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## SECTORAL COMPOSITION OF GOVERNMENT SPENDING AND MACROECONOMIC (IN)STABILITY

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*This article examines the quantitative interrelations between sectoral composition of public spending and equilibrium (in)determinacy in a two-sector real business cycle model with positive productive externalities in investment. When government purchases of consumption and investment goods are set as constant fractions of their respective sectoral output, we show that the public-consumption share plays no role in the model's local dynamics, and that a sufficiently high public-investment share can stabilize the economy against endogenous belief-driven cyclical fluctuations. When each type of government spending is postulated as a constant proportion of the economy's total output, we find that there exists a trade-off between public consumption versus investment expenditures to yield saddle-path stability and equilibrium uniqueness. (JEL E32, E62, O41)*

### I. INTRODUCTION

One of the central paradigms in modern macroeconomic research is known as the real business cycle (RBC) theory pioneered by Kydland and Prescott (1982). The basic idea of this agenda is to extend the Arrow-Debreu general equilibrium framework to the dynamic and stochastic analyses of aggregate fluctuations within a representative-agent macroeconomy.

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Under the assumptions of perfect competition and constant returns-to-scale in production, the standard RBC model exhibits an interior steady state that is a locally determinate or isolated saddle point around which there is a unique convergent equilibrium trajectory, as well as business cycles driven by exogenous shocks to economic fundamentals such as technology, preferences, and endowment. As it turns out, most of early RBC studies have been unnecessarily restrictive as they focus on environments in which decentralized equilibrium allocations are Pareto efficient, hence precluding the discussion on the role of government macroeconomic policies.

Starting with the work of Benhabib and Farmer (1994, 1996), considerable progress has been made in showing that small departures from the assumptions of prototypical one-sector or multi-sector RBC models can lead to significant deviations from their aforementioned conclusions. Specifically, by allowing for aggregate increasing returns-to-scale in production because of monopolistic competition or positive productive externalities, competitive equilibria become sub-optimal under laissez faire. Moreover, the

#### ABBREVIATIONS

CRRA: Constant-Relative-Risk-Aversion  
GDP: Gross Domestic Product  
MPK: Marginal Product of Capital  
RBC: Real Business Cycle

Benhabib-Farmer economy may possess an indeterminate steady state (i.e., a sink) and a continuum of stationary rational expectations equilibrium paths along which agents' animal spirits can be an independent impulse for generating endogenous business cycles.<sup>1</sup> In sharp contrast to a canonical RBC formulation, these "sunspot" models call for traditional Keynesian-type stabilization policies or "automatic stabilizers" intended to mitigate the magnitude of macroeconomic volatilities through insulating the economy from belief-driven cyclical fluctuations such that saddle-path stability and equilibrium uniqueness are achieved. It turns out that many existing studies have investigated the macroeconomic (in)stability effects of various tax policy rules,<sup>2</sup> and thus left the potential (de)stabilization role of government spending in a representative-agent setting underexplored.<sup>3</sup> Motivated by this gap in the literature, we examine the quantitative interrelations between public expenditures and equilibrium (in)determinacy in a modified Benhabib-Farmer RBC model with two distinct production sectors, consumption and investment. Our analysis is valuable not only for its theoretical insights but also for its broad implications for the design, implementation, and evaluation of the government's spending policies within a macroeconomy.

In this article, we incorporate government purchases of goods and services into a discrete-time two-sector RBC model, as in Harrison (2001), with positive productive externalities present in the investment sector. The government balances its budget each period by levying lump-sum taxes on the representative household to finance its consumption and investment expenditures. This analytical framework allows us to isolate how

sectoral distribution of public spending affects the model's local stability properties, as well as facilitates comparison with previous work that studies the (de)stabilizing effects of income and/or consumption taxation in a RBC-type economy. In particular, Guo and Harrison (2001, 2011) consider a progressive tax schedule *à la* Guo and Lansing (1998) within the same theoretical setup as ours, and postulate that all the government's tax revenues are returned to households as a lump-sum transfer.<sup>4</sup>

We show that under the postulated preference formulation which is logarithmic in consumption and additively separable with labor hours, the equilibrium dynamics of our model economy depends crucially on (a) how the division of public expenditures between consumption versus investment goods is specified, and (b) the degree of productive externalities in investment. Specifically, when government purchases from the consumption and investment sectors are set as constant fractions of their respective sectoral output, we first find that the public-consumption share plays no role in the model's local dynamics. Although changes in government spending on consumption goods yield a within-sector trade-off between private and public consumption expenditures, they do not influence the relative composition of productive resources across the two sectors that is crucial in validating agents' anticipation about a higher future return on capital.

Next, in a calibrated version of our baseline specification, we show that the economy is more susceptible to indeterminacy and sunspots when the output fraction of public expenditures in the investment sector is relatively small. To understand this result, start the model from its steady state, and suppose that agents become optimistic about the economy's future. Acting upon this belief, the representative household will consume less and invest more today, hence two opposing effects on the household's intertemporal Euler equation ensue. On the one hand, because of positive productive externalities in the investment sector, the social production possibility frontier that traces out the trade-off between private consumption and investment spending is convex to the origin. As a result, the relative price of investment goods will fall (*the price effect*) upon

1. While an indeterminate non-stochastic steady state is dynamically stable, the associated stochastic "sunspot" equilibria exhibit higher volatilities than their "fundamentals" counterparts. See the study by Benhabib and Farmer (1999) for a survey of this RBC-based indeterminacy literature in which the terms animal spirits, sunspots, and self-fulfilling prophecies are used interchangeably.

