Title

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ABSTRACT

This research seeks to provide an understanding of the capacity and throughput requirements of an automated highway system(AHS) via development of a computer model that represents system capacity and demand, operational characteristics, and defined infrastructure. To do that, a variety of scenarios representing traffic input to the system must be represented. Also, a variety of infrastructures representing different system configurations must be represented. This includes the locations of entrances and exits.

Detailed vehicle-based simulations have been used to investigate AHS capacity and throughput requirements in past studies(see [5]). In these studies, flows of 6000-8000 vehicles/lane/hour were achieved using the platoon organization of vehicles. However, disturbances to traffic flow due to lane changes were noted and also not all vehicles were able to reach their respective exits. This computer model addresses the issues of requiring all vehicles to reach their destinations and of achieving high flow rates while not allowing serious purturbations to existing traffic. However, instead of simulation, this model involves an analytic, flow-based optimization approach that involves solving a linear program(LP).

Keywords: Automated Highway Systems, simulations, platoons, vehicle-based, flow-based, optimization, linear programming.

EXECUTIVE SUMMARY

AHS capacity prediction via microscopic simulation of vehicle movement for particular AHS operating scenarios has been progressing for several years. Some macroscopic simulation of aggregate traffic flow for smoothing traffic via local regulation of lane change activities started more recently. Based on the workload concept, models for static AHS traffic assignment and optimization have been developed(see [2]).

Dynamic traffic assignment through analytical modeling and optimization has been widely accepted by the ITS R&D community as a promising traffic control tool for relieving traffic congestion. Examples of dynamic urban transportation network models can be found in [3]. Due to the completely controlled nature of AHS traffic, models for dynamic assignment of AHS traffic are even more applicable. [1] describes an AHS dynamic traffic assignment model.

The AHSCAP program described in this document implements the approach to AHS dynamic traffic assignment that is described in [1]. The AHSCAP program is summarized below.

The AHSCAP program determines an optimal flow with lane assignments determined such that no disturbance in longitudinal flow results from lane changes and such that all vehicles reach their destinations. In this case 'optimal' means the maximization of throughput through an AHS segment. This also means that all vehicles in an AHS lane keep the same velocity. It should be noted that the solution may not be realistic, i.e. too optimistic, if the model parameters allow for an excessive number of lane changes for the given traffic conditions. In this case 'excessive' means that longitudinal flow would have been impeded in a realistic situation.

Contents

1 Introduction

AHSCAP is a computer program used for performing dynamic traffic assignment for a hypothetical Automated Highway System segment using analytical modeling and optimization. AHSCAP is a macroscopic model. The purpose of AHSCAP is to maximize the throughput of traffic in an AHS segment for a user-specified traffic scenario and infrastructure. It uses a linear program to optimize the traffic flow where the objective function represents the throughput and the constraints represent the system demand(i.e. flow from upstream and entrances) and system capacity. The capacity constraints for vehicle movement are defined for entrances, exits, sections and lanes. They are also defined for vehicle lane changes as a function of vehicle density in the destination lane.

The optimal solution results in a traffic flow state for all time intervals that maximizes the throughput and guarantees that all vehicles reach their respective exits. Traffic flow in each section and lane is partitioned into lateral and longitudinal flows for each time period. The lateral flow, which represents vehicles that are scheduled to change lanes in a time period, is constrained so that flow in the destination lane is not impeded. Longitudinal flow represents vehicles that stay in their lanes for a given time period. Reference [1] contains a detailed description of the theory behind this approach to AHS dynamic traffic assignment.

There are several assumptions and limitations that are implied by the model. These are summarized below.

- All traffic is assumed to be automated, i.e. optimization is global.
- AHSCAP supports one vehicle type.
- The roadway consists of two automated lanes.
- Vehicles enter the automated traffic directly from on-ramps or upstream and leave the automated traffic directly into off-ramps, i.e., there is no transition lane.
- The distance a vehicle travels in one time period must be less than the length of any section.

- There can be at most one off-ramp in a section and it can be on either side of the highway. The off-ramp is considered to be at the end of a section.
- There can be an on-ramp on either side of a section. The on-ramp is considered to be at the beginning of a section.
- Only one direction for lane changes is allowed in a section.
- All lateral and longitudinal flow is assumed to be uniformly distributed in all sections and lanes.
- The defined speed for each lane is constant for all sections at all times.

