THE QUARTERLY JOURNAL OF EXPERIMENTAL PSYCHOLOGY
2012, 65 (4), 661–674

Psychology Press
Taylor & Francis Group

Mental representations of arithmetic facts: Evidence from eye movement recordings supports the preferred operand-order-specific representation hypothesis

Fan Zhou[1], **Qian Zhao**[1], **Chuansheng Chen**[2], **and Xinlin Zhou**[1]

[1]National Key Laboratory of Cognitive Neuroscience and Learning, Beijing Normal University, Beijing, China
[2]Department of Psychology and Social Behavior, University of California, Irvine, CA, USA

There are three main hypotheses about mental representations of arithmetic facts: the independent representation hypothesis, the operand-order-free single-representation hypothesis, and the operand-order-specific single-representation hypothesis. The current study used electrical recordings of eye movements to examine the organization of arithmetic facts in long-term memory. Subjects were presented single-digit addition and multiplication problems and were asked to report the solutions. Analyses of the horizontal electrooculograph (HEOG) showed an operand order effect for multiplication in the time windows 150–300 ms (larger negative potentials for smaller operand first problems than for larger operand first ones). The operand order effect was reversed in the time windows from 400 to 1,000 ms (i.e., larger operand first problems had larger negative potentials than smaller operand first problems). For addition, larger operand first problems had larger negative potentials than smaller operand first in the series of time windows from 300 to 1,000 ms, but the effect was smaller than that for multiplication. These results confirmed the dissociated representation of addition and multiplication facts and were consistent with the prediction of the preferred operand-order-specific representation hypothesis.

Keywords: Arithmetic facts; Electrooculograph; Mathematical cognition; Numerical processing.

After extensive practice, arithmetic facts are stored in long-term semantic memory. The nature of mental representation of arithmetic facts has long been a major research topic in the field of numerical processing. There are three main hypotheses: the independent representation hypothesis, the operand-order-free single-representation hypothesis, and the operand-order-specific single-representation hypothesis.

The independent representation hypothesis

According to the independent representation hypothesis, each arithmetic fact is stored in long-term

Correspondence should be addressed to Xinlin Zhou, National Key Laboratory of Cognitive Science and Learning, Beijing Normal University, Beijing 100875, China. E-mail: zhou_xinlin@bnu.edu.cn

Fan Zhou and Qian Zhao contributed equally to this work. This study was supported by Project 30870759 from the National Natural Science Foundation of China (NSFC), Project 60931003 from NSFC, and the programme New Century Excellent Talents at Universities (NCET-07-0101).

http://www.psypress.com/qjep
http://dx.doi.org/10.1080/17470218.2011.616213

memory. This would be true for facts of different operations (e.g., $4 \times 6 = 24$ vs. $24 \div 4 = 6$), also true for those with the same operations but with different operands, and even true for those with the same operations and the same operands but with different orders of the operands (e.g., $4 \times 6 = 24$ vs. $6 \times 4 = 24$). This has been the dominant view in the past. Robert and Campbell (2008) recently declared, "For much of the modern history of arithmetic cognition research, the assumption was that both orders of operands are represented (cf. the models proposed by Ashcraft, 1982; Campbell, 1995; Siegler & Shrager, 1984)" (p. 136).

A typical model of the independent representation hypothesis is the table search model in simple arithmetic. According to the table search model, there are mental tables of addition and multiplication facts. The rows correspond to one operand (e.g., from 0 to 12), and the columns correspond to the other operand (e.g., also from 0 to 12). The correct answer for a problem is stored at the intersection of the row and column corresponding to the two operands (Ashcraft & Battaglia, 1978; Widaman, Geary, Cormier, & Little, 1989). When subjects solve an arithmetic problem, they have to move down the rows to find the first operand and move across the column to find the second operand so they can get to the intersection and retrieve the answer. The table search model can explain commonly observed effects such as the problem-size effect. Given the layout of the table, it takes longer to search for answers that are farther away from the starting point of [0, 0]. Indeed, Geary, Widaman, and Little (1986) showed that the reaction time was a linear function of the number of steps. Widaman et al. (1989) asked undergraduate students to respond to addition and multiplication problems in a true–false reaction time paradigm and found that the table search model was the best predictor for mental addition and multiplication.

The single-representation hypotheses

In contrast to the independent representation hypothesis, the single-representation hypotheses (either operand-order-free or operand-order-specific) state that certain sets of arithmetic facts are stored as single representations. Specifically, because addition and multiplication follow the commutative law, commuted pairs of facts are stored as one representation. According to the single- representation hypotheses, arithmetic facts sharing the same operands and answer (e.g., $3 + 8 = 11$ and $8 + 3 = 11$; $4 \times 7 = 28$ and $7 \times 4 = 28$) would be represented as a single long-term memory node. Which of the commuted pairs is stored in the memory? There are two hypotheses. The identical element (IE) model proposed by Rickard (Rickard, 2005; Rickard & Bourne, 1996; Rickard, Healy, & Bourne, 1994) proposes operand-order-free single representations in the cognitive stage of simple arithmetic. They posit that "the cognitive stage involves access to semantic representations that are independent of the modality-specific representations within the perceptual stage" (Rickard & Bourne, 1996, p. 1281). For example, the problems "$4 \times 7 = 28$" and "$7 \times 4 = 28$" share the same memory (4, 7, $\times$) $\rightarrow$ 28. The representation does not contain any order information. In contrast, the COMP model for simple addition proposed by Butterworth, Zorzi, Girelli, and Jonckheere (2001) proposes operand-order-specific (max + min) single representations. In the following paragraphs, we provide some details about these two models.

According to the IE model (Rickard, 2005; Rickard & Bourne, 1996; Rickard et al., 1994), the problems consisting of the same operands and answer have a single long-term memory node. For example, multiplication problems "$6 \times 8 = 48$" and "$8 \times 6 = 48$" shared the same representation. The problems consisting of different operands and answers would have independent representations. For example, "$48 \div 8 = 6$" and "$48 \div 6 = 8$" are stored separately. To support the IE model, Rickard and colleagues (Rickard & Bourne, 1996; Rickard et al., 1994) trained subjects extensively on single-digit multiplication and division problems and then tested performance on the same problems, problems with the operand order reversed, inverse problems with the other operation, and unpractised control problems. There was strong positive transfer when the elements of the test problem matched exactly with those of a practice problem, but virtually no positive transfer when the presented elements of a test problem did not match exactly those of a practice problem. Moreover, Rickard et al. (Rickard & Bourne, 1996; Rickard et al., 1994) found that subjects took longer to answer the multiplication problems with reversed operand order than to answer the problems with no changes. Rickard and colleagues argued that this effect originated at the perceptual (visual memory) stage, not the cognitive stage. Therefore, it should be noted that the IE model is one form of single representation only at the cognitive stage. It actually allows for order specificity at the perceptual stage (Rickard & Bourne, 1996), a point to which we return in the Discussion section.

