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Combination Product Set of Erv Wilson

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UNIVERSITY OF CALIFORNIA SAN DIEGO

Rotations Ensemble: Building a Musical Vocabulary with the
1-3-5-7-9-11 Eikosany Combination Product Set of Erv Wilson

A dissertation submitted in partial satisfaction of the
requirement for the degree Doctor of Philosophy

in

Music

by

Daniel Corral

Committee in Charge:

Professor Rand Steiger, Chair
Professor Anthony Davis
Professor Nancy Guy
Professor Bobby McElver
Professor Karola Obermüller

2025

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University of California San Diego

2025

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I'd love to thank the many self-described microtonalists who I've talked with and interviewed over the years, but that list would be much too long so I'll save it for another time.

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PUBLICATIONS

“A Beginner’s Guide to Erv Wilson.” *California Festival*. Los Angeles Philharmonic, 2023a.

<https://www.cafestival.org/excursions/a-beginners-guide-to-erv-wilson-by-daniel-corrall/>.

“California Microtonal ’zines.” *California Festival*. Los Angeles Philharmonic, 2023b.

<https://www.cafestival.org/excursions/california-microtonal-zines/>.

“The Mystery of Mildred Couper - by Daniel Corral.” *California Festival*. Los Angeles Philharmonic, 2023c.

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“A History of Changes.” Liner notes for *Changes: 64 Studies for 6 Harps*, by James Tenney. New World Records 80810, 2019, compact disc.

FIELDS OF STUDY

Music Composition

ABSTRACT OF THE DISSERTATION

Rotations Ensemble: Building a Musical Vocabulary with the
1-3-5-7-9-11 Eikosany Combination Product Set of Erv Wilson

by

Daniel Corral

University of California San Diego, 2025

Professor Rand Steiger, Chair

This dissertation details the musical vocabulary I have developed around the 1•3•5•7•9•11 Eikosany, focusing on five pieces I have written for Rotations Ensemble, a group formed to play Eikosany-based music. The Eikosany is a Combination Product Set (CPS) created by Californian experimental theorist Erv Wilson as a method for building Just Intonation scales. To better understand this uncentered spatial metaphor for microtonal pitch organization, I catalogued the smaller CPS contained in the five-dimensional pitch lattice, built a framework for

functional harmony within it based on “Harmonic Class,” and codified a compositional process based on harmony studies that I call “Incomplete Open Forms.” Since Wilson’s theories have not received much scholarly attention, I consulted independent sources such as the online Wilson Archives, zine-like journals such as *Xenharmonikôn* and *1/1: The Journal of the Just Intonation Network*, archives of conversations on the Mills Tuning List listserv, and guidance from former students and colleagues of Wilson’s like Kraig Grady, Terumi Narushimi, John Chalmers, and Stephen James Taylor. Using what I learned, I created Rotations Ensemble to perform my Eikosany-based music. In this dissertation, I first give an introduction to Combination Product Sets as scale-building tools, followed by a dissection of the 1•3•5•7•9•11 Eikosany. After establishing the concepts and terminology, I turn to my development of Rotations Ensemble, detailing instrumentation, notational practice, techniques for generating form, and musical influences. In the final section, I describe five compositions of mine that were performed by Rotations Ensemble in Spring 2025, including important formal and harmonic elements, as well as any other relevant musical justifications for my compositional choices. Through this process, I’ve come to see Wilson’s theoretical ecosystem as a collection of tools for musical individuation rather than as a prescriptive methodology for high performance in institutionally validated metrics of success.

Overview

In 2019, I began learning to compose music using Combination Product Sets (CPS)—a series of multidimensional pitch lattices theorized by Californian microtonalist Erv Wilson. Fitting my compositional ideas into the new framework has required learning—and in some cases, inventing—a new musical vocabulary. In 2021, I decided to focus on the 1•3•5•7•9•11 Eikosany, a twenty note pitch lattice based on the lowest six odd-numbered partials of the harmonic series. I hoped that a deep dive into a single lattice would give me a conceptual framework for learning how to approach other musical systems.

I’ve received guidance on understanding the 1•3•5•7•9•11 Eikosany from former students of Erv Wilson, since there has been little scholarly attention to his ideas. Most of his theoretical work was published in small, independent, zine-like journals and is now available online at the Wilson Archives.¹ However, Wilson’s documents are often offered with little introduction and can be difficult to parse. At the time of this writing, discussions of his work can be found online in archives of the Mills Tuning List (1994-1998), in the “Wilson’s Garden” Discord group, and in the Erv Wilson Discussion Group on Facebook.²

I began to develop a version of functional harmony with the Eikosany by categorizing intervals and sub-lattices within it. I decided on a notational system and an approach to form, and organized a musical group called Rotations Ensemble to play my music. In order to contextualize my creative work with the 1•3•5•7•9•11 Eikosany, I’ll first give a brief introduction to Erv Wilson, explain the basics of CPS lattices, and present my understanding of the Eikosany. Once

¹ For more on these independent microtonal publications, see my article *California Microtonal Zines*, written for the 2023 California Festival.

² The most complete archive of the Mills Tuning List can currently be found at <https://yahootuninggroupsultimatebackup.github.io/mills-tuning-list/>

the theoretical vocabulary is established, I'll describe how I have used harmony studies to build an approach to form, then describe five pieces I've composed for Rotations Ensemble.

1. Introduction to The Eikosany

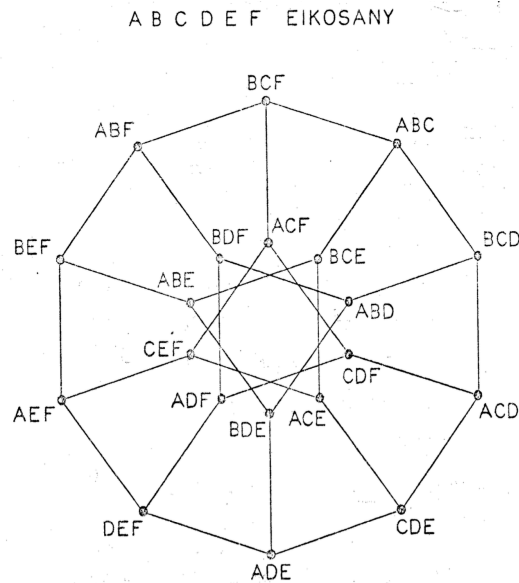


Figure 1: An Eikosany (Wilson 1989)

My interest in the Eikosany

I became interested in pitch lattices during my studies with James Tenney 2005-2006, who introduced me to Partch's tonality diamond. Tenney passed away in 2006, and in 2012 I began working for Tenney's widow Lauren Pratt, organizing his notes and scores to be sent to York University for archiving.³ Many of his notes were in unsorted piles, leaving me to decipher which piece each note was related to. Through this process, I got to know Tenney's catalog well enough to understand which composition a hand-drawn stochastic graph, BASIC printout, or pitch lattice might correspond to.

³ I took over this work from composer/violist Cassia Streb

In 2014, I started using Novation Launchpads as musical instruments, with the notes arranged similarly to one of Partch's Diamond Marimba. In 2016 I composed several pieces with the 7x3 lattice from LaMonte Young's *Well-Tuned Piano*. When I started composing a Launchpad quartet called *Polytope* in 2017, I designed several three-dimensional pitch lattices to map onto the Launchpads. However, as I explored these possibilities, they didn't sound how I had hoped. So, I reverted back to a 2D tonality diamond, but a 15-limit tonality diamond, since that's what fits on the 8x8 button grids of the Launchpads. Each row or column representing an otonality or utonality of 1, 3, 5, 7, 9, 11, 13, and 15.⁴

In 2019, I came across Terumi Narushima's book *Microtonality and the Tuning Systems of Erv Wilson*, which led to the beginning of my understanding of Erv Wilson's Combination Product Set (CPS). These seemed to be exactly what I was looking for when I was designing *Polytope*: a multidimensional JI lattice that also related to Partch's tonality diamond. In fact, Wilson has used the same the 5-dimensional geometry of the Eikosany to represent Partch's 11-limit tonality diamond.

Musicians who have connected with Wilson's ecosystem often describe having a transformative experience that changes their relationship to music and pitch. There is a surprising poetry in how his mathematical systems connect to the physicality of sound. His "Combination Product Sets" (CPS) are a good example of this. They are tools for generating Just Intonation (JI) scales that are also multidimensional pitch lattices. Under the hood is math—Pascal's Triangle and a series of multivariable multiplication tables—but the geometric diagrams that come out are eloquent JI structures akin to sacred geometry, which allow one to bypass the math once it's understood. Figure 2 shows Wilson's uncentered CPS lattices with up to twenty vertices,

⁴ Utonality is when a series of pitch ratios have the same numerator, while Otonality is when a series of pitch ratios have the same denominator. In *Genesis of a Music*, Partch calls the number that ratios in a Utonality or Otonality have in common a "Numerary Nexus" (Partch 1974, 72)

organized by complexity from top to bottom, with the single vertex Monony at the top and descending to the twenty-vertex Eikosany at the bottom.⁵ Once a lattice has been turned into a scale, it can be used as a visual reference for making music.

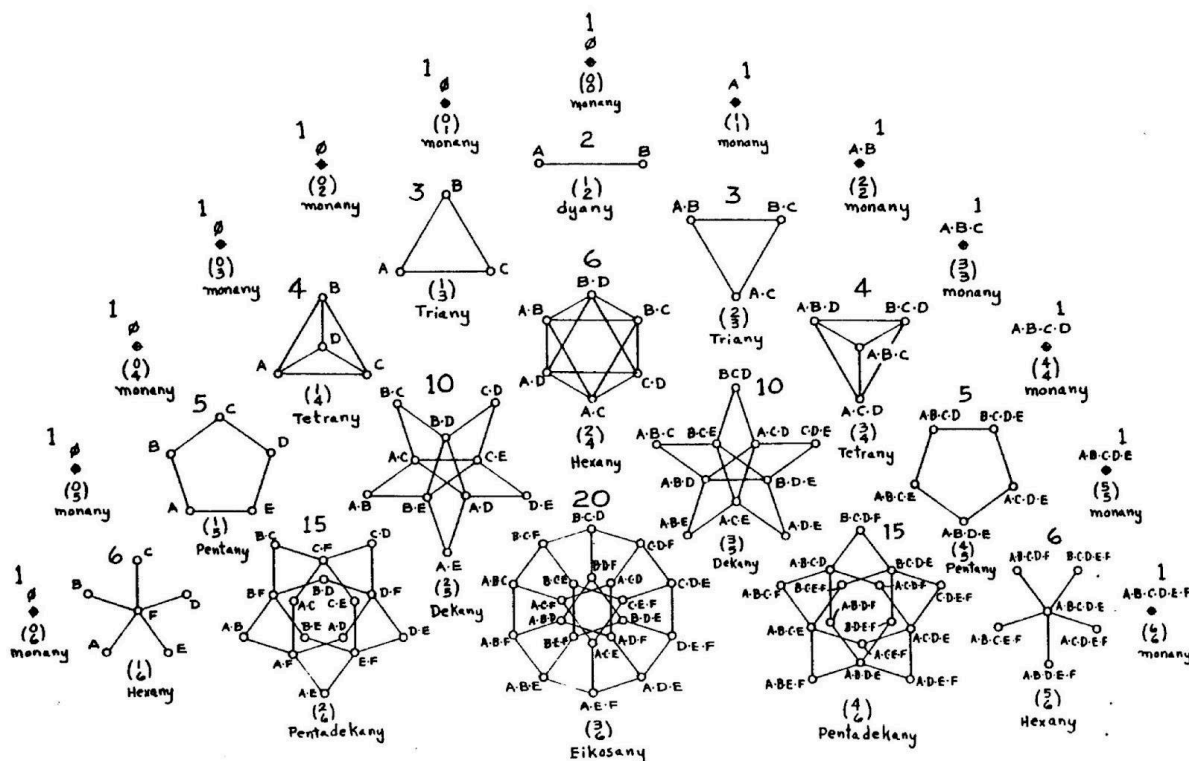


Figure 2: “A Pascal Triangle of Combination Set Lattices” (Wilson 1989)

A Brief Bio of Erv Wilson

Wilson’s background in agriculture informed how he approached music. A brief biography may shed some light on how these theories came to be. Ervin Wilson was born in Colonia Pacheco, Chihuahua, Mexico in 1928, to a family of Mormon missionaries from Utah. His childhood was a modest farm life, which set the tone for his later work. He learned to play the reed organ from his mother and became interested in South Indian music. As a teenager Wilson joined the US Navy, and during his stint in Japan he learned about the harmonic series

⁵ Though the Eikosany is the most complex lattice shown, the complexity can continue indefinitely, along with the great lattice names. An example of this is the Hebdomekontany, which John Chalmers describes as “a 70-tone harmonic structure in a space of eight harmonic axes.” (Chalmers 1974)

and Japanese pentatonic scales. After briefly studying at Brigham Young University, Wilson moved to LA, where he lived for most of his adult life and developed his microtonal theories while working with his father to distribute herbs from Northern Mexico to the US (Wilson 2009).

His connection to agriculture led him to think of musical scales in the same way he thought of plants. “He saw each kind of scale as a plant species that had the potential for growth and transformation...” (Narushima 2019). Wilson loved Southern California plants and typically maintained a flourishing garden wherever he was. However, his horticultural passion was in breeding high lysine yellow and white corn that could grow in the high altitude of the Sierra Madre Occidental mountain range where he grew up. Wilson took over the Colonia Pacheco family farm in 1970 and experimented with mixing local seed varieties with seeds from US and Mexican research institutions to provide higher-protein corn to feed northern Mexicans (Burns 1986).

In Los Angeles, Wilson worked as a draftsman and mechanical engineer and was part of a devoted community of microtonalists. He was economically among the artist class, and was often a roommate with musicians and students. In 1974, Wilson’s long-time friend John Chalmers began publishing Wilson’s work in his independent journal *Xenharmonikôn*, and for decades that was almost the only place it could be found.⁶ He taught numerous composers, musicians, and instrument builders—primarily from Los Angeles. Later in his life, these same students would take care of him and his archives, making sure he was comfortable and his archived material would be available.

Today, there is a cult-like following of musicians devoted to Wilson’s work, though his name is enigmatic to most outside of that circle and his ideas remain impenetrable to many who approach them independently. He did not seek to be a famous composer or performer, but rather

⁶ Sundry articles and diagrams could be found in *1/1: The Quarterly Journal of the Just Intonation Network*

a teacher whose approach to music seemed parallel to his love for gardening. His various musical systems fit into an interwoven ecosystem of number theory and geometry, with various genus and species. In addition, these theories were not hard-line rules but malleable tools for creating personally meaningful musical systems. He did not build musical scales but laid out a set of eloquent tools for others to build their own scales.

Wilson did not have an academic background and his students found him through word of mouth. His teaching/mentorship was tailored to each student's interests, and often involved conceptual puzzles just beyond the student's understanding. Several former students of Wilson have told me of receiving hand drawn diagrams with no explanation, leaving them to contemplate at great length what it might mean. Some of Wilson's former students include Kraig Grady, Paul Rapoport, Gary David, Emil Richards, Stephen James Taylor, Marcus Hobbs, Rod Poole, and Terumi Narushima. A great introduction to Wilson's concepts is the app *Wilsonic*, made by Marcus Hobbs. The iOS app allows a user to play and hear many of Wilson's ideas, while the *Wilsonic* MTS-ESP plugin expands on this by interfacing with a DAW.

Intro to Combination Product Sets

Of Wilson's musical theories, Combination Product Sets have gained the most interest. There are several essential resources for understanding CPS, both online and in print. I was first able to understand this information through Terumi Narushima's book *Microtonality and the Tuning Systems of Erv Wilson*. The most-referenced CPS document is Erv Wilson's 1989 article *D'allesandro, Like a Hurricane*, printed in *Xenharmonikon 12* and now available online on Kraig Grady's website dedicated to Wilson's archive alongside many more documents related to CPS.

D'Allessandro gives a brief overview of Wilson's exploration of CPS, including numerous examples of lattices and how they might map onto Wilsons' Generalized Keyboard—an isomorphic hexagonal keyboard design. Most of the diagrams in this writing come from the *D'Allessandro* article, including my own Eikosany diagrams which are annotated on Wilson's Eikosany lattice.

Figure 2 shows “A Pascal Triangle of Combination Set Lattices” from *D'Allessandro*, which contains various uncentered lattices, arranged in the same way as Pascal's triangle. With the exception of the Monony at the top, the two Tetranies in the fourth row, and the two Hexanies on the bottom row of Figure 2, there is no vertex in the center of any of the other diagrams.⁷

For a classically trained musician, Wilson's CPS terminology takes some getting used to—he refers to a dyad as a Dyany, a triad as a Triany, etc. This paper will use both terms, but with different meanings. Terms like “tetrad” or “hexad” will be used to refer to note groups within a larger CPS structure (like tetrads within a Hexany). Wilson's terms like Tetranity or Hexany will be capitalized and used to refer to isolated pitch lattices.

⁷ Since the Monony only has a single vertex, an exception can be easily made. The 1/6 Hexany and the 5/6 Hexany are the only lattices that are not contained in closed polygons.

The Hexany

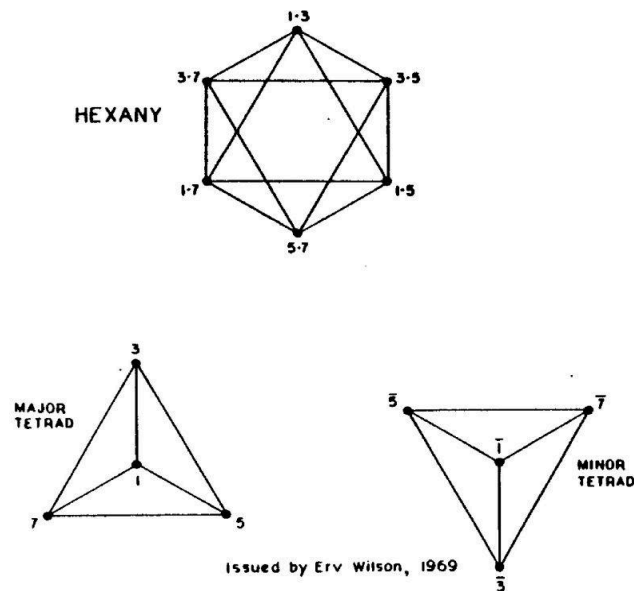


Figure 3: A 1•3•5•7 Hexany and its Generating Tetrads (Wilson 1989)

A Hexany is a CPS with six vertices as seen in Figure 3. When translated into a scale, each vertex becomes a note and each line (or edge if we think of it as a 3D shape) becomes an interval, just as they are in most other pitch lattices. In Figure 2, all of the vertices have two numbers in parenthesis below them, with one above the other like a ratio. The bottom number shows the number of variables in the lattice, and the top number shows how many of those variables will be multiplied together. In each row, the top number of the leftmost Monony is a zero. As you go across horizontally, this number increases by one. The bottom number increases as the number of the row increases. Thus, the Hexany has 2 on top because it is the second lattice from the left, and 4 on bottom because it is in the fourth row. For the Hexany, this means that two of the four variables (A, B, C, and D) will be multiplied against each other, as in Figure 4.

	A	B	C	D
A	AA	AB	AC	AD
B	BA	BB	BC	BD
C	CA	CB	CC	CD
D	DA	DB	DC	DD

Figure 4: The multiplication table for making a Hexany

If you eliminate the redundancies (like BA vs AB) and cases of a variable being squared, you are left with six pairs of variables: AB, AC, AD, BC, BD, and CD. These six pairs can then be arranged into a Hexany.⁸ Though Wilson's Hexany diagram looks like a star of David in two dimensions, in three dimensions it is also an octahedron, as shown in Figure 5.

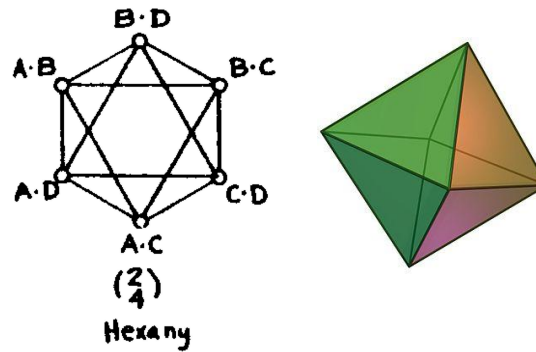


Figure 5:
Left: A Hexany as drawn by Erv Wilson.
Right: An Octahedron (Cyp 2004)

⁸ The process of placing the variables in the right place is detailed more in Narushima's book.

The letter variables can be replaced with numbers. One of the fascinating things about these CPS lattices is the capacity to use any variables. A single lattice can generate numerous JI scales, depending on the variables chosen. Using the lowest four odd-numbered partials (1, 3, 5, and 7) to replace A, B, C, and D respectively, multiplying the six pairs of variables results in the numbers 3, 5, 7, 15, 21, and 35. Each of these numbers corresponds to one of the axes in the Hexany. The next step is to choose one of the numbers to be the 1/1 of the scale by dividing all of the numbers by it.⁹ Wilson suggests using the largest of these numbers to make the math simpler (Narushima 2019). For this writing, I will use 7 (AD) as the 1/1. By dividing the six numbers by 7 and reducing the resulting ratios to the range between 1/1 and 2/1, the resulting ratios are 1/1 (AD), 15/14 (BC), 5/4 (CD), 10/7 (AC), 3/2 (BD), and 12/7 (AB). Figure 6 shows what the pitches would be if these ratios were turned into a musical scale starting on C.¹⁰

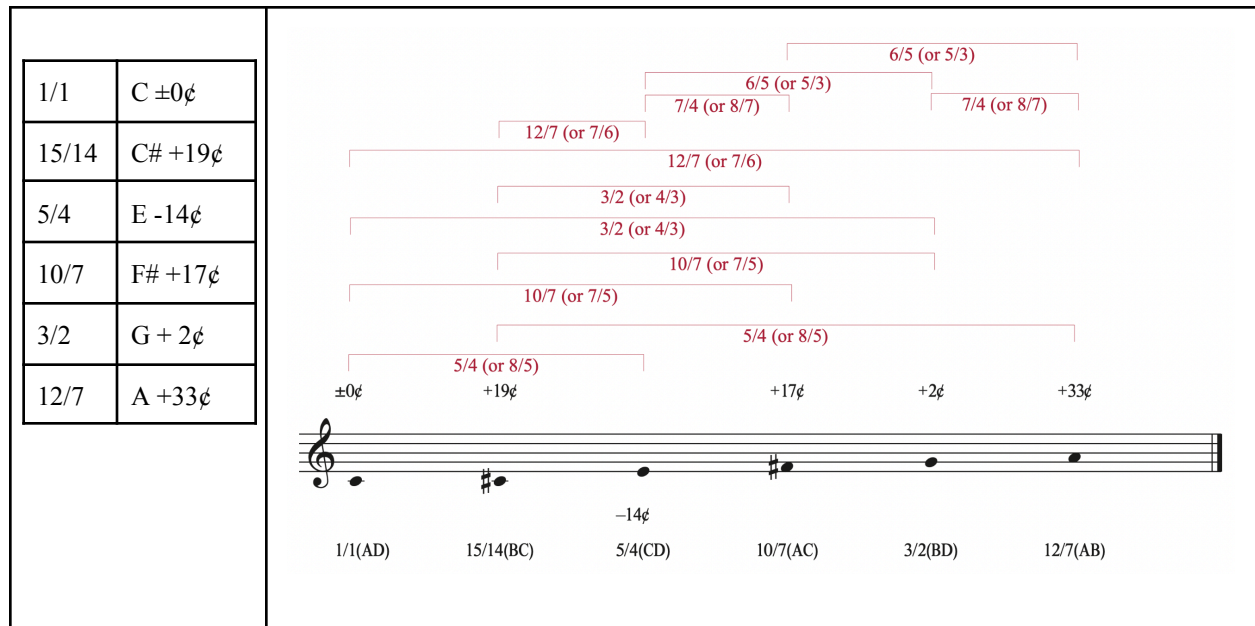


Figure 6:

Left: A chart of the notes and intervals of a 1•3•5•7 Hexany, with AD as 1/1, in C
Right: The same notes written in standard western notation

⁹ The resulting intervals will be the same regardless of which vertex becomes the 1/1.

¹⁰ Rounded to the nearest cent, and generated using the usual formula of $\log_2 x \cdot 1200$, with X being the JI interval.

A Hexany can be defined as a closed six-note shape with two complementary triads and 6 pairs of parallel intervals. In Figure 6, the intervals are drawn above the staff in red. In Figure 5, the “major” triad is the upright triangle formed by AD, CD, and BD in (C, E-14¢, G+2¢), while the “minor” triad is the upside-down triangle formed by AC, AB, and BC (F#+17¢, A+33¢, C#+19¢).

CPS as spatial metaphor

A CPS is a tool for generating musical scales, and once the work of constructing the scale is done, music can finally be made with it. Rather than using standard notation or graphs, the most streamlined way to interact with CPS lattices is to use the visual metaphor of the lattice directly.

Each line of the Hexany (which represents a vertex in the octahedron) acts like the lines in any pitch lattice: as an interval. Thus, all parallel lines are parallel intervals. The interval from AC to BC is parallel to that from AD to BD, so it must be a 3/2. In Figure 6, it is clear that the distance from F# +17¢ to C# +19¢ is indeed a 3/2.¹¹ By comparing the notated intervals in Figure 6 to the Hexany lattice in Figure 5, the interval relationships along the drawn lines of the Hexany should eventually be apparent without referencing Figure 6.¹²

All of the 2D triangles in the Hexany can be identified as part of the generating tetrad shown in Figure 3. Continuing in this way, this tetrad (and thus the hexany) can be further dissected into triads, dyads, and monads.

¹¹ With the 3/2 ratio of a just fifth being 702¢

¹² The intervals not represented by lines in the Hexany, like between AD and BC, will probably still take some calculation.

The Eikosany

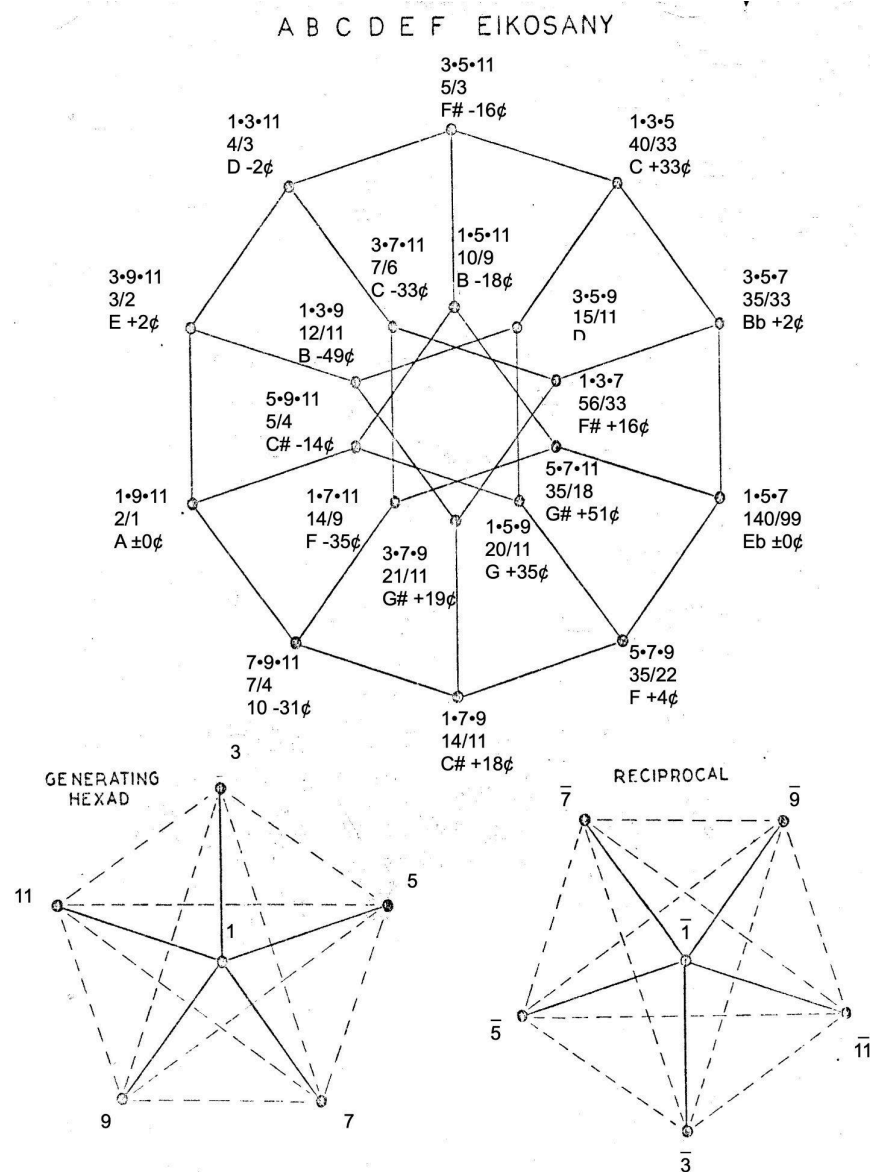


Figure 7: The 1·3·5·7·9·11 Eikosany I have been using, annotated on a diagram from *D'Allesandro*

The same rules and techniques used to generate a Hexany can be applied to constructing the 1·3·5·7·9·11 Eikosany in Figure 7. In the 2/4 Hexany, each vertex represents two out of four variables, but each vertex in the Eikosany represents 3 out of 6 variables. John Chalmers created

the multiplication table in Figure 8 with Erv Wilson to illustrate the intervals of a $1\cdot3\cdot5\cdot7\cdot9\cdot11$ Eikosany (Chalmers 2021).

Intervals of the $(\frac{3}{2})_{1357911}$ Eikosany

×	1·3·5	1·3·7	1·3·9	1·3·11	1·5·7	1·5·9	1·5·11	1·7·9	1·7·11	1·9·11	3·5·7	3·5·9	3·5·11	3·7·9	3·7·11	3·9·11	5·7·9	5·7·11	5·9·11	7·9·11
1·3·5	(1)	$\frac{7}{5}$	$\frac{9}{5}$	$\frac{11}{5}$	$\frac{7}{3}$	$\frac{9}{3}$	$\frac{11}{3}$	$\frac{7\cdot9}{3\cdot5}$	$\frac{7\cdot11}{3\cdot5}$	$\frac{9\cdot11}{3\cdot5}$	$\frac{7}{1}$	$\frac{9}{1}$	$\frac{11}{1}$	$\frac{7\cdot9}{1\cdot5}$	$\frac{7\cdot11}{1\cdot5}$	$\frac{9\cdot11}{1\cdot5}$	$\frac{7\cdot9}{1\cdot3}$	$\frac{7\cdot11}{1\cdot3}$	$\frac{9\cdot11}{1\cdot3}$	$\frac{7\cdot9\cdot11}{1\cdot3\cdot5}$
1·3·7	$\frac{5}{7}$	(1)	$\frac{9}{7}$	$\frac{11}{7}$	$\frac{5}{3}$	$\frac{5\cdot9}{3\cdot7}$	$\frac{5\cdot11}{3\cdot7}$	$\frac{9}{3}$	$\frac{11}{3}$	$\frac{9\cdot11}{3\cdot7}$	$\frac{5}{5}$	$\frac{5\cdot9}{3\cdot7}$	$\frac{5\cdot11}{3\cdot7}$	$\frac{9}{1}$	$\frac{11}{1}$	$\frac{9\cdot11}{1\cdot7}$	$\frac{5\cdot9}{1\cdot3}$	$\frac{5\cdot11}{1\cdot3}$	$\frac{9\cdot11}{1\cdot3}$	$\frac{5\cdot9\cdot11}{1\cdot3\cdot7}$
1·3·9	$\frac{5}{9}$	$\frac{7}{9}$	(1)	$\frac{11}{9}$	$\frac{5\cdot7}{3\cdot9}$	$\frac{5}{3}$	$\frac{5\cdot11}{3\cdot9}$	$\frac{7}{3}$	$\frac{7\cdot11}{3\cdot9}$	$\frac{11}{3}$	$\frac{5\cdot7}{1\cdot9}$	$\frac{5}{1}$	$\frac{5\cdot11}{1\cdot9}$	$\frac{7}{1}$	$\frac{7\cdot11}{1\cdot9}$	$\frac{11}{1}$	$\frac{5\cdot7}{1\cdot3}$	$\frac{5\cdot7\cdot11}{1\cdot3\cdot9}$	$\frac{5\cdot11}{1\cdot3}$	$\frac{7\cdot11}{1\cdot3}$
1·3·11	$\frac{5}{11}$	$\frac{7}{11}$	$\frac{9}{11}$	(1)	$\frac{5\cdot7}{3\cdot11}$	$\frac{5\cdot9}{3\cdot11}$	$\frac{5}{3}$	$\frac{7\cdot9}{3\cdot11}$	$\frac{7}{3}$	$\frac{9}{3}$	$\frac{5\cdot7}{1\cdot11}$	$\frac{5\cdot9}{1\cdot11}$	$\frac{5}{1}$	$\frac{7\cdot9}{1\cdot11}$	$\frac{7}{1}$	$\frac{9}{1}$	$\frac{5\cdot7\cdot9}{1\cdot3\cdot11}$	$\frac{5\cdot7}{1\cdot3}$	$\frac{5\cdot9}{1\cdot3}$	$\frac{7\cdot9}{1\cdot3}$
1·5·7	$\frac{3}{7}$	$\frac{3}{5}$	$\frac{3\cdot9}{5\cdot7}$	$\frac{3\cdot11}{5\cdot7}$	(1)	$\frac{9}{7}$	$\frac{11}{7}$	$\frac{9}{5}$	$\frac{11}{5}$	$\frac{9\cdot11}{5\cdot7}$	$\frac{3}{1}$	$\frac{3\cdot9}{1\cdot7}$	$\frac{3\cdot11}{1\cdot7}$	$\frac{3\cdot9}{1\cdot5}$	$\frac{3\cdot11}{1\cdot5}$	$\frac{3\cdot9\cdot11}{1\cdot5\cdot7}$	$\frac{9}{1}$	$\frac{11}{1}$	$\frac{9\cdot11}{1\cdot7}$	$\frac{9\cdot11}{1\cdot5}$
1·5·9	$\frac{3}{9}$	$\frac{3\cdot7}{5\cdot9}$	$\frac{3}{5}$	$\frac{3\cdot11}{5\cdot9}$	$\frac{7}{9}$	(1)	$\frac{11}{9}$	$\frac{7}{5}$	$\frac{7\cdot11}{5\cdot9}$	$\frac{11}{5}$	$\frac{3\cdot7}{1\cdot9}$	$\frac{3}{1}$	$\frac{3\cdot11}{1\cdot9}$	$\frac{3\cdot7}{1\cdot5}$	$\frac{3\cdot11}{1\cdot5}$	$\frac{3\cdot7\cdot11}{1\cdot5\cdot9}$	$\frac{7}{1}$	$\frac{7\cdot11}{1\cdot9}$	$\frac{11}{1}$	$\frac{7\cdot11}{1\cdot5}$
1·5·11	$\frac{3}{11}$	$\frac{3\cdot7}{5\cdot11}$	$\frac{3\cdot9}{5\cdot11}$	$\frac{3}{5}$	$\frac{7}{11}$	$\frac{9}{11}$	(1)	$\frac{7\cdot9}{5\cdot11}$	$\frac{7}{5}$	$\frac{9}{5}$	$\frac{3\cdot7}{1\cdot11}$	$\frac{3\cdot9}{1\cdot11}$	$\frac{3}{1}$	$\frac{3\cdot7\cdot9}{1\cdot5\cdot11}$	$\frac{3\cdot7}{1\cdot5}$	$\frac{3\cdot9}{1\cdot5}$	$\frac{7\cdot9}{1\cdot11}$	$\frac{7}{1}$	$\frac{9}{1}$	$\frac{7\cdot9}{1\cdot5}$
1·7·9	$\frac{3\cdot5}{7\cdot9}$	$\frac{3}{9}$	$\frac{3}{7}$	$\frac{3\cdot11}{7\cdot9}$	$\frac{5}{9}$	$\frac{5}{7}$	$\frac{5\cdot11}{7\cdot9}$	(1)	$\frac{11}{9}$	$\frac{11}{7}$	$\frac{3\cdot5}{1\cdot9}$	$\frac{3\cdot5}{1\cdot7}$	$\frac{3\cdot5\cdot11}{1\cdot7\cdot9}$	$\frac{3}{1}$	$\frac{3\cdot11}{1\cdot9}$	$\frac{3\cdot11}{1\cdot7}$	$\frac{5}{1}$	$\frac{5\cdot11}{1\cdot9}$	$\frac{5\cdot11}{1\cdot7}$	$\frac{11}{1}$
1·7·11	$\frac{3\cdot5}{7\cdot11}$	$\frac{3}{11}$	$\frac{3\cdot9}{7\cdot11}$	$\frac{3}{7}$	$\frac{5}{11}$	$\frac{5\cdot9}{7\cdot11}$	$\frac{5}{7}$	$\frac{9}{11}$	(1)	$\frac{9}{7}$	$\frac{3\cdot5}{1\cdot11}$	$\frac{3\cdot5\cdot9}{1\cdot7\cdot11}$	$\frac{3\cdot5}{1\cdot11}$	$\frac{3\cdot9}{1\cdot11}$	$\frac{3}{1}$	$\frac{3\cdot9}{1\cdot7}$	$\frac{5\cdot9}{1\cdot11}$	$\frac{5}{7}$	$\frac{5\cdot9}{1\cdot7}$	$\frac{9}{1}$
1·9·11	$\frac{3\cdot5}{9\cdot11}$	$\frac{3\cdot7}{9\cdot11}$	$\frac{3}{11}$	$\frac{3}{9}$	$\frac{5\cdot7}{9\cdot11}$	$\frac{5}{11}$	$\frac{5}{9}$	$\frac{7}{11}$	$\frac{7}{9}$	(1)	$\frac{3\cdot5\cdot7}{1\cdot9\cdot11}$	$\frac{3\cdot5}{1\cdot11}$	$\frac{3\cdot5}{1\cdot9}$	$\frac{3\cdot7}{1\cdot11}$	$\frac{3\cdot7}{1\cdot9}$	$\frac{3}{1}$	$\frac{5\cdot7}{1\cdot11}$	$\frac{5\cdot7}{1\cdot9}$	$\frac{5}{1}$	$\frac{7}{1}$
3·5·7	$\frac{1}{7}$	$\frac{1}{5}$	$\frac{1\cdot9}{5\cdot7}$	$\frac{1\cdot11}{5\cdot7}$	$\frac{1}{3}$	$\frac{1\cdot9}{3\cdot7}$	$\frac{1\cdot11}{3\cdot7}$	$\frac{1\cdot9}{3\cdot5}$	$\frac{1\cdot11}{3\cdot5}$	$\frac{1\cdot9\cdot11}{3\cdot5\cdot7}$	(1)	$\frac{9}{7}$	$\frac{11}{7}$	$\frac{9}{5}$	$\frac{11}{5}$	$\frac{9\cdot11}{5\cdot7}$	$\frac{9}{3}$	$\frac{11}{3}$	$\frac{9\cdot11}{3\cdot7}$	$\frac{9\cdot11}{3\cdot5}$
3·5·9	$\frac{1}{9}$	$\frac{1\cdot7}{5\cdot9}$	$\frac{1}{5}$	$\frac{1\cdot11}{5\cdot9}$	$\frac{1\cdot7}{3\cdot9}$	$\frac{1}{3}$	$\frac{1\cdot11}{3\cdot9}$	$\frac{1\cdot7}{3\cdot5}$	$\frac{1\cdot7\cdot11}{3\cdot5\cdot9}$	$\frac{1\cdot11}{3\cdot5}$	$\frac{7}{9}$	(1)	$\frac{11}{9}$	$\frac{7}{5}$	$\frac{7\cdot11}{5\cdot9}$	$\frac{11}{5}$	$\frac{7}{3}$	$\frac{7\cdot11}{3\cdot9}$	$\frac{11}{3}$	$\frac{7\cdot11}{3\cdot5}$
3·5·11	$\frac{1}{11}$	$\frac{1\cdot7}{5\cdot11}$	$\frac{1\cdot9}{5\cdot11}$	$\frac{1}{5}$	$\frac{1\cdot7}{3\cdot11}$	$\frac{1\cdot9}{3\cdot11}$	$\frac{1}{3}$	$\frac{1\cdot7\cdot9}{3\cdot5\cdot11}$	$\frac{1\cdot7}{3\cdot5}$	$\frac{1\cdot9}{3\cdot5}$	$\frac{7}{11}$	$\frac{9}{11}$	(1)	$\frac{7\cdot9}{5\cdot11}$	$\frac{7}{5}$	$\frac{9}{5}$	$\frac{7\cdot9}{3\cdot11}$	$\frac{7}{3}$	$\frac{9}{3}$	$\frac{7\cdot9}{3\cdot5}$
3·7·9	$\frac{1\cdot5}{7\cdot9}$	$\frac{1}{9}$	$\frac{1}{7}$	$\frac{1\cdot11}{7\cdot9}$	$\frac{1\cdot5}{3\cdot9}$	$\frac{1\cdot5}{3\cdot7}$	$\frac{1\cdot5\cdot11}{3\cdot7\cdot9}$	$\frac{1}{3}$	$\frac{1\cdot11}{3\cdot9}$	$\frac{1\cdot11}{3\cdot7}$	$\frac{5}{9}$	$\frac{5}{7}$	$\frac{5\cdot11}{7\cdot9}$	(1)	$\frac{11}{9}$	$\frac{11}{7}$	$\frac{5}{3}$	$\frac{5\cdot11}{3\cdot9}$	$\frac{5\cdot11}{3\cdot7}$	$\frac{11}{3}$
3·7·11	$\frac{1\cdot5}{7\cdot11}$	$\frac{1}{11}$	$\frac{1\cdot9}{7\cdot11}$	$\frac{1}{7}$	$\frac{1\cdot5}{3\cdot11}$	$\frac{1\cdot5\cdot9}{3\cdot7\cdot11}$	$\frac{1\cdot5}{3\cdot7}$	$\frac{1\cdot9}{3\cdot11}$	$\frac{1}{3}$	$\frac{1\cdot9}{3\cdot7}$	$\frac{5}{11}$	$\frac{5\cdot9}{7\cdot11}$	$\frac{5}{7}$	$\frac{9}{11}$	(1)	$\frac{9}{7}$	$\frac{5\cdot9}{3\cdot11}$	$\frac{5}{3}$	$\frac{5\cdot9}{3\cdot7}$	$\frac{9}{3}$
3·9·11	$\frac{1\cdot5}{9\cdot11}$	$\frac{1\cdot7}{9\cdot11}$	$\frac{1}{11}$	$\frac{1}{9}$	$\frac{1\cdot5\cdot7}{3\cdot9\cdot11}$	$\frac{1\cdot5}{3\cdot11}$	$\frac{1\cdot5}{3\cdot9}$	$\frac{1\cdot7}{3\cdot11}$	$\frac{1\cdot7}{3\cdot9}$	$\frac{1}{3}$	$\frac{5\cdot7}{9\cdot11}$	$\frac{5}{11}$	$\frac{5}{9}$	$\frac{7}{11}$	$\frac{7}{9}$	(1)	$\frac{5\cdot7}{3\cdot11}$	$\frac{5\cdot7}{3\cdot9}$	$\frac{5}{3}$	$\frac{7}{3}$
5·7·9	$\frac{1\cdot3}{7\cdot9}$	$\frac{1\cdot3}{5\cdot9}$	$\frac{1\cdot3}{5\cdot7}$	$\frac{1\cdot3\cdot11}{5\cdot7\cdot9}$	$\frac{1}{9}$	$\frac{1}{7}$	$\frac{1\cdot11}{7\cdot9}$	$\frac{1}{5}$	$\frac{1\cdot11}{5\cdot9}$	$\frac{1\cdot11}{5\cdot7}$	$\frac{3}{9}$	$\frac{3}{7}$	$\frac{3\cdot11}{7\cdot9}$	$\frac{3}{5}$	$\frac{3\cdot11}{5\cdot9}$	$\frac{3\cdot11}{5\cdot7}$	(1)	$\frac{11}{9}$	$\frac{11}{7}$	$\frac{11}{5}$
5·7·11	$\frac{1\cdot3}{7\cdot11}$	$\frac{1\cdot3}{5\cdot11}$	$\frac{1\cdot3\cdot9}{5\cdot7\cdot11}$	$\frac{1\cdot3}{5\cdot7}$	$\frac{1}{11}$	$\frac{1\cdot9}{7\cdot11}$	$\frac{1}{7}$	$\frac{1\cdot9}{5\cdot11}$	$\frac{1}{5}$	$\frac{1\cdot9}{5\cdot7}$	$\frac{3}{11}$	$\frac{3\cdot9}{7\cdot11}$	$\frac{3}{7}$	$\frac{3\cdot9}{5\cdot11}$	$\frac{3}{5}$	$\frac{3\cdot9}{5\cdot7}$	$\frac{9}{11}$	(1)	$\frac{9}{7}$	$\frac{9}{5}$
5·9·11	$\frac{1\cdot3}{9\cdot11}$	$\frac{1\cdot3\cdot7}{5\cdot9\cdot11}$	$\frac{1\cdot3}{5\cdot11}$	$\frac{1\cdot3}{5\cdot9}$	$\frac{1\cdot7}{9\cdot11}$	$\frac{1}{11}$	$\frac{1}{9}$	$\frac{1\cdot7}{5\cdot11}$	$\frac{1\cdot7}{5\cdot9}$	$\frac{1}{3}$	$\frac{3\cdot7}{9\cdot11}$	$\frac{3}{11}$	$\frac{3}{9}$	$\frac{3\cdot7}{5\cdot11}$	$\frac{3\cdot7}{5\cdot9}$	$\frac{3}{5}$	$\frac{7}{11}$	$\frac{7}{9}$	(1)	$\frac{7}{5}$
7·9·11	$\frac{1\cdot3\cdot5}{7\cdot9\cdot11}$	$\frac{1\cdot3}{9\cdot11}$	$\frac{1\cdot3}{7\cdot11}$	$\frac{1\cdot3}{7\cdot9}$	$\frac{1\cdot5}{9\cdot11}$	$\frac{1\cdot5}{7\cdot11}$	$\frac{1\cdot5}{7\cdot9}$	$\frac{1}{11}$	$\frac{1}{9}$	$\frac{1}{7}$	$\frac{3\cdot5}{9\cdot11}$	$\frac{3\cdot5}{7\cdot11}$	$\frac{3\cdot5}{7\cdot9}$	$\frac{3}{11}$	$\frac{3}{9}$	$\frac{3}{7}$	$\frac{5}{11}$	$\frac{5}{9}$	$\frac{5}{7}$	(1)

E. W. 1989

Figure 8: The multiplication table used to create a $1\cdot3\cdot5\cdot7\cdot9\cdot11$ Eikosany (Wilson 1989)

After eliminating all of the redundancies in this multiplication table, twenty numbers remain. For my own purposes, I made AEF the 1/1 and made that pitch A 440Hz.¹³ This resulted in the following twenty note scale:

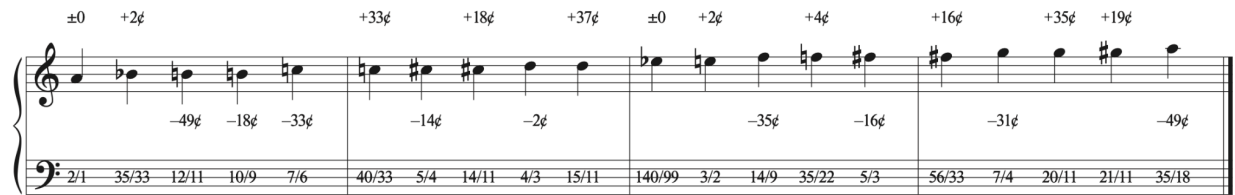


Figure 9: 1•3•5•7•9•11 Eikosany within one octave, written in standard notation with cent deviations

This is the version of the Eikosany I have used so far. I’m hesitant to use an Eikosany with higher variables until I am confident in my ability to hear and compose with the intervals of the 1•3•5•7•9•11 Eikosany. Harry Partch also chose an 11-limit for his tonality diamond, giving several justifications for it in *Genesis of a Music*. In brief summary, he argues that 11 is just one step beyond what Western 12TET music—and audiences used to hearing it—can accomplish harmonically, while 13 might still be a bridge too far for both performers and listeners (Partch 1974).

