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J. O. Rasmussen and T. T. Sugihara

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GAMMA-RAY YIELDS FROM NUCLEAR REACTIONS AND LEVEL DENSITIES OF DEFORMED NUCLEI*

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I. INTRODUCTION

There have been a number of studies of nuclear level densities based on the yields of isomeric pairs in compound nuclear reactions. The first analysis of such experiments was by Vandenbosch and Huizenga[1] and was intended to apply to systems involving relatively low angular momentum and excitation energy. Recently Dudey and Sugihara[2] have extended the statistical model formalism to cover much broader ranges of angular momentum and excitation energies such as one would encounter in alpha-particle-induced reactions at several tens of MeV.

The recently published work of Lark and Morinaga[3] may offer a more stringent test of this type of calculation. Following (α,2n) and (α,4n) reactions on even-even deformed nuclei, they were frequently able to resolve several prompt gamma rays of the ground rotational bands of the product nuclei and in a few cases delayed gamma rays from isomers. Stephens, Lark, and Diamond[4] have pursued similar studies using heavy-ion beams, usually with odd-Z projectiles on odd-even targets. Hansen et al.[5] have investigated gamma transitions in the ground band following (p,2n) reactions on odd-even rare earth targets.

II. ISOMER-YIELD THEORY APPLIED TO ^{180m}W

Let us examine the results of Lark and Morinaga[3] for intensities of gamma-ray cascades in ^{180}W. They give prompt gamma-ray yields, from which can be deduced independent yields, of 2^+, 4^+, 6^+, 8^+, 10^+, and 12^+ levels of the ground rotational band. This nucleus also has a 5 msec isomeric state, the yield of which they measured between beam pulses. Diamond, Stephens, and Burde[6] have restudied the ^{180}W levels and have located the isomer at 1525 keV, establishing an 8^- assignment on the basis of gamma-gamma angular correlations. The gamma-ray yields, which were studied for both $^{178}Hf(\alpha,2n)$ at 27 MeV and $^{180}Hf(\alpha,4n)$ at 52 MeV, are summarized in Tables I and II.

The yields have not been corrected for internal conversion and hence refer to the production of individual gamma rays rather than to the population of the corresponding energy levels.[3] For only the lowest energy gamma ray is internal conversion an important correction. The lowest energy gamma rays were also subject to the largest errors in determining intensities;[3] hence there will be no discussion of the cumulative or independent yield of the 2^+ level. According to Lark and Morinaga, crossover transitions, if present at all, were in much lower yield than cascade transitions. Furthermore, the yields were not corrected for gamma-ray anisotropy, which may well have been appreciable judging from more recent studies.[7,8] Thus the 90° measurements may have given lower than true yields for the prompt cascade gammas.

The data in Table I indicate that in the 27-MeV $^{178}Hf(\alpha,2n)^{180}W$ reaction, there is appreciable independent formation of ^{180}W nuclei of relatively low spin.

In the 52-MeV $^{180}Hf(\alpha,4n)^{180}W$ case, ^{180}W states of spin as high as 12^+ are populated; however, states of lower spin are produced with comparable independent yield. Because of the large experimental errors, quantitative judgements are difficult to make. It is interesting to note that even with a considerable increase in bombarding energy and hence input angular momentum, the prompt yield of high-spin states is not increased very much.

The yield of delayed ^{180m}W nuclei is perhaps more amenable to treatment. Evidently the decay of the 8^- level enters the ground-state rotational band at the 8^+ level. The cumulative yields of delayed $8^+\rightarrow 6^+$, $6^+\rightarrow 4^+$, and $4^+\rightarrow 2^+$ gamma rays should then be equal to each other and approximately to the sum of the $2^+\rightarrow 0^+$ plus X-ray yields. This is seen to be consistent with the delayed yields in Table II. The dependence on bombarding energy in this case is large. The average 8^- yields at 27 MeV and 52 MeV are $\approx$200 mb and $\approx$800 mb, respectively. Evidently the decay of high-spin states in the compound nucleus, which have a higher yield at higher bombarding energies, leads primarily to the 8^- levels in the residual ^{180}W nucleus.

We consider whether these data are interpretable in terms of the kinds of calculations used to analyze isomeric yield ratios. The method used for the $^{178}Hf(\alpha,2n)^{180}W$ reactions proceeds in the following manner:[2]

1. Partial-wave cross sections for compound nucleus formation in the reaction ^{178}Hf plus 27-MeV alpha particles were calculated by means of an optical model program.

2. In the decay of the compound nucleus the evaporation of neutrons, protons and alpha particles was considered. The spin distribution in ^{181}W residual nuclei was obtained at an average excitation energy corresponding to the emission of a neutron from ^{182}W with a kinetic energy equal to that averaged over the kinetic energy spectrum.

3. The decay of the excited ^{181}W nuclei was treated in a manner analogous to that of the compound nucleus, and the spin distribution of excited ^{180}W nuclei was determined. In this case the average kinetic energy of the evaporated neutrons was obtained by averaging over only that part of the kinetic energy spectrum which would result in the $(\alpha,2n)$ reactions. That is, evaporation paths which led to ^{180}W nuclei excited above the neutron separation energy were not included in the averaging; such events presumably lead to the $(\alpha,3n)$ reaction.

The details of the calculations, choices of parameters, etc. are described in an Appendix. Spin distributions calculated for the compound nucleus ^{182}W and the excited nuclei ^{181}W and ^{180}W are shown in Fig. 1. The calculations for the reaction $^{180}Hf(\alpha,4n)^{180}W$ with 52-MeV alpha particles were carried out in a manner similar to that for the $^{178}Hf(\alpha,2n)^{180}W$ reaction. Spin distributions for the $(\alpha,4n)$ case are shown in Fig. 2.

