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Author

Heard, Harry G.

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BEVATRON BEAM INDUCTION ELECTRODES

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UNIVERSITY OF CALIFORNIA
RADIATION LABORATORY

October 11, 1957

ERRATUM

To: Physics Distribution

Re: UCRL-3609, "Bevatron Beam Induction Electrodes" by Harry G. Heard

Please make the following changes in your copies of UCRL-3609:

Page 11, line 22 (last equation on page) should read

$$2A_1 = 2 \left[1.86 \times 1.63 \times 10^{-12} \times (10^{10}) \right] \text{ k volts} \approx 60 \frac{\text{millivolts peak to peak}}{10^{10} \text{ protons}}$$

Page 16, Fig. 7 (about center of figure)

Now reads: 160Ω , $1/2 \text{ w}$

Should read: $16\text{k}\Omega$, $1/2 \text{ w}$

Page 22, line 11 should read

$$R_0 = r_e + (r_c - r_m) \left[1 - \frac{r_c}{r_b + r_c + R_g} \right] \approx r_c (1 - a) \frac{1}{1 + r_c/R_g}$$

Page 22, line 12 should read

$$\text{For } R_g \approx 0, \quad R_0 \sim r_e + r_b (1 - a)$$

Page 22, line 14 (last paragraph)

Delete first sentence in paragraph -- "Since circuit."

Page 23, column 4 of the table should read

R_0
(ohms)
30
48
226
2,000
20,000

UCRL-3609

UNIVERSITY OF CALIFORNIA

Radiation Laboratory
Berkeley, California

Contract No. W-7405-eng-48

BEVATRON BEAM INDUCTION ELECTRODES

Harry G. Heard

February 12, 1957

Printed for the U.S. Atomic Energy Commission

BEVATRON BEAM INDUCTION ELECTRODES

Harry G. Heard

Radiation Laboratory
University of California
Berkeley, California

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ABSTRACT

A complete description is given of the nondestructive beam-induction electrode system used in the Bevatron to detect the magnitude and radial position of the primary proton beam. Design principles, frequency response, ultimate sensitivity, monitoring, and calibration techniques are disclosed.

A brief account is given of the problems associated with the development of induction electrodes. A quarter-scale model was used to develop a special electrode configuration which yielded a sum signal independent (within 6%) of beam position, and a maximum-output signal for small deviations of the proton beam from the radial centerline of the aperture.

The output signal of the beam-induction electrodes is monitored by a transistor connected as an emitter follower. The transistor is affixed to the induction electrode inside the vacuum tank of the Bevatron. An analysis is given which delineates the design principles involved in the transistor circuitry.

BEVATRON BEAM INDUCTION ELECTRODES

Harry G. Heard

Radiation Laboratory
University of California
Berkeley, California

February 12, 1957

INTRODUCTION

Protons are injected into the aperture of the Bevatron magnet by means of an 18-foot-radius 35° electrostatic analyzer located at a radius of 620 inches. Neglecting the bunching effect of the 202-Mc linear accelerator, a 500-microsecond pulse of 9.8-Mev protons, of nearly constant energy, starts to spiral into the aperture when the average field of the Bevatron magnet rises to approximately 297 gauss. Since the magnetic field is increasing at a rate of 8000 gauss/sec, the protons entering the aperture late in the injection cycle execute radial betatron oscillations having amplitudes as large as 20 to 25 inches. Vertical betatron oscillation amplitudes also develop during the injection period. They are mainly due to the divergence of the injected beam, gas scattering, and a number of slight defects in the magnet. By the time the acceleration cycle starts, the injected beam is quite diffuse, and has an approximate cross section of 1 by 4 feet. Since the injected beam is not bunched, its presence within the aperture can be detected only by charge collectors, fluorescent flags, or targets and counters. These methods of beam detection destroy the circulating beam and therefore preclude measurements during the acceleration cycle.

Energy is added to the circulating protons in the Bevatron by means of an 11-foot drift tube located in the north straight section. The radio-frequency voltage is applied to the drift tube as soon as the magnetic field rises to a value such that the instantaneous orbit for 9.8-Mev protons corresponds to a radius of $599\text{-}\frac{3}{8}$ inches, the centerline of the chamber. At the time the oscillator is turned on its frequency is approximately 354 kilocycles, corresponding to the period of rotation of $\beta = 0.145$ protons having a 394-foot orbit. When the oscillator is turned on, the aperture may be considered to be filled with protons having the same instantaneous orbit and all possible radial amplitudes from zero to the half width of the chamber. Only those particles within a stable azimuthal range of approximately 180° will start the acceleration cycle and become bunched; others are lost. Because of radial oscillations in the equilibrium orbit, additional losses will occur owing to collisions with aperture-limiting structures. The bunched protons will gain approximately 2 kilovolts of energy per revolution as they circulate through the drift tube. Oscillations in azimuth and radius occur, as not all the particles arrive at exactly the optimum phase with respect to the drift-tube voltage. Protons arriving at different phases receive energy increments different from the value necessary to maintain their orbit centered in the aperture. Therefore oscillations in phase occur. Only those particles whose phase excursions remain within phase-stable limits continue to be accelerated. At the end of $1\text{-}\frac{3}{4}$ sec the magnetic field has increased to 15,550 gauss; the radio-frequency has been increased to 2.5 Mc so that the 6.2-Bev particles remain nearly centered in the aperture.

BEAM DETECTION

There are two broad classes of schemes that may be used to detect the beam during the acceleration cycle: those which destroy the beam-- such as targets, fluorescent flags, and collectors--and nondestructive methods based upon electromagnetic interactions. It is with this latter method of detection that this report is primarily concerned. Either magnetic or electric induction may be used to detect the presence of the circulating beam, provided it is bunched. Although magnetic induction may be used for beam detection, there are three practical difficulties that make this approach unattractive. First, the induced emf varies as the square of the rotational period of the bunch. Therefore the available signal is small in the nonrelativistic portion of the acceleration cycle. Second, the output voltage of a pickup coil is reduced at the high-energy end of the acceleration cycle owing to the self-capacitance of the coil. Finally, the signal-to-noise ratio is poor owing to induced voltage from the pulsed magnet of the accelerator.