Campbell, Fuchs-Lacelle, and Phenix (2006) demonstrated that the IE model also applies to addition and subtraction by examining transfer of response time (RT) savings between prime and probe problems tested in the same block of trials. There were equivalent probe RT savings for addition with identical repetition (prime $6 + 9 \rightarrow$ probe $6 + 9$) or an order change ($9 + 6 \rightarrow 6 + 9$), but much greater savings for subtraction with identical repetition ($15 - 6 \rightarrow 15 - 6$) than with an order change ($15 - 9 \rightarrow 15 - 6$), and no savings with an operation change ($15 - 9 \rightarrow 6 + 9$, or $6 + 9 \rightarrow 15 - 6$).

In contrast to the IE model, the other single-representation model— Butterworth et al.'s (2001) COMP model (also called the min model) of simple addition—proposes operand-order-specific single-representations. The basic idea for the model is that people build addition fact memory on a max + min organization, with no separate representation for the min + max commuted version (e.g., $9 + 2 = 11$, but not $2 + 9 = 11$). When solving addition problems, people first compare operands to find the max (larger) and the min (smaller) of the two numbers. Then they use the max and min to access stored addition facts. Butterworth et al. (2001) asked college students to perform a number-naming task, a magnitude-comparison task, and an addition task. They found that performance on the naming and magnitude-comparison tasks accounted for 71% of the variance of the performance on the addition task. Verguts and Fias (2005) extended the COMP model to multiplication and built a similar operand-order-specific model for multiplication—that is, only max × min or min × max multiplication facts are stored. Consistent with that notion, Butterworth, Marchesini, and Girelli (2003) found that children responded progressively more quickly to multiplication problems with the larger digit as the first operand (e.g., $7 \times 5 = 35$) than to the problems with the smaller digit as the first operand ($5 \times 7 = 35$).

In contrast to the max × min order found by Butterworth et al. (2003), previous studies repeatedly found that Chinese subjects showed an opposite operand order effect for multiplication—that is, the smaller operand first problems (e.g., 7×9) took less time to solve and/or had fewer errors than the larger operand first problems (e.g., 9×7; e.g., Lefevre, Lei, Smith-Chant, & Mullins, 2001; Lefevre & Liu, 1997; Zhou & Dong, 2003). The main reason for this order effect is that a Chinese multiplication table has only smaller operand first and tie entries. It seems that Chinese subjects only have the representations for smaller operand first problems and ties but not those for larger operand first problems. They have to transform the larger operand first problems into smaller operand first problems in order to retrieve the relevant solutions.

Eye movement recording and representations of arithmetic facts

All three main hypotheses of mental representations of arithmetic facts received support from chronometric studies. Another approach to testing them is to use the eye movement recording technique. Previous studies have shown that eye movement recordings can indicate the magnitude of the number being processed (Loetscher, Bockisch, Nicholls, & Brugger, 2010). Because of these connections between eye movements and number processing, it is possible for us to infer how the mental representations of arithmetic facts are organized according to eye movement patterns. If the independent representation hypothesis is correct, we would expect no operand order effects in eye movements because the retrieval of each fact is directly mapped onto the given order of operands. Similarly, if the hypothesis of operand-order-free single-representations is correct, eye movements should not differ between smaller and larger operand first problems. On the other hand, if the hypothesis of operand-order-specific single-representations is correct, we would expect an operand order effect in eye movements because subjects had to compare the presented order of operands with the order of operands as stored in memory of arithmetic facts and, if necessary, manipulate the order of presented operands. For example, Chinese subjects should move their eyes from the smaller operand to the larger operand for a multiplication problem due to the smaller operand first verbal sequence for multiplication facts as mentioned above (e.g., "sān qī èr shí yī" for multiplication fact "$3 \times 7 = 21$", literally "three seven twenty one"). Thus, for the problem "3×7", their eyes should have a rightward movement, but for the problem "7×3", the eyes should have a leftward movement (see Figure 1, bottom panel, the retrieval stage).

For addition facts, the early acquisition experience for the Chinese is the same as that for children in other countries. That is, children typically use the min model (or the COMP model). The counter in the min model is set to the maximum digit and is then incremented by the minimum digit. Previous studies have shown that the min model is the best model to account for solution time on addition problems (e.g., Groen & Parkman, 1972). It has been found that even children as young as 4 years of age use the min model to solve addition problems (Groen & Resnick, 1977). Baroody and Ginsburg (1986) also found that 6-year-olds could understand addition and use the min model. As experience with addition increases, children and adults can directly retrieve answers from memory. As Butterworth et al. (2001) claimed, the counting procedure could affect the organization of facts in memory—that is, the preferred form is likely to be $5 + 3 = 8$ rather than $3 + 5 = 8$. In addition to the min model, children and adults also rely on

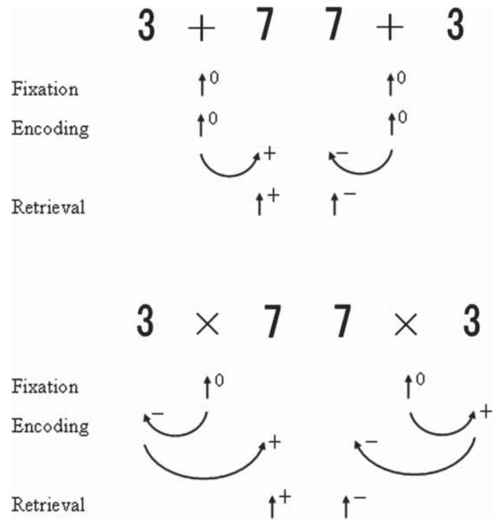

Figure 1. *Schematic representation of expected eye movement patterns and associated horizontal electrooculograph (HEOG). Arrows indicate directions of expected eye movements, and the signs indicate expected HEOG ("0" indicates no changes, "+" positive HEOG, and "–" negative HEOG). Tie problems are expected to show the same rightward pattern as the smaller operand first problems because of the natural left-to-right reading direction.*

transformation strategies to acquire and practise addition facts, such as, $8 + 5 = 8 + 2 + 3$, $5 + 8 = 3 + 2 + 8$ (e.g., Dehaene & Cohen, 1997; Geary, Hoard, Byrd-Craven, & DeSoto, 2004; LeFevre, Sadesky, & Bisanz, 1996; LeFevre, Smith-Chant, Hiscock, Daley, & Morris, 2003; Roussel, Fayol, & Barrouillet, 2002; Siegler, 1987). It seems that the larger operand is the basis for calculation in both the min model and the use of transformation strategy. With extensive practice on addition, older children and adults can directly retrieve the answer, which may decrease the operand order effect. However, it is possible that the remnants of the experience of using the min model or the transformation strategy may be stored in the long-term memory. Furthermore, these procedures may be a backup method when direct retrieval fails. In other words, the larger operand might play a central role in the representation of addition facts. Consequently, for the larger operand first addition, subjects' eyes should have a leftward movement, and for the smaller operand first addition, their eyes should have a rightward movement in order to focus on the larger operand (see Figure 1, top panel, the retrieval stage).