In the following sections, the AHSCAP user interface is followed by the design description. Sections **2-4** describe AHSCAP compilation, execution, data definitions and data formats. Sections **5-7** describe the AHSCAP design, which consists of the linear program problem definition and solution.

2 AHSCAP Compilation and Execution

The AHSCAP program requires the name of the input file containing model data and the base name for two output files. The program runs in a UNIX workstation environment. If *inputf* is the input file name and *outputf* is the base name for the two output files, then the following command will produce output files *outputf.lp* and *output.txt*.

ahscap *inputf outputf*

outputf.lp and *outputf.txt* are produced. *outputf.lp* contains a restatment of the LP used as input. *outputf.txt* contains the optimal solution which includes the value of the objective function and the values of all state variables. Examples of these files are contained in Section **4.**

The function ahscap above is a UNIX shell script. It executes the AHSCAP functions described in Section 5. AHSCAP LP Generation(Section **5.1)** is in an executable called "ahsdta". If it does not exist, then it must be recompiled using the makefile for the source code. The files containing the source code are identified below.

CPLEX LP Solution Generation(Section 5.2) uses the CPLEX Linear Optimizer to generate the optimal solution. The optimizer must be accessible by the workstation. If another optimizer is available and can also read an input file in MPS format, it can be used. The ahscap script would have to be modified to call another optimizer.

The following is a brief description of the source code(.c files).

main Main Program

parse Parses input and initializes parameters

write Initializes creation of MPS file defining the LP

writerows Writes the row/constraint data in MPS format

writecols Writes the column/variable data in MPS format

writerhs Writes the RHS column data in MPS format

3 AHSCAP Input Data Description

Table 1 lists the data required as input to the model. Items with an asterisk(*) are derived values that are not set by the user. **A** description of how these items are derived from the rest of the input is described in Section 7. Notes on setting the lane change parameters are in Section 7 also. Section **4.1** shows an example input file.

a	Section number(=0 for upstream)	
1	Lane number	1-2
s	Destination section number	
t	Time interval	

System Parameter Data

Name	Description	Units
AT	Time Interval Length	Seconds
T	Number of time intervals	
Vel_l	Defined velocity for lane 1	meters/sec

Highway Parameter Data

Name	Description	Units
L	Number of lanes	2
S	Number of sections	
L_a	Length of Section a	meters
Ent_{al}	=1 if entrance exists, =0 if not	0-1
$Exit_{al}$	=1 if exit exists, =0 if not	0-1
Lch_a	Left/Right indicator for dowable LC	L-R

Demand Data

Name	Description	Units
$X_{als}(t)$ $F_{als}(t)$	Number of vehicles existing at time 1 Upstream(a=0) and entrance demand for vehicles destined for section **s** at time t	veh/t

Supply Data

Name	Description	Units
$CN_{al}(t)$	Upstream(a=0) and entrance capacity	veh/t
$CF_{al}(t)$	Exit capacity	veh/t
$CL_{al}(t)$	Per unit length capacity(density)	vehjmeter
$a1, b1$	Parameters for linear function $a1 + b1 * d$ Used to constrain lane changes from one lane to another where d is the density	
$a2, b2$	Parameters for linear function a2 $+ b2 *$ d Forms a piecewise linear function with (a1,b1).	
$a10, b10$	Parameters for linear function a10 $+ b10 *$ d Used to constrain lane changes from an entrance to a lane with density d	
$a20$, b20	Parameters for linear function a20 $+ b20 *$ d Forms a piecewise linear function with (a10,b10).	
$PF_{als}*$	Proportion of vehicles that have destination **s** with respect to **all** destinations	0-1
$P1_{al}*$	Latitudinal Progression Factor	
$P0_{al}*$	Longitudinal Progression Factor	

Table 1: AHSCAP Input Data

4 AHSCAP Data Formats

The following subsections contain examples of AHSCAP input, MPS and output file formats. The MPS format is used to represent the LP and it is read by the CPLEX linear optimizer. One of the output files contains the solution to the LP.

