standard Bekenstein-Hawking entropy [154].

– The Barbero-Immirzi parameter can in principle take any value, but there is one "natural" value, $\gamma = i$. At this value, the Ashtekar-Sen connection of loop quantum gravity is self-dual, and the Hamiltonian constraint becomes much simpler. Loop quantum gravity is unfortunately much harder to work with when $\gamma = i$, because of the need for added constraints to ensure that the metric is real, but several recent computations [155–157] indicate that this choice also gives the correct Bekenstein-Hawking entropy, perhaps even more simply.

7.5 Induced gravity

Induced gravity, as first proposed by Sakharov [158], is a model in which the Einstein-Hilbert action is not fundamental, but appears a consequence of the presence of other fields. If one starts with a theory of quantum fields propagating in an initially nondynamical curved spacetime, counterterms from renormalization will automatically induce a gravitational action, which almost always includes an Einstein-Hilbert term at lowest order [159]. In Sakharov's language, the resulting spacetime dynamics is a kind of "metric elasticity" induced by quantum fluctuations of matter.

One can write down an explicit set of "heavy" fields that can be integrated out in the path integral to induce the Einstein-Hilbert action. The gravitational counterterms normally have divergent coefficients that must be renormalized, but by including an appropriately chosen collection of nonminimally coupled scalar fields, one can obtain finite values for Newton's constant and the cosmological constant [160].

Given such a model, one can then count quantum states of the "heavy" fields in a black hole background [160,161]. Some subtleties occur in the definition of entropy in the presence of nonminimally coupled fields, but in the end the computation reproduces the standard Bekenstein-Hawking expression, with the correct coefficient. Furthermore, if one dimensionally reduces to a two-dimensional conformally invariant system near the horizon, one can count states by standard methods of conformal field theory [162], as described below in section 9. This offers a new, and apparently completely different, picture of black hole microstates as those of the ordinary quantum fields responsible for inducing the gravitational action.

7.6 Other approaches

Just as in black hole thermodynamics, there are a number of other approaches to black hole statistical mechanics that also show promise. For example:

- Classical black holes do not have excited states, but they have characteristic damped oscillations, called quasinormal modes [111]. York has argued that these modes should be quantized [92], and as noted in section 5.11, obtains a value close to the Bekenstein-Hawking entropy with an approximate quantization.

- The "causal set" program is an attempt to quantize gravity that starts by replacing a continuous spacetime by a discrete set of points with specified causal relations [163]. While it is not yet known how to perform an exact calculation of black hole entropy, there are indications that one can obtain a reasonable value by counting the number of points in the future domain of dependence of a cross-section of the horizon [164] or the number of causal links crossing the horizon [165].

- Zurek and Thorne have shown that there is sense in which the entropy of a black hole counts the number of distinct ways in which a black hole with prescribed macroscopic properties such as mass and angular momentum can be formed from the collapse of quantum matter [166].

7.7 Logarithmic corrections

While many approaches to quantum gravity give the correct Bekenstein-Hawking entropy, they may differ on the next order corrections. In general, one expects

$$S_{\rm BH} = \frac{A_{\rm hor}}{4\hbar G} + \alpha \ln\left(\frac{A_{\rm hor}}{4\hbar G}\right) + \dots \tag{7.14}$$

where the coefficient α may depend on the quantum theory. There are some arguments for a "universal" answer that, as in section 9, depends on conformal field theory [167], but these are not conclusive, and particular models seem to give varying results [168–171]. The problem is further complicated by the fact that the logarithmic terms in (7.14) may differ depending on one's choice of thermodynamic ensemble [172].

8 The holographic conjecture

In ordinary thermodynamic systems, entropy is an extensive quantity, scaling with volume. For black holes, in contrast, the entropy scales with area. It is not so surprising that entropy is not quite extensive for a gravitating system: gravity cannot be shielded, so when one increases the size of a system one is necessarily changing the internal interactions. Nor is it surprising that entropy cannot be simply defined in terms of a density that can be integrated over a volume: there is a general result that, as a consequence of diffeomorphism invariance, observables in general relativity cannot be defined in terms of local densities [173]. Still, an entropy proportional to area is a rather dramatic departure.

't Hooft [174] and Susskind [175] have proposed that this feature is not unique to black holes, but is a general property of any gravitating system. They suggest that the entropy inside *any* region, whether or not it is a black hole, should scale as the area rather than the volume. This conjectured "holographic principle" would imply that our usual view of local physics drastically overcounts degrees of freedom.