I asked several former students of Wilson’s how they’ve used the Eikosany, and almost all suggested that I dissect it into the smaller lattices contained within the Eikosany. This is a similar process to how a major or minor scale is derived from a 12TET chromatic scale, then broken down into triads, etc. The Eikosany contains the following lattices from Figure 2: Dekanies, Hexanies, Tetranies, Trianies, and Dyanies.¹⁴

In order to create these subdivisions, I use what I refer to as “Primary” intervals. These are the intervals between vertices that are only one line (or vertex) away. For example, AEF-BEF (2/1 to 3/2 in Figure 7) is a primary interval but AEF-ABF (2/1 to 4/3 in Figure 7) is not. This

¹³ This allowed me to have the lowest 4 partials of the harmonic series as well as two 12TET pitch classes—A and Eb—to work with.

¹⁴ Mononies are present as well, but they won’t be addressed here except as vertices of a larger lattice.

approach is very much inspired by Kraig Grady's 1986 document "THE EIKOSANY VIEWED FROM THE CENTERED PENTAD LATTICE-with various patterns and structures illustrated," which can be found online at the Erv Wilson archives. In Grady's document, he color-codes various tetrads, hexanies, and dekanies to make the shapes clear. My approach is more granular, starting from Dyads and building up to Dekanies. In addition, I try to color code intervals, rather than entire shapes, to make the Primary intervals within each shape clear.

The goal of this dissection of the Eikosany was to understand how to assign some sort of value system or hierarchy to the twenty notes of the scale. It has been helpful to think of the Eikosany as a five-dimensional spatial metaphor rather than just a linear musical scale. In the beginning, this was quite disorienting, so I made a map of the Eikosany's landscape, starting from two-note groups and expanding to ten-note groups.

Dyads

The 1•3•5•7•9•11 Eikosany has many possibilities for resolution on a tonal center based on the concept of harmonic fusion, which is when frequencies combine to imply a lower fundamental. If harmonic fusion is used as a measure of consonance, then one could argue that Primary tetrads (tetrads made up of Primary intervals) from the Generating Hexad are the most consonant note groupings in the Eikosany, since all four notes reinforce harmonic fusion of a fundamental. Even if the Eikosany variables are not 1, 3, 5, 7, 9, and 11 are used, then these tetrads will likely be theoretically useful in establishing a harmonic vocabulary. In searching for a universal method for measuring consonance and dissonance in an Eikosany, an alternative criterion needs to exist, based on visual orientation rather than the specifics of one set of variables. To address this, the concept I explore here is "Harmonic Class," a criterion that measures how many Primary intervals it takes to get from one vertex to another.

Figure 10 contains all of the dyads in relation to AEF, which I treat as the 1/1 of the 1•3•5•7•9•11 Eikosany. The red intervals are the closest (and thus most “consonant”), the orange are the next closest, then green, blue, and violet, with violet being the furthest away and most “dissonant”. I refer to these respective intervals as Primary, Secondary, Tertiary, Quaternary, and Quinary.

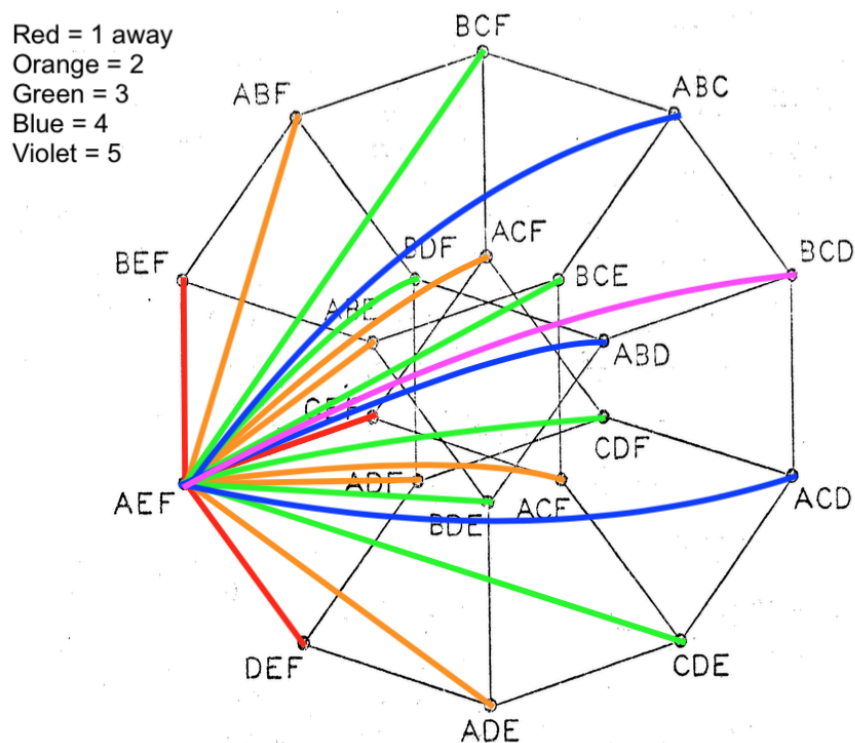


Figure 10: Dyads of varying Harmonic Class from AEF

So much of Wilson’s theoretical work is number theory applied to pitch relationships. While the Primary intervals of the Generating Hexad of the 1•3•5•7•9•11 Eikosany create the most acoustically smooth intervals (i.e. harmonic fusion), this may not be true if other variables are used. Even if the music being made doesn’t prioritize (or actively deprioritizes) this

smoothness, the concept of Harmonic Class could still prove useful for organizing pitches in a meaningful way within a composition.

Primary Dyads

Figure 11 shows all of the Primary intervals in an Eikosany, with each interval coded as a different color. The colors of the Generating Hexad and Reciprocal Hexad act as a guide, and every line in the Eikosany corresponds to one of these lines in the generating hexad.

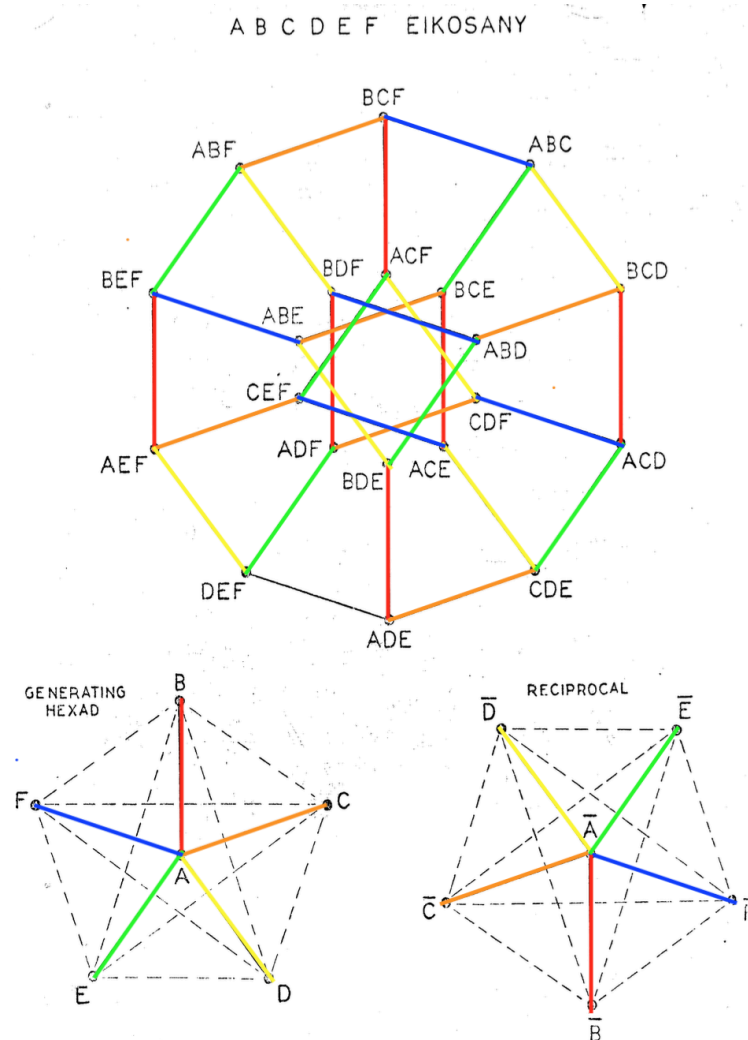


Figure 11: Color coded Primary intervals of an Eikosany

In a 1•3•5•7•9•11 Eikosany, all of the vertical RED lines (A-B) in Figure 11 correspond to a $3/2$ interval. The orange A-C lines represent $5/4$, the yellow A-D lines represent $7/4$, the green A-E lines represent $9/8$, and the blue A-F lines represent $11/8$. This is true in the generating hexad and in all parallel lines in the Eikosany.

All Primary intervals of the Eikosany fit into both the Generating Hexad and the Reciprocal Hexad. The Reciprocal Hexad represents the undertone series of the Generating Hexad—a concept that exists theoretically but does not have the same physical evidence of harmonic fusion that the overtone series does.

Secondary Dyads

Secondary dyads include notes that are two primary intervals away. They are visually represented in the Generating and Reciprocal Hexads in Figure 11, but they are the dotted lines which aren't represented visually in the Eikosany. Figure 12 shows a chart of all of the Secondary dyads of the 1•3•5•7•9•11 Eikosany, drawn in orange. Occasionally, dyads will be identical to a seemingly unrelated interval, and these are drawn in other colors. For example, the B-C dyad is equivalent to the AE-BC dyad, which is a Tertiary interval.

These intervals are specific to the 1•3•5•7•9•11 Eikosany. The concept of Harmonic Class is still applicable to other Eikosanies, but the specific intervals listed are not, and so the equivalent non-Secondary dyads will likely not be the same. However, these intervals will still hold true in a 1•3•5•7•9•11 Eikosany even if a different note is chosen as a $1/1$ instead of AEF.

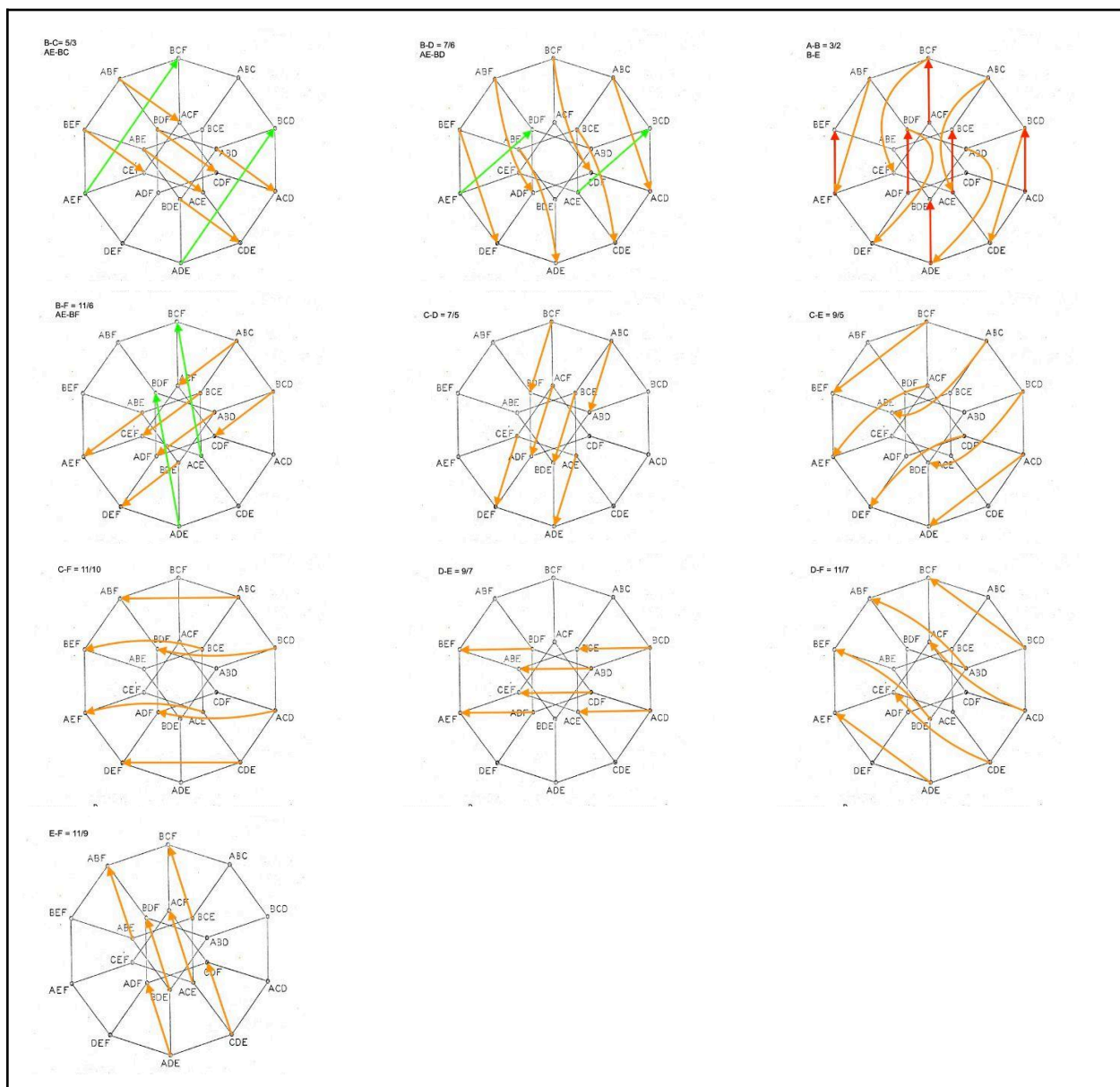


Figure 12: All Secondary dyads within a 1•3•5•7•9•11 Eikosany (points that are connected by two lines)

Appendix A expands on Figure 12 with diagrams of all of the non-Primary dyads within a 1•3•5•7•9•11 Eikosany, including dyads that are in a Secondary, Tertiary, Quaternary, and Quinary Harmonic Class.

Triads

Appendix B includes diagrams of Primary and Secondary triads present in the Generating Hexad. Each Primary triad exists three times in the Eikosany, as does its Reciprocal. Each non-Primary triad and its Reciprocal are listed together in one diagram.

When considering Harmonic Class in relation to these triads, it is best to think of the axis (the vertex at the center of the hexad) connected to two other notes as the starting point from which the intervals to the other two notes originate. In a 1•3•5•7•9•11 Eikosany, these would point toward varying degrees of harmonic fusion—depending on whether the triad is part of the Generating Hexad or the Reciprocal Hexad. Primary triads all include two primary intervals and 1 secondary interval (the interval between the outlying notes).

There are many other triads that include intervals not present in the generating hexad. An exhaustive list of chords and sub-scales would be worth creating in the style of Joseph Schillinger's book *Kaleidophone: Pitch Scales in Relation to Chord Structures*, but that is outside of the scope of this writing.¹⁵ Like the Hexanies and Tetranies, each triad can be extracted from the Eikosany and redrawn as an equilateral triangle Triany.

Tetrads

The most compact version of a tetrad involves one vertex in the center and three notes orbiting around it, like the Major Tetrad in Figure 3. A Primary tetrad, like a Primary triad, indicates the highest degree of harmonic fusion and lowest Harmonic Class. Each note in a 1•3•5•7•9•11 Eikosany is connected to three other notes by Primary intervals. So, any note could

¹⁵ *Kaleidophone* is a composing resource that gives all possible 12TET scales that fit within all possible chords from dyads to pentads.

be the “fundamental” of a Primary tetrad, meaning that there are twenty Primary tetrads in the Eikosany.¹⁶

Each tetrad can be redrawn as a triangular pyramid, like the major and minor tetrads in Figure 3. There are three Primary triads in each Primary tetrad. The fourth possible triad in a Primary tetrad includes only secondary intervals, so won’t be counted here. Diagrams of Primary tetrads in the Eikosany are listed in Appendix C.

Pentads

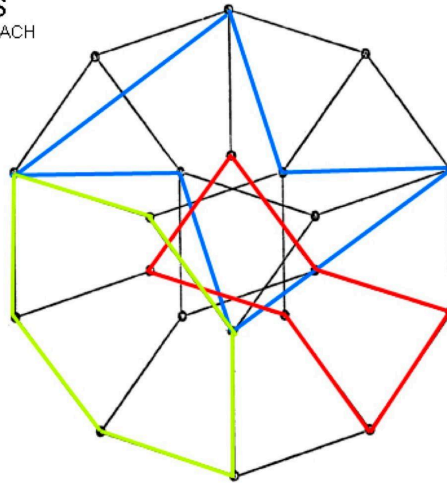
No Primary pentads exist in the Eikosany. However, pentads are worth mentioning briefly because the Hexadic Diamond in Figure 16 does include twelve Primary pentads.

Hexanies

The Hexany is the first closed CPS that appears in the Eikosany, so it’s referred to by Wilson’s name rather than as a hexad. The Eikosany includes two Primary Hexany shapes, and a third that can be considered Primary within one of the Dekany lattices in Figure 15.

¹⁶ I don’t differentiate here between Primary tetrads from the Generating and Reciprocal Hexad, in order to make allowances for Eikosanies with other variables.

HEXANY
SHAPES
10 ROTATIONS EACH



KRAIG GRADY 86

Figure 13: Hexanies within an Eikosany (Grady 1986b)

The Green and Red Hexanies in Figure 13 are made of Primary intervals, and I refer to them as Hexany shapes A and B, respectively. The Blue Hexany contains Secondary intervals, and I refer to it as shape C. It contains intervals that are Primary in the isolated Dekany lattice in Figure 15, but no intervals that are Primary within the Eikosany lattice.

Each of these Hexanies can be extracted from the Eikosany and viewed as an isolated lattice like Figure 3. Appendix D shows how these three Hexany shapes can each be rotated around the Eikosany five times. On a sixth rotation, hexanies are parallel, meaning that the intervals are identical to the first rotation—except the pitches are transposed. Appendix H also shows an interesting anomaly, which is that all of the notes of these parallel Hexanies are separated by a secondary Dyad. If one were to modulate via Primary intervals from one Hexany to a parallel Hexany, it can be done most efficiently through what I refer to as a “Transitional Hexany.” Since this process uses three hexanies without overlapping pitches, there are two pitches left unused in the Eikosany. While this is not of immediate value for my process here, it is worth noting in case it becomes useful.

Dekanies

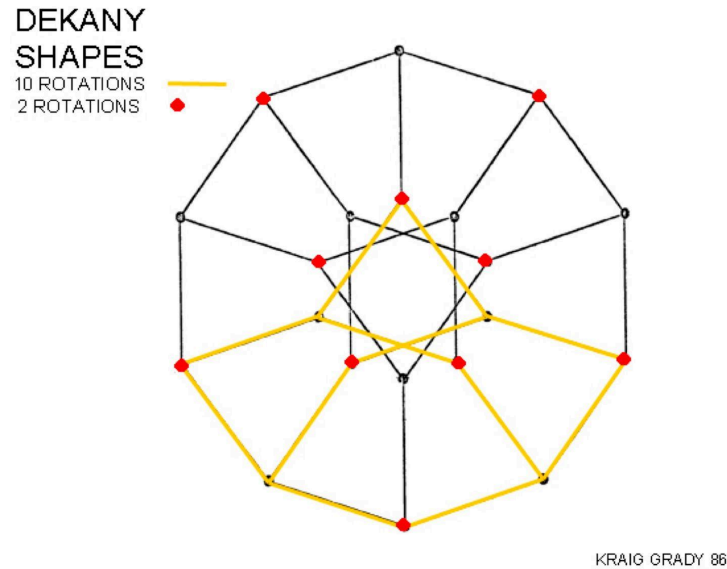


Figure 14: A Dekany within an Eikosany (Grady 1986b)

Dekanies are the largest closed CPS contained in the Eikosany. Because of this, I've focused my own compositional work with the Eikosany on using Dekanies. There are ten Primary Dekanies in the Eikosany, which are rotations of the yellow shape in Figure 14. Appendix E shows the ten Dekany Rotations contained within the $1\cdot3\cdot5\cdot7\cdot9\cdot11$ Eikosany in standard notation, then mapped onto the Eikosany lattice. I refer to these as Rotations 1-10, with the rotation movement following the outlying vertex in a clockwise direction.¹⁷

Each of these Dekanies includes four Primary Hexanies, one secondary Hexany, four Primary tetrads, and many triads. On its own, a Dekany can be viewed as one of the pentagram-like lattices in Figure 15, depending on whether it is a $3/5$ Dekany or a $2/5$ Dekany.

¹⁷ The outlying vertex of Rotation 1 is $10/9$, while the outlying note of Rotation 2 is $35/18$, etc.

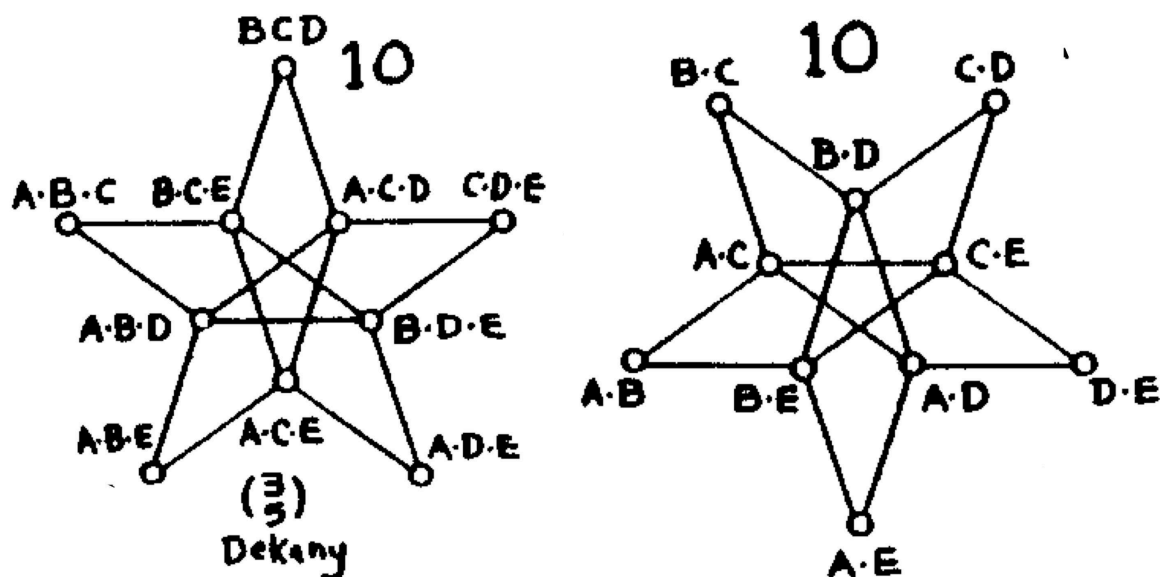


Figure 15: Two types of Dekanies (Wilson 1989)

Since I'm working with Dekanies in the context of the Eikosany, I don't take the extra step of translating Dekanies from the Eikosany into the geometry of the lattices in Figure 15. Rather, I use the Dekany shapes in Figure 14.

Other Eikosanies and Diamonds

Wilson reimagined the geometry of the Eikosany in several ways, though the version I've written about so far in this paper is the most commonly used. The alternative Eikosany on the left of Figure 16 uses almost the same Generating Hexad and has the same vertices, but it connects the five outer vertices of the Hexad to each other rather than connecting the five outer vertices to the center.

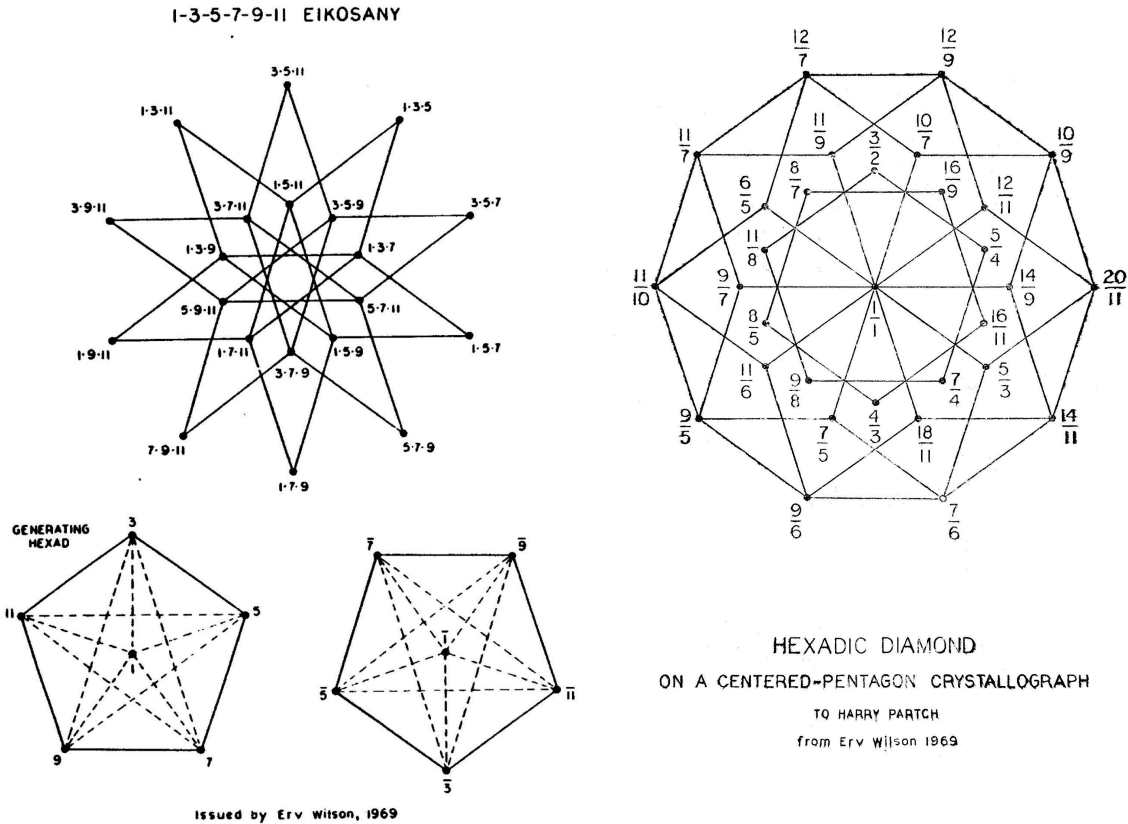


Figure 16:
Left: an alternative view of the 1•3•5•7•9•11 Eikosany
Right: A Hexadic Diamond made using the alternative Generating Hexad (Wilson 1989)

The numerical values and spatial placement of the vertices are the same in both versions of the Eikosany—only the connecting lines are different. This lattice is identical to both of the Dekany lattices in Figure 15 overlaid on top of each other. On the right of Figure 16 is a Hexadic Diamond that demonstrates how Wilson’s 5-dimensional geometry can represent the twenty-nine original pitches of Harry Partch’s 11-limit tonality diamond.¹⁸ The angles and line segment lengths are identical to the alternative Eikosany, but emit from a central vertex. Each of the pentagons in the Hexadic Diamond come from the Generating Hexad and reciprocal Hexad on the left of Figure 16. The underlying multiplication table used to create a diamond is different

¹⁸ 31 axes exist here, but 12/9 and 9/6 are equivalent to 4/3 and 3/2 respectively, so it can be assumed that they’re present for the sake of seeing the geometry. Figure 20 is a lattice that includes the additional 14 notes that make Partch’s signature 43-note scale.

from Wilson's CPS tables, but it is identical to how Harry Partch created tonality diamonds. Using the geometry of the alternative Eikosany and Hexadic Diamond in Figure 16, the image in Figure 17 shows the overlap between the two shapes.

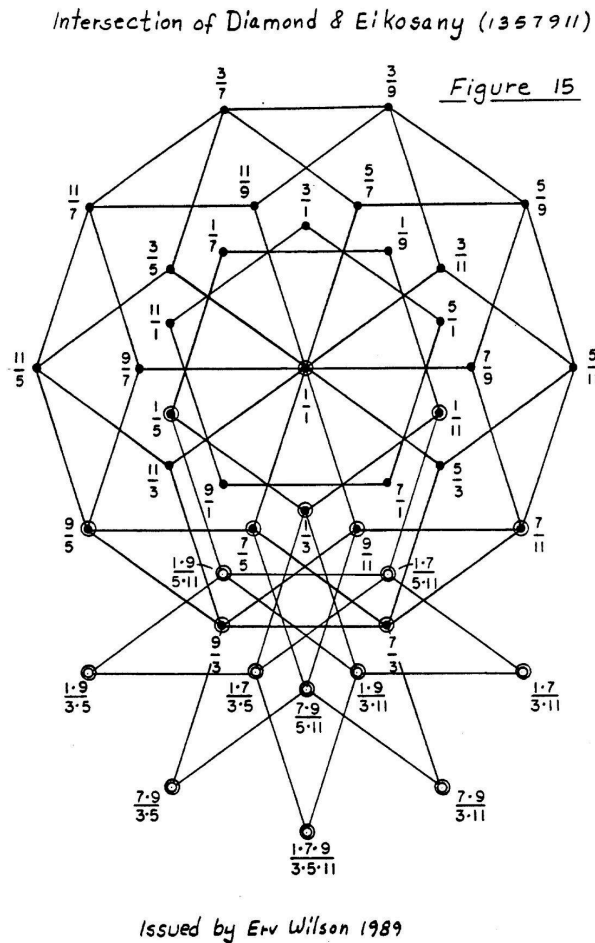


Figure 17: The Intersection of a 1•3•5•7•9•11 Eikosany and Hexadic Diamond (Wilson 1989)

The fact that the Diamond is physically above the Eikosany in Figure 17 is arbitrary. A diamond could be centered around any of the twenty pitches in the Eikosany. With a solid understanding of the Eikosany as a visual metaphor for a scale, it could potentially be used as a modulation device between Hexadic Diamonds, drastically expanding the harmonic possibilities of the Eikosany.

Stellation

When connecting the Hexadic Diamond and the alternative Eikosany, some intervals occur twice in a row. In the Eikosany, no Primary interval occurs twice in a row, or in opposite directions from a single vertex. However, this does happen in the Hexadic Diamond. The Eikosany includes the interval between $7/5$ and $1/1$, but the diamond also extends in the same direction with the reciprocal interval from $1/1$ to $5/7$. In geometry, this would be called stellation. Any CPS can be stellated, like the stellated Hexany in Figure 18.

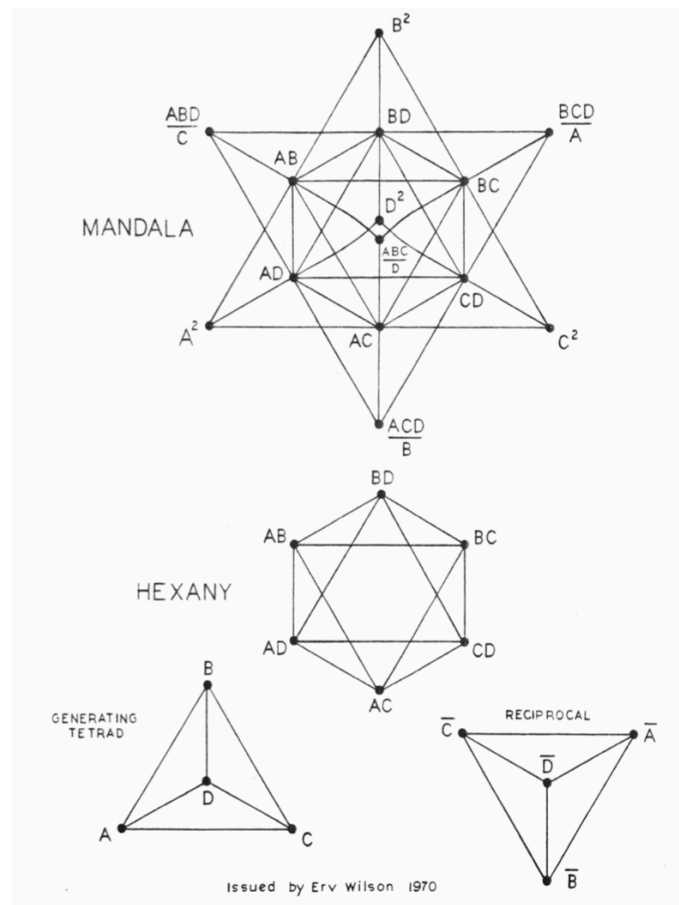


Figure 18: A Stellate Hexany (Wilson 1970)

When the vertices of the Hexany's octahedron are extended until they intersect with another line, the result is a fourteen-note scale that Wilson referred to as a Mandala or a Stellate

Hexany. All of the intervals of the Hexany exist within the Stellate Hexany, but there are more of them.

Music researcher Paul Rapoport created the Stellate Eikosany in Figure 19 using the same process. The uncentered Eikosany can be seen in the center, with vertices in black. The stellations expand outward with color-coded vertices and lines.

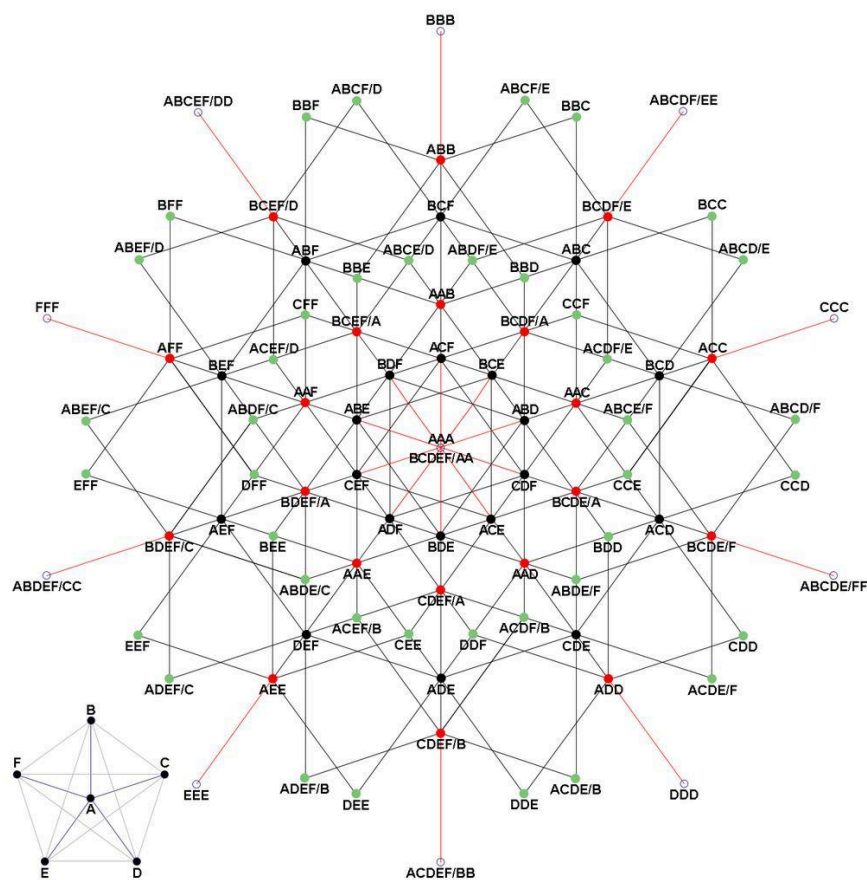


Figure 19: Stellate Eikosany designed by Paul Rapoport (Rapoport 2022)

Partch did something similar to achieve the forty-three notes of his signature scale, though he did not make a 5D lattice for it. He was unhappy with how uneven the adjacent intervals in his 11-limit tonality diamond were, so he added fourteen additional notes to his

The diagram illustrates a network of nodes and edges. Nodes are labeled with fractions, some in black and some in red. Red lines connect specific nodes, forming a path that includes the fraction $\frac{81}{80}$ at the bottom left and $\frac{160}{81}$ at the top right. The network is highly interconnected, with many nodes having multiple connections. The red path highlights a specific sequence of nodes and edges within the larger network.

2. Use of the Eikosany in Rotations Ensemble

One of my goals for pursuing a PhD in 2020 was to learn how to incorporate into my

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its Dekany Rotations 1-10 for ensemble writing. I chose an instrumentation of guitar, violin, viola, contrabass, and drumset, augmented by a set of twenty “Eikosany Chimes” I made in 2021. I named the group Rotations Ensemble after the Dekany rotations I was exploring. In addition, I had been introduced by composer Stephen James Taylor to the “space cages” and Gurule shells of Benjamin Gurule. The planes of these fascinating polyhedra have a spin—or rotation—that is theoretically related to quantum theory (Taylor 2022). While this remains a fascinating mystery to me, I found the idea poetically related to Wilson’s CPS and worth taking a name from.

Instrumentation

Eikosany Chimes

In 2021, I made twenty steel chimes to be used in a composition called *Eikosany Study* (2021), for contrabass, cello, violin, piano/synthesizer, and percussion. *Eikosany Study* was a harmony study of various Tetranies, Hexanies, and Dekanies around the Eikosany, using common tone modulations to transition between chords.

Since the 1•3•5•7•9•11 Eikosany I was working with includes two 12TET pitch classes (A and Eb), the piano part was restricted to those notes. The strings used scordatura and only played natural harmonics so that they could get as close to the exact pitches as possible. The pianist also had a MIDI synthesizer connected to a Pure Data patch that allowed them to play notes from the Eikosany. In order to fit a twenty note scale onto a 12TET keyboard, I divided the Eikosany into two Dekanies—Rotation 2 and Rotation 8—and had the octaves of the keyboard alternate between these Dekanies, as shown in Figure 21.



Figure 21: 1•3•5•7•9•11 Eikosany spread out over 2 octaves. Dekany Rotation 2 is represented in the lower octave, and Rotation 8 is in the higher octave.

The percussionist played the twenty “Eikosany Chimes” I created, which are divided into the same two Dekanies as the piano. I have since used my Eikosany Chimes in several pieces, including *Four of Wands*, *Two of Pentacles*, and *Six of Swords*.

Guitar

Between 2022 and 2023, I was in possession of a Lou Harrison Just-Tuned National Reso-Phonic Guitar (AKA Just National), on loan from John Schneider. This resonator guitar has a tuning designed by Harrison for his 2002 piece *Scenes from Nek Chand* (Fiore 2012). As shown in Figure 22, this tuning has two chains of just fifths—one being Eb [28/27], Bb [14/9], F [7/6], C [7/4] and the other being G [4/3], D [1/1], A [3/2], and E[9/8]—that are separated by a 7/4. The G and D are also each connected to two additional notes, one by a 5/4 and the other by an 11/8.¹⁹

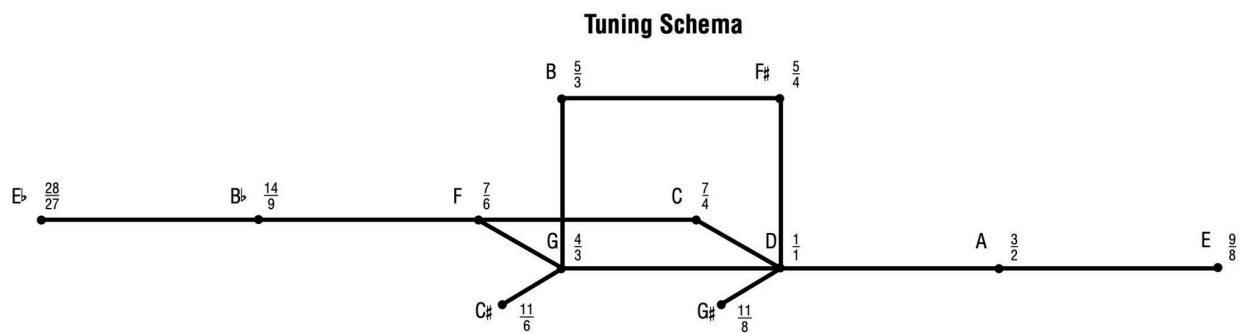


Figure 22: David B. Doty’s pitch lattice of the Lou Harrison Just-Tuned National Reso-Phonic Guitar (Doty 2006)

¹⁹ The 11/6 and 11/8 could also be connected by a horizontal line, representing a 3/2 interval.

With the Just National, I composed a piece in 2023 called *Two of Pentacles* for a City of Los Angeles' Individual Master Artist Project (COLA IMAP) grant. *Two of Pentacles* is a duet for one player, with the Just National guitar paired with my Eikosany Chimes, played robotically and controlled by Ableton Live via contral voltage.

Inspired by my work with the Just National guitar and wanting to continue performing *Two of Pentacles*, I looked for a way to keep playing just-tuned guitar after I returned the Just National to Schneider. I bought an electric guitar neck from Turkish guitarist Tolgahan Çoğulu's company Microtonal Guitars, who make string instruments featuring a fingerboard system with grooves under each string. These come with a box full of fretlets that fit into the grooves on the fingerboard, allowing individual fretlets to be easily placed under individual string and adjusted to achieve the desired pitch. I installed this neck on a Fender Player Pro Telecaster, and this is the guitar I play in Rotations Ensemble.

While it would be impractical to move frets in the middle of a performance, there is a wide range of customizability with this system. For Rotations Ensemble's first performance, the guitar was in a Just Intonation DADGAD tuning, which is standard for the Harrison Just National. All of the other music we played was in Dekany Rotation 9, so I placed twelve frets per octave on the guitar to accommodate both systems, as shown in Figure 23.

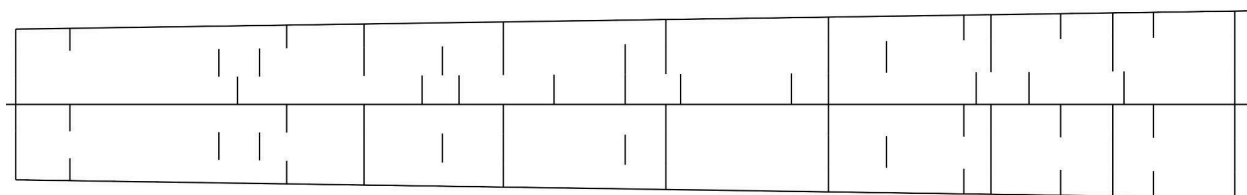


Figure 23: My original DADGAD fret placement for Dekany Rotation 9, exported from FretFind2D

For subsequent performances as of this writing, Rotations Ensemble has played music in Rotations 6 and 9. These Rotations share five notes, so I only needed a total of fifteen frets per

octave. Omitting the five extra frets needed for a full Eikosany makes the guitar easier to play—at least until I have music to play using those frets. As I explored, I found that tuning to a just open D chord felt more musical in practice.²⁰ The updated fifteen-note fret placement in open D is shown in Figure 24.

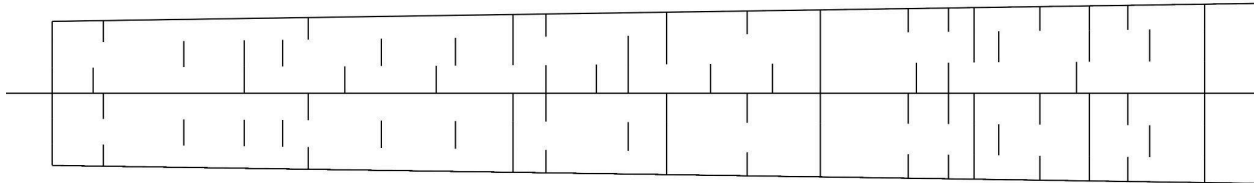


Figure 24: Guitar fret placement for Dekany Rotations 6 and 9 in an open D tuning (DADF#AD), exported from FretFind2D

Soon, I plan to add a few frets to the guitar so that I can switch between DADGAD and open D. This would allow me to perform *Two of Pentacles* on the Eikosany Guitar in DADGAD in a performance in which I’m also playing Rotations Ensemble music in open D.

Violin and Viola

There are many reasons to use string instruments for microtonal music. Non-fretted string instruments and slide-based brass do not have their pitches quantized in the same way that most woodwinds and keyboards do. Violin and Viola are such a common pairing that using them in Rotations Ensemble made perfect sense to me. In addition, violist Cassia Streb and violinist Myra Hinrichs were willing to play my music and able to read the HEJI accidentals I had started using.

²⁰ In this tuning, the notes from low to high are D-2¢, A±0¢, D-2¢, F#-14¢, A±0¢, D-2¢. A dilemma with DADGAD tuning in the 1•3•5•7•9•11 Eikosany I use is choosing which “G” to use. Harrison’s guitar is a more-or-less a just-tuned 12-note scale, and the G is a 4/3 of D. However, this note does not fit into my Eikosany. The notes that seemed to be the closest fit are the 7/4 (G -31¢) or the 20/11 (G +35¢) of A. Neither of these are a satisfactory surrogate for the 4/3 of D (G -4¢).

Bass and Drums

Inspired by Arnold Dreyblatt's combination of just intonation and repetitive rhythmic cells as compositional building blocks, I wanted much of Rotations Ensemble to be groove-oriented. To achieve the sound I imagined, I asked the upright bass player and drummer I had worked with for years in the band Timur and the Dime Museum—David Tranchina and Andrew Lessman—both of whom have backgrounds as jazz musicians.

Notation

Having chosen an ensemble, I needed to be able to communicate my microtonal ideas to them through notation. Deciding on a microtonal notation system is a fascinating challenge, as there is no universally accepted one. The closest thing to a universal language in microtonal accidentals is Johnny Reinhard's choice to use standard accidentals with cent deviations written above or below the staff. This could be the most streamlined process for communicating with musicians who have dedicated decades of their lives to reading standard western notation.