An electric induction system also has certain practical disadvantages. As it is a high-impedance monitoring system, it must be carefully shielded to reduce coupling to radio-frequency voltages and to induced voltages resulting from ignitron switching transients. The structure must be physically large if it is to be sensitive to circulating beam intensities as low as 10^5 protons. In addition, the design of the leads as well as the electrodes must be such that the beam bunches will not excite resonances in the system. Despite these disadvantages, the electric induction method of detection has been found to be quite satisfactory. It may be used for a continuous determination of beam intensity during the acceleration cycle, thereby facilitating observations of beam losses. The instantaneous density of the beam bunch, the decay and growth of phase oscillation, and the frequency of phase oscillations may be observed in detail. Special electrode designs yield radial as well as vertical beam-position information; the former is useful in manual or automatic beam tracking.

ELECTRIC INDUCTION ELECTRODE

Principle of Operation

In principle the electric induction electrode consists of an insulated conducting plate so placed within the aperture that it is in proximity to the circulating beam. As the bunched charge passes above the plate, charges are induced on the plate and a current flows through a suitably placed grounding resistor. If the system response is adequate, the voltage developed across the resistor is at every instant proportional to the charge in that portion of the beam in the immediate vicinity of the plate. Neglecting for the moment the internal fluctuations of charge density within the bunch, we can say that the peak-induced voltage is directly proportional to the density of charge in the circulating beam.

Beam Parameters

The average voltage developed by the induction electrode may be expressed in terms of the charge-density distribution within the bunch as

$$V_{\text{ind}} = \int_0^{2\pi} V(\phi) d\phi, \quad (1)$$

where V_{ind} is the average induced voltage for a bunch and $V(\phi)$ is the voltage developed at any time when the relative phase of the bunch with respect to an arbitrary reference is ϕ . If the effective length of the induction electrode is l , the mean radius of the accelerator is r , and the length of the straight section is L , this voltage may be expressed in terms of the charge $q(\phi)$ and the capacity C of the induction electrode to its surroundings as

$$V_{\text{ind}} = \int_0^{2\pi} \left(\frac{l}{2\pi r + 4L} \right) \frac{q(\phi)}{C} d\phi = \frac{l}{2\pi r + 4L} \left(\frac{Q}{C} \right), \quad (2)$$

where Q is the total charge in the bunch.

Ultimate Sensitivity

Since the induced voltage per unit beam charge varies inversely with the capacity of the induction electrode, it is important that this capacity be minimized. In order to collect a large fraction of the electric field associated with the beam charge, the electrodes should be long. The lower limit to the useful length of induction electrode is determined by the fraction of the total capacity of the electrode that is contributed by the insulating supports, whereas the upper limit to the length is usually determined by the available space.

Whereas thermal-agitation noise in the input resistance of the amplifier associated with the instrument determines the ultimate sensitivity of the induction electrode, practically, the maximum sensitivity is usually determined by induced signals from the radio-frequency system or switching transients from ignitrons. Voltage outputs of the order of 1 microvolt per 10^5 protons may be obtained with an electrode approximately 3 feet long.

Frequency Response

The frequency response of the induction-electrode system is limited at low frequencies by the value of the input impedance of the beam-measuring device. This frequency dependence is easily seen in the case of a resistive termination. Consider the equivalent circuit for the induction electrode as shown in Fig. 1. The magnitude of the voltage available at the resistor terminals is given by

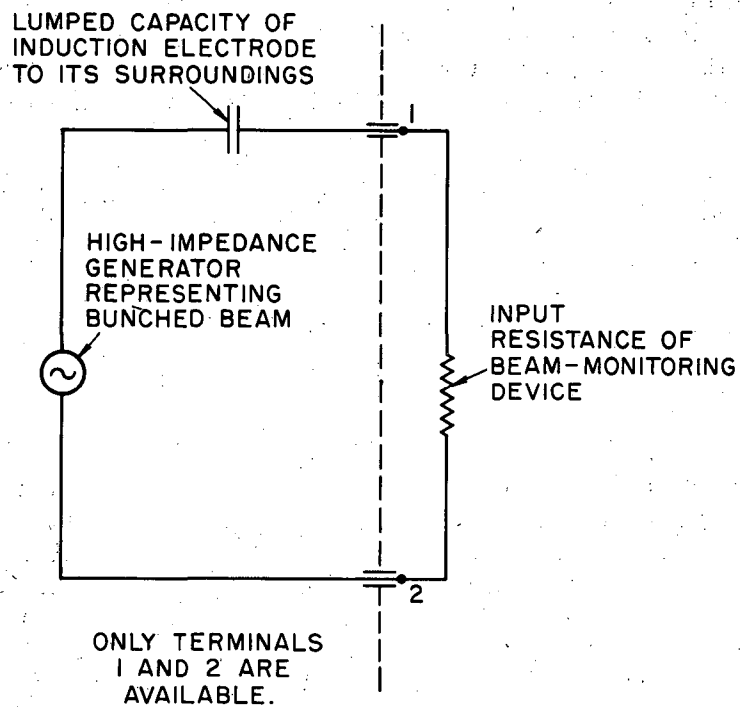


Fig. 1. Low-frequency equivalent circuit of beam induction electrode.

$$V = V_{\text{ind}}(R/Z) = V_{\text{ind}} \left(\frac{R}{R^2 + \frac{1}{\omega^2 C^2}} \right)^{1/2} = \frac{V_{\text{ind}}}{\left(1 + \frac{1}{\omega^2 R^2 C^2} \right)^{1/2}}, \quad (3)$$

where R is the value of the terminating resistance, C is the total capacity of the induction electrode, and ω is the angular frequency $2\pi f$. If an electrode is made large, so as to have a high enough output voltage to drive a transmission line directly, as in the electrode shown in Fig. 2 and described in Table I, then the low-frequency response is poor. Further, a calibration must be made for each energy. This same electrode will, of course, have excellent response if it drives a high-impedance load.

