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Authors
Plambeck, Erica L
Taylor, Terry A

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On the Value of Input Efficiency, Capacity Efficiency, and the Flexibility to Rebalance Them

Erica L. Plambeck
Graduate School of Business, Stanford University, Stanford, California 94305, elp@stanford.edu

Terry A. Taylor
Haas School of Business, University of California, Berkeley, Berkeley, California 94720, taylor@haas.berkeley.edu

A common characteristic of basic material manufacturers (which account for 85% of all industrial energy use) and of cleantech manufacturers is that they are price takers in their input and output markets. Variability in those prices has implications for how much a manufacturer should invest in three fundamental types of process improvement. Input price variability reduces the value of improving input efficiency (output produced per unit input) but increases that of capacity efficiency (the rate at which a production facility can convert input into output). Output price variability increases the value of capacity efficiency, but it increases the value of input efficiency if and only if the expected margin is small. Moreover, as the expected input cost rises, the value of input efficiency decreases. A third type of process improvement is to develop flexibility in input efficiency versus capacity efficiency (the ability to respond to a rise in input cost or fall in output price by increasing input-efficiency at the expense of capacity efficiency). The value of this flexibility decreases with variability in input and output prices if and only if the expected margin is thin. Together, these results suggest that a carbon tax or cap-and-trade system may reduce investment by basic material manufacturers in improving energy efficiency.

Key words: energy efficiency; process improvement; flexibility; environment

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1. Introduction and Overview

Every manufacturer strives to improve productivity in two key dimensions. The first, input efficiency, is the amount of saleable output produced per unit input. The second, capacity efficiency, is the rate at which a production facility can convert input into saleable output. This paper shows that variability in the market prices for a manufacturer’s input and output has substantial implications for whether the manufacturer should focus on improving input efficiency or capacity efficiency. Input cost variability reduces the value of improving input efficiency but increases that of capacity efficiency. Output price variability increases the value of capacity efficiency, but it increases the value of input efficiency if and only if the expected margin is small. Furthermore (because of variability in input and output prices), an increase in the expected input cost can reduce the value of improving input efficiency.

As an alternative to improving these two dimensions of efficiency, a manufacturer may develop flexibility to trade off the two types of efficiency. A manufacturer with flexibility in input efficiency versus capacity efficiency can adapt to a rise in input costs or fall in output prices by changing the production process to increase input efficiency at the expense of capacity efficiency. In so doing, the manufacturer gives up some output to reduce its variable cost of production. This paper shows that the value of flexibility in input efficiency versus capacity efficiency decreases with variability in input or output prices if and only if the expected margin is thin.

These insights are relevant to the wide swath of manufacturers that are price takers in their input and output markets. They are particularly important for cleantech and basic material manufacturers, whose output prices and input costs are highly variable and uncertain, in large part because of volatility in energy prices and environmental policy. When input costs rise above output prices, firms can suspend production—which is the phenomenon that drives our analytic results. For examples of basic material (e.g., aluminum and ammonia) manufacturers suspending production in response to increased energy prices, see U.S. Department of Energy (2007) and Seay (2012). For examples of cleantech firms (e.g., biofuel, solar photovoltaic, and wind turbine manufacturers) suspending production due to changes in government policy, see Wiser et al. (2007), Guzman et al. (2012), and Mufson (2012).

According to the Intergovernmental Panel on Climate Change (2007), one of the most important means
to mitigate climate change is to improve energy efficiency in the manufacturing of basic materials (chemicals, metals, minerals, paper, and petroleum products), which accounts for 85% of all industrial energy use and a quarter of all anthropogenic greenhouse gas emissions. However, our results suggest that a carbon tax or cap-and-trade system may discourage investment in energy efficiency by incumbent basic material manufacturers. Therefore, eliminating emissions from basic material manufacturing may require cleantech manufacturers to displace those incumbents or supply them with “clean” energy.

Cleantech firms’ judicious investments in process improvement will enhance their viability. This paper uses the example of a prominent cleantech manufacturer, Amyris, to illustrate that price variability has large—and directionally different—impacts on the value of improving input efficiency and the value of improving capacity efficiency. Further, developing flexibility in input efficiency versus capacity efficiency is highly valuable.

Flexibility in input efficiency versus capacity efficiency is particularly important for many cleantech manufacturers. Amyris, for example, uses genetically engineered yeast to ferment sugar into farnesene; farnesene is a precursor for various fuels and petrochemicals, hence its price is tied to the price of oil. When the price of sugar rises relative to the price of oil, Amyris can switch to a strain of yeast that produces more farnesene per unit of sugar input (increasing input efficiency) but multiplies and produces the farnesene more slowly (reducing capacity efficiency) (Lievense 2010). Many other cleantech firms use microorganisms to transform sugar or other biomass (from agricultural commodities, agricultural waste, or municipal waste) into fuels or chemicals. Characteristically, they all have the flexibility to increase input efficiency at the expense of capacity efficiency by modifying the microorganism or increasing the batch processing time (as in Ata et al. 2012). Alternatively, they may increase input efficiency (output per unit cost of input) by using a cheaper, lower-quality input that requires more processing or reduces expected yield and therefore reduces capacity efficiency. Similarly, in molding the blades for a wind turbine, manufacturers have the flexibility to use a cheaper resin that requires less heat but more time to infiltrate the mold and harden, which increases input efficiency (number of blades per unit cost of input) but reduces capacity efficiency (number of blades per unit time) (Stewart 2012). In another large category, solar photovoltaic manufacturers have the flexibility to increase input efficiency (module output per unit of silicon input) by applying more thinly sliced silicon, without reducing a module’s light conversion efficiency. However, this increase in input efficiency comes at the expense of capacity efficiency (module output per unit time) because thinly sliced silicon tends to break and require rework (Zuretti 2006).

