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NOVEL METHOD FOR CONSTRUCTING TETRAHEDRAL FRAMES

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INTRODUCTION

This paper is concerned with the design and construction of a frame that makes optimal use of modern materials together with optimal structural design. It is modular in format and can be constructed using puckered rings which combine to make a frame with tetrahedral nodes and cubic symmetry; analogous with the crystal structure of diamond. This frame can be extended indefinitely in three dimensions by adding modules to its core. Such a frame has high capacity to bear loads relative to its weight. For some types of loading it has the highest possible specific-structural strength (for a frame of struts). It is a structural analog of a foam.

The struts of a frame are commonly straight and joined together at nodes by means of various types of male and female coupling devices. Thus the struts are discontinuous at the nodes unless the material of construction can be welded. But some of the materials with the highest specific strengths are composites consisting of strong filaments embedded in less strong matrices, and such materials cannot be welded. They are difficult to join using coupling devices without adding substantial extra weight. Therefore, in building lightweight

structures, it is desirable to be able to use continuous strut-like members that do not need nodal coupling devices.

The frame described here uses continuous puckered rings as strut-like members. These rings provide maximum structural efficiency, and can be readily constructed from materials that have maximal specific stiffnesses and strengths, including filamentary composites.

In two dimensions the minimum number of struts needed to fully determine the position of a point is three. Therefore, frames are usually based on triangular configurations. But in three dimensions the minimum number of struts becomes four. For maximum symmetry, and therefore maximum specific-structural-strength, these four struts must be arrayed with equal angles between them at each node. Thus, they must have tetrahedral angles between them ($109^{\circ} 28'$). This allows the nodes to be connected in one of two ways yielding frames of either cubic or hexagonal symmetry. Of these two configurations, the cubic one has the most symmetry; and it is the one that will be emphasized here.

A tetrahedral frame constructed from puckered rings is shown in Figure 1. It is periodic in three dimensions. As a result of this periodicity the frame can be built up in a repetitive way by adding fundamental modular units to it until a desired set of dimensions is obtained. Its geometric form causes it to have three-dimensional stability without depending on lateral stabilizing members or complex networking. Also, this design takes advantage of the inherent rigidities of skeletal tetrahedra. Since the design achieves these features with maximum geometric efficiency (it uses the minimum number of struts per node to have

stability) it has a maximum amount of open space within it; thereby improving its usefulness. The open space is in the form of quasi-cylindrical channels that pass through it along three orthogonal directions. A sketch showing one set of these channels is presented in Figure 2.

FRAME CONSTRUCTION

The standard approach to constructing a tetrahedral frame would be to connect a set of straight struts of equal length with nodal connectors, or with welds (1). Welding could be used for some metals or alloys, but cannot be applied effectively to ceramics, aligned polymers, or composites. In general, there are two types of nodal connectors to be considered: male and female as illustrated in Figure 3. The female connector (Figure 3A) has the disadvantage of requiring excess materials on the outside of the strut which adds to the weight of the system. The male one (Figure 3B) is less weighty, but tends to splay the end of the hollow strut if there is a bending moment present. This is especially serious for filamentary composites because they have relatively little transverse strength, and is also a problem with brittle, notch-sensitive materials.

The approach proposed here uses puckered rings (2). The rings have six sides and have trigonal symmetry as shown in Figure 4. They can be constructed by winding up fibers to form skeins which can then be appropriately puckered and set into shape with an impregnated adhesive (or wound with the adhesive already on the fiber). Thus there need be no discontinuities in the rings.

The puckered rings stack well, so they can be stored compactly until the

time comes to construct the frame. Then they can be assembled using adhesives, brazements, mechanical connectors such as splines, or weldments. Even if they are not used for strength, splines may be used to aid in assembly because they minimize disruption of the continuity in the rings.

To maximize the packing density prior to assembly, standard elbows may be the structural unit (Figure 5). Six of them will form a ring, and such rings can be assembled into a frame.

The structural units can be made of metals, glasses, polymers, ceramics, or composites. Their cross-sections may be circular, but they will pack more densely if they are hexagonal or triangular. Also, they may be hollow, solid, or channels.

The first step in constructing a frame is to assemble four rings into a tetroid space-filling unit as shown in Figure 6. Then, either by joining tetroids together (Figure 7), or by adding rings to a tetroid, a frame can be built up (Figure 1). Such a frame uses a minimum amount of material to sustain a given set of applied forces. This was proven long ago by Michell (3).

Sometimes it is desirable to have a higher frame density on one or more sides of an underlying less dense frame. One example arises when a large beam is to have a frame as its substructure so more strength is needed at the outer surfaces than in the interior. The same may be true for large plates or shells. Another example arises when a thin skin is to be supported by a frame, so an increased density of support points is desired at the surface of an otherwise low density frame. Figure 8 shows how increased frame density can be obtained with

the present method of construction. In order for the smaller and larger rings to mesh coherently, the sizes of the smaller ones need to be integral sub-multiples of the larger ones.