As a first argument for the plausibility of this holographic viewpoint [176], consider an approximately spherical surface $\mathcal{S}$ with a surface area $A_{\mathcal{S}} = 4\pi R^2$. Let us try to fill the interior of this surface with quantum excitations. To be contained in the region, each excitation should have a wavelength no larger than $2R$, and thus an energy $E \gtrsim \hbar/2R$. N such excitations will have an energy $E \gtrsim \hbar N/2R$, and to avoid forming a black hole, we must have $R \gtrsim 2GE$. Hence

$$N \lesssim R^2/\hbar G \tag{8.1}$$

If we interpret N as the number of degrees of freedom in the system, we see that (8.1) implies a holographic bound.

As a second argument, suppose the surface $\mathcal{S}$ now contains *almost* enough matter to form a black hole. Now surround the region by a collapsing shell of matter, with enough additional mass to form a black hole. The initial state has an entropy

$$S_{\rm init} = S_{\rm interior} + S_{\rm shell}$$

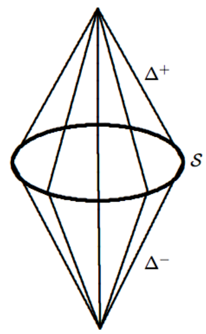


Figure 5: Future-directed null geodesics orthogonal to $\mathcal{S}$ form two light sheets, usually one expanding outward (not shown) and one collapsing inward (Δ^+). The same holds for past-directed null geodesics. In this figure, symmetry causes the light sheets to collapse to single points. In general, they will end at caustics where neighboring geodesics cross and the expansion becomes positive.

while the final state has an entropy

$$S_{\text{final}} = \frac{A_{\mathcal{S}}}{4\hbar G}$$

By the generalized second law of thermodynamics, $S_{\text{final}} \geq S_{\text{init}}$. Hence the matter initially inside $\mathcal{S}$ must have had no more entropy than that of a black hole.

Neither of these arguments is conclusive. The first implicitly assumed that the region inside $\mathcal{S}$ was flat; one may be able to fit more degrees of freedom into a curved space. There are, in fact, classical configurations — so-called "monsters" — in which the entropy within a region is greater than that of the corresponding black hole [177], although all known examples form from initial singularities (they cannot be "made in the laboratory") and collapse quickly to form black holes.

The second argument fails for a more subtle reason: the area of $\mathcal{S}$ is not a gauge invariant quantity [178, 179]. We are really imagining a timelike "world tube" traced out by $\mathcal{S}$ and asking its cross-sectional area at a fixed time. But this area depends on the choice of time-slicing, and by choosing a slice that "wiggles" enough in a timelike direction — and is therefore nearly null over much of its intersection with the world tube — we can make the area as small as we like.

One might worry that the same problem occurs for black holes, and that the "horizon area" might also not be well-defined. In fact, black holes evade the problem, essentially because the horizon is not timelike, but null (and, for stationary black holes, nonexpanding). Bousso [180] has proposed a covariant entropy bound that exploits a similar idea. He starts with a closed surface $\mathcal{S}$, no longer necessarily spherical, and extends it not as a timelike world tube, but as a "collapsing" light sheet, a null hypersurface of decreasing area extending to the past and future of $\mathcal{S}$, as illustrated in figure 5. Bousso's conjecture is that the total entropy flux through either light sheet — Δ^+ or Δ^- in the figure — is bounded by $A_{\mathcal{S}}/4\hbar G$. For the special case of a black hole, Δ^- may be interpreted as the event horizon, and the Bousso entropy bound is basically the generalized second law of thermodynamics.

Classically, Bousso's conjecture can be proven correct, given suitable energy conditions and bounds on the entropy flux and its gradient [178, 181]. Quantum mechanically, the situation is less clear, though there is strong evidence for the case of free fields with negligible gravitational backreaction [182].

Although the holographic principle in its larger sense remains a conjecture, there is one case in which it has been dramatically successful. The AdS/CFT correspondence of string theory [133, 134] describes the "bulk" physics of an asymptotically anti-de Sitter space in terms of a lower-dimensional "surface" conformal field theory. If this holographic viewpoint can be extended beyond the anti-de Sitter case, black hole thermodynamics will have had a fundamental impact on a far wider field, profoundly altering our approach to fundamental physics.

