UC Davis

UC Davis Previously Published Works

Title

New MIMO ARQ protocols and joint detection via sphere decoding

Permalink

<https://escholarship.org/uc/item/50t1w6hs>

Journal

IEEE Transactions on Signal Processing, 54(2)

ISSN

1053-587X

Authors

Samra, H

Ding, Z

Publication Date

2006-02-01

Peer reviewed

New MIMO ARQ Protocols and Joint Detection via Sphere Decoding

Harvind Samra, *Member, IEEE*, and Zhi Ding, *Fellow, IEEE*

Abstract—In this paper, we develop some new automatic repeat request (ARQ) protocols for multiple-input multiple-output (MIMO) flat-fading channels which adapt the bit-to-symbol mapping of each ARQ retransmission. We begin by defining a model for distinctly mapped transmissions through flat-fading MIMO channels. We characterize the effect that such a mapping diversity has on an integrated receiver utilizing sphere decoding. Varying the symbol mapping complicates the sphere decoding process, particularly for the enumeration of candidate solutions within the sphere. A technique that enables quick candidate enumeration is presented, utilizing concepts from existing closest point search schemes. The advantage of mapping diversity, in reducing bit error rate and reducing computational complexity, is presented along with simulation examples.

Index Terms—Automatic repeat request (ARQ), mapping diversity, multiple-input multiple-output, sphere decoding, space-time coding.

I. INTRODUCTION

CRITICAL figures of merit for a communication system often include low frame error rates and high data throughput to its end users. Effective handling and reduction of packet retransmissions are vital to performance improvement of wireless automatic repeat request (ARQ) networks. Generally, if errors remain after error correction in reception of a transmitted data packet, a request for retransmission is issued to the transmitter. Clearly, ARQ protocols and forward error-correction codes are two important types of error correction mechanism in wireless networks. As a result, the development and exploitation of ARQ protocols have been the subject of much research at both the network and the physical layers. A general discussion of ARQ systems from a coding perspective can be found in [1].

At the physical layer itself, various approaches have been proposed for both packet combining and improving diversity among these retransmissions. Chase developed a maximum likelihood combining scheme for an arbitrary number of coded packets, concatenating M copies of a codeword into a single codeword [2]. Harvey and Wicker proposed several ARQ

strategies, including an approach that combines soft-decoded codewords from multiple packet transmissions into a single soft codeword [3]. Others, including Hagenauer and Rowitch and Milstein, have developed schemes utilizing rate-compatible codes, where retransmitted copies of a packet are each uniquely punctured to improve throughput [4], [5]. Stuber and Narayanan developed an ARQ receiver using error-correcting codes where the extrinsic information from the decoding of previous packets is reused [6]. Zhang and Kassam outlined a hybrid ARQ protocol for rate-compatible codes in fading channels that selectively combines a subset of the M received transmissions [7]. Recently, an approach that adapts the bit-to-symbol mapping for each retransmission was developed for an integrated receiver, providing significant reduction in bit error rate (BER) while requiring minimal increase in system complexity [8], [9]. Also, approaches exist that exploit retransmission diversity at the output of intersymbol interference channels [10], [11].

This paper studies the effectiveness of packet retransmissions in multiple-input multiple-output (MIMO) systems, where multiple antennas may be present at the transmitter and/or receiver. The spatial dimension provided by multiple antennas is known to dramatically increase system capacity [12], [13]. As a result, MIMO systems have become important and popular research subjects. In particular, the combination of MIMO spatial diversity with temporal diversity has led to widespread development of space-time coding schemes. Many approaches have been proposed for space-time coding; some of the more popular are V-BLAST, space-time trellis coding, space-time bit-interleaving coded modulation (BICM), and space-time block coding [14]–[18].

As packet retransmissions in MIMO systems utilize spatiotemporal diversity, this work has many similarities to space-time coding. However, with ARQ we utilize incremental redundancy (i.e., retransmissions made only when necessary) to minimize the number of retransmissions for better throughput; with space-time coding, the redundancy or number of retransmissions (or code rate) is predetermined. Others have recently studied packet retransmissions in MIMO systems. Onggosanusi *et al.* introduced methods for combining packet transmissions by using zero-forcing and minimum mean squared error (MMSE) receivers [19]. Ding and Rice proposed a hybrid ARQ protocol involving spatiotemporal vector coding and multidimensional trellis-coded modulation (TCM) [20]. Nguyen and Ingram investigated hybrid ARQ protocols for systems that use recursive space-time codes [21]. Zheng *et al.* suggested separating the ARQ process into ARQ subprocesses that operate over an isolated pairing of a transmitter and receiver antenna [22].

Manuscript received August 3, 2004; revised March 12, 2005. This paper was presented in part at the International Conference on Acoustics, Speech, and Signal Processing, Montreal, PQ, Canada, in 2004. This work was supported in part by the National Science Foundation under Grants ECS-0121469, CCF-0515058, CNS-05020126, DMI-0400155, and U.S. Army Research Office Grant W911NF-05-1-0382. The associate editor coordinating the review of this manuscript and approving it for publication was Prof. Ioan Tabus.

H. Samra is with Kestrel Signal Processing, Inc., Fairfield, CA 94534 USA (e-mail: hssamra@kestrelsp.com; hssamra@ece.ucdavis.edu).

Z. Ding is with the Department of Electrical and Computer Engineering, University of California, Davis, CA 95616 (e-mail: zding@ece.ucdavis.edu).

Digital Object Identifier 10.1109/TSP.2005.861781

In this paper, we define an ARQ protocol for MIMO flat-fading channels where the bit-to-symbol mapping is adapted per retransmission using symbol mapping diversity. After reviewing mapping diversity, we define a model for distinctly mapped transmissions through MIMO channels and the impact that mapping diversity has on a sphere decoding receiver. Varying the symbol mapping complicates the sphere decoding process, mostly in the enumeration of candidates within the sphere. We propose a method that delivers fast candidate enumeration, using a technique developed for computational vision problems. An application of mapping diversity is provided, along with insight into its effect on BER improvement and computational complexity. A second, but very similar, ARQ protocol is described involving hybrid retransmissions that are composed of both retransmitted symbols and original symbols. Precoding (per retransmission) techniques are introduced to increase the diversity among retransmissions through identical MIMO channels. Also, a discussion of the relationship between these protocols and space-time coding is presented. We conclude with simulation results to validate the primary ARQ protocol.