Flexibility in input efficiency versus capacity efficiency has not previously been addressed in the economics or operations management literature, except in the model of a waste-to-energy firm in Ata et al. (2012). Surveys of the literature on manufacturing flexibility are in Sethi and Sethi (1990), Gerwin (1993), and Goyal and Netessine (2011).

This paper focuses on the value of improving input efficiency, capacity efficiency, and flexibility therein, but abstracts from the costs of doing so. For model-based and empirical research on how to improve input efficiency or capacity efficiency (and the associated costs), we refer the reader to Carrillo and Gaimon (2000), Lapre et al. (2000), Gaimon (2008), Tanrisever et al. (2012), and the papers surveyed therein. The potential for flexibility in input efficiency versus capacity efficiency may be inherent in a production technology. However, that flexibility typically must be developed through investments in research and development or experimentation in the operating mode of a production facility, which, as Carrillo and Gaimon (2000) and Terwiesch and Xu (2004) show, is costly.

Following Hicks’s (1932) observation that an increase in the cost of an input will spur innovation to use that input more efficiently, current models for energy and climate policy analysis that endogenize energy efficiency assume that firms will respond to an increase in energy prices by improving energy efficiency (Gillingham et al. 2008, 2012). Our paper helps to explain the contradictory empirical finding by Linn (2008) that when current or forecasted energy prices rise, incumbent firms in energy-intensive industries do not invest to improve energy efficiency. An extensive literature documents that firms commonly fail to make seemingly profitable investments in energy efficiency and provides various explanations (see Jira and Lee 2012 and references therein). For example, Metcalf and Hassett (1993) explain that investment in energy efficiency is irreversible, so uncertainty in future energy prices favors postponing such investments. We do not model optimal delay. Instead, we identify a different mechanism by which uncertainty in energy prices inhibits energy efficiency. An increase in the mean or variance of energy prices (which may be due to a carbon tax or cap-and-trade system) increases the likelihood that a firm will suspend production, which tends to reduce the expected value of energy efficiency.

2. Model Formulation
In our simple model, first, a manufacturer can choose to improve its process (by increasing capacity efficiency, increasing input efficiency, or developing
flexibility in input efficiency versus capacity efficiency). Then, the manufacturer realizes its input and output prices and chooses its production quantity (and mode, if it has developed flexibility) subject to its capacity constraint. The manufacturer is a price taker in its input and output markets. That is, the manufacturer’s cost per unit input and selling price per unit output are random variables that are not affected by the manufacturer’s input purchase and output sales quantities. We assume \( C \sim \text{Normal}(\mu_c, \sigma_c^2) \), \( P \sim \text{Normal}(\mu_p, \sigma_p^2) \), \( C \) and \( P \) have correlation \( \rho \), and that for each random variable the mean is sufficiently large relative to the standard deviation that the probability that the random variable is negative is negligible. Further, in expectation, the output price exceeds the input cost \( \mu_p > \mu_c \). Let \( \phi(c, p) \) denote the joint density of \( C \) and \( P \).

In our baseline scenario, without loss of generality, we normalize both capacity and input efficiency to one, so that if the manufacturer produces, it produces one unit of output and consumes one unit of input. Baseline expected profit is

\[
\Pi_O = \int_{-\infty}^{\infty} \int_{-\infty}^{\infty} \max(p-c, 0) \phi(c, p) \, dp \, dc
\]

\[
= \int_{-\infty}^{\infty} \int_{c}^{\infty} (p-c) \phi(c, p) \, dp \, dc. \tag{1}
\]

The optimal policy is to produce when \( p \geq c \) and otherwise to idle the production facility.

Through process improvement (e.g., cycle time reduction or elimination of downtime), the manufacturer might increase its capacity efficiency \( k \) above the baseline level 1, so that expected profit becomes

\[
\Pi = \int_{-\infty}^{\infty} \int_{-\infty}^{\infty} \max(k(p-c), 0) \phi(c, p) \, dp \, dc
\]

\[
= \int_{-\infty}^{\infty} \int_{c}^{\infty} k(p-c) \phi(c, p) \, dp \, dc. \tag{2}
\]

To increase input efficiency \( i \) above the baseline level 1, the manufacturer might focus on increasing the expected yield of output per unit input (e.g., through quality management, to ensure that input is converted into saleable output rather than waste), so that expected profit becomes

\[
\Pi = \int_{-\infty}^{\infty} \int_{-\infty}^{\infty} \max((ip-c), 0) \phi(c, p) \, dp \, dc
\]

\[
= \int_{-\infty}^{\infty} \int_{c/i}^{\infty} (ip-c) \phi(c, p) \, dp \, dc, \tag{3}
\]

or the manufacturer might focus on reducing the amount of input required to produce a unit output (e.g., by improving the energy efficiency of the production process), so that expected profit becomes

\[
\Pi = \int_{-\infty}^{\infty} \int_{-\infty}^{\infty} \max((p-c/i), 0) \phi(c, p) \, dp \, dc
\]

\[
= \int_{-\infty}^{\infty} \int_{c/i}^{\infty} (p-c/i) \phi(c, p) \, dp \, dc. \tag{4}
\]

We will investigate how variance and correlation in \( C \) and \( P \) influence the value of improving capacity efficiency \( \partial \Pi/\partial k \) and the value of improving input efficiency \( \partial \Pi/\partial i \) in the baseline scenario with \( k = i = 1 \).

