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Journal

Physics of Plasmas, 27(3)

ISSN

1070-664X

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Publication Date

2020-03-01

DOI

10.1063/1.5138129

Peer reviewed

A closer look at turbulence spreading: how bistability admits intermittent, propagating turbulence pulses

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APS DPP 2018

Supported by the Department of Energy under Award Number DE-FG02-04ER54738

What is turbulence spreading?

- Phenomenon in which turbulent fluctuations propagate radially [Garbet et al., 1994, Diamond and Hahm, 1995]
- Fluctuations can penetrate into linearly stable zone and excite turbulence there [Hahm et al., 2004, Naulin et al., 2005]
- Closely related conceptually to avalanching: both are mesoscale, nonlinear turbulent front propagation phenomena

Figure Cartoon depicting a turbulence pulse propagating into the stable zone and exciting turbulence there.

Why should we care about it?

- Believed to be key actor in various nonlocality phenomena [Ida et al., 2015]
- Crucial: spreading results in the fluctuation intensity being influenced by dynamics outside of the turbulence correlation length
- Result: fluctuation level, heat flux have *nonlocal dependence* on driving gradient, e.g.

$$Q(r) = -\chi \nabla T(r) \longrightarrow Q(r) = -\chi \int dr' K(r, r') \nabla T(r')$$

- Spreading also believed to be involved in the observed breakdown of gyro-Bohm transport scaling [Lin and Hahm, 2004], transport barrier formation, staircase formation

Conventional wisdom: Fisher equation

- Simplest, most common model:

$$\partial_t I = \underbrace{\gamma_0 I}_{\text{local lin. growth/decay}} - \underbrace{\gamma_{nl} I^2}_{\text{local nonlin. coupling to dissipation}} + \underbrace{\partial_x (D_0 I \partial_x I)}_{\text{nonlin. diffusion of turb. energy}}$$

- Spreading occurs in supercrit. ($\gamma_0 > 0$): traveling waves connecting the laminar and turbulent fixed points with constant speed

$$c = \sqrt{\frac{D_0 \gamma_0^2}{2 \gamma_{nl}}}$$

- No marginal or subcrit. spreading

Figure Fisher equation dynamics

Does it make sense?

- No. While partly successful (e.g. propagation speed), supercritical spreading is a cheat! Noise should excite the system in the first place
- For turbulent/laminar phase coexistence, a *subcritical bifurcation* is necessary
[Pomeau, 1986, Pomeau, 2015]
- Also, penetration into stable zone is *weak*. Turbulence level decays exponentially to finite depth $\sim \sqrt{D_0/\gamma_{nl}}$, i.e. just a few correlation length [Gürçan et al., 2005] — suggests Fisher may be insufficient to explain nonlocality!

Figure Penetration of Fisher front into stable zone

A new model is born

- We propose a new model:

$$\partial_t I = \gamma_1 I + \gamma_2 I^2 - \gamma_3 I^3 + \partial_x (D_0 I \partial_x I) \quad (*)$$

- Roughly anticipate $\gamma_i \sim \omega_*$, $D_0 \sim \chi_{GB} \sim c_s \rho_i^2 / a$
- Motivation: simplest, generic 1D model with subcritical bifurcation. Other forms possible, but qualitative features should be the same!
- Similar to [Barkley et al., 2015, Pomeau, 2015] models for onset of turbulence in pipe flow
- But is MF plasma turbulence actually subcritical?

Evidence for subcritical turbulence

- Experiments have clearly demonstrated hysteresis between fluctuation intensity and gradient in the L-mode (no ITB) [Inagaki et al., 2013] → bistable S-curve relation??
- In simulation, subcritical turbulence observed in the presence of magnetic shear damping [Biskamp and Walter, 1985, Scott, 1990] or strong perpendicular sheared flows [Barnes et al., 2011, van Wyk et al., 2016]

Figure Inagaki et al. 2013

Summary of model regimes

regime	stable roots	unstable roots	waves	comments
$\gamma_1 > 0$	I_+	0	forward-propagating	unstable similar to Fisher
$\gamma_1 < 0$ $ \gamma_1 \gamma_3/\gamma_2^2 < 15/64$	0, I_+	I_-	forward-propagating	$\alpha < \alpha^*$ turbulent root abs. stable
$\gamma_1 < 0$ $15/64 < \gamma_1 \gamma_3/\gamma_2^2 < 1/4$	0, I_+	I_-	receding	$\alpha > \alpha^*$ turbulent root metastable
$\gamma_1 < 0$ $ \gamma_1 \gamma_3/\gamma_2^2 > 1/4$	0	none	none	"strong damping"

Table Summary of features of the various parameter regimes in cubic model. Here $I_{\pm} = (\gamma_2 \pm \sqrt{\gamma^2 + \gamma_1\gamma_3})/2\gamma_3$.