In the current study, we used the tie problems (i.e., problems with the same digit as the operands, $3 + 3$, 3×3) as the baseline. Because no manipulation of the order of operands is needed for such problems, subjects are expected to follow a regular left-to-right reading direction and have rightward eye movements, which are the same as those for the smaller operand first problems.

We used the electrooculograph (EOG) recording technique, focusing on the horizontal EOG (HEOG). The HOEG can be recorded by placing two channels at the outer canthi of both eyes. The EOG recording has received much attention in the field of eye movement-controlled human–computer interface. Although in event-related potential studies, the EOG signal would always be removed from potentials recording on the scalp electrodes, the psychophysiological electric signals have temporal resolution and sensitivity that are sufficient for detecting potential deflection resulting from any changes in the eye position. This technique has several advantages over the traditional eye-tracking technique (e.g., video-oculography), such as better spatial resolution and the ability to record eye movements even when eyes are closed (Fenn & Hursh, 1936).

To clearly demonstrate the relation between deflection direction of EOG and eye movement, an additional eye movement task was conducted in the current study with a separate sample. Subjects were asked to move their eyes in specific directions, and their EOGs were recorded and analysed. Rightward eye movements were expected to elicit negative HEOG, and leftward eye movements positive HEOG.

Method

Subjects

Thirty-four healthy right-handed university students (16 males and 18 females) were recruited from Beijing Normal University, China. The average age of the subjects was 23.2 years, ranging from 19.8 to 27.8 years. Thirty-two students performed the arithmetic tasks. The remaining two students performed an eye movement task, which was used to validate that EOG deflections are directly linked to eye movements. They self-reported to have normal or corrected-to-normal eyesight. They had not participated in any experiments similar to the present one (i.e., involving simple arithmetic tasks of addition and multiplication) during the previous half a year. Informed written consent was obtained from each subject after procedures had been fully explained. The experiment on these subjects was approved by the State Key Lab of Cognitive Neuroscience and Learning at Beijing Normal University.

Experimental design

For the arithmetic tasks, we used a 2×3 within-subject design, with operation (one-digit addition vs. multiplication) and operand order (smaller first vs. larger first vs. tie) as independent variables. For the eye movement task, we used a 2×4 within-subject design, with distance of eye movement (far vs. near), and direction (left vs. right vs. up vs. down) as independent variables.

Materials

Sixty-four addition problems from $2 + 2$ through $9 + 9$ and 64 multiplication problems from 2×2 through 9×9 were used for the arithmetic task in this study (cf. Zhou, Chen, Zang, et al., 2007). Problems with 0 or 1 as an operand were excluded because they are rule-based problems. Due to the limited number of problems, we had to present each problem twice in different blocks to allow for enough trials for the event-related potentials recording. There were 128 trials for each type of operation. The addition and multiplication problems were divided into smaller operand first, larger operand first, and tie problems. For each operation (i.e., addition or multiplication), there were 56 larger operand first trials, 56 smaller operand first trials, and 16 tie problems.

For the eye movement task, there were eight conditions (four directions in each type of distance of eye movement and two types of distances of eye movement; see Procedure for details). There were 100 trials for each condition.

Procedure
Subjects were seated 105 cm away from the computer screen in a dimly lit, sound-attenuated room. For the arithmetic tasks, all stimuli were presented visually in white against black background at the centre of the computer screen. To reduce processing load, we presented addition and multiplication problems in different blocks so that subjects could focus on memory retrieval of arithmetic facts without the additional need to recognize type of operation. Before each block, subjects were instructed which type of operation they would perform. Subjects were asked to orally report the answer for each problem. Before each arithmetic problem was presented, a fixation sign "*" was presented in the centre of the screen. The asterisk's position was the operator's position at the centre of the arithmetic problem. Subjects were asked to focus on the asterisk for 500 ms. Each trial (i.e., a problem), consisting of two numbers and the operation sign " + " for addition or " × " for multiplication, remained on the centre of the screen until the subject responded. Following the response, a 2,000-ms blank screen was presented. During the experiment, an experimenter sat beside the subject to record what he or she reported. Before formal tests, subjects were given practice problems involving "0" and "1" as an operand. They were trained to respond orally so that their oral response could activate the voice-controlled switch. Subjects were also asked to keep their head steady and avoid eye blinks before an oral response. Eye blinks were allowed in the 2,000-ms blank interval after each oral response.

For the eye movement task, a fixation sign "*" was first presented in the centre of the screen for 500 ms, which was the same as in the experiment. Then the asterisk was randomly presented in one of four directions (i.e., up, down, left, and right) at one of two distances ("close" = the distance between an operand and the operation sign; "far" = twice the "close" distance) for 1,000 ms. During the experiment, subjects were asked just to focus on the asterisk, and they did not need to make any response.

HEOG recording and analysis
HEOG and scalp voltages for the arithmetic task were recorded using a SCAN system (Neurosoft, Inc., Sterling, USA) with a 64-channel Quick-cap. Linked ears served as reference, and the middle of the forehead served as ground. Two channels were placed at the outer canthi of both eyes to record the horizontal electrooculogram (HEOG; Figure 2), and another two channels above and below the left eye for the vertical electrooculogram (VEOG). The default algorithms were used, in which HEOG was calculated as the right eye minus the left eye, and VEOG as the top of the eye minus the bottom of the eye. Therefore, the wave was negative-going when the eyes moved towards the left, but positive-going when they moved towards the right. The electroencephalogram (EEG) was amplified online with a low-pass frequency filter of 30 Hz. The sampling rate was 1,000 Hz. The impedance of all electrodes was kept below 5 kΩ. The scalp EEG and VEOG were not analysed in the current study.

