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Tree-unitary sigma models and their application to strong WW scattering¹

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Abstract

Sigma models are exhibited which have tree amplitudes for Goldstone boson scattering that satisfy elastic unitarity exactly. The models have imaginary coupling constants and the scalar propagators have poles on the imaginary axis in the complex p^2 plane. They are equivalent to K-matrix models, which are *ad hoc* unitarizations of low energy theorems for Goldstone boson scattering that have been used recently to describe strong WW scattering. The sigma model formulation of the K-matrix models may be used to estimate directly the effect of strong WW scattering on low energy radiative corrections.

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Introduction

In perturbation theory unitarity relates terms of different orders and cannot be satisfied in tree approximation. This paper exhibits nonstandard sigma models interpreted as effective tree-level theories in which the Goldstone boson scattering amplitudes in tree approximation do satisfy elastic unitarity exactly. The models have imaginary coupling constants and the sigma propagators have poles on the imaginary axis in the complex p^2 plane. However, the models are not crossing symmetric, a given sigma model only represents a specific scattering process, and the discussion is restricted to s -wave scattering. Even with these restrictions the models are useful in the context in which they were obtained, to represent strong WW scattering in a dynamically broken electroweak gauge theory in a manner consistent with chiral symmetry and unitarity. In addition to reproducing models of high energy WW scattering, the effective sigma model formulation may be used to estimate the contribution of strong WW scattering to low energy radiative corrections, which will be considered elsewhere.

The tree-unitary sigma models are equivalent to K-matrix unitarization of the low energy theorems for Goldstone boson scattering. The K-matrix is an *ad hoc* unitarization prescription that has been used to construct models of strong WW scattering[1] at high energy colliders. K-matrix models are suitable for the purpose because their partial wave amplitudes are generically “strong” (i.e., tend to saturate unitarity) while respecting chiral symmetry low energy theorems and unitarity.

The equivalence of K-matrix models to tree-unitary Higgs/sigma models emerges naturally from a gauge invariant formulation of strong WW scattering[2, 3] in which models of s -wave scattering are represented by means of effective “Higgs boson” (or sigma) propagators.³ The method is defined by a Feynman diagram algorithm, introduced in [3], which determines the 4-body scattering amplitudes involving gauge and/or Goldstone bosons ($WWWW$, $wWWW$, $wwWW$, and $wwwW$) from a model of the Landau gauge Goldstone boson scattering amplitude (www). Tree amplitudes computed from the diagrammatic algorithm represent the initial models exactly. Strong scattering models, which

³Because of the equivalence theorem[4] we can refer interchangeably to strong scattering of longitudinal W 's or Goldstone bosons. We also refer interchangeably to Higgs or sigma bosons and to Higgs or sigma models.

are typically formulated in Landau gauge, can then be transcribed to unitary gauge or to any generalized renormalizable R_ξ gauge[5]. The initial motivation was to use the U-gauge transcription to compute strong WW scattering signals at high energy colliders without using the effective W approximation[6] (EWA), in order to obtain information not available from the EWA, such as jet distributions needed for jet tags and vetos. The gauge invariant formulation was checked by direct computation[2] and by explicitly verifying BRS invariance[3].

We consider $I = 0$ and $I = 2$ Goldstone boson scattering channels, which both have s -wave threshold behavior. To any model s -wave amplitude the gauge invariant formulation associates a corresponding effective scalar propagator and interaction. In general, for an arbitrarily complicated scattering amplitude, the corresponding scalar propagator is arbitrarily complicated and the coupling “constant” is not constant but is a function of the scattering energy. But for K-matrix models the transcription is especially simple: the scalar propagators have simple poles, like elementary Higgs scalars, and the coupling constants are indeed constant. However, the pole positions are on the negative (positive) imaginary axis for $I = 0$ ($I = 2$) scattering and the coupling constants are imaginary.

Interpreted naively, the poles correspond to Breit-Wigner resonances with decay widths twice as big as their masses. The imaginary coupling constant implies a non-Hermitian Hamiltonian and therefore suggests a nonunitary S-matrix, an apparent paradox since the models are unitary by construction. In fact chiral symmetry assures the cancellation of the potentially nonunitary terms in the tree amplitudes just as it assures the threshold behavior required by the low energy theorems.