Alternatively, the manufacturer may develop flexibility in input efficiency versus capacity efficiency, the ability to operate with higher input efficiency \( i > 1 \) at the expense of reducing its capacity efficiency to \( k < 1 \). The expected profit of a manufacturer with flexibility in input efficiency versus capacity efficiency is

\[
\Pi_F = \int_{-\infty}^{\infty} \int_{-\infty}^{\infty} \max(p-c, k(p-c/i), 0) \phi(c, p) \, dp \, dc. \tag{5}
\]

(Equation (5) implicitly interprets input efficiency as reducing the amount of input required to produce a unit of output as in (4). When input efficiency is interpreted as increasing the expected yield as in (3), the formulation is identical, except that \( k \) replaces \( i \) in (5), so that the max-operator’s second argument becomes \( k(p-c) \). If \( k_i \geq 1 \), then operating with higher input efficiency always dominates operating at the nominal input efficiency, so the issue of flexibility is irrelevant. If \( k_i < 1 \), then the two formulations are equivalent.) Let \( j = i(1-k)/(i-k) \), and observe that \( j < i \). The optimal policy is as follows: if \( p > c/j \), use the nominal process with input efficiency and capacity efficiency of 1; if \( p \in (c/i, c/j) \), then increase input efficiency to \( i > 1 \) and reduce capacity efficiency to \( k < 1 \); otherwise, idle the production facility. Therefore, (5) simplifies to

\[
\Pi_F = \int_{-\infty}^{\infty} \left( \int_{c/j}^{c/i} (k(p-c/i) \phi(c, p) \, dp \right. \\
\left. + \int_{c/i}^{\infty} (p-c) \phi(c, p) \, dp \right) dc.
\]

We will investigate how variance in \( C \) and \( P \) influence the value of flexibility in input efficiency versus capacity efficiency:

\[
\Delta = \Pi_F - \Pi_O. \tag{6}
\]

Our results and proofs hold when expected profit is maximized over a finite time horizon \([0, T]\); input cost and output price evolve according to a Brownian motion with initial values \((\mu_c, \mu_p)\), zero drift, instantaneous variance \((\sigma_c^2, \sigma_p^2)\), and correlation \( \rho \); and the firm chooses its production rate (and mode, if it has flexibility) at each instant in time \( t \in [0, T] \) and does not hold inventory. Our results hold, qualitatively, when the firm instead decides periodically\(^1\) whether

\(^1\)Specifically, the times at which a firm decides whether or not to initiate a batch process and the batch processing time are fixed. Hence an increase in capacity efficiency corresponds to an increase in output per batch. Note, however, that in efforts to increase output per unit time, reducing the batch processing time would be
or not to run a batch process, and our results regarding the impact of changes in $\mu_c$ and $\sigma_c$ hold, qualitatively, when the firm can hold inventory at some constant cost per unit, per unit time.

The standard assumption in the literature on commodity prices is that prices follow a mean-reverting or geometric Brownian motion, implying a lognormal distribution at each point in time. Our model is consistent with this literature in that a lognormal distribution is well approximated by a normal distribution whenever its mean is much larger than its standard deviation, as we have assumed regarding the input cost $C$ and output price $P$.

### 3. Results

One might think that as an input becomes more costly, it becomes more important to use that input more efficiently. Proposition 1 reveals that under a broad set of conditions, that conjecture is incorrect.

**Proposition 1.** The value of improving input efficiency in (3) (increasing yield) decreases with the expected input cost

$$(\partial^2/\partial \delta \mu_c) \Pi < 0. \quad (7)$$

The value of improving input efficiency in (4) (reducing input requirements) decreases with the expected input cost if and only if

$$\text{Prob}(P - C \geq 0) < \lim_{\delta \to 0} \left(\partial / \partial \delta\right) \text{Prob}(P - C / i \geq 0). \quad (8)$$

Inequality (8) tends to hold when the expected margin $m \equiv \mu_c - \mu_p$ is sufficiently small. More precisely, when the correlation between input cost and output price $\rho = 0$, inequality (8) holds if and only if the expected margin $m < \bar{m}$, where $\bar{m} > 0$. When $\rho \neq 0$, (8) holds if $m < \bar{m}$, where $\bar{m} > 0$ if the relatively mild condition that expected input cost is sufficiently large relative to the standard deviation of the contribution margin, $\mu_c > \sqrt{(\sigma_c^2 - 2\rho \sigma_c \sigma_p + \sigma_p^2)} \pi / 2$, is satisfied; (8) is violated when the expected margin $m$ is sufficiently large.

The intuition for Proposition 1 is as follows. Consider the setting in which input efficiency reduces the input requirement, which for concreteness we interpret as improving energy efficiency. An increase in the expected cost of the energy input has two opposing effects on the value of improving energy efficiency. First, an increase in the cost of energy decreases the probability that the manufacturer operates. Because improving energy efficiency is of value only in the event the manufacturer operates, decreasing this probability decreases the value of energy efficiency. Second, in the event that the manufacturer operates, increasing the cost of energy makes energy efficiency more valuable. The first (negative) effect dominates the second to the extent that the manufacturer expected margin $m$ is small, so that the probability of operating is small and more sensitive to an increase in the cost of energy. In the setting in which input efficiency increases the yield, only the first effect—increasing the input cost reduces the probability that the manufacturer operates—is at work, so increasing the input cost reduces the value of improving input efficiency.

For an input efficiency improvement at Amyris that takes the form of (4), we find that the company’s thin margins imply that (8) holds, as explained in the numerical example section below. Thus, Proposition 1 implies that when the expected cost of sugar rises, Amyris should invest less in process improvement efforts aimed at using less sugar.

