Title
Analytical Expressions for the Permeability of Random Two-Dimensional Poisson Fracture Networks Based on Regular Lattice Percolation and Equivalent Media Theories

Authors
Hestir, K.
Long, J.

Publication Date
1990-08-01

LBL-30289
UC-403

Lawrence Berkeley Laboratory
UNIVERSITY OF CALIFORNIA

EARTH SCIENCES DIVISION

Analytical Expressions for the Permeability of Random Two-Dimensional Poisson Fracture Networks Based on Regular Lattice Percolation and Equivalent Media Theories

K. Hestir and J. Long

August 1990

Prepared for the U.S. Department of Energy under Contract Number DE-AC03-76SF00098.

DISCLAIMER

Analytical Expressions for the Permeability of Random Two-Dimensional Poisson Fracture Networks Based on Regular Lattice Percolation and Equivalent Media Theories

Kevin Hestir[*,†] *and Jane Long*[*]

[*]Earth Sciences Division
Lawrence Berkeley Laboratory
University of California
Berkeley, California 94720

[†]Department of Mathematical Sciences
New Mexico State University
Las Cruces, NM 88003

August 1990

This report is published in abbreviated form as LBL-27625 in the *Journal of Geophysical Research*, vol. 95, no. B13, pp. 21,565–21,581.
This version contains the data on which the results are based.

This work was supported by the Manager, Chicago Operations, Repository Technology Program, Repository Technology and Transportation Division, of the U.S. Department of Energy under Contract No. DE-AC03-76SF00098.

Abstract

The permeability of random two-dimensional Poisson fracture networks can be studied using a model based on percolation theory and equivalent media theory. Such theories are usually applied on regular lattices where the lattice elements are present with probability, p. In order to apply these theories to random systems, we (1) define the equivalent to the case where $p = 1$, (2) define p in terms of the statistical parameters of the random network, and (3) define the equivalent of the coordination number, z. An upper bound for permeability equivalent to the case of $p = 1$ is found by calculating the permeability of a the fracture network with the same linear fracture frequency and infinitely long fractures. The permeability of networks with the same linear fracture frequency and finite fractures can be normalized by this maximum. An equivalent for p is found as a function of the connectivity, ζ, which is defined as the average number of intersections per fracture. This number can be calculated from the fracture density and distributions of fracture length and orientation. Then the equivalent p is defined by equating the average run length for a random network as a function of ζ to the average run length for a lattice as a function of p. The average run length in a random system is the average number of segments that a fracture is divided into by intersections with other fractures. In a lattice, it is the average number of bonds contiguous to a given bond. Also, an average coordination number can be calculated for the random systems as a function of ζ. Given these definitions of p and z, expressions for permeability are found based on percolation theory and equivalent media theory on regular lattices. When the expression for p is used to calculate the correlation length from percolation theory, an empirical formula for the size of the REV can be developed. To apply the models to random length systems, the expression for ζ must be modified to remove short fractures which do not contribute to flow. This leads to a quantitative prediction of how permeability decreases as one removes shorter fractures from a network. Numerical studies provide strong support for these models. These results also apply to the analogous electrical conduction problem.

1.0 Introduction

The calculation of flow through porous media is normally based on the assumption that the medium can be modeled as an equivalent continuum. Thus, in using a porous medium model, one makes the assumption that for some definable volume, the porous medium behaves like a continuum which can be characterized by a symmetric permeability tensor. This definable volume is at least as large as the REV, or representative elementary volume (Toth, 1967).

In recent years there has been a great deal of interest in extending the techniques used for porous media to flow through fractured rock. One of the first important contributions in this regard was developed by Snow (1965, 1969). Snow made the assumption that fractures are ''infinite'' in length. This assumption allows one to develop an analytical expression for the permeability of the fracture network because one can assume that the flow in each fracture is independent of flow in any other fracture. As a result the equivalent permeability can be found by simply summing the contribution of each of the fractures.

In reality, fractures in rock are not infinite. They occur on all scales, from the micro crack to the San Andreas Fault. So the permeability of the rock depends on the degree of interconnection in the fracture network. As Snow's infinite fractures are ''perfectly'' connected, the permeability of real ''partially'' connected fracture systems must tend to be less than that predicted by Snow. More importantly, the requirements for the continuum assumption may be invalid (Long *et al.*, 1982). Conductive fractures may be very erratically and sparsely connected such that the representative volume required for making the continuum assumption is extremely large. In fact the representative elementary volume (REV) may not exist if the scale of the observable features continues to increase with the scale of observation.

In order to study the theoretical aspects of this problem, we have used a simple two-dimensional Poisson network model where the fractures are essentially conducting pipes. In this model all the fractures are equally conductive and the location and orientation of each fracture is randomly chosen independent of every other fracture. We use this to study the permeability of partially connected networks as a function of the degree of interconnection and scale of measurement.

A previous paper (Long *et al.*, 1985) developed a relationship between average permeability and the density and length of fractures for the particular case of a Poisson fracture model where the fractures were all of the same length and conductivity and for a particular orientation distribution. This paper generalizes the relationship for permeability to any distribution of fracture length and orientation. Furthermore, these relationships are expressed analytically for all two-dimensional Poisson fracture networks with constant fracture conductivities. We also establish an analytical expression for the size of the REV.

Our approach has been to compare permeability data derived from the numerical parameter studies to mathematical models derived from a combination of spatial statistics, equivalent media and percolation theories. The parameter studies are designed to measure permeability as a function of (1) Degree of interconnection (''connectivity'') expressed as a function of the density of the fractures (number of fractures per unit area), the size distribution of the fractures (length distribution) and the orientation distribution, (2) Scale of Measurement and (3) Distance to the applied boundary conditions (degree of ''*in-situ*-ness'').

Equivalent media and percolation theories are used to find analytical expressions for permeability in terms of the connectivity. Both percolation theory and equivalent media theory are usually applied to regular lattices. In these problems either the bonds of the lattice (bond percolation) or the sites (site percolation) are allowed to be present with probability, p. Figure 1(a,b,c,d) shows some realizations of bond percolation networks for different values of p. As p increases, the connectivity of the network increases and therefore the permeability increases. These theories provide expressions for permeability as a function of p.

In a random network (Fig. 1 e, f, g, h) the orientation, density and length of the fractures determines the connectivity and as the connectivity increases the permeability increases. Application of expressions derived for percolation on a lattice to the problem of random networks comes down to finding a parameter, $\tilde{p}$, analogous to p, for the random network geometry. Further, to apply equivalent media theory, we need to know $\tilde{z}$, the coordination number. On a regular lattice, z is the number of bonds that can be attached to any site. In a random system, there is no clear equivalent for z.

The definition of $\tilde{p}$ is not obvious so we explore possible analytical expressions for $\tilde{p}$ as a function of the connectivity and substitute these expressions in the percolation equations. We choose one that appears to work the best and give evidence from our parameter studies that this expression holds for all homogeneous two-dimensional Poisson fracture networks with constant fracture conductivities. The permeability predicted by this expression is the permeability that one would measure at the scale of the REV. We propose a second expression for the scale of measurement needed to observe this permeability, i.e. the REV, and check this expression against the parameter study results. Finally, we check our definition of $\tilde{p}$ in the percolation theory equation near the critical limit.

The paper is organized as follows. We briefly describe the stochastic approach used to determine the fracture network to be modeled and the actual numerical calculations of flow and permeability. An expression for connectivity, ζ, is derived from the statistical parameters controlling the fracture network geometry. The probabilistic relationships between these parameters form the basis of the numerical parameter studies. Concepts from equivalent media and percolation theory are reviewed and alternative expressions for an analogous $\tilde{p}$ and $\tilde{z}$ on a random network are proposed. The results of the parameter studies are presented and an expression for permeability and REV are given as a function of connectivity, ζ, and as a function of $\tilde{p}(\zeta)$ and $\tilde{z}(\zeta)$, for the proposed equivalent model. We choose one set of expressions for $\tilde{p}$ and $\tilde{z}$ as superior to the others.

We have derived these relationships for random Poisson fracture systems where all the fractures have the same conductivity, but fractures in rock are not likely to be this unstructured. It is more likely that the fracture network will be complex. Fractures may occur in swarms or fracture zones and such complex features may occur on many scales. Furthermore, the density of fracturing may vary regionally. These heterogeneities are collectively known as "structure." All of the heterogeneities which occur in natural systems can also be thought of as statistical perturbations.

The effect of each perturbation is to increase the size of sample which is representative, i.e. to increase the REV to the point where no equivalent continuum can be defined. An REV determined by assuming that the system is structureless will therefore always be a minimum.

2.0 Fracture Flow Modeling Techniques

A numerical code has been developed to generate sample fracture systems in two dimensions and determine the permeability of such systems (Long *et al.*, 1982; Long, 1983). The two-dimensional mesh generator, FMG, produces random realizations of fractures (line segments) in a square region called a generation region (Fig. 2). Each fracture is generated independently and then the individual fractures are superimposed. The exact statistical model used for generating networks is detailed in Section 5.1. A constant value of conductance per unit length is assigned to each fracture. Intersections between the fractures are found and the network is coded for finite element analysis of fluid flow and the study of equivalent permeability.

2.1 Calculation of Permeability

When all the fractures have been generated, a flow region is selected for finite element analysis. The flow region is a square which is centered within the generation region at an arbitrary orientation (Fig. 2). A gradient (Fig. 3) is applied across the flow region by assigning appropriate values of hydraulic potential to the fractures where they intersect the boundaries of the flow region (see Long *et al.*, 1982, for details). For example, a hydraulic potential of one is assigned to all points where fractures intersect the inflow side, (Side 2 in Fig. 3). Zero hydraulic potential is assigned to the opposite side which is the outflow side, (Side 4 in Fig. 3). The other two sides (1 and 3) of the flow region are assigned hydraulic potentials which vary linearly from one to zero. These boundary conditions imitate those which would act on an arbitrary square region of arbitrary orientation in the interior of an anisotropic continuum where isopotentials and flow lines are not necessarily orthogonal.

The advantage of the linear boundary conditions described above (Fig. 3) is that they create a constant gradient field in a homogeneous medium of arbitrary anisotropy. In a generally anisotropic medium, closed boundaries on the lateral sides (Sides 1 and 3 in Fig. 3) would cause a perturbation of the gradient near the boundaries. Using the linear boundary conditions insures that each part of the fracture network is equally stressed. Because flow can enter Sides 1 and 3 under this choice of boundary conditions, flow into Side 2, Q_{in}, is not necessarily equal to flow out of Side 4 (Q_{out}).

Steady flux into the flow region in the direction of the gradient is calculated using a finite element program, LINEL which was developed by Wilson (1970). Flux in the elements is calculated under the assumption that flow in the fractures obeys Darcy's Law. The rock matrix is assumed to be impermeable so that there is only flow in the fractures. Steady state flow through the network is calculated by solving a series of equations which guarantee that mass balance is maintained at each node. These numerical methods also apply without change to electrical networks obeying Ohm's Law and Kirchhoff's Laws.

2.2 Directional Permeability

The permeability in the direction of the gradient, K_g, can then be calculated from J, the magnitude of the gradient applied across the flow region, and Q_{in}, the total flow into the flow region in the direction of the gradient:

$$K_g = \frac{Q_{in}}{J} \ .$$

Permeability can be measured in any direction, α, by rotating the boundaries of the flow region α degrees and consequently rotating the direction of the gradient. Note that Q_{out} for α is equal to Q_{in} for $\alpha+180°$.

For a homogeneous, anisotropic medium, permeability is expected to be a tensor property so that $1/\sqrt{K_g(\alpha)}$ versus α is an ellipse when plotted in polar coordinates (Marcus and Evanson, 1961; Marcus, 1962; Bear, 1972). However, for inhomogeneous fractured media, the tensor description fails and $1/\sqrt{K_g(\alpha)}$ may not plot as a smooth ellipse. In fact, the shape of a plot using measured values of $K_g(\alpha)$ for a given test volume of rock may be quite erratic. This plot can therefore be used as a test of whether or not the given volume can be approximated as a homogeneous porous medium.

A regression technique similar to that of Scheidegger (1954) is employed to quantitatively interpret the directional permeability data by determining a best-fit permeability tensor that in general will be anisotropic. In this technique we solve for the components of K_{ij} by minimizing the the normalized mean square error (NMSE) over a number of differently oriented measurements. The difference between the values of directional permeability (in the direction $n_i(\alpha)$ calculated using the tensor $K_{ij}n_i(\alpha)n_j(\alpha)$), and the actual values calculated from the model ($K_g(\alpha)$) is considered the "error." The NMSE which is minimized is then calculated via:

$$NMSE = \frac{R_{\min}}{\left[\dfrac{K_1 + K_2}{2}\right]^2} = \frac{\sum\limits_{\alpha}\left[K_g(\alpha) - K_{ij}n_i(\alpha)n_j(\alpha)\right]^2}{\left[\dfrac{K_1 + K_2}{2}\right]^2}$$

where K_1 and K_2 are the principal permeabilities and α ranges over a finite number of equally spaced angles from 0° to 180°. The NMSE approaches zero as the behavior of the fracture system approaches that of an anisotropic, homogeneous porous medium. The larger NMSE is, the less the fracture system behaves like a continuum. At the scale of the REV the NMSE should approach zero.

The above discussion is a synopsis of the more complete description found in Long *et al.* (1982) and Long (1983).

2.3 Proximity to the Boundary Conditions

Under linear boundary conditions, flow may be forced through fractures or fracture clusters that only transect the corner of the region. To see how such a flow can happen imagine a small cluster of fractures that intersects Sides 3 and 4 in Fig. 3. Because there is a non-zero potential difference between the intersections on Side 3 and the intersections on Side 4 there will be a net flux through the cluster. Now, if the boundary conditions are fixed further away, so that the cluster has much fewer intersections with Sides 3 and 4, the resulting flux will be lowered. This effect causes a consistent over-estimation of permeability which decreases in importance as the scale of measurement is increased. For similar reasons, closed lateral boundaries would cause a consistent under-estimation of permeability because clusters near Sides 1 and 3 are effectively turned off. This under-estimation effect from closed boundary conditions also decreases with scale of measurement.

To deal with the problem of over-estimation of permeability we have defined the study region (Fig. 2) which is centrally located inside the flow region. The width of the border between the flow region and the study region is called Δ. Now the boundary conditions are applied to the flow region and we examine the flux through the study region. Clearly this will decrease the corner flow effects discussed above. As Δ is increased, by fixing the size of the study region and enlarging the flow region, the flow through the study region will decrease. At the point where increases in Δ make no difference in flow through the study region we say that we are observing "*in situ*" behavior.

Use of the study region data rather than the flow region data creates some difficulties in the definition of the gradient, J. The gradient operating on the flow region is clearly defined by the applied boundary conditions. The gradient operating on the study region could be defined in two different ways. We can take J equal to the gradient operating on the flow region which we call the "regional gradient". Alternatively we can calculate a local gradient by calculating the hydraulic potential in each fracture at the point where the fracture crosses the boundary of the study region. The local gradient is calculated arbitrarily as an simple average based on these individual measurements. The local gradient is theoretically that which you would measure *in situ*. Problems with this kind of averaging have been discussed in Narasimhan (1980).

Clearly the regional gradient and the local gradient should be identical if the medium behaves as a continuum, but will be different in the non-continuum case. Because the medium behaves like a continuum on the scale of the REV, a comparison of permeabilities calculated with the two different gradients can also be used as a test of the REV.

2.4 Average Permeability

The parameter studies performed for this study deal with the change in behavior with scale of measurement. For each scale of observation, we record the average permeability and NMSE for the flow region as well as for each study region using both the local and the global gradient. The average permeability K, of a medium is the arithmetic mean of all the equally distributed measurements of directional permeability. This in turn is equal to the average (arithmetic mean) of the diagonal entries in the best fit permeability tensor, $K = \delta_{ij} K_{ij} / d$, where d is the dimension and δ_{ij} is the Kronecker delta. So average permeability is equivalent to taking the average of the principal permeabilities. We use only the average permeability as the measure of permeability in comparing systems of different connectivity and different scales. We have not studied the anisotropy of permeability in this paper.

2.5 Description of the REV

Given data on average permeability as a function of scale of measurement, we wish to define the REV for each degree of connectivity. Strictly speaking, the REV can be chosen at a scale which has these attributes:

(1) The permeability does not change with increase in scale of measurement.

(2) The value of NMSE is small (By experience, NMSE is much less than 1.0).

(3) The value of the permeabilities and NMSE's calculated with the global and local gradients are the same, ie. the local and global gradients are approximately equal.

When we wish to choose a value of "*in-situ*" permeability for the REV, we also wish to satisfy one other criterion:

(4) A decrease in the distance to the boundary conditions (Δ) does not change the calculated value of permeability.

Because of the limits of computation, it is not possible to find precise equality between calculations. Hence, some of the above criteria can be hard to verify numerically. To overcome this difficulty we essentially work backward We propose an expression for REV in terms of the connectivity of the fracture system. Then we check criteria 1–4 above at the scale of this calculated REV.

3.0 Percolation Theory and Equivalent Media Theory

Percolation theory and equivalent media theory are usually applied to problems on regular lattices ("percolation problems"). The amount of literature covering percolation problems is massive and one author points out that the number of publications per year is growing exponentially. Perhaps the most important reason for this is that the relationships derived from percolation theory are largely heuristic. There is a small but growing body of mathematical proof to support such conclusions (Kesten, 1987). However, it is mostly the weight of evidence from numerical studies that leads to the belief that these relationships are valid.

Two types of percolation problems can be defined: bond percolation or site percolation (Kesten, 1982). In either case, percolation theory and equivalent media theory describe the equivalent permeability of the lattices as the bonds or sites are randomly filled with probability, p. As p increases, clusters of bonds or sites are formed and these clusters increase in size with increases in p. At the critical probability, $p_{crit} < 1$, at least one cluster suddenly becomes infinite in size in what is called a critical phenomenon. Percolation theory looks at the conductance of these lattices, especially when $p \approx p_{crit}$. Equivalent media theory is applied to cases where $p >> p_{crit}$. These lattices are often studied with a Monte Carlo approach. For a given value of p, realizations of lattices of a given size are created and their properties studied. The expected value of permeability, K of such systems is a function of p and the size of the lattice used, i.e. scale of measurement. Our problem is to estimate the infinite scale permeability and the finite scale (REV) at which this permeability can be observed.

If we could observe these systems on an infinite scale, percolation theory tells us that we could immediately tell whether the value of p was above or below p_{crit}. Above critical, the lattice would be conductive, i.e. have at least one infinite cluster. Below critical, all clusters would be finite in size. However, we can only observe at finite scales. At finite scales some conductive graphs will be created even when $p < p_{crit}$. Also, for some cases that are above critical, the realization will not be conductive. As we increase the scale of measurement we should find that the frequency of conductive realizations decreases for systems that are below critical. On the other hand if the system is above critical, increases in scale will increase the percolation frequency. An estimate of p_{crit} is often made by finding the value of p for which the percolation frequency at any finite scale is 0.5 (Englman *et al.*, 1983).

Relationships between p and K have been deduced through Monte Carlo percolation and equivalent media studies. Orbach (1986) for one gives:

$$\frac{K}{K_{p=1}} \propto (p - p_{crit})^t$$

where $K_{p=1}$ is the permeability at $p = 1$.

For two-dimensional regular lattices, p_{crit} is between 0.35 to 0.65 depending on the lattice. The exponent t is thought to be independent of the lattice (for this reason t is called universal) and is about equal to 1.1 (Stauffer 1985). However, some studies give t as high as 1.3 but in any case we expect $t > 1.0$ (Zallen 1983). Because Equation 3.1 is a conjecture based on numerical study the symbol "$\propto$" is loosely defined and can be read as "goes as." It can mean anything from the ratio of the two sides tends to 1 to the ratio of the logarithms of the two sides tends to one (Kesten 1982). In practice we interpret "$\propto$" to be an approximate linear relationship:

$$\ln\left[\frac{K}{K_{p=1}}\right] = t \ln(p - p_{crit}) + constant \quad . \tag{3.1}$$

A second important relationship is for the correlation length, ξ, which is the scale on which a system looks homogeneous. Orbach (1986) gives:

$$\xi \propto | p - p_{crit} |^{-\nu}$$

where ν is thought to be exactly 4/3 for two-dimensional systems.

From the equivalent media literature, Kirkpatrick (1973) for example, one can find another relationship between K and p for the case where $p >> p_{crit}$ in a bond percolation model:

$$\frac{K}{K_{p=1}} = 1 - \frac{(1-p)}{(1-2/z)} \tag{3.2}$$

where z is the coordination number defined as the number of bonds coordinated with a site. For example, on a square lattice, z is four. This expression predicts a linear relationship between p and K. Kirkpatrick supports this relationship with perturbation analysis of the matrix equation for values of potential in the lattice.

In summary we have for bond percolation on a regular lattice:

(1) Permeability on the infinite scale becomes non-zero at $p > p_{crit}$;

(2) For $p > p_{crit}$ permeability initially increases exponentially in $p - p_{crit}$ (percolation theory) and then becomes linear in p (equivalent media theory);

(3) The REV increases exponentially as p approaches p_{crit}.

3.1 Application to Fracture Networks

In applying these theories to random fracture networks we try to resolve the following problem: What are p and z? The probability, p, is well defined for a regular lattice problem. In any given regular lattice it can be simply estimated by dividing the number of bonds (or sites) present by the total number available to fill. In random fracture networks it is not so clear what p is. In a regular lattice, z is an integer; 6 for a triangular lattice, 4 for a square lattice, and 3 for an hexagonal lattice. In a random line lattice, 4 line segments radiate from an intersection so one might expect that $z = 4$. However we suggest that for random lines $z < 4$. The reason is that there will always be pieces at the ends of every line that must necessarily hang unattached to the rest of the network. One can view this as having the effect of always making z less than 4.

In percolation problems on a regular lattice, an upper bound for conductivity exists for the case where $p = 1$. That is, all the lattice elements are present (Fig. 1d). For any case of $p \leq 1$, the resulting permeability can be normalized against the case of $p = 1$. Thus the normalized permeability will be 1 when $p = 1$. For random fracture networks, there is theoretically no end to the degree of fracturing. Each time a fracture is added to the system, the permeability increases *ad infinitum*. For fracture systems considered in this manner, there is no upper bound on permeability and it would be difficult to normalize the results of parameter studies. In other words, the fracture system which corresponds to the lattice with $p = 1$ can not be identified. We cannot determine how "relatively filled" a fracture network is because there is no obvious case which represents a "completely filled" lattice.

We solve the problem of defining p by studying systems where λ_l, the average frequency of fractures intersecting a sampling line, is held constant. Further, any network can be rescaled to a given constant value of λ_l. This way of looking at the system turns out to be the key to finding p. If λ_l is fixed, then the permeability of the fracture network will be a maximum if the fractures are infinite in length. This, in fact, is exactly the case studied by Snow (1969). Thus Snow's permeability can be taken as the permeability equivalent to the lattice case where $p = 1$.

4.0 An Upper Bound for Average Permeability Using Results of Snow

In this section we carefully define λ_l and Snow's upper bound. First, consider a two-dimensional Poisson network of line segments with random orientations given by the probability density function, $f(\phi)$. The Poisson process of fracture centers has density, λ_A and the average line length is $\bar{l}$. The rate of line segment intersections on a given reference line is $\tilde{\lambda}_l$. This rate is directly proportional to λ_A and $\bar{l}$. The effect of the orientation of the fractures relative to the reference line was given by Terzaghi (1965):

$$\tilde{\lambda}_l = \bar{l}\,\lambda_A\,E(\cos\phi) \ ,$$

where ϕ is the angle between the poles of the fractures and the reference line and

$$E(\cos\phi) = \int_0^{\frac{\pi}{2}} \cos\phi \, g(\phi) d\phi \ .$$

Therefore a line frequency corrected for orientation can be defined by:

$$\lambda_l = \frac{\tilde{\lambda}_l}{E(\cos\phi)} \ ,$$

which simplifies the above relationship to:

$$\lambda_l = \bar{l}\lambda_A \ . \qquad (4.1)$$

The quantity λ_l is equivalent to the average linear fracture frequency when all fractures are rotated so that they are perpendicular to the reference line, i.e. as if $\phi = 0$ for all fractures.

4.1 Snow's Permeability

If all fractures in a network extend to the boundaries, then the flow in each fracture does not depend on its intersections with other fractures. Hence, to calculate the total flux through such a system one can calculate the flux in each fracture individually and add the fluxes. Using superposition a simple expression for the permeability tensor of an infinite length fracture network can then be derived. The resulting tensor can be averaged over all directions which gives K_S, the average Snow's permeability. Let λ_l be the line rate of the system and t be the transmissivity of the fracture in its plane $[L^2/T]$, then K_S is given by (Snow 1965):

$$K_S = \frac{d-1}{d} t\lambda_l = k_0 \lambda_l \tag{4.2}$$

where k_0 is a constant and $d = 2$ or $d = 3$ dimensions. This expression is valid for 2-dimensional systems of linear fractures or 3-dimensional systems of planar fractures. The factor $\frac{d-1}{d}$ is required because there are $d-1$ directions where a fracture has transmissivity equal to t and one direction in which the fracture has a transmissivity of zero.

In summary, K_S is directly proportional to λ_l and does not depend on the orientation distribution. Computer studies support the statement that K_S is an upper bound but we have yet to verify this in a mathematically rigorous way.

As we have found a way to define a maximum permeability, $K_S = k_0 \lambda_l$, our work is reduced to finding a relationship between $\tilde{p}$, $\tilde{z}$, and the statistical parameters describing the Poisson network, such that the observed permeabilities can be explained with percolation and equivalent media theories. This we do in three steps. First, we define ζ, the connectivity of the network in terms of the statistical parameters. Then we propose relationships between ζ, $\tilde{p}$ and $\tilde{z}$. Finally, we use our parameter studies to verify that the proposed expressions for $\tilde{p}$ and $\tilde{z}$ yield relationships for permeability which fit Equations 3.1 and 3.2.