Standard notations are used in this paper. Vectors are denoted by lowercase boldface fonts ($\mathbf{a}$) and matrices by uppercase boldface fonts ($\mathbf{A}$). The transpose and Hermitian transpose operations are indicated by $(\cdot)^T$ and $(\cdot)^H$, respectively. The operators $\Re\{z\}$ and $\Im\{z\}$ provide the respective real and imaginary components of the complex argument z . A real-valued Gaussian random variable x with mean $\mu_x = E_x\{x\}$ and variance $\sigma_x^2 = E_x\{x^2\} - \mu_x^2$ is indicated by $\mathcal{N}(\mu_x, \sigma_x^2)$. The operator $E_x\{f(x)\}$ indicates the expected value of $f(x)$ with respect to the random variable x . A complex Gaussian random variable z with mean $\mu_z = E_z\{z\}$ and variance $\sigma_z^2 = E_z\{|z|^2\} - |\mu_z|^2$ is denoted by $\mathcal{CN}(\mu_z, \sigma_z^2)$. The real and imaginary components of z are assumed to each have variance $\sigma_z^2/2$.

II. ARQ AND MAPPING DIVERSITY

The ARQ protocol utilized herein is fairly straightforward: a packet of labels is initially transmitted, and subsequent retransmissions of this packet are made until the entire packet is received error-free. With regard to error-checking, generally the bits that comprise the packet undergo a cyclic redundancy and parity check at the receiver. If that check fails, the receiver saves the received packet and requests the transmitter to retransmit the packet itself or some additional information about the packet. Previous erroneous transmissions are saved so that they can aid in mitigating errors of the current transmission, typically through some type of packet combining.

Before describing an ARQ protocol for MIMO systems that employs mapping diversity, we review mapping diversity in single-input single-output flat-fading channels [9]. We begin with a set $\mathcal{C}$ of real- or complex-valued points that represents the points of a signal constellation, e.g., 16-QAM. Given a packet of bits, consecutive groups of $\log_2 |\mathcal{C}|$ bits are assigned to symbols in $\mathcal{C}$ via a symbol mapping function $\psi : \{0, 1, \dots, |\mathcal{C}| - 1\} \rightarrow \mathcal{C}$. Herein, these groups of $\log_2 |\mathcal{C}|$ bits are referred to as labels (which is consistent with [23]) and label $s \in \{0, 1, \dots, |\mathcal{C}| - 1\}$ represents the decimal equivalent of these bits.

Fig. 1. Flow diagram of ARQ protocol with mapping diversity.

The key concept of mapping diversity lies in each retransmission using a symbol mapping that differs from the mappings of previous transmissions in order to minimize the BER at the receiver. Fig. 1 illustrates the basic operational and informational flow of ARQ with mapping diversity. For M transmissions of the same packet, we define M symbol mapping functions $\psi_1, \dots, \psi_M$. The transmitter sends symbols $\psi_1[s], \dots, \psi_M[s]$, and the receiver obtains samples $z_m = g_m \psi_m[s] + v_m$, where $m = 1, \dots, M$ and v_m is $\mathcal{CN}(0, \sigma_v^2)$. The variable g_m represents the fading gain of the m th transmission and is Gaussian. The receiver stores the previously received transmissions, and thus uses a maximum likelihood (ML) receiver on samples $z_1, \dots, z_M$

$$\min_{\hat{s}=0,1,\dots,|\mathcal{C}|-1} \sum_{m=1}^M |z_m - g_m \psi_m[\hat{s}]|^2. \quad (1)$$

For choosing the appropriate mappings, optimality is defined as minimizing the BER. A generic BER upper bound was developed for M transmissions under symbol mapping diversity

$$\sum_{s=0}^{|\mathcal{C}|-1} \sum_{\substack{t=0 \\ t \neq s}}^{|\mathcal{C}|-1} \Pr\{s\} B[s, t] \Pr\{\alpha_M[t] < \alpha_M[s]\} \quad (2)$$

where $\Pr\{s\}$ is the probability of transmitting s (generally $1/|\mathcal{C}|$)

$$B[s, t] = \frac{(\text{number of differing bits between } s \text{ and } t)}{\log_2 |\mathcal{C}|}$$

is defined to account for the number of bit errors that result from a label misdetection and $\Pr\{\alpha_M[v] < \alpha_M[t]|s\}$ is the pairwise error probability (PEP) that t is more likely than s to be detected given that s is transmitted. Here, the metric $\alpha_M[s]$ is defined as

$$\sum_{m=1}^M |z_m - g_m \psi_m[s]|^2.$$

Rather than jointly computing the M mappings, we compute the M th mapping given the previous $M-1$ mappings, usually starting with any Gray mapping for $M=1$. This solution is optimal for ARQ-type applications which have a secondary objective to minimize the number of transmissions (mappings) needed to achieve a desired BER. In other words, it is desirable to choose mapping M without expecting or relying on future (re)transmissions. Our optimization problem then simplifies to

$$\min_{\psi_M \in \Psi} \sum_{s=0}^{|\mathcal{C}|-1} \sum_{\substack{t=0 \\ t \neq s}}^{|\mathcal{C}|-1} f[s, \psi_M[s], t, \psi_M[t]] \quad (3)$$

where Ψ is the set of all possible mappings and $f[s, a, t, b]$ is the pairwise BER that results from mapping label s to symbol $a \in \mathcal{C}$ and label t to symbol $b \in \mathcal{C}$ in the M th mapping

$$f[s, a, t, b] = \Pr\{s\}B[s, t]\Pr\{\delta < 0\}.$$

Though still computationally difficult, (3) falls into a category of combinatorial optimization problems referred to as the quadratic assignment problem, and exact and approximate solvers are available. For the optimization posed in (3)

$$\delta = (\alpha_{M-1}[k] - \alpha_{M-1}[s]) + |z_M - g_M b|^2 - |z_M - g_M a|^2$$

leading to a PEP $\Pr\{\delta < 0\}$ of

$$E_u \left\{ Q \left(\sqrt{\frac{1}{2\sigma_w^2} \left(\sum_{m=1}^{M-1} |g_m d_m[s, t]|^2 + |g_M(a - b)|^2 \right)} \right) \right\}$$

with $d_m[s, t] = \psi_m[s] - \psi_m[t]$ and $\mathbf{g} = \{g_1, \dots, g_M\}$. This PEP is numerically computed to provide the function $f[s, a, k, b]$ for solving (3), generating optimal mappings for fading channels [9].

Rather than computing mappings in real-time, we compute the mappings for a set of appropriate signal-to-noise ratio (SNR) values (and perhaps Rice factors) offline and store them in a table at both the transmitter and receiver. This avoids both the prohibitive computational costs and the need to transmit/receive mapping information.

III. SYSTEM MODEL

We now extend mapping diversity to MIMO scenarios. We consider M transmissions of a packet of labels, illustrated in Fig. 2, with a transmitter of N antennas and a receiver of K antennas. More specifically, we narrow our focus on blocks of the packet, with each block $\mathbf{s} = [s_1, \dots, s_N]^T$ containing N user labels, that are transmitted M times. Each transmission utilizes a unique symbol mapping ψ_m for all N labels. A $K \times N$ matrix $\mathbf{H}_m$ ($K \geq N$) represents the m th transmission channel response, with its (i, k) th element $h_{m,ik}$ indicating the fading coefficient between transmit antenna i and receive antenna k . This fading gain is Rayleigh distributed, so that $h_{m,ik}$ is a Gaussian variable of $\mathcal{CN}(0, 1)$.