In the usual application of this type of calculation to the analysis of isomeric yield ratios,[1,2] the excited residual nucleus is assumed to decay by successive electromagnetic transitions (usually dipole) to one or the other final isomeric states. Most of the experimental data are for spherical nuclei near shell edges since such nuclei most frequently have long-lived isomeric states. In the case of a deformed nucleus such as ^{180}W, electric quadrupole transitions may be more common than dipole transitions. A further discussion of a possible role of K-selection rules in gamma-ray deexcitation for deformed nuclei is reserved for a later section.

We restrict ourselves here to semi-quantitative considerations. We ask whether the yield of the delayed 8- isomer relative to the prompt yield in the two reactions can be reasonably explained by the calculated spin distributions. At 27 MeV the ratio of delayed to prompt is $\approx 200/490 = 0.4$. The prompt yield is taken to be the cumulative yield of the 4^+ level; this includes the contribution of levels as high as 10^+ and perhaps others still higher which have not decayed to the 8^- level. The experimental error in the ratio is of the order of 50%. If it is assumed that excited ^{180}W nuclei which are formed by neutron evaporation and which have spin 10 or higher decay to the 8- level and those which have spin 9 or lower decay to the ground-state rotational band, the spin distribution in Fig. 1 indicates that the delayed/prompt ratio is $0.32/0.68 = 0.47$. If the division is made at $J \geq 11$ and ≤ 10, which we shall denote (11, 10), the ratio is 0.28.

At 52 MeV the experimental ratio is $\approx 800/320 = 2.5$, again with about 50 % experimental error. Dividing the spin distribution of excited ^{180}W nuclei as in the 27-MeV case, we see from Fig. 2 that the calculated delayed/prompt ratio is $0.76/0.24 = 3.2$ for (10, 9); it is 2.3 for (11, 10). A single "dividing spin value" appears to account for the bombarding energy dependence of the delayed yield. However, the experimental errors are too large to permit a choice between (10, 9) and (11, 10).

The detailed shapes of the spin distributions in Figs. 1 and 2 depend on the values assigned to parameters such as the effective moment of inertia $\mathfrak{I}$, the spin cutoff parameter σ, and the limiting spin J' (see Appendix). The values used in the calculations are listed in Table III and IV. Since the values of these parameters have not been firmly established, we have checked whether it is only fortuitous that a single dividing spin is found for the two reactions. A separate calculation was made in which larger values were assigned to all of the parameters for the final evaporation step. In the 27-MeV $(\alpha,2n)$ case, when $\mathfrak{I}/\mathfrak{I}_R = 0.64$, $\sigma^2 = 25.3$, and $J' = 16\hbar$ in the second neutron-evaporation step, the delayed/prompt ratio for the division (10, 9) was 0.54 and for (11, 10) it was 0.34. At 52 MeV, when $\mathfrak{I}/\mathfrak{I}_R = 0.9$, $\sigma^2 = 35.5$, and $J' = 19\hbar$ in the fourth neutron-evaporation step, the ratio was 3.7 for (10, 9) and 2.8 for (11, 10). A single dividing spin value at the two bombarding energies is still consistent with the experimental results.

III. CONSIDERATIONS REGARDING THE GAMMA RAY CASCADE

We have seen above that ordinary isomer yield theory is capable of explaining the ratio of 8- isomer yield to lowest 4+ yield for two different reactions leading to widely different spin distributions. In this case it was necessary to assume that after the last neutron evaporation nuclei with spins greater than ~10 contribute to the 8- isomer and those with lower spins do not. It is not simple to rationalize why this particular dividing spin should apply. Clearly, the yield received by various low-lying levels is not simply a function of spin alone, for the yields to 8- and 8+ levels are quite different. The K quantum number (8 for the 8- level and 0 for the 8+), parity or details of feeder levels above must have an important influence in determining yields.

Nuclear spectroscopic studies on even-even nuclei in the deformed region can give us insight into the factors governing the paths of the gamma cascade in the lower energy region $\lesssim 4\Delta$ where Δ is the odd-even mass difference. Below 2Δ there is only the ground rotational band of K=0 and even parity, with some collective bands[9] moved down somewhat below 2Δ.

Above 2Δ will begin many two-quasi particle bands with K equal to sums or differences of Ω quantum numbers of two neutron (or two proton) Nilsson orbitals near the Fermi surface (cf. Gallagher and Soloviev).[10] In the specific case of ^{180}W the near-lying Nilsson orbitals are mostly Ω=5/2, 7/2, and 9/2. Thus, for this nucleus we expect a family of low-K bands (K=0, 1, 2) beginning above 2Δ and a family of high-K bands (K=6, 7, 8). The K- selection rules would effectively prevent fast gamma transitions between different families at whatever spin. Gamma ray cascades within the low-K family would constitute the prompt gamma yield, and cascades down the high-K family would all give the 8-isomer yield. For nuclei in another region of the Nilsson diagram where $\Omega=1/2$ or $3/2$ orbitals are prevalent, no such division of two-quasi-particle bands into families would occur, but the prompt gamma yield to various members of the ground rotational band could well vary from nucleus to nucleus, according to the particular K values of the lowest two-quasi-particle bands which would serve as principal "feeder" paths into the ground band.

Above an energy $\sim 4\Delta$, where four-quasi particle excitations could occur, it seems fruitless to speculate about gamma cascade paths, since it is not clear to what extent states will mix and the K quantum number, number of quasi-particles, etc. lose their validity. Such considerations suggest the particular desirability of additional experiments on gamma ray yields, performed at bombarding energies within 3 MeV of threshold. At such energies near threshold the whole region of greatest uncertainty for mapping the gamma ray cascade (i.e., between 3 MeV and the neutron binding energy of ~8 MeV) could be completely avoided. The last neutron would have evaporated to leave a final state of two-quasi-particle excitation or less. Of course, the fewer the number of neutrons evaporated the easier it will be to push the gamma ray measurements closer to threshold. Perhaps, (p,2n) and $(\alpha,2n)$ reactions are hopeful for carrying yield measurements toward threshold, and the behavior of gamma ray yields or anisotropies with energy might give really definitive information on the dependence of nuclear level density on J after the first neutron evaporation step.