5.0 Statistics of the Fracture Model

Given the fracture (line segment) density, length and orientation distributions and constant conductance per unit length, we can generate a series of fracture networks and calculate the permeability of these networks. What we would like to do is find a direct relationship between these parameters and the permeability. To set the stage for this, we first give an expression for connectivity, ζ based on the statistical parameters of the random line system. This expression forms the basis of the parameter studies which are presented later.

Because permeability is measured at the scale of the REV, the measurement is made over a large realization of the Poisson line system. This results in a spatial average that gives a permeability largely independent of the realization which obviates the need for extensive Monte Carlo analysis.

5.1 A Measure of Connectivity

Several authors have used the average number of intersections per fracture, ζ, as a measure of the connectivity in a random line processes (Robinson, 1984a, and Charlaix *et al.*, 1986). In this section we give an expression for ζ as a function of the statistical parameters governing the network and describe a few of its properties. First, we define ζ and our line process:

(i) Points are placed in the 2-dimensional plane with coordinates (X_i , Y_i) according to a Poisson process $\mathbf{P}$, with fixed density λ_A.

(ii) At each point (X_i , Y_i), a line segment, $\mathbf{L}_i$, is placed so that the center of the line segment lies on the point (X_i , Y_i). We refer to this line segment as a fracture.

(iii) Each line segment $\mathbf{L}_i$, is given a random length L_i and orientation Θ_i.

(iv) The Poisson process, lengths, and orientations above are all statistically independent. In particular, $\mathbf{P}$, L_1 , Θ_1 , L_2 , Θ_2 , . . . are independent.

(v) To define ζ, let the random quantity Ψ be constructed by the following experiment: Chose a line at random from the line process, let Ψ be the number of lines intersecting the chosen line. We define ζ to be the average value of Ψ, $\zeta = E(\Psi)$.

With this model it can be shown that:

$$\zeta = \lambda_A (\bar{l})^2 H(\Theta) = \lambda_l \bar{l} H(\Theta) \tag{5.1}$$

where

$$H(\Theta) = \int_0^{\pi}\int_0^{\pi} \sin|\theta_0 - \theta|\ g(\theta)\, g(\theta_0)\, d\theta\, d\theta_0 \ .$$

is the orientation correction factor. Note that $0 \le H(\Theta) < 1$ for any orientation distribution.

We give two examples of values of $H(\Theta)$ for different orientation distributions. When Θ is uniformly distributed on $[\,0\,,\pi\,]$ ie., $g(\theta) = \frac{1}{\pi}$, then:

$$H(\Theta) = \frac{2}{\pi} \ .$$

For two orthogonal sets of lines of equal frequency there is not a probability density but we can still evaluate the integral taking $g(\theta) = \frac{1}{2}\delta(\theta) + \frac{1}{2}\delta(\theta - \frac{\pi}{2})$ where δ is the Dirac delta function. This yields:

$$H(\Theta) = \frac{1}{2} \ .$$

6.0 Possible Expressions for Permeability

In this section we present functional forms for normalized permeability, K/K_S. The first is based on work presented by Englman *et al.* (1983). The second is from Robinson (1984a) and the third is our own. Each of the models give an expression for an analogous $\tilde{p}(\zeta)$ in a different way. A minimal requirement for the expression $\tilde{p}(\zeta)$ to work in both a percolation model and equivalent media model should be:

$$\lim_{\zeta \to 0} \tilde{p}(\zeta) = 0$$

and

$$\lim_{\zeta \to \infty} \tilde{p}(\zeta) = 1 \ .$$

Expressions for $\tilde{p}$ and $\tilde{z}$ are substituted into Equation 3.1 or 3.2 to obtain an expression for K/K_S in terms of ζ. In Section 9 we compare the models' predictions with numerical results.

6.1 Englman, Gur, and Jaeger Model

Englman *et al.* (1983) proposed

$$\tilde{p}(\zeta) = \tilde{p}_E(\zeta) = 1 - e^{-\zeta_E} \tag{6.1}$$

where $\zeta_E = \pi\lambda_A\overline{l}^2$. No explicit correction for orientation was included. Thus ζ_E is slightly different from our ζ. In the following, we substitute our expression for ζ rather than ζ_E because we wish to see if Equation 6.1 is reasonable for different orientation distributions.

The basis of Englman *et al.*'s relationship between ζ and p is that in the case of constant lengths, $1 - e^{-\zeta}$ is the probability that a line chosen at random from the system has at least one intersection. Substitution into Equation 3.1 gives:

$$\frac{K}{K_S} = \kappa\,[e^{-\zeta_{crit}} - e^{-\zeta}]^t \ . \tag{6.2}$$

Although Englman *et al.* only discussed using this expression with respect to percolation theory, we can try substituting $\tilde{z} = 4$ and Equation 6.1 into Equation 3.2:

$$\frac{K}{K_S} = 1 - 2e^{-\zeta} \ . \tag{6.3}$$

6.2 Robinson's Porosity Model

In this formulation (Robinson, 1984a), $\tilde{p}(\zeta)$ takes the role of the average fraction of a line segment which is available for flow. To briefly recount the derivation, consider a arbitrary line segment L with $n(L)$ intersections. For a system with constant line lengths, $n(L)$ will be a Poisson process with rate ζ. One can show that the average fraction of L available for flow (that fraction which is between the two end sites on L) is $\frac{n(L)-1}{n(L)+1}$. When averaged over all possible values of $n(L)$, this gives an average fraction of

$$\tilde{p}(\zeta) = \tilde{p}_R(\zeta) = \sum_{n=2}^{\infty} \frac{n-1}{n+1} P(n(L) = n) \tag{6.4}$$

$$= \sum_{n=2}^{\infty} \left(\frac{n-1}{n+1} \frac{\zeta^n}{n!} e^{-\zeta}\right) = \left(1 + \frac{2}{\zeta}\right)(1+e^{-\zeta}) - \frac{4}{\zeta} \ .$$

With this definition of $\tilde{p}$ Equation 3.1 gives:

$$\frac{K}{K_S} = \kappa\,[\tilde{p}_R - \tilde{p}_{crit}]^t \tag{6.5}$$

Robinson suggests using a coordination number of 4. Substituting this in Equation 3.2 we obtain the expression:

$$\frac{K}{K_S} = 1 - 2(1 - \tilde{p}_R(\zeta)) \ . \tag{6.6}$$

6.3 Average Run Length Model With Modified Coordination Number

In this model we find $\tilde{p}(\zeta)$ as a function of ζ by finding the average length of a line in the bond model as a function of p and the average length of a line in the random line model as a function of ζ and then equating the two averages. We describe this in detail below.

First consider the bond percolation model where p is the probability that a particular bond is present. Think of a given existing bond as a piece of a line in a random line system, Fig. 4a. To make this given bond a piece of a line which is k bonds long we would have to attach a total of $k-1$ bonds to the given bond followed by no bonds on the ends. This can be done a total of k different ways each with probability $p^{k-1}(1-p)^2$. Thus the probability that a given bond is in a line k bonds long is:

$$kp^{k-1}(1-p)^2\,, \quad k = 1\,, 2\,, \ldots\,.$$

The resulting average line length in units of bonds is:

$$\sum_{k=1}^{\infty} k(1-p)^2 p^{k-1} k = \frac{1+p}{1-p}\ .$$

Now consider an arbitrary line in the random line system. One can view that line as being made up of bonds, each bond being a piece of the line between two intersections, Fig. 4b. The average length of a line in units of bonds is then the average number of intersections per line plus 1, that is $\zeta+1$.

Equating the two average lengths described above gives:

$$\zeta+1 = \frac{1+\tilde{p}(\zeta)}{1-\tilde{p}(\zeta)}$$

where we use $\tilde{p}(\zeta)$ because we are in the random line segment domain. Finally, solving for $\tilde{p}(\zeta)$ gives:

$$\tilde{p} = \tilde{p}_{HL}(\zeta) = \frac{\zeta}{\zeta+2}\ . \tag{6.7}$$

The percolation model is then:

$$\frac{K}{K_S} = \kappa \left[\frac{\zeta}{\zeta+2} - \frac{\zeta_{crit}}{\zeta_{crit}+2} \right]^t . \tag{6.8}$$

One might argue that the number of bonds we calculate from intersections is too high in the random line system. For example, if a line segment has 2 intersections perhaps we should view it as being only 1 bond long (instead of 3 bonds long) because 2 bonds hang unattached at the ends. Heuristically, we compensate for this by modifying the coordination number. This modified coordination number $z(\zeta)$, for a random line system is given below.

The coordination number of a regular lattice is the number of bonds connected to a site. In the rectangular lattice in Fig. 1(a–d) the coordination number is $z = 4$. In a random line system we propose an average coordination number:

$$\tilde{z}(\zeta) = \tilde{z}(\zeta) = 4(\,1 - \frac{1}{\zeta}\,)\ . \tag{6.9}$$

This modified coordination number coupled with $\tilde{p}_{HL}(\zeta)$ in Equation 3.2 then yields:

$$\frac{K}{K_S} = 1 - \frac{\tilde{z}(\zeta)(1-\tilde{p}_{HL}(\zeta))}{\tilde{z}(\zeta)-2} = \frac{\zeta(\zeta-4)}{\zeta^2-4} \ . \tag{6.10}$$

The reason for a coordination number smaller than $\tilde{z} = 4$ is shown in Fig. 4b. Here we can see that the two intersections on the ends of a typical line only have coordination number $\tilde{z} = 3$; the ends of the line do not count because they will never contribute to flow. Clearly, other intersections could have coordination numbers ranging from 2 to 4. To find the average coordination number for all sites we can use the expression

$$\tilde{z}(\zeta) = \frac{\textit{total number of bonds radiating from sites}}{\textit{total number of sites}} = \frac{\sum (2n_i - 2)}{\frac{1}{2}\sum n_i} = 4(1-\frac{1}{\zeta})$$

where n_i denotes the number of intersections on the i^{th} fracture, $2n_i-2$ is the number of bonds that the i^{th} fracture radiates from these intersections, and

$$\zeta = \textit{average number of intersections per fracture} = \frac{\sum_{i==1}^{N} n_i}{N} \ .$$

The analogy we have used to derive $\tilde{p}$ and $\tilde{z}$ is clearly not the only one that can be made. So the question arises: what makes a good analogy? The obvious answer is that the model produced by the analogy must make successful predictions of permeability which ours does. Of course this does not prove that ours is only possible model that can give good predictions of permeability. There may be others, as yet undiscovered. A hint of this appears in Section 9 where we show that the very different model given by Robinson works as well as ours in the constant length cases but fails for random lengths.

7.0 Connectivity and Permeability for Variable Fracture Lengths.

The Englman *et al.* (1983) expression seems to account for random length in that ζ is given as a function of $\bar{l}^2$. This model implies that the only property of the length distribution important to permeability is mean length. For random length systems Robinson (1984a) proposed a modification of $\tilde{p}_R(\zeta)$ directly, without calculating a change in ζ. The change in $\tilde{p}_R(\zeta)$ can be calculated based on the fact that the number of intersections on a line segment of length L, $n(L)$ will have a Poisson distribution with frequency $\lambda(l) = l\lambda_A \bar{l} H(\Theta)$ where l is the length of L. This yields:

$$\tilde{p}_R(\zeta) = \int_0^\infty \left[(1 + \frac{2}{\lambda(l)})(1+e^{-\lambda(l)}) - \frac{4}{\lambda(l)} \right] f(l)dl \quad . \tag{7.1}$$

As will be seen in Section 8, these models do not give good results when the standard deviation of fracture length is large. We believe that the problem has to do with the fact that when the standard deviation is large, there are fractures which are relatively small. These fractures do not contribute to the permeability because they essentially provide no new connections. So one could remove these fractures without changing the permeability. However, the removal of small fractures will raise $\bar{l}$ and decrease λ_A which changes the value of ζ, K_S, and $\tilde{p}(\zeta)$. By removing various percentages of these unimportant fractures, we could have many networks with the same permeability but different connectivities which would imply a non-unique relationship between connectivity and permeability. In this section we show how to recalculate ζ for random length systems by eliminating all lines with length less than a cutoff c. The cutoff can be chosen such that we eliminate the maximum number of fractures without changing the permeability. In this way we obtain a unique relationship between connectivity and permeability in a random length system.

7.1 ζ and K_S as Function of Cutoff, c

When we eliminate the short fractures, λ_A, $\bar{l}$, K_S, and ζ will all be changed. Let $\lambda_A(c)$, $\bar{l}(c)$, $K_S(c)$, and $\zeta(c)$ denote the values of λ_A, $\bar{l}$, K_S, and ζ respectively when all lines shorter than a cutoff c are deleted from a system. In this section we find expressions for these quantities.

We begin by finding expressions for fracture density and average length as functions of c. A correction factor, $G(c)$, for the areal fracture density is given by:

$$G(c) = \int_c^\infty f(l)dl$$

where $f(l)$ is the probability density function of fracture length. So after truncation at c the density will be:

$$\lambda_A(c) = \lambda_A G(c) \ .$$

To find the new average length, calculate the correction factor $F(c)$:

$$F(c) = \int_c^\infty lf(l)dl \ .$$

This gives average line length after truncation at c to be:

$$\bar{l}(c) = \frac{F(c)}{G(c)} \ .$$

Hence, from Equation 5.1, truncation at c will yield a new value for ζ given by:

$$\zeta(c) = (\bar{l}(c))^2 \lambda_A(c) H(\Theta) = \lambda_A \frac{F^2(c)}{G(c)} H(\Theta) \ . \tag{7.2}$$

As K_S is proportional to λ_l (Equation 4.2) and $\lambda_l = \bar{l}\lambda_A$ (Equation 4.1), it is clear that truncation at c will also yield a new value for K_S given by:

$$K_S(c) = k_0 \lambda_l(c) = k_0 \bar{l}(c) \lambda_A(c) = k_0 F(c) \lambda_A \ . \tag{7.3}$$

7.2 Permeability in a Random Length System

For constant length systems we have defined the model via Equation 6.10 for cases above critical:

$$K = K_S \frac{\zeta(\zeta - 4)}{(\zeta^2 - 4)} \ .$$

Substitution of the expressions for $\zeta(c)$ and $K_S(c)$ into this model yield the expression:

$$\tilde{K}(c) = K_S(c) \frac{\zeta(c)(\zeta(c) - 4)}{(\zeta(c)^2 - 4)} \tag{7.4}$$

or

$$\tilde{K}(c) = k_0 F(c)\lambda_A \left[\frac{\lambda_A F^2(c)H(\Theta)\left[\lambda_A F^2(c)H(\Theta) - 4G(c)\right]}{\lambda_A{}^2 F^4(c)H^2(\Theta) - 4G^2(c)} \right] .$$

In order to pick the appropriate value of c, we examine the functions $\zeta(c)$, $K_S(c)$, and $\tilde{K}(c)$. As an example, these functions are plotted in Fig. 5 (with $k_0 = 1[L^2T^{-1}]$) for the set of parameters given below in Section 8, Case s. As shown below the qualitative behavior illustrated in Fig. 5 is quite general. Now, in Fig. 5 we see that $K_S(c)$ decreases with c as one would expect. Also, $\zeta(c)$ increases with c and then decreases. The interesting function is $\tilde{K}(c)$. We can see that $\tilde{K}(c)$ is first increasing in c, then decreasing attaining a maximum at a value denoted by c_0. It is obvious that removal of fractures from a network can not result in an increase in permeability. Therefore, the function $\tilde{K}(c)$ does not correspond to the permeability of our truncated random length systems for values of $c < c_0$. We assert however, that the maximum value of $\tilde{K}(c)$ (ie., $\tilde{K}(c_0)$) can be used to predict the permeability of the full untruncated random length system. The final model for permeability in a random length system will then be given by:

$$K = \tilde{K}(c_0) = K_S(c_0)\frac{\zeta_{HL}(\zeta_{HL} - 4)}{(\zeta_{HL}^2 - 4)} \tag{7.5}$$

where c_0 is a value of c that maximizes $\tilde{K}(c)$ and $\zeta_{HL} = \zeta(c_0)$. A general analytical solution for maximizing $\tilde{K}(c)$ is hard to find but a solution can easily be found numerically for any given case. It is interesting to note that in general $\zeta_{HL} = \zeta(c_0)$ is not the maximum value of ζ as one might expect.

Qualitative Properties of $K_S(c)$, $\zeta(c)$, $\tilde{K}(c)$, and Existence of c_0 in General

If we assume that $f(l) > 0$ for $l > 0$ and $\bar{l} < \infty$ then the qualitative aspects of Fig. 5, including the existence of c_0, are quite general. To see this first note that $\bar{l} < \infty$ implies that $F(c)$ is decreasing in c. Because $K_S(c)$ is proportional to $F(c)$, $K_S(c)$ is also decreasing in c.

To examine $\zeta(c)$ use the expression

$$\frac{d\zeta(c)}{dc} = \lambda_A H(\Theta)\frac{cF(c)f(c)}{G(c)}\left[\frac{F(c)}{cG(c)} - 2\right] .$$

Now the term

$$\eta(c) = \frac{F(c)}{cG(c)} - 2$$

clearly determines the sign of $\frac{d\zeta(c)}{dc}$. For $c \approx 0$ with $c > 0$ we have $F(c) \approx \bar{l}$ and $G(c) \approx 1$ so that $\eta(c) > 0$ and $\zeta(c)$ is increasing. Next we notice that $cG(c) < F(c)$ so $\lim_{c \to \infty} cG(c) = 0$. An application of l'Hospital's rule gives

$$\lim_{c \to \infty} \frac{F(c)}{cG(c)} = \lim_{c \to \infty} \frac{cf(c)}{G(c) + cf(c)} \leq 1 \ .$$

So for large enough c, $\eta(c) < 0$ and $\zeta(c)$ is decreasing. Therefore $\zeta(c)$ attains a maximum for some $c_\zeta > 0$.

Finally to show that c_0 exists examine the right hand side of Equation 7.4 to see that $\tilde{K}(c)$ is decreasing for $c > c_\zeta$. Hence, there is a $c_0 \leq c_\zeta$ where $\tilde{K}(c)$ attains a maximum and c_0 exists. In situations where c_0 is not unique we can take the largest maximizing value of c for the unique cutoff.

For length distributions concentrated on an interval $[a,b)$ with $0 < a$ and $b \leq \infty$ similar arguments can be made for the existence of c_0 which in this case may be equal to the lower bound on length, a.

Behavior of Permeability with Truncation of Shorter Fractures

The maximization principle from Equation 7.5 gives an expression for $K(c)$, the permeability of a system with all lines shorter than c removed:

$$K(c) = \begin{cases} \tilde{K}(c_0) & \text{if } c \leq c_0 \\ \tilde{K}(c) & \text{if } c \geq c_0 \end{cases} \qquad (7.6)$$

where $\tilde{K}(c)$ and c_0 are defined by the untruncated length distribution. Qualitatively, this says that for a given length distribution, orientation distribution and areal rate, there is a critical cutoff, c_0, so that permeability of a random line system will not change if we delete all lines with length less than c_0. To illustrate this concept Fig. 6a shows the behavior of $K(c)$ based on Equation 7.6

for the system from Case p described in Section 8, (with $k_0 = 1[L^2T^{-1}]$). Figures 6b–e show example networks taken from this case. The networks corresponding to the untruncated system and the system truncated at c_0 are shown in Figs. 6b and 6d. The respective conducting portions of these systems are shown in Figs. 6c and 6e. One can see by inspection that the reduced networks are nearly the same which explains why small fractures have a negligible effect on permeability.

7.3 Limitations of the Maximization Principle

Equation 7.5 has been derived for the realm of equivalent media theory, $\tilde{p} >> \tilde{p}_{crit}$ and may not apply for values of ζ close to ζ_{crit}. For this realm, an expression similar to Equation 7.5 could be derived by substituting Equations 7.2 and 7.3 into Equation 6.8 for percolation theory. However, our parameter studies show that the equivalent media expression (Equation 7.5) works well until ζ is fairly close to ζ_{crit} (see Section 9.0 and Fig. 11).

8.0 Parameter Studies

In this section we describe the parameter studies that verify the relationship between permeability and ζ. We also explain how finite scale calculations were used in conjunction with an expression for the REV to obtain estimates of permeability at the infinite scale. Finally, we explain how the data were used to estimate parameters for application of percolation theory.

8.1 Description of Parameter Studies to Estimate Permeability

Twenty-two parameter settings have been used to generate data for checking the proposed models for permeability. These are Cases a–v. Tables 1–5 give brief descriptions of each case. In each of the cases, a different seed was chosen for the pseudo-random number generator used to create the realization. The parameter values for the 22 different cases were chosen to illustrate the fact that ζ can be used to predict permeability for a wide variety of random line systems. In addition to these 22 cases, another series of cases (w, x and y) were generated to find a value for ζ_{crit}. The particulars are described below.

All parameter values are given in a dimensionless form. To illustrate what values one might get in cgs units we can look at Case a, Table 1. Here we would have $\lambda_A = 0.0576$ line centers per cm^2, l = 10cm, and $\lambda_l = 0.576$ lines per cm. The value of k_0 is obtained from the transmissivity of fractures, t, say 1×10^{-5}cm^2/sec, which gives

$$k_0 = \frac{d-1}{d} t = 5 \times 10^{-6} \text{cm}^2/\text{sec} \ .$$

Similarly, any other set of consistent units can be applied.

Cases a–h (Table 1) are called the length density study. All of these cases have approximately the same value of $\lambda_l = 0.576$ $[L^{-1}]$ with each case having different constant length line segments. So, in each of these cases we generate a system of constant length lines with l and λ_A adjusted so that λ_l remains at 0.576 $[L^{-1}]$. All of these cases have the same orientation distribution which is a mixture of two normals, one with an average of 0° and a standard deviation of 20°, and the other with an average of 90° and a standard deviation of 20°. This orientation distribution is equivalent to generating two sets of fractures where each set is assumed to have the same areal density and constant length. A numerical calculation gives $H(\Theta) = 0.624$ for this orientation distribution.

Cases i–k, (Table 2) are the orientation distribution study. Each case has the same area rate and constant line lengths giving a value of $\lambda_l = 0.576$ $[L^{-1}]$. These cases differ only in the standard deviation of their orientation distributions. Each of these cases has a normal orientation distribution with different standard deviation which has the effect of changing $H(\Theta)$ and hence ζ. This tests the effect of the orientation correction factor $H(\Theta)$.

Cases l–o (Table 3) are the normalization study. Each of these cases has a value of λ_l approximately equal to 0.288 $[L^{-1}]$ which is about half that of the other cases which gives a value for K_S that is half that of the other cases. This tests the effect of having a different normalization factor K_S. Each of these cases has constant length line segments with a normal orientation distribution with a standard deviation of 70° yielding $H(\Theta) = 0.583$.

Cases p–s (Table 4a,b) are the random length study. The first three cases (p, q and r) have lognormal distributions for length with different coefficients of variation (ratio of the standard deviation to the mean). Case s has an exponential distribution for length. Each of these cases has the same orientation distribution as that used in Cases a–h. Values for K_S and ζ_{HL} have been calculated using the truncation analysis described in Section 7. Approximate values for c_0, $\zeta_{HL} = \zeta(c_0)$, and $K_S(c_0)$ in the the random length cases we have studied are shown in Table 4b. These values were found by numerically maximizing $\tilde{K}(c)$ in Equation 7.4.

Cases t–v (Table 5) are the near critical study. Each of these cases has constant length lines, $\lambda_l \approx 0.576$ $[L^{-1}]$, and a uniform orientation distribution which gives $H(\Theta) = 2/\pi$. These cases are used to study permeability of system near ζ_{crit}, the critical value of ζ.

8.2 Parameter Studies to Estimate ζ_{crit}

A final parameter study, cases w, x and y, were used to find a value for ζ_{crit}. In this study, we generated many networks at different values of ζ close to the critical value. For each value of

ζ, we counted the percentage of connected graphs. The value of ζ_{crit} was estimated to be the value of ζ which produced 50% connected graphs and 50% non-connected. This study was repeated 3 times for 3 different orientation distributions which are representative of the orientation distributions we have studied. In Table 6 the three different cases are listed along with the estimated values of ζ_{crit}. The estimates were obtained by fitting data to a least squares line of the form $y = m\zeta + b$ where y is the percentage of connected networks in 20 trials. Using the least squares values of m and b the value of ζ that gives $y = 50\%$ can be calculated. This is the estimate given in Table 6.

In summary, we found $\zeta_{crit} \approx 3.6$ for all 3 cases. Hence, we take this as the value of ζ_{crit} in the cases we have studied. This value also agrees with that obtained by Pike and Seager (1973) where only the uniform orientation distribution was considered. There are orientation distributions where $\zeta_{crit} \approx 3.6$ is not a good estimate. For example, Robinson (1984b) has found $\zeta_{crit} \approx 3.1$ for an orientation distribution that gives two orthogonal sets of fractures.

8.3 An Expression for REV

The quantity ξ, called the correlation length, is fundamental in percolation theory. One can heuristically view ξ as the scale at which a percolation system for p near p_{crit} looks homogeneous (Orbach, 1986). The percolation scaling law for ξ is:

$$\xi \propto |p - p_{crit}|^{-\nu} \tag{8.1}$$

where ν is thought to be exactly 4/3 in two dimensions.

