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The Accelerating Universe, the Landscape, and the Swampland

by

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Professor Hitoshi Murayama, Chair
Professor Yasunori Nomura
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Abstract

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The accelerating expansion of the universe is a portal for us to understand the physics at the very fundamental level. It's a phenomena that is apparently IR but intrinsically UV. In this dissertation we investigate various aspects of the accelerating universe in the context of the string landscape and swampland. In the first part of the dissertation, we investigate if the observed small and nearly scale-invariant primordial cosmic perturbation is typical in the landscape of vacua after imposing anthropic selections on them. We propose a scenario that combines new-inflation-type models with the landscape, in which our universe had been trapped at a meta-stable vacuum and underwent a precedent inflation. We argue that the initial inflaton field value is typically non-zero because of the quantum fluctuation created during the precedent inflation. Imposing anthropic constraint on the initial condition, together with certain distributions of inflation model parameters that are physically well-motivated, makes the observed small and nearly scale-invariant spectrum typical. In a latter part of the dissertation, we discuss the quintessence model building in supergravity, in light of the recently proposed de Sitter swampland conjecture. Particularly, the conjecture claims that the scalar potential V in any consistent theory of quantum gravity should satisfy the constraint $|\nabla V| \geq cV$ where c is a positive number of order one. If true, positive cosmological constant (even metastable one) cannot be obtained in string theory and dark energy needs to be described by an evolving scalar field, *i.e.* quintessence, within supergravity. We demonstrate that by imposing a shift symmetry on the Kähler potential, one can embed any quintessence models into supergravity while avoiding the fifth force constraint and protecting the flatness of the quintessence potential from supersymmetry breaking, which are the two main obstacles when constructing quintessence models in supergravity. In addition, the small energy scale of quintessence is technically natural in this setup. In the last part of the dissertation, we discuss the phenomenological implications of swampland conjectures on both inflation and dark energy, using the fact that the conjectures are *universal* throughout the whole field space. We show that the refined de Sitter conjecture, along with multi-field inflation, opens up the opportunity for observations to determine if the dark energy equation of state deviates from that of a cosmological constant.

To My Family

Contents

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Chapter 1

Introduction

The main theme of this dissertation is about the accelerating expansion of the universe. In 1998, through supernovae observations, it was discovered that our universe is currently expanding at an increasing rate [196, 201]. Because gravitational interaction dominates the dynamics at the large scale and ordinary matter only attracts each other gravitationally, such *accelerating* expansion cries for new physics. The exotic matter or field that drives the accelerating expansion is often called *dark energy*. Despite more than two decades of active research, it is fair to say that we still do not fully understand the nature of dark energy. Particularly, even though adding a positive cosmological constant to Einstein's equation can help fitting the data, its minuscule value compared to the naive expectation from quantum theory, an 120 orders of magnitude difference, still remains an open challenge for physicists. On the other hand, mainly motivated by the difficulties of constructing de Sitter spacetime solution in string theory with fully controllable calculations, recently the *de Sitter swampland conjecture* has been proposed. Such conjecture forbids cosmological constant as an explanation for the current accelerating expanding universe, and a slow-rolling canonical scalar field minimally coupled to gravity, dubbed *quintessence*, may be the most economical alternative. Half of this dissertation will be devoted to the discussion of quintessence model-building in supergravity, the low energy effective field theory of string theory, and the phenomenological implications of de Sitter swampland conjecture.

In addition to the current accelerating expansion, observational evidences also support the idea that the universe underwent a much rapid accelerating expansion in the primordial era, called *inflation*. The inflationary paradigm not only explain the horizon and flatness problems, *i.e.* why two regions in the universe that seemingly do not have causal contact can have similar temperature and why the current universe is spatially flat, it also elegantly explains the approximately scale-invariant power spectrum of the primordial perturbation seen in the cosmic microwave background. But similar to dark energy, even though we have very successful phenomenological models for inflation, its physical nature is yet to be understood. In addition, we will argue that so far there are no completely satisfactory explanation for the values of the observed primordial scalar perturbation amplitude and spectral index. In many models, the observed perturbation amplitude is used to fix other model parameters, instead of being a prediction of the model. On the other hand, even though inflation generically predicts an approximately scale-invariant spectrum, to reach the observed value of scalar perturbation spectral index, one either needs to go to a realm where the effective field theory may not be controllable, or resort to fine-tuning. In Chapter 2 we will show that by incorporating anthropic principle with conventional model-building, these issues can be largely ameliorated.

In this chapter, we will first give a review of accelerating expanding spacetime and the generation of primordial perturbation from inflation. This paves the road toward our work in Chapter 2. In the second part of the introduction we will give a brief review on *swampland conjectures*, which are proposed criteria that determine if a low energy effective field theory can be UV completed into a consistent theory of quantum gravity. Different swampland conjectures are often interwoven with one another and can have strong phenomenological implications on model-buildings which we will explore in Chapter 3 and 4.

1.1 Accelerating Expanding Universe

One of the main pillars of modern cosmology is the homogeneity and isotropy of the universe at the very large scale. This means that at such scale the background spacetime can be described by the Friedmann-Robertson-Walker metric of the form

$$ds^2 = -dt^2 + a^2(t)d\boldsymbol{x}^2. \tag{1.1}$$

where t is the physical time coordinate and $\boldsymbol{x}$ are the spatial comoving coordinates. The expansion of the universe is described by the evolution of the scale factor $a(t)$. For the matter contents, at the large scale they can be approximated as a perfect fluid, characterized by their energy density ρ and isotropic pressure P, with energy-momentum tensor $T_{\mu\nu}$ of the form

$$T_{\mu\nu} = (\rho + P)U_\mu U_\nu + P g_{\mu\nu}. \tag{1.2}$$

where U^μ is the 4-velocity of the fluid and $U^\mu = (1, 0, 0, 0)$ in the comoving frame. With the above form of spacetime ansatz and energy-momentum tensor, one can obtain the Friedmann equations that determine the evolution of the background spacetime,

$$H^2 = \frac{8\pi G}{3}\rho \tag{1.3}$$

$$\frac{\ddot{a}}{a} = -\frac{4\pi G}{3}(\rho + 3P) \tag{1.4}$$

where $H = \dot{a}/a$ is the Hubble parameter and G is the Newton's constant. In many cosmologically interesting models, the equation of state $P(\rho)$ can be parametrized by the equation of state parameter w defined as

$$w = \frac{P}{\rho}. \tag{1.5}$$

From Eq. (1.4) one can see that an accelerating expansion requires an equation of state parameter $w < -1/3$, which is quite abnormal. For example, if one has a tank of exotic gas with a negative equation of state parameter, the gas will heat up if its volume expands, unlike ordinary gas which would cool down.

As bizarre as it is, there is actually a simple way to achieve negative w. In particular, one can add a cosmological constant Λ to the Einstein Hilbert action,

$$S_{\rm EH} + S_\Lambda = \frac{1}{16\pi G}\int \sqrt{-g}R - \frac{1}{8\pi G}\int d^4x\sqrt{-g}\Lambda, \tag{1.6}$$

where the cosmological constant term yields an energy-momentum tensor that is proportional to the metric $g_{\mu\nu}$ itself

$$T_{\mu\nu} = -\frac{2}{\sqrt{-g}}\frac{\delta S_\Lambda}{\delta g^{\mu\nu}} = -\frac{\Lambda}{8\pi G}\, g_{\mu\nu}. \tag{1.7}$$

This means the pressure of cosmological constant is equal to its energy density but with opposite sign,

$$P_\Lambda = -\rho_\Lambda, \tag{1.8}$$

i.e. $w = -1$ for a cosmological constant, which satisfies the criteria $w < -1/3$ to drive an accelerating expansion.

Another possible way to drive an accelerating expansion is to consider a canonical scalar field with a flat enough potential. Namely, consider the action of a scalar field φ

$$S_\varphi = \int d^4x\sqrt{-g}\left[-\frac{1}{2}\partial_\mu\varphi\partial^\mu\varphi - V(\varphi)\right], \tag{1.9}$$

which gives a energy-momentum tensor

$$T_{\mu\nu} = \partial_\mu\varphi\partial_\nu\varphi + \left[-\frac{1}{2}\partial_\mu\varphi\partial^\mu\varphi - V(\varphi)\right] g_{\mu\nu}. \tag{1.10}$$

Assuming the scalar field is spatially uniform $\varphi = \varphi(t)$, this yields an equation of state parameter

$$w = \frac{P}{\rho} = \frac{\frac{1}{2}\dot\varphi^2 - V(\varphi)}{\frac{1}{2}\dot\varphi^2 + V(\varphi)}, \tag{1.11}$$

where the dot on φ denotes derivative with respect to time. We see that if the potential is flat enough such that the field is rolling slowly, *i.e.* $\dot\varphi^2 \ll V$, then $w \approx -1$. In addition, cosmological constant can be thought of as a spatially homogeneous constant potential energy. If the scalar field is settled at a minima such that $\dot\varphi = 0$, then the constant potential energy yields an equation of state parameter $w = -1$.

One natural contribution to the cosmological constant is the vacuum energy. Naively, the vacuum energy originates from the quantum mechanical ground states. For a quantum harmonic oscillator, the ground state has an energy $\hbar\omega/2$. A quantum field can be decomposed into infinitely many momentum modes each acts like a quantum harmonic oscillator.

The vacuum energy is then the sum of the ground state energy of these momentum modes which is apparently infinite if one includes modes with infinitely large momentum. However, given an effective field theory, it is only valid up to a cutoff scale. For a theory involving gravity, the natural cutoff scale is the Planck scale, given by the reduced Planck mass $M_{P\ell} \equiv 1/\sqrt{8\pi G} \sim 10^{18}$ GeV. By simple dimensional analysis, the vacuum energy density is then, in natural unit, expected to be $\rho_{\rm vac} \sim M_{P\ell}^4$.

On the other hand, even though a cosmological constant can phenomenologically explain the current accelerating expansion of the universe, its observed value, in terms of energy density, is about $\rho_\Lambda \sim 10^{-120} M_{P\ell}^4$. The 120 orders of magnitude difference between $\rho_{\rm vac}$ and ρ_Λ remains one of the biggest mystery and challenge for physicists. The challenge actually has two folds. First off, the vacuum energy $\rho_{\rm vac}$ is only one kind of contribution to ρ_Λ. There might be other contributions, such as the classical potential energy of a scalar field. However, the fact that $\rho_{\rm vac}$ itself already exceeds ρ_Λ means that there might be some other negative contributions that result in a large cancellation, or the naive picture is just completely wrong. For the first scenario, one may invoke supersymmetry, where every bosonic field has a fermionic superparter and vice versa. While bosonic fields contribute positive vacuum energy, fermionic fields contribute negative ones and if supersymmetry is unbroken the two kinds of contribution will exactly cancel. However, this is only true for global supersymmetry. In theories involve gravity, supersymmetry needs to be gauged and one requires supergravity. In supergravity, at the classical level, the vacuum energy density is actually negative when supersymmetry is unbroken. This problem may be resolved by uplifting the negative vacuum energy with supersymmetry breaking. Yet, even if one manages to make $\rho_{\rm vac} \sim 0$, we still face the second challenge where ρ_Λ is not exactly 0 but a minuscule value comparing to any other scale in physics. In sum, the challenge has two aspects. The first is to make $\rho_{\rm vac} \sim 0$; the second is to explain why $\rho_\Lambda \sim 10^{-120} M_{P\ell}^4$ which seems to require extreme fine-tuning. Both of these problems are extremely challenging and it is very likely that a satisfactory answer will not be found until we have a full-fledged quantum theory of gravity and fully understand the nature of spacetime and vacuum.

Instead of adopting cosmological constant to explain the accelerating expansion, as we mentioned above, another economical alternative is to consider a scalar field Q called quintessence that is minimally coupled to gravity. In such setting, the problem is then phrased as why $\rho_{\rm vac} \sim 0$ and why ρ_Q, the energy density of the quintessence, is about $10^{-120} M_{P\ell}^4$. Later when we discuss quintessence model building in supergravity, we do not attempt to tackle this challenging problem fully. Since the nature of $\rho_{\rm vac}$ most likely requires a full treatment of quantum gravity, our working philosophy is to assume that $\rho_{\rm vac} = 0$ a priori and the energy density of dark energy is provided by the quintessence field. We argue that in models with shift symmetry, the smallness of quintessence energy density is technically natural. Namely, when its value is exactly zero, the shift symmetry is fully restored.

1.2 Inflation and the Primordial Perturbation

In this section we give a brief review on inflation and the primordial perturbation generated by inflaton. Our treatment follows two excellent lecture notes by Baumann [37] and Senatore [205].

When we look into the sky, the temperature of the cosmic microwave background (CMB) is strikingly uniform. The fact the universe is so big such that different parts of the universe did not have causal contact in the past makes us wonder why the current universe and the temperature of the CMB can be such uniform. It turns out that if the universe underwent a much rapid accelerating expansion in the primordial era, called inflation, we not only can explain the homogeneity of the universe, we can even further explain the tiny inhomogeneity, $\delta T/T \sim 10^{-5}$ in terms of CMB temperature, present in the universe.

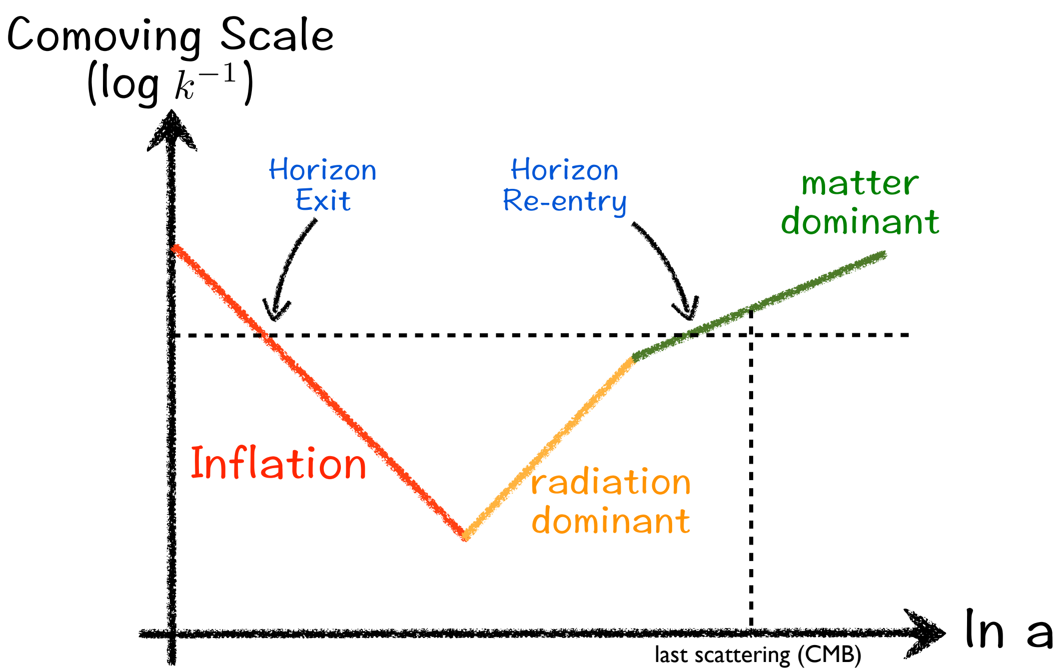

Figure 1.1: Evolution of comoving horizon and the horizon exit/re-entry of a comoving mode.

The essence of the *inflationary paradigm* can be encapsulated in Figure 1.1. If we start from present and go back with time, the comoving mode corresponding to the current cosmological scale was outside the comoving horizon, which means two parts of the universe that are presently separated by a distance of cosmological scale did not have causal contact in the past. Because during both matter and radiation dominant era the comoving horizon shrinks if we go back in time, there is no way for the comoving mode to be in casual contact before and the homogeneity of the CMB temperature cannot be explained. The key of the inflationary paradigm is to have a phase of accelerating expansion during which the comoving horizon expands if we go back in time, and therefore the comoving mode can be inside the comoving horizon in the far past. In other words, if we start from the far past and go forward in time, the comoving mode of the present cosmological scale was far inside the horizon at the beginning, which explains the homogeneity of current universe. In addition, the comoving modes of the inflaton underwent quantum fluctuations until the mode exited the horizon and froze. The frozen fluctuation thawed when the comoving mode re-enter the horizon and served as the "initial condition" for late-time fluctuation which later on formed into structures we see today. Because the fluctuation was originally frozen, different modes of the same wavelength started to oscillate *in the same phase* after horizon re-entry. Without this, one cannot obtain the clear peaks and troughs seen in the CMB spectrum [98]. In below we will derive the amplitude of the primordial inflaton fluctuation and its spectral index, whose observed values are one of the main focus of this dissertation.

Over the decades, various inflationary models have been proposed. But the main feature of the inflationary paradigm can be described by a single inflaton field ϕ with a flat potential. The flatness of the potential is characterized by two potential slow-roll parameters

$$\epsilon_V \equiv \frac{M_{P\ell}^2}{2}\left(\frac{V'}{V}\right), \qquad \eta_V \equiv \frac{M_{P\ell}^2 V''}{V}, \tag{1.12}$$

where the primes denote derivative with respect to the inflaton ϕ. The action of the inflaton is obtained by minimally coupling it to gravity,

$$S_\phi = \int d^4x\sqrt{-g}\left[-\frac{1}{2}g^{\mu\nu}\partial_\mu\phi\partial_\nu\phi - V(\phi)\right]. \tag{1.13}$$

As we mentioned in the previous section, when a scalar field such as inflaton slowly rolls down its potential, its potential energy can drive accelerating expansion. Particularly, if $\dot{\phi}^2 \ll V$, the Friedmann equation can be written as

$$H^2 \simeq \frac{V}{3M_{P\ell}^2}. \tag{1.14}$$

The time-variation of the Hubble scale during inflation is then parametrized by the slow-roll parameter,

$$-\frac{\dot{H}}{H^2} \simeq \epsilon_V, \tag{1.15}$$

where we have used the inflaton equation of motion

$$\ddot{\phi} + 3H\dot{\phi} + V' = 0 \tag{1.16}$$

and assuming $\ddot{\phi}$ is much smaller than the other two terms, i.e. $3H\dot{\phi} \approx -V'$. Therefore, for flat enough potential, i.e. small slow-roll parameter ϵ_V, the Hubble scale during inflation is nearly constant,

$$H \approx \text{constant}. \tag{1.17}$$

With $H = \dot{a}/a$, this implies that, at the zeroth order, the spacetime scale factor expands exponentially

$$a \propto e^{Ht}. \tag{1.18}$$

If the potential is exactly constant, then the Hubble scale is exactly constant, which yields the de Sitter spacetime solution. For inflation, the accelerating expansion must ends and so the potential must has a shallow slope. The background solution of inflationary spacetime is therefore *quasi-de Sitter*.

The zeroth order solution of the inflaton is only time-dependent, but its fluctuation has spatial dependence. Particularly, we have the form

$$\phi(t,x) = \bar{\phi}(t) + \delta\phi(t,x), \tag{1.19}$$

where $\bar{\phi}$ denotes the zeroth order background value, and $\delta\phi$ denotes the fluctuation. Because inflaton field value determines the value of the potential, and hence the value of the Hubble scale, the inflaton acts like the clock that determines at what stage the inflation is. The fluctuation $\delta\phi(t,x)$ can then be thought of as the fluctuation of the clock value – at some points in space the clock runs ahead of the background value $\bar{\phi}(t)$ while at some points it lags behind. It is actually this time-delay (and advance) that yields the leading contribution to the scalar curvature perturbation ζ, instead of the perturbation due to the energy density. Indeed, from the perturbed Einstein's equation, one can estimate the contribution of the energy density perturbation as

$$\frac{\delta\rho}{\rho} \simeq \frac{V'\,\delta\phi}{V} = \sqrt{2\epsilon_V}\frac{\delta\phi}{M_{P\ell}}, \tag{1.20}$$

which is suppressed by the slow-roll parameter ϵ_V. On the other hand, the scalar perturbation originated from time-delay is

$$\zeta \sim \frac{\delta a}{a} = H\,\delta t = \frac{H}{\dot{\bar{\phi}}}\,\delta\phi \simeq \frac{1}{\sqrt{2\epsilon_V}}\frac{\delta\phi}{M_{P\ell}}. \tag{1.21}$$

Due to this, we can proceed our calculation by considering the inflaton fluctuation around the background spacetime solution without worrying about the backreaction of $\delta\phi$ on the spacetime, at least at the leading order.[1]

Before considering the quantum fluctuation around the quasi-de Sitter background, it is useful to consider a scalar field on the flat spacetime,

$$S = \int d^4x \left(-\frac{1}{2}\partial_\mu\phi\,\partial^\mu\phi\right) = \int dt \int \frac{d^3k}{(2\pi)^3}\left(\frac{1}{2}\dot{\phi}_{\vec{k}}\,\dot{\phi}_{-\vec{k}} - \frac{1}{2}\omega_k^2\,\phi_{\vec{k}}\,\phi_{-\vec{k}}\right) \tag{1.22}$$

[1]Another way to interpret this is to consider the general metric perturbation constructed from 3-scalars,

$$ds^2 = -(1+2\Phi)dt^2 + 2a\partial_i B dx^i dt + a^2\left[(1-2\Psi)\delta_{ij} + 2\partial_i\partial_j E\right]dx^i dx^j$$

and work in the *spatially-flat gauge*, $\Psi = E = 0$. In this gauge, the gauge invariant curvature perturbation $-\zeta \equiv \Psi + \frac{H}{\dot{\bar{\phi}}}\delta\phi$ is then determined by $\delta\phi$ only.

where $\omega_k^2 \equiv |\vec{k}|^2$, and the Fourier mode $\phi_{\vec{k}}$ is defined by

$$\phi(t,\vec{x}) = \int \frac{d^3k}{(2\pi)^3}\phi_{\vec{k}}(t)e^{i\vec{k}\cdot\vec{x}}. \tag{1.23}$$

We see that each $\vec{k}$-mode is essentially a harmonic oscillator. Promoting $\phi_{\vec{k}}$ to an operator, expanding it in terms of creation and annihilation operators,

$$\phi_{\vec{k}} = \frac{1}{\sqrt{2\omega_k}}\left(a_{\vec{k}}\, e^{-i\omega t} + a^\dagger_{-\vec{k}}\, e^{+i\omega t}\right) \tag{1.24}$$

and using the commutation relation $[a_{\vec{k}}, a^\dagger_{\vec{k}}] = (2\pi)^3\delta^{(3)}(\vec{k}-\vec{k}\,')$ we obtain the amplitude

$$\langle 0|\, \phi_{\vec{k}}\phi_{\vec{k}\,'}\, |0\rangle = \frac{1}{2\omega_k}(2\pi)^3\delta^{(3)}(\vec{k}+\vec{k}\,'). \tag{1.25}$$

Coming back to inflation, expanding the inflaton action Eq. (1.13) to the second order in perturbation, one obtain the action for the inflaton fluctuation around the quasi-de Sitter spacetime

$$S_{\delta\phi} = \int d^4x\sqrt{-g}\left(-\frac{1}{2}\partial_\mu(\delta\phi)\partial^\mu(\delta\phi) - V''(\phi)\delta\phi^2\right) + \cdots \tag{1.26}$$

The mass term of the fluctuation has the order of $V'' = \eta_V V/M^2_{P\ell} \simeq 3\eta H^2$, which is suppressed by the slow-roll parameter η_V. The spatial derivative term in the kinetic term, after going to the Fourier space, has the form $\frac{1}{2}\frac{k^2}{a^2}(\delta\phi_{\vec{k}})^2$. We are interested in the value of the amplitude when the Fourier modes cross the horizon, i.e. when $k \simeq aH$. Hence, the spatial gradient term yields a mass of the order of H^2 for the Fourier modes and the mass term $V''(\delta\phi)^2$, which is suppressed by the slow-roll parameter, can be neglected. In terms of Fourier modes, the action can then be expressed as

$$S_{\delta\phi}\int dt\int\frac{d^3k}{(2\pi)^3}a^3(t)\left[\frac{1}{2}(\delta\dot{\phi}_{\vec{k}})(\delta\dot{\phi}_{-\vec{k}}) - \frac{1}{2}\frac{k^2}{a^2}(\delta\phi_{\vec{k}})(\delta\phi_{-\vec{k}})\right] \tag{1.27}$$

Comparing this with the canonically normalized field in flat spacetime Eq. (1.22), one observes that we can directly apply the result previously obtained in Eq. (1.25) and use the correspondence

$$a^{3/2}\delta\phi_{\vec{k}} = \delta\phi_{\vec{k},\mathrm{can}} \quad \text{and} \quad \frac{k^2}{a^2} = \omega_k^2 \tag{1.28}$$

to obtain the amplitude for $\delta\phi_{\vec{k}}$,

$$\langle\delta\phi_{\vec{k}}\,\delta\phi_{\vec{k}\,'}\rangle = \frac{1}{a^3}\langle\delta\phi_{\vec{k}\mathrm{can}}\delta\phi_{\vec{k}\,'\mathrm{can}}\rangle = \frac{1}{a^3}\frac{1}{2(k/a)}(2\pi)^3\delta^{(3)}(\vec{k}+\vec{k}\,'). \tag{1.29}$$

Evaluating the amplitude at horizon crossing $k = aH$, we then obtain

$$\langle\delta\phi_{\vec{k}}\,\delta\phi_{\vec{k}\,'}\rangle\Big|_{k=aH} = \frac{H^2}{2k^3}(2\pi)^3\delta^{(3)}(\vec{k}+\vec{k}\,'). \tag{1.30}$$

Recall the curvature perturbation ζ is related to inflaton fluctuation $\delta\phi$ by Eq. (1.21), the amplitude of the curvature perturbation is then

$$\langle \zeta_{\vec{k}}\, \zeta_{\vec{k}\,'} \rangle = \frac{H^2}{4M_{P\ell}^2 k^3 \epsilon_V} (2\pi)^3 \delta^{(3)}(\vec{k} + \vec{k}\,') \equiv (2\pi)^2 \delta^{(3)}(\vec{k} + \vec{k}\,') \Delta_\zeta(\vec{k}), \tag{1.31}$$

where the *power spectrum* of the curvature perturbation is given as

$$\Delta_\zeta(\vec{k}) = \frac{H^2}{4M_{P\ell}^2 k^3 \epsilon_V}. \tag{1.32}$$

The dimensionless power spectrum P_ζ is defined as

$$P_\zeta \equiv \frac{k^3}{2\pi^2} \Delta_\zeta(\vec{k}) = \frac{H^2}{8\pi^2 M_{P\ell}^2 \epsilon_V} \approx \frac{V}{24\pi^2 M_{P\ell}^4 \epsilon_V}. \tag{1.33}$$

We see that the dimensionless power spectrum Eq. (1.33) is *scale-invariant*. Physically this is because the Hubble scale remains nearly constant and hence different scales have the same history – they all started deep inside the horizon and exited the horizon $1/H$ as the universe expands. However, we know that the Hubble scale does vary over time as the potential of the inflaton is not exactly flat and the scale invariance of the power spectrum should only be an approximation. The *spectral index* n_s is defined by parametrizing the scale-dependence of the power spectrum as

$$P_\zeta = P_\zeta^{(0)} \left(\frac{k}{k_*} \right)^{1-n_s}. \tag{1.34}$$

Where k_* is a reference scale, or the *pivot scale*, and $P_\zeta^{(0)}$ is the amplitude of this scale. From the above parametrization, we see that if $n_s = 1$ the amplitude is exactly scale invariant. An equivalent definition of n_s is

$$n_s \equiv 1 + \frac{d \ln P_\zeta}{d \ln k}. \tag{1.35}$$

With the definition of slow-roll parameter ϵ_V, one can write

$$\frac{d \ln P_\zeta}{d \ln k} = \frac{d\phi}{d \ln k} \left(\frac{3V'}{V} - \frac{2V''}{V'} \right). \tag{1.36}$$

Since we evaluate the amplitude at the horizon crossing, we have $k = aH$, and

$$d \ln k = d \ln a + d \ln H = H\, dt + \frac{1}{H} dH = \left(\frac{H}{\dot{\phi}} + \frac{H'}{H} \right) d\phi. \tag{1.37}$$

Using $3H\dot{\phi} \simeq -V'$ and $3M_{P\ell}^2 H^2 = V$,

$$\frac{d \ln k}{d\phi} = -\frac{V}{M_{P\ell}^2 V'} + \frac{1}{2} \frac{V'}{V} = -\frac{V}{M_{P\ell}^2 V'} (1 - \epsilon_V). \tag{1.38}$$

Substitute this back into Eq. (1.36), and expand to first order in slow-roll parameters, we obtain

$$n_s = 1 + 2\eta_V - 6\epsilon_V . \tag{1.39}$$

This matches our expectation. Namely, if Hubble scale H is exactly constant during inflation, the spectrum is exactly scale-invariant. However, we expect the shallow slope of the potential to break scale invariance, and hence n_s should be able to be expanded by slow-roll parameters.