HEOG was processed in NeuroScan EDIT (Version 4.3). A direct current (DC) correction was first applied. The continuous EEG data were segmented into epochs starting from 200 ms before the onset of the second operand and continuing for 1,500 ms. The 200-ms prestimulus served as the baseline. Epochs exceeding the range of −100 ~ 100 μV were rejected as artefacts. A total of 93.5% of trials, from 86.0% to 98.8% for all subjects, were kept. The remaining trials were baseline corrected. The corrected data were

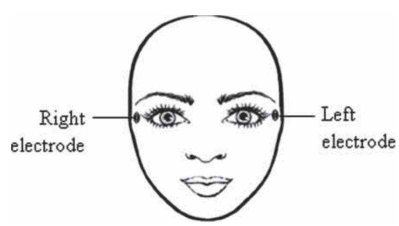

Figure 2. *Surface electrodes placed around the eyes for horizontal electrooculograph.*

averaged for each subject by conditions (i.e., those involving smaller operand first problems, larger operand first problems, and tie problems). A filter with a low pass of 30 Hz (12 dB/octave) was applied to the averaged results.

For the eye movement task, the EOG recording was made using the same procedure as that for the arithmetic task. The EOG analysis was similar to that for the arithmetic task with one exception. That is, epochs exceeding the range of –400 ~ 400 μV, other than –100 ~ 100 μV, were rejected as artefacts. Just three trials were discarded for 1 of the 2 subjects. The deflection of EOG as a result of direct eye movement seemed to be much larger than that induced by the mental processing in the arithmetic task.

Statistical analysis

One subject's EOG data for the arithmetic task were not further analysed because fewer than 50% of the trials were valid across all conditions. We conducted analyses of variance (ANOVAs) with the time window and operand order (smaller operand first vs. larger operand first) as independent variables for addition and multiplication separately. A separate set of ANOVAs were conducted comparing tie with nontie problems. ANOVAs were conducted on the mean amplitude for several small time windows. These time windows were selected on the basis of the step of 50 ms from 150 ms after the onset of stimulus to 1,000 ms. These time windows should have covered the whole period of arithmetic processing except for the early visual perception of numbers. The mean EOG data for each condition of the eye movement task were used to demonstrate the relation between deflection directions of EOG and eye movements.

For the behavioural data, any trial with a reaction time of more than 2 s was discarded. Reaction times were further trimmed with the three-standard-deviations convention (i.e., a trial with a reaction time three standard deviations above or below the mean was treated as an erroneous response). Only 2.3% of trials were discarded. Reaction times and error rates were subjected to repeated measures ANOVA as described above.

Results

Behaviour results

The grand mean error rates across all subjects were 3.7% for addition and 4.7% for multiplication. The error rates were low and thus were not further analysed. Mean reaction time (RT) for the arithmetic task is shown in Figure 3. To examine the operand order effect, RTs were analysed with two-factor repeated measures ANOVA with arithmetic operation (addition and multiplication) and operand order (smaller operand first problems and larger operand first problems) as the within-subject factors. The two-way interaction was significant, $F(1,\ 30) = 11.74$, $MSE = 770.57$, $p < .005$, $\eta^2 = .28$. Simple effects tests showed that the operand order effect was significant for multiplication problems, $F(1,\ 30) = 16.64$, $MSE = 1{,}725.18$, $p < .001$, $\eta^2 = .36$, but not for addition problems.

To examine how each operand order differs from the baseline (the tie problems), we conducted two additional two-factor repeated measures ANOVAs with arithmetic operation (addition and multiplication) and tie effect (smaller operand first and larger operand first problems vs. tie problems) as the within-subject factors as well as associated simple effects analyses. Results showed

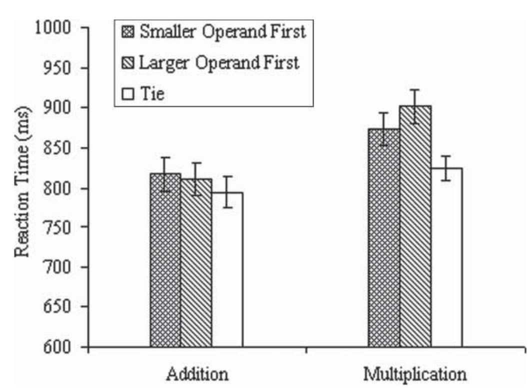

Figure 3. *Operation, operand order, and tie effects based on the reaction time. Error bars indicate the standard error.*

that for multiplication, RTs were longer for both smaller operand first and larger operand first problems than for the tie problems, $F(1, 30) = 28.03$, $MSE = 1{,}327.53$, $p < .001$, $\eta^2 = .48$, and $F(1, 30) = 55.73$, $MSE = 1{,}644.88$, $p < .001$, $\eta^2 = .65$, respectively. For addition, smaller operand first problems, but not larger operand first problems, took longer to solve than the tie problems, $F(1, 30) = 7.11$, $MSE = 1{,}073.18$, $p < .05$, $\eta^2 = .19$. It should be noted that we also conducted the above analyses with median RTs, and the results were the same as those from the analyses of mean RTs.

HEOG results

Figure 4 shows the HEOGs for simple addition and multiplication. The HEOG was first analysed in a series of time windows of 50 ms from 150 ms after the onset of stimulus to 1,000 ms. ANOVA showed significant operand order and tie effects in multiple time windows (see Figure 4, the horizontal solid grey bars). The operand order effect (i.e., smaller operand first problems had larger negative potentials than larger operand first problems) was first observed for multiplication in time window 150–300 ms. Then the reversed operand order effect was observed in the time windows from 400 to 1,000 ms. For addition, the operand order effect (i.e., larger operand first problems had larger negative potentials than smaller operand first problems and tie problems) was observed in time window 300–1,000 ms. As expected, tie problems showed the same HEOG deflection as the smaller operand first problems for both addition and multiplication, and they differed from larger operand first problems.

The operation effect occurred as early as 200 ms post stimulus. The operation effect was consistent till the last time window analysed. Generally, multiplication had greater negative HEOG than addition, which is consistent with our hypothesis.

Finally, as a verification of the HEOG as a direct and sensitive measure of eye movements, we collected HEOG data from 2 subjects who followed specific directions to move their eyes, as shown in Figure 5. The up–down eye movement seemed to have little effect on the HEOG according to the 2 subjects' results. The leftward eye movement led to greater negative potentials. The farther the eyes moved towards the left, the larger the negative potentials were. The rightward eye movement led to greater positive potentials, and the farther the eyes moved towards the right, the larger the positive potentials were.

