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Publication Date

1980-02-01

Peer reviewed

Presented at the American Society of Mechanical
Engineers 1980 Energy Technology Conference and
Exhibition, New Orleans, LA, February 3-7, 1980

LBL-10517

CONF-800204--10

ROCK PROPERTIES AND THEIR EFFECT ON THERMALLY-INDUCED DISPLACEMENTS AND STRESSES

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February 1980

MASTER

Prepared for the U.S. Department of Energy
under Contract W-7405-ENG-48

LAWRENCE BERKELEY LABORATORY
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INTRODUCTION

One of the main areas of concern regarding the disposal of radioactive waste materials in subsurface geologic repositories is the response of the in-situ rock mass, comprising the repository and the surroundings, to the thermomechanical loading that will be induced by the heat generating waste canisters. In order to assess quantitatively the response of a granitic rock mass to loading of this type a series of experiments currently is being conducted at the Stripa Mine in Sweden. Electrically heated canisters, with power levels and geometries representative of canisters of reprocessed spent fuel high level wastes, are used for these tests. These canisters are emplaced in boreholes which are drilled vertically into the floor of the drifts which comprise the experimental site. This site is located some 340 m below the surface. Instrumentation, in the form of thermocouples, rod extensometers and borehole deformation gauges, is installed in boreholes to monitor, respectively, the rock temperature, displacement and stress fields during the tests. A diagram illustrating schematically the layout of the heaters and some of the instruments in two of the experiments, is given in Figure 1. A more detailed description of the objectives and the design of the heater experiments is given elsewhere.[1,2]

PRELIMINARY THERMAL AND THERMOMECHANICAL MODELS

Prior to conducting these tests a series of calculations was made to predict the likely temperature, displacement and stress fields. At the time these calculations were carried out the thermal and thermomechanical properties for Stripa granite had been determined by laboratory testing using a very limited number of small intact rock specimens [3] Only the thermal conductivity had been measured as a function of temperature and was found to decrease almost linearly with increase of temperature at the rate of approximately 0.1 percent per °C, (Figure 2). Therefore, the initial models were linear thermoelastic in which the rock was assumed to be a homogeneous, isotropic and perfectly elastic continuum with constant material properties.

The thermal field was obtained using a closed form solution of the linear heat conduction equation for a finite length line source.[4] This temperature distribution then was applied as thermal loads to calculate thermoelastic displacements and stresses using a modified version of the finite element code SAP IV*.[5] The constant material properties used in these calculations are given in Table 1.

Table 1

Material Property	Symbol	Value	Units
Thermal Conductivity	k	3.2	W/m°C
Thermal Diffusivity	$\kappa = k/\rho c_p$	1.47×10^{-6}	m^2/s
Coefficient of Linear Thermal Expansion	α	11.1×10^{-6}	/°C
Young's Modulus	E	51.3	GPa
Poisson's Ratio	ν	0.23	

The model used for prediction of displacements and stresses in the experiments described in this paper was axi-symmetric and used boundary conditions which took account of the mine excavations. Details of these thermoelastic calculations are given in a report by Chan and Cook.[6]

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*The SAP IV code has been modified to achieve higher computational efficiency and to improve the accuracy of thermal strain and stress calculations. This Lawrence Berkeley Laboratory/Lawrence Livermore Laboratory version has been documented by Sackett.[7]

COMPARISON BETWEEN FIELD DATA AND PRELIMINARY CALCULATIONS

A consistent, and puzzling, feature of the results from the heater tests is that although the temperatures measured through the rock mass accord well with predictions, the displacements and stresses are less than those predicted by a factor of more than two. This result is difficult to explain since if the rock is being heated then expansion should be taking place. Checks were made of the performance of the instruments and the calculation routine but errors sufficient to explain this large discrepancy were not discovered. The analysis which has been performed to investigate reasons for this discrepancy has concentrated on the displacement measurements since these are easier to make and much easier to interpret than the stress measurements. Displacement readings by the extensometers demonstrate two distinct modes of behavior. First an initial phase is observed immediately after turn-on of the heaters when displacements are completely unpredictable and are very much less than the calculated values. Second, a phase when the measured displacements are less than the predicted values by a constant amount. For many of these instruments the ratio of the measured to the predicted displacements during this second phase is about 0.4.[1]

The initial, non-linear, phase is difficult to explain and possibly may be caused by absorption of the early expansions of the rock into pre-existing discontinuities within the mass. This hypothesis is supported by evidence from an independent experiment where an increase in the velocity of ultrasonic waves through the rock in the vicinity of the heater was observed.[8] The subsequent phase indicates that the rock is responding to the loading in a predictable manner but with a magnitude less than the calculations forecast. Behavior of this kind is consistent with a difference in values for the rock properties for an in-situ rock mass and for those used in the calculations.