IV. REMARKS ON GAMMA ANGULAR DISTRIBUTIONS

Further valuable information on the "spin history" in the neutron evaporation and gamma ray cascade can be provided by measurements of gamma ray angular distributions with respect to the beam. We referred earlier to measurements[7,8] indicating rather large anisotropies. Diamond, et al.[7] observed $P_4(\cos\theta)$ as well as $P_2(\cos\theta)$ terms in quadrupole gamma ray angular distributions following heavy-ion reactions on rare-earth targets. Thus, we are prompted to present some formulas and coefficients that may be helpful in analyzing such experiments.

For spin-zero target and projectile the spins of initial compound nuclei are perfectly aligned in the plane perpendicular to the beam axis (only $m_I=0$ states). Consider first the limiting case of such complete alignment at the top of a gamma-ray cascade. Expressing the angular distribution as

$$W(\theta) = 1 + A_2^0 P_2(\cos\theta) + A_4^0 P_4(\cos\theta) \tag{1}$$

we wish to calculate the coefficients for all quadrupole radiation in the "stretched" cascade $J \xrightarrow{L=2} J-2 \xrightarrow{L=2} J-4 \ldots 6 \xrightarrow{E2} 4 \xrightarrow{E2} 2 \xrightarrow{E2} 0$. The coefficients are the same for all the gammas. From formulas given for analysis of low-temperature nuclear alignment experiments[11] it is simple to derive the desired coefficients A_2^0 and A_4^0.

The angular distribution of a gamma ray of pure multipolarity λ emitted from an assembly of oriented nuclei is usually expressed as follows:

$$W(\theta) = 1 + B_2 F_2 P_2(\cos\theta) + B_4 F_4 P_4(\cos\theta) + \ldots \tag{2}$$

The coefficient F_k is taken from γ-γ angular correlation theory and depends on initial spin J, final spin J', and multipolarity λ as follows:

$$F_k(JJ'\lambda\lambda) = (-)^{J'-J-1}(2J+1)^{\frac{1}{2}}(2\lambda+1)(\lambda\lambda 1{-}1|k0)\ W(JJ\lambda\lambda;kj') \quad (3)$$

The coefficient B_k is determined by the initial populations $w(m)$ in the various 2J+1 magnetic substates of the gamma-emitting nucleus.

$$B_k(J) = (2k+1)^{\frac{1}{2}} \frac{\sum_m (Jkm0|Jm)w(m)}{\sum_m w(m)} \quad (4)$$

For the case we consider $w(m) = \delta_{m,0}$, so the sum for B_k reduces to a single term, and we get by substitution

$$A_2^0(J) = B_2F_2 = -5^{3/2}(2J+1)^{1/2}(J200|J0)(221{-}1|20)W(JJ22;\ 2\ J{-}2)$$

$$A_4^0(J) = B_4F_4 = -15(2J+1)^{1/2}(J400|J0)(221{-}1|40)W(JJ22;\ 4\ J{-}2) \quad (5)$$

The values of A_2^0 and A_4^0 for various initial I values are given in Table V. The formulas above include the Clebsch-Gordan coefficients and Racah coefficients in standard notation.

Any departure from complete stretching in the cascade results in lowered anisotropies for all succeeding transitions. In the notation of Ref. 11 one must insert U_k coefficients in Eq. (2) for each transition that intervenes between the spin state for which the B coefficients are determined and the gamma transition for which the angular distribution is desired.

The general formula[12] for attenuation caused by a preceding radiation step $J' \xrightarrow{L} J$, where L will be half integral for neutron evaporation and integral for gammas, is as follows:

$$U_k(J' \xrightarrow{L} J) = (2J'+1)^{\frac{1}{2}}(2J+1)^{\frac{1}{2}} W(J'kLJ;\, J'J) \qquad (6)$$

Table VI gives values for U_2 and U_4 for the special case of a $J \xrightarrow{L=2} J$ intervening transition. An L=2 transition other than $J \to J$ will (for large J) result in less attenuation than above. A dipole transition will cause less attenuation also at a given J.

A simple approximate expression for Eq. (6) can be given that is valid for large J and J'. We use the limiting expression (A2.3) of Edmonds[13] for Racah coefficients with large ar[illegible] to substitute into Eq. (6) and get

$$U_k(J \xrightarrow{L} J') \cong P_k(\cos\theta), \qquad (7)$$

where θ is the angle in the old vector model subtended by the short side $\sqrt{L(L+1)}$ with adjacent sides of length $\sqrt{J(J+1)}$ and $\sqrt{J'(J'+1)}$. The law of cosines can be used to evaluate the argument

$$\cos\theta = \frac{J(J+1) + J'(J'+1) - L(L+1)}{2[J(J+1)J'(J'+1)]^{\frac{1}{2}}}$$

The above formula was tested on some values from Table VI for L=k=2; i.e., at J=24, U_2(approx.) = 0.985, U_2(exact.) = 0.985; at J=10, U_2(approx.) = 0.9193, U_2(exact.) = 0.9189 at J=4, U_2(approx.) = 0.5838, U_2(exact.) = 0.5734.

If one knew the whole "spin history" from the completely aligned (m_J=0) compound nucleus to the top of the stretched gamma cascade, one could just take the product of the U factors for each preceding step. For multiple paths we could take a weighted average of the U products for each pair. At present such reliable detailed models have not been developed, and we shall be interested in approximate treatments.

Table V can also be used to calculate the A_2^m and A_4^m coefficients for the initial angular momentum vector not perpendicular to the beam axis (i.e. $m_J \neq 0$).