In this study we are faced with the problem of determining when a finite scale is large enough to give a good estimate of infinite scale permeability. This scale is what we call the REV. For a particular realization the REV is hard to determine precisely so we need a quantitative rule for finding it. To a hydrologist, the heuristic behind ξ is suggestive of the concept of REV so we use Equation 8.1 as a basis for an empirical formula for REV. Using $\tilde{p}_{HL}$ which fits the data best (as shown in Section 9) we obtain the following:

$$\text{REV} = a\frac{\bar{l}}{\zeta}\left[\tilde{p}_{HL}(\zeta) - \tilde{p}_{HL}(\zeta_c)\right]^{-4/3} \tag{8.2}$$

where a is a constant. The reason for the factor of $\bar{l}/\zeta$ is that $\bar{l}/\zeta$ is the average length between sites in the random line system and is therefore a natural length scale. The choice of the constant a is somewhat arbitrary. Generally, one wants to make a large enough so that the criteria in Section 2.5 are met approximately. We have found that $a = 15$ works well.

It is important to stress that Equation 8.2 is only an empirical formula used to define a scale of measurement. There are many unresolved issues concerning the application of Equation 8.2 for finding REV that are not addressed in this paper. For example, the authors first tried to estimate a and ν by first finding REV for each realization then solving Equation 8.2 for a and ν. This leads to inconsistent estimates because of the difficulty in precisely determining REV from a particular realization. Also, one reviewer has pointed out that $\bar{l}/\zeta$ is proportional to $1/\lambda_l$ (as can be seen from Equation 5.1) so that $1/\lambda_l$ may be a better natural length scale than $\bar{l}/\zeta$. We chose $\bar{l}/\zeta$ because it contains the orientation correction factor and $1/\lambda_l$ does not. However, its not clear which is better. For these reasons we reiterate that Equation 8.2 is merely a convenient guideline for finding REV that has worked well with the cases we have studied.

8.4 Estimation of Infinite Scale Permeability

Permeability, K, for each of these cases was found numerically by generating a realization and calculating the average of permeability in 24 directions, at larger and larger sizes (in increments of 60 or 20 dimensionless length units) for the flow and study regions. We will call these the scale effects studies. Figure 7 shows example normalized permeability for a few selected cases. Many intermediate values of Δ were used, but for simplicity only $\Delta = 0$ and $\Delta = 180[L]$ are shown on the plots. Permeabilities for $\Delta = 60[L]$ are plotted for Case h in Fig. 7 as an example comparison of different Δ's. The horizontal dotted lines in Fig. 7 are the predicted values of K/K_S given by Equation 7.5 (which is equivalent to Equation 6.10 in the constant length case).

For each of the Cases a–v, our goal is to determine a value for average permeability at the infinite scale from the finite scale effects studies. Practically, we have obtained estimates of infinite scale average permeability in the following steps:

(i) The expression, Equation 8.2, for REV based on percolation theory with $a = 15$ was applied. The resulting REV is given in Tables 1–5 and plotted in the example Cases of Fig. 7 with a dashed vertical line.

(ii) To minimize boundary effects we estimate average permeability at the infinite scale using a study region with a large value of Δ. To prevent single fracture connections between the study region and the boundary, the value of Δ should be at least as large as the average fracture length (and several times larger if possible). Because the largest average fracture length in the parameter studies was $124[L]$, and most were much shorter, we chose to use $\Delta = 180[L]$. In some of the cases it was within the capabilities of our computer to calculate average permeability in a study region at scales larger than the REV with $\Delta = 180[L]$ (e.g. Case h). In these cases the infinite scale average permeability was taken to be the average permeability of the study region (with $\Delta = 180[L]$) with size just above the REV.

(iii) In other cases, due to computer limitations, it was not possible to calculate average permeability in a study region (with $\Delta = 180[L]$) at the scale of the REV, (e.g. Case b). In these cases the average permeability at the infinite scale was estimated by the average permeability of the study region (with $\Delta = 180[L]$) at the largest possible scale.

Table 7 shows that in general, the scale effect data supports our choice of REV according to criteria 1–4 in Section 2.5. In Table 7 we can see that at the scale of the REV the NMSE was always small, on the order of 10^{-3}. Table 7 also shows that the difference in permeability measured with local and global gradients (as described in Section 2.3) is small at the scale of the REV. Also, an examination of scale effects plots, such as those plotted in Fig. 7, shows that at the REV permeability is not changing much with the scale of measurement.

8.5 Estimation of Percolation Parameters

Application of the expressions we have proposed for $\tilde{p}(\zeta)$ as a function of ζ to percolation theory, gives a relationship of the form

$$\frac{K}{K_S} = \kappa(\tilde{p}(\zeta) - \tilde{p}(\zeta)_{crit})^t \quad (8.3)$$

where we must find values for κ and t. Approximate values for t in regular two-dimensional lattices range from 1.1–1.3 (Zallen 1983 and Stauffer 1985).

To determine t and κ in our setting we have plotted $log(p - p_c)$ vs. $log(K/K_S)$ as shown in Fig. 8. In this figure the relationship given in Equation 8.3, should be a straight line with slope = t. The constants t and κ can be determined by fitting a least squares line to the data. Because Equation 8.3 applies to values of $\tilde{p}(\zeta)$ near $\tilde{p}(\zeta)_{crit}$ the least squares line should only be fit to data where $\tilde{p}(\zeta)$ is near $\tilde{p}(\zeta)_{crit}$. Table 8 shows the slopes of least squares lines applied to different ranges of data near $\tilde{p}(\zeta)_{crit}$. One can see the estimate of t using the Englman model is sensitive to the range of values of $\tilde{p}(\zeta)$ used. In our model (including random lengths) and in Robinson's model (excluding random lengths) an exponent of $t \approx 1.1$ holds for a range of values of $\tilde{p}(\zeta)$ even far away from $\tilde{p}(\zeta)_{crit}$. The lines shown on Fig. 8 are the least squares lines using data for $4.0 < \zeta < 10$. As ζ approaches ζ_{crit}, the scale of the REV increases very rapidly. Therefore, the data point at $\zeta = 3.725$ was excluded because its proximity to ζ_{crit} makes our estimate of its infinite scale permeability unreliable.

9.0 Evaluation of the Models

The permeability data from Section 8.1 is plotted on three sets of figures, one for Englman *et al.*'s (1983) model, one for Robinson's (1984a), and one for ours (Figs. 9, 10 and 11 respectively). On each figure, K/K_S versus ζ and K/K_S versus $\tilde{p}$ are shown. On Figs. 10 and 11, K/K_S versus $\tilde{p}$ are plotted on two scales, one for $\tilde{p}_{crit} < \tilde{p} < 1$ and one for $\tilde{p} \approx \tilde{p}_{crit}$. On all plots, the dashed curve represents the percolation model using the parameters derived for each model as shown in Table 8 for $M = 10$. The dotted line is the equivalent media model. The data points are differentiated in two ways. "Stars" are measurements that were made above the calculated value of REV. Inverted triangles are the best available estimates made below the REV. Points with boxes drawn around them are random length cases. The points $\zeta = \zeta_{crit}$, $K/K_S = 0$ and $\tilde{p} = \tilde{p}_{crit}$, $K/K_S = 0$ are shown on each plot with a circled "+" and should be distinguished from other data points.

In Fig. 9, we see that Englman *et al.*'s model does not work at all for values of ζ well above critical. It is particularly bad for random length cases. However, near the percolation limit, there is a reasonable match with the percolation model.

Figure 10, shows that Robinson's model is extremely good except for the random length cases which are conspicuously different from the model predictions. For these cases, ζ was calculated using no truncation analysis and $\tilde{p}_R$ was calculated using Equation 7.1. One nice feature of this model is that the percolation and equivalent media models are extremely close to each other.

In Fig. 11 the Hestir and Long percolation and equivalent media models are not as close to each other as Robinson's model. However, this model is shown to fit all the data, including the random length cases. The predictions for K/K_S given by this model are also plotted as a dashed horizontal lines in Fig. 7 showing how the scale effects plots converge to these values. In Fig. 11, we see a fairly distinct transition from percolation theory to equivalent media theory at ζ values of $\zeta \approx 12$ or $\tilde{p}_{HL} = 0.86$. In both the Robinson Model and the Hestir and Long model the data point closest to ζ_{crit} does not fit the percolation model well. The measured value of permeability for this point is most likely in error because of the large scale required for its measurement.

In summary, Englman's model has only been proven valid very near to ζ_{crit}, and Robinson's model has proven to be very good at predicting K/K_S in constant length cases. However, a comparison of model predictions as given in Table 9, shows that Robinson's model fails in the random length cases. We therefore recommend the Hestir and Long model. In all the cases we have studied it works quite well.

10.0 Summary and Conclusions

We have shown how percolation theory and equivalent media theory can be applied to random two-dimensional fracture networks in order to develop expressions for permeability and the REV. The key for doing this is to relate the lattice filling ratio, p and the coordination number, z to the parameters controlling the random system. We have explored three different relationships, two from the literature, and one our own development.

For constant length fractures, all the models can be made to work well near the critical limit. This is probably due to the fact that percolation theory allows the choice of two parameters (κ and t) in fitting the model. However, we did not test variable length fractures near the critical limit. For cases above the critical limit, the Englman *et al.* (1983) model fails to fit the data. The Robinson (1984a) model does very well in fitting the data in the constant length cases, but does not predict variable length cases well. The model developed in this paper (Hestir and Long model) is able to predict permeability for all cases of constant and variable length.

Based on percolation theory, we proposed an expression for the REV which compared well with the scale effect data. However, this formula should be considered as "convenient" rather than the last word on the subject. The basis of the equation is percolation theory which technically does not apply to the above critical cases where we applied it. However, our percolation based permeability model worked fairly well up to rather high values of ζ, so the REV model may also be a good approximation in this range.

An interesting extension of the model would include a prediction of the whole permeability tensor (i.e. the anisotropy) rather than just the average permeability. Also, it remains to extend the model to the case of variable fracture conductance. Variable conductance can also be thought of as a kind of connectivity in that fractures with small values of conductance form bottle necks and thus decrease the degree of connection. This is a problem that was partially solved by Charlaix *et al.* (1986) for constant length fractures. In this case, one can order the fractures from the largest to the smallest conductance. By placing the fractures in the network from the smallest to the largest, Charlaix *et al.* showed that the conductance of the fracture at the critical limit of connectivity controls the conductance of the network. However, the case of variable length combined with variable conductance is more complicated because the high conductivity fractures may be short and thus not contribute much to the connectivity. This problem has not to our knowledge been solved.

Although some fracture flow problems may be usefully modeled as two-dimensional Poisson systems, many real fracture networks are not two-dimensional Poisson networks and it is of interest to extend this work to more complex models. We think that this will be mathematically difficult. However, it may be possible to transform these complex models into equivalent Poisson models, and from there be able to predict the permeability structure. In any case, this model provides insight into the type of behavior to expect.

Comparing the Robinson model to the Hestir and Long model for the constant length cases, it is interesting to note that both predict the same critical exponent, $t \approx 1.1$, and that both work well in the equivalent media realm. Why is it that two completely different analogies between regular lattice systems and random line segment systems give such good results? For us, this is one of the most intriguing findings of this work.

Reviewers of this work have pointed out that the structure of fracture networks is more complex than can be represented by the Poisson model. On the other hand, our experience is that in some cases, the fracture structure seems to have little to do with the hydrology because even if the fractures are ubiquitous, only a very few seem to conduct fluids (Long *et al.*, 1989). Whether the Poisson model is appropriate in such cases is an open question, however, it is clear to us that one should not look at the validity of this work in terms of whether the Poisson model is a good model of fracture geometry. The important thing to remember is that we are not primarily trying to model the fracture geometry; our main problem is to model the hydrologic behavior. The real contribution of this exercise is that we have been able to map the hydraulic behavior of the more complex Poisson system into that of a simple lattice which has well understood universal behavior. If the Poisson model can be mapped into a lattice, then perhaps fracture systems with even more complex structure can also have their hydrologic behavior modeled using an equivalent lattice. If it does turn out that the lattice model has general utility it would provide a simple vehicle for modeling discontinuous systems without resorting to determining the details of the conducting fracture geometry. This paper should be seen as a step in the direction of finding simple equivalent models for complex, chaotic systems.

Acknowledgements

The authors would like to thank reviewers S. Cox, K. Ghosh, K. Karasaki, T. Madden and P. Robinson for careful reading and many helpful suggestions. This work was carried out under U. S. Department of Energy Contract No. DE-AC03-76SF00098, administered by the Manager, Chicago Operations, Repository Technology Program, Repository Technology and Transportation Division.

References

Bear, J., *Dynamics of Fluids in Porous Media*, Elsevier, New York, NY, 1972.

Charlaix, E., Guyon, E. and Roux, S., Permeability Effects in the Permeability of Heterogeneous Media, in Fragmentation, Form and Flow in Fractured Media, *Annals of Israël Physical Society*, 8, 316–324, 1986.

Englman, R., Gur, Y. and Jaeger, Z., Fluid Flow Through a Crack Network in Rocks, *J. Appl. Mech.*, 50, 707–711, 1983.

Kesten, H., *Percolation Theory for Mathematicians*, p. 5, Birkhauser, Boston, Mass., 1982.

Kesten, H., Percolation Theory and First-Passage Percolation, *The Annals of Probability*, 15(4), 1231–1271, 1987.

Kirkpatrick, S., Percolation and Conduction, *Reviews of Modern Physics*, 45(4), 574–588, 1973.

Long, J. C. S., Remer, J. S., Wilson, C. R. and Witherspoon, P. A., Porous Media Equivalents for Networks of Discontinuous Fractures, *Water Resour. Res.*, 18(3), 645–658, 1982.

Long, J. C. S, Investigation of Equivalent Porous Medium Permeability in Networks of Discontinuous Fractures, Ph.D. thesis, University of California, Berkeley, 1983.

Long, J. C. S. and Witherspoon, P. A., The Relationship of the Degree of Interconnection to Permeability in Fracture Networks, *J. Geophys. Res.*, 90(B4), 3087–3097, 1985.

Long, J. C. S., Hestir, K., Karasaki, K., Davey, A., Peterson, J., Kemeny, J. and Landsfeld, M., Fluid Flow in Fractured Rock: Theory and Application, in *Transport Processes in Porous Media*, J. Bear and M. Y. Corapcioglu eds., Kluwer Academic Publishers, The Netherlands, 1989.

Marcus, H. and Evanson, D. E., Directional Permeability in Anisotropic Porous Media, Water Resources Center Contribution No. 31, University of California, Berkeley, 1961.

Marcus, H., The Permeability of a Sample of an Anisotropic Porous Medium, *J. Geophys. Res.*, 67, pp. 5215–5225, 1962.

Narasimhan, T. N., A Note on Volume Averaging, *Advances in Water Research*, 3, 135–139, 1980.

Orbach, R., Dynamics of Fractal Networks, *Science*, 231, 814–819, 1986.

Pike, G. E. and Seager, C. H., Percolation and Conductivity: A Computer Study, *Physical Review B*, 10(4), 1421–1446, 1974.

Robinson, P., Connectivity, Flow and Transport in Network Models of Fractured Media, Ph.D. thesis, St. Catherine's College, Oxford, 1984a.

Robinson, P., Numerical Calculations of Critical Densities for Lines and Planes, *J. Phys. A: Math. Gen.* 17, 2823–2830, 1984b.

Scheidegger, A. E., Directional Permeability of Porous Media to Homogeneous Fluids, *Geofisica pura Applicata*, 28, 75–90, 1954.

Snow, D. T., A Parallel Plate Model of Fractured Permeable Media, Ph.D. thesis, University of California, Berkeley, 1965.

Snow, D. T., Anisotropic Permeability of Fractured Media, *Water Resour. Res.*, 5(6), 1273–1289, 1969.

Stauffer, D., *Introduction to Percolation Theory*, p. 52, Taylor and Francis Ltd, London, England, 1985.

Terzaghi, R., Sources of Error in Joint Surveys, *Geotechnique*, 15, 287–303, 1965.

Toth, J., Groundwater in Sedimentary (Clastic Rocks), Proceedings, National Groundwater Hydrology, San Francisco, CA, Nov. 6–8, 1967.

Wilson, C. R., An Investigation of Laminar Flow in Fractured Porous Rocks, Ph.D. thesis, University of California, Berkeley, 1970.

Zallen, R., *Physics of Amorphous Solids*, pp. 135–204, Wiley, New York, NY, 1983.

Table 1. Length-Density Study

Case	λ_A $[L^{-2}]$	l [L]	ζ	K_S/k_0 $[L^{-1}]$	REV [L]
a	.0576	10.00	3.59	0.576	*
b	.0288	20.00	7.19	0.576	577
c	.0188	30.55	10.95	0.574	352
d	.0114	50.00	17.78	0.570	259
e	.0082	69.43	24.67	0.569	228
f	.0058	97.21	34.20	0.564	211
g	.0046	124.98	44.84	0.575	196
h	.0082	70.00	25.07	0.574	225

* $\zeta < \zeta_{crit}$, so REV formulation not applied.

Table 2 Orientation Distribution Studies

Case	λ_A $[L^{-2}]$	l [L]	ζ	Standard Deviation of Orientation [deg]	$H(\Theta)$	K_S/k_0 $[L^{-1}]$	REV [L]
i	.0144	40.00	3.50	7.8	0.152	0.576	*
j	.0144	40.00	8.85	21.4	0.384	0.576	705
k	.0144	40.00	11.13	29.0	0.483	0.576	447

* $\zeta < \zeta_{crit}$, so REV formulation not applied.

Table 3 Normalization Study

Case	λ_A $[L^{-2}]$	l [L]	ζ	K_S/k_0 $[L^{-1}]$	REV [L]
l	.0146	19.63	3.28	0.287	*
m	.0073	39.27	6.56	0.287	1458
n	.0048	58.90	9.71	0.283	855
o	.0024	117.81	19.42	0.283	538

* $\zeta < \zeta_{crit}$, so REV formulation not applied.

Table 4(a) Variable Length Study

Case	λ_A $[L^{-2}]$	Length Distribution	Mean Length [L]	Standard Deviation of Length [L]	ζ Before Truncation	K_S/k_0 Before Truncation $[L^{-1}]$
p	.0420	lognormal	13.8	27.6	4.99	0.580
q	.0232	lognormal	24.8	74.4	8.90	0.575
r	.0232	lognormal	24.8	24.8	8.90	0.575
s	.0232	exponential	24.8	24.8	8.90	0.575

Table 4(b) Variable Length Study

Case	c_0 [L]	$\lambda_A(c_0)$ $[L^{-2}]$	$\bar{l}(c_0)$ [L]	$\zeta(c_0)$	$K_S(c_0)/k_0$ $[L^{-1}]$	REV [L]
p	8.54	0.0167	29.29	8.94	0.489	505
q	8.99	0.0107	49.67	16.47	0.531	289
r	7.71	0.0194	28.62	9.92	0.555	398
s	8.08	0.0167	32.88	11.27	0.549	359

Table 5 Near Critical Study

Case	λ_A $[L^{-2}]$	l [L]	ζ	K_S/k_0 $[L^{-1}]$	REV [L]
t	.0479	12.021	4.407	0.576	2559
u	.0528	10.908	3.999	0.576	5982
v	.0568	10.150	3.725	0.577	26349

Table 6 Estimation of ζ_{crit}

Case	Orientation	Estimated ζ_{crit}
w	Uniform as in cases t–v	3.58
x	Mixture of normals as in cases a–k	3.59
y	Normal with $\sigma = 7.80$ as in case i	3.62

Table 7 Measurements at the Scale of REV

Case	Global Gradient NMSE×10^{-3} $\Delta = 0$	Global Gradient NMSE×10^{-3} $\Delta = 180$	Global Gradient K/K_S	Local Gradient K/K_S
a	163.1*	246.7*	0.010*	0.009*
b	4.87	3.91**	0.420**	0.420**
c	2.15	1.96	0.640	0.633
d	4.06	1.17	0.765	0.764
e	6.83	3.22	0.832	0.839
f	4.41	3.50	0.879	0.866
g	6.76	6.18	0.923	0.934
h	3.65	1.76	0.800	0.839
i	56.83*	38.27*	0.084*	0.045*
j	3.04**	2.30**	0.525**	0.576**
k	0.80	0.45**	0.608**	0.604**
l	184.82**	2228.24**	0.011**	0.036**
m	2.24**	4.87**	0.413**	0.411**
n	2.33**	4.24**	0.597**	0.584**
o	5.13**	7.32**	0.787**	0.670**
p	2.29	2.44**	0.561**	0.543**
q	2.39	2.04	0.782	0.803
r	2.39	1.27	0.623	0.609
s	1.48	0.82	0.658	0.649
t	11.71**	7.59**	0.115**	0.110**
u	16.41**	28.85**	0.058**	0.058**
v	76.05**	55.54**	0.032**	0.029**

* $\zeta < \zeta_{crit}$, so REV formulation not applied and largest scale used instead.

** REV larger than scale of numerical simulation, and largest scale used instead.

Table 8 Least Squares Determination of t and κ
(using data with $4.0 \leq \zeta \leq M$)

Englman Model
(constant length cases only)

M	Number of Data Points	t	κ	Correlation Coefficient
6.0	2	1.31	27.38	1.0
8.0	4	1.89	384.80	0.9902
10.0	6	2.04	769.13	0.9845
20.0	10	2.24	1860.28	0.9678

Robinson Model
(constant length cases only)

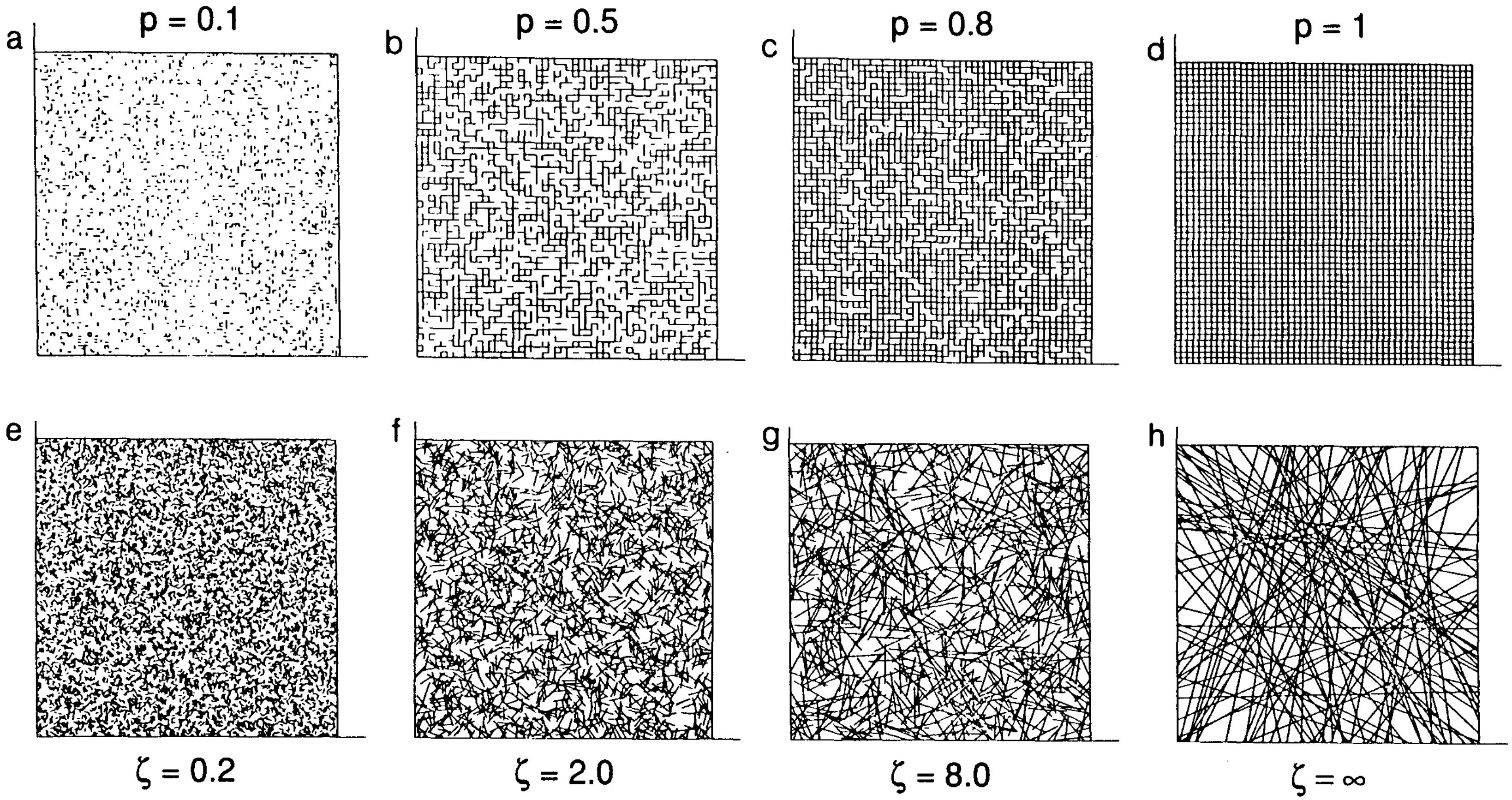
M	Number of Data Points	t	κ	Correlation Coefficient
6.0	2	1.06	1.72	1.0
8.0	4	1.16	2.36	0.9983
10.0	6	1.15	2.29	0.9987
20.0	10	1.14	2.21	0.9988

Hestir and Long Model
(including random lengths)

M	Number of Data Points	t	κ	Correlation Coefficient
6.0	2	1.07	3.12	1.0
8.0	4	1.16	4.33	0.9983
10.0	8	1.14	4.10	0.9987
20.0	14	1.11	3.69	0.9975

Table 9 Comparison of Random Length Cases

Case	Englman et al. Predicted K/K_S using Eqn. 6.3	Robinson Predicted K/K_S using Eqn 6.6 and 7.1	Hestir and Long Predicted K/K_S using Eqn. 7.11	Measured K/K_S as described in Section 4.3
p	0.986	0.0	0.582	0.561
q	1.000	0.0	0.768	0.782
r	1.000	0.270	0.622	0.623
s	1.000	0.168	0.666	0.658



XBL 896-2335A

Figure 1. Examples of percolation networks: (a) through (d) are regular lattices which correspond to the random Poisson networks (e) through (h) below them.