SENSITIVITY ANALYSIS FOR TEMPERATURE-INDEPENDENT ROCK PROPERTIES

For linear thermoelasticity in a homogeneous medium we can estimate the effects of changes in the thermal and thermomechanical properties on the temperatures and the induced displacements and stresses without repeating the numerical calculations using different sets of properties. The method for such "sensitivity" analysis, along with specific examples, is described briefly below.

Thermal conductivity, k, and thermal diffusivity, κ

For linear thermal conduction the temperature rise due to a constant power cylindrical heat source such as the heater canister is inversely proportional to the thermal conductivity provided that the time is properly scaled. The correct scaling can be found by inspection of the closed form solution for a finite-length line or cylindrical heat source. [4, 9, 10, 11] Thus, consider two different rock media with thermal conductivities and diffusivities, k_1, κ_1 and k_2, κ_2, respectively. The temperature rises, $\Delta T_1(t_1)$ at time t_1 and $\Delta T_2(t_2)$ at time t_2, caused by the same heat source in these two different media, are related by

$$\Delta T_2(t_2) = \frac{k_1}{k_2} \Delta T_1(t_1) \qquad (1)$$

where

$$t_2 = \frac{\kappa_1}{\kappa_2} t_1 \qquad (2)$$

For example, the effect of increasing k by 10 percent is to reduce the temperature rise by 9 percent. In addition the corresponding temperature rise occurs at an earlier time. Thermoelastic displacement and stress are directly proportional to mean temperature change.

Thermal expansion, α, Poisson's ratio, ν, and Young's modulus, E.

From the classical theory of linear thermoelasticity it can be shown[6,12,13] that, for an axially symmetric system, displacements and stresses are proportional, respectively, to the following combinations of thermomechanical properties

$$\alpha \frac{1 + \nu}{1 - \nu}$$

and

$$\frac{\alpha E}{1 - \nu}$$

Thus, for two rock media with material properties α_1, ν_1, E_1 and α_2, ν_2, E_2, respectively, the corresponding displacement, u_1, u_2, and stresses, σ_1, σ_2, are related by

$$u_2 = \frac{\alpha_2 \, (1 + \nu_2)(1 - \nu_1)}{\alpha_1 \, (1 + \nu_1)(1 - \nu_2)} \; u_1 \qquad (3)$$

and

$$\sigma_2 = \frac{\alpha_2 \, E_2 \, (1 - \nu_1)}{\alpha_1 \, E_1 \, (1 - \nu_2)} \; \sigma_1 \qquad (4)$$

Example

In the preliminary finite element calculations using values for the properties of Stripa granite given in Table 1, it was found that 200 days after the activation of the 5 kW heater the thermoelastic radial displacements and stresses at a radial distance 1 m from the heater axis and along the heater midplane are, respectively, u_1 = 0.71 mm, σ_1 = 51.5 MPa. Table 2 gives 3 sample calculations where one property at a time is varied and the corresponding effects of this on displacement and stress can be seen. From this it is evident that, as expected, u and σ scale proportionally with α; a large change in ν results in a relatively small change in either u or σ; and E causes no change in u and a proportional change in σ.

Table 2

	α (10^{-6}/°C)	ν	E GPa	u (mm)	σ (MPa)	δ property (%)	δu (%)	$\delta\sigma$ (%)
Base Case	11.1	0.23	51.3	0.71	51.5			
Case #1	8.0	0.23	51.3	0.51	37.1	-28	-28	-28
Case #2	11.1	0.12	51.3	0.57	44.9	-48	-20	-13
Case #3	11.1	0.23	37.0	0.71	37.1	-28	0	-28

To verify the equations (1) to (4) for the effects of material properties, and, incidentally, to convince skeptics, we have made a number of finite element runs with different sets of properties. The results have confirmed not only the temperatures, displacements and stresses quoted in the examples above, but also the validity of Equations (1)-(4) at various values of time, at various locations and for all components of displacements and stresses.