Most other microtonal notation systems are based on standard western notation, supplemented by a series of accidentals to imply specific cent deviations. A notable deviation from this is Harry Partch, who wrote his scores in tablature that is idiosyncratic to each of his instruments. His notation for fixed pitch percussion instruments like the Diamond Marimba, Boo, or Bass Marimba shows where the note is physically located on the instrument, but gives no information on what pitch that bar represents.²¹ In contrast, his Adapted Viola notation gives only ratios in relation to a 12TET G fundamental with stems for rhythmic clarity. Most Adapted Viola players I've met as of this writing have transcribed Partch's parts into something more closely

²¹ Players and composers must consult explanatory charts like those in Partch's *Genesis of a Music*.

resembling standard notation, using a notation system like Helmholtz-Ellis. Though Partch's tablature seems cumbersome and is only applicable to the world of his own instruments, it is surprisingly practical. If one were to communicate Partch's unique musical ideas to performers, it is hard to imagine a more effective way to do it. Partch's notation for western classical instruments or voice was equally centered on communicating to performers, using standard 12TET notation. If the cent deviation is important, he notates it as ratios, as he does in the vocal part of *The Potion Scene from Shakespeare's Romeo and Juliet*. In other cases, he writes with standard western accidentals and relies on the musicians' ears to adjust their intonation, as he does in the arrangements of *Ulysses* (Partch 2017, 2018).²²

Many other early twentieth century composers exploring microtonality also invented systems of accidentals, including Ivan Wyschnegradsky, Alois Hába, and Julián Carrillo. Some more recent examples include Ezra Sims, who used his own series of accidentals for his 72TET music, and Ben Johnston, who had his own collection of accidentals for his Extended Just Intonation music.

Sagittal notation is an option invented by microtonalists to most accurately express their musical ideas. It was designed by George Secor and Dave Keenan between 2001 and 2006, with feedback from users of microtonality-centric Yahoo mailing lists *tuning* and *tuning-math* (Secor 2006). The name ostensibly comes from Sagittarius—the bow-wielding centaur from Greek mythology—and is named such since it is based on a series of arrows. Sagittal was officially made available in 2004 with a website launch, and an introductory article was published in 2006 in the microtonal journal *Xenharmonikôn* 18 (Keenan 2004). While Sagittal has been recommended to me by other microtonalists, I have yet to meet any performing musicians who

²² The titles of the arrangements of Partch's *Ulysses* vary. Though he finally settled on *Ulysses at the Edge of the World*, various arrangements have also been called *Ulysses Departs from the Edge of the World* and *Ulysses Turns Homeward from the Edge of the World*—both 1955 (Gilmore 1998)

prefer it. This may be because of my musical community, whom I've observed using the Helmholtz-Ellis system far more often.

The Helmholtz-Ellis Just Intonation (HEJI) system was also designed in the early 2000s by Marc Sabat and Wolfgang Von Schweinitz. This is the system I am using for notating the 1•3•5•7•9•11 Eikosany. I've observed more professional musicians using HEJI than any other system, sometimes re-notating music from other systems so that it is legible to them. I'm able to notate the pitches of the 1•3•5•7•9•11 Eikosany using only four groups of accidentals: Natural, Syntonic, Quarter-tone, and Septimal.

HEJI Notation of the 1•3•5•7•9•11 Eikosany

Figure 25 shows the 1•3•5•7•9•11 Eikosany written with standard 12TET accidentals and cent deviations. Note that the pitches A and Eb are the same as in standard 12TET.

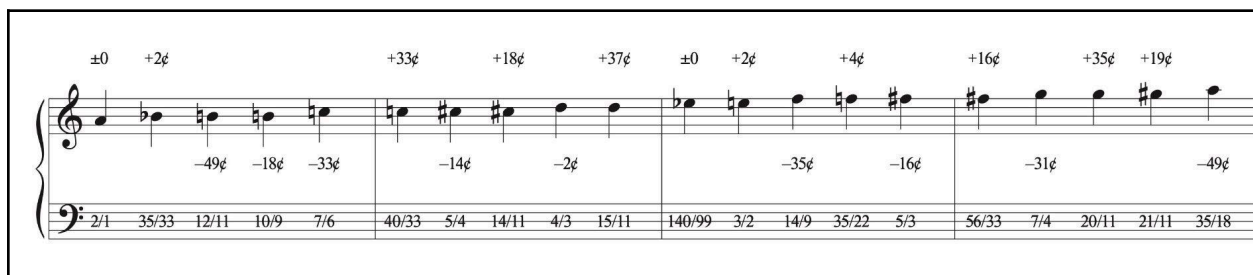


Figure 25: 1•3•5•7•9•11 Eikosany written with standard 12TET accidentals and cent deviations.

Figure 26 shows the same scale, but notated with HEJI accidentals.



Figure 26: The same 1•3•5•7•9•11 Eikosany, but written with HEJI accidentals

HEJI Accidentals I Use

I use four types of accidentals to notate the 1•3•5•7•9•11 Eikosany, shown in Figure 27. They may appear right-side up or upside down. HEJI can get much more sophisticated, but I made an effort to keep it as streamlined as possible.

Standard			
	A note with a natural with a flat bar above it means no cent deviation from 12TET tuning. There are only two pitches that use it: A and Eb.		A note with a natural or flat—without the bar on top— is $\pm 2\text{¢}$, depending on the context (more on this later). This is the cent deviation of a just fifth (3/2) or fourth (4/3)
Arrows (Syntonic)			
	A note with a natural or sharp with a downward arrow is lowered by about 14¢ from 12TET, again depending on context. Sabat’s justification for this accidental in HEJI is that the arrow represents the syntonic comma between the Pythagorean third (408¢) and the Just third (386¢)—approximately 22¢ when rounded to the nearest cent (Sabat 2009).		A note with a sharp with an upward arrow is raised by about 14¢, approximately the opposite of the downward arrow. The same justification applies as the downward arrows.
Quarter-tone			
	A backwards flat is a standard way of notating that a note is lowered by 50¢. There are only two notes that use this: A half-flat (35/18) and B half-flat (12/11)		
L-shape (Septimal)			
	In my system, an upright septimal accidental means to lower the note by approximately 31¢ (minus 2-4¢, depending on context).		An upside down septimal accidental means to raise the note by about 31¢—the opposite of the rightside-up accidental.

Figure 27: HEJI Accidentals I use to notate the 1•3•5•7•9•11 Eikosany

Context for specific cent deviations

Most of the pitches in this Eikosany can be organized in three-note groups of stacked just fifths ($3/2$), as shown in Figure 28. The exceptions are the two quarter tones (actually -49¢ but in practice they're quarter tones). In the first group, a natural sign is used for both the D and the E, though one is $+2\text{¢}$ and the other is -2¢ . Same with the Eb and Bb in the second group. Similarly, in the fifth group a septimal accidental on an F means -35¢ while on a G it means -31¢ .

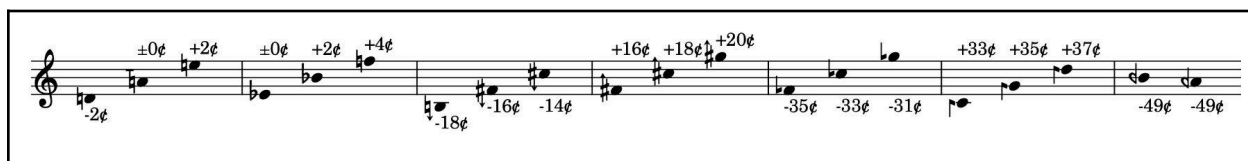


Figure 28: Pitches grouped into 6 groups of just fifths (and two quarter-tones) to show the groups of accidentals

For the sake of keeping the notation as simple as possible, I left it at this level of specificity, with fine tuning to be done by ear. This may be confusing at first, but I'm hoping it will be a reasonable compromise between accuracy and legibility.

Dekany Rotation 6

Figure 29 shows Dekany Rotation 6, which is a $1\cdot5\cdot7\cdot9\cdot11$ CPS. The absence of vertical red lines indicates that Rotation 6 contains no $3/2$ intervals. It does contain the Primary Intervals $5/4$, $7/4$, $9/8$, and $11/8$.

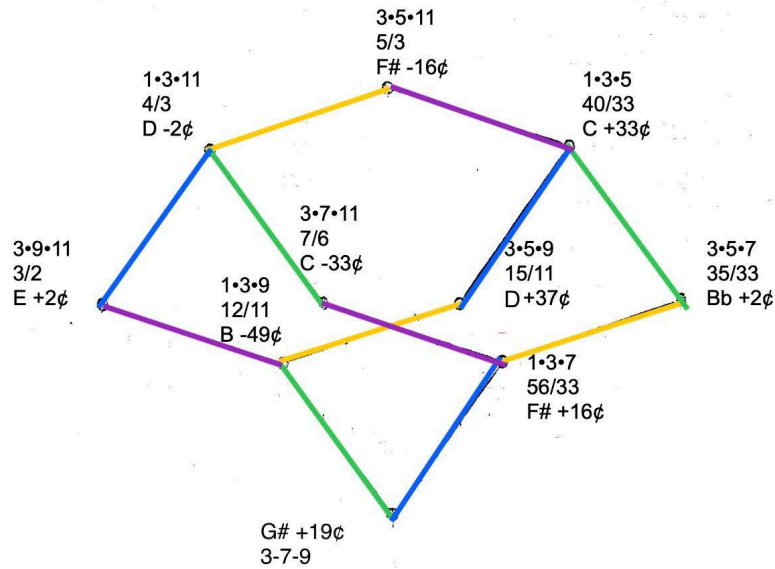


Figure 29: Dekany Rotaton 6 as part of the 1•3•5•7•9•11 Eikosany

Figure 30 shows the same scale in standard notation with HEJI accidentals. The bracketed lines are the Primary Intervals visible in Figure 29.

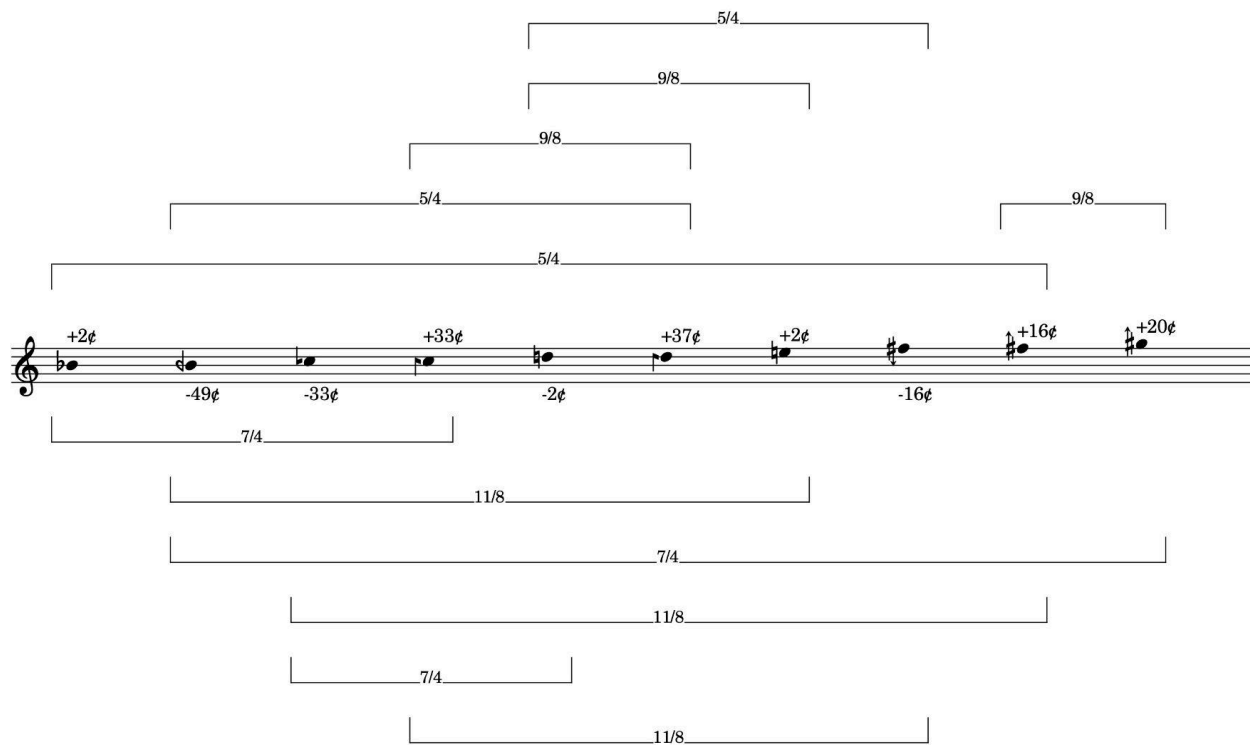


Figure 30: Rotation 6 with Primary Intervals marked with brackets

Some of the intervals in Figure 30 should show their reciprocal ratio, like the 5/4 interval from 35/33 (Bb +2¢) to 56/33 (F# +16¢), which is technically a 8/5. However, I leave these as the Primary intervals shown in Figure 29. In the case of the 35/33 and 56/33, it would be a 5/4 if it were to start at the 56/33 and move to the 35/33.

Dekany Rotation 9

Figure 31 shows Dekany Rotation 9, which is a 1•3•5•9•11 CPS. This Rotation does include red lines (indicating 3/2 Primary Intervals), but no green lines (thus, no 7/4 Primary Intervals). It contains the Primary Intervals 3/2, 5/4, 9/8, and 11/8.

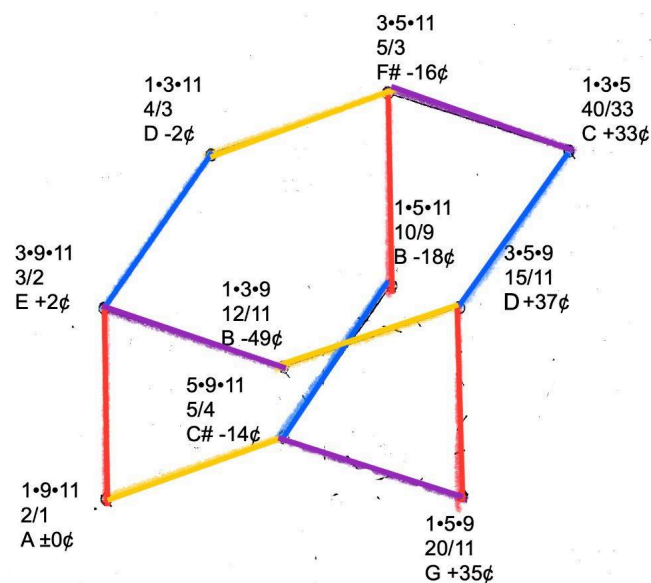


Figure 31: Dekany Rotaton 9 as part of the 1•3•5•7•9•11 Eikosany

Figure 32 shows the same scale in standard notation with HEJI accidentals. The bracketed lines are the Primary Intervals visible in Figure 31.

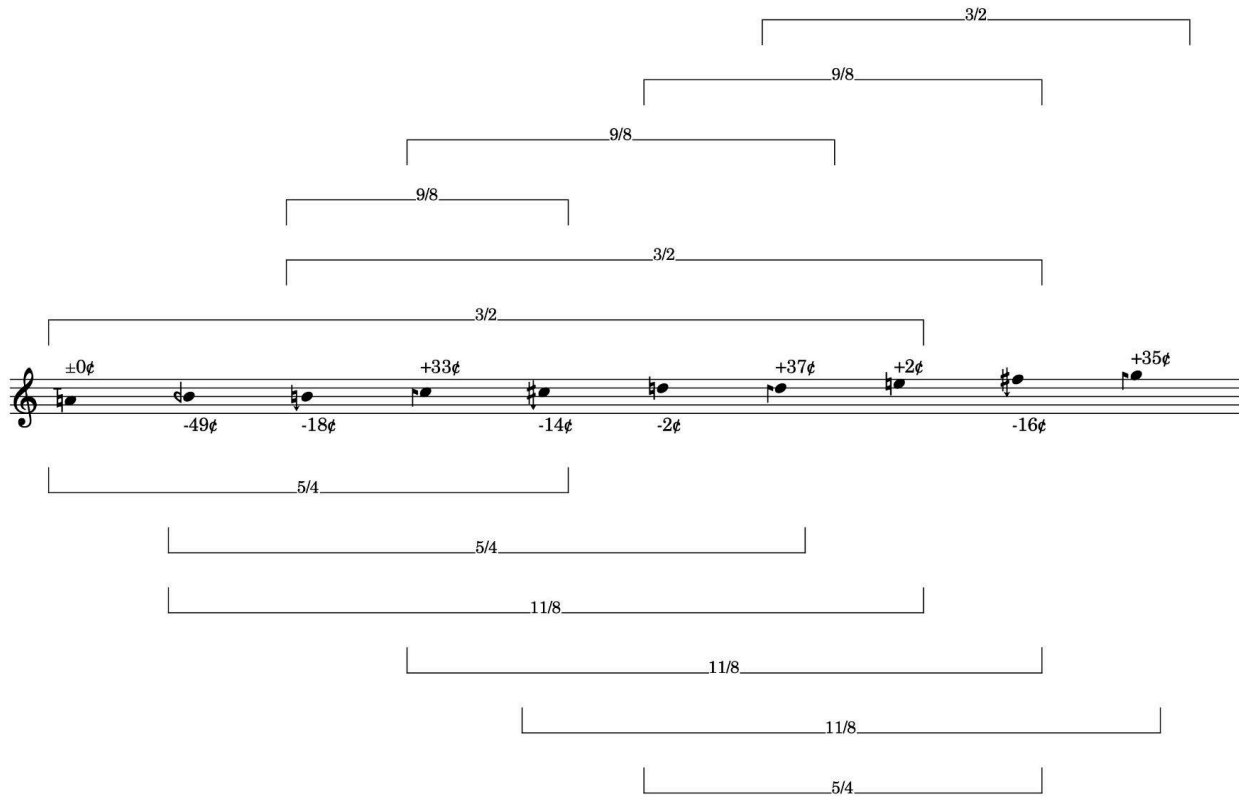


Figure 32: Rotation 9 with Primary Intervals marked with brackets.

Incomplete Open Forms

Since the Eikosany was still a relatively new musical system for me, I wanted a clear and systematic approach to understanding how the materials I had set up could translate into music. Thinking of my music with the Eikosany as a series of harmony studies has helped give me some direction. Besides breaking the Eikosany down into smaller CPS lattices, I took an approach I think of as “Incomplete Open Forms,” after Sol LeWitt’s 1974 sculpture series *Incomplete Open Cubes*. In LeWitt’s piece, he builds all of the possible wireframe structures that include three to eleven edges of a cube, most easily seen in the chart *Variations of Incomplete Open Cubes* (LeWitt 2001). I took a similar approach to understanding the Eikosany, using what I think of as *Incomplete Open Forms*. For my Incomplete Open Forms, I approached Wilson’s lattices with a

treatment similar to LeWitt's. A key geometric difference is that I added and subtracted vertices, which are notes in the Eikosany, rather than edges, which are intervals.

I have used this process in several pieces written with the 1•3•5•7•9•11 Eikosany. The first piece in which I used it deliberately was *Incomplete Open Hexany*, composed in 2022 for synthesizer, bass clarinet, upright bass, and drumset and premiered by Now Hear Ensemble at the Riverside Art Museum's Outpost Concert Series on April 6, 2022. *Incomplete Open Hexany* (*IOH*) is a harmony study of the six notes in the 1•3•7•9 Hexany shown in Figure 33.



Figure 33: The six pitches of the 1•3•7•9 Hexany used in *Incomplete Open Hexany*

Each of *IOH*'s five sections starts with dyads, then expands to a pentad before repeating the process of building up from dyads—never revealing the full Hexany. Besides LeWitt's work, this approach was also inspired by Tom Johnson's 1986 piano piece *Chord Catalogue*, in which the pianist plays the 8,178 possible combinations of notes within one octave. As shown in Figure 34, *Chord Catalogue* starts with all possible dyads in ascending order, then continues to all possible triads in ascending order, etc.²³

²³ Johnson's score also uses an "etc." to imply that the player should be able to infer the rest of the process without more written music as guidance.

The Chord Catalogue

(for keyboard)

TOM JOHNSON

The 78 two-note chords possible in one octave:

The 286 three-note chords possible in one octave:

The 715 four-note chords possible in one octave:

Etc., playing the 1287 five-note chords, the 1716 six-note chords, the 1716 seven-note chords, the 1287 eight-note chords, the 715 nine-note chords, the 286 ten-note chords, the 78 eleven-note chords, the 13 twelve-note chords, and the 1 thirteen-note chord. 8178 chords in all.

© 1986 BY TOM JOHNSON

Figure 34: The score for *Chord Catalogue* by Tom Johnson, 1986.

Rather than follow Johnson’s method of linearly presenting every permutation of the Hexany’s six notes in ascending order, I chose to have the harmonies expand and contract in complexity, making the music more sectional than a singular additive gestalt. My aims were to use these chord progressions as structural frameworks rather than directly as musical materials on their own, so the transparency of the process is less important in my work than in Johnston’s.

My 2023 piece *Two of Pentacles* uses only seven pitches and approaches them in a similar away. These were the only pitches that overlapped between my 1•3•5•7•9•11 Eikosany Chimes and the Just National guitar.²⁴ To write *Two of Pentacles*, I first translated Harrison’s

²⁴ See Section 2:Guitar for more information

scale into the 5D geometry of the $1\cdot3\cdot5\cdot7\cdot9\cdot11$ Eikosany so that I could see how the two structures might overlap. The lattices in Figure 35A and 35B show how the seven pitches in Figure 35C overlap between the two scales if the tuning of the Just National guitar is lowered by 2ϕ . So, I decided to compose with just those seven pitches.

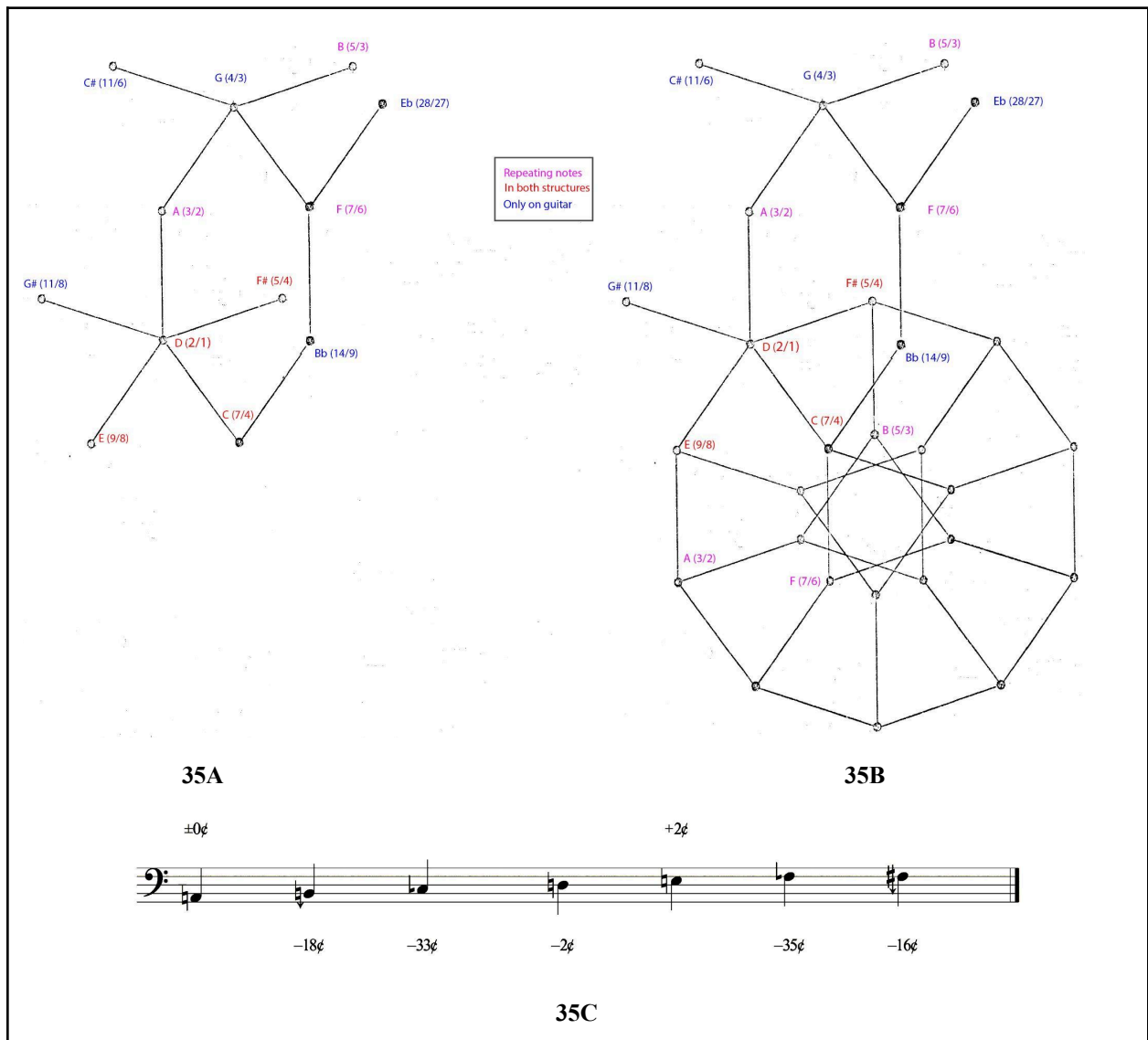


Figure 35:

- A.** The notes of the National Reso-Phonic Just Intonation Guitar in the 5D geometry of the Eikosany
- B.** The notes of the National Reso-Phonic Just Intonation Guitar overlaid on the $1\cdot3\cdot5\cdot7\cdot9\cdot11$ Eikosany
- C.** The 7 overlapping notes between the two lattices

In order to understand the materials I was working with, I made a chord catalogue in the style of Johnson's *Chord Catalogue*. However, I did not want to make something that was constantly additive. I preferred for *Two of Pentacles* to expand and contract, creating episodes of complexity that drifted back into relative simplicity. I re-ordered the notes of my seven-note chord catalogue so that it would start with a two-note chord, add one or two notes until there were five or six pitches, and then gradually retract to two different pitches. I ordered the chords so that harmonic movement was through common tones.

Arch Form Daniel Corral

© 2023 by Daniel Corral

Figure 36: Arch form of *Two of Pentacles*.²⁵ In the final piece, each chord here is approximately 30 seconds

²⁵ Notated here in 12TET. In practice, the cent deviations from Figure 35C still apply

The “Arch Form” chord progression in Figure 36 became the structure for *Two of Pentacles*, over which I composed a series of rhythmic cells for the Just National guitar and Eikosany Chimes. This way of composing laid a foundation and left room for play based on what worked with the materials, both aurally and physically.

While the harmony and form of *Two of Pentacles* were dictated by the overlap in pitches between the Just National Guitar and my Eikosany Chimes, other pieces explored Incomplete Open Forms in different ways. For my 2023 piece *Four of Wands*, I used a very similar approach, but applied to seven hexanies within the 1•3•5•7•9•11 Eikosany. The harmonic progression I made for the 2023 video piece *Candoro* is a study of the Primary tetrads within a Dekany. Using the geometry and vocabulary of the Eikosany, this “Candoro form” implies tonal centers by offering incomplete impressions of overtones or undertones.

Candoro Form

In the fall of 2022, I spent four weeks at the Loghaven Artist Residency in Knoxville, TN with collaborators Alexander Gedeon and DaEun Jung. While there, I drew the ten Dekany Rotations I’ve since focused on and made Scala files of each, so that I could later create MIDI demos of the music for Rotations Ensemble. I also made charts of all of the triads, quartads, and Hexanies within each Dekany.

We had an opportunity to make a movement-based video work called *Candoro*, released by Music for Your Inbox in March, 2023. The sound design for *Candoro* was based on a field recording, but with filtering to bring out specific frequencies and harmonies. The frequencies were pitches from Dekany Rotation 4 of my 1•3•5•7•9•11 Eikosany, and the harmonies slowly transform over thirteen minutes. The harmonic progression begins with a single pitch and adds pitches until a tetrad is formed. This tetrad contracts and expands between two and four pitches,

morphing into various tetrads of the Dekany that each imply a different fundamental of an overtone or undertone series.²⁶ Figure 37 shows how this harmonic progression traverses the entire Rotation 4 lattice in twenty-two steps. The Purple dots on the lattice show which notes are used in each section. The purple dots in the Generating and Reciprocal Hexads below each lattice show how the notes above relate to the intervals of the Generating or Reciprocal Hexad.

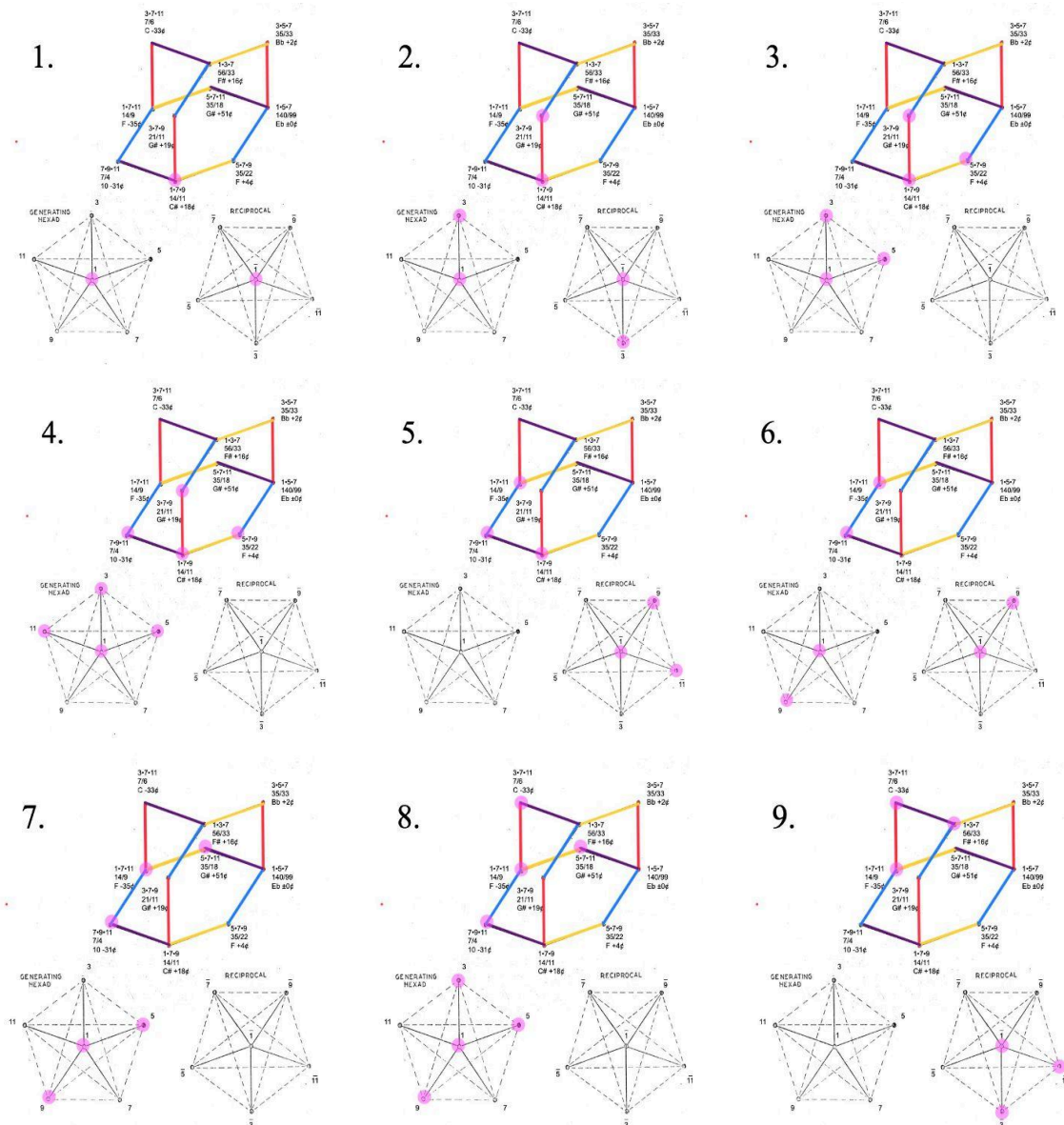


Figure 37: *Candoro* arch form in Dekany Rotation 4 of the 1•3•5•7•9•11 Eikosany

²⁶ This works theoretically, even though the undertone series does not have the same harmonic fusion as the overtone series.

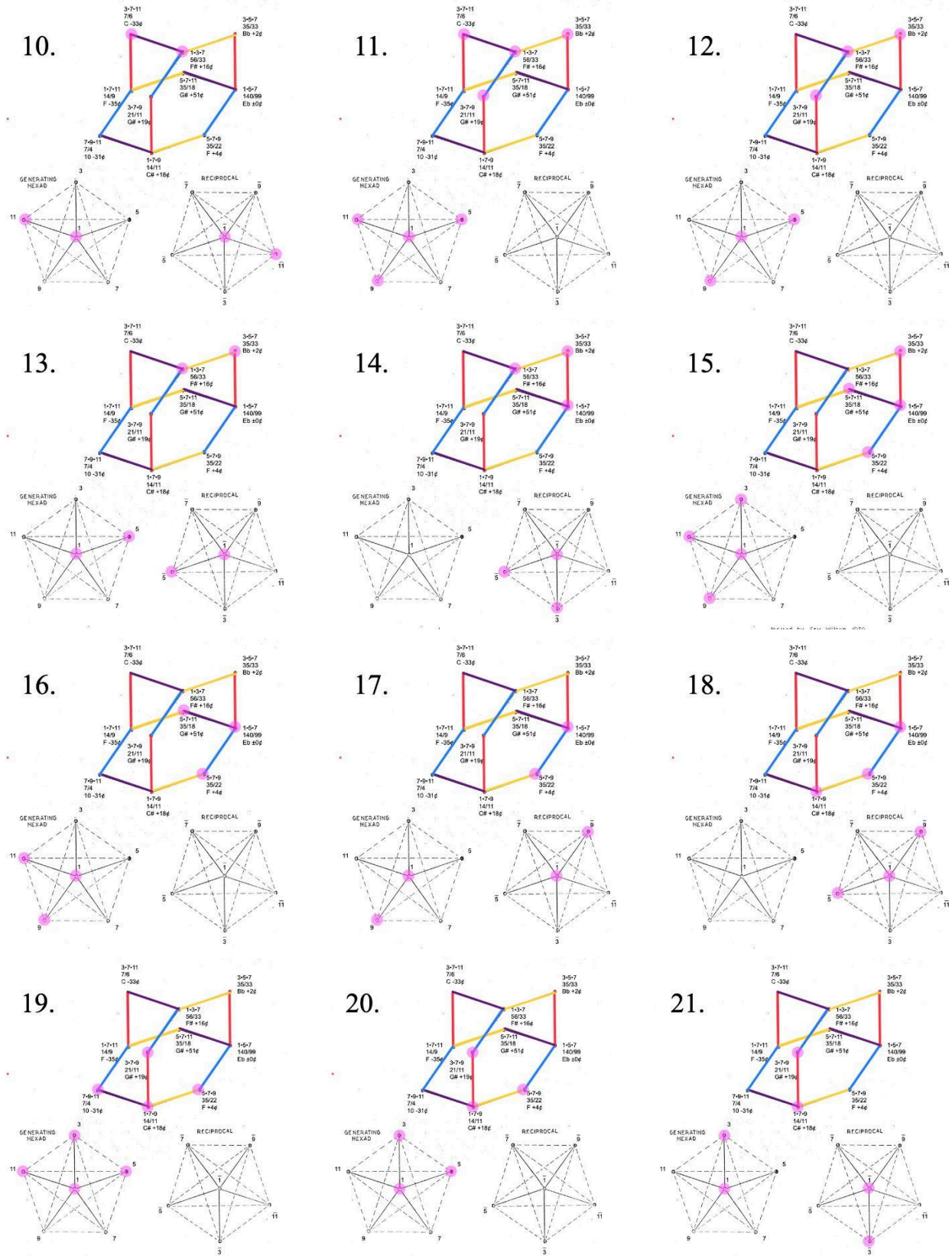


Figure 37: Candoro arch form in Dekany Rotation 4 of the $1 \cdot 3 \cdot 5 \cdot 7 \cdot 9 \cdot 11$ Eikosany (Continued)

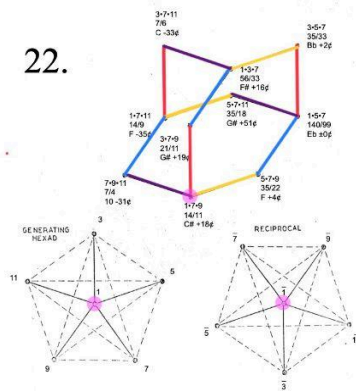


Figure 37: *Candoro* arch form in Dekany Rotation 4 of the 1•3•5•7•9•11 Eikosany (Continued)

Figure 38 shows the same progression as Figure 37, but as musical notes. As the progression moves from left to right, the 1/1 fundamental changes depending on the interval relationships in each measure. The 1/1 is always in a rectangular enclosure, overtones are always in ascending order above 1/1, and undertones are all in descending order from 1/1.



Figure 38: *Candoro* arch form in Rotation 4

The first measure contains only C#+18¢—just like the first image in Figure 37—establishing it as the fundamental. Mm. 2-4 add the third, fifth, and eleventh partials respectively. In m.5, G-31¢—the eleventh partial of C#+18¢—becomes the new 1/1. This happens because the fifth partial and third partial of C#+18¢ are removed, while the undertone equivalent of the ninth partial (F-35¢, the 16/9 of G-31¢) is added. With these changes, the strongest just relationships point to G-31¢ as the new fundamental of an undertone series (G -31¢ : F -35¢ : C# -18¢ = 99:176:144, or 1:9:11 within the undertone series). In other words, the music has modulated via common tones from the overtone series of C#+18¢ to the undertones of G-31¢. The fundamental of m.6 is ambiguous, since it is a dyad. However, it is fair to assume that the preceding focus on G-31¢ would prepare a listener’s ears for that pitch to be the fundamental by default. The addition of G#+51¢ in m.7 points to F-35¢ as a new fundamental, since it is the 5th partial of that note. In m.8, the C-33¢ cements F-35¢ as the fundamental before modulating again in m.9. The progression continues around the Dekany in a similar fashion before returning to C#+18¢ as a fundamental, ending on that single pitch.

The arch forms for my later pieces *Three of Pentacles* and *Six of Swords* are both based on versions of this same “Candoro Form,” but in different Dekany rotations. In *Three of Pentacles*, this Candoro form is rotated 216° around the Eikosany to Dekany rotation 6. Figure 39 shows the Candoro form in Rotation 6 in standard notation with HEJI accidentals, while Appendix F shows the Candoro form mapped onto the Rotation 6 lattice. In Dekany rotation 6, what was a 3/2 in Rotation 4 is now a 9/8. Just like in Figure 38, overtones in Figure 39 are always in ascending order from 1/1 while undertones are in descending order from 1/1.

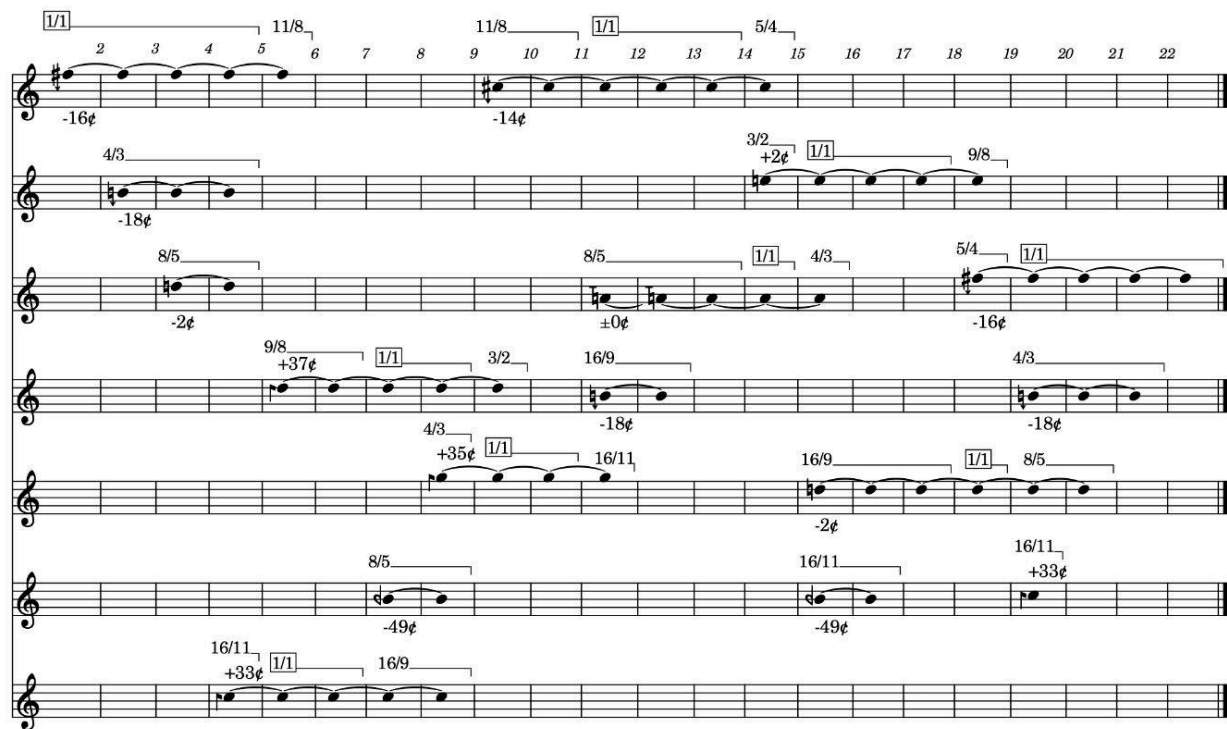


Figure 40: Candoro arch form of *Six of Swords* in Rotation 9

Influences on Incomplete Open Forms

The additive/subtractive processes in my Incomplete Open Forms draw influence from twentieth century process-based music. These processes can be found all over Steve Reich's work, in famous pieces like *Music for Pieces of Wood* or *Drumming*, in which players individually build an ostinato, adding single notes until the full ostinato is revealed. In my own past work, I used similar processes in my Launchpad pieces like *Polytope* (2018). *Polytope* is played by four musicians playing Launchpad Pros. The "score" is the buttons of the Launchpads, which light up for each scene. *Polytope* is performed in darkness, and the lights of the Launchpads are caught by an overhead camera pointing down, with the players' hands creating dancing silhouettes over the lights. This live-feed video is projected into the performance space in real time, so audiences can watch along as the colorful graphic score progresses.

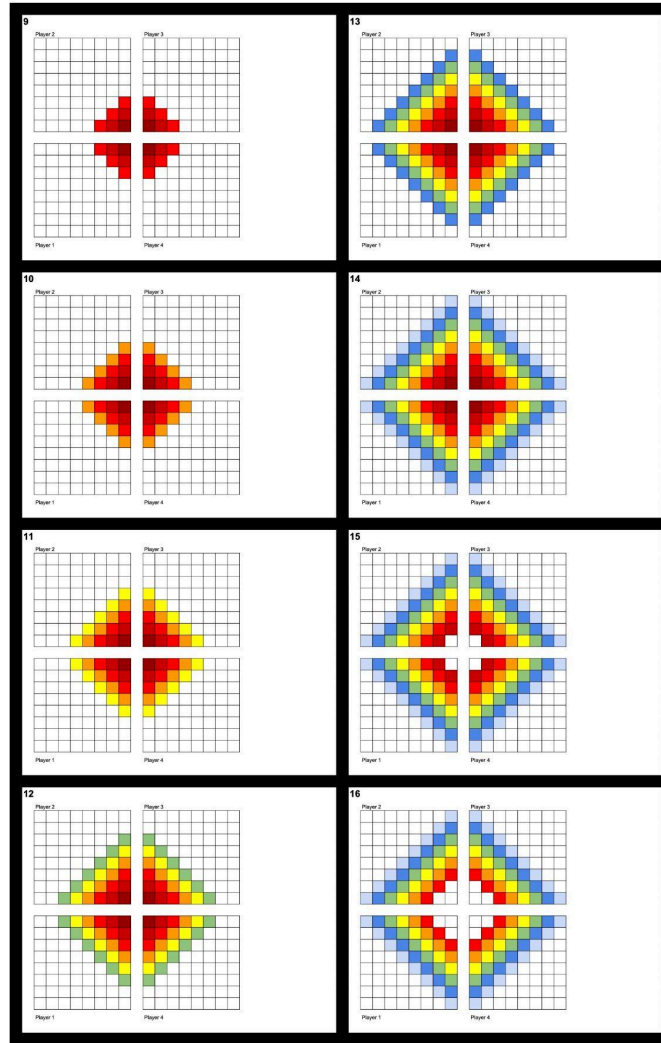


Figure 41: Scenes 9-16 of *Polytope*. The order of scenes here is top to bottom, then left to right.

The 8x8 grid of buttons makes a 15-limit tonality diamond, but only a limited amount of these notes are used at any given time. Figure 41 shows an excerpt from a section of *Polytope* called “Square 1.” Figure 41 is divided into eight “scenes,” numbered 9-16. The scenes can be followed from top to bottom, with the columns going from left to right, as a square-like shape is gradually revealed. The field of possible notes starts small in scene 9 (left column, top scene) and expands outward from the center until it is completely visible in scene 14 (right column, second down). In scene 15, the notes in the center of the square are removed, and continue to be removed until only a single diagonal row of buttons is lit up.

These processes can be reductionist, which runs parallel to my interest at the time in the Oulipo literary movement. The Oulipo wrote with game-like constraints, such as George Perec’s 1969 novel *La disparition* that was written without the letter E. Another Oulipo constraint is the “Snowball,” which involves writing a poem with a one-letter word in the first line, a two-letter word in the second, three-letter words in the third, and so on (Mathews et al 1998). Between 2017 and 2020, I wrote a series of compositional exercises I called *The Oulipo Composer*, which translates Oulipo constraints from Harry Mathews’ *Oulipo Compendium* into games for manipulating musical materials. I think of it less as a compositional method than as a tool for overcoming writer’s block and generating new material. In a similar way, the whole process of learning about the Eikosany has been about accumulating creative tools.

3. Rotations Ensemble Repertoire

I’ve been lucky to have musical friends willing to try out my untested new musical ideas. With them in mind, I’ve developed a series of pieces that are performable by Rotations Ensemble. The following pieces were performed together in a set by Rotations Ensemble in several concerts in Spring 2025.

1. *Three of Pentacles* (2025)
2. *Candoro* (video piece, 2023)
3. *Nine of Wands* (2023)
4. *Six of Swords* (2025)
5. *Ace of Pentacles* (2024)

In the following section, I’ll describe some of the important formal elements of each of these pieces, including how the Eikosany is used, how Incomplete Open Forms—including Candoro Form—are deployed, and any other relevant musical justifications for my compositional choices.