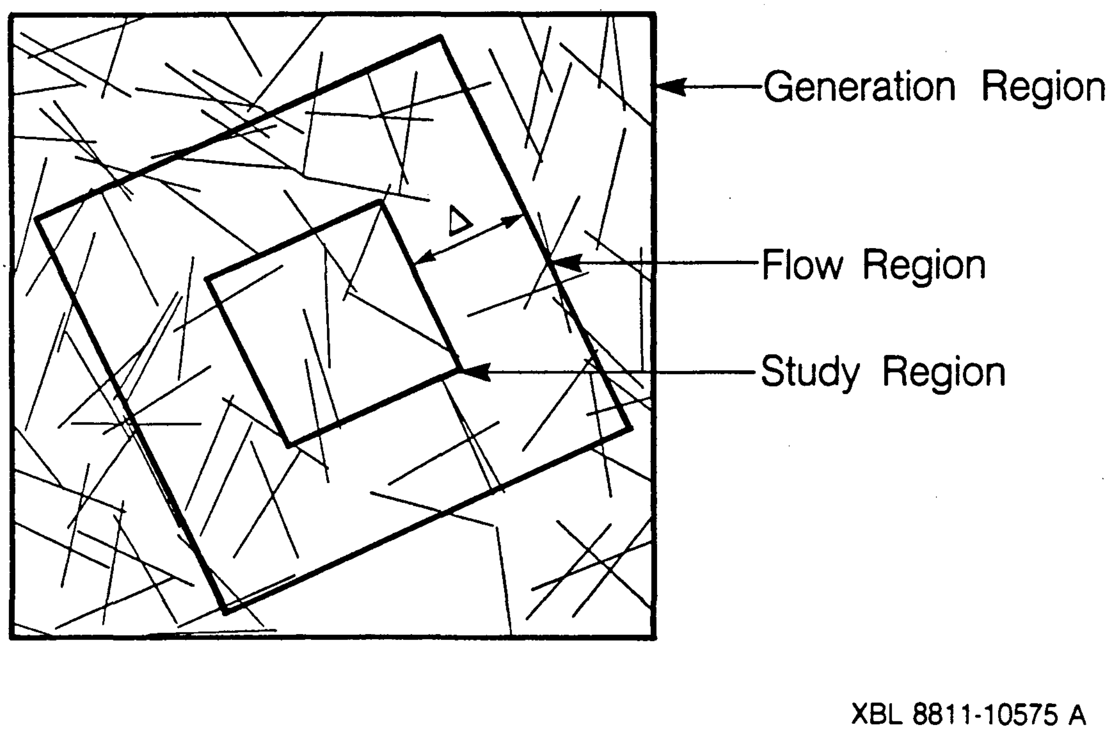


Figure 2. Illustration showing the generation region with respect to the subregions where flow is calculated.

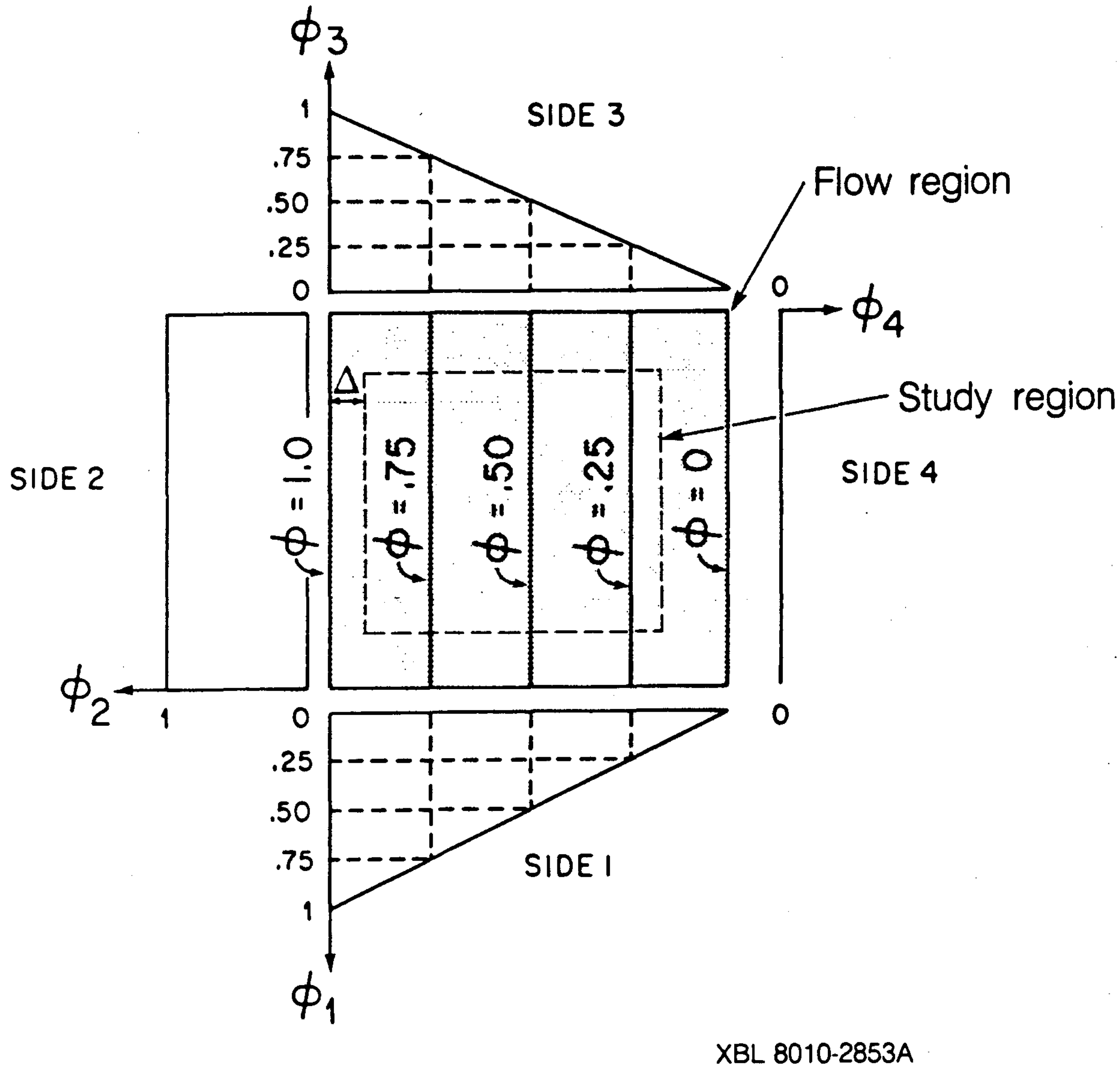


Figure 3. Boundary conditions on the flow region.

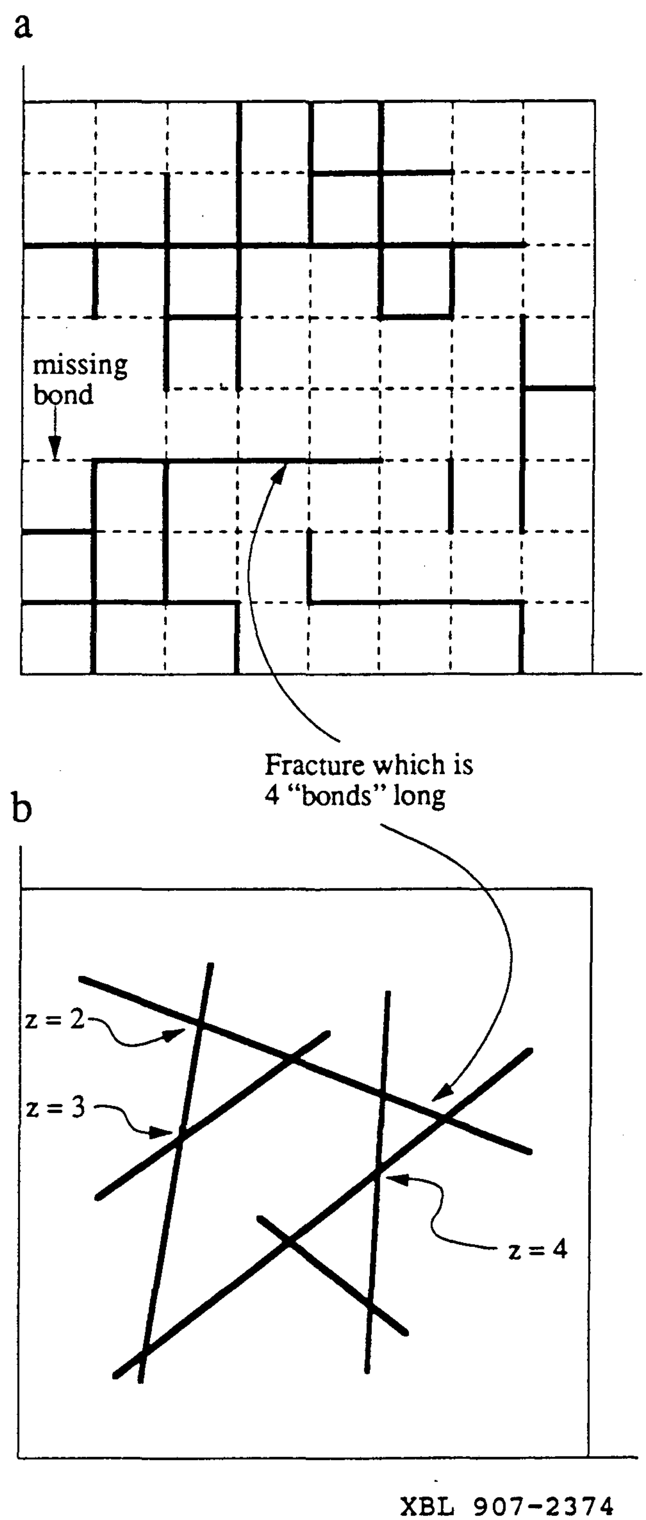
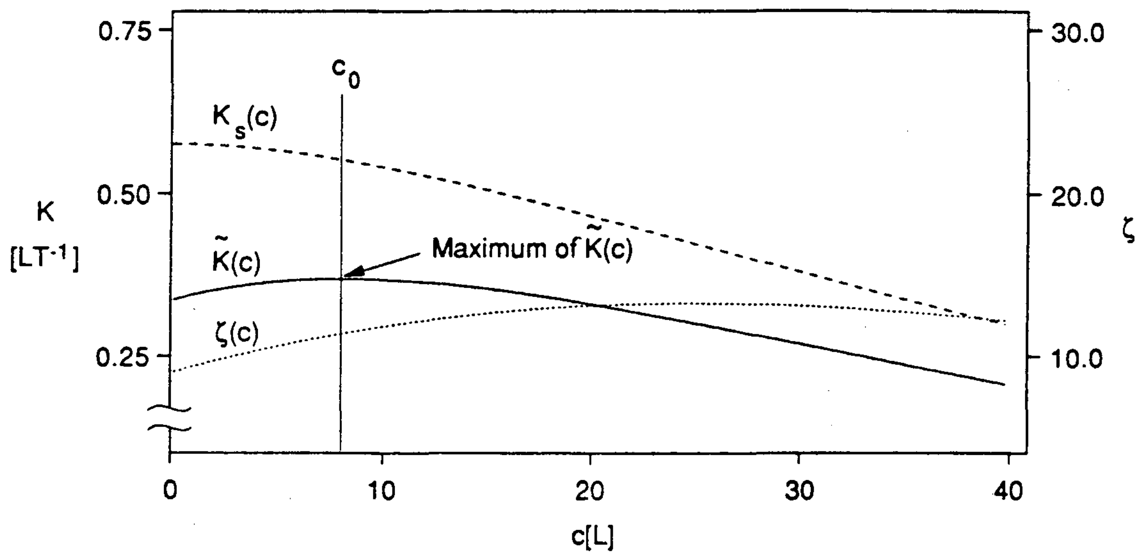
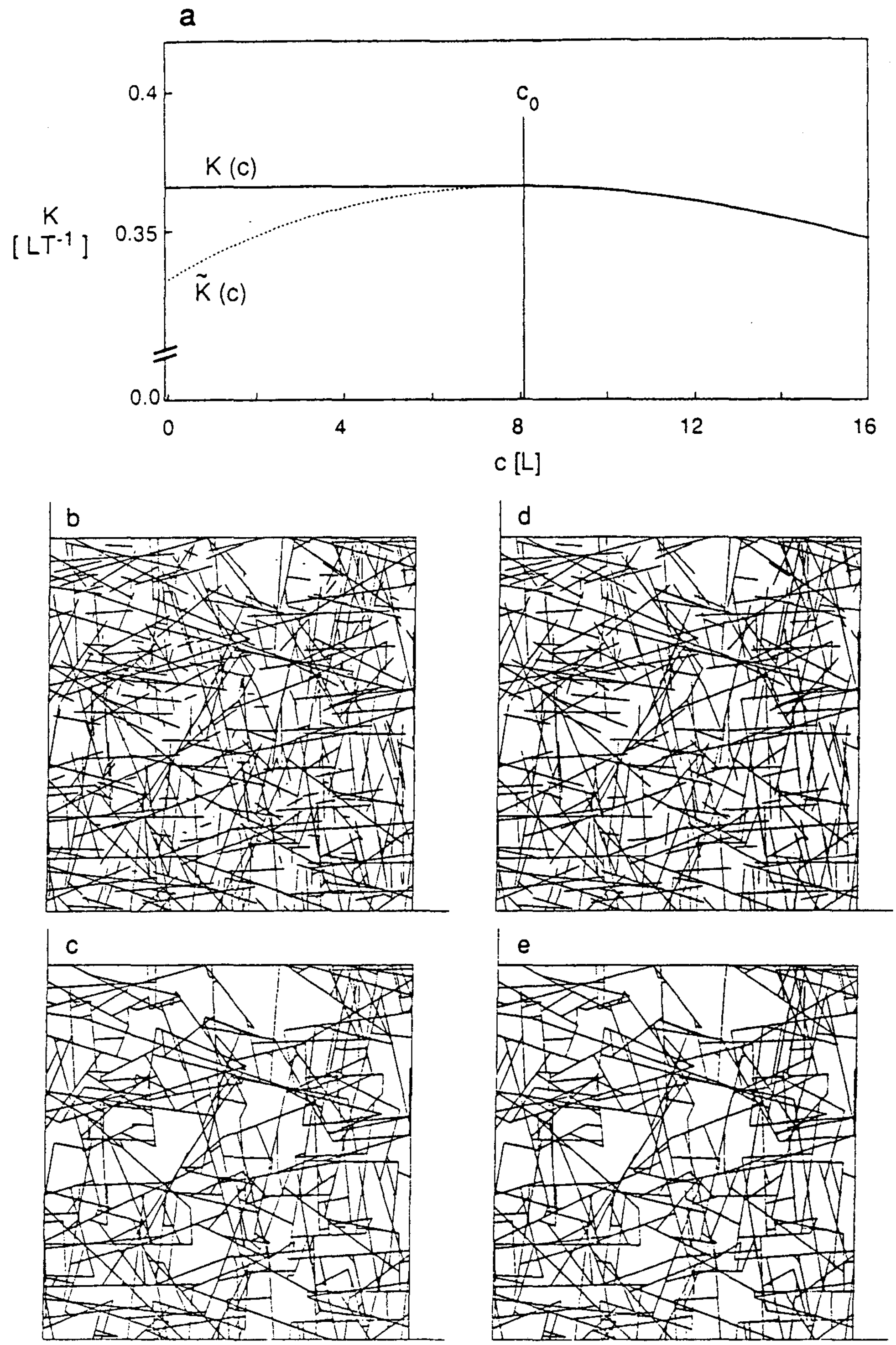


Figure 4. Correspondence of "average run length" between the regular lattice model and the Poisson model.



XBL 907-2373

Figure 5. Behavior of Snow's permeability K_S, connectivity ζ, and $\tilde{K}$ (defined by Equation 7.4) as functions of a cutoff c.



XBL 8811-10574 A

Figure 6. a) Permeability decrease as a function of fracture length cutoff c, b) untruncated network, c) conductive part of (b), d) system truncated at c_0, e) conductive part of (d).

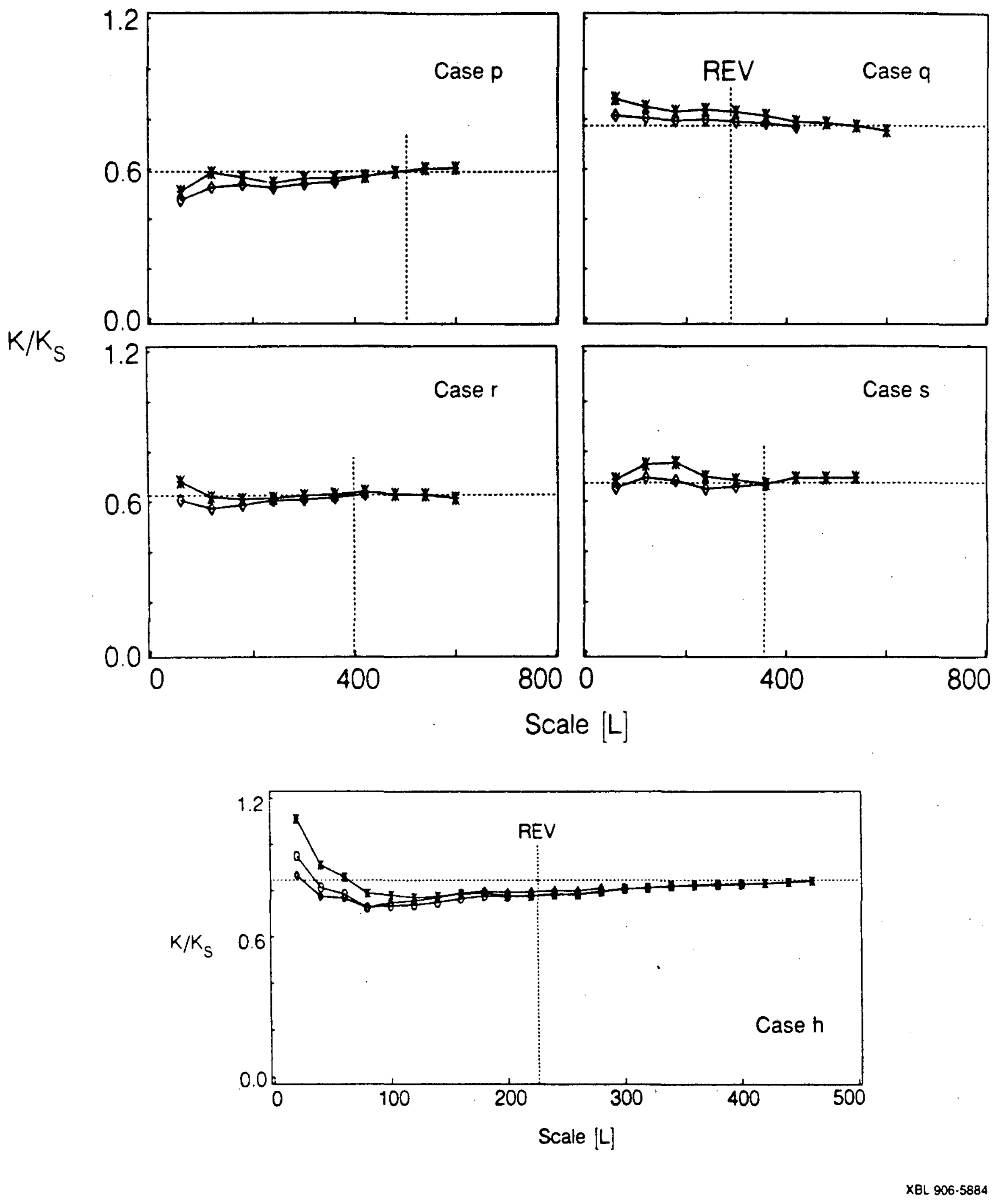


Figure 7. Scale effects plots. These show measurements of permeability at different scales (sizes of study region) for $\Delta = 0[L]$ (✱) and $\Delta = 180[L]$ (◊). Case (h) also shows permeabilities for $\Delta = 60[L]$ (O). The vertical dashed line is REV given by Equation 8.2 and the horizontal dashed line is predicted permeability at the REV based on Equation 7.5.

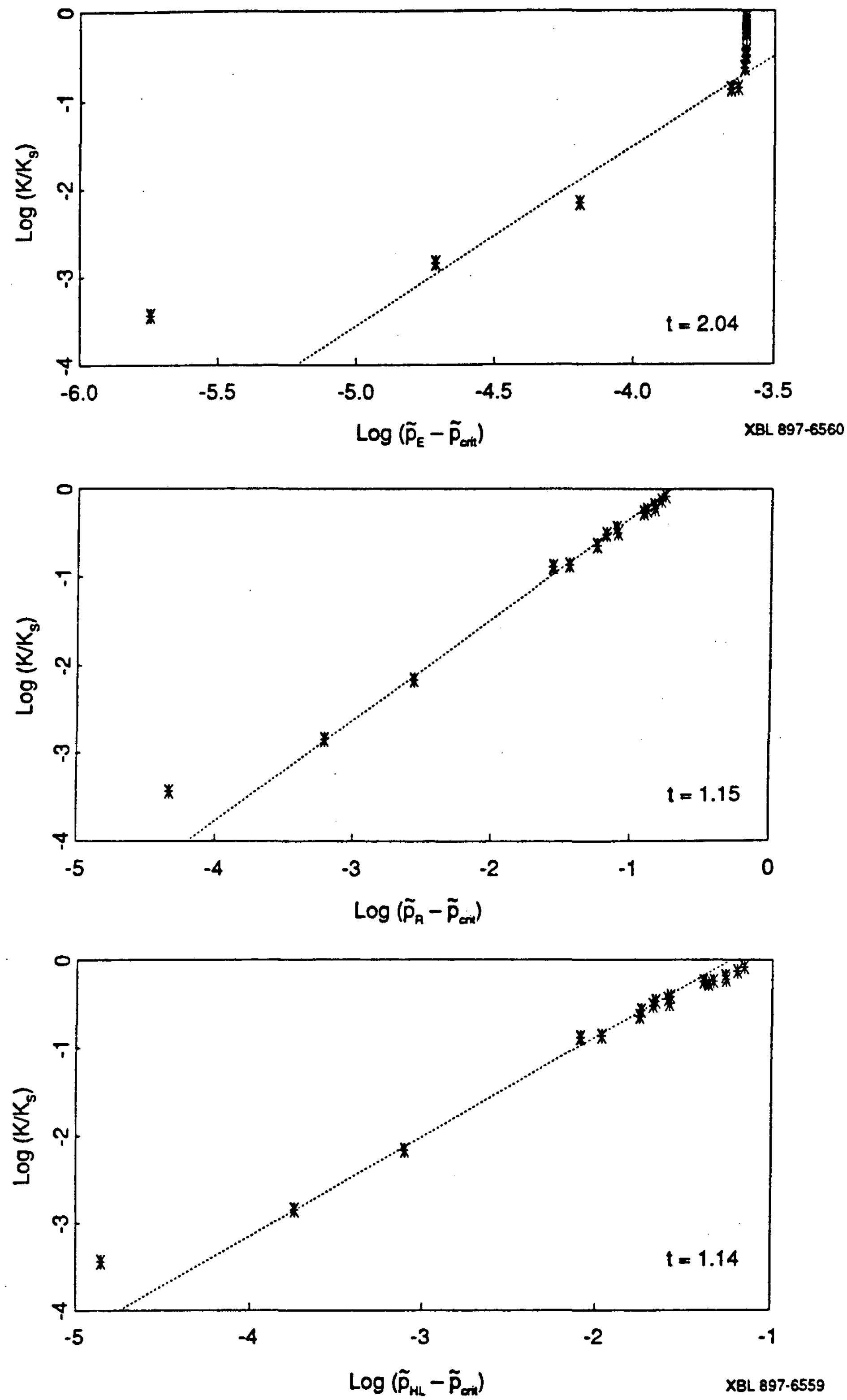


Figure 8. Determination of critical exponent t for the Englman *et al.*, Robinson, and Hestir and Long models. In log-log domain Equation 8.3 plots as a straight line with slope = t. The line shown is a least squares fit for data points with $4.0 < \zeta < 10.0$.