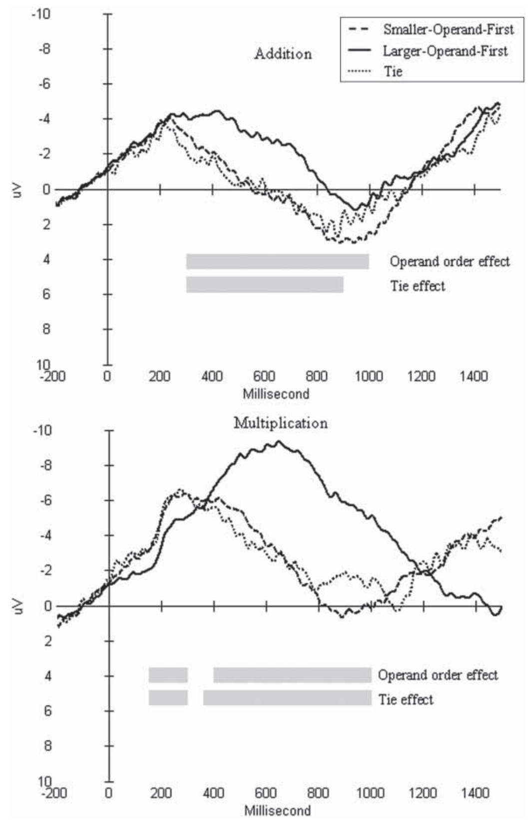

Figure 4. *Operand order effect in horizontal electrooculograph (HEOG) for single-digit addition and multiplication. The light grey bar shows the operand order effect and tie effect (the larger operand first vs. the tie problems) from the analysis of variance with time window (from 150 ~ 200, 200 ~ 250, . . . , to 950–1,000 ms) and operand condition as independent variables.*

Discussion

Eye movement patterns during arithmetic processing

The current study used the electrical recording method of eye movements to examine the organization of arithmetic facts in long-term memory. Our analyses of the HEOG in a series of small time

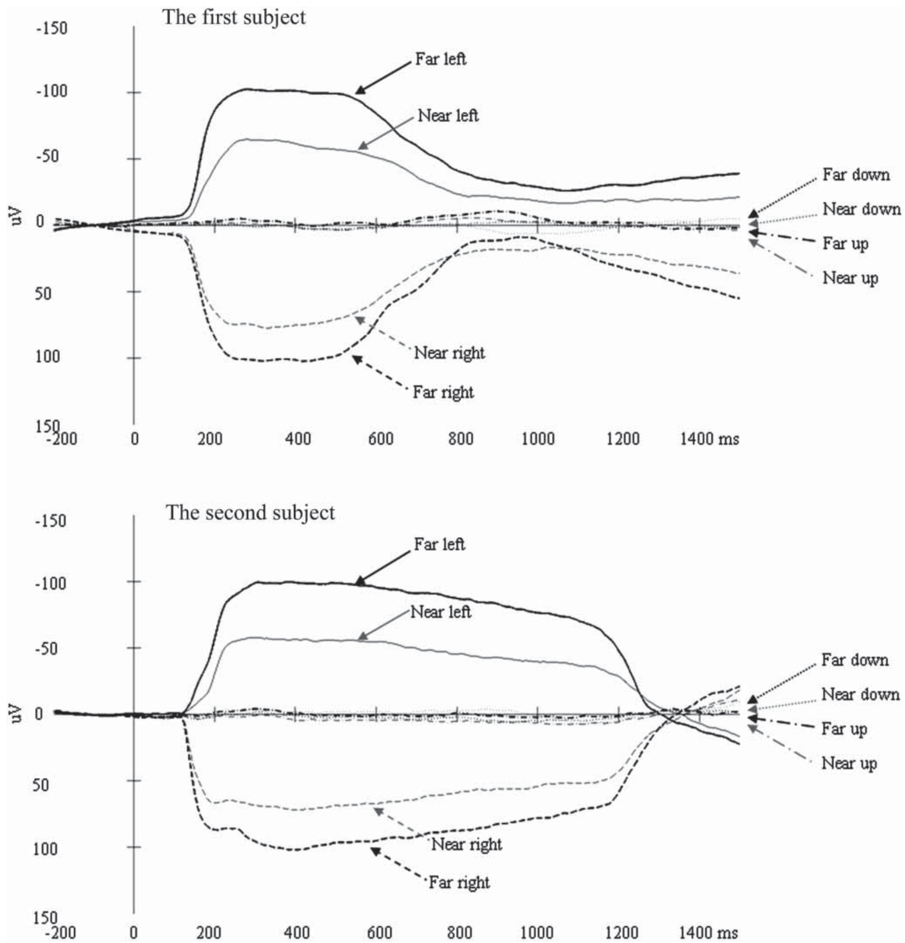

Figure 5. *Two subjects' horizontal electrooculograph (HEOG) for the eye movement task in four directions (left, right, up, and down) and two types of distances (near and far).*

windows of 50 ms showed that the operand order effect was first observed for multiplication in time window 150–300 ms (i.e., smaller operand first problems and tie problems had larger negative potentials than larger operand first problems). The operand order effect was reversed in the time windows from 400 to 1,000 ms (i.e., larger operand first problems had larger negative potentials than smaller operand first problems). For addition, larger operand first problems had larger negative potentials than smaller operand first problems in the series of time windows from 300–1,000 ms. For both addition and multiplication, no differences in HEOG were found between the tie problems and the smaller operand first problems in all of the time windows. These results were consistent with the preferred operand-order-specific representation hypothesis that eye movements would vary according to the order of operands in the long-term memory of arithmetic facts.

A special note is needed about the relevance of our results to the IE model. As mentioned in the introduction, the IE model focuses on single representations at the cognitive stage. It makes no strong prediction about order effect at the perceptual stage. As Rickard and Bourne (1996) showed, much of the learning with training occurs within the perceptual stage, and order specificity is possible at this stage. In that sense, if our results were based on the perceptual stage, they would appear consistent with the IE model (as discussed by Rickard & Bourne, 1996). There two main reasons, however, that we believe our findings were not limited to the perceptual stage, but were extended to the cognitive stage. First, the operand order effect in HEOG was observed from the time of the presentation of arithmetic problems to the time of oral reports of answers. The time course covered both the encoding and retrieval stages. Second, the dissociated eye movement patterns of HEOG between addition and multiplication also reflected the dissociation of the long-term memory of arithmetic facts (as discussed a following section).Taken together, although the IE model allows for order specificity at the perceptual stage, which is consistent with our results, the IE model's order-free representations in long-term memory were not supported by our results.

