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Finite element-based damage detection using expanded Ritz vector residuals

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ABSTRACT

The incomplete measurement problem poses a significant obstacle for both model correlation and structural health monitoring (SHM). In practice, information about the health of a structure must be ascertained using measurement data from only a limited number of sensors. Several approaches to this problem have been proposed. One approach involves a reduced order model, such as that obtained using methods such as the Guyan reduction, the Improved Reduced System Model, or the System Equivalent Reduction Process. A second approach, which this work considers, involves an expansion of the test data to match a higher-fidelity model. This work presents a damage detection method utilizing Ritz vectors and the Method of Expanded Dynamic Residuals (MEDR). Ritz vectors have several advantages over eigenvectors for application to damage detection, including lower sensitivity to noise, and, as a result of their load-dependent nature, a greater sensitivity to localized damage. The MEDR restricts identified damage locations to those where there is physical connectivity, which eliminates the "smearing" that plagues direct expansion methods, and provides a physically meaningful estimate of the damage location. *LA-UR-12-25460*.

Keywords: Ritz vectors, Damage detection, Matrix disassembly, NASA 8-bay truss, Expanded residuals

INTRODUCTION

Structural damage detection is an important issue for large civil structures, airplane wings, car frames, and any structure that regularly undergoes fatigue or impact loading. Damage detection becomes especially important when human lives are at stake, but improved damage detection methods would have financial benefits as well. Early detection of faults would allow repairs or replacements to be made before the damage becomes too costly to repair. Certain expensive components, such as carbon-fiber composite airplane wings, might enjoy a longer service life, rather than being replaced on an expensive time-based maintenance schedule.

Damage detection is often approached as a finite element model updating problem, either through updating the stiffness matrix or by physical parameter updating. Several methods for updating the mass and stiffness property matrices have been proposed, including the Minimum Rank Perturbation Theory (MRPT) [1]. Without *a priori* knowledge of the damage location, physical parameter updating requires intense manual interaction and heavy reliance on engineering experience. Genetic Algorithms can be used to provide accurate convergence for the nonlinear optimization problem posed by physical parameter updating, but knowledge of the damage location would significantly improve the results and reduce the time required for the algorithm to converge.

Several difficulties present themselves in the damage detection problem. One difficulty to overcome is the incomplete measurements problem, wherein it is desired to detect damage based on a finite element model using physical measurements taken at only a small subset of the degrees of freedom present in the full model. Various model reduction techniques have been developed to help address the incomplete measurements problem, including the Guyan static reduction [2], sometimes referred to as a static condensation, the Improved Reduced System [3], and the System Equivalent Reduction Expansion Process (SEREP) [4]. However, a study that attempted to utilize mode shape expansion methods for damage detection with reduced measurements showed that direct expansion of measured mode shapes is ineffective [5].

The Method of Expanded Dynamic Residuals (MEDR) [6] couples the concept of the damage residual in MRPT with the concept of stiffness matrix disassembly [7] to arrive at an expanded damage residual vector, which is used to determine the location of damage using limited measurement points. With the damage location identified, the task of determining damage extent is significantly simplified. Several studies exist to address the issue of damage extent given knowledge of the damage location. One such study proposed a method for reduced model updating using Ritz vectors to compute the stiffness change [8].

MEDR was originally developed using measured mode shapes. In this work, Ritz vectors are applied to MEDR to provide a comparison between the application of Ritz vectors and mode shapes to MEDR in the face of significant signal noise. Load-dependent Ritz vectors [9] are an alternative to mode shapes that contain information about the dynamics of a structure. The first Ritz vector is the structure's deflection under a static load, and the subsequent Ritz vectors are defined recursively based on the structure's mass and stiffness properties.

The prime advantage of Ritz vectors over mode shapes in damage detection is that, rather than each vector describing the structure's behavior at a given frequency, each Ritz vector describes some aspect of the structure's behavior across a broad frequency range. Ritz vectors can be used to detect local structural damage, which is generally a high frequency phenomenon, using only the first few global Ritz vectors. Ritz vectors are also less sensitive to noise than measured mode shapes, and confidence in extracted Ritz vectors can be further increased by the application of accuracy indicators [10].

An important step in the development of Ritz vectors and their application to damage detection was the development of a method to extract Ritz vectors from dynamic testing data. After characterizing the effects of noise on measured Ritz vectors [11] and applying them to structural damage detection [12], Cao and Zimmerman showed that, using a state-space system realization, Ritz vectors extracted from dynamic testing data agree with analytically calculated Ritz vectors [13, 14]. Cao, Zimmerman, and James applied this method to identify Ritz vectors using Space Shuttle data [15], and later combined Ritz properties with modal strain for experimental damage detection [16]. Jeancolas and Zimmerman [17], Boxoen and Zimmerman [10], and Taylor and Zimmerman [18] continued to develop improvements to Ritz vector extraction, with further applications to damage detection [19, 20]. More recently, Khoury and Zimmerman have developed pre-test planning methods to aid in the extraction of load dependent Ritz vectors [21].