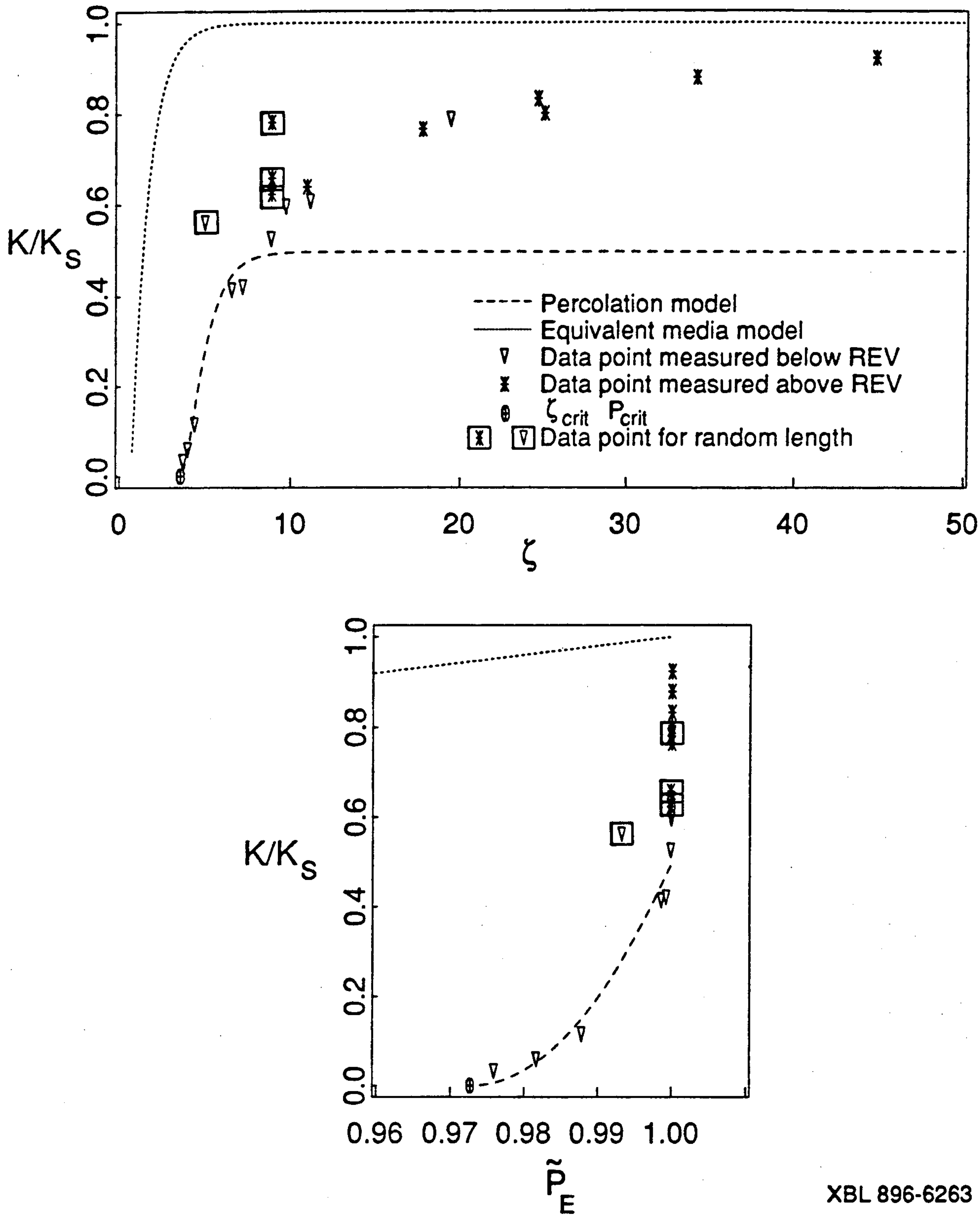
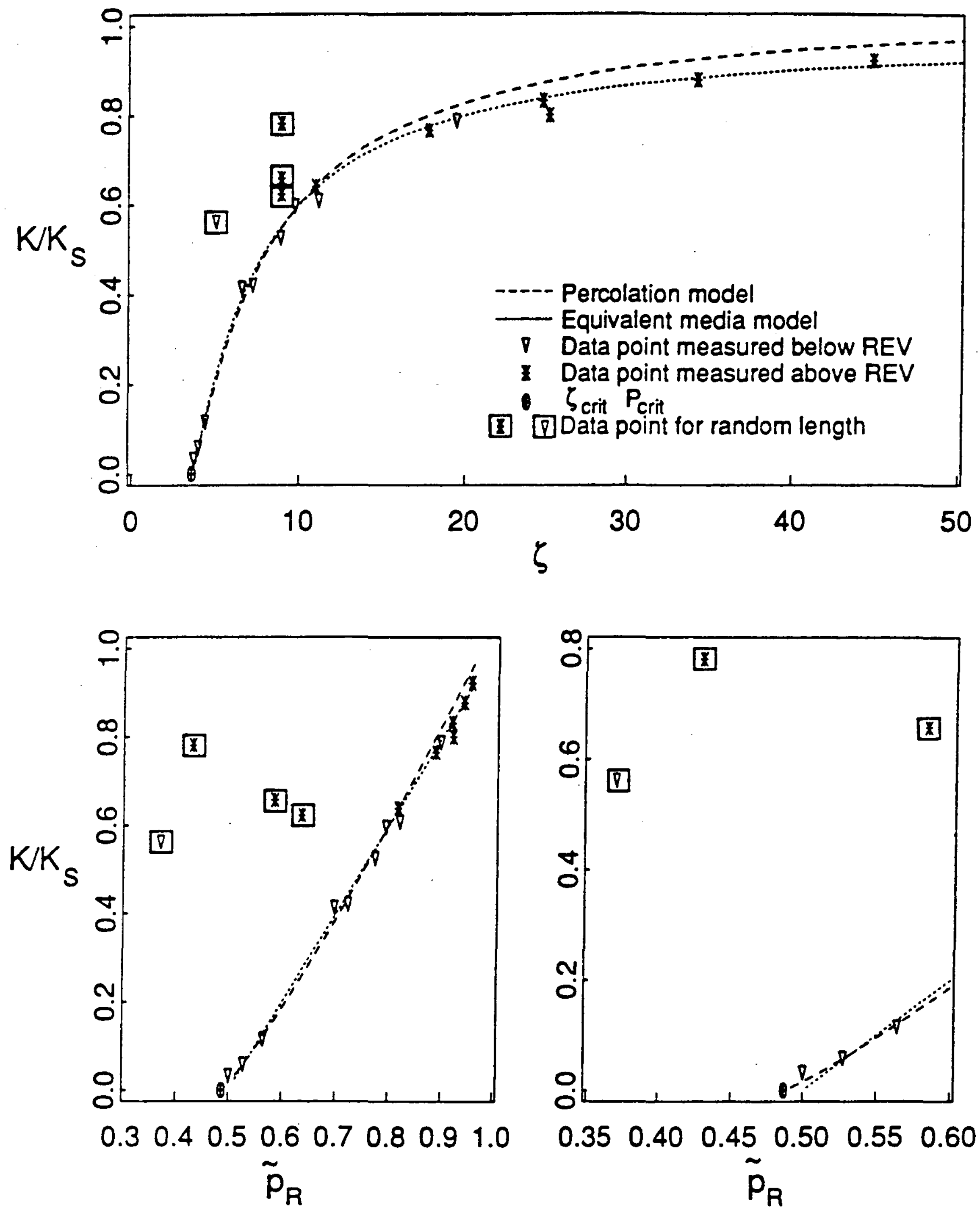


Figure 9. Results of parameter studies plotted against the Englman *et al.* model, Equation 6.1–3, in the ζ domain and $\tilde{p}_E$ domain.



XBL 907-2372

Figure 10. Results of parameter studies plotted against the Robinson model, Equation 6.4–6.6, in the ζ domain and $\tilde{p}_R$ domain. For the random length case ζ was calculated using the full length distribution with no truncation and $\tilde{p}_R$ was calculated using Equation 7.1.

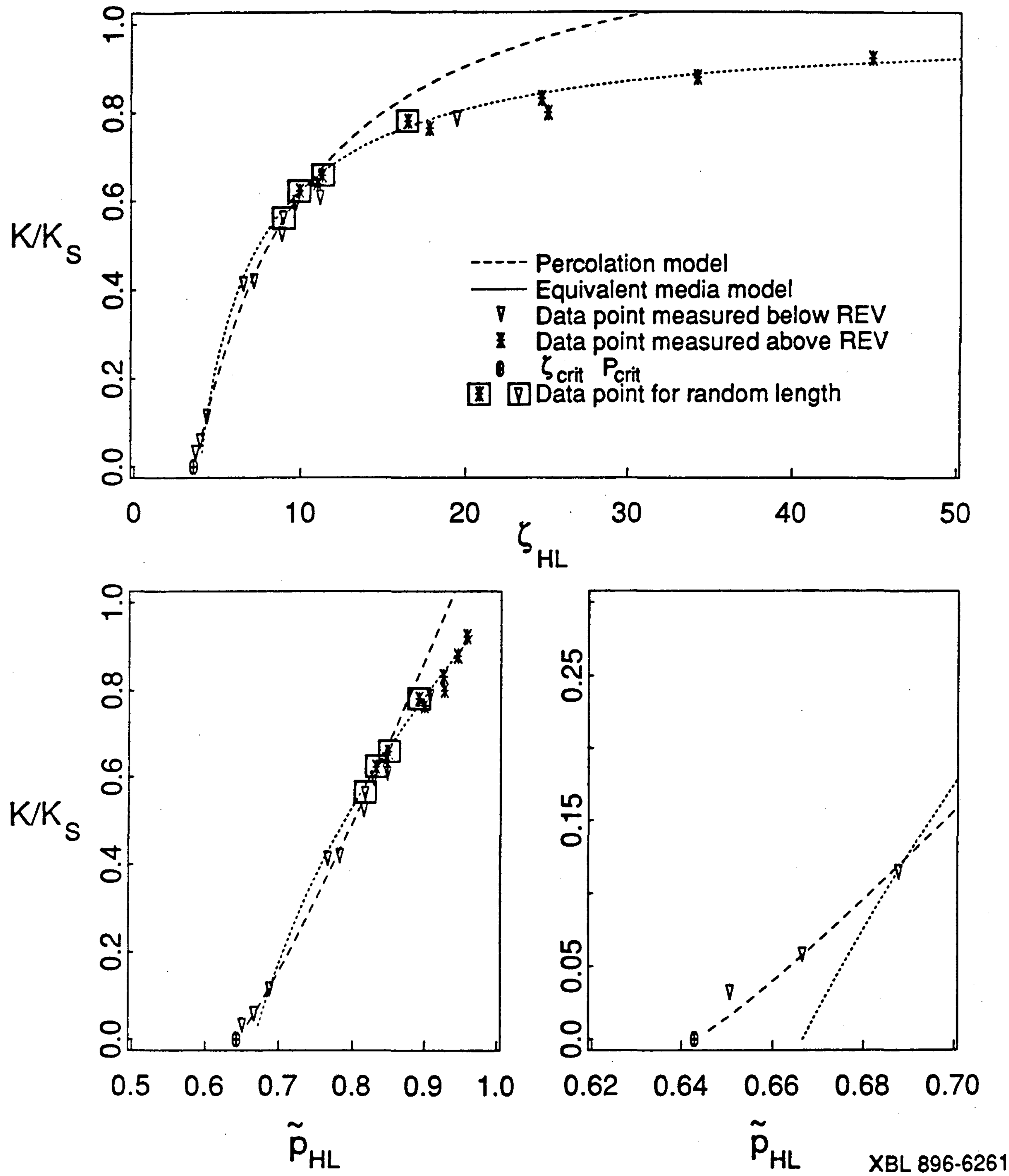


Figure 11. Results of parameter studies plotted against the Hestir and Long model, in the ζ_{HL} domain and $\tilde{p}_{HL}$ domain. Equations 6.8, 6.10 and 7.5.

Appendix

Some example plots of scale effects studies are shown in Section 8.0. This appendix contains a complete set of tables (Cases a–v) of data from all scale effects studies. For each case the tables give flow region size (flw), study region size (stdy), Δ (del), principal permeabilities (k1 and k2), average of the principal permeabilities (aver), NMSE for geometric mean (gnmse), and NMSE for arithmetic mean (anmse). Plots are shown for each case in Figures A1–A6. In these figures average permeability is plotted for Δ = 180 and Δ = 0. The plot for Case h also shows values for Δ = 60.

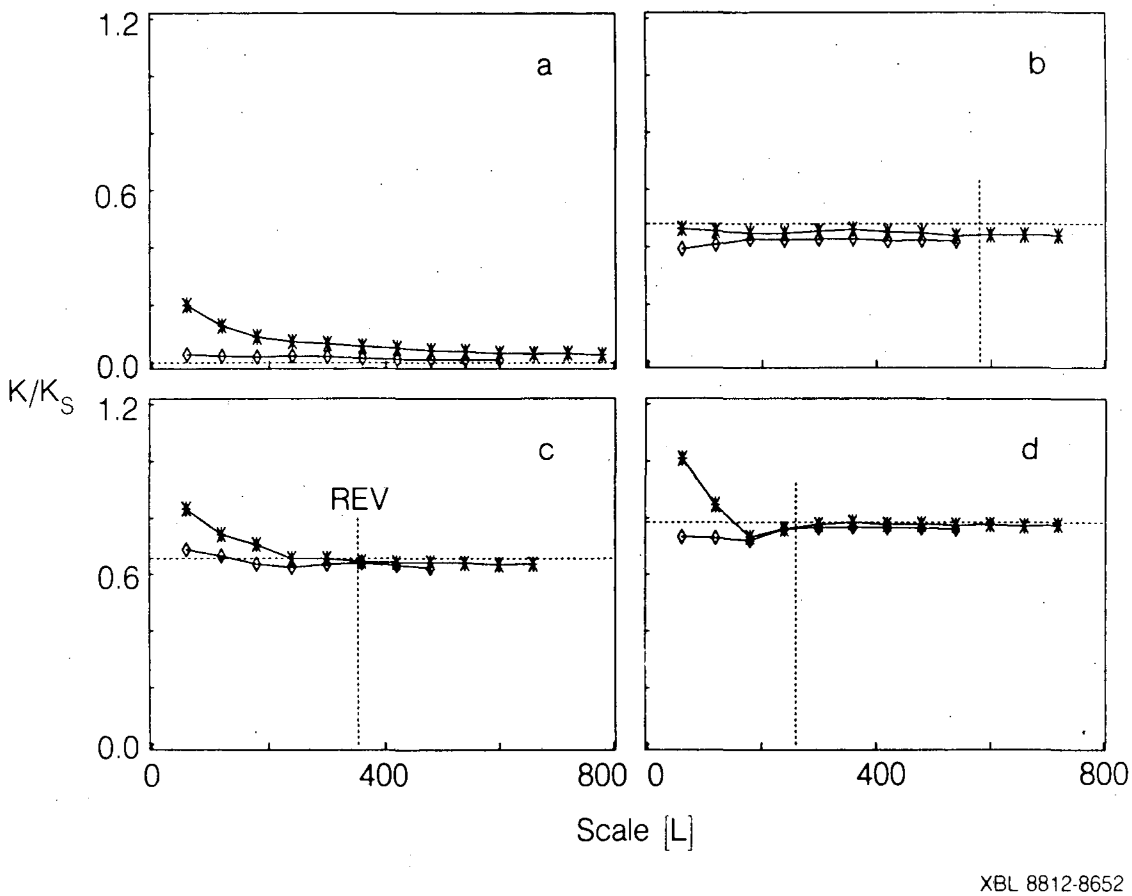


Figure A1. Scale effects plots. These show measurements of permeability at different scales (sizes of study region) for $\Delta = 0[L]$ (⋇) and $\Delta = 180[L]$ (◊). The vertical dashed line is REV given by Equation 8.2 and the horizontal dashed line is predicted permeability at the REV based on Equation 7.11.

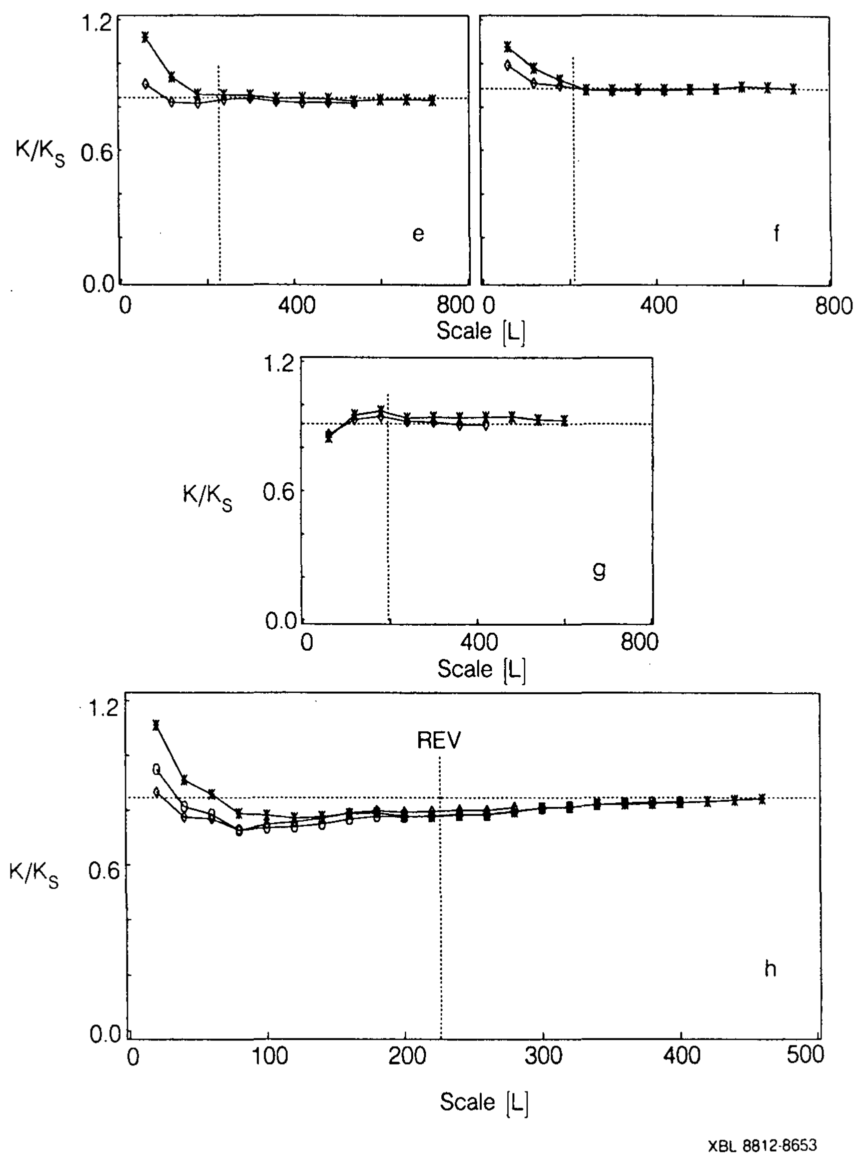


Figure A2. Scale effects plots. These show measurements of permeability at different scales (sizes of study region) for $\Delta = 0[L]$ (✱) and $\Delta = 180[L]$ (◊). Plot (h) also shows permeabilities for $\Delta = 60[L]$ (O). The vertical dashed line is REV given by Equation 8.2 and the horizontal dashed line is predicted permeability at the REV based on Equation 7.11.

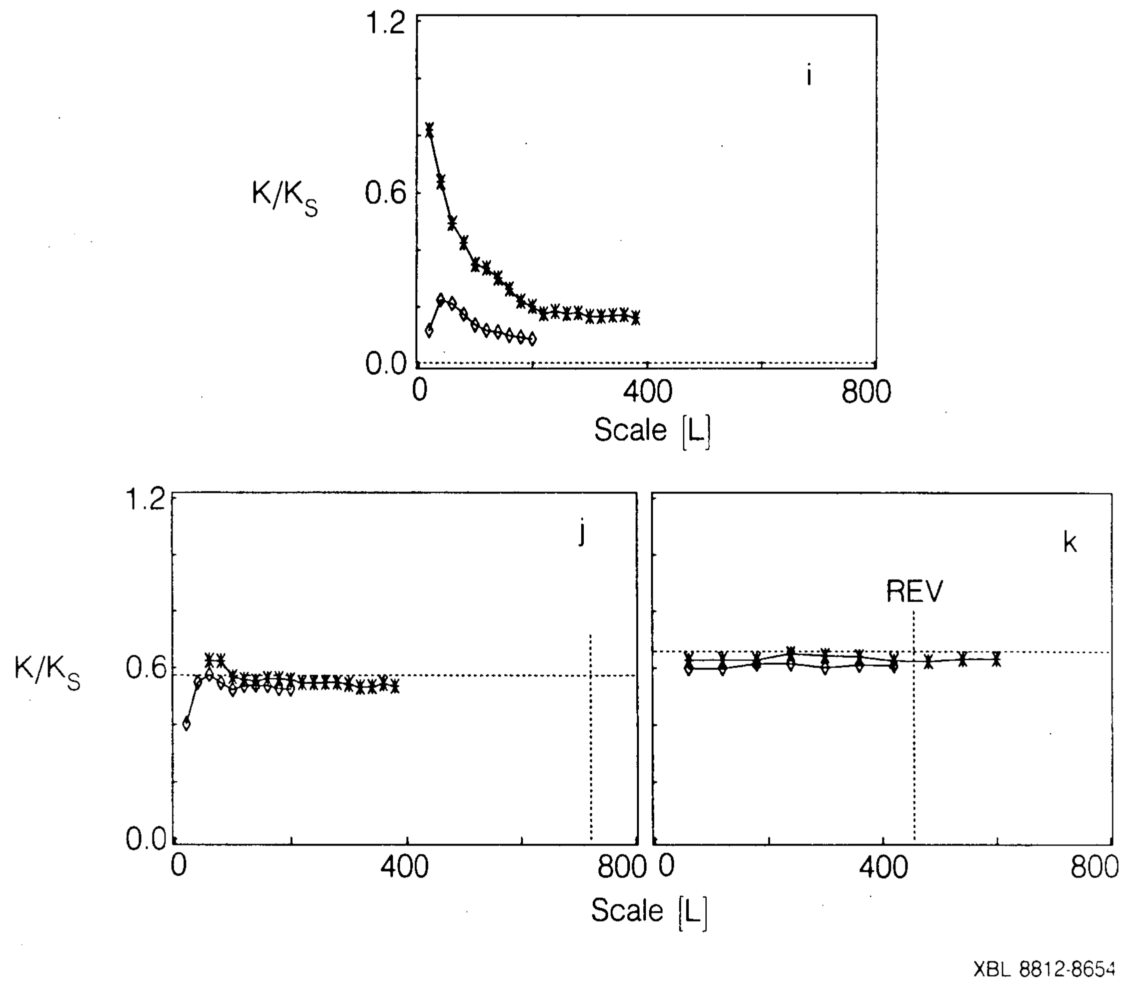


Figure A3. Scale effects plots. These show measurements of permeability at different scales (sizes of study region) for $\Delta = 0[L]$ (✱) and $\Delta = 180[L]$ (◊). The vertical dashed line is REV given by Equation 8.2 and the horizontal dashed line is predicted permeability at the REV based on Equation 7.11.

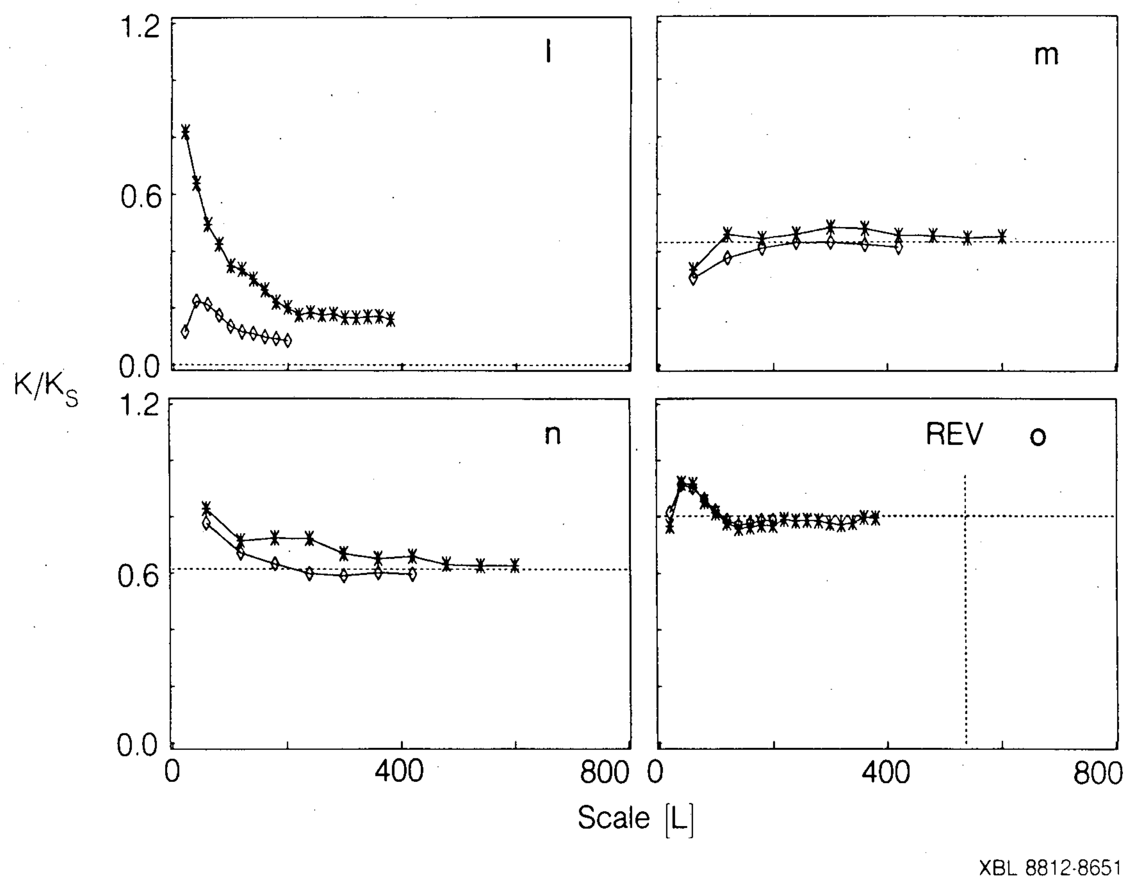


Figure A4. Scale effects plots. These show measurements of permeability at different scales (sizes of study region) for $\Delta = 0[L]$ (✱) and $\Delta = 180[L]$ (◊). The vertical dashed line is REV given by Equation 8.2 and the horizontal dashed line is predicted permeability at the REV based on Equation 7.11.

