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## Authors

Guo, Jang-Ting
Krause, Alan
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# Dynamic income taxation without commitment: Comparing alternative tax systems ${ }^{\boxed{2}}$ 

Jang-Ting Guo ${ }^{\mathrm{a}, 1}$, Alan Krause ${ }^{\mathrm{b}, *}$<br>${ }^{\text {a }}$ Department of Economics, 3133 Sproul Hall, University of California, Riverside, CA 92521, USA<br>${ }^{\text {b }}$ Department of Economics and Related Studies, University of York, Heslington, York YO10 5DD, UK

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#### Abstract

This paper addresses the question as to whether it is optimal to use separating or pooling nonlinear income taxation when the government cannot commit to its future tax policy. We also compare the levels of social welfare attainable under these tax systems versus that in the autarkic equilibrium. Both two-period and infinite-horizon settings are considered. Under empirically plausible parameter values, separating taxation is optimal in the two-period model, whereas autarky is optimal when the time horizon is infinite. The effects of varying the degree of wage inequality, the populations of low-skill and high-skill workers, and the discount rate are explored as well. For reasonable changes in these parameters, separating taxation remains optimal in the two-period model, while autarky remains optimal in the infinite-horizon model. Pooling is not optimal in either the two-period or infinite-horizon models for all parameter changes considered.


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## 1. Introduction

Traditionally, macro-style analyses of taxation have examined dynamic models, but the common assumption that all individuals are identical rules out a redistributive role for tax policy. On the other hand, micro-style analyses of taxation typically study models with heterogeneous agents, which allows for redistributive concerns, but these models tend to be static which rules out intertemporal considerations. In recent years, a literature known as the 'new dynamic public finance' has emerged that seeks to unite the macro- and micro-style approaches by extending the workhorse Mirrlees (1971) model of optimal nonlinear income taxation to a dynamic setting. ${ }^{2}$ For the most part, this literature has maintained the Mirrlees assumption that there is a continuum of skill types, and it assumes an infinite time horizon and that future wages are determined by random productivity shocks. Accordingly, the complexity of these models has led most researchers to make the simplifying assumption that the government can commit to its future tax policy. In particular, the government cannot use

[^0]skill-type information revealed in earlier periods to redesign the tax system and achieve a better allocation in latter periods.

The commitment assumption has been criticized as being inconsistent with the micro-foundations of the Mirrlees model. Specifically, a key feature of the Mirrlees approach to optimal taxation is that no ad hoc constraints be placed on the tax instruments available to the government-these are determined only by the information structure. In the Mirrlees model, the government cannot observe each individual's skill type, which is the reason it must adopt (the second-best) incentivecompatible taxation. But such taxation in earlier periods of a dynamic Mirrlees model results in skill-type information being revealed to the government, which would then enable it to implement (the first-best) personalized lump-sum taxes in latter periods. Therefore, ruling out lump-sum taxation in a dynamic Mirrlees model via a commitment assumption might be considered ad hoc, in much the same way as ruling out lump-sum taxation in representative-agent models is considered somewhat artificial. ${ }^{3}$ The commitment assumption has also been criticized as being unrealistic, since the present government cannot easily impose binding constraints on the tax policies of future governments. ${ }^{4}$

[^1]The difficulty with relaxing the commitment assumption is that it may no longer be social-welfare maximizing for the government to design a (separating) nonlinear income tax system in which individuals are willing to reveal their skill types. ${ }^{5}$ Instead, it may be optimal to pool the individuals so that skill-type information is not revealed. Similarly, the autarkic equilibrium of the economy may be preferred to both separating and pooling income taxation. Very little is known as to under what conditions separating taxation, pooling taxation, or autarky is most desirable from the perspective of maximizing social welfare. To the best of our knowledge, only two other papers have focused on the issue. Roberts (1984) finds that if the time horizon is infinite and there is no discounting, separating taxation will never occur. The intuition is fairly straightforward: if high-skill individuals live forever, they will forever face personalized lump-sum taxation if they reveal their type. Moreover, since they do not discount the future, they cannot be compensated in the present for the ever-lasting personalized lumpsum taxation they would face after revealing their type. As a result, separation is not possible. Berliant and Ledyard (2014) examine a two-period model with discounting. They conclude that separation occurs provided that the discount rate is high. Intuitively, when highskill individuals are not too concerned about their future welfare, there exists a relatively low level of compensation that they can be given in period 1 for revealing their type and facing personalized lump-sum taxation in period 2 . In this case, separation is not too costly from a social-welfare point of view, and is therefore desirable.

We think that the assumption made by Roberts (1984) that there is no discounting is extreme, and in Berliant and Ledyard (2014) it is not clear whether the 'high' discount rate that their conclusion requires is empirically plausible. Also, Roberts (1984) and Berliant and Ledyard (2014) do not consider the effects of other parameters on the relative desirability of separating taxation. ${ }^{6}$ Accordingly, the main objective of this paper is to investigate in much greater detail the conditions under which separating taxation, pooling taxation, or autarky is most desirable. ${ }^{7}$ To this end, we consider a simple two-type (low-skill and high-skill) version of the Mirrlees model, but we extend it to twoperiod and infinite-horizon settings. ${ }^{8}$ For analytical convenience, we further postulate that preferences take the additively-separable form, which allows us to conduct numerical simulations in a more transparent manner. Our main results can be summarized as follows. Under empirically plausible values of the model's parameters, separating taxation is optimal in the two-period model, whereas autarky is optimal in the infinite-horizon model. We then examine how the relative desirability of separating taxation, pooling taxation, and autarky is affected by changes in some key parameters, namely, the degree of wage inequality, the populations of low-skill and high-skill workers, and the discount rate. For reasonable changes in these parameters, it is shown that separating taxation remains optimal in the two-period model, while autarky remains optimal in the infinite-horizon model. Pooling is not optimal in either the two-period or infinite-horizon models for all parameter

[^2]changes considered. We therefore conclude that separating taxation is optimal in two-period settings (as it is in static nonlinear income tax models), but Mirrlees-style taxation is either not feasible or not desirable within infinite-horizon settings when the government cannot commit.