Differences between representations of addition and multiplication facts

Previous studies have shown the dissociated representations of addition, subtraction, and multiplication facts. Addition and subtraction have a greater reliance on visuospatial processing or magnitude manipulation, whereas multiplication has a greater reliance on verbal processing (e.g., Prado et al., 2011; Zhou et al., 2006; Zhou, Chen, Zang, et al., 2007). For example, Zhou et al. (2006) found that, compared to addition and subtraction, multiplication elicited a greater N300 at the left frontal electrodes, peaking around 320 ms (in the interval between 275 and 334 ms after the onset of the visually presented arithmetic problems). The greater N300 component could be interpreted as greater verbal processing. In another study, Zhou, Chen, Zang, et al. (2007) compared the patterns of brain activation elicited by single-digit addition and multiplication problems. They found that, compared to multiplication problems, addition problems elicited more activation in the intraparietal sulcus and the middle occipital gyri at the right hemisphere and the superior occipital gyri at both hemispheres. These brain regions are known to be involved in visual perception, visual mental imagery, visuospatial working memory, and spatial attention (e.g., Corbetta et al., 1998; Corbetta, Kincade, Ollinger, McAvoy, & Shulman, 2000; Postle, Awh, Jonides, Smith, & D'Esposito, 2004; Vingerhoets, de Lange, Vandemaele, Deblaere, & Achten, 2002; Zurowski et al., 2002). Thus, the greater activation for addition than for multiplication in these regions could be reasonably interpreted as a result of a greater reliance on visuospatial processing during the retrieval of addition facts. In contrast, multiplication had more activation in the precentral gyrus, supplementary motor areas, and posterior and anterior superior temporal gyrus at the left hemisphere. These brain regions are associated with verbal production, such as planning and execution of throat and tongue movements (e.g., Cowell, Egan, Code, Harasty, & Watson, 2000; Hanakawa, Honda, Okada, Fukuyama, & Shibasaki, 2003; Hickok et al., 2000; Paus, Perry, Zatorre, Worsley, & Evans, 1996; Riecker, Wildgruber, Dogil, Grodd, & Ackermann, 2002; Wildgruber, Ackermann, Klose, Kardatzki, & Grodd, 1996; Zhou et al., 2006; Ziegler, Kilian, & Deger, 1997).

The current study found two major differences in the eye movement data between addition and multiplication. First, operand order did not elicit differences in eye movements at the early stage (i.e., 150–300 ms) of solving addition problems, but it did for multiplication problems. Second, multiplication showed two types of eye movement patterns sequentially, which had opposite directions. As discussed in the introduction, the two operands of addition problems were most likely encoded simultaneously through their visual Arabic digit codes (Dehaene & Cohen, 1997; Zhou & Dong, 2003). Therefore, there was no operand order effect during the encoding stage. At the retrieval stage, subjects focused their attention on the larger operand, which resulted in the operand order effect in HEOG. In contrast, for multiplication, to access the linear verbal sequences of multiplication facts, subjects had to first move their eyes to the smaller operand from the fixation point (i.e., the centre of the problem or the position for operation sign) and then move their eyes from the smaller operand to the larger operand.

Consequently, we observed the two types of opposite patterns of eye movements.

Differences in the operand order effect between behavioural and HEOG measurements

The current study found that the operand order effect was evident for both addition and multiplication. In terms of the behavioural measurements, however, only multiplication showed the operand order effect. The finding that addition showed the operand order effect in eye movements, but not in the behavioural measurement, was probably due to the greater sensitivity of the HEOG measurements than of behavioural measurements. Previous research has shown that the operand order effect for addition was consistent but very weak (Butterworth et al., 2001; Zhou & Dong, 2003). Zhou and Dong found that smaller operand first problems took 8 ms longer to answer than did larger operand first problems. Butterworth et al. (2001) found a similar effect of 9 ms. However, due to differences in sample size and variance, the operand order effect was statistically significant in Zhou and Dong, whose sample size was 24, but not in Butterworth et al., whose sample size was 20. In the current study with 32 subjects, we found no significant differences in RT (in fact the RTs favoured the smaller operand first problems by 6 ms, with a standard error of 5). The HEOG measurements, however, were sensitive enough to detect a significant operand order effect. It should be noted that previous studies have also shown dissociations between behavioural data and eye movement data (e.g., Castelhano & Henderson, 2007). For example, Castelhano and Henderson recorded eye movements to explore whether the information acquired from a scene in an initial glimpse can facilitate subsequent eye movements, and they found significant effects in eye movement data, but not in reaction time data.

Experience during acquisition shapes preferred operand-order-specific representations of the commuted pairs of arithmetic facts

Operand order specificity appears to have resulted from the experience of acquiring these arithmetic facts. Children acquire the facts of single-digit arithmetic mainly through two types of strategies: procedural strategies and rote verbal memory (e.g., Dehaene & Cohen, 1997; Roussel, Fayol, & Barrouillet, 2002; Zhou & Dong, 2003). Procedural strategies, such as counting, transformation (e.g., $6 + 7 = 6 + 6 + 1$; $9 + 7 = 9 + 1 + 6$), and repeated addition, typically involve quantity manipulation along the mental number line. With the rote memory strategy, children repeatedly recite arithmetic facts so that the facts can be stored in memory as a type of modularized phonological association between a digit pair and their answer. Schoolchildren are usually taught to use procedural strategies for simple addition and subtraction, but to use verbal memory strategy to memorize multiplication facts (only the smaller operand first and tie entries in the case of mainland Chinese subjects; e.g., Dehaene & Cohen, 1997; Roussel et al., 2002; Zhou & Dong, 2003). These differential strategies during the acquisition of arithmetic facts may play an important role in shaping their mental representations (e.g., Siegler & Shipley, 1995; Siegler & Shrager, 1984) and the associated eye movements as summarized above (i.e., focusing on the larger operand for addition and moving from the smaller operand to the larger operand for multiplication).

It should be noted that our results for the eye movements associated with multiplication should be specific to subjects such as mainland Chinese who memorize only the smaller operand first and the tie entries of the multiplication table. Even within China, the Hong Kong and Macao Chinese used the multiplication table that included both smaller operand first and larger operand first entries. We found that these samples did not show the mismatch negative waveform for the larger operand first problems in an event-related potential (ERP) study (Zhou, Chen, Zhang, et al., 2007). We would speculate that Hong Kong and Macao subjects would not show the operand order effect in HEOG, because they have both smaller and larger operand first multiplication facts in their long-term memory.