MATHEMATICAL PRELIMINARIES

Analytical Ritz Vectors

A discrete n-degree-of-freedom (n-DOF) single-input undamped system model can be expressed as

$$\mathbf{M}\ddot{\mathbf{x}}(t)+\mathbf{K}\mathbf{x}(t)=\mathbf{b}u(t), \tag{1}$$

where $\mathbf{M}$ and $\mathbf{K}$ are the $(n\times n)$ mass and stiffness matrices, respectively, $\mathbf{x}(t)$ is an $(n\times 1)$ position vector, $\mathbf{b}$ is an $(n\times 1)$ force influence vector, and $u(t)$ is the force input signal. The first Ritz vector is the deflection of the structure under a unit static load; it is the mass-normalized static solution to Eq. (1), given by

$$\overline{\mathbf{v}}_1=\mathbf{K}^{-1}\mathbf{b} \quad \mathbf{v}_1=\overline{\mathbf{v}}_1/\Gamma \quad \Gamma_1^2=\overline{\mathbf{v}}_1^{\mathrm{T}}\mathbf{M}\overline{\mathbf{v}}_1 . \tag{2}$$

Subsequent Ritz vectors are computed in a recursive algorithm that includes the inertial. Each subsequent Ritz vector is computed as

$$\overline{\mathbf{v}}_i=\mathbf{K}^{-1}\mathbf{M}\mathbf{v}_{i-1}. \tag{3}$$

Upon computation, each Ritz new vector is orthogonalized using a Gram-Schmidt orthogonalization process and mass normalized as

$$\hat{\mathbf{v}}_i = \overline{\mathbf{v}}_i - \sum_{j=1}^{i-1}\left(\mathbf{v}_j^{\mathrm{T}}\mathbf{M}\overline{\mathbf{v}}_i\right)\mathbf{v}_j \qquad \mathbf{v}_i = \hat{\mathbf{v}}_i / \Gamma_i \quad \Gamma_i^2 = \hat{\mathbf{v}}_i^{\mathrm{T}}\mathbf{M}\hat{\mathbf{v}}_i \tag{4}$$

Ritz Damage Residual

Damage location can be estimated using a model updating technique and a Ritz vector-based damage residual matrix. The primary assumptions that this work applies for such a method are that (1) a finite element model given by mass and stiffness $\mathbf{M}$ and $\mathbf{K}$, with negligible damping accurately describes the structure of interest in an undamaged state; that (2) any damage to the structure only affects the stiffness property of the model; and that (3) the damaged structure continues to behave in a linear fashion described by an unknown stiffness matrix, $\mathbf{K}_d = \mathbf{K} - \Delta\mathbf{K}$. For a healthy system, Eq. (3) can be rewritten as the following equilibrium equation:

$$\mathbf{K}\overline{\mathbf{z}}_i - \mathbf{M}\mathbf{z}_{i-1} = \mathbf{0}. \tag{5}$$

The Ritz vectors are written in Eq. (5) with a ‘z’ to indicate that they are experimental Ritz vectors. The experimentally extracted Ritz vectors will satisfy Eq. (5) only if the structure remains in a healthy state. If the system becomes damaged, the equilibrium condition becomes

$$\left(\mathbf{K} - \Delta\mathbf{K}\right)\overline{\mathbf{z}}_i - \mathbf{M}\mathbf{z}_{i-1} = \mathbf{0}. \tag{6}$$

If the Ritz vectors are extracted from a damaged structure, the right hand side of Eq. (5) becomes nonzero, and it is defined as the dynamic residual vector as

$$\mathbf{d} \equiv \mathbf{K}\overline{\mathbf{z}}_i - \mathbf{M}\mathbf{z}_{i-1} = \Delta\mathbf{K}\overline{\mathbf{z}}_i, \tag{7}$$

If the test DOFs are the same as those for the model, the DOFs affected by damage will appear as nonzero elements of $\mathbf{d}$. Given *n* DOFs and *m* Ritz vectors, Eq. (7) can be formulated as

$$\mathbf{B}_d \equiv \mathbf{K}\overline{\mathbf{Z}} - \mathbf{M}\mathbf{Z} = \Delta\mathbf{K}\overline{\mathbf{Z}}, \tag{8}$$

where $\mathbf{B}_d$ is the $n \times \left(m-1\right)$ damage residual matrix. The stiffness change can then be obtained using minimum rank perturbation theory (MRPT) as

$$\Delta\mathbf{K} = \mathbf{B}_d\left(\mathbf{B}_d{}^{\mathrm{T}}\overline{\mathbf{Z}}\right)^{-1}\mathbf{B}_d{}^{\mathrm{T}}. \tag{9}$$

In the noise-free case, the formulation given in Eq. (9) would be sufficient, as each column of $\mathbf{B}_d$ would contain identical information. Practical implementations would implement a noise-resistant solution using the Singular Value Decomposition (SVD) as in [10].

Experimental Ritz Vectors

Although the preceding analytical formulation is useful, in order for damage detection using Ritz vectors to be practical, the vectors must be obtainable from dynamic testing data. Cao and Zimmerman [14] presented an algorithm using a system realization to extract Ritz vectors. The Eigenvalue Realization ion Algorithm (ERA) [22] uses dynamic testing data to identity a discrete time system model of the form

$$\begin{aligned} \mathbf{x}\left(k+1\right) &= \hat{\mathbf{A}}\mathbf{x}\left(k\right) + \hat{\mathbf{B}}\mathbf{u}\left(k\right) \\ \mathbf{y} &= \hat{\mathbf{C}}\mathbf{x}\left(k\right) \end{aligned}, \tag{10}$$

where $\hat{\mathbf{A}}$, $\hat{\mathbf{B}}$, and $\hat{\mathbf{C}}$ are the discrete time system matrices. In practice these system matrices can be converted to continuous time using a zero-order hold method. The resulting continuous-time state-space representation $\left[\mathbf{A,B,C}\right]$ can be used to extract experimental Ritz vectors utilizing the improved Ritz vector extraction method [18], which orthonormalizes Ritz vectors in physical coordinates to produce non-orthonormalized Ritz vectors that are consistent with those produced by the Wilson algorithm. This method is necessary for accurate damage residual estimates using experimental Ritz vectors.

