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How Numbers are Like the Earth (and Unlike Faces, Loitering or Knitting)

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Abstract

Different kinds of knowledge are acquired in different ways. In this chapter, I consider four kinds of knowledge: (1) Core knowledge, which is implicit, nonlinguistic and learned easily and automatically; (2) Explicit, linguistic knowledge that fits into an existing conceptual structure (i.e., a new word for an old concept); (3) Explicit, linguistic knowledge that does not fit into any prior conceptual structure (e.g., how to knit); and (4) Explicit, linguistic knowledge that is incompatible with some prior conceptual structure, making it very difficult to learn (e.g., knowing that the earth is a round ball floating in space.) It is this fourth type of learning that Carey (2009) calls ‘conceptual change.’ In this chapter I argue that mathematics includes all four types of knowledge, but that natural numbers are an example of the fourth and most interesting type—knowledge that is the result of conceptual change.

Key words: Concepts, Conceptual Change, Early Childhood, Learning, Number.

How Numbers are Like the Earth (and Unlike Faces, Loitering or Knitting)

To study cognitive development is to study learning. But different kinds of knowledge are learned in different ways. Some learning is implicit and non-linguistic, as when birds learn to navigate by the stars, or when babies learn to recognize the faces of their caregivers. Human and animal brains have evolved to maximize the chances that this type of information will be learned: Baby birds automatically look up to the sky for dots of light, and use those dots to build a star map. Baby humans automatically look for human-type faces, and fall in love with them.

A second type of learning happens when we acquire knowledge that is explicit and linguistic, but also nearly effortless because it fits into our existing conceptual structures. Last night, my nine-year-old son James was reading a book, and came across some words he did not understand.

“No sol-stice-ing?” he read aloud.

“No soliciting,” I said.

“What’s that?”

“It means, no people selling things.”

“Oh . . . No lo-ter-ing?”

“Loitering.”

“What’s that?”

“It means you can’t go there and hang around and do nothing.”

“Oh.”

Then he read the whole sentence and laughed. “Mom! This monster has a sign on his store! It says, ‘No soliciting; no loitering; no living!’”

That learning episode took no more than 30 seconds. James learned two new words ('soliciting' and 'loitering') but because he already understood what it means to sell things, or to hang around doing nothing, only the words themselves were new. Whether you call 'soliciting' and 'loitering' new concepts, or new words for old concepts, they were easy for him to acquire.

A third type of learning also involves explicit, linguistic knowledge, but this knowledge is more difficult to acquire because it does not fit into any conceptual structure you already have. Imagine that instead of 'soliciting' and 'loitering,' James had been reading one of my knitting books and asked me, "What is a 'provisional cast-on'?"

Now, I can define 'provisional cast-on,' but what makes this term different from 'soliciting' and 'loitering' is that I can only define it in terms that James does not understand. I could say, "A provisional cast-on is a way of casting on stitches using waste yarn, so that instead of a cast-on edge, you have live stitches that you come back and work later from the other side."

But unlike my definitions of soliciting and loitering, this definition would raise more questions than it answered. What is casting on? What is waste yarn? What is a cast-on edge? What are live stitches? If I tried to define the 'provisional cast-on' in terms of James's existing concepts, I might say something like, "It's a special way of starting a piece of knitting." But that doesn't really define it. In order for James to truly understand what 'provisional cast-on' means, James would also have to know a lot of other things about knitting. In other words, 'provisional cast-on' is part of a whole structured system of concepts (i.e., knitting knowledge) that are inter-defined.

The funny thing is that once a person acquires a highly structured conceptual system (like knitting or, as we will see below, numbers), the knowledge can be translated easily between languages. In a sense, the words are like scaffolding—they provide the initial structure of

conceptual frameworks, but can be kicked away after the structure is built. A couple of years ago, I used to wait for James's school bus in the afternoon, and I would bring my knitting to work on while I waited. There were a couple of older Chinese ladies who waited at the same stop for their grandchildren, and although we shared no common language, they would reach out for my knitting, examine it, smile and nod, and then modestly point to some hand-knit article they were wearing—a hat or socks or a scarf, and I understood them to be saying, *I knit, too. I made this*. And I would examine the article, and smile and nod in appreciation.

One day I was knitting a sock. Socks are traditionally started at the cuff and knit downward, but I was knitting this one from the toe upward. One of the grandmothers was watching my work with a furrowed brow. She called the other grandmother over and they peered intently at the sock, peppering me with questions (at least I assume they were questions), shaking their heads, and looking puzzled.

Finally, the first grandmother picked up a twig from the ground, took hold of my yarn, and started doing a provisional cast-on, using the twig as a knitting needle. She looked at me, pointed to her twig and yarn, and raised her eyebrows in a question.

I smiled and nodded, took a piece of scrap paper from my bag, and drew this:

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The grandmother laughed and nodded in understanding.