The two physical channels with pure s -wave threshold behavior are $W_L^+ W_L^- \rightarrow Z_L Z_L$ and $W_L^+ W_L^+ \rightarrow W_L^+ W_L^+$, where the subscript L denotes longitudinal polarization.³ The latter is pure $I = 2$ while the former is a superposition of $I = 0$ and $I = 2$. The Higgs boson representation of the K-matrix model for $W_L^+ W_L^+ \rightarrow W_L^+ W_L^+$ follows immediately from the transcription defined in reference [3], in which the amplitude is represented by a single effective (charge 2) scalar propagator. A valid representation of the K-matrix model for $W_L^+ W_L^- \rightarrow Z_L Z_L$ scattering can also be obtained using a single effective propagator, but the propagator does not have a single simple pole, the coupling “constant” is

not constant, and the theory does not have a Higgs/sigma model structure. A simple representation is obtained in this case by introducing two propagators, corresponding to the two isospin components of the s -channel amplitude, and the resulting effective theory is a two doublet Higgs boson model.

The next section explains the K-matrix prescription and presents the K-matrix amplitudes for $W_L^+ W_L^+ \rightarrow W_L^+ W_L^+$ and $W_L^+ W_L^- \rightarrow Z_L Z_L$ scattering. The third section describes the effective Higgs boson representation of the K-matrix model for $W_L^+ W_L^+ \rightarrow W_L^+ W_L^+$, while the fourth section considers representations of the $W_L^+ W_L^- \rightarrow Z_L Z_L$ scattering model with one or two effective scalar propagators. The BRS invariance of the effective two doublet model is illustrated by explicitly verifying one of the nontrivial BRS identities. The final section contains a brief discussion of the results and implications.

The K-matrix prescription

We consider elastic partial wave unitarity for massless particles since we are interested in Goldstone boson scattering or, correspondingly, in $W_L W_L$ scattering at high energy, $E \gg m_W$, where m_W can be neglected. Strictly speaking there is no domain of pure elastic scattering for massless particles, but inelastic scattering is strongly suppressed near threshold, and in practice $W_L W_L \rightarrow W_L W_L W_L W_L$ is negligible relative to $W_L W_L \rightarrow W_L W_L$ at the energies of interest to us[7], between 0.5 and 2 TeV. The unitarity constraint on the partial wave amplitude $a_{IJ}(s)$ for isospin I and angular momentum J (with $s = E^2$) is then

$$\text{Im } a_{IJ} = |a_{IJ}|^2. \quad (1)$$

A useful equivalent formulation of equation 1 is

$$\text{Im } \frac{1}{a_{IJ}} = -1. \quad (2)$$

The K-matrix prescription is defined by choosing an arbitrary real function $R_{IJ}(s)$ as the real part of the inverse of a_{IJ} ,

$$\text{Re} \left(\frac{1}{a_{IJ}} \right) = R_{IJ} \quad (3)$$

and then specifying the complete amplitude a_{IJ}^K by

$$\frac{1}{a_{IJ}^K} = R_{IJ} - i \quad (4)$$

which obviously assures equation 2.

For Goldstone boson scattering, we ensure consistency with the low energy theorems[8] that follow from chiral symmetry by appropriately choosing the real function R_{IJ} ,

$$R_{IJ} = \frac{1}{a_{IJ}^{\text{LET}}} \quad (5)$$

where a_{IJ}^{LET} is the low energy theorem amplitude. For the the s -wave channels the low energy theorem amplitudes are

$$a_{00}^{\text{LET}} = \frac{s}{16\pi v^2} \quad (6)$$

and

$$a_{20}^{\text{LET}} = -\frac{s}{32\pi v^2}. \quad (7)$$

At energies for which the $J = 0$ partial waves dominate, we have finally the K-matrix models for the $I = 0$ and $I = 2$ channels,

$$\mathcal{M}_0^K(s) = \frac{s}{v^2} \left(\frac{1 + ia_{00}^{\text{LET}}}{1 + (a_{00}^{\text{LET}})^2} \right) \quad (8)$$

and

$$\mathcal{M}_2^K(s) = -\frac{s}{2v^2} \left(\frac{1 + ia_{20}^{\text{LET}}}{1 + (a_{20}^{\text{LET}})^2} \right). \quad (9)$$

