Title
CHOICE OF IP GEOMETRY AND BEAM PARAMETERS FOR THE SSC

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Publication Date
1983-12-01

LBL-17052

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Accelerator & Fusion Research Division

Presented at the Ann Arbor Workshop on Accelerator Physics Issues for Superconducting Super Collider, Ann Arbor, MI, December 12-16, 1983; and to be published in the Proceedings

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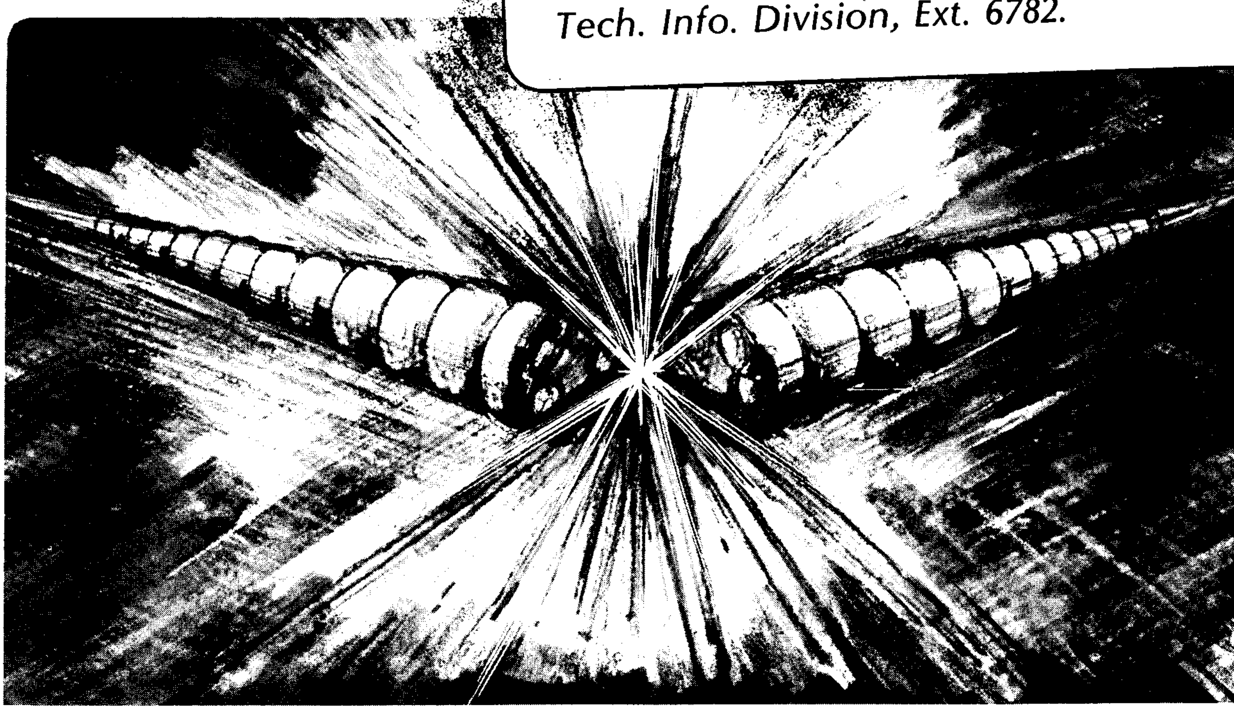

Prepared for the U.S. Department of Energy under Contract DE-AC03-76SF00098

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LBL-17052
SSC Note-16
December 8, 1983

CHOICE OF IP GEOMETRY AND BEAM PARAMETERS FOR THE SSC*

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Introduction

Beams consisting of closely spaced bunches crossing at a small angle are the most promising solution. Bunched beams allow to reach high luminosity with a moderate total charge in the machine while the close bunch spacing keeps the peak event rate low. The non-zero crossing angle is required to avoid unwanted collisions in the vicinity of each IP where the beams remain close.