What happened here? What information was communicated, and how was such communication possible without any shared language? The answer is that our shared knowledge of knitting did most of the work. The other knitter and I both know a lot of things that didn't have to be said. We know that although there are traditional ways to start a sock from the toe, the sock I was making didn't look like any of them. In fact, the sock I was making had no visible beginning at all, raising the question of how it was constructed. The way to construct a piece of knitting with no visible beginning is with a provisional cast-on.

When my fellow knitter picked up the twig and started working a provisional cast-on, I understood that she was asking, *Is this how you started the sock?* It was a reasonable question, being one of the few things that an experienced knitter couldn't tell about the work just by looking. My diagram answered the next logical question, by showing that I worked a provisional cast on, and then a short-row toe. (That's the hourglass shape.)

If you are not a knitter, this English explanation probably still doesn't make much sense, which is exactly my point. Knitting knowledge (like knowledge of numbers) is organized in a structure. Concepts are defined in terms of other concepts, and they only make sense in terms of each other. It takes 10 seconds to make another English speaker understand what 'loitering' means, but much longer to make a non-knitter understand what 'provisional cast-on' means, because that person would have to learn the basics of knitting first. Conversely, I could spend any amount of time at the bus stop with my Chinese grandmother friends and never learn the Mandarin word for 'loitering.' And yet we exchanged detailed information about the construction of a sock—something that I cannot do here because many readers, being non-knitters, will lack the conceptual framework needed to understand it.

So far, we've seen three different learning situations: Implicit, non-linguistic learning (as when babies learn to recognize the face of their caregivers); explicit learning of information that fits into existing conceptual structures (as when James learned 'soliciting' and 'loitering'), which is fast and easy; and explicit learning of information that doesn't fit into any existing conceptual structures (as when a non-knitter learns about the provisional cast-on), which is much slower and more difficult, because at least part of a new conceptual structure must be built before the new information can be understood (i.e., you need the scaffolding).

A fourth, and even more challenging kind of learning is when you must acquire a new conceptual structure, but you have an existing conceptual structure that *conflicts* with it. That is, in order to learn the new information, you have to overcome wrong ideas that you had before. In technical terms, the problem arises when Conceptual System 2 (the new information you are trying to learn) cannot be defined or expressed within the framework of Conceptual System 1 (the old system of ideas you already have). When that happens, the two systems are said to be *incommensurable*. (Carey, 2009).

For example, young children know (as adults knew, centuries ago) that the earth is a horizontal plane, and we live on top of it. The problem is, children today are told that the earth is actually round, like a ball. This seems like nonsense (kids can see that the earth is a flat plane) but the adults around them seem very sure that it's a ball. In one of my favorite developmental studies, Stella Vosniadou and William Brewer (1992) showed that many kids solve this problem (at least temporarily) by creating a model that reconciles what they can observe (i.e., the earth is flat) and what they've heard (i.e., the earth is a round ball).

For example, many children construct a mental model of the earth as something like a snow globe. In their snow globe model, there is a hemisphere of solid material on the bottom,

and a hemisphere of air (called ‘the atmosphere’) on the top. The flat plane in the middle (called ‘the surface’) is where the people live. This model allows children to make sense of statements that contradict with things they know. In their snow-globe model, ideas like, “The earth is round, like a ball”; “Spacemen wear helmets because they can’t breath outside the earth’s atmosphere”; and “People live on the surface of the earth” can co-exist with ideas that are really difficult to let go of, such as the idea that we live on a flat surface, and that gravity is a force pulling everything downward through space.

What makes the earth problem different from the knitting problem is the presence of an existing conflicting conceptual system that conflicts with the new information. In the case of knitting, people may have no knitting knowledge, but they probably don’t have any prior conceptual system that is incompatible with knitting either. In the earth problem, children have strong beliefs in a flat earth and in the notions of ‘up’ and ‘down,’ which actually make it hard to understand the new information. Carey (2009) calls the fourth kind of learning (the kind required by the earth problem) *conceptual change*, because the learner must change from one conceptual system to another.

What About Numbers?

Human number knowledge includes all four types of learning. The first type of learning—innate, implicit learning—certainly occurs for number (see below); and we can find instances of the second and third types if we think about the mathematics we learn in school. For example, it is easy to learn what ‘odd number’ and ‘even number’ mean when you already have a concept of ‘number,’ just as it is easy to learn what ‘solicit’ means when you already have a concept of ‘sell.’ And geometry, like knitting, requires the student to learn a whole bunch of new concepts at once, because their meanings are partially defined in terms of each other.

In this chapter, I am mostly concerned with the fourth type of learning, which I think is needed for young children to learn the exact meanings of cardinal number words like *eight*. You will recall that the fourth type of learning situation arises when there is not only a new conceptual system to be learned, but also a prior conceptual system, already in place, which cannot represent the new information.

For numbers that prior conceptual system is the result of the first type of learning: innate, implicit learning. Evolution has given humans the ability to represent several kinds of number information: We distinguish individuals from sets, we track several individuals at once, and we form mental representations of approximate numbers.