9 The problem of universality

For a physicist working on quantum gravity, black hole thermodynamics is, at first sight, a huge source of hope. The Hawking temperature and Bekenstein-Hawking entropy are "quantum gravitational," depending on both Planck's and Newton's constants. Moreover, as Wheeler's famous aphorism states, "a black hole has no hair" — a classical black hole has no distinguishing characteristics beyond its mass, charges, and spins. Hence if the entropy of a black hole is an ordinary statistical mechanical entropy, it seems that the only states it can be counting are quantum gravitational states. We may thus have a window into quantum gravity.

Upon closer examination, though, the situation is not so simple. As we saw in section 7, we suffer an embarrassment of riches: there seem to be many different quantum descriptions of black hole statistical mechanics, with very different quantum states, all giving the same entropy. Moreover, while the simple Bekenstein area law seems natural in some of these approaches, in others it seems almost miraculous.

The "problem of universality" is really two problems, which are logically distinct, although their solutions may be related. The first is that black hole entropy has such a simple, universal value, one-fourth of the horizon area in Planck units, regardless of the mass, spin, charges, or the number of dimensions. Even the horizon topology is irrelevant: the same area law holds for black holes, black strings, black rings, and black branes. The second is that so many different approaches to quantum gravity give the same result, regardless of any of the details of the black hole states.

It is tempting to address the first problem with a claim that the horizon area must be quantized, as in section 7.1. But such an answer is at best incomplete: it does not explain the universal coefficient of 1/4, nor does it tell us why no other features matter. There is an interesting explanation of the factor of 1/4 in terms of the topology of the Euclidean black hole [65], but it is not clear why this feature would be reflected in the counting of states.

A cynical answer to the second problem is that only quantum theories that give the "right" result are published. But this, too, is at best incomplete: it does not explain why any such models work at all. Consider, for example, the weakly coupled string approach of section 7.3.1. As we saw there, the theory gives the correct entropy for a large class of near-extremal black holes; but each new type of black hole requires a separate computation, and the relationship between entropy and area only appears at the very last step. Such miracles cry out for explanation.

In the remainder of this section, I will describe one attempt to explain these miracles. This is still very much a case of work in progress, but there have been some hopeful signs.

9.1 State-counting in conformal field theory

Statistical mechanical entropy is basically a measure of the number of quantum states, so we are looking for a universal mechanism to explain the density of states of a black hole. There is one context in which such a mechanism is known: that of two-dimensional conformal field theory.

The metric for a two-dimensional manifold can always be written locally as

$$ds^2 = 2g_{z\bar{z}}dzd\bar{z} \tag{9.1}$$

in terms of complex coordinates z and $\bar{z}$. Holomorphic and antiholomorphic coordinate changes $z \to z + \xi(z)$, $\bar{z} \to \bar{z} + \bar{\xi}(\bar{z})$ merely rescale the metric, and provide the basic symmetries of a conformal field theory. Unlike the conformal symmetry group in higher dimensions, the two-dimensional group has infinitely many generators, denoted $L[\xi]$ and $\bar{L}[\bar{\xi}]$. These satisfy a Virasoro algebra [48],

$$\begin{aligned}
[L[\xi], L[\eta]] &= L[\eta\xi' - \xi\eta'] + \frac{c}{48\pi}\int dz\left(\eta'\xi'' - \xi'\eta''\right)\\
[\bar{L}[\bar{\xi}], \bar{L}[\bar{\eta}]] &= \bar{L}[\bar{\eta}\bar{\xi}' - \bar{\xi}\bar{\eta}'] + \frac{\bar{c}}{48\pi}\int d\bar{z}\left(\bar{\eta}'\bar{\xi}'' - \bar{\xi}'\bar{\eta}''\right)\\
[L[\xi], \bar{L}[\bar{\eta}]] &= 0,
\end{aligned} \tag{9.2}$$

uniquely determined by the values of the two constants c and $\bar{c}$, the central charges. As in ordinary field theory, the zero modes L_0 and $\bar{L}_0$ of the symmetry generators are conserved quantities, the "conformal weights," which can be seen as linear combinations of mass and angular momentum.