Calibration

The absolute calibration of the induction electrode depends upon a knowledge of the effective length of the electrode, the capacity of the electrode to its surroundings, and the length of the orbit of the circulative beam. Although the capacity may easily be measured to 1%, the determination of the effective length is not so simple. The effective length of the induction electrode depends upon the configuration of the guard ring. It is approximately equal to

$$l = l' + h,$$

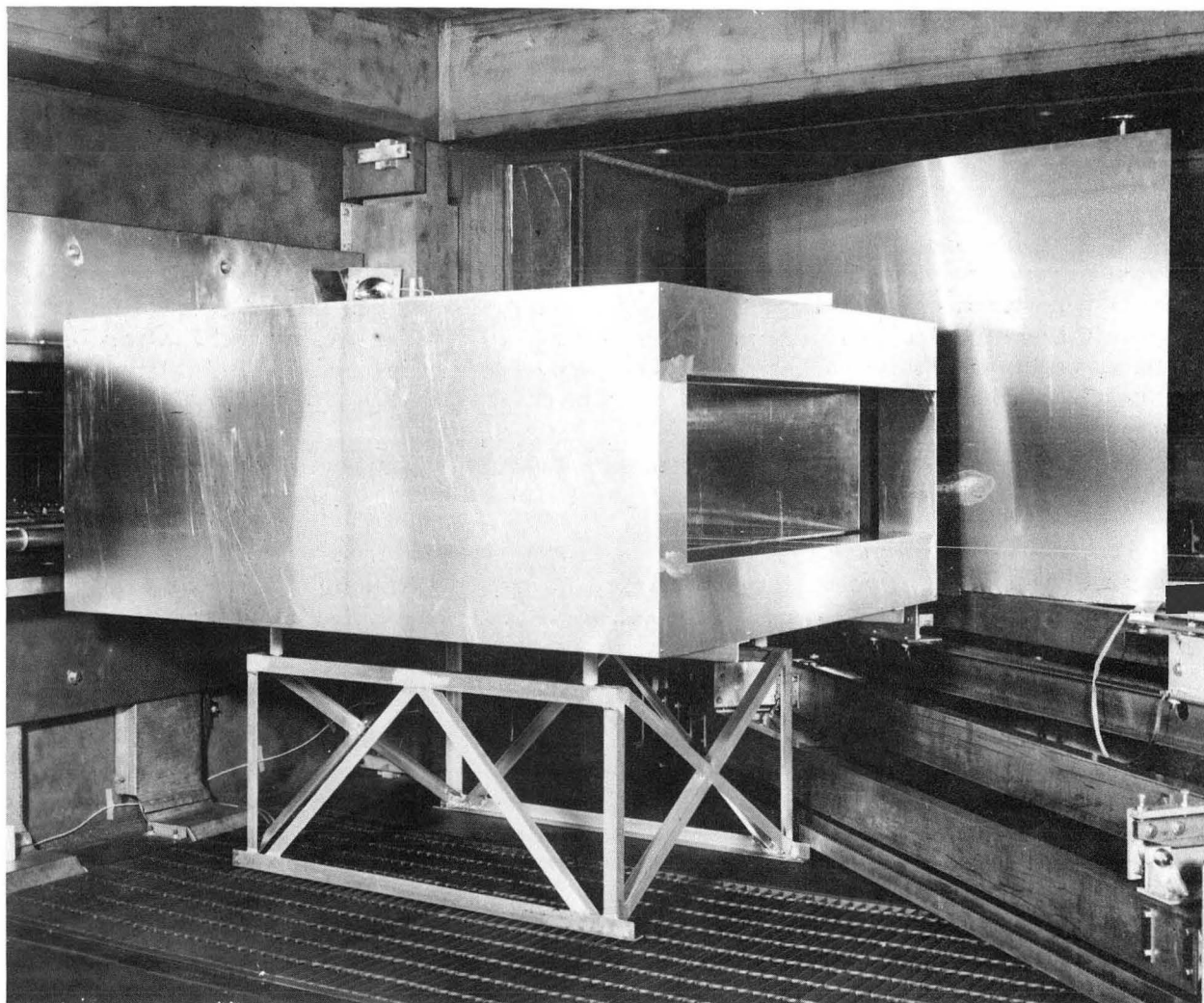
where l' is the physical length of the induction electrode and h is the width of the gap between the electrode and the guard ring. The total circulating beam may be related to the average value by the expression

$$V_{\text{ind}} = \frac{Nq\ell}{(2\pi M + 4L) C} = \frac{Nq\ell'k}{(2\pi M + 4L) C}, \quad (4)$$

where N is the number of protons of charge q circulating within the bunch and k is a correction factor due to end effects. Obviously, the longer the electrodes, the smaller the calibration error due to end effects.

One cannot use the peak amplitude of the induction electrode signal as a measure of the circulating charge because the peak value is also dependent upon the discrete charge-density distribution within the bunch. Since the density distribution varies markedly with drift-tube voltage as well as with the amplitudes of perturbed phase oscillations, errors of as much as 20% may occur.

If one neglects as second-order the changes in wave form and period of the beam bunch, one may calibrate the beam-induction electrode directly with a sine-wave oscillator whose frequency is the rotational frequency of the



ZN-1415

Fig. 2. Large induction electrode. This electrode is located in the east straight section of the Bevatron. Its output drives a transmission line having a characteristic impedance of 125 ohms.

Table I

Characteristics of Bevatron beam induction electrodes						
Location (straight section)	Facility	Measured capacity to surroundings (μpfd)	Measured length (inches)	Effective length (inches)	Calculated sensitivity $\left(\frac{\text{millivolts}}{10^{10} \text{ protons}}\right)$	Sensitivity compared with east electrode
East	beam amplitude	356	46.5	48.6	46.1	
South	beam amplitude	240	11.56	----	16.3k	35/88
South	beam radial	inner 105	11.56	----	5.2k	-----
	position	outer 103	11.56	----	5k	-----