**Proposition 2.** Suppose (9). The value of improving capacity efficiency increases with input cost variability

$$(\partial^2/\partial \delta \sigma_c) \Pi > 0. \quad (10)$$

The value of improving input efficiency decreases with input cost variability

$$(\partial^2/\partial \delta \sigma_c) \Pi < 0. \quad (11)$$

The value of improving capacity efficiency increases with output price variability

$$(\partial^2/\partial \delta \sigma_p) \Pi > 0. \quad (12)$$

---

2 Let the mean of a lognormal random variable approach infinity, while fixing its standard deviation at any strictly positive level. Its skew and excess kurtosis converge to zero. Hence when the mean is large, one can approximate the first four significant moment (hence, shape) characteristics of the lognormal with those of a normal distribution (mean, variance, and zero skew and excess kurtosis).
The value of improving input efficiency decreases with output price variability

\[
\left( \frac{\partial^2}{\partial \sigma_p^2} \right) \Pi < 0 \quad (13)
\]

if and only if the expected margin is large \( m > \bar{m} \), where \( \bar{m} \in (0, \infty) \). The value of improving capacity efficiency decreases with correlation between the input cost and the output price

\[
\left( \frac{\partial^2}{\partial k \partial \rho} \right) \Pi < 0. \quad (14)
\]

The value of improving input efficiency increases with that correlation

\[
\left( \frac{\partial^2}{\partial \sigma_p \partial \rho} \right) \Pi > 0 \quad (15)
\]

if and only if the expected margin \( m > \bar{m} \).

The closed-form expressions for \( \bar{m} \) and \( \bar{m} \) are evident in the proof of Proposition 2, and they give us the following observations. When input efficiency reduces the input requirement as in (4), \( m < 0 \), meaning that (15) always holds. When input efficiency increases the output yield as in (3), the following hold: The thresholds \( \bar{m} \) and \( \bar{m} \) are strictly positive and decreasing in \( \mu_c \), implying that (13) and (15) tend to hold when the expected output price \( \mu_c \) is large. As \( \sigma_c \to 0 \), \( m \to 0 \), and \( m \to 0 \), which implies that (13) and (15) hold when the variability in the input cost \( \sigma_c \) is sufficiently small. When \( \rho = 0 \), \( \bar{m} = \bar{m} = \sqrt{\frac{(\sigma_c^2 + \sigma_p^2)(\mu_c^2 + \sigma_c^2 + 4\sigma_p^2) - \mu_c(\sigma_c^2 + \sigma_p^2)}{2\sigma_c^2}} \), which is increasing in \( \sigma_p \), implying that (13) and (15) tend to hold when input price variability \( \sigma_p \) is small. (The results regarding the impact of \( \mu_c \) and \( \sigma_p \) on \( \bar{m} \) also apply when input efficiency reduces the input requirement.)

Proposition 2 reveals that variability in input prices has the opposite effect on the value of improving input efficiency as on the value of improving capacity efficiency. Variability in the output price and correlation also do so, provided that the expected margin \( m \) is large enough.

The results and intuition regarding capacity efficiency are straightforward. The manufacturer has the option to produce or not, which has value \( E[p - c] = \delta \Pi \delta k \). As is well known in finance, the value of \( E[p - c] \) increases with variability in \( p - c \), and hence with variability in input cost and output price (when correlation is not too large) and with reduced correlation.

The results and intuition regarding the value of improving input efficiency are sharpest when the expected margin is large \( m \gg 0 \). We begin with this case before turning to how the results change when the margin is small. The primary benefit of increasing input efficiency from the nominal value \( i = 1 \) is that doing so increases profit when it is economical to produce, an event that occurs with \( \text{Prob}(P - C > 0) \). When there is little variability in the contribution margin, it is almost always economical to produce \( (m \gg 0 \text{ implies } \text{Prob}(P - C > 0) \approx 1) \), and the manufacturer benefits the most from increasing input efficiency. As variability in the contribution margin increases, the probability that the manufacturer will be able to use the improved input efficiency decreases, as does the value of this efficiency. Consequently, improved input efficiency becomes less valuable with variability in input cost and output price (when the correlation is not too large), and with reduced correlation.

The results change when the expected margin is narrow for two reasons. First, the effect described above diminishes. In the limiting case with a zero expected margin, the probability that it is economical to produce is unaffected by the variability in the contribution margin. Consequently, a secondary benefit of increasing input efficiency—doing so increases the probability that it will be economical to produce—comes into play. This secondary effect can offset the primary effect, such that when the expected margin is small, input efficiency becomes more valuable with variability in the output price, and with reduced correlation.

The managerial contribution of Proposition 2 is to characterize how a manufacturer should respond in terms of efficiency-improvement efforts to changes in the variability and correlation in input and output prices. When input cost variability increases, a manufacturer should focus more effort on improving capacity efficiency and less effort on improving input efficiency. A large-margin manufacturer should respond in the same way to an increase in variability in the output price or a decrease in the correlation between the input cost and output price. A caveat is that improvements in input efficiency and capacity efficiency are complementary, so any improvement in capacity efficiency will increase the value of improving input efficiency.