The remainder of the paper is organized as follows. Section 2 describes the analytical framework that we consider. Section 3 describes the structure of the possible tax systems and autarky in the twoperiod model, while Section 4 discusses the corresponding numerical simulations. Section 5 extends the analysis to an infinite-horizon setting, and Section 6 presents numerical simulations for the infinitehorizon case. Section 7 discusses the challenges involved in possibly extending the model to more than two types, while Section 8 concludes.

## 2. Preliminaries

We first consider an economy that lasts for two periods, and then examine an extension to an infinite-horizon setting. There is a unit measure of individuals who live for the duration of the economy, with a proportion $\phi \in(0,1)$ being high-skill workers and the remaining $(1-\phi)$ being low-skill workers. The wage rates of the high-skill and low-skill types are denoted by $w_{H}$ and $w_{L}$, respectively, where $w_{H}>w_{L}>0$. Wages are assumed to remain constant over time and there are no savings by individuals or the government. We make these assumptions so that each period in our model is distinguished only by the extent of skill-type information available to the government.

Individuals have the same preferences over consumption and labor in each period, which are represented by the often-employed additivelyseparable utility function:
$\frac{\left(c_{i}^{t}\right)^{1-\sigma}}{1-\sigma}-\frac{\left(l_{i}^{t}\right)^{1+\gamma}}{1+\gamma}$
where $c_{i}^{t}$ denotes type $i$ 's consumption in period $t, l_{i}^{t}$ denotes type $i$ 's labor supply in period $t$, while $\sigma>0$ and $\gamma>0$ are preference parameters. When $\sigma=1$, the utility function becomes logarithmic in consumption. All individuals discount the future using the discount factor $\delta=\frac{1}{1+r}$, where $r>0$ is the discount rate. Type $i$ 's pre-tax income in period $t$ is denoted by $y_{i}^{t}$, where $y_{i}^{t}=w_{i} l_{i}^{t}$. Since there are no savings, $y_{i}^{t}-c_{i}^{t}$ is equal to total taxes paid by a type $i$ individual in period $t$.

As in the related literature, we assume that the government seeks to maximize social welfare over the duration of the economy and that social welfare is measurable by a utilitarian social welfare function. As the social welfare function will be strictly concave in consumption and leisure, the government will be using its taxation powers to redistribute from high-skill to low-skill individuals. However, the government cannot implement (the first-best) personalized lump-sum taxes in each period under the standard assumption that each individual's skill type is initially private information. In static models of this kind, it is well known that the best the government can do is implement (the second-best) incentive-compatible taxation in which each individual is willing to reveal their type (see, e.g., Stiglitz (1982)). But since our model is dynamic and the government cannot commit, each individual knows that if they reveal their type in period 1 they will be subjected to personalized lump-sum taxation thereafter. This generally implies that high-skill individuals must be offered a relatively favorable tax treatment in period 1 if they are to willingly reveal their type, in order to compensate them for the unfavorable tax treatments they will face in periods $t \geq 2$. From the government's point of view, the lack of redistribution it can undertake in period 1 if skill-type information is to be obtained may be very costly in terms of the level of social welfare attainable. Instead, a higher level of social welfare might be obtained if the government were to pool the individuals in period 1 so that skill-type information is not revealed, even though it is then constrained to use second-best taxation in period 2 in the two-period model or to keep
on pooling in the infinite-horizon model. Likewise, social welfare may actually be higher in the autarkic equilibrium, i.e., no government intervention may be optimal. It is the relative desirability of these alternative tax systems and autarky that we seek to analyze.

## 3. Two-period model: general structure

As our model has two types of individual, there are also two tax systems that the government may consider: (i) implement separating taxation in period 1, and then apply first-best taxation in period 2 , or (ii) implement pooling taxation in period 1 , and then apply standard (second-best) nonlinear income taxation in period 2. In general, whether separating or pooling taxation yields the higher level of social welfare depends upon the parameters of the model, ${ }^{9}$ thus theory cannot rule out either being optimal. The details of these tax systems and autarky are now described.

### 3.1. Autarky

If the government does not intervene in the economy, each individual $i$ will solve the following problem in each period. Choose $c_{i}^{t}$ and $l_{i}^{t}$ to maximize:

$$
\begin{equation*}
\frac{\left(c_{i}^{t}\right)^{1-\sigma}}{1-\sigma}-\frac{\left(l_{i}^{t}\right)^{1+\gamma}}{1+\gamma} \tag{3.1}
\end{equation*}
$$

subject to their period $t$ budget constraint:
$c_{i}^{t} \leq w_{i} l_{i}^{t}$.
The solution to program (3.1)-(3.2) will yield the functions $c_{\bar{i}}$ ${ }^{t}\left(\sigma, \gamma, w_{i}\right)$ and $l_{i}^{t}(\cdot)$. Substituting these functions into Eq. (3.1) yields each type's utility in each period, which can then be used to determine social welfare, which we denote by $W_{A}^{t}\left(\phi, \sigma, \gamma, w_{L}, w_{H}\right)$. Total social welfare in the autarkic equilibrium is equal to $W_{A}^{1}(\cdot)+\delta W_{A}^{2}(\cdot)$.