Representing individuals. Humans and other animals distinguish individuals from sets (Barner, Wood, Hauser, & Carey, 2008; Li, Ogura, Barner, Yang, & Carey, 2009) and create mental models of ongoing events in which each individual is represented by a single symbol (Chiang & Wynn, 2000; Feigenson & Carey, 2003; Feigenson, Carey, & Hauser, 2002; Feigenson, Dehaene, & Spelke, 2004; Mandler & Shebo, 1982; Scholl, 2001; Starkey & Cooper, 1980; Xu, 2003). For example, a group of three children could be represented as ‘Molly, Claire, Zoe.’ Note that this is not really a number system. Number is represented only implicitly, because ‘Molly, Claire, Zoe’ includes three symbols, but no symbol for ‘three.’

Nevertheless, our representations of individuals do support some number-relevant computations, because models can be compared on the basis of 1-1 correspondence. For example, if a baby sees two objects placed behind a screen, and then the screen is raised to reveal just one object, babies look at the scene longer (presumably indicating surprise) than at a similar scene where the screen is raised to reveal the expected two objects (Wynn, 1992a, see also Antell & Keating, 1983; Feigenson & Carey, 2003; Starkey & Cooper, 1980; Strauss & Curtis, 1981).

Despite allowing the child to represent number implicitly, this system cannot represent large, exact, cardinal numbers. First, it has no symbols for cardinal values. (Three symbols is not a symbol for ‘three’.) Second, the models can only represent up to three or four individuals. More items, and it’s too many to remember. So I can keep track of the three children I brought to the park, but not of the 17 undergraduates working in my lab.

The approximate number system (ANS). Humans and other animals form mental representations of the approximate ‘how many-ness’ of large sets, up to at least several hundred (Dehaene, 2011; Feigenson et al., 2004). The brain represents numerosity as a neural magnitude that gets bigger as the set being represented gets bigger. For this reason, the representations are sometimes called analog magnitudes, and the whole system is sometimes called the analog-magnitude number system. These representations can be compared in order to decide which of two sets has more; they can also be added, subtracted and multiplied (Barth et al., 2006; Dehaene, 2011; McCrink & Spelke, 2010; Nieder & Dehaene, 2009). Unlike the system of tracking individuals, the ANS truly represents number. But in the ANS, numbers are only approximate.

To understand how the two systems differ, imagine you have taken three children to a crowded playground. You must keep track of the three kids you brought with you. The ANS lets you quickly see that there are more children on the grass than in the sandbox, but the only information it holds about the kids you brought is ‘approximately three.’ So if you only used the ANS, at the end of the day you might just grab any random two to four kids from the playground to take home¹.

Because the ANS does not represent numbers exactly, it cannot represent 43 as something different from both 42 and 44. So cardinal number words cannot be mapped directly

¹ Note to self: Consider exchanging my children for better-behaved ones, blaming approximate number system?

onto ANS representations. Which brings us to the question of where those number concepts come from. If the innate systems don't allow us to represent a number like 43, how are we able to do it?

On some accounts, we are born with exact-number concepts and learning a counting system just gives us a handy way to keep track of them (Butterworth, 2010; Gelman & Butterworth, 2005; Gelman & Gallistel, 1978, 2004; Leslie, Gelman, & Gallistel, 2008). But if the description of the innate systems above is correct, then learning an exact-number system presents a real problem. The system of representing individuals is limited to small sets (and has no representations of number anyway), and the ANS is not precise enough. This problem, identified by Susan Carey (see her 2009 book for review), has motivated my research for over a decade.

Representing Large, Exact Numbers.

Of the different learning situations described at the beginning of this chapter, learning a number system (that is, learning how the counting system of your language represents exact numbers) is most like the fourth example—learning the shape of the earth. I don't think it is like the first example (recognizing your mother's face) because as explained above, our innate cognitive systems do not represent large, exact numbers such as 43. I also don't think that number learning is like the second example (learning the meaning of 'soliciting' and 'loitering'), because it's not quick and easy. Children take a long time (often more than a year) to figure out how the number system of their language works. (Later on in mathematics, there are plenty of examples of the second type of learning. For example, once you have exact-number concepts, it's easy to learn the meanings of *odd number*, *even number*, and *round number*.)

Numbers do have something in common with the third example, knitting, because

individual number words (like knitting terms) only make sense within the larger structure. *Five* is simply the name of the fifth word in the English counting list, and its cardinal meaning is defined by that position. If I tell you that the fifth word in the Russian counting list is *pyat*’, you know that *pyat*’ has to mean 5. The meanings of the individual words are completely defined by their place in the list. So to learn the word meanings, a child has to build the structure. That’s why so many researchers like me study how children learn number-word meanings. But the analogy with knitting breaks down when we think about humans’ innate systems of number representation (described above). Our systems of tracking individuals have no way of representing a number as large as 43, and the ANS can only represent such quantities approximately. This makes the system of exact numbers *incommensurable* with the innate systems. This incommensurability means that learning numbers is the fourth kind of learning situation—the kind that Carey (2009) calls conceptual change.

Exact numbers are incommensurable with the innate systems, but in a different way than a round earth is incommensurable with a flat earth. In the earth problem, the new concept is hard to learn because the old concept actually conflicts with it. The process of building the new conceptual system is the process of dismantling the old one.

