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Perspectives on Instructor Modeling in Mathematics Teacher Education

A dissertation submitted in partial satisfaction of the requirements for the
degree Doctor of Philosophy

in

Mathematics and Science Education

by

Cassandra Brown

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2009

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University of California, San Diego

San Diego State University

2009

DEDICATION

This is dedicated to all of those who have helped me along the way.

A special thanks to Dr. Susan Nickerson
for all of the time and energy she put in on my behalf.

To Susan, and all of the PDC staff, thank you for your support,
I could not have done it without you.

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ABSTRACT OF THE DISSERTATION

Perspectives on Instructor Modeling in Mathematics Teacher Education

by

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Doctor of Philosophy in Mathematics and Science Education

University of California, San Diego, 2009
San Diego State University, 2009

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Teachers' instructional practices are greatly shaped by their own learning experiences as students in K-12 and college classrooms, which for most teachers was traditional, teacher-centered instruction. One of the challenges facing mathematics education reform is that, traditional teaching is in contrast to reform student-centered instruction. If teachers learn from their experiences as mathematics students, mathematics teacher educators are encouraged to model practices they would like teachers to use.

In this study I examined the implications of instructor modeling in mathematics teacher education courses. More specifically, I investigated what practicing teachers gained from mathematics teacher educators' modeling in mathematics teacher education courses. My questions were: (1) *What do mathematics teacher educators believe they*

model about effective instructional practice? (2) What do practicing teachers notice about the mathematics teacher educators' pedagogy and identify as effective mathematics teaching? (3) In what ways do these perspectives align in mathematics courses for practicing teachers?

I drew upon three-step-design methodology with stimulated recall interviews and complementary accounts methodology to explicate three mathematics teacher educators' reports of the ways they model teaching practice. I utilized these methodologies to analyze the researcher's, and the practicing teachers' perspectives on the mathematics teacher educators' instruction.

In general the mathematics teacher educators and practicing teachers reported that the mathematics teacher educator modeled student-centered instruction as conveyed in the NCTM *Professional Standards for Teaching Mathematics* (1991), but also facets of the collaborative work of teachers outside of the classroom. This work outside of the classroom might include teachers collaboratively planning instruction, reflecting on practice, creating and reflecting on new practices, and supporting one another's professional growth.

This research informs the body of knowledge about teaching the practice of teaching mathematics in two ways. First, it highlights how explicit discussion about mathematics teaching and implicit modeling of instruction supports practicing teachers' noticing of the instruction they experience. And second, this research points to how engaging in facets of the teaching profession that take place outside of their interactions

with students has the potential to foster the enculturation of teachers into a professional learning community of mathematics teachers.

CHAPTER 1:

RATIONALE

In this chapter I begin with a description of an episode from a mathematics course for practicing teachers. In the episode the teachers and the mathematics teacher educator share the thoughts that underpin their actions while teaching. The contrast in reasons the teachers and the mathematics teacher educators gave, in part, provide a motivation for investigating perspectives on instructor modeling in mathematics teacher education. Next, I discuss calls for improving mathematics education. Then, I describe the vision and reality for mathematics education in the U.S. and the challenge for mathematics teachers and mathematics teacher education. Last, I discuss the influence of learning from the milieu, and related research.

In a mathematics class for practicing middle school mathematics teachers, a mathematics teacher educator had the goal of modeling effective mathematics teaching. The mathematics teacher educator organized teachers into small groups and asked them to work on an open-ended and challenging geometry task. He circulated around the room for several minutes listening in on the discourse in the small groups. He brought the whole class together to debrief the problem, but began by asking the teachers if they ever put their students into small groups to work and, if so, what they, the teachers, did as they circulated. Some teachers said they put students into groups to work on problems to encourage student engagement. Teachers' explanations for their activity as they circulated about the room included "classroom management", "looking to see if students had the correct answer" and "checking to make sure

students were on task.” After the teachers shared, the mathematics teacher educator shared his mathematical goals for the lesson and what mathematical discussions he expected to hear as he circulated among their small groups. He told the teachers several things he actually did hear and what mathematical activity he decided to pursue as a result of what he heard—reminding them of his mathematical goal (Masarik, Nickerson, & Brown, 2008).

This episode raises questions for mathematics teacher educators. What do teachers notice about the instruction they receive as they participate in teacher education mathematics content classes? What can teachers potentially learn about *mathematics teaching* from participation in teacher education mathematics content classes? How are teachers’ perceptions of the pedagogy being modeled aligned with what the mathematics teacher educator believes he or/she is modeling? These are important questions to explore if we are to support teachers in changing instructional practice.

Over the past decade there has been an increasing demand to improve the quality of mathematics education (cf. Conference Board of the Mathematical Sciences, 2001; Haselkorn & Harris, 1998; National Commission on Teaching and America’s Future, 2003; National Mathematics Advisory Panel, 2008; RAND Mathematics Study Panel, 2003; U.S. Department of Education, 1998; 2002). The implementation of recommendations for reforming mathematics education requires change in both the mathematics content taught in schools and the approaches to teaching that content (National Council of Teachers of Mathematics, 2000; National Research Council,

1996, 2001). The findings of the Third International Mathematics and Science Study (TIMSS) (U.S. Department of Education, 1996) support reform recommendations by highlighting the importance of curriculum that engages students with significant mathematical ideas, and learning environments that allow students to work as a community through complex and open-ended tasks (Boaler, 1998; Cogan & Schmidt, 1999; Goos, 2004; Hiebert, 1999; Hiebert & Stigler, 2000; Rasmussen, Kwon, Allen, Marrongelle & Burtch, 2006; Spillane, 2000; Spillane & Zeuli, 1999). Reform recommendations call for student-centered instruction with an emphasis on problem solving, mathematical reasoning, conceptual understanding, real-life applications, and a greater appreciation for mathematics as a cultural phenomenon (Bishop, 1988; Grouws, 1992; Lester, 2000; National Council of Teachers of Mathematics, 1991; 2000).

However, research has documented in U.S. schools the prevalence of mathematics lesson sequences in which the normative pedagogy is more traditional. In these mathematics lessons, instruction occurs primarily by means of the teacher's provision of information to the students (Cogan & Schmit, 1999; Hiebert, 1999; Hiebert & Stigler, 2000; Stigler & Hiebert, 1997; U.S. Department of Education, 1997/2000). In a traditional mathematics course, rote learning is often privileged and students are expected to learn procedures, algorithms, and definitions put forth by their teacher with little opportunity for students to make meaning. The vision of reform teaching is one of student-centered learning environments and students engaged in communities of inquiry. Student-centered learning environments, as suggested in

reform documents such as the National Council of Teachers of Mathematics (NCTM) *Professional Standards for Teaching Mathematics* (1991), focus on the development of the students' mathematics rather than the communication of the teachers' mathematics (Lobato, Ellis, & Muñoz, 2003). Student-centered instruction requires teachers to make use of teaching practices that engage students in inquiry to deepen mathematical discussion. The teaching practices in a reform classroom include selection of non-routine problems, probing student thinking, creating opportunities for student inquiry through pedagogic interventions during teaching, orchestrating discussion about mathematical ideas and structure, and de-briefing (NCTM, 2000). There is a stark contrast between the vision of effective teaching and the reality. The challenge for mathematics teacher educators lies in helping teachers construct a vision of reform teaching and in supporting teachers in making it a reality in U.S. schools.

The Vision

Gamoran and his colleagues (2003) argue that the basic purpose of the current reform can be characterized by a shift in our expectations for teaching mathematics and science with an aim toward "teaching for understanding." An emphasis on students learning *principled* mathematical knowledge suggests that the focus for students should be on learning key concepts and ideas that can be used to construct mathematical procedures. Spillane (2000) distinguishes procedural mathematical knowledge from principled mathematical knowledge, noting, "Procedural knowledge centers on computational procedures and involves memorizing and following predetermined steps to compute answers, (while) principled knowledge focuses on the

mathematical ideas and concepts that undergird mathematical procedures” (p. 144). Spillane and Zeuli (1999) write, “If all students are to have opportunities to master principled mathematical knowledge, core dimensions of instruction will have to change substantially, especially the mathematical tasks (i.e., the problems, questions, and exercises the students work on) and classroom discourse norms (i.e., the ways teachers and students interact with each other about mathematics)” (p. 4).

The NCTM *Professional Standards for Teaching Mathematics* (1991) outlines ways in which instruction must change. Teachers should pose worthwhile tasks, allow students time to grapple with significant mathematical ideas (both independently and collaboratively with peers), encourage students to explore and make conjectures, orchestrate discourse, and elicit student thinking and discussion while engaging in ongoing analysis of student learning. Such instruction is in stark contrast to traditional instruction and often requires teachers’ reconceptualization of what it means to teach and learn mathematics. Teachers educated in a traditional system have views of mathematics, mathematics learning, and mathematics teaching that are incompatible with those needed for the envisioned reforms (Simon, 2008).

The Reality

While research has pointed to the benefit of student-centered instruction and collaborative, inquiry-oriented learning environments in both K-12 and college mathematics courses (Boaler, 1998; Bowers & Nickerson, 2001; Goos, 2004; Rasmussen, Kwon, Allen, Marrongelle & Burtch, 2006), as suggested earlier, many mathematics courses are still taught rather traditionally. Borg (2004) points to what

Lortie (1975) calls the *apprenticeship of observation* to explain the prevalence of traditional mathematics instruction. With respect to teaching practices, the notion of apprenticeship of observation suggests that teachers learn about teaching long before they enter the classroom. Their education as teachers starts when they themselves are students and their experiences influence the ways in which they think about the teaching and learning of mathematics (Ball, 1988). Thus, it is widely accepted that, in general, teachers teach in the manner in which they were taught (Hiebert & Stigler, 2000; Sowder, 2007; Speer, 2004; Thompson, 1992). Many teachers' experiences as learners of mathematics, from elementary through college, primarily involved their teacher in the role of the provider of information, in what has been called a factory model of education (Callahan, 1962). Since teachers bring their own experiences as learners of mathematics to their practice, the way teachers teach mathematics is not often substantially different from how they learned mathematics. The fact that teachers were educated in a traditional system has been described as "...perhaps the greatest obstacle to these reforms" (Simon, 2008, p. 17).

The Challenge

As suggested, major shifts in the mathematics classroom environment require a reconceptualization of what it means to learn and teach mathematics. These include shifts: (1) from thinking of classrooms as made up of collections of individuals to thinking of classrooms as mathematical communities; (2) from having the teacher as the authority for right answers to anyone in the classroom community as a source of verification of logic and mathematical evidence; (3) from requiring students to

memorize procedures to develop mathematical reasoning; (4) from an emphasis on mechanistic answer-finding to an emphasis on conjecturing, inventing, and problem-solving; (5) from treating mathematics as a body of isolated concepts and procedures to connecting mathematics, its ideas, and its applications (NCTM, 1991). Such transitions are often difficult for teachers because these shifts involve more than the development of pedagogical skills. Mathematics teaching practice, as described by the NCTM teaching standards (1991) requires teachers to reconsider and challenge their current beliefs about the nature of mathematics, mathematics learning, and teaching. Changing teachers' beliefs has been widely recognized as one of the biggest challenges for teacher education.

In addition to changed beliefs, the development of content knowledge coupled with pedagogical skills is key in mathematics teacher reform. Simon and Schifter (1991) write, "[p]ositive reform of mathematics instruction will necessarily require significant new initiatives in teacher development" (p. 309). Teachers need deeper understanding of content in order to facilitate student-centered classrooms. For practicing teachers this often means engaging in professional development programs that are long-term, employ curriculum that deepens teachers' knowledge of the mathematics they teach, and makes connections between professional development and practice (Ball, 1996; Carpenter, Blanton, Cobb, Franke, Kaput & McClain, 2004; Nickerson, 2008; RAND, 2004). Furthermore, it has been suggested that if teachers learn from their experiences as mathematics students then "... good mathematics teaching should be modeled in teachers' professional development experiences"

(NCTM, 1991, p. 11). The implication is that a factor in helping teachers enact these shifts is to engage them as learners in inquiry-oriented mathematics communities. Researchers assert that mathematics teacher educators should model desired instructional practices in teacher education and professional development programs (Borasi, Fonzi, Smith, & Rose, 1999; Loughran & Berry, 2005; Simon & Schifter, 1991; Sowder, 2007).

In this study, I investigated the instructional and pedagogical implications of instructor modeling in mathematics courses for practicing teachers by unpacking the notion of modeling in mathematics teacher education. In general, the mathematics teacher education programs discussed in this study are driven by two primary objectives, (1) to deepen the pedagogical skills and mathematical understanding of the mathematics the teachers teach and (2) to model instructional practices by engaging the teachers as learners in instructional environments that promote teaching for understanding. The teacher education courses in this study were classroom sessions where the teachers engaged in and worked on mathematical tasks, analyzed the mathematics of students, and discussed literature related to teaching and student learning. The teachers solved mathematics problems and participated actively in small group or whole-class discussions about the mathematics they themselves were doing, discussed specific issues of curriculum and pedagogy as derived from research and instruction, and discussed issues of instruction and student learning of mathematics. The mathematics teacher educators worked to provide learning opportunities that

helped the teachers develop a deeper understanding of the mathematics, as well as, how students come to understand these topics.

The mathematics teacher educators in this study also had the stated objective of modeling instructional practices that promote teaching for understanding. This objective is based on the belief that in engaging in these mathematics classes, teachers learn from this experience not only about mathematics, but also about teaching practice. There are theories that suggest how this learning about teaching may occur. Though not fully elaborated until Chapter 2, the role of participation and observation in learning about teaching are discussed in what follows.

The Influence of Participation and Observation on Learning

A social perspective on learning emphasizes the influence of participation, observation, and listening in as learning and development are mediated by culture and communication (Brown, Collins & Duguid, 1989; Collins, Brown & Newman 1989; Lave & Wenger, 1991). The notion of perceptual lived experience (Loughran & Barry, 2005) suggests that some learning occurs by “living through” experiences. Loughran and Barry give an example in which student teachers develop perceptual rather than conceptual knowledge of a situation. In their example the student teachers, as students in a classroom setting experience emotions, images, needs, values, volitions, personal hang-ups, temper, character traits and so on of individuals in the situation, which develops their perceptual knowledge of learning environments instead of their conceptual knowledge. While this kind of learning is rather passive, the

learner gleans ideas about the “how to” of the activity by simply being a part of the situation.

Intent participation (Rogoff, Paradise, Arauz, Correa-Chávez & Angelillo, 2003), a related, more deliberate form of learning from experiences in a situation, describes learning from keen observation and listening to ongoing activities in which the learner participates or expects to participate in the future. Both perceptual lived experience and intent participation suggest that learning occurs through the (implicit) modeling of an expert, where the expert provides an example of the required performance. In mathematics teacher education, specifically mathematics courses for practicing teachers, the constructs suggest that teachers have the opportunity to learn about the instructional practice aspect of the teaching profession by being engaged as learners gleaning knowledge about instruction from the teaching and learning they experience.

Still, teachers’ learning about practice in mathematics teacher education is often not restricted to learning about classroom mathematics instruction. There are also opportunities for teachers to learn about other facets of the teaching profession. The current reform movement in mathematics education has a strong underlying theme of the professionalism of teaching. This view recognizes the teacher as a part of a learning community that continually fosters growth in knowledge, stature, and responsibility (Dufour, 2005; NCTM, 1991; Nickerson, 2008). Reform recommendations suggest that teachers ought to collaboratively plan instruction, reflect on practice, create and reflect on new practices, and support one another’s

professional growth (NCTM, 1991). Such interactions allow teachers in a school and its administrators to continuously seek and share learning and then act on what they learn. The goal of these interactions is to enhance teachers' effectiveness as professionals so that students benefit (Astuto, Clark, Read, McGree, Fernandez, 1993; Hord, 1997). This type of communication and collaboration with a focus on inquiry about student learning are important aspects of what researchers call a "professional learning community" (Dufour & Eaker, 1998; Astuto et al, 1993; Hord, 1997).

Another way teachers can learn about the practice of teaching through modeling in teacher education is that the teacher can learn as an apprentice to an "expert" teacher. Here the learner is not unaware of the intentional modeling of practice. Apprentices learn through methods of observation, scaffolding, and increasing independent practice (Collins, Brown & Newman, 1989; Lave & Wenger, 1998). Apprentices in teacher education, generally student teachers and observers, learn from master teachers, teacher coaches, and teacher educators. The learner can be seen as a cognitive apprentice (Collins et al, 1989; Schoenfeld, 1992) where instructional practices are learned through observation, guided experience and participation. The master teachers and coaches provide models of desired instructional practice, feedback, and contextual support for the classroom situation that are slowly removed, so that the learner can become more self-reliant.

However, apprentices are not always in the form of student teachers. In the mathematics teacher education courses in this study practicing teachers participated in mathematics classes, and thus learning about instructional practice looked quite

different. In mathematics teacher education like the courses in this study, mathematics teacher educators must make connections to K-12 classrooms through anticipation exercises, discussions of possible trajectories in classroom situations, and reporting back experiences. Such connections often takes the form of commentary by the mathematics teacher educator that is related to a mathematical activity the teachers themselves are engaged in and prompts discussion among the teachers anticipating how it might apply in a classroom situation. Also, the teachers might discuss their own teaching as they report on a predetermined task that all of the teachers in the class tried with their own students, called “try-ons” in this context. This anticipation and reporting back on teaching experiences with a more experienced mathematics teacher educator can provide a means of scaffolding the learning of reform teaching.

The constructs of perceptual lived experience, intent participation, and apprenticeship share a situated perspective on learning from the milieu. They all suggest that knowledge is developed and deployed in activity and is not separable from or ancillary to learning and cognition (Brown, Collins, & Duguid, 1989). Thus, learning the teaching profession stems, at least in part, from the teaching teachers see and experience as learners and the activity they engage in as professionals. Moreover, how teachers learn about practice affects how they view the practice of teaching.

Thus, in mathematics professional development courses for practicing teachers there are opportunities for learning to take place on two planes: (1) the teachers learn mathematics as students and (2) the teachers learn about teaching through the mathematics instruction they experience as mathematics teachers. It is then the charge

of the mathematics teacher educator to understand the nature of “expert” reform, inquiry-oriented instruction and model appropriate methods for teaching mathematics content and the practice of teaching mathematics for understanding.

Prior Research on Learning from Modeling

The idea of learning from participation has been explored in modeling the *doing* of mathematics, that is, engaging in mathematical activity. Schoenfeld (1992) reports on teaching college students mathematical problem solving through cognitive apprenticeship. In the course Schoenfeld taught, he intentionally and explicitly modeled the use of heuristic methods mathematics experts often employ while carrying out complex and/or realistic mathematical tasks. He also highlighted the thinking and reasoning skills involved in the mathematics problem solving he did. Much of the class time was devoted to higher-order problem solving activities that required students to not only actively integrate and appropriately apply sub-skills and conceptual knowledge, but also reflect on their activities, explore and discuss solution techniques, strategies for making judgments about how best to proceed, and alternative courses of action. Schoenfeld (1992) found that (implicitly) modeling and (explicitly) teaching heuristic and control strategies of expert practice showed students that doing mathematics is not only applying problem-solving procedures, but is also reasoning about and managing problems using heuristics and control strategies.

In the same way, others have suggested that practicing teachers can learn to teach differently by means of observing more experienced teachers’ implicit modeling and through explicit discussion and reflection on pedagogical decisions and

implications for instruction in mathematics teacher courses. In the mathematics education community it is taken to be somewhat of an axiom that teachers will gain insight into mathematics teaching and learning by experiencing reform instruction for themselves. With respect to teacher education, learning by participation suggests that mathematics teacher educators ought to teach in the same fashion in which they want their students to teach, but also integrate metacognitive activities so that teachers can reflect on the teaching and learning being experienced (Kosnik, 2001; Loughran & Berry, 2005; Mewborn, 1999). Researchers suggest that teacher educators must *implicitly* model student-centered, inquiry-oriented instruction, *explicitly* model or discuss the thoughts and actions that underpin their pedagogical approach, and provide opportunities for practicing teachers to *reflect* on practice and the mathematics learning experienced in mathematics courses for practicing teachers in order to promote change in teachers' practice.

Research Questions

This research extends the previous research by exploring the nature of modeling in mathematics courses for practicing teachers and draws from complementary accounts of the mathematics teacher educators' pedagogy to suggest opportunities for learning. More specifically, I investigated the kind of learning that potentially results from teacher modeling in mathematics courses for practicing teachers by answering the following questions:

- (1) *What do mathematics teacher educators believe they model about effective instructional practice?*

- (2) *What do practicing teachers notice about the mathematics teacher educators' pedagogy and identify as effective mathematics teaching?*
- (3) *In what ways do these perspectives align in mathematics courses for practicing teachers?*

The third research question is an important one, since practicing teachers may not see or notice in their experiences as learners in their mathematics courses what the mathematics teacher educator would like them to note. Without being apprenticed into systems of thinking, the ability to see a meaningful event is not a transparent, psychological process. DeGroot (1965) demonstrated that a given stimulus is perceived and understood differently, depending on the knowledge one brings to a situation. "Ways of seeing" are socially situated. Professional *ways of seeing* are part of historically constituted practice through which the objects of knowledge mediate the discourse of a profession (Goodwin, 1994). Goodwin (1994) points out how social and cognitive organization of a profession allows it to shape events for learners by focusing on specific aspects, events or objects on which the discourse of the profession is structured. Thus, the professional vision of mathematics teacher educators and practicing teachers may not be shared. To this end, it is necessary to investigate the ways in which mathematics teacher educators and practicing teachers perspectives on the mathematics teacher educator's actions are aligned. Specifically, in this research I sought to shed light on the relationship that exists between what mathematics teacher educators' attempt to convey about the practice of teaching mathematics and what practicing teachers take from those interactions.

To do this I drew upon a three-step-design methodology (Busse & Ferri, 2003) and complementary accounts research methodology (Clarke, 1997). The facets of the three-step-design and the complementary accounts research methodologies include: observation, interview, stimulated recall, and analysis of complementary perspectives. My data included classroom observations and video, interviews with the mathematics teacher educator, interviews with some of the practicing teachers who participated in the teacher education, and practicing teacher reflection surveys. This set of data sources allowed for analytical techniques sensitive to the multifaceted and multiple connected nature of classroom data. This integrated data provided the basis for an investigation of complementary interpretations of the mathematics teacher educator's pedagogy from the perspective of the researcher, the mathematics teacher educators, and the practicing teachers. These perspectives were then compared and contrasted across classroom settings and instances to identify and categorize shared episodes.

Changes in mathematics education depend greatly on the effectiveness of opportunities for ongoing mathematics teacher development. This work contributes to the relatively little research done regarding the mathematics teacher educator. Understanding the multiple perspectives of the mathematics teachers' modeling activity is critical to educating experienced teachers to transform their teaching practice to be more consistent with current reform recommendations.

Organization

In Chapter 2, I discuss a historical context that sheds light on the current state of education and surveys past research on modeling in teacher education. I also

summarize theories on learning from the milieu, and on enculturation to expert practice. Finally, I discuss how these theories might explain learning from modeling in mathematics teacher education. In Chapter 3, I outline the methodology for the investigation of the research questions by describing professional development settings for the study and mathematics teacher educators and practicing teachers of the study. In addition, I describe the data corpus, analytical framework, and the methods of analysis.

The results are described in Chapter 4 beginning with a description of the emergent analytic framework. What follows is an analysis of modeling with respect to the prior year of professional development from the perspectives of the mathematics teacher educators and the practicing teachers in their cohorts. Then, with a focus on two consecutive class sessions, I present the perspectives of researcher, mathematics teacher educator, and a few practicing teachers from the class. Chapter 4 ends with a synthesis in order to answer the research questions. Chapter 5 includes a summary of the results, the implications for and contributions to mathematics teacher education, limitations of the study, and possible further research.

CHAPTER 2: LITERATURE REVIEW

In the early 1900's, for the most part, schooling was strongly fashioned after a 19th-century efficiency expert's, Taylor, time-and-motion studies of steelworkers for industrial efficiency. Teachers were cast as technical workers who were supposed to insert information into receptacles of knowledge or skill, the students. The information was to be broken into small manageable bits and delivered in a specified sequence, like an assembly line. This view of teaching and learning was prevalent. A leading educational administration textbook in 1916 stated, "[o]ur schools are, in a sense, factories in which the raw products (children) are to be shaped and fashioned into products to meet the various demands of life... It is the business of the school to build its pupils according to the specifications laid down." (Cubberley, 1916, p. 338).

The tradition of teaching and learning by the provision of information is still evident in present day instruction (Cogan & Schmidt, 1999; Hiebert, 1999; Hiebert & Stigler, 2000; Stedman, 1997; Stigler & Hiebert, 1997; U.S. Department of Education, 1996). In general, traditional methods are based on direct instruction where students are shown standard methods of performing tasks in a standard sequence. Such classes are often geared toward the development of rote procedural skills. Competency is developed through "drill and practice" and the memorization of procedures, algorithms and facts put forth by the teacher. As manifested in contemporary mathematics courses, the teacher puts forth definitions, procedures and algorithms, and the students are to learn them and use them in the same manner exemplified by

their instructor leaving little room for the development of principled knowledge. Authentic mathematical activity entails more than memorization. Mathematicians make use of conjecture and collaboration, and provide justification and proof, as they work to solve problems. There is often little opportunity for this in present-day mathematics instruction. Brown, Collins and Duguid (1989) write:

Too often the practices of contemporary schooling deny students the chance to engage the relevant domain culture, because that culture is not in evidence. Although students are shown the tools of many academic cultures in the course of a school career, the pervasive cultures that they observe, in which they participate, and which some enter quite effectively are the cultures of school life itself. These cultures can be unintentionally antithetical to useful domain learning... Thus, students may pass exams (a distinctive part of school cultures) but still not be able to use a domain's conceptual tools in authentic practice. (p. 34)

Brown and his colleagues suggest that we engage students in practices that are not authentic mathematical activity. The culture they observe and in which they participate is not the culture of authentic mathematical practice. Such instruction provides for limited opportunities for students to observe and participate in mature activities of the mathematical community. This, in turn, may restrict learners' understanding of the mature roles of participants in their community (Rogoff, 1990). We want students to be better prepared to use their mathematical knowledge in flexible ways and make sound mathematical arguments, yet, for the most part current mathematics instruction, in which children are seen as receptacles for knowledge, does not foster such understanding (U.S. Department of Education, 1996).

In contrast, in reform methods of instruction students are asked to devise conjectures and methods for approaching tasks, make arguments and provide

justifications, like mathematicians authentic practice. In the mathematics community, problem solving, conjecturing, proving and peer review characterize activities of mathematicians in their mature roles. Like in a community of mathematicians, in a reform classroom community of mathematics learners, learning and development is driven by mathematical discussion and exploration. In such classroom environments, the teacher mediates discourse to foster students' active participation in the construction of their own knowledge.

If the goal is to structure mathematics classrooms so as to assist students in using conceptual tools in authentic practice, the teaching practices and the development of teachers must also change. However, many teachers learned mathematics in very traditional classes and thus are accustomed to the transmission model of mathematics instruction, which is often manifested in their own teaching. Lortie (1975) attributes this influence on teachers' practice to what he called "apprenticeship of observation" (Borg, 2004). That is, teachers' perceptions of mathematics and what it means to teach mathematics are influenced by their experiences as students. Huibregtse, Korthagen, and Wubbels (1994) showed that even with experienced teachers there is a strong relationship between their preferred way of teaching and the way they themselves are used to learning. Teachers' firm roots in the many years of experience as students in traditional education can make efforts to transform the structure of formal schooling a challenge (Simon, 2008). This difficulty stems in part from teachers having to learn to engage in radically different classroom participation structures (Rogoff et al, 2003). Having experienced a

participation structure of teacher as transmitter and arbiter of mathematical correctness, teachers must learn to engage in and mediate a community of mathematics learners.

Education reformers realize that professional development is essential to foster improvement of instructional practice of practicing teachers (Ball & Cohen, 1999; Elmore & Burney, 1999; Nelson & Hammerman, 1996; Sykes, 1999; Thompson & Zeuli, 1999). Prospective teachers in teacher education generally have preconceptions about learning and teaching that are in contrast to the theories taught in teacher education programs (Wubbels, 1992). Often teachers need to reconsider and challenge their current beliefs about mathematics learning, teaching and what mathematics content entails and their work in and outside of the classroom. This is something that has been widely recognized as one of the biggest challenges for teacher education (Philipp, 2007; Simon, 2008; Sowder, 2007; Thompson, 1992). Mathematics education researchers advocate that professional development with an aim toward changing practice should involve long-term engagement with teachers, curriculum that deepens teachers' knowledge of the mathematics they teach, and connections between professional development and practice (Ball, 1996; Carpenter, Blanton, Cobb, Franke, Kaput & McClain 2004; RAND, 2004).

The NCTM (1991) *Professional Standards for Teaching Mathematics* lists several assumptions that provide the foundation for the standards for the professional development of teachers of mathematics. One assumption is that teachers are influenced by the teaching they see and experience. That is, teachers' experiences

while engaging in and observing mathematics instruction convey messages about what constitutes appropriate teaching and learning, and may have a profound impact on their knowledge of, beliefs about, and attitudes toward the nature of mathematics learning and teaching. Thus, mathematics teacher educators ought to model reform mathematics instruction in teacher education to promote reform mathematics teaching (c.f. Borasi, Fonzi, Smith & Rose, 1999; Simon & Schifter, 1991; Loughran & Berry, 2005). Such instruction might include an inquiry-oriented or a student-centered approach where teachers are given opportunities to grapple with mathematical ideas, develop understanding of mathematical ideas and mathematical structure through exploration and discussion, just as they will engage their students.

As recognized in the mathematics teacher education literature, changes in the core dimension of mathematics instruction are not easy to accomplish, may take several years, and require appropriate professional development (Clarke, 1994; Friel & Bright, 1997; Fennema & Nelson, 1997; Loucks-Horsley, Hewson, Love & Stiles, 1998). In the *Professional Standards for the Teaching of Mathematics* (1991) it is noted that such experiences provide teachers with images and models (conscious or unconscious) of what it means to teach and learn mathematics. Lunenberg, Korthagan and Swennen (2007) assert that despite differences in the context between university and school settings, the model a teacher educator provides for learning can be a more important factor in shaping teacher practice than the content of the messages they think they are communicating about teaching. In order to learn to teach mathematics in a student-centered manner, teachers must first understand what constitutes reform

teaching. One can learn about a professional organizations' vision of effective mathematics teaching and learning through various means, however, observation of teaching is likely a critical part of constructing an understanding of teaching in a student-centered manner. Collins, Brown and Newman (1989) and Lave and Wenger (1991) hypothesize that through observation, learners develop a conceptual model that provides them with an advanced organizer and interpretive structure for reflecting on a given practice. It can be argued that observing reform teaching is important for practicing teachers because they can make use of this interpretation when reflecting on their own teaching. In mathematics teacher education, teachers could have the opportunity not only to learn mathematics, but also the practice of teaching mathematics from the mathematics instruction they observe as they experience it as learners. So just as the teaching of mathematics needs to change so that students come to understand their mature roles in a community, the teaching of mathematics teachers may also need to change to help teachers develop a conceptual model of effective inquiry-oriented teaching. Mathematics teacher educators can provide such a model of effective mathematics teaching.

Past Research on Modeling in Teacher Education

The assumption that experiences that engage teachers as learners in inquiry-oriented environments can challenge their conceptions of mathematics and its teaching and can lead to measurable changes in teachers' beliefs about teaching and learning mathematics (Borasi, Fonzi, Smith, & Rose, 1999; Simon & Schifter, 1991).

Lunenberg, Korthagen, and Swennen (2007) reviewed the literature on modeling by university-based teacher educators and found little had been written. The literature had a focus across content areas (more accurately, *sans* disciplines) on modeling pedagogy for student teachers. From this literature, Lunenberg and his colleagues (2007) describe four forms of modeling: 1) implicit modeling, 2) explicit modeling, 3) explicit modeling and facilitating translation to student teachers' practice, and 4) connecting exemplary behavior with theory. *Implicit modeling* involves teacher educator acting as examples of what the teacher educator was trying to promote. Though advocating the importance of implicit modeling, they describe a study in which implicit modeling of desired practices in a preservice program had limited effect on the teaching of these graduates. They suggest that time and degree of exposure as well as "fit" with a student teachers' concerns may well be influential in effect. *Explicit modeling* usually involves meta commentary on choices made while teaching or modeling and also debriefing. *Facilitating translation to teaching* is undertaken by explicit modeling accompanied by discussing directly how the commentary is related to teaching. They also note the apparent lack of *connecting exemplary practice with theory*.

Borasi and his colleagues (1999) report on a professional development program aimed to address the factors of time and "fit" with teachers' concerns. In the summer institute modeling served as a vehicle to reform the mathematics teaching practices of teachers. The authors write about a summer institute where teacher educators modeled teaching practices that could support an inquiry approach to

teaching mathematics. These strategies included, for example, “starting with meaningful and complex problems, accommodating different learning modalities and validating student contributions during discussions by recording them on large sheets of paper and referring back to them” (p. 56). The teacher educators modeled the teaching of instructional units the teachers themselves would be teaching. The researchers’ analysis of teacher participants’ journals, reflections and spontaneous comments revealed that engaging teachers in illustrative units as learners of mathematics is a powerful way to exemplify the advantages and disadvantages of inquiry-oriented activities and puts teachers in a better position to evaluate the potential value of such activity for their students.