Outline of Procedure

The quantities of interest are the following:

1. Major Input Data (Performance goals):

   Luminosity $\mathcal{L}$

   Beam Energy E

2. Critical assumptions:

   $\Delta Q(0)$ tolerable beam-beam tune shift at each IP

   $\Delta Q(LR)$ tolerable long range beam-beam tune shift in the vicinity of each IP

*This work was supported by the Director, Office of Energy Research, Office of High Energy and Nuclear Physics, High Energy Physics Division, U. S. Dept. of Energy, under Contract No. DE-AC03-76SF00098.

3. Crucial free (within some range) parameters:

S_B bunch separation (bunch to bunch distance)

β^* value of β-function at IP

4. Parameters describing beam and collision geometry:

N_B # of particles per bunch

ε_n normalized rms emittance, $\varepsilon_{nx} = \varepsilon_{ny}$

σ_z rms bunch length

σ_γ rms energy spread, $\sigma_\gamma = \sigma_E/m_p$

α crossing angle

To derive the quantities (4) we will proceed along the following lines:

From an assumed value $\Delta Q(0)$ we will derive ε_n, N_B as functions of the dimensionless quantity $r = \alpha\sigma_z/2\sigma^*$. From an upper limit on $\Delta Q(LR)$ we will derive $\alpha(r)$, therefore obtaining a one to one correspondence between r and σ_z. We will then use auxiliary considerations to narrow the choice of σ_z (and therefore r) as well as σ_γ.

Derivation of $\varepsilon_N(r)$, $N_B(r)$, $\sigma_z(r)$

The following relations hold:[1]

$$\mathcal{L} = \frac{C}{S_B} \frac{1}{4\pi} \frac{\gamma}{\beta^*} \frac{N_B^2}{\varepsilon_N} (1+r^2)^{-1/2} \tag{1}$$

$$\left|\Delta Q_y(0)\right| = \frac{1}{4\pi} \frac{r_o N_B}{\varepsilon_N} \, 2 \, \frac{(1+r^2)^{1/2} - 1}{r^2} \tag{2}$$

with $$r = \frac{\alpha\sigma_z}{2\sigma^*} .$$

ΔQy as given by (2) is the linear tune shift for a particle centered in its bunch and $|\Delta Q_y| \geqslant |\Delta Q_x|$ for crossing in the x–z plane. Expression (2) is valid for all r, $0 \leqslant r \leqslant \infty$. From (1) and (2) follow:

$$N_B = \beta^* S_B \frac{r_0}{c\gamma} \frac{\mathcal{L}}{|\Delta Q(0)|} \frac{g(r)}{f(r)} \qquad (3)$$

$$\epsilon_N = \beta^* S_B \frac{r_0^2}{c\gamma} \frac{\mathcal{L}}{|\Delta Q(0)|^2} \frac{}{4\pi} \frac{g^2(r)}{f(r)} \qquad (4)$$

with $$f(r) = (1+r^2)^{-1/2}, \quad g(r) = 2 \frac{(1+r^2)^{1/2} - 1}{r^2} \qquad (5a,b)$$

$\epsilon_N(r)$, $N_B(r)$ are plotted in Figs. (1) and (2).

To calculate $\alpha(r)$ we consider the long range beam-beam interaction due to the close encounters at distances $z = \pm n\ S_B/2$ away from the IP. A detailed investigation[(2)] has shown that:

1) For ratios σ_x/σ_y of "a few" and at distances $r \gtrsim 10$ Max (σ_x, σ_y) a simple $1/r$ dependence is a very good approximation for the fields created by a bi-gaussian charge distribution.

2) With typical insertion designs, with little betatron phase advance in the insertion quadrupoles, the resulting tune shift is, at least for antisymmetric configurations approximated to within ≈ 10% by

$$\left|\Delta Q_{z,y}(LR)\right| \cong \frac{r_o N_B}{4\pi} \frac{1}{\epsilon_N} \frac{8}{\eta^2} \frac{D}{S_B}$$

Where we have substituted a drift length D extending from the IP to the separating dipoles and where η is defined as:

$$\eta = \frac{\alpha z}{\sigma(z)} \cdot$$

From this we can evaluate $\alpha(r)$, $\sigma(r)$:

We obtain:

$$\alpha(r) = D^{1/2} \frac{r_o}{c\gamma} \left(\frac{2c}{\pi} \frac{\mathcal{L}}{|\Delta Q(0)|\,|\Delta Q(LR)|}\right)^{1/2} \left(\frac{g(r)}{f(r)}\right)^{1/2} \qquad (7)$$

$$\sigma_z(r) = \beta^* \left(\frac{S_B}{D}\right)^{1/2} 2r \left(\frac{|\Delta Q(LR)|}{8|\Delta Q(0)|} g(r)\right)^{1/2} \qquad (8)$$

Fig. (2) shows $\epsilon_n(r)$, $N_B(r)$, $\alpha(r)$, $\sigma_z(r)$ in the appropriate normalization for $\gamma = 2.13\text{x}10^4$, $\mathcal{L} = 10^{33}\ \text{cm}^{-2}\text{s}^{-1}$, $|\Delta Q(0)| = 0.003$, $|\Delta Q(LR)| = 0.00025$.

Carrying out these calculations we have implicitly assumed that a tolerable upper limit for $\Delta Q(0)$ is independent of r and that we should keep $|\Delta Q(LR)| \ll |\Delta Q(0)|$. In view of the fact that the nonlinear aspects of the beam-beam interaction are really what counts, this assumption will need closer inspection. The general structure of our argument would not be changed but the r dependence of the relations (5) through (8) might have to be modified.