In the number problem, the old information is whatever can be represented by the innate systems (i.e., approximate number information and tracking of individuals). But the innate systems aren’t wrong—they just aren’t powerful enough to represent large, exact numbers. So the child still has to build a new conceptual system, but doesn’t have to dismantle an old one.

When a child learns an exact number system, she doesn’t let go of her approximate number representations. Instead, the conceptual structure of exact numbers is built on top. And after using an exact-number system for a while, the child develops a ‘gut’ sense of how big the

numbers are. This gut sense reflects the calibration of the exact-number system with the ANS.

To understand the difference between knowing an exact-number system and having this system calibrated to your ANS, think about traveling to a country with a different currency. Perhaps you live in Europe or the U.S., and you travel to Japan. In one sense, the system of enumerating Japanese yen (¥) is perfectly familiar to you. You know that ¥10,000 is five times more than ¥2,000; you know that if you walk into a shop and pick up a toothbrush for ¥300 and a pair of sunglasses for ¥1,200, you will be charged ¥1,500 plus tax.

But in different way, using yen feels unfamiliar—as if the money were fake. Is ¥1,500 a lot to pay for a toothbrush and sunglasses? Are these cheap sunglasses or expensive sunglasses? The cab driver says the ride to your hotel will cost ¥6,500. Is that a rip-off, or a good deal? You wonder if you should take the subway. It's only after you've been in the country for a while that you develop an intuitive sense of how much money ¥1,500 or ¥6,500 really is.

Children face a similar problem when they first learn how their language's counting list represents exact numbers. They were born with an approximate number system, they learn the counting system, and it takes some practice to fully connect the two. The incommensurability between the ANS and the exact-number system arises because the ANS is not precise enough to represent exact numbers.

So, how do children learn a represent large, exact numbers? How do they overcome the problem of incommensurability? Carey (2009) has proposed that they do it through 'conceptual-role bootstrapping.' In this process, children first learn a structured set of symbols, which serves as a placeholder for the conceptual structure being built. In the case of counting, the placeholder structure is counting.

As early as 15-18 months of age, children start to recognize the counting list of their language (e.g., *one, two, three, . . .*) and the pointing gestures that go with it (Slaughter, Itakura, Kutsuki, & Siegal, 2011). But even after children learn to count (in the sense of reciting the list and pointing to things), they still don't understand what each individual number word means. In the studies reviewed below, we will see that the children learn number-word meanings gradually, over a period of months or years. Bit by bit, children fill the placeholder structure of the counting list. They learn the meanings of the first few number words (up to 3 or 4), and at the same time, they gradually learn how the counting list represents number. Eventually, children build a conceptual structure that can represent the meanings of larger, exact numbers like 9, 15 and 43.

Levels of early number knowledge. A note about the studies reviewed below: Most of them assess children's number knowledge using the Give-N task (also called the Give-a-Number task, Wynn, 1990, 1992b). In this task, the child is asked to produce sets of a particular number. For example, the child might be given a bowl of 15 small rubber fish, and asked to give one, two, five or some other number of them to a stuffed animal (e.g., 'Give *five* fish to the anteater.')

Although many 2–4-year-olds can recite the count list up to ten and point to objects just fine (e.g., when asked to count a row of six buttons, they say "one, two, three, four, five, six" and point to each button in turn), they are unable to give five fish to the anteater when asked. Instead of counting, they just grab a handful of fish and hand them to the anteater. Even when experimenters tell them to count, and even if they count the fish correctly, they don't know how counting relates to the number of fish they have given the anteater (e.g., Le Corre, Van de Walle, Brannon, & Carey, 2006; Sarnecka & Carey, 2008; Wynn, 1990, 1992b) In other words, the children don't initially understand that the last word of a count tells how many items are in the

whole set—they don't understand the cardinal principle of counting (Fuson, 1988; Gelman & Gallistel, 1978; Schaeffer, Eggleston, & Scott, 1974).

Children at the earliest level are called 'pre-number knowers,' because they treat all number words alike. For example, on the Give-N task, some pre-number knowers give one object no matter what they are asked for. (E.g., whether they are asked to "give one fish," "give two fish," "give three fish," or "give six fish," they give just one.) Other pre-number knowers give a handful of fish for every request. Pre-knowers consistently fail to give the number of items requested (but see Barner & Bachrach, 2010).

Children at the next level, called 'one-knowers,' give exactly one fish when asked for 'one,' and give more than one fish when asked for any other number. 'Two-knowers' give one fish when asked for 'one,' two fish when asked for 'two,' and more than two for higher numbers. This is followed by a 'three-knower' level and a 'four-knower' level. Le Corre and colleagues (2006) coined the term 'subset-knowers' to describe children at the one-, two-, three-, and four-knower levels, because although these children can typically count to ten or higher, they only know the cardinal meanings of a subset of those number words. After the four-knower level, children figure out the cardinal principle, and are called 'cardinal-principle knowers' or CP-knowers for short. The studies reviewed below mainly ask when (i.e., at what knower-level) children figure out different parts of the conceptual framework that is our number system.