LABORATORY DETERMINATION OF ROCK PROPERTIES

A problem that exists at the present time is that little information is published in the literature which describes the response of rock specimens to thermomechanical loading. Especially there is a lack of information of the coefficient of thermal expansion of rocks as a function of both temperature and confining pressure although very recently some workers have initiated studies of this type. [14,15]

A limited number of laboratory tests of intact small samples of Stripa granite were conducted in parallel with the *in-situ* heater experiments. These tests were carried out to determine the properties of this rock as functions of temperature, and in some cases, confining stress. The results of these tests for the rock properties of interest from the viewpoint of calculation of displacement and stress are illustrated in Figure 2.

CALCULATIONS USING TEMPERATURE-DEPENDENT PROPERTIES

Since the thermomechanical properties all exhibit significant variations with temperature, as shown in Figure 2, numerical calculations were carried out to investigate the influence of these temperature dependent properties on predicted displacements and stresses. To include temperature-dependent thermal conductivity[*] a nonlinear finite element code, DOT[+16] was utilized for the thermal modeling. These thermal calculations also account for the non-uniform initial temperature distribution in the rock and convective boundary conditions at the surfaces of the heater borehole and the

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[*] Specific heat and density were found by Pratt et al.[3] to vary insignificantly with temperature over the temperature range concerned.

[+] The DOT code has been modified at LBL to take advantage of large core memory of the CDC-7600 computer.

drifts. The results of these thermal calculations have been summarized by Javandel and Chan.[17]

Thermally induced displacements and stresses were calculated using the finite element code SAP IV. To delineate the effect of the temperature dependence of each individual rock property shown in Figure 2, several computer runs have been made in which one, two, three, or four temperature dependent properties are included. Figure 3A-D illustrates the influence of each parameter in turn on calculated displacement, and compares these results with the observed relative displacement. These displacements are for a vertical extensometer located at 2 m radial distance from the heater between anchor points 2.24 m above and below the midplane of the 5 kW heater.

From Figure 3A it can be seen that the weak temperature dependence of thermal conductivity of Stripa granite (Figure 2C) has only minor effects on the vertical displacements.

Figure 3B and 3C show that the temperature dependence of Young's modulus, E, affects the displacements more than the temperature dependence of Poisson's ratio, ν, although the latter property varies more rapidly with temperature. This is in contrast with the situation for constant rock properties where the magnitude of E does not affect the thermal displacements at all.

Finally, the very strong temperature dependence of the thermal expansion coefficient, α, of Stripa granite is reflected in its dramatic effect on the displacements, as depicted in Figure 3D. Note that the absolute values of the vertical displacements predicted using all temperature dependent properties are now somewhat smaller than the measured values.

At present only a small portion of the displacement data has been examined. For this limited amount of vertical displacement data the same general trend seems to hold.

In the thermal expansion coefficient measurements some hysteresis was observed during the heating-cooling cycle. It is of interest to determine whether hysteresis of this type is observed in the measurements of displacement for the in-situ rock mass during the cool-down phase of the heater experiments. This result is of importance in terms of the overall design of a repository because, as a result of the decay of thermal power from the waste canisters the rock surrounding the repository will undergo a thermal pulse.[18] The rock temperatures will reach a maximum within one or two centuries and after this cooling will occur. The behavior of the rock during both the heating and the cooling phases will govern the stress field induced, and the rates of groundwater flow through the rock mass. Demonstration of an ability to predict the rock behavior during both of these phases will be a pre-requisite for repository design.

Some preliminary work has been done in this direction. An example is given in Figure 4, in which the measured and calculated relative displacements between a pair of anchor points across the heater midplane on a vertical extensometer at radial distance of 1 m from the 3.6 kW heater has been plotted as function of time, including the cooling period, i.e., beyond 398 days. No apparent difference can be discerned at this moment between the heating and cooling portions of the curves. The measured curve still follows the predicted curve for temperature dependent properties quite closely, but the absolute values of the measured displacements are again slightly higher.

Interpretation of the measurements for changes in rock stress is still in progress. In the field better experience has been achieved with the vibrating-wire Creare gauges than with the Bureau of Mines borehole deformation gauges. For this reason analysis of results from the former is more advanced. Figure 5 illustrates the influence of temperature dependent properties on the calculations for radial stress at a point in the rock at radial distance 1.5 m from the 5 kW heater and 0.85 m above the midplane. From Figure 5A it is seen that, although using temperature dependent rock properties reduces the discrepancy between measured and calculated stresses, the improvement is not as impressive as that for displacements.