Including factors of two for states with identical particles the isospin decompositions of the physically relevant channels are

$$\mathcal{M}^K(w^+w^- \rightarrow zz) = \frac{2}{3}(\mathcal{M}_0^K - \mathcal{M}_2^K) \quad (10)$$

and

$$\mathcal{M}^K(w^+w^+ \rightarrow w^+w^+) = 2\mathcal{M}_2^K. \quad (11)$$

Effective Higgs boson model for $w^+w^+ \rightarrow w^+w^+$

Because it contains only a single isospin component in the s -channel the K-matrix model for $w^+w^+ \rightarrow w^+w^+$ scattering is easily expressed as an effective Higgs boson model simply by following the algorithm given in [3]. For an arbitrary model, labeled by X and specified by an R-gauge scattering amplitude \mathcal{M}_R^X , the corresponding effective s -channel propagator is

$$P^X(s) = -\frac{v^2}{s^2}(\mathcal{M}_R^X - \mathcal{M}^{\text{LET}}) \quad (12)$$

where \mathcal{M}^{LET} is the low energy theorem amplitude for the relevant channel. The corresponding Higgs sector coupling constant is

$$\lambda^X(s) = \frac{s}{2v^2} \frac{\mathcal{M}_R^X}{\mathcal{M}_R^X - \mathcal{M}_{\text{LET}}}, \quad (13)$$

The vertices that define the Feynman diagram algorithm are given in reference [3]. For $w^+w^+ \rightarrow w^+w^+$ scattering they differ in some instances from the standard model Feynman rules because the algorithm imposes an s -channel scalar exchange to represent interactions that arise from t - and u -channel exchanges in the standard model.⁴ The deviations from the standard model rules are specified in table 1 of reference [3]. In general P^X may have an arbitrarily complicated form depending on the form of \mathcal{M}_R^X , and the coupling “constant” is a function of the scattering energy, $\lambda^X = \lambda^X(s)$.

It is easy to obtain the effective scalar propagator corresponding to the K-matrix model for $w^+w^+ \rightarrow w^+w^+$. Substituting equation 11 and the low energy theorem

$$\mathcal{M}^{\text{LET}}(w^+w^+ \rightarrow w^+w^+) = -\frac{s}{v^2} \quad (14)$$

into equation 12 we find the effective propagator has the very simple form,

$$P^K(w^+w^+ \rightarrow w^+w^+) = \frac{-1}{s - m_{++}^2} \quad (15)$$

where

$$m_{++}^2 = 32\pi i v^2. \quad (16)$$

The propagator has a simple pole in the complex s plane, though at a peculiar location on the imaginary axis. Correspondingly, from equation 13 the coupling constant is in fact constant,

$$\lambda_{++}^K = 16\pi i \quad (17)$$

though with a peculiar imaginary phase. Notice that the algorithm is consistent with the standard model relation $m_{++}^2 = 2\lambda_{++}^K v^2$, which is essential for maintaining BRS invariance. The negative phase of the propagator arises because, as noted in [2, 3], we require an effective $I = 2$ s -channel exchange to represent forces due to $I = 0$ t - and u -channel exchanges in the standard model.

⁴For $w^+w^- \rightarrow zz$ the algorithm can be represented by an effective Lagrangian, but for w^+w^+ scattering the model is defined only by the Feynman diagram algorithm.

Effective Higgs boson model for $w^+w^- \rightarrow zz$

A valid BRS invariant representation of the K-matrix model for $w^+w^- \rightarrow zz$ scattering can also be obtained by substituting equation 10 and the low energy theorem

$$\mathcal{M}^{\text{LET}}(w^+w^- \rightarrow zz) = \frac{s}{v^2} \quad (18)$$

into equation 12. In this case the effective s -channel scalar exchange has the same topology as the standard model Higgs exchange and with the propagator and coupling constant of equations 12 and 13 the Feynman diagram algorithm agrees precisely with the standard model Feynman rules.[3] The effective propagator is then

$$P^K(w^+w^- \rightarrow zz) = \frac{2}{3} \left(\frac{1}{s - m_0^2} + \frac{1}{2} \frac{1}{s - m_2^2} \right) \quad (19)$$

where

$$m_0^2 = -16\pi iv^2 \quad (20)$$

$$m_2^2 = +32\pi iv^2 \quad (21)$$

and the coupling constant is

$$\lambda_{+-}^K = \frac{-16\pi i}{1 + ix} \quad (22)$$

where

$$x = \frac{s}{16\pi v^2}. \quad (23)$$

The propagator is the sum of two simple scalar poles but the coupling “constant” is not in fact constant. Although by the machinery of reference [3] this is a valid, BRS invariant representation of the model, it does not have a simple interpretation as a Higgs/sigma model.