**Flexibility in Input Efficiency vs. Capacity Efficiency**

A manufacturer that develops flexibility in input efficiency versus capacity efficiency can adapt to a rise in input costs or fall in output prices by increasing input efficiency at the expense of capacity efficiency. In many settings, flexibility is a tool to cope with variability in the external environment, and so investing in flexibility is sensible provided that the variability in the external environment is sufficiently high (Tombak and DeMeyer 1988, Mills 1984, Anupindi and Jiang 2008, Chod and Rudi 2005, Goyal and Netessine 2011). Proposition 3 identifies circumstances under which precisely the opposite is true regarding flexibility in input efficiency versus capacity efficiency.
Before stating the result, to build intuition, it is helpful to note under what price and cost realizations the flexibility in input efficiency versus capacity efficiency is of value. It is of no value when the input cost is very low or very high relative to the output price. If the input cost is very high \( c > ip \), then it is not economical to produce at all. If the input cost is low \( c < jp \), then it is optimal to squeeze every possible unit of output from the production facility by using as much of the input as possible (i.e., operating at the nominal input efficiency). Flexibility increases the manufacturer’s profit only when the input cost is moderately high relative to the output price \( c \in (jp, ip) \). Then, it is attractive to use the input more efficiently, at the expense of sacrificing some output.

**Proposition 3.** There exists \( \bar{m} > \mu_c (1 - j)/(1 + j) \) such that if \( \sigma_c = 0 \), then the value of flexibility in input efficiency versus capacity efficiency decreases with input cost variability

\[
\frac{\partial V}{\partial \sigma_c} \Delta < 0 \tag{16}
\]

if and only if the manufacturer’s expected margin \( m < \bar{m} \). There exists \( \bar{m} > \mu_c (1 - j)/(2j) \) such that if \( \sigma_c = 0 \), then the value of flexibility in input efficiency versus capacity efficiency decreases with output price variability

\[
\frac{\partial V}{\partial \sigma_p} \Delta < 0 \tag{17}
\]

if and only if the manufacturer’s expected margin \( m < \bar{m} \).

The value of flexibility in input efficiency versus capacity efficiency decreases with variability in the input cost or output price if and only if the expected margin is sufficiently small. Proposition 3 formalizes this result for the case variability is present on only one side (input cost or output price), but the result continues to hold when there is variability on the other side, provided that it is sufficiently small.

First, we discuss the intuition for why the value of flexibility is decreasing in input cost variability. This result occurs when the expected margin (and the variability in the output price) is not too large. The driving force behind the intuition is that flexibility in input efficiency versus capacity efficiency is of value only when the input cost is moderate \( c \in (jp, ip) \). When the expected margin is small, the effect of increasing variability in the input cost is to make moderate realizations of the input cost less likely and extreme realizations more likely. This shifts probability mass away from the realizations where flexibility in input efficiency versus capacity efficiency creates value, which reduces the expected value of this flexibility. (In contrast, when the expected margin is large, the effect of increasing variability in the input cost is to make moderate realizations of the input cost more likely, which increases the expected value of flexibility.) The intuition for the impact of output price variability on the value of flexibility in input efficiency versus capacity efficiency parallels the intuition for the impact of input cost variability.

We conclude that the manufacturer’s margin plays a critical role in determining whether increased variability increases the value of flexibility in input efficiency versus capacity efficiency. When variability is primarily on one side, the results are particularly sharp. Increased variability in that dimension increases the value of developing flexibility for a rich-margin manufacturer, but reduces the value for a thin-margin manufacturer. Because many price-taking manufacturers have relatively thin margins, the latter observation is especially relevant.

**Numerical Example: Amyris**

This section shows that for Amyris, the effects characterized in Propositions 1, 2, and 3—as well as the value of flexibility in input efficiency versus capacity efficiency—are large in magnitude.

Amyris’s production facility has nominal production capacity of 50,000 tons of farnesene per year, at an input efficiency of 1 ton of farnesene per 20 tons of crushed sugarcane. The output price for farnesene \( P \) in $/ton is equal to the price of West Texas Intermediate (WTI) crude oil in $/bbl, multiplied by 8.063. A ton of sugarcane contains 135 kg of raw sugar (Melo et al. 2011). According to guidelines from Consecana (a nonprofit association representing the sugar and ethanol industries in São Paulo State, Brazil, wherein the Amyris plant is located), a sugarcane farmer captures 60% of the value of that raw sugar (Consecana 2006). Therefore, we assume that the input price \( C \) in $/(20 tons sugarcane) is the price of raw sugar in $/kg multiplied by (135/0.6)/20; this is the variable cost of producing a ton of farnesene in the nominal mode of production. We also assume that the price of WTI oil and the price of raw sugar have a bivariate normal distribution with mean, standard deviation, and correlation calculated from the weekly prices for WTI oil and raw sugar #11 reported in the Global Financial Data database for the five years from June 2, 2007, through June 1, 2012. Hence the output price \( P \) and input cost \( C \) have a joint normal distribution with \( \mu_p = 685.5, \sigma_p = 168.7, \mu_c = 675.3, \sigma_c = 253.4, \) and \( \rho = 0.1415 \).

Figure 1 shows the value of improving capacity efficiency \( \partial V / \partial \epsilon \) in (2)) and the value of improving input efficiency \( \partial V / \partial \mu \) in (4)) as the standard deviation of the input cost \( \sigma_c \) varies from zero to \( \mu_c \) (left panel). Consistent with Proposition 2, variability in the input cost decreases the value of improving input efficiency and increases that of capacity efficiency. In contrast to the effect of input cost variability depicted in Figure 1, output price variability increases the value of improving input efficiency and that of capacity efficiency; this is consistent with Proposition 2 because
Amyris’s expected margin is thin $m < \tilde{m}$. The impact of output price variability on the value of improving capacity efficiency is pronounced: at the “true” $\sigma_c = 168.7$, the value of improving input efficiency is 7% higher and the value of improving capacity efficiency is 11% higher than in a scenario with no variability in output prices $\sigma_p = 0$. The impact of input cost variability is even more substantial: at the “true” $\sigma_c = 253.4$, the value of improving input efficiency is 25% lower and the value of improving capacity efficiency is 63% higher than in a scenario with no variability in input costs $\sigma_c = 0$. The large magnitude of the effect of variability in input costs and output prices shows the importance of accounting for such variability in deciding how much to invest in different forms of process improvement.