The general formulas for $A_k^m(J)$ differ from Eq. (5) only in replacement of the first Clebsch-Gordan coefficient by (Jkm0|Jm). The dependence of this Clebsch-Gordan coefficient on m can be expressed simply in terms of a Legendre polynomial

$$(J2M0|JM) = 4J(J+1)\left[\frac{(2J+1)(2J-2)!}{(2J+3)!}\right]^{\frac{1}{2}} P_2\left[\frac{m}{\sqrt{J(J+1)}}\right]$$

or more generally for all k.

$$\frac{(Jkm0|Jm)}{(Jkm'0|Jm')} = \frac{P_k\left(\frac{m}{\sqrt{J(J+1)}}\right)}{P_k\left(\frac{m'}{\sqrt{J(J+1)}}\right)}$$

From this relation and the expressions for the Legendre polynomials of order 2 and 4

$$P_2(x) = \frac{3x^2-1}{2}$$

$$P_4(x) = (35x^4 - 30x^2 + 3)/8$$

we get

$$A_2^m(J) = A_2^0(J)\ [1 - 3m^2/J(J+1)]$$

$$A_4^m(J) = A_4^0(J) \left[1 - 10\,\frac{m^2}{J(J+1)} + \frac{35}{3} \times \frac{m^4}{J^2(J+1)^2}\right] \tag{8}$$

The above formulas are exact, and they are simple enough to lend themselves to various approximate treatments of the spin history from the initial compound nucleus through successive neutron evaporation stages and gammas to the top of the stretched cascade.

For heavy ion reactions, including alpha reactions at the higher energies, the average orbital angular momentum carried in is much greater than the angular momenta carried off in individual neutron evaporation or gamma stages.

A different formal approach is to consider distributions in magnetic substates. For a normalized distribution $w(m)$ over magnetic substates at initial state J of the stretched gamma cascade we have the general formula

$$\bar{A}_k(J) = \frac{A_k^0(J)}{P_k(0)} \sum_{-J \le m \le J} w(m)\, P_k\left(m/\sqrt{J(J+1)}\right) \tag{9}$$

That the total angular momentum values in typical heavy-ion reactions are large compared to individual angular momentum changes may be seen from Fig. 3, which shows the distribution of spins for 56 MeV B^{11} on Ho^{165}. In such cases the neutron evaporation stages may have approximately the effect on alignment of a one-dimensional random walk in the projection m_J. If at each stage the average step length is μ, after n steps we approach a Gaussian distribution[14] in m.

$$w(m) \approx (2n\mu^2\pi)^{-1/2} e^{-\frac{m^2}{2n\mu^2}}$$

With such approximation and for $2n\mu^2 \gg 1$, we can make a further approximation of replacing the sum of Eq. (9) over magnetic substates by an integral. We then get the following:

$$\begin{aligned} A_2(J) &\approx A_2^0(J)\left[1 - \frac{3n\mu^2}{J(J+1)}\right] \\ A_4(J) &\approx A_4^0(J)\left[1 - \frac{10n\mu^2}{J(J+1)} + \frac{35n^2\mu^4}{J^2(J+1)^2}\right] \end{aligned} \tag{10}$$

If the target and projectile spins are not both zero, there will be an additional attenuation factor on the anisotropy due to these spins. This attenuation factor can be calculated by noting that there is the following distribution of magnetic substates in the initial compound nucleus

$$w_i(m) = \begin{cases} 0 & J_>+J_< <|m| \\ \dfrac{J_>+J_< - |m|+1}{(2J_>+1)(2J_<+1)}, & J_>-J_< \leq |m| \leq J_>+J_< \\ \\ \dfrac{1}{2J_>+1}, & |m|<J_>-J_< \end{cases} \qquad (11)$$

where $J_>$ is the greater of the two spins (target or projectile) and $J_<$ is the lesser.

If the above analysis were applied to experimental angular distributions, one could derive a μ value, the effective step length in the random walk of spin projection on the beam axis. For a succession of S-wave neutron evaporations the step length would be $\mu=1/2$. For larger angular momentum carried off, the μ value will depend both on the average value of angular momentum carried off and also upon the extent to which the spin history deviates from the completely stretched spin sequence, which minimizes μ.

The analysis of γ-anisotropies in Fig. 1 of Diamond et al.[7] nicely shows the validity of a Gaussian approximation to the distribution in magnetic substates for heavy ion reactions.

We would like to conclude with a sample calculation not making use of the Gaussian treatment and to suggest some consequences of the experimental angular distributions from the work of Ref. 7.

Consider $J^0=24$ compound nucleus completely aligned (M=0). Consider four successive neutron evaporation steps each carrying off 5/2 units of angular momentum through a stretched spin sequence to the top of the E2 cascade at spin $J=14$. We calculate the coefficient $\overline{A}_2$ for the cascade gamma angular correlation with respect to the beam direction, using Eq. (7) and Table V.

$$\overline{A}_2 = U_2(0\rightarrow I)U_2(I\rightarrow II)U_2(II\rightarrow III)U_2(III\rightarrow IV)A_2^0(14)$$

From Table V we read $A_2^0(14) = 0.39682$. From Eq. (7) we calculate

$$U_2(0\rightarrow I) = 0.9933$$

$$U_2(I\rightarrow II) = 0.9913$$

$$U_2(II\rightarrow III) = 0.9887$$

$$U_2(III\rightarrow IV) = 0.9849$$

$$\Pi U_2 = 0.9588$$

$$\overline{A}_2 = 0.3805$$

This value of A_2 should constitute an effective upper limit for typical heavy-ion reactions. It is remarkable to see in Fig. 1 of Diamond et al.[7] that the $12\rightarrow 10$ and $10\rightarrow 8$ gamma transitions show A_2 coefficients very close to this limit. Thus, we might at first be tempted to conclude that the feeding at the spin 10-12 level has come mainly via a stretched-spin sequence in the neutron evaporation process, and any gamma cascade preceding the rotational band. However, perusal of Table VI shows us that we can have several unstretched transitions with $\Delta J \approx 2$ in the higher spin levels of the spin history without appreciably lowering the measured A_2. Measurements of A_2 are thus not a sensitive indicator of spin history in the high-spin end of the process.