Also in contrast to mainland Chinese (with smaller operand first representations), Italian adults might have preferred larger operand first representations (i.e., max × min). Butterworth et al. (2003) found that Italian fourth- and fifth-graders had an advantage in their responses to the larger operand first problems as compared to the smaller operand first problems. Although Italian children are taught the whole multiplication table, they appear to convert multiplication to addition (i.e., dynamic reorganization), according to Butterworth et al. (2003). They seem to take the larger operand and perform multiplication as multiple addition problems (Butterworth et al., 2003). For Italian subjects, therefore, we would expect to find a pattern of HEOG for multiplication that is opposite of the one for mainland Chinese.

Limitations

Several limitations of the current study need to be noted. First, as mentioned earlier, theoretical models such as the IE model make different predictions about the order effect for different stages of processing. Unfortunately, our paradigm did not allow for a clear separation of encoding from the retrieval stage. Future research should develop ways to examine different stages separately.

Second, it is not clear exactly when the relative digit magnitude of the operand is determined (before or after any eye movement from fixation). If relative magnitude can be determined without eye movement, then the very first eye movement may reflect the type of memory storage (i.e., the retrieval stage processing). Although there have been debates regarding whether subjects could determine the relative digit magnitude before they retrieve arithmetic facts (Butterworth et al., 2001; Robert & Campbell, 2008), the most recent evidence (Robert & Campbell, 2008) is that there is no number comparison in simple addition and multiplication before the retrieval of arithmetic facts. Robert and Campbell (2008) asked 64 volunteers to complete a number comparison task, an addition task, and a multiplication task with both size-congruent and size-incongruent stimuli. The size congruity effect occurred only in the comparison task, not in the addition and multiplication tasks. They thus concluded that the participants did not compare operands before the retrieval of arithmetic facts. For the eye movement data to be a valid indicator of stored number facts, future research is needed to further substantiate that there is no early comparison stage.

Third, a frequently ignored but nevertheless important issue in number cognition is individual differences. For the current study, if groups of individuals had different preferred order of processing operands or natural tendency of eye movements, our results would have been affected. There exist prior data on this issue. Within our data (see Online Supplementary Material, Figures S1 and S2), we examined individual subjects' data and found variations around the group means. It is not clear whether they represented systematic variations in strategies or eye movement "habits" (in those cases, however, a bimodal distribution might have been expected) or mere measure errors. Future research may consider linking these variations to behavioural measures to see whether these variations were related to individual differences in behaviours.

CONCLUSION

In sum, mental representations of arithmetic facts (e.g., addition and multiplication) can be investigated through the eye movement technique. The eye movement patterns can directly reflect how arithmetic facts are stored in long-term memory. To our knowledge, this is the first study that used the direct eye movement recording technique to examine how arithmetic facts are represented. The results supported the preferred operand-order-specific representation hypothesis.

Original manuscript received 13 March 2011
Accepted revision received 3 August 2011
First published online 5 October 2011

REFERENCES

Ashcraft, M. (1982). The development of mental arithmetic: A chronometric approach. *Developmental Review*, *2*(3), 213–236.

Ashcraft, M., & Battaglia, J. (1978). Cognitive arithmetic: Evidence for retrieval and decision processes in mental addition. *Journal of Experimental Psychology: Human Learning and Memory*, *4*(5), 527–538.

Baroody, A., & Ginsburg, H. (1986). The relationship between initial meaningful and mechanical knowledge of arithmetic. In J. Hiebert (Ed.), *Conceptual and procedural knowledge: The case of mathematics* (pp. 75–112). Hillsdale, NJ: Erlbaum.

Butterworth, B., Marchesini, N., Girelli, L., Baroody, A., & Lea, A. (2003). Multiplication facts: Passive storage or dynamic reorganization. In A. J. Baroody & A. Dowker, (Eds.), *The development of arithmetical concepts and skills* (pp. 198–202). Mahwah, NJ: Lawrence Erlbaum.

Butterworth, B., Zorzi, M., Girelli, L., & Jonckheere, A. (2001). Storage and retrieval of addition facts: The role of number comparison. *The Quarterly Journal of Experimental Psychology A*, *54*(4), 1005–1029.

Campbell, J. (1995). Mechanisms of simple addition and multiplication: A modified network-interference theory and simulation. *Mathematical Cognition*, *1*(2), 121–164.

Campbell, J., Fuchs-Lacelle, S., & Phenix, T. (2006). Identical elements model of arithmetic memory: Extension to addition and subtraction. *Memory and Cognition*, *34*(3), 633.

Castelhano, M. S., & Henderson, J. M. (2007). Initial scene representations facilitate eye movement guidance in visual search. *Journal of Experimental Psychology: Human Perception and Performance*, *33* (4), 753–763.

Corbetta, M., Akbudak, E., Conturo, T. E., Snyder, A. Z., Ollinger, J. M., Drury, H. A., et al. (1998). A common network of functional areas for attention and eye movements. *Neuron*, *21*(4), 761–773.

Corbetta, M., Kincade, J. M., Ollinger, J. M., McAvoy, M. P., & Shulman, G. L. (2000). Voluntary orienting is dissociated from target detection in human posterior parietal cortex. *Nature Neuroscience*, *3,* 292–297.

Cowell, S., Egan, G., Code, C., Harasty, J., & Watson, J. (2000). The functional neuroanatomy of simple calculation and number repetition: A parametric PET activation study. *NeuroImage*, *12*(5), 565–573.

Dehaene, S., & Cohen, L. (1997). Cerebral pathways for calculation: Double dissociation between rote verbal and quantitative knowledge of arithmetic. *Cortex*, *33,* 219–250.

Fenn, W., & Hursh, J. (1936). Movements of the eyes when the lids are closed. *American Journal of Physiology*, *118*(1), 8.

Geary, D. C., Hoard, M. K., Byrd-Craven, J., & DeSoto, M. C. (2004). Strategy choices in simple and complex addition: Contributions of working memory and counting knowledge for children with mathematical disability. *Journal of Experimental Child Psychology*, *88*(2), 121–151.

Geary, D., Widaman, K., & Little, T. (1986). Cognitive addition and multiplication: Evidence for a single memory network. *Memory & Cognition*, *14*(6), 478–487.

Groen, G., & Parkman, J. (1972). A chronometric analysis of simple addition. *Psychological Review*, *79*(4), 329–343.

Groen, G., & Resnick, L. (1977). Can preschool children invent addition algorithms? *Journal of Educational Psychology*, *69*(6), 645–652.

Hanakawa, T., Honda, M., Okada, T., Fukuyama, H., & Shibasaki, H. (2003). Differential activity in the premotor cortex subdivisions in humans during mental calculation and verbal rehearsal tasks: A functional magnetic resonance imaging study. *Neuroscience Letters*, *347*(3), 199–201.