The value of improving input efficiency decreases in the expected input cost, which is consistent with Proposition 1 because Amyris’s thin expected margin implies that inequality (8) holds. The impact is large in magnitude: at the “true” $\mu_c = 675.3$, the value of improving input efficiency is 208% higher than at an expected input cost of $\mu_c = 1,008$, the average cost experienced in the last year of the five-year time frame.

The right panel depicts the value of flexibility in input efficiency versus capacity efficiency ($\Delta$ in (6)) as the standard deviation of the input cost $\sigma_i$ varies from 0 to $\mu_i$. We assume that Amyris has the flexibility to improve input efficiency to $i = 1.25$ by reducing its capacity efficiency to $k = 0.9$. This is “in the ballpark” but smaller than the magnitude of flexibility reported for other industrial processes that rely on metabolism by microorganisms (Bouallagui et al. 2003). Nevertheless, the value of that flexibility is large in magnitude. At the “true” $\mu_c = 253.4$, it increases expected profit by 43%. Consistent with Proposition 3, variability in the input cost decreases the value of flexibility. The impact is substantial. At the “true” $\sigma_c = 253.4$, the value of flexibility is 31% lower than in a scenario with no variability $\sigma_c = 0$.

4. Concluding Remarks

A tax on greenhouse gas emissions (or any other policy that increases the cost of fossil fuels) will tend to reduce investment in improving energy efficiency in basic material manufacturing. This observation follows from Proposition 1, because the tax will increase the expected energy cost and reduce expected margins, which are already thin for many basic material manufacturers.

In addition, any uncertainty and variability in the cost of greenhouse gas emissions (inherent in a cap-and-trade system, for example) will tend to further reduce investment in improving energy efficiency in basic material manufacturing. Variability in the cost of emissions translates into variability in energy costs for basic material manufacturers, which Proposition 2 suggests will reduce their investment in energy efficiency.

Moreover, for basic material manufacturers with thin margins, Proposition 3 shows that variability will tend to reduce investment in developing flexibility to operate with high energy efficiency by sacrificing some capacity efficiency. A manufacturer without that flexibility uses strictly more energy per unit output, in expectation.

An important caveat is that our analysis relies on the assumption that a firm does not produce when the input cost exceeds the output price. In reality, a firm might do so to meet a commitment to a customer, to build inventory (anticipating a higher selling price in the future), because of setup costs in starting and stopping production, or because the output price drops during a batch process. A firm that is more likely to operate has a greater motivation to improve its energy efficiency as energy become more costly. However, a setup cost or batch-processing time could motivate a firm to postpone the start of production until the output price substantially exceeds the input cost (as in the example of ammonia manufacturing (Seay 2012)) and thus reinforce the negative impact of a carbon tax on investment in energy efficiency.
In conclusion, our results suggest that a carbon tax or cap-and-trade system could backfire by reducing investment in energy efficiency by incumbent basic material manufacturers. Further, a tax that moved inversely with the market price of a fossil fuel (to reduce the variability in the effective price of that fuel) would be less of a deterrent to investment in energy efficiency.

Supplemental Material
Supplemental material to this paper is available at http://dx.doi.org/10.1287/msom.2013.0444.

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Appendix

Proof of Proposition 1. For (3), \( \frac{\partial}{\partial \mu} \Pi = -\text{Prob}(P - C \geq 0) \). Because by assumption the probability that \( P \) is negative is negligible, \( \text{Prob}(P - C \geq 0) \) is strictly increasing in \( i \), which implies (7). For (4), \( \frac{\partial^2}{\partial \mu \partial \mu} \Pi = -\text{Prob}(P - C /i \geq 0)/i \) and

\[
\lim_{i \to 1} \frac{\partial^2}{\partial \mu \partial \mu} \Pi = \text{Prob}(P - C \geq 0) = \lim_{i \to 1} \frac{\partial}{\partial i} \text{Prob}(P - C /i \geq 0). 
\]

We prove Proposition 2 under a very general formulation of input efficiency, which subsumes formulations (3) and (4) as extreme cases. Specifically, under input efficiency \( i \geq 1 \), the manufacturer's production process consumes \( f(i) \leq 1 \) units of input to produce ("yield") \( y(i) \geq 1 \) units of output, where \( y(\cdot) \) is increasing and \( f(\cdot) \) is decreasing, and at least one is strictly so. At the nominal input efficiency of \( i = 1 \), \( y(1) = f(1) = 1 \). The manufacturer's expected profit is

\[
\Pi = \int_{y(1)}^{\infty} \int_{f(1)}^{\infty} \text{Prob}(P - C) \psi(\mu, c, p) \phi(c, p) \, dc \, dp = \int_{y(1)}^{\infty} \int_{f(1)}^{\infty} \text{Prob}(P - C) \psi(\mu, c, p) \phi(c, p) \, dc \, dp.
\]