The illustrative units were generally followed by explicit reflections from complementary perspectives of the teachers and the mathematics teacher educators. These discussion included reflections on mathematical reasoning, discussions and aspects of mathematics content. This echoes another assumption the NCTM lists about the importance of reflection. The NCTM (1991) *Professional Standards for Teaching Mathematics* suggests learning to teach is a process of integration of theory and practice, and teachers should be afforded opportunities to comment and reflect on their own learning and teaching. Teacher development research notes the importance of reflective activities to make salient and highlight aspects of teachers’ educational experiences (Kosnik, 2001; Doyle, 1990; Loughran & Berry, 2005; Mewborn, 1999).

The literature suggests that, ideally, teachers should engage simultaneously in studies of mathematics content and mathematics pedagogy. As different instructional

strategies are modeled in teacher development, teachers should discuss implications for and the research that supports choices of strategies. Discussion allows teachers to identify what supported their learning as well as its effect on learners, draw from their experiences and others' perspectives of the larger implications for beliefs and practices. One participant in the Borasi et al study noted, "This [reflection] is a necessity because the process is critical but when you are part of the process you focus on learning rather than what the process has brought about the learning was. We need to have it pointed out that you were modeling various teaching methods, especially since many of these ideas were new to us" (p. 72).

Loughran and Barry (2005) discuss incorporating explicit modeling of pedagogy into a preservice course for student teachers. They describe modeling (or doing in practice) innovative teaching procedures rather than transmitting information about the teaching procedures. Yet, they also describe the need to offer their students access to reasoning and thoughts behind their practice through what they call explicit modeling. As one teacher educator modeled teaching practices, the other teacher educator "debriefed" the teaching, encouraging student teachers to consider the possibilities for learning embedded in their experience. They emphasized that explicit modeling is not just simply saying what you are doing but it also can include offering possible contexts for teachers to experience a situation. Merely providing an implicit model of desired teaching practices and then describing the situation does not explain the intended purpose or convey the implications and consequences for participants

(Loughran & Barry, 2005). Explicit modeling allows student teachers to deliberately reflect on intentions, purposes, and actions of the teaching experiences.

Learning from modeling in teacher education is often described in cycles of (implicit) modeling teaching practices and then (reflection) eliciting and sharing participants' perspectives as learners in the experience. In the literature modeling and reflection are discussed as key to changing teachers' practice (Kosnik, 2001; Doyle, 1990; Borasi, Fonzi, Smith, & Rose, 1999; Simon & Schifter, 1991; Loughran & Berry, 2005, Lunenberg, Korthagen, & Swennen, 2007). It is hoped that teachers' experiences in different participation structures both as learners and teachers in mathematics professional development serve as a catalyst for teachers to begin to transform their teaching practices. The hope is that teachers' experiences in inquiry-oriented learning enculturates them into a community of practice that is more consistent with current reform recommendations.

Perspectives on Learning from the Milieu

The literature of teacher education and investigations of learning instructional practice, in general, suggest the need to model (implicitly and explicitly) practice and provide opportunities for reflection on practice. Since the essence of this study is to understand what practicing teachers gain from implicit and explicit modeling in mathematics teacher education courses, a situated perspective provided the basis of my analysis. The situated theoretical framework is based on the assumption that knowledge is located in the social plane; that is, it is born through social interactions. From this prospective, knowledge is situated and is a product of activity in the context

of a culture (Brown, Collins, & Duguid, 1989). The situated cognition framework broadens the unit of analysis beyond the development or genesis of thought within the individual; instead such an analysis situates the development of an individual within a community. If we are to understand the learning phenomena we need to examine social and physical contexts, the milieu, in which activity is embedded. The milieu refers to the environment within a situation of action, everything that acts on the student or that on which he or she acts on (Brousseau, 1997). This includes, but is not restricted to, the teacher, the materials and the learning strategies chosen.

As part of this study, I propose to theoretically begin to unpack the notion of learning from instructor modeling in mathematics teacher education. Specifically, I consider theoretically how teachers learn about the practice of teaching by participating as learners in mathematics courses for practicing teachers. In what follows, I describe a situated perspective on learning from the milieu as found in the literature. In particular, I discuss the notions of perceptual lived experience, intent participation, apprenticeship and legitimate peripheral participation, and how they relate to mathematics teachers' learning about teaching.

Perceptual Lived Experience

Loughran and Berry (2005) argue that it is in “living through” experiences that real learning about teaching occurs. They suggest that there are insights that may not be fully understood if not experienced. Korthagen, Kessels, Koster, Lagerwerf and Wubbles (2001) define *phronesis* as the ability to think about how and why we should act, as perceptual instead of conceptual, born of experience rather than outside

it in some external conceptual form. Such knowledge or understanding is built by living through experiences. That is, the learner, rather passively, gleans ideas about the “how to” of the activity by simply being a part of the situation where the expert does their work. This is, in contrast to *epistme* or propositional, expert knowledge applied generally to many situations.

Intent Participation

While one may be learning something by “living through experiences” the individual may not be attending to specific features of the activity. The construct of intent participation, a related, more deliberate form of learning from experiences in a situation, suggests that learning stems from keen observation and listening in on ongoing activities in which the learner participates or expects to participate in the future (Rogoff, Paradise, Arauz, Correa-Chávez, & Angelillo, 2003). The Vygotskian school of thought presupposes that human thinking is inherently social in its origins and that learners’ conceptions evolve as a result of interactions with the milieu. Vygotsky (1978) suggests that there are two phases in development, social (interpsychological) and then individual (intrapsychological). A major theme of Vygotsky's theoretical framework is that experiences in social interactions play a fundamental role in development when novice practitioners engage in activity with experts. Rogoff et al note that such observation involves an attentiveness and intentionality central to this kind of learning. Though this kind of learning is not didactic, the execution of the tasks can be accompanied by “think aloud” commentary by the more capable other. This kind of learning embeds the development of skills

and knowledge in their social and functional context of the ongoing activity (Rogoff et al, 2003).

In general, learning from perceptual-lived experience and intent participation do not involve didactic teaching. Instead a knowledgeable practitioner executes a task, modeling the target processes while the learner attends to features involved in the activity and comes to understand processes involved in the execution of the task with respect to their interpretation of what was modeled. The distinction from perceptual-lived-experience lies in the intentionality of learning from the situation.

These theories suggest that participants in mathematics courses for practicing teachers have opportunities to learn about the instruction part of the teaching profession from perceptual-lived experience and intent participation of the teaching they experience as learners. Boaler (1998) studied the participation structures of two high school classes in England; one was a reform class and the other was rather traditional. She reported that when learners engage in (mathematical) activity it is not just the problem-solving techniques or the learner's conceptualization of the problem that changes, but also his or her view of the nature of the discipline (Boaler, 1998). Boaler's work suggests that just as high school students can learn from being a part of the classroom situation, teachers can develop ideas about alternative participation structures, and particular norms in mathematics teaching and learning. After a three-year case study, Boaler (1998) reports, "students who learned mathematics in an open, project-based environment developed a conceptual understanding that provided them with advantages in a range of assessments and situations" (p. 41), while those who

learned mathematics in an examination-oriented content-based class found it difficult to make use of school-learned methods and could not see connections between what they had done in the classroom and the demands of their lives outside the classroom. Boaler argues that the project students had been “apprenticed” into a system of thinking and using mathematics. In the same way, it can be argued that teachers who learned mathematics in an open, project-based environment can be apprenticed into a system of thinking about what mathematics teaching and learning entails, specifically with respect to participation structures and roles in the classroom community.

Increased participation and socially organized ways of seeing, however, are not immediate automatic phenomena. Cobb and Yackel (1996) argue that in classroom situations a student’s development is not only a product of their cognitive restructuring and conceptual reorganizations with a catalyst of social interactions, but also the negotiation of social norms and use of language. In the school setting, Cobb and Yackel (1991) describe norms as that which delineates the classroom participation structure. Cobb and Yackel argue norms are not psychological processes that can be attributed to any particular individual but they characterize regularities in communal or collective activity. Cobb and Yackel (1996) conjecture that as norms are negotiated “students reorganize their individual beliefs about their own role, others' roles, and the general nature of mathematical activity” (p.178).

Goos (2004), drawing on the Vygotskian school of thought, investigates specific actions a teacher in a secondary school mathematics classroom takes that facilitate the development of a culture of inquiry. In particular, Goos works to identify

how the teacher shaped the negotiation of norms and practices that emphasize mathematical sense making and justification of ideas and arguments, and illustrates the learning practices that students developed in response to their teacher's expectations. The students in the 11th-grade class Goos observed over the course of a year began to value paths to finding a solution over the solution itself and to view themselves and their peers as mathematical resources.

Cobb and Yackel (1996) and Goos (2004) report on studies in 2nd and 11th-grade classes respectively, in which the instructors' expectations helped guide the development of inquiry oriented mathematics in their classrooms. These expectations played a key role in developing social norms that prompted students to explain and justify solutions and thinking, work to make sense of explanations given by others and discuss differences in explanations and thinking. A major sociomathematical norm that arose in both classrooms was a shared understanding of what counts as an acceptable mathematical explanation. Goos (2004) points to an instance where the students are beginning to adhere to the instructor's expectations and in turn we see the emergence of the norms of explaining and justifying solutions. Goos explains that in the 17th week of the school year, one student, Adam, calculated the angle between two vectors and announced that he had the answer, his friend, Dean, then pointed out that the teacher "doesn't want the answer, he wants how you work out the angle" (p. 276).

Goos (2004) describes a somewhat generalized template for what a teacher might actually do to facilitate students' participation in a community of mathematical inquiry.

1. Present students with problems designed to engage them with a new mathematical concept.
2. Elicit their initial conjectures about the concept, withholding judgment to maintain an authentic state of uncertainty regarding the validity of these conjectures.
3. Orchestrate discussion or present further problems that would assist students to test their conjectures and justify their thinking to others.

Thus the teacher has a central role in guiding students to participate in a community of mathematical inquiry. Since from the sociocultural perspective, mathematics learning occurs through mathematical communication in social contexts, scaffolding, collaboration, and interweaving, theoretical concepts facilitated the co-construction of knowledge.

With respect to perceptual lived experience, teachers can learn about participation structures from being a part of the milieu. Whereas I suggest, teachers can learn about how the mathematics teacher educator fosters various participation structures and norms by intent participation.

Apprenticeship

Like intent participation, apprenticeship learning is a deliberate form of learning from experiences in a situation. Cognitive apprenticeship involves learning-through-guided-experience (Collins, Brown, & Newman, 1989). In apprenticeship learning, skills are not only continually in use by the novice practitioner, but are instrumental to the accomplishment of meaningful tasks. It is the job of the expert to present tasks in their entirety, then provide assistance or scaffolding so that novices can participate in the task within his/her zone of proximal development (ZPD). The ZPD is the distance between the actual development level as determined by

independent problem solving and the level of potential development as determined through problem solving with more knowledgeable others (Vygotsky, 1978). This theory of learning potentially describes more assistance in the “trying on” of practices than is typically associated with mathematics teacher education. However, in the studies mentioned above, the aspect of teachers going back to their own classes to “try on” practice is not addressed.

Legitimate Peripheral Participation

Lave and Wenger (1991) describe a related construct, legitimate peripheral participation (LPP), as the “engagement of a learner who participates in the actual practice of an expert, but only to a limited degree and with limited responsibility for the ultimate product as a whole” (p. 14). Once the learner has a grasp of the target skill, the expert reduces (or fades) his participation, providing only limited hints, refinements, and feedback to the learner, who practices by successively approximating smooth execution of the whole skill (Collins, Brown, & Newman, 1989). For example, Vai apprentice tailors have the opportunity to observe the whole process of making a piece of clothing while they contribute to aspects they can manage, such as ironing, hemming and tracing garments (Lave & Wenger, 1991). Legitimate peripheral participation apprenticeship highlights the potential of learning-through-guided-experience and participation, where learning is taken to be improved participation through interactive systems of communication (Greeno, 1992; Goos, 2004).

With respect to legitimate peripheral participation and cognitive apprenticeship, skills and learning stem from “authentic situations” where observation, coaching, and successive approximation develops ones understanding through continued situated use (Brown, Collins, & Duguid, 1989; Collins, Brown, & Newman, 1989). This kind of activity can be observed in mathematics courses for practicing teachers. For example, in the professional development discussed by Borasi et al (1999), the mathematics teacher educator modeled instruction displaying the kind of instruction promoted by the education program. Since the participants in the mathematics professional development are practicing teachers they can “try-on” aspects of the instructional pedagogy they observed while they were engaged as learners of mathematics. The teachers’ perceptions of the mathematics teacher educator’s model of targeted processes can be approximated and the teachers’ understanding of student centered instruction can be developed as the teachers make use of the pedagogy in classroom situations. It was not uncommon for the mathematics teacher educator to take an aside and embed the activity in the context of teaching children, prompting discussion of how particular activities or discussions might play out with their own students. In teacher education such interactions exemplify to the learner the “how to” and “under what condition to” in mathematics teaching situations.

Loughran and Berry (2005) discussed instructor modeling in their investigation of a preservice education program and noted the importance of explicitly modeling for their students the thoughts and actions that underpin the teacher

educator's pedagogical approach. In mathematics courses for practicing teachers' learning through guided experience and legitimate peripheral participation looks very different since the classes are comprised of practicing teachers who already have assumed the role of mathematics teacher. That is, the participants have already taken on the role of teacher and have full responsibility of all facets of practice. Learning by means of apprenticeship and legitimate participation presupposes that novices are embedded in a community where experts provide support for and multiple opportunities for developing competence in desired practice. In general, most practicing mathematics teachers do not have support that fosters the development of reform teaching practices. In mathematics teacher education, the mathematics teacher educator can implicitly model mathematics pedagogy that provides for scaffolding and coaching to foster the learning of mathematics.

Teacher educators can also model the kinds of discussion and reflection activities with members of the community of teachers that serve as support for teachers' professional practice. In general, teachers practice in isolation (Dufour, 2005) and given the opportunity to observe, most would find they are not embedded in a community of expert inquiry-oriented teachers. With respect to teachers engaging in activities related to facets of the mathematics teaching profession that go beyond classroom instruction, such as lesson planning or curriculum design, the mathematics teacher educator is in a position to help teachers to hone their skills and look to their colleagues as resources.

Enculturation

The above constructs are unified under the theme of enculturation, describing learners' adoption of the behavior and belief systems of social groups, and learning occurring through authentic activity. With this perspective, knowledge is situated. Learners are described as approximating practices toward the execution of the composite set. As an alternative to conventional educational practices, these situated perspectives on learning propose that learning occurs in situations where scaffolding allows the learner's scope of participation to expand until the learner can perform the whole task unassisted. This type of learning can be compared with open or process-based forms of mathematics instruction. Open-ended process-based work in mathematics allows students to do practical investigations that require them to use active and flexible thought, make decisions, plan routes through tasks, choose methods, apply their mathematical knowledge, and develop an ability to adapt and change methods to fit new situations (Boaler, 1998). Teaching mathematics in a manner described in reform recommendations requires inquiry and investigation of teaching in a similar way. Further, these perspectives on learning from the milieu highlight the importance of observation, modeling, coaching and scaffolding to support the devolution of the responsibilities for learning. Collins et al (1989) note "[t]he interplay between observation, scaffolding, and increasingly independent practice aids apprentices both in developing self-monitoring and self-correction skills and in integrating the skills and conceptual knowledge needed to advance toward expertise" (p. 458).

Such professionalism of mathematics teachers requires that they participate in a *professional learning community* (Dufour & Eaker, 1998; Astuto et al, 1993, Hord, 1997). A professional learning community is characterized by a supportive and shared leadership, collective creativity, shared values and vision, supportive conditions, and shared personal practice. In mathematics teacher education there are opportunities for teachers to engage in the collective inquiry process (Dufour & Eaker, 1998), an important aspect of a professional learning community. Dufour and Eaker summarize Ross, Smith, and Roberts' (1994) description of "the team learning wheel" to describe facets of the collective inquiry process:

1. Public reflection – members of the team talk about their assumptions and beliefs and challenge each other gently but relentlessly.
2. Shared meaning – the team arrives at common ground, shared insights.
3. Joint planning – the team designs action steps, an initiative to test their shared insights.
4. Coordinated action – the team carries out the action plan. This action need not be joint action but can be carried out independently by the members of the team. At this point, the team analyzes the results of its actions and repeats the four-step cycle. (Dufour & Eaker, 1998, p. 26)

Engaging in this process is not automatic; in reform-centered mathematics teacher education the hope is that the teachers are enculturated into a professional learning community of mathematics teachers. The teachers are in essence apprentices to the mathematics teacher educator, however instead of modeling mathematics pedagogy, the mathematics teacher educator is modeling participation in a community of mathematics teachers. Such an apprenticeship provides a catalyst for learning-

through-guided-experience and participation, where learning is taken to be improved participation through interactive systems of communication (Greeno 1992; Goos 2004), in the case of teacher education the collective inquiry process of the professional learning community of mathematics teachers.

Brown et al (1989) suggest that enculturation through engagement in authentic activity fosters the development of cultural tools such as shared language or vocabulary as a means to discuss, reflect upon, evaluate, and validate ideas in a collaborative process. They further argue that cultural tools “can only be fully understood through use, and using them entails both changing the user's view of the world and adopting the belief system of the culture in which they are used” (Brown et al, 1989, p. 33). The authors note it is possible to acquire a tool and be unable to use it. Actively using tools rather than acquiring them has the capacity to build an increasingly rich implicit understanding of its domain. In order “...to learn to use tools as practitioners use them, a student, like an apprentice, must enter that community and its culture” (Brown et al, 1989, p. 33). Thus, it is important that teachers engage in authentic practice and build a conceptual model of reform teaching to transform teachers’ understanding of mathematics instruction or beliefs about teaching, learning, development and discourse.

Cohen (1990) reported on a case study of an elementary school teacher. Mrs. Oublier, the teacher in the study, willingly participated in a workshop that was built around the 1985 Mathematics Framework for California Public Schools. She eagerly adopted new curriculum materials and activities introduced to her during professional

development. While Mrs. Oublier certainly embrace the framework's doctrine of inquiry instruction, it was not reflected in her practice. Cohen (1990) noted that she had simply adapted these new materials and activities to her traditional teaching style. She continued to teach mathematics very traditionally, and discouraged exploration and discourse. Cohen reflected that, because of her superficial understanding of the mathematical and pedagogical infrastructure, she could not engage the students in open discourse and exploration that would reveal students' understanding.

Mathematics courses for practicing teachers serve as one way to help teachers move forward in their mathematical understanding of the mathematics they teach and develop pedagogical skills, both of which are necessary to promote teaching for understanding through inquiry. In many courses the instructors model desired classroom approaches to project a clearer vision of the student-centered learning. It can be argued that without being apprenticed into systems of thinking “the ability to see a meaningful event is not a transparent, psychological process. Ways of seeing and understanding is a socially situated, historically constituted practice through which the objects of knowledge mediate the discourse of a profession (Goodwin, 1994). Goodwin (1994) discusses how social and cognitive organization of a profession shape events for the learner by focusing on specific aspects, events or objects on which the discourse of the profession is structured. He discusses this in terms of professionals working in an archaeological field excavation and in a courtroom. Goodwin notes “[t]hrough the structure of talk in interaction members of a profession hold accountable for, and contest, the proper constitution and perception of the objects that

define their professional competence” (Goodwin, 1994, p. 606). Speech, in conjunction with tools, representations, coding schemes, and highlighting produce perceptual fields allow learners to “see” relevant objects of scrutiny in a domain (Goodwin, 1994). Understanding these socially organized ways of “seeing” and understanding is necessary for learners to become competent practitioners.

In the context of mathematics courses for practicing teachers, if we take learning to be improved participation through interactive systems of communication (Greeno, 1992; Goos, 2004), then the notions of perceptual lived experience, intent participation, apprenticeship, and legitimate peripheral participation allow for a first hand view of student-centered inquiry-oriented learning environments that the teachers may themselves foster in their own teaching.

Modeling and Reflection in Learning

Implicit Modeling. Implicit modeling in mathematics courses for perspective or practicing teachers can project a clearer vision of the proposed changes to teaching practices. Implicit modeling provides learning opportunities through participation in reform-oriented mathematics classrooms. This learning is enabled through aspects of perceptual lived experience, intent participation, apprenticeship, and legitimate peripheral participation. In such environments mathematics teacher educators can solicit teachers’ conscious commitment to participate actively in class discussions. Teachers can engage as learners in a setting that fosters the learning of mathematical ideas through inquiry: exploring mathematical problem situations, looking for patterns, generating, verifying or rejecting ideas and hypotheses, generalizing, and

justifying in collaborative activity. In mathematics courses for practicing teachers, inquiry-learning experiences have the capacity to provide insights into, and perhaps challenge, pedagogical beliefs. Mathematics teaching and learning that are apprehended in practice might otherwise not be fully appreciated or understood if the individual did not experience such learning (Simon & Schifter, 1991).

Explicit Discussion of Modeling. There are often stark contrasts in perceptions of an event. There are also aspects of teaching decisions that cannot be observed, such as reasons for actions. One way to make salient reasons that underpin a pedagogical approach and bring forth implications for learning and teaching is explicit discussion of modeling by the mathematics teacher educator. Applying participation and apprenticeship methods to largely cognitive skills, such as those involved in teaching, requires the externalization of processes that are usually carried out internally. Loughran and Berry (2005) point out, “being explicit requires a sensitivity to the ongoing tensions associated with balancing student teachers’ perceived needs and concerns and their teacher educator’s beliefs about what they need to know and be able to do” (p. 194). Cognitive apprenticeship teaching methods are designed to bring these tacit processes into the open, where students can observe, enact, and practice them with help from the teacher and from other students. The expression of such pedagogy to be conveyed requires considerable awareness of oneself, pedagogy and students (Rogoff, 2003). However in order for an instructor to guide the student in the conceptual organization of experiences, the instructor must have an adequate idea of where the student is and the goals of instruction.

Goodwin's (1994) study of the discursive practices used by members of a profession to shape events and focus the attention of novice explains how graphic representations, talk and diagrams have the capacity to highlight, guide the perception and make salient specific events and objects of scrutiny while further reifying the objects. Speech in conjunction with tools, representations, coding schemes, and highlighting produce perceptual fields that allow learners to "see" relevant objects of scrutiny in a domain (Goodwin, 1994). With respect to instruction this suggests,

[o]ne dimension of acquiring greater competence appears to be the increased ability to segment the perceptual field (learning how to see). Research on expertise suggests the importance of providing students with learning experiences that specifically enhance their abilities to recognize meaningful patterns of information (e.g., Simon, 1980; Bransford et al., 1989)... (An expert's) knowledge is organized around core concepts or "big ideas" that guide their thinking about their domains (National Research Council, 2000, p. 36).

It can be argued that it is essential that some aspects that guide an expert's thinking must be explicated for the learner. This gives the learners insights into approaches to a task that are not directly observable. For example, as the opening of Chapter 1 conveyed, in a mathematics course for practicing teachers during small group discussions some of the teachers expressed that they thought the intent of the teacher educator's roaming around the room listening was to make sure everyone was on task, and expressed that this was an effective method to monitor students that they might employ in their own classes. When asked about his intent, the teacher educator noted that he listened in on the discussions of the groups to get an idea of how they were thinking about the mathematics and how their understanding could inform whole class discussion in the debrief of the activity. Two very different perspectives were present.

Reflection. Implicit modeling can be made explicit through reflection and discussion about the questioning, probing and inquiry created through pedagogic interventions during teaching and de-briefing of shared teaching and learning experiences. Loughran and Berry (2005) note that “[a]ccessing these views we see as offering participants a form of ‘metalearning’” (p. 194). Reflection (Mewborn, 1999; Simon & Schifter, 1991) or meta-learning (Loughran & Berry, 2005) has long been highlighted by many teacher education and professional development programs as a means to learn about teaching and learning. As early as 1904, Dewey advocated the importance of reflective thinking, an “active, persistent, and careful consideration of any belief or supposed form of knowledge in the light of the grounds that support it and the further conclusions to which it tends” (Dewey, 1933, p. 9) in teacher education. He suggested teachers need to be assisted in developing habits of reflection, and the lack of such habits leads to prescriptive propositional teaching practices and an intellectual dependency on “those persons who give them clear-cut and definite instructions as to just how to teach this or that” (p. 152).

A common theme in the literature is that reflection is both an individual and a shared experience. While reflection involves introspection in early stages, it also requires outside prompting and probing. Reflection is not a natural state of mind, nor is it easy to make reflective thinking a habit (Floden & Buchmann, 1990; Goodman, 1984; Harris, 1989; MacKinnon, 1987). If learners’ conceptions evolve as a result of interactions with the milieu, whenever possible, discussions should be based on teachers’ experiences in teaching settings. Because examination of specific

problematic situations could lead to deeper discourse and, Mewborn (1999) argues possibly, to the generation of alternatives and constraints to those alternatives.

In Simon's (1994) reports of the model of mathematics teachers' professional development in his study, reflection is an integral component of each of the learning cycles. The teachers' reflections can address various aspects of the experience, the mathematics content learned, the teacher educator's pedagogy, how the experience relates to practice and student thinking, or complementary aspects of the teacher's own practice. Mewborn (1999) argues that teachers need opportunities to reflect on their experiences and on the beliefs that have resulted from their experiences in order to become aware of how these beliefs and experiences influence their views of teaching and student learning. Doyle (1990) argued that reflective thinking is a means to help teachers begin to view the claims from research or theory as avenues for inquiry in the face of a classroom situation rather than as definite answers to all educational problems, thus bridging the gap between practice and educational theory.

The role of the teacher educator is to help teachers explore and refine their perceptions through the opportunity to systematically reflect on the details of their practical experiences (Korthagen et al., 2001). This reflective activity allows for learning beyond the immediate experience uncovering understanding of the learning and teaching being experienced (Loughran & Berry, 2005), and how this knowledge can be transformed in their classrooms. Borasi et al. (1999) note that providing teachers with multiple opportunities to reflect on their experiences as learners and as teachers, by means of both discussions and writing tasks allow teachers to better

appreciate the significance of their learning experiences and the implications for their beliefs and teaching practices. “In order to promote reflective teacher education, a clear conceptual grasp is required of what the processes of reflection involve, what students might usefully reflect about, and how their reflection is going to be influenced by the nature of the tasks they are set and the kind of teacher education context in which they work” (Calherhead, 1989, p. 49).

This reflective discussion is a practice that can be observed in groups of mathematics teachers who participate in a professional learning community.

Each word of the phrase "professional learning community" has been chosen purposefully. A "professional" is someone with expertise in a specialized field... (and) is also expected to remain current in its evolving knowledge base... "Learning" suggests ongoing action (practice) and perpetual curiosity (study)... the term "community" suggests a group linked by common interests... In a professional learning community, all of these characteristics are evident. Educators create an environment that fosters mutual cooperation, emotional support, and personal growth as they work together to achieve what they cannot accomplish alone (p. xii).

In mathematics courses for practicing teachers, the teachers engage as learners in a classroom community of learners, and as professionals in a community of mathematics teachers. In both levels of engagement the teachers with the mathematics teacher educator construct a culture that values the strengths of all participants. Teacher educators and teachers shift among the roles of expert, researcher, learner, and teacher, supporting themselves and each other.

Theoretical Framework

I view learning as inherently social in origin, but here I seek to account for individual perspectives within evolving social practices. I portray the learning process

of an individual situated within a complex social setting. Furthermore, within the social setting of classrooms, I see teachers and students engaged in the negotiation of meaning (Cobb & Bauersfeld, 1995).

The first research question focuses on what mathematics teacher educators believe about the ways in which they model effective mathematics instruction and the kinds of messages they believe they convey through their pedagogical approach about teaching, learning and the teaching profession. The second research question seeks to identify what practicing teachers notice and identify as effective mathematics teaching in their mathematics professional development. And the third question is an attempt to understand the ways these perspectives were aligned in mathematics courses for practicing teachers. In order to answer these questions it is necessary to situate individuals' interpretations of classroom events in a framework that encompasses both the individual and social perspectives.

This study combines cognitive and social perspectives on mathematics teacher development. Therefore, I focus on individuals' perceptions of the mathematics teacher educator's pedagogy and meaning conveyed in classroom discourse, whereas the social perspective on mathematics teacher development tries to articulate theories of the conditions for the possibilities of noticing aspects of practice. If we would like to give an account of what mathematics teacher educators believe their instruction conveys, what individual teachers notice and describe the social conditions in which the teaching acts of note were situated, neither the social processes nor the individual's interpretations can be considered without the other.

As a learner of mathematics, apprenticeship of observation can account for one's development of a perceptual sense of the role of the teacher, the role of the student, and beliefs about what it means to do mathematics. In inquiry-based learning environments participation structures, norms and how they are different than those in traditional classes can become salient for teacher. And as just as students Boaler reported on were "apprenticed" into a system of thinking and using mathematics, teachers can also adopt, re-construct and reflect on the new participation structures they experience and what it means for their own practice both in and outside of the classroom.

As part of building an image of the nature of classroom learning environments the learner gleans ideas about how to teach without necessarily attending to particular aspects of the teachers pedagogy. But, as a prospective or practicing teacher engages as a learner they may be more deliberate in attending to aspects of instruction they experience as it may be useful to draw upon in their own practice. For example, Borasi et al (1999) describes mathematics teacher educators that modeled units that the participants might use in their own classes. Still building a conceptual model of practice can go beyond re-enacting units derived from professional development. Such intent participation not only provides a model for teaching specific content or activities but also can generalize a style of teaching and engaging learners. In the study the authors reported that the units were followed by explicit reflections from complementary perspectives of reasoning, reflections, discussions and aspects of

mathematics content they learned. Such activities allow teachers the opportunity to unpack the pedagogy they experienced and reflect on their own teaching.

Still as novices learn from experts, perceptions of a situation may be perceived differently and some facets of a situation may be invisible to the novice and thus must be pointed out or explicated by the expert. The expert must highlight features of the perceptual field to allow the learner to “see” objects of scrutiny. The teacher educator plays a key role in developing the teachers’ professional visions. The mathematics teacher educator can point out the kinds of discussion that are fruitful as teachers engage as professionals and reflect on practice or explicate the underlying intentions and purpose for pedagogical moves as the teachers are engaged as learners. Such aspects may be invisible to teacher but may play an important part of becoming an effective teacher.

In the next chapter, I describe the methodology used for understanding these questions based on coordinating the researcher’s, mathematics teacher educators’ and the practicing teachers’ perspectives on what is being modeled in mathematics classes for practicing teachers.

CHAPTER 3

METHODOLOGY

The analytic methodology for this study combines analysis of interviews, videotape data and participants' (both mathematics teacher educators' and practicing teachers') reconstructions of classroom events. This methodology allowed for the creation of integrated data sets to be coordinated and connections to be made between the mathematics teacher educators' and the practicing teachers' perspectives of shared classroom experiences. In this chapter, I will discuss the setting, describe the participants (mathematics teacher educators and practicing teachers), and elaborate on the nature of data sources. I then provide an elaboration of the interpretive framework and methods of analysis as they relate to the data sources.

Setting

The mathematics courses for practicing teachers discussed in this dissertation are offered by a university-based professional development group. The university professional development is housed within a large urban university in the southwest United States. The university partners with local school districts (both rural and urban) in the county to provide two types of teacher education: (1) mathematics professional development and (2) mathematics courses for practicing teachers. The mathematics courses for practicing teachers facilitated by the university professional development group are semester-long courses taken for university credit and are a part of a series of related mathematics courses required for certification as a mathematics teacher. The certificate takes two years to earn and focuses in depth on the content

classes and pedagogy classes for a given grade range. This certificate is not recognized by the state but is recognized by districts. The teachers in this certificate program participate in 180 hours of coursework in mathematics courses for practicing teachers on a voluntary basis over two consecutive years. For upper elementary teachers (grades 4-6), the certificate includes six units of university course credit in undergraduate mathematics courses focusing on the mathematics of the upper elementary grades (Number, Geometry, and Algebraic Thinking) and six units of university course credit in graduate teacher education courses focusing on the teaching of mathematics in these grades. Typically, teachers participate in an intensive three-day institute in the summer followed by weekly three-hour class meetings during the school year. The classes during the academic school year alternate between mathematics content and teaching and evaluation. A similar certificate is offered to primary teachers (grades k-3), but the mathematics courses are primarily focused on the mathematics of primary grades (upper grades content is also discussed, but to a lesser extent).

The needs and resources of the district generally drive the mathematics professional development facilitated by the university professional development group. District personnel are an intrinsic part of professional development geared toward their particular needs. Thus, the content and time commitment required of the teachers varies by partnership. The contact time and range of topics is often smaller than that of the mathematics courses for practicing teachers. However, in general, the models of mathematics teacher education offered by the university professional

development group have two primary objectives, (1) to deepen mathematical understanding of the mathematics teachers teach, and (2) to model instructional practices that promote teaching for understanding (Nickerson & Brown, 2008). As with many mathematics professional development programs, the goal is to move teachers forward in their thinking about content and student learning so teachers can work to help increase student achievement in mathematics (Nickerson, 2008; Sowder, 2007). Thus, the professional development programs are designed to provide extra preparation for teaching mathematics not only by communicating pedagogical knowledge, but also by providing opportunities for teachers to reexamine the mathematics they teach by deepening their content knowledge.

At the time of this study, the university professional development group was partnered with five local school districts encompassing grades k-12. Due to differences in the districts the professional development design for the districts varies in scope, duration and focus of mathematics content discussed in the professional development. At the time of this study, there were four models of professional development offered to the teachers in the five districts, here referred to as districts A through E.