Keep in mind that children at each of these knower levels typically know how to count (if counting just means reciting the counting list and pointing to items). But they do not understand how counting represents number. They don't know what exact set size each number word picks out, nor do they know how counting can be used to generate sets of a given number or to find out how many items are in a set.

In order to understand these things, children must fill in the ‘placeholder’ structure (i.e., the counting words and gestures) with meaning (i.e., the meanings of the individual number words, and the conceptual understanding of how counting represents number). Carey (2009) calls this process ‘conceptual-role bootstrapping.’ In the rest of this chapter, we will look at some studies that explore this process.

Figuring Out What Number Words Refer To

Numbers are about things, not stuff.

An important part of understanding numbers is knowing that numerosity is a property of sets of individuals, not continuous masses of stuff. As adults, we know that a flock of geese on a lake has numerosity: there is some number of geese. If we counted them, we would find out what it was. But the lake itself does not have numerosity. It cannot be counted; there is no ‘how many’ to the water. If we want to use numbers to quantify the amount of water in the lake, we have to impose some unit of measurement, such as cups or gallons.

To explore children’s understanding of this aspect of numbers, Emily Slusser, Annie Ditta and I tested preschoolers on what we called the Blocks and Water Task (Slusser, Ditta, & Sarnecka, 2013). The goal was to find out whether subset knowers know that ‘five’ and ‘six’ refer to sets of discrete individuals, rather than masses of continuous stuff.

In the Blocks and Water task, children are shown two cups—one containing objects, the other a continuous substance. For example, one cup might have five yellow blocks, while the other has green-colored water. Then the child is either asked a number question (e.g. “Which cup has five?”) or a color question (e.g. “Which cup has green?”).

Children at all number-knower levels did very well on the color questions, and also did well on the number questions when the cup with objects had only one or two items in it. (In this

case, the number question was “Which cup has one?” or “Which cup has two?”). But children’s performance on questions about ‘five’ and ‘six’ was mixed. On these questions the CP-knowers did very well, and the three- and four-knowers also performed above chance, indicating that they understood ‘five’ and ‘six’ to be numbers of discrete objects, even though they could not yet identify or generate a set of five or six items on the Give-N task.

The one- and two-knowers were the most interesting. They pointed to the cup with objects for ‘five’ and ‘six’ more often than chance would predict, but only if they were asked about ‘one’ and ‘two’ first. Children who were given the trials in the opposite order (i.e., they were first asked about sets of ‘five’ and ‘six,’ then about ‘one’ and ‘two’) chose at random. That is, they didn’t know whether the blocks or the water were a better example of ‘five’ or ‘six.’

This result suggested to us that one- and two-knowers have some fragile understanding that higher numbers (i.e., ‘five’ and ‘six’) apply to things, not stuff. But they do not understand exactly how it works. By doing the task with known numbers (‘one’ and ‘two’) first, children were primed and this fragile knowledge was brought to the fore, which allowed them to succeed on the trials with higher numbers.

Numbers are about ‘more’ and ‘less’

One of the earliest things children might figure out about numbers and counting is that these things are related to quantity—to how big something is, or how much of something there is. The first time my older son Teddy ever used a number word was when, at 18 months old, we were visiting the home of some friends, who had tall stacks of DVDs sitting on the floor in their living room. Teddy toddled over to one stack and pointed to it repeatedly, saying, “onetwothree, onetwothree, onetwothree” each time. I think he was trying to convey something like, *Hey, look at all those DVDs!* He didn’t know yet that ‘one,’ ‘two’ and ‘three’ were separate words, but he

knew that when we say the string ‘onetwothree . . .’ and point to things, we are saying that it’s a lot of things.

A slightly older child’s understanding that numbers are about quantity might be expressed by the idea that e.g., a set five objects will always be five, unless something is added to it or taken away. The five objects can be shaken up, turned upside-down, stirred with a spoon, arranged in a line and so on, but it will still be a set of five objects. The number only changes when the quantity changes.

To find out when (i.e., at what number-knower level) children understand this, Susan Gelman and I tested preschoolers on what we called the Transform-Sets task (Sarnecka & Gelman, 2004). In this task, we showed the child a box containing five or six objects and said, e.g. “I’m putting *six* buttons in this box.” Then, we did something to the box. We shook it, rotated it 360 degrees on the table, added an item to it, or took an item out. Then we asked the child, “Now how many buttons? Is it five or six?” We did not test one-knowers in that study, but all the groups we did test (which included two-knowers and higher knower-levels) said that there were still six items after the box had been shaken or rotated, but there were no longer six when an item had been added or removed. In other words, before they know exactly what quantities the words *five* and *six* denote, children already know that those words *are about quantity*. They should only change when things are added or removed from the set. This gives us a sort of snapshot of children in the process of building the conceptual structure of the number system. They are in the process of figuring out what numbers are.

Numbers refer to ‘how many-ness’ (not area, contour length, etc.)