For the m th transmission, the receiver obtains

$$\begin{aligned} \mathbf{y}_m &= \begin{bmatrix} y_{m,1} \\ \vdots \\ y_{m,K} \end{bmatrix} = \mathbf{H}_m \begin{bmatrix} \psi_m[s_1] \\ \vdots \\ \psi_m[s_N] \end{bmatrix} + \begin{bmatrix} w_{m,1} \\ \vdots \\ w_{m,K} \end{bmatrix} \\ &= \mathbf{H}_m \vec{\psi}_m[\mathbf{s}] + \mathbf{w}_m. \end{aligned} \quad (4)$$

The noise vectors $\{\mathbf{w}_m\}$ are independent identically distributed with Gaussian distribution $\mathcal{CN}(\mathbf{0}_{K \times 1}, \sigma_w^2 \mathbf{I}_K)$. Note that each label s_n is distinctly mapped via M mapping functions $\psi_1, \dots, \psi_M$. Later in this paper, we shall discuss the selection of optimal mappings.

The receiver employs a joint ML decoder that integrates multiple received signal vectors $\mathbf{y}_1, \dots, \mathbf{y}_M$ to generate more reliable source signal estimates $\hat{\mathbf{s}} = [\hat{s}_1, \dots, \hat{s}_N]^T$. Assuming that

Fig. 2. Block diagram of multiple MIMO transmissions of a packet with mapping diversity.

the channels are perfectly known to the receiver, the ML decoder leads to

$$\hat{\mathbf{s}} = \arg \min_{\mathbf{s}} \sum_{m=1}^M \|\mathbf{y}_m - \mathbf{H}_m \vec{\psi}_m[\mathbf{s}]\|^2. \quad (5)$$

At first glance, minimizing the metric in (5) appears to require an exhaustive search over all $|\mathcal{C}|^N$ candidates of $\hat{\mathbf{s}}$. Fortunately, through algebraic manipulations, the metric can be rewritten to avoid an exhaustive search. By defining $\mathbf{p}_m = \mathbf{H}_m^H \mathbf{y}_m$, the metric is

$$\sum_{m=1}^M (\vec{\psi}_m[\mathbf{s}] - \mathbf{p}_m)^H \mathbf{H}_m^H \mathbf{H}_m (\vec{\psi}_m[\mathbf{s}] - \mathbf{p}_m).$$

Let $\mathbf{U}_m$ be an upper triangular matrix that satisfies $\mathbf{U}_m^H \mathbf{U}_m = \mathbf{H}_m^H \mathbf{H}_m$. This is typically accomplished with a Cholesky decomposition [24], [25]. The elements of $\mathbf{U}_m$ are indicated using $u_{m,ik}$ (or $u_{m,i,k}$), and the metric becomes

$$\begin{aligned} \sum_{n=1}^N \sum_{m=1}^M u_{m,nn}^2 \left| \psi_m[s_n] - p_{m,n} \right. \\ \left. + \sum_{k=n+1}^N \frac{u_{m,nk}}{u_{m,nn}} (\psi_m[s_k] - p_{m,k}) \right|^2. \end{aligned} \quad (6)$$

This metric bears many similarities with the metric that is minimized using sphere decoding. The primary difference involves the inclusion of multiple transmissions and their different mappings. We now discuss the application of sphere decoding to minimize our metric while maintaining a low computational burden.

IV. APPLYING SPHERE DECODING

Recently, sphere decoding has become a low-cost alternative to exhaustive, brute-force ML searches [26], [27]. The primary strategy in search reduction is the removal of all points outside of a hypersphere centered about the received data point $\mathbf{p}$. Beginning with s_N , labels whose mapped symbols fall inside the hypersphere are sequentially selected to compose a data estimate $\hat{\mathbf{s}}$. Several techniques exist that quickly enumerate all symbols/labels within the hypersphere [17], [27]. This estimate is saved, and the radius of the hypersphere is decreased to the distance between the symbols of $\hat{\mathbf{s}}$ and $\mathbf{p}$. This process is repeated until no candidates exist within the hypersphere, and the most recent data estimate $\mathbf{s}$ becomes the ML estimate. The expected

complexity of sphere decoding is cubic (N^3) in most instances [26].

In applying sphere decoding to our multiple-transmission scenario, we first define a hypersphere of radius r_N , with a modification of the rule proposed by Hochwald and ten Brink [17]

$$r_N^2 = \sum_{m=1}^M 2\sigma_v^2 \xi K - \mathbf{y}_m^H \left(\mathbf{I}_K - \mathbf{H}_m (\mathbf{H}_m^H \mathbf{H}_m)^\dagger \mathbf{H}_m^H \right) \mathbf{y}_m.$$

The parameter ξ is chosen to ensure that the hypersphere contains some candidates. From the metric (6), we focus on the label s_N by looking only at the $n = N$ term of the summation. Thus, we seek to choose a label s_N that satisfies the inequality

$$\sum_{m=1}^M u_{m,NN}^2 |\psi_m[s_N] - p_{m,N}|^2 < r_N^2. \quad (7)$$

The set of all candidate labels that satisfy (7) is denoted by $\mathcal{S}_N$. With $M = 1$, techniques exist to enumerate candidates that satisfy (7) [17]. These techniques lead to *fast* enumeration, one of the critical benefits of sphere decoding. Unfortunately, these techniques do not extend feasibly to cases where $M > 1$. One solution is to directly compute (7) for all $|\mathcal{C}|$ possible labels and sort these in ascending order to produce a list of viable candidates for s_N . However, this is an exhaustive solution and renders sphere decoding less useful. The issue of quickly enumerating these labels is discussed shortly.

Given a method for fast candidate enumeration, the sphere decoding algorithm is easily applied. With the set $\mathcal{S}_N$, a candidate label $\hat{s}_N$ is chosen and removed from $\mathcal{S}_N$, and we proceed in finding a candidate for s_{N-1} . The process described above is repeated, where the $n = N - 1$ term of the summation in (6) is isolated, and $\hat{s}$ is substituted into the $n = N$ term. A candidate $\hat{s}_{N-1}$ is chosen and removed from $\mathcal{S}_{N-1}$, and we proceed to find $\mathcal{S}_{N-2}$ and $\hat{s}_{N-2}$, etc. In constructing the candidate set $\mathcal{S}_n$, the inequality of interest becomes

$$\sum_{m=1}^M u_{m,nn}^2 |\psi_m[s_n] - a_m|^2 < r_n^2 \quad (8)$$

with

$$\begin{aligned} a_m &= p_{m,n} - \sum_{k=n+1}^N \frac{u_{m,nk}}{u_{m,nn}} (\psi_m[\hat{s}_k] - p_{m,k}), \\ b_m &= \sum_{k=n+1}^N u_{m,kk}^2 \left| \psi_m[\hat{s}_k] - p_{m,k} + \right. \\ &\quad \left. \times \sum_{t=k+1}^N \frac{u_{m,kt}}{u_{m,kk}} (\psi_m[\hat{s}_t] - p_{m,t}) \right|^2 \end{aligned}$$

and radius

$$r_n^2 = r_N^2 - \sum_{m=1}^M b_m.$$

The variable a_m is the center of the region defined by (8), and b_m adjusts the radius of this region by accounting for the $n+1$ through N terms of the summation in (6).