From CMB observations, the observed value of perturbation amplitude and spectral index are $P_\zeta^{(0)} = (2.101^{+0.031}_{-0.034}) \times 10^{-9}$ and $n_s = 0.965 \pm 0.004$ [13]. The question, then, naturally arises – can we predict these observed values from dynamical models. Unfortunately, even though various inflationary models have been constructed, we still don't have a completely satisfactory answer to this question. Specifically, in many models the observed perturbation amplitude P_ζ is often used to fix other model parameters,[2] instead of being a prediction from the models. On the other hand, even though inflationary paradigm generically predict a nearly scale invariant spectrum, one might expect a larger deviation from $n_s = 1$, say $n_s = 0.7 \sim 0.8$. The observed value $n_s \simeq 0.96$ seems to require a certain amount of fine-tuning. Namely, why should η_V and ϵ_V be at the order of 1 percent? This is particularly true for small field inflation models, where inflaton field displacement is much smaller than $M_{P\ell}$. In these models, one needs to fine-tune the potential to achieve the observed spectral index. On the other hand, for large field inflation models, where field displacement large exceeds Mp, the spectral index is associated with the number of efolds of inflation. For instance, for chaotic inflation with potential $V(\phi) = \mu^{4-p}\phi^p$, the spectral index is given by $n_s = 1 - \frac{(2+p)}{2\mathcal{N}_e^*}$. If we require about 60 e-folds to solve the horizon problem, then $1 - n_s \sim 10^{-2}$. However, it is not clear if the effective field theories describing large field inflation are theoretically consistent when the field displacement is larger than Planck scale.

In sum, so far we do not have a scenario that can simultaneously explain the observed value of perturbation amplitude and spectral index. In Chapter 2 we will discuss a new inflation scenario that naturally builds on the notion of string landscape. We will see that by combining the idea of dynamical model building with anthropic principle, we can make a big step toward this goal.

1.3 Swampland Conjectures

The full quantum theory of gravity has been one of the holy grail in physics research which, despite decades of research, still remains elusive. Nonetheless, even if we do not have a full-fledged theory, along the way we have learned some generic properties that we believe quantum gravity should possess. This set of properties are sometimes referred as *swampland conjectures*. The reason behind this nomenclature is that in string theory, which is believed to be a promising candidate as a theory of quantum gravity, one has a space of consistent low energy effective field theories derived from string theory which are called

[2] Since the amplitude is nearly scale-invariant, we simply write $P_\zeta^{(0)}$ as P_ζ.

the landscape. On the other hand, we have another set of effective field theories (EFTs), dubbed *the swampland*, that are seemingly consistent at the low energy but cannot be UV completed into the consistent theory of quantum gravity. Swampland conjectures are then the proposed criteria that determines the boundary of these two sets of low energy EFTs.

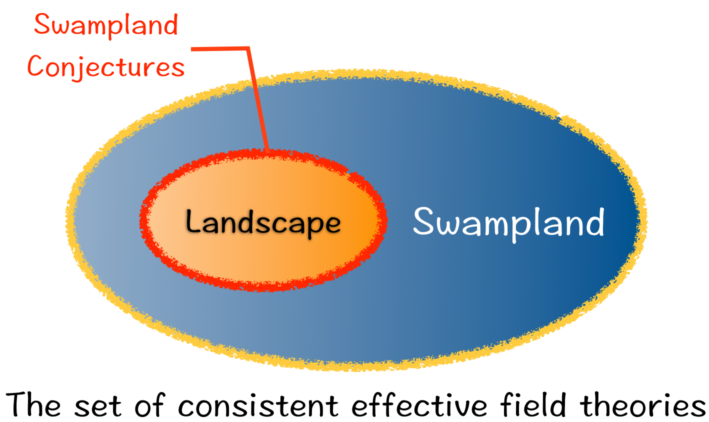

Figure 1.2: Swampland conjectures form the boundary between the swampland and the landscape.

Some well-known swampland conjectures include the weak gravity conjecture, swampland distance conjecture, no continuous global symmetry conjecture, etc. One might think that these conjectures are just some various speculations. However, different swampland conjectures are often interwoven with each other and form an intricate web, which we will discuss below. The connections among different conjectures make the swampland program an exciting and intriguing subject, and their phenomenological implications are worth investigation. Below we will give a very light review on some of these conjectures, which mainly follows the excellent review by Eran Palti [192].

The No Continuous Global Symmetry

One of the oldest folklore of quantum gravity is that it does not permit continuous global symmetry [30, 31]. This conjecture can be motivated by many arguments. Here we give an example using black holes. Recall that Hawking radiation can be thought of as a result of particle-antiparticle pair production near the event horizon where one of the particles falls into the black hole while its partner escapes. Given a continuous symmetry, there is an associated charge. Suppose we have a black hole that carries a net positive charge $Q > 0$. If the symmetry is global, then there is no associated gauge field that couples to the charges and mediates force. Hence. for pair productions near the horizon, particles and antiparticles have equal chances to escape from the black hole. As a result, the charge of the black hole is conserved under Hawking radiation process if the continuous symmetry is global. The story is different if the symmetry is a gauge symmetry, which entails gauge fields that can mediate forces between charges. Due to the gauge field, for a black hole that carries a positive charge, it will repel the particle (assuming to carry a positive charge) while attract the antiparticle (assuming to carry a negative charge). As a result, particle and antiparticle have different chance to escape from the black hole and the black hole losses charges. This is summarized in Figure 1.3.

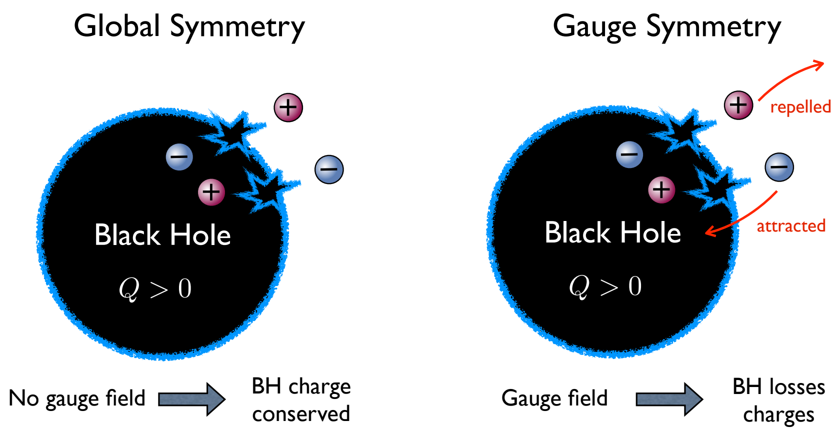

Figure 1.3: Global/gauge symmetry and charge conservation.

The fact that the associated charge is conserved during the Hawking radiation process if the continuous symmetry is global leads to contradictions. In particular, suppose the black hole can completely evaporate due to Hawking radiation, then the charge it carries is also gone, contradicting with the fact that charge should be conserved when the continuous symmetry is global. On the other hand, if the black hole cannot totally evaporate but ends up with a remnant of mass, say, $M_{P\ell}$, then one can prepare an infinite amount of black holes with different charge Q_i and after Hawking radiation we obtain infinitely numbers of black hole with the same mass $M_{P\ell}$ but different charge Q_i. However, by no-hair theorem, one cannot distinguish these black holes from outside. In other words, a black hole of mass $M_{P\ell}$ can have infinitely many possible microstates, leading to the violation of entropy bound. We see that in either case, whether the black hole can evaporate completely or not, continuous global symmetry leads to contradictions.

The Swampland Distance Conjecture

Swampland distance conjecture [187] concerns about the moduli fields, i.e. scalar fields with no potential. Consider a theory with moduli space $\mathcal{M}$ parametrized by the expectation value of some fields ϕ_i. The conjecture states that when one moves from a point $P \in \mathcal{M}$ to Q such that the distance $d(P, Q)$ between them approaches infinity, there exists an infinite tower of states, with associate mass scales, such that

$$M(Q) \sim M(P)e^{-\alpha\, d(P,Q)}. \tag{1.40}$$

where α is some positive constant. In other words, we have an infinite tower of states that becomes exponentially light when one has a large excursion in the moduli space. Later on this conjecture has been refined such that the constant α is explicitly connected to $M_{P\ell}$, and it was proposed that the conjecture also applies to scaler fields with a potential [41, 158].

Although this conjecture definitely needs to be tested in a quantum gravity theory such as string theory, it can be well-motivated by Kaluza-Klein compactification in general relativity. Particularly, consider a $D = d + 1$ dimensional spacetime with the dth spatial dimension being compactified,

$$X^d \simeq X^d + 1. \tag{1.41}$$

One can parametrized the D-dimensional metric in the following form,

$$ds^2 = G_{MN} dX^M dX^N = e^{2\alpha\phi} g_{\mu\nu} dX^\mu dX^\nu + e^{2\beta\phi}(dX^d)^2. \tag{1.42}$$

where the μ, ν indices are for the uncompactified dimensions. Note that the size of the compactified direction is determined by ϕ,

$$2\pi R \equiv \int_0^1 \sqrt{G_{dd}}\, dX^d = e^\beta \phi. \tag{1.43}$$

With this parametrization, the Einstein-Hilbert action in the higher dimension the reduce to lower dimensional theory which has gravity plus a moduli field ϕ,

$$\int d^D X \sqrt{-G} R^{(D)} = \int d^d X \sqrt{-g} \left[R^d - \frac{1}{2}(\partial\phi)^2 \right]. \tag{1.44}$$

Suppose we have another massless scalar field $\Psi(X^M)$ in this compactified theory, we can expand it in terms of Kaluza-Klein (KK) mode $\psi_n(X^\mu)$,

$$\Psi(X^M) = \sum_{n=-\infty}^{\infty} \psi_n(X^\mu) e^{2\pi i n X^d}. \tag{1.45}$$

The equation of motion of the KK modes is then

$$\left[\partial^\mu \partial_\mu - \left(\frac{n}{R}\right)^2 \left(\frac{1}{2\pi R}\right)^{\frac{2}{d-2}} \right] \psi_n = 0. \tag{1.46}$$

We see that the KK modes form an infinite tower of states with KK mass given by

$$M_n^2 = \left(\frac{n}{R}\right)^2 \left(\frac{1}{2\pi R}\right)^{\frac{2}{d-2}}. \tag{1.47}$$

Since $R = e^{\beta\phi}/(2\pi)$, we see that if the moduli field $\phi \to +\infty$, we have an infinite tower of KK modes that becomes exponentially light.

Note that in the above field theory example, we only have light states when ϕ goes to $+\infty$ but not when $\phi \to -\infty$. However, when one consider closed string theory, in addition to usual KK modes, we also have *winding modes* that corresponds to the string winding around the compactified dimension multiple times. For the winding modes, the larger the compactified dimension, the longer the string is stretched, and hence the effective mass is larger. Particularly, we now have modes labeled by n, the KK index, and w, the number of times where the string winds around the compactified dimension. The mass square of these states is given by

$$M_{n,w}^2 = \left(\frac{1}{2\pi R}\right)^{\frac{2}{d-2}} \left(\frac{n}{R}\right)^2 + (2\pi R)^{\frac{2}{d-2}} \left(\frac{wR}{\alpha'}\right)^2, \tag{1.48}$$

where α' is related to the string tension T by $T = 1/(2\pi\alpha')$. We now see that if $\phi \to -\infty$ so that the size of the compactified dimension shrinks, the winding modes, characterized by the second term in the above equation, become exponentially light. So no matter which direction ϕ goes, as long as it approaches infinite distance, either the KK modes, or the winding modes become exponentially light. In this string theory example, one can notice the close relation between the distance conjecture and the well-known T-duality, namely the theory is the same under the duality transformation

$$R \longleftrightarrow \frac{\sqrt{\alpha'}}{R}. \tag{1.49}$$

This gains our confidence on the conjecture.

The Weak Gravity Conjecture

The weak gravity conjecture (WGC) was proposed about a decade ago [27] and has been a topic of active research for the past few years. Intuitively, the conjecture essentially states that the gravity is the weakest force. The conjecture actually has multiple closely related versions. For instance, one version of WGC states that a theory with $U(1)$ gauge symmetry must contain a charged particle with mass m and charge q such that

$$m \leq \sqrt{2} g q M_{P\ell}. \tag{1.50}$$

where g is the gauge coupling. Another version of WGC states that the cutoff scale Λ of the EFT is bounded by the gauge coupling g

$$\Lambda \lesssim g M_{P\ell}. \tag{1.51}$$

Note that WGC is closely related to the no continuous global symmetry conjecture. Particularly, when $g \to 0$, the gauge symmetry becomes global and there is no energy regime where the EFT is valid according to the WGC.

There are several ways to motivate the WGC. But to make connection with the distance conjecture, let us consider the KK compactification again, but this time with a little bit different parametrization for the higher dimensional metric,

$$ds^2 = e^{2\alpha\phi} g_{\mu\nu} dX^\mu dX^\nu + e^{2\beta\phi} \left(dX^d + A_\mu dX^\mu \right)^2 . \tag{1.52}$$

After dimensional reduction we have

$$\int d^D x \sqrt{-G} R^{(D)} = \int d^d X \sqrt{-g} \left[R^{(d)} - \frac{1}{2} (\partial\phi)^2 - \frac{1}{4} e^{-2(d-1)\alpha\phi} F_{\mu\nu} F^{\mu\nu} \right], \tag{1.53}$$

where we can see that in the lower dimensional theory, A_μ becomes the gauge field with a gauge coupling determined by the moduli field ϕ,

$$g = e^{-2(d-1)\alpha\phi}. \tag{1.54}$$

We see that when $\phi \to \infty$, the gauge coupling goes to zero, which implies the breakdown of the EFT from WGC. We see that WGC is indeed closely related to the distance conjecture.

The de Sitter Swampland Conjecture

Ever since the discovery of current accelerating expansion, constructing de Sitter solution in string theory has become an important topic for string theorists. On the other hand, as inflation opens the window to probe physics at very high energy regime, it is also tempting to construct inflationary models in string theory. For a review for these topics, see [38]. The attempts to construct de Sitter solutions in string theory [58, 111, 146] have lead to the notion of the string landscape. The landscape consists of an enormous number of vacua, each described by different low-energy EFTs of different fields and parameters.

Constructing de Sitter/inflationary solution is string theory is a rather difficult task. Part of the reason is that in supergravity, which is the low energy EFT of string theory, the vacuum of the scalar fields is generically negative. In order to lift up the vacuum, one requires supersymmetry breaking, which often involves non-perturbative effects whose calculations are not fully controllable. In addition, compactification of the extra dimensions entails many moduli fields, just like the KK compactification we previously discussed. There are not so many known mechanisms to achieve moduli stabilization and so far there is still no de Sitter solution in string theory that is technically exact without making certain assumptions, even though in some well-known constructions the assumptions are reasonable. For a recent discussion on the obstruction of constructing de Sitter solution in string theory, see [87].

In light of this, one may adopt the view that perhaps there is no de Sitter solution in string theory. Recently, a quantitative version of this statement, dubbed the *de Sitter swampland conjecture*, was proposed [185], which states that the scalar potential of a low-energy limit of quantum gravity must satisfy

$$M_{P\ell} |\nabla V| \geq c\, V, \qquad c \approx O(1) > 0 \tag{1.55}$$

where ∇ denotes the gradient with respect to the field space, and the norm of the gradient is defined by the metric on field space. Certainly, the validity of this conjecture is under heated debate, and its relation with other swampland conjectures is still unclear. Since the intricate web formed by different conjectures is one of the main reason that makes the whole swampland program attractive, it's important to find such connection. One possible connection [130] with the distance conjecture is that when the field goes to large value, the mass of an infinite tower of states becomes exponentially light. One can then think that the potential of the scalar field, which is limited by the cutoff scale Λ or the mass of the lightest heavy particle, has the asymptotic behavior $V \sim e^{-\alpha|\phi|}$ when $|\phi| \to \infty$. In this region, the dS conjecture is then satisfied if the constant α is of order one. However, this argument only applies to the asymptotic region and local minimum that violates the dS conjecture is still possible when $|\phi|$ is not large. Certainly, there is still a lot of research need to be done in order to establish a more concrete connection between the dS conjecture and other swampland conjectures.

That being said, if dS swampland conjecture holds true, it has dramatic phenomenological consequence. An immediate phenomenological implication of the dS conjecture is that the potential does not allow a local minimum of positive potential energy. In other words, a positive cosmological constant is forbidden by the conjecture. As an alternative, the current accelerating expansion can be explained by a canonical scalar field, dubbed quintessence, rolling down the non-zero slope of the potential. Whether one can construct quintessence models in string theory, or at least in supergravity, then becomes an important non-trivial test of the dS swampland conjecture. The second part of this dissertation will focus on quintessence model building in supergravity, and the phenomenological implications of swampland conjectures on inflation and dark energy.

Chapter 2

New Inflation and Typicality of the Observed Cosmic Perturbation

This chapter is based on the work with Keisuke Harigaya [69]. I would like to thank Keisuke for his guidance and collaboration throughout this project and beyond.

2.1 Introduction

Inflationary paradigm not only solves the horizon and flatness problem [118] (see also [154]), but also elegantly explains the nearly scale-invariant and Gaussian cosmic perturbation imprinted in the cosmic microwave background (CMB) and the large scale structure of the universe [180, 128, 210, 120, 33], given that inflation is driven by a scalar field with a very flat potential [165, 20] (see also [209]). However, despite the phenomenological success of the generic paradigm, the underlying physical origin of cosmic inflation is still an open problem.

We investigate the inflation paradigm in the view point of the string landscape (see [211] for a review). The string theory predicts that there are numerous vacua, and each vacuum yields an effective field theory with a different set of fields and parameters. An example leading to various cosmological constants is given in [58]. The landscape of vacua supports the notion of the anthropic principle. The parameters of the nature which we observe is not necessarily explained by the dynamics of the theory, but may be chosen so that the human civilization can exist. There would be multiple vacua on which we can live. We can calculate the distribution function of the parameters sampled from those habitable vacua weighted by the number of observers in the vacua. The parameter we observed would be around the most plausible one (the principle of mediocrity [220]). This notion succeeded in predicting a rough value of the cosmological constant [224].

In the landscape the expected inflationary dynamics is the following [103, 119]. The universe would be initially inhomogeneous, with length/energy scales set by the fundamental scale. A scalar field resides in a meta-stable vacuum and the potential energy eventually dominates the universe, driving a *precedent inflation* which erases the inhomogeneity. The scalar field tunnels toward the vacuum with a small potential energy, and the universe becomes open and curvature dominated [79, 114]. For habitability, inflation with a sufficient number of e-foldings must occur afterward, since otherwise the galaxy formation is prevented [167, 221]. Then the flatness of the inflaton potential is not necessarily the one to be explained by the property of the theory, but may be as a result of the anthropic selection. Still, we should ask if the small, $P_\zeta \sim 10^{-9}$, and nearly scale-invariant, $n_s \sim 0.96$, cosmic perturbation [13] is a plausible one. We investigate this question by considering the inflationary dynamics as well as the post-inflationary evolution of the universe.

Anthropic arguments from the post-inflationary evolution alone do not seem strong enough to enforce the amplitude of primordial perturbation power spectrum $P_\zeta \sim 10^{-9}$. A larger energy density from cosmological constant ρ_Λ requires a larger primordial perturbation so that structure can be formed in our universe. In particular, the density contrast at the time of matter-dark energy equality needs to be larger than a certain threshold to allow structure formation [224, 173, 35]

$$\left(\frac{\delta\rho}{\rho}\right)_{\text{M-DE eq}} = \left(\frac{\delta\rho}{\rho}\right)_{\text{R-M eq}} \left(\frac{a_{\text{M-DE eq}}}{a_{\text{R-M eq}}}\right) \propto \left(\frac{\delta\rho}{\rho}\right)_* \left(\frac{\Omega_M^{(0)}}{\Omega_\Lambda^{(0)}}\right)^{1/3} > \left(\frac{\delta\rho}{\rho}\right)_{min}. \tag{2.1}$$

Here the subscripts R-M eq, M-DE eq, and $*$ denote the time of radiation-matter equality, matter-dark energy equality, and the time of horizon re-entrance respectively. We approximated the density contrast $\delta\rho/\rho$ at the time of radiation-matter equality by that at horizon re-entrance because the density contrast only evolves logarithmically during radiation dominant era. With $(\delta\rho/\rho)^2 \sim P_\zeta$, this means that for a given P_ζ, the maximum energy density the cosmological constant can have is then[1]

$$\rho_\Lambda^{max} \propto P_\zeta^{3/2}. \tag{2.2}$$

Assuming the energy density of the cosmological constant follows a uniform probability distribution

$$\int_0^{\rho_\Lambda^{max}} d\rho_\Lambda, \tag{2.3}$$

this translates to a contribution to the probability distribution of P_ζ of the form $P_\zeta^{3/2}$ which biases toward large P_ζ.

A universe with a very large P_ζ may be anthropically disfavored by the property of the galaxy [214, 215]. If $(\delta\rho/\rho)$ is too large, the galaxy would be too dense such that the time scale of orbital disruption and close encounter with nearby planets is too short. Although it is not clear what kind of encounter kills the earth-like planet, and the corresponding bound on $\delta\rho/\rho$ is uncertain, we adopt the bound of $(\delta\rho/\rho) < \mathcal{O}(10^{-4})$ [214, 215]. Although the

[1] There are other criteria proposed for the anthropic conditions for the dark energy density (see, e.g., [36, 57]), which can lead to different powers than 3/2. In this dissertation we consider the original criterion in [224, 173].

typical value of $\delta\rho/\rho$ is $P_\zeta^{1/2}$, even if $P_\zeta > \mathcal{O}(10^{-8})$, it is still possible that we live in a part of the universe with a small energy contrast. Assuming the probability distribution to populate at the region with a density contrast $\delta \equiv \delta\rho/\rho$ in a universe with a primordial perturbation amplitude P_ζ is Gaussian, the probability to be in the habitable region is[2]

$$\int_0^{10^{-4}} d\delta \, \frac{1}{\sqrt{2\pi P_\zeta}} e^{-\frac{\delta^2}{2P_\zeta}} \simeq \int_0^{10^{-4}} d\delta \, \frac{1}{\sqrt{2\pi P_\zeta}} \propto P_\zeta^{-1/2}. \tag{2.4}$$

In Figure 2.1 we schematically summarize the probability distribution $\mathcal{P}_{\rm post}(P_\zeta)$ of having a universe with primordial perturbation amplitude P_ζ based on the anthropic consideration of post-inflationary evolution we discussed. We see that by just considering the post-inflationary evolution, the observed value $P_\zeta \sim 10^{-9}$ already require a fine-tuning of about a few percent. In addition, to obtain the full probability of having a P_ζ, one also needs to consider the probability stemming from inflation dynamics. Particularly, the net probability distribution is of the form

$$\mathcal{P}_{\rm net} = \mathcal{P}_{\rm post}\mathcal{P}_{\rm inf}. \tag{2.5}$$

In doing so, as most of the realistic measures suggest [166, 55, 93, 56, 184, 108], we do not weight the increase of the volume due to inflation. If the inflation scale V is simply given by a mass parameter, it is biased toward the fundamental scale. Then $P_\zeta = \frac{V}{24\pi^2\epsilon}$ is also biased toward larger values. For example, in Sec.2.4 we consider a generic small field inflation model with Z_2 symmetry and find that the probability from anthropic consideration on inflation alone strongly bias toward large P_ζ with $P_{\rm inf}(P_\zeta) \propto P_\zeta^{19/4}$. In this type of model the small primordial perturbation is highly implausible. An inflation model with P_ζ not biased toward large values is required.

A spectral index close to unity is apparently challenging. The spectral index n_s is given by[3]

$$n_s = 1 + 2\eta - 6\epsilon, \;\; \eta \equiv \frac{V''}{V}, \;\; \epsilon \equiv \frac{1}{2}\left(\frac{V'}{V}\right)^2. \tag{2.6}$$

Having a spectral index $n_s \sim 0.96$ requires $\eta \sim 0.02 \ll 1$. In order to explain the nearly-scale-invariant spectrum, we essentially need to solve the η-problem [190, 137, 113, 84, 83]. It is not obvious if the requirement of the large enough number of e-foldings can ensure such small η parameter.

Ref. [213] investigates the distributions of P_ζ and n_s, assuming that the inflaton potential obeys a Gaussian distribution, and find that the observed values are highly implausible unless the inflaton field value is as large as the Planck scale. Ref. [174] investigates the inflection

[2]The property of the galaxy may depend on P_ζ. For example, for larger P_ζ the formation of proto-galaxies occurs earlier, which will change the initial metallicity of the galaxy. We do not consider this effect in this dissertation.

[3]In this chapter we will dropped the subscript and denote potential slow-roll parameters as ϵ and η.

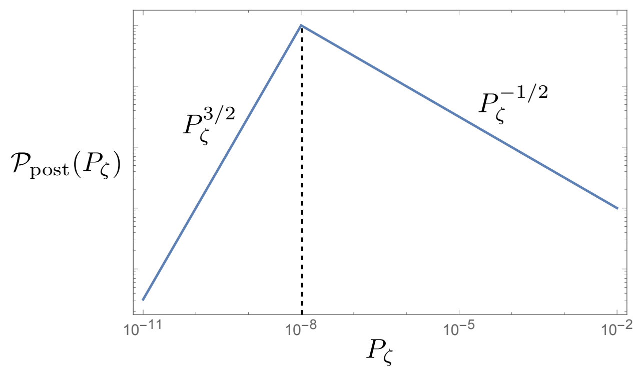

Figure 2.1: The schematic summary of the probability distribution $\mathcal{P}_{\rm post}(P_\zeta)$ of having a primordial perturbation amplitude P_ζ from anthropic consideration of post-inflationary evolution.

point inflation also assuming the Gaussian distribution. In this set up the number of the e-folding tends to be larger for a small and positive η parameter. After imposing the anthropic requirement, the spectrum tends to be blue, but the probability of $n_s < 0.97$ is found to be about 0.2, which is reasonably high. However, the distribution of P_ζ is not discussed.