A child may understand that numbers have something to do with quantity, and even that they are about discrete items, but not identify numerosity (as distinct from other dimensions of

experience) as the thing that numbers pick out. Besides having numerosity, sets have other quantitative dimensions such as area, contour length, and so forth. Emily Slusser and I did a study asking when (i.e., at what knower-level) children identify numerosity as the dimension that number words are about (Slusser & Sarnecka, 2011).

To answer this question, we tested young children on a Match-to-Sample task. In this task, we showed the child a sample picture and told them its number (e.g., “This picture has eight turtles.”) We then placed two more pictures on the table, saying, “Find another picture with eight turtles.” One picture had the same number of items as the sample. The other had a different number (either half or twice as many as the sample picture), but matched the sample either in total summed area or in total summed contour length of the items. The experiment also included control trials, where children were asked to match pictures according to mood (happy or sad) or color. For example, “This picture has happy turtles. Find another picture with happy turtles” or, “This picture has green turtles. Find another picture with green turtles.”

We found that CP-knowers extended the words ‘four,’ ‘five,’ ‘eight,’ and ‘ten’ to another set of the same number (even though they were not allowed to count the items), but subset-knowers did not. Children in all groups did fine on the mood and color trials, showing that the subset-knowers understood what we were asking. But the subset-knowers did not seem to realize that two sets of the same numerosity are labeled by the same number word, whereas sets of different numerosities have different number words. CP-knowers were the only group who seemed to recognize numerosity as the dimension of experience that number words pick out. Of course, a child has to recognize this in order to *become* a CP-knower. Because our data are cross-sectional (that is, they are snapshots of the different groups’ knowledge, and cannot show knowledge changing over time), we can only infer how identifying numerosity is related to

understanding cardinality. One possibility is that children get to the four-knower level and then identify numerosity as the relevant dimension. Having done that, they can grasp the logic of cardinality and soon become CP-knowers.

Figuring Out How Counting Represents Number

Numbers in the list start small and get bigger.

To find out when children understand that numbers occurring later in the count list represent larger cardinalities, Susan Carey and I tested preschoolers on what we called the Direction task (Sarnecka & Carey, 2008). All of these children could count at least to ten. The question was whether they understood that each word in the list denotes a larger set than the word before it.

In the Direction task, children were shown two plates, each containing six items (e.g. six red erasers on one plate, six yellow erasers on the other), and told them, “This plate has six, and that plate has six. And now I’ll move one.” Then we picked up an item from one plate and moved it to the other plate, and said, “Now there’s a plate with five, and a plate with seven. Which plate has five?” (On half the trials the question was, “Which plate has seven?”). Only four-knowers and CP-knowers succeeded on this task; one-, two-, and three-knowers performed at chance. In other words, the one-, two- and three-knowers did not say that the set that gained something should now have a number from later in the list (e.g., ‘seven’ instead of ‘six’), while the set that lost something should have a number from earlier in the list (e.g., ‘five’ instead of ‘six.’)

The numbers grow by exactly one.

To understand how counting represents number, children must also understand the successor principle. This is the principle stating that each number has a ‘next’ number (i.e., a

successor), which is exactly one more, and which is named by the next word in the list. In the same study described above, children were also given a task designed to measure successor-principle knowledge; it was called the Unit task. In this task, the child was shown a box containing (for example) five small plastic bananas and said, “There are five bananas in this box.” Then we added either one or two more bananas and asked, “Now how many? Is it six or seven?” Subset-knowers performed at chance on this task, meaning that they seemed to guess at random. CP-knowers performed above chance: if they saw one banana added to the box, they said the number should change to six; whereas if they saw two items added, the number should change to seven. We concluded that CP-knowers understood the successor principle.

Looking back, this conclusion was overstated. First, although the CP-knowers performed above chance, they were nowhere near perfect. Moreover, we claimed rather optimistically (and without evidence) that CP-knowers understand the successor principle to be true for all the number words they know, as high as they can count. Although that made for a satisfying story, more recent work suggests that the process is more gradual. It seems that children initially understand the successor principle for a few, relatively familiar number words (e.g., *five* and *six*) but take some time to apply it to all numbers (Davidson, Eng, & Barner, 2012). In other words, children’s big achievement when they come to understand the cardinality principle is that they see for the first time how counting can represent *any* numbers. The generalization to *all* numbers comes later.

Understanding what it means for two sets to have the same number

Another difference between subset-knowers and CP-knowers is that CP-knowers demonstrate a much clearer understanding of ‘just as many,’ or exact equality (Izard, Pica, Spelke, & Dehaene, 2008). Subset knowers do have some understanding of ‘just as many.’ For

example, Izard, Streri and Spelke (2014) found that subset-knowers use one-to-one correspondence to tell when an object is missing from a set of five or six. In that study, five or six finger puppets were placed on a display stand with exactly six spaces, so that either all the spaces were filled (in trials with six puppets), or one space was left empty (in trials with five puppets). The experimenters did not use any number words, but they drew children's attention to whether every space was occupied by a puppet, or one space was left empty. The puppets were then removed from the scene, and a few seconds later they were returned to it. After five puppets had been retrieved from the box, the children had the opportunity to reach in to the box (they could not see into it, but they could reach in) and look for another puppet if they thought there was one in there. Children searched longer in the trials where the display had been full (the trials with six puppets) than in the trials where a space had been empty at the start (the trials with five puppets), showing that they used the information from the scene (i.e., the fact that either all the spaces were full or one space was empty) to reason about whether there was another puppet left in the box.