As a sensitivity analysis, the calculations using temperature dependent rock properties were repeated with a scaled down Young's modulus

$$E(T) = \frac{37}{69} \times E_{lab} (T) \quad .$$

The rationale for this choice of scaling factor is that a suite of measurements were made of the <u>in situ</u> rock modulus, at the Stripa heater test site using the Colorado School of Mines cell.[19] Results of these measurements, which showed considerable scatter, showed a mean value for the modulus of 37 GPa compared to the laboratory value of 69 GPa at 10°C.[20] As expected the calculated radial stress was found to be scaled down by the same ratio, Figure 5B.

Incidentally, scaling down Young's modulus does not affect the displacements even for a temperature dependent modulus. This should be contrasted to the effect of temperature dependent Young's modulus on displacements shown in Figure 3B. Apparently, the functional dependence on temperature affects the displacements, whereas a constant of proportionality does not.

CONCLUSIONS

The work described in this paper must be regarded as preliminary since data from laboratory tests for the properties of Stripa granite at elevated temperatures and pressures are limited and only a small portion of the field data has been compared with the modified finite element calculations. Nevertheless the calculations for the displacements and stresses that are induced in the in-situ rock mass during the heater experiments demonstrate the dependence and the sensitivity of these quantities on the rock properties, k, ν, E, and, α. The calculations using constant material properties validate the standard thermomechanical equations for the dependence of displacement and stress on material properties. Also, calculations using temperature-dependent material properties demonstrate the fairly significant dependence of the rock displacements on Young's modulus. This is in contrast with the situation where the modulus is independent of temperature when calculations of rock displacements remain independent of changes in rock modulus. The material property that affects the predicted rock displacements most significantly is the variation of the coefficient of thermal expansion with temperature. Variations of thermal conductivity and Poisson's ratio with temperature have little effect on the predicted rock displacements. Changes in the rock properties with temperature have less effect overall on the stress field that is induced in the rock mass than incorporating into the model a value for the measured in-situ Young's modulus.

RECOMMENDATIONS FOR FUTURE WORK

The importance of accurate knowledge of material properties as functions of temperature and confining stress has been demonstrated. An extensive program of laboratory testing is required to collect this data. It is recommended that these measurements of rock properties be made both for intact and fractured specimens. Also it is recommended that tests be conducted to determine the effect of rock sample size on these properties. It is planned to conduct some work of this kind at the University of California, Berkeley under contract to the Lawrence Berkeley Laboratory. Concurrently, further finite element analysis should be performed to minimize the inaccuracy introduced by the geometrical approximation of the models, to study the influence of stress dependent properties, and to determine more precisely the appropriate ranges of temperatures and confining stresses as well as the required accuracy of laboratory rock property measurements.

ACKNOWLEDGEMENT

This work was prepared under the auspices of the U. S. Department of Energy under Contract W-7405-ENG-48. This project is managed by the Office of Nuclear Waste Isolation, Battelle, Columbus, Ohio.

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FIGURE LEGENDS

Figure 1. Diagram illustrating part of the Stripa experimental site. This sketch shows the two full scale heaters emplaced in vertical boreholes from the floor of the drifts and the location of the rod extensometers relative to these heaters. Not shown in this diagram are the gauges used for monitoring changes in the thermally induced stress field. These gauges are emplaced in vertical and in horizontal boreholes in the vicinity of the heater canisters.

Figure 2. Results from the limited amount of laboratory testing showing the temperature dependence of material properties of Stripa granite. Results given in Plots A and B are after Swan[20] and results given in Plot C are after Pratt *et al*.[3]

Figure 3. A series of graphs comparing measured rock displacements in the vertical plane between anchor points 2.24 m above and below the heater midplane for an extensometer at a radial distance 2 m from the 5 kW heater with calculated values for these displacements. The plots illustrate the sensitivity of these predicted displacements to the temperature dependence of the material properties. The Plots A-D illustrate the effect of incorporating into the model an additional material property as a function of temperature.

Figure 4. A plot comparing measured and predicted rock displacements in the vertical plane between anchor points 2.24 m above and below the heater midplane for an extensometer at a radial distance of 1 m from the 3.6 kW heater. This graph shows the predicted displacements for both temperature dependent and temperature independent material properties. Also illustrated in this Figure are the displacements, both measured and predicted during the cool down after the heater was turned off.