Two-dimensional conformal symmetry is remarkably powerful. In particular, with a few mild restrictions, Cardy has shown that the asymptotic density of states of any two-dimensional conformal field theory is given by [183, 184]

$$\begin{aligned}
&\ln\rho(L_0) \sim 2\pi\sqrt{\frac{cL_0}{6}}, \qquad \ln\bar{\rho}(\bar{L}_0) \sim 2\pi\sqrt{\frac{\bar{c}\bar{L}_0}{6}} \qquad \text{(microcanonical)}\\
&\ln\rho(T) \sim \frac{\pi^2}{3}cT, \qquad \ln\bar{\rho}(T) \sim \frac{\pi^2}{3}\bar{c}T \qquad \text{(canonical)}
\end{aligned} \tag{9.3}$$

where T is the temperature. The entropy is thus uniquely determined by a few parameters, regardless of the details of the conformal field theory — just the kind of universal behavior we would like for black holes.

9.2 Application to black holes

Black holes are neither conformally invariant nor two-dimensional, so one might wonder whether the preceding results are relevant. There is a sense, however in which the near-horizon region is *nearly* conformally invariant: the extreme red shift washes out dimensionful quantities such as masses for an observer who remains outside the black hole [185]. The same red shift also makes transverse excitations negligible relative to those in the $r - t$ plane, effectively reducing the problem to two dimensions; this is the same dimensional reduction that allowed many of the anomaly-based calculations described in section 5.5.

Conformal techniques of this type were first applied to the (2+1)-dimensional BTZ black hole [86, 87]. Here, the connection to two-dimensional conformal field theory is clear: the asymptotic boundary at infinity is a two-dimensional cylinder, and the canonical generators of diffeomorphisms obey a Virasoro algebra. Moreover, as Brown and Henneaux showed long ago [85], this Virasoro algebra has a classical central charge, whose presence can be traced back to the need to add boundary terms to the canonical generators. The conformal weights L_0 and $\bar{L}_0$ for the BTZ black hole have simple expressions in terms of charge and mass, and the microcanonical form of the Cardy formula (9.3) then gives the standard Bekenstein-Hawking entropy.

The generalization to higher dimensional black holes [186–188] is more subtle, and not so firmly established, but there has been some significant progress.‡ The key trick is to treat the horizon as a sort

‡See my review [189] for a more thorough treatment.

of boundary — not in the sense that matter cannot pass through it, but in the sense that it is a place where one must impose "boundary conditions," namely the condition that it is a horizon. As in the BTZ case, the canonical generators of diffeomorphisms acquire boundary terms, and for a number of sensible "stretched horizon" boundary conditions these pick out a Virasoro subalgebra with a calculable central charge. Similar techniques have been used for the near-horizon geometry of the near-extremal Kerr black hole [190], again giving the expected Bekenstein-Hawking entropy.

9.3 Effective descriptions

While conformal methods may allow a "universal" computation of Bekenstein-Hawking entropy, they do not tell us a lot about the quantum states of the black hole. In one sense, this is a good thing: the point, after all, is to explain why many different descriptions of the states yield the same entropy. Still, one may be able to learn something useful about "effective" descriptions.

It is well known that the presence of a boundary, either at infinity or at a finite location, alters the canonical generators of diffeomorphisms by requiring the addition of boundary terms [64]. The exact form of these boundary terms depend on the choice of boundary conditions, and the resulting generators necessarily respect these boundary conditions. As a result, some of the "gauge transformations" of the theory without boundary — those that change the boundary conditions — are no longer invariances of the theory. This boundary symmetry-breaking increases the number of states in the theory: states that would previously have been considered gauge-equivalent are now distinct [191]. Moreover, these new states appear only at the boundary, since the gauge symmetry remains unbroken elsewhere.

This phenomenon is not quite the same as the Goldstone mechanism [192], but it is similar in spirit. By analogy, it suggests a possible effective description of black hole horizon states in terms of the parameters that label the boundary-condition-breaking "would-be diffeomorphisms." For the case of the BTZ black hole, such a description can be made explicitly [88,193]. The effective field theory is a particular conformal field theory, Liouville theory, with a central charge that matches the Brown-Henneaux value. The counting of states in this theory is poorly understood: technically, the relevant states are in the "nonnormalizable sector," which is not under good control. But as discussed in section 5.10, one can couple the Liouville theory to external matter and recover a correct description of Hawking radiation.

The effective description described here is holographic, and is reminiscent of the "membrane paradigm" for black holes [194,195], in which a black hole is also described by a collection of degrees of freedom at the horizon. It is possible that a better understanding of these degrees of freedom could help us understand the information loss problem, the next topic of this review.

10 The information loss problem

Let us turn finally to one of the most puzzling aspects of black hole thermodynamics [196], the "information loss paradox." This is a topic that is very much in flux, with nothing near a consensus concerning its resolution. The purpose of this section is therefore simply to introduce some of the central ideas.