The form of the propagator suggests that we instead consider an ansatz with two Higgs scalars. In particular, consider the two doublet model with complex doublets Φ_0 and Φ_2 , corresponding to the $I = 0$ and $I = 2$ components of the $w^+w^- \rightarrow zz$ amplitude. The most general potential with appropriate vacuum is[9]

$$\begin{aligned}
V(\Phi_0, \Phi_2) = & \sum_{a=0,2} \lambda_a (\Phi_a^\dagger \Phi_a - v_a^2)^2 \\
& + \lambda_3 [(\Phi_0^\dagger \Phi_0 - v_0^2) + (\Phi_2^\dagger \Phi_2 - v_2^2)]^2 \\
& + \lambda_4 [(\Phi_0^\dagger \Phi_0)(\Phi_2^\dagger \Phi_2) - (\Phi_0^\dagger \Phi_2)(\Phi_2^\dagger \Phi_0)] \\
& + \lambda_5 [\text{Re}(\Phi_0^\dagger \Phi_2) - v_0 v_2 \cos \xi]^2 \\
& + \lambda_6 [\text{Im}(\Phi_0^\dagger \Phi_2) - v_0 v_2 \sin \xi]^2
\end{aligned} \tag{24}$$

The vacuum expectation values satisfy $v^2 = v_0^2 + v_2^2$ with $m_W = gv/2$ and the Goldstone bosons are

$$w^a = \cos \beta \phi_0^a + \sin \beta \phi_2^a \tag{25}$$

where $\phi_{0,2}^a$ are the appropriate components of the complex doublets $\Phi_{0,2}$. The angle is determined by the ratio of the vev's, $\tan \beta = v_2/v_0$.

The pole positions in equation 19 correspond precisely to the $I = 0$ and $I = 2$ amplitudes,⁵ therefore we want the scalar eigenstates to be unmixed, $\alpha = 0$ in the conventional notation. Since we are only constructing an effective tree-level theory to replicate the K-matrix amplitude, equation 10, we are free to fine-tune the potential shamelessly. We therefore choose $\lambda_3 = \lambda_5 = 0$ so that H_0 and H_2 are the eigenstates with

$$m_a^2 = 2\lambda_a v_a^2 \tag{26}$$

To determine the angle β we proceed as in [2, 3] and use the equivalence theorem[4] to determine the U-gauge Higgs sector contribution,

$$\mathcal{M}_{U,H}^K = \mathcal{M}_R^K - \mathcal{M}_{\text{gauge sector}} \tag{27}$$

As always in discussions of strong WW scattering we neglect corrections of order g^2 and m_W/\sqrt{s} . In that approximation $\mathcal{M}_{\text{gauge sector}} \simeq s/v^2$ and substituting \mathcal{M}_R^K from equation 10 we find

$$\mathcal{M}_{U,H}^K = \frac{2}{3} \frac{s^2}{v^2} \left(\frac{1}{s - m_0^2} + \frac{1}{2} \frac{1}{s - m_2^2} \right). \tag{28}$$

⁵That is, they are the poles that would emerge by substituting equations 8 and 9 into equation 12.

This is to be compared with the contribution from exchange of the two Higgs scalars in the two doublet model,

$$\mathcal{M}_{U,H}^{2\text{-doublet}} = \epsilon_{L1} \cdot \epsilon_{L2} \epsilon_{L3} \cdot \epsilon_{L4} \sum_{a=0,2} \left(\frac{g^2 v_a}{2} \right)^2 \frac{1}{s - m_a^2} \quad (29)$$

where ϵ_{L_i} are the longitudinal polarization tensors for the four gauge bosons. Indices 1 and 2 refer to W^\pm and indices 3 and 4 to the final state Z bosons. For simplicity, here and in the discussion of BRS invariance below, we assume the gauge group is just $SU(2)_L$; I have verified that the conclusions are the same for $SU(2)_L \times U(1)_Y$. Approximating $\epsilon_{L_i} = p_i/m_W$ we find that equations 28 and 29 are consistent if

$$\tan\beta = \frac{1}{2}, \quad (30)$$

that is, $v_0^2 = 2v^2/3$ and $v_2^2 = v^2/3$. With the masses, equation 20 and 21, this in turn fixes the coupling constants,

$$\lambda_0 = -12\pi i \quad (31)$$

and

$$\lambda_2 = +48\pi i. \quad (32)$$

It is now straightforward to close the circle by using the parameters determined above to verify that the tree amplitude $\mathcal{M}(w^+w^- \rightarrow zz)$ computed from the two doublet model is indeed the K-matrix amplitude of equation 10.