In this chapter we investigate the distributions of P_ζ and n_s for a new inflation model where the inflaton is trapped around the origin during the precedent inflation by a Hubble induced mass. Although the field value of the inflaton is homogeneous inside the horizon because of the damping during the precedent inflation, quantum fluctuation of long-wave length modes is produced, which effectively works as a homogeneous but non-zero initial condition of inflation, unless the Hubble induced mass is much larger than the Hubble scale to suppress the quantum fluctuation. We show that this probabilistic nature of inflaton initial condition is an important key to understand n_s close to unity. We focus on a supersymmetric model. As we will see, the smallness of P_ζ is then also explained, as some of the parameter of the theory can be biased toward small values or logarithmically distributed in supersymmetric theories. Our results are summarized in Figure 2.9, where we show the probability distribution $P_\zeta \mathcal{P}_{\rm net}(P_\zeta, k)$ of P_ζ and $k \simeq -\eta$, taking into account of both inflationary and post-inflationary dynamics. In the contour plot we can see that the probability distribution is biased toward smaller k value and hence the η-problem is solved. In addition, with an anthropic bound on the density contrast $\delta\rho/\rho < \mathcal{O}(10^{-4})$, the observed universe with $P_\zeta \sim 10^{-9}$ and $n_s \simeq 0.965$, which is marked by the blue star in Figure 2.9, is actually a typical one.

This chapter is organized as follows. In the next section we first elaborate on the necessity of including the probability distribution of inflaton initial condition in generic new inflation models. We then consider a supersymmetric model and parametrize the probability distributions of the couplings. We find that, for a certain distribution of the couplings of the model, the observed small ($P_\zeta \sim 10^{-9}$) and scale-invariant ($n_s \simeq 0.96$) curvature perturbation is probabilistically favorable. In Sec.2.4 we show our study on the general new-inflation-type model with Z_2 symmetry, where observed spectral index n_s is probabilistically favored but the smallness of primordial perturbation cannot be explained. We then discuss and summarize the results of this chapter in Sec.2.5.

2.2 New Inflation in the Landscape after Quantum Tunneling

In this section we consider new-inflation-type models in the landscape. We assume that the last inflation which explains the flatness of the universe and the observed cosmic perturbation is a new-inflation-type model with an inflaton ϕ. In the theory with multiple vacua, it is expected that a singlet scalar field χ stays at its metastable vacuum and drives a precedent inflationary expansion, leading to a homogeneous universe. After the quantum tunneling of the singlet scalar field, the universe becomes an infinite open curvature dominated Friedmann-Robertson-Walker (FRW) universe while the scalar field rolls down to a local minimum with a small potential energy. The universe is eventually dominated by the potential energy of the inflaton (Figure 2.2). One may naively expect that a coupling between the χ field and the inflaton (leading to so-called the Hubble induced mass) can trap the inflaton to the origin and the initial inflaton field value ϕ_i is automatically small enough to initiate the last inflation. This is generically not true. As we will see, after the tunneling the Hubble induced mass of the inflaton is not effective. Therefore the inflaton fluctuation mode that just exited the horizon before quantum tunneling may survive. Although the inflaton field value is homogeneous inside the horizon, the field value must be fine-tuned for the last inflation to occur and last long enough. We investigate the impact of this observation by computing the distribution function of the curvature perturbation P_ζ and the spectral index n_s (equivalently the η parameter) after requiring enough number of e-folds $\mathcal{N}_e^{tot}$ during the last inflation. We find that P_ζ as well as η may be biased toward small values, explaining the observed very small ($P_\zeta \sim 10^{-9}$) and scale-invariant ($n_s \simeq 0.96$) curvature perturbation.

Hubble Induced Mass and the Initial Condition after Tunneling

Let us follow the dynamics of the singlet scaler field χ and the inflaton ϕ before the inflation starts. The mass of the inflaton in general depends on the energy density of the universe. Its evolution is summarised in Figure 2.3.

When the singlet field is at its metastable vacuum $\chi_{\rm pre}$, the potential energy V_χ of the singlet field dominates and the universe is in a precedent inflationary expansion. In the meanwhile, the inflaton acquires a Hubble induced mass and can be driven toward $\phi = 0$. For example, in supergravity when the potential energy is dominated by the potential of the

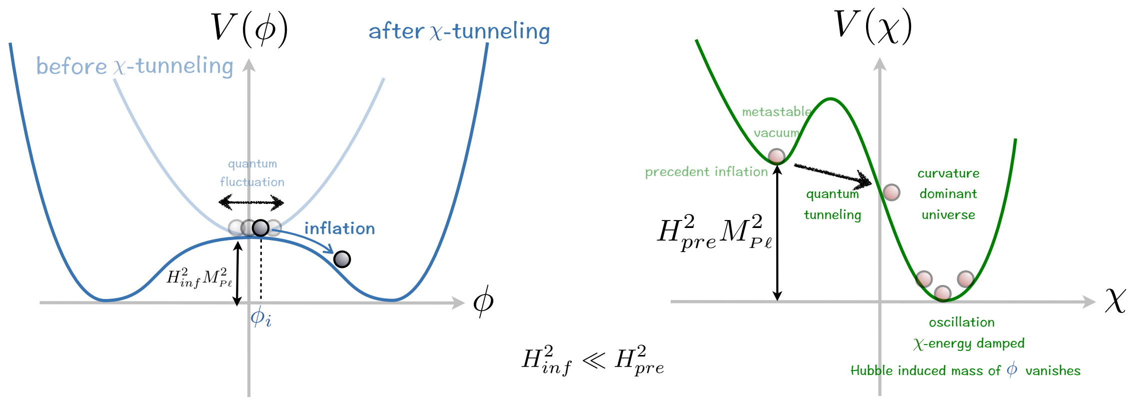

Figure 2.2: Schematic figure of the proposed scenario.

moduli field V_χ, the potential includes

$$V \supset \frac{V_\chi}{M_*^2}|\phi|^2, \tag{2.7}$$

where M_* is the cutoff scale. We expect that $V_\chi \sim M_*^4$, and hence the Hubble induced mass of the inflaton, $m_\phi(\chi)$, during the precedent inflation is as large as M_*. The similar is true for non-supersymmetric theories. We expect a coupling of the form

$$M_*^2 f(\frac{\chi}{M_*})\phi^2, \tag{2.8}$$

where f is some function, leading to the Hubble induced mass of $\mathcal{O}(M_*)$. The Hubble scale $H_{\rm pre}$ is on the other hand of $\mathcal{O}(M_*^2/M_{P\ell}) \lesssim M_*$. If the Hubble induced mass is positive, the inflaton is driven toward $\phi = 0$.

After the quantum tunneling of the singlet field χ (denoted as a_0 in Figure 2.3), the universe is dominated by the curvature energy density ρ_K and the singlet field χ is fixed by the Hubble friction. Because the Hubble induced mass m_ϕ^2 is proportional to ρ_χ, we have $m_\phi^2 \lesssim H^2 = \rho_K/3M_{P\ell}^2$ right after the tunneling. Note that the curvature energy density alone does not give a Hubble induced mass term.[4] As the universe expands, ρ_K decreases and when it becomes smaller than $m_\chi^2 M_{P\ell}^2 \sim M_*^2 M_{P\ell}^2$, the singlet field χ starts to roll down to the global minimum and oscillates.[5] At this point the mass of the inflaton is as large as the Hubble scale. However, since the energy density of the singlet field decreases as a^{-3}, the mass of the inflaton $m_\phi \sim \rho_\chi^{1/2}/M_{P\ell}$ does not exceed the Hubble scale of the expansion and

[4]The coupling $|\phi|^2 R$, where R is the Rich scalar, gives a Hubble induced term through a potential energy of the universe.

[5]When ρ_K is larger than $M_*^2 M_{P\ell}^2$, the inverse of the size of the horizon exceeds the cut off scale M_* and the validity of the effective field theory is questionable. The discussion here is applicable even if ρ_K after the tunneling is as small as $M_*^2 M_{P\ell}^2$.

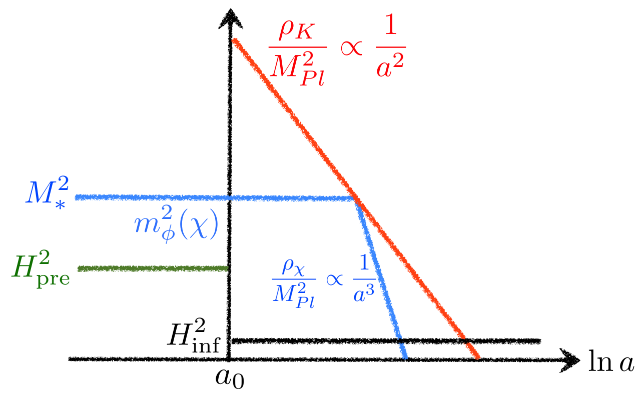

Figure 2.3: The evolution of inflaton's Hubble induced mass.

hence the inflaton can be regarded as massless after the tunneling. When ρ_K drops below the inflaton potential energy $H_{\rm inf}^2 \sim \rho_\phi / M_{P\ell}^2$, the inflation begins.

Let us discuss the evolution of the fluctuation of the inflaton based on the above observation. Expanding the field into comoving momentum modes $\phi = \int \frac{d^3k}{(2\pi)^3} \phi_{\vec{k}} e^{i\vec{k}\cdot\vec{x}}$, the modes fluctuate with decreasing amplitude as the spacetime expands. During the precedent inflation era, after a mode exit the comoving horizon, $k = aH_{\rm pre}$, the amplitude is continually damped because of the Hubble induced mass and eventually vanishes in the superhorizon limit $aH_{\rm pre} \gg k$. Hence, right after tunneling, the superhorizon mode that has the largest amplitude is the one that exited the horizon right before tunneling. This mode has an amplitude

$$\delta\phi_{\rm pre} \simeq H_{\rm pre}{pre}\sqrt{\frac{H_{\rm pre}}{m_\phi(\chi_{\rm pre})}} \simeq \frac{M_*^{5/2}}{M_{P\ell}^{3/2}}. \tag{2.9}$$

The horizon after the tunneling resides inside the horizon before the tunneling. The comoving horizon $1/aH$ remains constant during curvature dominant era, and hence there is no horizon entrance nor exit. Thus inflaton fluctuation modes inside the horizon continue to be suppressed, while the long wavelength superhorizon modes are frozen as the inflaton is essentially massless during the curvature dominant era. Those frozen modes effectively work as the zero mode ϕ_i which obeys a Gaussian distribution with a zero mean and a variance $(\delta\phi_{\rm pre})^2$. ϕ_i is nothing but the initial condition of the new inflation.

In order to wipe out the curvature energy density and to have structures on the galaxy scale, the inflation needs to last long enough with an anthropic bound $\mathcal{N}_e^{tot} \gtrsim \mathcal{N}_e^{\rm ant}$. In [103], it is found that in order to have typical galaxies being formed, the comoving Hubble scale at the time of photon decoupling should satisfy

$$\frac{a_{dc}H_{dc}}{a_t H_t} > 30, \tag{2.10}$$

where the subscript t denotes the time right after the quantum tunneling.[6] With some manipulation we have

$$\frac{a_{dc}H_{dc}}{a_{ent}H_{ent}}\frac{a_*H_*}{a_{end}H_{end}}\frac{a_{end}H_{end}}{a_iH_i}\frac{a_iH_i}{a_tH_t} > 30. \tag{2.11}$$

Here $a_{ent}H_{ent}$ denotes the coving Hubble scale of the horizon re-entrance of the CMB scale, which is equal to that of horizon exit a_*H_*. $a_{end}H_{end}$ and a_iH_i are the comoving Hubble scales at the end and beginning of the inflation respectively. Because the period between the time right after quantum tunneling and the beginning of inflation is curvature dominant, the comoving Hubble scale remains the same, i.e. $(a_iH_i)/(a_tH_t)$=1. We assume the Hubble scale during the inflation is nearly constant, $H_i \simeq H_* \simeq H_{end}$. Also, the comoving Hubble scale does not evolve much between the horizon re-entrance of the CMB scale and the photon decoupling, so $(a_{dc}H_{dc})/(a_{ent}H_{ent}) \simeq 1$. Putting everything together, we then have a constraint

$$\mathcal{N}_e^{tot} > \mathcal{N}_e^{\rm ant} \simeq \mathcal{N}_e^* + 3.4\,, \tag{2.12}$$

where $\mathcal{N}_e^*$ is the number of e-foldings between the end of the inflation and the horizon exits of the CMB pivot scale.

For potentials of a new inflation type, the initial field value ϕ_i must be close to zero to have long enough inflation. For a large enough M_*, $\delta\phi_{\rm pre}$ is larger than the required initial field value and hence some tuning of the initial field value is required. As ϕ_i obeys a Gaussian distribution, which is flat for small ϕ_i, the probability distribution of ϕ_i in the region of interest is approximately uniform;

$$\mathcal{P}_{\phi_i}d\phi_i \propto d\phi_i. \tag{2.13}$$

The anthropic constraint $\mathcal{N}_e^{tot} \gtrsim \mathcal{N}_e^{\rm ant}$ leads to the upper bound $\phi_i < \phi_{\rm ant}$, where $\phi_{\rm ant}$ is the field value of the inflaton such that the number of e-foldings after the inflaton pass the field value is $\mathcal{N}_e^{\rm ant}$.

The fact that the initial condition ϕ_i has a probability distribution over a certain range instead of having to start at $\phi_i \simeq 0$ plays an important role to solve the η-problem in new inflation. Particularly, as now the inflaton tends to start from an initial condition away from zero, the anthropic constraints $\mathcal{N}_e^{tot} > \mathcal{N}_e^{\rm ant}$ requires the potential around the origin to be flatter. The η parameter is biased toward smaller values after the anthropic constraint is imposed. On the contrary, for a small enough M_* so that $\phi_i \simeq 0$ is forced, then the inflation can easily last longer than $\mathcal{N}_e^{\rm ant}$ e-folds and the anthropic constraints on $\mathcal{N}_e^{tot}$ plays no significant role. We will see this point quantitatively in the following.

A Supersymmetric New Inflation Model

In Sec.2.4 we will study a new inflation model with Z_2 symmetry, assuming that the parameters of the potential are uniformly distributed. We will find that the resultant P_ζ is

[6] The effect of spatial curvature on structure formation is also discussed in [34].

strongly biased toward a large value, and the observed one is probabilistically disfavored. Here we instead investigate a supersymmetric model where it is sensible that the parameters of the model, including the scale of the inflation, obey distributions different from uniform ones. We expect that for certain distributions of the parameters, P_ζ is biased toward small ones. In particular, we consider an R-symmetric single field new inflation model [159, 141, 126] with a discrete R-symmetry Z_{2N} is present and the superpotential

$$W = v^2\Phi - \frac{g}{N+1}\Phi^{N+1}, \tag{2.14}$$

where Φ is a chiral superfield while v and g are constants. Here and hereafter, we work in the unit where the reduced Planck scale is unity. The Kähler potential is

$$K = \Phi^\dagger\Phi + \frac{1}{4}k(\Phi^\dagger\Phi)^2\cdots, \tag{2.15}$$

where the ellipses denote higher order terms that are irrelevant to the inflationary dynamics. From Eqs.(2.14) and (2.15), the potential of the scalar component of Φ which we call φ is given by

$$\begin{aligned} V(\varphi) &= |v^2 - g\varphi^N|^2 - kv^4|\varphi|^2 + \cdots \\ &= v^4 - kv^4|\varphi|^2 - \left(gv^2\varphi^N + \text{ h.c.}\right) + \cdots . \end{aligned} \tag{2.16}$$

In terms of the radial and angular components, $\varphi = \frac{\phi}{\sqrt{2}}e^{i\theta}$, the potential can be rewritten as

$$V = v^4 - \frac{1}{2}kv^4\phi^2 - \frac{g}{2^{\frac{N-2}{2}}}v^2\phi^N\cos(N\theta). \tag{2.17}$$

For simplicity we assume that the inflaton has an initial condition around $\theta = 0$ mod $2\pi/N$ (which are minima along the angular direction) and focus only on the radial direction.

In Sec.2.4 we study general new-inflation-type models with Z_2 symmetry. Here the resulting potential has the form of Eq.(2.78) without the c_m perturbation term. Using Eqs.(2.85) and (2.86) with $a = v^4$, $b = kv^4$, $c_n = gv^2/2^{(N-2)/2}$ and $c_m = 0$, we have

$$n_s \simeq 1 - 2k - 2N(N-1)kf_N^* \quad , \tag{2.18}$$

$$P_\zeta = \frac{\left[g^2v^{4(N-3)}k^{-2(N-1)}f_N^{*\,-2}\right]^{\frac{1}{N-2}}}{24\pi^2\left(1+Nf_N^*\right)^2}, \tag{2.19}$$

where

$$f_N \equiv f_N(k,\mathcal{N}_e) = \frac{1}{N}\frac{1}{\left(\left[1+(N-1)k\right]e^{(N-2)k\mathcal{N}_e}-1\right)} \tag{2.20}$$

and $f_N^* = f_N(k,\mathcal{N}_e^*)$ in which $\mathcal{N}_e^*$ is the number of e-folds between the horizon-exit of the CMB scale and the end of inflation.

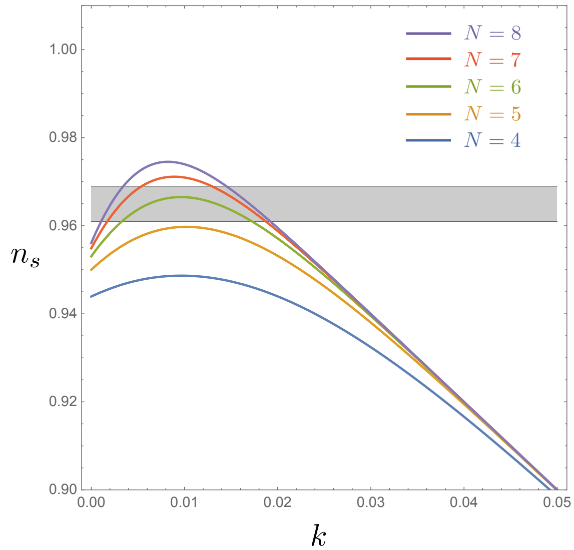

Figure 2.4: The $n_s - k$ plot for $N = 4, 5, 6, 7$ and 8, with $\mathcal{N}_e^* = 52$.

The spectral index n_s, as given in Eq.(2.18), is a function of the parameter k, and the number of e-folds $\mathcal{N}_e^*$ between the horizon exit of the CMB scale and the end of inflation. Assuming instant reheating, this is determined by the inflation energy scale,

$$\mathcal{N}_e^* \simeq 62 + \ln\left(\frac{v}{M_{P\ell}}\right). \tag{2.21}$$

Using Eq.(2.19) with $k = 0.01$ and the observed P_ζ to get an estimate on v, we have $\mathcal{N}_e^* \simeq 52$. Setting $\mathcal{N}_e^* = 52$, we plot the spectral index n_s as a function of k in Figure 2.4. One can see that $N = 4$ and 5 are ruled out by the observation, while $N = 6$ fits the observation very well in a certain region of k. Even if we relax the assumption of instant reheating, such that $\mathcal{N}_e^* < 52$, the maximum $n_s(k)$ of $N = 6$ case lies in the observational allowed region unless the reheating temperature is very small. In below we will frequently use $N = 6$, i.e. the model with discrete Z_{12} R-symmetry, as a reference point.

Probability Distribution of the Observables

A natural question now arises: what is the probability for k to lie in the region that yields observationally allowed n_s? From Figure 2.4 we see that the observed n_s requires k to be of the order of 0.01, while in general k would be much larger. Note that the slow-roll parameter η is related to k by $\eta = -k$. Therefore, making the observed spectral index $n_s \sim 0.96$ probabilistically favorable is equivalent to solving the η-problem, and this requires a probability distribution that biases toward small k.

To investigate the probability distribution function of the observables, we need to first make assumptions on the probability distribution of the Lagrangian parameters. As k is a coupling in the Kähler potential, it would be natural that k obeys a uniform probability distribution $\mathcal{P}(k)dk \propto dk$. The parameters v^2 and g are superpotential couplings and may obey distributions different from uniform ones. Furthermore, those parameters are related with the vacuum expectation value (vev) of the superpotential $W_0 \equiv \langle W \rangle \sim v^{2(1+1/N)} g^{-1/N}$, which is related with the electroweak scale and the cosmological constant. Let us start from the distribution of v^2, g, the supersymmetry breaking scale F and the μ term of the electroweak Higgs,

$$dv^2 v^{2p'} \times dg g^{q'} \times dF F^r \times d\mu \mu^s. \tag{2.22}$$

If a parameter is given by a dimensional transmutation, the index (p', q', r, s) of the distribution is -1, while it is 1 if the parameter is a complex parameter biased toward a large value. The electroweak scale $v_{\rm EW}^2$ is given by

$$v_{\rm EW}^2 \simeq F^2/M_{\rm med}^2 - \mu^2, \tag{2.23}$$

where $M_{\rm med}$ is the mediation scale of the supersymmetry breaking. We assume that the electroweak scale must be in a certain range close to the observed one[7] as is argued in [16, 123],

$$c v_{{\rm EW},obs}^2 < v_{\rm EW}^2 < c' v_{{\rm EW},obs}^2, \tag{2.24}$$

where c and c' are constants which we do not have to specify. The scanning over the μ parameter yields

$$\int_{c v_{{\rm EW},obs}^2 < v_{\rm EW}^2 < c' v_{{\rm EW},obs}^2} d\mu \mu^s \simeq v_{{\rm EW},obs}^2 \left(\frac{F}{M_{\rm med}} \right)^{s-1} \propto F^{s-1}, \tag{2.25}$$

where we have used $F/M_{\rm med} \simeq \mu \gg v_{{\rm EW},obs}$ as is suggested by the non-discovery of supersymmetric particles so far. The cosmological constant is given by

$$\rho_\Lambda = F^2 - 3 W_0^2 M_{P\ell}^2, \tag{2.26}$$

where W_0 is the vev of the superpotential. A change of variables gives gives

$$\int dF F^{r+s-1} \propto W_0^{r+s-2} d\rho_\Lambda. \tag{2.27}$$

Here we have used $F^2 \simeq 3 W_0^2 M_{P\ell}^2 \gg \rho_\Lambda$. We omit the measure $d\rho_\Lambda$, which leads to the uniform distribution of the cosmological constant, in the following. Using the relation $W_0 \sim v^{2(1+1/N)} g^{-1/N}$, the distribution of v^2 and g are given by

$$dv^2 v^{2p} \times dg g^q, \quad p = p' + (1 + \frac{1}{N})(r + s - 2), \quad q = q' - \frac{1}{N}(r + s - 2). \tag{2.28}$$

For $-1 \leq p', q', r, s \leq 1$, a wide range of (p, q) can be obtained.

[7] This assumption is not crucial for our discussion. Without the anthropic constraint on the electroweak scale, the distribution is given by Eq.(2.28) with $s = 1$.

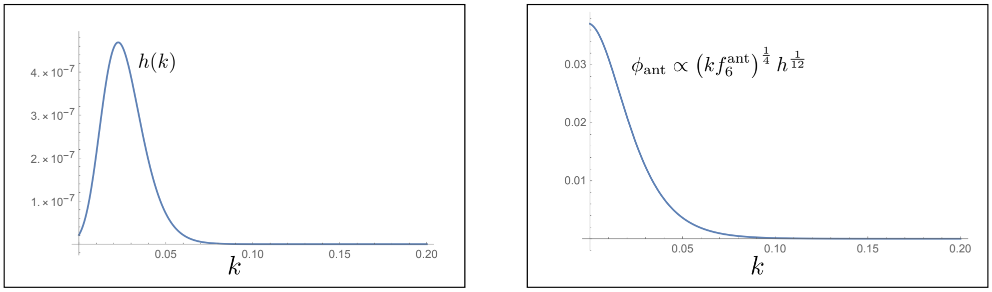

Figure 2.5: The function $h(k)$ (left) and the contribution of $\phi_{\rm ant}$ to the probability distribution of k (right). We take $N = 6$ and $\mathcal{N}_e^* = 52$.

Putting everything together, the probability distribution of the parameters to start with is

$$\int d\phi_i \, dk \, dg \; dv^2 \; g^q \, v^{2p}. \tag{2.29}$$

The distribution of the initial condition ϕ_i is uniform as discussed in Sec.2.2. If the anthropic bound $\phi_{\rm ant}$ is smaller than the amplitude of quantum fluctuation $\delta\phi_{\rm pre}$ given by Eq.(2.9), then the integration of $d\phi_i$ ranges from 0 to $\phi_{\rm ant}$. On the other hand, if $\phi_{\rm ant} > \delta\phi_{\rm pre}$, the integration of $d\phi_i$ is capped by $\delta\phi_{\rm pre}$ and the anthropic constraint is no longer effective. In other words, integrating out $d\phi_i$ yields

$$\int dk \, dg \; dv^2 \; g^q \, v^{2p} \phi_{\rm bound}, \qquad \phi_{\rm bound} \equiv \min[\phi_{\rm ant}, \delta\phi_{\rm pre}]. \tag{2.30}$$

Using Eqs.(2.82), (2.87) and (2.19), the field value $\phi_{\rm ant}$ is given by

$$\begin{aligned} \phi_{\rm ant} &= \sqrt{2}\, g^{-\frac{1}{N-3}} k^{\frac{2}{N-3}} f_N^{*\,\frac{1}{(N-3)(N-2)}} \left[24\pi^2 \left(1 + N f_N^*\right)^2\right]^{\frac{1}{2(N-3)}} f_N^{{\rm ant}\,\frac{1}{N-2}} P_\zeta^{\frac{1}{2(N-3)}} \\ &= \sqrt{2} g^{-\frac{1}{N-3}} f_N^{{\rm ant}\,\frac{1}{N-2}} k^{\frac{1}{N-2}} h^{\frac{1}{(N-3)(N-2)}} P_\zeta^{\frac{1}{2(N-3)}}, \end{aligned} \tag{2.31}$$

where we have defined

$$h(k) \equiv k^{N-1} f_N^* \left[24\pi^2 (1 + N f_N^*)^2\right]^{\frac{N-2}{2}} \tag{2.32}$$

for future convenience. It is instructive to understand the behavior of $h(k)$ and the contribution of $\phi_{\rm ant}$, if $\phi_{\rm ant} < \delta\phi_{\rm pre}$, to the final probability distribution of k which we plot in Figure 2.5. Most importantly, we can see that $\phi_{\rm ant}$ gives a bias toward small k, which is the key of solving the η-problem and making observed spectral index n_s probabilistically favorable.