4.1 AHSCAP Input File Format

```
//This file contains the input data for the AHS DTA model
SYSTEM PARAMETER DATA
15 15 28 30
//Time interval length(seconds), max number of time periods,
//velocity of lane 1, velocity of lane 2
HIGHWAY INFRASTRUCTURE DATA
2 6
500 0 0 1 0 L
500 1 0 0 0 R
500 0 0 1 0 L
500 1 0 0 0 R
500 0 0 0 0 L
500 1 1 0 0 R
//Number of lanes and sections
//Section data(1st section on bottom)
//Length(meters),number of left/right entrances, left/right exits(0 or 1)
//lane change direction allowed(L=left, R=right)
DEMAND INPUT PARAMETERS
x 1 1 2 0
x 1 2 2 0
//Section, lane, destination section, number of vehicles at time period 1.
//X[a][l][s] (time 1)
F 0 2 4
0 1 0 1 0 1 0 1 0 1 0 0 0 0 0 0 0 0 0 0
```

F 0 1 4

0 1 0 1 0 1 0 1 0 1 0 0 0 0 0 0 0 0 0 0

F 1 2 4

1 0 1 0 1 0 1 0 1 0 1 0 0 0 0 0 0 0 0 0 0

F 1 1 4

1 0 1 0 1 0 1 0 1 0 1 0 0 0 0 0 0 0 0 0 0

F 1 2 6

1 0 1 0 1 0 1 0 0 0 0 0 0 0 0 0 0 0 0 0

F 1 1 6

1 0 1 0 1 0 1 0 0 0 0 0 0 0 0 0 0 0 0 0

F 3 2 6

10 10 10 10 10 10 10 10 10 0 0 0 0 0 0

F 5 2 6

10 10 10 10 10 10 10 10 10 0 0 0 0 0 0

//Upstream demand and entrance demand. Vehicles/time.

//F[a][l][s][t[

SUPPLY INPUT PARAMETERS

CN 0 1

30 **30** 30 30 30 30 30 30 30 **30** **30** 30 30 **30** 30

CN 0 2

30 30 30 30 30 **30** 30 30 **30** 30 **30** **30** 30 30 30

CN 1 1

30 **30** 30 **30** 30 30 30 **30** 30 30 30 **30** 30 30 30

CN 1 2

30 30 30 30 30 30 30 30 30 30 **30** **30** 30 30 **30**

CN 3 2

30 30 30 30 30 30 30 30 30 30 30 **30** 30 30 30

C N 5 2

30 **30** 30 30 30 **30** 30 30 30 **30** 30 **30** 30 30 30

//Upstream capacity, Entrance capacity

//CN[a][l][t]

CF 4 2

30 30 30 30 30 30 30 30 30 30 30 30 30 30 30

CF 6 2

30 30 30 30 30 30 30 30 30 30 30 30 30 30 30

//Exit capacity

//CF[a][l][t]

CL 1 1

.16 .16 .16 .16 .16 .16 .16 .16 .16 .16 .16 .16 .16 .16 .16

CL 1 2

.16 .16 .16 .16 .16 .16 .16 .16 .16 .16 .16 .16 .16 .16 .16

CL 2 1

.16 .16 .16 .16 .16 .16 .16 .16 .16 .16 .16 .16 .16 .16 .16

CL 2 2

.16 .16 .16 .16 .16 .16 .16 .16 .16 .16 .16 .16 .16 .16 .16

CL 3 1

.16 .16 .16 .16 .16 .16 .16 .16 .16 .16 .16 .16 .16 .16 .16

CL 3 2

.16 .16 .16 .16 .16 .16 .16 .16 .16 .16 .16 .16 .16 .16 .16

CL 4 1

.16 .16 .16 .16 .16 .16 .16 .16 .16 .16 .16 .16 .16 .16 .16

CL 4 2

.16 .16 .16 .16 .16 .16 .16 .16 .16 .16 .16 .16 .16 .16 .16

CL 5 1

.16 .16 .16 .16 .16 .16 .16 .16 .16 .16 .16 .16 .16 .16 .16

CL 5 2

.16 .16 .16 .16 .16 .16 .16 .16 .16 .16 .16 .16 .16 .16 .16

CL 6 1

.16 .16 .16 .16 .16 .16 .16 .16 .16 .16 .16 .16 .16 .16 .16

CL 6 2

.16 .16 .16 .16 .16 .16 .16 .16 .16 .16 .16 .16 .16 .16 .16

//Section/lane maximum density in vehicles/meter

//CL[a][l][t]