In the improved Ritz vector extraction method, the first Ritz vector in state-space is computed as

$$\overline{\mathbf{x}}_1 = -\mathbf{A}^{-1}\mathbf{B} . \tag{11}$$

This vector is immediately transformed to physical coordinates and mass-normalized as

$$\begin{aligned} \overline{\mathbf{z}}_1 &= \mathbf{C}\overline{\mathbf{x}}_1 \\ \mathbf{z}_1{}^T\mathbf{M}\mathbf{z}_1 &= 1 \end{aligned} . \tag{12}$$

The first orthonormalized Ritz vector in state space is obtained by transforming the normalized Ritz vector back into state-space as

$$\mathbf{x}_1 = \mathbf{C}^{+}\mathbf{z}_1 , \tag{13}$$

where the '+' is the pseudo-inverse operator. Subsequent Ritz vectors are computed and transformed to physical coordinates directly as

$$\overline{\mathbf{z}}_i = -\mathbf{C}\mathbf{A}^{-2}\mathbf{x}_{i-1} , \tag{14}$$

where they are orthogonalized and mass-normalized as

$$\begin{aligned} \mathbf{z}_i &= \overline{\mathbf{z}}_i - \sum_{j=1}^{i-1}\left(\mathbf{z}_j{}^{\mathrm{T}}\mathbf{M}\overline{\mathbf{z}}_i\right)\mathbf{z}_j \\ \mathbf{z}_i{}^{\mathrm{T}}\mathbf{M}\mathbf{z}_i &= 1 \end{aligned} . \tag{15}$$

At each step, the orthonormalized Ritz vectors are obtained in state-space as

$$\mathbf{x}_i = \mathbf{C}^{+}\mathbf{z}_i . \tag{16}$$

Matrix Disassembly

Matrix disassembly is a process that decomposes a structural matrix into a matrix representation of the connectivity between DOFs and a matrix containing the magnitude information [7]. This application of matrix disassembly reduces any stiffness matrix into a set of equivalent springs. For a truss structure, this disassembly is exact, but for structures made up of more complicated elements, the disassembly is only approximate. Although some advanced applications utilize a disassembly of the stiffness matrix into the elemental stiffness matrices, the matrix disassembly method produces a general technique that can be applied to any model without detailed knowledge into the actual elements used. in. The matrix C is an $(n \times m)$ matrix, where n is the matrix dimension of K, and m is equal to the total number of unique entries in K. If K is symmetric, this number amounts to the number of nonzero entries in the upper triangular portion of the matrix. The diagonal matrix P is calculated as

$$\begin{aligned} \mathbf{P}(i,i) &= \sum_{j=1}^{n}\mathbf{K}(i,j) \qquad i = 1:n \\ \mathbf{P}(i,i) &= \mathbf{K}(j,k) \qquad i = (n+1):m \end{aligned} . \tag{17}$$

The $(n \times m)$ matrix $\mathbf{C}$ can be written as $[\mathbf{C}_1 \quad \mathbf{C}_2]$, where $\mathbf{C}_1$ is the identity matrix. The elements of the $n \times (m-1)$ matrix $\mathbf{C}_2$ are defined according to the element locations of the unique entries of the stiffness matrix $\mathbf{K}$. For each element $\mathbf{K}(j,k)$ used to define the element $\mathbf{P}(i,i)$ with $i = (n+1):m$, the i^{th} column of $\mathbf{C}$ is given as

$$\begin{aligned} \mathbf{C}(j,i) &= 1 \qquad i = (n+1):m \\ \mathbf{C}(k,i) &= -1 \qquad i = (n+1):m \end{aligned} . \tag{18}$$

Damage Residual Estimation with Reduced Measurements

If an $(n \times p)$ transformation matrix **T**, where n is the number of model DOFs and p is the number of measurement DOFS, relates the reduced measurement system to the full system as

$$\mathbf{x}_{full} = \mathbf{T}\mathbf{x}_{red}, \tag{19}$$

then the damage residual matrix of Eq. (8) can be rewritten as

$$\mathbf{B}_d = \mathbf{K}\mathbf{T}\overline{\mathbf{Z}}_r - \mathbf{M}\mathbf{T}\mathbf{Z}_r. \tag{20}$$

This work utilizes the transformation matrix **T** derived from the Guyan model reduction [2]. Premultiplying Eq. (20) by $\mathbf{T}^{\mathrm{T}}$, the reduced damage residual matrix can be defined as

$$\mathbf{B}_{d,r} = \mathbf{K}_r\overline{\mathbf{Z}}_r - \mathbf{M}_r\mathbf{Z}_r, \tag{21}$$

where $\mathbf{B}_{d,r} = \mathbf{T}^{\mathrm{T}}\mathbf{B}_d$, $\mathbf{K}_r = \mathbf{T}^{\mathrm{T}}\mathbf{K}\mathbf{T}$, and $\mathbf{M}_r = \mathbf{T}^{\mathrm{T}}\mathbf{M}\mathbf{T}$.