Figure 5. Shows two sets of curves both of which give the stress measured in a direction radial to the 3.6 kW heater with a vibrating-wire Creare gauge. Also given in these curves are calculations for the stress at this point demonstrating the effect of temperature independent and temperature dependent material properties together with the effect of incorporating a scaled down value for the Young's modulus to approximate more closely the rock modulus *in-situ*.

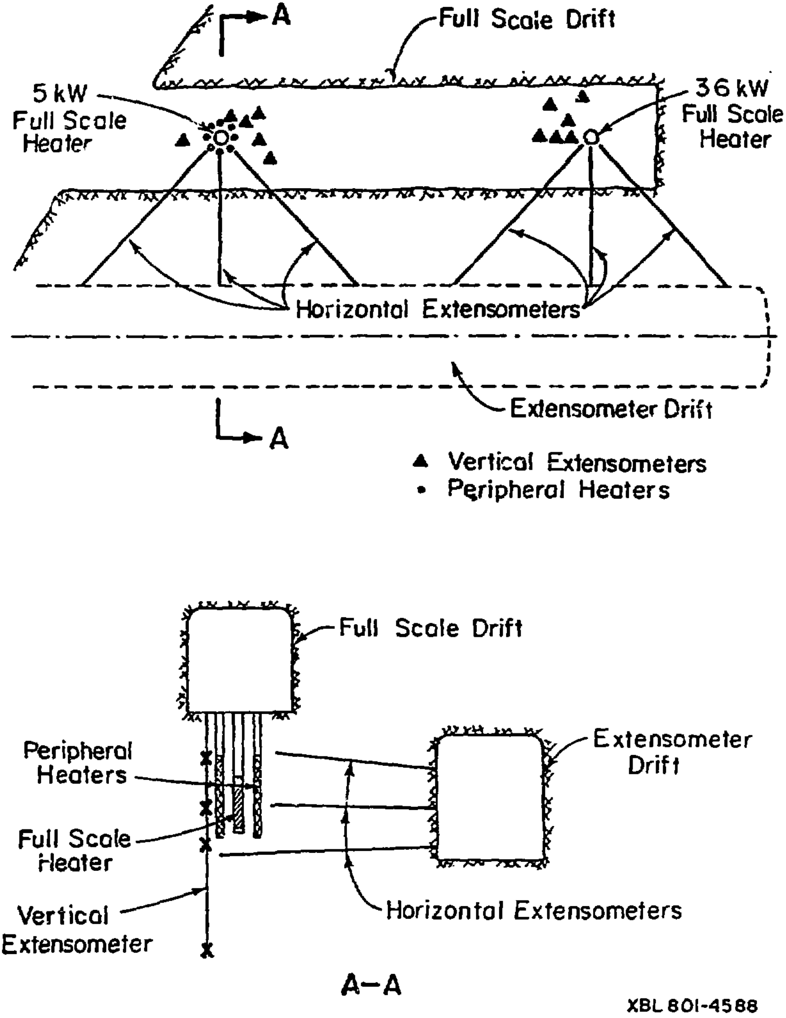
A
Full Scale Drift
5 kW
Full Scale
Heater
36 kW
Full Scale
Heater
Horizontal Extensometers
Extensometer Drift
A
▲ Vertical Extensometers
• Peripheral Heaters
Full Scale Drift
Peripheral
Heaters
Full Scale
Heater
Vertical
Extensometer
Extensometer
Drift
Horizontal Extensometers
A–A
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Figure 1.