### 3.2. Separating taxation

If the tax system in period 1 were designed to separate the individuals, the government can use skill-type information revealed in period 1 to implement (the first-best) personalized lump-sum taxes in period 2. In this case, the government's behavior in period 2 can be described as follows. Choose tax treatments $\left\langle c_{L}^{2}, y_{L}^{2}\right\rangle$ and $\left\langle c_{H}^{2}, y_{H}^{2}\right\rangle$ for the low-skill and high-skill individuals, respectively, to maximize:

$$
\begin{equation*}
(1-\phi)\left[\frac{\left(c_{L}^{2}\right)^{1-\sigma}}{1-\sigma}-\frac{\left(\frac{y_{L}^{2}}{w_{L}}\right)^{1+\gamma}}{1+\gamma}\right]+\phi\left[\frac{\left(c_{H}^{2}\right)^{1-\sigma}}{1-\sigma}-\frac{\left(\frac{y_{H}^{2}}{w_{H}}\right)^{1+\gamma}}{1+\gamma}\right] \tag{3.3}
\end{equation*}
$$

subject to:
$(1-\phi)\left(y_{L}^{2}-c_{L}^{2}\right)+\phi\left(y_{H}^{2}-c_{H}^{2}\right) \geq 0$
where Eq. (3.3) is the second-period utilitarian social welfare function, while Eq. (3.4) is the government's second-period budget constraint. ${ }^{10}$ The solution to program (3.3)-(3.4) yields functions for the choice variables $c_{L}^{2}\left(\phi, \sigma, \gamma, w_{L}, w_{H}\right), y_{L}^{2}(\cdot), c_{H}^{2}(\cdot)$, and $y_{H}^{2}(\cdot)$. Substituting these functions into Eq. (3.3) yields the level of social welfare in period 2 when the individuals have been separated in period 1 , which we denote by $W_{S}^{2}(\cdot)$.

[^3]All individuals know that if they reveal their type in period 1, the government will solve program (3.3)-(3.4) in period 2. Therefore, in order to induce each individual to reveal his/her type in period 1 , the government chooses tax treatments $\left\langle c_{L}^{1}, y_{L}^{1}\right\rangle$ and $\left\langle c_{H}^{1}, y_{H}^{1}\right\rangle$ to maximize:

$$
\begin{equation*}
(1-\phi)\left[\frac{\left(c_{L}^{1}\right)^{1-\sigma}}{1-\sigma}-\frac{\left(\frac{y_{L}^{1}}{w_{L}}\right)^{1+\gamma}}{1+\gamma}\right]+\phi\left[\frac{\left(c_{H}^{1}\right)^{1-\sigma}}{1-\sigma}-\frac{\left(\frac{y_{H}^{1}}{w_{H}}\right)^{1+\gamma}}{1+\gamma}\right] \tag{3.5}
\end{equation*}
$$

subject to:

$$
\begin{equation*}
(1-\phi)\left(y_{L}^{1}-c_{L}^{1}\right)+\phi\left(y_{H}^{1}-c_{H}^{1}\right) \geq 0 \tag{3.6}
\end{equation*}
$$

$$
\begin{align*}
& \frac{\left(c_{H}^{1}\right)^{1-\sigma}}{1-\sigma}-\frac{\left(\frac{y_{H}^{1}}{w_{H}}\right)^{1+\gamma}}{1+\gamma}+\delta\left[\frac{\left(c_{H}^{2}(\cdot)\right)^{1-\sigma}}{1-\sigma}-\frac{\left(\frac{y_{H}^{2}(\cdot)}{w_{H}}\right)^{1+\gamma}}{1+\gamma}\right] \geq \frac{\left(c_{L}^{1}\right)^{1-\sigma}}{1-\sigma}-\frac{\left(\frac{y_{L}^{1}}{w_{H}}\right)^{1+\gamma}}{1+\gamma} \\
& \quad+\delta\left[\frac{\left(c_{L}^{2}(\cdot)\right)^{1-\sigma}}{1-\sigma}-\frac{\left(\frac{y_{L}^{2} \cdot(\cdot)}{w_{H}}\right)^{1+\gamma}}{1+\gamma}\right] \tag{3.7}
\end{align*}
$$

where Eq. (3.5) is the first-period social welfare, Eq. (3.6) is the government's first-period budget constraint, and Eq. (3.7) is the highskill type's incentive-compatibility constraint. ${ }^{11}$ To ensure that highskill individuals choose $\left\langle c_{H}^{1}, y_{H}^{1}\right\rangle$ rather than $\left\langle c_{L}^{1}, y_{L}^{1}\right\rangle$, the utility they obtain in period 1 from choosing $\left\langle c_{H}^{1}, y_{H}^{1}\right\rangle$ and thus revealing their type, plus the utility they obtain from $\left\langle c_{H}^{2}, y_{H}^{2}\right\rangle$ which they are forced to accept in period 2 , must be greater than or equal to the utility they could obtain in period 1 from choosing $\left\langle c_{L}^{1}, y_{L}^{1}\right\rangle$ and thus pretending to be low skill, plus the utility they would then obtain from the low-skill tax treatment $\left\langle c_{L}^{2}, y_{L}^{2}\right\rangle$ in period 2.

The solution to program (3.5)-(3.7) yields the functions $c_{L}^{1}(\phi,-$ $\left.\sigma, \gamma, w_{L}, w_{H}, \delta\right), y_{L}^{1}(\cdot), c_{H}^{1}(\cdot)$, and $y_{H}^{1}(\cdot)$. Substituting these functions into Eq. (3.5) yields the level of social welfare in period 1 under separating taxation, which we denote by $W_{S}^{1}(\cdot)$. Total social welfare under separating taxation is then equal to $W_{S}^{1}(\cdot)+\delta W_{S}^{2}(\cdot)$.