Using the relation between v^2 and P_ζ derived from Eq.(2.19),

$$v^2 = g^{-\frac{1}{N-3}} h^{\frac{1}{N-3}} P_\zeta^{\frac{N-2}{2(N-3)}}, \tag{2.33}$$

we can perform a change of variable to obtain the probability distribution in terms of cosmic perturbation P_ζ. For the parameter region where $\phi_{\rm ant} < \delta\phi_{\rm pre}$, we have

$$\int dk\, dg\, dv^2\; g^q\, v^{2p}\phi_{\rm ant} \tag{2.34}$$

$$= \frac{(N-2)}{\sqrt{2}(N-3)}\int dk\,\frac{dP_\zeta}{P_\zeta}\,dg\,g^{\frac{q(N-3)-p-2}{N-3}}h^{\frac{(p+1)}{(N-3)}}P_\zeta^{\frac{(N-2)(p+1)}{2(N-3)}}\left[\left(kf_N^{\rm ant}\right)^{\frac{1}{N-2}}h^{\frac{1}{(N-3)(N-2)}}P_\zeta^{\frac{1}{2(N-3)}}\right], \tag{2.35}$$

where we have left the contribution from $\phi_{\rm ant}$ to k- and P_ζ-distribution in the square brackets for future convenience. We now need to integrate out g to obtain the final probability distribution. When $q(N-3) - p + N - 5 \neq 0$, the integration yields

$$\frac{(N-2)}{\sqrt{2}\,[q(N-3)-p+N-5]}\int dk\,\frac{dP_\zeta}{P_\zeta}\left[g^{\frac{q(N-3)-p+N-5}{(N-3)}}\right]_{g_{min}}^{g_{max}}h^{\frac{(p+1)}{(N-3)}}P_\zeta^{\frac{(N-2)(p+1)}{2(N-3)}}\left[\cdots\right] \tag{2.36}$$

where the ellipses in the square brackets represent the contribution from $\phi_{\rm ant}$ as in Eq.(2.35). The lower cutoff of the integral g_{min} is given by Eq.(2.33) with the natural requirement $v^2 \leq M_*^2$, i.e. the energy scale v should not be larger than the cut off scale. Particularly, we have

$$g_{min} = hP_\zeta^{\frac{N-2}{2}}M_*^{-(N-3)} \ll 1. \tag{2.37}$$

On the other hand, g_{max} is determined by the cutoff scale M_*. After restoring $M_{P\ell}$ and M_* to the superpotential, we have

$$W \supset -\frac{1}{N+1}\frac{g}{M_{P\ell}^{N-2}}\Phi^{N+1} \equiv -\frac{1}{N+1}\frac{c_g}{M_*^{N-2}}\Phi^{N+1}. \tag{2.38}$$

Assuming the dimensionless coupling c_g is bounded by unity, the coupling g is bounded by

$$g < g_{max} = \left(\frac{M_{P\ell}}{M_*}\right)^{N-2}. \tag{2.39}$$

Therefore, if $q(N-3)-p+N-5 > 0$, the $g-$integration merely gives a proportional constant and does not affect the probability distribution of k and P_ζ. We therefore have

$$\mathcal{P}_{\rm inf}(k, P_\zeta)dk\,dP_\zeta = \mathcal{P}_k\mathcal{P}_{P_\zeta}dk\,dP_\zeta = \mathcal{P}_k P_\zeta\mathcal{P}_{P_\zeta}dk\,d{\rm ln}P_\zeta \tag{2.40}$$

where $\mathcal{P}_{\rm inf}$ is the probability distribution from inflationary dynamics with

$$\mathcal{P}_k \propto h^{\frac{(p+1)}{(N-3)}}\left[\left(kf_N^{60}\right)^{\frac{1}{N-2}}h^{\frac{1}{(N-3)(N-2)}}\right], \tag{2.41}$$

$$P_\zeta\mathcal{P}_{P_\zeta} \propto P_\zeta^{\frac{(N-2)(p+1)}{2(N-3)}}P_\zeta^{\frac{1}{2(N-3)}}. \tag{2.42}$$

The quantity $P_\zeta \mathcal{P}_{P_\zeta}$ can be understood as the relative probability to obtain the curvature perturbation of $\mathcal{O}(\mathcal{P}_\zeta)$. When $q(N-3)-p+N-5=0$, Eq.(2.36) does not apply and the integration over g yields a logarithmic contribution $\ln(g_{min})$ instead. This logarithmic contribution changes the probability distribution of k and P_ζ only slightly, and we may use Eqs.(2.41) and (2.42) as a good approximation.

If $\phi_{\rm ant} > \delta\phi_{\rm pre}$, then $\phi_{\rm ant}$ plays no role and the integration of ϕ_i and g merely yields a proportional constant which do not affect the probability distribution of the observables:

$$\int dk\, dg\, dv^2\; g^q\, v^{2p} \phi_{\rm bound} \propto \int dk\, \frac{dP_\zeta}{P_\zeta} h^{\frac{(p+1)}{(N-3)}} P_\zeta^{\frac{(N-2)(p+1)}{2(N-3)}}. \tag{2.43}$$

Thus for $\phi_{\rm ant} > \delta\phi_{\rm pre}$, we obtain

$$\mathcal{P}_k \propto h^{\frac{(p+1)}{(N-3)}}, \tag{2.44}$$

$$P_\zeta \mathcal{P}_{P_\zeta} \propto P_\zeta^{\frac{(N-2)(p+1)}{2(N-3)}}. \tag{2.45}$$

Note that in the parameter region $q(N-3)-p+N-5 \geq 0$, the probability distribution is parametrized only by p but not by q.

In the parameter region where $q(N-3)-p+N-5 < 0$, the integration over g is dominated by the lower cutoff contribution, where $g = g_{min} \ll 1$. Recall that $\phi_{ant} \propto g^{-1/(N-3)}$, and hence for such a small g, ϕ_{ant} is much larger than $M_{P\ell}$. This means the inflaton field value at the CMB scale will also be much larger than $M_{P\ell}$, and therefore the assumption of small field inflation breaks down.

In Figure 2.6 we plot the distribution function using Eqs.(2.41), (2.42), (2.44) and (2.45) in the parameter region $q(N-3)-p+N-5 \geq 0$ for both $\phi_{\rm ant} < \delta\phi_{\rm pre}$ and $\phi_{\rm ant} > \delta\phi_{\rm pre}$. Let us first look at the upper left panel of Figure 2.6. We see that $\mathcal{P}_k$ is largely suppressed for large k as long as $p \geq -2$. To understand how the parameter p alters the probability distribution at large k, note that the functions f_N and h behave as

$$f_N \propto k^{-1} e^{-(N-2)k\mathcal{N}_e} \tag{2.46}$$

$$h \propto k^{N-2} e^{-(N-2)k\mathcal{N}_e} \tag{2.47}$$

when $k \to \infty$, and hence

$$\mathcal{P}_k \propto k^{\frac{(N-2)(p+1)+1}{N-3}} e^{-\frac{(N-2)(p+2)}{N-3}k\mathcal{N}_e} \tag{2.48}$$

for large k. It is therefore clear that in order to solve the η-problem, one requires $p \geq -2$ so that the distribution is suppressed for large k.

The behavior of $\mathcal{P}_k$ for negative p can be understood as follows. Recall that the primordial perturbation is given as

$$P_\zeta \sim \frac{V}{\epsilon} \sim \frac{v^4}{k^2 \phi_{\rm cmb}^2}. \tag{2.49}$$

where $\phi_{\rm cmb}$ is the inflaton field value when the CMB scale exited the horizon. Therefore, for a fixed P_ζ, smaller v^2 requires smaller $k\phi_{\rm cmb}$. For a given e-folds $\mathcal{N}_e^*$, the field value $\phi_{\rm cmb}$

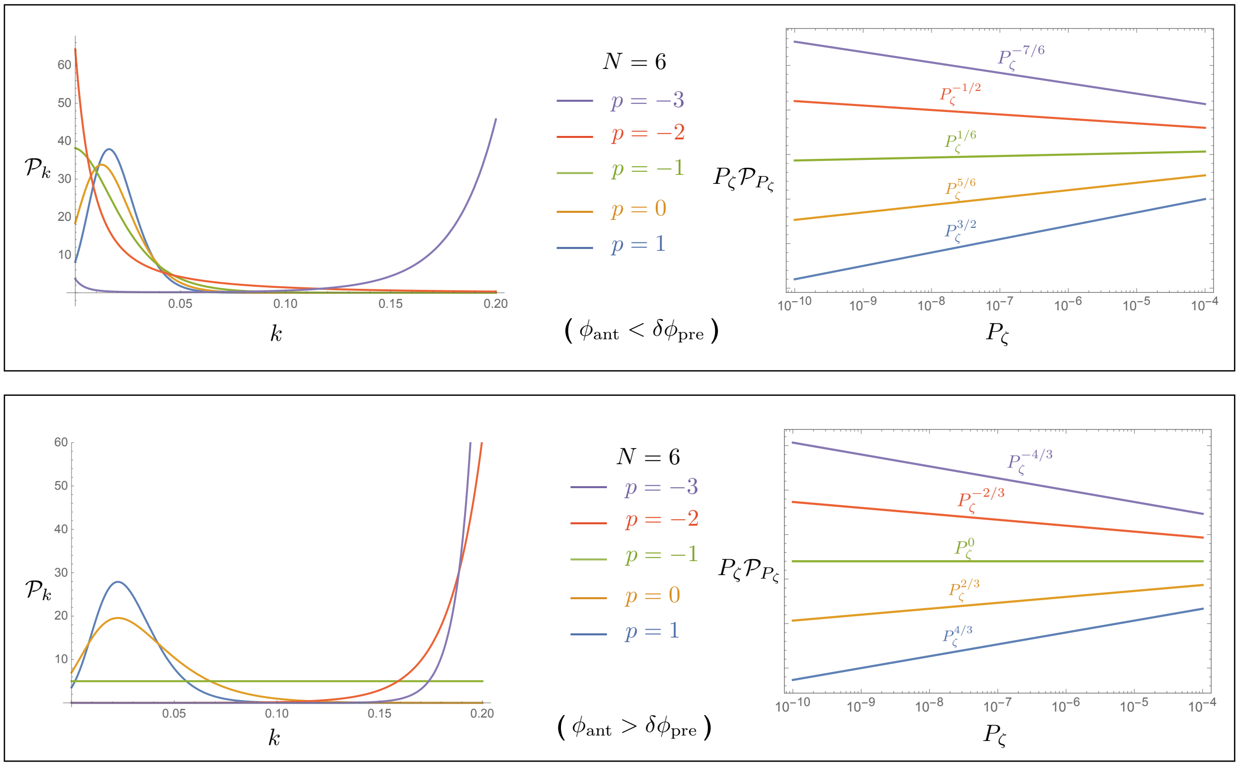

Figure 2.6: The probability distribution functions in the parameter region $q(N-3)-p+N-5 \geq 0$ when $\phi_{\rm ant} < \delta\phi_{\rm pre}$ (upper panel) and $\phi_{\rm ant} > \delta\phi_{\rm pre}$ (lower panel), respectively. We take $N = 6$ and $\mathcal{N}_e^* = 52$.

is in fact related to the parameter k. For larger k, the potential is steeper and hence $\phi_{\rm cmb}$ has to be smaller to maintain the same number of e-folds. (This relation is explicitly shown in Figure 2.13 in the Sec.2.4.) The full k-dependence of the denominator $k^2\phi_{\rm cmb}^2(k)$ is thus nontrivial, and is worked out in Eq.(2.82). It turns out that when k decreases, $k^2\phi_{\rm cmb}^2(k)$ increases. Therefore, for a given P_ζ, if v^2 is biased toward small values as when p is negative, then k is biased toward large values. This is why large k is favored when p is too negative, where the bias toward small k from $\phi_{\rm ant}$ is defeated.

So far we have only discussed the probability distribution in terms of k but not the observable n_s. Since the spectral index n_s is a function of k only, as given in Eq.(2.18), the probability distribution of k is sufficient to give the probabilistic information of n_s. That being said, the probability distribution of n_s displays an important feature of R-symmetry new inflation which we now discuss. To this end, we perform a change of variable from k to n_s,

$$\int dk\, \mathcal{P}_k = \int dn_s \left|\frac{\partial k}{\partial n_s}\right| \mathcal{P}_k(k(n_s)) \equiv \int dn_s\, \mathcal{P}_{n_s}. \tag{2.50}$$

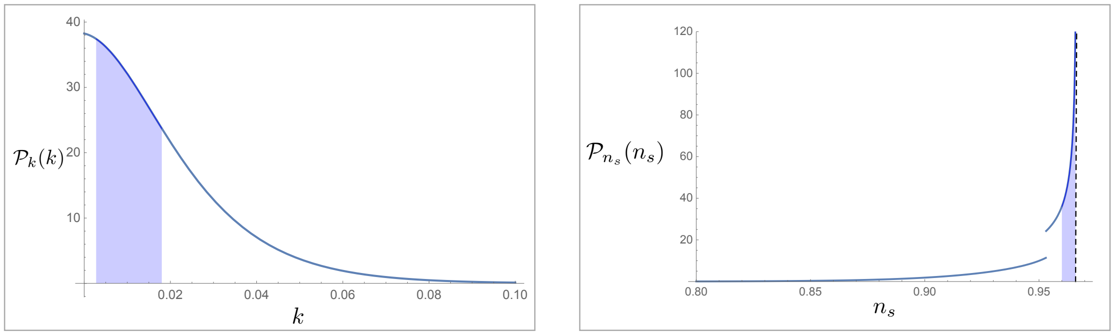

Figure 2.7: The normalized probability distribution function for k (left) and n_s (right). The shaded areas corresponds to the parameter region where $n_s > 0.96$.

As shown in Figure 2.4, the function $n_s(k)$ is not monotonic and has a maximum n_s^{max} very close to the observed value $n_s \simeq 0.965$. These two properties have important implications in $\mathcal{P}_{n_s}$ we show in Figure 2.7. The first feature of $\mathcal{P}_{n_s}$ is the jump due to non-monotonicity of $n_s(k)$. The second, and probably more important, feature of $\mathcal{P}_{n_s}$ is that n_s will never reach $n_s = 1$ and $\mathcal{P}_{n_s}$ diverges at n_s^{max} because the Jacobian factor $\left|\frac{\partial k}{\partial n_s}\right|$ diverges at n_s^{max}. Once the probability distribution of large k ($n_s \ll 1$) is suppressed due to the probabilistic nature of initial field value ϕ_i, in R-symmetry new inflation we not only can explain why n_s is very close to one, but can also predict an $n_s \neq 1$ that is near the observed value $n_s \simeq 0.965$. For $\mathcal{N}_e^* = 52$ and $p = -1$, the probability for $0.96 < n_s < n_s^{max}$ is

$$P_{0.96<n_s<n_s^{max}} = \frac{\displaystyle\int_{0.96}^{n_s^{max}} \mathcal{P}_{n_s}\, dn_s}{\displaystyle\int_{-\infty}^{n_s^{max}} \mathcal{P}_{n_s}\, dn_s} \simeq 0.48\,, \tag{2.51}$$

and the probability distribution diverges at $n_s^{max} \simeq 0.966$. Note that $p = -2$ yields a similar result.

Move on to the probability distribution of P_ζ, the upper right panel of Figure 2.6 shows that P_ζ is biased toward smaller values for a sufficiently negative p. This is simply because P_ζ is proportional to the inflation scale, and hence a bias toward small v^2 results in a bias toward small P_ζ. For $p = -2$, the perturbation P_ζ is biased toward small value strongly. For $p = -1$, P_ζ is biased toward large values only mildly. The power of $P_\zeta \mathcal{P}_{P_\zeta}$, as shown in Eq.(2.42), is given by

$$\frac{p(N-2)+(N-1)}{2(N-3)} = \frac{(p+2)(N-2)}{2(N-3)} - \frac{1}{2}. \tag{2.52}$$

In order to solve the η-problem simultaneously, we need $p \geq -2$. The most negative power we can get for the probability of P_ζ from inflationary dynamics is then $-1/2$.

As we mentioned in the introduction, the anthropic consideration on the post-inflation dynamics gives an additional bias on $P_\zeta \mathcal{P}_{P_\zeta}$ that scales as $P_\zeta^{3/2}$ for small P_ζ, and scales as $P_\zeta^{-1/2}$ for large P_ζ where the turning point is at $P_\zeta \sim 10^{-8}$. See Figure 2.1. Combining this with the contribution from inflationary evolution, we see that the power of $P_\zeta \mathcal{P}_{P_\zeta}$ should be smaller or equal to $\frac{1}{2}$, since otherwise $P_\zeta \sim O(1)$ is much more favored than $P_\zeta \sim 10^{-9}$. This requires

$$p \leq \frac{-2}{N-2}. \tag{2.53}$$

Recall that in order to solve the η-problem, one requires $p \geq -2$. Hence, to *simultaneously* solve the η-problem and explain the smallness of perturbation power spectrum, we need

$$-2 \leq p \leq \frac{-2}{N-2}. \tag{2.54}$$

The probability to obtain the observed value $P_\zeta \simeq 2.1 \times 10^{-9}$ is maximized when $p = -2$: With an anthropic bound on the density contrast $\delta\rho/\rho < \mathcal{O}(10^{-4})$ from the property of galaxies, the probability is $O(10)\%$, which is reasonable.

To show the impact of the bias from $\phi_{\rm ant}$, in the lower panel of Figure 2.6, we show the distribution functions *without* the $\phi_{\rm ant}$ contribution originated from the probabilistic nature of the initial field value ϕ_i, which is the case when $\phi_{\rm ant} > \delta\phi_{\rm pre}$. Comparing with the upper panel, we see that this does not affect the distribution of P_ζ much. However, for the distribution of k, the probability for large k is suppressed only for $p = 0$ and 1. For $p = -1$, without the additional suppression at large k from $\phi_{\rm ant}$ as illustrated in the right figure of Figure 2.5, we have a uniform distribution in k and hence it is more likely to find k to be of order 1, instead of order 0.01. Comparing both distributions in the lower panel of Figure 2.6, it is clear that without scanning the initial condition ϕ_i, it is impossible to simultaneously solve the η-problem and explain the smallness of P_ζ.

We summarize the discussion of the (p, q) parameter space for $N = 6$ in Figure 2.8. The gray-shaded region is where $q(N-3) - p + N - 5 < 0$ and the small field assumption breaks down; the red region is where the probability distribution of P_ζ biases toward large value even though the parameter k tends to be small; the orange region is the opposite, where the η-problem persists despite the smallness of the perturbation power spectrum can be explained. In between the two regions we have parameter sets that can solve both problems. Those values of (p, q) can be obtained by appropriate choice of (p', q', r, s).

In claiming the existence of viable (p, q), we assume the contribution to the distribution of P_ζ from the post-inflationary dynamics shown in Figure 2.1. If the power of the distribution at large P_ζ increases/decreases because of possible biases we have not considered, the white region in Figure 2.8 shrinks/expands. The white region exists as long as the power is smaller than 1/2.

It is worth emphasizing again that the viable parameter sets, the white region in Figure 2.8, exists because of the probabilistic nature of the inflaton initial field value ϕ_i. Without this contribution, the window between the red and orange regions is closed. We examine

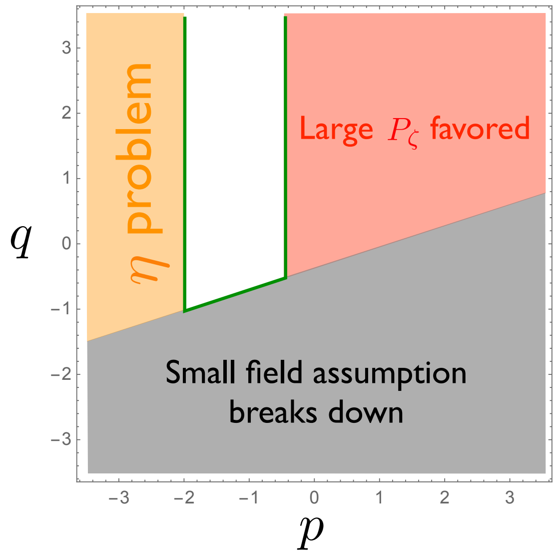

Figure 2.8: Illustration of the result in (p, q) parameter space for $N = 6$. The white region represents the parameter space where both the η-problem and the smallness of cosmic perturbation can be explained.

in which part of the parameter space is $\phi_{\rm ant} < \delta\phi_{\rm pre}$ and does the contribution of $\phi_{\rm ant}$ kick in. Recall that the amplitude of the quantum fluctuation $\delta\phi_{\rm pre}$ is proportional to $M_*^{5/2}$ as given in Eq.(2.9). On the other hand, $\phi_{\rm ant}$ also depends on M_* through the superpotential coupling g. The probability distribution then has the form

$$\begin{aligned}&\int dk\, dg\ dv^2\ g^q\, v^{2p}\, \phi_{\rm bound}(k, P_\zeta, g; M_*)\\ =&\frac{(N-2)}{\sqrt{2}(N-3)}\int dk\,\frac{dP_\zeta}{P_\zeta}\,dg\, g^{\frac{q(N-3)-p-1}{N-3}}\, h^{\frac{(p+1)}{(N-3)}}\, P_\zeta^{\frac{(N-2)(p+1)}{2(N-3)}}\ \phi_{\rm bound}(k, P_\zeta, g; M_*). \end{aligned} \tag{2.55}$$

The parameter region of interest is $q(N-3) - p + N - 5 > 0$. Therefore after integrating out g, the integration is dominated by $g_{max} = (M_{P\ell}/M_*)^{N-2}$, at which

$$\phi_{\rm bound}(k, P_\zeta; M_*) = \min\left[\phi_{\rm ant}(k, P_\zeta, g_{max}), \delta\phi_{\rm pre}\right]. \tag{2.56}$$

Combining with the contribution from the anthropic constraint discussed in the introduction,

$$\mathcal{P}_{\rm post}\,(P_\zeta) = \min\left[(P_\zeta/10^{-8})^{3/2}, (P_\zeta/10^{-8})^{-1/2}\right], \tag{2.57}$$

the net probability distribution $P_\zeta \mathcal{P}_{\rm net}(k, P_\zeta)$ in the (k, P_ζ) space that includes both inflationary and post-inflationary dynamics is proportional to

$$h^{\frac{(p+1)}{(N-3)}}\, P_\zeta^{\frac{(N-2)(p+1)}{2(N-3)}}\ \min\left[\phi_{\rm ant}(k, P_\zeta, g_{max}), \delta\phi_{\rm pre}\right] \min\left[(P_\zeta/10^{-8})^{3/2}, (P_\zeta/10^{-8})^{-1/2}\right]. \tag{2.58}$$

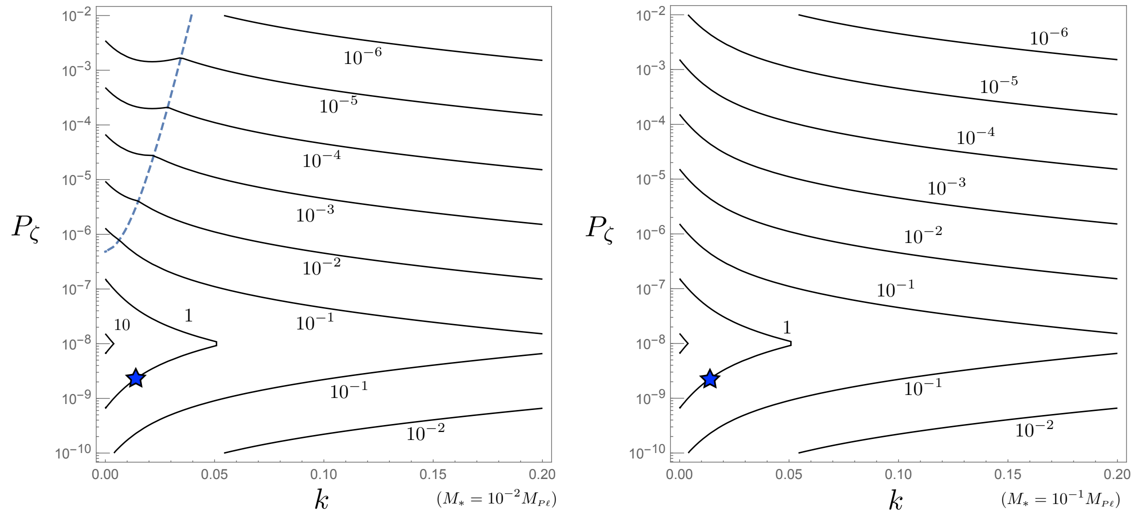

Figure 2.9: The contour plot of $\frac{P_\zeta \mathcal{P}_{\rm net}(k,P_\zeta)}{P_\zeta \mathcal{P}_{\rm net}\big|_{k_{obs},P_\zeta^{obs}}}$ for $N = 6$, $p = -2$ and $M_* = 10^{-2}M_{P\ell}$ ($M_* = 10^{-1}M_{P\ell}$) on the left (right). To the right of the blue dashed line, $\phi_{\rm ant} < \delta\phi_{\rm pre}$ and hence the anthropic constraints $\phi_i < \phi_{\rm ant}$ contributes to the probability distribution. The blue star marks the point (k_{obs}, P_ζ^{obs})

The distribution $P_\zeta \mathcal{P}_{\rm net}$ normalized with respect to $P_\zeta \mathcal{P}_{\rm net}\big|_{k_{obs},P_\zeta^{obs}}$ for $N = 6$, $p = -2$ and $M_* = 10^{-2}M_{P\ell}$ is given in the left panel of Figure 2.9, where $k_{obs} \simeq 0.0134$ and $P_\zeta^{obs} \simeq 2.2 \times 10^{-9}$ are the observed value for the parameter k and the cosmic perturbation P_ζ respectively and (k_{obs}, P_ζ^{obs}) is marked by the blue star. To the right of the blue dashed line, $\phi_{\rm ant} < \delta\phi_{\rm pre}$ and the anthropic constraints $\phi_{\rm ant}$ plays an important role to solve the η-problem. The observed point (k_{obs}, P_ζ^{obs}) lies deep inside the region and hence the proposed scenario can indeed explain the nearly scale invariant small cosmic perturbation. Note that the cusps at $P_\zeta \sim 10^{-8}$ originate from the turning point of $\mathcal{P}_{\rm post}(P_\zeta)$, while the cusps at the blue dashed line are due to $\min\left[\phi_{\rm ant}(k, P_\zeta, g_{max}), \delta\phi_{\rm pre}\right]$. The distribution for $M_* = 10^{-1}M_{P\ell}$ is given in the right panel, where the blue dashed line is absent because $\delta\phi_{\rm pre} \propto M_*^{5/2}$ is always larger than $\phi_{\rm ant}$.