2. See, for example, Guo and Lansing (1998), Christiano and Harrison (1999), Guo and Harrison (2001, 2011), Greiner (2006), Giannitsarou (2007), Dromel and Pintus (2007, 2008), Mino and Nakamoto (2008), Chen and Guo (2013a, 2013b), Gokan (2013), and Nourry, Seegmuller, and Venditti (2013), among others.

3. Previous research that examines the (de)stabilizing effects of government spending within dynamic general equilibrium macroeconomic models include Schmitt-Grohé and Uribe (1997, Section III), Raurich (2001), Gokan (2006), and Lloyd-Braga, Modesto, and Seegmuller (2008), among others.

4. Guo and Harrison (2011) correct an error in Guo and Harrison's (2001) description of the household's and government's budget constraints, and then show that all the authors' earlier results are qualitatively unchanged.

agents' optimism that shifts more capital and labor inputs into producing investment goods. On the other hand, an increase in today's private investment expenditures will reduce next period's real interest rate because of diminishing marginal product of capital (*the MPK effect*). Our analysis finds that for a sufficiently low output fraction of government spending on investment goods, the price effect outweighs the MPK effect such that the household's initial rosy expectation can be justified as a self-fulfilling equilibrium. Moreover, since higher increasing returns-to-scale in the production of investment goods induce agents to move more productive resources out of the consumption sector, we find that the threshold level of the public-investment share needed to stabilize the economy against sunspot-driven cyclical fluctuations will rise as well.

For sensitivity analysis, we examine the formulation in which public expenditures of consumption and investment goods are postulated as constant proportions of the economy's total output. In sharp contrast to the benchmark configuration, the GDP fraction of government purchases from the consumption sector is found to appear in the model's Jacobian matrix and thus affects its local stability properties. Intuitively, changes in the public-consumption share not only generate a within-period switch between private and public consumption spending, they also influence the representative household's intertemporal investment decision through its consumption Euler equation. We also find that saddle-path stability is *ceteris paribus* more likely to occur under a sufficiently high level of the public-investment or public-consumption share. It follows that there exists a trade-off between the government's investment versus consumption expenditures to suppress endogenous business cycles within the alternative version of our model economy. In sum, this article shows that in the context of a two-sector RBC model with positive investment externalities, it is the sectoral composition, rather than the total amount, of public spending that matters for the design of the government's stabilization policies.

The remainder of this article is organized as follows. Section II describes the model and analyzes its equilibrium conditions. Section III undertakes a quantitative investigation of local dynamics in a calibrated version of our benchmark specification. Section IV explores the model economy under a different formulation

of the government's spending policy rules. Section V concludes.

## II. THE ECONOMY

We incorporate government spending on goods and services into the discrete-time two-sector RBC model *à la* Harrison (2001). Households live forever, and derive utility from consumption and leisure. The production side of the economy consists of two distinct sectors, consumption and investment. On the basis of the empirical findings of Harrison (2003), competitive firms in each sector produce output with identical Cobb-Douglas technologies, but positive productive externalities are limited to the investment sector. The government balances its budget each period by levying lump-sum taxes on the representative household to finance its expenditures. We also postulate that there are no fundamental uncertainties present in the economy.

### A. Firms

In the consumption sector, output is produced by competitive firms using the following constant returns-to-scale technology:

$$(1) \quad Y_{ct} = K_{ct}^\alpha L_{ct}^{1-\alpha}, 0 < \alpha < 1,$$

where  $K_{ct}$  and  $L_{ct}$  are the capital and labor inputs utilized in the production of consumption goods. Under the assumption that factor markets are perfectly competitive, the first-order conditions for profit maximization are

$$(2) \quad r_t = \alpha Y_{ct} / K_{ct},$$

$$(3) \quad w_t = ((1 - \alpha) Y_{ct} / L_{ct}),$$

where  $r_t$  is the capital rental rate, and  $w_t$  is the real wage rate. Similarly, investment goods are produced by competitive firms using the technology

$$(4) \quad Y_{it} = X_t K_{it}^\alpha L_{it}^{1-\alpha}.$$

Here,  $K_{it}$  and  $L_{it}$  are physical capital and labor hours in the investment sector, and  $X_t$  represents productive externalities that each individual firm takes as given. Moreover,  $X_t$  is postulated to take the following form:

$$(5) \quad X_t = \left( \bar{K}_{it}^{-\alpha} \bar{L}_{it}^{1-\alpha} \right)^\theta, \theta \geq 0,$$

where  $\bar{K}_{it}$  and  $\bar{L}_{it}$  denote the within-sector average levels of capital and labor devoted to producing investment goods, and  $\theta$  measures the degree

of sector-specific externalities in the investment sector. In a symmetric equilibrium, all firms in the investment sector take the same actions such that  $K_{It} = \bar{K}_{It}$  and  $L_{It} = \bar{L}_{It}$ , for all  $t$ . Our analysis is restricted to cases with  $\alpha(1 + \theta) < 1$  such that the model economy does not exhibit sustained economic growth.

The first-order conditions that govern the demand for capital and labor in the investment sector are

$$(6) \quad r_t = p_t (\alpha Y_{It} / K_{It}),$$

$$(7) \quad w_t = p_t ((1 - \alpha) Y_{It} / L_{It}),$$

where  $p_t$  denotes the relative price of investment to consumption goods at time  $t$ . Notice that firms in each sector face the same factor prices in equilibrium as capital and labor inputs are assumed to be perfectly mobile across the two sectors.