A1B1 .002 -.0125

A2B2 .002 -.0125

A10B10 2. -12.5

A20B20 2. -12.5

//Lane change parameters a1, b1, a2, b2, a10, b10, a20, b20

4.2 MPS File Format

For the simple linear program listed below, the equivalent program in MPS file format is shown.

$$\text{Maximize } x_1 + 2x_2 + 3x_3$$

subject to the following constraints:

$$-x_1 + x_2 + x_3 <= 20$$

$$x_1 - 3x_2 + x_3 <= 30$$

$$0 <= x_1 <= 40$$

The corresponding data in the column-oriented MPS format follows:

```
NAME example
ROWS
N obj
L c1
L c2
COLUMNS
 x1  obj  -1  c1  -1
 x1  c2   1
 x2  obj  -2  c1  1
 x2  c2   -3
 x3  obj  -3  c1  1
```

x3 c2 1
RHS
rhs c1 20 c2 30
BOUNDS UP bnd x1 40 ENDATA

The constraints in the above MPS format are named c1 and c2. For a complete description of the MPS format see [6].

4.3 Output File Formats

For the example problem in the previous section, the following describes the format of the .lp and .txt files. File *.lp is a restatment of the original LP and *.txt displays the optimal solution. For a complete description of these formats, see [6]. The following is an example of a .lp file.

Minimize
obj: - x1 - 2 x2 - 3 x3
Subject To
c1: - x1 + x2 + x3 <= 20
c2: x1 - 3 x2 + x3 <= 30
Bounds
0 <= x1 <= 40
End

The solution data for all variables is in a .txt file. Each variable in the LP is represented by a character stream whose components represent the variable name and values of the subscripts and superscripts. The character name for a given variable is given by $vaalssftttt$ where leading zeros are used so that all variable names have the same length. v represents the variable name, aa is the entrance section(0 for upstream), l is the lane number, ss is the destination section, f is the within/cross section indicator(within=0, cross=1, both=2), and $tttt$ is the time interval. For example, variable $w_{31}^{51}(0035)$ is represented by $w0310510035$ where a $= 3$, l $= 1$, $s = 5$, $f = 1$, and $t = 0035$. The format for file .txt is described in [6].

For a particular study it is likely that graphical data is desired instead of the raw data in a .txt file. To generate the data in graphical form a tool such as MATLAB can be used. MATLAB requires an input(.m) file containing data to be plotted. For example, the cross-sectional lateral or longitudinal flow can be plotted as a function of time for each section, lane and destination section. The sum of these flows over all time periods represent the LP objective function value.

5 AHSCAP Design Description

The following is a description of each of the AHSCAP functions. Figure 1 shows the AHSCAP Data Flow Diagram.

5.1 AHSCAP LP Generation

Table 1 in Section 3 lists the data required as input to the model. This data is used to define coefficients and RHS(right-hand-side) values for the LP. The LP is translated into MPS format, which is described in Section 4.2. The LP is defined in Section 6.

5.2 CPLEX LP Solution Generation

The CPLEX Linear Optimizer is a tool that determines the optimal solution to a linear program. It reads a LP defined in MPS format and generates an output file containing the optimal solution with values for the objective function and state variables. For this model, CPLEX is used to determine the maximum throughput subject to the demand and supply constraints. The throughput is defined in terms of lateral and longitudinal flows and the corresponding variables are represented in the objective function. The demand constraints represent the flows into the defined AHS segment from upstream and from on-ramps. The supply constraints represent the capacities for segments, lanes, on-ramps and off-ramps and traffic density in the destination lane for lane changes.