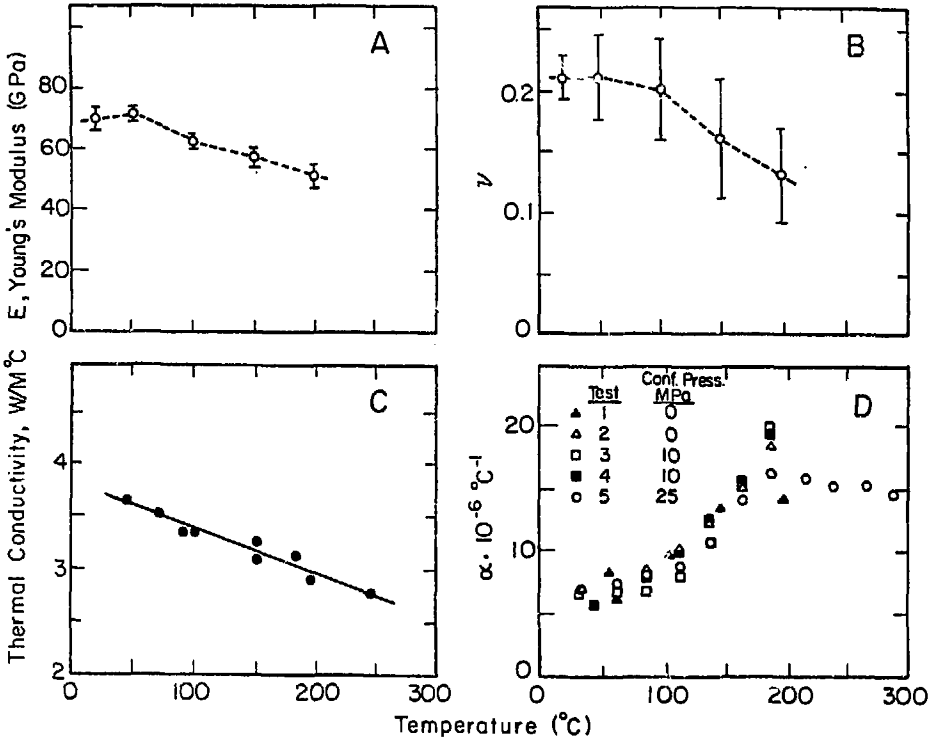

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Figure 2.

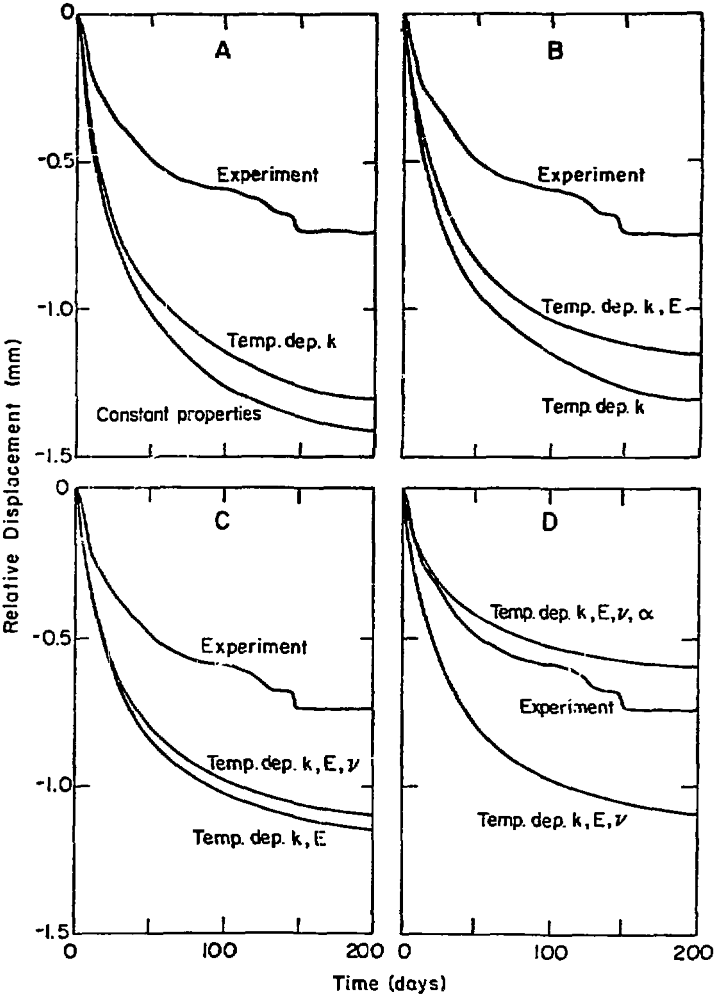

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Figure 3.