When the decoder finishes with the $n = 1$ stage, we have a complete estimate $\hat{\mathbf{s}}$. This estimate is not necessarily the true ML estimate, as other points may still lie within the hypersphere. The radius r_N is updated to the distance between $\hat{\mathbf{s}}$ and $\mathbf{p}$, which is computed using (6). The decoding process starts over using this updated radius to determine if a better estimate exists.

When the set $\mathcal{S}_n$ is empty, then at least one of the previously chosen candidates $\hat{s}_{n+1}, \dots, \hat{s}_N$ is incorrect and should be changed. The decoder moves sequentially through the existing candidate sets starting from $\mathcal{S}_{n+1}$ to $\mathcal{S}_N$ until a nonempty set $\mathcal{S}_c$ is encountered. It then chooses and removes a new candidate $\hat{s}_c$ from $\mathcal{S}_c$ and continues the decoding process with the creation of a new $\mathcal{S}_{c-1}$ and $\hat{s}_{c-1}$, etc. When the sets $\mathcal{S}_{n+1}, \dots, \mathcal{S}_N$ are all empty, there are two possible actions. If estimates for $\hat{\mathbf{s}}$ already exist, then the most recent estimate is the true ML estimate. Otherwise, the initial radius r_N is too small and is increased; the decoding process starts over with this new radius.

V. CANDIDATE ENUMERATION

We propose an enumeration method that incorporates concepts applied in many computational vision problems that require pattern matching using eigendecompositions. These problems frequently require closest point searches to find the point q , in a fixed set of points $\mathcal{Q}$ embedded in a d -dimensional space, that is closest in Euclidean distance to a query point x . To accomplish this search efficiently, an algorithm was introduced where the points of $\mathcal{Q}$ are sorted along each dimension, creating d sorted lists [28]. Given a point x , a hypercube $\mathcal{D}$ is defined with center x , and points $q \in \mathcal{Q}$ within the hypercube are quickly identified from the sorted lists.

The region specified by (8) can be rewritten, separating the real and imaginary components, as

$$\sum_{m=1}^M \left(\frac{\Re\{\psi_m[s_n]\} - \Re\{a_m\}}{r_n/u_{m,nn}} \right)^2 + \left(\frac{\Im\{\psi_m[s_n]\} - \Im\{a_m\}}{r_n/u_{m,nn}} \right)^2 < 1.$$

This form follows that of a $2M$ -dimensional hyperellipsoid region $\mathcal{E}_n$ with axes of length $r_n/u_{m,nn}$ along the m th real and imaginary components and center

$$c_n = (\Re\{a_1\}, \dots, \Re\{a_M\}, \Im\{a_1\}, \dots, \Im\{a_M\}).$$

To find the candidate values of s_n inside $\mathcal{E}_n$, an orthotope (or hyperbox) region $\mathcal{D}_n$ is defined that has center c_n and edge length $2r_n/u_{m,nn}$ along the m th real and imaginary components. Since $\mathcal{D}_n$ tightly bounds $\mathcal{E}_n$, all points in $\mathcal{E}_n$ will also be in $\mathcal{D}_n$. Assuming that all symbols $\psi_m[s_n] \in \mathcal{C}$ are sorted along the M real and M imaginary dimensions, a set of labels $\mathcal{S}'_n$ inside $\mathcal{D}_n$ is easily created. The set $\mathcal{S}_n$ of candidate labels that fall in $\mathcal{E}_n$ are discovered by evaluating (7) for all labels in $\mathcal{S}'_n$. The details of using sorted lists to create $\mathcal{S}'_n$ are found in the work of Nene and Nayar, who indicate that the computational complexity of this search algorithm is generally independent of the problem dimensionality [28]. We apply and review the basic principles of their search algorithm to enumerate labels in the orthotope $\mathcal{D}_n$.

A. Sorting and Searching

The indexes of the labels sorted along the m th real and imaginary dimensions are indicated by the sets $\mathcal{J}_{R,m}$ and $\mathcal{J}_{I,m}$, respectively, so that $\mathcal{J}_{R,m}[1]$ is the label with the smallest real component in mapping ψ_m and label $\mathcal{J}_{R,m}[|C|]$ has the largest real component. Thus, the sorted list $\mathcal{L}_{R,m}$ is organized so that $\mathcal{L}_{R,m}[t] = \Re\{\psi_m[\mathcal{J}_{R,m}[t]]\}$. Conversely, the sets $\mathcal{J}_{R,m}^{-1}$ and $\mathcal{J}_{I,m}^{-1}$ contain the indexes that are the reverse of $\mathcal{J}_{R,m}$ and $\mathcal{J}_{I,m}$, where $\mathcal{J}_{R,m}^{-1}[t]$ is the location of label t in the list $\mathcal{L}_{R,m}$, so that $\mathcal{J}_{R,m}[\mathcal{J}_{R,m}^{-1}[t]] = \mathcal{J}_{R,m}^{-1}[\mathcal{J}_{R,m}[t]] = t$. We note that these sorted lists and sets can be created off-line and stored in memory at the receiver.

Using $\Re\{a_m\}$, $\Im\{a_m\}$ and the axes $x_m = r_n/u_{m,nn}$, the labels that fall in the intervals $[\Re\{a_m\} - x_m, \Re\{a_m\} + x_m]$ and $[\Im\{a_m\} - x_m, \Im\{a_m\} + x_m]$ are quickly identified. More specifically, a search of $\mathcal{L}_{R,m}$ is employed to find the index $\text{lft}_{R,m}$ of the smallest element in $\mathcal{L}_{R,m}$ greater than $\Re\{a_m\} - x_m$. Similarly, the index $\text{rgt}_{R,m}$ indicates the largest element in $\mathcal{L}_{R,m}$ less than $\Re\{a_m\} + x_m$. This search is closely related to the well-known binary search, with worst case complexity $\mathcal{O}(\log_2 |C|)$ [29]. Moreover, finding these boundary indexes for all $2M$ dimensions implies a worst case complexity $\mathcal{O}(M \log_2 |C|)$. From the dimension d that has the minimum $\text{rgt}_{R,m} - \text{lft}_{R,m}$ (or $\text{rgt}_{I,m} - \text{lft}_{I,m}$), the set of labels is initialized to

$$\mathcal{S}'_n = \{\mathcal{J}_{R,d}[\text{lft}_{R,d}], \mathcal{J}_{R,d}[\text{lft}_{R,d} + 1], \dots, \mathcal{J}_{R,d}[\text{rgt}_{R,d}]\}.$$