If one assumes that the connectivity information is invariant with respect to the damage, then the changes in the disassembly detailed in the previous section would be contained in the diagonal magnitude matrix **P**. The connectivity matrix would be an invariant that could be used as a linearly dependent set of basis vectors to approximate the true damage residual. Choosing a reduced target vector, $\mathbf{u}_{tar}$, to be the first left singular vector of the reduced damage residual matrix $\mathbf{B}_{d,r}$ the reduced target vector and the full target vector, similarly defined as the first left singular vector of the full damage residual matrix $\mathbf{B}_d$, can ideally be related as

$$\mathbf{u}_{red} = \mathbf{T}^{\mathrm{T}}\mathbf{u}_{full}, \tag{22}$$

where the unknown full target vector is approximated as a linear combination of the columns of the connectivity matrix **C**, as

$$\mathbf{u}_{full} = \mathbf{C}\boldsymbol{\alpha}, \tag{23}$$

where $\boldsymbol{\alpha}$ is a vector of constants defining the contribution of each column. The reduced target vector can then be related to the full target vector through the connectivity matrix as

$$\mathbf{u}_{red} = \mathbf{T}^{\mathrm{T}}\mathbf{C}\boldsymbol{\alpha}. \tag{24}$$

Choosing the values for $\boldsymbol{\alpha}$ that best satisfy (24) permits the approximation of the full target vector using (23). The number of columns of C will exceed its rank, so many of the components of $\boldsymbol{\alpha}$ must be zero. The full combinatorial optimization problem that arises in choosing the nonzero elements of $\boldsymbol{\alpha}$ would be infeasible even for modestly sized models, so a "best subspace" selection algorithm [23] is implemented to select the columns of **C** that minimize

$$\min_{\text{wrt } \gamma} \quad \varepsilon = \left(\mathbf{T}^{\mathrm{T}}\mathbf{c}_i - \gamma\mathbf{u}_{tar}\right)^{\mathrm{T}}\left(\mathbf{T}^{\mathrm{T}}\mathbf{c}_i - \gamma\mathbf{u}_{tar}\right). \tag{25}$$

At each selection of a column of **C** to include, the target vector $\mathbf{u}_{tar}$ is updated to remove the portion provided by that column, as

$$\mathbf{u}_{tar}^{i} = \tfrac{1}{\gamma}\mathbf{T}^{\mathrm{T}}\mathbf{c}_i - \mathbf{u}_{tar}^{i-1}. \tag{26}$$

The process can be repeated until the error ε in Eq. (25) drops below some threshold. Once the nonzero elements of $\boldsymbol{\alpha}$ are identified, the full damage residual vector can be estimated as

$$\mathbf{u}_{full} = \mathbf{C}\boldsymbol{\alpha}^{*}, \tag{27}$$

where $\boldsymbol{\alpha}^{*}$ is the least squares solution to (24) using only the identified nonzero elements.

NUMERICAL EXAMPLES

Description of Test Model

The NASA 8-bay truss was part of the Dynamic Scale Model Technology Program at NASA Langley Research Center [24]. This truss has been used extensively for damage location and extent studies [25]. A schematic of the 96-DOF cantilevered truss is shown in Fig 1, with several experimental damage cases highlighted. Zimmerman and Bartkowicz [6] applied MEDR to several of these damage cases using measured mode shapes. The case considered in this work is case H, in which the longeron connecting DOFs 62 and 74 was removed.

This work seeks to demonstrate the efficacy of Ritz vector-based methods using examples that are relevant to the community, which is the motivation for choosing the NASA 8-bay truss for an example case. While some works have computed Ritz vectors using measured flexibility matrices, [26], the benefits of Ritz vectors cannot fully be realized if they are limited to spanning the space of the measured modes. As such, experimental Ritz vector extraction requires time-domain data; however, at the time of this writing, the only data available from tests conducted on this structure were the measured modes. In order to make a valid comparison between the application of mode shapes and Ritz vectors to MEDR, each vector set was obtained using the simulated system response. In an effort to remain true to the original experimental test bed, no more than the first five modes of vibration are used in this work, because individual members' local modes of vibration made it impossible to experimentally measure beyond the fifth global mode.