References to Tarot

Except for the video work *Candoro*, all of these pieces are named after Minor Arcana Tarot cards.²⁷ I've always found it challenging to title instrumental pieces that aren't programmatic. Some of my past works had more mathematical names, like the original working title of my piece *Four of Wands*, which was "4 for 20." Switching from mathematical names to Tarot card names bridges a gap between number theory and numerology, adding a vaguely programmatic element that hopefully pulls listeners away from feeling a need to understand the underlying microtonal theory.

So far, I've chosen cards intuitively to represent pieces. For example, I chose the name for my piece *Three of Pentacles* because the image on the three of Pentacles card in the popular Rider-Waite Tarot deck contains arches that are vaguely similar to the carport of the Candoro Marble Building, and it was the first instrumental piece I completed using my Candoro form. There are many ways to interpret this card's meaning and how it relates to my music composition. A performance containing my Tarot pieces could even be interpreted as a musical divination. Rotations Ensemble has played the following three pieces attacca: *Nine of Wands*, *Six of Swords*, and *Ace of Pentacles*. Thought of as a divination, they could be seen as a three-card spread—a popular type of Tarot spread that can be interpreted many different ways. By invoking this type of mysticism, I don't try to express specific meaning, but rather point to the fact that there is subjective meaning to be found.

²⁷ Minor Arcana are the equivalent of modern-day play cards: Ace, numbers 2-10, Page, Knight, Queen, and King

Three of Pentacles (2025)

Three of Pentacles treats the Candoro form in Rotation 6 as an arch form, using the moments of Dyads as section dividers. This creates five sections, which I gave different meters as shown in Figure 42.

Part 1 5/4	Part 2 3/4	Part 3 5/4	Part 4 4/4	Part 5 5/4
---------------	---------------	---------------	---------------	---------------

Figure 42: Candoro form of *Three of Pentacles* with the sections marked below.

Having laid a formal foundation for the piece, I overlaid musical gestures based on the pitch material in each measure of the Candoro form. Parts 1, 3, and 5 are all in 5/4 and explore variations of the phrase in Figure 43.

Figure 43 displays the musical score for measures 27 through 30 of the piece *Three of Pentacles*. The score is written for five instruments: E. Gtr (Electric Guitar), Vln (Violin), Vla (Viola), D. B. (Double Bass), and Dr. (Drum). The key signature is one sharp (F#) and the time signature is 4/4. A rehearsal mark 'F' is placed above measure 27. The dynamics are marked as *pp* (pianissimo) in measures 27 and 30, and *mf* (mezzo-forte) in measures 28 and 29. The E. Gtr part features a melodic line with a sharp bend in measure 27. The Vln and Vla parts play a rapid sixteenth-note figure in measure 27, which then transitions to a slower, more melodic line in measures 28 and 29. The D. B. part provides a steady bass line, and the Dr. part plays a simple, rhythmic pattern.

Figure 43 continues the musical score for measures 31 through 33 of the piece *Three of Pentacles*. The score is written for the same five instruments: E. Gtr, Vln, Vla, D. B., and Dr. The key signature remains one sharp (F#) and the time signature is 4/4. The dynamics are marked as *mf* (mezzo-forte) in measures 31 and 32, and *fp* (fortissimo) in measure 33. The E. Gtr part continues its melodic line, featuring a sharp bend in measure 31. The Vln and Vla parts play a rapid sixteenth-note figure in measure 31, which then transitions to a slower, more melodic line in measures 32 and 33. The D. B. part provides a steady bass line, and the Dr. part plays a simple, rhythmic pattern.

Figure 43: Mm.27-32 of *Three of Pentacles*

The pitch material at every rehearsal letter is different, depending on where it is in the Candoro form. The music in Figure 43 uses the harmonies in m.3 of the Candoro Form in Figure 42. Throughout sections 1, 3, and 5, the melody played here by the violin and viola is sometimes played in canon, sometimes unison (or octaves), and sometimes harmonized in contrary motion. By revisiting this material in sections 3 and 5, I hoped to give the piece an altered version of a symmetrical ritornello form.

The image displays a musical score for measures 67 through 74 of the piece 'Three of Pentacles'. The score is arranged in two systems, each containing five staves. The instruments are Electric Guitar, Violin, Viola, Double Bass, and Drum Set. The key signature is one flat (B-flat major or D minor), and the time signature is 3/4. Measure 67 is marked with a tempo of quarter note = 110. The score includes dynamic markings such as *f* (forte) and *mp* (mezzo-piano). The Electric Guitar part features a melodic line with a box labeled 'A' above measure 67. The Violin and Viola parts play a similar melodic line. The Double Bass part provides a rhythmic foundation with alternating notes. The Drum Set part features a complex rhythmic pattern with various note values and rests.

Figure 44: Mm.67-74 of *Three of Pentacles*

Part 2 is in 3/4 and explores variations on the gesture in Figure 44, in which the audible pulse alternates between dotted 8th notes (mm.67-68 and mm.71-72) and quarter notes (mm.69-70 and mm.73-74). In mm.69-70, the alternating notes in the bass create an audible 2/4 meter, while in mm.73-74 the bassline highlights the written 3/4 meter.

In Part 4, the meter is a very straightforward 4/4 that focuses on the quarter note pulse, as shown in Figure 45.

Figure 45: Mm.123-126 of *Three of Pentacles*

Candoro (2023)

Candoro is a video work that was shot in fall 2022 in the driveway and carport of the Candoro Marble Building in Knoxville, TN, near the Loghaven Artist Residency. I drew a series of routes weaving through the marble columns of the Candoro carport, and then my collaborators Alexander Gedeon, DaEun Jung, and I filmed each of us individually walking those routes, following the pace of a metronome at 110 BPM. These routes were chopped into reusable clips that could be collaged in the style of film multiple exposures, using masks within Final Cut Pro.

Many of our collaborative discussions at Loghaven orbited around the theme of systems—both the joy and the oppression that they bring. I had also recently read Henri Lefebvre’s *Rhythmanalysis: Space, Time and Everyday*, in which Lefebvre describes urban space

and architecture as having rhythm, expanding outward to consider every aspect of life as rhythmic. He defines “presence” in time as a constantly evolving continuum of being, contrasting it with the “present,” a frozen moment in time that is identifiable and commodifiable. In *Candoro*, these ideas of human embodiment within architectural rhythm meld with structuralist film and parametric counterpoint.

number	time														people		
1	0:00														order (a)	Alex	A
2	0:32.5	Db														DaEun	
3	1:05	Db	Ab													Daniel	
4	1:37.5	Db	Ab	F												ALL	
5	2:10	Db	Ab	F	G										DaEun		
6	2:42.5	Db			G	E									Alex		
7	3:15				G	E									ALL	B	
8	4:47.5				G	E	A								Daniel		
9	4:20				G	E	A	C							Alex		
10	4:52.5					E		C	Gb						Daniel		
11	5:25							C	Gb						DaEun		
12	5:57.5		Ab					C	Gb	Bb					ALL		
13	6:30		Ab						Gb	Bb				disrupt (b)	ALL	A'	
14	7:02.5								Gb	Bb					DaEun		
15	7:35								Gb	Bb	Eb				Daniel		
16	8:07.5			F			A			Bb	Eb				Alex		
17	8:40			F			A					Eb			Daniel		
18	9:12.5			F								Eb			ALL		
19	9:45	Db		F								Eb			Alex	B'	
20	10:17.5	Db	Ab	F	G										DaEun		
21	10:50	Db	Ab	F											ALL		
22	11:22.5	Db	Ab												Daniel		
23	11:55	Db													DaEun		
24	12:27.5													order (a)	Alex		

Figure 46: Formal chart for the music and video of *Candoro*.

Both the visual and audio components of the thirteen-minute video follow a strict form based on twenty-four equal divisions of 32.5 seconds. The black and white chart on the left of Figure 46 is the same as the Candoro form in Figure 38—written with pitch class names without cent deviation because they corresponded to notes in a Scala file for MIDI playback. The soundtrack for Candoro starts with 32.5 seconds of unaltered field recording. After that, different

filters fade in and out every 32.5 seconds, following the Candoro form in figure 38. To do this, I set up ten buses which would receive audio from that field recording. Each bus had a drastic EQs that would accentuate specific frequencies—one for each note of Dekany Rotation 4—while cutting out all other frequencies. The send values from the field recording to the various busses are automated, so that only the frequencies related to the harmonic progression are sent through the busses. There is a noise gate on each bus. The release of the noise gate is automated so that when the bus is receiving audio, the release of the gate gradually increases, peaks halfway through the bus' duration receiving audio, and then decreases again. This was done to keep the sound elements in constant flux. Around the halfway mark (approximately 6:00-7:00), these buses and the sound of the metronome disappear, leaving only the field recording and low-pitched sine waves. These sine waves act as a symmetrical axis, after which the metronome and filtered busses return before ending in unaltered field recording again.

The colorful charts on the right of Figure 46 were a formal guide for video editing. In the first column, I gave myself an ABA form, with the subjective prompt of moving from order to disruption and returning to order. The second column shows which of my collaborators would be most present onscreen at any given moment—Alex, Daeun, myself, or all three of us. This is also symmetrical over time. It is divided into six groups, and each of us (and a cell for all of us) appear once in every group. I left the specific meaning of the ABAB' form in the third column open, thinking it might be useful. However, in the end, I didn't use it because the elements I wanted to serialize were addressed in the other columns.

Nine of Wands (2023)

Of the repertoire that Rotations Ensemble has performed, *Nine of Wands* was the first piece I composed. In fall 2022, I returned home from Loghaven with Scala files of all ten Dekany Rotations, which I could use to make pitch-accurate MIDI demos with any MPE-enabled synth inside of a DAW. After composing some ensemble gestures in Rotation 9, I fashioned two of them into *Nine of Wands*.

Nine of Wands: Intro

Nine of Wands has two sections: “Intro” and “Canon.” The Intro is in the style of the many micropolyphony-inspired pieces I have made that ask an ensemble to play repeated phrases at individual pulses to create a sound texture that is repetitive but never verbatim. *Neotrope* was an early piece of mine in this style, composed in 2010 for an accordion orchestra riding on the Santa Monica Pier Carousel.²⁸

In the score shown in Figure 47, the ensemble is divided into two groups, with one group playing the top staff and the second group playing the bottom staff. The large numbers between the staves are minute markings. Each player follows a stopwatch and moves from one measure to the next at the appropriate minute. Within each measure, the players repeat the written phrase at any speed they choose, as long as it does not line up with the pulse of another player. When there is a solid vertical line—like in m.5—the players all sustain the full chord in that measure. When there is a thick horizontal line—like in mm.6-8—the players alternate at will between repeating the phrase in that measure and sustaining any combination of those notes.

²⁸ Commissioned by Machine Project and the City of Santa Monica for the 2010 Glow Festival

NEOTROPE

Daniel Corral

© 2010 by Daniel Corral

Figure 47: *Neotrope* (2010)

I revisited this cloud-like approach for the Intro of *Nine of Wands*, having the players loop gestures in individual pulses. I instructed the violin and viola, to start pizzicato and gradually add more arco notes until they are completely arco by rehearsal letter I. The notes of letter I are the opening B and F# of the Canon, and the two parts are played attacca.

Nine of Wands: Canon

The Canon in *Nine of Wands* interpolates ideas from James Tenney's *Chromatic Canon*. Tenney's piece was dedicated to his friend Steve Reich, and combines Reich's signature gradual processes with dodecaphony. As Rob Wannamaker notes in *The Music of James Tenney Vol. 2: A Handbook to the Pieces*, Tenney composed *Chromatic Canon* with a derived row in the style of Webern's *Concerto Op.24*. However, the row in *Chromatic Canon* is based on a minor triad, a

sort of musical joke about how tonal the “minimalism” of composers like Reich or Philip Glass is. Like Webern’s *Op.24*, Tenney’s twelve-tone row for *Chromatic Canon* includes three-note cells that are Prime, Retrograde, Inversion, and Retrograde Inversion, as shown in Figure 48 (Wannamaker 2021, 185-188).

Bb — F — Db	Prime
G — B — E	Retrograde
Eb — Ab — C	Inversion
F# — D — A	Retrograde Inversion

Figure 48: The four segments of the 12-tone row in Tenney’s *Chromatic Canon*

Though *Chromatic Canon* is typically played on two 12TET grand pianos, Tenney’s original score was intended to be in JI (like most of his later work). The original performance notes included the tuning system shown in Figure 49.

Prepare the following notes with the sostenuto pedal:

Figure 49: Tuning from Tenney’s original performance notes for *Chromatic Canon*

The symmetry of Tenney’s twelve-tone row’s partitions can be seen in the ratios between the notes, which are also written within the staff in his original notes. With the 189/100 as an axis of symmetry, the interval on either side of it is 2/3, then 5/4, then 10/7.²⁹ After a 10/7 to the left and a 5/7 to the right of the axis, the last two intervals in either direction are 4/5 and 3/2. Tenney

²⁹ Written as 5/7 between the C and F# only because it goes down rather than up. This might also be written as a 7/5, if the interval went in the opposite direction from F# to C. In most of his music, Tenney followed Partch’s convention of writing ratios between 1/1 and 2/1.

clearly intended for the 8 notes prepared with the sostenuto pedal to resonate sympathetically with the ostinati being played an octave higher. These eight notes omit the four middle notes of the twelve-tone row, including the opening just Bb-6 chord and the just D7 chord that ends the row. At the end of the piece, the seventh and fifth partials are removed one-by-one, leaving only the first and third partials.

While harmonic symmetry is not out of character for Tenney's work, the way it is written is. In most of his scores, he simply wrote the intended ratios or cent deviations. This style of writing the interval ratios between successive pitches seems to me more in the style of Lou Harrison, whose *Music Primer* includes pages of pentatonic scales notated in a similar way (Harrison 1971, 27-37).³⁰

The fact that these just-intonation performance notes were present inspired my use of Tenney's process within the context of the Eikosany. The additive/subtractive process of the *Nine of Wands'* Canon is almost identical to Tenney's, but the pitch material is different and I don't use a derived row. However, almost all of the intervals between successive notes are Primary intervals. This is true except for the interval between the fifth and sixth notes, which is a 5/3 Secondary interval as shown in Figure 50. In addition, both of the pentads on either side of the 5/3 axis of symmetry contain all four of the Primary intervals present in Dekany Rotation 9 (3/2, 5/4, 9/8, 11/8 or their inversions 4/3, 8/5, 16/9, 16/11). In the spirit of Morton Feldman's rug-inspired love of disproportionate symmetry, I was happy with the ten-note row I had chosen (Feldman 2000).

³⁰ Though in Harrison's book, the notes are in ascending order and the notes aren't written in standard music notation.

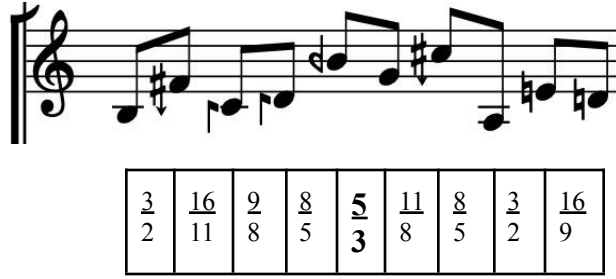


Figure 50: Intervals between notes in the 10-note row of *Nine of Wands*.

Figure 51 shows the opening of the Canon from *Nine of Wands*, in which the repeated cell at rehearsal letter A is similar to the opening fifths of *Chromatic Canon*. Rehearsal letter B is similar to the second section of *Chromatic Canon*, but with a 12/11 of the perceptible tonal center of B -18¢ rather than a minor third. In addition, the bass plays a two-bar phrase that doubles the first and last notes of each measure of the guitar part. At rehearsal measure C, the violin and viola play a phrase of five notes that is the length of six eighth notes, creating a polyrhythm against the 2/4 pulse of the bass and guitar. The violin and viola's phrases are identical, but an octave apart and offset by three eighth notes. The violin and viola each play their respective ostinati twice within the three-measure repeat, while the guitar ostinato plays three times and the bass line happens once. The bass, within their ostinato connected to the guitar's four-beat pattern, accents the repetition of the violin and viola's ostinato with the B on beat two of the second measure of C. The written meter at each rehearsal letter always follows the meter of the bass part. The violin and viola part at rehearsal C might be more easily read in 3/8 like the shorthand version of *Chromatic Canon* or the simultaneous notated meters that Morton Feldman uses in scores like *For Philip Guston* (Feldman 1985). However, in rehearsal the ensemble found it more useful to keep all of the meters unified.

Figure 51 shows a musical score for four instruments: Violin, Viola, Electric Guitar 1, and Contrabass. The score is divided into three sections labeled A, B, and C. Section A is marked 'Allegretto' with a tempo of 130. The key signature has two flats. The time signature changes from 4/4 to 3/4 to 2/4. The dynamics are marked with 'f' (forte). The Violin and Viola parts are in treble and alto clefs respectively, while the Electric Guitar 1 and Contrabass parts are in treble and bass clefs respectively. The score shows the first three repeated cells of the piece.

Figure 51: The first three repeated cells of *Nine of Wands*

The Canon in *Nine of Wands* continues in this way until all ten notes of the Dekany are played by both groups at rehearsal letter J. At that point, the ten-note row and rhythmic cells shrink until there is just the final sustained D in m. Up until that point, the bass has primarily sustained the B -18¢ and doubled the last eighth note of the guitar part. At rehearsal K, this changes as the guitar's 10-note ostinato removes notes from the beginning of the row instead of adding notes to the end. At rehearsal K, the bass mostly plays F# -16¢. At rehearsal L, the bass plays C +33¢. The predominant bass note continues to change at every rehearsal letter, moving through the ten-note row until only the D is left. This increased motion contrasts the sustain of the B -18¢ from the first half of the piece and gives the impression of increased temporal density of harmonic changes until the pulse implodes in the piece's final cells.

Six of Swords (2025)

Six of Swords uses the Candoro Form in Dekany Rotation 9, as shown in Figure 40. In performance with *Nine of Wands*, *Six of Swords* follows *Nine of Wands* attacca. To make this

transition work, I instruct the ensemble to rearrange the first three measures so that the piece starts with a D -2♭, introduces the B -18♭ in m.2 and the F# -16♭ in m.3. This change allows for a smoother transition from *Nine of Wands*, which ends with a sustained D -2♭. Though this deviates from the Candoro Form, it serves the music better. One of my goals with generating Rotations Ensemble Repertoire is to create musical modules that can be transitioned between to make a concert-length performance, so these sorts of deviations are acceptable.

I made *Six of Swords* to contrast my other Rotations Ensemble pieces. It is a very placid piece, in contrast to the minimalist rhythmic drive of the other music. In addition, there is room for choice and improvisation, while the other Rotations Ensemble pieces thus far are very strictly notated. The level of choice in *Six of Swords* harkens back to Tenney's use of "available pitches" in several of his *Harmonium* pieces.³¹ In these scores, the players are given a choice of pitches in each measure and instructed to play a swell on their chosen pitch $ppp < X > ppp$, with X being a dynamic indicated in the score. My score for *Six of Swords* acts similarly, giving string players specific available pitches within each measure (following Candoro Form in Rotation 9). This includes the bass player, and this is the only Rotations Ensemble piece thus far that includes arco in the bass part. However, I ask them to sustain long tones on their chosen pitches until they choose to change pitch. I ask the drummer to play Eikosany Chimes for this piece, giving them the same Candoro form and instructions as the string players.

The guitar part is a series of chord voicings of the notes of the Rotation 9 Candoro form. They are played sparsely, and sustained with a delay pedal and a Freeze Pedal. This placid, slow motion harmony study offers a contrasting way of listening to the intervals of the Eikosany. While my other Rotations Ensemble pieces show the Eikosany in the context of moving lines (often in repeated cells), *Six of Swords* lingers on each harmony, partially inspired by my

³¹ Specifically, *Harmonium* #1, *Harmonium* #4, *Harmonium* #6, and *Harmonium* #7.

experiences playing accordion in the music of Chiyoko Szlavnic at the wulf. in Los Angeles. Each measure is cued in *Six of Swords*, making its total duration malleable. So far, I've performed it with the measures being forty-five seconds or sixty seconds.

In performance, I've had the end of *Six of Swords* dovetail with the beginning of *Ace of Pentacles*, making a sort of three-movement suite including *Nine of Wands*, *Six of Swords*, and *Ace of Pentacles*. The transition from *Six of Swords* to *Ace of Pentacles* requires fewer changes, with mm.20-22 of *Six of Swords* being replaced by m.1 of *Ace of Pentacles*.

Ace of Pentacles (2024)

Like *Six of Swords*, this piece is a departure from the strictly composed ensemble writing of *Three of Pentacles* and *Nine of Wands*. At the time, I was reading *Lou Harrison: American Musical Maverick* and thinking about how to incorporate more melody-centric music into Rotations Ensemble, having for a number of years focused on music that did not prioritize melody. The homophony of *Ace of Pentacles* is what came out of that inquiry. It begins with a meterless, Taqsim-inspired improvisation, drawing influence from my time playing accordion and synthesizer in the Arabic-music ensembles Qamar and Saffron Ensemble. Once the tempo sets in, the rest of the piece is a through-composed homophonic composition with a main melody and chord progressions. At rehearsal D, there is a solo section that gives players a chance to improvise within Dekany Rotation 9. Since improvising in a microtonal system that was new to the players was a large request, I gave them the lead sheet in Figure 52, which includes chords and scales that might work with the chord tones.

D 2. Solo chart

F#(-14 ϕ) minor

26 $+35\phi$ $+37\phi$ $+2\phi$

$\pm 0\phi$ -18ϕ -14ϕ

28

30

32 (4)

E

A mixolydian

34 $+2\phi$ $+35\phi$

$\pm 0\phi$ -18ϕ -14ϕ -2ϕ -16ϕ

mf

36 "C" $+33\phi$ $+37\phi$ $+2\phi$ $+35\phi$ $+2\phi$

$\pm 0\phi$ -16ϕ

38 A mixolydian

$+2\phi$ $+35\phi$

$\pm 0\phi$ -18ϕ -14ϕ -2ϕ -16ϕ

mf

40 "C" $+33\phi$ $+37\phi$ $+2\phi$ $+35\phi$ $+2\phi$

$\pm 0\phi$ -16ϕ

Figure 52: Lead sheet for the solo section of *Ace of Pentacles*

After the extended placidity of *Six of Swords*, the energy of *Ace of Pentacles* comes as a surprise. They are both examples of thoughtfully loosening control of all elements of a composition while remaining adherent to the harmonic explorations of the Eikosany. Somewhere between these extremes, I see a path forward for Rotations Ensemble.

4. Future Developments

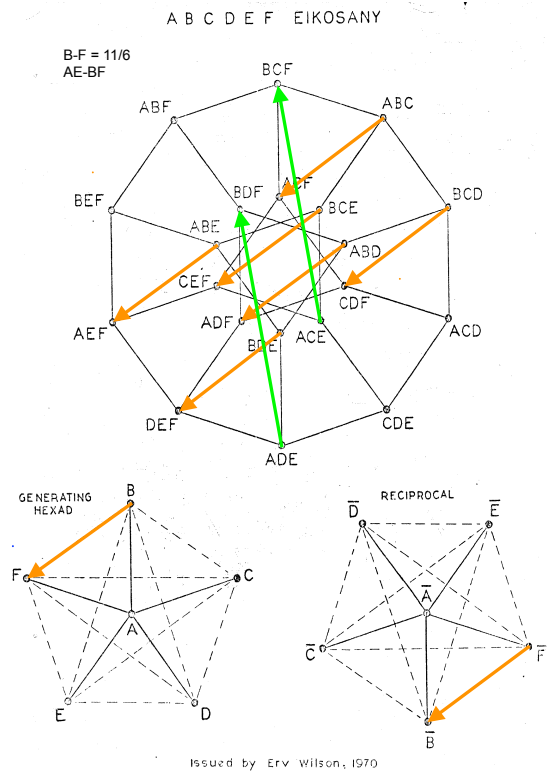
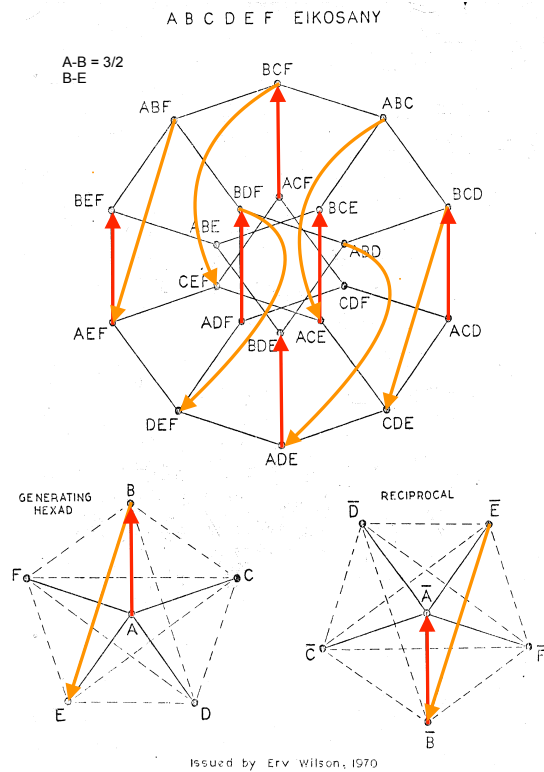
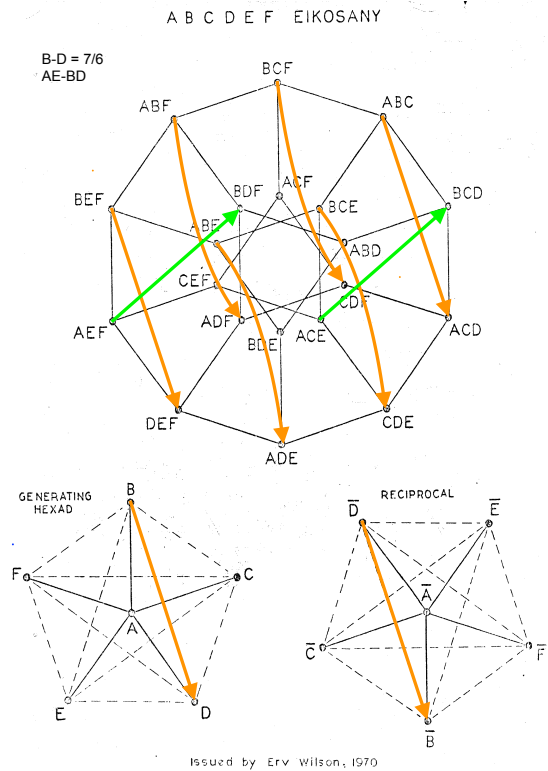
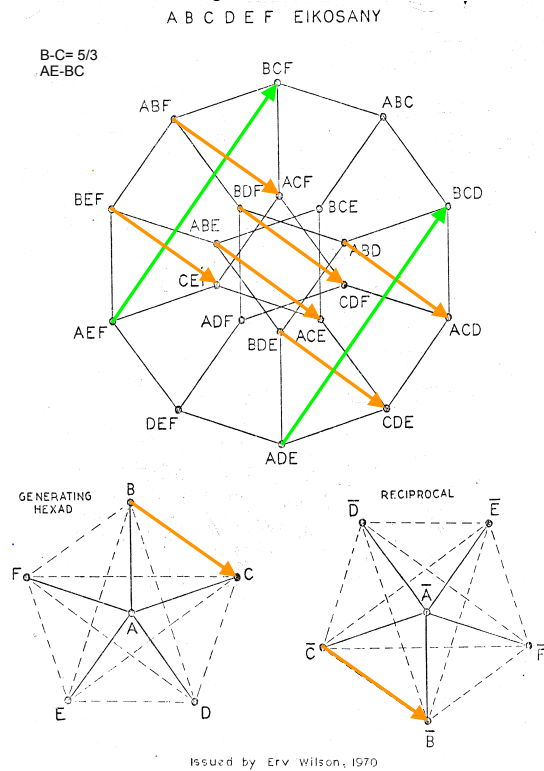
As of this writing, I am using what I've learned from the Eikosany lattice to write a piece for the Los Angeles-based PARTCH Ensemble. To do this, I'm using the Hexadic Diamond in Figure 16 and the lattice in Figure 20 to map out Hexanies and centered hexads that exist on or connect to Partch's instruments. I'm combining this Wilson-inspired harmonic vocabulary with Partch's use of "multiple exposures," drawing influence from well-known Partch works like *Castor & Pollux* and *And on the Seventh Day, Petals Fell in Petaluma*.³² In this way, I'm hoping to compose a piece that combines and pays homage to both Wilson and Partch's respective musical worlds.

I've spent a lot of time thinking about how the rules and systems that govern an artist's practice expand outward to influence how they interact with the world at large. What I find most interesting about Wilson's theoretical ecosystem is that there is no single way to use it. Even the fact that CPS have variables waiting for values to be assigned to them indicates the creation of space for someone inclined to do so to seek musical individuation there. This is why I like that an Eikosany has no central vertex. As mentioned in my discussion of Hexadic Diamonds, any vertex of the Eikosany could be expanded to become the central axis of a Hexadic Diamond. There is no hierarchy to the system besides what a user brings with them. This can be seen in contrast to the work of Harry Partch, whose life work revolved around his presence within it just as his tuning system revolved around the 1/1 of G. In the uncentered Eikosany, every vertex is equally verdant with possibility.

³² In Partch's music, a multiple exposure is a series of duos or trios that can be played simultaneously. His 1952 arrangement of *Castor & Pollux* is a great example of this: First, three duos are played, one after the other. After that, They're played simultaneously as a sextet (in the 1968 arrangement, the first group is a trio not a duo).

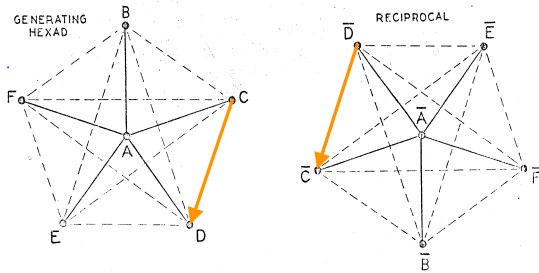
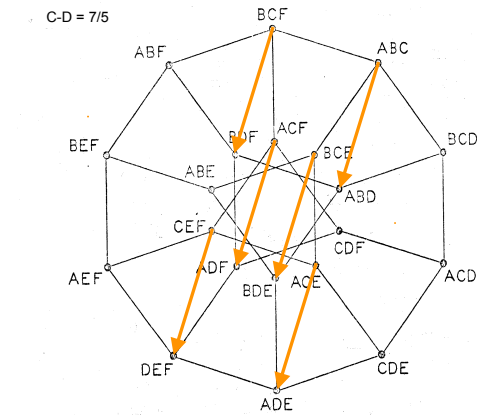
I don't know whether I will stay loyal to the $1\cdot3\cdot5\cdot7\cdot9\cdot11$ Eikosany indefinitely into the future, but for now, there is more to explore in it. Among other possibilities, I have performance repertoire for Rotations Ensemble and more Dekany Rotations to familiarize myself with. Going deep into this one lattice has been very fruitful for me, and I believe that it will inform how I approach other tuning systems—and musical situations generally—in the future.

Appendix A: Dyads of the Eikosany



ABCDEF EIKOSANY

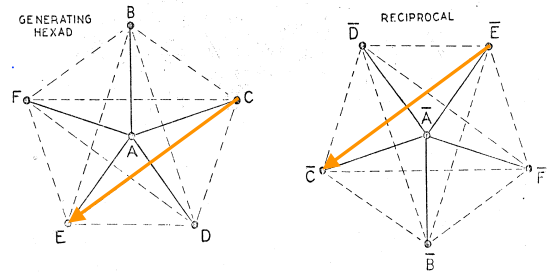
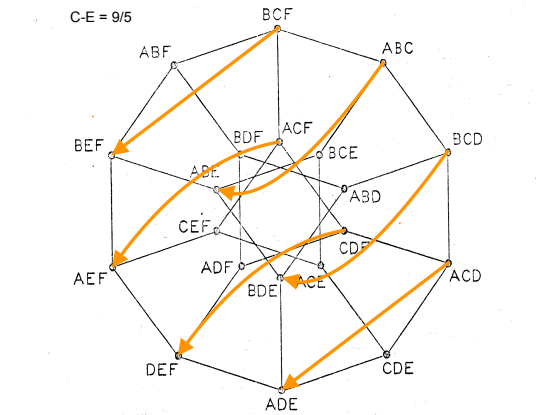
C-D = 7/5



Issued by Erv Wilson, 1970

ABCDEF EIKOSANY

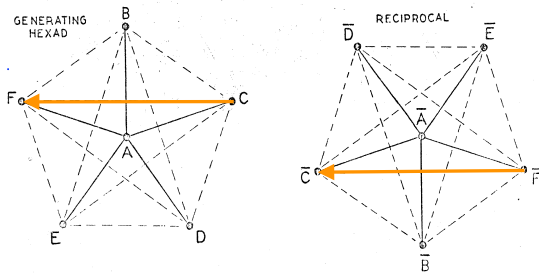
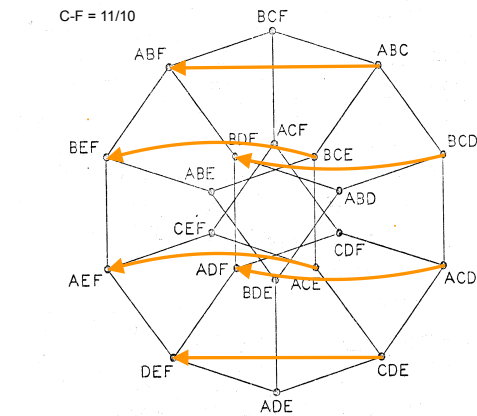
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ABCDEF EIKOSANY

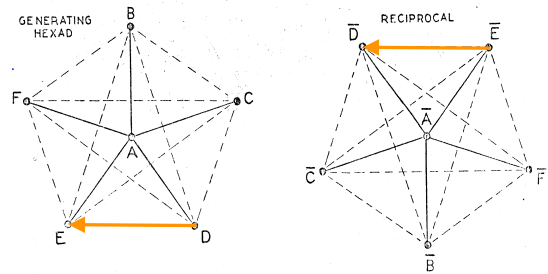
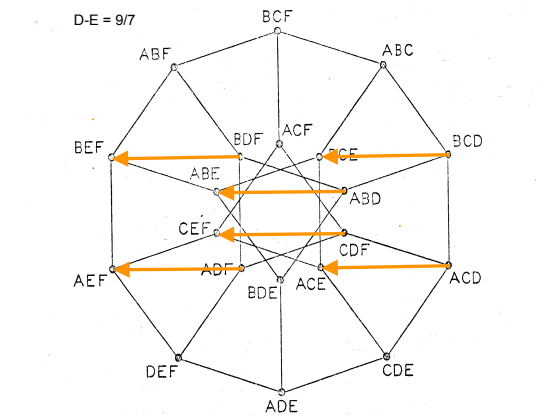
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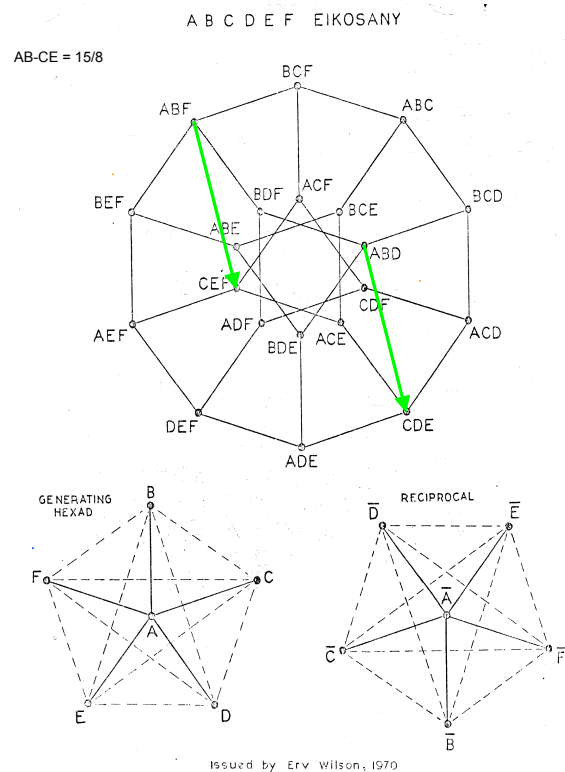
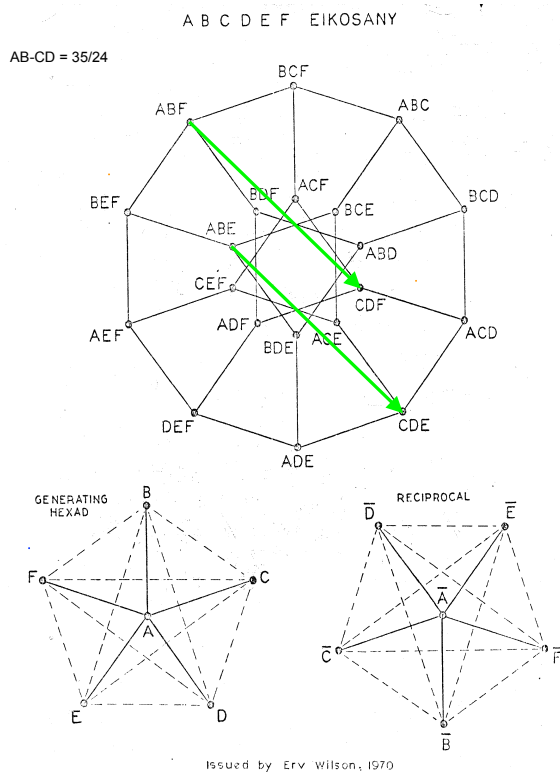
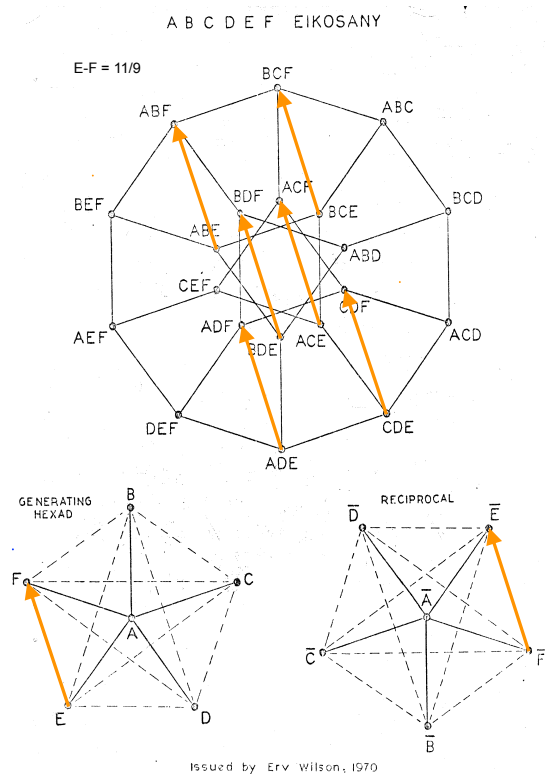
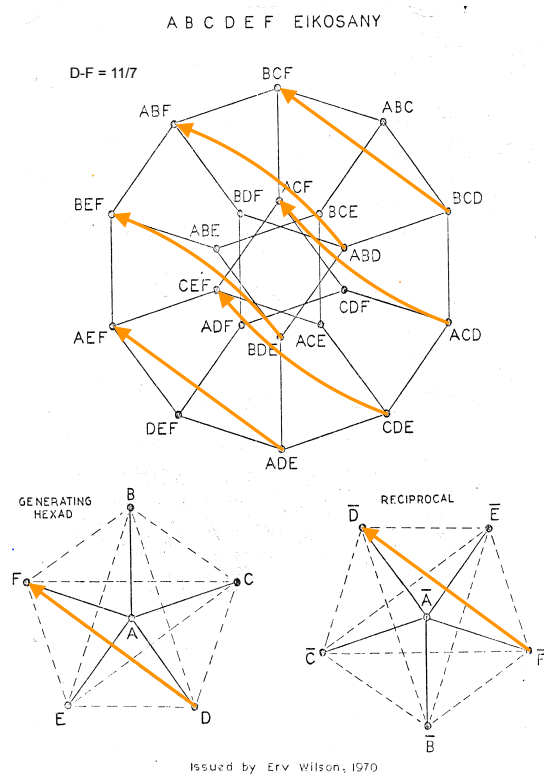
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ABCDEF EIKOSANY

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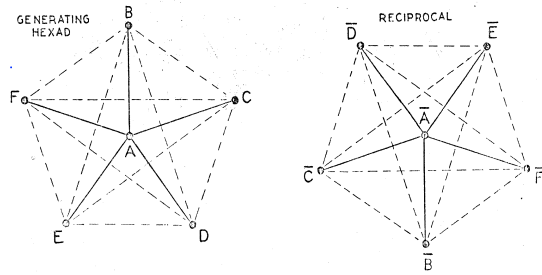
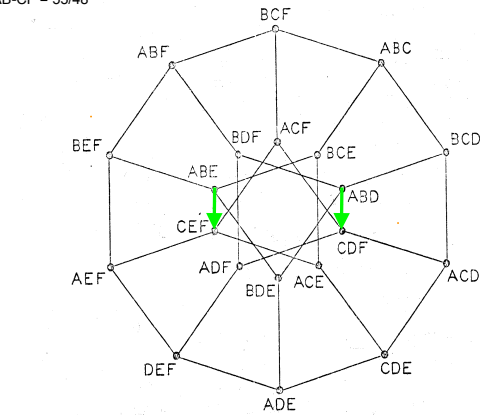


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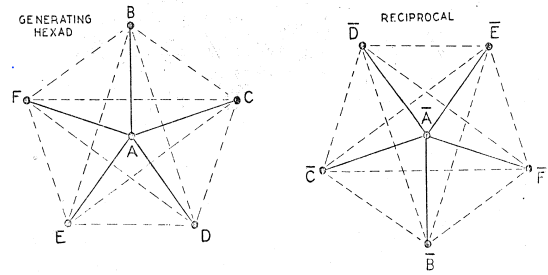
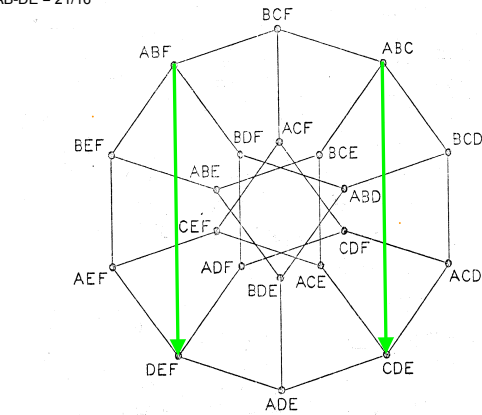
AB-CF = 55/48