Consider a process in which a shell of quantum matter in a pure state is allowed to collapse to form a black hole, which then evaporates completely into Hawking radiation. It appears that this process has allowed an initial pure state to evolve into a final highly mixed (thermal) state. But this is not possible in quantum mechanics: unitarity requires that a pure state evolve to a pure state. If one thinks of the

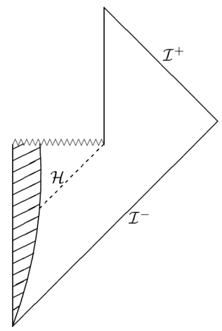


Figure 6: A possible Carter-Penrose diagram for an evaporating black hole

von Neumann entropy as a measure of lack of information, this process has led to an unallowable loss of information.

This argument contains several hidden assumptions, of course, any one of which could point to a resolution. We have assumed that a pure state *can* collapse to form a black hole; that the final state consists only of Hawking radiation, with no long-lived "remnants" of the black hole; that the final spacetime is simply nearly flat space, with no residue of the singularity and no "baby universes" that could swallow up lost correlations; and that the Hawking radiation is genuinely thermal, with no hidden correlations. Let us consider each of these in turn.

10.1 Nonunitary evolution

One simple answer to the information loss problem is simply that quantum mechanics is not quite correct, and the evolution is not unitary. Figure 6 shows a proposed Carter-Penrose diagram for an evaporating black hole. In this picture, it is clear where the "missing" information has gone: it has simply vanished into the singularity. Of course, one might hope that quantum gravity would eliminate the singularity, but that need not guarantee that the lost correlations would have to reappear for a future observer.

Restricted to black holes, such a loss of unitarity might seem innocuous. But there are presumably quantum amplitudes for any physical process that include virtual black holes, so the effect, though perhaps tiny, would be pervasive. It has been argued that such violations of unitarity lead to either violations of causality or energy nonconservation at an unacceptable level [197], but there are known counterexamples [198], and the possibility remains open.

10.2 No black holes

We really understand the formation of a black hole only as a classical process, in which the collapsing matter is typically in a complicated thermal state. It is possible that matter in a pure state simply does not collapse to form a black hole. In the fuzzball picture of section 7.3.2, for instance, the microscopic states are not black holes; properties such as horizons appear only statistically as features of ensembles [131].

One can also write down "phenomenological" metrics for which the black hole singularity is replaced by a nonsingular de Sitter-like region [199–202]. In such models, a true horizon never forms, but there can be a "quasi-horizon" that looks much like an event horizon for a long period of time. Recall from section 5.1 that a horizon is not required for Hawking radiation — it is enough to have exponential blue shift and a slowly varying effective surface gravity — so for the most part, the thermodynamics will not change. But there is no longer a singularity at which correlations can be lost, and when the quasi-horizon eventually disappears, the information in the interior becomes accessible.

Such models share some similarities with the "remnant" scenarios discussed below. In particular, there are unsettled issues concerning how fast the information can "escape" after the disappearance of a quasi-horizon. Nevertheless, they capture a feature that must be shared by almost any model of unitary evolution: if the black hole eventually disappears without any loss of information, then almost by definition there cannot have been a true event horizon.

10.3 Remnants and baby universes

Calculations of Hawking radiation seem likely to be reliable down to the length scales at which quantum gravitational effects become important. Beyond that scale, though, we do not know what to expect. In particular, it is possible that black hole evaporation might stop [203], leaving a black hole "remnant."

If remnants exist, they offer a new resolution to the information loss problem. A remnant could be correlated with the Hawking radiation, allowing the combined state to remain pure. Of course, this requires that remnants have a huge entropy, that is, a huge number of possible states. This is generally understood to be incompatible with the AdS/CFT correspondence, and there is a danger of remnants being overproduced by pair production in the vacuum [204]. On the other hand, particular exact models of black hole evaporation *do* avoid the information loss problem through a remnant mechanism [205], providing at least an existence proof.