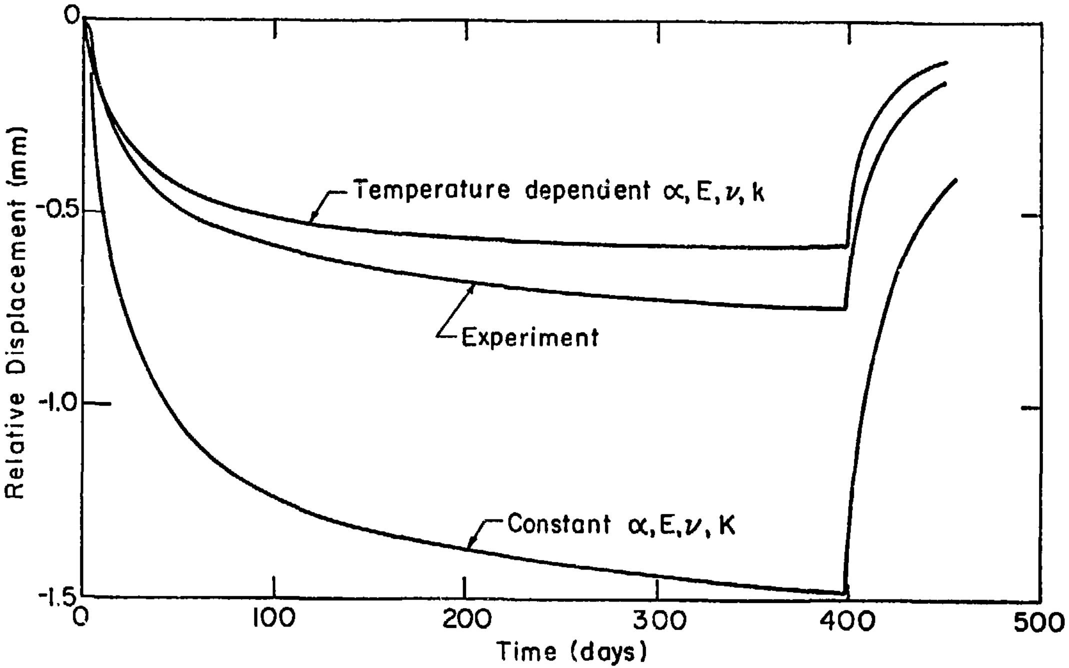

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Figure 4.

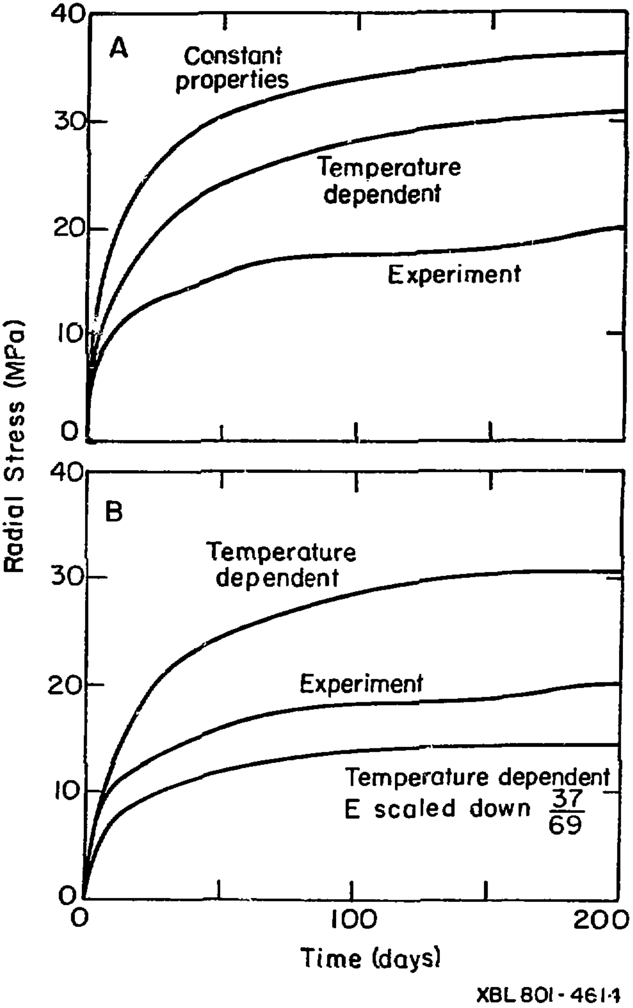
A
Constant properties
Temperature dependent
Experiment
B
Temperature dependent
Experiment
Temperature dependent
E scaled down $\frac{37}{69}$
Radial Stress (MPa)
0
10
20
30
40
0
100
200
Time (days)
XBL 801-4614

Figure 5.