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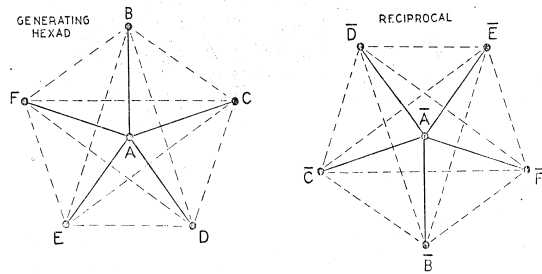
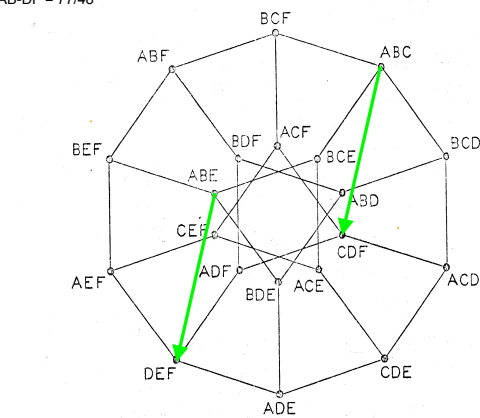
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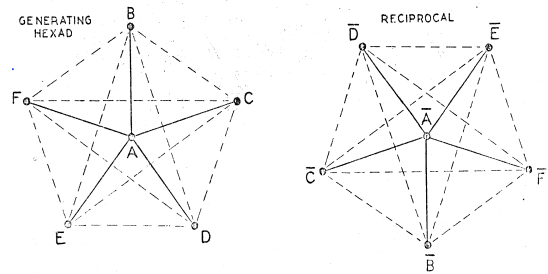
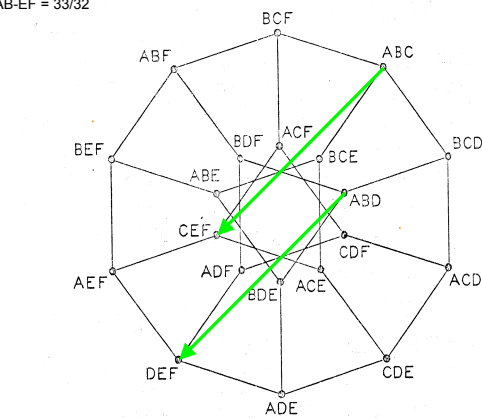
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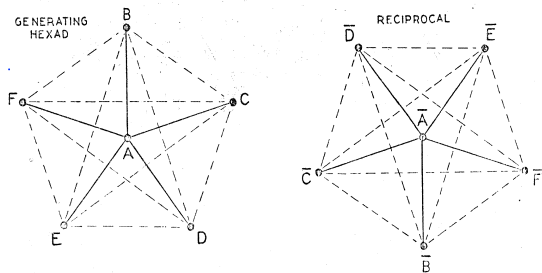
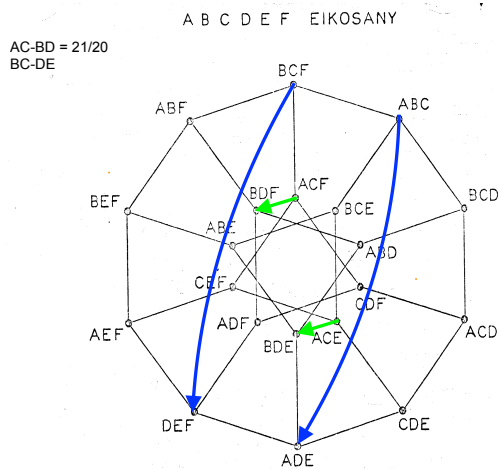
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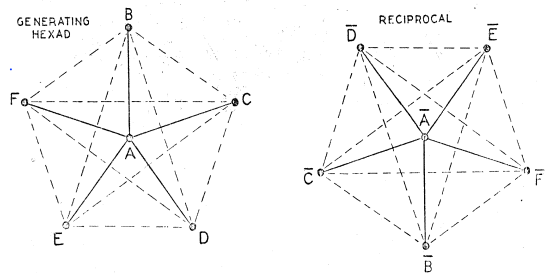
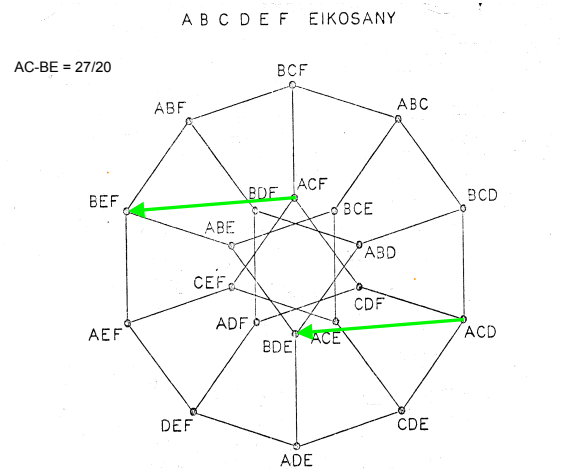
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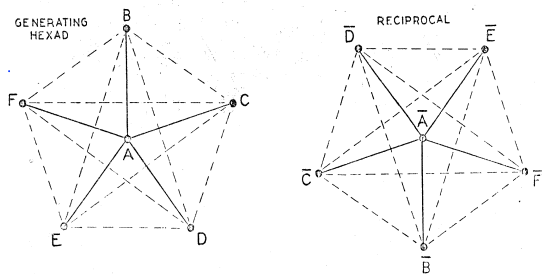
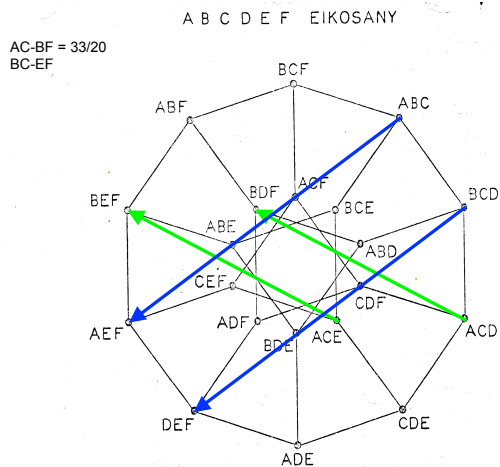
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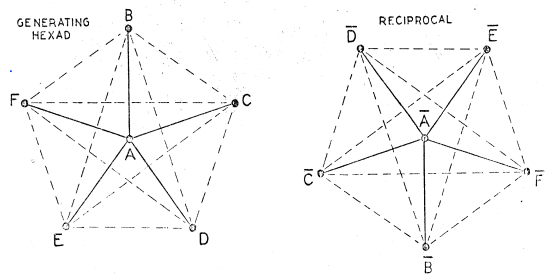
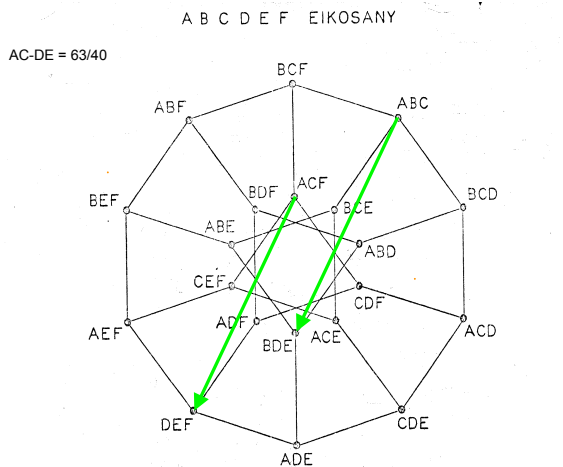
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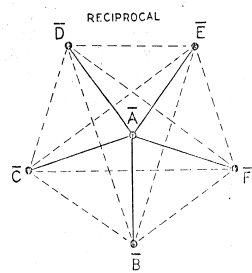
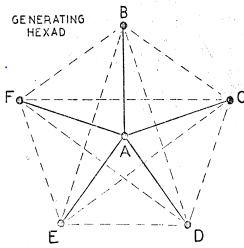
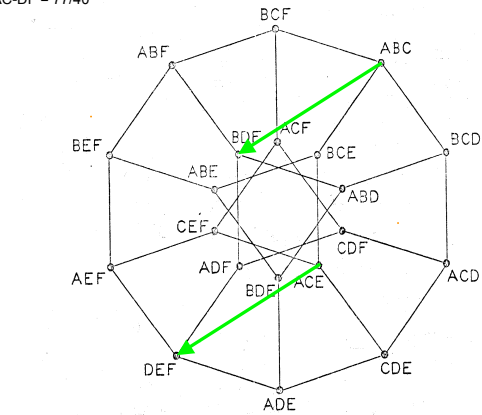
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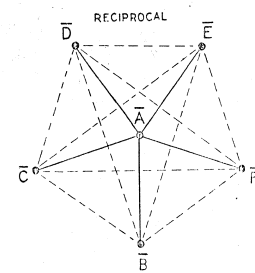
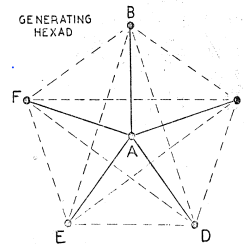
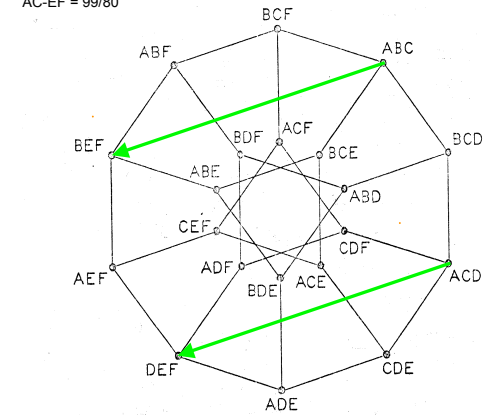
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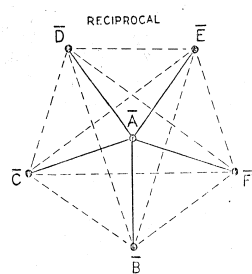
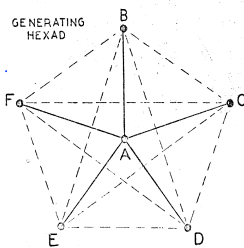
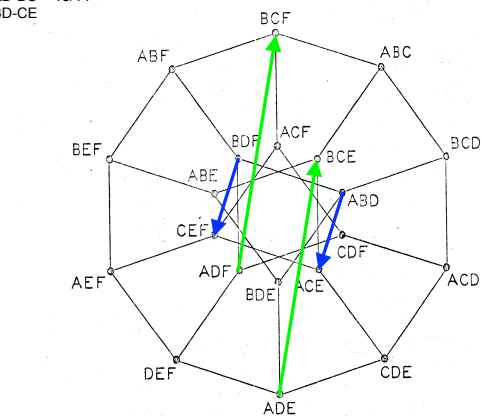
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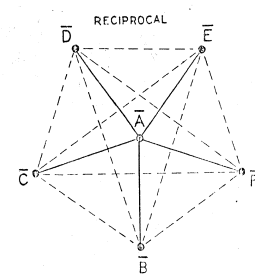
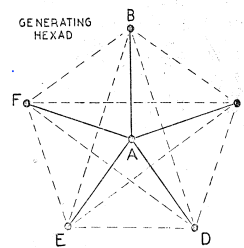
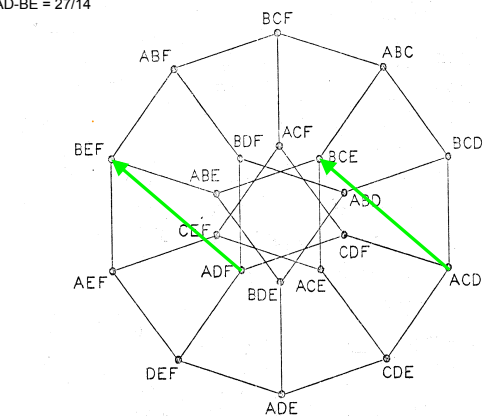
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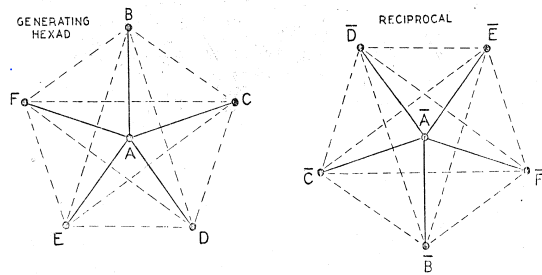
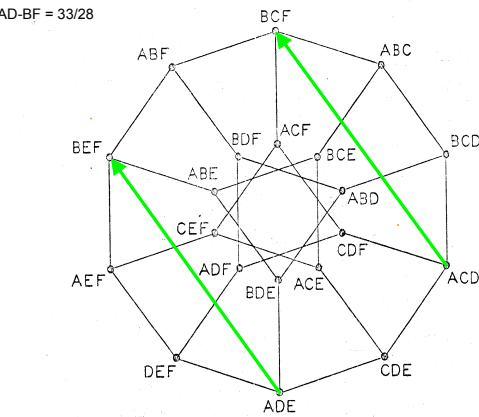
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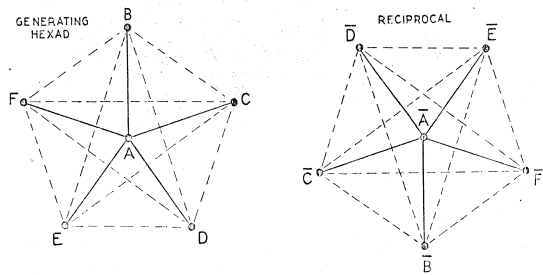
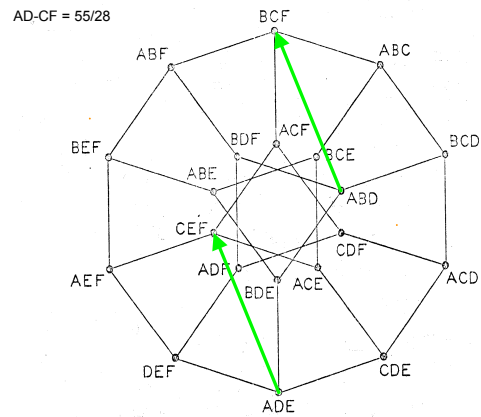
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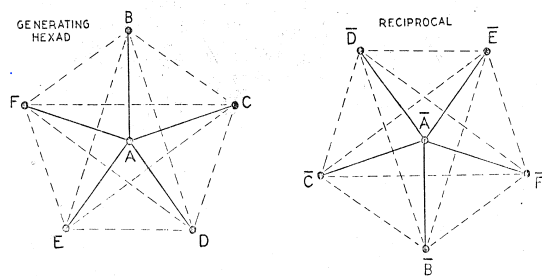
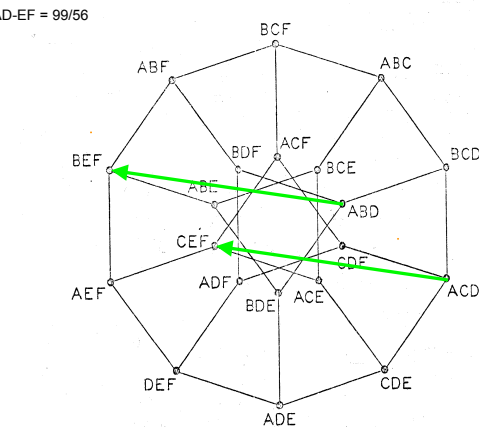
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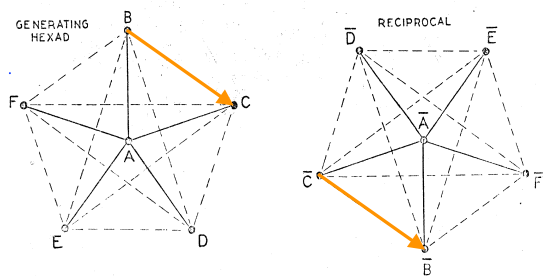
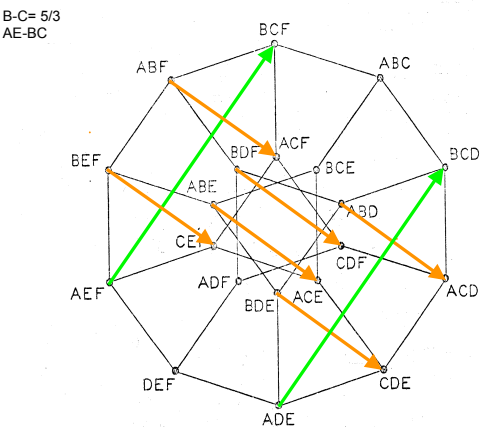
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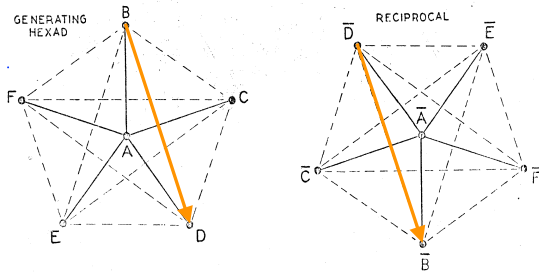
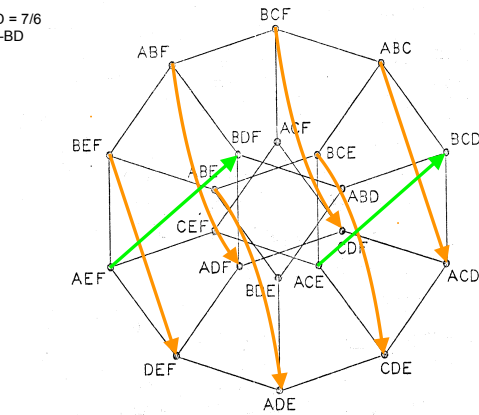
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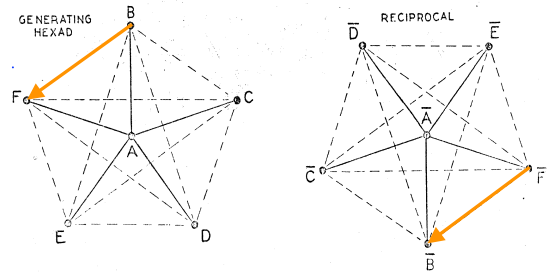
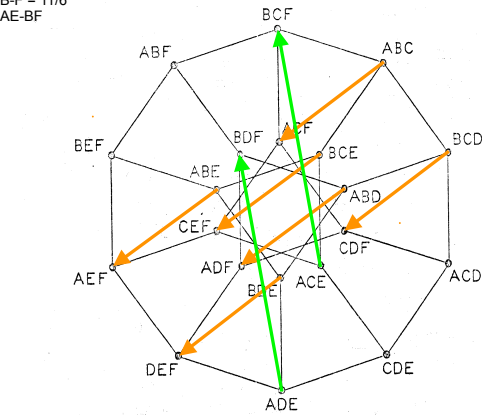
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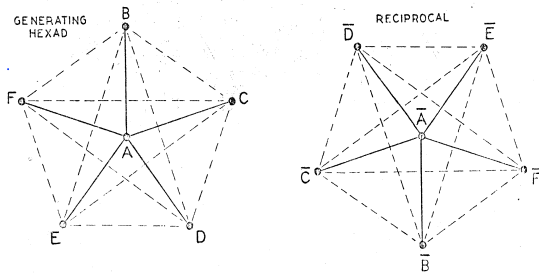
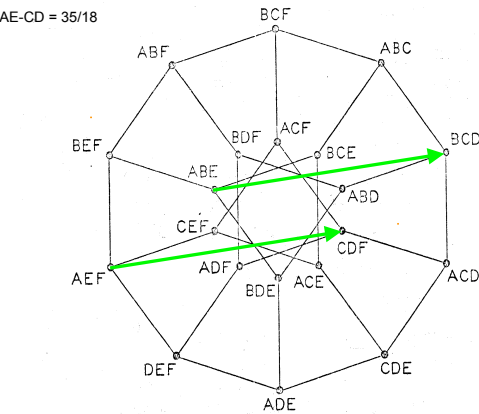
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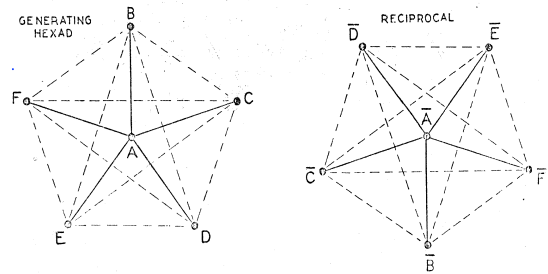
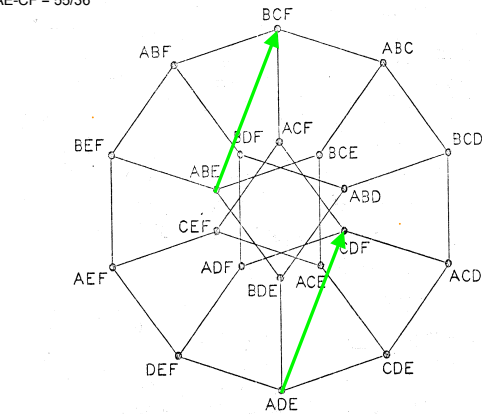
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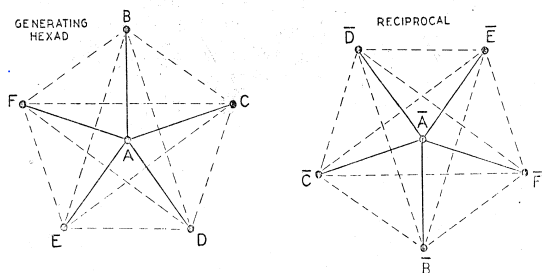
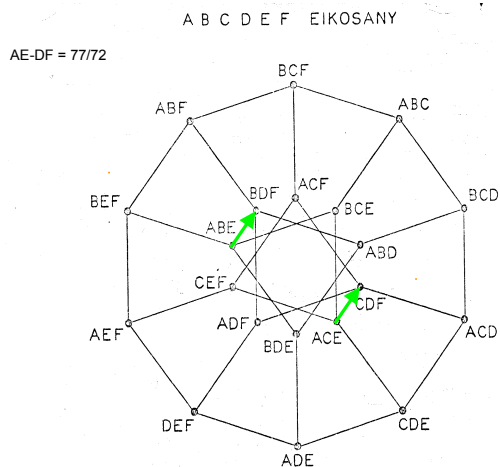
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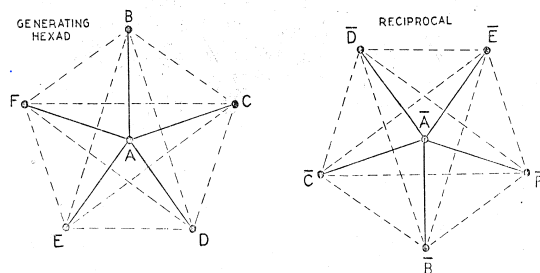
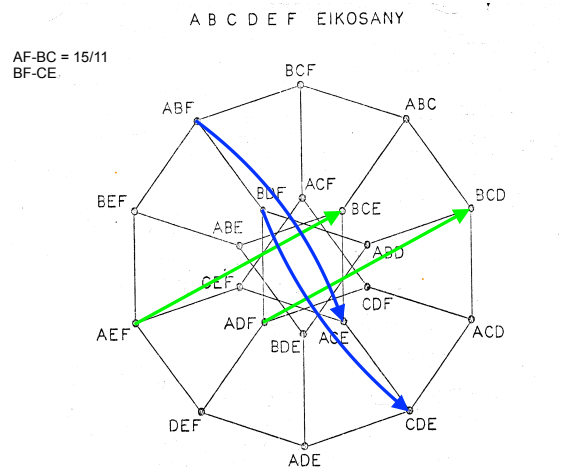
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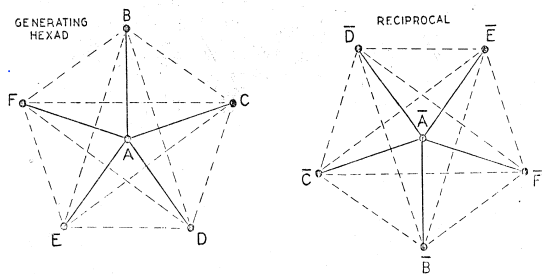
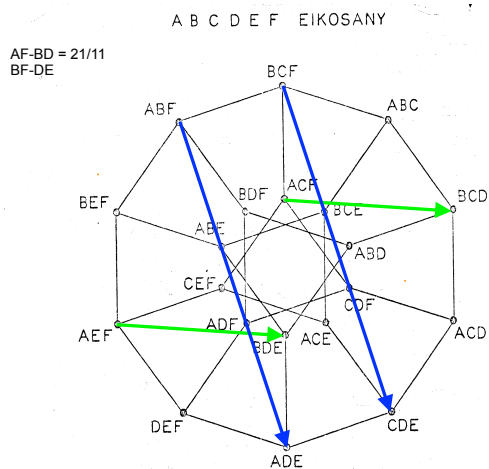
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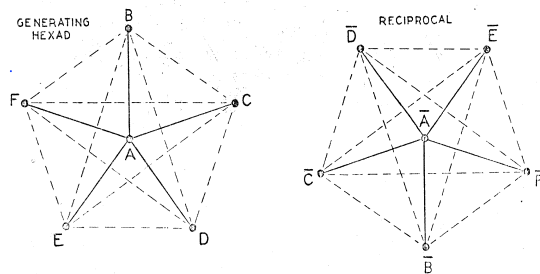
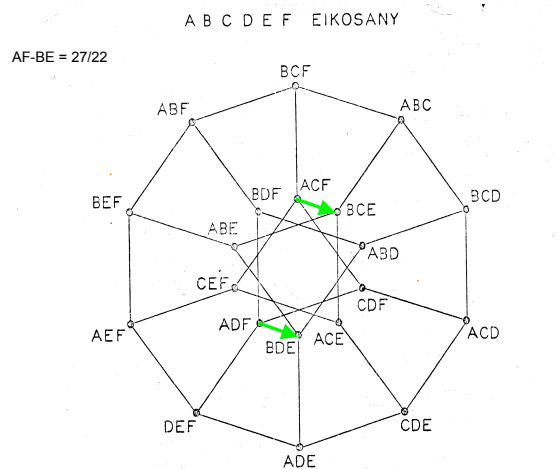
Issued by Erv Wilson, 1970



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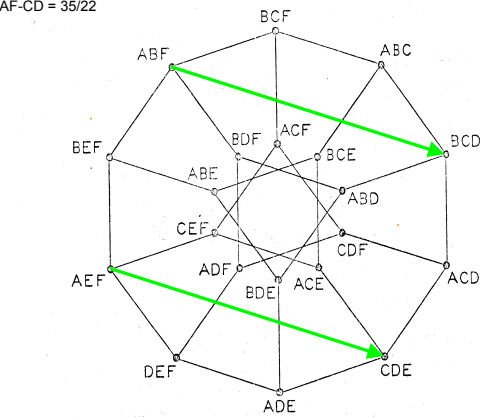
Issued by Erv Wilson, 1970



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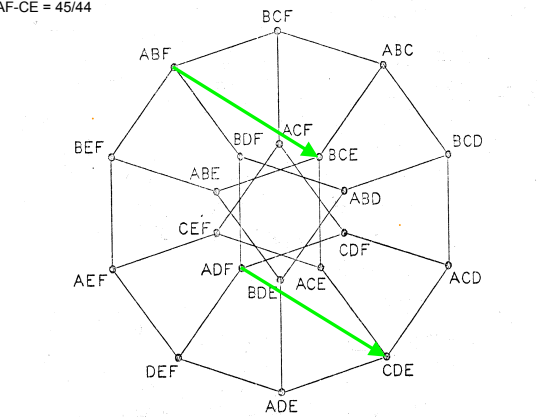
AF-CD = 35/22



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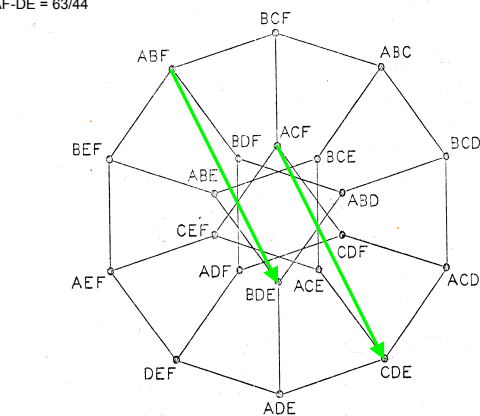
AF-CE = 45/44



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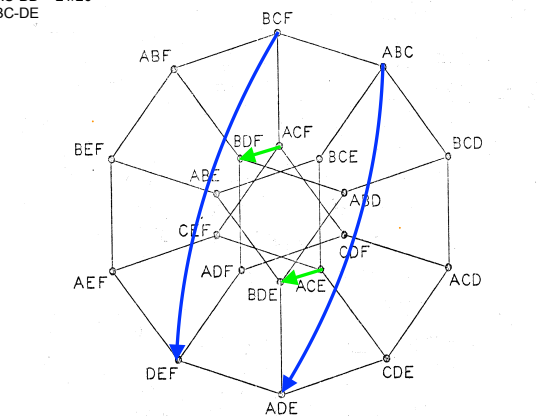
AF-DE = 63/44



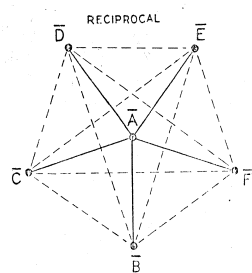
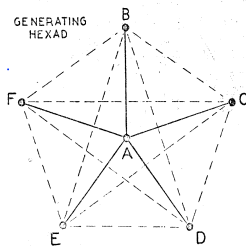
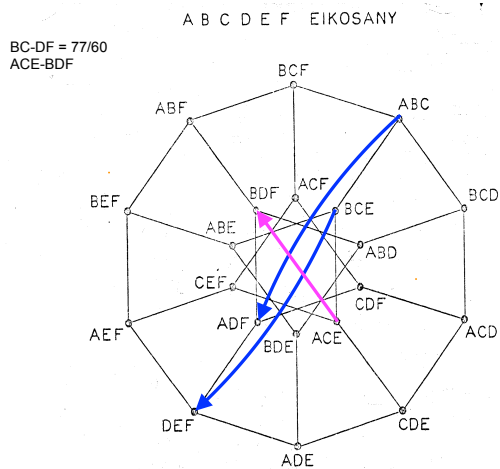
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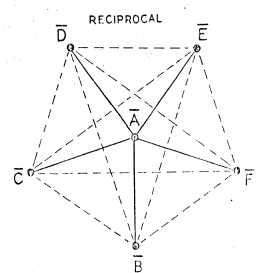
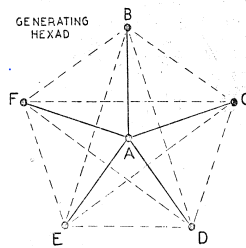
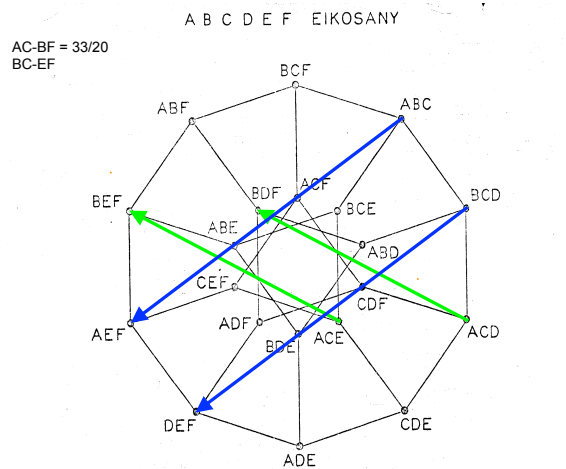
AC-BD = 21/20
BC-DE



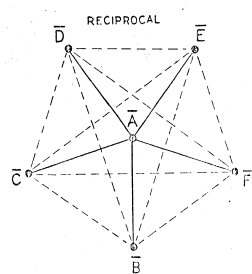
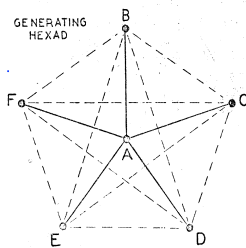
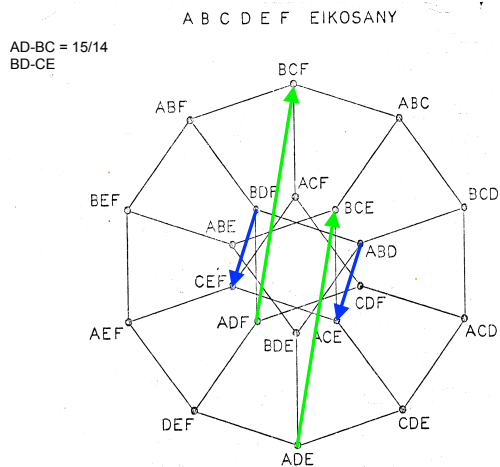
Issued by Erv Wilson, 1970



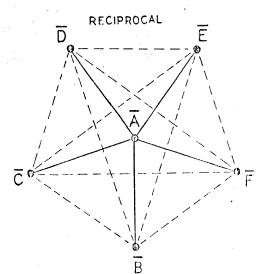
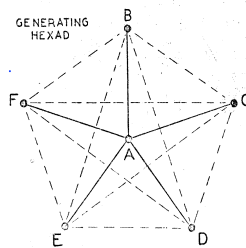
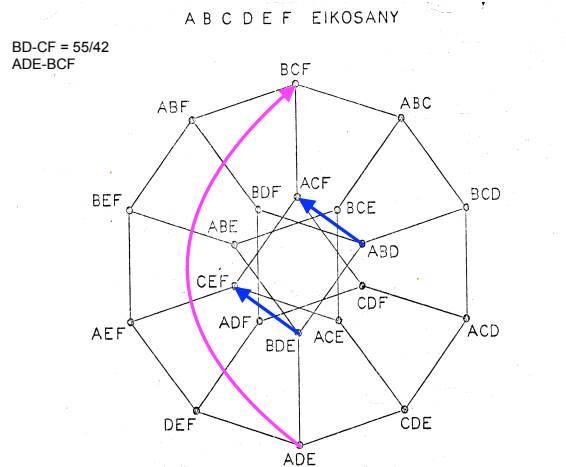
Issued by Erv Wilson, 1970



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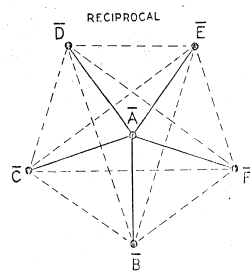
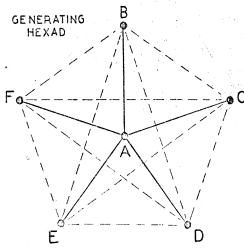
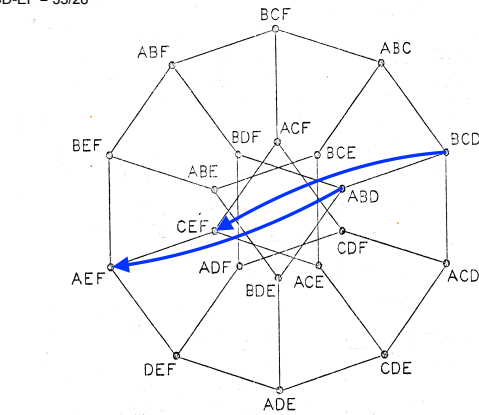
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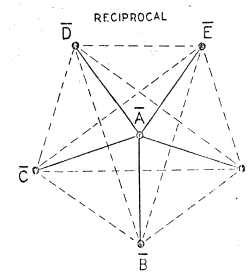
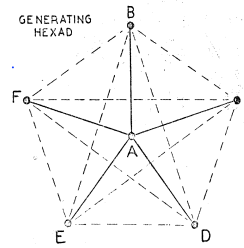
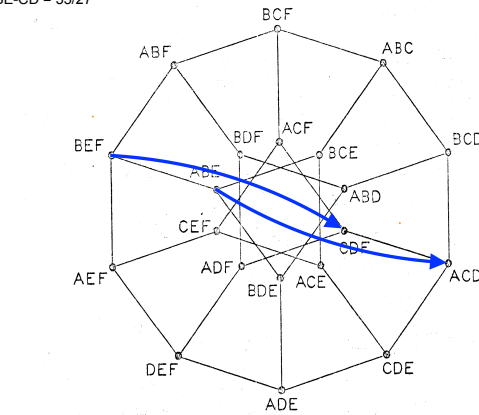
BD-EF = 33/28



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A B C D E F EIKOSANY

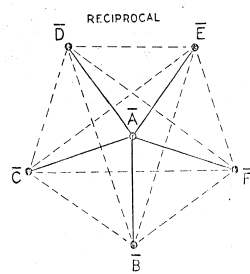
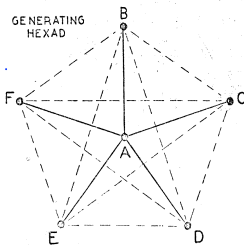
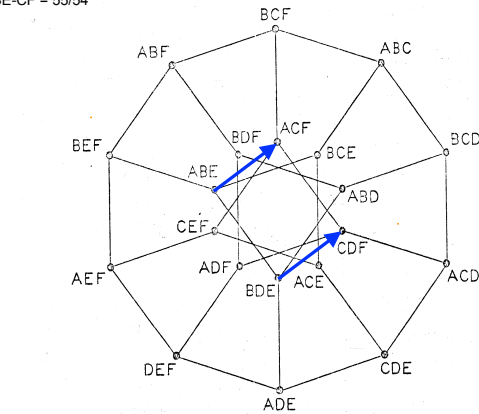
BE-CD = 35/27



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A B C D E F EIKOSANY

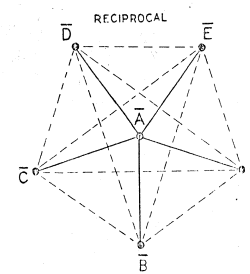
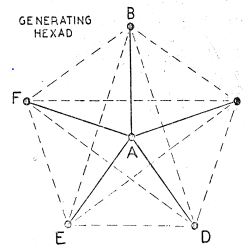
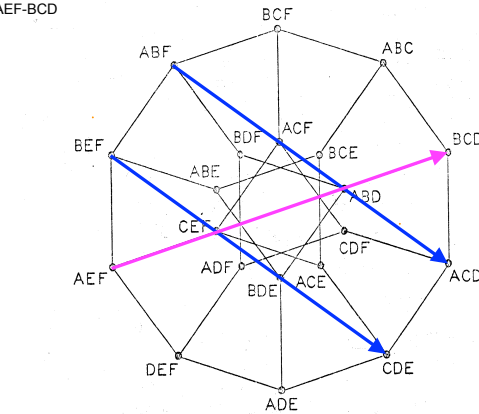
BE-CF = 55/54



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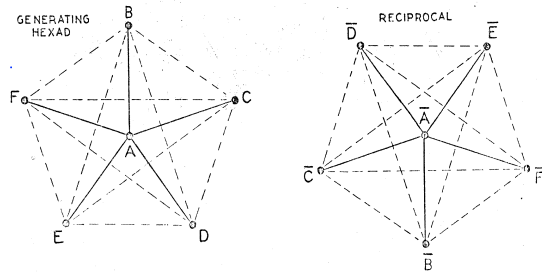
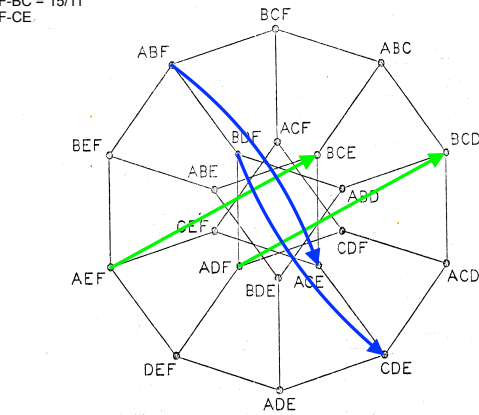
BF-CD = 35/33
AEF-BCD



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A B C D E F EIKOSANY

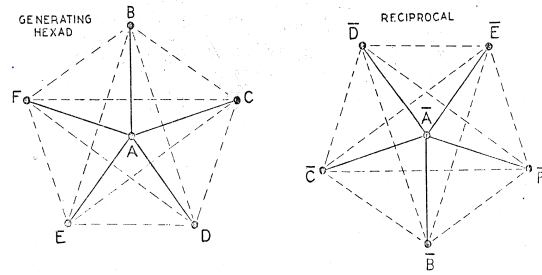
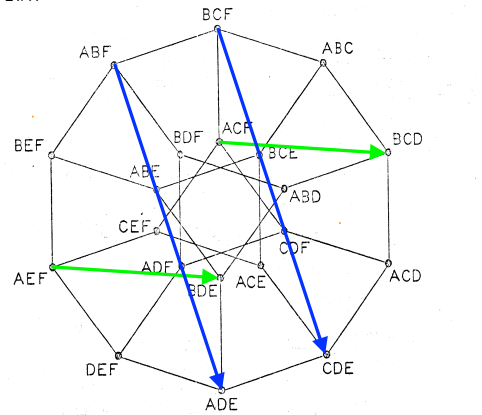
AF-BC = 15/11
BF-CE



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A B C D E F EIKOSANY

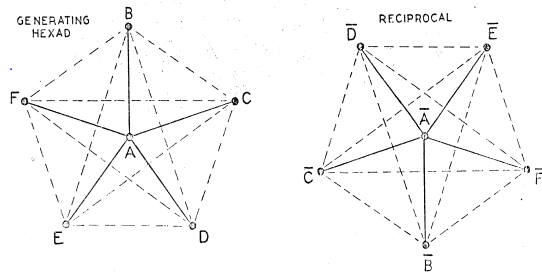
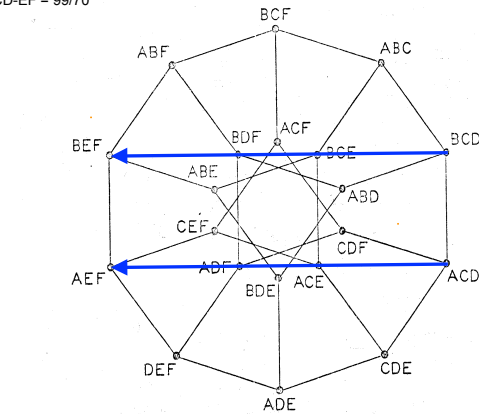
AF-BD = 21/11
BF-DE



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A B C D E F EIKOSANY

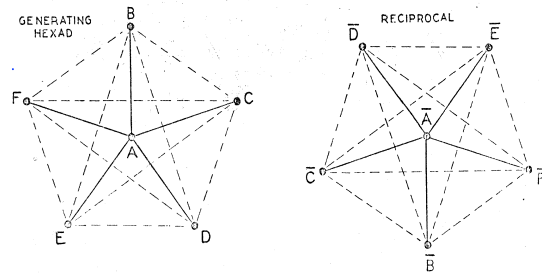
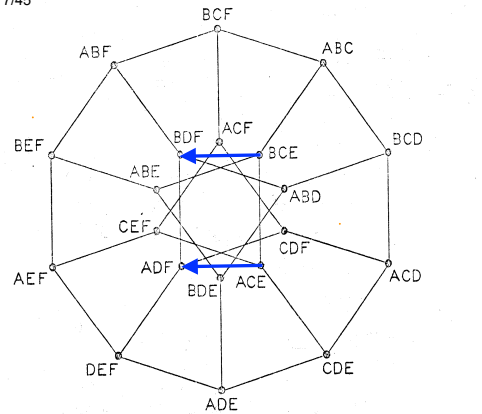
CD-EF = 99/70



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A B C D E F EIKOSANY

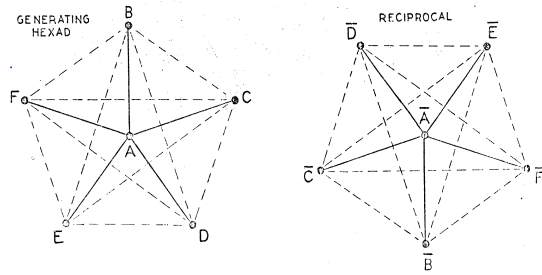
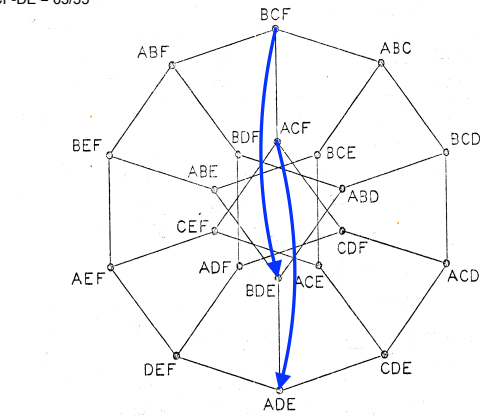
CE-DF = 77/45



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A B C D E F EIKOSANY

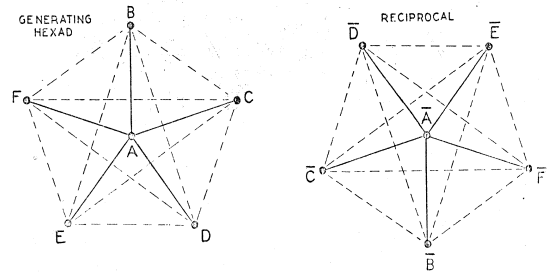
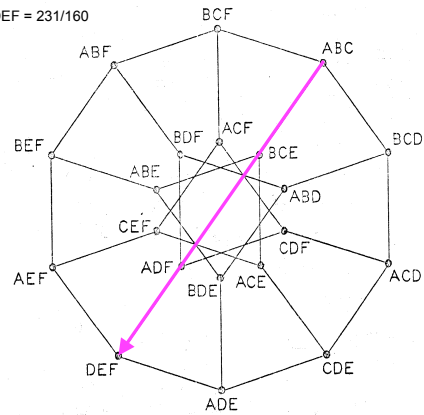
CF-DE = 63/55



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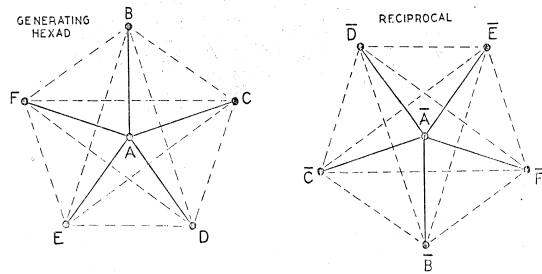
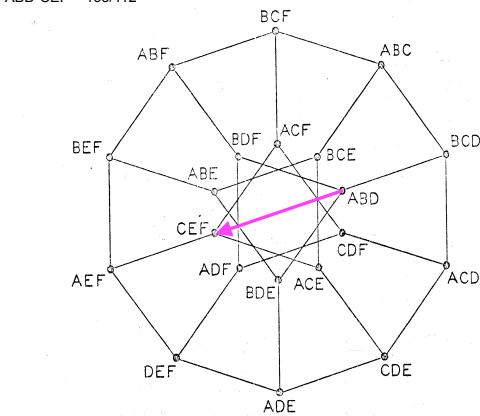
ABC-DEF = 231/160



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A B C D E F EIKOSANY

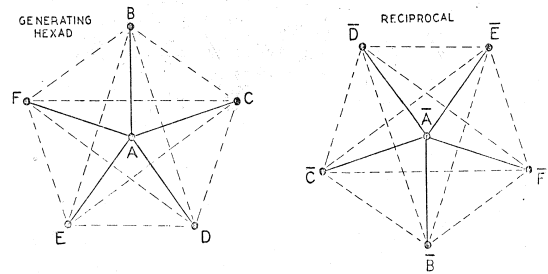
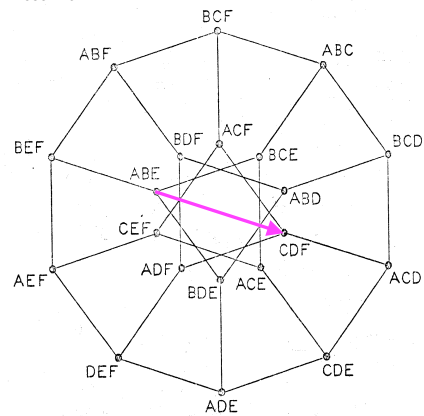
ABD-CEF = 165/112



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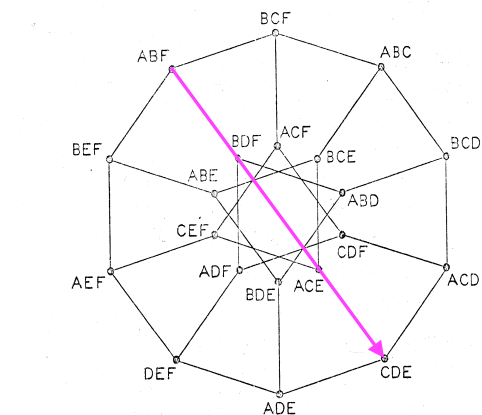
A B C D E F EIKOSANY

ABE-CDF = 385/216



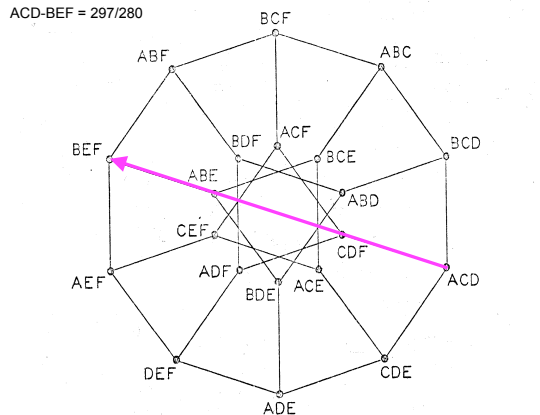
Issued by Erv Wilson, 1970

ABF-CDE = 105/88
A B C D E F EIKOSANY



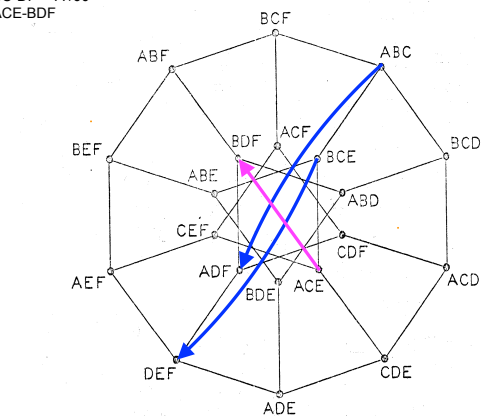
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ACD-BEF = 297/280
A B C D E F EIKOSANY