### 3.3. Pooling taxation

If the individuals were pooled in the first period, the government has no skill-type information that it can use in the second period. Therefore, because period 2 is the last period, the government simply solves a standard static optimal nonlinear income tax problem. That is, the government chooses tax treatments $\left\langle c_{L}^{2}, y_{L}^{2}\right\rangle$ and $\left\langle c_{H}^{2}, y_{H}^{2}\right\rangle$ to maximize:
$(1-\phi)\left[\frac{\left(c_{L}^{2}\right)^{1-\sigma}}{1-\sigma}-\frac{\left(\frac{y_{L}^{2}}{w_{L}}\right)^{1+\gamma}}{1+\gamma}\right]+\phi\left[\frac{\left(c_{H}^{2}\right)^{1-\sigma}}{1-\sigma}-\frac{\left(\frac{y_{H}^{2}}{w_{H}}\right)^{1+\gamma}}{1+\gamma}\right]$
subject to:
$(1-\phi)\left(y_{L}^{2}-c_{L}^{2}\right)+\phi\left(y_{H}^{2}-c_{H}^{2}\right) \geq 0$
$\frac{\left(c_{H}^{2}\right)^{1-\sigma}}{1-\sigma}-\frac{\left(\frac{y_{H}^{2}}{w_{H}}\right)^{1+\gamma}}{1+\gamma} \geq \frac{\left(c_{L}^{2}\right)^{1-\sigma}}{1-\sigma}-\frac{\left(\frac{y_{L}^{2}}{w_{H}}\right)^{1+\gamma}}{1+\gamma}$
where Eq. (3.8) is the second-period social welfare, Eq. (3.9) is the government's second-period budget constraint, and Eq. (3.10) is the

[^4]Table 1
Baseline parameter values for numerical simulations: two-period model ${ }^{*}$.

| $\varphi$ | 0.30 | $\delta$ | 0.46 | $w_{L}$ | 1.00 |
| :--- | :--- | :--- | :--- | :--- | :--- |
| $\sigma$ | 1.00 | $r$ | 0.04 | $w_{H}$ | 1.60 |
| $\gamma$ | 2.00 |  |  |  |  |
|  |  | Separating | Pooling | Autarky |  |
|  | -0.189 | -0.248 | -0.192 |  |  |
|  |  | -0.155 | -0.170 | -0.192 |  |
| First-period social welfare | -0.260 | -0.325 | -0.280 |  |  |
| Second-period social welfare |  |  |  |  |  |
| Discounted total |  |  |  |  |  |

* Each period is assumed to be 20 years in length. The levels of social welfare are the values of the utilitarian social welfare function under each tax regime. For example, under separating taxation, first-period social welfare ( -0.189 ) corresponds to Eq. (3.5), secondperiod social welfare $(-0.155)$ corresponds to Eq. (3.3), and the discounted total $(-0.260)$ is first-period social welfare plus the discounted value (using $\delta=0.46$ ) of second-period social welfare.
high-skill type's incentive-compatibility constraint. The solution to program (3.8)-(3.10) yields the functions $c_{L}^{2}\left(\phi, \sigma, \gamma, w_{L}, w_{H}\right), y_{L}^{2}(\cdot), c_{H}^{2}(\cdot)$, and $y_{H}^{2}(\cdot)$. Substituting these functions into Eq. (3.8) yields the level of social welfare in period 2 after the individuals were pooled in period 1 , which we denote by $W_{P}^{2}(\cdot)$.

If the government decides to pool the individuals in period 1 , it chooses a single tax treatment $\left\langle c^{1}, y^{1}\right\rangle$ for both types to maximize first-period social welfare:
$(1-\phi)\left[\frac{\left(c^{1}\right)^{1-\sigma}}{1-\sigma}-\frac{\left(\frac{y^{1}}{w_{L}}\right)^{1+\gamma}}{1+\gamma}\right]+\phi\left[\frac{\left(c^{1}\right)^{1-\sigma}}{1-\sigma}-\frac{\left(\frac{y^{1}}{w_{H}}\right)^{1+\gamma}}{1+\gamma}\right]$
subject to the first-period budget constraint:
$y^{1}-c^{1} \geq 0$.
As the budget constraint will bind, the solution to program (3.11)-(3.12) will involve $c^{1}=y^{1}=y^{1}\left(\phi, \sigma, \gamma, w_{L}, w_{H}\right)$. Substituting this function into Eq. (3.11) yields the level of social welfare in period 1 under pooling, which we denote by $W_{P}^{1}(\cdot)$. Total social welfare under pooling is equal to $W_{P}^{1}(\cdot)+\delta W_{P}^{2}(\cdot)$.

## 4. Two-period model: numerical simulations

As the relative desirability of each tax system and autarky depends upon the model's parameters, we begin by identifying a set of baseline parameter values that are reasonable. These are presented in Table 1. The OECD (2013) reports that on average across OECD countries, approximately $30 \%$ of all adults have attained tertiary level education. We therefore assume that $30 \%$ of individuals are high-skill workers, i.e., we set $\phi=0.3$. Chetty (2006) concludes that a reasonable estimate of the coefficient of relative risk aversion is one. We therefore set $\sigma=1$, so that the utility function becomes logarithmic in consumption. Furthermore, we set $\gamma=2$ as this implies a labor supply elasticity of $0.5,{ }^{12}$ which is consistent with empirical estimates (see, e.g., Chetty et al. (2011)). We assume an annual discount rate of $4 \%$, which is in line with standard practice. As most individuals work for around 40 years of their lives, we take each period to be 20 years in length. ${ }^{13}$ An annual discount rate of $4 \%$ then corresponds to a 20 -year discount factor of $\delta=0.46$. Fang (2006) and Goldin and Katz (2007) estimate that the college wage premium, i.e., the average difference in the wages of university graduates over high-school graduates, is approximately $60 \%$. We therefore normalize the low-skill type's wage rate to