In bistable case can transform to FitzHugh-Nagumo form

$$\partial_t I = f(I) + \partial_x(DI\partial_x I)$$

with $f(I) = \gamma I(I - \alpha)(1 - I)$, $\gamma = \gamma_3 I_+^2$, $D = I_+ D_0$, $\alpha = I_-/I_+$

Free energy picture

- Dynamics governed by dissipation of free energy: can rewrite

$$D(I)\partial_t I = -\frac{\delta \mathcal{F}}{\delta I}$$

with free energy functional

$$\mathcal{F} = \int dx \underbrace{\left[\frac{1}{2} (D(I)\partial_x I)^2 \right]}_{\text{kinetic/flux}} - \underbrace{\int_0^I dl' D(I')f(I')}_{\text{potential}}$$

and $d\mathcal{F}/dt \leq 0$

Figure Plot of potential part of free energy $\mathcal{V}(I) = -\int_0^I dl' D(I')f(I')$

Key predictions of bistable model

- In marginal and weakly subcritical regime, again have propagating turbulence fronts with speed $\sim \sqrt{D\gamma}$ (coeff. depends on α)
- If turbulence level driven globally above “potential barrier” at α (say by external flux), system relaxes to turbulent root $\rightarrow$ explains hysteresis in Inagaki
- There is also *local* threshold behavior: a sufficiently large slug of turbulence will grow and propagate. How to determine threshold size? Spreading of a turbulent spot is classic problem in turbulence

Figure A slug will either grow into a wave (above) or collapse (below)

Threshold for spreading of a slug of turbulence

- Threshold for amplitude is clear: intensity must exceed $I = \alpha$ somewhere
- Otherwise effective linear growth $\gamma_{eff} = (I - \alpha)(1 - I)$ is negative everywhere
- What about threshold in spatial extent? Question seems largely unexplored in literature!

Figure Plot of effective local linear growth as function of turbulence intensity

Lengthscale threshold

- Can estimate by assuming initial growth of turbulent mass in “cap” (part $> \alpha$) of slug governs asymptotic spreading
- Threshold then determined by competition between outgoing diffusive flux from cap and local growth in cap
- This competition suggested by form of free energy functional
- Leads to power law $L_{min} \sim (I_0 - \alpha)^{-1/2}$. Excellent agreement with simulation of PDE

Figure Illustration of slug's “cap”

Lengthscale threshold: analytical vs. simulation

Figure Numerical result for threshold at $\alpha = 0.3$ for three types of initial condition (Gaussian (I_1), Lorentzian (I_2), parabola (I_3)), compared with analytical estimate

From slugs to avalanches

- So: an initially localized turbulent slug with amplitude exceeding I_- and spatial extent exceeding L_{min} will spread and excite the system to turbulence
- This closely resembles an avalanche. Note the similarity of our model to [Gil and Sornette, 1996] model for sandpile avalanching
- Diffusion both provides mechanism for turbulence to topple from one region to next and limits avalanching by setting minimum scale length
- Near marginal linear stability, threshold is “small”:

$$I_- \sim \frac{|\gamma_1|}{\gamma_2} \ll 1, \quad L_{min} \sim \left(\frac{\chi_{GB}}{\omega_*} \right)^{1/2} \sim \rho_i$$

- Suggests that noise (e.g. background sub-ion-scale turbulence) can *intermittently* excite turbulence pulses. Related: turbulence transition in fluids is highly intermittent [Pomeau and Manneville, 1980]

Penetration into bistable zone

- Finally, let's revisit the problem of spreading from weakly supercritical into weakly subcritical ($\alpha < \alpha^*$), now with nonlinear instability
- Amplitude of wave in unstable region always exceeds amplitude threshold in stable region
- Thus, another wave forms in second region! Turbulence front propagates at constant speed (instead of finite depth), as long as weakly subcritical
- Conclude: delocalization effect much stronger than in Fisher

Figure A wave develops in the unstable zone, penetrates into the bistable zone, and forms a new traveling wave with reduced speed and turbulence level.