### B. Households

The economy is populated by a unit measure of identical infinitely lived households. Each household maximizes its present discounted lifetime utility

$$(8) \quad \sum_{t=0}^{\infty} \beta^t \left[ \log C_t - \left( L_t^{1+\gamma} / (1+\gamma) \right) \right],$$

$0 < \beta < 1$ , and  $\gamma \geq 0$ ,

where  $C_t$  and  $L_t$  are the household's consumption and hours worked, respectively;  $\beta$  is the discount factor, and  $\gamma$  denotes the inverse of the wage elasticity for labor supply. Notice that the period utility function in Equation (8) is consistent with balanced long-run growth, a feature that is commonly adopted in the RBC literature.<sup>5</sup> The budget constraint faced by the representative agent is given by

$$(9) \quad C_t + p_t I_t + T_t = Y_t = r_t K_t + w_t L_t,$$

where  $I_t$  is gross investment,  $T_t$ , a non-distortionary lump-sum tax,  $Y_t$ , national income or GDP, and  $K_t$ , the household's capital stock that evolves according to the law of motion

$$(10) \quad K_{t+1} = (1 - \delta) K_t + I_t, \quad K_0 > 0 \text{ given,}$$

5. It can be shown that all our results (reported in Sections III and IV below) are qualitatively robust to a generalized preference formulation that is isoelastic with respect to the household's consumption (i.e., the coefficient of relative risk aversion is not equal to one) and additively separable with labor hours, regardless of whether the degree of productive externalities in the consumption sector is zero or positive. The details for these findings are available upon request.

where  $\delta \in (0, 1)$  is the capital depreciation rate.

The first-order conditions for the household's dynamic optimization problem are

$$(11) \quad C_t L_t^\gamma = w_t,$$

$$(12) \quad 1/C_t = [\beta/C_{t+1}] [r_{t+1} + (1 - \delta)p_{t+1}] / p_t$$

$$(13) \quad \lim_{t \rightarrow \infty} \beta^t (K_{t+1}/C_t) = 0,$$

where Equation (11) equates the slope of household's indifference curve to the real wage rate, Equation (12) is the Euler equation for intertemporal choices of private consumption, and Equation (13) is the transversality condition.

### C. Government

The government spends its (lump – sum) tax revenues  $T_t$  on goods and services produced by the consumption and investment sectors, and balances its budget each period. Hence, its period budget constraint is given by

$$(14) \quad T_t = G_{ct} + p_t G_{It},$$

where  $G_{ct}$  and  $G_{It}$  are quantities of consumption and investment goods, respectively, purchased by the government. In order to ensure that the economy exhibits a well-defined competitive equilibrium, we close the benchmark model by postulating

$$(15) \quad G_{ct} / Y_{ct} = \phi_c$$

and

$$(16) \quad G_{It} / Y_{It} = \phi_I,$$

where  $0 \leq \phi_c, \phi_I < 1$ . That is, government purchases from the consumption and investment sectors are set as constant fractions of their respective sectoral output.<sup>6</sup> When  $\phi_c = \phi_I = 0$ , we recover the laissez-faire model of Harrison (2001). Finally, combining Equations (9) and (14) yields the following aggregate resource constraint for the economy:

$$(17) \quad C_t + p_t I_t + G_t = Y_t,$$

where  $G_t = G_{ct} + p_t G_{It}$  denotes total government spending.

6. Section IV examines the alternative specification whereby government purchases of consumption and investment goods are postulated as constant proportions of the economy's GDP. When public expenditures of each type are assumed to be a constant fraction of total government spending, our current setup with lump-sum taxation will not have enough equations to pin down the model's equilibrium allocations. It would be a worthwhile topic for future research to investigate this formulation under distortionary income tax policies.

#### D. Equilibrium

As firms use identical production technologies and face equal factor prices across the two sectors, the fractions of capital and labor inputs utilized in the consumption sector are the same,

$$(18) \quad K_{ct}/\bar{K}_t = L_{ct}/L_t \equiv \mu_t.$$

Using Equations (1) and (2), (4)–(6), and (18), the equilibrium relative price of investment goods can be expressed as

$$(19) \quad p_t = 1 / \left[ (1 - \mu_t) K_t^\alpha L_t^{1-\alpha} \right]^\theta.$$

The equalities of demand by households and the government versus supply by firms in the consumption and investment sectors are given by

$$(20) \quad C_t + G_{ct} = Y_{ct},$$

$$(21) \quad I_t + G_{It} = Y_{It}.$$

Moreover, both the capital and labor markets clear whereby

$$(22) \quad K_{ct} + K_{It} = K_t,$$

$$(23) \quad L_{ct} + L_{It} = L_t.$$

We focus on symmetric perfect-foresight competitive equilibria, which consist of a set of prices  $\{r_t, w_t, p_t\}_{t=0}^\infty$  and allocations  $\{C_t, L_t, K_{t+1}, Y_t, Y_{ct}, K_{ct}, L_{ct}, Y_{It}, K_{It}, L_{It}, X_t, I_t, T_t, G_{ct}, G_{It}, \mu_t\}_{t=0}^\infty$  that satisfies Equations (1)–(7), (9)–(12), (14)–(16), (18), and (20)–(23), together with the given initial aggregate capital stock  $K_0$  and the transversality condition (13).<sup>7</sup>