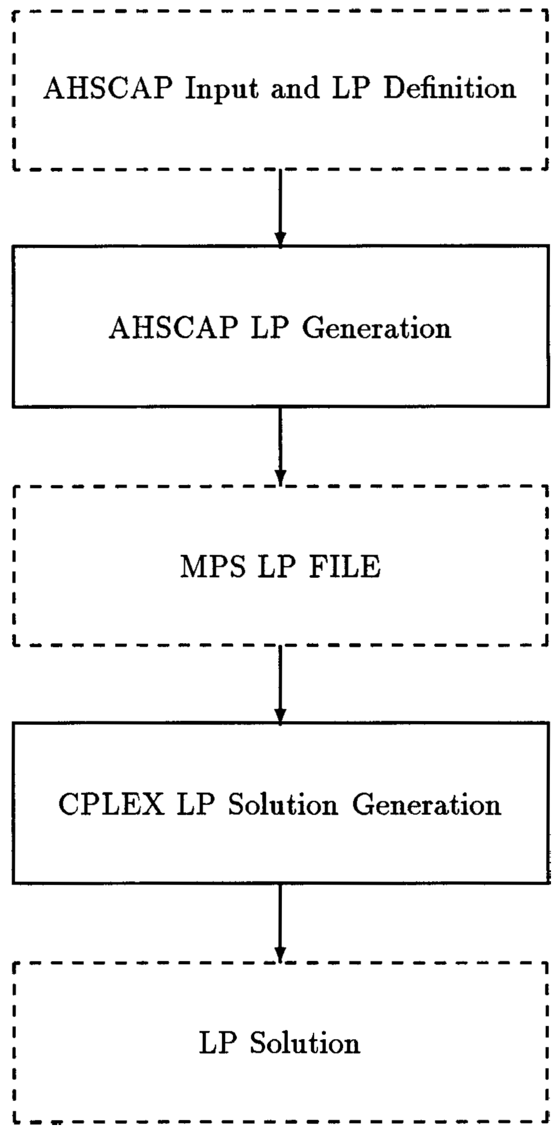

Figure **1:** AHSCAP Data Flow Diagram

6 AHSCAP LP Definition

The input data described in Section 3 is used to define the coefficients for variables and RHS values for the constraints. See Reference [1] for a more detailed description of the LP. The LP is defined below.

6.1 Decision/Control Variables

There are two types of metered inflow variables, metered inflow from on-ramps and metered inflow from upstream.

$u_{al}(t)$: When $a = 0$, this denotes the number of vehicles entering lane l of section **1** during time period t from upstream. When $a = 1, 2, ..., N$, this denotes the total number of vehicles entering lane l of section a, through metering (without disturbing the speed of the existing traffic) from the on-ramp during time period t. If lane l is not equipped with an on-ramp, then $CN_{al}(t)$ is set to 0.

$v^s_{al}(t)$: the number of vehicles that are traversing lane l of section a at the beginning of time period t with destination s and change lanes from lane l to lane $\bar{l}$ (the complement of lane l or the lane other than lane l) during time period t. ($\bar{1} = 2$ and $\bar{2} = 1$.) If lane changing from lane l to lane $\bar{l}$ is disallowed, this variable is set to 0 explicitly in the formulation.

6.2 Derived Variables

This subsection defines variables that can be derived from the decision variables. The derivation is defined in the linear programming formulation.

Metered inflow from an on-ramp destined for section s down the highway can be obtained by multiplying the total metered inflow by the corresponding time-varying OD proportion:

$u^s_{al}(t)$: When $a = 0$, this denotes the number of vehicles entering lane l of section **1** during time period t from upstream with destination s. When $a = 1, 2, ..., N$, this denotes the total number of vehicles entering lane l of section a, through metering from the on-ramp during time period t that are destined for section s.

Two types of lateral flow can be derived - lateral flow that also reaches the next section by the end of the time period and lateral flow that cannot reach the next section by the end of the time period.

$v_{al}^{s1}(t)$: the number of vehicles traversing lane I of section a at the beginning of time period t with destination s that change lanes from lane I to lane $\bar{l}$ and move from section a to section $a+1$ during time period t.

$v_{al}^{s0}(t)$: $v_{al}^{s}(t) - v_{al}^{s1}(t)$, i.e. the number of vehicles traversing lane I of section a at the beginning of time period t with destination s that change lanes from lane I to lane $\bar{l}$ during time period t but cannot reach section $a+1$ during time period t. These two types of lateral flow are referred to as cross-section lateral flow and within-section lateral flow respectively. The superscripts 1 and 0 signify cross-section and within-section flow respectively.

There are three other major types of derived variables, the outflow, longitudinal flow and existing traffic. There are two types of longitudinal flow: cross-section and within-section.

$w_{al}^{s1}(t)$: the number of vehicles with destination s that are traversing lane I of section a at the beginning of time period t and are able to to move from section a to section $a+1$ on lane I during time period t.