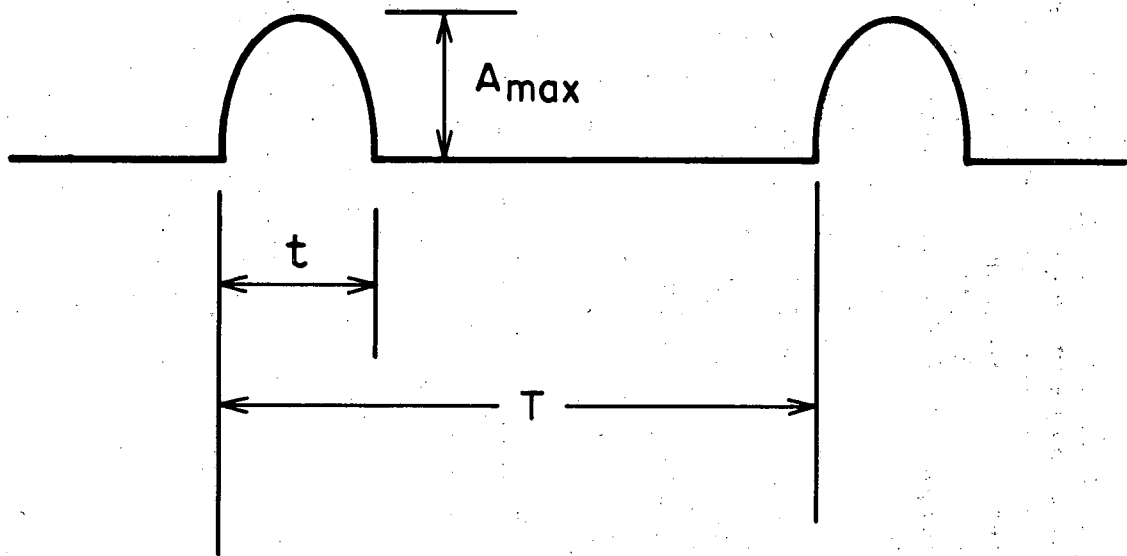


Fig. 3. Half sine-wave pulses simulating the beam induction-electrode signal.

bunch and whose amplitude is normalized to the ratio of amplitude of the first harmonic to the average value of the wave form of the resultant bunch. The amplitude of the first harmonic may be determined from a Fourier analysis of the wave form. If one assumes that the beam-induction electrode signal is of the form of a series of half sine-wave pulses¹ as shown in Fig. 3, then the ratio of the nth harmonic to the average value of the wave form is

$$A_n/A_0 = \pi/2 \left[\frac{\sin \pi/2 (1 - \frac{2nt}{T})}{\pi/2 (1 - \frac{2nt}{T})} + \frac{\sin \pi/2 (1 + \frac{2nt}{T})}{\pi/2 (1 + \frac{2nt}{T})} \right] \quad (5)$$

If $t = T/4$, assuming a 90° bunch and $n = 1$, the ratio of the first harmonic to the average value becomes

$$A_1/A_0 = \pi/2 \left[\frac{\sin \pi/4}{\pi/4} + \frac{\sin 3\pi/4}{3\pi/4} \right] \approx 1.86 \quad (6)$$

While this may not be the best model of the signal, reference to the general expression makes it clear that the ratio is not too sensitive to the general shape of the pulse.

As a sample calculation, consider the calibration of the shielded induction electrode described in Table I and shown in Fig. 4 (DWG 7M 1916). The constants in Eq. (4) are $q = 1.60 \times 10^{-19}$ coulombs, $l' = 11.56$ inches, $L = 240$ inches, $r = 600$ inches, $C = 240 \times 10^{-12}$ farad. Then

$$V_{ind} = \frac{Nql'}{(2\pi r + 4L)C} \quad k = \frac{N(1.60 \times 10^{-19})(11.56)k}{(1200\pi + 960)(2.4 \times 10^{-10})} = 1.63 \times 10^{-12} N \cdot k \text{ (volts),}$$

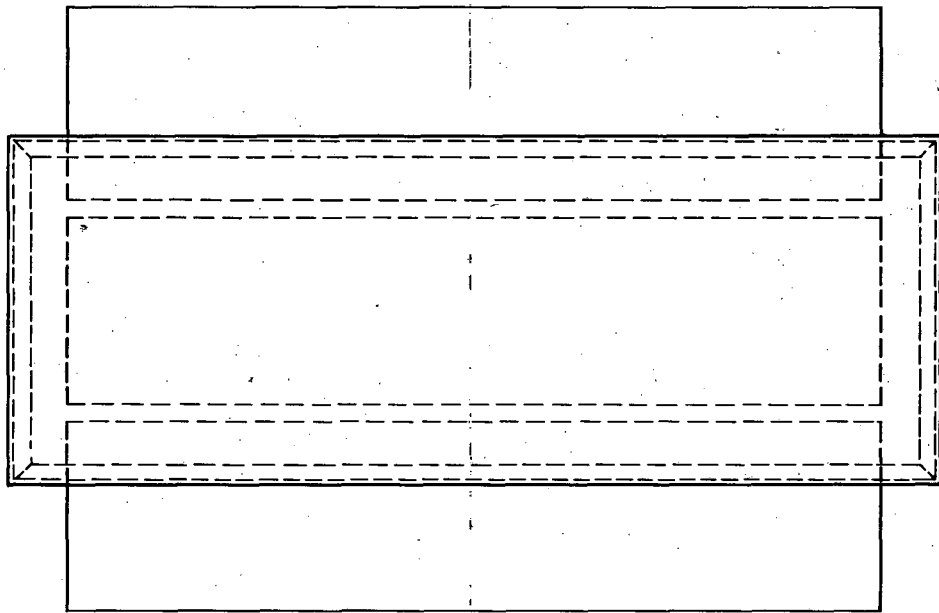
or an output voltage of $16.3 \cdot k$ millivolts for 10^{10} protons. Now since $k \sim 1$, an output voltage on the order of 20 millivolts is to be expected.

A calibration oscillator should have an output of

$$2A_1 = 2[1.86 \times 1.63 \times 10^{-12} \times (10^{10})] k \text{ volts} \approx \frac{60}{30} \text{ millivolts peak-to-peak}/10^{10} \text{ protons}$$

if it is assumed that the wave-form model is a series of half sine-wave pulses. The value of k has been found by comparison to be approximately 1.12 for this electrode.

¹William A. Wenzel, Calibration of the Bevatron Internal Beam, Bevatron Report No. 174, October 15, 1956.