2.3 Fine-tuning in General New Inflation

In this section we study fine-tuning problems in general new-inflation-type models. The only symmetry we impose here is Z_2 symmetry, where the most generic potential is of the form

$$V(\phi) = M_{P\ell}^4 \left[V_0 + \sum_{n=1}^{\infty} c_{2n} \left(\frac{\phi}{M_{P\ell}} \right)^{2n} \right]. \tag{2.59}$$

Hereafter we will work in the Planck unit where the reduced Planck mass $M_{P\ell}$ is set to unity. We study how much fine-tuning is required to yield the observed perturbation amplitude and spectrum. We assume that the probability distributions for the dimensionless coefficients V_0 and $|c_{2n}|$'s are uniform between zero and one, and vanish outside this interval. Certainly, not all possible values of c_{2n} allow inflation, as slow-roll conditions

$$\epsilon = \frac{1}{2}\left(\frac{V'}{V}\right)^2 \simeq \frac{1}{2}\left(\frac{\sum_{n=1}^{\infty} 2n\, c_{2n}\phi^{2n-1}}{V_0}\right)^2, \tag{2.60}$$

$$\eta = \frac{V''}{V} \simeq \frac{\sum_{n=1}^{\infty} 2n(2n-1)c_{2n}\phi^{2n-2}}{V_0}, \tag{2.61}$$

are violated if coefficients are too large. The primes in the above equations denote derivative with respect to ϕ. The parameter region in the $\{c_2, c_4, c_6, ...\}$-space where inflation can occur and generate the observed power spectrum is bounded by some $c_{2n}^{\max}$ for each c_{2n}. As we will see below, $c_{2n}^{\max}$'s are determined by the energy scale parameter V_0. As we assume the probability distribution is uniform, the probability to have the inflation to occur around the energy scale V_0 is

$$P \propto \int_0^{\phi_{\rm ant}} d\phi_i \int dV_0 \prod_{n=1}^{\infty}\left(\int_0^{c_{2n}^{\max}} dc_{2n}\right) \propto V_0\phi_{\rm ant}\prod_{n=1}^{\infty} c_{2n}^{\max}. \tag{2.62}$$

Note that we have simplified the problem by assuming $c_{2n}^{\max}$'s are independent on each other and a more detailed treatment will result in a probability slightly smaller than Eq.(2.62). Nevertheless, the main takeaway we can learn from such analysis will not be affected as we will explain below. Also note that we have included the probability distribution of the inflaton initial field value as advocated in Sec.2.2, and impose the anthropic constraint that inflation needs to last for more than $\mathcal{N}_e^{\rm ant}$.

To find what $c_{2n}^{\max}$ is, we need to first know the field value $\phi_{\rm end}$ when the inflation ends. This is determined by the number of e-folds $\mathcal{N}_e^*$ between horizon exit and the end of the inflation, the inflation energy scale V_0, and the perturbation power spectrum P_ζ. In particular, one has

$$\begin{aligned}\mathcal{N}_e &= \int H\, dt = \int \frac{H}{\dot\phi}\, d\phi = -\int \frac{V}{V'}\, d\phi = \int \frac{d\phi}{\sqrt{2\epsilon}} \simeq \frac{\phi_{\rm end}-\phi_{\rm cmb}}{\sqrt{2\epsilon}} = \frac{\Delta\phi}{\sqrt{2\epsilon}},\\ \Delta\phi &\equiv \phi_{\rm end}-\phi_{\rm cmb},\end{aligned} \tag{2.63}$$

where we assumed the ϵ parameter to be nearly constant over the period of inflation. Its value can be determined by the perturbation power spectrum,

$$\epsilon = \frac{1}{24\pi^2}\frac{V_0}{P_\zeta}. \tag{2.64}$$

For potentials of a new inflation type, we typically have $\Delta\phi \simeq \phi_{\rm end}$. For example, for the potential

$$V = a - \frac{1}{2}b\,\phi^2 - c_n\phi^n, \tag{2.65}$$

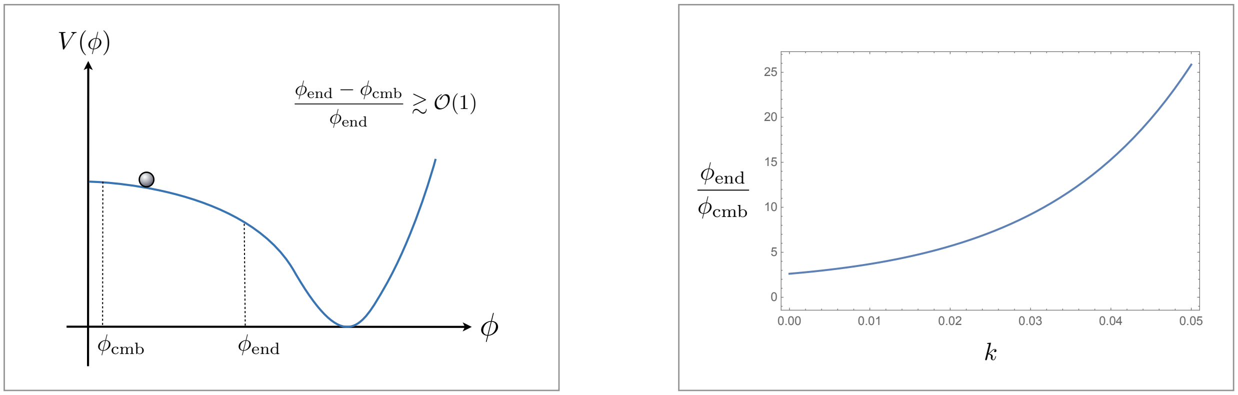

Figure 2.10: For new inflation, the inflationary expansion occur mostly on the hilltop where ϕ is close to the origin. The inflation ends at $\phi_{\rm end}$ where the potential violates the slow-roll condition and $\phi_{\rm end} \gg \phi_{\rm cmb}$.

for which the relation between $\phi_{\rm end}$ and $\phi_{\rm cmb}$ is explicitly computed in Sec.2.4, the ratio $\phi_{\rm end}/\phi_{\rm cmb}$ depends only on the parameter $k \equiv b/a$ and is plotted in the right of Figure 2.10. We see that the ratio grows for larger k, which is not surprising as a larger k requires a smaller $\phi_{\rm cmb}$ to maintain the same e-folds of inflation. As $\Delta\phi = \phi_{\rm end} - \phi_{\rm cmb}$ and $\phi_{\rm end} \gtrsim \phi_{\rm cmb}$, we actually have $\Delta\phi \simeq \phi_{\rm end}$ and hence

$$\phi_{\rm end} \simeq \frac{\mathcal{N}_e^*}{\sqrt{12\pi}} \sqrt{\frac{V_0}{P_\zeta}}. \tag{2.66}$$

We define c_{2n}^{ϵ} to be the value of c_{2n} such that the $c_{2n}\phi_{\rm end}^{2n}$ term alone in Eq.(2.60) can violate the ϵ slow-roll condition, i.e.

$$\frac{1}{2}\left(\frac{2nc_{2n}^{\epsilon}\phi_{\rm end}^{2n-1}}{V_0}\right)^2 = 1, \tag{2.67}$$

which yields

$$c_{2n}^{\epsilon} = \frac{V_0}{\sqrt{2}n}\left(\frac{\mathcal{N}_e^{*2}V_0}{12\pi^2 P_\zeta}\right)^{\frac{1-2n}{2}}. \tag{2.68}$$

Similarly, we define c_{2n}^{η} such that the $c_{2n}\phi_{\rm end}^{2n}$ term alone in Eq.(2.61) can violate the η slow-roll condition, which yields

$$c_{2n}^{\eta} = \frac{V_0}{2n(2n-1)}\left(\frac{\mathcal{N}_e^{*2}V_0}{12\pi^2 P_\zeta}\right)^{1-n}. \tag{2.69}$$

We then define $c_{2n}^{\rm max}$ to be the minimum of c_{2n}^{ϵ}, c_{2n}^{η} and one,

$$c_{2n}^{\rm max} = {\rm Min}\left[c_{2n}^{\epsilon}, c_{2n}^{\eta}, 1\right]. \tag{2.70}$$

Lastly, we estimate $\phi_{\rm ant}$ originating from scanning over the initial inflaton field value. From the total number of e-folds, we have

$$\mathcal{N}_e^{tot} = -\int \frac{V}{V'}\, d\phi \simeq \int \frac{V_0}{\sum 2n\, c_{2n}\, \phi^{2n-1}} d\phi \simeq \int \frac{V_0}{\sum 2n\, c_{2n}^{\eta}\, \phi^{2n-1}}\, d\phi \simeq \int \frac{V_0}{2n\, c_{2n}^{\eta}\, \phi^{2n-1}}\, d\phi \tag{2.71}$$

where in the third equality we used c_{2n}^{η} to replace c_n because when the potential terms are relevant to the inflationary dynamics, their coefficients will be bounded by c_{2n}^{max} and $c_{2n}^{\eta} < c_{2n}^{\epsilon}$ when the energy scale V_0 is small. In the last equality, we approximated the summation by the n-th term because all the relevant potential terms are comparable. After performing the integral from ϕ_i to ϕ_{end}, because the integral is dominated by the ϕ_i term, we obtain

$$\mathcal{N}_e^{tot} \simeq \frac{2n-1}{2(n-1)} \left(\frac{12\pi^2 P_\zeta}{\mathcal{N}_e^{*2} V_0} \right)^{1-n} \phi_i^{2(1-n)}, \tag{2.72}$$

which gives

$$\phi_{\rm ant} \propto \sqrt{\frac{V_0}{P_\zeta}}. \tag{2.73}$$

The probability $P \propto V_0 \phi_{\rm ant} \prod_{n=1}^{\infty} c_{2n}^{\rm max}$ is a function of V_0 and P_ζ. We plot the unnormalized probability function in the left panel of Figure 2.11 for three different P_ζ's. One can see that the probability is strongly biased toward large perturbation. In addition, there are several kinks along each curves. To understand these kinks more, it is illustrative to plot the first few $c_{2n}^{\rm max}$'s. In the right panel of Figure 2.11, we see that the coefficients for higher dimensional operators, those with $n \geq 3$, have $c_{2n}^{\rm max} = 1$ for small V_0. This is because $\phi_{\rm end} \propto \sqrt{V_0}$ as shown in Eq.(2.66) and hence for small V_0, the higher-dimensional operators are Planck-suppressed and irrelevant to inflationary dynamics. This is also the reason why assuming $c_{2n}^{\rm max}$'s are independent on each other does not change the qualitative result. Only the lower dimensional coefficients can affect the higher ones but not vice versa. As we increase the inflation energy scale, the field displacement becomes larger and hence higher-dimensional operators become relevant and their required coefficients start to decrease from one. This translates to the kinks shown in the left panel of Figure 2.11. For instance, in Figure 2.11 we see that, for $P_\zeta = 10^{-9}$, $V_0 \sim 10^{-17}$ indicated by the orange dashed line is precisely the scale where the octet term starts to be relevant and require fine-tuning. Also note that, regardless of the value of P_ζ, the amount of fine-tuning is minimal when the octet operator just became relevant.

We compute the probability to obtain a cosmic perturbation P_ζ. As we have observed, the probability P peaks at the point when the octet becomes relevant, i.e. when $c_8^{\eta} = 1$. This gives us the energy scale V_0^{max} where the fine-tuning is minimal,

$$V_0^{max} = \frac{1}{2\sqrt{14}} \left(\frac{\mathcal{N}_e^{*2}}{12\pi^2 P_\zeta} \right)^{-\frac{3}{2}}. \tag{2.74}$$

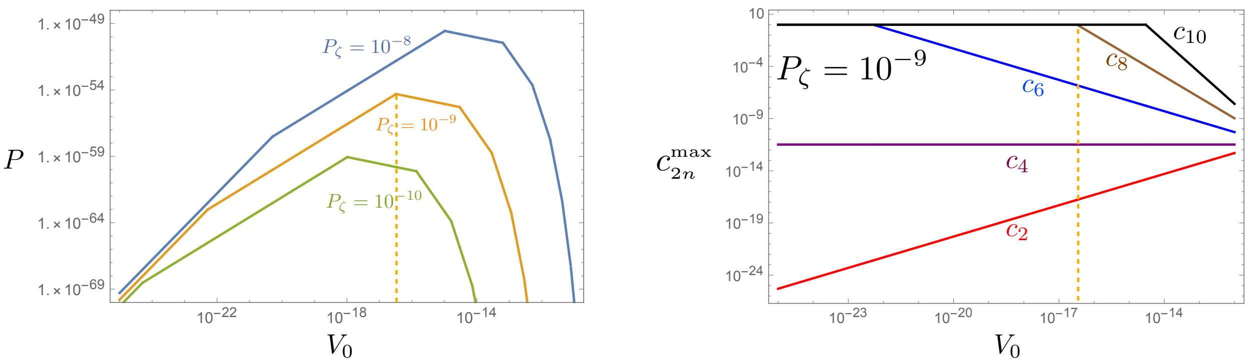

Figure 2.11: **Left:** The probability $\mathcal{P}$ for different primordial perturbation amplitude P_ζ. **Right:** The values of $c_2^{\max}, c_4^{\max}, c_6^{\max}, c_8^{\max}$ and $c_{10}^{\max}$ as a function of V_0 with $P_\zeta = 10^{-9}$. Notice that $c_8^{\max}$ starts to decrease from 1 at $V_0 \sim 10^{-16}$, where $\mathcal{P}$ reaches maximum.

When we integrate out V_0 the integration is dominated by the region around V_0^{max}, we therefore have

$$P \propto V_0^{max} \phi_{\rm ant} c_2^{max} c_4^{max} c_6^{max} \propto P_\zeta^{\frac{19}{4}}. \tag{2.75}$$

This can be understood as P_ζ obeying the distribution

$$P(P_\zeta) dP_\zeta \propto P_\zeta^{\frac{15}{4}} dP_\zeta. \tag{2.76}$$

The probability is strongly biased toward large P_ζ. Unless there exists a strong anthropic bound disfavoring P_ζ larger than the observed one, it is unlikely that general new inflation with Z_2 symmetry results in our observed universe. Our analysis is also applicable to the case with $U(1)$ symmetry because the radial direction is essentially Z_2 symmetric, while the angular direction is flat and does not affect the inflationary dynamics at the background level.

For completeness, we continue our further analysis of general new inflation with Z_2 symmetry in the next section. In particular, assuming that the perturbation amplitude P_ζ is fixed to the observed value for some reason, we investigate the probability distribution of spectral index n_s. We will find that it is probabilistically favored to have a spectral index $n_s \simeq 0.96$, which is quite remarkable.

2.4 New Inflation and the Most Probable Spectral Index

In Sec.2.3 we found that when considering inflation with Z_2 symmetry, we need to fine-tune terms at least up to the octet order,

$$V = V_0 + c_2\phi^2 + c_4\phi^4 + c_6\phi^6 + c_8\phi^8. \tag{2.77}$$

Nevertheless, for simplicity we will consider potential of the form

$$V = a - \frac{1}{2} b\,\phi^2 - c_n \phi^n - c_m \phi^m \tag{2.78}$$

where the c_n term dominates over the c_m term and the latter is treated perturbatively. Namely, we consider the case where c_m is sufficiently small and $c_m\phi^m < c_n\phi^n$. We can then extract the physics of the complete octet model, Eq.(2.77), by extrapolation. In Eq.(2.78) we make quadratic term explicitly negative as we now consider cases where the inflaton rolls down from $\phi = 0$.

The number of e-folds $\mathcal{N}_e$ which the inflation would last before it ends and the corresponding field value $\phi_{\mathcal{N}_e}$ has the relation

$$\begin{aligned}
\mathcal{N}_e &= \int_{\phi_{end}}^{\phi_{\mathcal{N}_e}} \frac{V}{\partial V/\partial\phi}\, d\phi \\
&\simeq \int_{\phi_{end}}^{\phi_{\mathcal{N}_e}} \frac{a}{-b\phi - nc_n\phi^{n-1} - mc_m\phi^{m-1}}\, d\phi \\
&\simeq a\int_{\phi_{end}}^{\phi_{\mathcal{N}_e}} \left[\frac{1}{b\phi + nc_n\phi^{n-1}} - \frac{m\phi^{m-1}}{b\phi + nc_n\phi^{n-1}}\, c_m\right] d\phi \\
&= \frac{1}{(n-2)k} \ln\left(\frac{k\phi_{\mathcal{N}_e}^{2-n} + \frac{c_n}{a}n}{k\phi_{end}^{2-n} + \frac{c_n}{a}n}\right) - \frac{1}{k}\left[\mathcal{F}(\phi_{end}) - \mathcal{F}(\phi_{\mathcal{N}_e})\right] c_m
\end{aligned} \tag{2.79}$$

where we defined $k \equiv b/a$ and

$$\mathcal{F}(\phi) \equiv \frac{m}{n-2}\frac{\phi^{m-2}}{ak}\left\{\left(\frac{n-m}{m-2}\right) {}_2F_1\left(1, \frac{m-2}{n-2}; \frac{n+m-4}{n-2}; -\frac{nc_n}{ak}\phi^{n-2}\right) + \frac{ak}{ak + nc_n\phi^{n-2}}\right\}. \tag{2.80}$$

Here ${}_2F_1$ is the hypergeometric function and ϕ_{end} is the field value where the inflation ends, determined by the slow-roll condition. In particular, the inflation ends when the η-parameter reaches -1, which yields

$$\phi_{end}^{n-2} \simeq \frac{a}{n(n-1)c_n}. \tag{2.81}$$

Solving Eq.(2.79) for $\phi_{\mathcal{N}_e}(\mathcal{N}_e)$ perturbatively in c_m, that is, with $\phi_{\mathcal{N}_e}(\mathcal{N}_e) = \phi_{\mathcal{N}_e}^{(0)} + c_m\,\phi_{\mathcal{N}_e}^{(1)}$, one has

$$\phi_{\mathcal{N}_e}^{(0)} = \left[\frac{a}{c_n} k f_n(k)\right]^{\frac{1}{n-2}}, \quad \text{and} \quad \phi_{\mathcal{N}_e}^{(1)} = \phi_0\, c_n^{-\frac{m-2}{n-2}}\, a^{\frac{m-n}{n-2}}\, g_{n,m}(k) \tag{2.82}$$

where

$$f_n(k) \equiv \frac{1}{n}\frac{1}{\left([1+(n-1)k]\, e^{(n-2)k\mathcal{N}_e} - 1\right)} \tag{2.83}$$

and

$$g_{n,m}(k) \equiv \frac{m(n-m)}{(n-2)(m-2)}\,(1+nf_n)\times \left\{ k^{\frac{m-n}{n-2}} f_n^{\frac{m-2}{n-2}} \left[{}_2F_1\left(1, \frac{m-2}{n-2}; \frac{n+m-4}{n-2}, -nf_n\right) + \frac{1}{1+nf_n}\right] - \left(\frac{1}{n(n-1)}\right)^{\frac{m-2}{n-2}} \frac{1}{k}\left[{}_2F_1\left(1, \frac{m-2}{n-2}; \frac{n+m-4}{n-2}, -\frac{1}{(n-1)k}\right) + \frac{1}{1+\frac{1}{(n-1)k}}\right]\right\}. \tag{2.84}$$

The spectral index n_s and perturbation power spectrum P_ζ when inflation can last another $\mathcal{N}_e$ of e-folds before it ends is then given by

$$\begin{aligned} n_s &= 1 + 2\eta - 6\epsilon \\ &= 1 - 2k - 2n(n-1)kf_n - 3a^{\frac{2}{n-2}} c_n^{-\frac{2}{n-2}} k^{\frac{2n-2}{n-2}} \left(1+nf_n\right)^2 f_n^{\frac{2}{n-2}} \\ &\quad + \left\{ 2a^{\frac{m-n}{n-2}} c_n^{-\frac{m-2}{n-2}} \left[m(1-m)k^{\frac{m-2}{n-2}} f_n^{\frac{m-2}{n-2}} - n(n-1)(n-2)k\, f_n\, g_{n,m}\right] \right. \\ &\quad \left. - 6a^{\frac{m-n+2}{n-2}} c_n^{-\frac{m}{n-2}} k^{\frac{n}{n-2}} f_n^{\frac{2}{n-2}} (1+nf_n) \left(mk^{\frac{m-2}{n-2}} f_n^{\frac{m-2}{n-2}} + k\, g_{n,m} + n(n-1)k\, f_n\, g_{n,m}\right)\right\} c_m, \end{aligned} \tag{2.85}$$

$$\begin{aligned} P_\zeta &= \frac{1}{24\pi^2}\frac{V}{\epsilon} \\ &= \frac{a^{\frac{n-4}{n-2}} k^{\frac{-2n+2}{n-2}} f_n^{-\frac{2}{n-2}}}{12\pi^2\left(1+nf_n\right)^2} \left[c_n^{\frac{2}{n-2}} - c_n^{\frac{4-m}{n-2}}\, c_m\, \frac{2a^{\frac{m-n}{n-2}}\left(mk^{\frac{m-n}{n-2}} f_n^{\frac{m-2}{n-2}} + g_{n,m} + n(n-1)\, f_n\, g_{n,m}\right)}{(1+nf_n)}\right]. \end{aligned} \tag{2.86}$$

By taking the inverse of Eq.(2.86) perturbatively in c_m, one obtain $c_n = c_n^{(0)} + c_m\, c_n^{(1)}$ with

$$c_n^{(0)} = \left(12\pi^2\right)^{\frac{n-2}{2}} a^{\frac{4-n}{2}} k^{n-1} f_n \left(1+nf_n\right)^{n-2} P_\zeta^{\frac{n-2}{2}}, \tag{2.87}$$

and

$$c_n^{(1)} = \frac{n-2}{2} c_n^{(0)\,\frac{n-m+2}{n-2}} \frac{2a^{\frac{m-n}{n-2}}\left(mk^{\frac{m-n}{n-2}} f_n^{\frac{m-2}{n-2}} + g_{n,m} + n(n-1)f_n\, g_{n,m}\right)}{1+nf_n}. \tag{2.88}$$

With Eq.(2.85), (2.87) and (2.88), the relation between the spectral index n_s and the parameter k is plotted in Figure 2.12 for various set of (n, m). We set the parameter $a = 10^{-16}$, the number of e-folds $\mathcal{N}_e = 55$, and the perturbation power spectrum to the observed value, $P_\zeta = 2.2 \times 10^{-9}$. The parameter c_m is bounded by the requirement that the perturbation holds, i.e. $c_m \phi^m < c_n \phi^n$. As $\phi_{end} > \phi_{\mathcal{N}_e}$, the bound of c_m is therefore

$$|c_m| < \begin{cases} c_n \phi_{\mathcal{N}_e}^{n-m} & (n > m), \\ \\ c_n \phi_{end}^{n-m} & (n < m). \end{cases} \tag{2.89}$$

We first look at the case with quartic and sextic terms, shown on the left of Figure 2.12. Note that the quartic-dominant case is excluded by observation. One generic feature that appears for all (n, m) is that when $n < m$, a positive perturbation ($c_m > 0$) leads to a spectral index closer to scale-invariance, i.e. $n_s = 1$, while a negative perturbation ($c_m < 0$) makes n_s deviate away from 1. This might be counterintuitive as the potential

$$V = a - \frac{1}{2} b \phi^2 - c_n \phi^n - c_m \phi^m, \tag{2.90}$$

is flatter when $c_m < 0$ and we would have expected a spectral index closer to 1. However, a flatter potential also means the inflaton moves slower before it reaches ϕ_{end}, and hence the field value can be closer to ϕ_{end} (but farther away from zero) while giving the same number of e-foldings as shown in the left of Figure 2.13. As the field is farther away from zero where the potential is the flattest, the spectral index can deviate from -1. The two effects, flatter potential and larger $\phi_{\mathcal{N}_e}$, compete with each other and the latter wins when $n < m$. On the other hand, for $n > m$ the effect of flatter potential dominates and a negative perturbation ($c_m < 0$) leads to spectral index closer to 1. The fact that the two cases, $n > m$ and $n < m$, behave oppositely and the region of positive perturbation lies between two unperturbed curves makes us confident that one can extrapolate our perturbative treatment to the case where the c_n term and c_m term comparable – it simply lies between the two perturbative regions where one dominates the other, as shown in the right in Figure 2.13.

In Figure 2.12 we see that sextic dominant models, $n = 6$, with either quartic perturbation ($m = 4$) or octet perturbation ($m = 8$), fit the observation quite well when $k \lesssim 0.018$. We discuss the chance for k to lie in this region. We will focus on the following analysis without perturbation,

$$V = a - \frac{1}{2} b\, \phi^2 - c_n \phi^n \tag{2.91}$$

as additional perturbation $c_m \phi^m$ does not change the end result significantly. We assume the parameter in the Lagrangian a, b and c_n has uniform probability distribution and also take the probabilistic nature of the initial condition into account. Using the definition $b = ak$, Eq.(2.82) and Eq.(2.87), one has

$$\int_0^{\phi_{\rm ant}} d\phi_i \int da\, db\, dc_n = \int da\, dP_\zeta\, dk\, (12\pi^2)^{\frac{n-3}{2}} a^{\frac{7-n}{2}} P_\zeta^{\frac{n-5}{2}} k^{n-2} f_n \,(1 + n\, f_n)^{n-3}\,, \tag{2.92}$$

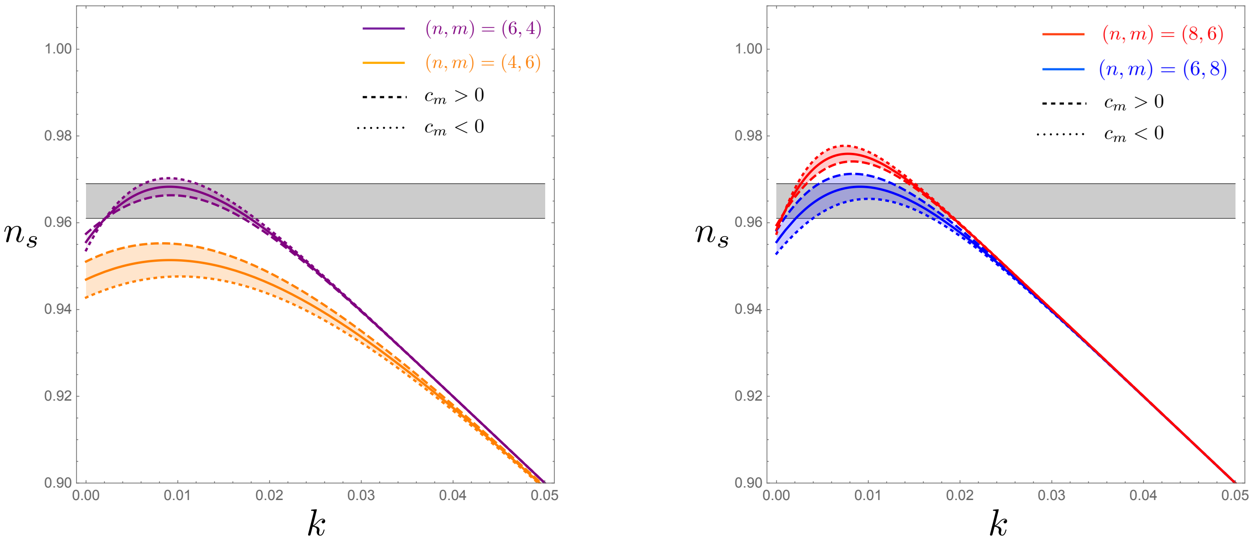

Figure 2.12: The $n_s - k$ plot for different set of (n, m). The dashed lines corresponds to the boundary of positive perturbation, $c_m > 0$, while the dotted ones represents negative perturbation, $c_m < 0$. The grey region shows the current observational value.

where we need to integrate over a to obtain the probability distribution of k for a given P_ζ. For $n = 6$ and 8, the integration over a is divergent with an upper bound a_{max} that is around 10^{-17} where only terms below the octec order require fine-tuning. The exact upper bound a_{max} is correlated with the upper bound for c_n, but most importantly a_{max} is independent on k and hence the integration over a does not give an additional k-dependence. In sum, for the cases of interest, the probability distribution of k is

$$\mathcal{P}_k\, dk = k^{n-2} f_n \left(1 + n\, f_n\right)^{n-3}\, dk, \tag{2.93}$$

where $f_n(k)$ is defined in Eq.(2.83), and the plot for $n = 6$ is given in Figure 2.14. The shaded area corresponds to the interval $\mathcal{I}$ of k that yields spectral index $n_s > 0.96$. The probability for k to lie in this region for $n = 6$ is

$$P_{n_s>0.96} = \frac{\int_{\mathcal{I}} \mathcal{P}_k\, dk}{\int_0^\infty \mathcal{P}_k\, dk} \simeq 0.49. \tag{2.94}$$

and the distribution $\mathcal{P}_k$ peaks at $k \simeq 0.016$, which yields a spectral index of $n_s = 0.963$. Overall it is quite remarkable that once one matches the observed perturbation power spectrum P_ζ, there is about a few ten chance to achieve the observed spectral index without much further fine-tuning in general new inflation with Z_2 symmetry. But as we discussed in Sec.2.3, the observed P_ζ can be obtained without significant fine-tuning only if there is a strong anthropic bound on P_ζ right at the observed value which seems to be unlikely.