Choice of r and $\sigma_z(r)$

The choice of σ_z, and therefore r, is to some extent arbitrary. Low values are favored because they result in lower currents and larger emittances, thus reducing total synchrotron radiation power and the demands on injector brightness. Considerations of RF-technology and collective stability also enter. We will not discuss these but look at intra beam scattering (IBS). IBS growth rates depend critically on the six-dimensional phase space density. Fig. (3) shows growth rates for two different cases ($\beta^* = 2m$, $\beta^* = 1m$) as function of r for different energy spreads σ_γ. IBS essentially determines a lower limit for σ_γ. Fig. (3) shows rapid increase of growth rates for very low values of σ_z, a minimum and then again a slow increase for large σ_z (corresponding to small ε_n).

References

1) Christoph Leemann, SSC Note-3.

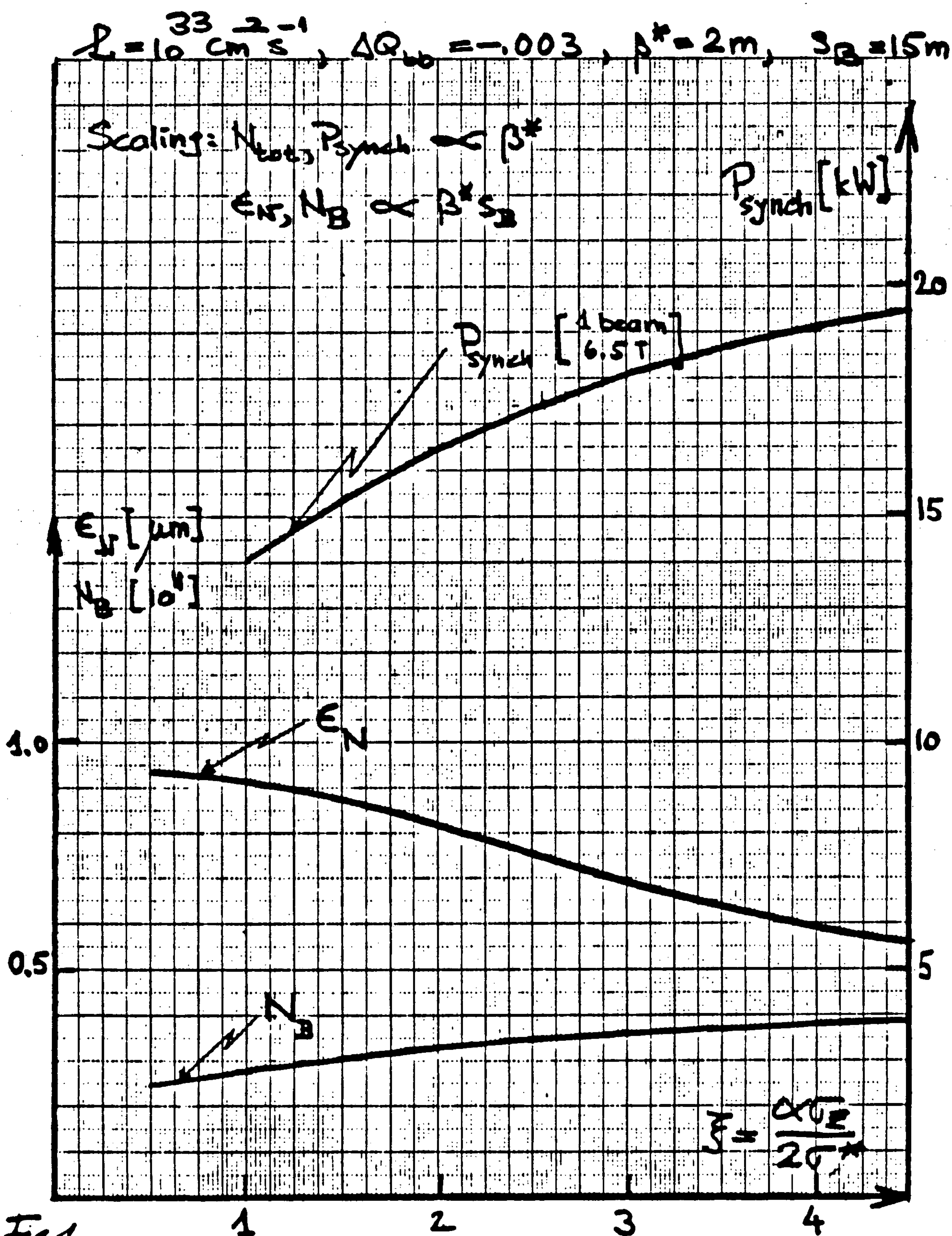
2) Christoph Leemann, SSC Note-13.