Hickok, G., Erhard, P., Kassubek, J., Helms-Tillery, A. K., Naeve-Velguth, S., Strupp, J. P., et al. (2000). A functional magnetic resonance imaging study of the role of left posterior superior temporal gyrus in speech production: Implications for the explanation of conduction aphasia. *Neuroscience Letters*, *287*(2), 156–160.

LeFevre, J., Lei, Q., Smith-Chant, B., & Mullins, D. (2001). Multiplication by eye and by ear for Chinese-speaking and English-speaking adults. *Canadian Journal of Experimental Psychology*, *55*(4), 277–284.

LeFevre, J., & Liu, J. (1997). The role of experience in numerical skill: Multiplication performance in adults from Canada and China. *Mathematical Cognition*, *3* (1), 31–62.

LeFevre, J.-A., Sadesky, G. S., & Bisanz, J. (1996). Selection of procedures in mental addition: Reassessing the problem size effect in adults. *Journal of Experimental Psychology: Learning, Memory & Cognition*, *22,* 216–230.

LeFevre, J., Smith-Chant, B. L., Hiscock, K., Daley, K. E., & Morris, J. (2003). Young adults' strategic choices in simple arithmetic: Implications for the development of mathematical representations, In A. J. Baroody & A. Dowker (Eds.), *The development of arithmetic concepts and skills* (pp. 203–228). Mahwah, NJ: Erlbaum.

Loetscher, T., Bockisch, C., Nicholls, M., & Brugger, P. (2010). Eye position predicts what number you have in mind. *Current Biology*, *20*(6), R264–R265.

Paus, T., Perry, D. W., Zatorre, R. J., Worsley, K. J., & Evans, A. C. (1996). Modulation of cerebral blood flow in the human auditory cortex during speech: Role of motor-to-sensory discharges. *European Journal of Neuroscience*, *8*(11), 2236–2246.

Postle, B., Awh, E., Jonides, J., Smith, E., & D'Esposito, M. (2004). The where and how of attention-based rehearsal in spatial working memory. *Cognitive Brain Research*, *20*(2), 194–205.

Prado, J., Mutreja, R., Zhang, H., Mehta, R., Desroches, A. S., Minas, J. E., et al. (2011). Distinct representations of subtraction and multiplication in the neural systems for numerosity and language. *Human Brain Mapping*.

Rickard, T. (2005). A revised identical elements model of arithmetic fact representation. *Journal of Experimental Psychology: Learning, Memory & Cognition*, *31*(2), 250–257.

Rickard, T., & Bourne, L., Jr. (1996). Some tests of an identical elements model of basic arithmetic skills. *Journal of Experimental Psychology: Learning, Memory & Cognition*, *22*(5), 1281–1295.

Rickard, T., Healy, A., & Bourne, L. (1994). On the cognitive structure of basic arithmetic skills: Operation, order, and symbol transfer effects. *Journal of Experimental Psychology: Learning, Memory & Cognition*, *20*(5), 1139–1153.

Riecker, A., Wildgruber, D., Dogil, G., Grodd, W., & Ackermann, H. (2002). Hemispheric lateralization effects of rhythm implementation during syllable repetitions: An fMRI study. *NeuroImage*, *16*(1), 169–176.

Robert, N., & Campbell, J. (2008). Simple addition and multiplication: No comparison. *European Journal of Cognitive Psychology*, *20*(1), 123–138.

Roussel, J. L., Fayol, M., & Barrouillet, P. (2002). Procedural vs. direct retrieval strategies in arithmetic: A comparison between additive and multiplicative problem solving. *European Journal of Cognitive Psychology*, *14*(1), 61–104.

Siegler, R. S. (1987). The perils of averaging data over strategies: An example from children's addition. *Journal of Experimental Psychology: General*, *116*(3), 250–264.

Siegler, R. S., & Shipley, E. (1995). Variation, selection, and cognitive change. In G. Halford & T. Simon (Eds.), *Developing cognitive competence: New approaches to process modeling* (pp. 31–76). Hillsdale, NJ: Lawrence Erlbaum Associates, Inc.

Siegler, R., & Shrager, J. (1984). *Strategy choices in addition and subtraction: How do children know what to do*, In C. Sophian (Ed.), The origins of cognitive skills (pp. 229–293). Hillsdale, NJ: Erlbaum.

Verguts, T., & Fias, W. (2005). Interacting neighbors: A connectionist model of retrieval in single-digit multiplication. *Memory & Cognition*, *33*(1), 1.

Vingerhoets, G., de Lange, F. P., Vandemaele, P., Deblaere, K., & Achten, E. (2002). Motor imagery in mental rotation: An fMRI study. *NeuroImage*, *17* (3), 1623–1633.

Widaman, K., Geary, D., Cormier, P., & Little, T. (1989). A componential model for mental addition. *Journal of Experimental Psychology: Learning, Memory & Cognition*, *15*(5), 898–919.

Wildgruber, D., Ackermann, H., Klose, U., Kardatzki, B., & Grodd, W. (1996). Functional lateralization of speech production at primary motor cortex: A fMRI study. *Neuroreport*, *7*(15–17), 2791.

Zhou, X., Chen, C., Dong, Q., Zhang, H., Zhou, R., Zhao, H., et al. (2006). Event-related potentials of single-digit addition, subtraction, and multiplication. *Neuropsychologia*, *44*(12), 2500–2507.

Zhou, X., Chen, C., Zang, Y., Dong, Q., Qiao, S., & Gong, Q. (2007). Dissociated brain organization for single-digit addition and multiplication. *NeuroImage*, *35*(2), 871–880.

Zhou, X., Chen, C., Zhang, H., Zhou, R., & Dong, Q. (2007). The operand-order effect in single-digit multiplication: An ERP study of Chinese adults. *Neuroscience Letters*, *414*(1), 41–44.

Zhou, X., & Dong, Q. (2003). The representation of addition and multiplication. *Acta Psychologica Sinica*, *35*, 345–351.

Ziegler, W., Kilian, B., & Deger, K. (1997). The role of the left mesial frontal cortex in fluent speech: Evidence from a case of left supplementary motor area hemorrhage. *Neuropsychologia*, *35*(9), 1197–1208.

Zurowski, B., Gostomzyk, J., Grön, G., Weller, R., Schirrmeister, H., Neumeier, B., et al. (2002). Dissociating a common working memory network from different neural substrates of phonological and spatial stimulus processing. *NeuroImage*, *15*(1), 45–57.