[^5]Table 1.1
High-skill country parameter values: two-period model.

| $\varphi$ | 0.50 | $\delta$ | 0.67 | $w_{L}$ | 1.00 |
| :--- | :--- | :--- | :--- | :--- | :--- |
| $\sigma$ | 1.00 | $r$ | 0.02 | $w_{H}$ | 1.40 |
| $\gamma$ | 2.00 |  |  |  |  |
|  |  | Separating | Pooling | Autarky |  |
|  |  | -0.173 | -0.206 | -0.165 |  |
|  |  | -0.144 | -0.154 | -0.165 |  |
| First-period social welfare | -0.270 | -0.310 | -0.276 |  |  |
| Second-period social welfare |  |  |  |  |  |

unity ( $w_{L}=1$ ), and set the high-skill type's wage rate at $w_{H}=1.6$. For these parameter values, Table 1 shows that separating taxation is social-welfare maximizing, autarky is ranked second, while pooling is third. Pooling is worse than autarky, even though pooling in period 1 allows standard nonlinear income taxation to be used in period 2 (which is better than autarky). However, pooling in the first period is very costly, as reflected in the low level of social welfare.

To obtain further insights about the preceding quantitative results, ${ }^{14}$ Table 1.1 presents parameter values for "high-skill" countries, i.e., countries in which a large proportion of individuals have attained tertiary education. Based on the OECD general statistics, around $50 \%$ of all adults in the Russian Federation, Japan, and Canada have attained tertiary level education, so we set $\phi=0.5$. There is also a body of evidence which suggests that high-skill individuals tend to be more patient than low-skill individuals, ${ }^{15}$ so for these countries we use a lower discount rate (2\%). Finally, it is natural to expect that the skill premium is lower in countries which have a large proportion of skilled individuals, so for these countries we set $w_{H}=1.4$. As it turns out, Table 1.1 shows that separating taxation remains social-welfare maximizing, autarky remains second, while pooling remains third. Analogously, countries such as Brazil, Turkey, and Italy are "low-skill" in that a lower proportion (around $15 \%$ ) of adults have attained tertiary level education. For these countries, we set $\phi=0.15$, and use a higher discount rate (6\%) and a higher wage premium ( $w_{H}=1.8$ ). Table 1.2 shows that for these countries separating taxation is still social-welfare maximizing, autarky is still second, and pooling is still third.

Fig. 1 shows the effects of relatively large variations in the size of the high-skill population ( $\phi$ ), the discount rate ( $r$ ), and the wage premium $\left(w_{H}\right)$, while holding all other parameters at their baseline levels reported in Table 1. The social-welfare ranking of separating taxation, autarky, and pooling remains unchanged for the variations considered, except for when $w_{H}=1.2$ in which case autarky becomes (marginally) the best option. Separation increases its advantage over autarky and pooling as $\phi$ increases. An increase in $\phi$ implies that high-skill individuals receive a greater weight in the social welfare function, which means redistribution in period 2 under first-best taxation becomes less severe. This in turn implies that high-skill individuals require less compensation in period 1 to reveal their type, thus making separation more attractive. Autarky also increases its advantage over pooling as $\phi$ increases. Increases in $\phi$ exacerbate the redistributive inefficiency of pooling in period 1 , since the greater weight high-skill individuals receive in the social welfare function, combined with the pooling restriction that both types receive the same allocation, imply that high-skill individuals are made better-off and low-skill individuals are made worse-off in period 1. This inefficiency is partly reversed in period 2 when nonlinear income taxation is used after pooling, since nonlinear income taxation is effective in achieving redistribution. But the benefit is not sufficient to overcome the increased inefficiency of pooling in the first period.

Higher values of $r$ increase the advantage that separation has over autarky and pooling. As $r$ increases, high-skill individuals become less concerned with the low level of utility they obtain under first-best

[^6]Table 1.2
Low-skill country parameter values: two-period model.

| $\varphi$ | 0.15 | $\delta$ | 0.31 | $w_{L}$ | 1.00 |
| :--- | :--- | :--- | :--- | :--- | :--- |
| $\sigma$ | 1.00 | $r$ | 0.06 | $w_{H}$ | 1.80 |
| $\gamma$ | 2.00 |  |  |  |  |
|  |  | Separating | Pooling | Autarky |  |
|  |  | -0.234 | -0.289 | -0.245 |  |
|  |  | -0.205 | -0.221 | -0.245 |  |
| First-period social welfare | -0.298 | -0.358 | -0.322 |  |  |
| Second-period social welfare |  |  |  |  |  |

taxation in period 2 . Accordingly, the utility they require in period 1 as compensation for revealing their type decreases, making separation less costly. Increases in $r$ also make autarky more attractive than pooling. Since pooling in period 1 is less desirable than autarky, but nonlinear income taxation in period 2 is better than autarky, increases in $r$ make pooling in period 1 along with nonlinear income taxation in period 2 less attractive because an increase in $r$ implies a relatively higher concern for first-period social welfare and a lower concern for secondperiod social welfare.


Fig. 1. Social welfare: two-period model.

Separation increases its advantage over autarky and pooling as $w_{H}$ increases. Given the government's redistributive concerns, autarky in both periods or pooling in period 1 along with standard nonlinear income taxation in period 2 are not as powerful as separating the individuals in period 1 and then being able to use first-best taxation in period 2 . Moreover, the relative desirability of separating taxation is naturally increasing in the degree of wage inequality, since the need for redistribution rises. As $w_{H}$ increases, autarky is also increasingly preferred to pooling. On the one hand, an increase in wage inequality exacerbates the inefficiency of pooling in period 1 , but on the other hand higher wage inequality increases the desirability of using nonlinear income taxation in period 2 . However, on balance our numerical simulations indicate that pooling becomes increasingly less desirable than autarky as $w_{H}$ increases.