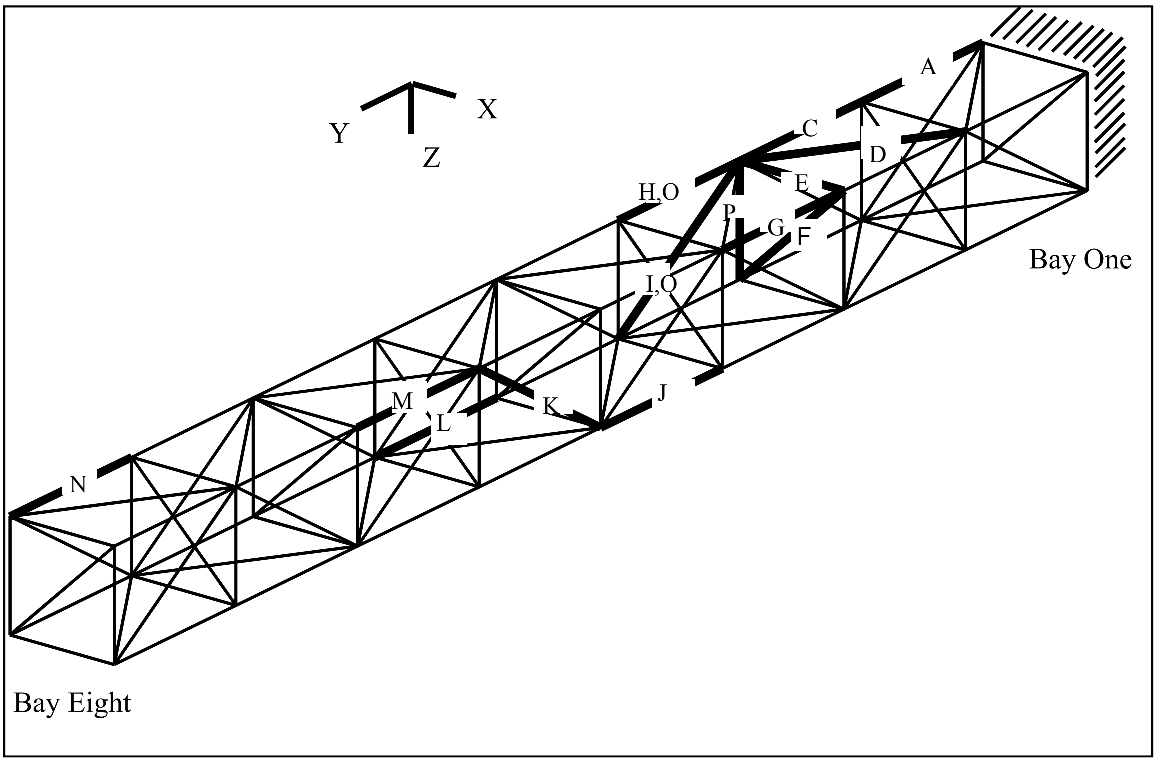

Fig 1 NASA eight-bay truss

Damage Residual Estimation

The impulse response of the damaged truss system was simulated using the mass and stiffness for the damaged case, and ERA was used to identify a state-space model of the structure. The full analytical model of the damaged structure was used to generate the impulse response because this data is closest to what would be measured in an actual test, but the output from only 10 sensors were utilized, their locations chosen using a QR decomposition method to maximize linear independence of the selected DOFs. Applying the MEDR, estimates of the full damage residual vector were separately obtained using mode shapes and Ritz vectors. Estimates of the full damage residual vector were obtained each for the mode shapes and Ritz vectors. The estimates were normalized to unit length. The results are shown in Fig 2, which contains two bar plots. The upper plot shows the absolute value of the damage residual estimates obtained using the ERA-identified mode shapes, while the lower plot shows the estimates obtained using the extracted Ritz vectors. The actual damaged DOFs (62 and 74) are indicated with asterisks.

For the noise-free case, the results obtained using mode shapes and Ritz vectors are nearly identical to one another. This result is expected, because Ritz vectors should see no advantage over mode shapes in the absence of noise. Note that in both cases, DOFs 50 and 62 were indicated as damaged, rather than DOFs 62 and 74, as was the case. This results from the fact that, for particular reduced sensor sets, certain damage cases are indistinguishable from one another. In this case, removing the longeron connecting DOFs 50 and 62 has exactly the same effect on the reduced dynamic system behavior as removing the longeron connecting DOFs 62 and 74.

The process was repeated for noisy data, where the signals at the 10 output DOFs were corrupted with 5% RMS noise. The noisy data was generated as

$$\mathbf{y}_n = \mathbf{y} + \frac{x}{100}\mathrm{RMS}(\mathbf{y}) \cdot \mathbf{r}, \tag{28}$$

where $\mathbf{y}_n$ is the corrupted time response vector, $\mathbf{y}$ is the noise-free time response vector, and x is a scalar that determines the noise level, expressed as a percentage of the root-mean-square (RMS) value of the noise-free signal. The vector $\mathbf{r}$ is generated from a pseudo-random uniform distribution over $[-1,1]$.

Using identified mode shapes and extracted Ritz vectors from the noisy data, the MEDR was applied once more to obtain estimates of the full damage residual vector. The absolute values of the normalized damage residual estimates are shown in Fig 3, where the upper plot shows the estimates obtained using mode shapes, and the lower plot shows the estimates obtained using Ritz vectors. The residual estimate using mode shapes primarily indicates DOF 50, while the estimate using Ritz vectors indicates DOFs 50 and 62, as in the noise-free case. While the mode shapes' result indicating DOF 50 is not entirely incorrect, because the damage cases are indistinguishable by the reduced model, the mode shapes do appear to be more significantly affected by the presence of noise.