Measurements of nuclear-reaction gamma ray intensities and angular distributions at various bombarding energies seem indeed a hopeful and powerful tool for study.

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APPENDIX

The equations used in the calculation of spin distributions are summarized below.[2] The partial cross section $\sigma_c(E_c,J_c)$ to form a compound nucleus of excitation energy E_c and spin J_c, for the case of alpha-particle bombardment, is given by

$$\sigma_c(E_c,J_c) = \pi\lambda\!\!\!^{-2} \sum_{\ell=|J_c-S|}^{J_c+S} \frac{2J_c+1}{2J_t+1} T_\ell(\epsilon) \qquad \text{(A1)}$$

where $\lambda\!\!\!^{-}$ is the reduced de Broglie wavelength of the incoming alpha particle, J_t is the spin of the target nucleus, S is the entrance channel spin, and $T_\ell(\epsilon)$ is the transmission coefficient of the incident alpha particle of channel energy ϵ and orbital angular momentum ℓ.

The normalized probability for the compound nucleus to decay by emission of a particle ν(neutron, proton, or alpha particle) to a final state of average excitation energy $\overline{E}_f$ and spin J_f is given by

$$P_\nu^{(1)}(\overline{E}_f,J_f) = \frac{\sum_{J_c} \Omega_\nu(\overline{E}_f,J_f) P_c(E_c,J_c) \sum_{S_\nu=|J_f-s_\nu|}^{J_f+s_\nu} \sum_{\ell_\nu=|J_c-S_\nu|}^{J_c+S_\nu} T_\ell(\overline{\epsilon}_\nu)}{\sum_\nu \sum_{J_f} \Omega_\nu(\overline{E}_f,J_f) \sum_{S_\nu} \sum_{\ell_\nu} T_\ell(\overline{\epsilon}_\nu)} \, . \qquad \text{(A2)}$$

Here $\Omega_\nu(\overline{E}_f,J_f)$ is the level density in the residual nucleus formed by emission of particle $\underline{\nu}$; the functional form of $\Omega_\nu(\overline{E}_f,J_f)$ is discussed below. Also $P_c(E_c,J_c)$ is a normalized form of $\sigma_c(E_c,J_c)$, that is

$$P_c(E_c,J_c) = \sigma_c(E_c,J_c) \Big/ \sum_{J_c=0}^{\infty} \sigma_c(E_c,J_c); \tag{A3}$$

s_ν and ℓ_ν are the intrinsic spin and orbital angular momentum of particle ν; S_ν the exit channel spin. The quantity in the denominator of Eq. (A2) is a normalization constant which accounts for all of the original compound nuclei.

If more than one particle is evaporated in cascade, the distribution $P_\nu^{(1)} E_f, J_f)$ is used instead of $P_c(E_c,J_c)$ in Eq. (A2) to generate another distribution $P_\nu^{(2)}(\overline{E}_f,J_f)$. This is continued for as many steps as necessary.

The density of levels of energy E and spin J is taken to be

$$\Omega(E,J) = \frac{(2J+1)a^{1/4}}{2(2\pi\sigma^2)^{3/2}E^{3/4}} \exp\left[2(aE)^{1/2} - J(J+1)/2\sigma^2\right] , \tag{A4}$$

where $a = A/10.7\ \mathrm{MeV}^{-1}$ for a residual nucleus of mass number A. The spin cutoff parameter σ^2 is formally related to the effective moment of inertia $\Im$ by

$$\sigma^2 = \Im T/\hbar^2 \tag{A5}$$

where the nuclear temperature T is obtained from

$$1/T = (a/E)^{1/2} - 3/4E. \tag{A6}$$

The analysis of isomeric yield ratios[2] indicated that a consistent fit to experiment was obtained if $\Im$ was chosen to be the moment of inertia $\Im_R$ of a rigid sphere of radius $1.2A^{1/3}$ F when $E \geq 10$ MeV and less than $\Im_R$ at lower excitation energies. The rough correlation of $\Im/\Im_R$ with E when $E \leq 10$ MeV, as given in Ref. 2, was used for calculations here.

Equation (A4) was considered to give the level density for J values up to a limiting J value given by

$$J' = 1.37(\sigma^2\hbar/\pi)(6a/\Im)^{1/2} \quad . \tag{A7}$$

For $J > J'$, $\Omega(E,J)$ was taken to be zero.

In all of the calculations the excitation energy E was corrected for the odd-even character of residual nuclides. For odd-odd nuclei, the correction was zero; for even-odd or odd-even, E was reduced by 1.0 MeV; and for even-even nuclei, the reduction was 2.0 MeV.

Transmission coefficients for neutrons, protons, and alpha particles were calculated with an optical model program of Glendenning.[7] The parameters assumed for a nuclear potential of Woods-Saxon form

$$V_{nuc} = -\frac{V_0 + iW}{1 + \exp\left[(r - r_0A^{1/3} - r_1)/d\right]} \tag{A8}$$

are given in Table III.

The average kinetic energy of an emitted neutron was assumed to be 2T if essentially all of the energy spectrum could lead to the desired final nuclide. If some fraction of the spectrum had to be excluded, the average energy was calculated for the available part assuming the spectrum shape was Maxwellian. In the case of charged particles the average kinetic energy was assumed to be B+T where B is an effective barrier energy. It was assumed to be that kinetic energy for which the penetrability of an $\ell=0$ wave was 0.5 by the optical model calculation. Typical values are B_α = 19.6 MeV for alpha particles when the residual nucleus is ^{178}Hf and B_p = 10.1 MeV for protons when the residual nucleus is ^{179}Ta.

Table IV summarizes the values of the quantities $\Im/\Im_R$, σ^2, J', and $\overline{E}_f$ as used in various stages of the calculation. The distributions in Figs. 1 and 2 result from these choices. As indicated earlier, the ratio of delayed to prompt gamma rays is not a sensitive function of the values assumed for any of these quantities.