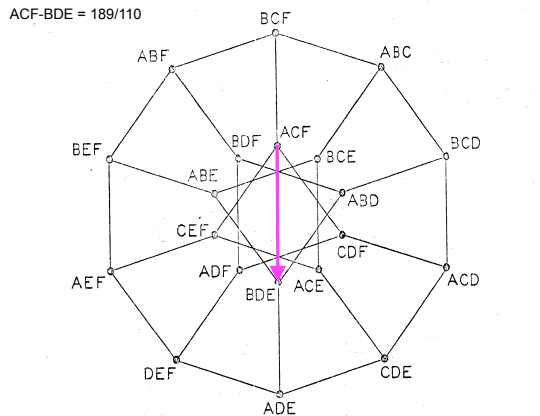
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BC-DF = 77/60
ACE-BDF
A B C D E F EIKOSANY

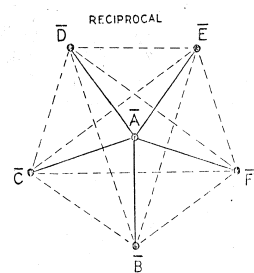
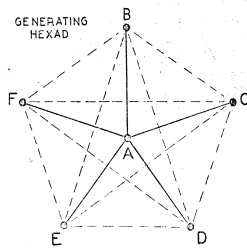
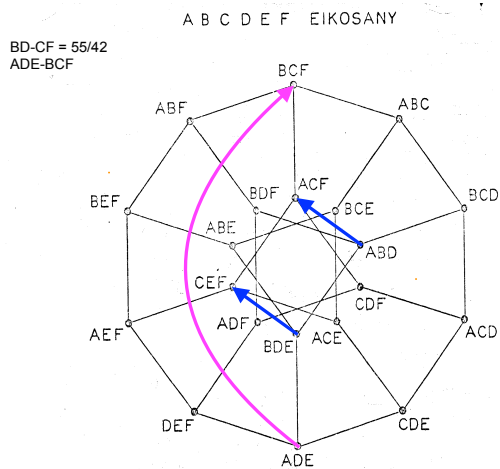


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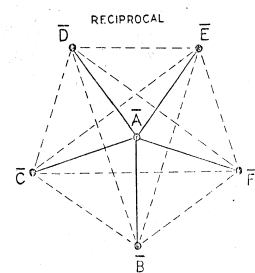
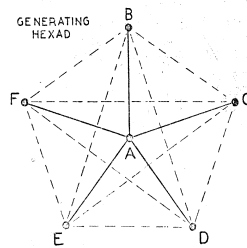
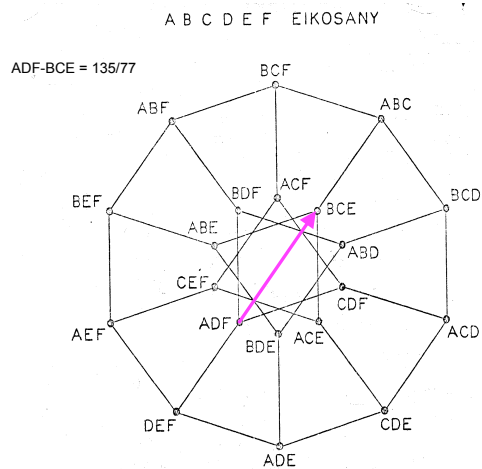
ACF-BDE = 189/110
A B C D E F EIKOSANY



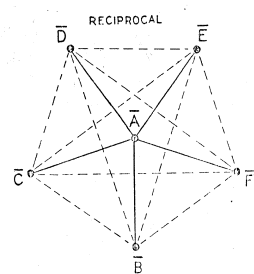
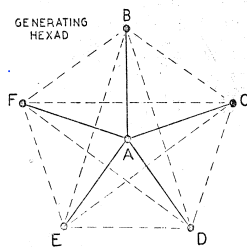
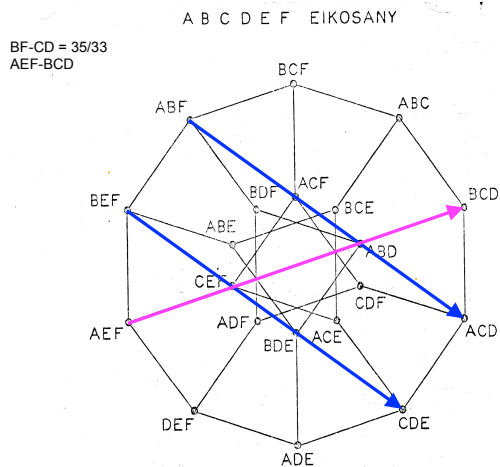
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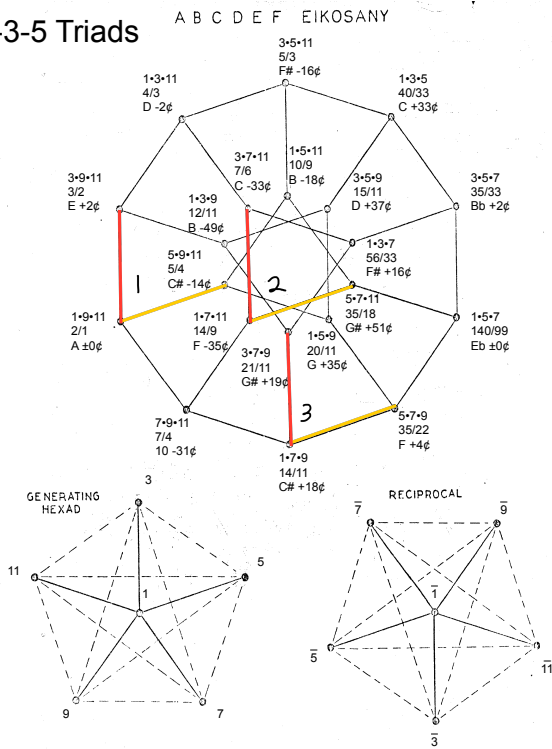
Issued by Erv Wilson, 1970



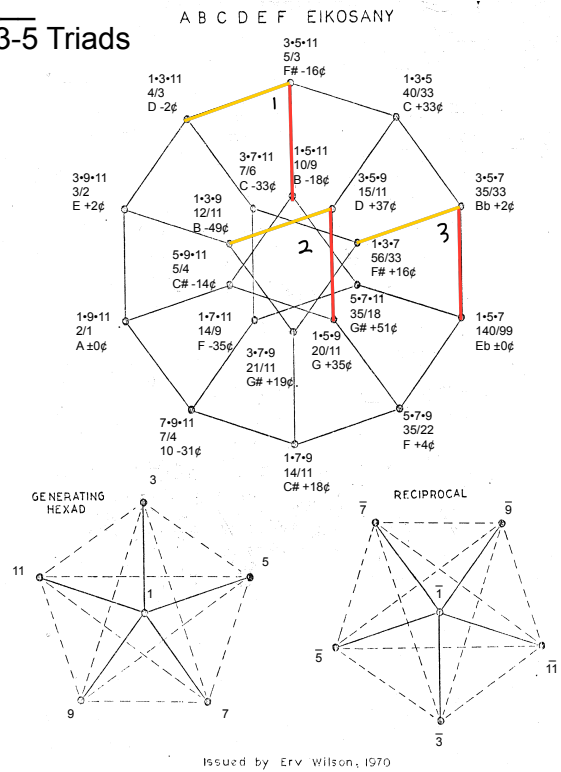
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Appendix B: Triads of the Eikosany

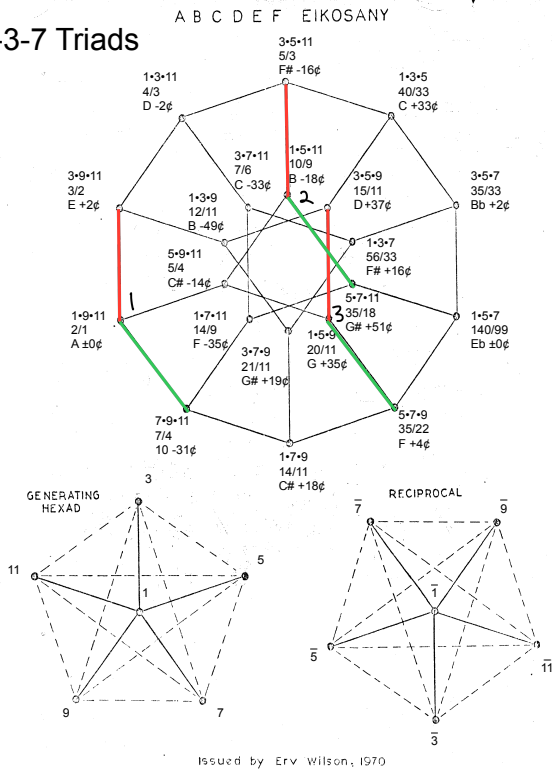
1-3-5 Triads



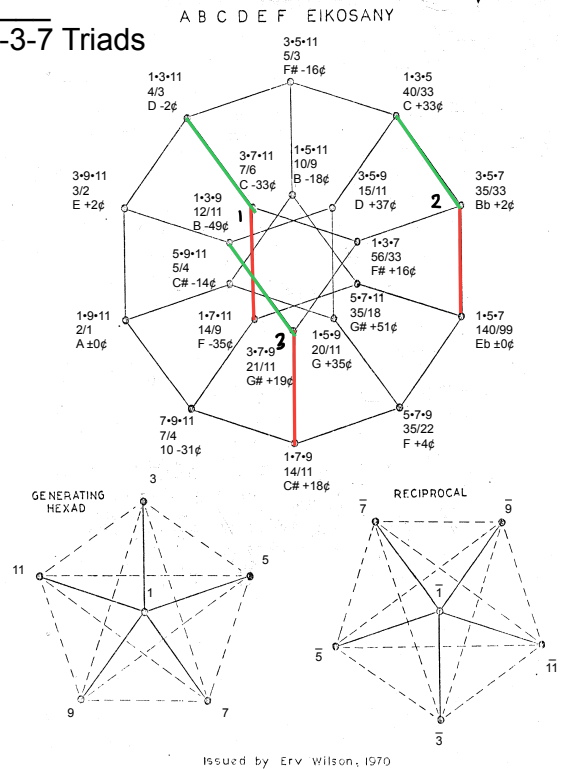
1-3-5 Triads



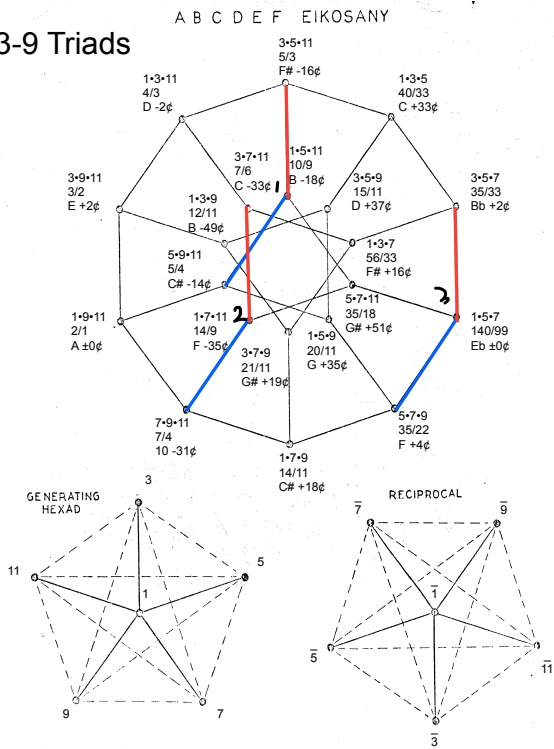
1-3-7 Triads



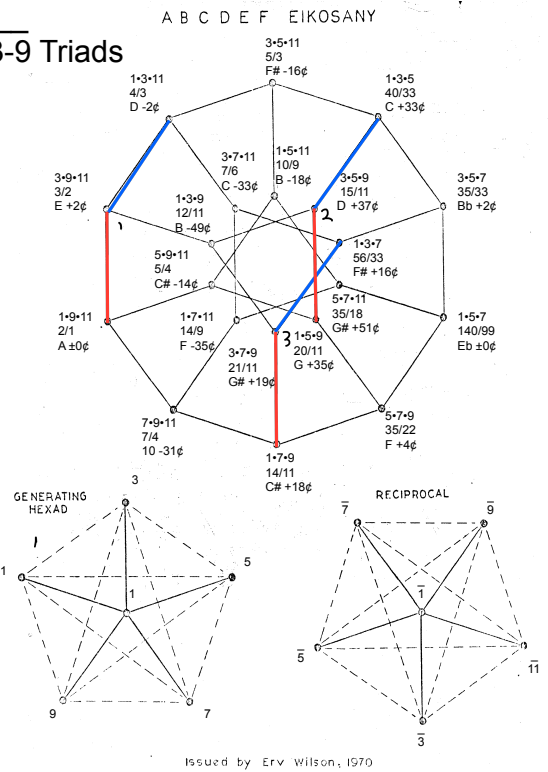
1-3-7 Triads



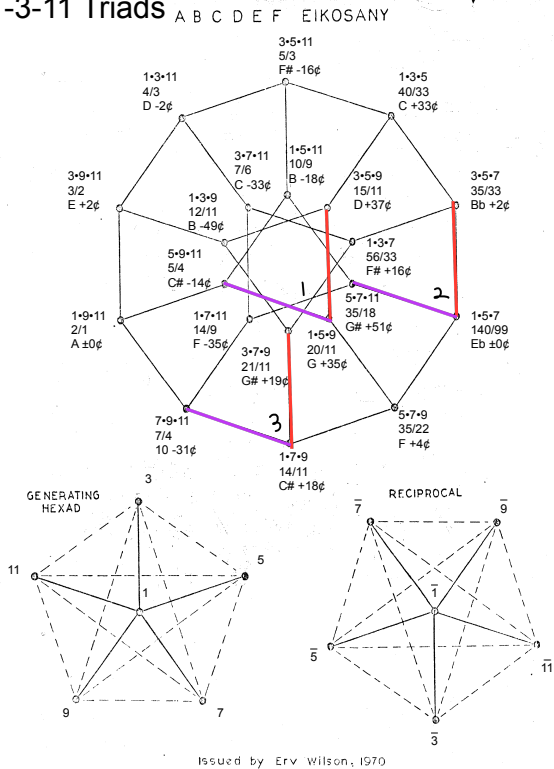
1-3-9 Triads



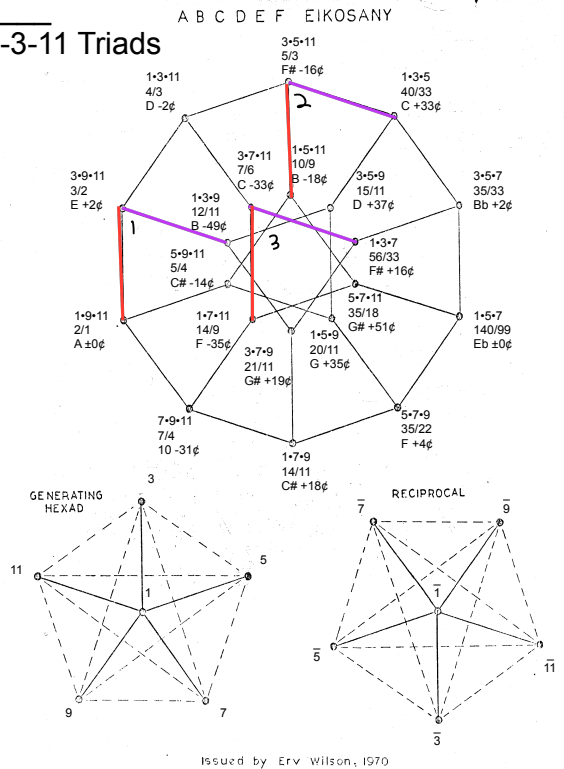
1-3-9 Triads



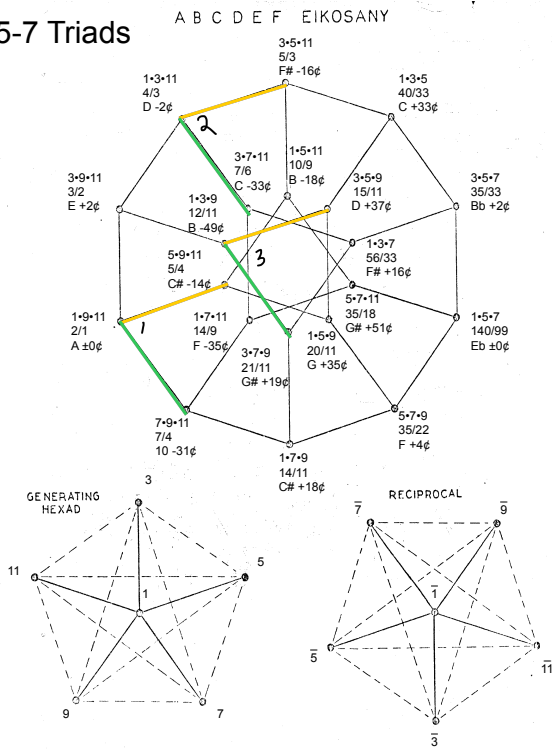
1-3-11 Triads



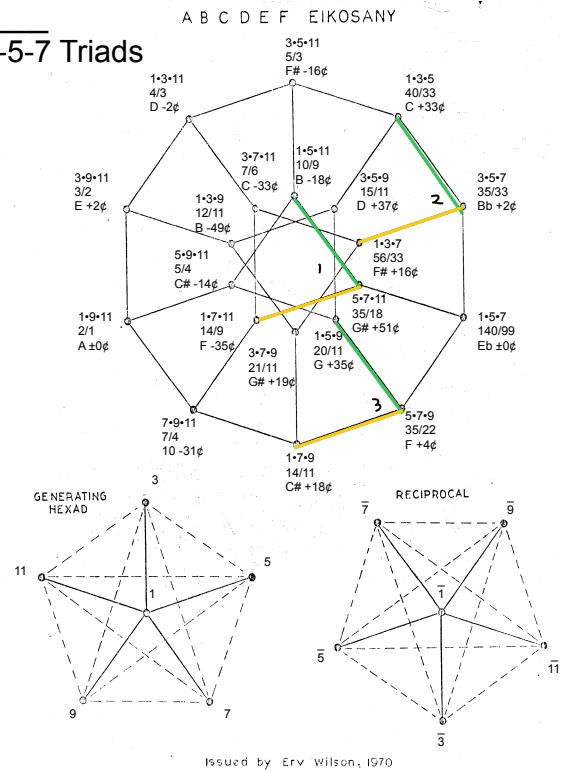
1-3-11 Triads



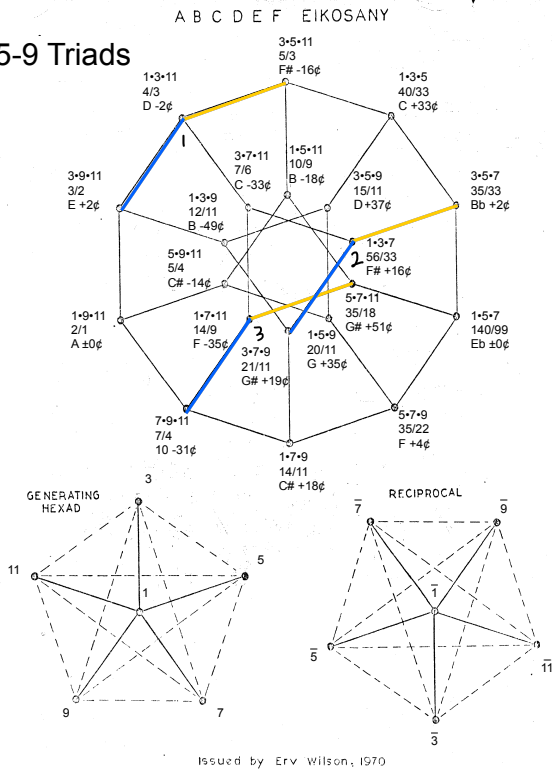
1-5-7 Triads



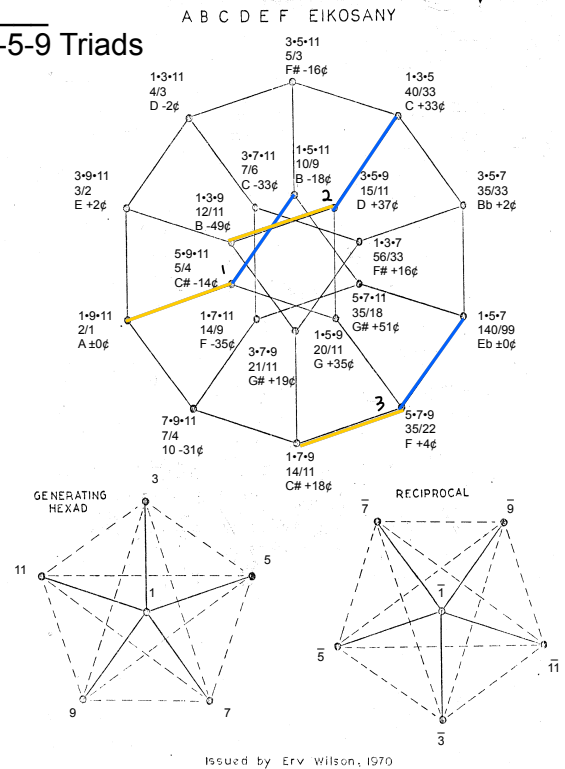
1-5-7 Triads

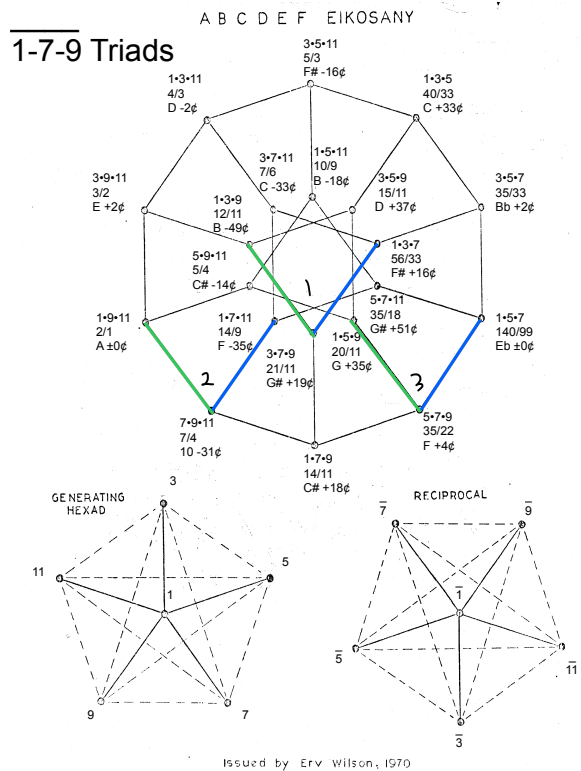
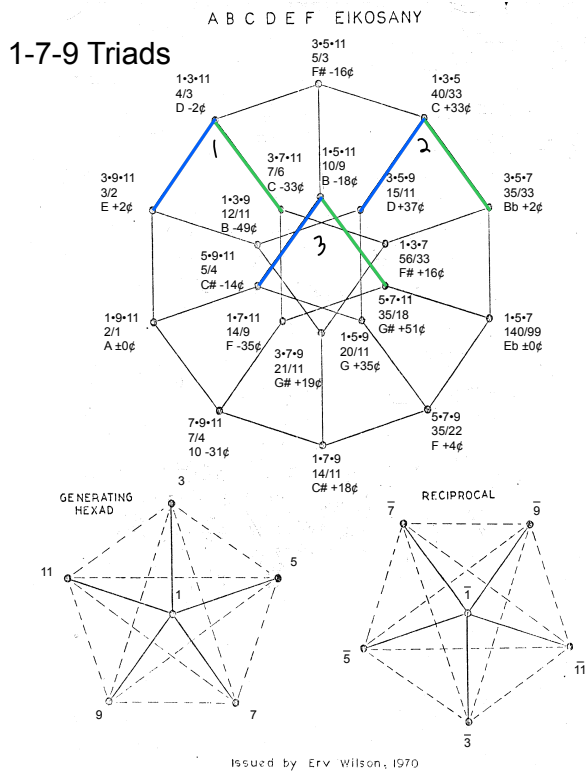
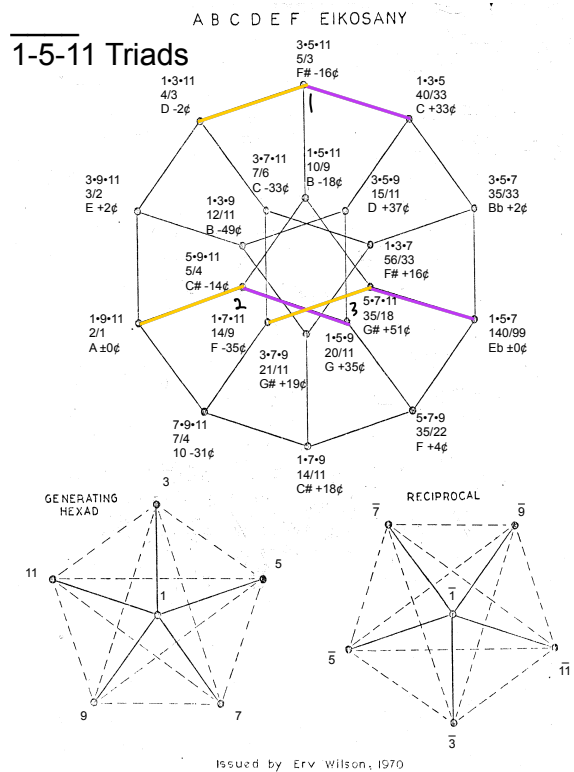
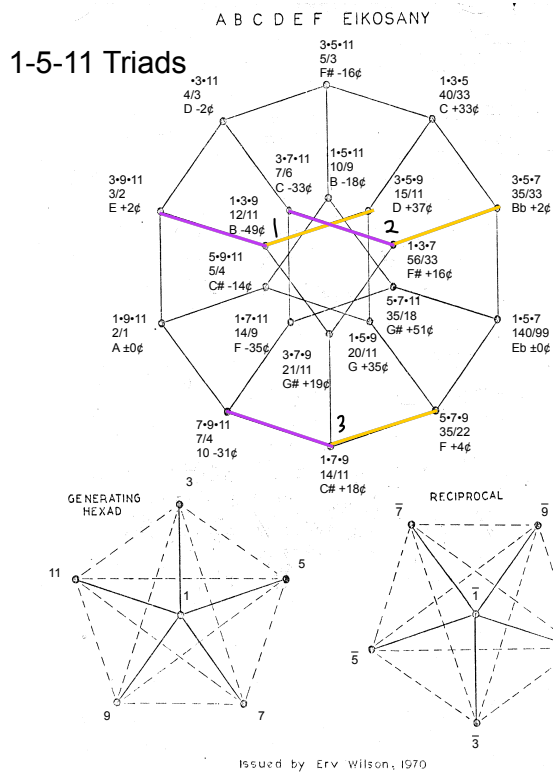