District A was the second largest district in the state, and the eighth largest urban district in the country. The student population was extremely diverse, and represented more than 15 ethnic groups and over 60 languages and dialects. The university professional development group had worked with District A since the 2000-01 academic year. District A was a k-12 district. The certificate in mathematics

teaching was originally developed for the teachers in District A. The teachers participated in the mathematics specialist certificate program at either the primary elementary (grades K-3) or upper elementary (grades 4-6) level. The teachers in this study from District A participated in the mathematics teacher education provided by the university on a voluntary basis.

District B had a one-year partnership with the university professional development group. District B was an urban k-8 district. The participants in the mathematics professional development were middle school teachers and were a part of a systemic effort to improve student achievement in middle grades. Thus all of the middle school teachers with responsibilities for teaching mathematics were required to participate in the professional development offered by the university as a part of this systemic effort. There were two cohorts of teachers in this model of professional development, cohort-M and N. The cohorts differed only in the frequency and duration of mathematics professional development sessions. For both cohorts the professional development was focused on middle school mathematics content, algebra in the middle grades and task analysis. The teachers in cohort-M attended three eight-hour sessions in the summer before the academic year and twelve four-hour sessions during the school year for a total of 64 hours professional development for the year. The teachers in cohort-N had eight eight-hour sessions during the academic year for a total of 64 hours of professional development. The teachers in both cohorts in District B had the option to use the mathematics professional development toward college credit.

District C was a rural district similar to District B with respect to the content taught in the classes. However, while District B was making a systemic effort to improve student achievement through teacher education, teachers in District C participated on a voluntary basis. District C was a k-12 district. It was in its third year of a five-year partnership at the time of this study. In the first and second year approximately half of the teachers from grades 4-6 participated in the professional development. The participating teachers in the third year were middle school algebra teachers. This cohort of teachers attended twelve six-hour mathematics professional development sessions per year for a total of 72 hours of professional development per year.

District D was in the third year of a five-year partnership. District D was an urban grade 7-12 district. In this partnership all teachers with responsibilities for teaching Pre Algebra, Elementary Algebra and/or Algebra I were required to participate in the mathematics professional development offered by the university professional development group over the five years. The first year had a focus on algebra content. In the second year the focus of the professional development was on task analysis. In both of these years the mathematics professional development for the teachers in District D included four six-hour professional development sessions per year. In the third year the professional development was customized for different schools in the district. The mathematics teacher educators took on a coach or facilitator role at the school site. For example, at one high school in the district, the high school's leadership and entire math department teamed with the university

professional development group to implement Lesson Study activities aimed at improving instruction in Algebra in order to increase student achievement.

District E, an urban k-8 district, was in its third year of a five-year partnership with the university professional development group. In the first academic year the professional development was provided exclusively to primary teachers, k-2 on a systemic basis. In that year, three professional development sessions were held in the summer and there was one six-hour meeting per month during the school year, for a total of 72 hours of professional development in the first year. Continuing professional development sessions for the second academic year was provided as 18 three-hour meetings after-school, about two times per month for K-2 teachers on a voluntary basis. The teachers received a stipend for attending these after school sessions. The mathematics content curriculum for the K-2 teachers included (whole) number and operations, some geometry and measurement and fractions as they apply to primary grades. At the time of this study, the model of professional development in District E was a blended in-class and online model for grade 3-5 teachers.

In all of the cohorts discussed in this study there was a well-articulated mission anchored in student learning of core disciplines and skills. The goal was to have the teachers rethink and reconceptualize the mathematics they teach. The focus of each cohort dealt with specific issues of curriculum and pedagogy as derived from research and instruction, and issues of instruction and student learning of mathematics. While many topics may have been appropriate and important to discuss in the classes, because of the limited time the mathematics teacher educators have with participants,

decisions had to be made about the focus of the mathematics content. In general three questions guided decisions about what to include or omit in these courses: (1) What concepts were the teachers responsible for teaching? (2) What mathematics were the teachers able to learn from the curriculum they were using? (3) With what topics might teachers have had only superficial experiences? (Nickerson, 2010; Nickerson & Brown, 2008).

One of the strengths of an investigation in this study is the diversity of instructional settings for observing the mathematics teacher educators. Table 3.1 below summarizes the facets of the five cohorts with respect to curriculum and topics, grade level of the participating teachers, time commitment, and whether or not their participation was voluntary and college credit was available.

Table 3.1: Description of Districts A-E.

	District A	District B	District C	District D	District E
Curriculum/ Topics	Year 1 Rational number; Year 2 Geometry & Probability & Stats	Algebra in the middle grades and task analysis	Year 1 Rational Number, Numbers and Operations & Algebraic reasoning; Year 2 Rational Number, Quantitative reasoning & Geometry	Year 1- rational number; algebraic thinking; Year 2- Task level analysis; maintaining/ declining	Year 1 Number; Year 2 Number and some Geometry, Measurement & Fractions
Grade Level Taught by Teachers	K-6	6-8	4-6	7-12	K-2
Time Commitment	2 years	1 year	2 years	5 years	2 years
Frequency/ Session Duration	School Year: 3-hour classes once a week for 16 weeks Summer: Five 7-hour sessions for one week	School Year: Group 1 – twelve 4-hour sessions Group 2 – eight 8-hour sessions Summer: Group 1 – three 8-hour sessions	School Year: Year 1 & 2, once a month for 6 hours Summer: Five 6-hour sessions for one week	School Year 1 & 2: one day (6 hours) 4 times a year Summer: 0 meetings	School Year: Year 1, once a month for 6 hours. Year 2, 18 three-hour sessions (about 2 per month) Summer: Three 6 hour sessions
Total Hours of PD 2006-07	180	64	90	24	72
Voluntary/ Systemic	Voluntary	Systemic	Voluntary	Systemic	Year 1 systemic Year 2 voluntary
Certificate in Mathematics Teaching or College Credit	Yes	Optional (No teachers were working towards credit)	Optional (7 were working towards credit)	No	Optional

Mathematics Teacher Education Sessions. As mentioned earlier, the mathematics professional development programs that took the form of classroom

sessions were made up of two components: content and pedagogy. In this study, I observed the mathematics teacher education in two cohorts from District A and one from District B. In District A, I studied the modeling in two mathematics courses for practicing teachers. One was a primary elementary cohort (grades K-3) and the other was an upper elementary cohort (grades 4-6). In the primary elementary cohort, topics included rational numbers, the real number system, geometric shapes and measurement. The upper elementary cohort content included similar content but also algebra, the quantifying of change, and probability.

In District B, I studied modeling in cohort-M, henceforth referred to as the middle school cohort. In the middle school cohort the mathematics professional development focused on middle school mathematics and the content included, task analysis and algebra in the middle grades.

The primary elementary, upper elementary and middle school cohorts were chosen for this study mainly because all of the participant teachers were in their first year of professional development. Also, the model of professional development is comparable as all three come in the form of three-hour classroom sessions. I sought to study classroom sessions because this dissertation has a focus on what mathematics teacher educators model in mathematics courses for practicing teachers. While the number of contact hours for primary and upper elementary cohorts differ from those of the middle school cohort, I conjectured that since they were all in their first year, they were not so far embedded in the milieu that they have ceased to notice what seemed significant about the learning they were experiencing. I chose cohorts that were not so

new to the university professional development so as to have had little time for reflection, but were immersed enough to notice facets of the instruction they were experiencing. These cohorts were comprised of a set of teacher participants from a wide range of grade levels – primary elementary, upper elementary and middle schools. Further note that the teacher educators that facilitated the professional development in these cohorts are knowledgeable in content and pedagogy of mathematics teaching and learning and the teaching of teachers.

Participants

The aim of this study is to explore the types of instructor modeling in a variety of mathematics courses for practicing teachers and practicing teachers' perceptions of the mathematics teacher educators' pedagogy. An important aspect of this is understanding the perspectives of the persons involved in these classes: the mathematics teacher educator and the practicing teachers. In this section, I describe the two categories of participants in this study, the mathematics teacher educators who model reform instruction and the practicing teachers who participate in the mathematics courses.

Mathematics Teacher Educators

While in this study I focus on three mathematics teacher educators that teach three different courses, it is important to note that each of the mathematics teacher educators teach in more than one partnership. This allowed them to work within a wide range of teacher education models. All of the mathematics teacher educators have K-12 teaching experience, experience teaching prospective and practicing

teachers and have strong mathematics backgrounds. The mathematics teacher educators in this study apprenticed with the university based mathematics teacher educators to develop their practice of teaching teachers. Here I describe each mathematics teacher educator using gender-preserving pseudonyms.

At the time of this study, Karla had 4½ years experience teaching grade 7-12 students. She also had 2½ years experience teaching college courses like mathematics for elementary school teachers, statistics, calculus and developmental mathematics. Karla had worked as a mathematics teacher educator for eight years, and has a B.A. and M.A. in mathematics, and a clear Single Subject Mathematics Credential. Her single subject credential authorizes Karla the authorization to teach secondary school mathematics in this state. In this academic year, Karla team-taught the mathematics professional development for districts B and C.

Tony team-taught the mathematics professional development with Karla in District B. He had 19 years of experience teaching k-12 students. Tony had spent the last seven years working as a teacher educator. Tony has a B.A. in Education with an emphasis in Special Education and mental retardation, an M.A. in Education with a focus on computers in education, a Multiple subject credential, and a certificate for Developing Mathematical Ideas (DMI) leadership training. Tony's Multiple Subject Credential authorizes him to teach grades k-8 in this state. Beyond his work with District B this academic year, Tony worked with another mathematics teacher educator in District E as well. Tony was a teacher-on-loan from a partner school district.

Scott had 28 years of experience in teaching grades k-6. Over that time, he also had some experiences at both high school and middle school. Scott had spent 20 years teaching teachers. Scott was a teacher-on-loan and had taught both content and pedagogy courses for the university. At the time of the study, Scott was teaching mathematics courses for practicing primary teachers in District A required for the certificate in mathematics teaching. He has a B.A. in Science and the Social Sciences, a Master of Arts in Education with a specialization in Mathematics Education, and a Lifetime Multiple Subjects Credential.

Lisa taught the mathematics professional development with Karla for District C. Lisa had 22 years of experience working with grade 7-12 students. Lisa had worked as a mathematics teacher educator for three years, and has a B.A. in mathematics, an M.A. in Curriculum and Educational Leadership, and a clear Single Subject Mathematics Credential. In addition to her work with District C, Lisa also worked with District E on their professional development efforts. Lisa was a teacher-on-loan from a partnership district.

Max had four years of experience teaching grade 7-12 students. He had worked as a mathematics teacher educator for 9 years teaching classes for both pre-service and in-service teachers. At the time of the study he was teaching two mathematics courses for practicing teachers both of which are required for the certificate in mathematics teaching in the elementary cohorts in District A. He also was working with Lisa and Karla on District D's professional development activities.

Max has a B.A. and M.A. in mathematics, and a Single Subject Mathematics Credential.

Donna had 13 years of experience teaching grade 3-6 students. She also had 10 years experience as a mathematics teacher educator. Donna has an M.S. in Curriculum Design and a Multiple Subject Mathematics Credential. In this academic year Donna team-taught the mathematics course for practicing teachers with Scott in District A.

Table 3.2 summarizes the mathematics teacher educators teaching experience, educational history and which districts they worked with at the time of this study.

Table 3.2: Demographics of mathematics teacher educators.

MTE	Teaching Experience	Experience as a Teacher Educator	Degrees/ Credentials	District
Karla	4 years: grades k-12 2 ½ years: College	8 years	Single Subject Credential BA, Mathematics M.A. Mathematics	B, C
Tony	19 years: grades k-12	7 years	Multiple Subject Credential B.A. Education M.A. Education	A(upper), B, E
Scott	28 years: grades k-6	20 years	Multiple Subject Credential B.A. Science and Social Science M.A. Education	A(primary)
Lisa	22 years: grades 7-12	3 years	Single Subject Credential B.A. Mathematics M.A. Curriculum Development	C, D, E
Max	4 years: grades 7-12	9 years	Single Subject Credential B.A. Mathematics M.A. Mathematics	A(upper), D
Donna	13 years: grades 3-6	10 years	Multiple Subject Credential M.A. Curriculum Design 3 years of CGI training	A(primary)

As mentioned above, the professional development programs in the form of classroom sessions offered by the university professional development group have two components, content and pedagogy. In the mathematics courses required for the certificate in mathematics teaching, the weekly classes alternate between content and pedagogy. Thus while for the primary and upper elementary cohort Scott and Donna, and Max and Lisa team-taught the mathematics and pedagogy course respectively, the interviews and class sessions discussed in this study will be Max and Scott's. While, in the mathematics professional development for the middle school cohort, Karla and Tony team-taught the sessions in general, Tony's role in the two sessions under scrutiny was subtle and he was not present for much of them. Though Karla and Tony were an integrated team, wherein Karla's instructional focus was on content component, Karla is the mathematics teacher educator in this study.

Practicing Teachers

All of the practicing teachers in this study are drawn from the three courses described above; there were 49 teachers in all. They were in their first year of instruction in professional development provided by the university professional development group, and came from a wide range of elementary and middle schools in the county. The practicing teachers vary in the number of years of teaching experience with an average of 12 years. Their current grade-level assignment ranged from kindergarten to 8th-grade. The practicing teachers from District A were taking the course for college credit. While the teachers from District B were offered the option to earn college credit, none opted to do so.

The primary elementary cohort was made up of 18 teachers (16 females and 2 males). The grades the teachers were teaching ranged from kindergarten to 1st grade. One teacher was a physical education teacher and worked with students from kindergarten to 6th-grade during physical education. All of the teachers in the primary elementary cohort, except for the physical education teacher, had a teaching credential. The average number of years experience was about 9 years for the teachers in the primary elementary cohort.

The upper elementary cohort was made up of 22 teachers (21 females and 1 male). The grades the teachers taught ranged from 3rd to 5th-grade. All of the teachers in upper elementary cohort had teaching credentials and the average number of years of experience was about 7 years. The middle school cohort was made up of 9 teachers (5 females and 4 males). The grades the teachers were taught ranged from 6th to 8th-grade. All of the teachers in the middle school cohort had teaching credentials and the average number of years experience, as a teacher was about 15 years.

A subset of the teachers from each cohort was interviewed for the study. In this report these teachers are referred to using gender-preserving pseudonyms. These teachers are described below. All other teachers discussed in this study are referred to with pseudonyms like Ms. A and Mr. B. These teachers were not interviewed, but are key to the descriptions of classroom events. Note that A, B, C and so on are used to distinguish one speaker or contributor from another and are not necessarily the same person from episode to episode.

Three teachers from the primary elementary cohort were interviewed: Rita, Eve, and Anya. At the time of this study Rita taught a combination first and second grade class. She had worked for 14 years as a second grade teacher and three years as a first grade teacher. Rita has a B.A. in Liberal Studies and has a Multiple Subject Credential. She also has a Master of Arts in Education. Erin was a first grade teacher at a private school where she was the Elementary Program Coordinator. Erin had worked as a first grade teacher for 18 years. She has a B.A. in German with a minor in education and was working towards a Master's Degree in Human Development. Erin has teaching credentials from two states and achieved National Board Certification three years prior to this study. Anya was a kindergarten teacher and had worked as a teacher for 30 years, 20 of which were spent teaching preschool students. In previous years she had taught 2nd grade. She has an M.A. in Childhood Development, has a Multiple Subject Credential and a Cross-cultural Language and Academic Development (CLAD) certificate.

Three teachers from the upper elementary cohort were interviewed: Sandy, Kelly, and Tina. Sandy was a third grade teacher and has worked as a teacher for five years. In previous years she had taught first grade. She has a B.A. in Law & Society and Communication, an M.A. in Elementary Education, and holds a Clear Multiple Subject Credential. Kelly was a sixth grade teacher and has worked as a teacher for 10 years. She has a B.A. in Liberal Studies and holds a Multiple Subjects Clear credential with Specially Designed Academic Instruction in English (SDAIE) training. Kelly has been trained in AB 466 HOLT training. Tina was a 4th-grade teacher. She

teaches Math, Science and Social Studies. Tina had worked as a teacher for 9 years and during that time has taught kindergarten, 1st and 5th-grade. She has a B.A. in Humanities and holds a Multiple Subject Credential and a National Board Certificate.

Two teachers from the middle school cohort were interviewed: Cathy and Victor. Cathy was a 6th-grade mathematics and science teacher and has worked as a teacher for 10 years. In previous years, she had taught 3rd and 5th-grade. She has a B.A. in Liberal Studies, a Multiple Subject Teaching Credential, a CLAD certificate, GATE certification, an M.A. in Education with an emphasis in Educational Leadership, and an Administrative Credential. Cathy was also a first year Education Doctoral student in Teaching and Learning. Victor was a 7th-grade teacher and had worked as a teacher for 17 years. In previous years he has taught grades 6 through 10. Victor has a B.S. in Mathematics, a Single Subject Credential and a Bilingual Certificate in Language Development.

Table 3.3 summarizes the eight participating teachers that were interviewed as a part of this study. The table notes the cohort the teachers were in, teaching experience and education.

Table 3.3: Demographics of the eight participating teachers interviewed.

Name	Cohort	Currently Teaches	Has Taught	Years of Experience	Degree/ Credential
Rita	Primary	1 st / 2 nd grade combination	2 nd , 3 rd	18 years	B.A. in Liberal Studies Multiple Subject Credential. M.A. in Education.
Eve	Primary	1 st grade	1 st	18 years	B.A. in German, minor in Education Teaching Credential
Anya	Primary	Kindergarten	Preschool, kindergarten and 2 nd grade	30 years	Multiple Subject Credential M.A. Child Development CLAD
Sandy	Upper	3 rd grade	1 st	5 years	B.A. in Law & Society and Communication Clear Multiple Subject Credential M.A. in Elementary Education,
Kelly	Upper	6 th -grade	6 th	10 years	B.A. in Liberal Studies Multiple Subject Clear credential SDAIE. Ms. K has been trained in AB 466 HOLT training.
Tina	Upper	4 th -grade	Kindergarten, 1 st and 5 th -grade	9 years	B.A. Humanities Multiple Subject Credential National Board Certificate.
Cathy	Middle	6 th -grade	3 rd and 4 th -grade	10 years	B.A. Liberal Studies, Multiple Subject Credential CLAD certificate GATE certification M.A. in Education with an emphasis in Educational Leadership Administrative Credential.
Victor	Middle	7 th -grade	6 th -10 th -grade	17 years	B.S. Mathematics Single Subject Credential Bilingual Certification

Methodological Analysis

The essence of this study was to explore what practicing teachers gain from implicit and explicit modeling in mathematics teacher education courses. Because the assumption is that prospective and practicing teachers learn to teach from their own experiences as learners, my focus was on the perceptions and descriptions of

mathematics teacher educators' and practicing teachers' interpretations of classroom events. The research methods are both qualitative and quantitative in character.

In mathematics courses for practicing teachers, while the mathematics teacher educators' teaching actions are directly observable, the internal processes that govern these acts -- and what practicing teachers discern from them -- cannot be directly accessed. In order to understand aspects of mathematics teacher educator modeling in mathematics teacher education courses and the ways in which practicing teachers learn from these pedagogies, I draw upon the three-step-design methodology (Busse & Ferri 2003) and complementary accounts methodology (Clarke, 1997). Classrooms are complex social settings. The facets of the three-steps-design and the complementary accounts research methodologies I employed in this study provide for analytical techniques sensitive to the multifaceted and connected nature of classroom data.

The three-step-design methodology incorporates stimulated recall interview in conjunction with interview and observation to elicit perceptions and thoughts entertained while participating in activity. Events are video recorded (in some cases the audio-video recording device is head-mounted to capture visual and auditory cues attended to by the subject). Then, soon after the event, the recording is replayed for the subject to immerse them in the past event. At this step the participant is directed to freely express any recollections in response to the recording while analysis and evaluation by the researcher is suspended. Last, the subject is interviewed, to probe him or her about fundamental processes including errors and decision strategies as

he/she review statements or actions from steps 1 and 2 (Busse & Ferri, 2003; Miller, 2004).

In this study in both the mathematics teacher educator and teacher interviews I made use of the first step in the three-step design methodology and combine the second and third steps in a single interview. Since I seek to coordinate and make connections between mathematics teacher educators' and practicing teachers' accounts of shared experiences the stimulated recall interview (the second step) was coupled with a semi-structured interview protocol derived from the data collected prior to a given interview. For example, while Scott, a mathematics teacher educator in this study, was instructed to freely express any recollections in response to the recording, the interviewer at times interjected with questions. The questions in general were for clarification and to ask for further elaboration. There were also questions driven by comments Scott made in the pre-session interview, the researcher's fieldnotes of the session and the reflection survey completed by the teachers in Scott's class.

The complementary accounts methodology also employed in this research is intended to study learning in legitimate classroom settings while minimizing the need for researcher inference regarding participant thought processes and maximizing the richness of the research data base (Clarke, 1997). Like the three-step-design methodology, the complementary accounts methodology combines videotape and interview data to construct integrated data sets that reconstruct classroom events. The complementary accounts methodology, however, not only includes the reflective voice of the mathematics teacher educator, but also the reflective voice of the participant

students and observers. The analytical approach utilizes a research team with complementary but diverse areas of expertise thus enabling a multifaceted analysis of a common body of classroom data (Clarke, 1997).

Both methodologies make use of stimulated (video-cued) recall (Wanger, 1977), a form of retrospective reporting to collect verbal data. Commonly, it is used for investigating human cognitive processes in decision-making. The three-step-design allows for the combination of the observation of events, (video) stimulated recall to re-immense the subjects in the situation, and interview to get more information on the factors that contribute to the mathematics teacher educators' actions and statements and how practicing teachers perceive them. Specifically, the stimulated recall interviews were used to gather information about the mathematics teacher educator's thoughts and actions that underpin their pedagogical approach and to understand practicing teachers' perceptions of the mathematics teacher educator's pedagogy. The complementary accounts methodology allowed for analysis of complementary interpretations of the mathematics teacher educator's pedagogy from various perspectives. This integrated data set then provided the basis for complementary interpretations of the mathematics teacher educator's pedagogy from the perspective of the mathematics teacher educator and the practicing teachers, which can then be compared and contrasted to characterize the mathematics teacher educators' pedagogy from the various perspectives.

Data Corpus

The data set is made up of:

- 1) Audio recorded pre-sessions interviews with the three teacher educators.
- 2) Video of the six individual classroom sessions (two consecutive classes for each of the three cohorts) and the associated timelines.
- 3) Stimulated recall interviews (SRI) with the mathematics teacher educators (MTEs), one for each session.
- 4) Reflection surveys, given to all 49 practicing teachers in the three cohorts at the end of the first session.
- 5) Quantitative surveys, given to all 49 practicing teachers in the three cohorts at the end of the second session.
- 6) Semi-structured interviews with a subset of the practicing teachers, part of which will include stimulated recall interview.
- 7) Researcher fieldnotes of the six classroom session and interviews.

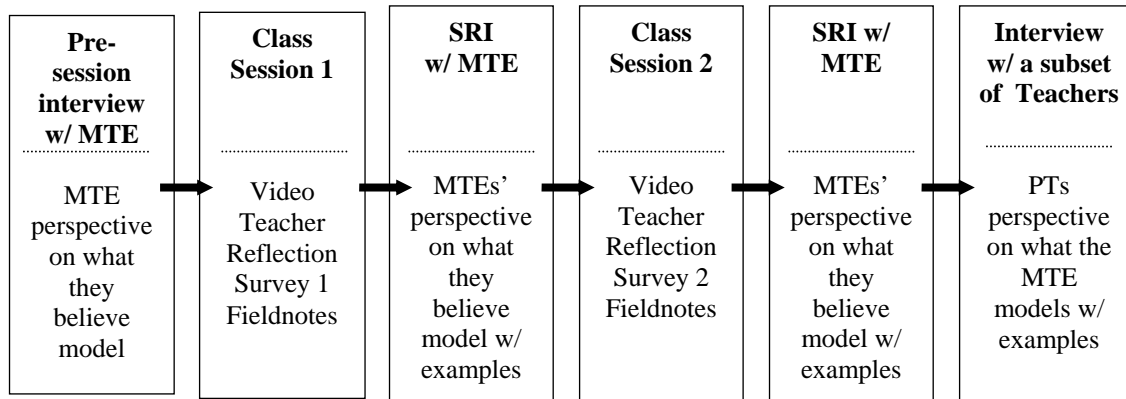


Figure 3.1: Sequence of data collection: The figure above illustrates the sequence of data collection for each of the three cohorts.

Before classroom data was collected for each cohort the mathematics teacher educators were interviewed regarding the lesson plan for the two sessions and the ways in which they feel they model mathematics teaching. The videotaped classroom data was collected in two consecutive classes for each of the three cohorts. The

researcher observed all sessions and took fieldnotes. From this data timelines were created to describe the progression of classroom events. This data, coupled with informal discussions with the mathematics teacher educators and other researchers from the university professional development group staff, informed the development of the second reflection survey (see Appendix D) and my observation of the class and thus analysis of the fieldnotes. After each session, the fieldnotes and video of the lesson were used to create a timeline of the classroom sessions (for an example of a timeline see Appendix G). The timelines were used in the stimulated recall interviews with the mathematics teacher educator's and the practicing teachers.

During the first session all of the practicing teachers responded to a written reflection survey, (for Reflection Survey 1 see Appendix C). This survey was intended to elicit information about what they had observed from the course in terms of modeling effective practice. At the end of the second session, the practicing teachers took another written reflection survey (for Reflection Survey 2 see Appendix D). This second survey was intended to collect demographic data and quantify teaching practice that they noticed in their classes. The pre-session interviews with the mathematics teacher educators, classroom data, and reflection surveys were used to refine the protocol for the semi-structured stimulated recall interview for the mathematics teacher educator and the practicing teachers. All interviews were videotaped and portions were transcribed. The instructor data was used to reconstruct what the mathematics teacher educator identified as the significant events in his or her classroom and to understand the mathematical and pedagogical reasons for observed

pedagogy. Moreover, this data allowed me to understand (1) what the mathematics teacher educators believe they model, and (2) how these instructional practices were modeled in the classroom session under review.

Individual interviews with eight of the practicing teachers from the class sessions (three from the primary elementary cohort and three from the upper elementary cohort and two from the middle school cohort) were conducted using semi-structured stimulated recall interview protocol regarding their perspectives on what their mathematics teacher educator models about effective practice. The interview data and the two reflections surveys allowed me to gather information about what participants note about teaching from instruction they had experienced and why.

The data allowed for analysis on two levels, (1) globally on modeling throughout the year of professional development and (2) locally on modeling in two consecutive class sessions. The pre-session interviews with the mathematics teacher educators, the reflection surveys and the individual interviews with eight practicing teachers provided data on perspectives on what the mathematics teacher educators modeled about practice in general. The stimulated recall interviews with the mathematics teacher educators and the practicing teachers provided data on specific events. Both levels were important to examine. Globally, with respect to the year, because though a specific instructional act may not have occurred in a particular session or one may not have noted certain aspects of instruction in a given event does not imply the action is not common or that it has not been noted elsewhere. Locally, with respect to two consecutive sessions, is important to examine because

interpretations of a specific event may vary. An analysis of both levels has the capacity to highlight how implicit and explicit modeling conveys teaching practices.

Analytical Framework

The NCTM *Professional Standards for Teaching Mathematics* (1991) served as a starting point for coding significant events identified by the mathematics teacher educators and the practicing teachers. The *Professional Standards for Teaching Mathematics* identifies six principles on which K-12 school mathematics should be grounded. The elements of teachers' work highlighted in the standards are based on emerging research that suggests such elements have a significant impact on the mathematics learning that takes place in a classroom. The teaching standards state that teachers of mathematics should:

- 1) Pose worthwhile mathematical tasks
- 2) Orchestrate class discourse
- 3) Promote student discourse
- 4) Encourage the use of tools to enhance discourse
- 5) Create a learning environment that fosters the development of each student's mathematical power
- 6) Engage in ongoing analysis of teaching and learning

(For an extended summary of the teaching standards see Appendix A.). The Standards are organized under 4 categories tasks (item 1), discourse (items 2-4), environment (item 5) and analysis (item 6). In general, the Standards suggest that teachers should pose worthwhile tasks, allow students to grapple with significant mathematical ideas

(both independently and collaboratively with peers), explore conjectures, orchestrate discourse and elicit student thinking and discussion.

Analysis

An initial analysis of the classroom videos and the interviews began with the NCTM *Professional Standards for Teaching Mathematics* with respect to the four categories: tasks, discourse, environment and analysis. Using a cyclical process for confirming and disconfirming evidence (Strauss & Corbin, 1990) the pre-session interview data and the classroom data (fieldnotes and classroom video) informed the elaboration of the four categories derived from the NCTM *Professional Standards for Teaching Mathematics*. This elaboration led to the emergence of the analytical framework used to describe the mathematics teacher educators' pedagogy.

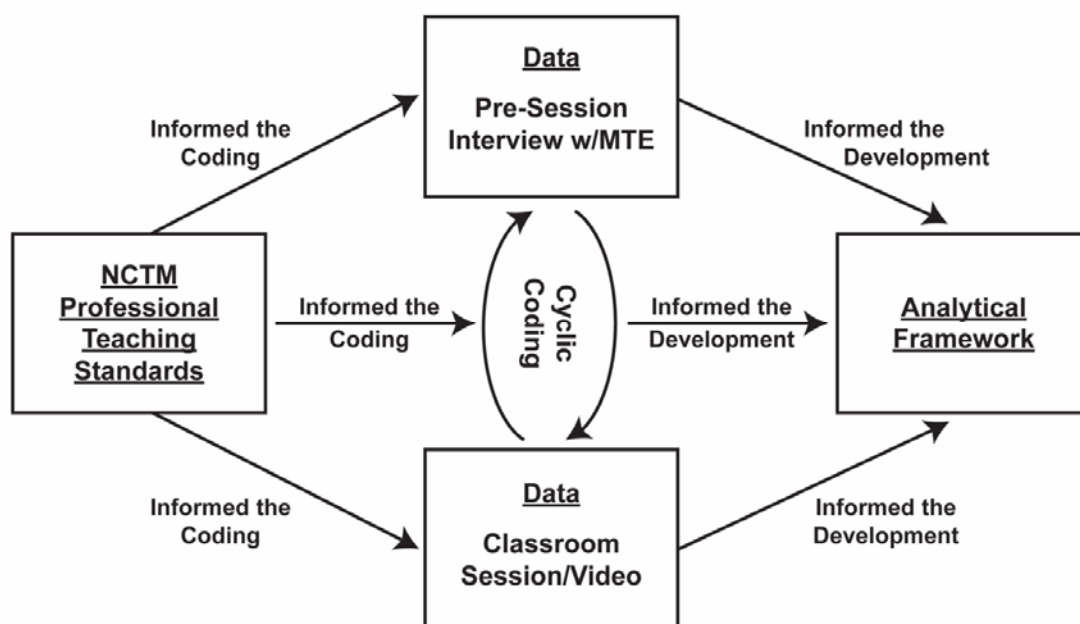


Figure 3.2: An illustration of how the analytical framework emerged. The arrows show what informed the analysis.

Once the emergent analytical framework proved stable, it was used to characterize (1) *events* as outlined in the timeline that describe the mathematics teacher educators' pedagogy in each of the six class sessions from the researcher's perspective, (2) the researcher's interpretations of what the mathematics teacher educators' and practicing teachers' *utterances* conveyed about classroom events in the interviews (pre-session, stimulated recall) and (3) the written utterances in the teacher Reflection Survey 1. Here events refer to the summary of utterances (verbal and non verbal), and a set of events (one or more events) is an episode. The analysis of the timelines led to the development of an integrated data set from which complementary accounts of the classroom events were analyzed. To create full integrated data sets of each of the cohort, I drew on approximately:

- 1) Twelve hours of classroom video data for the two sessions (three cohorts, two hours per session, two sessions).
- 2) Ten hours of video of stimulated recall interviews with the mathematics teacher educators (three mathematics teacher educators, two interviews each, each interview about 1 hour 40 minutes).
- 3) Eight hours of video of the interviews with the practicing (eight teachers 1onehour each).

Episode 3.1, below, is a portion of the integrated data set from the first session of the upper elementary cohort is provided to illustrate one aspect of aligning perspectives. The episodes are displayed in a table with time in the first column to indicate time in class relative to the start and elapsed time. The strings of letters ending with a number reflect the coding of the event from the various perspectives. Under the Researcher column, I coded the events from my perspective. Under the MTE-Max and Kelly columns, I coded the utterance the mathematics teacher educator and teacher made

with respect to the event. The meaning of the code will be elaborated on in Chapter 4.

(For a summary of the codes and their associated meaning see Appendix B).

Episode 3.1: Upper Elementary Cohort, Session 1.