$w_{al}^{s0}(t)$: the number of vehicles with destination s that are traversing lane I of section a at the beginning of time period t and moved forward on lane l of section a without being able to reach section $a+1$ during time period t.

$x_{al}^{s}(t)$: the number of vehicles that are destined for section s and are traversing lane I of section a at the beginning of time period t.

$y_{al}(t)$: the number of vehicles leaving the highway from lane I of section a during time period t. This variable applies only when lane l **is** an exit lane.

6.3 The Objective Function

The objective function represents the total cross-sectional flow to be maximized:

$$\textbf{maximize} \sum_{t=1}^{M} \sum_{a=1}^{N} [w_{a1}^{\cdot 1}(t) + w_{a2}^{\cdot 1}(t) + v_{a1}^{\cdot 1}(t) + v_{a2}^{\cdot 1}(t)]$$

$w_{al}^{\cdot 1}$ and $v_{al}^{\cdot 1}$ represent the longitudinal and lateral cross-section flows for section a and lane 1. Cross-section flow is flow that reaches the next section by the end of time t.

6.4 The on-ramp capacity constraints

The total metered inflow from an on-ramp is limited by its capacity as follows:

$$u_{al}^{\cdot}(t) <= CN_{al}(t)$$

Note: $CN_{al}(t) = 0$ if there is no on-ramp for lane l in section a.

6.5 The on-ramp merging constraints

The total metered inflow from an on-ramp has to merge into the existing traffic on the receiving lane at the merge point. Since the speed of the existing traffic is not to be interrupted, the inflow is further limited by the existing flow on the receiving lane.

$$u_{al}^{\cdot}(t) <= [a_1^0 + b_1^0 \frac{x_{al}^{\cdot}(t)}{L_a}]\Delta T$$
$$u_{al}^{\cdot}(t) <= [a_2^0 + b_2^0 \frac{x_{al}^{\cdot}(t)}{L_a}]\Delta T$$

These constraints apply only if lane l is an entry lane of of section a.

6.6 The on-ramp demand constraint

The total on-ramp metered inflow cannot exceed the demand at the ramp:

$$u_{al}^{\cdot}(t) <= \sum_{i=0}^{t} F_{al}^{\cdot}(i) - \sum_{i=1}^{t-1} u_{al}^{\cdot}(i)$$

This constraint applies ***only*** if lane l is an entry lane of section a.

6.7 Capacity of upstream section/lane

The upstream metered inflow is limited by the capacity of the upstream section/lane as follows:

$$u^{\cdot}_{0l}(t) <= CN_{0l}(t)$$

$l = 1, 2,$ and $t = 1, 2, ...,$ M. Also, a=0 means that the traffic is from upstream instead of an on-ramp.

6.8 The upstream demand constraints

The upstream metered inflow cannot exceed the demand.

$$u^{\cdot}_{0l}(t) <= \sum_{i=0}^{t} F^{\cdot}_{0l}(i) - \sum_{i=1}^{t-1} u^{\cdot}_{0l}(i)$$

$l = 1, 2,$ and $t = 1, 2, ...,$ M. Also, a=0 means that the traffic is from upstream instead of an on-ramp.

6.9 The demand distribution

The total on-ramp metered inflow as well as the total upstream metered inflow are segregated into that for each of the downstream sections according to the time-varying OD proportion matrix.

$$u^{s}_{al}(t) = u^{\cdot}_{al}(t) PF^{s}_{al}(t)$$

$$u^{s}_{0l}(t) = u^{\cdot}_{0l}(t) PF^{s}_{0l}(t)$$

These constraints apply only when lane l **is** an entry lane of section $\boldsymbol{a}$**.**

6.10 Outflow in Destination Section

For the exit lane only, we have the following constraint that ensures that all the traffic destined for the section exit the highway from the off-ramp.