Table I. Prompt gamma-ray yields[a] σ_γ in the reactions ^{178}Hf(α,2n)^{180}W at 26 MeV and ^{180}Hf(α,4n)^{180}W at 52 MeV and independent yields[b] σ_I for residual ^{180}W nuclei of spin I.

			(α,2n)		(α,4n)	
E_γ (keV)	Level Energy (keV)	Spin and Parity of Level	σ_γ (mb)	σ_I (mb)	σ_γ (mb)	σ_I (mb)
102±5	102±5	2^+	200±30	----	----	----
234±5	336±7	4^+	490±70	220±100	320±120	90±140
354±7	690±10	6^+	270±70	90±90	230±70	60±80
457±7	1147±15	8^+	180±60	50±90	170±40	-10±90
520±10	1667±20	10^+	130±70	130±70	180±80	100±100
585±15	2252±25	12^+	-----	-----	80±50	80±50

[a] From Ref. 3.

[b] Calculated by difference.

Table II. Delayed gamma-ray yields[a] σ_γ^{d} in the reactions $^{178}Hf(\alpha,4n)^{180m}W$ at 27 MeV and $^{180}Hf(\alpha,4n)^{180m}W$ at 52 MeV.

E_γ (keV)	Level Energy (keV)	Spin and Parity of Level	$(\alpha,2n)$ σ_γ^{d} (mb)	$(\alpha,4n)$ σ_γ^{d} (mb)
54	-----	K X-ray	230±60	590±270
102±5	102±5	2^+	----	110±80
234±5	336±7	4^+	230±90	1130±180
354±7	690±10	6^+	170±90	610±200
457±7	1147±15	8^+	210±70	720±270

[a] From Ref. 3.

Table III. Parameters[a] used in optical-model calculation of penetrabilities.

Parameter	Alpha	Proton	Neutron
V_o (MeV)	50.0	57.0	57.0
W (MeV)	21.0	20.0	20.0
r_o (F)	1.17	1.25	1.25
r_1 (F)	1.77	0.00	0.00
d (F)	0.576	0.650	0.650

[a]See Eq. (A8) for definition of symbols.

Table IV. Values of effective moment of inertia $\mathscr{I}$ (units are rigid-body moment $\mathscr{I}_R$), spin cutoff parameter σ^2, limiting spin J', and average energy of residual nucleus $\bar{E}_f$. Step 1 refers to evaporation from the compound nucleus; steps 2, 3, and 4 refer to further evaporation from residual nuclei formed by neutron emission in the previous step.

Reaction	Step	Particle Emitted	$\mathscr{I}/\mathscr{I}_R$	σ^2	J' ($\hbar$)	$\bar{E}_f$ (MeV)
$^{178}Hf(\alpha,2n)^{180}W$ 27-MeV α	1	n	1.0	81.4	40.5	14.1
		p	0.64	25.1	15.5	5.9
		α	0.64	24.8	16.0	6.1
	2	n	0.5	19.4	14.0	6.3
		p	---	---	---	---
		α	---	---	---	---
$^{180}Hf(\alpha,4n)^{180}W$ 52-MeV α	1	n	1.0	130	64.5	38.3
		p	1.0	113	55.5	29.7
		α	1.0	111	55.0	29.9
	2	n	1.0	114	57.0	29.5
		p	1.0	92	45.0	19.5
		α	1.0	90	43.5	19.5
	3	n	1.0	93	46.5	19.1
		p	1.0	71	35.5	11.1
		α	1.0	70	35.0	11.2
	4	n	0.7	27.6	16.0	8.7
		p	0.25	9.9	10.0	1.4
		α	0.25	9.5	9.5	1.3

Table V. Gamma Ray Angular Coefficients for Stretched Quadrupole Cascades Initially Perfectly Aligned at Spin J.

I	$A_2^0(J)$	$A_4^0(J)$
2	.71429	-1.71429
4	.51020	- .36735
6	.45455	- .24242
8	.42857	- .19780
10	.41353	- .17514
12	.40372	- .16149
14	.39682	- .15238
16	.39170	- .14588
18	.38776	- .14100
20	.38461	- .13721
22	.38206	- .13419
24	.37994	- .13171

Table VI. Angular Attenuation Coefficients for Gammas Following a $J \xrightarrow{2} J$ Quadrupole Transition in the Cascade.

J	$U_2(J \xrightarrow{2} J)$	$U_4(J \xrightarrow{2} J)$
2	-.21429	.28571
4	.57338	-.14935
6	.79091	.36364
8	.87676	.60965
10	.91893	.73851
12	.94267	.81326
14	.95735	.86021
16	.96703	.89153
18	.97376	.91343
20	.97862	.92934
22	.98224	.94124
24	.98504	.95038