Remnants need not have infinite lifetimes; they, too, may eventually decay into something like Hawking radiation. There are, however, constraints on such a process, if unitarity is to be preserved [203, 206]. As an estimate of the lifetime [207], suppose the remnant has mass m_R, and decays by emitting N photons. Each photon carries roughly one bit of information, and if the process is to preserve unitarity, their total entropy must be on the order of that of the original black hole. We thus require $N \sim GM^2/\hbar$, so each photon should have an energy and wavelength

$$E \sim \hbar m_R/GM^2, \qquad \lambda \sim GM^2/m_R.$$

For the photons to be uncorrelated, they should be emitted far enough apart that their wave packets do not significantly overlap. The total time for emission is thus

$$\tau \sim N\lambda \sim \left(\frac{M}{m_{Pl}}\right)^4 \frac{\hbar}{m_R}, \tag{10.1}$$

typically far longer than the age of the Universe. By carefully analyzing the entanglement entropy at future null infinity, Bianchi has found a similar time scale, although with a bit more flexibility depending on the details of the evaporation [208].

A related approach to the problem is the "baby universe" scenario [210]. While we might expect quantum gravity to eliminate the singularity in figure 6, the result could plausibly be a new region of the Universe that is causally disconnected from $\mathcal{I}^+$ — something one could perhaps think of as a peculiar, large remnant. In this case, unitarity is technically maintained, but it cannot be tested by a single observer.

10.4 Hawking radiation as a pure state

The most widely held expectation among physicists in this field is that the information loss problem will be resolved by a demonstration that Hawking radiation has subtle hidden correlations and is actually in a

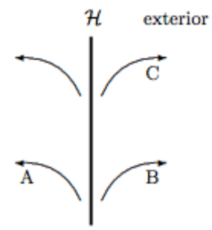


Figure 7: Early and late Hawking quanta emitted from near the horizon $\mathcal{H}$.

pure state. This belief gains support from the AdS/CFT correspondence: in the setting in which we think we best understand quantum gravity, the boundary conformal field theory is unitary, and there does not seem to be room in its Hilbert space for remnants. At first sight, this idea does not seem unreasonable; the density matrix of a pure state can be very close to a thermal density matrix [209], differing only by terms of order e^{-S}.

But while the difference between a pure state and a thermal state may be very small, it can still be large enough to cause trouble. Consider two Hawking quanta, one "early" and one "late," emitted by a black hole, as shown in figure 7. As in section 4.2, each escaping particle is accompanied by a "partner" inside the horizon. In order for a freely falling observer to see the standard vacuum near $\mathcal{H}$, particle B must initially be entangled with its "partner" A. This correlation is part of the basic structure of the quantum vacuum; it can be violated only at the cost of introducing large vacuum expectation values of such quantities as the stress-energy tensor.

On the other hand, if the Hawking radiation is to be in a pure state in the far future, particle A must be entangled with other particles in the radiation — for simplicity, say particle C. But this violates a fundamental result of quantum mechanics, "monogamy of entanglement," which states that a state cannot be simultaneously maximally entangled with two others [211]. More technically, the problem can be restated as a violation of the strong subadditivity of entropy [5].

Of course, two quantum states that are initially entangled need not remain so, and it might be possible to transfer the entanglement of particle B from A to C. But A and C are not causally connected — they are separated by a horizon — so such a process would have to be nonlocal [212]. One might respond that quantum gravity is always nonlocal [213, 214], but it remains unclear whether "enough" nonlocality can be made compatible with local effective field theory without doing violence to our notions of the scales at which quantum gravitational effects ought to be important.

For a slightly different perspective [215], consider the "membrane" description of effective degrees of freedom of section 9.3, or more generally a holographic picture in which the black hole is described by horizon degrees of freedom. As particle A enters the black hole, its state should be captured by these horizon degrees of freedom, which could then be correlated with the later emission of particle C. An infalling observer, on the other hand, should be able to see particle A inside the horizon. This would seem to violate the "no cloning" theorem of quantum theory [216], which prohibits the duplication of a state. The notion of "black hole complementarity" is that this need not be a problem, because no single observer can measure both states. But in a much-cited paper, Almheiri *et al.* have disputed this [5], arguing instead that the entanglement across the horizon is likely to be broken; see also Braunstein [217] and Mathur [218] for similar arguments. The resulting large expectation values of the stress-energy tensor would then form a "firewall" for any infalling observer. As Almheiri *et al.* point out, this implies a breakdown of the principle of equivalence near a horizon, a rather dramatic claim. At this writing, the controversy is far from being settled.

11 Conclusion

Classically, black holes are very nearly the simplest structures in general relativity. With the advent of black hole thermodynamics, however, we have come to see them as highly complex thermal systems, rarely at equilibrium, with a truly remarkable number of internal states.