But CP-knowers do something more. They use number words to infer one-to one correspondence between sets. For example, CP-knowers say that if you have ten flowers and ten vases, you must have exactly one flower for each vase (Muldoon, Lewis, & Freeman, 2009). I have sometimes used the term 'equinumerosity' to refer to this knowledge, but of course children do not conceive of equinumerosity in the explicit way that philosophers do (e.g., children do not talk about bijective, surjective functions). CP-knowers seem to understand something more modest: That any set of, e.g., six items has 'just as many' as any other set of six.

My colleagues and I have explored this by giving preschoolers a scenario in which two stuffed animals (a frog and a lion) were given 'snacks' that were actually laminated cards with

pictures of food (Sarnecka & Gelman, 2004; Sarnecka & Wright, 2013). In half the trials, the snacks were identical (e.g., Frog's and Lion's pictures each showed six peaches); on the other trials, the snacks differed by one (e.g., Frog's picture showed five cupcakes while Lion's showed six). The food in the pictures was arranged in rows, and the sameness or difference of the two snacks was easy to see.

At the beginning of each trial, we asked the child whether the animals' snacks were 'just the same.' Children usually (i.e., about 95% of the time) answered this question correctly. When they did not, the experimenter said, e.g., "Well, they both have cupcakes, but look! Lion has a cupcake here, and Frog doesn't. Oh no, I forgot one of Frog's cupcakes! I'm so silly! I made a mistake!" Then we asked the child, e.g., "Frog has five cupcakes. Do you think Lion has five or six?" (Children were not allowed to count the items.)

Only the CP-knowers succeeded on this task. That is only the CP-knowers knew that if Frog had six, and the sets were identical, then Lion must also have six. And they knew that if the sets were different, then Lion must have a different number than Frog. This is not a rule about how to recite the counting list or how to point to objects; it is an insight about what numbers are, and only children who understand cardinality (as measured by the Give-N task) seem to grasp it.

This question is further explored in recent work by Mathieu Le Corre (2015), who confirms that CP-knowers understand 'same number' and 'different number,' in the sense that they know every set of (e.g.) 7 can be mapped onto every other set of 7, but no other number. Le Corre shows that children grasp cardinality and 'same number/different number' as logical principles at a time when they are still quite unfamiliar with specific numbers. For example, a CP-knower probably knows that a set of 7 cannot map onto a set of 8, but may not know which set is larger.

*Number Learning in Different Languages**Grammatical number systems and early number learning.*

The same number-knower levels found in English-speaking children are also found in children who speak Arabic, Japanese, Mandarin Chinese, Russian, Slovenian and even Tsimane', the language of a farming and foraging society in rural Bolivia. (Almoammer et al., 2013; Barner, Libenson, Cheung, & Takasaki, 2009; Li, Le Corre, Shui, Jia, & Carey, 2003; Piantadosi, Jara-Ettinger, & Gibson, 2014; Sarnecka, Kamenskaya, Yamana, Ogura, & Yudovina, 2007).

Although children everywhere seem to go through the same series of levels (pre-knower, one-knower, two-knower, three-knower, etc.), the relative time children spend at each number-knower level may differ according to the grammatical number marking system of their language. For example, English requires singular/plural marking on almost every sentence. So children learning English must constantly pay attention to the difference between one thing and more than one thing. The same is true for children learning Russian. On the other hand, Japanese and Mandarin rarely mark the singular/plural distinction, so children learning those languages might not pay so much attention to it.

Perhaps that means English and Russian speakers should reach the one-knower level earlier than Japanese and Mandarin speakers. On the other hand, humans and other animals do have an innate ability to distinguish between individuals and sets ((Barner et al., 2008 see the discussion of innate systems, above) and the one-knower level could presumably reflect just that distinction, so perhaps the grammars of individual languages would not matter.

It turns out that the language environment does make a difference. English speakers and Russian speakers do learn the meaning of 'one' earlier than Japanese and Mandarin speakers (Li et al., 2003; Sarnecka et al., 2007). Similarly, children who speak Slovenian or Saudi Arabic

(languages that mark a three-way distinction between singular, dual and plural) learn the meaning of ‘two’ earlier than English-speaking children, and remain at the ‘two’ level longer. (Almoammer et al., 2013). In other words, children are quicker to learn the numbers that correspond to distinctions in their grammar-- ‘one’ for English and Russian speakers; ‘one’ and ‘two’ for Slovenian and Arabic speakers. This is not definitive evidence for any particular theory of number-concept construction, of course, but it seems to underline the idea that building number concepts is a long and difficult process, and that children use evidence from multiple sources in figuring out what the first few number-words mean.