Time h/m/s	Researcher	MTE-Max	Kelly	Description of events	MTE Utterances	Teacher Utterances
0.47.34	SLE3 SCD5			Mr. B from group 5 asks for clarification of the question. MTE-Max clarifies that he is asking if it represents $\frac{1}{2} \times \frac{1}{3}$. There are murmurs of discussion		
0.48.01	SCD4 SLE3	SCD4		MTE-Max asks Heather is she can repeat to the class what she is discussing. Heather shares her thinking. MTE-Max asks the teachers to comment on Heather's thoughts. No one responds.	48 – I wanted them to take a moment and really think about what she said.	
0.48.15	SMA2 STD1 SCD1 SOA1 SLE1 SLE2	SMA2 STD1 SCD1 SOA1 SLE1 SLE2 SOA2 SOA4	SMA2 STD1 SCD1 SLE2 SOA4	MTE-Max asks them to consider the meaning of the $\frac{1}{2}$ and the $\frac{1}{3}$ in both drawings. He guides them in trying to think through Sandy's work and discussing the referent unit of each of the numbers sentence Sandy has drawn. And then ask the teachers to discuss the drawing in their groups to convince themselves. As the groups discuss MTE-Max walks around and listens in on and discusses with the groups	44 – There is so much mathematics and representation in Sandy's thinking so I want to give them time to think about it. 49 – I am asking questions to check for understanding and listening for that "choral" response, I am not getting it so I ask them to discuss in groups, during this time they get to talk to each other and I get to hear where they are struggling	37 – Kelly: He asks the teachers to discuss the drawing in our groups, then in whole class. The discussion with peers, it is team work the process helps us work it out, then he brings it together

As the researcher, I was able to examine and re-examine events in the timeline for as long as I needed. In the stimulated recall interviews the mathematics teacher educators examined and commented on the two-hour sessions in interviews that lasted about 1 hour 40 minutes, and the practicing teachers examined and commented on the four hours of classroom video data in their hour long interviews. The timelines were given to the mathematics teacher educators and the practicing teacher to use as a reference to aid them in pointing out instances in the video they found to be noteworthy. In Chapter 4, I will elaborate on the similarities and differences in interpretations of classroom events.

In this study, the stimulated recall interviews were used to gather information about the mathematics teacher educator's thoughts and actions that underpin his or her pedagogical approach. The stimulated recall interviews for the practicing teachers were used to understand their perceptions of the mathematics teacher educators' pedagogy. And the teacher reflection surveys were to gain further insight on what the participants notice about the mathematics teacher educators' pedagogy.

The key to the validity of approach described here is to ground practicing teachers' accounts of the mathematics teacher educators' pedagogy, including thoughts, motivations, and construed meanings, in a videotape record of shared classroom events and to supplement the practicing teachers' account with an associated data base of other practicing teachers', the mathematics teacher educator's account, researcher fieldnotes, and descriptions of sessions.

The analysis was done on two levels: (1) within each class, I compiled an integrated account of the interpretations of the mathematics teacher educator's pedagogy from the perspective of the mathematics teacher educator and the practicing teachers to categorize perspectives on shared events and (2) between classes, among the mathematics teacher educators and then among the practicing teachers. Reports of the mathematics teacher educators' pedagogy as gleaned from the interviews, surveys and classroom observation data were identified and categorized. The mathematics teacher educators' perspectives provided insight into both what they believe they model in instruction and the internal processes that govern their actions. The practicing teachers' perspective reflected what they discern about teaching and learning mathematics from their learning experience in the mathematics courses for practicing teachers and mathematics professional development.

The pre-session interviews and classroom data were analyzed in a cyclical process of coding and search for confirming and disconfirming evidence (Strauss & Corbin, 1990) to characterize the mathematics teacher educators' pedagogy. This process allowed for the delineation of the NCTM *Professional Standards for Teaching Mathematics* (1991) and the emergence of parallel categories for describing the mathematics teacher educators' pedagogy. The parallel categories characterize the mathematics teacher educators' pedagogy as the participants are engaged as mathematics learners and as mathematics teachers.

In various stages of the development of the analytical framework two other researchers coded portions of the class session and interview data. After a few cycles

of training and refining the codes there was about 80% agreement on coding portions of the class session as described in the timeline and transcript of interviews. The main source of discrepancy of the coding related to subtleties in the subcategories. Also, at times, the other two researchers characterized events or utterances with fewer codes. However upon review of the video data and discussion most differences were reconciled.

The emergent code was used to categorize and coordinate the mathematics teacher educators' and the practicing teachers' perceptions of classroom events. Analysis of pre-session interviews, classroom video, session timelines, fieldnotes, and audio recordings and the stimulated recall interviews culminated both within and across classes to identify characteristics of the ways the perspectives of the mathematics teacher educators' and the participants perspectives are aligned.

Incorporating facets of three-step design methodology and complementary accounts methodology in the data collection and analysis process, classroom videotape analysis and semi-structured stimulated recall interview combine to reciprocally validate and illuminate perceptions of the mathematics teacher educator and the practicing teachers. This design suited this study well because observation, stimulated recall, or interview alone is insufficient to make inferences about the interaction of internal and external processes that govern pedagogy and what participants gain from those acts. Observation allows pedagogy to be directly observed, however the internal processes that govern these acts and what participants discern from them cannot be directly accessed.

Concurrent interview methods are impractical (and inappropriate) to gain such insight since probing the instructors' about reasons underlying their actions during a class session would interfere with the flow of the class session. Retrospective reporting after the class session has the risk that the participant will forget details and that their recall will be interpretively filtered. Miller (2004) notes that self-reports risk distortion due to subjective biases such as perceptual and motivational states may be omitted from recall, a reluctance to negatively assess ones self, and the fact that recall is subject to hindsight reflection. However observation and interview combined with stimulated recall helps to avoid these pitfalls by using video to help the participant recall details of their experiences and avoid selective interpretation. It re-immerses subjects in an event to stimulate more representative recollections with less self-defensive justification and editing than memory alone. And the complementary account allows for triangulation of characterizations of the mathematics teacher educators' pedagogy to be categorized from various perspectives. In this study, stimulated recall methods provide a way of being close to the classroom session without disrupting the progression of the class.

The table below summarizes the data and the research questions the data addresses.

Table 3.4: Research questions and data sources.

Research Questions	Data Sources
(1) What do mathematics teacher educators believe they model about effective instructional practice?	- Pre-Session Interviews - SRIs after each class session
(2) What do practicing teachers notice about the mathematics teacher educators' pedagogy and identify as effective mathematics teaching?	- Reflection Surveys - SRIs with a 8 teachers from the cohort
3) In what ways do these perspectives align in mathematics courses for practicing teachers?	- Integrated data set created from analysis of SRI - Teacher Reflection Surveys and Pre-Session Interviews

In Chapter 4, I report on the emergent framework for coding the data derived from the NCTM *Professional Standards for Teaching Mathematics* (1991). The re-defined and elaborated categories discussed in Chapter 4 provide insight into what mathematics teacher educators believe they model and what practicing teachers notice about the mathematics teacher educators' pedagogy. I then report on the way in which my research questions were answered.

CHAPTER 4: RESULTS

Starting with the notion that teachers learn about the practice of teaching from the instruction they experience, I examined modeling in mathematics teacher education for practicing teachers. In this study I examined the instructional and pedagogical implications of instructor modeling in three mathematics teacher education courses by unpacking the notion of modeling in mathematics teacher education. In part 1 of the results, I discuss the emergence of a framework used to characterize facets of the pedagogy that is modeled by mathematics teacher educators, in order to unpack the notion of modeling in this chapter. Then in part 2 of the results, I discuss what practicing teachers gain from mathematics teacher educators' modeling in mathematics teacher education courses, with respect to the analytical framework, by comparing and contrasting the mathematics teacher educators' and the practicing teachers' perspectives on what is modeled in the mathematics teacher education courses.

The episode describe in Chapter 1 suggests that there may be some discrepancies in the motivation for a given instructional move, and thus also in interpretations of an instructional act from the perspectives of the participants in a classroom situation. In particular, what the mathematics teacher educator intends to model about the practice of teaching for the teachers in his or her class may not be conveyed to the teachers. In this chapter I shed light on (1) *What do mathematics teacher educators believe they model?* (2) *What do practicing teachers notice about the mathematics teacher educators' pedagogy and identify as effective mathematics*

teaching? and (3) *In what ways are these perspectives aligned in mathematics courses for practicing teachers?*

Part 1: The Emergent Analytical Framework

The NCTM *Professional Standards for Teaching Mathematics* (1991) served as a lens for describing the accounts of the mathematics teacher educators' mathematics pedagogy. The *Professional Standards for Teaching Mathematics* (1991) was a suitable starting point because the university professional development group that coordinates the teacher education programs from which the three cohorts in this study derive, has a well-articulated mission anchored in student learning of core disciplines and skills that is consistent with research and reform recommendations. The *Professional Standards for Teaching Mathematics* expresses the vision of the National Council of Teachers of Mathematics (NCTM, 1991) for teachers who are well prepared to teach mathematics. This vision highlights the importance of teachers ability to (1) select tasks that engage students' intellect and deepen students' understanding, (2) orchestrate mathematical discourse, (3) use technology and tools to pursue mathematical investigations, (4) make connections to previous or developing knowledge, and (5) guide individual, small group and whole class work. (For a summary that further elaborates the NCTM *Professional Standards for Teaching Mathematics* see Appendix A.) In general, the Standards advocate a shift in the mathematics classroom environment from an emphasis on mathematics as an individual pursuit that privileges the memorization of algorithms and procedures to an emphasis on mathematics as a collaborative endeavor among members of the

classroom community in which collaborative activity, logical reasoning and argumentation are used to solve problems.

In what follows I redefine and elaborate on the NCTM *Professional Standards for Teaching Mathematics* (1991) as they apply to mathematics teacher education with respect to teachers' engagement as both learners of mathematics and as mathematics teachers in mathematics courses for practicing teachers.

An initial analysis of the classroom videos and the interviews revealed that the NCTM *Professional Standards for Teaching Mathematics* in its current form were insufficient to describe all of the classroom events. The primary reason was that using the NCTM *Professional Standards for Teaching Mathematics* as a lens for categorizing interactions between the mathematics teacher educators and the teachers only allowed for coding instances of the mathematics teacher educator modeling mathematics teaching as the participants are engaged as learners. It became evident that in the mathematics courses for practicing teachers in this study, the teachers were engaged on two levels: (1) as learners of mathematics in a classroom community and (2) as teachers, that is, professionals in a community of mathematics teachers. From the perspectives of the researcher and the mathematics teacher educator, the mathematics teacher educators modeled both mathematics instruction and the work of mathematics teachers. In the next section, I discuss the emergence of a parallel framework for describing the teachers' engagement as professionals in mathematics teacher education.

The NCTM *Professional Standards for Teaching Mathematics* (1991) describe six standards organized under four categories: tasks, discourse, environment, and

analysis. Analysis of the pre-session interviews and classroom data began with these four categories and were coded in a cyclical process for confirming and disconfirming evidence (Strauss & Corbin, 1990) to elaborate on and redefine the NCTM *Professional Standards for Teaching Mathematics* so that they apply to mathematics teacher education. Cyclic coding and analysis of the pre-session interview data and the classroom data (fieldnotes and classroom video) led to a series of refinements of the NCTM categories used to describe the mathematics teacher educator pedagogy.

A refinement of the categories drawn from the standards used to examine the data was needed to reflect that the students in the sessions are, in fact, practicing teachers; the facilitators of the sessions are mathematics teacher educators. Henceforth, the participants in the mathematics teacher education sessions in this study are referred to as either participants or teachers, the facilitators as mathematics teacher educators, and the term student is reserved for the children whom the teachers teach.

The data suggested a distinct category for tools. In the NCTM *Professional Standards for Teaching Mathematics* tools are in the category of classroom discourse. While tools often mediate discourse and have the capacity to enhance discussion, in reform instruction tools are also used in problem-solving, to make sense of the mathematics. The use of tools to solve problems and mediate discussion appeared to be noteworthy in the data beyond the category of discourse alone and thus called out as a new category. The five categories used to characterize the mathematics teacher educators pedagogy in this study are:

- 1) Worthwhile Tasks

- 2) Discourse
- 3) Tools
- 4) Learning Environment
- 5) Ongoing Analysis

In this research I use the term discourse to describe the mathematics teacher educators' discursive moves, that is, the way the mathematics teacher educator orchestrates mathematical discussion in whole class and groups settings. The term tools describe objects of reference (tangible or intangible), such as manipulatives, diagrams, written work or hypotheses and so on, used in mathematical conversations and problem-solving. Later in this chapter, the categories will be elaborated.

While the NCTM *Professional Standards for Teaching Mathematics* required very little adaptation for categorizing the actions of the mathematics teacher educator as the teachers were engaged as learners of mathematics, instances where the participants were engaged as mathematics teachers were not captured with these descriptions alone. Thus, parallel categories for mathematics teachers engaging in the collective inquiry process facilitated by the teacher educator emerged from the data. The mathematics teacher educators modeled the collective inquiry process of a professional learning community of mathematics teachers by engaging teachers in activities that are a part of the practice of teaching related to lesson planning, thinking about student thinking, understanding and learning, evaluating and so on. In general, analysis of the pre-session interviews and class sessions suggests that the mathematics teacher educators model the collective inquiry process of a professional learning

community of mathematics teachers by engaging teachers in activities that are a part of the practice of teaching mathematics. In the mathematics teacher education programs in this study, these activities were mediated by the mathematics teacher educator and provided opportunities for teachers to increase their participation in the collective inquiry process in a community of mathematics teachers.

The categories, tasks, discourse, tools, environment and analysis in the analytical framework describe ways in which mathematics teacher educators model facets of the mathematics teaching profession with respect to the two levels of engagement. The constructs of perceptual lived experience, intent participation, apprenticeship and legitimate peripheral participation situates opportunities to learn about reform mathematics pedagogy and participation structures from the teachers' experience. The constructs also situate teachers' enculturation into a professional learning community of mathematics teachers by engaging teachers in facets of the Collective Inquiry Process. Table 4.1 below summarizes the emergent analytical framework under the headings of tasks, discourse, tools, environment and ongoing analysis with respect to the two levels of engagement, as learners (left column) and as professionals (right column).

Table 4.1: Summary of analytical framework.

	Classroom Community of Mathematics Learners	Community of Mathematics Teachers
Tasks	Worthwhile Tasks are the projects, questions, problems, constructions, applications, and exercises in which teachers engage intended to develop teachers' mathematical content knowledge.	Tasks are the projects, questions, problems, constructions, applications, and exercises in which teachers engage intended to develop the teachers' understanding of the students' mathematics.
Discourse	Classroom Discourse refers to the ways that mathematics teacher educators mediate discourse about mathematical ideas to focus discussion on concepts and solution paths instead of answers.	Classroom Discourse refers to the ways that mathematics teacher educators mediate discourse about practice to focus discussion on teaching, student learning and/or thinking.
Tools	Tools are objects, tangible or intangible that the MTE and/or teacher's use in learning situations to reason, make connections, solve problems, communicate and enhance discourse.	Tools are objects, tangible or intangible that the MTE and/or teachers use to reason or enhance learning about or discussion of lesson planning, student learning, thinking and understanding, and so on.
Environment	Environment represents the setting for which the development of each teachers mathematical power is fostered.	Environment represents the setting for which the development of each teacher's knowledge for teaching is fostered.
Analysis	Ongoing Analysis is the systematic reflection in which mathematics teacher educators engage. It entails the ongoing monitoring of classroom life-how well the tasks, discourse, and environment foster the development of every student's mathematical literacy and power. Through this process, teachers examine relationships between what they and their students are doing and what students are learning.	Ongoing Analysis is the systematic reflection It entails the ongoing monitoring of classroom life-how well the tasks, discourse, and environment. The mathematics teacher educators engage in such analysis to foster teachers' participation in the Collective Inquiry Process of a professional learning community of mathematics teachers. The teachers and the mathematics teacher educators engage in this analysis to gather information about students to assess what they are learning, examine the effects of the tasks, discourse, and learning environment on students' mathematical knowledge, skills, and dispositions and consider implications for instruction.

The distinction between the parallel categories is subtle. It is important to note that, in general, mathematics teacher educators' teaching actions are multi-faceted and

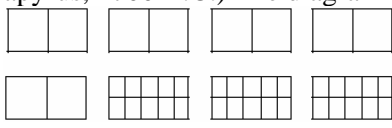
do not often fall rigidly under one category or another. Instead, classroom events can be categorized under a variety of headings to best describe the given instance. That is, teaching actions may fall under a number of categories both within and across levels of engagement. For example, a mathematics teacher educator might pose a group task that both motivates the development of mathematical ideas and making sense of the mathematics of students. In what follows, I elaborate on the way mathematics teacher educators model tasks, discourse, tools, environment and analysis with respect to the two levels of engagement and provide examples to highlight the differences. In the next section I summarize segments of the class in episodes. These episodes are displayed in a table with time in the first column to indicate time in class relative to the start and elapsed time. The descriptions below highlight one facet at a time to draw attention to features of each of the categories in turn with an example.

Tasks

When teachers are engaged as learners, tasks refer to activities that are intended to develop teachers' mathematical content knowledge. Such tasks generally, (1) motivate the development of mathematical ideas through exploration, (2) problematize the mathematics to promote inquiry, call for problem formulation or problem solving, (3) stimulate teachers to make connections between mathematical ideas, and/or (4) promote discussion about mathematics. To illustrate the distinction between modeling the use of worthwhile tasks as the participants are engaged as learners and as teachers, consider the task in the following Episodes 4.1 thru 4.3 in turn and their respective descriptions.

Episode 4.1 begins with a review of the questions from the teachers' homework that is due later on in the session. The question illustrated in figure 4.1, is the topic of discussion for this episode.

Eight loaves of bread are to be shared equally among 10 men. How might this be done? (This problem is from the Rhind papyrus, 1700 B.C.) The diagram ...suggests one way



Each man gets $\frac{1}{2} + \frac{3}{10}$.

- What part of a loaf does each man get in all?
- How else might the sharing be carried out?
- Does each person get the same total amount, in each situation? Justify that each person gets the same amount in each of your cuttings.

(Sowder, J., Sowder L. & Nickerson, 2007).

Figure 4.1: A homework question from the Primary Cohort, Session 1.

Episode 4.1: Primary Elementary Cohort, Session 1:

Time h/m/s	Description
0.04.28	MTE-Scott asked the teachers to consider homework question 2 (see figure 4.1 above) MTE-Scott pointed the teachers to the solution given in the book and placed it on the document camera and said "each person gets a half and 3/10" and said that there are many ways to think about how to get that. He pointed out this that this problem situation represents a sharing problem. MTE-Scott asked the teachers what part of each loaf does each person get?
0.05.40	After a pause, various teachers responded 8/10.
0.06.07	MTE-Scott asked if anyone was willing to share his or her solution, how they arrived at the solution 8/10. No one volunteered. MTE-Scott asked the teachers to take a few minutes to share their responses with their groups and said "then there is always someone who will volunteer someone else's really good paper" The groups discussed. MTE-Scott walks about the room and listens in on the groups' conversations.

The homework item discussed in Episode 4.1 exemplifies some ways that MTE-Scott models the use of mathematical tasks intended to develop teachers' mathematical content knowledge in two main ways. First, the work that the teachers

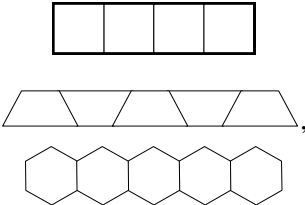
did prior to coming to class required that they problem-solve and draw upon previously learned ideas like meanings for fractions, part-whole and division, and fraction addition, discussed in the previous session. Second, during the discussion of the homework item, MTE-Scott problematized and promoted inquiry into the task by prompting the teachers to rethink their solution, explain it to others and make sense of the solution paths of their peers.

In the pre-session interview MTE-Scott noted that he typically would spend at least the first half hour to 40 minutes of the class session discussing the homework. He further noted that while this may seem like a long time to spend on homework in a 3-hour class session, homework questions were chosen to serve as a starting point for discussion about the mathematical ideas of the upcoming lesson. MTE-Scott reports, “Homework itself is less about practicing something they did and more about setting them up for something that I want to do with them later on.” MTE-Scott notes that he consciously chooses homework questions that require the teachers to go beyond what they discussed in the previous session, thus providing for exploration, inquiry and problem solving, and then in class discussion about the mathematics and the understanding that came from them.

Now consider Episode 4.2 below. In this session with MTE-Karla, the teachers have brought in student work drawn from a predetermined task that all of the teachers in the class tried with their own students, called “try-ons” in this context. The “try-ons” are generally driven by mathematical activity the teachers themselves have

engaged in and discussed during a previous session. This particular try-on was a “develop a rule for a pattern” problem. (See Figure 4.2.)

A banquet hall has a huge supply of various shaped tables (square, trapezoidal and hexagonal). Only one person can sit on each side of a table, except the longest side of the trapezoid table, which can seat two people. The same shape tables must be used for each banquet. The banquet rooms are long and narrow, so the tables can only be put together as shown below



For a given table shape, develop a rule or formula for the number of people that can be seated at 1 table, 2 tables, 5 tables, 100 tables, or n tables.
(Adapted from Burns 1992)

Figure 4.2: Try-on task from the Middle School Cohort Session 1.

Episode 4.2: Middle School Cohort, Session 1

Time h/m/s	Description
0.10.00	Karla began the class by asking the teachers to discuss in their groups the student work that they brought from their own students and choose a few examples that they thought would be interesting to share with the class. She noted that a student’s solution might be interesting to share if it was representative of many students’ responses or if it stood out from the rest in some way. As the groups discussed their students’ work, Karla walked about the room, listened in on the groups’ conversations and briefly joined the discussions of each group in turn. Much of the discussion in groups focused on trying to understand what the students were thinking, the reasonableness of the students’ approach, common approaches the students took, etc. After a few more minutes Karla brought the class back together so the teachers could discuss the task as a class.

The try-on task in Episode 4.2 was derived from work the teachers had done in a previous session. While the task has the capacity to develop the teachers’ understanding of functional and recursive relationships, I argue that the primary goal was to develop teachers’ pedagogical content knowledge since the task as framed for this session was for the teachers to reflect on student work. In Episode 4.1, the focus

was for the teachers to make sense of sharing division. In contrast, during Episode 4.2 the mathematics teacher educator modeled the coordinated action and analysis of the collective inquiry process. This task and tasks like it provide participants the opportunity to participate in authentic activity of mathematics teachers. In particular, this task, and other tasks that fall under this categorization have the capacity to develop teachers' knowledge about mathematics pedagogy generally these tasks:

- 1) motivate the development of the teachers' understanding of the students' mathematics through exploration and/or experimentation with students
- 2) problematize the students' mathematics to promote inquiry, call for problem formulation, problem-solving, and mathematical reasoning
- 3) stimulate teachers to discuss and make connections between the math they themselves are learning and teaching and/or
- 4) promote discussion about students' mathematics.

The teachers in the elementary cohorts engaged in similar work. In the second observed session of the primary cohort, the teachers analyzed student work from an interview with a student as he divided fractions. The teachers in the upper elementary cohort were to analyze the work of their peers in the first session. In the observed session in all three cohorts, there were instances the task required the teachers to interpret the student's work and thinking to develop their pedagogical content knowledge.

Now we will consider Episode 4.3 below from the upper elementary cohort. In this episode MTE-Max prompts the teachers to discuss a set of word problems intended to represent the number sentence "one half minus one-third".

Episode 4.3: Upper Elementary Cohort, Session 1

Time h/m/s	Description of Events
0.00.10	MTE-Max has just returned the homework and asked them to look at problem #7. He explained to the class that for their first activity that he has chosen 5 word problems intended to represent the number sentence one half minus one-third $\frac{1}{2} - \frac{1}{3}$ that various teachers in the class gave. He noted that the five responses are representative of the type of answers the class gave in general, some of which are correct and incorrect. MTE-Max asked the teachers to examine and think about how others thought about the task in their groups. He noted that an important aspect of understanding is the ability to think about how someone else thought about solving. The Teachers begin to work.
0.02.40	MTE-Max walks around and passes out the pink sheet. The teachers start to work on the activity in their groups as MTE-Max walked about the room listened in on the groups
0.04.07	MTE-Max asked the teachers to take some time and think about and rethink the original task since they have not met or thought about this assignment for almost 2 months
0.06.26	The teachers began to work independently at first then started to discuss in their groups (there are about 2-4 teachers in each group)
0.07.14	MTE-Max listened in and discussed with the groups as they work and took notes.

The task in Episode 4.3, I argue, has the capacity to develop teacher knowledge about both content and pedagogy. Having to consider the work of their peers and reconsider their own work regarding which word problem correctly represent the subtraction $\frac{1}{2} - \frac{1}{3}$ and which do not, can help teachers see the kinds of errors students make in reasoning about word problems, and this requires the teachers to re-evaluate what they know about subtraction in context. The mathematics in the word problems given in the activity problematizes the mathematics both for them as teachers analyzing student work and as learners making sense of the context in a word problem. Thus, the task in Episode 4.3 is categorized with respect to both teachers engagement as learners and professionals.

Discourse

When teachers are engaged as learners classroom discourse refers to the orchestration of mathematical discussion mediated by the mathematics teacher educator. This mediation is intended to focus discussion on ideas, concepts and solution paths. The mathematics teacher educator can orchestrate such discussion by:

- 1) directing the teachers to work or discuss in small groups,
- 2) asking questions that guide or shape thinking, promote inquiry or challenge the teachers' thinking,
- 3) prompting teachers to share solutions, provide justifications or explanations,
- 4) using teachers' contributions or work to shape discussion, or
- 5) providing information to add to, or move mathematical discussion along.

Consider Episode 4.4 below from the primary cohort. This is a continuation of Episode 4.1. Recall that Episode 4.1 ended with MTE-Scott, the primary elementary school cohort facilitator, asking the teachers to discuss in their groups their solution paths for the equal sharing problem prior to a whole class discussion. Episode 4.4 takes place after the teachers have had some time to discuss in small groups. A large segment of the class is provided to situate the discussion piece of interest that begins at 0.11.19.

Episode 4.4: Primary Elementary Cohort, Session 1

Time h/m/s	Description of Events
0.08.00	After a few moments, MTE-Scott brought the class back together to discuss. MTE-Scott asked someone to share his or her solution. A teacher, Ms. A, volunteered to share her solution with the class. MTE-Scott put her solution on the document camera and Ms. A explained that she cut each loaf into 10 pieces so each person got $\frac{8}{10}$.

Episode 4.4 continued: Primary Elementary Cohort, Session 1

Time h/m/s	Description of Events
0.08.53	MTE-Scott asked for someone else to explain Ms. A's thinking. After a pause MTE-Scott noted that the way we model problem situations can shape ones understanding. Then asked if anyone can describe Ms A's thinking from her model. A teacher, Ms. B, explained Ms. A's solution and said each loaf is cut into 10 pieces and shared 8 times
0.09.50	MTE-Scott rephrased Ms. B's contribution as a question and with the help of the teachers further elaborated on Ms. A's solution
0.10.27	MTE-Scott asked for a different solution. Ms. C offered her solution and said that she wished that she could have seen Ms. A's solution before she solved it because she thought Ms. A's solution explained it easily. MTE-Scott said that it is ok to have different ways of seeing it. MTE-Scott put Ms. C's solution on the document camera and she explained her work. She said that she knew the answer was $\frac{4}{5}$ so she cut the loaves into 5 pieces and gave each person 4, so $\frac{4}{10}$
0.11.19	MTE-Scott asked the class if they understood Ms. C's answer. One teacher noted that she did it the same way and does not know why. Another teacher added that she did solved in a similar way as well but only considered 4 loaves and that she knew that the $\frac{2}{5}$ should be doubled so $\frac{4}{5}$ but also didn't recall why she did it that way. MTE-Scott rephrased and summarized the teachers contributions, "to see if he understood their solutions" and used the work on the document camera to exemplify his summary, covering half of the "loaves" and then asked the class how is it that this works?
0.12.31	Ms. S explains that it is because it is half of the loaves. Another teacher offered that two loaves could be the bigger unit. MTE-Scott added that experience re-unitizing may help us. Then he asked if they looked at it as $\frac{8}{10}$ being $\frac{4}{5}$ then doing it or did you decide to deal with just half of it first?
0.13.31	MTE-Scott asked Maria to share her answer and Maria placed her solution on the document camera, Maria explained that she did not really understand, but knew each person should get at least a half, then she cut the remainder of the loaves in to fourths and was left with a half of a loaf. So she cut that half into 10 pieces that she originally called tenths, but after check by adding the $\frac{1}{4}$, $\frac{1}{4}$, $\frac{1}{4}$ and $\frac{1}{10}$ realized that those pieces were twentieths. MTE-Scott pointed out that that was because the half was cut into tenths but the whole would be cut into twentieths.
0.15.15	MTE-Scott compared Maria's solution with the example. He pointed out a difference and asked the class why Maria's solution still works
0.16.00	Ms. G explained that because everyone gets the same amount. MTE-Scott reiterated Ms. G's response and asked how do they come to get that same amount?
0.16.14	One teacher explained how Maria added the pieces. Pointing to Maria's picture, MTE-Scott reiterated explaining that everyone gets 2 fourths or a half, then there is another round so everyone gets more, then we end up with $\frac{1}{2}$ of a loaf and 10 people to share it, and suggested that maybe that is where the twentieths come in.
0.17.10	After a pause, he returned Maria's paper to her and thanked her. And then discussed how there are many ways to think about this and some are more or less elaborate than others.

In Episode 4.4, MTE-Scott modeled the use of discursive moves to orchestrate mathematical discussion. He prompted the teachers to discuss their solutions in their groups, and then when it was time for the teacher to share out with the whole class, he pressed the teachers for elaborations and justifications of their responses with questions. For example, he asked why Ms. C's alternate solution works as well (0.11.19) and when MTE-Scott compared Maria's solution with the example and pointed out a difference, he then asked the class why Maria's solution still works (0.15.15). In the Episode MTE-Scott also highlighted the contributions made by the teachers by repeating or summarizing them or requesting that someone else do so.

In the pre-session interview MTE-Scott reported that he chooses homework questions that lend themselves to having mathematical discussions “as things unfold.” He noted that “questioning is a big part of what we do... they know part of it is that they are not going to get an automatic answer from me. It's going to be another question that makes them think about, you know, clarify their own question or at least get them to think about something”. This really highlights what teachers might note about the norms of discourse in the mathematics class simply by engaging in the class as a learner.

Mathematics teacher educators also mediate classroom discourse about practice to model teachers' participation in the community of teachers in the public reflection and sharing out phases of the collective inquiry process. Just as when teachers are engaged as learners, the MTE can (1) make use of small group discussion, (2) ask questions that guide or shape thinking and promote inquiry, (3) prompt sharing out, (4)

use teachers' contributions to shape discussions and (5) provide additional information; all of which are intended to enhance discussion about practice. The distinction here is that the discourse is intended to focus discussion on teaching, student learning or student thinking and not so much the teachers' mathematical development.

Consider Episode 4.5 below. Recall Episode 4.2 ended with MTE-Karla, the middle school cohort facilitator, bringing the teachers together to discuss the interesting solutions their students had given for the banquet hall problem.

Episode 4. 5: Middle School Cohort, Session 1

Time h/m/s	Description																																				
0.12.22	<p>The class began to discuss student work related to using tables to determine linear relationships. One teacher, Ms. A, gave a student's work to MTE-Karla to be placed on the document camera. MTE-Karla asked Ms. A what she could tell the class about the work. (See Figure 4.4 below)</p> <div data-bbox="618 997 1227 1371" style="border: 1px solid black; padding: 10px; margin: 10px auto; width: fit-content;"> <p style="text-align: center;">Square</p> <table style="width: 100%; border-collapse: collapse;"> <tr> <td style="border-right: 1px solid black; padding: 2px 5px;">Tables</td> <td style="padding: 2px 5px;">1</td> <td style="border-right: 1px solid black; padding: 2px 5px;">2</td> <td style="padding: 2px 5px;">5</td> <td style="border-right: 1px solid black; padding: 2px 5px;">100</td> <td style="padding: 2px 5px;">n tables</td> </tr> <tr> <td style="border-right: 1px solid black; padding: 2px 5px;">People</td> <td style="padding: 2px 5px;">4</td> <td style="border-right: 1px solid black; padding: 2px 5px;">6</td> <td style="padding: 2px 5px;">12</td> <td style="border-right: 1px solid black; padding: 2px 5px;">202</td> <td style="padding: 2px 5px;">Add 2 each time</td> </tr> </table> <p style="text-align: center;">Trapezoid</p> <table style="width: 100%; border-collapse: collapse;"> <tr> <td style="border-right: 1px solid black; padding: 2px 5px;">Tables</td> <td style="padding: 2px 5px;">1</td> <td style="border-right: 1px solid black; padding: 2px 5px;">2</td> <td style="padding: 2px 5px;">5</td> <td style="border-right: 1px solid black; padding: 2px 5px;">100</td> <td style="padding: 2px 5px;">n tables</td> </tr> <tr> <td style="border-right: 1px solid black; padding: 2px 5px;">People</td> <td style="padding: 2px 5px;">5</td> <td style="border-right: 1px solid black; padding: 2px 5px;">8</td> <td style="padding: 2px 5px;">17</td> <td style="border-right: 1px solid black; padding: 2px 5px;">302</td> <td style="padding: 2px 5px;">Add 3 each time</td> </tr> </table> <p style="text-align: center;">Hexagon</p> <table style="width: 100%; border-collapse: collapse;"> <tr> <td style="border-right: 1px solid black; padding: 2px 5px;">tables</td> <td style="padding: 2px 5px;">1</td> <td style="border-right: 1px solid black; padding: 2px 5px;">2</td> <td style="padding: 2px 5px;">5</td> <td style="border-right: 1px solid black; padding: 2px 5px;">100</td> <td style="padding: 2px 5px;">n tables</td> </tr> <tr> <td style="border-right: 1px solid black; padding: 2px 5px;"></td> <td style="padding: 2px 5px;">6</td> <td style="border-right: 1px solid black; padding: 2px 5px;">10</td> <td style="padding: 2px 5px;">22</td> <td style="border-right: 1px solid black; padding: 2px 5px;">402</td> <td style="padding: 2px 5px;">Add 4 each time</td> </tr> </table> </div> <p style="text-align: center;">Figure 4. 3: A recreation of a student's work that was shared in the professional development class.</p>	Tables	1	2	5	100	n tables	People	4	6	12	202	Add 2 each time	Tables	1	2	5	100	n tables	People	5	8	17	302	Add 3 each time	tables	1	2	5	100	n tables		6	10	22	402	Add 4 each time
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0.12.47	<p>Ms. A explained that the student could determine the number of seats at the banquet by her rule of adding a certain number for each table added, but could not generalize that statement with a formula. Ms. N, a teacher in Ms. A's group, further elaborated on the student's work and explained that the student could reason about the situation additively but had not yet transitioned her thinking about repeated addition as multiplication or make use of variables to further generalize the situation.</p>																																				

Episode 4.5 continued: Middle School Cohort, Session 1

Time h/m/s	Description
0.13.49	MTE-Karla summarized what Ms. N and Ms. A said. Ms. N reiterated and said that the student could not think about it with respect to a variable expression. Karla pointed out that the student is making use of a recursive relationship to solve the task but not the functional relationship. Ms. N noted that the students are sixth graders so it is to be expected.
0.14.32	MTE-Karla asked if there are any other thoughts comments or observations before the move on. Ms. N expressed that they chose to share that example of student work because many of their students thought about the task in a similar fashion. They discussed another student's work where the student did use multiplication to do the task but did not generalize the linear relationship with a function and was unsure how to think about n tables. This particular student chose n to be 200 and found the number of seats available if there were 200 tables.