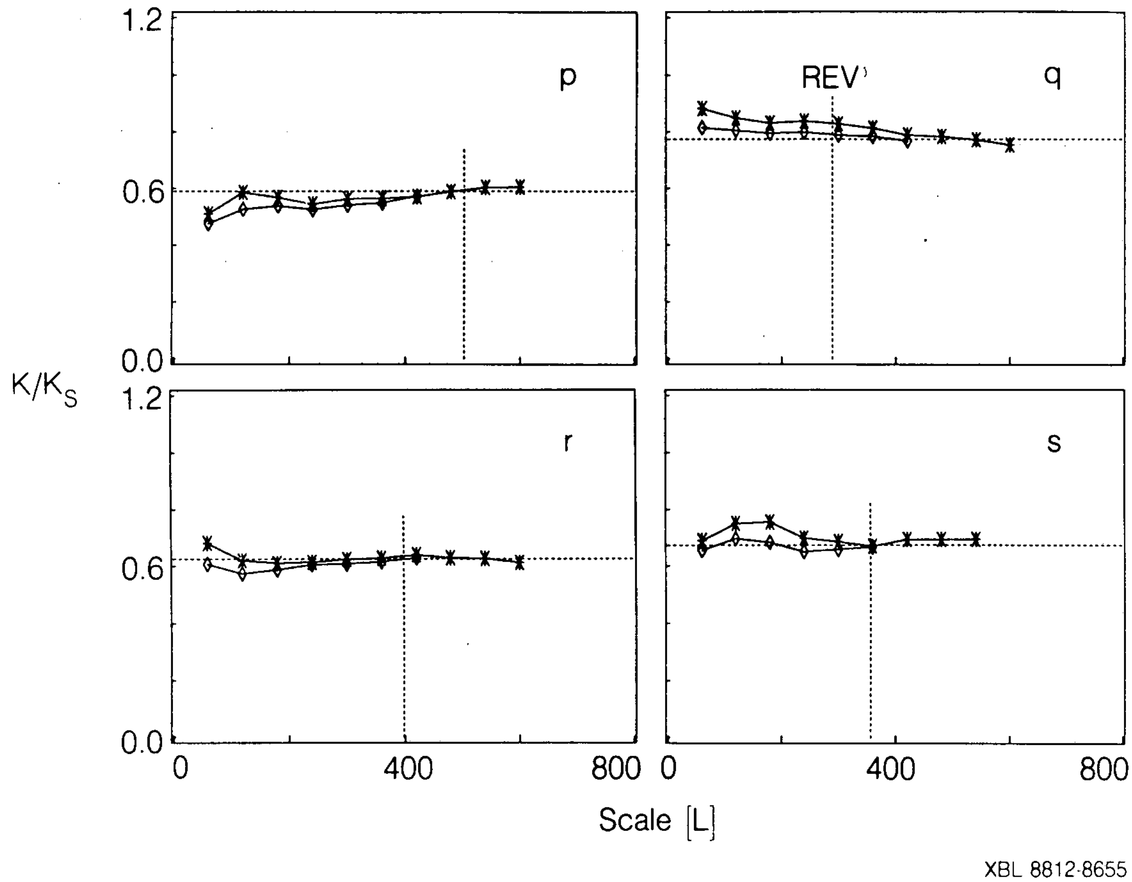
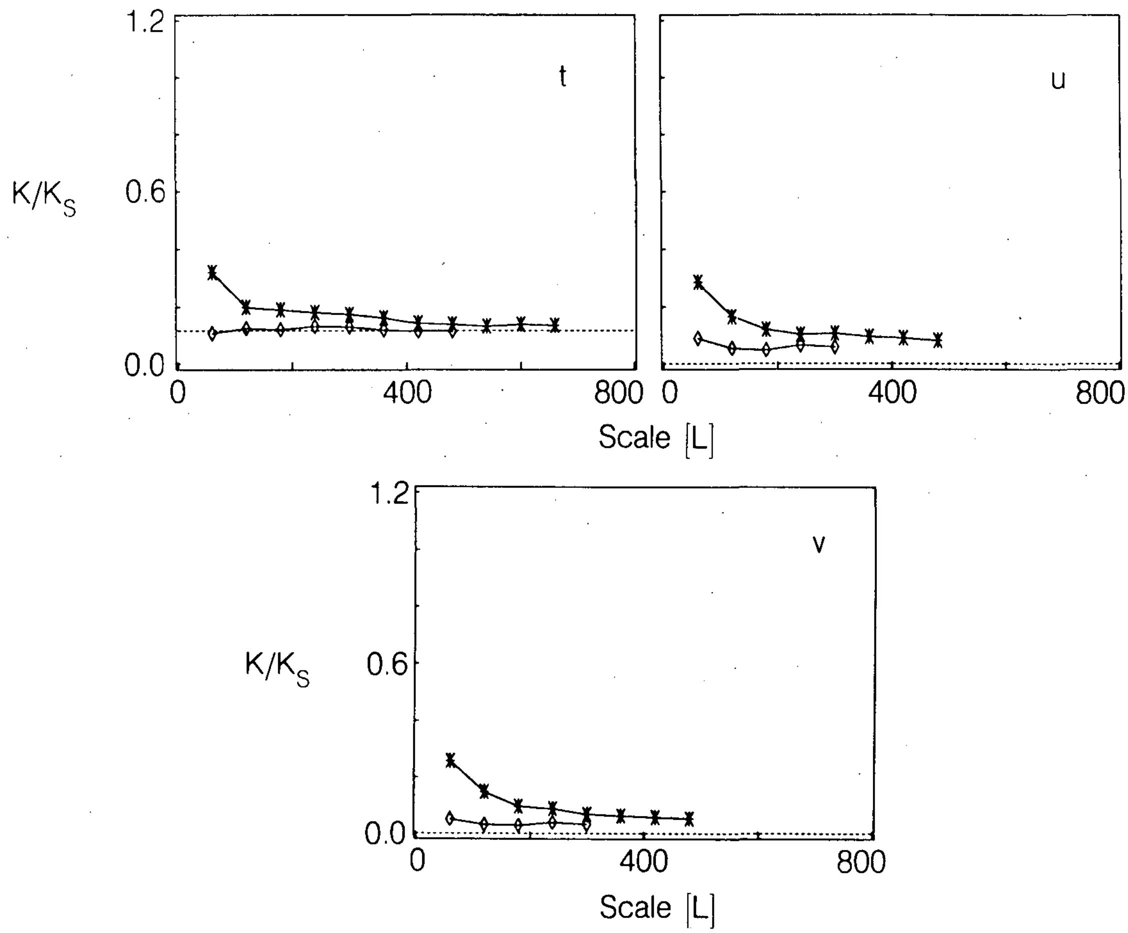


Figure A5. Scale effects plots. These show measurements of permeability at different scales (sizes of study region) for $\Delta = 0[L]$ (✱) and $\Delta = 180[L]$ (◊). The vertical dashed line is REV given by Equation 8.2 and the horizontal dashed line is predicted permeability at the REV based on Equation 7.11.



XBL 891-6101

Figure A6. Scale effects plots. These show measurements of permeability at different scales (sizes of study region) for $\Delta = 0[L]$ (✱) and $\Delta = 180[L]$ (◊). The horizontal dashed line is predicted permeability at the REV based on Equation 7.11.

Case a

flw	stdy	del	k1	k2	aver	gnmse	anmse
60.	60.	0.	0.522612E-06	0.413974E-06	0.468293E-06	0.164988E+00	0.162768E+00
120.	120.	0.	0.334686E-06	0.267130E-06	0.300908E-06	0.105666E+00	0.104335E+00
180.	180.	0.	0.235706E-06	0.185675E-06	0.210690E-06	0.971761E-01	0.958062E-01
240.	240.	0.	0.189526E-06	0.151628E-06	0.170577E-06	0.150237E+00	0.148383E+00
240.	60.	180.	0.102945E-06	0.301235E-07	0.665340E-07	0.590326E+00	0.413535E+00
300.	300.	0.	0.166057E-06	0.144021E-06	0.155039E-06	0.135314E+00	0.134631E+00
300.	120.	180.	0.897086E-07	0.162479E-07	0.529783E-07	0.576867E+00	0.299579E+00
360.	360.	0.	0.151579E-06	0.115632E-06	0.133605E-06	0.857350E-01	0.841834E-01
360.	180.	180.	0.781083E-07	0.144744E-07	0.462913E-07	0.376035E+00	0.198393E+00
420.	420.	0.	0.130693E-06	0.106476E-06	0.118585E-06	0.953909E-01	0.943963E-01
420.	240.	180.	0.874794E-07	0.224988E-07	0.549891E-07	0.114752E+00	0.746914E-01
480.	480.	0.	0.110884E-06	0.859547E-07	0.984192E-07	0.743158E-01	0.731238E-01
480.	300.	180.	0.707398E-07	0.291838E-07	0.499618E-07	0.000000E+00	0.208473E+00
540.	540.	0.	0.980002E-07	0.844801E-07	0.912402E-07	0.819680E-01	0.815180E-01
540.	360.	180.	0.409120E-07	0.294557E-07	0.351839E-07	0.281437E+00	0.273978E+00
600.	600.	0.	0.909772E-07	0.616590E-07	0.763181E-07	0.158507E+00	0.152659E+00
600.	420.	180.	0.351142E-07	0.210991E-07	0.281066E-07	0.000000E+00	0.227629E+00
660.	660.	0.	0.793791E-07	0.700488E-07	0.747140E-07	0.685494E-01	0.682821E-01
660.	480.	180.	0.327772E-07	0.152186E-07	0.239979E-07	0.000000E+00	0.242906E+00
720.	720.	0.	0.848557E-07	0.689639E-07	0.769098E-07	0.118001E+00	0.116741E+00
720.	540.	180.	0.314827E-07	0.167875E-07	0.241351E-07	0.209675E+00	0.190242E+00
780.	780.	0.	0.701627E-07	0.606350E-07	0.653989E-07	0.163951E+00	0.163081E+00
780.	600.	180.	0.279334E-07	0.198491E-07	0.238912E-07	0.253968E+00	0.246698E+00

Case b

flw	stdy	del	k1	k2	aver	gnmse	anmse
60.	60.	0.	0.126462E-05	0.912042E-06	0.108833E-05	0.395382E-01	0.385008E-01
120.	120.	0.	0.120129E-05	0.939966E-06	0.107063E-05	0.103817E-01	0.102271E-01
180.	180.	0.	0.110757E-05	0.994930E-06	0.105125E-05	0.693397E-02	0.691407E-02
240.	240.	0.	0.111861E-05	0.980495E-06	0.104955E-05	0.812129E-02	0.808614E-02
240.	60.	180.	0.114752E-05	0.700909E-06	0.924215E-06	0.513897E-01	0.483896E-01
300.	300.	0.	0.111955E-05	0.101638E-05	0.106796E-05	0.602112E-02	0.600707E-02
300.	120.	180.	0.104335E-05	0.879336E-06	0.961343E-06	0.259332E-02	0.257445E-02
360.	360.	0.	0.112137E-05	0.104648E-05	0.108392E-05	0.250151E-02	0.249853E-02
360.	180.	180.	0.106649E-05	0.940755E-06	0.100362E-05	0.333823E-02	0.332513E-02
420.	420.	0.	0.110529E-05	0.102289E-05	0.106409E-05	0.486427E-02	0.485698E-02
420.	240.	180.	0.106400E-05	0.930242E-06	0.997123E-06	0.269105E-02	0.267895E-02
480.	480.	0.	0.107210E-05	0.103849E-05	0.105530E-05	0.209907E-02	0.209854E-02
480.	300.	180.	0.103758E-05	0.972274E-06	0.100493E-05	0.164767E-02	0.164593E-02
540.	540.	0.	0.106362E-05	0.999165E-06	0.103140E-05	0.309677E-02	0.309375E-02
540.	360.	180.	0.103236E-05	0.982822E-06	0.100759E-05	0.188244E-02	0.188131E-02
600.	600.	0.	0.105557E-05	0.102182E-05	0.103869E-05	0.487079E-02	0.486950E-02
600.	420.	180.	0.100517E-05	0.975942E-06	0.990557E-06	0.284501E-02	0.284439E-02
660.	660.	0.	0.104707E-05	0.103147E-05	0.103927E-05	0.448035E-02	0.448009E-02
660.	480.	180.	0.101367E-05	0.978428E-06	0.996050E-06	0.408097E-02	0.407970E-02
720.	720.	0.	0.105175E-05	0.101877E-05	0.103526E-05	0.756677E-02	0.756485E-02
720.	540.	180.	0.101383E-05	0.961874E-06	0.987851E-06	0.391818E-02	0.391547E-02

Case c

flw	stdy	del	k1	k2	aver	gnmse	anmse
60.	60.	0.	0.212562E-05	0.178306E-05	0.195434E-05	0.176485E-01	0.175130E-01
120.	120.	0.	0.184449E-05	0.163490E-05	0.173970E-05	0.446093E-02	0.444474E-02
180.	180.	0.	0.172725E-05	0.158351E-05	0.165538E-05	0.413048E-02	0.412269E-02
240.	240.	0.	0.155476E-05	0.152557E-05	0.154017E-05	0.548045E-02	0.547995E-02
240.	60.	180.	0.193652E-05	0.129019E-05	0.161335E-05	0.127657E-01	0.122535E-01
300.	300.	0.	0.155994E-05	0.151785E-05	0.153890E-05	0.255918E-02	0.255870E-02
300.	120.	180.	0.165815E-05	0.146375E-05	0.156095E-05	0.368571E-02	0.367142E-02
360.	360.	0.	0.152275E-05	0.150435E-05	0.151355E-05	0.214558E-02	0.214550E-02
360.	180.	180.	0.151532E-05	0.147012E-05	0.149272E-05	0.396726E-02	0.396635E-02
420.	420.	0.	0.153483E-05	0.147464E-05	0.150473E-05	0.193777E-02	0.193699E-02
420.	240.	180.	0.153590E-05	0.139463E-05	0.146527E-05	0.233164E-02	0.232622E-02
480.	480.	0.	0.154315E-05	0.146973E-05	0.150644E-05	0.217652E-02	0.217523E-02
480.	300.	180.	0.152905E-05	0.145294E-05	0.149099E-05	0.249588E-02	0.249425E-02
540.	540.	0.	0.153329E-05	0.147049E-05	0.150189E-05	0.222103E-02	0.222006E-02
540.	360.	180.	0.153540E-05	0.146603E-05	0.150072E-05	0.196206E-02	0.196101E-02
600.	600.	0.	0.151636E-05	0.145745E-05	0.148690E-05	0.293785E-02	0.293670E-02
600.	420.	180.	0.151508E-05	0.145177E-05	0.148343E-05	0.244272E-02	0.244161E-02
660.	660.	0.	0.151610E-05	0.147118E-05	0.149364E-05	0.519331E-02	0.519213E-02
660.	480.	180.	0.149164E-05	0.142875E-05	0.146020E-05	0.353780E-02	0.353616E-02

Case d

flw	stdy	del	k1	k2	aver	gnmse	anmse
60.	60.	0.	0.251387E-05	0.219037E-05	0.235212E-05	0.175740E-01	0.174909E-01
120.	120.	0.	0.205814E-05	0.188173E-05	0.196993E-05	0.982870E-02	0.980899E-02
180.	180.	0.	0.174962E-05	0.166150E-05	0.170556E-05	0.726427E-02	0.725942E-02
240.	240.	0.	0.181510E-05	0.172805E-05	0.177157E-05	0.353780E-02	0.353566E-02
240.	60.	180.	0.186812E-05	0.154817E-05	0.170815E-05	0.140873E-01	0.139637E-01
300.	300.	0.	0.184325E-05	0.177710E-05	0.181018E-05	0.406216E-02	0.406080E-02
300.	120.	180.	0.171812E-05	0.168062E-05	0.169937E-05	0.552842E-02	0.552775E-02
360.	360.	0.	0.185733E-05	0.179552E-05	0.182643E-05	0.171100E-02	0.171052E-02
360.	180.	180.	0.171997E-05	0.163027E-05	0.167512E-05	0.292103E-02	0.291893E-02
420.	420.	0.	0.183211E-05	0.178739E-05	0.180975E-05	0.149384E-02	0.149361E-02
420.	240.	180.	0.180758E-05	0.173717E-05	0.177238E-05	0.125561E-02	0.125512E-02
480.	480.	0.	0.182981E-05	0.179572E-05	0.181277E-05	0.110267E-02	0.110257E-02
480.	300.	180.	0.180918E-05	0.175575E-05	0.178247E-05	0.116497E-02	0.116471E-02
540.	540.	0.	0.181659E-05	0.178841E-05	0.180250E-05	0.156687E-02	0.156678E-02
540.	360.	180.	0.182430E-05	0.175042E-05	0.178736E-05	0.113702E-02	0.113654E-02
600.	600.	0.	0.183731E-05	0.177889E-05	0.180810E-05	0.224551E-02	0.224492E-02
600.	420.	180.	0.182925E-05	0.173443E-05	0.178184E-05	0.233110E-02	0.232945E-02
660.	660.	0.	0.180421E-05	0.178717E-05	0.179569E-05	0.276411E-02	0.276405E-02
660.	480.	180.	0.179491E-05	0.176306E-05	0.177898E-05	0.214366E-02	0.214349E-02
720.	720.	0.	0.181441E-05	0.179959E-05	0.180700E-05	0.399608E-02	0.399602E-02
720.	540.	180.	0.177986E-05	0.176268E-05	0.177127E-05	0.246064E-02	0.246058E-02

Case e

flw	stdy	del	k1	k2	aver	gnmse	anmse
60.	60.	0.	0.280657E-05	0.239215E-05	0.259936E-05	0.175479E-01	0.174364E-01
120.	120.	0.	0.222134E-05	0.212871E-05	0.217502E-05	0.457446E-02	0.457239E-02
180.	180.	0.	0.201415E-05	0.196536E-05	0.198975E-05	0.538658E-02	0.538578E-02
240.	240.	0.	0.201031E-05	0.195781E-05	0.198406E-05	0.683172E-02	0.683052E-02
240.	60.	180.	0.233949E-05	0.186053E-05	0.210001E-05	0.115318E-01	0.113819E-01
300.	300.	0.	0.199103E-05	0.197767E-05	0.198435E-05	0.550984E-02	0.550978E-02
300.	120.	180.	0.196177E-05	0.185197E-05	0.190687E-05	0.278864E-02	0.278633E-02
360.	360.	0.	0.198868E-05	0.191750E-05	0.195309E-05	0.322221E-02	0.322114E-02
360.	180.	180.	0.193163E-05	0.185698E-05	0.189430E-05	0.199264E-02	0.199187E-02
420.	420.	0.	0.198725E-05	0.192488E-05	0.195607E-05	0.180112E-02	0.180066E-02
420.	240.	180.	0.198473E-05	0.188644E-05	0.193558E-05	0.322372E-02	0.322165E-02
480.	480.	0.	0.198094E-05	0.191671E-05	0.194883E-05	0.148488E-02	0.148447E-02
480.	300.	180.	0.199390E-05	0.190852E-05	0.195121E-05	0.282643E-02	0.282508E-02
540.	540.	0.	0.195850E-05	0.189259E-05	0.192555E-05	0.171834E-02	0.171784E-02
540.	360.	180.	0.197038E-05	0.187483E-05	0.192260E-05	0.114648E-02	0.114577E-02
600.	600.	0.	0.198059E-05	0.190411E-05	0.194235E-05	0.183591E-02	0.183520E-02
600.	420.	180.	0.194915E-05	0.186778E-05	0.190847E-05	0.117871E-02	0.117818E-02
660.	660.	0.	0.196843E-05	0.191755E-05	0.194299E-05	0.178318E-02	0.178288E-02
660.	480.	180.	0.195383E-05	0.186841E-05	0.191112E-05	0.187158E-02	0.187064E-02
720.	720.	0.	0.196914E-05	0.190766E-05	0.193840E-05	0.357482E-02	0.357392E-02
720.	540.	180.	0.194076E-05	0.186533E-05	0.190305E-05	0.222140E-02	0.222052E-02

Case f

flw	stdy	del	k1	k2	aver	gnmse	anmse
60.	60.	0.	0.292519E-05	0.201601E-05	0.247060E-05	0.778790E-02	0.752423E-02
120.	120.	0.	0.248113E-05	0.201417E-05	0.224765E-05	0.581860E-02	0.575581E-02
180.	180.	0.	0.227426E-05	0.197173E-05	0.212299E-05	0.586876E-02	0.583896E-02
240.	240.	0.	0.210859E-05	0.193895E-05	0.202377E-05	0.441802E-02	0.441026E-02
240.	60.	180.	0.252829E-05	0.202951E-05	0.227890E-05	0.809387E-02	0.799693E-02
300.	300.	0.	0.204911E-05	0.199710E-05	0.202310E-05	0.320244E-02	0.320191E-02
300.	120.	180.	0.216454E-05	0.201779E-05	0.209116E-05	0.628337E-02	0.627564E-02
360.	360.	0.	0.203687E-05	0.203113E-05	0.203400E-05	0.343522E-02	0.343521E-02
360.	180.	180.	0.208868E-05	0.203785E-05	0.206327E-05	0.413589E-02	0.413526E-02
420.	420.	0.	0.204252E-05	0.201474E-05	0.202863E-05	0.273786E-02	0.273773E-02
420.	240.	180.	0.205997E-05	0.198873E-05	0.202435E-05	0.349720E-02	0.349612E-02
480.	480.	0.	0.205304E-05	0.201065E-05	0.203184E-05	0.212555E-02	0.212532E-02
480.	300.	180.	0.208145E-05	0.195115E-05	0.201630E-05	0.226314E-02	0.226078E-02
540.	540.	0.	0.204367E-05	0.202744E-05	0.203555E-05	0.326851E-02	0.326846E-02
540.	360.	180.	0.203252E-05	0.200360E-05	0.201806E-05	0.333381E-02	0.333364E-02
600.	600.	0.	0.207260E-05	0.205141E-05	0.206200E-05	0.182027E-02	0.182022E-02
600.	420.	180.	0.202924E-05	0.200432E-05	0.201678E-05	0.215988E-02	0.215980E-02
660.	660.	0.	0.207194E-05	0.203243E-05	0.205219E-05	0.236512E-02	0.236490E-02
660.	480.	180.	0.204869E-05	0.201352E-05	0.203111E-05	0.137973E-02	0.137963E-02
720.	720.	0.	0.205371E-05	0.202824E-05	0.204097E-05	0.171216E-02	0.171209E-02
720.	540.	180.	0.208169E-05	0.198726E-05	0.203448E-05	0.136975E-02	0.136901E-02

Case g

flw	stdy	del	k1	k2	aver	gnmse	anmse
60.	60.	0.	0.252953E-05	0.144851E-05	0.198902E-05	0.236391E-01	0.218934E-01
120.	120.	0.	0.255772E-05	0.191333E-05	0.223552E-05	0.130909E-01	0.128190E-01
180.	180.	0.	0.256612E-05	0.199322E-05	0.227967E-05	0.760877E-02	0.748864E-02
240.	240.	0.	0.242344E-05	0.198174E-05	0.220259E-05	0.683334E-02	0.676463E-02
240.	60.	180.	0.273989E-05	0.130105E-05	0.202047E-05	0.182606E-01	0.159455E-01
300.	300.	0.	0.238167E-05	0.204865E-05	0.221516E-05	0.464156E-02	0.461533E-02
300.	120.	180.	0.257828E-05	0.179487E-05	0.218657E-05	0.816023E-02	0.789836E-02
360.	360.	0.	0.229316E-05	0.212038E-05	0.220677E-05	0.322556E-02	0.322062E-02
360.	180.	180.	0.244985E-05	0.199267E-05	0.222126E-05	0.695450E-02	0.688085E-02
420.	420.	0.	0.229086E-05	0.213593E-05	0.221340E-05	0.289966E-02	0.289611E-02
420.	240.	180.	0.228378E-05	0.205148E-05	0.216763E-05	0.619552E-02	0.617773E-02
480.	480.	0.	0.229465E-05	0.214590E-05	0.222027E-05	0.280580E-02	0.280265E-02
480.	300.	180.	0.223273E-05	0.209513E-05	0.216393E-05	0.336582E-02	0.336241E-02
540.	540.	0.	0.226284E-05	0.210640E-05	0.218462E-05	0.155876E-02	0.155676E-02
540.	360.	180.	0.217543E-05	0.208164E-05	0.212854E-05	0.247884E-02	0.247764E-02
600.	600.	0.	0.224414E-05	0.210982E-05	0.217698E-05	0.240509E-02	0.240280E-02
600.	420.	180.	0.218716E-05	0.207272E-05	0.212994E-05	0.203852E-02	0.203705E-02