## 5. Infinite-horizon model: general structure

In this section, we describe how the general structure of autarky, separating taxation, and pooling taxation changes when the model is extended from two periods to an infinite-horizon setting.

### 5.1. Autarky

If the government does not intervene, individuals will solve program (3.1)-(3.2) in each period. Total social welfare under autarky is therefore simply equal to $\sum_{t=1}^{\infty} \delta^{t-1} W_{A}^{t}(\cdot)$.

### 5.2. Separating taxation

If the individuals were separated in period 1 , the government can implement personalized lump-sum taxation from period 2 onwards. That is, the government will solve program (3.3)-(3.4) in periods $2, \ldots, \infty$. Let $u_{i F}^{t}\left(\phi, \sigma, \gamma, w_{L}, w_{H}\right)$ denote the utility type $i$ obtains under first-best taxation in each period, let $\hat{u}_{H F}^{t}(\cdot)$ denote the utility the highskill type obtains from the low-skill type's first-best tax treatment in each period, and let $W_{F}^{t}(\cdot)$ denote the level of social welfare under first-best taxation in each period.

If skill-type information is revealed in period 1 , everyone knows that the government will solve program (3.3)-(3.4) in periods $2, \ldots, \infty$. Therefore, in order to induce individuals to reveal their types in period 1 , the government chooses tax treatments $\left\langle c_{L}^{1}, y_{L}^{1}\right\rangle$ and $\left\langle c_{H}^{1}, y_{H}^{1}\right\rangle$ for the low-skill and high-skill types, respectively, to maximize:

$$
\begin{equation*}
(1-\phi)\left[\frac{\left(c_{L}^{1}\right)^{1-\sigma}}{1-\sigma}-\frac{\left(\frac{y_{L}^{1}}{w_{L}}\right)^{1+\gamma}}{1+\gamma}\right]+\phi\left[\frac{\left(c_{H}^{1}\right)^{1-\sigma}}{1-\sigma}-\frac{\left(\frac{y_{H}^{1}}{w_{H}}\right)^{1+\gamma}}{1+\gamma}\right] \tag{5.1}
\end{equation*}
$$

subject to:

$$
\begin{equation*}
(1-\phi)\left(y_{L}^{1}-c_{L}^{1}\right)+\phi\left(y_{H}^{1}-c_{H}^{1}\right) \geq 0 \tag{5.2}
\end{equation*}
$$

$\frac{\left(c_{H}^{1}\right)^{1-\sigma}}{1-\sigma}-\frac{\left(\frac{y_{H}^{1}}{w_{H}}\right)^{1+\gamma}}{1+\gamma}+\sum_{t=2}^{\infty} \delta^{t-1} u_{H F}^{t}(\cdot) \geq \frac{\left(c_{L}^{1}\right)^{1-\sigma}}{1-\sigma}-\frac{\left(\frac{y_{l}^{1}}{w_{H}}\right)^{1+\gamma}}{1+\gamma}+\sum_{t=2}^{\infty} \delta^{t-1} \hat{u}_{H F}^{t}(\cdot)$
where Eq. (5.1) is the first-period social welfare, Eq. (5.2) is the government's first-period budget constraint, and Eq. (5.3) is the highskill type's incentive-compatibility constraint. If high-skill individuals are willing to reveal their type, the utility they obtain from choosing $\left\langle c_{H}^{1}, y_{H}^{1}\right\rangle$ in period 1 and thus revealing their type, plus the discounted sum of utilities they obtain under first-best taxation from period 2 onwards, must be greater than or equal to the utility they could obtain by pretending to be low skill. The solution to program (5.1)-(5.3) yields

Table 2
Baseline parameter values for numerical simulations: infinite-horizon model*.

| $\varphi$ | 0.30 | $\delta$ | 0.96 | $w_{L}$ | 1.00 |
| :--- | :--- | :---: | :---: | :---: | :---: |
| $\sigma$ | 1.00 | $r$ | 0.04 | $w_{H}$ | 1.60 |
| $\gamma$ | 2.00 |  |  |  |  |
|  |  | Separating |  | Pooling |  |
|  |  | Not feasible |  | -6.438 | Autarky |
| Social welfare |  |  |  | -5.001 |  |

* Each period is assumed to be one year in length.
functions for the choice variables $c_{L}^{1}\left(\phi, \sigma, \gamma, w_{L}, w_{H}, \delta\right), y_{L}^{1}(\cdot), c_{H}^{1}(\cdot)$, and $y_{H}^{1}(\cdot)$. Substituting these functions into Eq. (5.1) yields the level of social welfare in period 1 under separating taxation, which we denote by
$W_{S}^{1}(\cdot)$. Total social welfare under separating taxation is then equal to $W_{S}^{1}$
$(\cdot)+\sum_{t=2}^{\infty} \delta^{t-1} W_{F}^{t}(\cdot)$.


### 5.3. Pooling taxation

In the two-period model, the government can solve a standard optimal nonlinear income tax problem in period 2 after pooling in period 1 , because there are no later periods in which the government can take advantage of skill-type information acquired in period 2. In the infinite-horizon model, however, there is no last period in which the government can solve a standard nonlinear income tax problem. Therefore, pooling in the infinite-horizon model means pooling in every period, i.e., the government solves program (3.11)-(3.12) in each period. Total social welfare under pooling is therefore equal to $\sum_{t=1}^{\infty} \delta^{t-1} W_{P}^{t}\left(\phi, \sigma, \gamma, w_{L}, w_{H}\right)$, where $W_{P}^{t}(\cdot)$ is the level of social welfare associated with program (3.11)-(3.12).