Another method that can be used to "close-off" a surface on a tetrahedral frame is to use triangular rings in a two-dimensional array. This is the most efficient two-dimensional frame.

The tetroid units of Figure 6 are related to the "diamond saddle polyhedra" described by Pearce (4). However, they differ in being skeletal rather than blocky. Also, the method of constructing them from rings as described here is novel.

MECHANICAL RESPONSE OF TETRAHEDRAL FRAMES

Tetrahedral frames are not isotropic. They may have either cubic or hexagonal symmetry. In the case of the cubic ones, their structural stiffnesses in response to distributed applied loads, require three stiffness constants for their description; thus they are orthotropic. Three constants analogous with the elastic stiffnesses of cubic crystals can be used (5). Their conventional designations are: C_{11} , C_{12} , and C_{44} . The resistance of the frame to a uniform dilatational strain (its bulk modulus) can be described in terms of these constants. Its resistance to uniaxial stretching along its three characteristic directions: $\langle 100 \rangle$, $\langle 110 \rangle$, and $\langle 111 \rangle$ can also be described in terms of them. The first of these characteristic directions lies along any edge of the reference cube sketched in Figure 9. The second lies along any face-diagonal; and the third along any body-diagonal.

Stiffness

In order to develop a description of the frame stiffness, the first step is to define strut constants for the elements that connect the nodes. These elements consist of bundles of six of the sides of adjacent puckered rings. The strut lengths, l approximately equal the distances between nodes. The ring sides may be assumed to be cylindrical, but they might have hexagonal or other more complex cross-sections. Their cross-sectional shapes will determine their section moduli and hence their resistance to bending. Then, if the axial stiffness of the material in the struts is described by Young's Modulus, E , and the cross-sectional area is A , the stretching constant for a strut is:

$$C_s = AE \quad (1)$$

and the total strain energy in a strut with a strain, ϵ is:

$$C_s \epsilon^2 / 2$$

In shear a strut is like two face-to-face cantilevers with each end "built-in". If the strut has an effective diameter, d and d/l is small, the strain-energy can be determined for a given deflection angle, θ (7). Then, since the total energy is:

$$C_b \theta^2 / 2$$

the bending constant becomes

$$C_b = \frac{6EI}{l} \quad (2)$$

where I is the section modulus for a circular bar and equals: $\pi d^4/64$.

To design a frame in terms of the stretching and bending constants, expressions that relate them to the stiffness constants, C_{ij} are needed. Then a frame with desired properties can be designed through proportioning of the strut dimensions, and selection of the materials of construction. The analysis used here parallels that for tetrahedrally-bonded crystals given by Harrison (6). It will not be repeated here except for the results.

The elastic stiffness constants in terms of the strut constants are given by the following expressions:

$$C_{11} = D(Cs/6 + 2C_b) \quad (3)$$

$$C_{12} = D(Cs/6 - C_b) \quad (4)$$

$$C_{44} = 3DC_s C_b / (C_s + 8C_b) \quad (5)$$

where $D = \sqrt{3}/2l^3$

In terms of these C_{ij} , the bulk stiffness, B which measures the resistance of the frame to uniform volume changes is:

$$B = \frac{1}{3}(C_{11} + 2C_{12})$$

or, in terms of the stretching constant:

$$B = DC_s/6$$

and substituting for C_s

$$B = (d/l)^2 E / 9 \quad (6)$$

This expression indicates a reduction from the Young's modulus of the material in proportion to the slenderness ratios of the struts. This is just as expected from qualitative considerations.

If B is divided by the density of the structure, the specific bulk modulus of the frame is obtained. The frame density in terms of the material density, ρ is:

$$\rho^* = (d/l)^2 \rho$$

and this can be very low. For example, if the slenderness ratio is 0.1, the frame density is only one per cent of the material density.

The specific bulk modulus for the frame is $B^* = B / \rho^*$

or

$$B^* = (E / \rho) / 9 \quad (7)$$

Note that this is independent of the slenderness ratio (for moderately high ratios). Also note how relatively high the specific modulus is; or in other words how efficient this design is in using the specific stiffness of the material to full advantage. Another way of looking at this is that this frame design allows very large and quite open frames to be constructed with minimum sacrifice of specific stiffness. As Gordon (8) has pointed out, this is the optimum approach to the design of large structures. It may be noted in this connection that a tetrahedral frame is a kind of foam built of linear struts rather than thin films.

In shear this frame is also efficient because no diagonal stiffeners are needed.

A simple cubic frame is much heavier because it requires shear stiffeners. For the case of dilatation, the bulk modulus is the same for both the simple cubic configuration and the tetrahedral one. However, there are six struts at each node so the weight is 50% more. Adding diagonal shear stiffeners increases the weight even more.