1-5-9 Triads

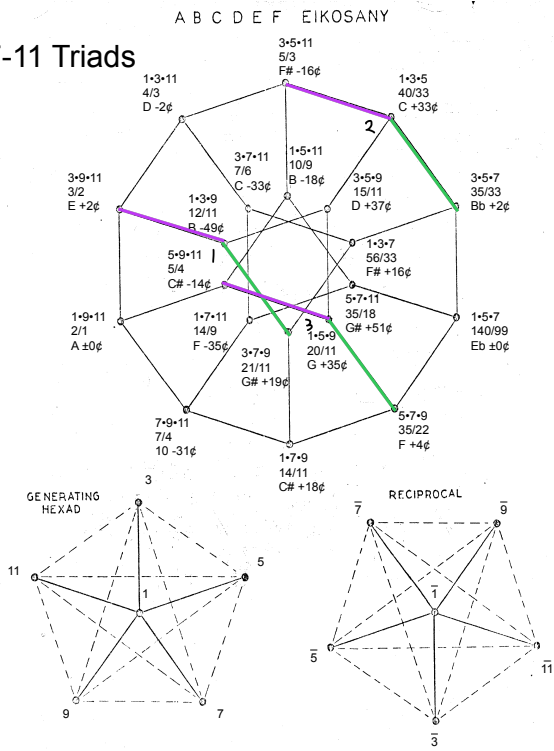


1-5-9 Triads

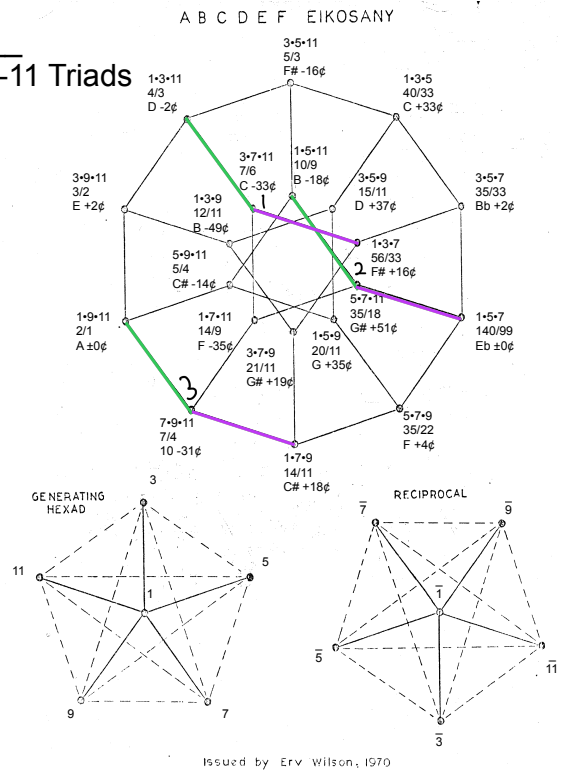




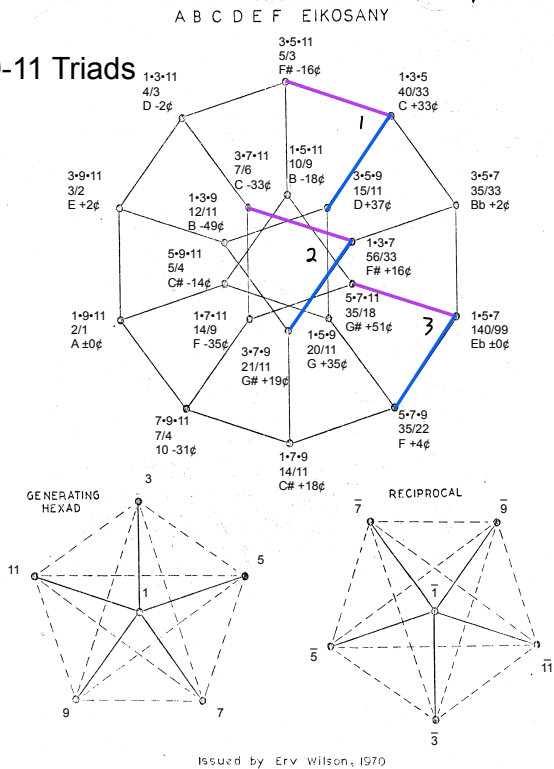
1-7-11 Triads



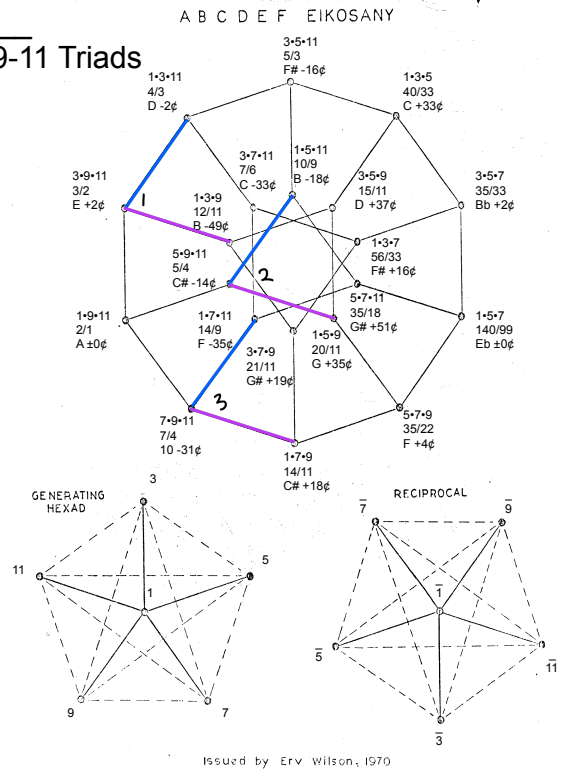
1-7-11 Triads



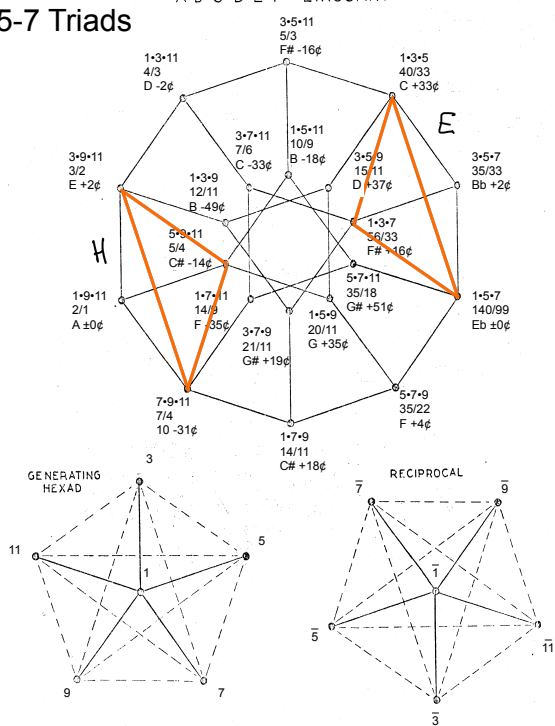
1-9-11 Triads



1-9-11 Triads

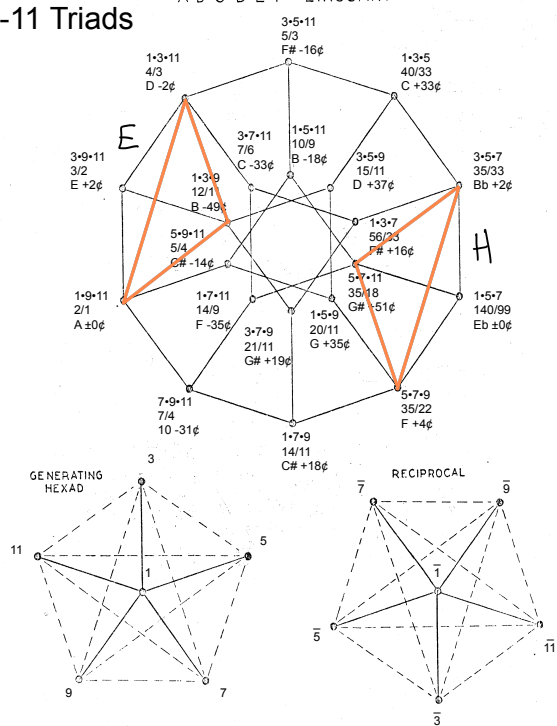


A B C D E F EIKOSANY



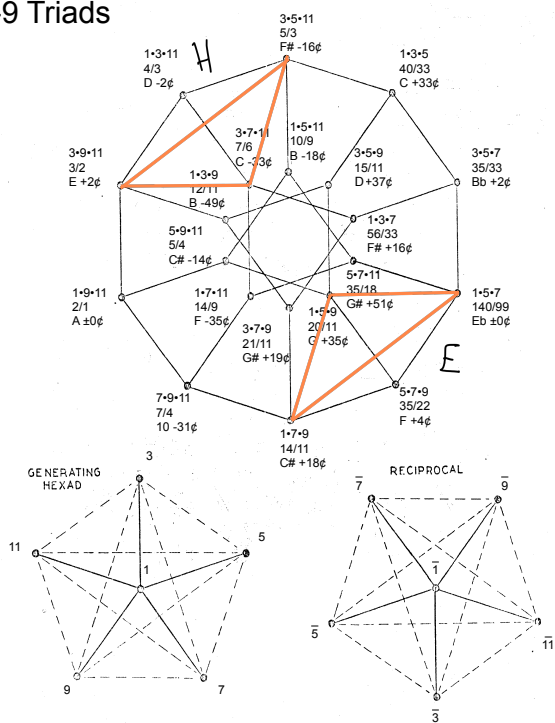
Issued by Erv Wilson, 1970

A B C D E F EIKOSANY



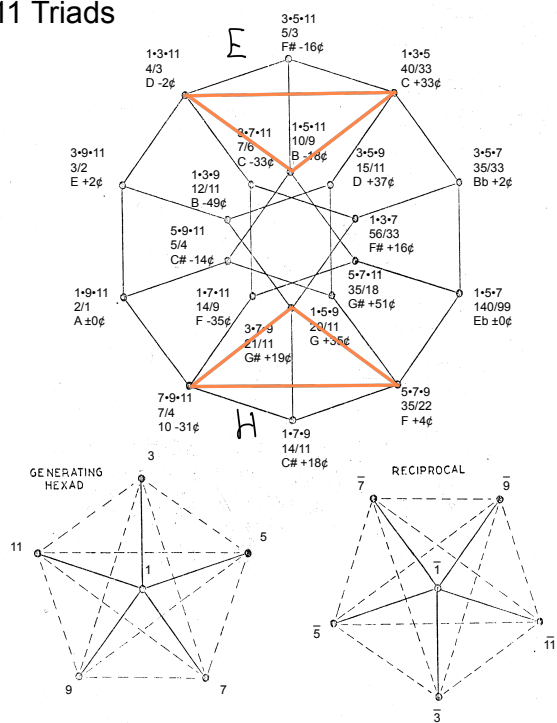
Issued by Erv Wilson, 1970

A B C D E F EIKOSANY



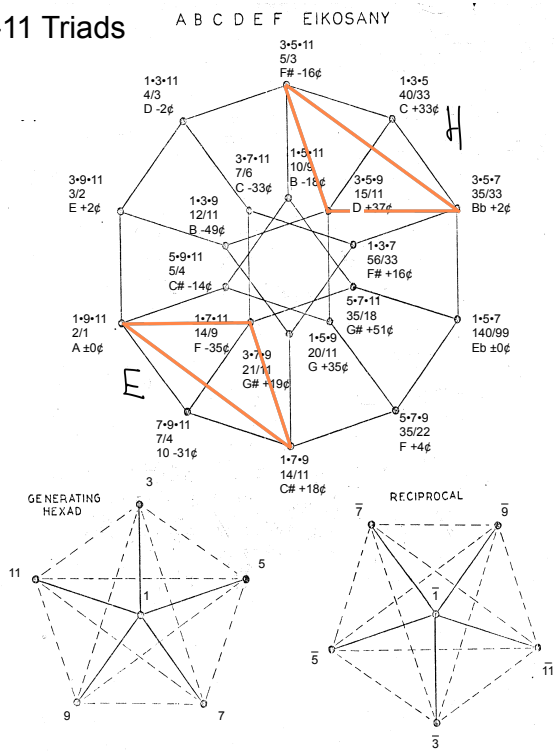
Issued by Erv Wilson, 1970

A B C D E F EIKOSANY

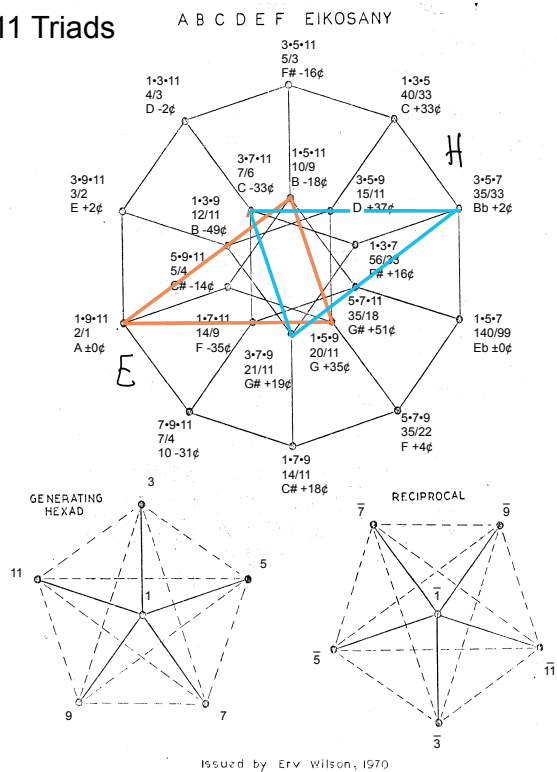


Issued by Erv Wilson, 1970

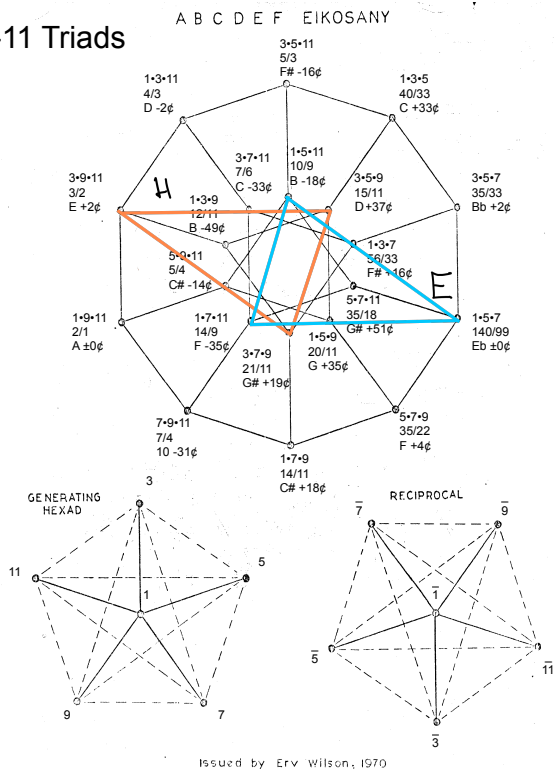
7-9-11 Triads



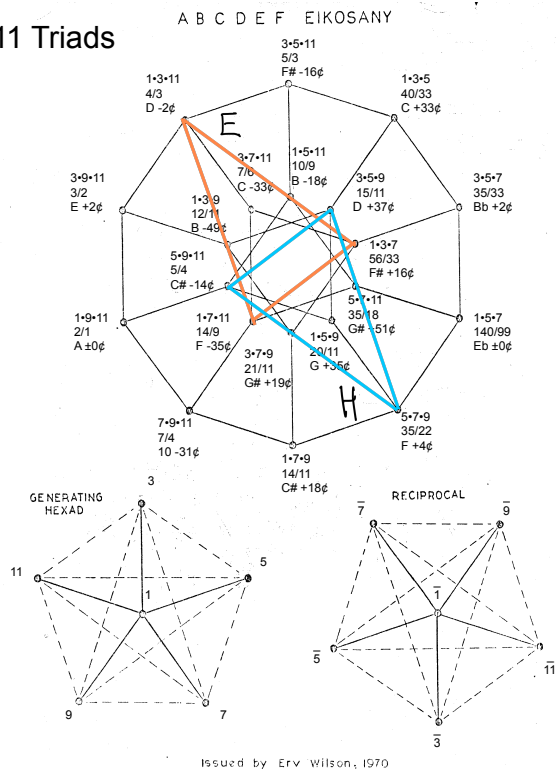
5-9-11 Triads



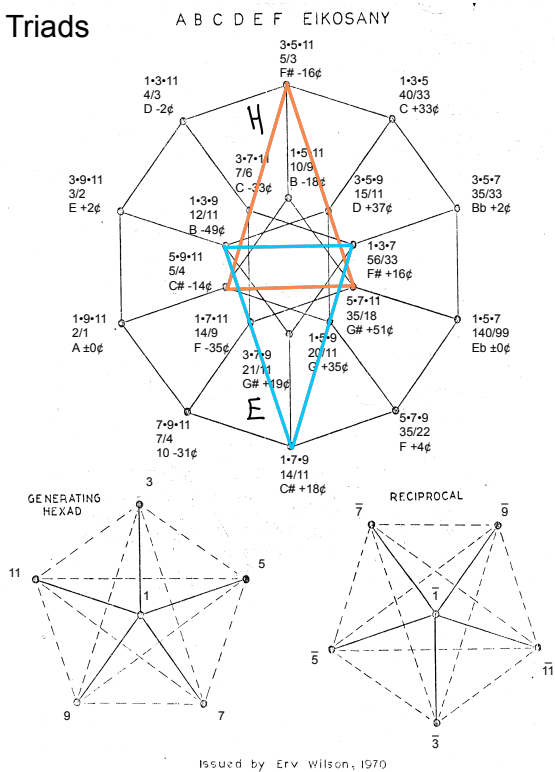
5-7-11 Triads



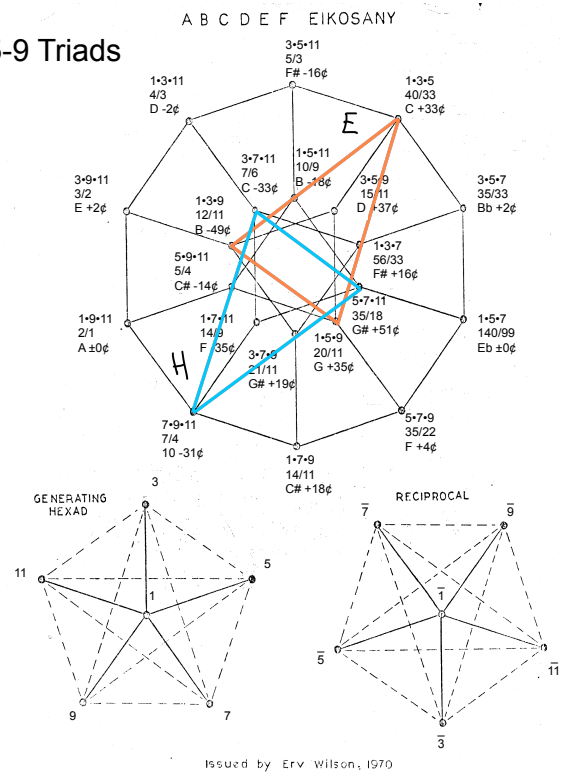
3-7-11 Triads



3-7-9 Triads

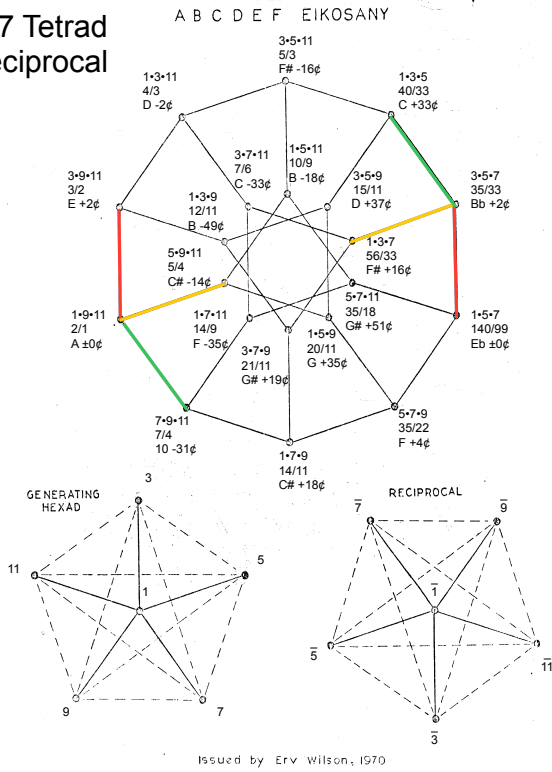


3-5-9 Triads

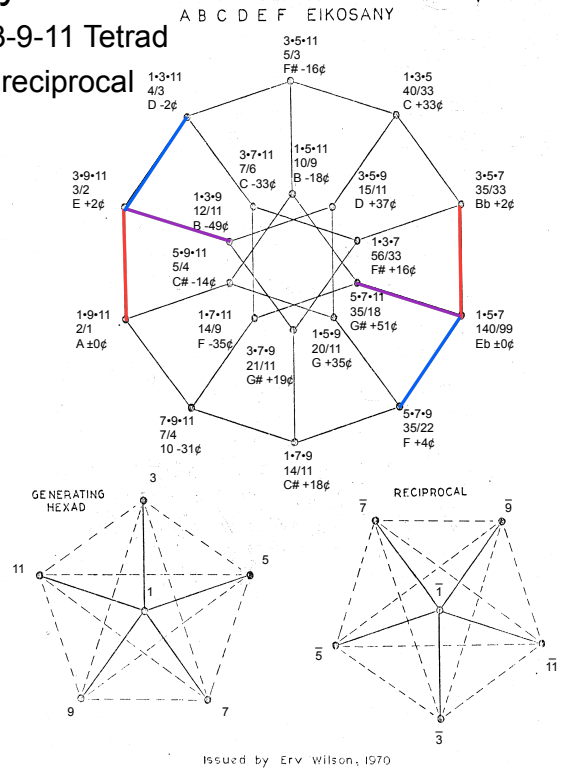


Appendix C: Tetrads of the Eikosany

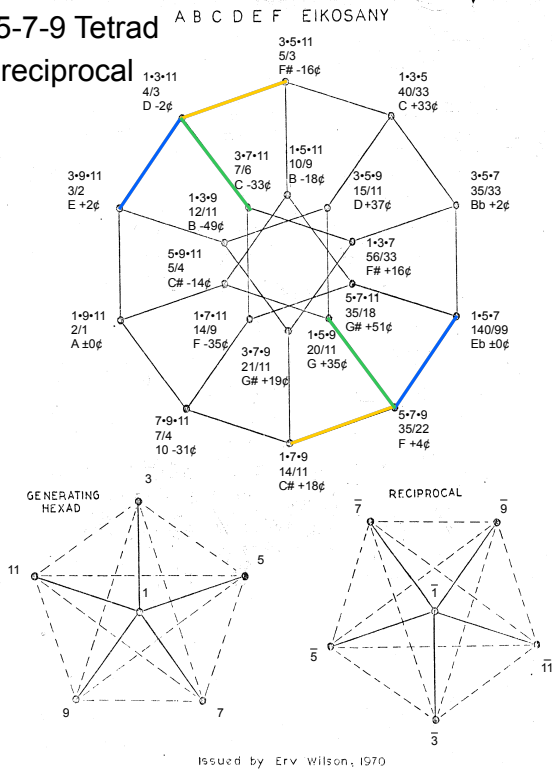
1-3-5-7 Tetrad and reciprocal



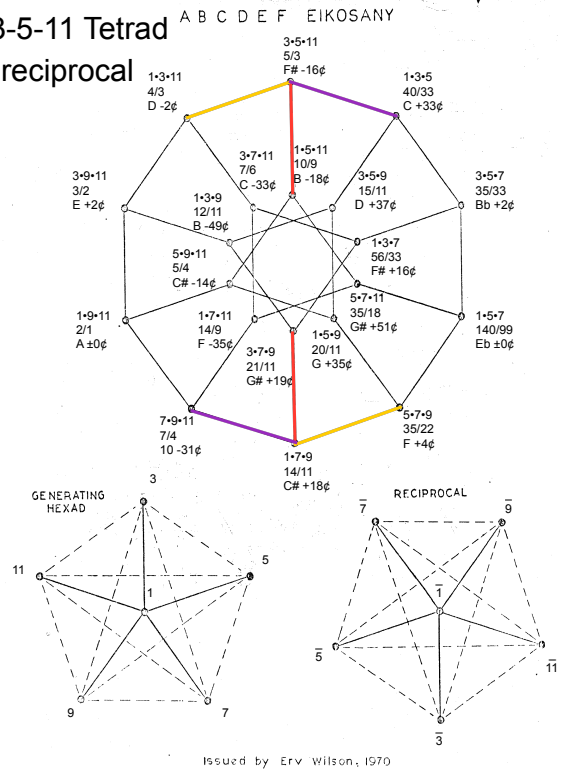
1-3-9-11 Tetrad and reciprocal



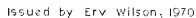
1-5-7-9 Tetrad and reciprocal



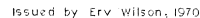
1-3-5-11 Tetrad and reciprocal



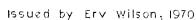
1-7-9-11 Tetrad
and reciprocal 1



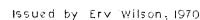
1-5-9-11 Tetrad



1-5-7-11 Tetrad

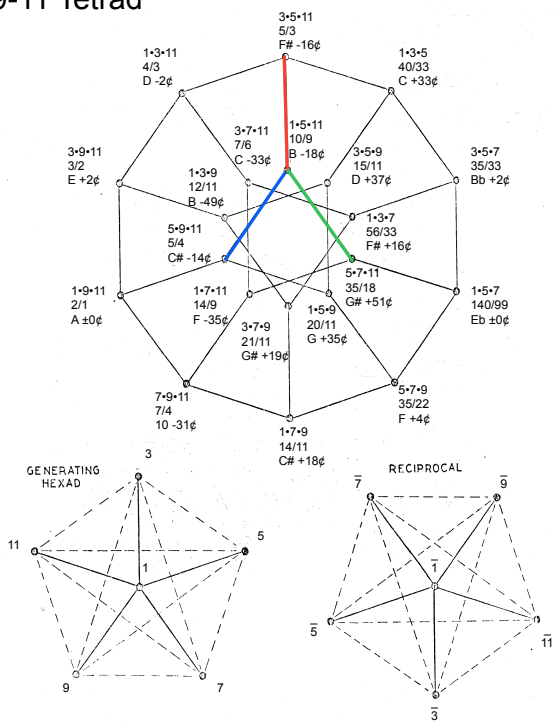


1-3-7-11 Tetrad



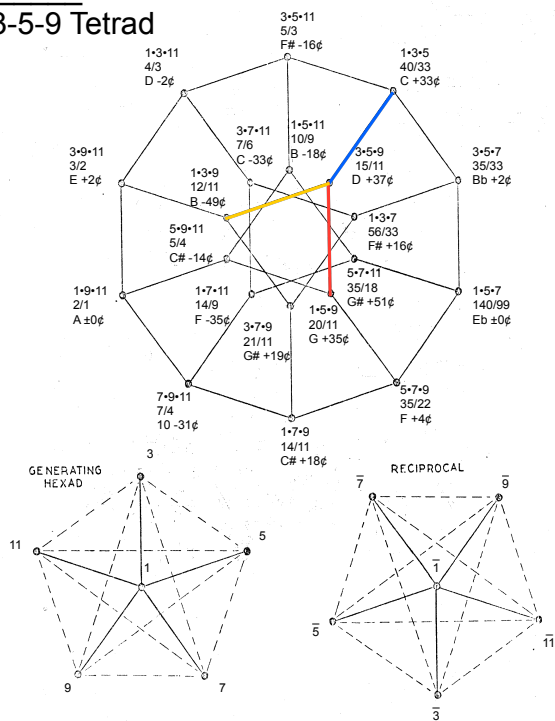
1-3-9-11 Tetrad

A B C D E F EIKOSANY



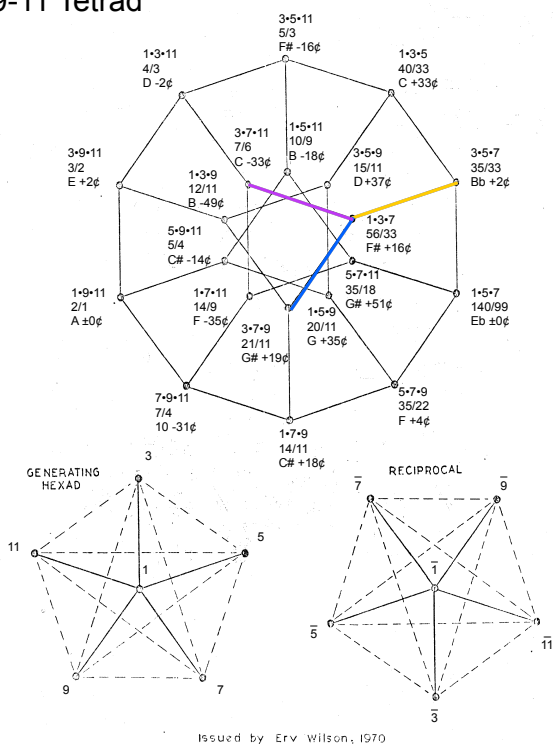
1-3-5-9 Tetrad

A B C D E F EIKOSANY



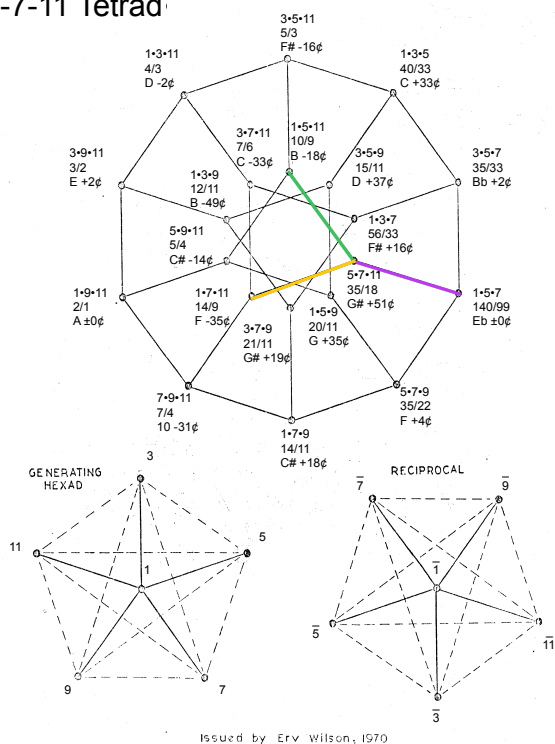
1-5-9-11 Tetrad

A B C D E F EIKOSANY

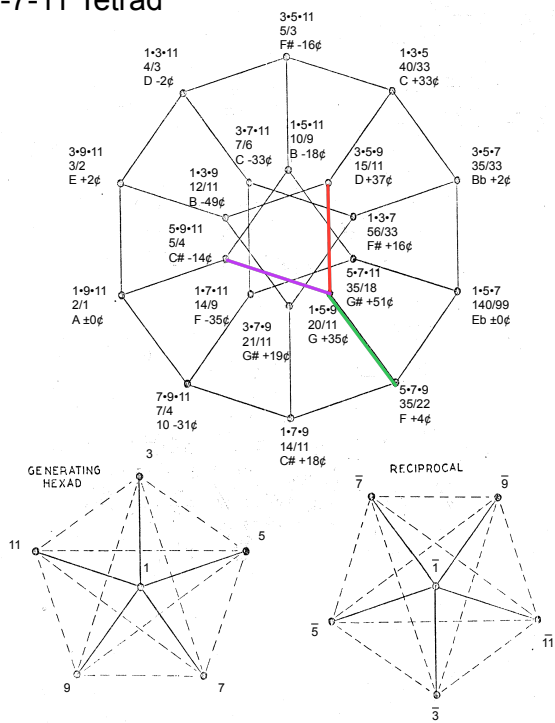


1-5-7-11 Tetrad

A B C D E F EIKOSANY

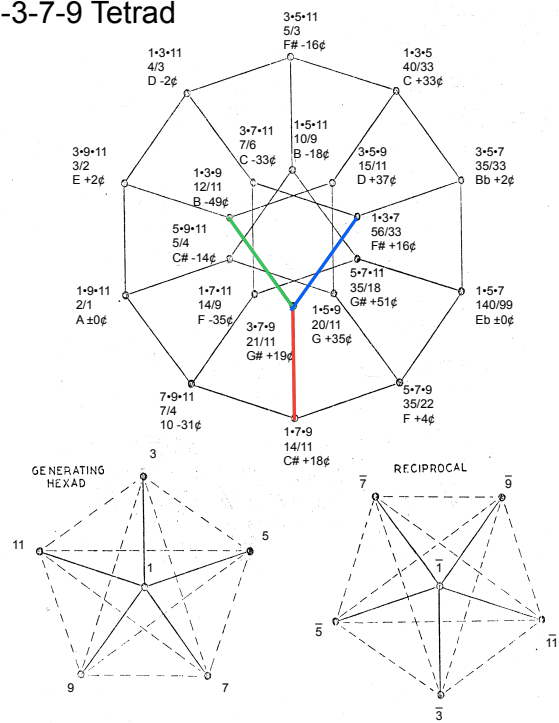


1-3-7-11 Tetrad A B C D E F EIKOSANY



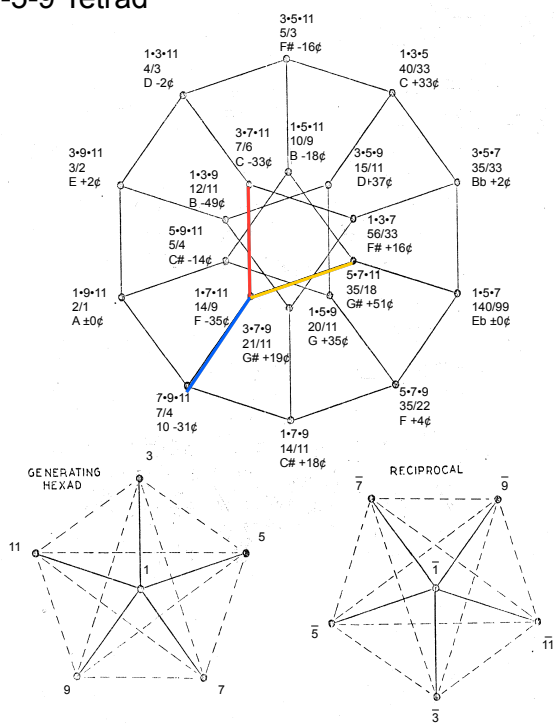
Issued by Erv Wilson, 1970

1-3-7-9 Tetrad A B C D E F EIKOSANY



Issued by Erv Wilson, 1970

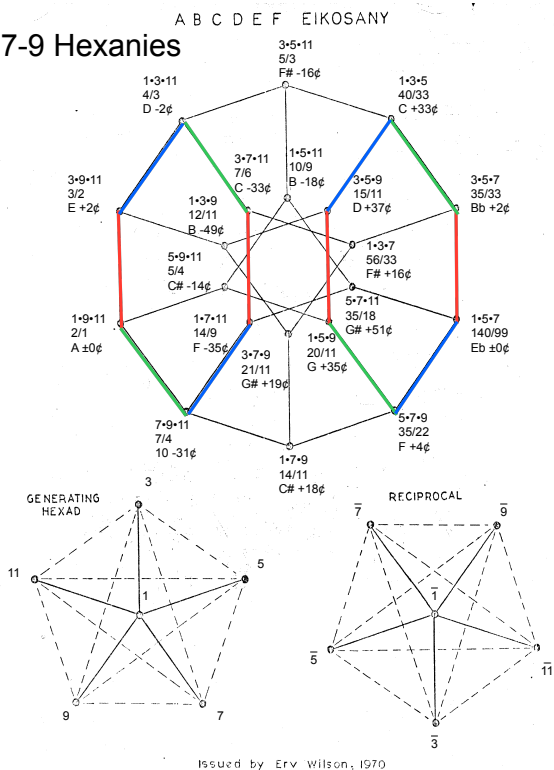
1-3-5-9 Tetrad A B C D E F EIKOSANY



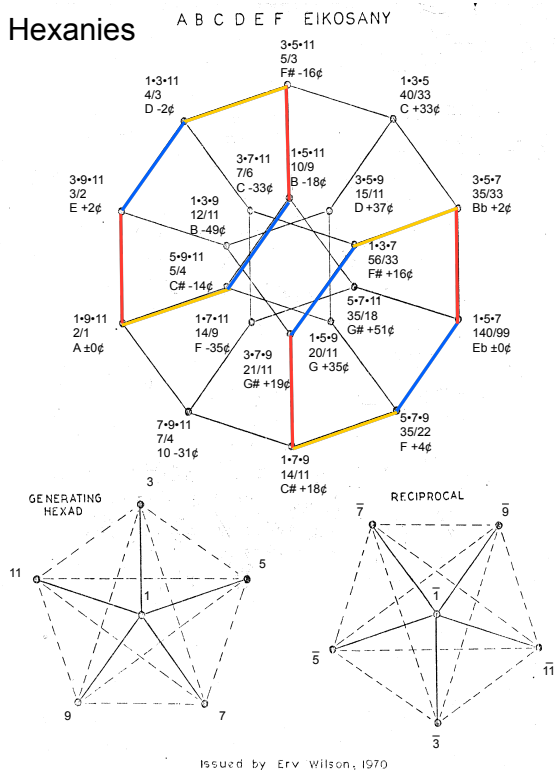
Issued by Erv Wilson, 1970

Appendix D: Hexanies of the Eikosany

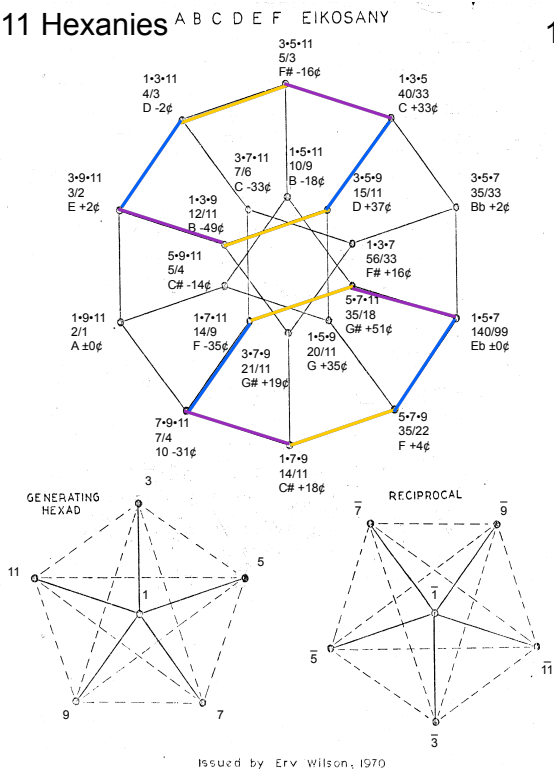
1-3-7-9 Hexanies



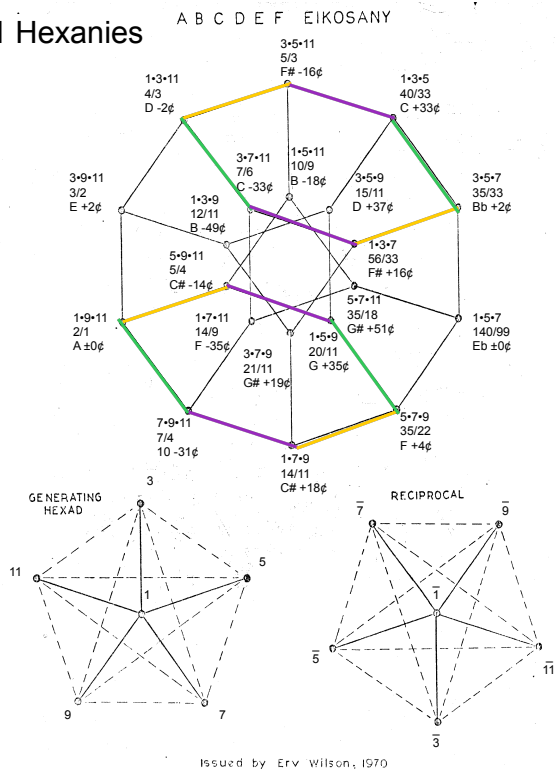
1-3-5-9 Hexanies



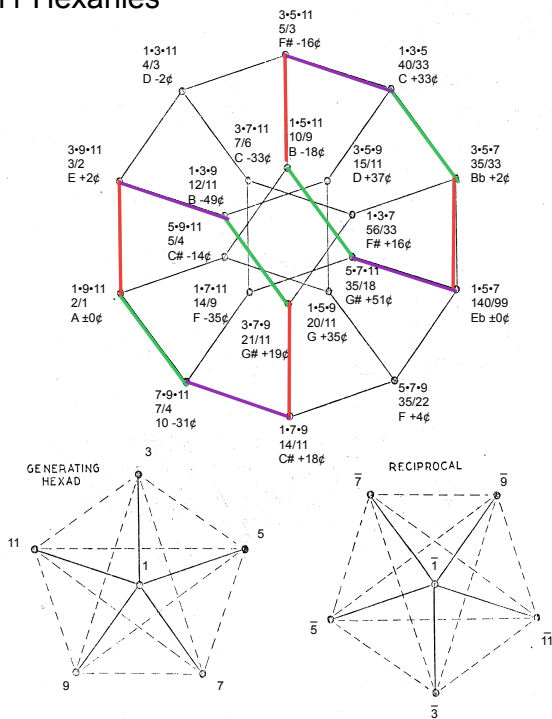
1-5-9-11 Hexanies



1-5-7-11 Hexanies

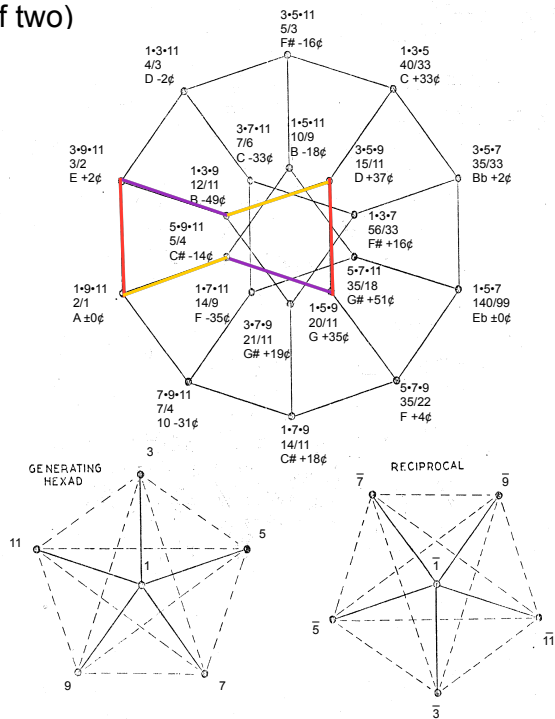


1-3-9-11 Hexanies A B C D E F EIKOSANY



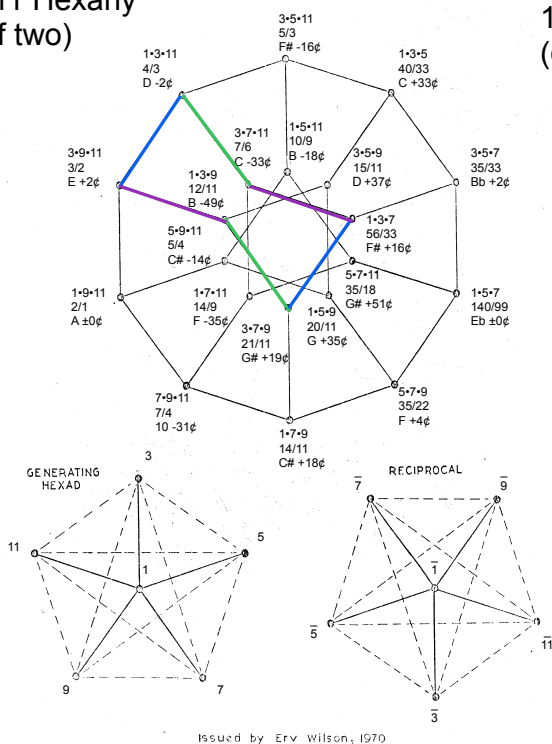
1-3-5-11 Hexany (one of two)

A B C D E F EIKOSANY



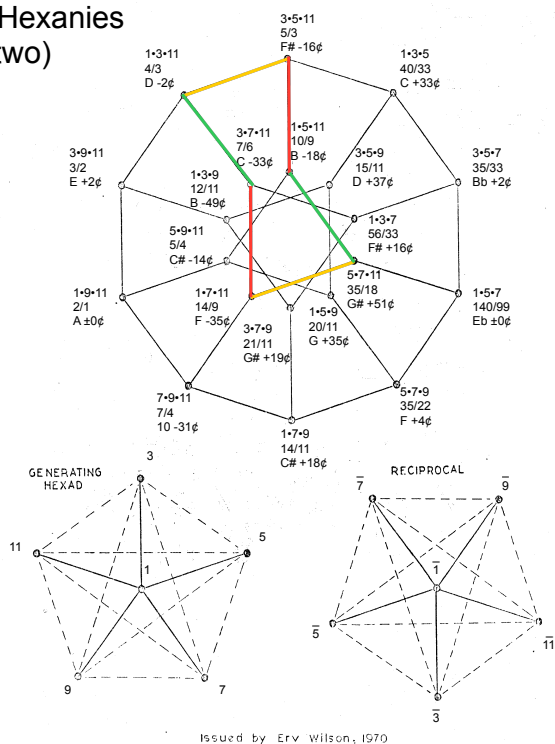
1-7-9-11 Hexany (one of two)

A B C D E F EIKOSANY



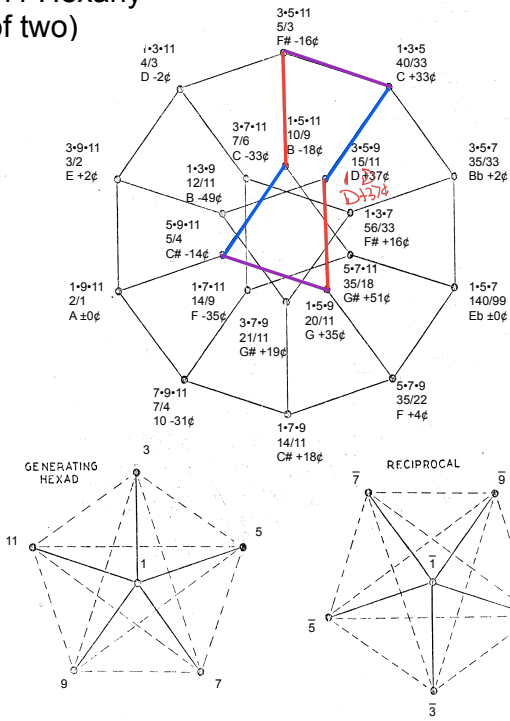
1-3-5-7 Hexanies (one of two)

A B C D E F EIKOSANY



1-3-9-11 Hexany
(one of two)

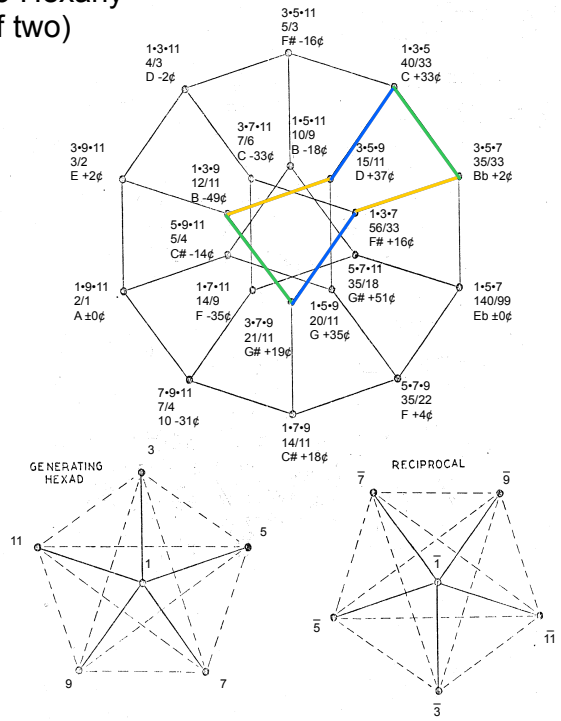
A B C D E F EIKOSANY



Issued by Erv Wilson, 1970

1-5-7-9 Hexany
(two of two)

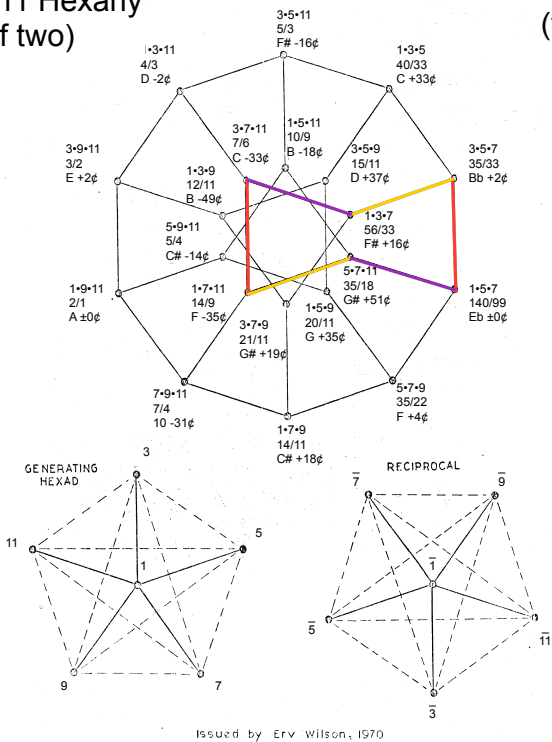
A B C D E F EIKOSANY



Issued by Erv Wilson, 1970

1-3-5-11 Hexany
(two of two)

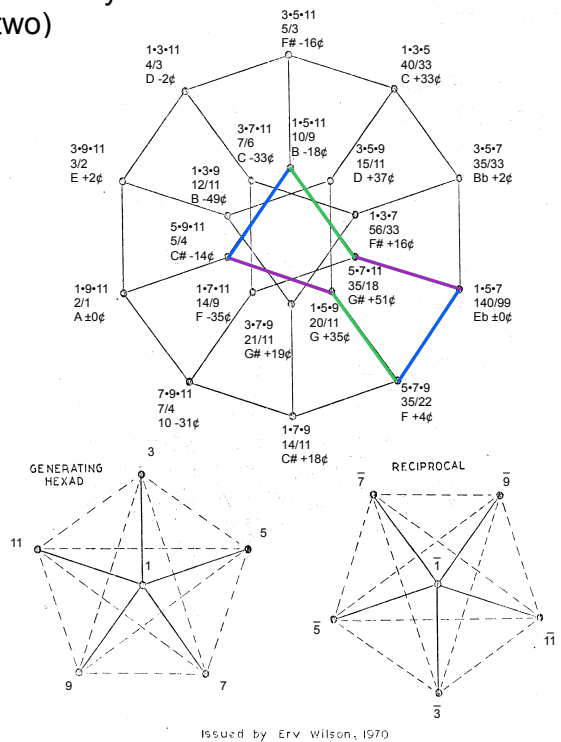
A B C D E F EIKOSANY



Issued by Erv Wilson, 1970

1-7-9-11 Hexany
(two of two)

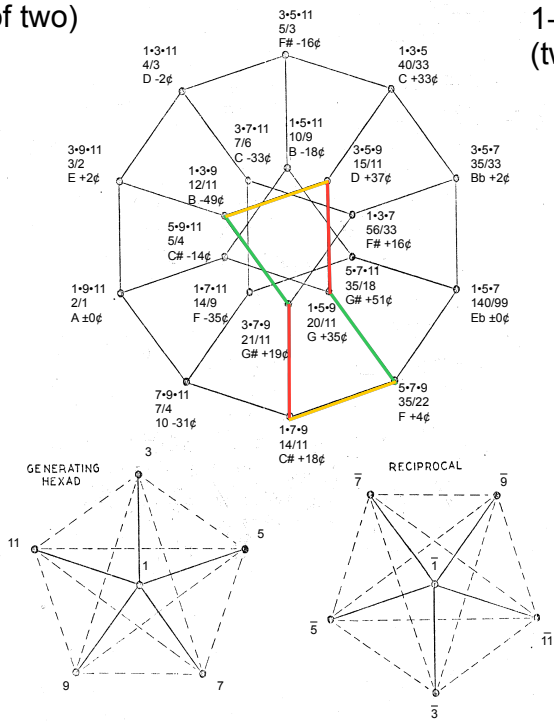
A B C D E F EIKOSANY



Issued by Erv Wilson, 1970

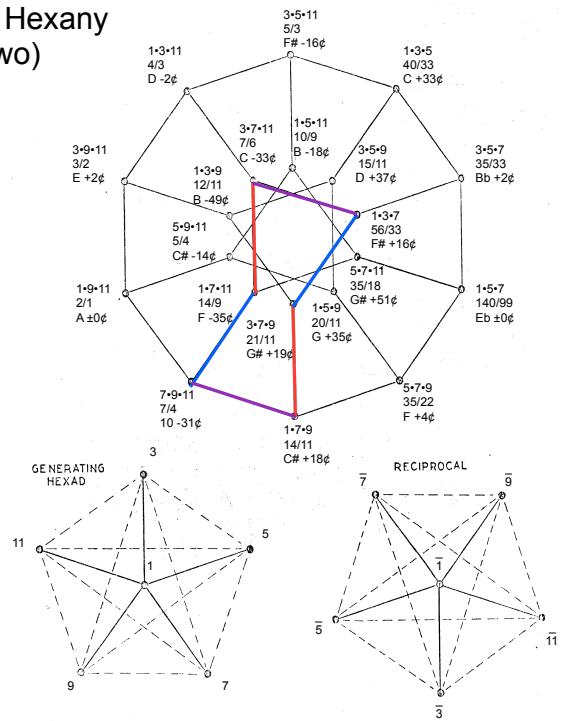
1-3-5-11 Hexany (two of two)

A B C D E F EIKOSANY



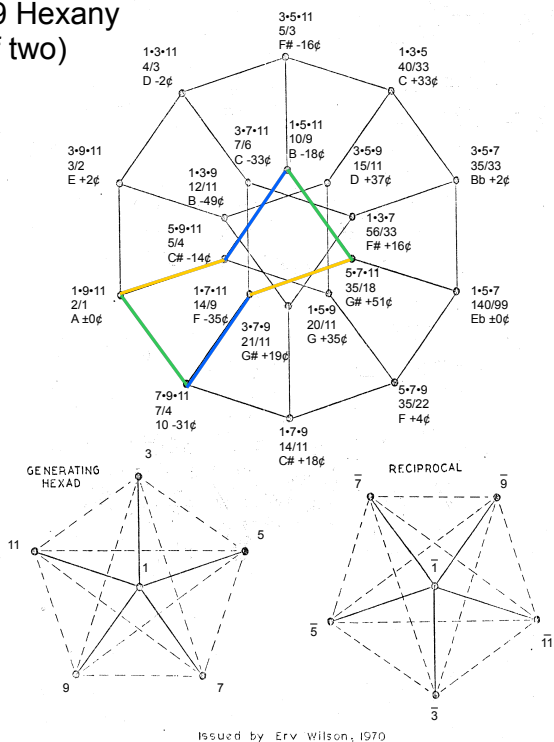
1-3-9-11 Hexany (two of two)

A B C D E F EIKOSANY



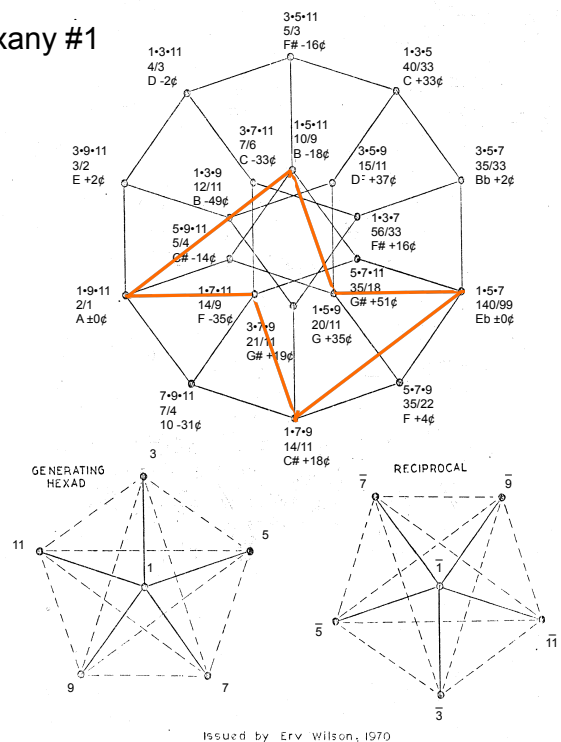
1-5-7-9 Hexany (two of two)

A B C D E F EIKOSANY

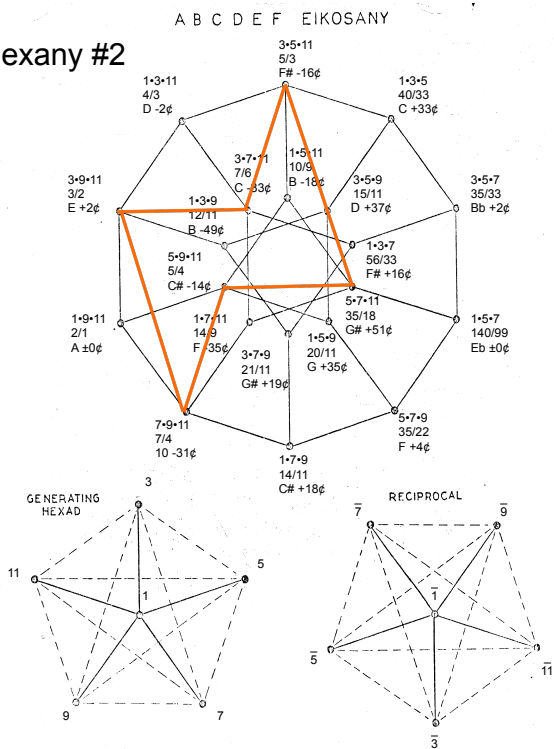


C Hexany #1

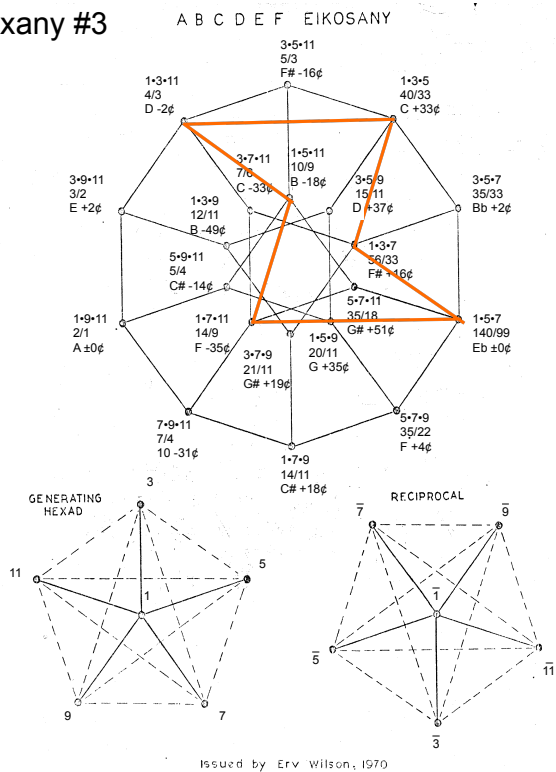
A B C D E F EIKOSANY



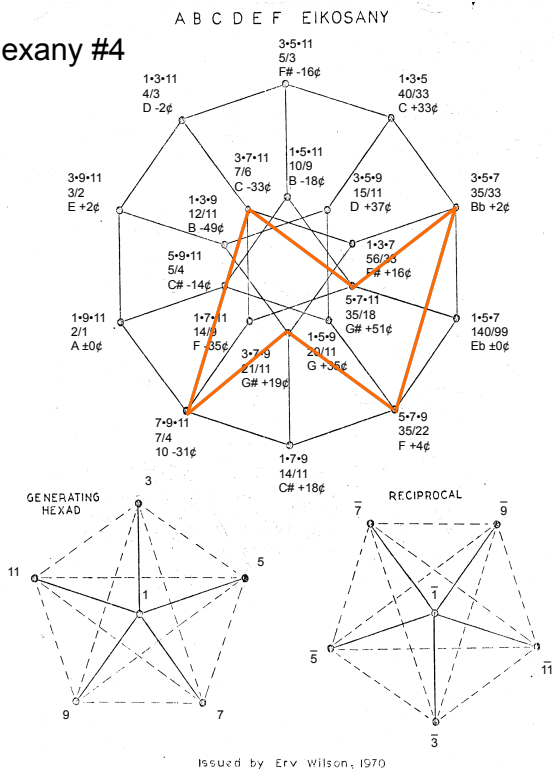
C Hexany #2



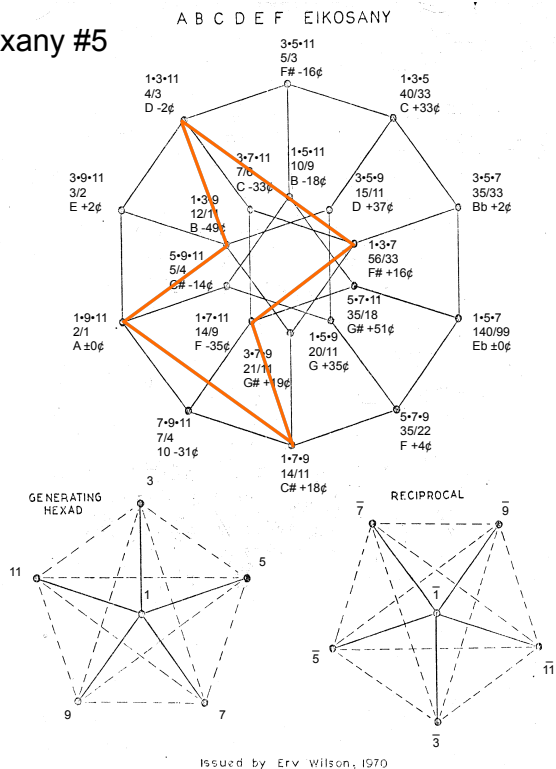
C Hexany #3



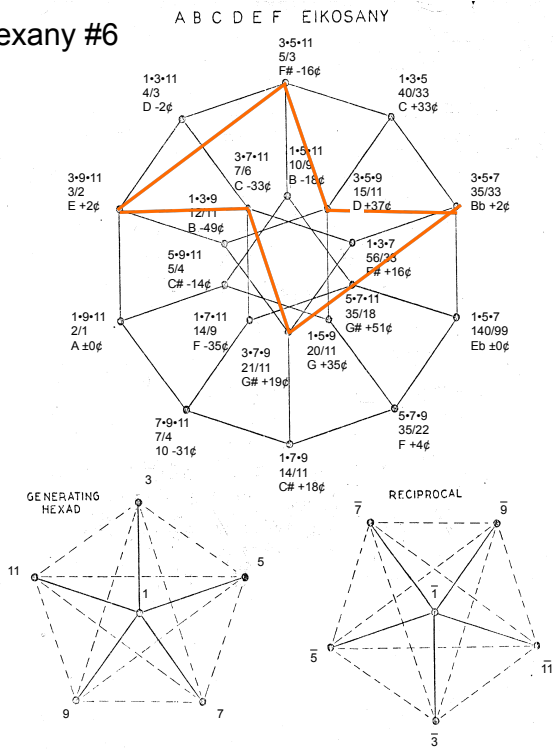
C Hexany #4



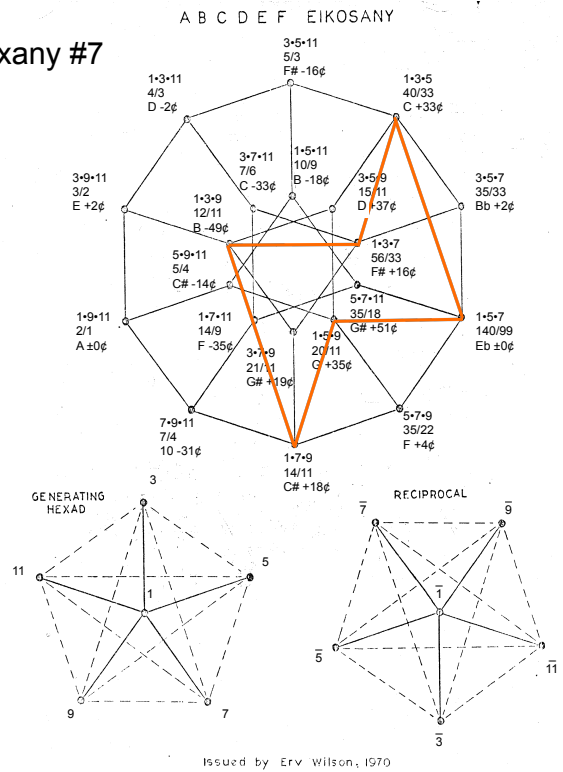
C Hexany #5



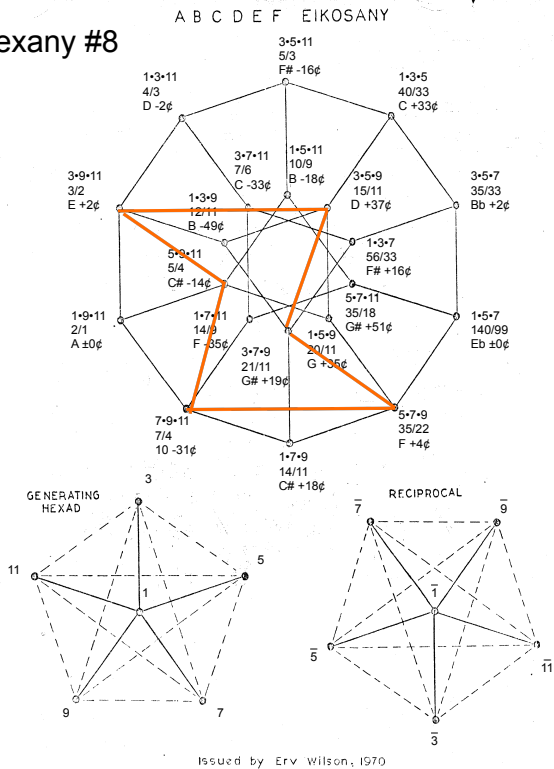
C Hexany #6



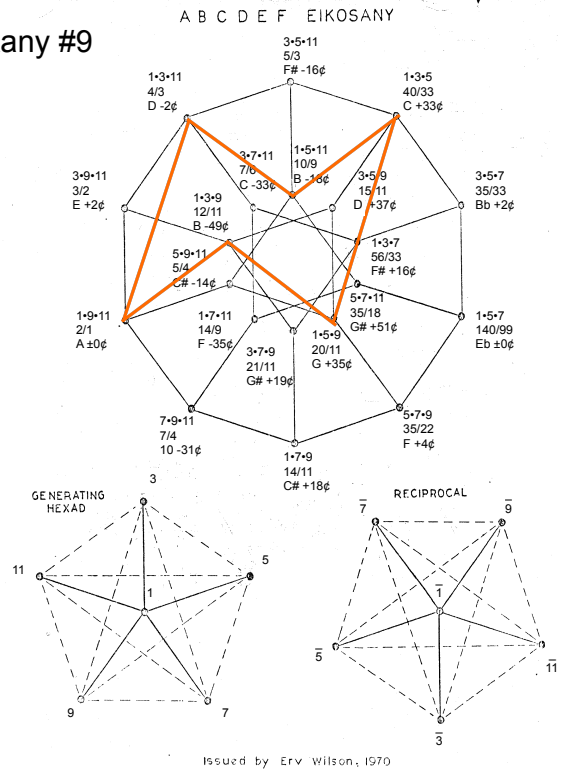
C Hexany #7



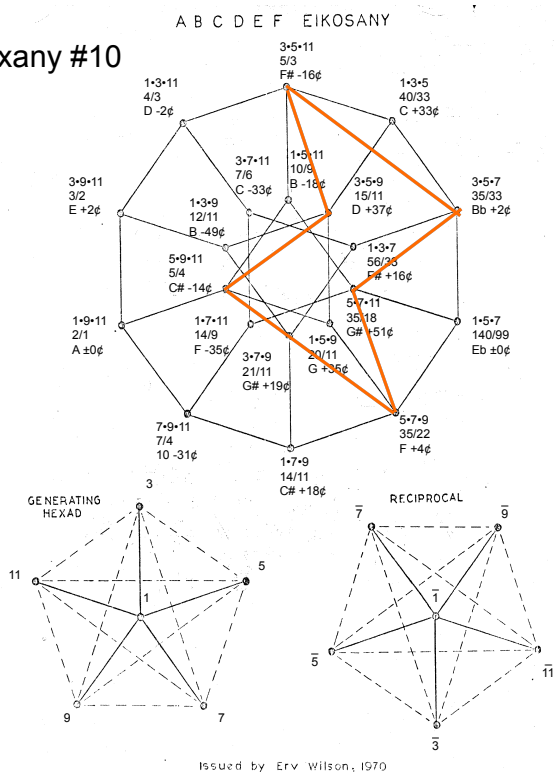
C Hexany #8



C Hexany #9



C Hexany #10



Appendix E: Dekanies of the 1•3•5•7•9•11 Eikosany

Rotation 1

Keyboard

±0 +18¢ ±0 +4¢ +35¢

–18¢ –14¢ –35¢ –31¢ –49¢

2/1 10/9 5/4 14/11 140/99 14/9 35/22 7/4 20/11 35/18

Rotation 2

Pno.