### 5.4. Pooling and separating taxation

In the infinite-horizon model, there exists a third tax system in which the government uses pooling taxation for $T-1$ periods (where $T \geq 2$ ), separates the individuals in period $T$, and then implements first-best taxation from period $T+1$ onwards. In this case, total social welfare with separation in period $T$ can be written as:
$W_{S e p}^{T}=\sum_{t=1}^{T-1} \delta^{t-1} W_{P}^{t}(\cdot)+\delta^{T-1} W_{S}^{T}(\cdot)+\sum_{t=T+1}^{\infty} \delta^{t-1} W_{F}^{t}(\cdot)$
where $W_{P}^{t}(\cdot)$ is again the level of social welfare associated with pooling taxation in period $t, W_{S}^{T}(\cdot)$ is the level of social welfare in the separating period $T$, and $W_{F}^{t}(\cdot)$ is again the level of social welfare under first-best taxation in period $t$.

## 6. Infinite-horizon model: numerical simulations

Table 2 presents baseline parameter values for our infinite-horizon model. These are identical to those for the two-period model, except that we now take each period to be one year in length. This implies that an annual discount rate of $4 \%$ corresponds to a one-year discount factor of $\delta=0.96$. For the baseline parameter values, separating

Table 2.1
High-skill country parameter values: infinite-horizon model.

| $\varphi$ | 0.50 | $\delta$ | 0.98 | $w_{L}$ | 1.00 |
| :--- | :--- | :--- | :--- | :--- | ---: |
| $\sigma$ | 1.00 | $r$ | 0.02 | $w_{H}$ | 1.40 |
| $\gamma$ | 2.00 |  |  |  |  |
|  |  | Separating |  | Pooling | Autarky |
| Social welfare | Not feasible |  | -10.499 | -8.420 |  |

Table 2.2
Low-skill country parameter values: infinite-horizon model.

| $\varphi$ | 0.15 | $\delta$ | 0.94 | $w_{L}$ | 1.00 |
| :--- | :---: | :---: | :---: | :---: | ---: |
| $\sigma$ | 1.00 | $r$ | 0.06 | $w_{H}$ | 1.80 |
| $\gamma$ | 2.00 |  |  |  |  |
|  |  | Separating |  | Pooling | Autarky |
| Social welfare | Not feasible | -5.107 | -4.331 |  |  |

taxation is not feasible. That is, the compensation high-skill individuals would require for revealing their type in period 1 and forever-after facing personalized lump-sum taxation is so large that it would necessitate that low-skill individuals face an average tax rate in period 1 of more than $100 \%$. Likewise, pooling for $T-1$ periods before separating in period $T$ is not feasible because this case also involves high-skill individuals facing first-best taxation for an infinite number of periods. The intuition is similar to that for the result of Roberts (1984) that separation never occurs if there is no discounting and the time horizon is infinite. Therefore, the only options available to the government in the infinitehorizon model are to pool the individuals in every period or to not intervene, thus allowing the autarkic equilibrium to be realized. Table 2 shows that since pooling is extreme in that it imposes the same consumption/pre-tax income allocation on both types, the autarkic solution is better in the infinite-horizon model. Similar to the additional analyses in Section 4, Tables 2.1 and 2.2 present the results using parameter values for "high-skill" and "low-skill" countries, respectively. For these countries, it is still the case that separating taxation is not possible, and that autarky is better than pooling.

Fig. 2 shows the effects of varying the high-skill population $\phi$, the discount rate $r$, and the wage premium $w_{H}$ on the relative desirability of autarky and pooling. Autarky remains clearly better than pooling for all variations considered. ${ }^{16}$ When $w_{H}=1.2$ separating taxation becomes possible, but as the bottom panel of Fig. 2 shows that it yields a level of social welfare that is much lower than that under autarky and pooling, and therefore separating taxation would never be implemented.

## 7. Discussion: more than two types

Extending our model to more than two types is quite challenging because the number of tax regimes that must be considered increases exponentially. For example, considering the economy with three types (low-skill, middle-skill, and high-skill) means that there are five possibilities in the two-period setting: (1) implement separating taxation in period 1 , and then apply first-best taxation in period 2 ; (2) implement pooling taxation in period 1 , and then apply second-best taxation in period 2; (3) pool the low-skill and middle-skill types in period 1, and then apply first-best taxation to high-skill individuals in period 2; (4) pool the middle-skill and high-skill individuals in period 1, and then apply first-best taxation to low-skill individuals in period 2; and (5) pool the low-skill and high-skill individuals in period 1, and then apply firstbest taxation to middle-skill individuals in period 2 . Which of these tax systems is socially optimal will, again, generally depend upon the parameters of the model.

Nevertheless, we have examined the three-type case, and found that the main findings are qualitatively the same as in the two-type model. That is, separating taxation is optimal in the two-period model, while autarky is optimal when the time horizon is infinite. The tax systems that involve complete or partial pooling are not optimal for all reasonable changes in the model's parameters. ${ }^{17}$ Accordingly, our main

[^7]

Fig. 2. Social welfare: infinite-horizon model.
conclusion that separating taxation is optimal in two-period settings, but Mirrlees-style taxation is either not feasible or not desirable in infinite-horizon settings, remains intact when there are three types of individual.

## 8. Summary and conclusion

A fundamental feature of the Mirrlees approach to nonlinear income taxation is that the optimal tax system is determined endogenously and no ad hoc constraints are placed on its design. As individuals are assumed to be distinguished only by their skill levels, it is only the government's inability to observe each individual's skill type that prevents it from implementing (the first-best) personalized lump-sum taxation. Given recent interest in dynamic Mirrlees taxation, it is natural to examine the implications of the government obtaining skill-type information over time. Most of the literature has assumed that the government can commit to ignore this information, which seems to be inconsistent with the spirit of the Mirrlees approach in that the tax system is determined endogenously.