Figure Captions

Fig. 1 Shown are ϵ_N, N_B as well as the approximate synchrotron radiation power in a 6.5T ring. Values are calculated for $\mathcal{L} = 10^{33}cm^{-2}s$, $|\Delta Q(0)| = 0.003$, $\beta^* = 2$ m and $S_B = 15$ m.

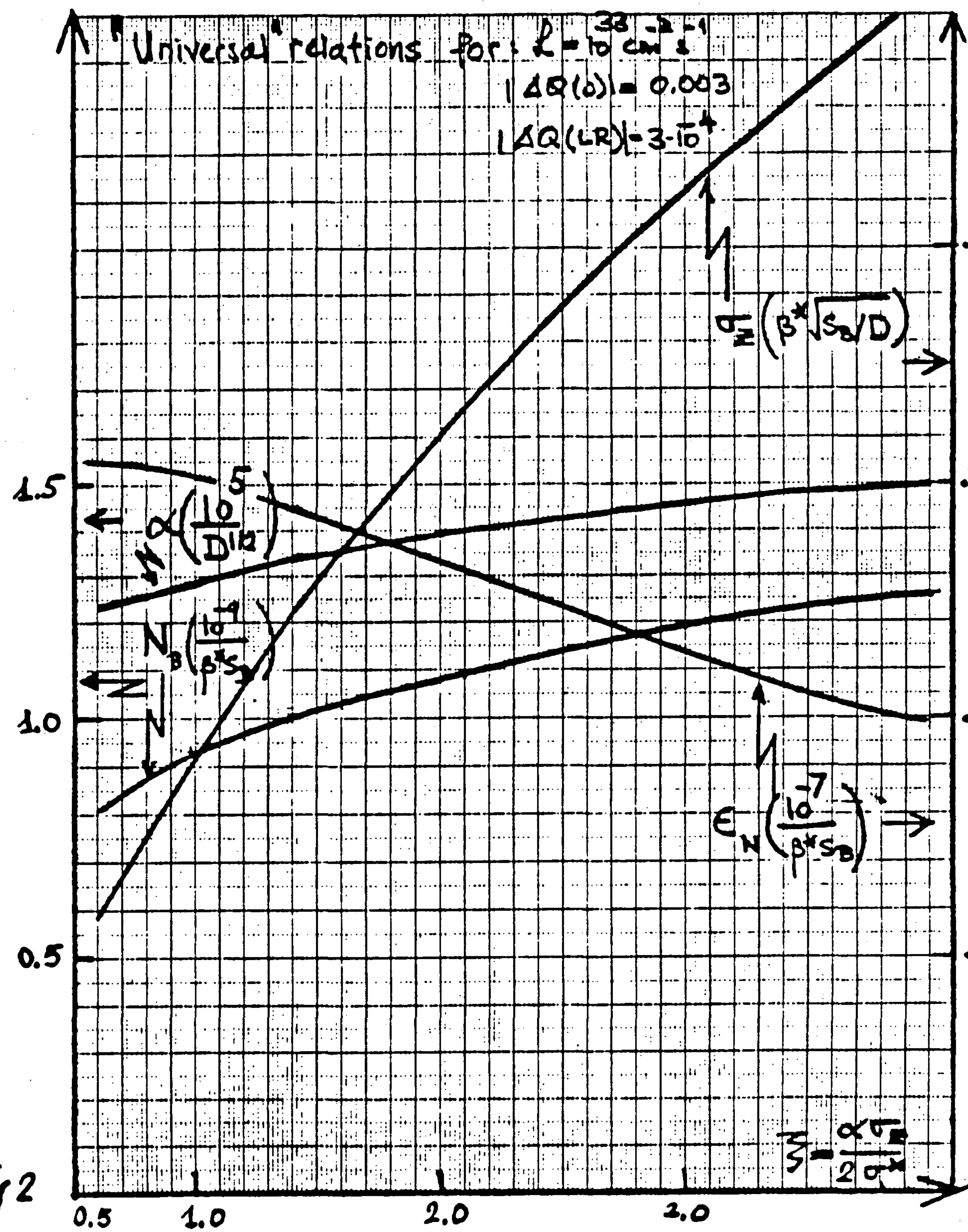
Fig. 2 Beam parameters ϵ_N, N_B, σ and α normalized with the appropriate scaling factors. Assumed is $\mathcal{L} = 10^{33}cm^{-2}s$, $|\Delta Q(0)| = 0.003$, $|\Delta Q(LR)| = 2.5x10^{-4}$.

Fig. 3 IBS growth rates calculated following Bjorken and Mtingwa's theory for $\beta^* = 2$ m and 1 m and $S_B = 15$ m. Calculations are performed on the basis of the regular arcs only ($L_c = 160$ m, $\mu = 60°$, $\phi \simeq 0.71°$).