±0 +2¢

–18¢ –33¢ –14¢ –2¢ –35¢ –16¢ –31¢ –49¢

2/1 10/9 7/6 5/4 4/3 3/2 14/9 5/3 7/4 35/18

Rotation 3

Pno.

+2¢ +33¢ ±0 +16¢

–18¢ –33¢ –2¢ –35¢ –16¢ –49¢

35/33 10/9 7/6 40/33 4/3 140/99 14/9 5/3 56/33 35/18

Rotation 4

Pno.

±0 +18¢ ±0 +4¢ +16¢ +19¢

–33¢ –35¢ –31¢ –49¢

35/33 7/6 14/11 140/99 14/9 35/22 56/33 7/4 21/11 35/18

Rotation 5

Pno.

±0 +18¢ +2¢ +16¢ +19¢

–49¢ –33¢ –2¢ –35¢ –31¢

2/1 12/11 7/6 14/11 4/3 3/2 14/9 56/33 7/4 21/11

Rotation 6

Pno.

35/33 12/11 7/6 40/33 4/3 15/11 3/2 5/3 56/33 21/11

Rotation 7

Pno.

35/33 12/11 40/33 14/11 15/11 140/99 35/22 56/33 20/11 21/11

Rotation 8

Pno.

2/1 12/11 5/4 14/11 15/11 3/2 35/22 7/4 20/11 21/11

Rotation 9

Pno.

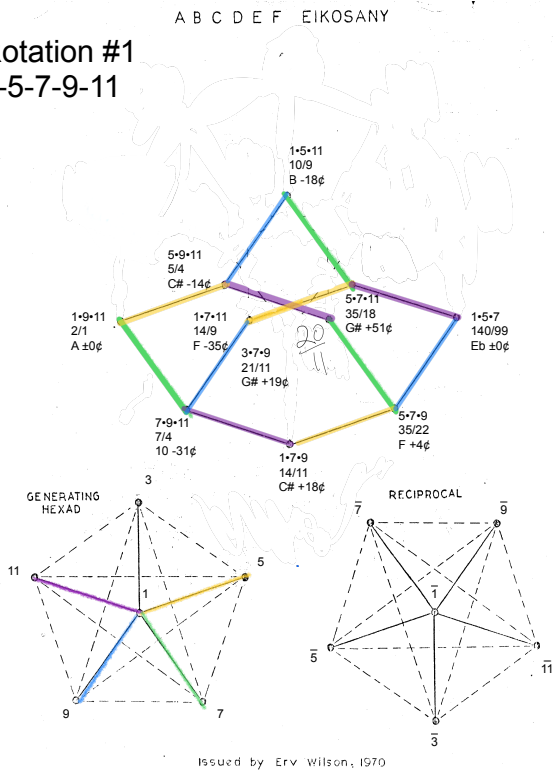
2/1 12/11 10/9 40/33 5/4 4/3 15/11 3/2 5/3 20/11

Rotation 10

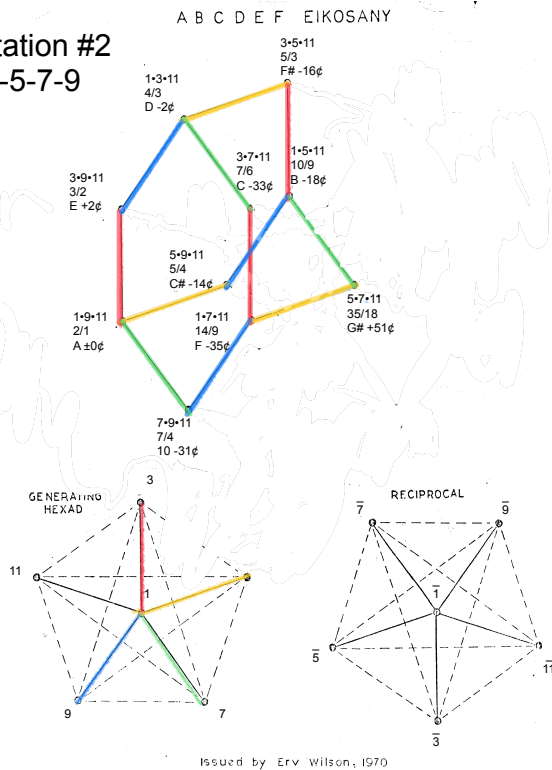
Pno.

35/33 10/9 40/33 5/4 15/11 140/99 35/22 5/3 20/11 35/18

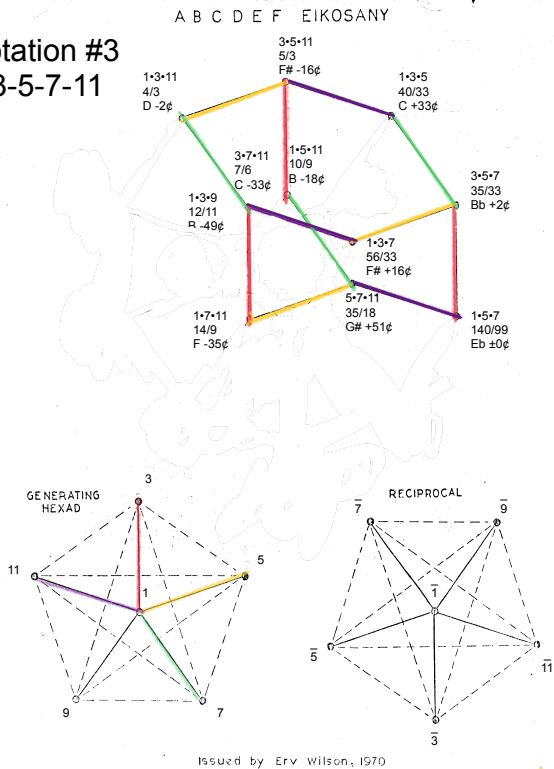
Rotation #1
1-5-7-9-11



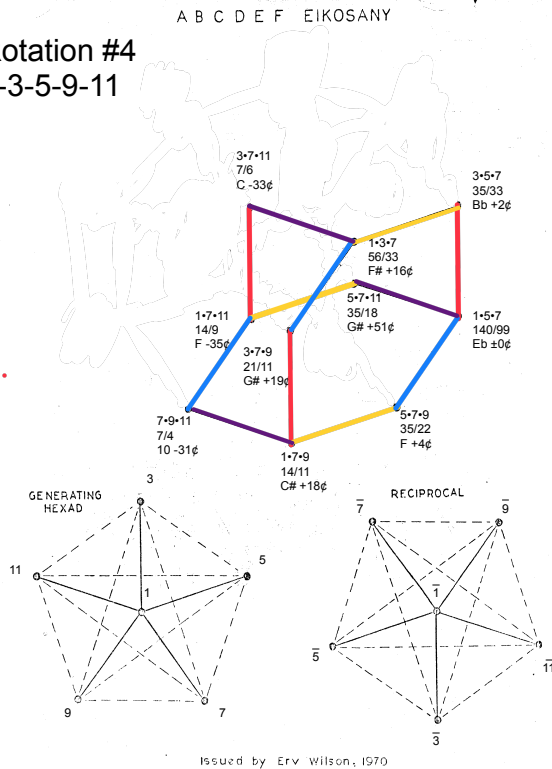
Rotation #2
1-3-5-7-9



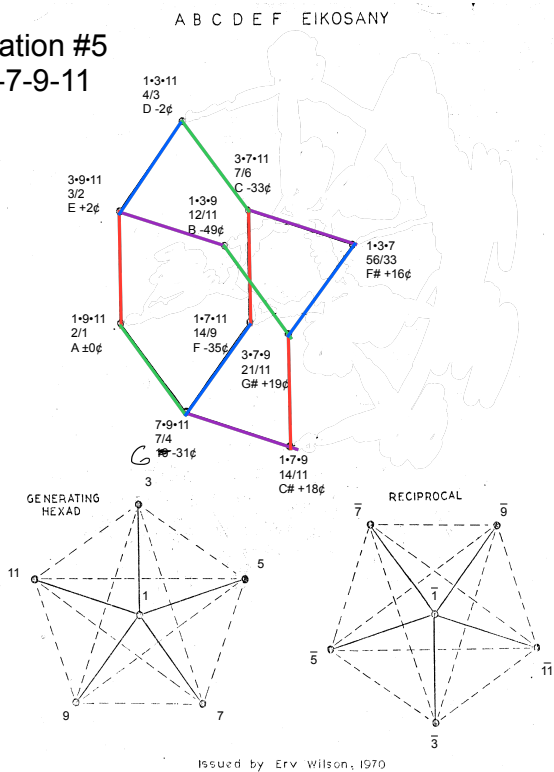
Rotation #3
1-3-5-7-11



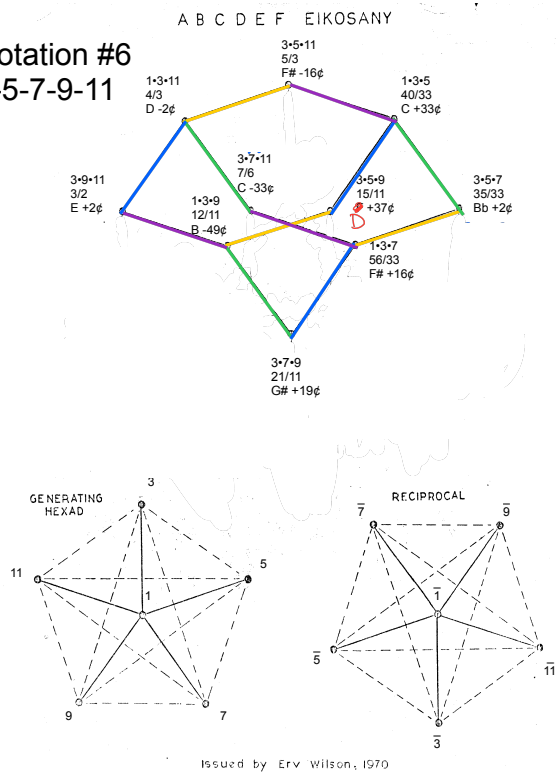
Rotation #4
1-3-5-9-11



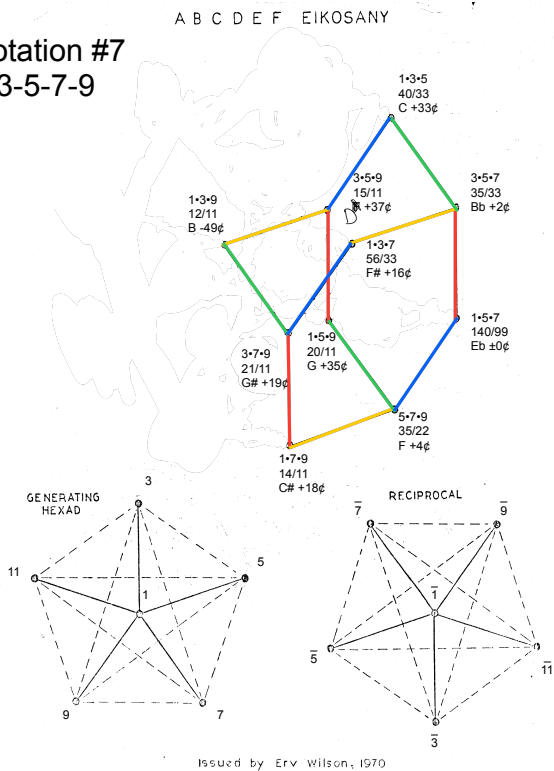
Rotation #5 1-3-7-9-11



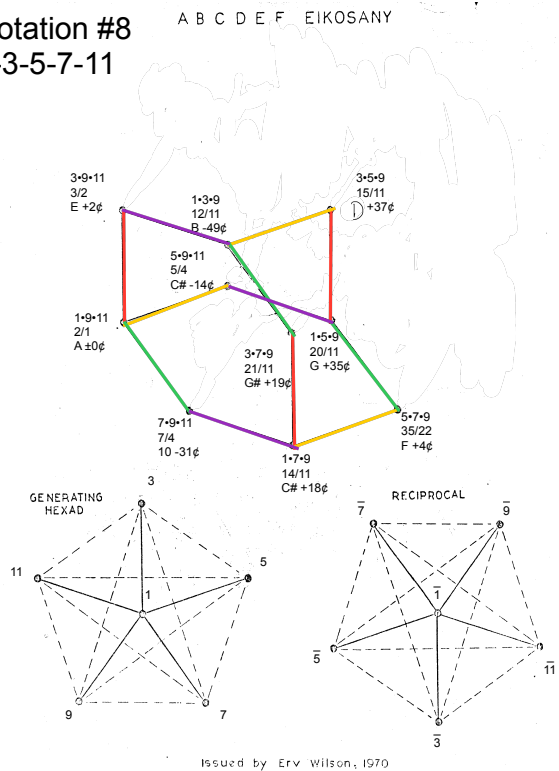
Rotation #6 1-5-7-9-11



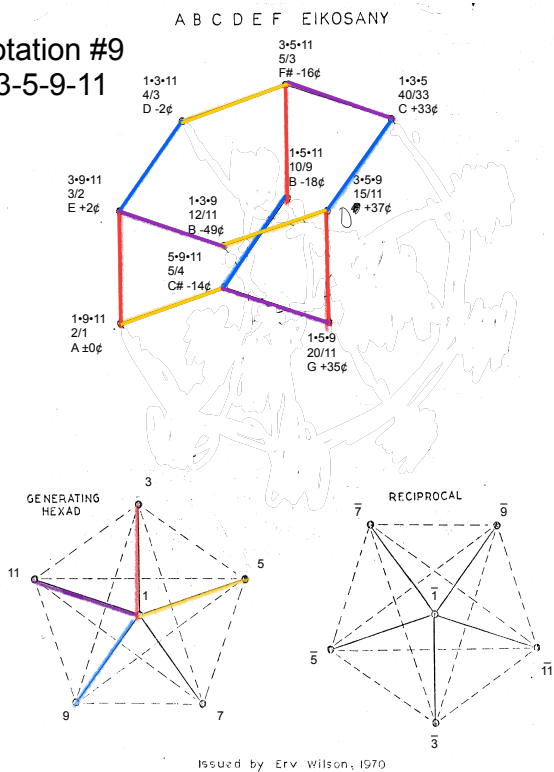
Rotation #7 1-3-5-7-9



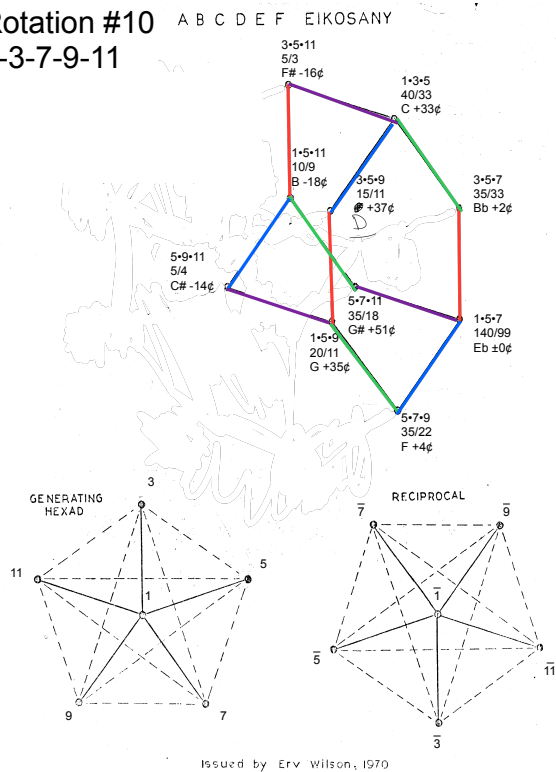
Rotation #8 1-3-5-7-11



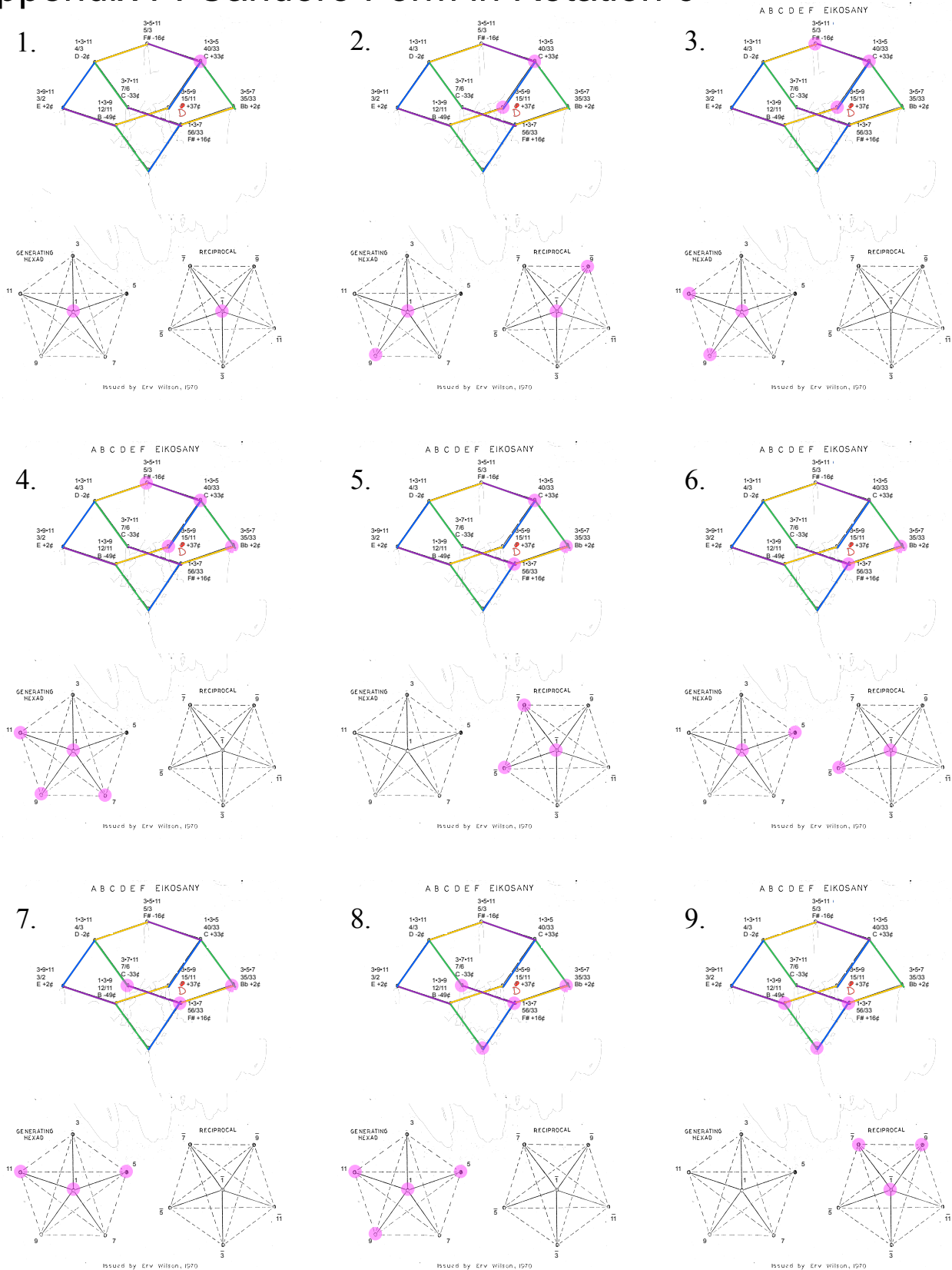
Rotation #9 1-3-5-9-11

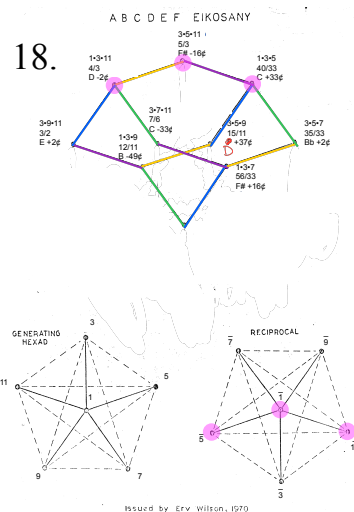
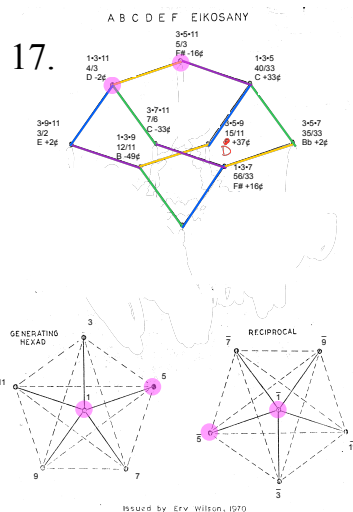
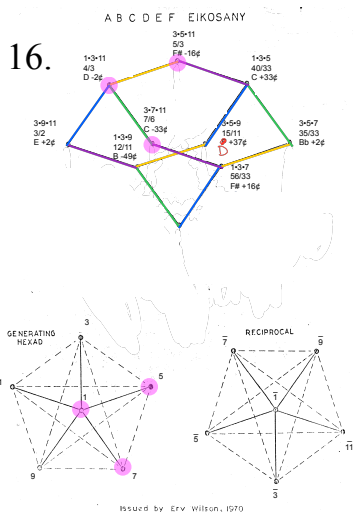
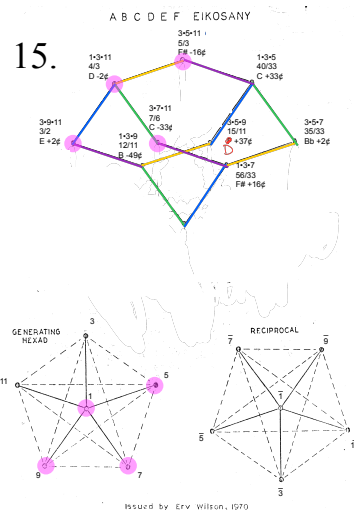
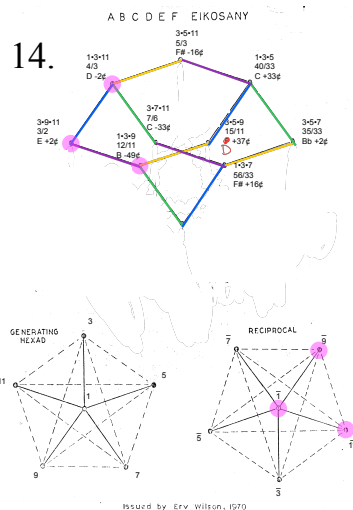
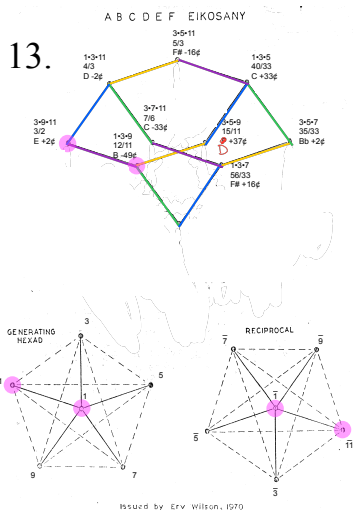
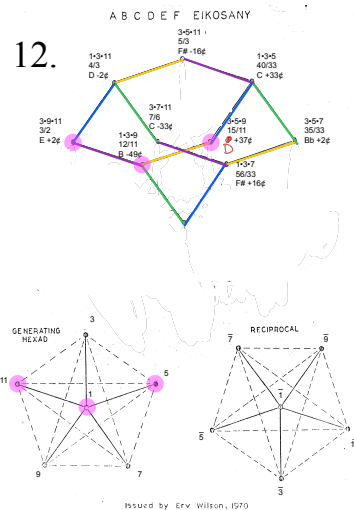
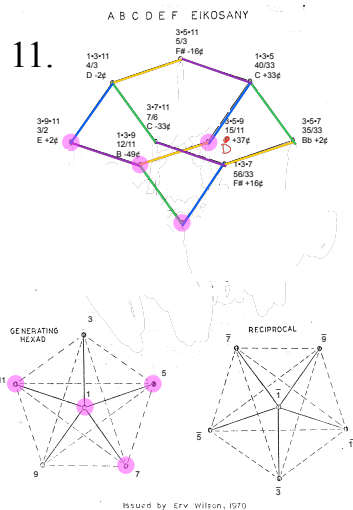
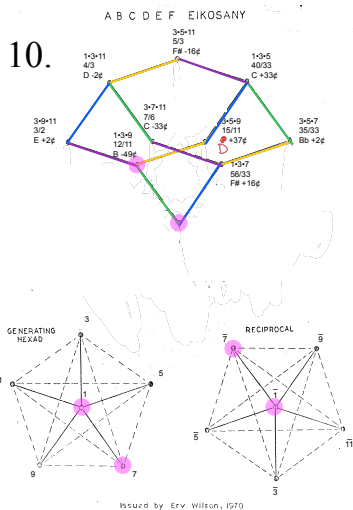


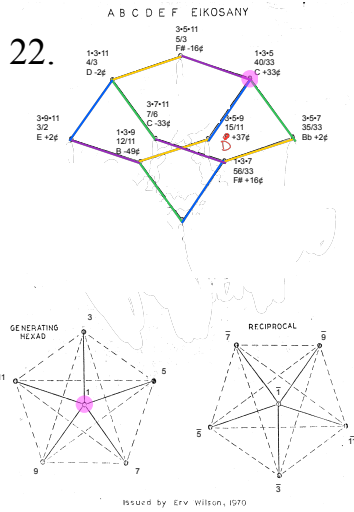
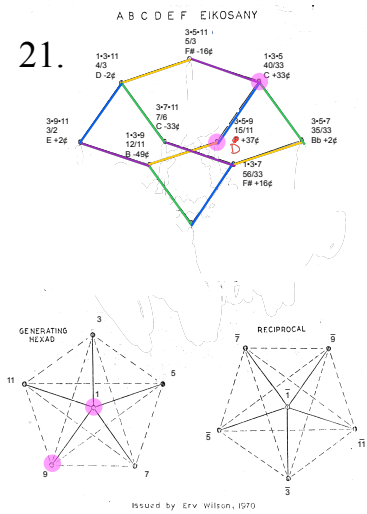
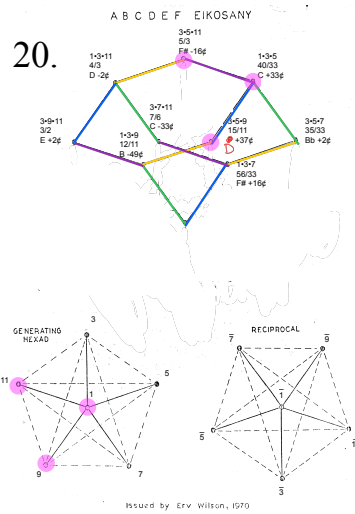
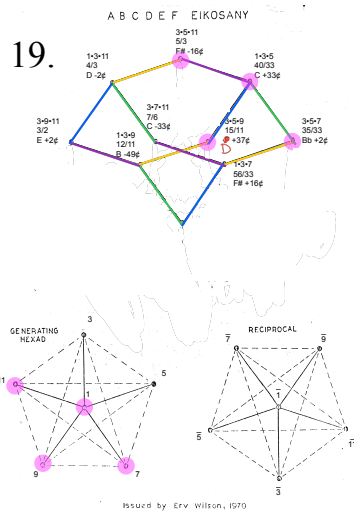
Rotation #10 1-3-7-9-11



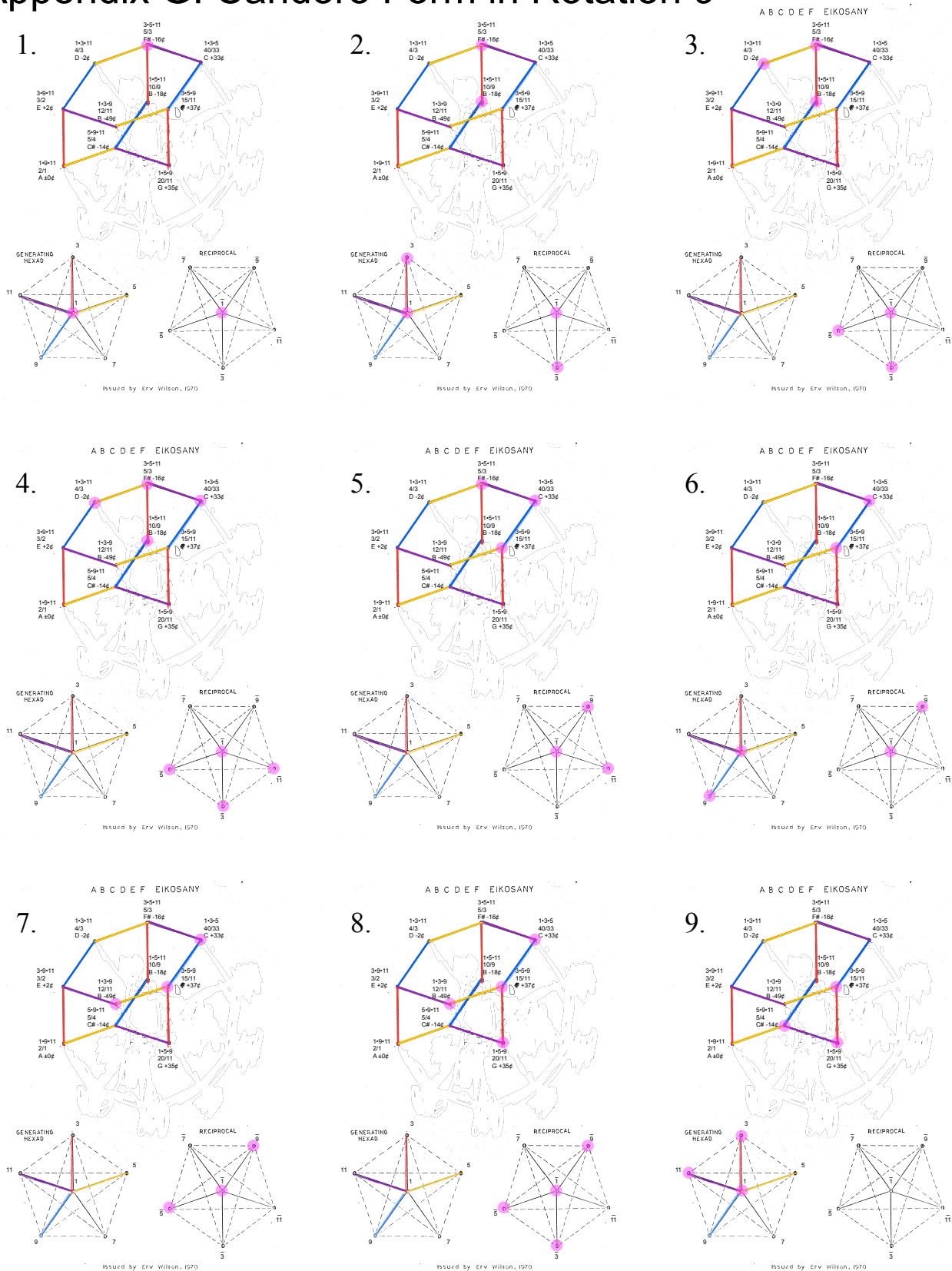
Appendix F: Candoro Form in Rotation 6

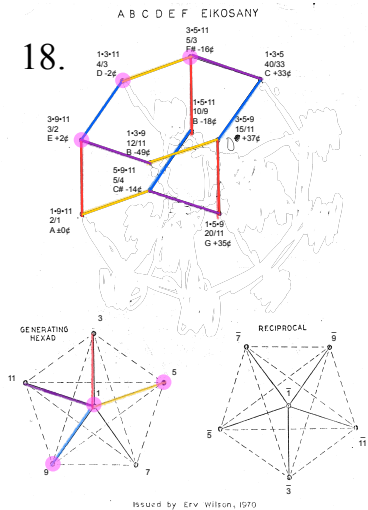
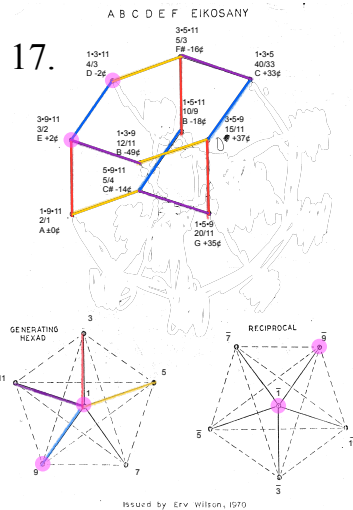
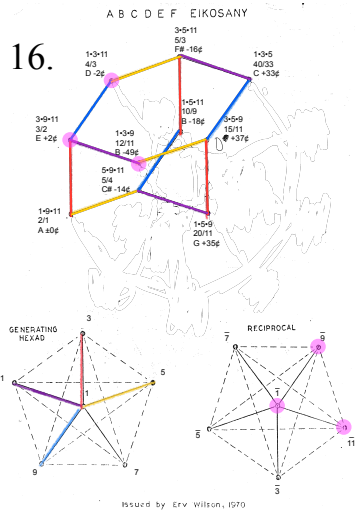
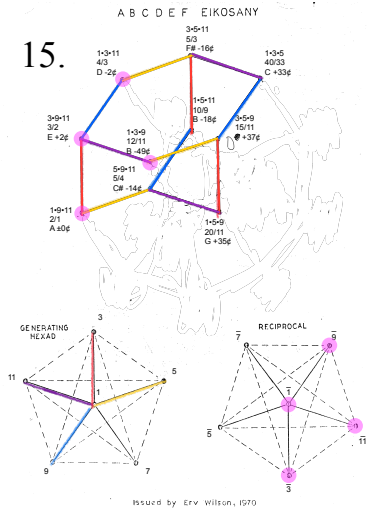
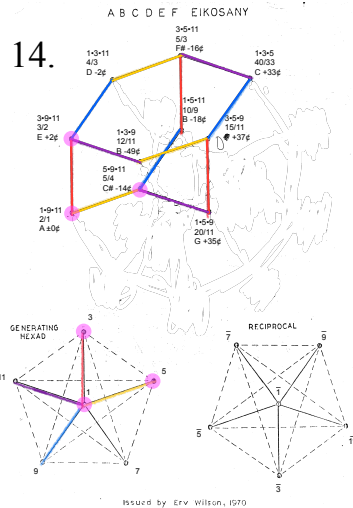
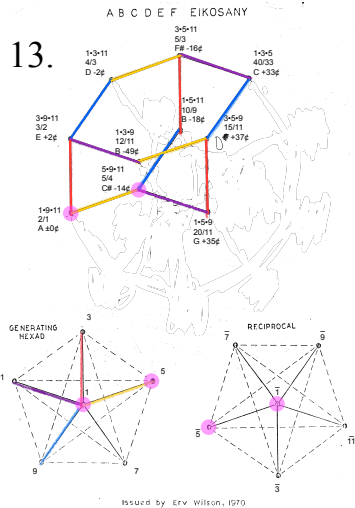
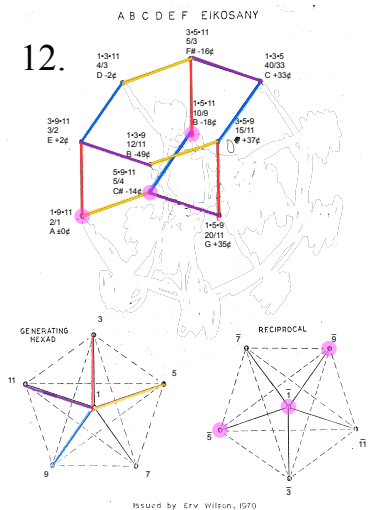
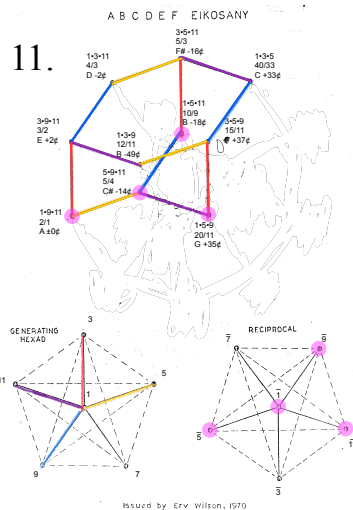
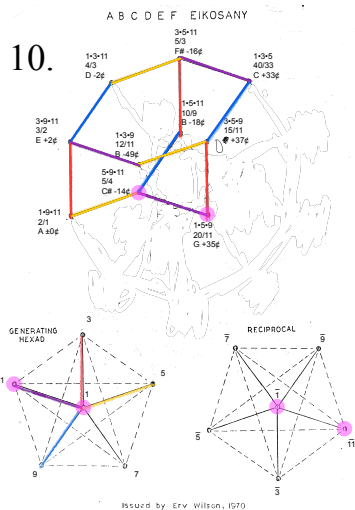






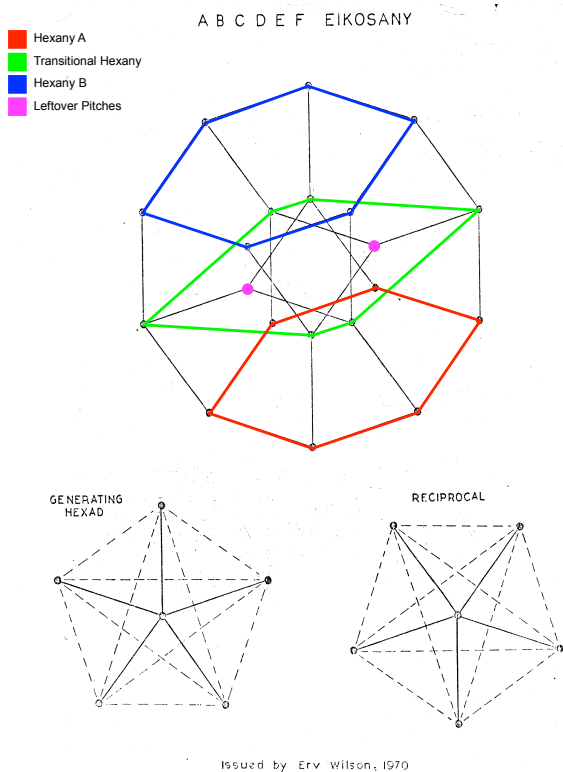
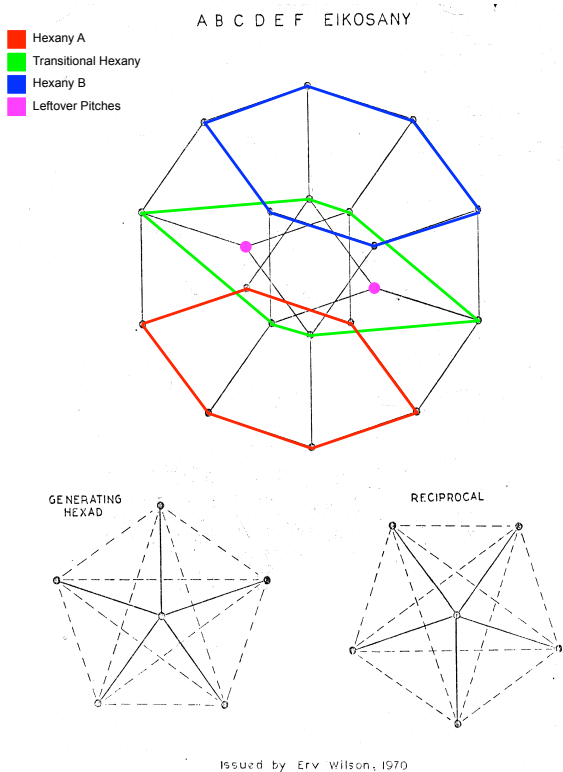
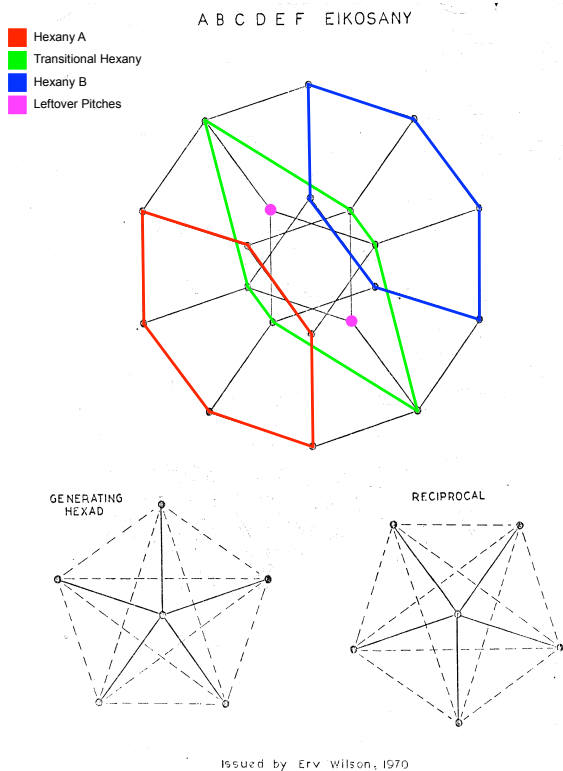
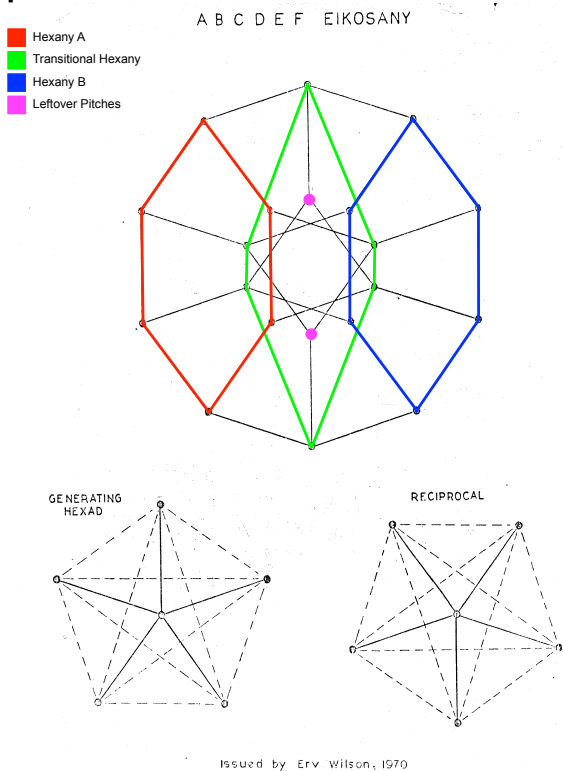
Appendix G: Candoro Form in Rotation 9

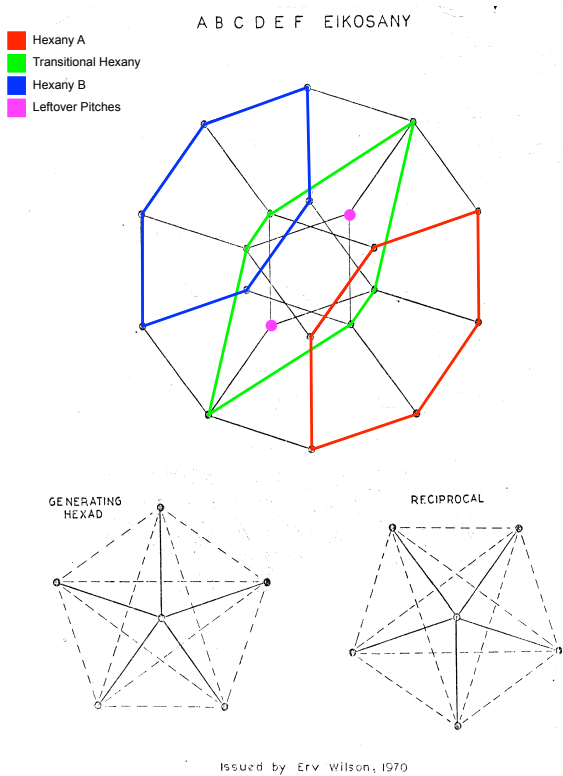






Appendix H: Hexanies and “Transitional Hexanies”





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