In the next example it was assumed that 12 sensors were available, their locations having again been chosen using the QR decomposition method. The response of each DOF was corrupted with 5% RMS noise as above, and ERA was used to identify five separate state-space systems utilizing 8, 9, 10, 11 and 12 of the available sensors. The MEDR was then applied to obtain estimates of the full damage residual. In Fig 4 and Fig 5 there are each five plots: one for each estimate obtained using the different sensor sets. Fig 4 contains the damage residual estimates obtained using mode shapes, while Fig 5 contains the estimates obtained using Ritz vectors. The correct damaged DOFs, 62 and 74, are indicated with asterisks. In this case, the three DOFs 50, 62, and 74 could all be indicated, because more columns of the connectivity matrix were utilized in computing the residual estimates. Observing Fig 4, only the sensor set with 9 sensors indicates the correct damaged DOFs, although for the sensor sets with 10 and 12 sensors, DOF 50 is indicated. It is clear that while the mode shapes are capable of indicating the correct damaged DOF, particular sensor sets allow them to become overwhelmed by signal noise. Observing Fig 5, it is apparent that, while the presence of noise affects the results, the Ritz vectors are able to produce a more consistently correct estimate of the damage location across the range of sensor sets.

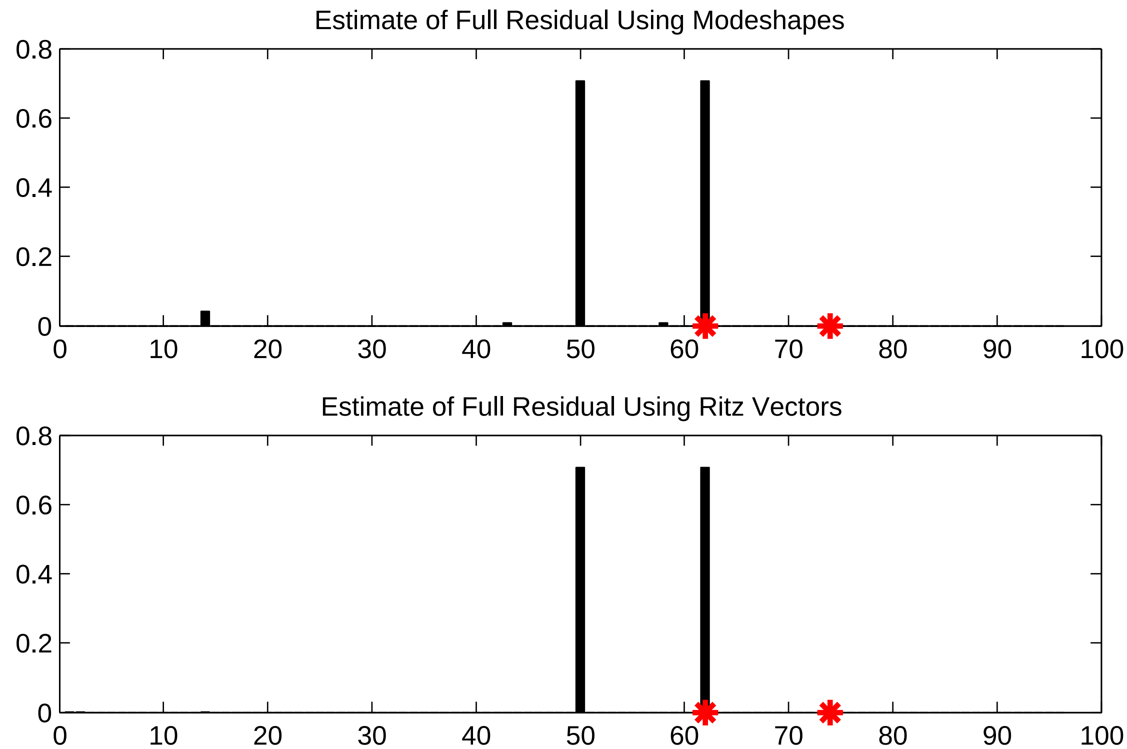

Fig 2 Normalized Estimates of the Full Damage Residual in the Noise-Free Case

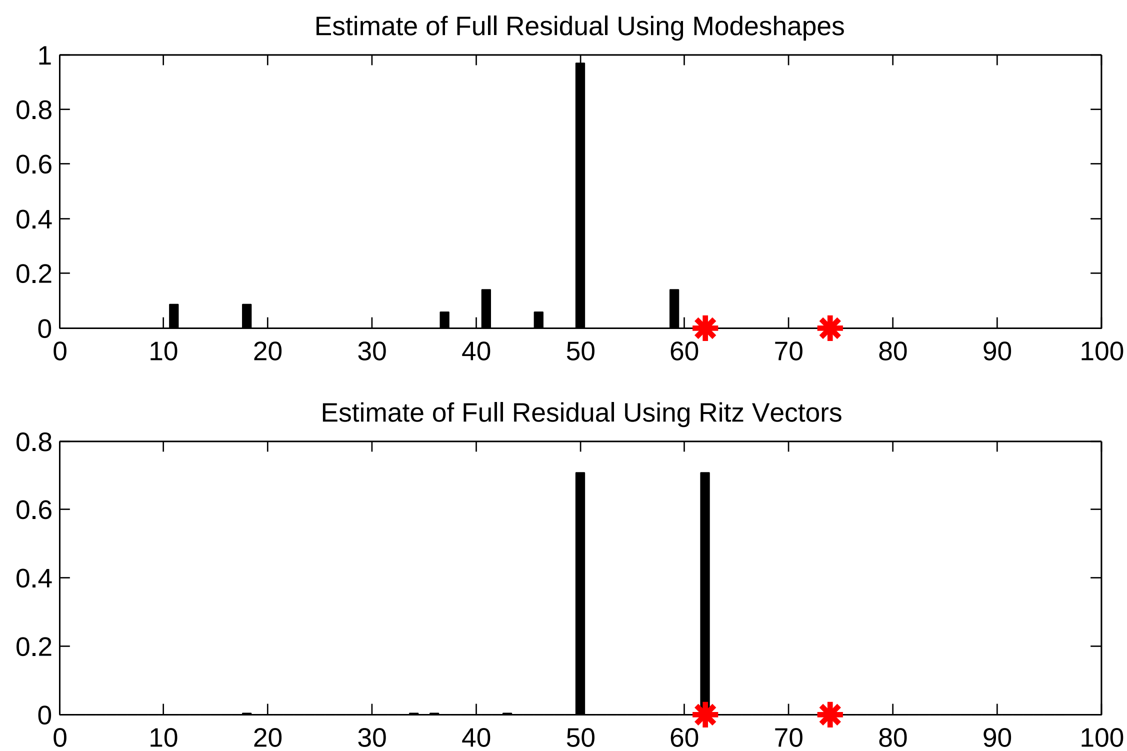

Fig 3 Normalized estimates of the full damage residual in the noisy case

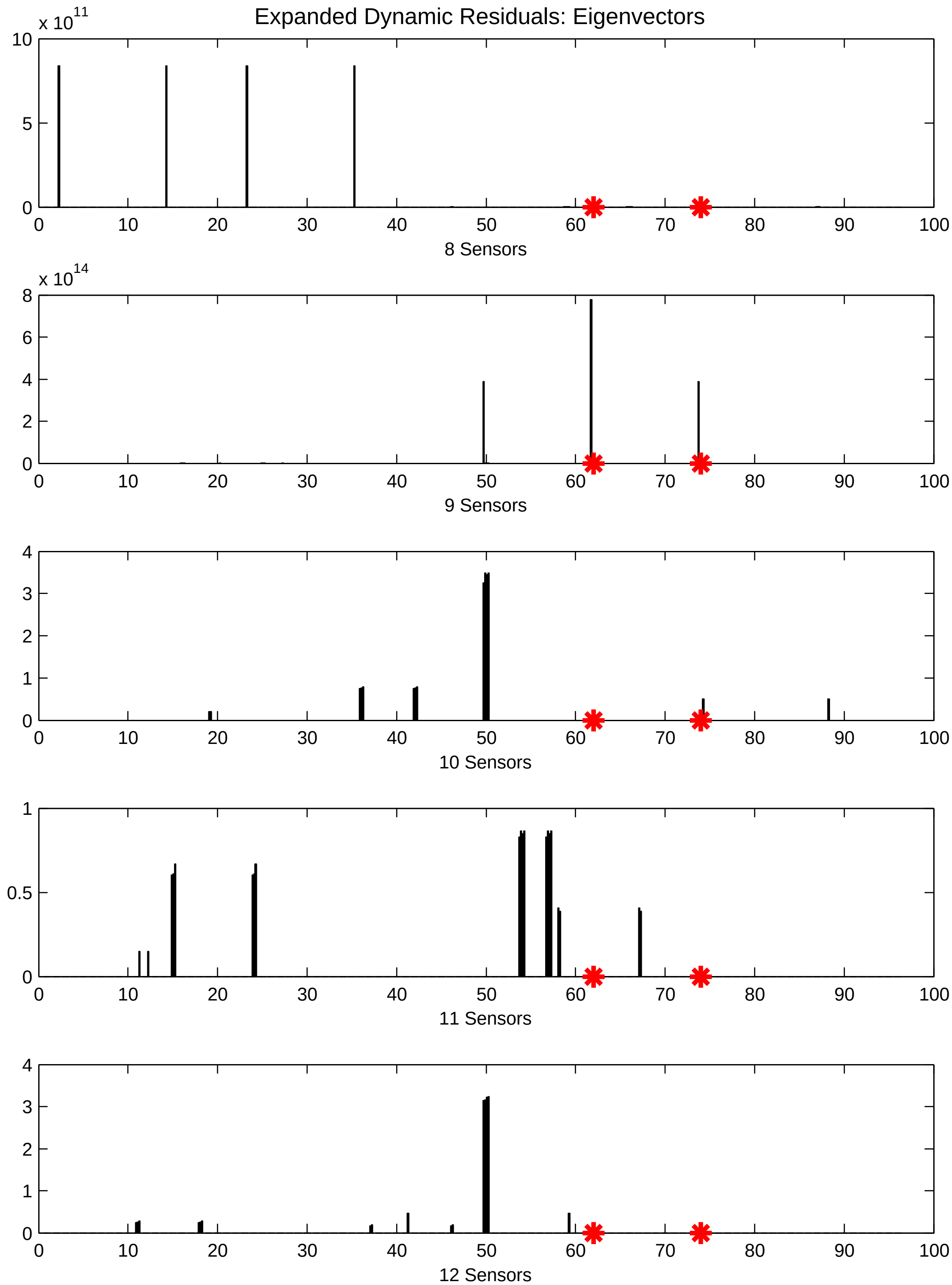

Fig 4 Damage residual estimates for different sensor sets using eigenvectors

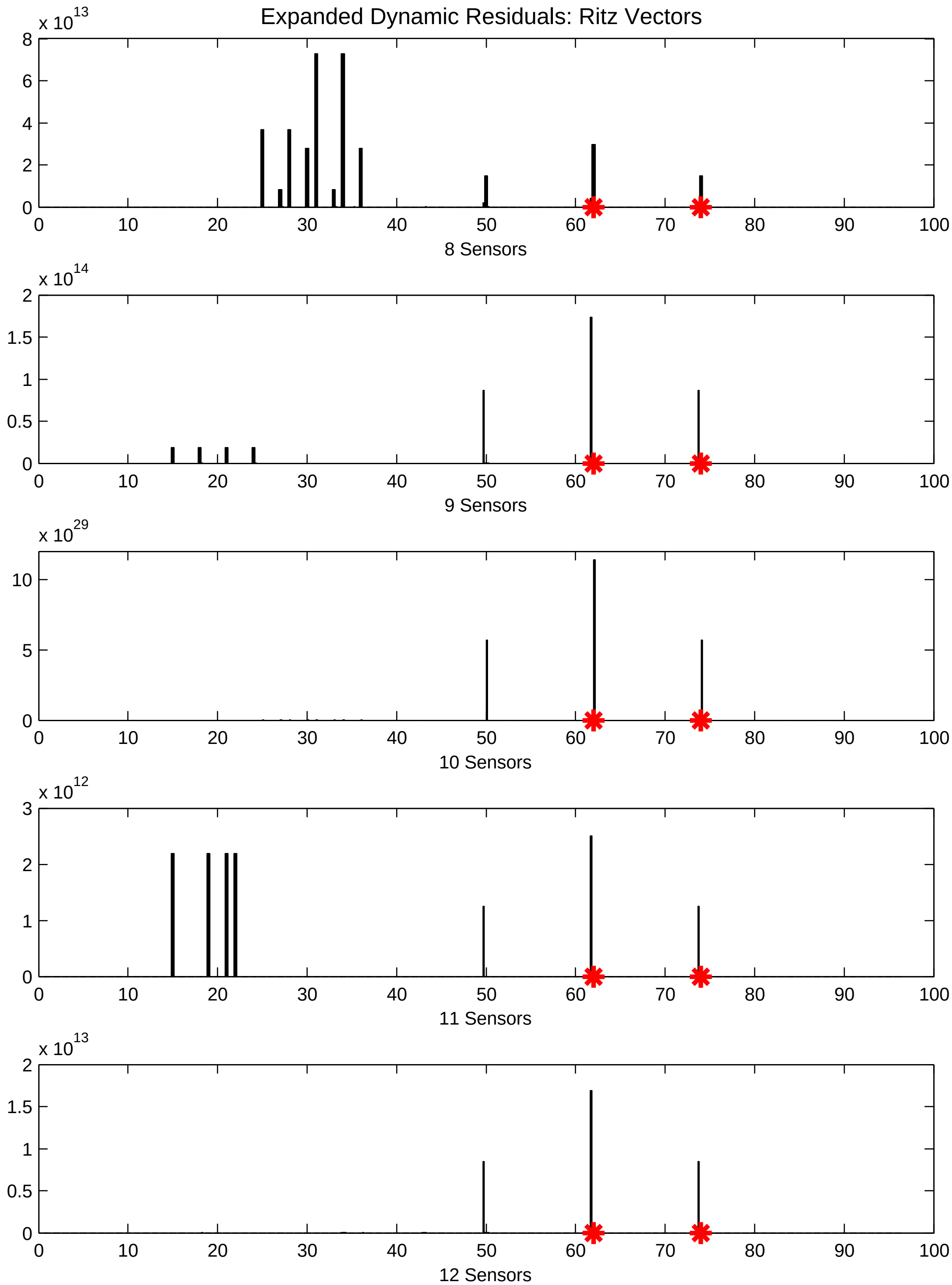

Fig 5 Damage residual estimates for different sensor sets using Ritz vectors

SUMMARY

The method of expanded dynamic residuals was found to be robust to noise, even with a significantly reduced measurement set. The application of both measured mode shapes and extracted Ritz vectors successfully located damage for the test cases considered, although in the case considered, the extracted Ritz vectors provided a more consistently correct result in comparison to mode shapes. In noisy cases, varying the reduced sensor set affected the ability to obtain consistently correct results using mode shapes. However, in spite of some incorrect estimates provided by particular sensor sets, the application of the singular value decomposition to several estimates obtained using slightly different sensor sets successfully located the correct damaged degrees of freedom. By comparison, the application of Ritz vectors yielded the correct damaged degrees of freedom for each sensor set considered. Because multiple estimates may not be required when using Ritz vectors and MEDR, it would be more advisable to utilize Ritz vectors in this application.

It is difficult to extend directly the results obtained using expanded residuals to compute the damage extent in terms of a stiffness update matrix. Methods such as the minimum rank perturbation theory require system vectors (either mode shapes or Ritz vectors) at the full system DOFs in order to obtain a quantifiable stiffness property change. Rather than seeking an update matrix, once the damage location is known, damage extent can more easily be determined using parameter updating methods and optimization schemes such as genetic algorithms. However, Ritz vectors lend themselves very well to static reduction, with the first Ritz vector being the static deflection itself.

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