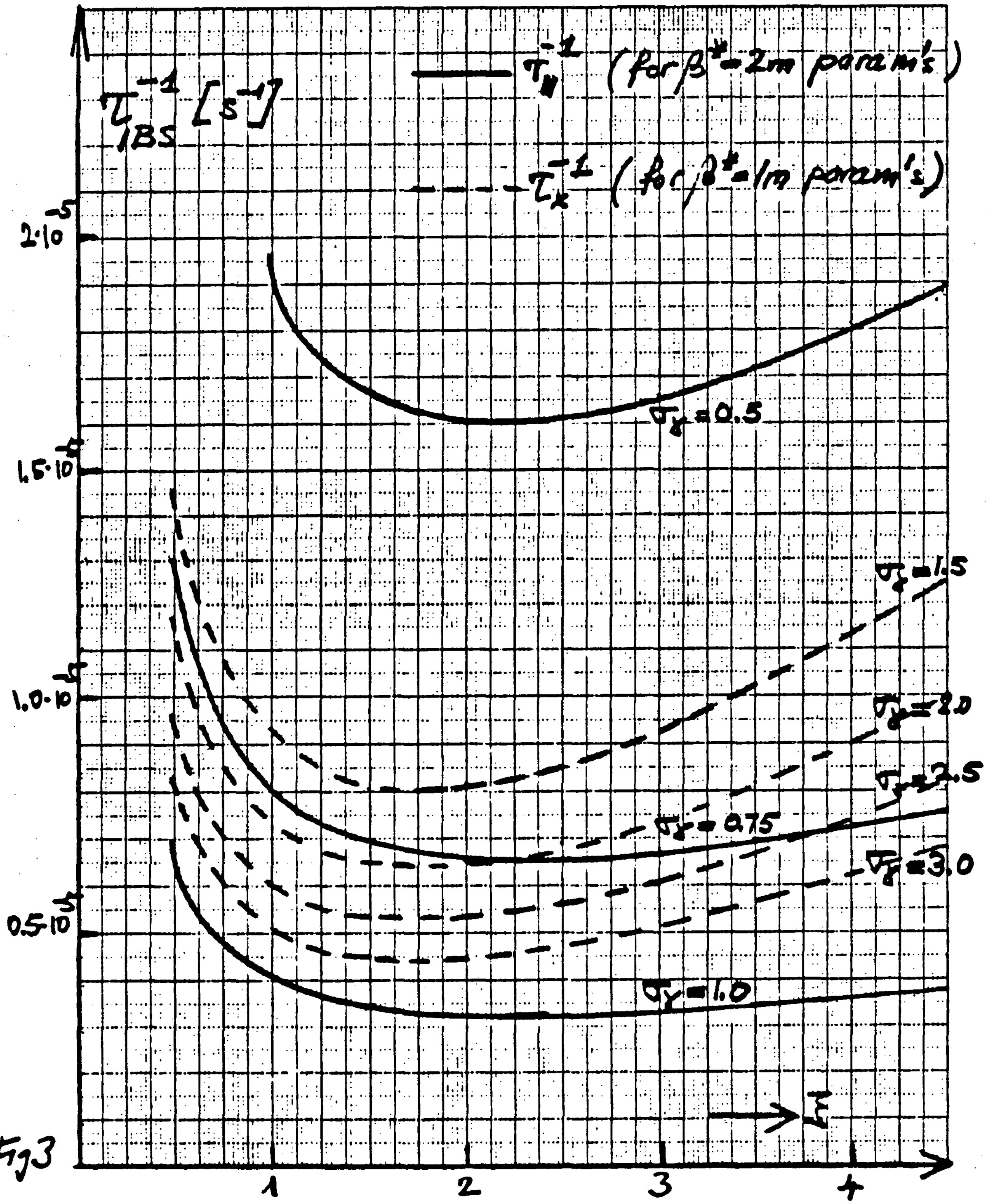
$\mathcal{L} = 10^{33}\ cm^{-2} s^{-1}$, $\Delta Q_{bb} = -.003$, $\beta^* = 2m$, $s_B = 15m$
Scaling: $N_{tot}, P_{synch} \propto \beta^*$
$\epsilon_N, N_B \propto \beta^* s_B$
P_{synch} [kW]
P_{synch} [1 beam, 6.5 T]
ϵ_N [μm]
N_B [10^{11}]
ϵ_N
N_B
$\xi = \frac{\alpha \sigma_z}{2\sigma^*}$
1.0
0.5
20
15
10
5
1
2
3
4

Fig 1



"Universal" relations for: $\mathcal{L} = 10^{33} cm^{-2} s^{-1}$
$|\Delta Q(0)| = 0.003$
$|\Delta Q(LR)| = 3 \cdot 10^{-4}$
$\sigma_z (\beta^* \sqrt{S_B/D})$
$\alpha \left(\frac{10^5}{D^{1/2}}\right)$
$N_B \left(\frac{10^{-9}}{\beta^* S_B}\right)$
$\epsilon_N \left(\frac{10^{-7}}{\beta^* S_B}\right)$
$\xi = \frac{\alpha \sigma_z}{2 \sigma^*}$
1.5
1.0
0.5
.4
.3
.2
.1
0.5
1.0
2.0
3.0

Fig 2



τ^{-1}_{IBS} [s^{-1}]
$\tau^{-1}_{\parallel}$ (for β^*=2m param's)
$\tau^{-1}_{\parallel}$ (for β^*=1m param's)
$2\cdot10^{-5}$
$1.5\cdot10^{-5}$
$1.0\cdot10^{-5}$
$0.5\cdot10^{-5}$
σ_y=0.5
σ_y=1.5
σ_y=2.0
σ_y=2.5
σ_y=0.75
σ_y=3.0
σ_y=1.0
ξ
1
2
3
4

Fig 3