FOOTNOTES AND REFERENCES

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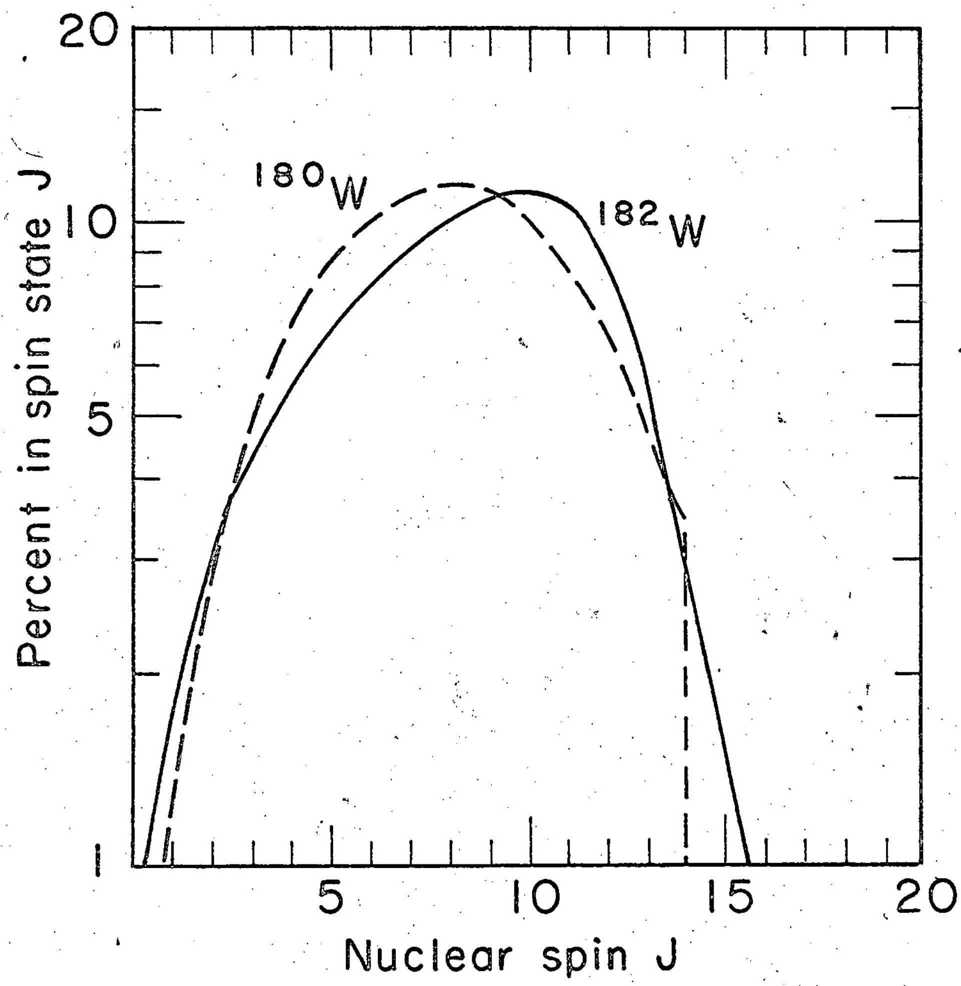
14. See also the somewhat related approach by Barlit, (E. M. Barlit, "Concerning the Angular Distribution of Rotational γ-Quanta." Dubna preprint 2197 (1965).) He treats the situation where at the beginning of the quadrupole gamma cascade there is a Gaussian distribution of spin vectors about the initial compound nucleus spin vectors, such as would result from a 3-dimensional random walk in J-space from the compound nucleus $\vec{J}_0$.

FIGURE CAPTIONS

Fig. 1. Calculated spin distribution of the compound nucleus ^{182}W formed by 27-MeV alpha particles on ^{178}Hf. Also plotted is the spin distribution of the residual nucleus ^{180}W formed by neutron evaporation from ^{182}W. The distribution for each nucleus is normalized to unity. The distributions should be histograms but for convenience in graphing J has been taken to be a continuous variable.

Fig. 2. Spin distribution of ^{184}W formed by 52-MeV alpha particles on ^{180}Hf. Also plotted are distributions for residual nuclei ^{181}W and ^{180}W. See also caption of Fig. 1. The unusual shape of the distribution for ^{180}W results from the sharp cutoff in $\Omega(E,J)$ at $J' = 16\hbar$. The division of the spin distribution into those excited nuclei which decay to the 8- isomer and those which enter the ground rotational band promptly is not a sensitive function of the shape of the spin distribution.