#### E. Steady State and Local Dynamics

It is straightforward to show that our model possesses a unique interior steady state at which the fraction of factor inputs allocated to producing consumption goods is

$$(24) \quad \mu = 1 - (\alpha\delta / (1 - \phi_I)) ((1/\beta) - 1 + \delta),$$

where time subscript is left out to denote the steady-state value. Given Equation (24), the

7. It is straightforward to show that our benchmark model can be analytically equivalent (with identical equilibrium conditions and local dynamics) to an economy in which the government levies constant tax rates on the household's consumption and investment expenditures, and then returns all its tax revenues to households as a lump-sum transfer. This analytical equivalence was pointed out to us by an anonymous referee, to whom we are grateful.

steady-state expressions of all other endogenous variables can be easily derived.<sup>8</sup>

We then take log-linear approximations to the model's equilibrium conditions in a neighborhood of this steady state to obtain the following dynamical system:

$$(25) \quad \underbrace{\begin{bmatrix} \hat{K}_{t+1} \\ \hat{C}_{t+1} \end{bmatrix}}_{\mathbf{J}} = \underbrace{\begin{bmatrix} \mathbf{J}_{11} & \mathbf{J}_{12} \\ \mathbf{J}_{21} & \mathbf{J}_{22} \end{bmatrix}}_{\mathbf{J}} \begin{bmatrix} \hat{K}_t \\ \hat{C}_t \end{bmatrix}, \quad \hat{K}_0 \text{ given},$$

where hat variables represent percentage deviations from their respective steady-state values, and  $\mathbf{J}$  is the Jacobian matrix of partial derivatives of the transformed dynamical system (see Appendix A). The expressions for the trace and determinant of the Jacobian matrix are

$$(26) \quad \text{Tr} = \mathbf{J}_{11} + \mathbf{J}_{22},$$

and

$$(27) \quad \text{Det} = \mathbf{J}_{11}\mathbf{J}_{22} - \mathbf{J}_{12}\mathbf{J}_{21}.$$

According to Azariadis (1993, 93), the economy exhibits saddle-path stability and equilibrium uniqueness if and only if

$$(28) \quad -(\text{Tr} + 1) < \text{Det} < \text{Tr} - 1 \quad \text{or}$$

$$\text{Tr} - 1 < \text{Det} < -(\text{Tr} + 1);$$

i.e., when one eigenvalue of  $\mathbf{J}$  lies inside and the other outside the unit circle. Our model possesses an indeterminate steady state (a sink) if and only if

$$(29) \quad -1 < \text{Det} < 1 \text{ and } -(\text{Det} + 1) < \text{Tr} < \text{Det} + 1,$$

hence both eigenvalues are inside the unit circle. The steady state becomes a totally unstable source, whereby both eigenvalues are outside the unit circle, if and only if

$$(30) \quad \text{Det} > 1 \text{ and } -(\text{Det} + 1) < \text{Tr} < \text{Det} + 1.$$

### III. MACROECONOMIC (IN)STABILITY

This section examines the quantitative interrelations between sectoral composition of public spending, governed by  $\phi_c$  and  $\phi_I$ , and equilibrium (in)determinacy in the benchmark version of our two-sector RBC model with productive externalities in the investment sector. Our first finding is

8. For example, the (aggregate) labor hours and capital stock provided by the household are given by  $L = [(1 - \alpha)/\mu(1 - \phi_c)]^{1/(1+\gamma)}$  and  $K = [(1 - \phi_I)(1 - \mu)^{1+\theta} L^{(1-\alpha)(1+\theta)/\delta}]^{1/(1-\alpha(1+\theta))}$ .

**RESULT 1.** *The output fraction of government purchases in the consumption sector  $\phi_c$  has no effect on the model's local stability properties, whereas the output fraction of government purchases in the investment sector  $\phi_I$  does.*<sup>9</sup>

The intuition for the above result can be understood as follows. Start the economy from its steady state, and consider an anticipated increase in the rate of return on capital caused by agents' optimistic expectation about an expansion of future economic activities. Acting upon this belief, the representative household will consume less and invest more today, which in turn reduces the amount of capital and labor inputs allocated to the production of consumption goods. As it turns out, although changes in government spending on consumption goods generate a within-sector trade-off between private and public consumption expenditures, they do not affect the relative composition of productive resources across the two sectors that is crucial in validating agents' initial optimism. Specifically, the public-consumption share  $\phi_c$  does not enter Equation (24), which governs the steady-state intensity of factor inputs  $\mu$ ; or the elements that make up the Jacobian matrix  $\mathbf{J}$  shown in Appendix A.<sup>10</sup> This implies that the government's purchases of consumption goods play no role in the equilibrium dynamics of our benchmark model.

However, Equation (24) together with (A.1)–(A.6) reported in Appendix A show that sectoral distribution of productive resources at the model's steady state, as well as the resulting local stability properties, are influenced by the output fraction of government purchases in the investment sector  $\phi_I$ . For example, an increase in public spending on investment goods not only crowds out private investment expenditures, it also raises the steady-state proportion of factor inputs that will be shifted toward the investment sector ( $\partial(1 - \mu)/\partial\phi_I > 0$ ) upon agents' optimistic expectation. It follows that the public-investment share may (de)stabilize the macroeconomy since  $\phi_I$  affects the accumulation of physical capital,

9. It can be shown that this result continues to hold when a generalized constant-relative-risk-aversion (CRRA) utility formulation in consumption and/or positive productive externalities in the consumption sector are considered. We thank an anonymous referee for pointing this out.