Compliance

The stiffness constants, C_{ij} when multiplied by a set of strains yield the corresponding stresses. But frames are usually subjected to tractions (stresses) rather than strains, and one wishes to know the corresponding strains (deflections). Such responses are conveniently described by a set of compliance constants, S_{ij} which yield the strains when they are multiplied by the stresses. They are inversely related to the C_{ij} 's (5).

Anisotropy

Since a tetrahedral frame does not have isotropic properties in general, its resistance to shear depends on the direction and plane of shear. Two shear constants are required. For convenience, one of these is C_{44} which describes shearing parallel to the faces of the cubic unit cell along the directions of the cube edges. The other is $1/2 (C_{11} - C_{12})$ which describes shearing parallel to diagonal planes of the unit cell containing a cube edge and a face-diagonal. The shear direction is parallel to the face-diagonal. This constant, in terms of the strut constants is: $3DC_b/2$.

The ratio of the two shear constants measures the elastic anisotropy, of the frame. It is:

$$A = 2C_{44}/(C_{11}-C_{12}) = GC_s/(C_s+8C_b)$$

If the anisotropy ratio equals unity, the frame is elastically isotropic. This condition is realized if:

$$C_s = 8C_b/5$$

Thus, if an isotropic frame is desired, it can be obtained by adjusting the strut design to satisfy this condition. Similarly, by adjusting the ratio of C_s to C_b the Poisson contractions of the frame can be designed to be positive, negative, or zero.

Strength

Of prime importance for a frame is its strength. This determines the load at which it begins to have excessive deflections, or at which collapse begins. One virtue of tetrahedral frames is that because of their high symmetry only one element needs to be designed for strength; namely, the strut that connects two nodes. However, these units must be designed to resist: buckling, inelastic bending, tension, and compression. If the frame is subject to fluctuating loads they must also resist fatigue; and if impacts are applied resistance to dynamic fracture also becomes important. Little can be said in general about these resistances until particular materials of construction have been specified, so this topic will not be pursued here.

APPLICATIONS

Tetrahedral frames can be used in the same situations as other space frames, but they have two advantages that may make them especially attractive for some kinds of aerospace systems. One is their exceptionally high specific structural stiffness and strength. The other is the simple module that can be used repetitively to construct them. Among the systems for which they might be useful are: the frames of lighter-than-air vehicles; space colony frames; the support for a solar sail; and the frames for large optical systems.

Pertinent to the last of these is the fact that graphite reinforcing fibers can be aligned with the axes of all of the struts if they are aligned in the puckered rings. Then the thermal expansion coefficient along the strut axes will be very low. In turn, by symmetry, the thermal expansion coefficient for the frame as a whole will be low. This will help provide the stability needed for optical systems.

Some other structures that might be built from these frames are: large boats; houses and other buildings; unsupported domes; large beams or plates; climbing frames for recreation; and large storage tanks.

It is not necessary to completely fill a section of space with a tetrahedral frame. They can be constructed containing internal cavities or channels. Thus they can be a variety of shapes, including cylinders, plates, and tubes. Just as exterior surfaces of them can be closed off with triangular rings, so can internal surfaces. Plates or membranes can be readily attached to their external surfaces.

A rather different application opportunity is that of embedded frames. For example, steel reinforcing bars might be cut and welded in the puckered ring

configuration of Figure 4. These might then be assembled as a tetrahedral frame inside a mold which is to be filled with concrete. The re-bar frame would efficiently reinforce the subsequent concrete casting (in terms of the amount of steel needed to obtain a given stiffness or strength). Ceramic and polymeric objects might similarly be reinforced. Also, one metal might be used to reinforce another.

ACKNOWLEDGMENT

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FIGURE CAPTIONS

- Figure 1. Model of tetrahedral frame constructed of puckered rings. Since the frame is periodic it can be extended indefinitely in three-dimensions. Note the continuity of the material at the nodes.
- Figure 2. One set of channels through a tetrahedral frame. There are six equivalent sets through the structure. On each of four planes, sets of three channels from equilateral triangular arrays.
- Figure 3. Schematic nodal couplers for the struts of a tetrahedral frame.
- Figure 4. Puckered ring that can be used to build up a tetrahedral frame. The sides are all of equal length and the angles of the bends are $109^{\circ} 28'$. Two views are shown: A - angled top view; B - side view.
- Figure 5. A standard elbow from which the ring of Figure 4 can be built up.
- Figure 6. Assembly of four puckered rings into a tetroid unit. The view at A shows the rings prior to assembly;
- Figure 7. Assembly of three tetroid units into a section of frame. At A the three tetroids approach each other; at B the completed frame section is shown.

Figure 8. Schematic drawing of a higher density frame that meshes coherently with the surface of a lower density frame. The ratio of the ring sizes must be an integer.

Figure 9. Coordinate frame of reference for a tetrahedral frame.

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Figure 1.

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Figure 2.

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Figure 3.

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Figure 4.

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Figure 5.

Figure 6.

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Figure 7.

Figure 8.

-- XBL 873-756 --

Figure 9.

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