Perhaps most surprisingly, we have learned that ordinary methods of general relativity and quantum theory — quantum field theory in curved spaces, WKB approximations, semiclassical path integrals, and the like — allow us to probe properties of quantum gravitational states. Such a claim should be greeted with skepticism, of course. But over the past few decades we have gathered such a weight of self-consistent results, based on enough different and independent approximations, that it seems almost certain we are seeing something real.

At the same time, black hole thermodynamics is having a profound impact on the rest of physics. The holographic conjecture suggests that our fundamental notions of local physics, and perhaps of space and time, are only low energy approximations. Attempts to understand black hole universality, while less sweeping, also point to the key role of "boundary" states. The information loss problem has led us to question basic aspects of quantum mechanics, and has exhibited remarkable connections among very different aspects of physics. And perhaps in the future, black hole thermodynamics will tell us something profound about quantum gravity. We have interesting times ahead.

12 Acknowledgments

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Appendix. Classical black holes

The well-known Schwarzschild and Kerr solutions are the archetypes of black holes. For black hole thermodynamics, though, we need more general configurations: "dirty" black holes whose horizons are distorted by nearby matter, dynamical black holes formed from collapsing matter, black holes in other spacetime dimensions. For this, we need a generalization of the notion of a horizon.

A black hole is defined as a "region of no return," an area of spacetime from which light can never escape. This idea can be made more precise—see Hawking and Ellis [219] for details—by considering a spacetime whose Carter-Penrose diagram looks roughly like that of figure 1(a), in the sense that it contains a future null infinity $\mathcal{I}^+$ that describes the "end points at infinity" of light rays. Points in the causal past of $\mathcal{I}^+$ are ordinary events from which light can escape to infinity. Points that do not lie in the past of $\mathcal{I}^+$ are cut off from infinity, and form a black hole region. The dividing line—the boundary of the past of $\mathcal{I}^+$—is the event horizon; in figure 1(a), it is the dashed line labeled $\mathcal{H}$.

The event horizon defined in this way has many interesting global properties: for instance, it cannot bifurcate or decrease in area [219]. But it might be argued that this definition does not quite capture the right physics. The problem can be traced to the word "never" in the phrase "light can never escape"—to truly make such a statement, one must know about the entire future. The event horizon is teleological in nature: it is determined by "final causes," events in the indefinite future.

Imagine, for instance, that the Earth is sitting at the center of a highly energetic incoming spherical shell of light, one so energetic that it has a Schwarzschild radius of one light year. To be sure, this is not likely, but we also cannot rule out the possibility observationally: no signal can propagate inward faster than such a shell, so we would not know of its existence until it reached us. Suppose now that the shell is presently two light years away, collapsing inward toward us at the speed of light. Step outside and point a flashlight up into the sky. One year from now, the light from your flashlight will have traveled one light year, where it will meet the collapsing shell just as the shell reaches its Schwarzschild radius. At that point, the light will be trapped at the horizon of the newly formed Schwarzschild black hole, and will be unable to travel any farther outward. In other words, in this scenario we are *already* at the event horizon of a black hole, even though we will see no evidence for this fact until we are suddenly vaporized two years from now.

This seem odd; surely Hawking radiation "now" should not need to know about the infinite future. A number of attempts have been made to find more local versions of the event horizon [220], leading to a variety of more or less useful definitions. The most useful for our purposes is the "isolated horizon" [9], a locally defined surface with properties suitable for describing equilibrium black holes.

An isolated horizon is a null surface—a surface traced out by light rays—whose area remains constant in time, as the horizon of a stationary black hole does. A thought experiment may again be helpful. Picture a spherical lattice studded with flashbulbs, set to all flash simultaneously in the lattice rest frame. The bulbs will produce two spherical shells of light, one traveling inward and one traveling outward. As we know from our experience in nearly flat spacetime, the area of the ingoing shell will normally decrease with time, while the area of the outgoing shell will increase. If the lattice is placed at the horizon of a Schwarzschild black hole, though, it is not hard to show that the area of the outgoing shell will remain constant. Similarly, if the lattice is placed inside the horizon, *both* shells will decrease in area.[§]

To express this idea mathematically, let us first define a nonexpanding horizon $\mathcal{H}$ in a d-dimensional spacetime to be a $(d-1)$-dimensional submanifold such that [9, 136]

1. $\mathcal{H}$ is null: that is, its normal vector ℓ_a is a null vector;
2. the expansion of $\mathcal{H}$ vanishes: $\vartheta_{(\ell)} = q^{ab}\nabla_a \ell_b = 0$, where q_{ab} is the induced metric on $\mathcal{H}$;
3. the field equations hold on $\mathcal{H}$, and $-T^a{}_b\ell^b$ is future-directed and causal.