10. It is straightforward to show that the household's consumption expenditures can be expressed as  $C_t = (1 - \phi_c)\mu_t Y_t$ . As a result, the public-consumption share  $\phi_c$  drops out from the Jacobian matrix when we log-linearize the model's equilibrium conditions.

and thus the intertemporal trade-off between consumption choices at different time periods.

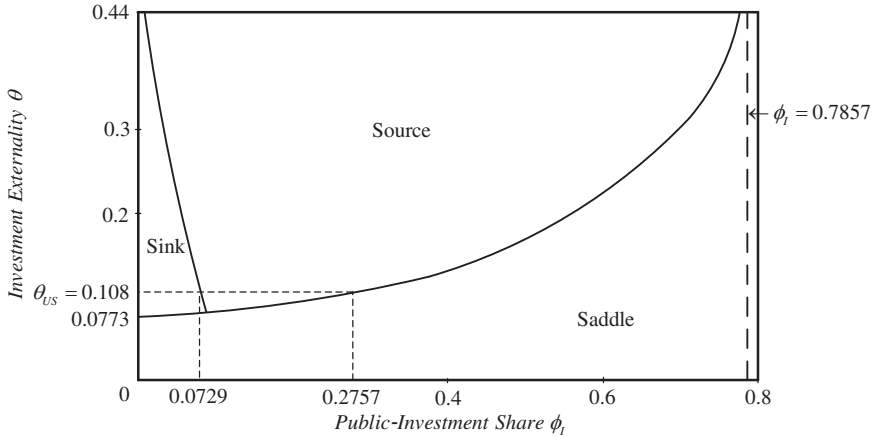
Next, we investigate how the government's spending policies affect equilibrium dynamics of competitive equilibria for combinations of calibrated parameters that are consistent with post-Korean war U.S. data. Each period in the model is taken to be one quarter. As in Benhabib and Farmer (1996), Harrison (2001) and many previous studies in the RBC literature, the labor share of national income,  $1 - \alpha$ , is chosen to be 0.7; the discount factor,  $\beta$ , is set to be 1/1.01; the labor supply elasticity,  $\gamma$ , is equal to 0 (i.e., indivisible labor *à la* Hansen (1985) and Rogerson (1988)); and the capital depreciation rate,  $\delta$ , is fixed at 0.025. In addition, Result 1 allows us to ignore  $\phi_c$  within the benchmark version of our model.

Given the above parameterization, Figure 1 depicts the local stability properties of our baseline model as a function of the output fraction of government purchases in the investment sector and the degree of productive externalities in investment. In particular, the  $\phi_I - \theta$  space is divided into regions of "Saddle," "Sink," and "Source." When  $\phi_I \rightarrow 0.7857$ , the steady-state fraction of capital and labor inputs utilized in the consumption sector  $\mu$  will approach zero (see Equation (24)), and hence an equilibrium is no longer feasible. Using durables as a proxy for the investment goods in our model, we follow Guo and Harrison (2010, 298) and set the upper bound of investment externalities  $\theta$  to be 0.44, which is the highest possible value that is regarded as empirically plausible, in Figure 1. This is one standard deviation above Basu and Fernald's (1997, Table III) aggregation-corrected point estimate for returns-to-scale in the U.S. durables manufacturing industry.

**RESULT 2.** *When  $\phi_I = 0$  as shown in the vertical axis of Figure 1, the model's steady state is a saddle point for  $0 \leq \theta \leq 0.0773$ ; or a sink for  $\theta \geq 0.0774$ .*

This result illustrates that irrespective of the value of  $\phi_c$ , our baseline model without public spending on investment goods ( $\phi_I = 0$ ) displays exactly the same local dynamics as those in Harrison (2001, section 4) under *laissez faire*. Intuitively, when agents anticipate a higher future return on today's investment, they need incentive to give up current consumption in exchange for more capital accumulation. As long as they are rewarded with productive investment, in the form of sufficiently high increasing returns-to-scale within that sector, it will be worthwhile for

**FIGURE 1**  
Benchmark Model with  $G_{It}/Y_{It} = \phi_I$



them to do so. Specifically in our calibrated economy, the rate of return on capital will rise to fulfill agents’ rosy expectation provided the external effects in the investment firms’ production processes are strong enough with  $\theta \geq 0.0774$ .

**RESULT 3.** *When  $\phi_I > 0$  and  $0 \leq \theta \leq 0.0773$ , the economy always exhibits saddle-path stability and equilibrium uniqueness.*

Substituting  $\theta = 0$  into Equation (19) shows that the relative price of investment to consumption goods becomes  $p_t = 1$  for all  $t$ . As a result, our model without any investment externality collapses to a canonical one-sector RBC formulation with aggregate constant returns-to-scale in production, except that we have added a government sector which does not change its saddle-path stability. It follows that equilibrium indeterminacy and belief-driven cyclical fluctuations can never arise within this setting. We also obtain the same finding under a “low” degree of investment externality ( $0 < \theta \leq 0.0773$ ) because it is not sufficiently strong to raise the equilibrium rate of return on capital, regardless of how public expenditures are divided between the two sectors.<sup>11</sup>

**RESULT 4.** *For a given level of  $\theta \geq 0.0841$ , the model’s local stability property switches from being*

11. By contrast, Guo and Harrison (2001, 2011) show that a regressive income tax policy may destabilize the economy by inducing endogenous cyclical fluctuation in an otherwise identical two-sector RBC model with zero or low investment externality.