Case h

flw	stdy	del	k1	k2	aver	gnmse	anmse
20.	20.	0.	0.311212E-05	0.209574E-05	0.260393E-05	0.474594E-01	0.456518E-01
40.	40.	0.	0.245316E-05	0.181581E-05	0.213449E-05	0.188182E-01	0.183987E-01
40.	20.	20.	0.294575E-05	0.191040E-05	0.242807E-05	0.489046E-01	0.466816E-01
60.	60.	0.	0.222543E-05	0.179779E-05	0.201161E-05	0.155969E-01	0.154207E-01
60.	20.	40.	0.277773E-05	0.181668E-05	0.229720E-05	0.429319E-01	0.410534E-01
60.	40.	20.	0.229916E-05	0.172881E-05	0.201398E-05	0.965389E-02	0.946033E-02
80.	80.	0.	0.202656E-05	0.167138E-05	0.184897E-05	0.103014E-01	0.102064E-01
80.	60.	20.	0.220093E-05	0.166330E-05	0.193212E-05	0.110656E-01	0.108514E-01
80.	40.	40.	0.226799E-05	0.163094E-05	0.194947E-05	0.697525E-02	0.678904E-02
80.	20.	60.	0.274229E-05	0.170932E-05	0.222581E-05	0.355207E-01	0.336081E-01
100.	100.	0.	0.198968E-05	0.167179E-05	0.183073E-05	0.998357E-02	0.990832E-02
100.	80.	20.	0.200357E-05	0.154891E-05	0.177624E-05	0.117745E-01	0.115816E-01
100.	60.	40.	0.220162E-05	0.157308E-05	0.188735E-05	0.111081E-01	0.108001E-01
100.	40.	60.	0.225635E-05	0.155875E-05	0.190755E-05	0.725084E-02	0.700841E-02
100.	20.	80.	0.269959E-05	0.163902E-05	0.216930E-05	0.329071E-01	0.309407E-01
120.	120.	0.	0.196397E-05	0.164838E-05	0.180618E-05	0.822809E-02	0.816529E-02
120.	100.	20.	0.194951E-05	0.159408E-05	0.177180E-05	0.999644E-02	0.989587E-02
120.	80.	40.	0.198937E-05	0.146903E-05	0.172920E-05	0.148384E-01	0.145025E-01
120.	60.	60.	0.218159E-05	0.149849E-05	0.184004E-05	0.125726E-01	0.121395E-01
120.	40.	80.	0.223674E-05	0.148982E-05	0.186328E-05	0.816666E-02	0.783859E-02
120.	20.	100.	0.265502E-05	0.157424E-05	0.211463E-05	0.315704E-01	0.295087E-01
140.	140.	0.	0.197124E-05	0.166468E-05	0.181796E-05	0.736583E-02	0.731347E-02
140.	120.	20.	0.195542E-05	0.158085E-05	0.176814E-05	0.751674E-02	0.743241E-02
140.	100.	40.	0.193881E-05	0.155322E-05	0.174602E-05	0.937072E-02	0.925646E-02
140.	80.	60.	0.196648E-05	0.144158E-05	0.170403E-05	0.154880E-01	0.151206E-01
140.	60.	80.	0.215151E-05	0.147520E-05	0.181335E-05	0.125879E-01	0.121502E-01
140.	40.	100.	0.219965E-05	0.147575E-05	0.183770E-05	0.824017E-02	0.792052E-02
140.	20.	120.	0.258845E-05	0.156700E-05	0.207772E-05	0.313305E-01	0.294374E-01
160.	160.	0.	0.201567E-05	0.167164E-05	0.184366E-05	0.323075E-02	0.320263E-02
160.	140.	20.	0.194144E-05	0.160912E-05	0.177528E-05	0.539840E-02	0.535111E-02
160.	120.	40.	0.194449E-05	0.153765E-05	0.174107E-05	0.669529E-02	0.660390E-02
160.	100.	60.	0.192514E-05	0.152325E-05	0.172420E-05	0.940832E-02	0.928053E-02
160.	80.	80.	0.193749E-05	0.143073E-05	0.168411E-05	0.152570E-01	0.149116E-01
160.	60.	100.	0.211547E-05	0.146871E-05	0.179209E-05	0.123166E-01	0.119155E-01
160.	40.	120.	0.215824E-05	0.147542E-05	0.181683E-05	0.892489E-02	0.860973E-02
160.	20.	140.	0.252038E-05	0.157203E-05	0.204621E-05	0.311868E-01	0.295120E-01
180.	180.	0.	0.202711E-05	0.167449E-05	0.185080E-05	0.195748E-02	0.193972E-02
180.	160.	20.	0.198239E-05	0.163259E-05	0.180749E-05	0.220587E-02	0.218522E-02
180.	140.	40.	0.193987E-05	0.158215E-05	0.176101E-05	0.415637E-02	0.411349E-02
180.	120.	60.	0.195099E-05	0.151194E-05	0.173147E-05	0.642203E-02	0.631880E-02
180.	100.	80.	0.192956E-05	0.150370E-05	0.171663E-05	0.980332E-02	0.965250E-02
180.	80.	100.	0.193218E-05	0.142164E-05	0.167691E-05	0.155754E-01	0.152145E-01
180.	60.	120.	0.210557E-05	0.146204E-05	0.178381E-05	0.125426E-01	0.121345E-01
180.	40.	140.	0.214441E-05	0.147399E-05	0.180920E-05	0.933916E-02	0.901857E-02
180.	20.	160.	0.249221E-05	0.157185E-05	0.203203E-05	0.307421E-01	0.291655E-01
200.	200.	0.	0.199573E-05	0.165250E-05	0.182412E-05	0.274559E-02	0.272128E-02
200.	180.	20.	0.199491E-05	0.164699E-05	0.182095E-05	0.131971E-02	0.130767E-02
200.	160.	40.	0.197419E-05	0.161601E-05	0.179510E-05	0.233637E-02	0.231312E-02
200.	140.	60.	0.194027E-05	0.156923E-05	0.175475E-05	0.392254E-02	0.387869E-02
200.	120.	80.	0.195755E-05	0.149837E-05	0.172796E-05	0.694042E-02	0.681789E-02
200.	100.	100.	0.193700E-05	0.149435E-05	0.171567E-05	0.103611E-01	0.101887E-01
200.	80.	120.	0.193632E-05	0.141641E-05	0.167637E-05	0.161865E-01	0.157973E-01
200.	60.	140.	0.210738E-05	0.145999E-05	0.178368E-05	0.126813E-01	0.122636E-01
200.	40.	160.	0.214583E-05	0.147434E-05	0.181008E-05	0.938206E-02	0.905927E-02
200.	20.	180.	0.248787E-05	0.157407E-05	0.203097E-05	0.307018E-01	0.291479E-01
220.	220.	0.	0.198021E-05	0.165945E-05	0.181983E-05	0.426658E-02	0.423344E-02
220.	200.	20.	0.198130E-05	0.163153E-05	0.180641E-05	0.183184E-02	0.181467E-02
220.	180.	40.	0.199674E-05	0.163583E-05	0.181628E-05	0.120550E-02	0.119360E-02
220.	160.	60.	0.198386E-05	0.160809E-05	0.179597E-05	0.242875E-02	0.240217E-02
220.	140.	80.	0.195382E-05	0.155769E-05	0.175575E-05	0.408576E-02	0.403377E-02
220.	120.	100.	0.197753E-05	0.148596E-05	0.173175E-05	0.764220E-02	0.748825E-02
220.	100.	120.	0.195860E-05	0.148308E-05	0.172084E-05	0.107742E-01	0.105686E-01

220.	80.	140.	0.195402E-05	0.140961E-05	0.168182E-05	0.166717E-01	0.162350E-01
220.	60.	160.	0.212231E-05	0.145722E-05	0.178977E-05	0.126626E-01	0.122254E-01
220.	40.	180.	0.215864E-05	0.147608E-05	0.181736E-05	0.960600E-02	0.926725E-02
240.	240.	0.	0.199207E-05	0.167334E-05	0.183270E-05	0.368190E-02	0.365406E-02
240.	220.	20.	0.198724E-05	0.164679E-05	0.181702E-05	0.344758E-02	0.341732E-02
240.	200.	40.	0.198739E-05	0.163169E-05	0.180954E-05	0.132705E-02	0.131423E-02
240.	180.	60.	0.200650E-05	0.163704E-05	0.182177E-05	0.124942E-02	0.123657E-02
240.	160.	80.	0.199699E-05	0.161091E-05	0.180395E-05	0.258087E-02	0.255132E-02
240.	140.	100.	0.196988E-05	0.155705E-05	0.176347E-05	0.454759E-02	0.448529E-02
240.	120.	120.	0.199613E-05	0.148382E-05	0.173998E-05	0.794299E-02	0.777084E-02
240.	100.	140.	0.197859E-05	0.148231E-05	0.173045E-05	0.110720E-01	0.108443E-01
240.	80.	160.	0.197202E-05	0.141118E-05	0.169160E-05	0.169636E-01	0.164975E-01
240.	60.	180.	0.213877E-05	0.146056E-05	0.179966E-05	0.126945E-01	0.122438E-01
260.	260.	0.	0.197275E-05	0.169768E-05	0.183521E-05	0.658061E-02	0.654365E-02
260.	240.	20.	0.199215E-05	0.166564E-05	0.182890E-05	0.355676E-02	0.352842E-02
260.	220.	40.	0.200301E-05	0.164130E-05	0.182216E-05	0.303873E-02	0.300880E-02
260.	200.	60.	0.200372E-05	0.163028E-05	0.181700E-05	0.112106E-02	0.110922E-02
260.	180.	80.	0.202584E-05	0.163778E-05	0.183181E-05	0.132824E-02	0.131333E-02
260.	160.	100.	0.201813E-05	0.161294E-05	0.181553E-05	0.259087E-02	0.255860E-02
260.	140.	120.	0.199169E-05	0.155942E-05	0.177555E-05	0.463627E-02	0.456757E-02
260.	120.	140.	0.202059E-05	0.148443E-05	0.175251E-05	0.818623E-02	0.799468E-02
260.	100.	160.	0.200242E-05	0.148408E-05	0.174325E-05	0.111682E-01	0.109214E-01
260.	80.	180.	0.199277E-05	0.141547E-05	0.170412E-05	0.169547E-01	0.164683E-01
440.	440.	0.	0.205535E-05	0.186708E-05	0.196121E-05	0.292844E-02	0.292169E-02
440.	420.	20.	0.203934E-05	0.186183E-05	0.195058E-05	0.202921E-02	0.202501E-02
440.	400.	40.	0.204410E-05	0.184145E-05	0.194277E-05	0.169165E-02	0.168705E-02
440.	380.	60.	0.205456E-05	0.183167E-05	0.194312E-05	0.225452E-02	0.224711E-02
440.	360.	80.	0.204867E-05	0.183294E-05	0.194080E-05	0.259171E-02	0.258370E-02
440.	340.	100.	0.204140E-05	0.182574E-05	0.193357E-05	0.315953E-02	0.314970E-02
440.	320.	120.	0.204622E-05	0.179647E-05	0.192135E-05	0.292169E-02	0.290935E-02
440.	300.	140.	0.203911E-05	0.179708E-05	0.191810E-05	0.302479E-02	0.301275E-02
440.	280.	160.	0.202104E-05	0.177341E-05	0.189722E-05	0.295355E-02	0.294097E-02
440.	260.	180.	0.200071E-05	0.174438E-05	0.187255E-05	0.305838E-02	0.304405E-02
280.	280.	0.	0.200154E-05	0.172317E-05	0.186235E-05	0.541449E-02	0.538424E-02
280.	260.	20.	0.196046E-05	0.169179E-05	0.182612E-05	0.578962E-02	0.575829E-02
280.	240.	40.	0.199755E-05	0.166555E-05	0.183155E-05	0.330514E-02	0.327799E-02
280.	220.	60.	0.201514E-05	0.163865E-05	0.182690E-05	0.280503E-02	0.277525E-02
280.	200.	80.	0.201963E-05	0.162853E-05	0.182408E-05	0.116144E-02	0.114809E-02
280.	180.	100.	0.204452E-05	0.163688E-05	0.184070E-05	0.144575E-02	0.142802E-02
280.	160.	120.	0.203894E-05	0.161228E-05	0.182561E-05	0.270230E-02	0.266540E-02
280.	140.	140.	0.201233E-05	0.155783E-05	0.178508E-05	0.475283E-02	0.467581E-02
280.	120.	160.	0.204084E-05	0.148221E-05	0.176153E-05	0.846403E-02	0.825122E-02
280.	100.	180.	0.202387E-05	0.148226E-05	0.175307E-05	0.112763E-01	0.110072E-01
300.	300.	0.	0.203374E-05	0.176119E-05	0.189747E-05	0.466282E-02	0.463877E-02
300.	280.	20.	0.199807E-05	0.172768E-05	0.186287E-05	0.481259E-02	0.478725E-02
300.	260.	40.	0.196522E-05	0.170061E-05	0.183292E-05	0.524696E-02	0.521962E-02
300.	240.	60.	0.200577E-05	0.167325E-05	0.183951E-05	0.302020E-02	0.299553E-02
300.	220.	80.	0.202740E-05	0.164444E-05	0.183592E-05	0.266125E-02	0.263231E-02
300.	200.	100.	0.203308E-05	0.163693E-05	0.183501E-05	0.115640E-02	0.114292E-02
300.	180.	120.	0.205917E-05	0.164525E-05	0.185221E-05	0.145027E-02	0.143217E-02
300.	160.	140.	0.205518E-05	0.162026E-05	0.183772E-05	0.262670E-02	0.258992E-02
300.	140.	160.	0.202948E-05	0.156433E-05	0.179690E-05	0.473370E-02	0.465440E-02
300.	120.	180.	0.205873E-05	0.148728E-05	0.177300E-05	0.843103E-02	0.821208E-02
320.	320.	0.	0.203416E-05	0.176810E-05	0.190113E-05	0.466207E-02	0.463924E-02
320.	300.	20.	0.202686E-05	0.175256E-05	0.188971E-05	0.457202E-02	0.454794E-02
320.	280.	40.	0.200284E-05	0.172909E-05	0.186597E-05	0.451389E-02	0.448960E-02
320.	260.	60.	0.197219E-05	0.170222E-05	0.183720E-05	0.493234E-02	0.490571E-02
320.	240.	80.	0.201299E-05	0.167355E-05	0.184327E-05	0.276702E-02	0.274356E-02
320.	220.	100.	0.203644E-05	0.164476E-05	0.184060E-05	0.257463E-02	0.254549E-02
320.	200.	120.	0.204273E-05	0.163915E-05	0.184094E-05	0.116842E-02	0.115438E-02
320.	180.	140.	0.206974E-05	0.164854E-05	0.185914E-05	0.141857E-02	0.140037E-02
320.	160.	160.	0.206605E-05	0.162443E-05	0.184524E-05	0.257567E-02	0.253879E-02
320.	140.	180.	0.204129E-05	0.156906E-05	0.180518E-05	0.474556E-02	0.466437E-02
340.	340.	0.	0.204993E-05	0.179569E-05	0.192281E-05	0.453008E-02	0.451028E-02
340.	320.	20.	0.204137E-05	0.176164E-05	0.190151E-05	0.431597E-02	0.429262E-02
340.	300.	40.	0.202924E-05	0.175626E-05	0.189275E-05	0.423444E-02	0.421242E-02

340.	280.	60.	0.200531E-05	0.173439E-05	0.186985E-05	0.408524E-02	0.406380E-02
340.	260.	80.	0.197764E-05	0.170608E-05	0.184186E-05	0.439801E-02	0.437411E-02
340.	240.	100.	0.201916E-05	0.167844E-05	0.184880E-05	0.247198E-02	0.245099E-02
340.	220.	120.	0.204409E-05	0.164803E-05	0.184606E-05	0.242382E-02	0.239592E-02
340.	200.	140.	0.205090E-05	0.164267E-05	0.184678E-05	0.111982E-02	0.110614E-02
340.	180.	160.	0.207814E-05	0.165174E-05	0.186494E-05	0.130275E-02	0.128572E-02
340.	160.	180.	0.207492E-05	0.162764E-05	0.185128E-05	0.255397E-02	0.251670E-02
360.	360.	0.	0.204580E-05	0.180806E-05	0.192693E-05	0.368203E-02	0.366802E-02
360.	340.	20.	0.203376E-05	0.179107E-05	0.191241E-05	0.394682E-02	0.393093E-02
360.	320.	40.	0.203629E-05	0.176218E-05	0.189923E-05	0.396280E-02	0.394217E-02
360.	300.	60.	0.202752E-05	0.175933E-05	0.189343E-05	0.396416E-02	0.394428E-02
360.	280.	80.	0.200546E-05	0.173774E-05	0.187160E-05	0.380928E-02	0.378979E-02
360.	260.	100.	0.198113E-05	0.170985E-05	0.184549E-05	0.407538E-02	0.405337E-02
360.	240.	120.	0.202415E-05	0.168170E-05	0.185292E-05	0.224469E-02	0.222552E-02
360.	220.	140.	0.204993E-05	0.165034E-05	0.185014E-05	0.222591E-02	0.219995E-02
360.	200.	160.	0.205790E-05	0.164428E-05	0.185109E-05	0.107442E-02	0.106101E-02
360.	180.	180.	0.208664E-05	0.165256E-05	0.186960E-05	0.126043E-02	0.124344E-02
380.	380.	0.	0.204862E-05	0.181224E-05	0.193043E-05	0.330150E-02	0.328913E-02
380.	360.	20.	0.203829E-05	0.180987E-05	0.192408E-05	0.334678E-02	0.333499E-02
380.	340.	40.	0.203000E-05	0.180313E-05	0.191656E-05	0.362087E-02	0.360818E-02
380.	320.	60.	0.203601E-05	0.177288E-05	0.190445E-05	0.359612E-02	0.357896E-02
380.	300.	80.	0.202984E-05	0.176944E-05	0.189964E-05	0.369666E-02	0.367929E-02
380.	280.	100.	0.200924E-05	0.174624E-05	0.187774E-05	0.358639E-02	0.356880E-02
380.	260.	120.	0.198677E-05	0.171746E-05	0.185211E-05	0.379344E-02	0.377339E-02
380.	240.	140.	0.202987E-05	0.168960E-05	0.185974E-05	0.207727E-02	0.205989E-02
380.	220.	160.	0.205586E-05	0.165794E-05	0.185690E-05	0.210477E-02	0.208061E-02
380.	200.	180.	0.206488E-05	0.165092E-05	0.185790E-05	0.103022E-02	0.101744E-02
400.	400.	0.	0.204643E-05	0.183570E-05	0.194106E-05	0.221689E-02	0.221036E-02
400.	380.	20.	0.205361E-05	0.181983E-05	0.193672E-05	0.288288E-02	0.287238E-02
400.	360.	40.	0.204232E-05	0.181882E-05	0.193057E-05	0.305664E-02	0.304639E-02
400.	340.	60.	0.203511E-05	0.181143E-05	0.192327E-05	0.344940E-02	0.343773E-02
400.	320.	80.	0.204009E-05	0.178231E-05	0.191120E-05	0.331899E-02	0.330389E-02
400.	300.	100.	0.203376E-05	0.178073E-05	0.190725E-05	0.346395E-02	0.344871E-02
400.	280.	120.	0.201404E-05	0.175722E-05	0.188563E-05	0.329848E-02	0.328319E-02
400.	260.	140.	0.199297E-05	0.172807E-05	0.186052E-05	0.346973E-02	0.345215E-02
400.	240.	160.	0.203726E-05	0.170077E-05	0.186901E-05	0.194068E-02	0.192496E-02
400.	220.	180.	0.206276E-05	0.166941E-05	0.186608E-05	0.197743E-02	0.195547E-02
420.	420.	0.	0.203995E-05	0.185996E-05	0.194996E-05	0.216764E-02	0.216302E-02
420.	400.	20.	0.204155E-05	0.183556E-05	0.193855E-05	0.186571E-02	0.186044E-02
420.	380.	40.	0.205156E-05	0.182554E-05	0.193855E-05	0.229423E-02	0.228643E-02
420.	360.	60.	0.204450E-05	0.182724E-05	0.193587E-05	0.261192E-02	0.260370E-02
420.	340.	80.	0.203784E-05	0.181975E-05	0.192879E-05	0.318047E-02	0.317030E-02
420.	320.	100.	0.204372E-05	0.179043E-05	0.191708E-05	0.302518E-02	0.301198E-02
420.	300.	120.	0.203668E-05	0.179045E-05	0.191356E-05	0.315803E-02	0.314496E-02
420.	280.	140.	0.201776E-05	0.176764E-05	0.189270E-05	0.304517E-02	0.303187E-02
420.	260.	160.	0.199742E-05	0.173848E-05	0.186795E-05	0.315908E-02	0.314391E-02
420.	240.	180.	0.204282E-05	0.171110E-05	0.187696E-05	0.177601E-02	0.176215E-02
460.	460.	0.	0.206789E-05	0.187787E-05	0.197288E-05	0.322444E-02	0.321697E-02
460.	440.	20.	0.205571E-05	0.185854E-05	0.195713E-05	0.261807E-02	0.261142E-02
460.	420.	40.	0.204223E-05	0.186520E-05	0.195372E-05	0.186765E-02	0.186381E-02
460.	400.	60.	0.204711E-05	0.184366E-05	0.194538E-05	0.148012E-02	0.147608E-02
460.	380.	80.	0.205701E-05	0.183251E-05	0.194476E-05	0.202571E-02	0.201896E-02
460.	360.	100.	0.205238E-05	0.183325E-05	0.194281E-05	0.237453E-02	0.236698E-02
460.	340.	120.	0.204602E-05	0.182531E-05	0.193567E-05	0.291359E-02	0.290412E-02
460.	320.	140.	0.205246E-05	0.179696E-05	0.192471E-05	0.275362E-02	0.274148E-02
460.	300.	160.	0.204475E-05	0.179853E-05	0.192164E-05	0.288959E-02	0.287773E-02
460.	280.	180.	0.202685E-05	0.177513E-05	0.190099E-05	0.281457E-02	0.280223E-02

Case i

flw	stdy	del	k1	k2	aver	gnmse	anmse
300.	300.	0.	0.657828E-06	0.111502E-06	0.384665E-06	0.177173E+00	0.878269E-01
300.	280.	20.	0.582382E-06	0.733601E-07	0.327871E-06	0.201965E+00	0.802672E-01
300.	260.	40.	0.524369E-06	0.358689E-07	0.280119E-06	0.140571E+00	0.336950E-01
300.	240.	60.	0.507287E-06	0.244605E-07	0.265874E-06	0.189672E+00	0.332945E-01
300.	220.	80.	0.485113E-06	0.688218E-08	0.245998E-06	0.506763E+00	0.279583E-01
300.	200.	100.	0.466561E-06	0.000000E+00	0.233280E-06	0.000000E+00	0.252028E-01
300.	180.	120.	0.485431E-06	0.000000E+00	0.242716E-06	0.000000E+00	0.525183E-01
300.	160.	140.	0.503518E-06	0.000000E+00	0.251759E-06	0.000000E+00	0.644494E-01
300.	140.	160.	0.532133E-06	0.000000E+00	0.266067E-06	0.000000E+00	0.481236E-01
300.	120.	180.	0.542277E-06	0.000000E+00	0.271139E-06	0.000000E+00	0.197744E-01
320.	320.	0.	0.663737E-06	0.105551E-06	0.384644E-06	0.137689E+00	0.651990E-01
320.	300.	20.	0.549350E-06	0.782296E-07	0.313790E-06	0.190745E+00	0.832521E-01
320.	280.	40.	0.534881E-06	0.587787E-07	0.296830E-06	0.188945E+00	0.674213E-01
320.	260.	60.	0.501194E-06	0.332739E-07	0.267234E-06	0.166932E+00	0.389823E-01
320.	240.	80.	0.490731E-06	0.231655E-07	0.256948E-06	0.216360E+00	0.372538E-01
320.	220.	100.	0.468260E-06	0.616727E-08	0.237214E-06	0.603482E+00	0.309717E-01
320.	200.	120.	0.445031E-06	0.000000E+00	0.222516E-06	0.000000E+00	0.174392E-01
320.	180.	140.	0.462188E-06	0.000000E+00	0.231094E-06	0.000000E+00	0.418228E-01
320.	160.	160.	0.483087E-06	0.000000E+00	0.241544E-06	0.000000E+00	0.522895E-01
320.	140.	180.	0.511776E-06	0.000000E+00	0.255888E-06	0.000000E+00	0.356364E-01
340.	340.	0.	0.676599E-06	0.112645E-06	0.394621E-06	0.126430E+00	0.618771E-01
340.	320.	20.	0.585761E-06	0.747287E-07	0.330245E-06	0.118549E+00	0.475810E-01
340.	300.	40.	0.514552E-06	0.602797E-07	0.287416E-06	0.161809E+00	0.607550E-01
340.	280.	60.	0.507756E-06	0.494576E-07	0.278607E-06	0.216859E+00	0.701590E-01
340.	260.	80.	0.476089E-06	0.274362E-07	0.251763E-06	0.215727E+00	0.444564E-01
340.	240.	100.	0.468702E-06	0.199095E-07	0.244306E-06	0.260975E+00	0.408028E-01
340.	220.	120.	0.447893E-06	0.494899E-08	0.226421E-06	0.749594E+00	0.324103E-01
340.	200.	140.	0.422054E-06	0.000000E+00	0.211027E-06	0.000000E+00	0.279171E-01
340.	180.	160.	0.437321E-06	0.000000E+00	0.218661E-06	0.000000E+00	0.458853E-01
340.	160.	180.	0.457009E-06	0.000000E+00	0.228505E-06	0.000000E+00	0.620672E-01
360.	360.	0.	0.676060E-06	0.123924E-06	0.399992E-06	0.117826E+00	0.616991E-01
360.	340.	20.	0.571627E-06	0.848975E-07	0.328262E-06	0.117193E+00	0.527799E-01
360.	320.	40.	0.542582E-06	0.610401E-07	0.301811E-06	0.118552E+00	0.431040E-01
360.	300.	60.	0.494922E-06	0.564819E-07	0.275702E-06	0.177155E+00	0.651509E-01
360.	280.	80.	0.491595E-06	0.473035E-07	0.269449E-06	0.253726E+00	0.812666E-01
360.	260.	100.	0.461535E-06	0.271376E-07	0.244336E-06	0.236886E+00	0.496979E-01
360.	240.	120.	0.455912E-06	0.205856E-07	0.238249E-06	0.272628E+00	0.450770E-01
360.	220.	140.	0.432793E-06	0.495426E-08	0.218874E-06	0.754601E+00	0.337745E-01
360.	200.	160.	0.411192E-06	0.000000E+00	0.205596E-06	0.000000E+00	0.268416E-01
360.	180.	180.	0.427325E-06	0.000000E+00	0.213663E-06	0.000000E+00	0.435972E-01
380.	380.	0.	0.640006E-06	0.109695E-06	0.374851E-06	0.113738E+00	0.568279E-01
380.	360.	20.	0.565587E-06	0.920492E-07	0.328818E-06	0.124488E+00	0.599425E-01
380.	340.	40.	0.514637E-06	0.660002E-07	0.290319E-06	0.961607E-01	0.387520E-01
380.	320.	60.	0.506926E-06	0.490919E-07	0.278009E-06	0.868777E-01	0.279734E-01
380.	300.	80.	0.473305E-06	0.471990E-07	0.260252E-06	0.149490E+00	0.493058E-01
380.	280.	100.	0.469977E-06	0.385597E-07	0.254268E-06	0.221138E+00	0.619854E-01
380.	260.	120.	0.443195E-06	0.214478E-07	0.232321E-06	0.243399E+00	0.428665E-01
380.	240.	140.	0.439137E-06	0.156582E-07	0.227397E-06	0.308763E+00	0.410579E-01
380.	220.	160.	0.413235E-06	0.486210E-08	0.209048E-06	0.627763E+00	0.288617E-01
380.	200.	180.	0.395585E-06	0.000000E+00	0.197792E-06	0.000000E+00	0.382666E-01
280.	280.	0.	0.735309E-06	0.987255E-07	0.417017E-06	0.139247E+00	0.581270E-01
280.	260.	20.	0.607865E-06	0.529102E-07	0.330388E-06	0.179076E+00	0.527637E-01
280.	240.	40.	0.565167E-06	0.296093E-07	0.297388E-06	0.213547E+00	0.404066E-01
280.	220.	60.	0.535339E-06	0.248687E-08	0.268913E-06	0.114630E+01	0.211037E-01
280.	200.	80.	0.509216E-06	0.000000E+00	0.254608E-06	0.000000E+00	0.234191E-01
280.	180.	100.	0.527310E-06	0.000000E+00	0.263655E-06	0.000000E+00	0.522498E-01
280.	160.	120.	0.545723E-06	0.000000E+00	0.272862E-06	0.000000E+00	0.769734E-01
280.	140.	140.	0.571265E-06	0.000000E+00	0.285632E-06	0.000000E+00	0.522122E-01
280.	120.	160.	0.585254E-06	0.000000E+00	0.292627E-06	0.000000E+00	0.275533E-01
280.	100.	180.	0.633453E-06	0.000000E+00	0.316727E-06	0.000000E+00	0.831179E-01
260.	260.	0.	0.742762E-06	0.757044E-07	0.409233E-06	0.160452E+00	0.538733E-01
260.	240.	20.	0.626772E-06	0.309097E-07	0.328841E-06	0.207341E+00	0.371465E-01