- 1. COVER
- 2. GROUND PLANE
- 3. FRAME
- 4. INDUCTION ELECTRODE
- 5. GROUND PLANE EXTENDED
- 6. SIDE PLATE
- 7. BOTTOM PLATE
- 8. END PLATE
- 9. MACHINE SCREW
- 10. PORCELAIN STANDOFF
- 11. POLYETHYLENE WASHER

20"

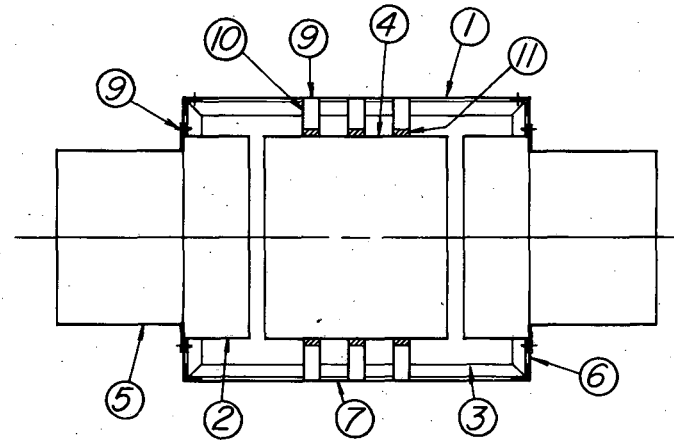
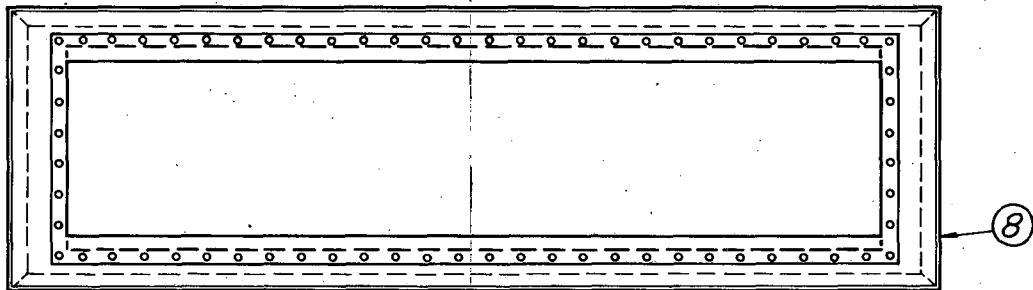


Fig. 4. Shielded sum-induction electrode.

DEVELOPMENT OF BEAM INDUCTION ELECTRODES

Radio-Frequency Model Test

A quarter-scale electrostatic model was constructed to determine the effect of beam position on the voltage induced on the induction electrode. A 2.0-Mc signal was applied to a stiff wire of small diameter, which was placed in the geometric median plane of the model electrodes. The magnitude of the signal induced on the electrode was recorded as a function of the position of the wire in the horizontal plane.

Many electrode geometries were studied in an attempt to obtain a configuration that had strong radial dependence but whose sum signal was independent of radial position. An electrode of the form of Fig. 5 (DWG 7M 5486) was found to be optimum. The response of this electrode for radial and sum signals is shown in Fig. 6.

Shielding of Induction Electrodes

One of the most common sources of extraneous signal in the Bevatron monitoring systems is due to induced magnet voltage. This signal may be eliminated from the induction-electrode monitoring system, neither the electrodes nor the monitoring circuits are grounded at the straight section. A one-point ground is established in the main control room. Insulation of the electrodes, cables, and electronic circuits from the ground at the straight section together with adequate shielding, has resulted in a system having a minimum of extraneous interference.

Monitoring of Induced Voltage

Several attempts were made to use conventional techniques to monitor the voltage on induction electrodes. Vacuum-tube cathode followers having adequate band width were excessively complicated, and were unreliable in that proper operation was strongly dependent upon vacuum-tube transconductance. Since the tubes required frequent substitution, they could not be placed in the vacuum chamber in proximity to the electrodes. An alternate solution to this problem resulted from the use of a transistor-type cathode follower (for analysis see Appendix). The unit shown in Fig. 7 was mounted directly on the induction electrode in the vacuum system. The circuit configuration used has an effective input impedance of about 10,000 ohms, so that the effective loading of the induction electrode down to 20 kc is negligible. The transistor drives a 200-ohm line directly with a peak-to-peak voltage in excess of 2 volts. The band width depends upon the transistor characteristics and can be in excess of 15 Mc. The gain of the system was stabilized at approximately 0.95. A large amount of dc feedback was used to preclude temperature sensitivity. Since the Hall-effect coefficient and the active transistor volume are both small, the Bevatron magnetic field causes negligible interference ($\sim 0.02\%$). The performance of these units has not varied measurably since their installation (~ 7000 hours of continuous operation).

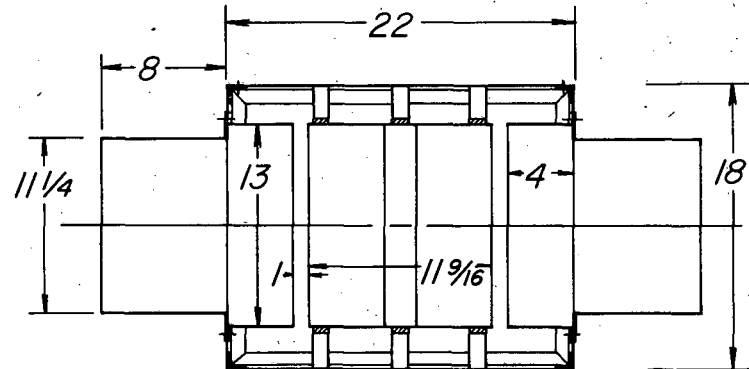
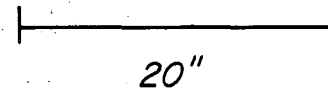
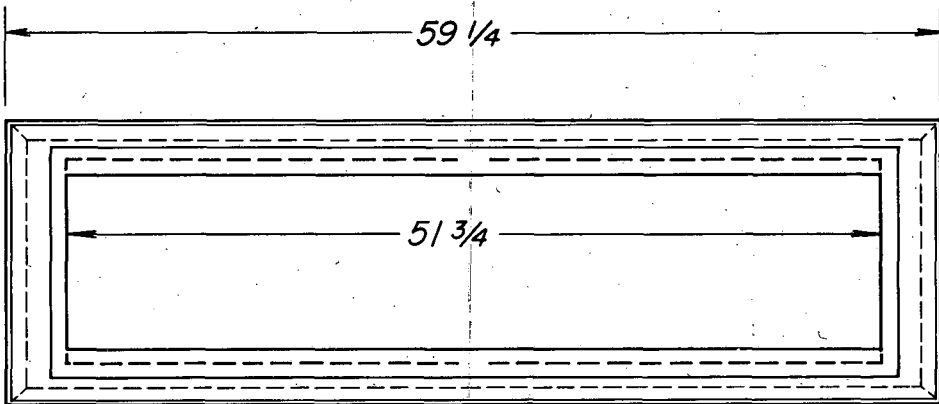
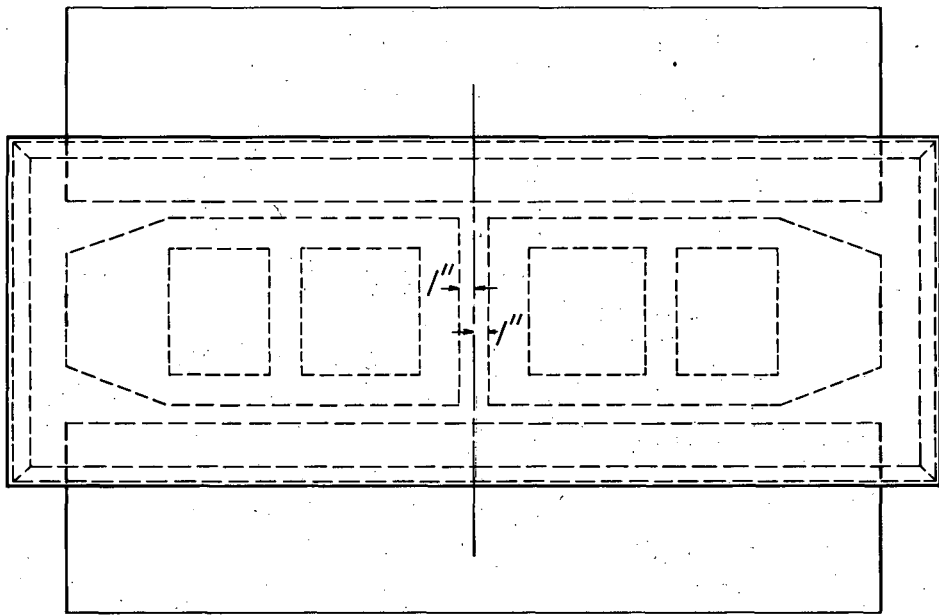