*a sink to a source, and then to a saddle point as the public-investment share  $\phi_I$  increases.*<sup>12</sup>

As an example, when the investment externality takes on an empirically realistic value of  $\theta_{US} = 0.108$  (Harrison 2003), local indeterminacy occurs for public-investment shares in the range  $0 \leq \phi_I \leq 0.0728$ . The steady state turns into a source when  $\phi_I$  is raised to the interval of  $[0.0729, 0.2756]$ . In this case, any trajectory that diverges from this completely unstable steady state may settle down to a limit cycle or to some complicated attracting sets. The model exhibits a locally unique equilibrium (a saddle path) for  $\phi_I \geq 0.2757$ . The policy implication of this result is that the economy is more susceptible to indeterminacy and endogenous business cycles when the output fraction of government purchases in the investment sector is zero (see Result 2) or “relatively small.”

**RESULT 5.** *When  $\phi_I > 0$  and  $\theta \geq 0.0774$ , the threshold level for the output fraction of public spending in the investment sector, denoted as  $\phi_I^{min}$ , that leads to saddle-path stability is monotonically increasing with respect to the degree of investment externalities, i.e.,  $\partial \phi_I^{min} / \partial \theta > 0$ .*

To understand the intuition for the previous results, we rewrite the intertemporal

12. When the investment externality  $\theta$  lies within the interim interval  $[0.0774, 0.0841]$ , the steady state changes from being a sink to a saddle point as  $\phi_I$  rises.



Euler equation for private consumption Equation (12) as

$$(31) \quad C_{t+1} / C_t = \beta [r_{t+1} + (1 - \delta)p_{t+1}] / p_t$$

Start from the model's steady state at period  $t$ , and suppose that households become optimistic about the economy's future. In response to this change in non-fundamental expectations, they sacrifice consumption ( $C_t$  falls) for higher investment today (raising  $K_{t+1}$ ), which in turn raises next period's hours worked via firms' labor demand, thereby producing more output and higher consumption in period  $t+1$  ( $C_{t+1}$  rises). It follows that the left-hand side of Equation (31) will rise. For this alternative dynamic path to be justified as a self-fulfilling equilibrium, the (price-weighted) rate of return on  $K_{t+1}$  net of depreciation, i.e., the right-hand side of Equation (31), needs to increase as well.

As it turns out, the quantitative interdependence between  $\theta$  and  $\phi_I$  that governs our model's local stability properties depends crucially on the relative strength of two opposing forces. On the one hand, due to the presence of positive externalities in producing investment goods, the social production possibility frontier which traces out the trade-off between private consumption and investment expenditures is convex to the origin. As a result, the relative price of investment  $p_t$  falls (*the price effect*) upon agents' optimism that shifts more capital and labor inputs into the investment sector—this effect causes the right-hand side of Equation (31) to rise. On the other hand, an increase in today's private investment expenditures that raises  $K_{t+1}$  will result in a lower real interest rate  $r_{t+1}$  because of diminishing marginal product of capital (*the MPK effect*)—this effect causes the right-hand side of Equation (31) to fall.

Results 2 and 4, together with footnote 12, illustrate that for a given degree of investment externalities  $\theta \geq 0.0774$ , the price effect outweighs the MPK effect provided the output fraction of government spending on investment goods is zero or relatively low. In either formulation, equilibrium indeterminacy results as the right-hand side of Equation (31) will rise to validate the initial anticipated increase in the return on capital. It follows that raising the public-investment share to the corresponding critical level  $\phi_I^{mim} (=0.2757$  when  $\theta_{US} = 0.108)$  or above is able to stabilize the economy against sunspot-driven business cycles because of a dominating MPK effect.

Finally, when  $\theta \geq 0.0774$  and rises, higher increasing returns-to-scale in investment induce households to move more productive resources out of the consumption sector, which in turn generate larger decreases in  $p_t$  and  $r_{t+1}$ , i.e., the price and MPK effects both become stronger. Per the same reasoning from the previous paragraph, Result 5 finds that  $\phi_I^{mim}$  will increase as well ( $\partial \phi_I^{mim} / \partial \theta > 0$ ) such that the reduction in  $r_{t+1}$  dominates that in  $p_t$  to render the equilibrium unique and determinate. In sum, within the benchmark version of our two-sector RBC model with positive productive externalities in the investment sector, we show that the output fraction of public expenditures on investment goods needs to be sufficiently high in order to suppress endogenous business cycles driven by agents' animal spirits. Since there are no shocks to economic fundamentals present in the economy, the resulting unique equilibrium with  $\phi_I > \phi_I^{mim}$  will not display any cyclical fluctuations.