The minimizing dimension d is used to initialize $\mathcal{S}'_n$ since it contains the smallest number of labels. Cases where no labels exist in $\mathcal{D}_n$ are easily noticed by cases such as $\text{lft}_{R,m} > \text{rgt}_{R,m}$, $\text{lft}_{I,m} > \text{rgt}_{I,m}$, or when $\text{lft}_{R,m}, \text{rgt}_{R,m}, \text{lft}_{I,m}$, and/or $\text{rgt}_{I,m}$ do not exist because the real or imaginary interval does not intersect the space inhabited by $\mathcal{C}$. Each label $t \in \mathcal{S}'_n$ is then checked among the remaining real and imaginary dimensions by simple integer comparisons. The label t is eliminated from $\mathcal{S}'_n$ if $\mathcal{J}_{R,m}^{(-1)}[t] < \text{lft}_{R,m}$, $\mathcal{J}_{R,m}^{(-1)}[t] > \text{rgt}_{R,m}$, $\mathcal{J}_{I,m}^{(-1)}[t] < \text{lft}_{I,m}$, and/or $\mathcal{J}_{I,m}^{(-1)}[t] > \text{rgt}_{I,m}$. This elimination stage has worst case complexity $\mathcal{O}(Mq)$, with q as the number of labels initially in $\mathcal{S}'_n$. Hence, the worst case complexity for creating $\mathcal{S}'_n$ is

$$\mathcal{O}(M(q + \log_2 |C|)). \quad (9)$$

The creation of $\mathcal{S}_n$ involves evaluating (8) over all elements in $\mathcal{S}'_n$, leading to a final enumeration complexity of

$$\mathcal{O}(M(q + \log_2 |C|) + |\mathcal{S}'_n|).$$

Since $|\mathcal{S}'_n| = q$, the enumeration complexity simplifies back to (9). Clearly, this is linear in the number of transmissions (M), but the nature of q is not so easily apparent.

B. Other Complexity Issues

To fully analyze the complexity, a key geometrical issue arises regarding the intersection of $\mathcal{D}_n$ and the mappings of all $|C|$ possible labels. This obviously depends upon the mappings $\psi_1, \dots, \psi_M$, as they may produce a sparser distribution the labels among the $2M$ dimensions. This desire of a sparser distribution is related to the objective of reducing BER, and

is discussed in the following section. As M increases, proper mappings should result in a more sparser distribution of the labels. The intersection is also dependent upon the center and eccentricity of the region $\mathcal{E}_n$; both are highly dependent upon the channel and thus difficult to directly analyze. However, as M increases, the likelihood of one or more large values among the $u_{m,nn}$, $\Re\{a_m\}$, and $\Im\{a_m\}$ variables increases. A large $u_{m,nn}$ results in a high eccentricity, while a large $\Re\{a_m\}$ or $\Im\{a_m\}$ (in terms of magnitude) offsets the center of $\mathcal{D}_n$. Either one of these scenarios will shrink the intersection space and produce a smaller q . We thus hypothesize that as M increases, the intersection space will contain fewer labels, as will $\mathcal{S}'_n$.

A second geometrical issue of interest is the volumetric relationship between $\mathcal{D}_n$ and $\mathcal{E}_n$. Comparing the volume $V_{\mathcal{E}}$ of a hyperellipsoid $\mathcal{E}_n$ with the volume $V_{\mathcal{D}}$ of orthotope $\mathcal{D}_n$ that bounds it provides insight into the number of labels in $\mathcal{S}'_n$ that are not in $\mathcal{S}_n$. The ratio R_V of these volumes is

$$\begin{aligned} R_V = \frac{V_{\mathcal{E}}}{V_{\mathcal{D}}} &= \frac{\frac{\pi^M}{\Gamma(M+1)} \prod_{m=1}^M \left(\frac{r_n}{u_{m,nn}} \right)^2}{\prod_{m=1}^M \left(\frac{2r_n}{u_{m,nn}} \right)^2} \\ &= \left(\frac{\pi}{4} \right)^M \frac{1}{\Gamma(M+1)} \\ &= \left(\frac{\pi}{4} \right)^M \frac{1}{M!}. \end{aligned}$$

The function $\Gamma(M+1)$ is the gamma function, and when M is an integer, $\Gamma(M+1) = M!$. For $M = 1$, this ratio implies that approximately 78.5% of the labels in $\mathcal{D}_n$ are also in $\mathcal{E}_n$. With this in mind, an alternative to evaluating all labels in $\mathcal{D}_n$ may be to directly assign $\mathcal{S}_n = \mathcal{S}'_n$. Any label selected from $\mathcal{D}_n$ that is not in $\mathcal{E}_n$ will produce an r_{n-1} that is negative, so $\mathcal{S}_{n-1}$ is empty, and the decoder will backtrack to select another label from $\mathcal{S}_n$. This alternative is only practical for small M ; for $M = 2, 3$, and 4 , the respective ratios are 30.8%, 8.1%, and 1.6%. As M increases, R_V approaches zero, the unavoidable consequence of $V_{\mathcal{E}}$ approaching zero.

VI. APPLYING MAPPING DIVERSITY

We now discuss the selection of effective mapping functions $\psi_1, \dots, \psi_M$. Previously we discussed systems where the transmitter and receiver each had a single antenna. Thus, every transmitted label was considered individually, without any interference from other labels within the packet. With multiple antennas, clearly this is not the case, as all N labels are simultaneously transmitted and will interfere with each other.

However, as illustrated in (8), there are still advantages to adaptive mapping. In the initial mapping $\psi_1[s_n]$, consider any pair of labels that are mapped to a pair of closely spaced symbols in $\mathcal{C}$. Additional mappings $\psi_2, \dots, \psi_M$ can increase the Euclidean distance between this pair of labels by mapping them to symbols in $\mathcal{C}$ much further apart. This obviously reduces the probability of label misdetection during the decoding process. In some sense, the sphere decoding process allows us to consider each label as though it were transmitted independently of the other $N-1$ labels. Given this observation, the variables $u_{1,nn}, \dots, u_{M,nn}$ effectively act as fading gains on M transmissions of the label s_n . Hence, the mappings

previously devised for flat-fading channels are highly appropriate for the MIMO version of mapping diversity. Also, as previously discussed, mapping diversity may also distribute the points $\psi_1[s_n], \dots, \psi_M[s_n]$ more evenly (sparsely), so that fewer candidates will fall inside $\mathcal{E}_n$ (and $\mathcal{D}_n$), thereby reducing the computational complexity of the sphere decoder.