Fig. 5. Induction electrodes having large radial position discrimination.

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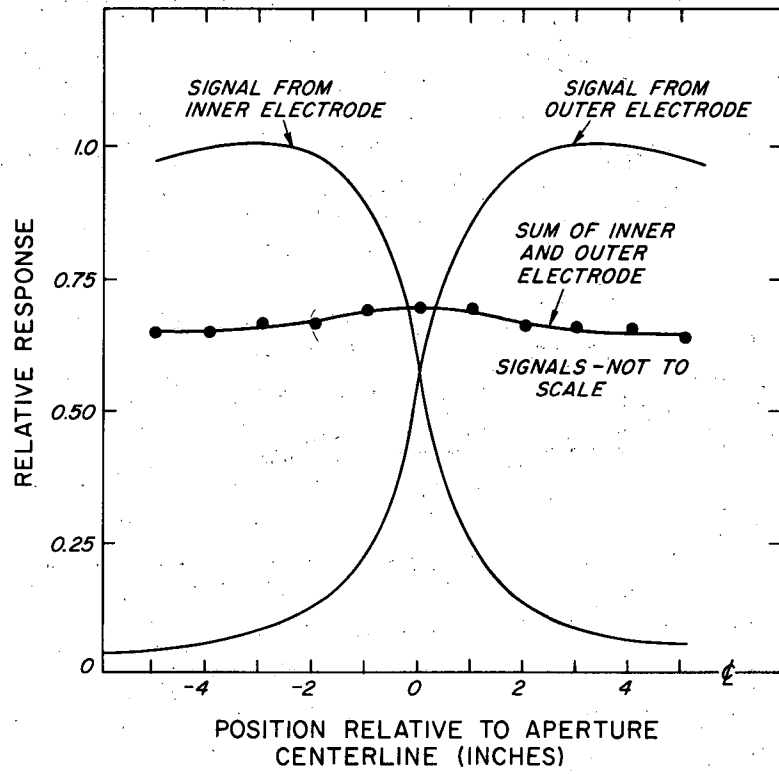
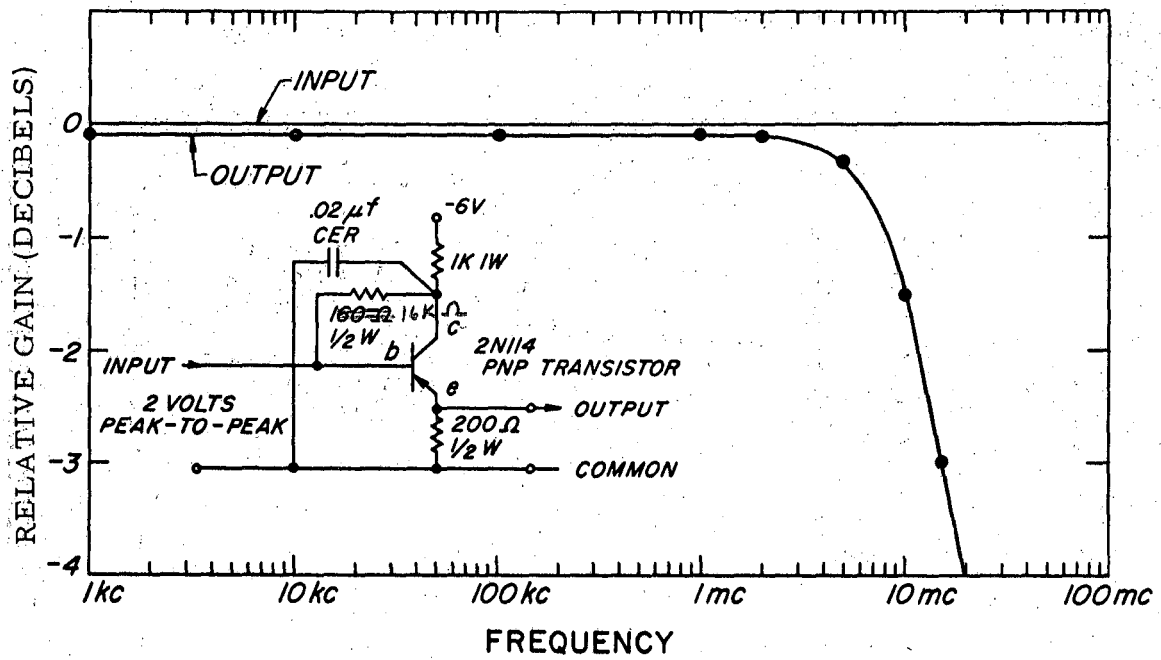


Fig. 6. Radial and sum signal response of split induction electrodes.