#### IV. ALTERNATIVE SPECIFICATION ON GOVERNMENT SPENDING

For sensitivity analysis, this section examines our two-sector RBC model under a different formulation of the government's spending policy rules. Specifically, public expenditures of consumption and investment goods are postulated as constant fractions of the economy's total output:

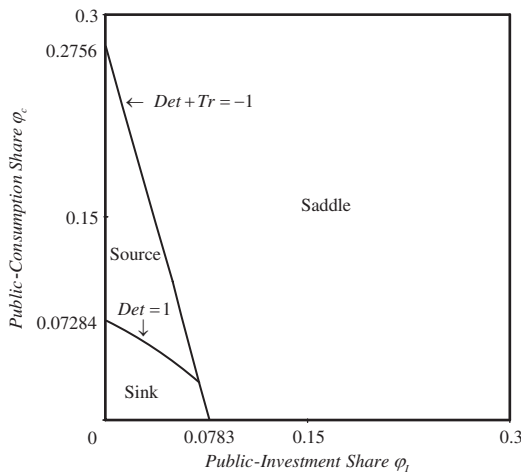
$$(32) \quad G_{ct}/Y_t = \varphi_c \quad \text{and} \quad p_t G_{It}/Y_t = \varphi_I,$$

where  $0 \leq \varphi_c, \varphi_I < 1$  and  $0 \leq \varphi_c + \varphi_I < 1$ . In this case, it is straightforward to show that the steady-state proportion of capital and labor inputs allocated to producing consumption goods is given by

$$(33) \quad \mu = 1 - \varphi_I - \alpha\delta / [(1/\beta) - 1 + \delta]$$

and that all other endogenous variables at the steady state can be derived accordingly. We then log-linearize the model's equilibrium conditions around this unique interior steady state to obtain the associated Jacobian matrix  $\mathbf{J}$  shown in Appendix B, as well as its trace and determinant given by Equations (26) and (27). In sharp contrast to Result 1 of our baseline specification, equations (B.1)–(B.7) show that the GDP fraction of government purchases in the consumption sector  $\varphi_c$ , as well as the public-investment share  $\varphi_I$ , now appears in the Jacobian matrix and thus affects the economy's local stability properties.

**FIGURE 2**  
 Alternative Model with  
 $G_{ct}/Y_t = \varphi_c$  and  $p_t G_{It}/Y_t = \varphi_I$



To understand this finding, we note that the household’s consumption spending within the alternative model can be expressed as

$$(34) \quad C_t = (\mu_t - \varphi_c) Y_t.$$

It follows that unlike in the benchmark counterpart (see footnote 10), the public-consumption share  $\varphi_c$  does not drop out from the log-linearization procedure and will play a role in the equilibrium dynamics. Intuitively, changes in  $\varphi_c$  not only yield a within-period trade-off between private and public consumption expenditures, they also influence the factor intensity across the two production sectors and the representative agent’s intertemporal investment decision through substituting Equation (34) into its consumption Euler Equation (12).

Under the same calibrated values of  $\alpha$ ,  $\beta$ ,  $\gamma$ , and  $\delta$  as those in our baseline configuration, together with the investment externality fixed at  $\theta_{US} = 0.108$ , Figure 2 summarizes the model’s local stability properties with public (consumption and investment) expenditures given by Equation (32). Since the post-Korean war average of government spending to output ratio is around 0.2 in United States, we set the individual upper bound of  $\varphi_c$  and  $\varphi_I$  to be 0.3 for the purpose of clear illustration.

Using conditions (28)–(30), Figure 2 shows that given the above parameterization over the feasible  $\varphi_I - \varphi_c$  space: (a) the most-binding necessary and sufficient condition which separates

the areas labeled “Saddle” versus “Sink” and “Source” turns out to be  $\text{Det} + \text{Tr} = -1$ ; (b) the downward-sloping curve that divides the regions of “Sink” and “Source” is characterized by  $\text{Det} = 1$ ; (c) when  $\varphi_c = 0$ , the minimum level for the GDP fraction of public spending in the investment sector that stabilizes the economy against belief-driven fluctuations is  $\varphi_I^{\text{min}} = 0.0783$ , below which local indeterminacy results; and (d) when  $\varphi_I = 0$ , the steady state is a sink under  $0 \leq \varphi_c \leq 0.07283$ , turns into a source as  $\varphi_c$  lies within the interval  $[0.07284, 0.2755]$ , and becomes a saddle point at  $\varphi_c^{\text{min}} = 0.2756$ .

Figure 2 also depicts that similar to our benchmark specification, saddle-path stability is *ceteris paribus* more likely to occur under a “sufficiently high” value of  $\varphi_I$  or  $\varphi_c$  within the alternative model. For a given level of  $\varphi_c$ , the intuition for why higher public-investment shares make equilibrium uniqueness easier to obtain is the same behind Result 4 of the preceding section: increases in  $\varphi_I$  will eventually generate a stronger MPK effect to invalidate the household’s initial optimism. For a given level of  $\varphi_I$ , a higher  $\varphi_c$  shifts more productive resources into the consumption sector, which in turn raises the relative price of investment goods  $p_t$  along the convex social production possibility frontier and thus prevents agents’ optimistic expectation from becoming self-fulfilling. Finally, combining these results on  $\varphi_I$  and  $\varphi_c$  implies that the dividing locus of  $\text{Det} + \text{Tr} = -1$ , above which the economy does not exhibit indeterminacy and sunspots, is negatively sloped. That is, there exists a trade-off between the public-investment versus the public-consumption shares for the design and implementation of stabilization policies in our model economy with government spending described by Equation (32).