We conclude this section by considering one of the BRS identities that is nontrivial in the sense that it probes the consistency of gauge and Higgs sector interactions,

$$\epsilon_{3\alpha}\epsilon_{4\beta} \left(k_{1\mu}k_{2\nu}\mathcal{M}^{\mu\nu\alpha\beta} + im_W(k_{1\mu}\mathcal{M}_{w^-}^{\mu\alpha\beta}k_{2\nu}\mathcal{M}_{w^+}^{\nu\alpha\beta}) - m_W^2\mathcal{M}_{w^+w^-}^{\alpha\beta} \right) = 0, \quad (33)$$

where subscripts 1,2,3,4 refer to W^+, W^-, Z, Z respectively. The subscripts w^\pm indicate amplitudes in which gauge boson W^\pm is replaced by Goldstone boson w^\pm . Using the Feynman diagram algorithm, which for $W^+W^- \rightarrow ZZ$ is just the standard model Feynman rules, to evaluate the amplitudes in R_ξ gauge we find after canceling identical terms that the left side of equation 33 is

$$\delta_{\text{BRS}}^2 = \frac{g^2}{8} \epsilon_3 \cdot \epsilon_4 \left(-v^2 + \sum_{a=0,2} \frac{v_a^2}{s - m_a^2} (s - 2\lambda_a v_a^2) \right) \quad (34)$$

which vanishes by equation 26. Consistency is assured for this and the other BRS identities because for $W^+W^- \rightarrow ZZ$ our diagrammatic algorithm is just the usual Feynman rules for the two doublet model. (BRS invariance of the algorithm for the w^+w^+ channel is less obvious because of departures from the usual Feynman rules in that case — see [3].)

Discussion

A similar result to the peculiar pole positions found here was obtained in a study of the $I, J = 0, 0$ channel in the $O(2N)$ Higgs/sigma model solved to leading order in the $N \rightarrow \infty$ limit.[10] Evaluating the solution for $N = 2$ (only 33% worse than standard operating procedure for large N QCD) the authors found a “Higgs remnant” far from the real axis in the fourth quadrant of the complex s plane. In the strong coupling limit the pole position tended to $-16\pi i v^2/3$, a factor 3 smaller than our K-matrix value for m_0^2 (though, as observed by Einhorn[10], the limit is actually outside the domain of validity of the model).

For a heuristic interpretation of the pole positions on the imaginary axis in the complex s plane we can consider the Breit-Wigner form,

$$P_{BW} = \frac{1}{s - (m - i\Gamma/2)^2} \quad (35)$$

where m and Γ are real. For the pole to occur on the imaginary axis, the width must be twice the mass, $\Gamma = \pm 2m$.

The imaginary coupling constants suggest a gross violation of unitarity, since a non-Hermitian Hamiltonian implies a nonunitary S-matrix, but by construction the tree amplitudes satisfy partial wave unitarity exactly. The explanation is that chiral symmetry protects the unitarity of the tree amplitudes just as it assures the threshold behavior required by the low energy theorems. Explicitly, in tree approximation the scattering amplitude is the sum of the constant, imaginary 4-point contact interaction, $-2\lambda_a$, and the s -channel Higgs/sigma exchange term which contains a canceling imaginary constant. Chiral symmetry requires the amplitudes to vanish at threshold and therefore enforces the cancellation of the constants regardless of their phase.

It is unexpected and interesting that scattering mediated by scalar exchanges with poles on the imaginary axis in the m^2 plane corresponds to tree

amplitudes that precisely follow the trajectory of the Argand circle characterizing exact elastic unitarity. That observation is useful to estimate directly the effect of strong WW scattering on low energy radiative corrections. Most discussions of low energy radiative corrections in theories with dynamical electroweak symmetry breaking have focused on the effects of specific quanta in specific models, for instance, the large oblique corrections from techni-quarks or technicolor pseudo-Goldstone bosons.[11] In the same spirit as the analysis of strong WW scattering at high energy experiments, which emphasizes model independent aspects of electroweak symmetry breaking by a strong force, it would be useful to estimate the effect on low energy corrections of just the strong scattering in the WW channels, without reference to specific model dependent features. Such an estimate can be made using the Higgs boson representation of the K-matrix models and will be considered in a subsequent paper.

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