In Episode 4.2 MTE-Karla prompted the teachers to go through the student work from their classes and pick out ones of interest. She noted that a student's solution might be interesting to share if it was representative of many students' responses or if it stood out from the rest in some way. Here she highlighted for the teachers the kinds of discussion that are fruitful in developing teachers' knowledge about the mathematics of students. The discourse in Episode 4.5 is embedded in the teachers' classroom experiences; MTE-Karla modeled the public reflection and shared insights. These are aspects of the collective inquiry process within a community of teachers. Questions like the question at 0.12.22 in Episode 4.5, when MTE-Karla asked a middle school teacher, Ms. A, what she could tell the class about the students thinking, focused teachers' discussion on student thinking. At 0.13.49, MTE-Karla added to the conversation, acting as a fellow (perhaps more knowledgeable) peer in a community of teachers, adding to the shared insight about student learning and thinking.

Tools

In Episode 4.5, the tools, or objects of reference, were the students' work. Tools used in the collective inquiry process refer to objects in the classroom environment (tangible or intangible) that the mathematics teacher educators and teachers make use of to enhance learning and/or discourse about lesson planning, student learning, thinking and understanding, and so on. Such tools may include (1) student work or classroom video (2) records of ideas or concepts that emerge, and/or mathematical notation and language attached to students' ideas and/or (3) literature, research, articles, and/or texts. MTE-Max models the use of tools, student work, in Episode 4.3 in a similar way. However unlike Episode 4.5, the work the teachers were to examine was the work of their peers. Still it was treated in the same manner; that is, the teachers were prompted to analyze and make sense of others' work.

While teachers are engaged as learners of mathematics tools are slightly different and used to enhance mathematical discussion and learning instead of discussion related to the practice of teaching. Here tools refer to objects in the classroom environment (tangible or intangible) that the mathematics teacher educator and teachers make use of to reason, make connections, solve problems, communicate and enhance discourse. Such tools may include:

- 1) the teachers' work, hypotheses, explanations and arguments,
- 2) records of ideas or concepts that emerge often coupled with mathematical notation and language, and
- 3) manipulatives, pictures, diagrams, tables, graphs, and so forth.

In the pre-session interview, MTE-Scott noted that he uses the teachers' homework responses to prompt mathematical discussion. In Episode 4.1, MTE-Scott prompts the teachers to use their written responses to motivate discussion about different solution paths. These written solutions served as a tool of discourse during the discussion of the homework. There are various kinds of tools that MTE-Scott and the teachers employed in both the mathematics teacher education sessions, such as pattern blocks, fraction circles and strips, and diagrams.

Environment.

The learning environment mediates the teachers' development. It is the charge of the mathematics teacher educator to create a learning environment that fosters the development of each teacher's knowledge base. This can include how time in the class session is structured and how the classroom is arranged. For teachers' engagement, as both teachers and learners, the teaching actions look very similar, since the learning environment mediates the teachers' development, the difference lies in the mathematics teacher educators' purpose for fostering such an environment, be it for developing teachers' mathematics content or pedagogical knowledge. Ways to support development of each might include:

- 1) small group discussion with the instructor,
- 2) time to work independently or in small groups on new material, allowing for think time or reflection and
- 3) making teachers feel comfortable to share or question and providing opportunities to do so.

The data pointed to two additional pedagogical moves the mathematics teacher educators used in mediating the learning environment,

- 4) differentiating instruction and
- 5) letting teachers struggle with a difficulty.

In Episodes 4.2 and 4.3, one of the goals of the interaction was to create a learning environment that fostered increased participation in the collective inquiry process. In order for teachers to participate in the collaboration involved in the collective inquiry process, teachers must be comfortable and open discussion must be commonplace. Collaboration with peers is not an automatic phenomena; in traditional teaching the work of teachers outside of the classroom is seen as a solitary endeavor (DuFour, 2005). In Episodes 4.2 and 4.3, the mathematics teacher educators modeled the collaborative work of teachers by engaging them in reform-oriented activity, prompting the teachers to share their thinking about student work thinking and understanding.

In Episode 4.1, the mathematics teacher educator modeled “wait time”, that is allowing for a pause in which the teachers have an opportunity to consider, reconsider, reflect and formulate a response to a recently made utterance. In Episode 4.1 when no one volunteered a solution (0.06.07), MTE-Scott asked the teachers to take a few minutes to share their responses with their groups allowing the teacher to struggle with difficulty or uncertainty collaboratively. During the group discussion, the teachers were given think time, the opportunity to struggle with a difficulty, discuss their thinking with their peers and build confidence in their solution before sharing out with the whole class. As MTE-Scott walked about the room he joined the discussions of various groups, which gave the teachers the opportunity to have instructor-small group

discussion. As groups engage in instructor-small group discussion the mathematics teacher educator often makes use of discursive moves to help the group to move forward in their thinking. During group or whole class discussion the mathematics teacher educator can also foster the development of each teachers' mathematical power by differentiating instruction and letting teachers struggle with a difficulty.

Analysis

Like the environment category, facets of the mathematics teacher educator's pedagogy related to ongoing analysis as the teachers engage as professionals and as learners look very similar. Analysis is the systematic reflection in which the mathematics teacher educators engage to monitor classroom life, for example, how well tasks, discourse, and environment foster learning. Through this process mathematics teacher educators examine relationships between what they and their teachers, are doing and what students are learning. Such analysis might include:

- 1) listening in on teacher discussion or looking in on their work,
- 2) checking for understanding,
- 3) monitoring the teachers' participation in discussions and deciding when and how to encourage each teacher to participate,
- 4) probing teacher's thinking, and
- 5) polling to gauge where the teachers are in their activity.

Again the intent of the teacher engagement and thus the analysis determines the distinction between the levels of engagement. The mathematics teacher educator might engage in ongoing analysis to foster teachers' enculturation into a professional learning

community of mathematics engaging in the collective inquiry process, and/or to foster teachers' mathematical development and learning.

In Episodes 4.1, 4.2, and 4.3, the mathematics teacher educators engaged in analysis when listening in on the conversations of the groups. This action allows the mathematics teacher educators to monitor the teachers' participation in discussions, glean insight into how the teachers approached and made sense of the task and developed an idea of the content of their discussions. Another way to monitor the classroom situation is exemplified in Episode 4.4, where MTE-Scott's check for understanding by asking the class if anyone could explain Ms. A's thinking as she presented her solution (0.08.53). This question sets up an opportunity for the mathematics teacher educator to probe the teachers' understanding and thinking about Ms. A's response.

With respect to the collective inquiry process, the mathematics teacher educator acting more as a peer or fellow teacher also engages the teachers in analysis of learning situations. While this analysis does not take place concurrently in a classroom setting with students, the analysis is related to the coordinated action phase of the collective inquiry process where the teachers (and mathematics teacher educator) analyze the results of a planned action to consider or discuss for example:

- 1) student thinking and learning,
- 2) the effects of the tasks, discourse, and learning environment on students' mathematical knowledge and development, and
- 3) skills, and dispositions and
- 4) implications for instruction.

Consider Episode 4.6 below from the primary cohort. Recall, Episode 4.5 ended with the discussion of the student's work for the banquet hall problem.

Episode 4. 6: Middle School Cohort, Session 1

Time h/m/s	Description of Events
0.16.55	MTE- Karla stated if we had to order the students in their level of understanding it seems this student seems to exhibit a little less understanding than the one in their last example. Karla noted that the students were from 6 th -grade and that they might not expect that all students at this level would be able to come up with a function to express the relationship. Karla then asked the teachers to think about what would be next for these students to push them further in their thinking if they were going to be their teacher next year. Ms. N suggested that they could give the kids practice translating words into variable expressions; and once they have practice with that they could go back to the banquet hall task and ask them how they could use their experience translating words into variable expressions to determine the number of seats when there are n tables are put together. Karla reiterated what Ms. N said and said that the students need to have some experiences translating variable expressions. Ms. A added that the practice could start with simple translations that yield expressions like $2x$ and $3x$ and so on. Karla suggested another possibility could be to ask the students express the relationship in words and later connect the words to the algebra
0.18.00	Karla then opened the floor for additional question or comments before moving on.

The teachers' analysis of where the students are in their understanding in Episode 4.5, the discussion in Episode 4.6 where MTE-Karla prompted about the levels of sophistication of the students' thinking, and the ways they as teachers could move this student forward in their thinking gave the teachers opportunities to practice the analysis of student thinking and to consider implications for instruction.

Rita, a primary elementary school teacher, noted observing how her mathematics teacher educator structured the sessions and the year has informed the way she plans lessons and will structure the upcoming lessons and school year. However this was not the norm.

In summary, the mathematics teacher educators modeled the collective inquiry process of a professional learning community of mathematics teachers by engaging teachers in activities that are a part of the practice of teaching related to collaboration among peers, lesson planning, thinking about and reflection on student thinking, understanding, learning, evaluating and so on. Similarly, the mathematics teacher educator modeled teaching practices consistent with reform recommendations by engaging the teachers as learners of mathematics. Here, for both levels of engagement, tasks, discourse, environment and analysis were discussed to exemplify the distinctions between the categories. Again it is important to note that in general, teaching actions are multi-faceted and do not often fall rigidly under one category or another. Instead, classroom events in mathematics teacher education can be categorized under a variety of headings both within and across levels of engagement to best describe the given event.

Part 2: The Analytical Framework and the Research Questions

As outlined in Table 3.4, analysis of the interviews, both the pre-session and the stimulated recall with the mathematics teachers educators provided opportunities to gain insight into (1) *what the mathematics teacher educators believe they model*, and the analysis of the interview data and survey data from the teachers provided opportunities to gain insight in to (2) *what practicing teachers notice about the mathematics teacher educators' pedagogy and identify as effective mathematics teaching*. Integrating these perspectives highlighted the (3) *ways these perspectives are aligned in mathematics courses for practicing teachers*. In this section, I answer

these research questions by first discussing the results with respect to the year of mathematics teacher education and then with respect to two consecutive class sessions for each cohort before synthesizing them. Throughout the discussions of the perspectives on the year and the two consecutive sessions I report on similarities and differences across cohorts. Lastly, I summarize comparisons and highlight aspects of the perspectives of the mathematics teacher educators and the teachers.

Perspectives on Modeling with Respect to the Year

Recall that the mathematics professional development was a yearlong program and the mathematics course for practicing teachers in a two-year program and this study takes place toward the end of the year of the former and the end of the first year of the latter. In this section I report on the analysis of pre-session interviews and teacher surveys to describe the mathematics teacher educators' perspectives and the participating teachers' perspectives on what they believe the mathematics teacher educators model with respect to the year of mathematics education programs. I draw on this analysis to discuss the results for research question 3, the ways the perspectives of the mathematics teacher educator and the teachers are aligned in this study.

Mathematics Teacher Educators

The study took place towards the end of a year of mathematics teacher education sessions. In the semi-structured pre-session interviews, each of the mathematics teacher educators were asked to reflect on the year to discuss what they believed they modeled about practice. The discussions were driven by what the mathematics teacher educators felt were important aspects of their work and on

average lasted about 45 minutes. Each mathematics teacher educator was asked the following questions to prompt discussion about what they believe they model:

- 1) What are the overarching goals of the courses you teach?
- 2) It was noted in a previous discussion about the teacher education programs that an important aspect is that mathematics pedagogy is modeled for the teachers. Do you feel this applies to the courses you teach?
- 3) In what way do you feel you model effective mathematics instructional practice?
- 4) In what ways do you feel you model practice with respect to the nature of classroom activity?
- 5) After spending almost a year working with you in this course, what might your students take away from it in terms of their developing understanding of teaching and learning mathematics?
- 6) What are some things that teachers might take away from the mathematics teacher education that you might not see right away because they are harder to implement?

Beyond asking the mathematics teacher educators to specifically describe aspects of their instructional practice, these questions were intended to elicit what the mathematics teacher educators thought teachers might notice about their practice. In the pre-session interviews, the pedagogy the mathematics teacher educators suggested they model was consistent with instruction as described in the NCTM *Professional Standards for Teaching Mathematics* (1991). This was not a surprising result since the university professional development group that coordinates the teacher education programs, from which the three cohorts in this study derive, has a well-articulated mission anchored in student learning of core disciplines and skills that is consistent with research and reform recommendations like those noted in the NCTM *Professional*

Standards for Teaching Mathematics. However, the *Professional Standards for Teaching Mathematics* alone were insufficient to categorize all of the ways the mathematics teacher educators suggest they model the practice of mathematics teachers. In the pre-session interviews the mathematics teacher educators described the pedagogy they model with respect to the participants' engagement as both learners of mathematics and teachers, thus leading to the emergent analytical framework described above.

The analytical framework was used to code the pre-session interview data. Specifically, the utterances the mathematics teacher educators made to describe their pedagogy were coded to characterize the content of their discussion. The utterances were coded with respect to the level of engagement (learner or teacher), category (task, discourse, tools, environment, or analysis) and also with the related subcategory. For example, MTE-Karla, in discussing how she uses the teachers work and conclusions from small group discussion said, "I think one of the things as the instructor or facilitator you're looking for (are) certain things to bring out... I already know what I want (to be) shared but I want it to come from them so I am looking for it when I am circulating." This utterance was coded with respect to the teachers' engagement as learners of mathematics under the categories of classroom discourse and ongoing analysis. With respect to the category of the classroom discourse, this utterance related subcategory to using the teacher(s) contributions/work to shape discussion. With respect to the category ongoing analysis, this utterance related to the subcategory listening in on teacher discussion.

Like the utterance described above, many of the mathematics teacher educators' utterances were coded by several categories and subcategories. Not all utterances were coded because they were outside of the focus of this study. For example, in the pre-session interview MTE-Scott said,

There is no question in my mind; even in some of the most recalcitrant teachers who have made changes and two years is enough time for them to... make those changes. I mean, because it's not the final change. You know, if you went back two years later it would be interesting to see does it last? But most programs are not funded for more than two years at a time, so you never get that kind of data.

While this utterance was meaningful and pointed to his beliefs about what teachers gain by engaging in professional development, such utterances were generally transitional and led up to the main idea of their exposition, of which only the main idea of the utterance was coded. The coded utterances specifically related to the research questions and allowed for characterizing specific pedagogical moves. In the tables below, I quantify the mathematics teacher educators' discussion of the ways they believe they model practice with respect to the analytical framework. Recall the analytical framework consists of parallel categories that describe pedagogy under the categories of tasks, discourse, tools, environment and analysis with respect to the participants' engagement as either learners or teachers. For each cohort, there are two tables below. The first table enumerates the number of times the mathematics teacher educator highlighted a particular instructional act in his or her discussion and the second displays the related percentages. The frequency with which an instructional act is discussed suggests the prevalence or relative importance of a particular act for the interviewee.

Table 4.2: The enumeration and related percentages by category for the coded utterances in the pre-session interview with the primary elementary cohort MTE Scott.

Engagement	Tasks	Discourse	Tools	Environment	Analysis	Total
Learners	28	20	5	12	0	65
Teachers	12	6	4	1	3	26
Total	40	26	9	13	3	91
Engagement	Tasks	Discourse	Tools	Environment	Analysis	Total
Learners	31%	22%	5%	13%	0%	71%
Teachers	13%	7%	4%	1%	3%	29%
Total	44%	29%	10%	14%	3%	100%

Table 4.3: The enumeration and related percentages by category for the coded utterances in the pre-session interview with the upper elementary cohort MTE Max.

Engagement	Tasks	Discourse	Tools	Environment	Analysis	Total
Learners	12	6	3	5	3	29
Teachers	4	0	0	0	0	4
Total	16	6	3	5	3	33
Engagement	Tasks	Discourse	Tools	Environment	Analysis	Total
Learners	36%	18%	9%	15%	9%	88%
Teachers	12%	0%	0%	0%	0%	12%
Total	48%	18%	9%	15%	9%	100%

Table 4.4: The enumeration and related percentages by category for the coded utterances in the pre-session interview with the middle school cohort MTE Karla.

Engagement	Tasks	Discourse	Tools	Environment	Analysis	Total
Learners	25	9	3	2	3	42
Teachers	18	6	8	1	4	37
Total	43	15	11	3	7	79
Engagement	Tasks	Discourse	Tools	Environment	Analysis	Total
Learners	32%	11%	4%	3%	4%	53%
Teachers	23%	8%	10%	1%	5%	47%
Total	54%	19%	14%	4%	9%	100%

In the pre-session interviews, modeling engaging the teachers in *worthwhile tasks* was the most noted pedagogical move in the discussions of all three mathematics teacher educators. In the pre-session interviews, all of the mathematics teacher educators put the second greatest emphasis on the way they feel they model productive classroom discourse, which describes discursive moves the mathematics teacher educator employs to mediate discourse to focus discussion on (1) the practice of mathematics teaching, student learning or student thinking or (2) mathematical ideas, concepts and solution paths instead of answers. As the teachers are engaged as learners of mathematics the mathematics teacher educators primarily noted that they:

- 1) Question to promote inquiry into the mathematics
- 2) Promote discussion among participants by prompting group discussion
- 3) Ask the teachers to share their solutions with their group or the class
- 4) Push the teachers to provide explanations and justification for responses

MTE-Scott, the mathematics teacher educator for the primary elementary cohort, noted in the pre-session interview,

“... because they (the teachers) know more of the elementary mathematics piece they are able to zero in on ... what question do I want to ask [the students]. It’s not this free-for-all ‘How do you feel about math today?’ but (questions like) ‘What do you think about the differences between $\frac{2}{3}$ times $\frac{3}{4}$ and $\frac{3}{4}$ times $\frac{2}{3}$ ’ ‘Is there a difference?’ ... ‘I thought they were equal and they are but they don’t look the same?’. So how do we know when we are dealing with one or the other?”

The elementary (primary and upper) mathematics teacher educators’ perspectives on pedagogical moves aligned with respect to all five categories in the analytical framework. The emphasis was placed on tasks, discourse, environment,

tools and analysis, in order from most to least emphasized. Note, for the upper elementary cohort there was not an emphasis on tools over analysis; they were rated the same. Nevertheless, the similarity enabled comparison with the middle school cohort mathematics teacher educator. The middle school cohort facilitator mathematics teacher educator, Karla, put an emphasis on tasks, discourse, tools, analysis and environment in that order from most to least emphasized. Table 4.5 shows the hierarchies of the pedagogy noted by the mathematics teacher educators and highlights the difference in emphasis the elementary cohort facilitators and the middle school facilitator put on the pedagogy described in the analytical framework.

Table 4. 5: Pre-session interviews w/ MTEs: hierarchy from most to least emphasized.

Elementary Cohorts	Tasks	Discourse	Environment	Tools	Analysis
Middle School Cohort	Tasks	Discourse	Tools	Analysis	Environment

MTE-Karla's differing emphasis on pedagogy is noteworthy because this points to possible differences due to the target objectives of the mathematics teacher education programs. Recall that the primary and upper elementary cohorts are mathematics courses for practicing teachers, where the primary focus of the sessions is on developing teachers' mathematical content knowledge, while the middle school cohort was mathematics professional development program that is a part of a certificate program designed to develop the participants' understanding of both mathematics content and pedagogy. MTE-Karla suggested that her responses might be different if

she were teaching one of the mathematics courses for practicing teachers as part the certificate program. She noted that the mathematics learning in the middle school cohort is necessarily couched in terms of how the teachers might use the mathematics in their own classroom lessons to encourage engagement in the mathematics professional development. This might also provide an explanation for why tools were emphasized above environment. In the pre-session interview with MTE-Karla, she noted she used tools, like algebra tiles and hundreds charts, to engage the participants in mathematical discussion or problem solving can readily be used in the teachers' own classrooms.

In general, the majority of mathematics teacher educators' discussion of the ways they feel that they model practice related to the teachers' engagement as learners: 71%, 88%, 53% of the discussion for MTE-Scott, MTE-Max and MTE-Karla respectively. MTE-Karla's, the middle school cohort facilitator, discussion with respect to engaging the teachers as learners or in collective inquiry as teachers, the emphasis was about equal, but slightly more of the discussion had to do with the teachers engaged as learners (about 53% as learners and 47% as teachers). If we compare the categories, tasks, discourse, tools, environment and analysis for each level of engagement, there is little difference in the order from most to least emphasized pedagogy. Again, since there is a dual focus on content and pedagogy in the mathematics professional development that MTE-Karla facilitates, this was not a surprising result.

If we consider MTE-Scott's, the primary elementary facilitator, order of relative importance from most to least emphasized with respect to the two levels of teachers' engagement, there is also little difference. However, there is a stark difference between MTE-Max's, the upper elementary cohort facilitator, discussion with respect to the ways he models practice as the teachers are engaged as learners of mathematics as compared to the ways he models the collective inquiry process as the teachers are engaged as professionals. Since both the primary and upper elementary cohorts have a focus on mathematics content, there may be little expectation that the mathematics teacher educators would be planning to engage the teachers as professionals. Yet MTE-Scott emphasized engaging the teachers as professionals to a greater extent than mathematics teacher educator from the upper elementary cohort. By way of explanation, in the pre-session interview MTE-Scott noted that since, at times, the content in the sessions is more advanced than the mathematics the k-3 teachers currently teach, he consciously discusses and points the teachers to implications for instruction as they apply to primary elementary school mathematics so that the teachers see benefit in what they are learning and remain engaged in the course.

The tables below summarize the collective responses from the pre-session interviews with the mathematics teacher educators. Table 4.6 is made up of two sections. The first section enumerates the number of times the mathematics teacher educators highlighted a particular pedagogical move in their discussion and the second displays the related percentages. As noted, I interpret the frequency or prevalence with

which a pedagogical move is discussed as a suggestion of its an importance. In Table 4.7 the cells represent the average percentage by category for the coded utterances in the pre-session interviews with the mathematics teacher educators. Thus the number of coded utterances of one mathematics teacher educator does not skew the fractional part.

Table 4.6: The enumeration and related percentages by categories for the coded utterances in all of the pre-session interviews with the MTEs.

Engagement	Tasks	Discourse	Tools	Environment	Analysis	Total
Learners	65	35	11	19	7	137
Teachers	34	12	12	2	7	67
Total	99	47	23	21	14	204
Engagement	Tasks	Discourse	Tools	Environment	Analysis	Total
Learners	32%	17%	5%	9%	3%	67%
Teachers	17%	6%	6%	1%	3%	33%
Total	49%	23%	11%	10%	7%	100%

Table 4.7: The average percentage by category for the coded utterances in the pre-session interviews with the MTEs.

Engagement	Tasks	Discourse	Tools	Environment	Analysis	Total
Learners	33%	17%	6%	10%	5%	71%
Teachers	16%	5%	5%	1%	3%	29%
Total	49%	22%	11%	11%	8%	100%

Notice there is little difference in the fraction of utterances that were coded under each category in Tables 4.6 and 4.7. Also, there is little difference if we compare Tables 4.6 and 4.7 with the table for the pre-session interviews for each cohort. In general, the mathematics teacher educators suggest they model tasks, discourse, tools, environment, and analysis in order from most to least emphasized. This overall ranking is fairly consistent with each mathematics teacher educators responses in the pre-session

interviews in order of emphasis on the practice they feel the model. The elementary cohort mathematics teacher educators, Scott and Max, emphasize environment over tools but as illustrated in Table 4.6 the difference between the emphases in the overall ranking is negligible, less than 1%. In Tables 4.2 and 4.3 for MTE-Scott and MTE-Max respectively the difference between the emphases on environment over tools is about 14% compared to 9%. MTE-Karla emphasized analysis over learning environment about 9% to 4%, as illustrated in Table 4.4, but again the difference in Table 4.6 is negligible, less than 4%. Like the individual discussions the table reflects a dominance, of the mathematics teachers engaging the teachers as learners of mathematics as they model practice. Each of the categories is discussed in greater detail in the following.

Tasks. In the pre-session interviews the category of worthwhile tasks was coded in about 49% of the mathematics teacher educators' discussion about the way they model the teaching practice. About 67% of those utterances related to the teachers engagement as learners in the sessions. MTE-Scott, the primary elementary school cohort facilitator, suggested that he models the use of tasks that promote communication about the mathematics that they were learning and then later discusses implications for teaching that content. He said "I've learned over time to choose homework questions that will allow me to, you know, have discussions with them as, you know, things unfold." MTE-Karla, the middle school cohort facilitator, also emphasized the way she models tasks that prompt the teachers to make connections between the mathematics they themselves are learning and teaching. Much of MTE-

Karla's discussion about tasks as the participants were engaged as learner related to stimulating teachers to make connections between mathematical ideas, sense making and considering alternate solutions paths. She models mathematics pedagogy by

[T]rying to have them engage with the math, trying to make connections between ideas, of trying to give them models and representations that can be used to make meaning of the math... we have done a lot of work with the secondary teachers is the procedures with connections from the task analysis guide ... I think we are always living in that realm, procedures with connections...

Like MTE-Karla, MTE-Max emphasized modeling tasks that prompt teachers to make connection between representations noting,

I'm always trying to develop those concepts first and foremost and making connections... One of the big things that I have done in this class is making connections between the naked number sentence, the word problems, and the pictures that they draw to go with them. And sometimes they even use pattern blocks with it so that it can be another connection ... and making connections between those representations, and then being able to go either direction with them.

The mathematics teacher educators' utterances illuminate the way they feel they model the use of tasks intended to develop teachers' content and pedagogical knowledge.

Discourse. In the pre-session interviews, the category of discourse was coded in about 23% of the mathematics teacher educators' discussion about the way they model the teaching practice. About 74% of those utterances related to the teachers engagement as learners in the sessions. Mathematics teacher educators Scott, Max and Karla put a lot of emphasis on discourse, the way they feel they model pedagogy related to small group and whole class discussion specifically with respect to questioning and sharing out. MTE-Scott noted that "questioning is a big part of what we do... I don't think that one gets there overnight with questioning, so having teachers

be able to know that certain things are just automatically going to be questioned.” MTE-Karla spoke also of developing the norm of providing justification and explanations for ones answers. She pointed to sharing out and said “one of the ways I would characterize it [the way she models practice]... is giving them the opportunity to make public their mathematics thinking, is a really important component of what we do.” The mathematics teacher educators also pointed to modeling the orchestration of discourse by using contributions by the teachers to shape discussion. MTE-Max noted he tries to move conversations along based on what the teachers are saying and thinking.

Tools and Analysis. Here I discuss tools and analysis together because in the pre-session interviews, the category of discourse was coded in about 11% of the mathematics teacher educators’ discussion about the way they model the teaching practice and about 52% of those utterances related to the participants’ engagement as teachers in the sessions. This suggests that while the mathematics teacher educators model use of tools as the teachers engage as learners in mathematical tasks and discussion, tools play an almost equal role in how they model the teachers engagement in the collective inquiry process. In the pre-session interviews the category of analysis was coded in about 7% of the mathematics teacher educators’ discussion about the way they model the teaching practice. About 50% of those utterances related to the participants engagement as teachers in the sessions. The mathematics teacher educators’ utterances suggest they model the use of tools such as, student work, classroom video, research, articles and student textbooks to develop teachers’

pedagogical content knowledge as they engage in various phases of the collective inquiry process. MTE-Scott noted:

Once you [the teachers] understand the content at a deeper level and you [they] understand what you [they] need to get across [to the students] that gives you [the teachers] all kinds of ideas for experimenting... I think that in large part comes through the discussion with each other during classes. They actually get to do the try it on pieces in their classes, bring back the work, talk about how it went. So it is almost a practice of what we are talking about in smaller chunks so they kind of get to see what happens when you do something outside the textbook... So they have an opportunity to do that a number of times throughout the 2 years in the program...

MTE-Scott's comments suggest that the teachers get to use tools that they themselves have used in the mathematics teacher education to facilitate problem solving and to enhance discourse in their own classes. They then analyze the lessons and the students' work and engagement, collectively as a group. Such experiences give teachers the opportunity to participate in facets of collective inquiry.

Environment. In the pre-session interviews, the category of discourse was coded in about 10% of the mathematics teacher educators' discussion about the way they model the teaching practice. About 93% of those utterances related to the teachers' engagement as learners in the sessions. The mathematics teacher educators pointed to modeling the creation of a learning environment that fosters the development of each teacher's mathematical power by allowing think time (individually or in groups) and small group discussion with the instructor. MTE-Max noted he would give, "a lot of time to work with others...[and then] bringing it [the class] back together me asking questions. Hopefully that furthers their thinking with both me asking questions to the individual group and me asking questions to the whole

group.” MTE-Scott and MTE-Max especially pointed to letting teachers struggle with difficulty. MTE-Scott reported that the teachers in his class say things like “I know what you are going to tell me isn’t an answer, but I have to ask anyway... I know you’re going to ask me another question, but I just have to ask anyway” he continued, “So... the notion is that they know part of it is that they are not going to get an automatic answer from me. It’s going to be another question that makes them think about, (or) clarify their own question or at least get them to think about something.”

The mathematics teacher educators put the most emphasis on the way they feel they model pedagogy with respect to the nature of tasks. As the teachers engage as learners, the mathematics teacher educators reported that they model tasks that introduce mathematical ideas with challenging activities that promote inquiry. The mathematics teacher educators reported such activities often require the teachers to make connections between mathematical concepts and representations. As the teachers engage as professionals, the mathematics teacher educators reported that the model tasks that allow for multiple entry points and connections to be made between the concepts the participants learned in the sessions and their own practice.

Second to tasks, the mathematics teacher educators emphasized the way they believe they model discourse. As the teachers engage in tasks, the mathematics teacher educators reported that they promote discourse between participants in whole class and group discussion. The mathematics teacher educators reported that they prompt the teachers to share their thinking and provide justifications and explanations for their answers, method strategies, and results. The mathematics teacher educators reported

that such discussions often include presentations of multiple solution paths and how the solutions relate to one another. Also, as the teachers engage in tasks the mathematics teacher educators reported the ways they model the use of questions to prompt the teachers to elaborate on and rethink not only their solution but that of their peers and to move the teachers forward in their thinking.

As the teachers engaged in tasks and discussions about tasks the mathematics teacher educators emphasized that they model the use of tools to enhance problem solving and discussion. As the teachers are engaged as learners these tools often make use of symbolic, pictorial, graphical and tangible representations which students use to make sense of mathematical ideas. When the participants are engaged as teachers, the tools generally included video of students sharing their thinking, student work and mathematics education literature to help teachers develop their knowledge about student thinking.

As the teachers engaged in tasks and discourse the mathematic teacher educator reported that they allow for adequate wait time so that the teacher can think through mathematical ideas independently or in groups to promote sense making for mathematical ideas. The mathematics teacher educators also suggested that they model ongoing analysis as they listen in on the teachers' discussions to check of understanding and listen for teachers' comments or conclusions that might be important to share with the class. The mathematics teacher educators also report that they probe their understanding so they can ask the right kinds of questions to move them for in their thinking.

In summary, reflecting on the year, the mathematics teacher educators reported that they model the use of worthwhile mathematical tasks, classroom discourse, tools, learning environment and analysis from most to least emphasized. In general their discussion related to the teachers' engagement as learners in the mathematics teacher education and the mathematics teacher educator implicitly modeling practice.

Teachers

The reflections surveys 1 and 2 asked the teachers, near the end of one year of mathematics teacher education, to reflect on the year and were intended to highlight what teachers note about the mathematics teacher educators' pedagogy and identify as effective mathematics instruction. Reflection Survey 1 was qualitative, and read:

Reflect through writing a brief response to the sessions:

Drawing on what you have learned in prior mathematics content sessions, what have you done to support student thinking in your classroom? If possible, please share a specific example in context (e.g., grade level of students, course and topic)

The analytical framework was used to code their written utterances. In the tables below, I quantify the teachers' responses on Reflection Survey 1 with respect to the analytical framework. Recall the analytical framework consists of parallel categories that describe pedagogy under the categories of tasks, discourse, tools, environment and analysis with respect to the participants' engagement as either learners or teachers. For each cohort, there are two tables, the first enumerated the number of times the teachers highlighted a particular pedagogical move that they have drawn from their experience in the class and the second displays the related percentages. The frequency or

prevalence with which a pedagogical move was written about suggests the importance of a particular act for the teachers. Note there are more pedagogical moves enumerated than there are teachers because most teachers listed more than one thing and some pedagogical moves can be described with respect to more than one characterization.

Table 4.8: Coded responses for reflection 1 by category for the primary elementary cohort enumerated and by percentage (n=16).