With \( y(1) = 1 \) and \( f(1) = 1 \), (18) simplifies to (3); with \( y(1) = 1 \) and \( f(1) = 1/i \), (18) simplifies to (4). For use in the proof of Proposition 2, define

\[
\alpha(\mu, \mu, \sigma, \sigma) = -\mu^2 \sigma_1 (\sigma - p \sigma)^2 + \mu_1 \mu_2 (\sigma - p \sigma)(\sigma^2 - \sigma_2^2) + \mu_1^2 \sigma_2 [(1 + \rho^2) \sigma_1 (\sigma - p \sigma^2) + \rho (\sigma^2 + \sigma_2^2) - (1 - \rho^2) \sigma_2^2 (\sigma^2 - 2 \rho \sigma_1 \sigma_2 + \sigma_2^2) - \rho \sigma_1^2 (\sigma^2 - \rho \sigma_1 \sigma_2 + \sigma_2^2)],
\]

\[
\beta(\mu, \mu, \sigma, \sigma) = (\sigma - p \sigma)(\sigma^2 - 2 \rho \sigma_1 \sigma_2 + \sigma_2^2) - \alpha(\mu, \mu, \sigma, \sigma),
\]

\[
\gamma(\sigma, \sigma) = \sigma_1^2 - 3 \rho \sigma_1 \sigma_2 + (2 + \rho^2) \sigma_2^2 - \rho \sigma_3^2,
\]

\[
\rho(\sigma, \sigma) = \left(3 \sigma_1^2 + \sigma_2^2 - 5 \sigma_1^2 - 2 \sigma_0^2 \sigma_1^2 + \sigma_2^2 \right) / (2 \sigma_0^2 \sigma_1),
\]

\[
s = \sqrt{\sigma_0^2 - 2 \rho \sigma_0 \sigma_1 + \sigma_2^2},
\]

\[
\lambda(m) = [(\mu + m)^2 - m^2] \sigma_1 (\sigma - p \sigma) - (\mu + m) \mu_1 (\sigma_1^2 - \sigma_2^2) + \mu_2^2 (\rho \sigma - \sigma_1) \sigma_2,
\]

\[
\theta(m) = -\lambda(m) - s^2.
\]

For use in the proofs of Propositions 2 and 3, let \( \psi(X) \) denote the density of the random variable \( X \) evaluated at 0.

Proof of Proposition 2. First, we establish the results involving \( \sigma \). For (2), observe that \( \lim_{i \to 1} (\partial^2 / \partial \mu \partial \sigma) \Pi = (\sigma_i - p \sigma) \psi(P - C) \), where the inequality follows because \( p < \sigma_i / \sigma_j \); this establishes (10). For (18), observe that \( (\partial / \partial \mu_j) \Pi = (f(i) \sigma_i - y(i) p \sigma)(\psi(y(i) P - f(i)) C) \). Further, \( \lim_{i \to 1} (\partial^2 / \partial \sigma \partial \mu) \Pi = \tau_i (m) (\psi(P - C) / s^4) \), where

\[
\tau_i (m) = \alpha(\mu, c, c, p) y(1) + \beta(\mu, c, c, p) f(1).
\]

Because \( (\partial^2 / \partial \mu^2) \tau_i (m) = 2 \sigma_1 (\sigma_i - p \sigma)^2 y(1) - f(1) < 0 \), and \( \lim_{m \to 0} (\partial / \partial \mu) \tau_i (m) = -\mu_1 (\sigma_i - p \sigma) s^2 y(1) - f(1) < 0 \), \( \tau_i (m) \) is concave decreasing in \( m \) on \( m \geq 0 \). To establish (11), it is sufficient to show that

\[
\tau_i (0) < 0 \quad \text{(19)}
\]

for \( \rho < \min(\sigma_i / \sigma_p, \sigma_p / \sigma_i) \), because \( \tau_i (0) > \tau_i (m) \) for \( m \geq 0 \). Note that

\[
\tau_i (0) = s^2 \gamma(\sigma_i, \sigma_i) y(1) - (1 - \rho^2) \sigma_i^2 f(1).
\]

Therefore, to establish (19), it is sufficient to show that

\[
\gamma(\sigma_i, \sigma_i) > 0 \quad \text{(20)}
\]

for \( \rho < \min(\sigma_i / \sigma_p, \sigma_p / \sigma_i) \). It is straightforward to show that (20) holds if and only if \( \rho < \rho(\sigma_i, \sigma_i) \). Further, if \( \sigma_i < \sigma_p \), then \( \rho(\sigma_i, \sigma_p) \in (\sigma_i / \sigma_p, 1) \); otherwise, \( \rho_i \geq 1 \). Therefore, because \( \rho < \min(\sigma_i / \sigma_p, \sigma_p / \sigma_i) \leq \rho(\sigma_i, \sigma_i) \), (20) and hence (19) hold.

Second, we establish the results involving \( \sigma \). For (2), observe that \( \lim_{i \to 1} (\partial^2 / \partial \mu \partial \sigma) \Pi = (\sigma_i - p \sigma) \psi(P - C) \), where the inequality follows because \( p < \sigma_i / \sigma_j \); this establishes (12). For (18), observe that \( (\partial / \partial \mu_j) \Pi = (y(i)(p \sigma)(\sigma_i - f(i) p \sigma)(\psi(y(i) P - f(i) C)) \). Further, \( \lim_{i \to 1} (\partial^2 / \partial \sigma \partial \mu) \Pi = \tau_i (m) \psi(P - C) / s^4 \), where

\[
\tau_i (m) = \beta(\mu, c, c, p, \sigma_p) y(1) + \alpha(\mu, c, c, p, \sigma_p) f(1).
\]