VII. GENERALIZATIONS

A. Hybrid Retransmissions

Thus far in this paper, only full retransmissions of packets have been discussed. In other words, when a block (of a packet) of N labels is originally transmitted, the entire block is always retransmitted. For bandwidth efficiency, it may be advantageous to not retransmit all N labels, but instead to puncture or retransmit a subset of these labels. A simple puncturing technique is to use only a subset of the transmit antennas. However, the full capacity of the MIMO channel is not utilized by this technique. An alternative is using the remaining antennas to transmit new labels. The m th transmitted block would then consist of N_m of the N initially transmitted labels, while $N - N_m$ new labels are yet to be transmitted later. The modified sphere decoder can accommodate hybrid retransmissions with some additional indexing. Rather than considering only the N original labels, one would consider $N' \geq N$ labels over the M transmissions.

There are potential advantages to these hybrid retransmissions. First, since new labels are included in the retransmissions, the overall throughput is increased. Second, the new labels are interfered by the N_m retransmitted labels, but additional information about these retransmitted labels exists from previous transmissions. Ergo, detection of the new labels is likely to improve in comparison to transmitting them separate of previously sent labels.

B. Precoding in Static Channels

In discussing multiple transmissions for MIMO systems, the effect of the channels $\mathbf{H}_1, \dots, \mathbf{H}_M$ has been ignored. Intuitively, more variation among these channels results in a stronger diversity effect, improving BER performance. Yet in many cases, the channel variation between retransmissions may be negligible. In these instances, some type of linear precoding can be significantly beneficial.

When $\mathbf{H}_1 = \mathbf{H}_2 = \dots = \mathbf{H}_M$, we propose distinct, linear precoding of each retransmission using an $N \times N$ matrix $\mathbf{Q}_m$. Fig. 3 illustrates M transmissions through a static channel with linear precoding. This precoding is easily incorporated into the receiver by replacing $\mathbf{H}_m$ with $\tilde{\mathbf{H}}_m = \mathbf{H}_1 \mathbf{Q}_m$ for $m \geq 2$. The objective in selecting $\mathbf{Q}_M$ is to modify $\tilde{\mathbf{H}}_M$ so that it appears independent of $\mathbf{H}_1, \tilde{\mathbf{H}}_2, \dots, \tilde{\mathbf{H}}_{M-1}$. A trace constraint $\text{tr}\{\mathbf{Q}_m \mathbf{Q}_m^H\} = N$ is necessary to maintain the transmitted signal power. Also, we seek generic precoders that are not specific to any particular channel $\mathbf{H}_1$, removing the requirement that the transmitter have some knowledge of $\mathbf{H}_1$. Finally, the selection of $\mathbf{Q}_M$ must only consider the previous precoders $\mathbf{Q}_2, \dots, \mathbf{Q}_{M-1}$, since future transmissions are neither guaranteed nor desired.

Onggosanusi *et al.* studied these types of precoders in their work on combining packet transmissions using zero-forcing and

Fig. 3. Block diagram of multiple precoded MIMO transmissions of a packet with mapping diversity in static channels.

MMSE receivers [19]. They described their approach as *basis hopping*, with the purpose of precoding being the avoidance of correlated interference between transmissions. They suggest unitary precoders that are parameterized, as is the case with Givens rotations. Another precoding design was first proposed by Giraud *et al.* and later studied by Xin *et al.*, effectively employing a Vandermonde matrix as the precoder in both single- and multiple-antenna systems [30]–[32].

Several simpler precoders are available to enhance the channel diversity. The simplest precoder is a permutation matrix, which simply shuffles the label-transmit antenna assignments for each transmission. A second option is a fast Fourier transform (FFT) [or inverse FFT (IFFT)] matrix, which spreads the symbol energy evenly among the N transmit antennas so that the effects of any deep fades (i.e., small values in $\mathbf{H}_m$) are alleviated. We combine these two options and suggest constructing $\mathbf{Q}_m$ by uniquely permuting the rows and columns of an FFT matrix. This corresponds to permuting the vector $\vec{\psi}$, multiplication with an FFT matrix, and again permuting the multiplied result. These permutation and FFT matrix multiplication operations are of low complexity.

C. Using Retransmission Diversity as Space–Time Coding

By combining MIMO retransmissions at the receiver, the spatial diversity of MIMO systems is combined with the temporal diversity of retransmissions. As previously mentioned, space–time coding similarly seeks to jointly exploit spatial and temporal diversity. The ARQ mapping diversity protocol proposed in this paper is akin to a space–time block code of rate N/M . Since the M block transmissions are made using M consecutive blocks, the MIMO channel does not vary, and precoding should be employed. The fundamental difference lies in the mapping design itself. The mappings are optimized iteratively, with each mapping ignoring the existence of future transmissions. In a space–time coding scheme, the number of blocks M is guaranteed, and the M mappings should be selected jointly using an optimization like that in (2). Unfortunately, this optimization problem is presently intractable. Fortunately, the iteratively obtained mappings are still quite effective in yielding significant coding gains. In the following section, we present some simulations illustrating the coding gains provided when the ARQ protocol is viewed as a space–time code.

Fig. 4. BER simulation results of $M = 2, 3, 4$ transmissions through a 4×4 MIMO channel using (a) 16-QAM and (b) 64-QAM, comparing mapping diversity against Gray mappings.

VIII. SIMULATION RESULTS

We now present simulation results that demonstrate the effectiveness of mapping diversity in MIMO environments. The modulations of interest are 16- and 64-QAM, in a system with $N = 4$ transmit and $K = 4$ receiver antennas. With a packet size of 800 bits (50 blocks), we compare retransmissions made using mapping diversity against those using identical Gray mappings, for $M = 2, 3, 4$ transmissions. Results are obtained via Monte Carlo simulation of 2000 packets, with SNR defined as $E\{|s_n|^2\}/\sigma_w^2$. We define three different relationships between channels $\mathbf{H}_1, \dots, \mathbf{H}_4$. Fig. 4 contains BER simulation results from retransmissions through independent channels. Fig. 5 deals with retransmissions through identical channels ($\mathbf{H}_1 = \dots = \mathbf{H}_4$). Fig. 6 contains results for precoded retransmissions through identical channels. For precoded retransmissions, the precoding strategy in Section VII-B is applied.

Fig. 5. BER simulation results of $M = 2, 3, 4$ transmissions through a static 4×4 MIMO channel using (a) 16-QAM and (b) 64-QAM, comparing mapping diversity against Gray mappings.