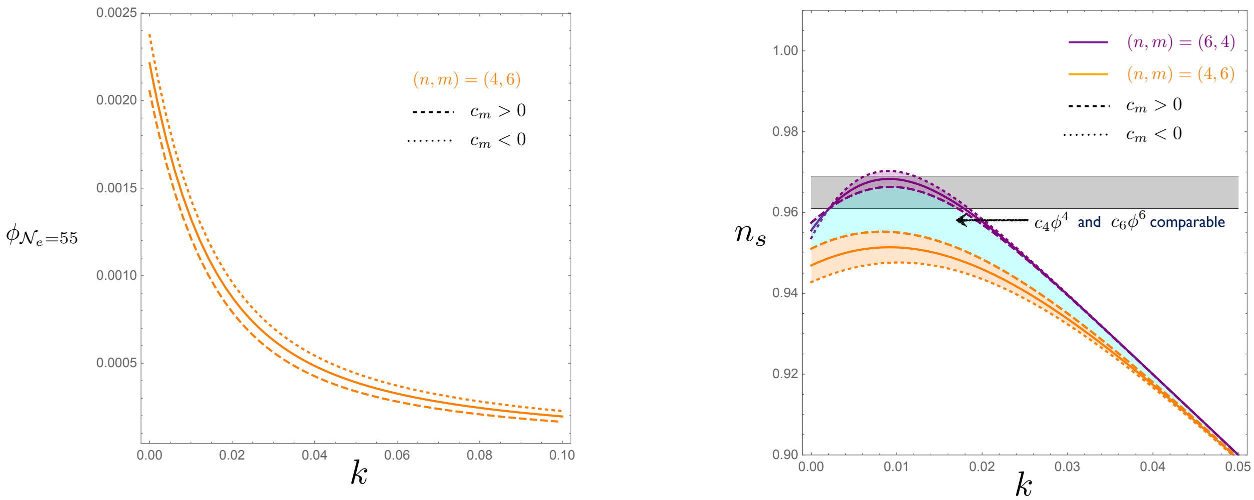

Figure 2.13: (Left) The $\phi - k$ plot for $(n,m) = (4,6)$. One can see that for the same k, ϕ is larger when $c_m < 0$. (Right) Even though our treatment is perturbative, the case where $c_n\phi^n$ and $c_m\phi^m$ are comparable should continuously connect the two perturbative regions.

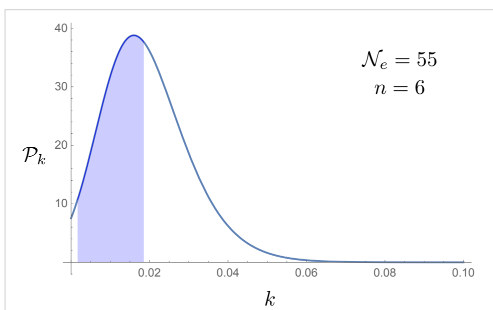

Figure 2.14: The normalized probability distribution of k for $n = 6$. The shaded area corresponds to the interval of k that yields spectral index $n_s > 0.96$.

2.5 Discussion

In this work we have investigated the typicality of the small and nearly scale-invariant perturbation in the landscape. Anthropic consideration of the cosmological constant yields a probability of P_ζ that biases toward large perturbation, until the anthropic constraint due to the density of the galaxy kicks in. In order for the observed small $P_\zeta \sim 10^{-9}$ to be typical, the inflationary evolution has to give a bias toward small P_ζ. Closeness of the spectral index to the unity should be also explained.

We consider the following scenario that naturally fits into the landscape scenario: The inflaton is coupled to a singlet scalar field that was initially trapped in a metastable vacuum and drove a precedent inflation. After the quantum tunneling of the singlet field, the universe became an open FRW universe dominated by the curvature energy density while the singlet field rolled down to a stable vacuum with negligible energy density. After a sufficient period of cosmic expansion, when the curvature energy density dropped below the potential energy of the inflaton, the inflation which explains the flatness of the universe and the cosmic perturbation occurred.

In this scenario, the inflaton field value is homogeneous inside the horizon because of the trapping during the precedent inflation. However, after the quantum tunneling the universe is curvature dominated and the trapping is no longer effective. As a result quantum fluctuation of long wavelength modes produced during the precedent inflation survive, which leads to a probabilistic nature of the initial field value of the inflaton. As the inflaton tends to start from the an initial condition away from the origin, the anthropic lower bound on the total number of e-folding during inflation favors the inflaton potential flatter around the origin, namely a smaller η parameter.

We investigated a supersymmetric new inflation model in detail. We find that for certain distributions of the parameters, the probability to obtain $P_\zeta \sim 10^{-9}$ is $\mathcal{O}(10)\%$, while the observed n_s is favored. We emphasize that both the model-building and the anthropic selection from the landscape play important roles in explaining the observed properties of the cosmic perturbation, P_ζ and $n_s - 1$. From the model-building side, the distribution function of the model parameters, which is not uniform owing to the supersymmetry and the R symmetry, yields the distribution of P_ζ not biased toward large values. From the landscape side, the requirement of large enough number of e-foldings and the probabilistic nature of the initial inflaton field value set by the precedent inflation dynamics favor small η parameter, thereby explaining the observed n_s.

The result is encouraging for the project on understanding the universe by the anthropic principle in the landscape. Further study is required toward this goal. For instance, in this work we assume the contribution to the distribution of P_ζ from the post-inflationary dynamics shown in Figure 2.1. As we comment in Sec. 2.2, our result holds qualitatively as long as the power of the distribution at large P_ζ is smaller than 1/2. It will be important to investigate the distribution at large P_ζ more carefully, taking into account the effect of e.g. the behavior of proto-galaxies.

Chapter 3

Quintessence Model Building in Supergravity

This chapter is based on the work with Hitoshi Murayama [71]. I would like to thank Hitoshi for his guidance, encouragement, and collaboration throughout finishing this project.

3.1 Introduction

Ever since its discovery [196, 201], the current accelerating expansion of the universe has been one of the major puzzles of modern physics and its cause is often dubbed dark energy as its very nature is still a mystery. The simplest solution may be adding a pure cosmological constant to the Einstein-Hilbert action and indeed the ΛCDM model has described our universe quite well [13]. Nevertheless, the physical origin of cosmological constant has remained obscured and the naïve theoretical expectation is about 120 orders of magnitude larger than the observed value [225]. To explain the value of cosmological constant, one may appeal to anthropic arguments [224, 107], whose recent resurgence stems from the string theory landscape [90, 58, 111, 146, 211]. To date, cosmological constant problem remains one of the most challenging problem in fundamental physics.

Cosmological constant problem aside, over the years many alternatives have been proposed to account for the accelerating expansion. Among various proposals, there is a class of models where the dark energy is attributed to a canonical scalar field named quintessence [200, 227, 229]. For a review see [217]. Some early models of this kind posses tracker behavior where the evolution of the field at late time is insensitive to initial conditions and hence make them rather attractive. Yet, as the observations have significantly improved for the past decades, now such models are under strong pressure from the observational constraints [168]. But regardless the initial condition problem and/or cosmic coincidence problem (why the energy density of matter and dark energy are comparable at present time) can be solved or not, one basic question we wish to know is whether dark energy is purely a constant or if it is dynamical and evolves over time. Thanks to the advancement in many cosmological observations like eBOSS, SuMIRe (HSC and PFS on Subaru), DESI, Euclid, WFIRST and many others in the near future, we will have better sensitivity to see if the equation of state parameter w of dark energy has any deviation from -1, which is the case if dark energy is not a pure cosmological constant. From this perspective, quintessence models are phenomenological tools that help us describe dark energy if it is dynamical with $w > -1$ and very often the vacuum energy contribution is assumed to be zero due to other mechanism. Certainly, regardless dark energy is a pure cosmological constant or not, one still needs to answer if vacuum energy contributes to dark energy and if so, how large it should be. Yet, these are ambitious problems and very likely a full theory of quantum gravity is required to completely solve the cosmological constant problem. On the other hand, recently a constraint on scalar field potential from quantum gravity was proposed in [185] which suggests that the de Sitter vacuum may belongs to the "swampland", where models cannot be UV completed with consistent theory of quantum gravity. Whether this dS swampland conjecture holds true has raised a lot of discussions [42, 176, 43, 206, 178, 87, 182, 144, 150, 102, 47, 48, 127, 148, 197, 149, 8, 77, 92, 202, 22, 24, 81, 19, 152, 145]. Meanwhile, its various phenomenological implications are also worth investigation [73, 133, 132, 172, 181, 157, 169, 46, 175, 80, 94, 95, 155, 105, 10, 15]. In particular, the conjecture gives another motivation to reexamine quintessence model-building. For discussions of embedding quintessence into string theory, see [72, 134, 156, 194, 117, 74].

Obviously, the string theory requires supersymmetry and hence its low-energy limit must be studied within the supergravity (SUGRA). Therefore, quintessence models must be formulated within SUGRA. For example, in [18, 151] a class of models were constructed by utilizing a nilpotent superfield. Also see [62, 82, 63, 61, 64] for some earlier works of constructing quintessence models in SUGRA. One particular point we would like to emphasize and is the focus of this paper is that when building quintessence model in supergravity, it is necessary to consider the effect of supersymmetry (SUSY) breaking on the quintessence sector because even if one successfully constructs a quintessence model alone, the SUSY breaking effect will spoil the flatness of the potential. In particular, the mass scale of quintessence is at the order of current Hubble parameter $H_0 \sim 10^{-33}$ eV. On the other hand, quite often quintessence will acquire a mass that has the same order as the gravitino mass $m_{3/2}$ which, for example, is about TeV in gravity mediation models, way much larger then the mass scale of quintessence. Even with low scale SUSY breaking, like vector mediation [138, 139] where $m_{3/2} \sim \mathcal{O}(1)$ eV, the hierarchy between H_0 and $m_{3/2}$ is still huge. This steepens the quintessence potential, yielding the field settles at the minimum in early time and one cannot distinguish it from a pure cosmological constant. Such minimum is also inconsistent with the swampland conjecture.

To be more concrete, let us consider a simple model where the hidden and quintessence sector are separated in the Kähler potential with the canonical form,

$$K = z^* z + Q^* Q, \tag{3.1}$$

where z and Q are the chiral superfields of the hidden and quintessence sector respectively. This is a natural assumption in the sense that one would expect the interaction between the hidden and quintessence sector is as minimal as possible so there should be no cross terms in the Kähler potential. Similarly, we assume the two sectors are separated in the superpotential as well,

$$W = W_0(z) + W_1(Q). \tag{3.2}$$

Given the Kähler potential and superpotential, the F-term scalar potential then reads

$$V_F = e^{K/M_{P\ell}^2}\left[D_i W K^{i\bar{j}} D_{\bar{j}} W^* - \frac{3}{M_{P\ell}^2}|W|^2\right], \tag{3.3}$$

where i and j sum over the two sectors and

$$D_i W \equiv \frac{\partial W}{\partial \Phi^i} + \frac{W}{M_{P\ell}^2}\frac{\partial K}{\partial \Phi_i}. \tag{3.4}$$

Here $M_{P\ell}$ is the reduced Planck mass $M_{P\ell} \equiv 1/\sqrt{8\pi G}$. Among various terms in the potential, there is a quadratic term of quintessence that couples to the superpotential of the hidden sector,

$$V \supset \frac{|W_0|^2}{M_{P\ell}^4}|\partial_Q K|^2 = \frac{|W_0|^2}{M_{P\ell}^4}|Q|^2. \tag{3.5}$$

As the gravitino mass $m_{3/2}$ is related to the superpotential by $\langle |W_0|^2\rangle \sim m_{3/2}^2 M_{P\ell}^4$, we see that

$$V \supset m_{3/2}^2 |Q|^2. \tag{3.6}$$

Due to the large hierarchy between the gravitino mass scale and the current Hubble scale, such term will make quintessence roll down to the minimum and stick at there at a very early time, regardless how flat the potential is in the quintessence sector *alone*. Observationally, quintessence then acts like a non-dynamical cosmological constant.

If we wish to construct a quintessence model that can be observationally distinguishable from a pure cosmological constant, for example having a time-varying equation of state in the present epoch, then one needs to prevent the quintessence sector from acquiring such gravitino mass. One known method is to impose shift symmetry to the quintessence sector [64]. We will review this in Sec.3.2, emphasizing that one can incorporate quintessence with all kinds of potential into supergravity using shift symmetry while remain radiatively stable. In addition, the energy scale of the quintessence field is connected with the gravitino mass, opening the possibility to relate electroweak scale with dark energy in supergravity. As a particular example, we will show that hidden supersymmetric QCD [26] can naturally generate the observed quintessence energy scale and be embedded into SUGRA. The cosmic coincidence problem is also ameliorated in such scenario. After reviewing the case with shift symmetry, in Sec.3.3 we will show our attempt to construct a quintessence model where the quintessence and hidden sector are *sequestered*, inspired by the brane-world scenario [199]. In such sequestered scenario, quintessence is protected from the SUSY breaking at least at the tree level, and it is possible to construct quintessence models of the small field type where the quintessence was frozen by Hubble damping for most of the time and only thawed recently. Yet, the constraint from the fifth force remains strong in this case and quintessence field value is limited in a tiny range, rendering it challenging to observationally distinguish such model from cosmological constant. However, in the phenomenological allowed range, exactly because of the small field displacement, the quantum correction beyond the tree level is well suppressed and the model is consistent from the effective field theoretic point of view. On the other hand, in the case with shift symmetry, the fifth force constraint is avoided.

3.2 SUGRA Quintessence with Shift Symmetry

We first review the construction of quintessence model in SUGRA where a shift symmetry is imposed on the imaginary part of the quintessence sector. Particularly, the Kähler potential has the form

$$K = z^* z + h(Q + Q^*). \tag{3.7}$$

where h is an arbitrary function of $Q + Q^*$ with nonvanishing second derivative, and we assume h starts at quadratic order in its argument. We also make the two sectors separated in the superpotential,

$$W = W_0(z) + W_1(Q). \tag{3.8}$$

The F-term potential then has the form

$$V_F = e^{\frac{K}{M_{P\ell}^2}} \left\{ \left| \frac{\partial W_0}{\partial z} + \frac{1}{M_{P\ell}^2} z^* (W_0 + W_1) \right|^2 + \frac{1}{h''} \left| \frac{\partial W_1}{\partial Q} + \frac{1}{M_{P\ell}^2} h' (W_0 + W_1) \right|^2 - \frac{3}{M_{P\ell}^2} |W_0 + W_1|^2 \right\} \tag{3.9}$$

where the prime on h denotes the derivative with respect to its argument. Below we will denote the real and imaginary part of the quintessence sector as r and q respectively,

$$Q = r + iq. \tag{3.10}$$

The field r is the scalar partner of the quintessence field and may be called squintessence, as analogous to sinflaton in supergravity inflation model. Note that because of the large hierarchy between the SUSY breaking scale and the energy scale of dark energy, the dynamics of SUSY breaking will not be affected by the quintessence sector. To be more precise, we consider the superpotential of the quintessence sector of the form

$$W_1(Q) = \Lambda^3 \, \mathcal{W}_1 \left(\frac{Q}{M_{P\ell}} \right), \tag{3.11}$$

where Λ is the energy scale of quintessence and $\mathcal{W}_1$ is a holomorphic function of $Q/M_{P\ell}$. Note that the shift symmetry is broken by the superpotential which is inevitable as superpotential has to be holomorphic. This gives quantum corrections to the Kähler potential that breaks shift symmetry. However, radiative stability is controlled by the smallness of Λ. We can also consider superpotentials that involve more parameters, as long as these parameters are smaller than Λ. For simplicity and minimality, we consider superpotentials of the form of Eq.(3.11). The coupling between the hidden sector and W_1 will not affect the dynamics of hidden sector because of the smallness of Λ. The only interaction between the hidden and quintessence sector that does not involve $W_1(Q)$ has the form

$$\frac{1}{h''}\frac{|W_0|^2}{M_{P\ell}^4}h'^2 \sim m_{3/2}^2\frac{h'^2}{h''}. \tag{3.12}$$

As h only depends on the real part r and the gravitino mass $m_{3/2}$ is much greater than the dark energy energy scale, if we assume h starts at quadratic order in its argument and hence h' does not have a constant piece, then this term sets the vacuum expectation value (vev) of r such that $\langle r\rangle = \langle h'\rangle = 0$. On the other hand, the interaction terms between the hidden sector and quintessence sector that involve W_1 are suppressed by Λ. Hence, when determining the vev of the hidden sector, it is sufficient to only consider the terms depending on z only,

$$V_{\not{\mathrm{SUSY}}} = e^{K/M_{P\ell}^2}\left\{\left|\frac{\partial W_0}{\partial z}\right|^2 + \frac{1}{M_{P\ell}^2}\left(z^*\frac{\partial W_0^*}{\partial z^*}W_0 + z\frac{\partial W_0}{\partial z}W_0^*\right) + \frac{1}{M_{P\ell}^4}|z|^2|W_0|^2 - \frac{3}{M_{P\ell}^2}|W_0|^2\right\}. \tag{3.13}$$

Assuming $\langle z\rangle$ and $\langle W_0\rangle$ are real, the potential of the quintessence sector then has the form

$$\begin{aligned} V_{\mathrm{Quin}} = e^{\frac{\langle z\rangle^2+h}{M_{P\ell}^2}}\Bigg\{ & \frac{|\langle W_0\rangle|^2}{M_{P\ell}^4}\frac{h'^2}{h''} + \frac{1}{M_{P\ell}^2}\left(\left\langle z\frac{\partial W_0}{\partial z}\right\rangle + \frac{\langle |z|^2\rangle}{M_{P\ell}^2}W_0 - 3\langle W_0\rangle\right)(W_1+W_1^*) \\ & + \frac{1}{M_{P\ell}^2}\frac{h'}{h''}W_0\left(\frac{\partial W_1}{\partial Q}+\frac{\partial W_1^*}{\partial Q^*}\right) + \frac{W_0}{M_{P\ell}^4}\frac{h'^2}{h''}(W_1+W_1^*) \\ & + \left|\frac{\partial W_1}{\partial Q}\right|^2 + \left(\frac{|\langle z\rangle|^2}{M_{P\ell}^4} + \frac{1}{h''}\frac{h'^2}{M_{P\ell}^4} - \frac{3}{M_{P\ell}^2}\right)|W_1|^2 \\ & + \frac{1}{M_{P\ell}^2}\frac{h'}{h''}\left(\frac{\partial W_1}{\partial Q}W_1^* + \frac{\partial W_1^*}{\partial Q^*}W_1\right)\Bigg\} \end{aligned} \tag{3.14}$$

Despite many terms shown in the equation above, it can be largely simplified. In the first line, the first term is exactly Eq.(3.12) which has a energy scale much larger than the other terms as $\langle W_0\rangle \sim m_{3/2}M_{P\ell}^2$. This term sets the vev of r such that $\langle h'\rangle = 0$, and hence the second line can be dropped. For the terms in the third line, they are all at the order of $\mathcal{O}(W_1^2)$, which has an energy scale of $\Lambda^6/M_{P\ell}^2$, where Λ is defined in Eq.(3.11). As we wish the potential to be at the order of current dark energy scale, and the leading term is the second term in the first line, we have

$$m_{3/2}\Lambda^3 \sim \frac{\langle F\rangle\Lambda^3}{M_{P\ell}} \sim M_{P\ell}^2H_0^2 \quad \Rightarrow \Lambda \sim M_{P\ell}\left(\frac{H_0^2}{\langle F\rangle}\right)^{1/3}. \tag{3.15}$$

where $\sqrt{\langle F\rangle}$ is the SUSY breaking scale. Note that there is a large hierarchy between Λ and $M_{P\ell}$. Because of this large hierarchy, the third line in Eq.(3.14) is largely suppressed.

Particularly, in the case of gravity mediation [68, 32, 140, 121], we have $m_{3/2} \sim$ TeV, $\langle F \rangle \sim 10^{21}(\text{GeV})^2$ such that $\Lambda \sim 10^{-35} M_{P\ell}$ and the third line in Eq.(3.14) has a energy scale of $10^{-210} M_{P\ell}^4$, which is 90 orders of magnitude smaller than the leading term. For anomaly mediation [199, 112], the gravitino mass and SUSY breaking scale is even higher, with $m_{3/2} \sim 100\,\text{TeV}$ and $\langle F \rangle \sim 10^{23}(\text{GeV})^2$. On the other hand, for low scale SUSY breaking models with gauge mediation [97, 183, 21], it typically requires $\sqrt{\langle F \rangle} \gtrsim \mathcal{O}(10^3)$ TeV and $m_{3/2} \gtrsim$ keV to explain the observed Higgs mass of 125 GeV. Yet, this is in tension with the cosmological bound $m_{3/2} < 4.7$ eV from CMB lensing and cosmic shear [189]. To satisfy this constraint, one may consider *vector mediation* [138, 139] where $\sqrt{\langle F \rangle} \sim \mathcal{O}(10)$ TeV and $m_{3/2} \sim \mathcal{O}(1)$ eV, which leads to $\Lambda \sim 10^{-31} M_{P\ell}$.

In conclusion, we can drop terms in the third line of Eq.(3.14) for all mediation scenarios and the potential of the quintessence sector has the following simple form when shift symmetry is imposed on the Kähler potential of the quintessence sector,

$$V_{\text{Quin}} = e^{\frac{\langle z \rangle^2 + h}{M_{P\ell}^2}} \frac{1}{M_{P\ell}^2} \left(\left\langle z \frac{\partial W_0}{\partial z} \right\rangle + \frac{\langle |z|^2 \rangle}{M_{P\ell}^2} W_0 - 3 \langle W_0 \rangle \right) (W_1 + W_1^*) . \tag{3.16}$$

For example, suppose the hidden sector have a superpotential of Polonyi type,

$$W_0 = \mu M_{P\ell}(z + \beta). \tag{3.17}$$

where μ is a parameter of mass dimension one. Requiring the hidden sector contribute zero vacuum energy $V_{\not{\text{SUSY}}}(\langle z \rangle) = 0$, one finds

$$\langle z \rangle = (\sqrt{3} - 1) M_{P\ell} \quad \text{and} \quad \beta = (2 - \sqrt{3}) M_{P\ell}. \tag{3.18}$$

The gravitino mass is given by

$$m_{3/2} = e^{2-\sqrt{3}} \mu. \tag{3.19}$$

Assuming the quintessence sector has the canonical Kähler potential with shift symmetry,

$$h = \frac{1}{2} \big(Q + Q^* \big)^2, \tag{3.20}$$

the potential of the quintessence then has a very simple form

$$V_{\text{Quin}} = -\sqrt{3}\, e^{2-\sqrt{3}} m_{3/2} (W_1 + W_1^*). \tag{3.21}$$

Note that because the scalar partner r is already stabilized by Eq.(3.12) with zero vev $\langle r \rangle = 0$, the superpotential in the above equation is no longer holomorphic. This means for any quintessence model with a potential $V_{\text{Quin}Quin}$, one can embed it into supergravity by making the real part of the superpotential proportional to the potential. Shift symmetry in the superpotential is *not* necessary. In addition, the energy density of the dark energy is related to SUSY breaking by $\rho_{\text{DE}} \sim m_{3/2} \Lambda^3$. This opens up the possibility to construct supergravity quintessence models that relate the electroweak scale to the scale of dark energy.

For example, in the case of vector mediation where $m_{3/2} \sim \mathcal{O}(1)$ eV, the scale $\Lambda \sim 10^{-31} M_{P\ell}$ can be obtained by $\Lambda \sim \langle H \rangle^2 / M_{P\ell}$, where $\langle H \rangle$ is the Higgs vev $\langle H \rangle \sim 246$ GeV.

Note that the shift symmetry of the Kähler potential will be inevitably broken by superpotential, as superpotentials are holomorphic. We can estimate the effect of this shift symmetry breaking by considering the quantum correction to the Kähler potential from superpotential coupling. In particular, considering the leading order correction, the cubic interaction term in the superpotential $W_1 \supset (\Lambda^3/M_{P\ell}^3)Q^3$ yields a loop correction to the Kähler potential $K_{\text{q.c.}}$ of the form

$$K_{q.c.} \sim \frac{1}{16\pi^2}\frac{\Lambda^6}{M_{P\ell}^6}|Q|^2. \tag{3.22}$$

Even though such correction breaks the shift symmetry in the Kähler potential, because of the small coupling in the superpotential, i.e. the large hierarchy between Λ and $M_{P\ell}$, this does not spoil the flatness of the quintessence potential. Indeed, as the scalar potential has the form $e^{K/M_{P\ell}^2}[\cdots]$ as shown in Eq.(3.3), the shift asymmetric correction Eq.(3.22) leads to a mass term

$$V \supset \frac{1}{16\pi^2}\frac{\Lambda^6}{M_{P\ell}^6}\frac{V_{\text{Quin}}}{M_{P\ell}^2}|Q|^2 \sim H_0^2\left(\frac{\Lambda^6}{M_{P\ell}^6}\right)|Q|^2, \tag{3.23}$$

where the mass $(\Lambda^3/M_{P\ell}^3)H_0$ is much smaller than the current Hubble scale H_0 and is therefore harmless. One may also worry the coupling to the SUSY breaking sector like Eq.(3.5) which yields

$$V \supset \frac{|W_0|^2}{M_{P\ell}^4}|\partial_Q K|^2 \sim m_{3/2}^2\left(\frac{\Lambda^6}{M_{P\ell}^6}\right)^2|Q|^2 \sim H_0^2\left(\frac{H_0^2}{M_{P\ell}^2}\right)\left(\frac{\Lambda^6}{M_{P\ell}^6}\right)|Q|^2, \tag{3.24}$$

where we have used $m_{3/2}\Lambda^3 \sim H_0^2 M_{P\ell}^2$ as given in Eq.(3.15). We see that this contribution is even smaller than that in Eq.(3.23) with an additional suppression $H_0^2/M_{P\ell}^2$. Overall we see that even though the shift symmetry of the Kähler potential will be radiatively broken by the superpotential, the effect is negligible and the potential Eq.(3.21) is protected from quantum corrections due to the smallness of Λ.

On the other hand, the smallness of Λ is technically natural in the t'Hooft sense, as the shift symmetry of the quintessence sector is restored when $\Lambda \to 0$. Note that this does not solve the cosmological constant problem. Like many other quintessence models, we assume the vacuum energy density (nearly) vanishes by some other mechanism, instead of tackling the cosmological constant problem directly. However, technical naturalness means that once the assumption of vanishing vacuum energy density is made, an extra fine-tuning of the quintessence energy scale parameter Λ is not required.