Engagement	Tasks	Discourse	Tools	Environment	Analysis	Total
As learners	12	11	5	4	0	32
As teachers	0	0	0	0	4	4
Total	12	11	5	4	4	36
Engagement	Tasks	Discourse	Tools	Environment	Analysis	Total
Learners	33%	31%	14%	11%	0%	89%
Teachers	0%	0%	0%	0%	11%	11%
Total	33%	31%	14%	11%	11%	100%

Table 4.9: Coded responses for reflection 1 by category for the upper elementary cohort enumerated and by percentage (n=19).

Engagement	Tasks	Discourse	Tools	Environment	Analysis	Total
Learners	23	7	12	2	0	44
Teachers	4	1	2	0	3	10
Total	27	8	14	2	3	54
Engagement	Tasks	Discourse	Tools	Environment	Analysis	Total
Learners	43%	13%	22%	4%	0%	81%
Teachers	7%	2%	4%	0%	6%	19%
Total	50%	15%	26%	4%	6%	100%

Table 4.10: Coded responses for Reflection Survey 1 by category for the middle school cohort enumerated and by percentage (n=6).

Engagement	Tasks	Discourse	Tools	Environment	Analysis	Total
Learners	5	3	4	4	0	16
Teachers	2	0	0	0	0	2
Total	7	3	4	4	0	18
Engagement	Tasks	Discourse	Tools	Environment	Analysis	Total
Learners	28%	17%	22%	22%	0%	89%
Teachers	11%	0%	0%	0%	0%	11%
Total	39%	17%	22%	22%	0%	100%

Like the mathematics teacher educators, the majority of their responses had to do with mathematics tasks. The teachers reported that their participation in the mathematics teacher education has influenced the kinds of tasks they engage their students in during mathematics lessons. Also like the mathematics teacher educators, analysis was among the least emphasized in their reflections on the year.

There was much variation in the hierarchies of the other three categories across cohorts however these differences were negligible. Consider Table 4.11 below. Note the categories that are bold-italicized-underlined categories suggests that within that cohort the order of the hierarchy may have been rearranged to highlight the similarities in the across cohort hierarchies (the determination of the across cohort hierarchy is elaborated below). The largest difference between the bold-italicized-underlined categories per cohort was at most 5%, which is negligible for groups of this size.

Table 4.11: Reflection survey 1, hierarchy per cohort and across cohorts in general.

Primary Elementary	Tasks	Discourse	<u>Tools</u>	<u>Environment</u>	<u>Analysis</u>
Upper Elementary	Tasks	Tools	Discourse	<u>Environment</u>	<u>Analysis</u>
Middle School	Tasks	<u>Discourse</u>	<u>Tools</u>	<u>Environment</u>	Analysis
Across Cohorts	Tasks	<u>Discourse</u>	<u>Tools</u>	Environment	Analysis

In the tables below I quantify all 41 teachers' reflection surveys from all three cohorts and then discuss the differences between the cohorts. The first table enumerates the number of times the teachers highlighted particular aspects of instruction that they have noted from participation in the mathematics teacher education, and the second displays the related percentages.

Table 4.12: The enumeration and related percentages by category for the coded responses for Reflection Survey 1 from all cohorts together.

Engagement	Tasks	Discourse	Tools	Environment	Analysis	Total
Learners	40	21	21	10	0	92
Teachers	6	1	2	0	7	16
Total	46	22	23	10	7	108
Engagement	Tasks	Discourse	Tools	Environment	Analysis	Total
Learners	37%	19%	19%	9%	0%	85%
Teachers	6%	1%	2%	0%	7%	15%
Total	43%	20%	21%	9%	7%	100%

Since teachers in different cohorts might focus on different pedagogy, partly perhaps due to having different mathematics teacher educators and the target content

differed per cohort, it became important to consider the emphasis teachers put on pedagogy in each cohort primary, upper and middle, so that the numbers per cohort, 16, 19, and 6 teacher responses respectively, do not skew the hierarchy. Per cohort the coded responses were counted and related percentages were determined to quantify the practices the teachers noted they have gained from engaging in the mathematics teacher education. The hierarchies from this analysis are reflected in Table 4.11. The table below shows the average of the percentages of the responses from the three cohorts with respect to the analytical framework.

Table 4.13: Reflection Survey 1, average percent

Engagement	Tasks	Discourse	Tools	Environment	Analysis	Total
Learners	35%	20%	19%	12%	0%	86%
Teachers	6%	1%	1%	0%	6%	14%
Total	41%	21%	21%	12%	6%	100%

Notice there is little difference between the related percentages in Table 4.12 and 4.13. The hierarchy of the practices the teachers report they have enacted as a result of participating in the mathematics teacher education is for the most part the same in both tables. The teachers' written utterances emphasized tasks, discourse, tools, environment and analysis, from most to least emphasized. Notice that this is the same hierarchy that described the mathematics teacher educators' perspective on what they believe they model about practice in the pre-session interviews.

As mentioned above, tasks were the most mentioned in the teachers' reflections. About 40% of the teachers' responses related to mathematical tasks. A 3rd grade teacher from the upper elementary cohort wrote, "We do a lot of 'meaning

making'. Before taking this class, I would have just taught algorithms w/out necessarily teaching the meaning." Many of the teachers in the cohorts noted the use of tasks they themselves had engaged in as learners during the sessions. In Reflection Survey 1, about 20% of the responses reported as a result of engaging in the mathematics teacher education, they now make use of mathematical discussion in their classes. For example, one teacher wrote that in their lessons they now allow "students (to) work in pairs to support each other and help each-other through problems." Across cohorts the teachers expressed that they make use of discursive moves specifically related to questioning and sharing out in whole class or group discussion. The teachers reported that they promote classroom discussion by asking open-ended questions, and providing opportunities for students to share their strategies and thinking with the class. Tools were noted in about 20% of their responses. A teacher in the upper elementary cohort wrote:

I teach (3 groups of 17 students) I have learned to use manipulatives in more engaging ways that support my students better in their understanding of math concepts. For example, I never thought of using the pattern block to help the 3rd graders understand fractions. I have also learned to use technology to help them. The geometer's sketchpad allows me to teach them with visuals that match the pattern blacks to help them understand and visualize what they will be working with.

In general, the teachers noted that, drawing on what they learned in the mathematics teacher education, they now choose tasks that support the development of mathematical ideas and sense making. As the teachers engage the students in tasks they suggested they make use of discursive moves like questioning and prompting whole class and group discussions. The teachers also noted that they make use of a

variety of tools to that have the capacity to help students reason, make connections, solve problems, communicate and enhance discourse and support student thinking.

Further information about the perspectives of the teachers came from a second survey constructed after an initial analysis of the pre-session interviews with the mathematics teacher educators. The pre-session interviews and informal discussions with the mathematics teacher educators and other staff members of the university professional development group informed the development of the reflection survey given during the second session of the two sessions from which I collected data (for Reflection Survey 2 see Appendix D). The purpose of Reflection Survey 2 was to gather demographic data about the teacher populations in each cohort and quantitative data on the frequency of the mathematics teacher educator's pedagogy. The teachers were directed to describe the frequency of their mathematics teacher educator's pedagogy in the class sessions by circling the options, 4, 3, 2, or 1 to indicate often, sometimes, seldom, or never, respectively for each item in a list that describe various pedagogical moves. The statements on the survey reflect pedagogy that would fall under the categories of tasks, classroom discourse and tools of discourse, environment, and analysis as described in the analytical framework. Before the surveys were given to the teachers, the mathematics teacher educators from the three cohorts reviewed the scaled questions and suggested that the items reflect teaching actions they privilege and the kind of teaching they believe they employ. The mathematics teacher educators suggested that they would, in general, score each item with a 4 to describe their own teaching acknowledging with some error since one could not guarantee that they

employed all of the teaching actions all of the time. There were 49 surveys collected and the average response was scored 3.8 with a standard deviation of 0.09, fairly close to the rating the mathematics teacher educators would have given themselves.

Since the 19 scaled questions in Reflection Survey 2 were all positively worded with respect to the categories, a modified version of the survey was given to a subset of the teachers approximately two months after completing their first year of mathematics teacher education with the university group. The elementary cohort teachers had just begun their second year of mathematics teacher education program to work toward their certificate in mathematics teaching. Due to transience of the teacher population and absences the day the survey was given, fewer teachers from the primary and upper elementary cohorts took the survey. The middle school cohort's mathematics teacher education was a yearlong program and the group had disbanded by the time Reflection Survey 2b was given; however, two of the six teachers submitted responses to the Reflection Survey 2b. Thus of the 49 teachers who took Reflection Survey 2, 39 took Reflection Survey 2b. Reflection Survey 2b contained the same 19 scaled questions as Reflection Survey 2 with the addition of 4 negatively worded statements that would be in contrast to the categories in the analytical framework. For example, item 15 read "the mathematics teacher educator accepts answers without justification." In the pre-session interview, MTE-Scott said that while, in general, the teachers are required explicate their reasoning and provide justifications for answers there are instances where he just wants an answer to move the discussion along. Upon reviewing Reflection Survey 2b the mathematics teacher educator and the university professional

development staff suggested items like item 15 are not out of the realm of possibility in the mathematics teacher education so the frequency would be scored a 1 or 2 for such items. So, unlike Reflection Survey 2, the expected average was 3.57 instead of 4. There were 39 surveys collected and the average response was scored 3.53. This is fairly consistent with the perspectives of the mathematics teacher educators. In general, the 4 items that were expected to be scored low (about a 1.5) were scored low by the teachers with an average of 1.8 and a standard deviation of .26. The average on the 19 items that were also on Reflection Survey 2 was scored 3.89 with a standard deviation of 0.11. This suggests the perspectives on the mathematics teacher educator's pedagogy are aligned.

Reflection Survey 2b also allowed for input from the teachers. Item 29 on the survey asked the teachers if there were any instructional acts not listed above (referring to the 23 scaled items) that they have noted about their mathematics teacher educator's instruction and to scale the frequency as before. Only 14 teachers, 4, 9 and 1 from the primary elementary, upper elementary and middle school cohort respectively, responded to this item. The 14 teacher responses included 25 utterances. The written utterances were coded with respect to the analytical framework. Only 20 were codeable. The other five responses noted that the mathematics teacher educator was knowledgeable, full of energy and patience, and reiterated item 19 on the survey "the mathematics teacher educator discusses multiple solution paths". Half of the 20 coded responses related to discourse, specifically with respect to questioning and sharing out. A few of the teachers wrote:

- [MTE-Max] Guides instruction through relevant questioning
- [MTE-Max] He was great at asking meaningful questions to clarify or guide our thinking
- [MTE-Scott] When we share ideas you always feel good about them even if you realize you had misconceptions
- [MTE-Scott] When we shared our thinking it helped you to think of new ways
- [MTE-Karla] Encourages dialogue between groups to discuss methods used

For a complete list of the teachers' responses to item 29 on Reflection Survey 2b see

Appendix F. On Reflection Survey 2b item 30, read:

Of the instructional acts listed above (including the ones you noted on item 29) list the item number (or letter) associated with what you feel were the 5 most effective instructional acts of your mathematics teacher educator.

Thirty-eight 38 of the 39 teachers responded to this item. The table below organizes the most frequently noted item called out by the teachers. The percent of teachers adds to more than 100% because, as directed the teachers put more than one response.

Table 4.14: Reflection Survey 2b, item 30.

Percent of Teachers	Item	
47%	24	Makes use of multiple representations in class discussion (e.g. symbolic, pictorial, graphical etc.)
42%	9	Promotes discourse/conversations between participants
37%	6	Introduces mathematical ideas/concepts by first presenting a challenging, related mathematical task
37%	19	Discusses multiple solution paths
34%	27	Promotes sense-making of mathematical ideas

The teachers' responses to Reflection Survey 1 highlight the kinds of pedagogical moves the mathematics teacher educator made that the participants identified as effective mathematics instruction over the course of the year. The most

prevalent or emphasized mathematics pedagogies the teachers reported on related to worthwhile mathematical tasks and discourse. The responses to Reflection Surveys 2 and 2b suggests that the teachers feel that the mathematics teacher educators' model instruction that is consistent with the mathematics teacher educators' views of their pedagogy, as the mathematics teacher educators' input helped develop the survey and it is also consistent with the pedagogy characterized in the analytical framework.

Perspectives on Modeling with Respect to Two Consecutive Class Sessions

In this section the focus is on two classes for each cohort. Here I report on the analysis of stimulated recall interviews with the mathematics teacher educators and teachers and the researcher's observation, I discuss the ways the perspectives on the mathematics teacher educators' pedagogical moves were aligned. Each of the mathematics teacher educators participated in two stimulated recall interviews. The three mathematics teacher educators were interviewed within about a day and a half of each observed session. Three teachers from the primary cohort, three teachers from the upper elementary cohort and two teachers from the middle school cohort participated in a stimulated recall interview in the week following the second of the two consecutive sessions from which data was collected from their respective cohorts.

The video of the sessions was used in the stimulated recall interviews to allow the mathematics teacher educators and teachers to re-immense themselves in the class session to reflect on and point to instances where they felt effective mathematics pedagogy was being modeled. The discussions in the stimulated recall interviews for both the mathematics teacher educators and the practicing teachers were driven by

what they felt were important aspects of the mathematics teacher educator's practice in the two classes under study. Upon review of the classroom video of the session, the interviewees were prompted to point to instances where they felt that effective mathematics pedagogy was being modeled. The interviewees were given a timeline of the sessions and free reign to play, scan, pause or stop the video as they pointed to instances and commented on the effective pedagogy that they felt was being modeled. The timeline aided the interviewees in navigating the video of the classroom sessions. The timeline allowed the interviewees to determine when to skip over certain events in order to focus on the ones that they felt were important. Thus, the teaching practices the interviewees noted were acts they found to be most effective and those that were frequently used in the class sessions were pointed out many times. The interviewees' descriptions of the pedagogy that was being modeled in the class sessions were then coded using the analytical framework. The videos of the class sessions were also coded from the researcher's perspective to further develop the analytical framework.

In general, the mathematics teacher educators and practicing teachers noted instances where they felt the mathematics teacher educator modeled effective mathematics pedagogy. These reflected the teaching actions described in the analytical framework under the five categories, tasks, discourse, tools, environment and analysis with respect to participants' engagement as either learners or teachers, however with varying degrees of frequency. In this section, I compare and contrast the mathematics teacher educator and the teachers' perspectives within each cohort on what was being modeled in the two class sessions. I report on the frequency that teaching actions

related to the analytical framework were mentioned, and report on common themes and differences within and across cohorts. Again, it is suggested here that the prevalence with which a pedagogical move was discussed helps us understand what type of acts are noticed by teachers and what is of importance to the mathematics teacher educators. Note again that in the analysis some pedagogical moves can be coded in more than one category in the analytical framework.

For each cohort I quantify the mathematics teacher educator's and the teachers' discussion about the way the mathematics teacher educators' model practice. For each mathematics teacher educator there are two tables. The first of the two tables enumerates the number of times a particular instructional act was highlighted by the mathematics teacher educator during the stimulated recall interview and the second displays the related percentages. For each cohort, the responses of the teachers were additively combined to create similar tables. These combined responses are intended to describe what the teachers collectively notice about the sessions. Instances where two or more teachers pointed to the same instructional act in a portion of the video to discuss a specific event were not double counted. Counting it once reflects the relative weight of the characterization of the event from the teachers' collective perspective.

Primary Elementary Cohort. Consider the tables below.

Table 4.15: The enumeration and related percentages by category for the coded utterances from the SRI with the primary elementary school cohort MTE Scott.

Engagement	Tasks	Discourse	Tools	Environment	Analysis	Total
Learners	13	31	6	12	6	68
Teachers	3	3	2	0	0	8
Total	16	34	8	12	6	76
Engagement	Tasks	Discourse	Tools	Environment	Analysis	Total
Learners	17%	41%	8%	16%	8%	89%
Teachers	4%	4%	3%	0%	0%	11%
Total	21%	45%	11%	16%	8%	100%

Table 4.16: The enumeration and related percentages by category for the coded utterances from the SRIs with the primary elementary school teachers: Rita, Erin and Anya.

Engagement	Tasks	Discourse	Tools	Environment	Analysis	Total
Learners	11	45	7	10	6	79
Teachers	1	2	1	0	0	4
Total	12	47	8	10	6	83
Engagement	Tasks	Discourse	Tools	Environment	Analysis	Total
Learners	13%	54%	8%	12%	7%	95%
Teachers	1%	2%	1%	0%	0%	5%
Total	15%	57%	10%	12%	7%	100%

Comparing the totals for the mathematics teacher educator and the teachers, I found the ranking order of discourse, tasks, environment, tools and then analysis best captures both the mathematics teacher educator's and teachers' emphasis on pedagogy modeled in the two primary cohort sessions.

The same hierarchy describes the mathematics teacher educator's and the teachers' emphasis on the pedagogy with respect to the teachers' engagement as learners. Both, the mathematics teacher educator's and the teachers' discussions

describe the mathematics teacher educator's pedagogy predominately in terms of the teachers' engagement as learners about 89% and 95% of the discussion respectively. This is reasonable since the mathematics teacher education for the primary elementary cohort is a mathematics course where the focus is on developing the teachers' mathematical content knowledge.

In the stimulated recall interviews, the greatest emphasis for both MTE-Scott and the teachers was on discourse, approximately 45% and 57% respectively, of their discussion of effective pedagogy. The teachers' responses on reflection 2b also highlighted this emphasis. About 38% of the practicing teachers from the primary cohort noted, "The mathematics teacher educator promotes discourse/conversations between participants" as one of the top five effective instructional acts by the mathematics teacher educator. (See item 30 discussed above).

In the stimulated recall interviews much of the teacher's emphasis on discourse related to questioning. Erin, a 1st grade teacher who participated in the teacher education primary elementary cohort, referring to MTE-Scott noted "He would pose questions that would really make us think...His role was to ask questions that led us to make our own discoveries... By asking good questions, by his questioning strategies, he really pulled the information out of us and really helped us to develop our own thinking." Another teacher, Rita, a 1st and 2nd grade teacher from the same cohort noted, "He always asks us questions like 'Does that work all of the time?'" She further explained, "He facilitates the class and pushes us (with questions) where he wants us to go and he builds the discussion on what we have... by questioning, engaging the

students, probing, using us and our ideas to get us to his instructional goal... (and) push[es] our thinking in the right direction.” The teachers reflected this sentiment in Reflection Survey 2b. In the survey 38% of the teachers in the primary cohort selected, “The mathematics teacher educator asks participants to think through ideas or concepts in groups” and 38% of the teachers selected, “The mathematics teacher educator uses participants’ contributions to guide class discussion” as one of the top five most effective instructional acts by the mathematics teacher educator. Rita provided an illustration like the one below (Figure 4.4) and explained that though they deviated from the line of reasoning he wanted them to take to reach the instructional goal, MTE-Scott’s questions directed them back to the path and they eventually reached the learning goal. In the illustration, the straight line represents the learning trajectory MTE-Scott would like the teachers to follow. The squiggly line represents the deviations the teachers take. MTE-Scott’s questions direct the teachers back toward the learning goals and keep the teachers from going too off course, eventually leading them to the desired outcome.

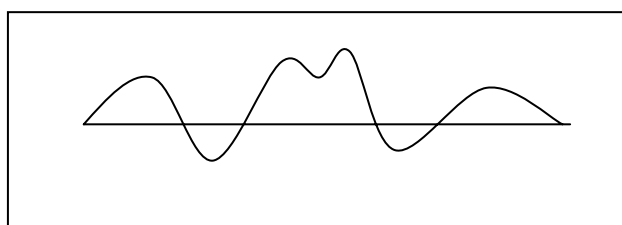


Figure 4.4: A recreation of Rita’s illustration of the learning path.

With respect to the participants’ engagement as teachers, the ranking order discourse, tasks, and then tools bests capture both the mathematics teacher educator’s and teachers’ emphasis on pedagogy modeled in the two sessions, though the

mathematics teacher educator pointed to mathematical task as often as classroom discourse. Notice that modeling the facilitation of learning environment and of ongoing analysis as the participants were engaged as teachers were not discussed at all by the mathematics teacher educator or by teachers. Again, since this is a mathematics course there is little expectation that the mathematics teacher educator would explicitly or implicitly foster a setting for teacher participation in the collective inquiry process of a professional learning community of mathematics teachers and monitor that engagement.

The perspectives on what MTE-Scott models about practice are aligned with respect to the emphasis both MTE-Scott and the teachers put on what they believe the mathematics teacher educator modeled about effective mathematics pedagogy.

Upper Elementary Cohort. Consider the tables below.

Table 4.17: The enumeration and related percentages by category for the coded utterances from the SRI with the upper elementary school cohort MTE Max.

	Tasks	Discourse	Tools	Environment	Analysis	Total
Learners	4	30	10	14	15	73
Teachers	8	1	0	0	3	12
Total	12	31	10	14	18	85
	Tasks	Discourse	Tools	Environment	Analysis	Total
Learners	5%	35%	12%	17%	18%	86%
Teachers	9%	1%	0%	0%	4%	14%
Total	14%	37%	12%	17%	21%	100%

Table 4.18: The enumeration and related percentages by category for the coded utterances from the SRIs with the upper elementary school teachers: Sandy, Tina, and Kelly.

	Tasks	Discourse	Tools	Environment	Analysis	Total
Learners	5	42	5	11	9	72
Teachers	1	1	0	0	0	2
Total	6	43	5	11	9	74

	Tasks	Discourse	Tools	Environment	Analysis	Total
Learners	7%	57%	7%	15%	12%	97%
Teachers	1%	1%	0%	0%	0%	3%
Total	8%	58%	7%	15%	12%	100%

The totals for the mathematics teacher educator and the teachers in the upper elementary cohort are shown in table 4.18. The ranking order discourse, environment, analysis, tools and then tasks best captures both the mathematics teacher educator's and teachers' emphasis on the pedagogy modeled in the two sessions.

The same hierarchy describes the mathematics teacher educator and the teachers' emphasis on the pedagogy with respect to the participants' engagement as learners, though in both cases the mathematics teacher educator emphasized ongoing analysis over learning environment but the differences were negligible. Both, the mathematics teacher educator and the teachers' discussions describe the mathematics teacher educator pedagogy predominately in terms of the teachers' engagement as learners about 86% and 97% of the discussion respectively.

In the pre-session interview MTE-Max responded to the question "What are the overarching goals of the courses you teach?" by explaining that the goal is "Developing conceptual understanding and making connections, so that procedural

fluency comes out of the conceptual understanding... In terms of almost whatever we are doing as trying to make connections even between representations or connection impact to a mathematical concept we're developing.” In this exposition, MTE-Max is specifically referring to the participants' mathematical development. Like this response, almost all of MTE-Max's responses in the pre-session interview related to engaging the teachers as learners and developing their conceptual understanding. This supports the intended purpose of the mathematics teacher education sessions for the upper elementary cohort, as it is a mathematics course.

Like in the primary cohort, the greatest emphasis for both MTE-Max and the teachers was on discourse, approximately 36% and 58% respectively, of their discussion of effective pedagogy. The teachers' responses on reflection 2b also highlighted this emphasis. 42% of the practicing teachers in the upper elementary cohort noted, “The mathematics teacher educator promotes discourse/conversations between participants” as one of the top five most effective pedagogical moves by the mathematics teacher educator.

Also like the primary cohort, the majority of the discussion about discourse related specifically to questioning. Sandy, a 3rd grade teacher, reflected that MTE-Max, the facilitator for the upper elementary school cohort, “... is deliberate, methodical about the questions he asks...He is very Socratic in his method.”

With respect to the participants' engagement as teachers about 14% of the mathematics teacher educator's discussion related to the teachers' engagement in the collective inquiry process while only about 3% of the participants' discussion related to

their engagement as teachers. Like the primary cohort, it is not unexpected that very little of their discussion had to do with their engagement as teachers since the upper elementary cohort is a mathematics course for practicing teachers where the focus is developing teachers' mathematics content knowledge. Notice that modeling the categories for learning environment and tools as the participants were engaged as teachers were not discussed at all by the mathematics teacher educator or teachers. MTE-Max pointed to the projects, questions, problems, constructions, applications, and exercises in which teachers engage intended to develop the teachers' understanding of the students' mathematics in about 9% of his discussion about the two sessions. In contrast, the teachers only noted such engagement in 1% of their discussion.

Both MTE-Scott and MTE-Max highlighted the way they feel they model the creation of a learning environment that fosters the development of each teacher's mathematical power with pedagogical moves such as think time and allowing the teacher to struggle with difficulty. The teachers from the primary cohorts that were interviewed noted this as well. In the stimulated recall interview, Kelly, a 6th-grade teacher in the upper elementary cohorts, said, referring to MTE-Max, "He does not give us the answer, he gives us a question to ponder and prompted us to use a picture to help me think about the rules I know, and you feel empowered because I figured it out by myself." Erin, a 1st grade teacher from the primary elementary cohort, noted that MTE-Scott "...wouldn't give us an answer, and he would make us really work it out ourselves and figure it out ourselves, and kind of always keep us wondering, which

kept us thinking, which is what we want the kids to be doing. So not only was he trying to make us think but he was also modeling for us.”

The teachers from the elementary school cohorts responses to Reflection Survey 1, suggest that drawing on their experiences in the sessions they now incorporate some aspects of this into their practice. Specifically, they describe (1) small group discussion with peers and the instructor and (2) allowing for think time or time for reflection or time to work independently or in small groups on new material to support student learning in their lessons. In the stimulated recall interview, Tina, a 3rd-grade teacher from the upper elementary cohort, noted that she observed how MTE-Max facilitated the pacing of topics and wait time and said that it made her become conscious of incorporating it into her practice. In general, the perspectives on what practices the mathematics teacher educators from the elementary cohorts felt they modeled were fairly aligned with respect to relative importance of the mathematics pedagogy the teachers felt were being modeled.

Middle School Cohort. Consider the tables below.

Table 4.19: The enumeration and related percentages by category for the coded utterances from the SRI with the middle school cohort MTE Karla.

Engagement	Tasks	Discourse	Tools	Environment	Analysis	Total
Learners	11	18	9	4	11	53
Teachers	8	8	3	5	12	36
Total	19	26	12	9	23	89
Engagement	Tasks	Discourse	Tools	Environment	Analysis	Total
Learners	12%	20%	10%	5%	12%	60%
Teachers	9%	9%	3%	6%	14%	40%
Total	21%	29%	14%	10%	26%	100%

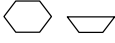
Table 4.20: The enumeration and related percentages by category for the coded utterances from the SRIs with the middle school teachers, Cathy and Victor.

Engagement	Tasks	Discourse	Tools	Environment	Analysis	Total
Learners	2	9	7	3	1	22
Teachers	4	1	3	0	2	10
Total	6	10	10	3	3	32
Engagement	Tasks	Discourse	Tools	Environment	Analysis	Total
Learners	6%	27%	21%	9%	3%	67%
Teachers	12%	3%	9%	3%	6%	33%
Total	18%	30%	30%	12%	9%	100%

Comparing the totals for the mathematics teacher educator and the teachers in the middle school cohort, the ranking order of discourse, tasks, and then environment bests capture both the mathematics teacher educator's and teachers' emphasis on pedagogy as modeled in the two sessions. Notice that analysis and tools are not in the ranking. The ranking above represents the first, third and fifth positions in frequency of notation, there was a discrepancy in the emphasis placed on analysis and tools. The mathematics teacher educator noted ongoing analysis as second, after tasks in the hierarchy, while the teachers emphasized the use of tools of discourse after tasks.

One reason that might account for such a discrepancy has to do with the fact that the middle school cohort is mathematics professional development and not a mathematics course for teachers. Teachers generally approach professional development with the expectation that there will be something that they can take back to their students or apply to their teaching. In other words, they may be looking for tools. For example in Reflection Survey 1, two of the six teachers in the middle school cohort, in response to the prompt "Drawing on what you have learned in prior mathematics content sessions, what have you done to support student thinking in your

classroom”, listed a variety of activities they, the teachers, had done during the sessions, and reported that they use them in their own classes. In fact about 35% of the 41 teachers surveyed mentioned either a specific activity or tool that was used in the mathematics teacher education. Two teachers from the upper elementary cohort noted in Reflection Survey 1 that they had never considered using pattern blocks to help students make sense of fractions. Another teacher from that cohort also noted the use of pattern block wrote:

“When discussing fractions with my 3rd grade class, we have been looking at parts of a whole and varying what that whole is. This has strengthened their thinking about what a fraction is and has made them more aware of what the ‘whole’ is. For example, they do not automatically think of the ‘trapezoid’ as $\frac{1}{2}$ (pattern blocks), they can see it as a whole, or $\frac{1}{3}$ of  (a whole made up of a hexagon and a trapezoid).”

Re-unitizing and making sense of fractions is a major component of curriculum of the elementary cohorts. The mathematics teacher educators made use of manipulatives like pattern blocks, empty number lines, fraction strips, fraction circles, and so on in tasks intended to develop the teachers understanding of fractions. Teachers’ responses like the one above suggests that they noticed aspects that were helpful to their own learning and thus tried to adapt it to their own teaching. Furthermore, they may be attending to something they can take away.

Victor, a teacher from the middle school cohort also pointed to the importance of tools in discussing the overall goals of the mathematics teacher education and the nature of the tasks. He explained, “The overall goals are to give us examples of alternative methods... Most of the tasks are hands on. They want to put us in

situations of what the students might do in the classroom, that way when we try to do something similar within the classroom we have an understanding of what the kids are going to go through.” In the pre-session interview MTE-Karla often noted she modeled the use of a variety of tools to communicate and enhance discourse about lesson planning, student learning, thinking and understanding, and so on. Discussions about how the tasks they were doing might apply to student learning and thinking, and their own teaching often permeate class activity.

In the pre-session interview MTE-Karla explained that the intent of many of the activities, “... is to deepen their content knowledge, it is always couched more in terms of topics, you know in terms of student learning... (There are teachers)... who have supplementals¹ (supplemental mathematics credentials) who don’t feel all that confident about what they are doing...” Cathy, a teacher from the cohort, echoes MTE-Karla’s position and explained that she learned a lot from participating in the mathematics teacher education and said “I think I did (learn a lot), I also feel, since I am a 6th-grade teacher I have a k-8 credential, I felt like I knew less than the single subject people. Like I didn’t have a in depth knowledge...” MTE-Karla noted that a major distinction between the mathematics course for practicing teachers and the mathematics professional development is that the teachers in the mathematics professional development do not expect to “feel” like they are in a mathematics class, thus the approach is different.

¹ Supplemental Mathematics Credentials can be granted to holders of Multiple Subject or Standard Elementary Teaching Credentials who have 20 semester units (or 10 upper-division semester units) of non-remedial course work in mathematics from a regionally accredited community college, college, or university.

This difference in approach is highlighted in how the teachers described the mathematics teacher educator's role in the mathematics teacher education. Victor, a 7th-grade teacher explained, "They are not really treating us as students per se... it is blended. They see us as a class but also as equals... A lot of the teachers leave with an 'ah-ha' moment, (like) 'I never thought of it that way' or 'I can see why kids have trouble with that'..." This response is in stark contrast to the kinds of responses the teachers from the primary cohorts gave; they describe the mathematics teacher educators' role as a facilitator (teacher) to aid them in developing their own learning and understanding of mathematics content. Sandy, a 3rd grade teacher in the upper elementary cohort in describing MTE-Max's role said, "He is the authority and his role is to guide us." These differences in how the teachers perceive the courses and the roles the mathematics teacher educator and the teachers feel they have in the courses may account for the teachers being more attuned to tools and less attuned to the ways in which the mathematics teacher educator monitored their mathematical development. Consider the episode below and the comments from MTE-Karla, and Cathy, a teacher in the cohort. In this episode the teachers are using algebra tiles to factor polynomials.

Episode 4.7: Middle School Cohort, Session 1

Time H/m/s	Description	Mathematics Teacher Educator's Comments	Teachers' Comments
0.59.32	MTE-Karla directs them to work thought the rest of the task in their groups and sketch their answers. MTE-Karla and MTE-Tom walk around and discuss with the groups.	When I see one teaching, "helping" another I listen to the nature of the discussion.	

Episode 4.7 continued: Middle School Cohort, Session 1

1.00.21	MTE-Karla joins the discussion of Chuck's group. She sits and joins them.	Here I wanted to see if she was following his explanation, and how what he was saying was supported by the blocks rather than the factoring rules he knows.	
1.02.53	MTE-Karla and Chuck discuss his solution	<p>I wanted to make sure both were involved in the conversation and not just him telling her what to do, and I wanted to see how what he was saying was supported by the blocks. So I am modeling making connections between representations and getting him to think about what students might observe about the blocks not knowing the rules for factoring</p> <p>I was trying to get them to make sense of the rules</p>	Cathy: I had to be very methodical; I did the algorithm to support my understanding of the tiles... My colleagues were helpful in helping me understand and it was comfortable because they were my peers

Cathy, the 6th-grade teacher that commented on the episode, described everyone at her table, including MTE-Karla, as her peers. Like Victor, Cathy notes that learning is a part of what is happening as they engage in tasks, but didactic teaching, whether inquiry based or not, is not at the forefront of her interpretation of what is happening. This may have been a product of the way MTE-Karla interacts with the groups. MTE-Karla often sat at the table with the groups as she listened in and joined the groups' discussion. In contrast MTE-Max and MTE-Scott often stood over the group as they listened in and commented on the group's discussion. Consider the framed taken from video of Episodes 4.3 and 4.7 in figure 4.4 below.



Figure 4.5: Below are screen grabs from episodes 3 and 7 respectively. The circles highlight the MTEs positions. April 2008.