## V. CONCLUSION

This article has shown that in the context of a two-sector RBC model with positive productive externalities in investment, it is the sectoral composition, rather than the total amount, of government spending that governs the economy’s macroeconomic (in)stability. Specifically in our benchmark specification whereby public expenditures from the consumption and investment sectors are set as constant fractions of their respective sectoral output, the public-consumption share plays no role in the model’s local dynamics. We also find that the economy is more susceptible to indeterminacy and

sunspots when the output fraction of government purchases of investment goods is relatively small; and that the threshold level for the public-investment share which yields saddle-path stability and equilibrium uniqueness is monotonically increasing with respect to the degree of investment externalities. When each type of public spending is postulated as a constant proportion of the economy's total output, we show that in sharp contrast to the baseline configuration, changes in the GDP fraction of government purchases in the consumption sector affect the local stability properties of competitive equilibria. In addition, there exists a trade-off between public consumption versus investment expenditures to stabilize the economy against endogenous belief-driven cyclical fluctuations within the alternative version of our model economy.

In terms of possible extensions, it would be worthwhile to study our model economy with the monetary authority that follows an interest rate rule *à la* Dressler (2011), or useful government spending that contributes to the household's utility *à la* Guo and Harrison (2008, section 3.2) or the firm's productivity *à la* Barro (1990), or a generalized non-separable preference formulation *à la* Jaimovich and Rebelo (2009) that considers different degrees of income effect on labor supply, or a non-balanced budget with national debt *à la* Schmitt-Grohé and Uribe (1997, 990). Incorporating these features will allow us to examine the robustness of this article's quantitative results and policy implications, as well as further enhance our understanding of the equilibrium dynamic effects of public expenditures in a representative-agent model with multiple production sectors. We plan to pursue these research projects in the future.

#### APPENDIX A

It can be shown that the elements which make up the benchmark model's Jacobian matrix  $\mathbf{J}$  as in Equation (25) are

$$(A.1) \quad \mathbf{J}_{11} = 1 - \delta + \alpha\delta(1 + \theta)(1 + \gamma) / (\alpha + \gamma)(1 - \mu),$$

$$(A.2) \quad \mathbf{J}_{12} = -\delta(1 + \theta) [1 - \alpha + \mu(\alpha + \gamma)] / (\alpha + \gamma)(1 - \mu),$$

$$(A.3) \quad \mathbf{J}_{21} = 1/\Pi [\alpha\theta(1 + \gamma) / (\alpha + \gamma)(1 - \mu) + \Psi\mathbf{J}_{11}],$$

$$(A.4) \quad \mathbf{J}_{22} = 1/\Pi \{[(\alpha + \gamma)(1 - \mu(1 + \theta)) - \theta(1 - \alpha)] / (\alpha + \gamma)(1 - \mu) + \Psi\mathbf{J}_{12}\}$$

where  $\mu$  is given by Equation (24) together with

$$(A.5) \quad \Pi = 1 + \{(1 - \alpha) [1 - \beta(1 - \delta)(1 + \theta)] - \beta\theta\mu(1 - \delta)(1 + \gamma) / (1 - \mu)\} / (\alpha + \gamma)$$

and

$$(A.6) \quad \Psi = \beta(1 - \delta) [1 - \alpha(1 + \theta)] - (1 - \alpha) + \alpha\{(1 - \alpha) [1 - \beta(1 - \delta)(1 + \theta)] - \beta\theta\mu(1 - \delta)(1 + \gamma) / (1 - \mu)\} / (\alpha + \gamma)$$

#### APPENDIX B

It can be shown that the log-linearized dynamical system for the alternative specification of our model economy exhibits the same representation as in Equation (25), and that the elements that make up the corresponding Jacobian matrix  $\mathbf{J}$  are

$$(B.1) \quad \mathbf{J}_{11} = 1 - \delta + \alpha\delta(1 + \theta) + \alpha [\delta\mu(1 - \alpha)(1 + \theta) + \Omega(1 + \gamma)(\mu - \varphi_C)] / \mu(\alpha + \gamma)$$

$$(B.2) \quad \mathbf{J}_{12} = -[\delta\mu(1 - \alpha)(1 + \theta) + \Omega(1 + \gamma)(\mu - \varphi_C)] / \mu(\alpha + \gamma)$$

$$(B.3) \quad \mathbf{J}_{21} = 1/\Delta \{ \alpha\theta + \alpha\theta[(1 + \gamma)(\mu - \varphi_C) / (1 - \mu) + 1 - \alpha] / (\alpha + \gamma) + \Lambda\mathbf{J}_{11} \}$$

$$(B.4) \quad \mathbf{J}_{22} = 1/\Delta \{ 1 - \theta(1 + \gamma)(\mu - \varphi_C) / (\alpha + \gamma)(1 - \mu) - \theta(1 - \alpha) / (\alpha + \gamma) + \Lambda\mathbf{J}_{12} \}$$

where  $\mu$  is given by Equation (33) together with

$$(B.5) \quad \Omega = (\delta\mu [(1 - \mu)(1 + \theta) - \theta\varphi_I] / (1 - \mu)(1 - \mu - \varphi_I)),$$

$$(B.6) \quad \Delta = 1 - \{(1 + \gamma)(\mu - \varphi_C)\beta\theta\mu(1 - \delta) / (1 - \mu) - \mu(1 - \alpha) [1 - \beta(1 - \delta)(1 + \theta)]\} / \mu(\alpha + \gamma)$$

and

$$(B.7) \quad \Lambda = \beta(1 - \delta) [1 - \alpha(1 + \theta)] - (1 - \alpha) - \alpha\{(1 + \gamma)(\mu - \varphi_C)\beta\theta\mu(1 - \delta) / (1 - \mu) - \mu(1 - \alpha) [1 - \beta(1 - \delta)(1 + \theta)]\} / \mu(\alpha + \gamma)$$

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