Conclusions

- Updating the unstable Fisher model to a bistable model simultaneously resolves several issues
 - ① Subcritical/marginal spreading properly supported
 - ② Can account for hysteresis in fluctuation intensity
 - ③ Reflects the emerging understanding that MF turbulence is subcritically unstable, at least in certain scenarios
 - ④ Allows for stronger penetration into stable zone via ballistic spreading
- Also functions as a basic model for avalanching by local excitation
- Future directions:
 - ① Full model needs to incorporate coupling to zonal flow and/or profiles
 - ② Avalanche threshold can be tested by initializing seed fluctuations in simulation and observing response
 - ③ Should also test for ballistic spreading into stable zone numerically. Possible inspiration: [Yi et al., 2014]

Experimental ideas

- Extensions of Inagaki
 - Better resolution of dependence of fluctuation intensity on the input power. Are there any jumps?
 - More careful study of relaxation after ECH is turned off. How does relaxation time compare to other timescales of interest?
 - More information on fluctuation field. E.g. spatial correlations?
 - Include spatiotemporal measurement of zonal flow pattern via Thomson scattering, heavy ion beam probe, etc.
- To investigate avalanches: perturb plasma locally, observe spatiotemporal response à la [Van Compernelle et al., 2015]. Compare near-marginal, far above stability threshold to rule out possibility of linear mode response

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Avalanching 101

- Observed in MFE plasma [Politzer, 2000]
- Basic picture: a sufficiently large, localized increase in the turbulence level radially cascades into neighboring regions, ultimately causing a sudden burst of transport
- Closely related to turbulence spreading: avalanching and (subcritical) spreading essentially two ways of looking at same phenomenon
- Associated with self-organized criticality (occurs near marginal, $1/f$ spectra)
- Intermittent (long tails)

Bistable case: reduction to FitzHugh-Nagumo

- (*) is bistable for weak damping $\gamma_1 < 0$ and $\gamma_2^2 > 4|\gamma_1|\gamma_3$
- Roots: $I = 0$, $I_{\pm} = (\gamma_2 \pm \sqrt{\gamma_2^2 - 4|\gamma_1|\gamma_3})/2\gamma_3$. $0, I_+$ stable (note: nonzero for marginal γ_1), I_- unstable
- If $\gamma_1 < 0$ and γ_2 sufficiently large, can be written

$$\partial_t I = f(I) + \partial_x(D(I)\partial_x I)$$

with $f(I) = \gamma I(I - \alpha)(1 - I)$ by defining

$$|\gamma_3|I_+^2 \rightarrow \gamma, \quad \frac{I_-}{I_+} \rightarrow \alpha, \quad I_+ D_0 \rightarrow D$$

- This is a version of the Nagumo equation, a simplification of the FitzHugh-Nagumo model for excitable media [FitzHugh, 1961, Nagumo et al., 1962]

Lengthscale threshold (details)

- Strategy: assume initial slug is even, has single max at l_0 and single lengthscale L
- Expand intensity curve about max to quadratic order, plug into dynamical equation, integrate over extent of cap
- Result: growth if

$$L > L_{\min} = \sqrt{\frac{\lambda D(\alpha) l_0}{f(l_0) - \frac{1}{3}(l_0 - \alpha)f'(l_0)}} = \sqrt{\frac{3\lambda D\alpha l_0}{\gamma(l_0 - \alpha)((1 - 2\alpha)l_0 + \alpha)}}$$

$E \times B$ staircase

- $E \times B$ staircase: quasiperiodic shear flow pattern observed in GK simulation [Waltz et al., 2006]
- [Guo and Diamond, 2017] showed that in mean field approx., result is additional nonlinear drive term, equation of the type (*) $\rightarrow$ global bistability
- Basic physics: inhomogeneous turbulence mixing. Shear suppression of turb. heat flux $\rightarrow$ effective negative turbulent heat diffusion $\rightarrow$ temperature corrugations $\rightarrow$ critical gradient locally exceeded $\rightarrow$ turbulence growth $\rightarrow$ further profile roughening

Figure Profile corrugations correlate with $E \times B$ shear (from [Dif-Pradalier et al., 2010])