Another difference between the elementary cohorts and the middle school cohorts that might account for this difference in the mathematics teacher educator's perceived role in the class may have to do with class size. The middle school cohort had only about 8 teachers participating (making two groups of teachers) and two mathematics teacher educators present during most of the sessions. In contrast there were five groups of three or four teachers working in the primary cohort sessions and seven groups of three or four teachers working in the upper elementary cohort. In both cases only one mathematics teacher educator was present. Perhaps the group to mathematics teacher educator ratio in this cohort afforded MTE-Karla the time to sit and discuss with the groups.

The researcher, mathematics teacher educator and teachers identified aspects of ongoing analysis in Episode 4.3. At 0.07.14, MTE-Max noted, "I am looking for work that is beneficial for the rest of the class to see. I am being purposeful in what I want students to share that might help bring out the mathematical idea of topic." In

reference to the same instance, Kelly, a 6th-grade teacher from the upper elementary cohort stated, “[sic] Him listening is a way for him to create a group discussion in the end.” While both the researcher and mathematics teacher educator pointed to aspects of ongoing analysis in Episode 4.7, the teacher, Cathy, in this episode did not note such pedagogy. Perhaps from the teachers’ perspective MTE-Karla’s interaction with the group may have seemed more like orienting herself to the group’s discussion as opposed to probing their thinking. MTE-Karla’s description of her actions included listening in on teacher discussion and checking for teacher understanding (0.59.32 and 1.00.21), monitoring the teacher’s participation in discussions (0.59.32 and 1.02.53) and probing teacher’s thinking/ understanding (1.02.53).

Both the mathematics teacher educator’s and the teachers’ discussions in the simulated recall interviews for the middle school cohort describe the mathematics teacher educator’s pedagogy predominately in terms of the teachers’ engagement as learners about 60% and 67% of the discussion respectively, which is substantially less often than what the elementary cohorts expressed. The hierarchies that describe the mathematics teacher educator and the teachers’ emphasis on the pedagogy with respect to the participants’ engagement as learners in the two class sessions, and the hierarchies with respect to the participants’ engagement as teachers, are quite different. The tables below summarize the hierarchies for each.

Table 4.21: Middle school cohort, participants engaged as teachers hierarchy from most to least emphasized.

MTE	Analysis	Discourse	Tasks	Environment	Tools
Teacher	Tasks	Tools	Analysis	Discourse	Environment

Table 4.22: Middle school cohort, participants engaged as learners hierarchy from most to least emphasized.

MTE	Discourse	Tasks	Analysis	Tools	Environment
Teacher	Discourse	Tools	Environment	Tasks	Analysis

If we take into account the teachers' expectation that there will be something that they can take back to their students or apply to their teaching it would make sense that they would give greater importance to the way in which the mathematics teacher educator models collective inquiry process with respect to tasks and tools. Similarly, when the participants are engaged as learners, the discussions and the tools help them make sense of the tasks. As Victor, one teacher pointed out, they (the teachers) can draw upon their experiences in learning a particular mathematical idea in the teacher education, which involved discussions and the use of tools they used in making sense of the concepts in their own teaching. Recall, Victor noted the importance of tasks in helping teachers experience what their students might experience. Victor's discussion echoes MTE-Karla's comments in the pre-session interview describing how the participants engage in mathematical tasks (as learners) and then discuss the task or engage in a new task (as teachers) that stimulates the teachers to discuss and make

connections between the math they themselves are learning and teaching. MTE-Karla said:

For instance in the class tomorrow... we engage in the mathematics and then we do always come to use this catch-all term of curriculum connection and sometimes that means they actually look in their curriculum to see how it's treated there... sometimes curriculum connections just means... while seated at your table you talk about which, for the grade level you teach, which pieces of this are a fit and where? Sometimes the curriculum connections is where we get them to try it on to actually get them to actually go and try them in their class and ask them to do and usually there is some time for them to talk about how they feel and how they might modify it.

A major difference between the elementary and the middle school cohorts is that in the elementary school cohorts mathematics teacher educator more often than not the mathematics learning is couched in terms of their own understanding and then student understanding whereas the middle school cohort considers the students' understanding first. In the pre-session interview, MTE-Max describes a typical learning sequence as follows:

“The problem given out that's going to be challenging for them... they're going to be given a little bit of time to think by themselves... a lot of time to work with others (in their groups), with me asking questions. Hopefully that furthers their thinking with both me asking questions to the individual group and me asking questions to the whole group... and that furthers their thinking about where they are with these ideas. So I think that idea of starting with a challenging problem to the students that gets them think about the conceptual mathematics, giving them to think by themselves, also gives them time to hear what others think after organizing their own thoughts. Coming back together and comparing ideas about what went around different groups and then me orchestrating that conversation in such a way to help them build those connections because a lot of times maybe not go in the direction that is as fruitful...”

However, in the middle school cohort the mathematics teacher educator discusses a typical sequence of event in the reverse order. In the pre-session interview MTE-Karla noted, “Even when our motive is to deepen their content knowledge, it is always couched... in terms of student learning...” In the pre-session interview MTE-Karla noted that an instructional goal for the first session of the middle school cohort was to think about factoring polynomials using algebra tiles. During the first session she reminded the teachers of the article they discussed in the previous session, “Using Algebra Tiles to Multiply Monomials and Binomials: The case of Monique Butler” (Stein, Smith, Henningsen, & Silver, 2000) and noted difficulties the teacher in the article faced in helping her students make sense of factoring with the tiles because they were first presented a procedure for factoring. MTE-Karla asked the teachers to try to factor in their groups using only the tiles, and not their knowledge of factoring. This instance highlights how the teachers’ mathematics learning was couched in better supporting their students’ learning to factor with respect to an area model.

In general, the perspectives on what MTE-Karla models about practice in the two sessions were not aligned with respect to emphasis both MTE-Karla and the teachers put on what they believe the mathematics teacher educator modeled about effective mathematics pedagogy. Yet, teachers in the middle school cohort, like the teachers in the primary cohort, report having incorporated some aspects of the instruction they experienced into their own instructional practice. Though MTE-Karla, the middle school cohort facilitator, did not emphasize explicitly modeling creation of a learning environment that fosters the development of each teacher’s mathematical

power, specifically with respect to “wait time” and “letting teacher struggle with difficulty”, the teachers, in Reflection Survey 1, noted that they now incorporate group discussion and think time. The differences in perspectives may be due to teachers’ expectations for mathematics professional development and the way the mathematics teacher educator engages with the teachers. The former and the later will be elaborated on in the next section.

Perspectives on the Mathematics Teacher Education in General

In the pre-session interviews the mathematics teacher educators from all three cohorts, primary elementary, upper elementary and middle school, suggested that an overarching goal of the mathematics teacher education programs they facilitate is to deepen the teachers’ mathematical content knowledge by engaging the teachers in the mathematics as learners. There is a sentiment among the mathematics teacher educators that if the teachers have a deeper understanding of the mathematics content they will be better prepared to present challenging tasks and ask questions that help students move forward in their thinking. The practicing teachers interviewed echo this sentiment. When asked about the educational goals of the mathematics teacher educator, Rita, a 1st and 2nd grade teacher from the primary cohort, noted that the goal of this course for teachers is, “to understand the math at a higher level and how numbers work so that we can see how that relates to the kids and how they learn.” Cathy, a 6th-grade teacher from the middle school cohort suggested that in order to transform what she knows about mathematics content and research into practice she has to know the math “at a deeper level” and analyze her own understanding. Cathy

noted her role in the class is to “develop my understanding of mathematics so I can be a better teacher for my students...”

The table below generalizes the mathematics teacher educators’ responses from the pre-session interview and the teacher responses on Reflection Survey 1 with respect to the yearlong mathematics teacher education.

Table 4.23: Hierarchy for the year of mathematics teacher education.

Reflection Survey #1	Tasks	Discourse	Tools	Environment	Analysis
Pre-Session Interview w/ MTES	Tasks	Discourse	Tools	Environment	Analysis

In the pre-session interviews about half of the mathematics teacher educators’ utterances related to engaging the teachers in tasks. Across cohorts the mathematics teacher educators and the teachers put great emphasis on discourse as the teachers were engaged as learners in those tasks, specifically with respect to questioning. In the pre-session interview MTE-Scott, the primary elementary school cohort facilitator, when asked to elaborate on the kinds of differences he would expect in the teachers’ classes in years following their participation in the mathematics, he noted, “I think questioning would be one... The style of questioning that teachers employ with their students.” Eve, a teacher from the primary cohort explained “He would pose questions that would really make us think... [He would] kind of always keep us wondering, which kept us

thinking, which is what we want the kids to be doing. So not only was he trying to make us think but he was also modeling for us, asking questions.”

The practicing teachers in their responses on Reflection Survey 1 also emphasized discourse; it was the second, in the hierarchy from most to least emphasized responses. About 20% of the teachers’ responses related to discourse. In general their responses related to prompting discussion of various solution paths and sharing out, and asking open-ended questions, all of which were implicitly modeled in the sessions. The practicing teachers also pointed to the prevalence of engaging them in discourse in Reflection Survey 2b. On Reflection Survey 2b the teachers responded to a series of statements related to inquiry-based or reform instruction and scaled each based on the frequency of their MTE engaging in such acts. Approximately 42% of the 38 teachers who responded noted the item that read “The mathematics teacher educator promotes discourse/conversations between participants” as one of the aspects of his pedagogy they find to be most effective. The table below generalizes the mathematics teacher educators’ and the teachers’ responses from the stimulated recall interviews with respect to the two sessions for each cohort.

Table 4.24: Hierarchy for the six sessions (two for each cohort).

SRI w/ PTs	Discourse	Tools	Tasks	Environment	Analysis
SRI w/ MTEs	Discourse	Analysis	Tasks	Environment	Tools

In the hierarchy discourse, tasks, and environment from most to least emphasized is consistent for the mathematics teacher educators and the teachers.

Discourse, tasks, and environment represent the first, third and fourth positions in the hierarchy, there was discrepancy in the second and last position. The mathematics teacher educators emphasized analysis second most often while the teachers emphasized the use of tools. Again, this difference might point to practicing teachers' expectations of a "take-away" from mathematics teacher education. Another reason might be related to the fact that when the mathematics teacher educators engage in ongoing analysis, it is an internal process and is generally not explicitly pointed to while the teachers are engaged in activity. In the stimulated recall interviews, the mathematics teacher educators called out various subcategories related to ongoing analysis while the practicing teachers pointed to one or none. In the researchers coding of the sessions it was also the case that ongoing analysis was not apparent. Consider the episodes below coupled with commentary from the mathematics teacher educator and the teachers. The highlighted codes refer to categories related to ongoing analysis as the teachers are engaged as learners. The subcategories for ongoing analysis are as follows:

The MTE engages in ongoing analysis (OA) of teaching and learning by

- (1) Listening in on teacher discussion or looking in on student's work.
- (2) Checking for teacher understanding.
- (3) Monitoring the teacher's participation in discussions and deciding when and how to encourage each teacher to participate.
- (4) Probing teacher's thinking/ understanding. Trying to understand the teacher's thinking.
- (5) Poll to gauge where teachers are in their activity

Episode 4.8: Upper Elementary Cohort, Session 1

Time h/m/s	Researcher	MTE-Max	PT-Kelly	Description	MTE Comments	PT Comments
0.48.15	SMA2 STD1 SCD1 SOA1 SLE1 SLE2	SMA2 STD1 SCD1 SOA1 SLE1 SLE2 SOA2 SOA4	SMA2 STD1 SCD1 SLE2 SOA4	MTE-Max asks them to consider the meaning of the $\frac{1}{2}$ and the $\frac{1}{3}$ in both drawings. He guides them in trying to think through Sandy's work and discussing the referent unit of each of the numbers sentence Sandy has drawn. And then ask the teachers to discuss the drawing in their groups to convince themselves. As the groups discuss MTE-Max walks around and listens in on and discusses with the groups	44 – There is so much mathematics and representation in Sandy's thinking so I want to give them time to think about it. 49 – I am asking questions to check for understanding and listening for that "choral" response, I am not getting it so I ask them to discuss in groups, during this time they get to talk to each other and I get to hear where they are struggling	37 – Kelly: He asks the teachers to discuss the drawing in our groups, then in whole class. The discussion with peers, it is team work the process helps us work it out, then he brings it together

Episode 4.9: Upper Elementary Cohort, Session 2

Time h/m/s	Researcher	MTE-Max	PT-Sandy	Description	MTE Comments	PT Comments
0.24.00	SMA1 SLE2 SCD1 SOA1 SLE1	SLE2 SOA1 SOA4	SLE2 SOA1	MTE-Max asks the teachers to write the number sentence that describes the student's thinking and discuss in their groups. MTE-Max gives the teachers a few minutes to think about it and discuss with their groups. MTE-Max walks around and listens in on and discusses with the groups	24 – I give them the 2 min because I am not getting a good read on that. I go around to find out what they are thinking.	50 – Sandy: A few minutes to think about is helpful, some people need quiet time, it also preserves different learning styles 51 – Sandy: He is walking around and listening

Episode 4.10: Middle School Cohort, Session 1

Time h/m/s	Researcher	MTE-Karen	PT-Cathy	Description	MTE Comments	PT Comments
0.59.32	SCD1 STD1 SLE1 SOA1 SLE2	SOA1 SOA2 SOA3		MTE-Karen directs them to work thought the rest of the task in their groups and sketch their answers. MTE-Karen and MTE-Tom walk around and discuss with the groups.	1.01 – When I see one teaching, "helping" another I listen to the nature of the discussion.	
1.00.21	SOA1	SOA1 SOA2		MTE-Karen joins the discussion of Chuck's group. She sits and joins them.	1.01 – Here I wanted to see if she was following his explanation, and how what he was saying was supported by the blocks rather than the factoring rules he knows.	
1.02.53	SLE1	SOA3 SOA4 STD3 SMA3 SMA2	STD3 SMA3	MTE-Karen and Chuck discuss his solution	1.03 – I wanted to make sure both were involved in the conversation and not just him telling her what to do, and I wanted to see how what he was saying was supported by the blocks. So I am modeling making connections between representations and getting him to think about what students might observe about the blocks not knowing the rules for factoring 1.04 – I was trying to get them to male sense of the rules	23 – Cathy: I had to be very methodical; I did the algorithm to support my understanding of the tiles... My colleagues were helpful in helping me understand and it was comfortable because they were my peers

In the episodes above there are many instances where events were coded with the string of letters SOA and followed by a number. This code characterizes the mathematics teacher educator's instructional act as ongoing analysis where the teachers were engaged as learners. Notice that the mathematics teacher educator notes this instructional move under a variety of subcategories (indicated by the numbers 1 through 4 after the code SOA) more frequently than the researcher and the practicing teachers. In such instances the mathematics teacher educator suggested that part of listening sometimes includes (a non verbal) checking for teacher understanding like in Episode 4.8 and Episode 4.10 (0.59.32 and 1.00.21). Listening also included trying to understand what the teachers were thinking as in Episode 4.8, Episode 4.9 and Episode 4.10 (1.02.53). Likewise, listening may have been to monitoring the teacher's participation in discussions and decide when and how to encourage each teacher to participate as in Episode 4.10 (0.59.32 and 1.02.53). Sandy, an upper elementary school teacher, reported that her mathematics teacher educator took notes as he listened in on the discussions of the groups, but suggested that at this point in her development as a teacher she did not have enough experience and the vast mathematical background to attend to student thinking in the same way in her own classes.

Also notice that SOA1, which characterizes events where the mathematics teacher educator listens in on teacher discussion or is looking in on the teachers' work, seemed most apparent from all perspectives. While instances where the mathematics teacher educator engaged in ongoing analysis were not implicitly noted by the teachers nor explicitly pointed out by the mathematics teacher educators, teachers may learn

about this aspect of teaching by being engaged in the class over the course of a year. In the pre-session interview MTE-Scott, reported that a teacher expressed to him that now he really tries to make sense of his students' work.

In Reflections Survey 1 and the stimulated recall interviews with the teachers, the participants noted that they now sought the input of their colleagues in lesson planning, presenting content and reflecting on the effectiveness of lessons. It is important to point out that neither the mathematics teacher educators nor the teachers suggested that such interactions were explicitly discussed as something they should do in their professional work. It is significant that the mathematics teacher educators did not mention in the pre-session interviews the modeling of this aspect of teaching practices; it became salient for the mathematics teacher educators in the stimulated recall interviews. Some teachers report having adopted some of these collective inquiry activities. For both teachers and mathematics teacher educators, this aspect of practice does not seem to be in the foreground, thus suggesting the teachers may have learned this through lived experience.

Summary

As part of this study, I worked to theoretically unpack the notion of instructor modeling in mathematics teacher education, specifically, what teachers learn about the practice of teaching by participating in mathematics courses for practicing teachers. The first result highlighted that the participants in the mathematics teacher education in this study are engaged as both learners and as teachers of mathematics, which led to the emergence of the analytical framework. The analytical framework elaborates on the

NCTM *Professional Standards for Teaching Mathematics* as they apply to mathematics teacher education under the categories of tasks, discourse, tools, environment and analysis, with respect to participants' engagement as learners of mathematics and as teachers of mathematics in mathematics courses for practicing teachers.

Analysis of the interviews, both the pre-session and the stimulated recall, with the mathematics teacher educators provided opportunities to gain insight in to *what the mathematics teacher educators believe they model*. The analysis of the interview data and survey data from the teachers provided opportunities to gain insight into *what practicing teachers notice about the mathematics teacher educators' pedagogy and identify as effective mathematics teaching*. In general, the mathematics teacher educators and the teachers noted that the mathematics teacher educator modeled instruction that is consistent with the categories in the analytical framework, but with varying degrees of emphasis. Integrating these perspectives highlighted the *ways these perspectives are aligned in mathematics courses for practicing teachers*.

Across Cohorts. With respect to the year of mathematics teacher education, the pre-session interviews with the mathematics teacher educators and the participant Reflection Survey 1 highlighted the importance and emphasis the mathematics teacher educators and the teachers put on instructional acts that they felt modeled effective mathematics pedagogy across cohorts. The hierarchies of emphasis on the teaching actions described in the analytical framework were the same for both mathematics teacher educators and teachers. Across cohorts the mathematics teacher educators and

the teachers highlighted tasks, discourse, tools, environment and analysis being modeled from most to least emphasized.

Across cohorts with respect to the two sessions the stimulated recall interviews revealed similar hierarchies for both the mathematics teacher educator, but there was a discrepancy in the emphasis on instances where the mathematics teacher educator modeled ongoing analysis and the use of tools of discourse.

Table 4.25: Hierarchy for the six (two from each cohort).

SRI w/ PTs	Discourse	Tools	Tasks	Environment	Analysis
SRI w/ MTEs	Discourse	Analysis	Tasks	Environment	Tools

One reason for this discrepancy may be related to the idea that the processes that govern the mathematics teacher educators' pedagogy associated with ongoing analysis are not directly observable and therefore are often invisible to the observer. That is, while it may be clear that the MTE is listening in on teacher discussion, looking in on or even interacting with teachers as they work, the reasons that underpin such actions may not be obvious. In general, the mathematics teacher educators pointed to reasons for such actions like checking for teacher understanding, monitoring the teachers' participation in discussions and deciding when and how to encourage each teachers to participate, probing teacher's thinking/understanding to try to understand the teachers' thinking and looking of noteworthy ideas to be shared and so on. However, the teachers (and the researcher) described the same instances as only listening in. Another reason that might point to this difference is the use of tools in

mathematical investigation and discussion may be novel to some teachers, and called out more readily because it is in contrast to their previous learning and instruction.

Within Each Cohort. The design of the various mathematics teacher education programs seemed to have had some effect on the ways in which the perspectives are aligned within a class session. This difference may have to do with teachers' expectations for the sessions. In the pre-session interviews with the mathematics teacher educators and in the stimulated recall interviews with the teachers, both the mathematics teacher educators and the teachers had an expectation of developing the teachers' content knowledge (as it was a mathematics course). Thus, the perspectives on the ways in which the mathematics teacher educators model effective pedagogy were for the most part aligned in the primary and upper elementary cohorts. In the pre-session interview, the mathematics teacher educator of the middle school cohort noted the dual focus of the mathematics teacher education content and pedagogy. She noted that the mathematics is often couched in terms of student learning. The teachers in the middle school cohort likely had an expectation of a "take-away", which refers to ideas for lessons, tools, tasks and so on, drawn from the mathematics professional development.

Table 4.26 below, summarizes the results with respect to each research question.

Table 4.26: Research questions and findings.

Research Questions	Findings
(1) What do mathematics teacher educators believe they model about effective instructional practice?	<ul style="list-style-type: none"> - The MTEs believe they model mathematics instruction as they engage the PTs as learners of mathematics. Also the MTEs believe they model the collective inquiry process as they engage the PTs as professionals. - In both cases the pedagogical moves the MTEs described can be characterized with respect to worthwhile tasks, discourse, tools, learning environment and ongoing analysis - Reflecting on the year the MTEs put the most emphasis on modeling the use of worthwhile tasks and discourse. - The elementary MTEs discussed engaging the PT as learners in the majority of their discussion while the MTE for the middle school cohort discussion was about equal - The middle school MTE emphasized the use of tools over environment and analysis. While the elementary MTEs emphasized environment over tools and analysis. - The differences between the elementary and middle school MTEs was perhaps due to variables like grade level taught by the PTs, model of mathematics teacher education and the PTs and the MTE view of their respective roles in the class
(2) What do practicing teachers notice about the mathematics teacher educators' pedagogy and identify as effective mathematics teaching?	<ul style="list-style-type: none"> - The PTs note that the MTEs model mathematics instruction. - The PTs note that the MTEs model the collaborative work of teachers - In both cases the pedagogical moves the PTs reported the MTEs employed can be characterized with respect to worthwhile tasks, discourse, tools, learning environment and ongoing analysis - The PTs primarily noted pedagogy related to tasks, discourse, and learning environment.
3) In what ways do these perspectives align in mathematics courses for practicing teachers?	<ul style="list-style-type: none"> - MTEs and the PTs report the MTEs model both mathematics pedagogy and the collective work of teachers - The pedagogical moves the MTEs and the PTs reported on can be characterized with respect to worthwhile tasks, discourse, tools, learning environment and ongoing analysis. The emphasis on each varied per cohort - The way the MTE modeled the use of tasks, discourse, and learning environment was emphasized most in the MTEs and PTs discussion - The MTEs and the PTs perspectives in the elementary cohorts aligned more than the perspectives of the MTE and the PTs in the middle school cohort. - Instances where the MTE believed they were modeling facets of Ongoing Analysis are not obvious to the PTs

In the next chapter I discuss, the research question, implications for mathematics teacher education, limitations of study and future research and the contributions this study makes to mathematics teacher education.

CHAPTER 5: CONCLUSION

In earlier chapters, I argued that teachers faced considerable challenges when learning to teach in a manner recommended by mathematics education reform documents. One of these challenges was that mathematics teachers, who were likely taught in traditional classrooms, tend to teach in the manner in which they were taught. Teachers need support to construct a new conceptual model of the nature of mathematics, mathematics learning and mathematics teaching in reform classrooms. It has been suggested that in order to develop a conception of reform instruction, teachers need to experience reform instruction. Therefore, mathematics teacher educators are encouraged to teach in a manner in which they would like their teachers to teach; in other words, mathematics teacher educators are encouraged from many sources to model effective mathematics teaching.

In Chapter 2, I presented a survey of the literature on modeling. I found very few articles written about modeling practice. There were few studies in mathematics and fewer studies in professional development and mathematics education settings. I also shared theoretical constructs that were useful for organizing thinking about what individual teachers might learn from participation in student-centered, inquiry-oriented mathematics classrooms. In Chapter 4, I presented the results of an in-depth inquiry in three mathematics courses for practicing teachers. The participants were teachers in a primary elementary, upper elementary or middle school cohort. This thesis compares what mathematics teacher educators believe they are model about the teaching practice, what teachers in their classes notice being modeled and identify as effective

mathematics instruction, and what alignment there is among perspectives of mathematics teacher educators and participating teachers.

In this chapter, I present a summary of major results, reflections on learning from modeling by mathematics teacher educators, major contributions, implications for instruction, limitations of the study, and directions for future research.

Summary of Results

Result 1: As discussed in Chapter 4, an initial analysis of the pre-session interview data revealed that the mathematics teacher educators modeled more than the teaching of mathematics, they also modeled participation in the collective inquiry process of a community of mathematics teachers. The mathematics teacher educators were not explicit in their discussion about modeling the collective inquiry part of the teaching profession; however, their discussion related to developing the teachers' pedagogical content knowledge pointed to the teachers' engagement as professionals in the mathematics teacher education sessions. These results were part of the development of the analytical framework. Though the *Professional Standards for Teaching Mathematics* served as a lens for the initial analysis of the data, refinements were necessary to specifically call out and characterize the mathematics teacher educators' pedagogy as described by the mathematics teacher educators and participating teachers and as observed by the researcher. The data drawn from the class sessions and the interview data allowed me to further develop the analytical framework discussed in part 1 of Chapter 4.

The framework characterizes the mathematics teacher educators' pedagogical moves with respect to the participants' engagement as learners and teachers.

Result 2: Teachers can learn about the practice of teaching from MTE modeling. In the Reflection Survey 1 and teacher interviews, the participants noted teaching actions related to the categories worthwhile tasks, discourse, tools, and learning environment in the analytical framework. Recall, Reflection Survey 1 asked the teachers to describe what they have done in their classes to support student learning drawing from what they have learned in prior mathematics content sessions. The teachers reported that they had incorporated specific activities and tools that they found useful in their own learning in the sessions. The teachers reported that they had adopted practices like engaging students in mathematical conversation (discourse), specifically, through questioning and prompting students to share out and provide justification. The teachers also reported incorporating wait-time and letting students struggle with difficulty, facets of learning environment from the analytical framework.

Result 3: While both the mathematics teacher educators' and the participants' utterances point to the mathematics teacher educators' pedagogy as the participants were engaged as both learners of mathematics and as mathematics teachers, the emphasis was on participants' engagement as learners, as opposed to their engagement as teachers. This was more so the

case for the teachers than for the mathematics teacher educators. The teachers pointed to the pedagogy as they were engaged as a learner in about 86% in their reflections on the year and 88% in their reflections on the two sessions.

The teachers in the primary and upper elementary cohorts noted pedagogy related to their engagement as learners almost exclusively, while the middle school teachers reported it in only about 71% of their coded utterances. On average in the pre-session interviews where the mathematics teacher educators reflected on the year, they emphasized mathematics pedagogy they believe they model as the teachers were engaged as learners in about 71% of the coded utterances. In the stimulated recall where the mathematics teacher educators reflected on the two sessions noted such engagement in about 78% of the coded utterances. Like the teachers, the mathematics teacher educators from the elementary cohorts more frequently discussed instructional acts where the participants were engaged as learners. The middle school cohort facilitator, MTE-Karla, emphasized pedagogy related to the participants' engagement as teachers and as learners almost equally, reflecting the dual purpose of the mathematics teacher education. This points to the likelihood that the difference is due to the intended purposes of the mathematics professional development sessions and the mathematics courses.

Result 4: The pedagogical moves most noticed by both the mathematics teacher educators and the teachers were characterized under the categories of tasks and discourse. The mathematics teacher educators and the teachers highlighted the way the mathematics teacher educator modeled the use of tasks, specifically noticing tasks that problematized the mathematics and prompted the teachers to make connections between mathematical ideas and among representations. These tasks were seen as promoting discussions of the mathematics. Both the mathematics teacher educators and the teachers pointed to the scope of tasks, particularly the large amount of time devoted to developing mathematical concepts in learning situations. While the teachers pointed to tasks and discourse, some pointed to how the coordination of the pedagogical moves related to structuring learning environment, specifically structuring the sessions. The teachers' discussion about discourse often pointed to discourse in task situations, collaborative work on tasks and sharing ideas. The teachers specifically mentioned how the mathematics teacher educator pushed for justification and asked questions as opposed to giving answers. As a part of this, the teachers did interject a facet of the learning environment category, reporting that the mathematics teacher educators allowed them to struggle with difficulty. The teachers characterized the mathematics teacher educators' instruction as Socratic, that is, the mathematics teacher educators asked questions that helped them shape their thinking and move them forward in their thinking.

The teachers reported that they try to incorporate these facets of the mathematics teacher educators' pedagogy in their own teaching.

Result 5: There are aspects of ongoing analysis that the mathematics teacher educators believe they model that are not noticed by the teachers. There is a belief among the mathematics teacher educators and the teachers that a deeper understanding of the mathematics content better prepares teachers to present challenging tasks and ask questions that help students move forward in their thinking. However, the mathematics teacher educators' report that one of the purposes of questioning as the teachers are engaged in tasks is ongoing analysis of the teachers' thinking. The mathematics teacher educators note that their questions are driven by the need to develop conceptual models of the teachers' thinking. The teachers did not note developing conceptual models as a reason for being able to ask "good" questions, the teachers pointed to experience, mathematical knowledge, and being well prepared for the lesson as key to the mathematics teacher educators' ability to ask questions that move them forward in their thinking. This perhaps points to one of the reasons for differences in the emphasis the mathematics teacher educators and the teachers put on the category of ongoing analysis.

Reflections on Learning from Modeling

As teachers are engaged as learners in mathematics teacher education classes, teachers can develop a conceptual model for what mathematics instruction entails and expectations for how individuals (teachers and students) in the community behave. In

the case of the mathematics teacher education classes in this study, the mathematics teacher educators reported trying to work to engage the teachers in what they view as inquiry-oriented mathematics instruction. In both the stimulated recall interviews and the reflection surveys, the teachers' discussion of the pedagogy related to learning environment and discourse were cast in terms of norms of the class and pointed to the nature of the roles of the teachers (students) and mathematics teacher educators (instructors). Much of the teachers' utterances related to MTEs allowing think time or time for reflection, making the learners feel comfortable with sharing ideas or asking questions, and letting the learners struggle with a difficulty. Teachers also noted as a departure from past experiences the norm that the students are to provide justification or input during small group or whole class discussions. That is, the teachers report having learned about the collaborative nature of mathematical activity and that the authority for mathematical correctness derives from the mathematics that is developed in a community through conjecture, discussion, argumentation, and justification and not solely the mathematics teacher educator (the instructor). None of the mathematics teacher educators or teachers reported this as an explicit topic of discussion but it is possible that this discussion was part of the first few classes.

The mathematics teacher educators modeled engaging in the collective inquiry process of a professional learning community of mathematics teachers by engaging teachers in activities that are a part of the practice of teaching beyond classroom instruction. The teachers in this study after a year of engagement in the mathematics teacher education pointed to ways they incorporated aspects of the collaborative work

of teachers in their practice as a result of participating in the sessions. By virtue of engaging in collaborative lesson planning and discussion about student thinking and teaching, the teachers seem to note and adopt facets of the collective work of teachers into their own practice.

Some aspects of modeled practice seemed to have received particular attention because of the participants' dual role as teacher and learner. The teachers in mathematics teacher education engaged as learners reported attending to features of the instruction particularly as it related to their own practice. That is, teachers make note of pedagogy they experience as learners because it has the potential to inform their own professional work. In the reflection surveys and interviews the teachers reported that they attended to mathematical tasks, discursive moves and the use of tools in activity and discourse. The teachers noted that they drew from their learning experience the way the mathematics teacher educator modeled the use of tasks that motivate the development of mathematical ideas through challenging task that engage the learners intellect and provide opportunities to extend one's thinking or that allow for exploration and promote discussion about mathematics. Teachers report having adopted practices such as directing students to work and discuss in small groups, wait time, asking questions to challenge student thinking, having students share out, and the use of tools (manipulatives) in problem-solving and discussion. Specifically, they cite having adopted them because they were useful to their own learning. Whereas, a conceptual model of reform instructional practice seems to have formed as a result of lived experience with regards to classroom participation structure, teachers' intent

participation seemed to contribute to their learning about practice given what worked for them as learners.

One can also consider the apprenticeship role of the teachers. The apprenticeship role suggests that leaning practice is in the foreground. The mathematics teacher educators report explicitly and intentionally asking participating teachers to make connections between the mathematics instruction they were receiving and the teaching they would be doing. This anticipation of practice, I argue, is an intentional form of scaffolding instructional practice.

While in a class setting, the practicing teachers also have the opportunity to legitimately participate in the authentic activity of a community of mathematics teachers related to collaboration among peers, lesson planning, thinking about and reflection on student thinking, understanding, learning, and evaluating. Such activity can be couched in actual student work as derived from the classrooms of the teachers, video of teacher or learning episodes, work drawn from peers and so on. While the mathematics teacher educator facilitates the sessions, the mathematics teacher educator also acts as a fellow (perhaps more knowledgeable) teacher.

Implications for Instruction

Implication 1: Teachers can notice *and learn* about the practice of teaching from MTE modeling. Specifically, implicit modeling of the use of worthwhile tasks, discourse, and learning environment is made salient for teachers as they are engaged in mathematics teacher education. After a year of mathematics teacher education the teachers in this study

were able to develop their knowledge about content and inquiry-oriented mathematics pedagogy by being engaged as learners in the mathematics teacher education. This suggests the importance of long-term engagement in teacher education to make salient desired practices.

Implication 2: Teachers can notice *and learn* about the practice of teaching from their own experiences as learners in mathematics teacher education sessions. Teachers can draw on aspects of learning environment that facilitated their mathematical development and adopt them in their own teaching. Reflective activity might help bring specific pedagogical moves to the forefront, particularly as teachers reflect on “what works” for them as learners.

Implication 3: A mathematics teacher educator should be aware that he or she is modeling aspects of practice that go beyond the teacher-student interactions of a particular mathematics lessons. Participating teachers reported an awareness of the collective work of teachers and of the need to plan lessons differently than for their current practice. Not all mathematics classes for teachers would be structured as these were. Yet, the message is that the potential exists for forming an image of the professional practice of a teacher beyond classroom instruction. And, in fact, teachers are forming an image of the professional practice of

teachers whether or not the mathematics teacher educators are consciously modeling this.