MU-12935

Fig. 7. Induction electrode emitter follower response.

MU-12935

The semiconductor noise of the transistor has an rms value of about 200 microvolts, and therefore sets the limiting sensitivity of the induction electrode at about 5×10^7 protons. Since most experiments require beam intensities on the order of 10^{10} protons and higher, this noise figure is adequate. (If greater sensitivity were required, the output could be filtered to reduce the noise components below 2 Mc. This technique could be used to increase the sensitivity by approximately two orders of magnitude, as semiconductor noise decreases in amplitude as $1/f$.)

ACKNOWLEDGMENTS

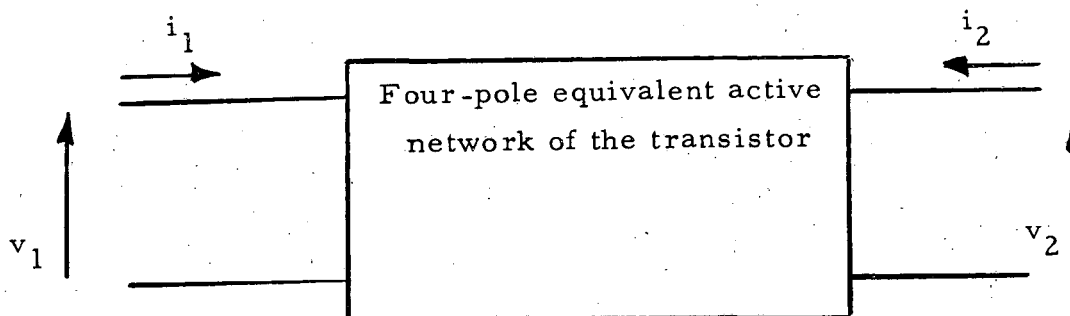
The author wishes to thank Dr. William Wenzel for his interest and encouragement in this work. Charles Neuman, and Fred Lothrop assisted in the assembly of test data.

APPENDIX

Emitter Follower Analysis

It will be convenient to analyze a transistor circuit in terms of the four-pole network theory of active transducers. Justification for this approach can be obtained from treatments by Feldtkeller¹ and Peterson.²

To this end let the transistor be represented by a "black box" with the positive directions of currents and voltages as shown below.



The lower-case letters refer to the small signal or ac parameters. In order to simplify the analysis, consider the low-frequency case, wherein the network parameters can be represented by pure resistances. The high-frequency case can be developed with a modified equivalent circuit in which the network elements become complex impedances.

One can write the open-circuit resistance parameters in the matrix for the four-pole as

$$\begin{bmatrix} v_1 \\ v_2 \end{bmatrix} = \begin{bmatrix} r_{11} & r_{12} \\ r_{21} & r_{22} \end{bmatrix} \begin{bmatrix} i_1 \\ i_2 \end{bmatrix}$$

¹R. Feldtkeller, Einführung in Die Vierpoltheorie, S. Hirzel, Leipzig.

²L. C. Peterson, Equivalent Circuits of Linear Active Four Terminal Networks, B. S. T. J., Vol. 27, October 1948.

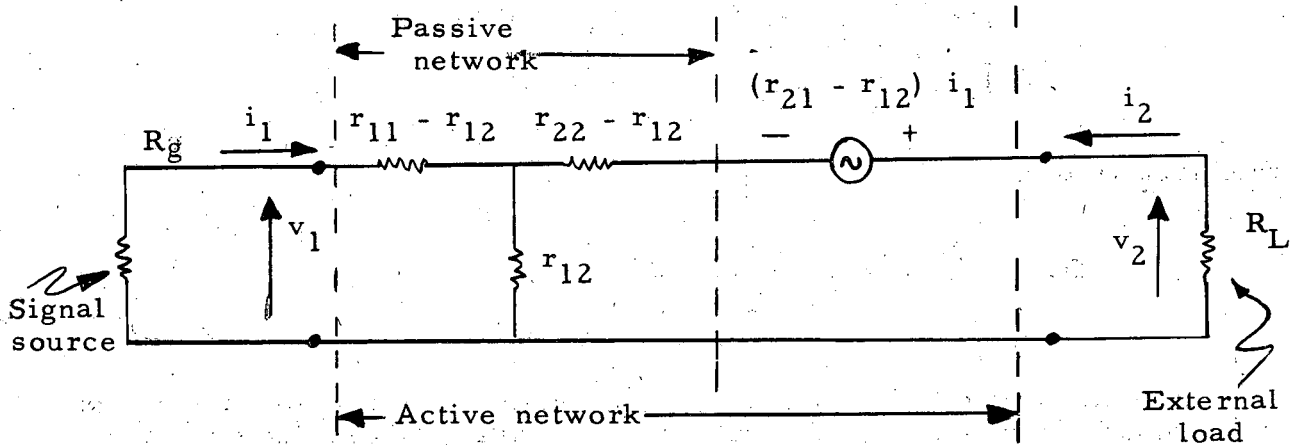
where the small signal parameters are just

- r_{11} = input impedance, output open circuited;
- r_{22} = output impedance, input open circuited;
- r_{21} = forward transfer impedance, output open circuited;
- r_{12} = reverse transfer impedance, input open circuited.

For any active network reciprocity does not exist, therefore

$$r_{12} \neq r_{21}$$

It can be shown that the equivalent circuit for such cases can be obtained by the construction of a suitable passive network and at least one generator. An example of such a network, including the input and output (external) circuits, is shown below.