Numbers are translatable across languages

Because numbers are a highly structured conceptual system, they are trivially easy to translate between languages. So easy, in fact, that expressing a number in a different language hardly seems to deserve the label ‘translation.’ There’s just no wiggle room in the meaning of *twenty-six*. The conceptual structure itself constrains the meanings of these words so tightly that they are completely defined by it, just as the conceptual structure of knitting placed tight constraints on the possible questions that the Chinese grandmother at the bus stop could have been asking about my sock.

If we think about numbers that way, let’s imagine what it means to be a young child learning more than one language. For example, imagine that you are four years old, and you speak Spanish at home and English in your Head Start classroom. You should generally know the meanings of the same number words in both languages, because counting has provided an analogous placeholder structure in each language, and those placeholder structures scaffold your number learning. Thus, if you know the meanings of ‘one,’ ‘two’ and ‘three’ (but not ‘four,’ ‘five’ or higher number words), you should know the meanings of ‘*uno*,’ ‘*dos*’ and ‘*tres*’ (but not

‘*cuatro*,’ ‘*cinco*,’ or higher number words). This cross-linguistic concordance is not predicted for other kinds of words (such as color words or common nouns) because those are not learned via a placeholder structure.

In a recent study, Meghan Goldman, Ted Wright and I tested three groups of bilingual preschoolers living in Southern California (Goldman et al., 2012; Sarnecka, Goldman, & Wright, under review; Sarnecka, Wright, & Goldman, 2011). Each child either spoke Spanish and English, Mandarin and English, or Korean and English. They were given a modified version of the Give-N task, which asked them not only about number words, but also about a set of six color words and 18 common nouns (six farm animals, six wild animals and six vehicles) in each of their languages. Results showed that children’s number-word knowledge was more closely correlated across their two languages than their knowledge of either color words or common nouns. That is, most children knew the meanings of the same number words in each of their two languages, but did not know the meanings of the same color words or common nouns. This finding is explained by the idea that number learning, unlike color or noun learning, is scaffolded by the placeholder structure of counting.

Conclusions

Different kinds of knowledge are learned in different ways. Some knowledge (e.g., what Mom looks like) is easy to learn, because our brains are set up to learn it from the beginning. Other information (e.g., the meanings of the words ‘soliciting’ and ‘loitering’) is not in any way innate or universal, but is still relatively easy to learn because it fits neatly into some conceptual system that we already have. Things get more difficult when we have to learn a whole new conceptual system (as in the case of knitting); and the fourth and most difficult case is when we

must learning a whole new conceptual system that conflicts with a conceptual system we already have (e.g., that the earth is a round ball floating in space, not a flat surface that we live on top of).

The number knowledge of an educated person includes all four types of learning, but the focus in this chapter has been on a case of the fourth type— the learning of exact, cardinal number concepts by young children. This exact-number knowledge is built on top of earlier knowledge that evolution has given us (i.e., knowledge of the first type), including the ability to distinguish individuals from sets, to track small numbers of individuals, and to perceive approximate numbers.

Building a conceptual structure to represent exact numbers takes a long time. Children first learn to recite the count list up to 5 or 10, then learn the meanings of the first few numbers (*one, two, three, and four*) one at a time, and in order. Along the way, they must figure out what dimension of experience numbers refer to, and they must also learn how the counting list represents number. That is, they must learn that each counting word corresponds to a specific, unique set size, that the numbers in the list start small and get bigger, and finally that each number in the list is exactly one more than the number before it. When children put all the pieces of this puzzle together, they can understand the cardinality principle-- at least for numbers that are relatively familiar, such as *five* and *six*. That is, they understand that *six cookies* is the number of cookies you have if you count *one, two, three, four, five, six*, adding one cookie to the plate with each word. They also know that this set of cookies can be matched up one-to-one with any other set of six things. This knowledge is a real achievement; it is the basis for representing natural numbers (i.e., positive integers), which in turn lay the groundwork for representing rational numbers, real numbers, and so on.

Children learning number concepts seem to follow a remarkably similar pattern across languages. So far, it seems that speakers of all languages learn the numbers one at a time and in order. But different language environments may affect the timing of number learning differently: If your language requires you to distinguish singular from plural in almost every sentence (as English and Russian do), you may have an easier time learning the difference between ‘one’ and ‘two’ than someone whose language does not mark plurality (e.g., Japanese or Mandarin). Similarly, if your language has a singular/*dual*/plural distinction (as Slovenian and Saudi Arabic do), you may learn the meaning of ‘two’ more easily than speakers of non-dual-marking languages.

But for those who have acquired the conceptual structure to represent exact numbers, the numbers themselves are perfectly translatable across languages. The number 8 (no matter whether you call it *eight*, *ocho*, *huit*, *vocem*’ or *hachi*) is the same everywhere; it’s just the 8th member of the counting list. In this way numbers are a conceptual structure for which language (in the form of a counting list) provides the initial scaffolding; but which in its final form (the natural numbers) transcends language.

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