Implication 4: Important aspects of a reform teacher's instructional practice are largely invisible to teachers and should probably be made explicit. Building a conceptual model of student thinking often requires listening in on teachers' discussion or looking in on their work, deciding when and how to encourage each teachers to participate and probing teacher's thinking/ understanding, trying to understand the teachers' thinking and so on. Such actions are a part ongoing analysis. In the stimulated recall interviews with the mathematics teacher educators, instances when the mathematics teacher educator pointed to their questioning activity often were preceded or concurrent with discussion of ongoing analysis. For the mathematics teacher educators, ongoing analysis was emphasized second after discourse in the stimulated recall interviews. In contrast, the teachers noted occurrences of ongoing analysis least in the stimulated recall interviews. Instances where the mathematics teacher educators pointed to ongoing analysis were not obvious from the perspectives of the participants and the researcher. Though it may be impractical to make salient during activities where the mathematics teacher educator in engaging in ongoing analysis, it might be helpful to debrief and have a discussion about the teaching and learning that was experienced. That is, *explicitly* model or discuss the thoughts and

actions that underpin their pedagogical approach, and provide opportunities for practicing teachers to *reflect* on practice of the mathematics teacher educators' practice and the mathematics learning the teachers experienced.

Contributions

This research informs the body of knowledge about teaching the practice of teaching mathematics. The work described in this thesis makes recommendations regarding modeling by mathematics teacher educators based on an analysis of empirical data. It provides insight into *what* and *how* teachers learn from modeling reform teaching. Specifically, the study draws on an analysis of data from three classrooms to illustrate how explicit discussion about mathematics teaching and implicit modeling of instruction supported practicing teachers' noticing of the instruction they experienced, and how engaging in facets of the teaching profession that take place outside of their interactions with students has the potential to foster the enculturation of teacher into a professional learning community of mathematics teachers.

The study derives its strength from the fact that the participants were teaching at three levels (primary, upper elementary, and middle school). The classes were both mathematics classes and professional development classes. These differences provided some contrasts that proved to be useful in the analysis. For example, the mathematics teacher educator of the primary cohort explained that he made more aspects of his practice explicit and this may account for the better alignment between his perspective

of what was modeled and the participating teachers' perspective of what was modeled. Collectively, this analysis helps mathematics teacher educators understand what aspects of practice need to be made explicit.

Limitations of Study and Future Research

For the most part, the perspectives of the mathematics teacher educators and the teachers on their shared experiences were aligned. This study represents a snap shot of the end of the first of a two-year program for the elementary school cohorts and the end of the yearlong program for the middle school cohort. It is possible that the teachers' perspectives could have been colored by explicit discussion in previous sessions. Another reason could be that all of the teacher education programs facilitated by same professional development group and perhaps what the teachers notice is a product of the setting.

However, there were discrepancies in the perspectives across cohorts, specifically related to analysis and ongoing analysis. Across cohorts the participants did not emphasize the pedagogy that the mathematics teacher educator described as modeling ongoing analysis and fostering and monitoring the learning environment in the same way. It is unclear if the sole reason for this differing perspective is that the internal processes are hard for an observer to gauge. It is possible that years of experience engaging in inquiry learning can be a factor in ones' perspective, thus it would be interesting to consider elementary groups perspectives at the end of their second year. Another factor could be the grade level that the teachers teach. In the pre-session interview, MTE-Scott alluded a difference in working with primary verses

elementary school teacher and perhaps such difference extend middle school teachers as well. Also, class size may also play a factor.

The teachers in the elementary cohorts were finishing the first of a two-year program. It would be interesting to investigate in what ways the perspectives are aligned with respect to a given set of classroom sessions at the end of two years. Further with respect to teachers' years of engagement in teacher education for practicing teachers, a future study could consider perspectives of teachers at the beginning, middle and end of their engagement in mathematics teacher education. It also, would be interesting to investigate teacher education programs from different professional development groups. Such a study might include how perspectives are aligned for teachers and mathematics teacher educators per program and what pedagogical moves, if any, are common and identified across programs.

Mathematics teacher education pedagogy is multifaceted. The mathematics teacher educator must devise ways to prompt teachers' development of content and pedagogical knowledge, and their enculturation into the collective work of teachers. To do this the mathematics teacher educator must take on a few roles. The mathematics teacher educator is a content expert, teaching expert and a fellow teacher. Depending on the focus of the class and the needs of the teachers, some roles are more predominant than others. This study provides a step towards providing support for the hard work of mathematics teacher educators.

APPENDIX

Appendix A: NCTM *Professional Standards for Teaching Mathematics* (1991)

1. Worthwhile Mathematical Tasks

The teacher of mathematics should pose tasks that are based on

- Sound and significant mathematics
- Knowledge of students' understandings, interests, and experiences
- Knowledge of the range of ways that diverse students learn mathematics

and that

- Engage students' intellect
- Develop students' mathematical understandings and skills
- Stimulate students to make connections and develop a coherent framework for mathematical ideas
- Call for problem formulation, problem solving, and mathematical reasoning
- Promote communication about mathematics
- Represent mathematics as an ongoing human activity
- Display sensitivity to, and draw on, students' diverse background experiences and dispositions
- Promote the development of all students' dispositions to do mathematics

2. Teacher's Role in Discourse

The teacher of mathematics should orchestrate discourse by

- Posing questions and tasks that elicit, engage, and challenge each student's thinking
- Listening carefully to students' ideas
- Asking students to clarify and justify their ideas orally and in writing
- Deciding what to pursue in depth from among the ideas that students bring up during a discussion
- Deciding when and how to attach mathematical notation and language to students' ideas
- Deciding when to provide information, when to clarify an issue, when to model, when to lead, and when to let a student struggle with a difficulty
- Monitoring students' participation in discussions and deciding when and how to encourage each student to participate

3. Students' Role in Discourse

The teacher of mathematics should promote classroom discourse in which students

- Listen to, respond to, and question the teacher and one another
- Use a variety of tools to reason, make connections, solve problems, and communicate
- Initiate problems and questions
- Make conjectures and present solutions
- Explore examples and counterexamples to investigate a conjecture
- Try to convince themselves and one another of the validity of particular representations, solutions, conjectures, and answers
- Rely on mathematical evidence and argument to determine validity

4. Tools for Enhancing Discourse

The teacher of mathematics, in order to enhance discourse, should encourage and accept the use of

- Computers, calculators, and other technology
- Concrete materials used as models
- Pictures, diagrams, tables, and graphs
- Invented and conventional terms and symbols
- Metaphors, analogies, and stories
- Written hypotheses, explanations, and arguments
- Oral presentations and dramatizations

5. Learning Environment

The teacher of mathematics should create a learning environment that fosters the development of each student's mathematical power by

- Providing and structuring the time necessary to explore sound mathematics and grapple with significant ideas and problems
- Using the physical space and materials in ways that facilitate students' learning of mathematics
- Providing a context that encourages the development of mathematical skill and proficiency
- Respecting and valuing students' ideas, ways of thinking, and mathematical dispositions

and by consistently expecting and encouraging students to

- Work independently or collaboratively to make sense of mathematics
- Take intellectual risks by raising questions and formulating conjectures
- Display a sense of mathematical competence by validating and supporting ideas with mathematical argument

6. Analysis of Teaching and Learning

The teacher of mathematics should engage in ongoing analysis of teaching and learning by

- Observing, listening to, and gathering other information about students to assess what they are learning
- Examining effects of the tasks, discourse, and learning environment on students' mathematical knowledge, skills, and dispositions

in order to

- Ensure that every student is learning sound and significant mathematics and is developing a positive disposition toward mathematics
- Challenge and extend students' ideas Adapt or change activities while teaching Make plans, both short- and long-range
- Describe and comment on each student's learning to parents and administrators, as well as to the students themselves

Appendix B: Analytical Framework

<u>Professional Community of Mathematics Teachers (T)</u>	<u>Mathematics Classroom Community of Teachers (S)</u>
The Mathematics Teacher Educators (MTEs) model the Collective Inquiry Process of a professional learning community of mathematics teachers by engaging teachers in activities that are a part of the practice of teaching related to lesson planning, thinking about student thinking, understanding, learning, evaluating and so on. And increasing their participation in the community	The Mathematics Teacher Educators Model Mathematics instruction (or the teaching of mathematics) by engaging teachers in mathematical activity as learners.

Utterances and events are characterized by noting the level of engagement (as teachers or learners), the category (task, discourse, tools, environment or analysis) and the associated subcategory.

Type of engagement, (T) – Teacher (S) – Learner	Category, (MA) –Mathematical Activity/Task (CD) – Classroom Discourse (TT) – Tools of Discourse (LE) – Learning Environment (OA) – Ongoing Analysis	Subcategory
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EX: (T, CD, 5)

The participants are engaged as teachers in the collective inquiry process. The mathematics teacher educator is orchestrating discourse and the development of ideas by providing information

<u>W o r t h w h i l e M a t h e m a t i c a l T a s k s / A c t i v i t y (M A)</u>	
<u>Professional Community of Mathematics Teachers (T)</u>	<u>Mathematics Classroom Community of Teachers (S)</u>
The MTE poses activities intended to develop teachers' pedagogical content knowledge. Such tasks	The MTE poses tasks intended to develop teachers' mathematics content knowledge. Such tasks
(1) Motivate the development of the teachers' understanding of the students' mathematics through exploration and/or experimentation with students.	(1) Motivate the development of mathematical ideas through challenging task that engage the learners intellect and provide opportunities to extend ones thinking or for exploration
(2) Problematize the students mathematics to promote inquiry, call for problem formulation, problem solving, and mathematical reasoning	(2) Problematize the mathematics to promote inquiry and/or call for problem formulation, problem solving, and mathematical reasoning, often involving context
(3) Stimulate teachers to discuss an make connections between the math they themselves are learning and teaching	(3) Stimulate teachers to make connections between mathematical ideas, make sense of or consider alternate solutions paths
(4) Promote discussion about students' mathematics	(4) Promote discussion about mathematics

C l a s s r o o m D i s c o u r s e (C D)	
<u>Professional Community of Mathematics Teachers (T)</u>	<u>Mathematics Classroom Community of Teachers (S)</u>
<p>The MTE mediates discourse about practice to focus discussion on teaching, student learning or student thinking by</p> <p>(1) Directing the teaches to work/discuss in small groups</p> <p>(2) Questioning to</p> <ul style="list-style-type: none"> • Guide or Shape Thinking – consider “x” • Promote inquiry – “why” questions • Challenge student thinking – “how” questions <p>(3) Sharing Out, the MTE might ask the teachers (during class discussion) to:</p> <ul style="list-style-type: none"> • Rephrase/Repeat what was said • Share solutions, or offer another solution • Provide input, justification or explanation <p>(4) Rephrasing (MTE or Teacher) and perhaps adding on to</p> <ul style="list-style-type: none"> • Use teacher(s) contributions to shape discussion • Summarize or highlight a recently made point or discussion <p>(5) Providing Information to</p> <ul style="list-style-type: none"> • Guide discussion, give directions, focus attention or guide thinking • Provide a timeline for class session or sessions • Review, to remind teachers of previous learn ideas in the present or past meeting sessions • Provide information or insight in the form of an explanation. • Offer alternate solution paths not brought up by the teachers 	<p>The MTE orchestrates discourse about mathematical ideas to focusing discussion on concepts and solution paths instead of answers by</p> <p>(1) Directing the teaches to work/discuss in small groups</p> <p>(2) Questioning to</p> <ul style="list-style-type: none"> • Guide or Shape Thinking – consider “x” • Promote inquiry – “why” questions • Challenge teachers’ thinking – “how” questions <p>(3) Sharing Out, the MTE might ask the teachers (during class discussion) to:</p> <ul style="list-style-type: none"> • Rephrase/Repeat • Solution, or another solution • Input, justification or explanation <p>(4) Rephrasing (MTE or Teacher) and perhaps adding on to</p> <ul style="list-style-type: none"> • Use teacher(s) contributions/work to shape discussion • Summarizing or highlighting a recently made point or discussion <p>(5) Providing Information to</p> <ul style="list-style-type: none"> • Guide discussion, give directions in order to focus attention, guide thinking or solve a problem • Provide a timeline for class session or sessions • Review, to remind teachers of previous learn ideas in the present or past meeting sessions • Provide information in the form of a hint, answer, explanation, etc. • Offer alternate solution paths not brought up by the teachers

<u>T o o l s o f D i s c o u r s e (T D)</u>	
<u>Professional Community of Mathematics Teachers (T)</u>	<u>Mathematics Classroom Community of Teachers (S)</u>
<p>The MTE and Teachers make use of a variety of tools to communicate and enhance discourse about lesson planning, student learning, thinking and understanding, and so on. Tools Such as:</p> <p>(1) Student work/Classroom video</p> <p>(2) Records of ideas/concepts that emerge, and/or mathematical notation and language attached to students' ideas</p> <p>(3) Literature, research, articles, texts</p>	<p>The MTE and Teachers make use of a variety of tools to reason, make connections, solve problems, communicate and enhance discourse. Tools Such as:</p> <p>(1) The teachers' work, hypotheses, explanations and arguments</p> <p>(2) Records of ideas/concepts that emerge and/or mathematical notation and language that shape academic language to teachers' ideas – 5, 6</p> <p>(3) Manipulatives, Pictures, diagrams, tables, graphs, etc. --- 7</p>
<u>Learning Environment (LE)</u>	
<u>Professional Community of Mathematics Teachers (T)</u>	<u>Mathematics Classroom Community of Teachers (S)</u>
<p>The MTE creates a learning environment that fosters the development of each teacher's knowledge for teaching</p> <p>(1) Small group discussion <u>with</u> the instructor</p> <p>(2) Time for reflection or to work independently or in small groups on <u>new</u> material</p> <p>(3) Make opportunities to share or question/ open the floor</p> <ul style="list-style-type: none"> ○ Teachers have the opportunity to <ul style="list-style-type: none"> ▪ Pose questions to the MTE or other Teachers ▪ Provide justification or elaboration for questions conjectures or answers put forth by their peers ▪ Share ideas, thinking or solutions with class 	<p>The MTE creates a learning environment that fosters the development of each teachers mathematical power by</p> <p>(1) Small group discussion <u>with</u> the instructor</p> <p>(2) Allowing think time, time for reflection or time to work independently or in small groups on <u>new</u> material</p> <p>(3) Making teacher feel comfortable to share or question and opportunities to do so</p> <ul style="list-style-type: none"> ○ Teachers have the opportunity and feel comfortable as they <ul style="list-style-type: none"> ▪ Pose questions to the MTE or other Teachers ▪ Provide justification or elaboration for questions conjectures or answers put forth by their peers ▪ Share ideas, thinking or solutions with class <p>(4) Differentiation instruction</p> <p>(5) Let teachers struggle with a difficulty</p>

Ongoing Analysis (OA)	
<u>Professional Community of Mathematics Teachers (T)</u>	<u>Mathematics Classroom Community of Teachers (S)</u>
<p>Examining effects of the tasks, discourse, and learning environment</p> <p>(1) The <u>MTE</u> engages in ongoing analysis to promote teaches participation in the Collective Inquiry Process of a professional learning community of mathematics</p> <ul style="list-style-type: none"> ○ Listening in on teacher discussion or looking in on student's work ○ Poll to gauge where teachers are in their activity ○ Monitoring the teaches' participation in discussions and deciding when and how to encourage each teacher to participate <p>(2) The <u>teachers and MTE</u> gather information about students to discuss/consider, what students are learning and how learning can be facilitated, and to examine the effects of the tasks, discourse, and learning environment on students' mathematical knowledge, skills, and dispositions</p>	<p>The MTE engages in ongoing analysis of teaching and learning by</p> <p>(1) Listening in on teacher discussion or looking in on student's work</p> <p>(2) Checking for teacher understanding</p> <p>(3) Monitoring the teaches' participation in discussions and deciding <u>when</u> and <u>how</u> to encourage each teachers to participate</p> <p>(4) Probing teacher's thinking/ understanding. Trying to understand the teachers' thinking</p> <p>(5) Poll to gauge where teachers are in their activity</p>

Appendix C: Reflection Survey 1

Reflection Survey 1

Date _____

Directions: Reflect through writing a brief response to the sessions

Drawing on what you have learned in prior mathematics content sessions, what have you done to support student thinking in your classroom? If possible, please share a specific example in context (e.g., grade level of students, course and topic)

Appendix D: Reflection Survey 2

Reflection Survey 2

Date _____

- 1) Male ____ Female ____
- 2) How many years have you been a teacher? _____
- 3) What grade(s) do you currently teach? _____
- 4) What grades have you taught in the past? _____
- 5) What credentials do you hold? _____
- _____

In this section describe the frequency of your mathematics teacher educator's instructional acts in the class sessions. Circle the options, 4 = Often, 3 = Sometimes, 2 = Seldom, 1 = Never

The Mathematics Teacher Educator:	Never	Seldom	Sometimes	Often
6) Introduces mathematical ideas concepts by first presenting a challenging related mathematical task	1	2	3	4
7) Asks participants for justification and explanations for answers, methods, and results	1	2	3	4
8) Promotes discourse/conversations between participants	1	2	3	4
9) Problematizes or promotes inquiry into the mathematics	1	2	3	4
10) Creates/Sustains a challenging task environment	1	2	3	4
11) Allows for adequate wait time	1	2	3	4
12) Presents tasks with multiple entry points	1	2	3	4
13) Asks for elaboration of participants' thinking/responses	1	2	3	4
14) Encourages class discussion of non-standard solution paths	1	2	3	4
15) Asks participants to work in small groups to complete tasks	1	2	3	4
16) Discuss multiple solution paths	1	2	3	4
17) Asks participants to think through ideas or concepts independently	1	2	3	4
18) Asks participants to think through ideas or concepts in groups	1	2	3	4
19) Asks participants to reflect on mathematics concepts	1	2	3	4

in writing or in group discussion

20) Makes use of multiple representations in class discussion (e.g. symbolic, pictorial, graphical etc.)	1	2	3	4
21) Uses participants' contribution to guide class discussion	1	2	3	4
22) Incorporates mathematical language in class discussions	1	2	3	4
23) Promotes sense making of mathematical ideas	1	2	3	4
24) Focuses on the mathematical ideas and concepts that underlie mathematical procedures and algorithms	1	2	3	4

Appendix E: Reflection Survey 2b

Reflection Survey 2b

Date _____

- 1) Male ____ Female ____
- 2) How many years have you been a teacher? _____
- 3) What grade(s) did you teach in the 2007-08 school year? _____
- 4) What grades have you taught in the past? _____
- 5) What degree(s)/credential(s) do you hold? _____

In describing the class for the last school year, describe the frequency of your mathematics teacher educator's instructional acts in the class sessions. Circle the options, 1 = Never, 2 = Seldom, 3 = Sometimes, 4 = Often

The Mathematics Teacher Educator:	Never	Seldom	Sometimes	Often
6) Introduces mathematical ideas/concepts by first presenting a challenging, related mathematical task	1	2	3	4
7) Emphasizes the importance of memorizing procedures and algorithms	1	2	3	4
8) Asks participants for justification and explanations for answers, methods, and results	1	2	3	4
9) Promotes discourse/conversations between participants	1	2	3	4
10) Presents a specific procedure for a problem type and expects to see you practice that procedure in successive problems	1	2	3	4
11) Creates/Sustains a challenging task environment	1	2	3	4
12) Promotes inquiry into the mathematics	1	2	3	4
13) Allows for adequate wait time	1	2	3	4
14) Presents tasks with multiple entry points	1	2	3	4
15) Accepts answers without justification	1	2	3	4
16) Encourages class discussion of non-standard solution paths	1	2	3	4
17) Asks participants to work in small groups to complete tasks	1	2	3	4
18) Asks for elaboration of participants' thinking/responses	1	2	3	4
19) Discusses multiple solution paths	1	2	3	4

20) Asks participants to think through ideas or concepts independently	1	2	3	4
21) Limits your time spent grappling with mathematical ideas in order to introduce standard methods of solving mathematical tasks	1	2	3	4
22) Asks participants to think through ideas or concepts in groups	1	2	3	4
23) Asks participants to reflect on mathematics concepts in writing or in group discussion	1	2	3	4
24) Makes use of multiple representations in class discussion (e.g. symbolic, pictorial, graphical etc.)	1	2	3	4
25) Uses participants' contributions to guide class discussion	1	2	3	4
26) Incorporates mathematical language in class discussions	1	2	3	4
27) Promotes sense-making of mathematical ideas	1	2	3	4
28) Focuses on the mathematical ideas and concepts that underlie mathematical procedures and algorithms	1	2	3	4

29) Are there any instructional acts not listed above that you have noted about your mathematics teacher educator's instruction? If so, list them and describe their frequency as 1 = Never, 2 = Seldom, 3 = Sometimes, 4 = Often

a.

b.

c.

d.

⋮

30) Of the instructional acts listed above (including the ones you noted on item 29) list the item number (or letter) associated with what you feel were the 5 **most** effective instructional acts of your mathematics teacher educator

31) Of the instructional acts listed above (including the ones you noted on item 29) list the item number (or letter) associated with what you feel were the 5 **least** effective instructional acts of your mathematics teacher educator

Appendix: F: Teachers' responses from Reflection Survey 2b, item 29

Primary Elementary Cohort (4 out of the 13 teachers responded)

- Teacher 1
 - o When we share ideas you always feel good about them even if you realize you had misconceptions
 - o When we shared our thinking it helped you to think of new ways
 - o "Never told us right or wrong
 - o Made us slow down, take time to think
 - o Discussing how problems are solved
- Teacher 10
 - o Class work usually related to homework and vice versa
- Teacher 15
 - o Uses manipulatives materials to helps students work
- Teacher 16
 - o Asks guiding questions
 - o Introduces concepts in meaningful contexts

Upper Elementary Cohort (9 out of the 24 teachers responded)

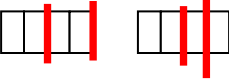
- Teacher 1
 - o Positive Feedback
- Teacher 10
 - o Uses more than one way of working through problems (independent and group work)
 - o Patience
 - o Energy
 - o Well-versed-able to explain and make concepts understandable
- Teacher 11
 - o Takes notes about student work to guide discussion
- Teacher 12
 - o Creates a challenging but safe learning environment 4
- Teacher 13
 - o Guides instruction through relevant questioning
- Teacher 14
 - o Introduces practices and other materials that are immediately useful in my classroom
- Teacher 16
 - o He was great at asking meaning questions to clarify or guide our thinking
 - o Was very well planned in his lessons
 - o He knew what he wanted to teach and how to get us where we needed to be
- Teacher 18
 - o Uses real life problems to stir students interest
- Teacher 22
 - o Strategically decides who will share out to ensure multiple representations/ strategies are shared 4

Middle School Cohort (1 out of the 2 teachers responded)

- Teacher 2
 - o Allows for multiple ways to arrive at a conclusion
 - o Encourages dialogue between groups to discuss methods used

Appendix: G: An example of a timeline.

The timeline below is from the upper elementary cohort, Session 2

Time h/m/s	Description
0.00.07	MTE-Max says that he will give the teachers 5 minutes to talk about the solution they were to examine for homework $1\frac{3}{4} \div \frac{1}{2}$. He directs them to share with their groups why the reasoning for the given solution is incorrect. MTE-Max informs them that he will ask a group to present their answer
0.01.11	MTE-Max walks around, listens in on, discusses with the groups, as he passes out papers
0.10.51	MTE-Max discusses with group 5. He asks group 5 to really think about this student's thinking. Then asks the same of group 2, and then continues to listen in and discuss with the other groups
0.15.25	MTE-Max brings the class together, acknowledges that many are not done discussing, and suggests that it might be more productive if they discuss the solution as a class. MTE-Max asks why is it that while the student's solution path is incorrect their answer is correct
0.16.23	MTE-Max says that he wants at least 3 groups to contribute before he interjects and encourages the class to give input on the solutions they hear
0.17.28	MTE-Max asks if anyone made use of a story problem to think about the task
0.17.41	Blanca from group 2 volunteers her way of thinking about the division. MTE-Max asks her to come to the board and draw it so that they can refer to it in their discussion. Blanca draws a picture to illustrate the division and explains that she drew two wholes and the notes that she only has 1 and $\frac{3}{4}$ and that she needs to count the number of half she has. So $3\frac{1}{2}$ halves 
0.18.56	MTE-Max asks if anyone has any questions about Blanca's picture
0.19.08	MTE-Max asks for a story problem. Ms. T from group 7 volunteers her story problem. Ms. T reads her story problem: Farmer John has one and three quarter of liquid fertilizer concentrate. It takes half of a gallon to make a tank. How many tanks can he make?
0.19.35	MTE-Max asks Ms. T to re-read her story problem because sometimes it takes more than one time to really understand what was going on. Ms. T reads it again
0.20.10	MTE-Max asks the class what this question is saying wrt to the picture, the story and the number sentence. The class responds, how many halves are in $1\frac{3}{4}$
0.20.25	MTE-Max writes this sentence on the board
0.20.51	MTE-Max asks the class to finish his sentence. They conclude the question is saying that there are $3\frac{1}{2}$ halves in $1\frac{3}{4}$
0.21.19	MTE-Max points them to the student work displayed on the document cam and says they are trying to mathematize the student's thinking on this problem. He asks them to characterize the student's thinking
0.21.39	MTE-Max asks a group to volunteer what they think the student is thinking. No one responds. After a pause, one teacher asks him to rephrase the question. MTE-Max re reads the question and asks them to write a number sentence that describes their thinking

0.24.00	MTE-Max asks the teachers to write the number sentence that describes the student's thinking and discuss in their groups. MTE-Max gives the teachers a few minutes to think about it and discuss with their groups. MTE-Max walks around and listens in on and discusses with the groups
0.26.34	MTE-Max brings the class back together and asks group 5 to explain their understanding of the solution. They suggest that the student's work reflect the division $1\frac{3}{4}$ divided by 2
0.27.40	The teachers discuss how while the solution to that division problem is $\frac{7}{8}$ yet the student wrote the answer as $3\frac{1}{2}$
0.28.00	A teacher from group 4 says that the student is answering how many fourths are in $\frac{7}{8}$ *.*
0.28.28	MTE-Max rephrases what was said, then asks what number sentence does this represent. As a class they decide that the student is counting the number of $\frac{1}{4}$ are in $\frac{7}{8}$. A discussion ensues about the meaning of division in this case and the related number sentence
0.31.28	MTE-Max asks about the division by $\frac{1}{4}$. Ms. D from group 2 explains that they are counting $\frac{1}{4}$ pieces. Another teacher from group 7 further elaborates that they did not explain where the $3\frac{1}{2}$ came from
0.32.35	MTE-Max summarizes the discussion and notes that instead of dividing by $\frac{1}{2}$ they divided by 2,
0.33.40	MTE-Max asks about the correspondence between $3\frac{1}{2}$ and $\frac{1}{2}$, and $\frac{7}{8}$ and $\frac{1}{4}$ and asks them discuss in their groups for 2 minute. MTE-Max walks around and listens in on the groups
0.35.50	MTE-Max brings the class together and asks why the division in both number sentences yield the same answer $3\frac{1}{2}$ divided by $\frac{1}{2}$, and $\frac{7}{8}$ divided by $\frac{1}{4}$
0.36.15	Dana asks if there is a relationship between splitting something into two groups and how many halves are in the same whole
0.37.00	MTE-Max repeats Dana's question in the first $1\frac{3}{4}$ dividing by 2 and the other is $1\frac{3}{4}$ divided by $\frac{1}{2}$. He asks how are these different and how do the answers compare. *.* They decide that one is twice than the dividend and the other is twice the dividend, MTE-Max summarizes as they go.
0.39.42	One teacher suggest because it is doubled. *.* As a class they determine it is because both the dividend and the divisor are doubled and thus the quotients are the same. MTE-Max asks the class if this is always true. The class says yes. MTE-Max asks if they all convinced.
0.40.23	MTE-Max relates this to addition and subtraction by the same amount, but in this case we are multiplying by the same factor, MTE asks why?
0.41.30	A teacher from group 1 suggests they consider $\frac{8}{4}$ and $\frac{4}{2}$, and they discuss. MTE-Max writes it on the board. The teachers note the other way works the same, if the numbers get smaller.
0.42.30	MTE-Maxs notes that they are using the idea of sharing and explains that we can generalize that idea and justify why 1 divided by $\frac{1}{2}$ is 2
0.44.15	MTE-Max asks if there are any question --- MTE-Max says that he will now collect the quizzes and move on to the next portion of the class.
0.44.30	MTE-Max collects the quizzes, and then passes out and explains the syllabus
0.46.54	MTE-Max orients the teachers to the new text and asks them to change a type-o on page 7
0.48.31	MTE-Max asks the teachers to independently read pages 7 and 8 then think about the questions at the end of the reading and explains the benefit of independent reading. He further instructs them to discuss their group members the difference between the term "value" and "quantity". MTE-Max walks around and listens in on the groups discussions
0.59.27	MTE-Max brings the group back together and asks Ms. C from group 6 to describe the difference between the terms "value" and "quantity". Ms. C explains
1.00.04	One teacher at group 7 describes her understanding of the difference using the table she is sitting at as a referent

1.00.43	Another teachers at group 4 explained her understanding of the difference
1.01.43	MTE-Max asks about the difference with respect to the two cupcakes on the front table and asks what is the quantity. *-* He points out the number of cupcakes vs. the cupcakes themselves
1.03.12	MTE-Max asks it there are any questions on the items A-D they discussed in their groups. Ms. D from group 6 voices her groups discussion and a class discussion ensues between the teachers
1.04.11	MTE-Max discusses the importance of being specific about the referent, number of cupcakes, height of the table and how some things are easier to measure than others
1.06.09	MTE-Max points the class to page 9 and asks for thumbs up or thumbs down whether or not each item is easy to measure There is some disagreement on whether or not the GNP is easy to hard to measure. MTE-Max chooses a teacher from the minority that suggests it is hard to measure. MTE-Max notes that it might be hard for us to work with that data but perhaps a computer might make the work easy
1.07.42	There is some disagreement on whether or not student achievement is easy to hard to measure. MTE-Max chooses a teacher that suggests it is easy to measure.
1.08.12	Blanca from group discusses why one might think it was not easy to answer. Discussion ensues about how it might depend on improvement, language, rubrics, and subjectivity. MTE-Max prompts each teacher to explain his or her reasoning. Then he sums up the discussion before moving on
1.13.36	MTE-Max shares the finding of the NCTM related to the use of MCTM text vs. standard text and student achievement
1.16.33	They discuss the next item, recently discovered measured. The class notes earthquakes, air quality, and speed of information. MTE-Max asked about the units for each
	--- Break ---
0.04.35	MTE-Max asks for a volunteer to read the next 2 pages to the class. Ms. T from group 7 reads
0.08.05	MTE-Max asks the teacher to look over what was just read and identify 1 or 2 sentences that rings true to them to later be shared with the whole class and why the sentence was chosen
0.09.22	MTE-Max asks them to share in their groups. MTE-Max walks around and listens in on and discusses with the groups
0.12.05	MTE-Max brings the class together. He asks them to consider the hotdog problem. He asks the teachers to list all of the quantities involve with the hotdog problem individually then compare their list with their group members. MTE-Max walks around and listens in on and discusses with the groups
0.16.51	MTE-Max brings the groups together and they share their lists as MTE-Max records on the document camera and presses them to be specific about the units and referents. As the teachers list the quantities MTE-Max often attaches units of specific language
0.22.41	MTE-Max asks for the associated values for the quantities they listed. *-* As a class the fill in the values. MTE-Max highlights that some values of quantities are given and others are not, and some are useful to the quantitative structure of the problem and others are not
0.26.40	MTE-Summarizes what they have done so far.
0.27.00	MTE-Max reads some text then discusses what they need to do to do the quantitative analysis and reads the explanation in the text defining quantitative analysis. He notes that an important aspect of a QA is the ability to represent the situation with a picture
0.29.10	MTE-Max asks the teachers to think through and do a quantitative analysis of the brothers and sisters problem. MTE-Max walks around and listens in on and discusses with the groups

0.41.01	MTE-Max asks Ms. D from group 6 to share with the class their discussion once the class comes back together
0.44.13	MTE-Max asks the teachers to describe their diagrams to each other. MTE-Max walks around and listens in on and discusses with the groups
0.49.28	MTE-Max asks Ms. M to describe her diagram to him
0.50.26	MTE-Max brings the class together. Ms. A from group 6 shares her work with the class. She uses the document camera to display and explain her list the quantities and describes her thinking process as she solved the problem
0.53.53	MTE-Max asks the teachers about the advantages of both teachers and students to list separately the values and the quantities
0.54.18	Blanca from group 1 responds. MTE-Max reiterates and points them to Chapter 1 and student strategies for solving word problems
0.56.20	MTE-Max explains the how the teachers in another class noted the importance of questioning instead of telling, to move students forward
0.57.49	MTE-Max asks Ms. M to share her diagram. Ms. M from group 2 shares her diagram with the class
0.58.38	MTE-Max asks for advantages for the diagram over a table. There are a few mumber from the class, then explains the learning trajectory for students linking the picture to the equation
0.59.46	MTE-Max asks the class if anyone has a diagram where the value of 17 is shown in the picture. Ms. M from group 5 shares her picture. MTE-Max re-explains her drawing
1.01.12	Ms. A at group 7 compares it to the number line
1.01.28	MTE-Max points them to a solution given in the book and passes out and explains the homework

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