Note that so far we have assumed, for simplicity, that h starts at quadratic order in its argument and hence h' does not have a constant piece. One can actually relax this assumption, and then the second line of Eq.(3.14) will also contribute to the quintessence potential, which now has the form

$$V_{\text{Quin}} = m_{3/2}\Lambda^3\left[c_1\left(\mathcal{W}_1 + \mathcal{W}_1^*\right) + c_2\left(\mathcal{W}_1' + \mathcal{W}_1'^*\right)\right] \tag{3.25}$$

where c_1 and c_2 are order one coefficients determined by the details of the SUSY breaking sector and the prime on $\mathcal{W}_1$ denotes derivative with respect to its argument.

With this simple approach in hand, let us discuss the realization of some quintessence potentials in this framework. For instance, the negative exponential potential

$$V_{\text{Quin}} = V_0\, e^{-\lambda \frac{q}{M_{P\ell}}} \tag{3.26}$$

discussed in [15, 200, 227] can be embedded in SUGRA with the superpotential

$$W = \Lambda^3 e^{i\lambda \frac{Q}{M_{P\ell}}} \tag{3.27}$$

and shift symmetric Kähler potential in the quintessence sector, using the general recipe Eq.(3.21). Note that the above superpotential does not posses shift symmetry, which is an example that although the construction requires a shift symmetric Kähler potential, the superpotential need not be so. Current observational constraint from dark energy equation of state parameter requires $\lambda \lesssim 0.6$ [15]. Also see [132, 133, 19]. As the swampland conjecture poses an upper bound on the parameter λ while observational constraints yields a lower bound, future observational data can play an important role in testing the swampland conjecture.

Since in this construction the Kähler potential already possesses a shift symmetry in the quintessence field, it is natural, although not necessary, to consider models where the superpotential is also shift symmetric in q, which is the case in axion-like models. An example where the superpotential of the quintessence sector possesses shift symmetry and the right energy scale can naturally arise [26, 122], based on the observation that the energy scale of dark energy is related to the electroweak scale M_{EW} and Planck scale by

$$\rho_{DE}^{1/4} \sim \frac{M_{EW}^2}{M_{P\ell}}. \tag{3.28}$$

In particular, assume SUSY is broken at the TeV scale by an order parameter chiral superfield $\langle S \rangle = \theta^2 M_{EW}^2$ and there is a hidden supersymmetric QCD (SQCD) sector Ψ with $SU(N_c)$ gauge group and N_f flavors that couples to SUSY breaking sector and the observable sector only through Planck-suppressed interactions. Once the SUSY is broken, the hidden sector quarks acquire a mass through the operator

$$\int d^4\theta \frac{S^*}{M_{P\ell}} \tilde{\Psi}\Psi \tag{3.29}$$

and hence the masses of the hidden quarks are of the order of $m_\Psi \sim M_{EW}^2/M_{P\ell}$. Similarly, the hidden gluino acquires a mass of the order of $m_\lambda/g^2 \sim M_{EW}^2/M_{P\ell}$ through the operator

$$\int d^2\theta \frac{S}{M_{P\ell}} \mathcal{W}_\alpha \mathcal{W}^\alpha = \frac{M_{EW}^2}{M_{P\ell}} \lambda\lambda. \tag{3.30}$$

Assuming the gluino mass is somewhat smaller than the others and $3N_c - N_f \ll N_f$, then the strongly coupled scale of the hidden SQCD is about the same as the hidden quark mass,

$$\Lambda \sim m_\Psi \sim \frac{M_{EW}^2}{M_{P\ell}}, \tag{3.31}$$

because the sector becomes strongly coupled quickly after the hidden quarks decoupled. With the gluino condensation, the axion Q associated with the SQCD then has a superpotential

$$W_{\text{axion}} = \Lambda^3 e^{-\frac{8\pi^2 Q}{N_c M_{P\ell}}}. \tag{3.32}$$

Plugging this back to Eq.(3.21) and an appropriate tuning of the cosmological constant contribution yield the usual cosine-type potential

$$V = \left(\frac{M_{EW}^2}{M_{P\ell}}\right)^4 \left[1 - \cos\left(\frac{q}{f_q}\right)\right], \tag{3.33}$$

with the decay constant $f_q = \frac{N_c}{8\pi^2} M_{P\ell}$. With large enough N_c, we may have super-Planckian decay constant. But note that for axion quintessence model, a super-Planckian decay constant is not strictly required to pass the cosmological test [101]. This is because quintessence models do not require accelerating expansion as long as that of inflation, and hence even if the decay constant is sub-Planckian, sufficient e-folds of expansion can still be achieved if the quintessence starts from near the top of the cosine potential.

The above paradigm is intriguing as it naturally connects the dark energy energy scale with the other two important physical scales, M_{EW} and $M_{P\ell}$, and the cosmic coincidence problem can also be explained [26]. In addition, as the quintessence sector is essentially the hidden SQCD axion, we have the shift symmetry to protect quintessence from acquiring large gravitino mass when we embed it into SUGRA. Yet, one should notice that such paradigm requires the SUSY breaking scale to be exactly at the TeV scale, which means the gravitino mass is at the order of meV. An explicit construction of such kind of model is challenging and yet to be done.

Lastly, it should be emphasized that even though in the above example the superpotential Eq.(3.32) is shift symmetric from $U(1)$ R-symmetry, shift symmetric superpotential is *not* necessary for general constructions. One only requires shift symmetries in Kähler potential. It may be possible and interesting to construct models where the energy scale Λ does not arise from mechanism that involves shift symmetry. Given that there is a simple framework to realize any quintessence potential within supergravity, the low-energy limit of string theory, the swampland conjecture passes the first non-trivial test.

3.3 Quintessence in Sequestered Supergravity

In this section we show another attempt of constructing quintessence model in supergravity. In particular, we consider the case where the hidden and quintessence sectors are *sequestered* [199], yielding a Kähler potential of the form

$$K = -3M_{P\ell}^2 \ln\left(1 - \frac{f(z,z^*)}{3M_{P\ell}^2} - \frac{g(Q,Q^*)}{3M_{P\ell}^2}\right), \tag{3.34}$$

where f and g are real functions of z and Q respectively. This form of Kähler potential can be originated from higher dimensional theory where the two sectors live on two separate 3-branes. For the superpotential, we have the same form as before,

$$W = W_0(z) + W_1(Q). \tag{3.35}$$

Like in the case of shift symmetry, we first work out potential of the hidden sector, which will tell us how SUSY breaking affects quintessence. Working out the scalar potential, we have

$$V_F = \frac{\mathcal{K}^{-2}}{\mathcal{D}}\left\{ g_{QQ^*}\left[\mathcal{K}_0\left|\frac{\partial W_0}{\partial z}\right|^2 + \frac{1}{M_{P\ell}^2}\left(f_z\frac{\partial W_0^*}{\partial z^*}W_0 + c.c.\right) - \frac{3}{M_{P\ell}^2}f_{zz^*}|W_0|^2\right] \right.$$
$$\left. + \frac{(|g_Q|^2 - g\, g_{QQ^*})}{3M_{P\ell}^2}\left|\frac{\partial W_0}{\partial z}\right|^2 + \mathcal{O}(W_1)\right\} \tag{3.36}$$

where

$$\mathcal{K} \equiv 1 - \frac{f}{3M_{P\ell}^2} - \frac{g}{3M_{P\ell}^2}, \qquad \mathcal{K}_0 \equiv 1 - \frac{f}{3M_{P\ell}^2}, \qquad \mathcal{D} = \mathcal{K} f_{zz^*} g_{QQ^*} + \frac{f_{zz^*}|g_Q|^2}{3M_{P\ell}^2} + \frac{g_{QQ^*}|f_z|^2}{3M_{P\ell}^2}. \tag{3.37}$$

The subscripts of f and g denotes derivative with respect to the respective variables. Because of the large hierarchy between the energy scale of dark energy and SUSY breaking scales, terms proportional to W_1 are negligible when considering the dynamics of SUSY breaking. From Eq.(3.36) we can see that *if* the condition

$$\left(|g_Q|^2 - g\, g_{QQ^*}\right) = 0 \tag{3.38}$$

is satisfied and the square bracket in the first line vanishes when z lies at its minimum such that the hidden sector contributes zero vacuum energy, then the quintessence sector will not acquire a mass of SUSY breaking scale. The condition Eq.(3.38) means that $g(Q,Q^*)$ has to be in the canonical form

$$g(Q,Q^*) = QQ^*. \tag{3.39}$$

Indeed, if we expand g to higher orders in the form of

$$g(Q,Q^*) = |Q|^2\left(1 + \alpha_1\frac{|Q|^2}{M_{P\ell}^2} + \alpha_2\frac{|Q|^4}{M_{P\ell}^4} + \cdots\right), \tag{3.40}$$

where α_i's are dimensionless coefficients, then the first term of the second line in Eq.(3.36) will then be

$$-\frac{|Q|^2}{3M_{P\ell}^2}\left[\alpha_1\frac{|Q|^2}{M_{P\ell}^2}+4\alpha_2\frac{|Q|^4}{M_{P\ell}^4}\cdots\right]\left|\frac{\partial W_0}{\partial z}\right|^2\sim-\frac{1}{3}\left[\alpha_1\frac{|Q|^2}{M_{P\ell}^2}+4\alpha_2\frac{|Q|^4}{M_{P\ell}^4}\cdots\right]m_{3/2}^2|Q|^2. \quad (3.41)$$

Unless $|Q|^2/M_{P\ell}^2 \ll 1$, we see that the quintessence sector acquires an effective quadratic term at the order of gravitino mass squared. In general, $g(Q,Q^*)$ need not be in the canonical form as Kähler potential would be renormalized when quantum effect is taken into account. Even if the two sectors live on separate branes, gravity still mediates between the two and quantum gravity effect generically spoils the tree level Kähler potential. Nonetheless, this can be regarded as another manifestation of the cosmological constant problem in the sense that quantum effect naively yields an energy scale much larger than the dark energy energy scale. In fact, when we make the hidden sector contributes zero vacuum energy, it is also controlled only at the tree level. As tackling quantum gravity effect fully remains challenging, we choose to proceed with the assumption that the canonical form of $g(Q,Q^*)$ is preserved by some unknown mechanism.

Moving ahead, let us workout the potential of the hidden sector more explicitly. For the superpotential we adopt the Polonyi type as we did in the shift symmetry case,

$$W_0 = \mu M_{P\ell}(z+\beta), \quad (3.42)$$

where μ is a mass dimension one parameter of the SUSY breaking scale. For the Kähler potential, we consider the form

$$f = |z|^2 + \lambda\frac{|z|^4}{3M_{P\ell}^2}. \quad (3.43)$$

The quartic term is included because with the quadratic term alone the potential will not be bounded from below. In fact, to make the potential bounded from below, λ has to be negative and we choose $\lambda = -1/4$ for convenience. Demanding the hidden sector contributes zero vacuum energy, one finds

$$\beta = \frac{\left(3-\sqrt{5}\right)^{3/2}}{2\sqrt{6}}M_{P\ell} \quad (3.44)$$

and the hidden sector field lies at the vev

$$\langle z\rangle = \sqrt{\frac{6}{5}\left(3-\sqrt{5}\right)}\,M_{P\ell}. \quad (3.45)$$

With the hidden sector settles at its vev and taking $g(Q,Q^*) = |Q|^2$, the potential for the quintessence sector then reads

$$V_{\text{Quin}} = \frac{1}{\langle \mathcal{D} \rangle \left(\langle \mathcal{K}_0 \rangle - \frac{|Q|^2}{3M_{P\ell}^2} \right)^2} \left\{ \left\langle \mathcal{K}_0 f_{zz^*} + \frac{|f_z|^2}{3M_{P\ell}^2} \right\rangle |W_{1Q}|^2 \right.$$
$$+ \frac{\langle f_{zz^*} \rangle}{3M_{P\ell}^2} \left(3W_1 Q^* W_{1Q}^* + \text{c.c.} - |Q|^2 |W_{1Q}|^2 - 9|W_1|^2 \right)$$
$$\left. \frac{1}{3M_{P\ell}^2} \left[\langle 3 f_{zz^*} W_0 - f_z W_{oz}^* \rangle \left(Q W_{1Q} - 3W_1 + \text{c.c.} \right) \right] \right\}. \tag{3.46}$$

Note that the potential approaches infinity when $Q \to \pm\sqrt{3\langle K_0 \rangle} M_{P\ell} \simeq 1.47 M_{P\ell}$, and hence the field displacement of the quintessence field is confined within this range, rendering large-field type model impossible in the sequestering setup. Even though such singularity can be apparently removed when we make the field redefinition so that the kinetic term in the Lagrangian becomes canonical, the prefactor outside the curly brackets still dominates and one can check that the slow-roll parameters are larger than unity in the large field region. In fact, since part of the prefactor is originated from the e^K prefactor of the scalar potential, this is similar to the η-problem in inflationary model building in supergravity. We therefore focus on small-field type potential, where, for instance, the quintessence field rolls on a plateau. These types of models are generally sensitive to initial conditions, unlike tracker-type quintessence models where the field evolutions with wide range of initial conditions converge to a common trajectory. Despite this shortcoming, our goal is to investigate the possibility if quintessence models can be built in sequestered supergravity that lead to observational signature distinguishable from cosmological constant, hence we will bear with this initial condition problem.

The first thing we would like to ensure is that the potential is bounded from below. In Eq.(3.46) the first and second line are at the order of $\mathcal{O}(W_1^2/M_{P\ell}^2)$, while the third line is at the order of $\mathcal{O}(\mu^2 W_1/M_{P\ell})$. Because of the SUSY breaking scale μ, the third line generically has a much larger energy scale and hence dominates the potential. However, the third line of Eq.(3.46) is not positive-definite and there is always a direction in the complex field space where the potential approaches negative infinity when $|Q| \to \sqrt{3\langle K_0 \rangle} M_{P\ell}$. It seems that we need multiple parameters in the superpotential with a large hierarchy among them to make the potential bounded from below.

Consider a superpotential for the quintessence sector of the form

$$W_1(Q) = \Lambda^3 \left(\frac{Q}{M_{P\ell}} \right)^n, \tag{3.47}$$

which yields

$$V_{\text{Quin}} = \frac{1}{\langle\mathcal{D}\rangle\left(\langle\mathcal{K}_0\rangle - \frac{|Q|^2}{3M_{P\ell}^2}\right)^2}\Bigg\{ -\frac{\langle f_{zz^*}\rangle}{3}(n-3)^2\frac{\Lambda^6}{M_{P\ell}^2}\left(\frac{|Q|^2}{M_{P\ell}^2}\right)^n + \left\langle \mathcal{K}_0 f_{zz^*} + \frac{|f_z|^2}{3M_{P\ell}^2}\right\rangle n^2 \frac{\Lambda^6}{M_{P\ell}^2}\left(\frac{|Q|^2}{M_{P\ell}^2}\right)^{n-1} \frac{\Lambda^3}{3M_{P\ell}^2}\langle 3f_{zz^*}W_0 - f_z W_{oz}^*\rangle (n-3)\left[\left(\frac{Q}{M_{P\ell}}\right)^n + \left(\frac{Q^*}{M_{P\ell}}\right)^n\right]\Bigg\}. \tag{3.48}$$

We see that the potential is positive-definite only when $n = 3$. Hence when we consider superpotential of polynomial form, the highest order must be truncated at $n = 3$. Going beyond cubic order will yield potential that is unbounded from below. We therefore consider

$$W_1(Q) = \tilde{a}\left(\frac{Q}{M_{P\ell}}\right)^3 + \tilde{b}\left(\frac{Q}{M_{P\ell}}\right)^2 + \tilde{c}\left(\frac{Q}{M_{P\ell}}\right) + \tilde{d}, \tag{3.49}$$

which gives the potential of the form

$$V_{\text{Quin}} = \frac{1}{\langle\mathcal{D}\rangle\left(\langle\mathcal{K}_0\rangle - \frac{|Q|^2}{3M_{P\ell}^2}\right)^2}\Bigg\{ 9\left\langle \mathcal{K}_0 f_{zz^*} + \frac{|f_z|^2}{3M_{P\ell}^2}\right\rangle \frac{|\tilde{a}|^2}{M_{P\ell}^2}\frac{|Q|^4}{M_{P\ell}^4} - \frac{1}{3M_{P\ell}^2}\langle 3f_{zz^*}W_0 - f_z W_{oz}^*\rangle \left[\tilde{b}\left(\frac{Q}{M_{P\ell}}\right)^2 + 2\tilde{c}\left(\frac{Q}{M_{P\ell}}\right) + 3\tilde{d} + \text{c.c.}\right]\Bigg\} \tag{3.50}$$

Note that in order to make all the terms to have the same energy scale and describe dark energy, we need $\tilde{a} \sim 10^{-60}M_{P\ell}^3$ and $\tilde{b} \sim \tilde{c} \sim \tilde{d} \sim 10^{-105}M_{P\ell}^3$. There is a large hierarchy between $\tilde{a}$ and the other three parameters because of the coupling of the latter three with the SUSY breaking scale μ, where $\mu^2/M_{P\ell} \sim$ TeV. Also note that because of this large hierarchy, we have neglected the cross terms in the potential like $\tilde{a}\tilde{b}$, $\tilde{b}\tilde{c}$, etc. Define the real and imaginary part of $Q/M_{P\ell}$,

$$r \equiv \mathfrak{Re}\frac{Q}{M_{P\ell}}, \qquad s \equiv \mathfrak{Im}\frac{Q}{M_{P\ell}}, \tag{3.51}$$

and assume the parameters are real, the potential has the form

$$V_{\text{Quin}} = \frac{1}{\langle\mathcal{D}\rangle\left(\langle\mathcal{K}_0\rangle - \frac{r^2+s^2}{3M_{P\ell}^2}\right)^2}\Bigg\{ 9\left\langle \mathcal{K}_0 f_{zz^*} + \frac{|f_z|^2}{3M_{P\ell}^2}\right\rangle \frac{|\tilde{a}|^2}{M_{P\ell}^2}(r^2+s^2)^2 + \frac{2}{3M_{P\ell}^2}\langle 3f_{zz^*}W_0 - f_z W_{oz}^*\rangle \left[\tilde{b}\,s^2 - \tilde{b}\,r^2 - 2\tilde{c}\,r - 3\tilde{d}\right]\Bigg\}. \tag{3.52}$$

Notice that the imaginary part has even power and global minimum at $s = 0$. Hence we can assume the imaginary part lies at its minimum and focus on the potential for the real part only.

Lastly, note that in supergravity the kinetic term of the scalar field is given by

$$\mathcal{L}_{kin} = -K_{QQ^*}\partial_\mu Q \partial^\mu Q^* = -\frac{\mathcal{K}_0}{\left[\mathcal{K}_0 - \frac{|Q|^2}{3M_{P\ell}^2}\right]^2}\partial_\mu Q \partial^\mu Q^* \quad (3.53)$$

Because the imaginary part lies at the $s = 0$, from now on we will take Q as real. The kinetic term will be canonical after we make the field redefinition

$$\tilde{Q} = \sqrt{3}M_{P\ell}\tanh^{-1}\left[\frac{Q}{\sqrt{3\mathcal{K}_0}M_{P\ell}}\right]. \quad (3.54)$$

In terms of the canonically normalized field, we arrive at the general potential of the quintessence field in the setup of sequestered supergravity:

$$V_{\text{Quin}} = \rho_{DE}\cosh^4\left(\frac{\tilde{Q}}{\sqrt{3}M_{P\ell}}\right)\left\{a^2\tanh^4\left(\frac{\tilde{Q}}{\sqrt{3}M_{P\ell}}\right) - b\tanh^2\left(\frac{\tilde{Q}}{\sqrt{3}M_{P\ell}}\right) + c\tanh\left(\frac{\tilde{Q}}{\sqrt{3}M_{P\ell}}\right) + d\right\} \quad (3.55)$$

Here a, b, c, and d are dimensionless parameters where $b > 0$ to ensure the imaginary part of the field lies at zero.

One possible scenario from this general form is a potential with long plateau on which the quintessence field slow-rolls. To obtain such kind of potential requires some fine-tuning of the parameters is required. In the left of Figure 3.1 we show an explicit example of such kind.

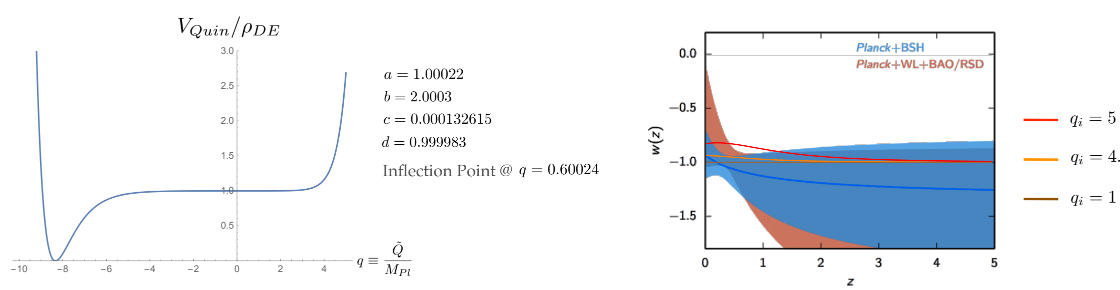

Figure 3.1: (Left): Plot of the quintessence potential Eq.(3.55) with the given parameters. The potential has an inflection point at $q = 0.60024$. The parameters are chosen in a way such that at the global minimum has $V = 0$, while the quintessence field contributes the right amount of energy when it slow rolls on the plateau. (Right): The evolution of the equation of state parameter w for the potential shown in the left panel with various initial conditions, in comparison with the $w(z)$ given in [11] which was reconstructed from observational data. We see that, for instance, with the quintessence field starting at $q_i = 4.5$, one can have a interesting deviation from $w = -1$ that still satisfies current constraints.

With a given potential of a long plateau, one also has to choose the initial condition for the field. The evolution of the quintessence field at late time, say redshift $z < 5$, is insensitive to the initial velocity of the field. This is because no matter how large the initial velocity is, it would soon be damped away by the Hubble friction and remain frozen near at its initial position until the matter and radiation energy density become low enough. After the field started to roll, its equation of state parameter w would gradually increase from $w = -1$. If the field started with an initial position q_i on the steep slope, say $q_i = 5$ for the potential shown in Figure 3.1, w may increase too much and exceed the observational bound. On the other hand, if the field started on the flat plateau, say $q_i = 1$, w would not deviate from $w = -1$ too much and act like a cosmological constant. The phenomenologically most interesting case happens when the field has an initial position at the junction of the flat plateau and the steep slope, say $q_i = 4.5$. The time evolution of the equation of state parameter with these three different initial conditions is shown on the right in Figure 3.1.

Although considering the quintessence and SUSY breaking sector alone can lead to interesting observational signatures in the context of sequestered supergravity, things would unfortunately change when we consider the coupling between the quintessence and matter. Specifically, the constraint from the fifth force would require the quintessence field to have a nearly zero field value, resulting a equation of state parameter with little deviation from $w = -1$ and hence cannot be observationally distinguishable from a pure cosmological constant. To elaborate more on this, note that the fermion mass has the form

$$m_{u,d} = y\, e^{K/2M_{P\ell}^2}\, v_{u,d}, \tag{3.56}$$

where y is the Yukawa coupling, $m_{u,d}$ are the mass of the u-type and d-type particles, and $v_{u,d}$ are the vev of u-type and d-type Higgs field H_u and H_d. With the $\tilde{Q}$-dependence in the Kähler potential, we effectively have a coupling between the quintessence and the matter sector, whose strength is determined by

$$\alpha \equiv M_{P\ell}\frac{\partial \ln m}{\partial \tilde{Q}}. \tag{3.57}$$

This interaction is often dubbed "the fifth force". In sequestered scenario, we have $K = -3M_{P\ell}^2 \ln \mathcal{K}$, where $\mathcal{K}$ is defined in Eq.(3.37). Hence the strength of the fifth force is given by

$$\alpha = -\frac{3}{2}M_{P\ell}\frac{dQ}{d\tilde{Q}}\frac{d\ln \mathcal{K}}{dQ} = \sqrt{3}\tanh\left(\frac{\tilde{Q}}{\sqrt{3}M_{P\ell}}\right). \tag{3.58}$$

Observational constraints on α from radar time-delay effect give stringent bounds on the strength of the fifth force. For instance, measurements made by Cassini spacecraft yield a bound of $\alpha^2 \lesssim 10^{-5}$ [49]. By Eq.(3.58), this translates to the bound on the quintessence field

$$\left(\frac{\tilde{Q}}{M_{P\ell}}\right)^2 \lesssim 10^{-5}. \tag{3.59}$$

Such a tiny small range means the the equation of state parameter w cannot be largely deviated from -1. To see this, note that the current field velocity is related to the field displacement by

$$\dot{\tilde{Q}} \sim \Delta\tilde{Q} H_0. \tag{3.60}$$

This is of course a rough approximation since the field doesn't roll for the whole time of the age of the universe. Yet the the difference should only be an order of one tenth. The equation of state parameter w at late time can therefore be approximated as

$$w = \frac{\frac{1}{2}\dot{\tilde{Q}}^2 - V}{\frac{1}{2}\dot{\tilde{Q}}^2 + V} \sim \frac{\frac{1}{2}\Delta\tilde{Q}^2 H_0^2 - 3M_{P\ell}^2 H_0^2}{\frac{1}{2}\Delta\tilde{Q}^2 H_0^2 + 3M_{P\ell}^2 H_0^2} \sim -1 + \frac{\Delta\tilde{Q}^2}{3M_{P\ell}^2}. \tag{3.61}$$

Hence, combining with the fifth force constraint, the deviation of equation of state parameter from -1 in sequestered supergravity is less than 10^{-5}, making it challenging to be detected in current and near future observations.

Recently, a swampland conjecture regarding the shape of the scalar potential in any consistent theory of quantum gravity was put forward in [185], where the authors suggested that the potential of the scalar fields φ_i's should satisfy the criterion

$$M_{P\ell}|\nabla V| > cV, \tag{3.62}$$

where ∇V is the gradient with respect to the scalar fields φ_i's, and c is a number of order $\mathcal{O}(1)$. Due the fifth force constraint, the field value of $\tilde{Q}$ is confined to be closed to the origin and hence the potential Eq.(3.55) can be approximated as a linear potential of the form

$$V_{\text{Quin}} = \rho_{DE}\left(1 + c\,\frac{\tilde{Q}}{M_{P\ell}}\right). \tag{3.63}$$

With this simple form, one can solve the evolution of $\tilde{Q}$ and the displacement $\Delta\tilde{Q}$ is given by

$$\Delta\tilde{Q} \sim c\,M_{P\ell}. \tag{3.64}$$

This means that in order to satisfy the fifth force constraint Eq.(3.59), we not only need to have an initial condition $\tilde{Q}_i^2 < 10^{-5}M_{P\ell}^2$, the slope of the potential also needs to satisfy $c^2 \lesssim 10^{-5}$. This will violate the swampland conjecture Eq.(3.62) if the number c in the conjecture is of order one, and the sequestered scenario will be theoretically ruled out. Nonetheless, as pointed out in [95], the swampland conjectures should be regarded *parametrically* and the number c does not have to be of order one. Indeed, using effective field theoretic arguments, the authors in [95] argued that c should be the order of m_ℓ/m_h, where m_ℓ is the mass of light particle considered, and m_h is the mass of the lightest heavy particle that one integrates out. Therefore, if the hierarchy between m_ℓ and m_h is large enough, namely $(m_\ell/m_h)^2 \lesssim 10^{-5}$, the sequestered scenario can still satisfy the swampland conjecture.