Fig. 3. Spin distribution of ^{176}Hf formed by 56 MeV ^{11}B ions on ^{165}Ho. Also plotted are distributions for residual nuclei ^{173}Hf and ^{172}Hf.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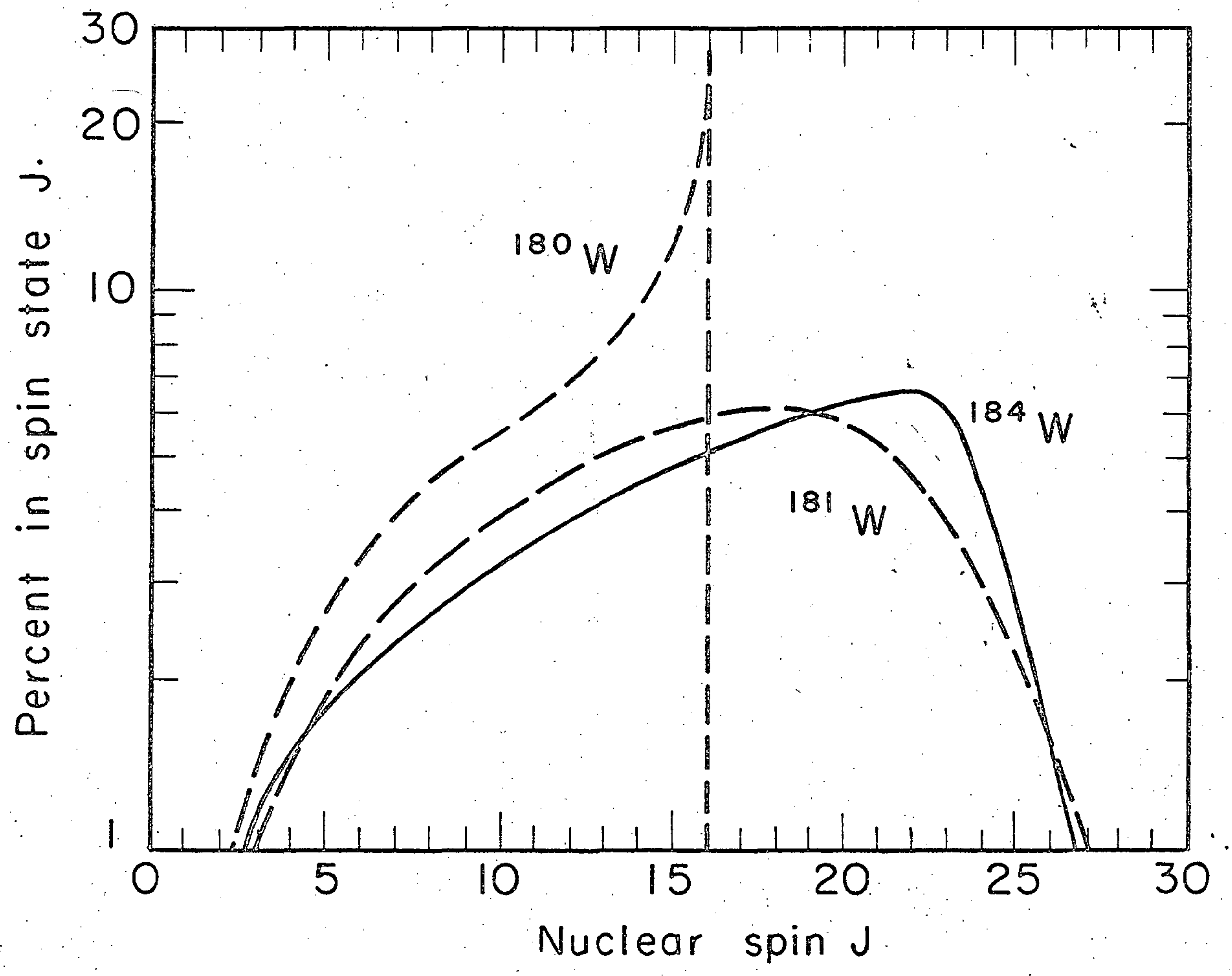
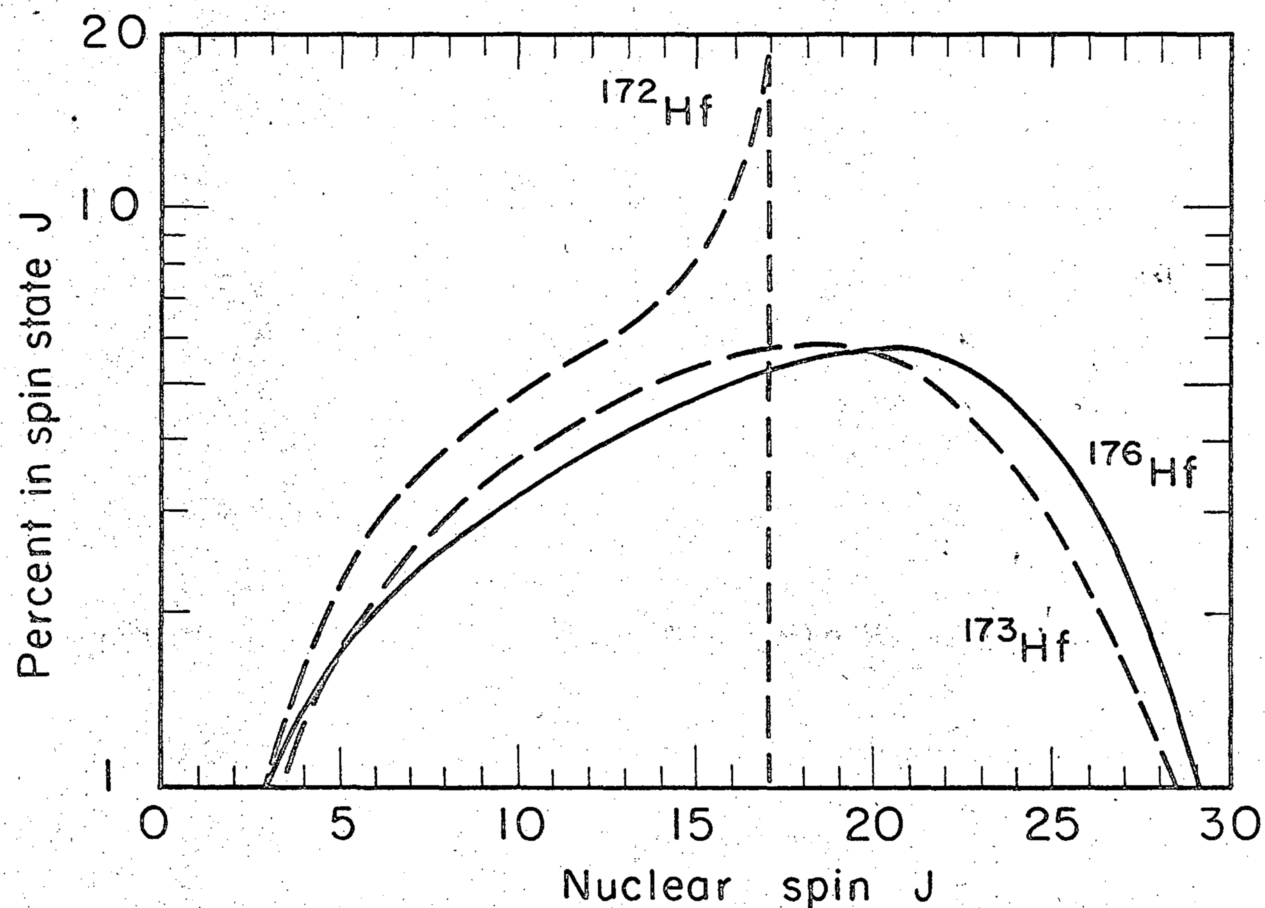
10
5
1
Percent in spin state J
180W
182W
5
10
15
20
Nuclear spin J

MUB-11193


30
20
10
1
0
5
10
15
20
25
30
Percent in spin state J.
Nuclear spin J
180 W
184 W
181 W

MUB-11192


20
10
1
Percent in spin state J
0
5
10
15
20
25
30
Nuclear spin J
^{172}Hf
^{176}Hf
^{173}Hf

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