The first condition expresses the fact that the horizon is traced out by light rays. The expansion $\vartheta_{(\ell)}$ is the logarithmic derivative of the area of a cross-section of $\mathcal{H}$ [221], so the second condition implies time independence of the horizon area. The third condition is a relatively weak prohibition of negative energy at the horizon.

These three conditions imply that

$$\nabla_a \ell^b = \omega_a \ell^b \quad \text{on } \mathcal{H}$$

for some one-form ω_a. The surface gravity $\kappa_{(\ell)}$ for the normal ℓ^a—the quantity that appears in the Hawking temperature (1.1)—is then defined as

$$\kappa_{(\ell)} = \ell^a \omega_a. \tag{A.1}$$

The normal ℓ^a is not quite unique, though: if ℓ^a is a null normal to $\mathcal{H}$ and λ is an arbitrary function, then $e^{\lambda}\ell^a$ is also a null normal to $\mathcal{H}$. We can reduce this ambiguity by demanding further time independence: a weakly isolated horizon is one for which, as an added condition, we require that

[§] The outgoing shell must still be outgoing with respect to the lattice, of course; as the lattice itself collapses inward, its area will decrease even faster than that of the "outgoing" shell of light. This is one way of understanding why any object inside a black hole must necessarily collapse inward.

4. $\mathcal{L}_\ell \omega = 0$ on $\mathcal{H}$,

where $\mathcal{L}$ is the Lie derivative. This constraint implies the zeroth law of black hole mechanics, that the surface gravity is constant on the horizon.

Even with this fourth condition, the null normal ℓ^a may be rescaled by an arbitrary constant. This also rescales the surface gravity, so the numerical value of $\kappa_{(\ell)}$ is not uniquely determined. At first sight this seems to be a terrible feature, but it reflects a genuine physical ambiguity: even at a horizon, the choice of time coordinate is not completely fixed. In fact, the first law of black hole mechanics (2.1) *requires* such an ambiguity: the scale of the mass M is also fixed only after one has normalized the scale of time at infinity.

To clarify this issue, it will help to take a small detour. For a *stationary* black hole, another type of a horizon can be defined. A Killing horizon is a $(d-1)$-dimensional submanifold $\mathcal{H}_K$ such that

1. some Killing vector χ^a is null, that is, $\chi_a\chi^a = 0$; and

2. $\mathcal{H}_K$ is itself a null surface, that is, its normal vector is null.

If both conditions hold, it follows that χ^a is itself a null normal to $\mathcal{H}_K$, and that

$$\chi^a\nabla_a\chi^b = \kappa_{(\chi)}\chi^b \quad \text{on } \mathcal{H}_K. \tag{A.2}$$

Note also that the expansion $q^{ab}\nabla_a\chi_b$ is automatically zero on $\mathcal{H}_K$, simply by virtue of the Killing equation $\nabla_a\chi_b + \nabla_b\chi_a = 0$.

In this case, the null vector ℓ^a that defines an isolated horizon may be chosen to equal χ^a at $\mathcal{H}_K$, and the two horizons coincide. This does not quite solve the problem of normalization: a Killing vector also requires normalization, since if χ^a is a Killing vector, so is $c\chi^a$. But in the stationary case, the normalization of χ^a can be fixed at infinity, for example by requiring that $\chi^t \sim 1$. In other words, for stationary black holes one can use global properties of the spacetime to adjust clocks at the horizon by comparing them with clocks at infinity.

If, on the other hand, one is only concerned with physics at or near the horizon, the normalization of $\kappa_{(\ell)}$ becomes more problematic. One can use known properties of exact solutions to express the surface gravity in terms of other quantities at the horizon [222], thereby fixing ℓ^a, but so far the procedure seems rather artificial. Alternatively, one can simply accept the ambiguity, and note that other quantities such as quasilocal masses defined near $\mathcal{H}$ require a similar choice of normalization.

As noted in section 2, weakly isolated horizons obey the four laws of black hole mechanics, the second law in the equilibrium form that the horizon area remains constant. Generalizations to dynamical, evolving horizons are also possible [222], and may be used to prove the inequality version (2.2) of the second law. These generalizations also provide a potential setting for nonequilibrium black hole thermodynamics, allowing us, for instance, to describe flows of energy and angular momentum that include the contribution of the gravitational field.

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