The matrix representation for this case becomes

$$\begin{bmatrix} v_1 \\ v_2 \end{bmatrix} = \begin{bmatrix} r_{11} + R_g & r_{12} \\ r_{21} & r_{22} + R_L \end{bmatrix} \times \begin{bmatrix} i_1 \\ i_2 \end{bmatrix}$$

To obtain the input impedance R_i , let $v_2 \rightarrow 0$;
then

$$\begin{bmatrix} v_1 \\ 0 \end{bmatrix} = \begin{bmatrix} r_{11} & r_{12} \\ r_{21} & r_{22} + R_L \end{bmatrix} \times \begin{bmatrix} i_1 \\ i_2 \end{bmatrix}$$

$$R_i = \frac{v_1}{i_1} = \frac{v_1}{i_1} \left| \begin{array}{cc|cc} r_{11} & r_{12} & & \\ r_{21} & r_{22} + R_L & & \\ \hline v_1 & r_{12} & & \\ 0 & r_{22} + R_L & & \end{array} \right| = r_{11} - \frac{r_{12} r_{21}}{r_{22} + R_L}$$

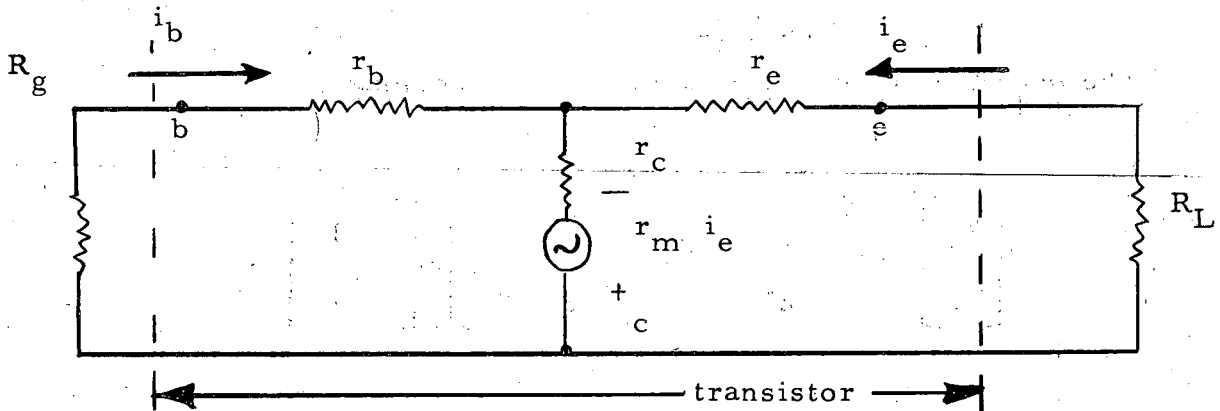
The output impedance R_0 can be obtained by a similar manipulation as indicated in the matrix

$$\begin{bmatrix} 0 \\ v_2 \end{bmatrix} = \begin{bmatrix} r_{11} + R_g & r_{12} \\ r_{21} & r_{22} \end{bmatrix} \times \begin{bmatrix} i_1 \\ i_2 \end{bmatrix}$$

which solves to give

$$R_0 = \frac{v_2}{i_2} = r_{22} - \frac{r_{12} r_{21}}{r_{11} + R_g}$$

It remains to develop the above expressions in terms of the common collector configuration. For brevity it will be assumed that the one generator equivalent circuit for the common collector stage is as shown below. (These circuit aspects are developed by Wallace and Pietenpol,³ for example.)



³R. L. Wallace and W. J. Pietenpol, Bell System Tech. J. 30, July 1951.

In terms of the familiar four-pole parameters, we have

$$r_{11} = r_b + r_c,$$

$$r_{12} = r_c - r_m,$$

$$r_{21} = r_c,$$

$$r_{22} = r_e + r_c = r_m,$$

wherein

$$r_b = \text{base resistance,}$$

$$r_c = \text{collector resistance,}$$

$$r_e = \text{emitter resistance,}$$

$$r_m = \text{mutual resistance,}$$

$$a = \frac{r_m + r_b}{r_c + r_b}.$$

The input impedance expression for this configuration reduces to

$$R_i = r_b + r_c - \frac{r_c (r_c - r_m)}{r_e + r_c - r_m + R_L},$$

whereas the output impedance becomes

$$R_o = r_e + r_c - r_m - \frac{r_c (r_c - r_m)}{r_b + r_c + R_g}.$$

For a 2N114 transistor, the measured values of the small-signal parameters are as follows:

$$r_b \cong 75 \text{ ohms,}$$

$$r_c \cong 10^7 \text{ ohms,}$$

$$r_e \cong 26 \text{ ohms,}$$

$$r_m \cong 9.8 \times 10^6 \text{ ohms,}$$

$$a = 0.98.$$

Before starting calculations, we simplify the expressions for R_i and R_o by approximations:

$$R_i \cong \frac{r_c (r_c - r_m) + r_c (r_e + R_L) - r_c (r_c - r_m)}{r_c - r_m + r_e + R_L} = \frac{r_e + R_L}{(1 - \alpha) + r_e + R_L} \cdot r_c$$

now $\alpha \cong \frac{r_m}{r_c}$, as $r_c, r_m \gg r_b$,

so that we have

$$R_i \cong \frac{r_e + R_L}{1 - \alpha}$$

Therefore, the input impedance of a common collector stage is approximately equal to the emitter load impedance multiplied by the base-to-collector current gain.

The corresponding expression for the output impedance reduces to

$$R_o = r_e + (r_c - r_m) \left[1 - \frac{r_c}{r_b + r_c + R_g} \right] \cong r_c (1 - \alpha) \frac{1}{1 + R_g/r_c + r_c/R_g}$$

for $R_g \cong 0$, ~~$R_o \cong r_c(1 - \alpha)$~~ , $R_o \sim r_e + r_b(1 - \alpha)$

for $R_g \cong r_c$, $R_o \sim r_c(1 - \alpha)/2$.

~~Since the output impedance of the transistor is always quite large in comparison with the emitter load resistance, the output resistor in the emitter circuit determines the actual impedance of the circuit.~~ Some calculated input and output impedance levels are tabulated below.

R_L Emitter load resistance (ohms)	R_i input resistance (ohms)	R_g (ohms)	R_0 (ohms)
100	6300	100	2×10^5 30
200	11300	1000	2×10^5 48
500	26300	10,000	2×10^5 226
1,000	51300	100,000	2×10^5 2,000
2,000	101,300	1,000,000	2×10^5 20,000
5,000	250,000		
10,000	500,000		

Except for noise level ($\sim 200\mu$ volts), it thus appears that the common collector transistor configuration is an ideal impedance transformer. This circuit is capable of "resistance transformations" on the order of 50:1 over band widths on the order of 20 Mc.

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