Note that \( (\partial^2 / \partial \mu^2) \tau_i (m) = -2 \sigma_1 (\sigma_i - p \sigma)(\sigma_i - f(i) p \sigma) \psi(y(i) P - f(i) C) \). Further, \( \lim_{m \to 0} (\partial / \partial \mu) \tau_i (m) = -\mu_1 (\sigma_i - p \sigma) s^2 y(1) - f(1) \), where the inequalities hold because \( \rho < \min(\sigma_i / \sigma_p, \sigma_p / \sigma_i) \). Therefore, \( \tau_i (m) \) is concave decreasing in \( m \) on \( m \geq 0 \). Further, \( \lim_{m \to 0} \tau_i (m) < 0 \). To establish that (13) holds if and only if \( m > \bar{m} \), where \( \bar{m} \in (0, \infty) \), it is sufficient to show that

\[
\tau_i (0) > 0 \quad \text{(21)}
\]

for \( \rho < \min(\sigma_i / \sigma_p, \sigma_p / \sigma_i) \). Note that

\[
\tau_i (0) = s^2 \gamma(\sigma_i, \sigma_i) y(1) - (1 - \rho^2) \sigma_i^2 f(1).
\]

Therefore, to establish (21), it is sufficient to show that

\[
\gamma(\sigma_p, \sigma_p) > 0 \quad \text{(22)}
\]
for $p < \min(\sigma/\sigma_p, \sigma_r/\sigma_r)$. It is straightforward to show that (22) holds if and only if $p < p(\sigma, \sigma_r)$. Further, if $\sigma_r > \sigma_r$, then $p(\sigma, \sigma_r) \in (\sigma_r/\sigma_r, 1)$; otherwise, $p(\sigma, \sigma_r) \geq 1$. Therefore, because $p < \min(\sigma/\sigma_p, \sigma_r/\sigma_r) \leq p(\sigma, \sigma_r)$, (22) and hence (21) hold. (Note that because $\tau_p(m)$ is quadratic in $m$ and $\tilde{m}$ is the larger root of $\tau_p(m) = 0$, $\tilde{m}$ is readily available in closed form.)

Third, we establish the results involving $p$. For (2), observe that $\lim_{m \to 0}(\hat{\theta}/\partial \theta) = \tau_p(m) = -\sigma(\sigma_r - \rho \sigma_p)(y(1) - f'(1)) > 0$ and $\lim_{m \to \infty}(\hat{\theta}/\partial \theta) = \tau_p(m) = \mu(s^2/y(1) - f'(1)) > 0$. Therefore, $\tau_p(m, \rho)$ is convex increasing in $m$ on $m > 0$. Further, $\lim_{m \to \infty}(\hat{\theta}/\partial \theta) = \tau_p(m) > 0$. Thus, $\tau_p(m) > 0$ and hence (15) if and only if $m > m$. Note that $m < 0$ if $\tau_p(0) > 0$. (Note that because $\tau_p(m)$ is quadratic in $m$ and $\tilde{m}$ is the larger root of $\tau_p(m) = 0$, $\tilde{m}$ is readily available in closed form.) $\square$

Proof of Proposition 3. First, we establish the results regarding (16). Suppose $\sigma_r = 0$. Then $(\hat{\theta}/\partial \theta) = \sigma = f_r(m)/\sqrt{2}\pi$, where $f_r(m) = (k/\pi)e^{-(m+1)(\mu_p)/(2\pi^2)} + (1 - k)e^{-(m+1)(\mu_p)/(2\pi^2)} - e^{-m^2/(2\pi^2)}$. If $m \leq m, (1 - j)(1 + j)$, which implies $m > 0$. Because $f_r(\cdot)$ is continuous, if $f_r(m) = 0$ has no positive root, then the result regarding (16) holds with $\tilde{m} = \infty$. Suppose instead that $f_r(m) = 0$ has at least one positive root. We will show that only one root exists. Let $\tilde{m}_r$ denote a root $f_r(m) = 0$, and index the roots so that $0 < \tilde{m}_1 < \tilde{m}_2 < \cdots$. Then $(\hat{\theta}/\partial \theta) = g_r(\tilde{m}_r) = g_r(\tilde{m}_r) = \mu(1 - j)(1 + j)$, where the first inequality is strict if and only if the second is strict. Because $f_r(m) = 0$, for $m \leq \mu_I, (1 - j)/(2j - k)$, where the first inequality is strict if and only if the second is strict. Because $f_r(m) = 0$, for $m \leq \mu_I, (1 - j)/(2j - k)$, $f_r(\cdot)$ is continuous, it must be that $(\hat{\theta}/\partial \theta) = g_r(\tilde{m}_r) > 0$, which implies $m > 0$. Suppose there exists a second root $\tilde{m}_r^*$. Then because $\tilde{m}_r^* > \tilde{m}_r > 0$, a contradiction because $f_r(\cdot)$ is continuous. Therefore, $f_r(m) = 0$ has one root, and $\tilde{m} = \tilde{m}_r$. Second, we establish the results regarding (17). Suppose $\sigma_p = 0$. Then $(\hat{\theta}/\partial \theta) = f_p(m)/\sqrt{2}\pi$, where $f_p(m) = k^2e^{-(m+1)(\mu_p)/(2\pi^2)} + (1 - k^2)e^{-(m+1)(\mu_p)/(2\pi^2)} - e^{-m^2/(2\pi^2)}$. If $m \leq m, (1 - j)/(2j)$, then $m < m$. The remainder of the proof follows by argument parallel to that above, where $\tilde{m}_p$ is defined analogously to $\tilde{m}_r$. Further, $g_p(m) = e^{-(m+1)(\mu_p)/(2\pi^2)} - e^{-(m+1)(\mu_p)/(2\pi^2)}$ is defined analogously to $g_r(m)$. Then $(\hat{\theta}/\partial \theta) = g_p(\tilde{m}_p) = (1 - j)/(\sigma_p^2)$; further, $g_p(m) \geq 0$ if and only if $m \leq \mu_I, (1 - j)/(2j - k)$, where the first inequality is strict if and only if the second is strict. $\square$

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