Clearly, mapping diversity provides significant gains in reducing BER. For $M = 2$ retransmissions, there is a 2 dB improvement for 16-QAM and more than a 4 dB gain for 64-QAM under independent retransmission channels. With identical retransmission channels, the gains are substantially larger; for $M = 2$, there is a 4 dB improvement for 16-QAM and about a 7 dB improvement for 64 QAM. A useful observation from comparing Figs. 4 and 5 is the inherent gain achieved from the channel variation between retransmissions, regardless of the mapping strategy employed. The suggested precoding strategy produces results in Fig. 6 that closely resemble those in Fig. 4. Thus, it is as expected that retransmission precoding transforms identical channels into nearly independent channels.

We repeated the experiment as in Fig. 4 but considered spatial correlation amongst the MIMO channel elements for another realistic scenario. In other words, the transmission channel is $\tilde{\mathbf{H}}_m = \mathbf{R}_{RX}^{-1/2} \mathbf{H}_m \mathbf{R}_{TX}^{-1/2}$, with $\mathbf{H}_m$ being the MIMO channel defined in Section III. The $\mathbf{R}_{RX}$ and $\mathbf{R}_{TX}$ matrices represent the correlation matrices at the receiver and transmitter antenna

(a)

(b)

Fig. 6. BER simulation results of $M = 2, 3, 4$ transmissions, with retransmission precoding, through a static 4×4 MIMO channel using (a) 16-QAM and (b) 64-QAM, comparing mapping diversity against Gray mappings.

arrays, respectively. We apply the following measured 4×4 matrices from the METRA project (environment FB7B2) [33]:

$$\mathbf{R}_{TX} = \begin{bmatrix} 1 & 0.3169 & 0.3863 & 0.0838 \\ 0.3169 & 1 & 0.7128 & 0.5626 \\ 0.3863 & 0.7128 & 1 & 0.5354 \\ 0.0838 & 0.5626 & 0.5354 & 1 \end{bmatrix}$$

$$\mathbf{R}_{RX} = \begin{bmatrix} 1 & 0.1317 & 0.1992 & 0.2315 \\ 0.1317 & 1 & 0.1493 & 0.1907 \\ 0.1992 & 0.1493 & 1 & 0.1996 \\ 0.2315 & 0.1907 & 0.1996 & 1 \end{bmatrix}.$$

The gains, as shown in Fig. 7, are very similar to those found in Fig. 4. This indicates that the benefits of mapping diversity are broad and are not significantly affected by the spatial correlations among antenna elements.

As a virtual space-time code/diversity, we compare the mapping diversity results in Fig. 6 to the Gray mapping results in Fig. 5. This corresponds to the comparison of a space-time code

Fig. 7. BER simulation results of $M = 2, 3, 4$ transmissions through a 4×4 correlated MIMO channel using 16-QAM, comparing mapping diversity against Gray mappings.

Fig. 8. Median number of candidates evaluated for sphere decoding of $M = 2, 3, 4$ transmissions through a 4×4 MIMO channel using 64-QAM constellation, comparing mapping diversity against Gray mapped retransmissions.

that utilizes mapping diversity and precoding against a simple repetition code. With $M = 4$, we have a full-rate space-time code. The coding gains, with respect to the repetition code, are approximately 8 dB for 16-QAM and 12 dB for 64-QAM.

Figs. 8 and 9 contain some computational complexity results pertaining to the 64-QAM simulations for independent retransmission channels. Specifically, the result of interest is the median number of candidates evaluated in the candidate enumeration stage of sphere decoding. For any value of M , BER measurement is rejected whenever fewer than 200 packets require an M th transmission. Fig. 8 compares the numbers of evaluated candidates for decoding mapping diversity retransmissions against those for decoding identical Gray mapped retransmissions. The primary observation here is the ability of mapping diversity to reduce the number of candidates within

Fig. 9. Median number of candidates evaluated for sphere decoding of $M = 2, 3, 4$ transmissions through a 4×4 MIMO channel with 64-QAM constellation and mapping diversity, comparing exhaustive enumeration against the proposed enumeration.

any given hyperellipsoid. Fig. 9 considers only mapping diversity retransmissions and compares an exhaustive enumeration approach against the proposed enumeration method using a bounding hyperbox. This figure validates the usefulness of a bounding hyperbox $\mathcal{D}_n$ in eliminating the majority of potential candidates, thereby greatly lowering the receiver complexity.

IX. CONCLUSION

We propose a new ARQ protocol for MIMO flat-fading channels that incorporates mapping diversity and an integrated receiver. By combining distinctly mapped transmissions through MIMO channels, we can achieve significant performance gains. We develop an integrated receiver with the application of sphere decoding that requires an important modification for the enumeration of candidates within the sphere. Borrowing concepts from an existing closest point search technique used for pattern matching applications, we propose a fast enumeration method. Additionally, we note that this retransmission protocol serves as a flexible and effective space-time coding technique.