$$y_{al}t) = (x^{a}_{al}(t)P^{0}_{al} + v^{a}_{a\bar{l}}(t)P^{1}_{al}$$

This constraint applies only if lane l is an exit lane of section ***a*.**

The following constraints ensure that there is no cross-sectional flow into the next section if the present section is the destination:

$$w_{al}^{a1}(t) = 0$$

$$v_{al}^{a1}(t) = 0$$

$$v_{al}^{a}(t) = v_{al}^{a0}(t)$$

Also, make all the vehicles that are destined for the section and are traversing the non-exit lane of the section change lanes to the exit lane for exiting as follows:

$$v_{al}^{a}(t) = x_{al}^{a}(t)$$

$$w_{al}^{a}(t) = 0$$

These constraints apply only if lane l is the non-exit lane of section ***a*.**

Also, allow no lane changes from the exit lane to the non-exit lane for those vehicles that are destined for the section and are traversing the exit lane of the section. In other words, set

$$v_{al}^{a}(t) = 0$$

Also, ensure that all the vehicles exit if they are in the exit lane and are in the destination section:

$$w_{al}^{a0}(t) = x_{al}^{a}(t)$$

This constraint applies only if lane l is the exit lane of section *a*.

These equations, together with the flow conservation equations to be provided later, ensures that all traffic destined for a section cannot travel beyond it.

6.11 Outflow to downstream sections

If the destination section is not the same as the present section, then both the cross-section and within-section lateral flows are defined as outlined below.

The cross-section and within section lateral flows are determined from the overall lateral flow and the lateral progression factor:

$$v_{al}^{s1}(t) = v_{al}^{s}(t)P_{al}^{1}$$

$$v_{al}^{s0}(t) = v_{al}^{s}(t)(1 - P_{al}^{1})$$

The cross-section and within section longitudinal flows are determined from the overall longitudinal flow and the longitudinal progression factor:

$$w_{al}^{s1}(t) = (x_{al}^{s} - v_{al}^{s}(t))P_{al}^{0}$$

$$w_{al}^{s0}(t) = (x_{al}^{s} - v_{al}^{s}(t))(1 - P_{al}^{0})$$

6.12 Off-ramp capacity constraints

Although the outflow comprises all the traffic destined for that section, it may exceed the off-ramp capacity. To prevent this, we need the off-ramp capacity constraints:

$$y_{al}(t) <= CF_{al}(t)$$

l is the exit lane of section a. This constraint applies only if lane l is the exit lane of section a. Note that spillback onto the AHS is not allowed. In other words, too small a value for $CF_{al}(t)$ could cause the LP to be infeasible.

6.13 Initial Traffic

The following equation serves to initialize the number of vehicles for the 1st time period for all sections and lanes.

$$x_{al}^{s} = X_{al}^{s}$$

6.14 Flow conservation and propagation constraints

The following constraints relate the existing traffic on lane l of section a at the beginning of time period $t+1$ to its counterpart at the beginning of time period t. We first deal with a $\neq$ 1. There are four different cases:

Case 1: Section a $\neq$ 1 is not the destination, i.e. $s \neq$ a.

$$x^s_{al}(t+1) = x^s_{al}(t) + u^s_{al}(t) + w^{s1}_{a-1,l}(t) + v^{s1}_{a-1,\bar{l}}(t) + v^{s0}_{a\bar{l}}(t) - w^{s1}_{al}(t) - v^s_{al}(t)$$

Case **2:** Section a $\neq$ 1 is the destination, i.e. $s =$ a, and l is the exit lane.

$$x^a_{al}(t+1) = x^a_{al}(t) + u^a_{al}(t) + w^{a1}_{a-1,l}(t) + v^{a1}_{a-1,\bar{l}}(t) + v^a_{a\bar{l}}(t) - x^a_{al}(t)P^0_{al} - v^a_{a\bar{l}}(t)P^1_{al}$$

Case **3:** Section a $\neq$ 1 is the destination, i.e. $s =$ a, and l is not the exit lane.

$$x^a_{al}(t+1) = w^{a1}_{a-1,l}(t) + v^{a1}_{a-1,\bar{l}}(t)$$

Note that traffic entering the **AHS** and exiting it in the same section but on the opposite side is not allowed because no lane changing is allowed in the section in which the vehicle enters the highway. Case **4:** a = 1. The conservation equations for all three cases above remain valid except that the two terms $w^{a1}_{a-1,l}(t) + v^{a1}_{a-1,\bar{l}}(t)$ should be replaced by $u^s_{0l}(t)$.

6.15 Longitudinal capacity

The number of vehicles on lane l of section a is limited by its capacity:

$$x^{\cdot}_{al}(t) <= L_a CL_{al}(t)$$

$a = 1, 2, ..., N$, and $l = 1, 2$.