3.4 Discussion

In this note we have discussed quintessence model building in supergravity. We stressed that there are two main issues when trying to construct supergravity quintessence models that are observationally distinguishable from a pure cosmological constant. Firstly, for any realistic models, it is necessary to consider the effect of SUSY breaking which often gives quintessence a mass at the scale of the gravitino mass, which is much larger than that of the current Hubble scale. This renders the potential too steep such that quintessence settles at the minimum in the very early time and acts like a pure cosmological constant. One can avoid this problem by imposing shift symmetry on the Kähler potential of the quintessence sector, and an advantage of this approach is that it is much easier to embed any quintessence potential in this framework – the quintessence potential is simply proportional to the real part of the superpotential. In addition, even though the shift symmetry is broken by the superpotential, because of the hierarchy between the quintessence energy scale and the Planck scale, such effect does not affect the quintessence dynamics. In addition, the energy scale of the quintessence potential is related to the gravitino mass, and hence one can consider models that relates electroweak physics and quintessence. As an example, we considered the scenario where SUSY is broken at the TeV scale and there is a hidden SQCD axion that plays the role of quintessence. In such scenario, the dark energy scale is given by the electroweak scale and Planck scale, and the cosmic coincidence problem can be ameliorated [26].

We also proposed another way to circumvent this issue, namely by sequestering the SUSY breaking and quintessence sectors. This approach is based on the picture of a higher dimensional theory where two sectors live on different 3-branes and only communicate with each other through gravity. We showed that indeed quintessence does not acquire a gravitino mass at least at the tree level. Once higher order terms kick in this generally no longer hold and one needs to assume some mechanism in quantum gravity preserves the form of the Kähler potential as Eq.(3.40). However, for models with small field displacement, these higher order terms are Planck suppressed and hence do not disturb the dynamics of the quintessence field.

The second main issue one needs to consider is the observational constraints from various gravitational tests like the fifth force constraint. In particular, the strongest source of the coupling between quintessence and matter stems from the exponential factor in the fermionic mass term Eq.(3.56). In most models, this gives a strong constraint on the quintessence field range, and as shown in Eq.(3.61), how much the equation of state parameter can deviate from -1 is constrained by the quintessence field displacement. Therefore, in order to build quintessence models that can be observationally distinguishable from pure cosmological constant, it seems that one needs to ensure quintessence field does not appear in the exponential factor. In the case with shift symmetry, because the Kähler potential Eq.(3.7) does not depend on the imaginary part of the quintessence superfield which plays the role of the slow-roll quintessence field, the quintessence field does not appear in the exponential factor and hence the observational constraint on matter-quintessence coupling, α, defined in Eq.(3.57) can be satisfied even for large field displacement. In the sequestered scenario, because the quintessence field still appear in the exponential factor, field displacement is strictly limited by the fifth force constraint and it would be a challenging task to observationally distinguish such models from pure cosmological constant through equation of state parameter.

To conclude, any generic quintessence potential can be realized in supergravity with shift symmetric Kähler potential in the quintessence sector, which avoids quintessence from obtaining a mass of the order of gravitino mass $m_{3/2}$ and evades fifth-force constraint. Given this simple framework to realize quintessence potential in SUGRA, the low energy effective theory of string theory, the swampland conjecture passes its first non-trivial test.

Chapter 4

What does Inflation say about Dark Energy given the Swampland Conjectures?

This chapter is based on the work with Jacob Leedom and Hitoshi Murayama [70]. I would like to thank both of them for the collaboration of this project as well as many stimulating discussions about the swampland program in general.

4.1 Introduction

The discovery of the accelerating expansion of the Universe [196, 201] was a huge surprise to the community. Because gravity only *pulls*, it should put a brake on the expansion of the Universe after the Big Bang and hence the expansion should decelerate. Acceleration implies there is a substance in the Universe that *pushes* the expansion. It was dubbed *dark energy*. The most discussed candidate for dark energy is the cosmological constant Λ, a finite energy density of the vacuum, due to the simple way it can be implemented into cosmological models based on general relativity. However, despite being consistent with data [13], the 120 orders of magnitude difference between the observed vacuum energy density ($\rho \approx (\text{meV})^4$) and the naïve theoretical expectation ($\rho \approx M_{P\ell}^4$) still remains the most challenging problem in modern physics [225].

Since dark energy and the cosmological constant problem inevitably involve quantum gravity, string theory, as a theory of quantum gravity, should address these topics. The attempts to construct de Sitter solutions (spacetime solutions to general relativity with a positive Λ) in string theory [58, 111, 146] have lead to the notion of the string landscape. The landscape consists of an enormous number of vacua, each described by different low-energy effective field theories (EFTs) of different fields and parameters. String theory therefore supports the anthropic argument [224], namely that the value of the observed dark energy density is what it is because otherwise human civilization could not exist. If we really live in a (meta-)stable vacuum in the string landscape where a constant vacuum energy explains dark energy, then there is no point in measuring the dark energy equation of state parameter $w = p/\rho$, where p and ρ are the pressure and energy density of the dark energy, respectively.

String theory seems to lead to many possible low-energy EFTs, so conversely one can ask what criteria a given low-energy EFT should satisfy in order to be contained in the string landscape. For the last decade, several criteria of this kind, dubbed *swampland conjectures*, have been proposed [218, 187, 27]. These can have important cosmological implications. For instance, one of the relatively well-established conjectures is the *distance swampland conjecture* [187, 191, 41, 158, 219, 53, 193, 170, 129, 76, 116, 131, 54, 162] which implies that scalar fields in a low-energy EFT of a consistent theory of quantum gravity cannot have field excursions much larger than the Planck scale since otherwise an infinite tower of states becomes exponentially light and the validity of the EFT breaks down. In other words, one has the constraint

$$\Delta\phi \lesssim \alpha M_{P\ell}, \qquad \alpha \approx O(1). \tag{4.1}$$

In the context of inflation, field excursions are related to the tensor-to-scalar ratio r by the Lyth bound [83],

$$\frac{\Delta\phi}{M_{P\ell}} \simeq \sqrt{\frac{r}{8}}\mathcal{N} \tag{4.2}$$

where $\mathcal{N}$ is the number of e-folds of inflationary expansion. Clearly the distance conjecture, Eq. (4.1), limits the possibility of measuring tensor modes and hence primordial B-modes in the cosmic microwave background (CMB). Naively, with $\mathcal{N} \gtrsim 50$, we find $r \lesssim 0.003$, which is on the edge of observability for future experiments [9, 212].

The attempts to construct de Sitter solutions or inflationary models in string theory [146, 147, 208, 29, 39, 226, 40, 99, 203, 52, 78, 75] have sparked discussions on various issues with such constructions, as well as no-go theorems [171, 216, 135, 85, 176, 66, 67, 65, 228, 207, 115, 110, 44, 50, 43, 86, 160, 198, 91, 143, 142, 25, 178, 206, 23, 87]. Motivated by the obstructions encountered in various attempts, the *de Sitter swampland conjecture* was proposed [185], which states that the scalar potential of a low-energy limit of quantum gravity must satisfy

$$M_{P\ell}|\nabla V| \geq c\, V, \qquad c \approx O(1) > 0 \tag{4.3}$$

where ∇ denotes the gradient with respect to the field space, and the norm of the gradient is defined by the metric on field space. Whether the conjecture holds true is still an open debate [150, 102, 47, 48, 127, 148, 197, 149, 8, 77, 92, 202, 22, 24, 81, 19, 152, 145, 177, 45, 109]. Yet, even before the debate is settled, it is interesting and important to investigate both its consequences in cosmology and potential modifications or extensions [73, 133, 132, 172, 181, 157, 169, 46, 175, 80, 94, 95, 155, 105, 10, 88, 222, 59, 125, 60, 96, 100, 164, 124, 153, 179, 28, 89, 223, 104, 130, 186, 106, 195, 51, 204, 163]. The primary implication of this condition is that the observed positive energy density of our Universe should correspond to the potential of a rolling quintessence field rather than a positive Λ [15]. The fact that one can easily embed any quintessence model into supergravity [64, 71] in a rather simple fashion, despite the difficulty that supersymmetry breaking generically spoils the flatness of the quintessence potential, is also encouraging. This raises the hope that $w \neq -1$ might be detected.

The de Sitter conjecture forbids (meta-)stable vacua with positive energy density, so it is not surprising that the inflationary paradigm has apparent conflicts with the conjecture and one may call for a paradigm shift. Nonetheless, one can also adopt a conservative approach and regard the conjecture as a parametric constraint where the inequality holds but the number c may not be strictly $\mathcal{O}(1)$ [95]. From this perspective, constraints on inflation can then be used to constrain c.

However, if we follow this route, the optimism that one can observe $w \neq -1$ is greatly diminished. To see this, recall that in single-field slow-roll inflation, the slow-roll parameters of the potential are defined as

$$\epsilon_V \equiv \frac{M_{P\ell}^2}{2}\left(\frac{V'}{V}\right)^2, \quad \eta_V \equiv M_{P\ell}^2 \frac{V''}{V}, \tag{4.4}$$

where the primes denote derivatives with respect to the inflaton. The distance conjecture limits the inflaton field excursion $\Delta\phi \approx \sqrt{2\epsilon_V}\,\mathcal{N} \lesssim \mathcal{O}(1)$ and therefore the necessary number of e-folds $\mathcal{N} \approx 50$ forces $c \lesssim \sqrt{2\epsilon_V} \lesssim \mathcal{N}^{-1} \sim 0.02$. On the other hand, the number c in Eq. (4.3) is meant to be *universal* in a given EFT. Therefore, the current accelerating expansion must involve a quintessence field Q whose potential V_Q must satisfy

$$1 + w = \frac{2(V_Q')^2}{(V_Q')^2 + 6V_Q^2} > \frac{2c^2}{6+c^2} \equiv \Delta \gtrsim 1.33 \times 10^{-4}. \tag{4.5}$$

Although this does not exclude observable quintessence, given the fact that so far almost all observations are consistent with a cosmological constant, such a small lower bound on possible deviation of w from -1 makes it questionable if it is worthwhile to push the sensitivity of the observations further. We may never know whether the Universe is de Sitter or quintessence.

However, the original de Sitter conjecture, Eq. (4.3), was so strong that even the Higgs potential was in tension with it [94]. The conjecture was also in tension with the well-understood supersymmetric AdS solutions [81]. Recently the *refined de Sitter swampland conjecture* was proposed [105, 188], which states that the scalar potential of a low-energy theory that can be consistently coupled to quantum gravity should satisfy *either*

$$M_{P\ell}|\nabla V| \geq c\,V, \qquad c \approx O(1) > 0, \tag{4.6}$$

or

$$M_{P\ell}^2 \min(\nabla_i \nabla_j V) \leq -c' V, \qquad c' \approx O(1) > 0, \tag{4.7}$$

where min(...) denotes the minimum eigenvalue of the Hessian $\nabla_i\nabla_j V$ in an orthonormal frame of the scalar field space. With this refinement, the aforementioned conflicts with the Higgs potential and the SUSY AdS solutions are resolved. The refined conjecture also raises new possibilities for inflation. In particular, one can evade the strict bound on c arising from the distance conjecture by having the scalar potential satisfy the second condition Eq. (4.7) of the new conjecture during part (or all) of inflation. As such, one may regain the hope that observable time-varying dark energy with $w \neq -1$ can be obtained. See also [14] for a recent discussion on w in consideration of the refined dS conjecture.

4.2 Single-Field Slow-Roll Inflation Models

Due to the above tension between the de Sitter conjecture and the requirements of inflation, we assume that the inflaton potential switches from one de Sitter condition to another as the inflaton rolls, an idea also utilized in [104]. To be specific, we take the following step-function approach to keep the discussion general and simple: we apply the first condition, Eq. (4.6), for the initial $\mathcal{N}_1$ e-folds and apply the second condition, Eq. (4.7), for the remaining $\mathcal{N}_2 = \mathcal{N}_{tot} - \mathcal{N}_1$ e-folds. In our analysis we set $\mathcal{N}_{tot} = 50$. We assume ϵ_V and η_V are approximately constant for each interval so that we have

$$\sqrt{2\epsilon_V^{(1)}} \geq c \text{ and } \eta_V^{(2)} \leq -c'. \tag{4.8}$$

Additionally, Eq. (4.1) requires that

$$\sqrt{2\epsilon_V^{(1)}}\mathcal{N}_1 + \sqrt{2\epsilon_V^{(2)}}\mathcal{N}_2 \leq \alpha \sim O(1). \tag{4.9}$$

To maximize c, we assume $\epsilon_V^{(2)} < 10^{-4}$ so that the contribution of the second era to Eq. (4.1) is negligible. Combining Eq. (4.8) and Eq. (4.9), we have

$$c < \frac{\alpha - \sqrt{2\epsilon_V^{(2)}}\mathcal{N}_2}{\mathcal{N}_1} . \tag{4.10}$$

We can also obtain a bound for c' from the spectral tilt $n_s = 1 - 2\epsilon - \eta$, where the Hubble slow-roll parameters are

$$\epsilon = -\frac{\dot{H}}{H^2}, \qquad \eta = \frac{\dot{\epsilon}}{H\epsilon} . \tag{4.11}$$

For single-field inflation models, these are related to the slow-roll parameters of the potential as $\epsilon_V = \epsilon$ and $\eta_V = 2\epsilon - \frac{1}{2}\eta$. Therefore, we can constrain η_V and hence the second parameter of the refined de Sitter conjecture as

$$c' < \frac{1}{2}\left(1 - n_s(k) - 6\epsilon_V^{(2)}\right) , \tag{4.12}$$

where we are allowing for a k-dependent spectral tilt. Since we assume $\epsilon_V^{(2)}$ is small, our bounds simplify to

$$(c', c) < \left(\frac{1 - n_s(k)}{2}, \frac{\alpha}{\mathcal{N}_1}\right) . \tag{4.13}$$

Eq. (4.13) is valid until $\mathcal{N}_1 = \mathcal{N}_{tot}$, at which point the derivation on the bound of c' above no longer applies, and the only constraint one finds is that $c < \alpha/\mathcal{N}_{tot}$. To proceed, we utilize the Planck analysis based on TT, TE, EE, lowE, lensing and BAO [13], which gives

$$dn_s/d\ln k = -0.0041 \pm 0.0067, \tag{4.14}$$
$$n_s = 0.9659 \pm 0.0040, \tag{4.15}$$

at $k_* = 0.05\text{Mpc}^{-1}$. We add errors in quadrature, ignoring correlations, and use

$$n_s(k) = 0.9659 - 0.0041 \ln \frac{k}{k_*} \pm \sqrt{(0.0040)^2 + \left(0.0067 \ln \frac{k}{k_*}\right)^2} . \tag{4.16}$$

A smaller n_s allows for larger c' in Eq. (4.13), so we take the 1σ allowed lower end in order to place our bounds. The weak correlation between n_s and $dn_s/d\ln k$ we see in Fig. 26 of [13] actually works in our favor and ignoring correlation is therefore the more conservative approach (*i.e.*, gives a smaller allowed range) [1]. Using the simple relationship $\mathcal{N}_1 = \ln(k/a_0 H_0)$, where a_0 is the present scale factor and H_0 is the present Hubble scale, we can constrain the swampland parameters in single-field inflation as shown in Fig. 4.1. The current CMB constraints on the spectral index and its running are limited to $\mathcal{N}_1 \lesssim 10$. This range is denoted by the solid lines in Fig. 4.1. Beyond this there are no strong observational constraints and we extend our analysis by extrapolating Eq. (4.16) to $\mathcal{N}_1 \geq 10$ shown by the dashed lines in Fig. 4.1. The unshaded regions indicate values of (c', c) that satisfy the above inequalities. The vertical asymptotes correspond to satisfying Eq. (4.7) for the entirety of the inflationary epoch, $\mathcal{N}_1 = 0$, so that c is left completely arbitrary but c' has a strict upper bound that is much less than the $\mathcal{O}(1)$ expectation. The horizontal dotted lines correspond to satisfying the first constraint Eq. (4.6) for all of inflation, $\mathcal{N}_2 = 0$, which leaves c' arbitrary but severely limits c. The horizontal black dashed lines indicate the lowest values of c that yield the given Δ defined in Eq. (4.5) as the lower bound on $1 + w$ from the constraint Eq. (4.6). Finally, the grey region excludes values of c that may satisfy Eq.(4.13), depending on the value of α, but conflicts with the constraint $r_{0.002} < 0.064$ [17], as $r = 16\epsilon \geq 8c^2$. The grey excluded region has a left vertical boundary since the constraint applies only to $k > 0.002\ \text{Mpc}^{-1}$.

We also comment on the observability of the tensor mode r. The swampland distance conjecture, Eq. (4.1), combined with the Lyth bound, Eq. (4.2), is normally believed to

[1]The Planck 2018 paper [13] also shows the analysis where they allow for the running of running $d^2 n_s/d\ln k^2$. Unfortunately they do not show the correlation and we cannot use it for our purposes. In fact, the extrapolation of $n_s(k)$ to small scales from the Planck data is most likely too restrictive, as the allowed range for the primordial power $P_\zeta(k)$ blows up for $k \gtrsim 0.2\ \text{Mpc}^{-1}$ (see Fig. 20 in [17]).

disfavor observably large r, assuming $\alpha \approx 1$. The best sensitivity anticipated in the future is $r \sim 10^{-3}$ [9, 212]. There is a parameter region in Figure 4.1 where $r \geq r_{\min} \equiv 8c^2$ is close to the current observational bound. Physically this is because, in our spirit of a step function approximation, we can allow for a brief initial period, say $\mathcal{N}_0 \sim 4$, where the upper bound on ϵ from the distance conjecture, $\epsilon \lesssim \mathcal{N}_0^{-2}/2 \sim 0.03$, is relaxed. Thus it is possible to have r large enough to saturate the observational bound at low ℓ. This is encouraging, especially for space-born CMB B-mode experiments such as LiteBIRD [212].

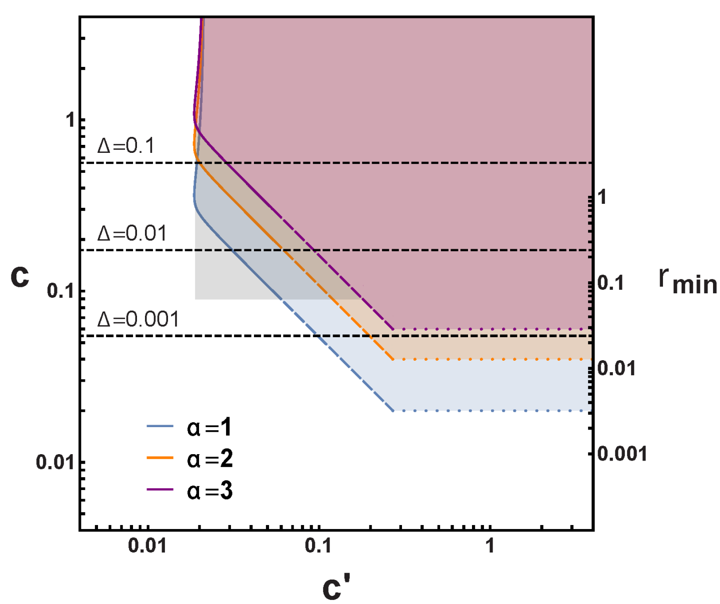

Figure 4.1: Bounds on swampland parameters for generic single-field inflation models at the 1σ level assuming the running of n_s can be extended to $\mathcal{N}_{tot} = 50$ e-folds. The unshaded region is the allowed parameter space. The solid lines are for $\mathcal{N}_1 \leq 10$; the dashed lines are for $10 < \mathcal{N}_1 < 50$, and the horizontal dotted lines correspond to $\mathcal{N}_1 = 50$, i.e. the first constraint Eq. (4.6) applies to the whole inflationary period. The values of c excluded by [17] are shaded in grey. We required the distance conjecture with $\Delta\phi \leq \alpha M_{Pl}$, and display the minimum values for $1 + w \geq \Delta$ with black dashed lines. With the original de Sitter conjecture, c had to be below the dotted horizontal lines but there were no constraints on c'.

4.3 Multi-Field Slow-Roll Inflation Models

The constraints discussed above are due to the tight relations between n_s, ϵ_V, η_V, and r in single-field slow-roll inflation models. It is natural to ask whether the constraints can be relaxed in multi-field models. In our analysis below, we take the conservative assumption that the swampland distance conjecture applies to the proper length of the trajectory, instead of the geodesic distance between the starting and ending points in the field space.

We discuss here a class of multi-field models where directions orthogonal to the slow-roll direction are massive, $M \gtrsim H$. The inflaton therefore rolls near the bottom of the valley, which has "bends" in the multi-dimensional field space. The main difference here is that the local angular velocities of the inflaton around the bends can modify the effective sound speed c_s of fluctuations. As a result, we have the modified relation [136]

$$\begin{aligned}12\eta_V = &(c_s^{-2}-1)\frac{M^2}{H^2} + 2\frac{M^2}{H^2} + 3(4\epsilon-\eta)\\ &- 2\sqrt{\left(\frac{M^2}{H^2} - \frac{3}{2}(4\epsilon-\eta)\right)^2 + 9(c_s^{-2}-1)\frac{M^2}{H^2}}\ .\end{aligned} \tag{4.17}$$

Here, η_V is the minimum eigenvalue of the Hessian and M is the effective mass of the field orthogonal to the slow-roll direction, and c_s is given by

$$c_s^{-2} = 1 + \frac{4\Omega^2}{M^2}\ , \tag{4.18}$$

where Ω is the local angular velocity describing the bend of the inflaton trajectory in the potential. Note that in the limit $\Omega \to 0$, the sound speed reduces to unity and η_V to the expression of the single-field models. Allowing for a significant deviation of c_s from unity relaxes the constraints on (c, c'), as shown in Fig. 4.2, where we set $M = H$. This allows for larger values of c and c' compared to the single-field case, which are preferred by the swampland conjecture. Note that lowering the sound speed further will not achieve $\mathcal{O}(1)$ values for c' because our scenario relies on having negative η_V. As c_s is reduced from unity, η_V initially becomes more negative and widens the allowed parameter space. Beyond some critical value $c_s \approx 0.3$, further reduction of c_s makes η_V less negative, thereby narrowing the allowed parameter space. For $c_s \lesssim 0.2$, η_V becomes positive and our analysis no longer holds. Empirically, we find that $c_s \sim 0.24$ maximizes the allowed parameter region in the (c', c)-plane. The grey shaded regions again correspond to experimental constraints on $r = 16\epsilon c_s$, but their area is greatly reduced as c_s decreases.

It is also interesting to note that we expect primordial equilateral and orthogonal non-Gaussianities once $c_s \neq 1$ in this class of models [136],

$$f_{\rm NL}^{\rm equil} = -(c_s^{-2}-1)(0.275 + 0.078c_s^2), \tag{4.19}$$

$$f_{\rm NL}^{\rm ortho} = (c_s^{-2}-1)(0.0159 - 0.0167c_s^2). \tag{4.20}$$

Here we have ignored the third order parameter. The current observational constraint on the sound speed is $c_s \geq 0.024$ (see Eq. (89) of [12]), which is an order of magnitude below the limit we can reach in our setup, as shown in Fig. 4.2. Future observations combining CMB lensing, galaxy and 21cm surveys, Lyman α forest, etc. have the potential to improve the constraint on $f_{\rm NL}$ by an order of magnitude or more [161].

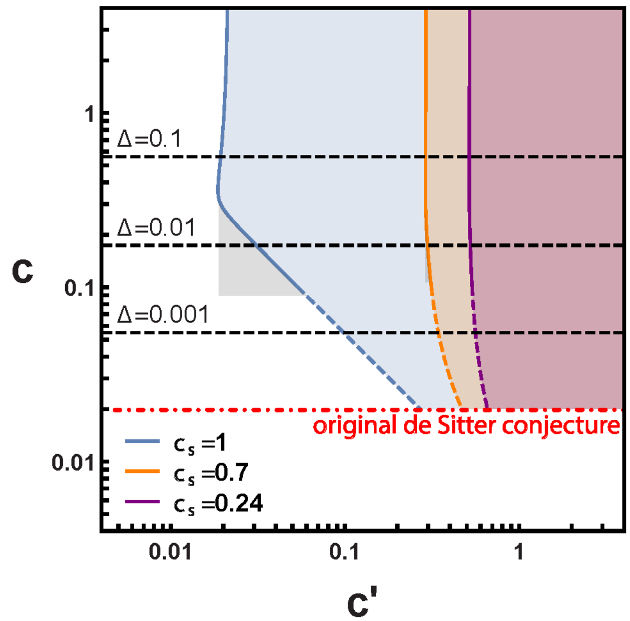

Figure 4.2: Bounds on swampland parameters for generic multi-field inflation models. We took $\alpha = 1$ and $M = H$. c_s is the sound speed for fluctuations, and the rest is the same as in Fig. 4.1. With the original de Sitter conjecture, Eq. (4.3), and single-field slow-roll models, c had to be below the red dot-dashed horizontal line.

4.4 Implications for Dark Energy

The de Sitter conjecture states that constants c and c' are *universal* and should apply to all sectors in a given EFT. Therefore, we can use inflationary physics to get a handle on the values of c and c' and apply this knowledge to the quintessence potential V_Q. When this argument is applied to single-field inflation models with conjectures Eq. (4.3) and Eq. (4.1), one deduces that there may be little hope in finding $w \neq -1$ due to the small lower bound seen in Eq. (4.5). This depressing outlook is drastically changed in light of Eqs. (4.6) and (4.7), as Fig. 4.1 illustrates. We see that the refined de Sitter conjecture has allowed for the possibility of having Δ bounded from below such that it must be larger than a few per cent and should be observable to experiments. Current and future experiments, such as DES [1], HSC [2], DESI [3], PFS [4], LSST [5], Euclid [6], and WFIRST [7], are aiming for an accuracy of about a percent in w. The cost for this is that c' must be much lower than the $\mathcal{O}(1)$ expectation of [105, 188] in the single-field case. This seems to indicate that single-field inflation falls more in line with the modified de Sitter conjecture discussed in [181], where the smallest Hessian eigenvalue needs only be negative when $|\nabla V| < cV$.

This state of affairs is altered by considering multi-field inflation models. Not only could Δ be forced to be as large as several per cent, it is also possible to have both c and c'

approximately $\mathcal{O}(1)$ as long as the sound speed is low enough, as seen in Fig. 4.2. In either the single-field or multi-field scenario, a better theoretical understanding of the magnitude of c' is essential to understand the consistency of the swampland conjectures and inflation.

4.5 Discussion

In this chapter, we studied the consequences of the latest swampland conjecture on inflation and dark energy. The original de Sitter conjecture raised the hope that measuring the dark energy equation of state w would be promising while simultaneously dashing that hope since consistency with single-field inflation suggests that the deviation from $w = -1$ would likely be unobservable. As we have shown, this situation is much more encouraging with the refined de Sitter conjecture. Not only could $w \neq -1$ be observable even with a single-field inflationary scenario, but tensor modes could be as well. If one considers multi-field inflationary scenarios, then the prospect for observing $w \neq -1$ is better and one gains improved agreement with the swampland conjectures.

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