REFERENCES

- [1] S. B. Wicker, *Error Control Systems for Digital Communications and Storage*. Englewood Cliffs, NJ: Prentice-Hall, 1994.
- [2] D. Chase, "Code combining—A maximum-likelihood decoding approach for combining an arbitrary number of noisy packets," *IEEE Trans. Commun.*, vol. COM-33, pp. 385–393, May 1985.
- [3] B. Harvey and S. Wicker, "Packet combining systems based on the Viterbi decoder," *IEEE Trans. Commun.*, vol. 42, pp. 1544–1557, Feb./Mar./Apr. 1994.
- [4] J. Hagenauer, "Rate-compatible punctured convolutional codes (RCPC codes) and their applications," *IEEE Trans. Commun.*, vol. 36, pp. 389–400, Apr. 1988.
- [5] D. Rowitch and L. Milstein, "On the performance of hybrid FEC/ARQ systems using rate compatible punctured turbo (RCPT) codes," *IEEE Trans. Commun.*, vol. 48, pp. 948–959, Jun. 2000.
- [6] K. Narayanan and G. Stuber, "A novel ARQ technique using the turbo coding principle," *IEEE Commun. Lett.*, vol. 1, pp. 49–51, Mar. 1997.
- [7] Q. Zhang and S. A. Kassam, "Hybrid ARQ with selective combining for fading channels," *IEEE J. Sel. Areas Commun.*, vol. 17, pp. 867–880, May 1999.
- [8] H. Samra, Z. Ding, and P. M. Hahn, "Optimal symbol mapping diversity for multiple packet transmissions," in *Proc. IEEE ICASSP'03*, vol. 4, 2003, pp. 181–184.
- [9] H. Samra and Z. Ding, "Symbol mapping diversity design for packet retransmissions through fading channels," in *Global Telecommunications Conf.*, San Francisco, CA, 2003.
- [10] —, "Precoded integrated equalization for packet retransmissions," in *Conf. Rec. 36th Asilomar Conf. Signals, Systems, Computers*, vol. 2, 2002, pp. 1938–1942.
- [11] D. N. Doan and K. R. Narayanan, "Iterative packet combining schemes for intersymbol interference channels," *IEEE Trans. Commun.*, vol. 50, no. 4, pp. 560–570, Apr. 2002.
- [12] G. J. Foschini and M. J. Gans, "On limits of wireless communication in a fading environment when using multiple antennas," *Wireless Personal Commun.*, vol. 6, no. 3, pp. 311–315, Mar. 1998.
- [13] E. Telatar, "Capacity of multi-antenna Gaussian channels," *Eur. Trans. Telecom.*, vol. 10, no. 6, pp. 585–595, Dec. 1999.
- [14] S. M. Alamouti, "A simple transmit diversity technique for wireless communications," *IEEE J. Sel. Areas Commun.*, vol. 16, pp. 1451–1458, Oct. 1998.
- [15] G. J. Foschini, "Layered space-time architecture for wireless communication in a fading environment when using multi-element antennas," *Bell Labs Tech. J.*, vol. 1, no. 2, pp. 41–59, 1996.
- [16] V. Tarokh, N. Seshadri, and A. R. Calderbank, "Space-time codes for high data rate wireless communications: Performance criterion and code construction," *IEEE Trans. Inf. Theory*, vol. 44, pp. 744–765, Mar. 1998.
- [17] B. M. Hochwald and S. ten Brink, "Achieving near-capacity on a multiple-antenna channel," *IEEE Trans. Commun.*, vol. 51, pp. 389–399, Mar. 2003.
- [18] V. Tarokh, H. Jafarkhani, and A. R. Calderbank, "Space-time block codes from orthogonal designs," *IEEE Trans. Inf. Theory*, vol. 45, pp. 1456–1467, Jul. 1999.
- [19] E. N. Onggosanusi, A. G. Dabak, Y. Hui, and G. Jeong, "Hybrid ARQ transmission and combining for MIMO systems," in *Proc. IEEE Int. Conf. Commun.*, vol. 5, May 2003, pp. 3205–3209.
- [20] Z. Ding and M. Rice, "Type-I hybrid-ARQ using MTCM spatio-temporal vector coding for MIMO systems," in *Proc. IEEE Int. Conf. Commun.*, vol. 4, May 2003, pp. 2758–2762.
- [21] A. V. Nguyen and M. A. Ingram, "Hybrid ARQ protocols using space-time codes," in *Proc. 54th IEEE Veh. Tech. Conf.*, vol. 4, Oct. 2001, pp. 2364–2368.
- [22] H. Zheng, A. Lozano, and M. Haleem, "Multiple ARQ processes for MIMO systems," in *Proc. 13th IEEE Int. Personal, Indoor, Mobile Radio Commun.*, vol. 3, Sept. 2002, pp. 1023–1026.
- [23] R. D. Wesel, X. Liu, J. M. Cioffi, and C. Kominakis, "Constellation labeling for linear encoders," *IEEE Trans. Inf. Theory*, vol. 47, pp. 2417–2431, Sep. 2001.
- [24] G. H. Golub and C. F. Van Loan, *Matrix Computations*, 3rd ed. Baltimore, MD: The John Hopkins Univ. Press, 1996.
- [25] L. N. Trefethen and D. Bau III, *Numerical Linear Algebra*. Philadelphia, PA: Society for Industrial and Applied Mathematics, 1997.
- [26] B. Hassibi and H. Vikalo, "On the expected complexity of integer least-squares problems," in *Proc. IEEE ICASSP*, vol. 2, May 2002, pp. 1497–1500.
- [27] E. Agrell, T. Eriksson, A. Vardy, and K. Zeger, "Closest point search in lattices," *IEEE Trans. Inf. Theory*, vol. 48, pp. 2201–2214, Aug. 2002.
- [28] S. A. Nene and S. K. Nayar, "A simple algorithm for nearest neighbor search in high dimensions," *IEEE Trans. Pattern Anal. Mach. Intell.*, vol. 19, pp. 989–1003, Sep. 1997.
- [29] T. H. Cormen, C. E. Leiserson, and R. L. Rivest, *Introduction to Algorithms*. New York: McGraw-Hill, 1990.
- [30] X. Giraud, E. Boutillon, and J. C. Belfiore, "Algebraic tools to build modulation schemes for fading channels," *IEEE Trans. Inf. Theory*, vol. 43, pp. 938–952, May 1997.
- [31] Y. Xin, Z. Wang, and G. B. Giannakis, "Space-time constellation-rotating codes maximizing diversity and coding gains," in *Proc. GLOBECOM'01*, vol. 1, Nov. 2001, pp. 455–459.
- [32] —, "Space-time diversity systems based on linear constellation precoding," *IEEE Trans. Wireless Commun.*, vol. 2, pp. 294–309, Mar. 2003.
- [33] L. Schumacher, J. P. Kermaol, F. Frederiksen, K. I. Pedersen, A. Algans, and P. E. Mogensen. (2001, Feb.) MIMO channel characterization. [Online]. Available: <http://www.ist-imetra.org/metra/deliverables/AAU-WP2-D2-V1.1.pdf>

Harvind Samra (M'92) received the B.S. degree in electrical engineering and in computer engineering from the University of Kansas, Lawrence, the M.S. degree in electrical engineering from Georgia Institute of Technology, Atlanta, and the Ph.D. degree in electrical engineering from the Department of Electrical and Computer Engineering, University of California, Davis, in 2004.

From 1998 to 2000, he was a Research Engineer with Statistical Signal Processing, Inc., developing software and algorithms toward various communications and signal intelligence applications. From 2000 to 2003, he was a Research Assistant at the University of California, Davis, developing signal-processing algorithms to exploit packet retransmissions. From 2004 to 2005, he was a Senior Engineer with the Rosum Corporation, responsible for system and software design toward the development of location and estimation technologies that use television signals. He is currently a partner in Kestrel Signal Processing, providing signal processing research and development services. His interests and experience generally involve communications and signal processing, ranging from algorithm and system development to software and hardware design.

Zhi Ding (F'03) received the B.Eng. from Nanjing Institute of Technology, China, the M.A.Sc. degree from the University of Toronto, Toronto, ON, Canada, and the Ph.D. degree from the School of Electrical Engineering, Cornell University, Ithaca, NY, in 1990.

He has been a Professor of Electrical and Computer Engineering, University of California, Davis, since 2005. From 1990 to 1998, he was a Faculty Member in the Department of Electrical Engineering, Auburn University, Auburn, AL, first as an Assistant Professor and later as an Associate Professor. From 1999 to 2000, he was with the University of Iowa. He has held visiting positions with Australian National University, Hong Kong University of Science and Technology, the NASA Lewis Research Center, and the USAF Wright Laboratory. His research interests include wireless communications, signal processing, and cross-layer optimization.

Dr. Ding served as an Associate Editor of the IEEE TRANSACTIONS ON SIGNAL PROCESSING from 1994 to 1997 and from 2001 to 2004.