Analyzing dynamic nonlinear income taxation without commitment presents a number of challenges, the main one being that the optimal tax system may involve separation of types to obtain skill-type information, or pooling of types to prevent skill-type information being revealed. Accordingly, in a model with many types and many periods, there are numerous tax systems to be considered. To make the problem manageable, this paper has assumed that there are only two types, and examined just two-period and infinite-horizon settings. This means, of course, that we have only considered a part of the general problem, but nevertheless we think that the key insights obtained from our analysis will carry over to more general settings. In the two-period model we show that separating taxation is optimal, as it is when the government can commit. However, social welfare in the 'present' (period 1 ) is much lower than in the 'future' (period 2). Indeed, this is true for both separating and pooling taxation. Extending the model to an infinite-horizon setting allows the effects of the 'very long run' to be considered. In this setting, we find that the government cannot improve upon the autarkic equilibrium; that is, the optimal policy is to do nothing. For empirically plausible parameter values, separating taxation is not possible because once a high-skill individual reveals her type, she will forever-after face personalized lump-sum taxation. Therefore, high-skill individuals require substantial compensation in the short run to reveal their type, but such compensation is too excessive to make separating taxation feasible. The choice then reduces to pooling taxation versus autarky, and our numerical experiments show that autarky is clearly better.

There are a number of possible extensions of our paper, but the one we think most worth pursuing is the introduction of savings by individuals and/or the government. By ignoring savings, our paper has the advantage of isolating the effects of the possible revelation and use of skill-type information from the effects of any other dynamic links. A key finding is that implementation of either separating or pooling taxation involves a short-run welfare cost. Savings would allow agents to transfer resources over time, and therefore might be used to bring forward some of the benefits and delay some of the costs associated with each tax system. However, attempts by governments to bring forward some of the benefits (through government debt) may be undermined by individuals' savings behavior, which in turn raises a number of interesting issues regarding the optimal taxation of savings. These seem particularly interesting avenues for future research.

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    * Corresponding author. Tel.: +441904323 572; fax: +44 1904323759.

    E-mail addresses: guojt@ucr.edu (J.-T. Guo), alan.krause@york.ac.uk (A. Krause).
    ${ }^{1}$ Tel.: +1 951827 1588; fax: +1 19518275685 .
    ${ }^{2}$ Surveys of the new dynamic public finance literature are provided by Golosov et al. (2006) and Golosov et al. (2011). For a textbook treatment, see Kocherlakota (2010).

[^1]:    ${ }^{3}$ Indeed, one of the motivations behind the new dynamic public finance literature is to remove the need for ad hoc constraints on the tax instruments available to the government, which must be imposed in standard macro-style dynamic models. See Golosov et al. (2006) for further discussion.
    ${ }^{4}$ To be fair, one could argue in favor of the commitment assumption on the basis that real-world tax systems are not frequently redesigned. Gaube (2007), for example, makes this argument.

[^2]:    ${ }^{5}$ This issue is not specific to nonlinear income taxation, but also applies to other applications of principal-agent theory in dynamic settings. See, e.g., Laffont and Martimort (2002, Section 9.3) for further discussion.
    ${ }^{6}$ Similar nonlinear income tax models without commitment have been used by Apps and Rees (2006), Bisin and Rampini (2006), Brett and Weymark (2008), Krause (2009), and Guo and Krause (2011, 2013, 2015), among others. However, none of these papers addresses the issue of whether separating or pooling is optimal, with most simply considering in turn separating and pooling taxation.
    ${ }^{7}$ While the existing literature has assumed either full commitment or no commitment, Guo and Krause (2014) consider a third possibility of 'loose commitment' in which the government can commit only with some probability. Specifically, they examine a twotype infinite-horizon model in which loose commitment is modeled as a Markov switching process, whereby in each period there is some probability that the government can and cannot commit. However, it is assumed that the probability of commitment is sufficiently high so that separating taxation is always optimal, as under full commitment, and the focus is on how loose commitment affects the utility of low-skill individuals versus high-skill individuals.
    ${ }^{8}$ Since much of the related literature has considered either two-period or infinitehorizon settings, we also focus on these time horizons.

[^3]:    ${ }^{9}$ There does exist an additional tax system in which the government plays a 'mixed strategy' by pooling a fraction of the high-skill individuals with the low-skill individuals. However, for the sake of simplicity we do not consider this mixed tax system.
    ${ }^{10}$ We normalize the government's revenue requirement to zero, so the tax system is purely redistributive.

[^4]:    ${ }^{11}$ We omit the low-skill type's incentive-compatibility constraint, because the government's redistributive goals imply that it will not be binding. We will continue to omit the low-skill type's incentive-compatibility constraint throughout the remainder of the paper.

[^5]:    ${ }^{12}$ To see this, note that the first-order conditions corresponding to program (3.1)-(3.2) can be manipulated to yield $\left(c_{i}^{t}\right)^{\sigma}\left(l_{i}^{t}\right)^{\gamma}=w_{i}$ or $\sigma \ln \left(c_{i}^{t}\right)+\gamma \ln \left(l_{i}^{t}\right)=\ln \left(w_{i}\right)$, which implies a labor supply elasticity of $1 / \gamma$.
    ${ }^{13}$ Kocherlakota (2010) also assumes that each period consists of twenty years when individuals work for two periods.

[^6]:    ${ }^{14}$ We thank an anonymous referee for making this suggestion.
    ${ }^{15}$ See, for example, Diamond and Spinnewijn (2011) and Golosov et al. (2013).

[^7]:    ${ }^{16}$ When there is no discounting $(r=0)$, aggregate social welfare over the infinite horizon is not defined.
    ${ }^{17}$ Details for the results with three types of individual are available upon request.