6.16 Lateral/longitudinal flow interaction

Since traffic speed is not to be disrupted, the lateral capacity is limited by the longitudinal flow:

$$v^{\cdot}_{al}(t) <= [a_1 + b_1 \frac{x^{\cdot}_{a\bar{l}}(t)}{L_a}] L_a \Delta T$$

$$v^{\cdot}_{al}(t) <= [a_2 + b_2 \frac{x^{\cdot}_{a\bar{l}}(t)}{L_a}] L_a \Delta T$$

These constraints apply only if lane changing from lane l to lane $\bar{l}$ is allowed in section a. If such lane changing is not allowed, then

$$v^{s}_{al}(t) = 0$$

If there is no exit from lane $\bar{l}$ then

$$v^{a}_{al}(t) = 0$$

6.17 Definitional Constraints

The following definitional constraints are needed to complete the definition of all variables in the LP:

$$v^{\cdot 1}_{al} = \sum_{i=a}^{N} v^{i1}_{al}(t)$$

$$v^{\cdot}_{al}(t) = \sum_{i=a}^{N} v^{i}_{al}(t)$$

$$w^{s}_{al}(t) = w^{s0}_{al}(t) + w^{s1}_{al}(t)$$

$$w^{\cdot 1}_{al}(t) = \sum_{i=a}^{N} w^{i1}_{al}(t)$$

$$w^{\cdot}_{al}(t) = \sum_{i=a}^{N} w^{i}_{al}(t)$$

$$x^{\cdot}_{al}(t) = \sum_{i=a}^{N} x^{i}_{al}(t)$$

6.18 Non-negativity Constraints

$$u_{al}(t) >= 0$$

$$u_{0l}(t) >= 0$$

$$v^{s}_{al}(t) >= 0$$

7 Detailed Design Notes

The following is a description of how parameters PF_{als}, $P1_{al}$, and $P0_{al}$ are determined from the input data described in Section **3.**

PF_{als} is the proportion of vehicles that are in section a, lane 1 at time t that have section s as the destination. It is defined as follows:

$$PF_{als}(t) = \frac{F_{als}(t)}{\sum_{s=a}^{S} F_{als}(t)}$$

The Latitudinal Progression Factor $P1_{al}$ is the proportion of vehicles in a section and lane that changes lanes and can reach the next section in one time period. The Longitudinal Progression Factor $P0_{al}$ is the proportion of vehicles that stay in the same lane and can reach the next section in one time period. The traffic assignment of vehicles to sections and lanes is performed by the LP optimization. This determines whether or not the vehicle is to change lanes in a given time period for a particular section and lane. $P1_{al}$ and $P0_{al}$ are defined as follows:

$$P1_{al} = \frac{Vel_1 + Vel_2}{2} * \frac{\Delta T}{L_a}$$

$$P0_{al} = \frac{Vel_l * \Delta T}{L_a}$$

The lane change parameters are calculated based on several assumptions. A maximum lateral flow is assumed for the case where there are no vehicles in the destination lane. Also, the maximum lateral flow is 0 if the desination lane is at maximum density. This determines the end points of the linear function that defines the maximum lateral flow based on traffic density in the destination lane(See Sections *6.5, 6.16).*

References

[1] Tsao, H. S. J. *Dynamic Traffic Assignment for Automated Highway Systems: A Two-lane Highway with Speed Constancy,* 1996

[2] Hall, R.W. *Longitudinal and Lateral Throughput on an Idealized Highway,* Transportation Science, Vol. 29, No. 2, 1995.

[3] Ran, B. and Boyce, D.E. *Dynamic Urban Transportation Network Models: Theory and Implications for IVHS,* Springer-Verlag, 1994.

[4] Tsao, H. S.J. and Hall, R.W. *Analytical Models for Vehicle/Gap Distribution on Automated Highway Systems*

[5] Tsao, H. S.J. and Hall, R.W. and Hongola, B., *Capacity of Automated Highway Systems: Effect of Platooning and Barriers* UCB-ITS-PRR-93-26

[6] *Using the CPLEX Linear Optimizer,* CPLEX Optimization Inc., Incline Village, NV, (702) 831-7744, techsupport@cplex.com