260.	220.	40.	0.581608E-06	0.529640E-08	0.293452E-06	0.433699E+00	0.155140E-01
260.	200.	60.	0.556181E-06	0.000000E+00	0.278091E-06	0.000000E+00	0.213669E-01
260.	180.	80.	0.557464E-06	0.000000E+00	0.278732E-06	0.000000E+00	0.543387E-01
260.	160.	100.	0.585202E-06	0.000000E+00	0.292601E-06	0.000000E+00	0.805970E-01
260.	140.	120.	0.616822E-06	0.000000E+00	0.308411E-06	0.000000E+00	0.576610E-01
260.	120.	140.	0.635361E-06	0.000000E+00	0.317680E-06	0.000000E+00	0.381953E-01
260.	100.	160.	0.693247E-06	0.000000E+00	0.346624E-06	0.000000E+00	0.902358E-01
260.	80.	180.	0.808296E-06	0.319800E-08	0.405747E-06	0.500855E+01	0.786412E-01
240.	240.	0.	0.791303E-06	0.684684E-07	0.429886E-06	0.175768E+00	0.515308E-01
240.	220.	20.	0.676864E-06	0.140387E-07	0.345451E-06	0.448600E+00	0.357202E-01
240.	200.	40.	0.634902E-06	0.000000E+00	0.317451E-06	0.000000E+00	0.469153E-01
240.	180.	60.	0.621494E-06	0.000000E+00	0.310747E-06	0.000000E+00	0.586851E-01
240.	160.	80.	0.646637E-06	0.000000E+00	0.323319E-06	0.000000E+00	0.736340E-01
240.	140.	100.	0.682311E-06	0.000000E+00	0.341156E-06	0.000000E+00	0.569568E-01
240.	120.	120.	0.691185E-06	0.000000E+00	0.345592E-06	0.000000E+00	0.310813E-01
240.	100.	140.	0.754721E-06	0.000000E+00	0.377360E-06	0.000000E+00	0.636210E-01
240.	80.	160.	0.883119E-06	0.374450E-08	0.443432E-06	0.336790E+01	0.566395E-01
240.	60.	180.	0.990977E-06	0.139590E-08	0.496186E-06	0.000000E+00	0.609564E-01
220.	220.	0.	0.770863E-06	0.533414E-07	0.412102E-06	0.199248E+00	0.482420E-01
220.	200.	20.	0.680246E-06	0.156405E-07	0.347943E-06	0.000000E+00	0.606162E-01
220.	180.	40.	0.654797E-06	0.000000E+00	0.327398E-06	0.000000E+00	0.704929E-01
220.	160.	60.	0.668381E-06	0.000000E+00	0.334191E-06	0.000000E+00	0.746927E-01
220.	140.	80.	0.695825E-06	0.000000E+00	0.347912E-06	0.000000E+00	0.545933E-01
220.	120.	100.	0.702882E-06	0.000000E+00	0.351441E-06	0.000000E+00	0.280264E-01
220.	100.	120.	0.771407E-06	0.000000E+00	0.385703E-06	0.000000E+00	0.598802E-01
220.	80.	140.	0.901941E-06	0.394990E-08	0.452946E-06	0.324042E+01	0.562695E-01
220.	60.	160.	0.992957E-06	0.747156E-08	0.500214E-06	0.000000E+00	0.541332E-01
220.	40.	180.	0.931511E-06	0.121683E-06	0.526597E-06	0.318732E+00	0.130283E+00
200.	200.	0.	0.872163E-06	0.714051E-07	0.471784E-06	0.199241E+00	0.557468E-01
200.	180.	20.	0.740596E-06	0.149344E-07	0.377765E-06	0.000000E+00	0.674117E-01
200.	160.	40.	0.724907E-06	0.000000E+00	0.362454E-06	0.000000E+00	0.771728E-01
200.	140.	60.	0.753022E-06	0.000000E+00	0.376511E-06	0.000000E+00	0.538527E-01
200.	120.	80.	0.755447E-06	0.000000E+00	0.377723E-06	0.000000E+00	0.373913E-01
200.	100.	100.	0.836814E-06	0.000000E+00	0.418407E-06	0.000000E+00	0.542423E-01
200.	80.	120.	0.973086E-06	0.721410E-08	0.490150E-06	0.186228E+01	0.544153E-01
200.	60.	140.	0.102278E-05	0.386987E-07	0.530741E-06	0.000000E+00	0.361525E-01
200.	40.	160.	0.950295E-06	0.139146E-06	0.544721E-06	0.210428E+00	0.937747E-01
200.	20.	180.	0.439881E-06	0.989583E-07	0.269420E-06	0.000000E+00	0.885634E+00
180.	180.	0.	0.983989E-06	0.486473E-07	0.516318E-06	0.161744E+00	0.290432E-01
180.	160.	20.	0.891807E-06	0.769170E-08	0.449750E-06	0.138419E+01	0.469404E-01
180.	140.	40.	0.906324E-06	0.000000E+00	0.453162E-06	0.000000E+00	0.255555E-01
180.	120.	60.	0.907335E-06	0.000000E+00	0.453668E-06	0.000000E+00	0.251533E-01
180.	100.	80.	0.979553E-06	0.000000E+00	0.489777E-06	0.000000E+00	0.703215E-01
180.	80.	100.	0.111274E-05	0.117010E-07	0.562219E-06	0.137021E+01	0.564408E-01
180.	60.	120.	0.115373E-05	0.257056E-07	0.589717E-06	0.000000E+00	0.319233E-01
180.	40.	140.	0.112638E-05	0.988109E-07	0.612595E-06	0.234212E+00	0.694626E-01
180.	20.	160.	0.625997E-06	0.923319E-07	0.359164E-06	0.863210E+00	0.386771E+00
160.	160.	0.	0.117322E-05	0.605281E-07	0.616872E-06	0.204408E+00	0.381455E-01
160.	140.	20.	0.106184E-05	0.169787E-07	0.539412E-06	0.537894E+00	0.333290E-01
160.	120.	40.	0.102887E-05	0.000000E+00	0.514436E-06	0.000000E+00	0.103662E-01
160.	100.	60.	0.104294E-05	0.000000E+00	0.521468E-06	0.000000E+00	0.685873E-01
160.	80.	80.	0.118937E-05	0.138340E-07	0.601602E-06	0.125874E+01	0.572247E-01
160.	60.	100.	0.123950E-05	0.957635E-08	0.624540E-06	0.000000E+00	0.333486E-01
160.	40.	120.	0.123056E-05	0.816005E-07	0.656081E-06	0.272279E+00	0.635177E-01
160.	20.	140.	0.702243E-06	0.937373E-07	0.397990E-06	0.851890E+00	0.354029E+00
140.	140.	0.	0.134318E-05	0.756661E-07	0.709422E-06	0.161186E+00	0.325502E-01
140.	120.	20.	0.117091E-05	0.250810E-07	0.597995E-06	0.126284E+00	0.103710E-01
140.	100.	40.	0.114119E-05	0.000000E+00	0.570596E-06	0.000000E+00	0.482875E-01
140.	80.	60.	0.129108E-05	0.167990E-07	0.653940E-06	0.897895E+00	0.455393E-01
140.	60.	80.	0.132071E-05	0.218999E-07	0.671304E-06	0.000000E+00	0.190429E-01
140.	40.	100.	0.131239E-05	0.112614E-06	0.712500E-06	0.135047E+00	0.393159E-01
140.	20.	120.	0.785584E-06	0.152998E-06	0.469291E-06	0.240721E+00	0.131374E+00
120.	120.	0.	0.149080E-05	0.880860E-07	0.789441E-06	0.959900E-01	0.202261E-01
120.	100.	20.	0.128027E-05	0.000000E+00	0.640135E-06	0.000000E+00	0.397416E-01
120.	80.	40.	0.139179E-05	0.166900E-07	0.704242E-06	0.819473E+00	0.383815E-01
120.	60.	60.	0.146182E-05	0.322729E-08	0.732525E-06	0.000000E+00	0.199995E-01

120.	40.	80.	0.148598E-05	0.108467E-06	0.797223E-06	0.130605E+00	0.331215E-01
120.	20.	100.	0.939176E-06	0.155902E-06	0.547539E-06	0.346577E+00	0.169265E+00
100.	100.	0.	0.157125E-05	0.617488E-07	0.816499E-06	0.183967E+00	0.267734E-01
100.	80.	20.	0.156667E-05	0.409463E-07	0.803811E-06	0.141540E+00	0.140529E-01
100.	60.	40.	0.154895E-05	0.650283E-08	0.777725E-06	0.000000E+00	0.111201E-01
100.	40.	60.	0.159411E-05	0.116884E-06	0.855495E-06	0.144619E+00	0.368180E-01
100.	20.	80.	0.107644E-05	0.170523E-06	0.623482E-06	0.375155E+00	0.177148E+00
80.	80.	0.	0.182744E-05	0.168191E-06	0.997817E-06	0.212279E+00	0.655316E-01
80.	60.	20.	0.155070E-05	0.226387E-07	0.786671E-06	0.000000E+00	0.125051E-01
80.	40.	40.	0.158610E-05	0.146888E-06	0.866493E-06	0.895891E-01	0.277999E-01
80.	20.	60.	0.110206E-05	0.183612E-06	0.642834E-06	0.366780E+00	0.179603E+00
60.	60.	0.	0.222196E-05	0.100645E-06	0.116130E-05	0.855490E-01	0.141857E-01
60.	40.	20.	0.210131E-05	0.764149E-07	0.108886E-05	0.192949E+00	0.261316E-01
60.	20.	40.	0.193962E-05	0.460265E-07	0.992821E-06	0.343848E+00	0.311421E-01
40.	40.	0.	0.290693E-05	0.930246E-07	0.149998E-05	0.124863E+00	0.150071E-01
40.	20.	20.	0.306896E-05	0.112906E-06	0.159093E-05	0.108601E+00	0.148675E-01
20.	20.	0.	0.367706E-05	0.177340E-06	0.192720E-05	0.102152E+00	0.179350E-01

Case j

flw	stdy	del	k1	k2	aver	gnmse	anmse
20.	20.	0.	0.228259E-05	0.324146E-06	0.130337E-05	0.000000E+00	0.195512E+00
40.	40.	0.	0.232472E-05	0.501711E-06	0.141322E-05	0.357432E-01	0.208738E-01
40.	20.	20.	0.160562E-05	0.304864E-06	0.955242E-06	0.000000E+00	0.335641E+00
60.	60.	0.	0.238326E-05	0.555908E-06	0.146958E-05	0.391592E-01	0.240226E-01
60.	20.	40.	0.130922E-05	0.292211E-06	0.800718E-06	0.000000E+00	0.408437E+00
60.	40.	20.	0.195331E-05	0.486312E-06	0.121981E-05	0.488805E-01	0.312059E-01
80.	80.	0.	0.243714E-05	0.495859E-06	0.146650E-05	0.479892E-01	0.269661E-01
80.	20.	60.	0.140006E-05	0.322812E-06	0.861435E-06	0.000000E+00	0.413628E+00
80.	40.	40.	0.204431E-05	0.449374E-06	0.124684E-05	0.547204E-01	0.323356E-01
80.	60.	20.	0.238353E-05	0.482226E-06	0.143288E-05	0.478541E-01	0.267899E-01
100.	100.	0.	0.237053E-05	0.300159E-06	0.133535E-05	0.594086E-01	0.237060E-01
100.	80.	20.	0.233009E-05	0.330155E-06	0.133012E-05	0.399381E-01	0.173658E-01
100.	60.	40.	0.245595E-05	0.358689E-06	0.140732E-05	0.244588E-01	0.108790E-01
100.	40.	60.	0.216245E-05	0.393555E-06	0.127800E-05	0.449552E-01	0.234244E-01
100.	20.	80.	0.150639E-05	0.314291E-06	0.910339E-06	0.000000E+00	0.388047E+00
120.	120.	0.	0.230192E-05	0.326772E-06	0.131435E-05	0.507694E-01	0.221064E-01
120.	100.	20.	0.221596E-05	0.205515E-06	0.121074E-05	0.388767E-01	0.120780E-01
120.	80.	40.	0.229887E-05	0.236482E-06	0.126768E-05	0.395616E-01	0.133835E-01
120.	60.	60.	0.243890E-05	0.273504E-06	0.135620E-05	0.300786E-01	0.109085E-01
120.	40.	80.	0.221967E-05	0.341419E-06	0.128054E-05	0.509820E-01	0.235615E-01
120.	20.	100.	0.154620E-05	0.298780E-06	0.922489E-06	0.000000E+00	0.371590E+00
140.	140.	0.	0.226829E-05	0.333578E-06	0.130093E-05	0.252973E-01	0.113100E-01
140.	120.	20.	0.217083E-05	0.293516E-06	0.123218E-05	0.488074E-01	0.204833E-01
140.	100.	40.	0.217845E-05	0.196913E-06	0.118768E-05	0.228215E-01	0.694013E-02
140.	80.	60.	0.227537E-05	0.232174E-06	0.125377E-05	0.394529E-01	0.132589E-01
140.	60.	80.	0.238912E-05	0.267557E-06	0.132834E-05	0.314575E-01	0.113962E-01
140.	40.	100.	0.219045E-05	0.330299E-06	0.126037E-05	0.553712E-01	0.252189E-01
140.	20.	120.	0.153636E-05	0.292863E-06	0.914614E-06	0.000000E+00	0.360672E+00
160.	160.	0.	0.226631E-05	0.373136E-06	0.131972E-05	0.102262E-01	0.496520E-02
160.	140.	20.	0.219310E-05	0.331499E-06	0.126230E-05	0.158805E-01	0.724570E-02
160.	120.	40.	0.218076E-05	0.300736E-06	0.124075E-05	0.407772E-01	0.173718E-01
160.	100.	60.	0.220200E-05	0.211231E-06	0.120662E-05	0.200442E-01	0.640360E-02
160.	80.	80.	0.230598E-05	0.238189E-06	0.127208E-05	0.391055E-01	0.132734E-01
160.	60.	100.	0.241250E-05	0.267636E-06	0.134007E-05	0.278347E-01	0.100079E-01
160.	40.	120.	0.220514E-05	0.332619E-06	0.126888E-05	0.525444E-01	0.239369E-01
160.	20.	140.	0.155456E-05	0.297541E-06	0.926053E-06	0.000000E+00	0.363732E+00
180.	180.	0.	0.227618E-05	0.365257E-06	0.132072E-05	0.686723E-02	0.327315E-02
180.	160.	20.	0.219291E-05	0.344813E-06	0.126886E-05	0.335907E-02	0.157759E-02
180.	140.	40.	0.218360E-05	0.323879E-06	0.125374E-05	0.123594E-01	0.556084E-02
180.	120.	60.	0.219843E-05	0.299241E-06	0.124884E-05	0.386023E-01	0.162830E-01
180.	100.	80.	0.223242E-05	0.214917E-06	0.122367E-05	0.201808E-01	0.646631E-02
180.	80.	100.	0.234107E-05	0.238187E-06	0.128963E-05	0.387471E-01	0.129910E-01
180.	60.	120.	0.244903E-05	0.266617E-06	0.135782E-05	0.280862E-01	0.994693E-02
180.	40.	140.	0.224013E-05	0.330767E-06	0.128545E-05	0.531647E-01	0.238402E-01
180.	20.	160.	0.158892E-05	0.295152E-06	0.942037E-06	0.000000E+00	0.360536E+00
200.	200.	0.	0.224217E-05	0.380274E-06	0.131122E-05	0.116649E-01	0.578486E-02
200.	180.	20.	0.217591E-05	0.328497E-06	0.125221E-05	0.773113E-02	0.352424E-02
200.	160.	40.	0.217524E-05	0.317055E-06	0.124615E-05	0.758904E-02	0.337046E-02
200.	140.	60.	0.217622E-05	0.309082E-06	0.124265E-05	0.107847E-01	0.469771E-02
200.	120.	80.	0.220232E-05	0.287427E-06	0.124487E-05	0.365312E-01	0.149219E-01
200.	100.	100.	0.224940E-05	0.206760E-06	0.122808E-05	0.245435E-01	0.756861E-02
200.	80.	120.	0.236074E-05	0.224646E-06	0.129269E-05	0.417405E-01	0.132469E-01
200.	60.	140.	0.247343E-05	0.250527E-06	0.136198E-05	0.321369E-01	0.107354E-01
200.	40.	160.	0.226249E-05	0.314402E-06	0.128844E-05	0.579541E-01	0.248327E-01
200.	20.	180.	0.160669E-05	0.283986E-06	0.945338E-06	0.000000E+00	0.353571E+00
220.	220.	0.	0.217983E-05	0.388773E-06	0.128430E-05	0.929216E-02	0.477421E-02
220.	200.	20.	0.215253E-05	0.363616E-06	0.125807E-05	0.579470E-02	0.286557E-02
220.	180.	40.	0.214118E-05	0.323688E-06	0.123243E-05	0.710434E-02	0.324173E-02
220.	160.	60.	0.216704E-05	0.322014E-06	0.124453E-05	0.712610E-02	0.321059E-02
220.	140.	80.	0.217207E-05	0.318333E-06	0.124520E-05	0.969365E-02	0.432279E-02
220.	120.	100.	0.220055E-05	0.294810E-06	0.124768E-05	0.341866E-01	0.142470E-01
220.	100.	120.	0.224492E-05	0.211412E-06	0.122816E-05	0.215480E-01	0.677989E-02

220.	80.	140.	0.235583E-05	0.226950E-06	0.129139E-05	0.401072E-01	0.128582E-01
220.	60.	160.	0.246978E-05	0.253248E-06	0.136151E-05	0.301838E-01	0.101844E-01
220.	40.	180.	0.226288E-05	0.315971E-06	0.128943E-05	0.567327E-01	0.243977E-01
240.	240.	0.	0.218154E-05	0.390543E-06	0.128604E-05	0.639963E-02	0.329669E-02
240.	220.	20.	0.212093E-05	0.381042E-06	0.125098E-05	0.549262E-02	0.283644E-02
240.	200.	40.	0.212121E-05	0.364658E-06	0.124293E-05	0.433288E-02	0.216945E-02
240.	180.	60.	0.212013E-05	0.328290E-06	0.122421E-05	0.571943E-02	0.265620E-02
240.	160.	80.	0.215410E-05	0.329916E-06	0.124201E-05	0.555989E-02	0.256146E-02
240.	140.	100.	0.215447E-05	0.328758E-06	0.124161E-05	0.100177E-01	0.460267E-02
240.	120.	120.	0.217988E-05	0.306876E-06	0.124338E-05	0.341309E-01	0.147685E-01
240.	100.	140.	0.221869E-05	0.220927E-06	0.121981E-05	0.171035E-01	0.563440E-02
240.	80.	160.	0.233234E-05	0.234149E-06	0.128325E-05	0.374791E-01	0.124295E-01
240.	60.	180.	0.244527E-05	0.260769E-06	0.135302E-05	0.290499E-01	0.101185E-01
260.	260.	0.	0.217172E-05	0.404927E-06	0.128832E-05	0.506520E-02	0.268366E-02
260.	240.	20.	0.213220E-05	0.373451E-06	0.125283E-05	0.402246E-02	0.204067E-02
260.	220.	40.	0.210753E-05	0.375703E-06	0.124162E-05	0.432836E-02	0.222313E-02
260.	200.	60.	0.211707E-05	0.363650E-06	0.124036E-05	0.450392E-02	0.225379E-02
260.	180.	80.	0.211956E-05	0.330572E-06	0.122506E-05	0.555449E-02	0.259320E-02
260.	160.	100.	0.215374E-05	0.334515E-06	0.124413E-05	0.489938E-02	0.228045E-02
260.	140.	120.	0.215231E-05	0.335835E-06	0.124407E-05	0.102788E-01	0.480045E-02
260.	120.	140.	0.217647E-05	0.315147E-06	0.124581E-05	0.330791E-01	0.146190E-01
260.	100.	160.	0.221467E-05	0.229365E-06	0.122202E-05	0.147020E-01	0.500103E-02
260.	80.	180.	0.232946E-05	0.242895E-06	0.128618E-05	0.363831E-01	0.124443E-01
280.	280.	0.	0.217737E-05	0.401371E-06	0.128937E-05	0.560933E-02	0.294872E-02
280.	260.	20.	0.213384E-05	0.364118E-06	0.124898E-05	0.361492E-02	0.180050E-02
280.	240.	40.	0.212889E-05	0.358148E-06	0.124352E-05	0.318036E-02	0.156815E-02
280.	220.	60.	0.211145E-05	0.365142E-06	0.123830E-05	0.378590E-02	0.190354E-02
280.	200.	80.	0.212586E-05	0.354857E-06	0.124036E-05	0.440930E-02	0.216204E-02
280.	180.	100.	0.213171E-05	0.324624E-06	0.122817E-05	0.603122E-02	0.276693E-02
280.	160.	120.	0.216648E-05	0.330998E-06	0.124874E-05	0.496694E-02	0.228416E-02
280.	140.	140.	0.216567E-05	0.334352E-06	0.125001E-05	0.100623E-01	0.466302E-02
280.	120.	160.	0.219036E-05	0.313947E-06	0.125215E-05	0.315414E-01	0.138337E-01
280.	100.	180.	0.223014E-05	0.227839E-06	0.122899E-05	0.140278E-01	0.471905E-02
300.	300.	0.	0.217818E-05	0.369472E-06	0.127382E-05	0.725412E-02	0.359783E-02
300.	280.	20.	0.215048E-05	0.356288E-06	0.125338E-05	0.604418E-02	0.294785E-02
300.	260.	40.	0.213411E-05	0.339618E-06	0.123686E-05	0.467671E-02	0.221567E-02
300.	240.	60.	0.214792E-05	0.341498E-06	0.124471E-05	0.369310E-02	0.174848E-02
300.	220.	80.	0.213450E-05	0.352837E-06	0.124367E-05	0.382532E-02	0.186264E-02
300.	200.	100.	0.214757E-05	0.344989E-06	0.124628E-05	0.438939E-02	0.209375E-02
300.	180.	120.	0.215487E-05	0.316391E-06	0.123563E-05	0.648447E-02	0.289563E-02
300.	160.	140.	0.219148E-05	0.323722E-06	0.125760E-05	0.547549E-02	0.245611E-02
300.	140.	160.	0.219362E-05	0.327910E-06	0.126077E-05	0.108775E-01	0.492237E-02
300.	120.	180.	0.221945E-05	0.307331E-06	0.126339E-05	0.311653E-01	0.133183E-01
320.	320.	0.	0.215776E-05	0.338323E-06	0.124804E-05	0.631080E-02	0.295775E-02
320.	300.	20.	0.213615E-05	0.334488E-06	0.123532E-05	0.623778E-02	0.292068E-02
320.	280.	40.	0.213887E-05	0.337959E-06	0.123841E-05	0.505100E-02	0.238064E-02
320.	260.	60.	0.213844E-05	0.332316E-06	0.123538E-05	0.501265E-02	0.233408E-02
320.	240.	80.	0.215387E-05	0.333382E-06	0.124362E-05	0.377697E-02	0.175358E-02
320.	220.	100.	0.214293E-05	0.346167E-06	0.124455E-05	0.436393E-02	0.209001E-02
320.	200.	120.	0.215801E-05	0.338953E-06	0.124848E-05	0.497325E-02	0.233383E-02
320.	180.	140.	0.216680E-05	0.310133E-06	0.123847E-05	0.737440E-02	0.323091E-02
320.	160.	160.	0.220420E-05	0.318188E-06	0.126119E-05	0.619557E-02	0.273182E-02
320.	140.	180.	0.220796E-05	0.321872E-06	0.126492E-05	0.127655E-01	0.567007E-02
340.	340.	0.	0.217306E-05	0.333368E-06	0.125321E-05	0.463169E-02	0.213641E-02
340.	320.	20.	0.213016E-05	0.312091E-06	0.122112E-05	0.344877E-02	0.153758E-02
340.	300.	40.	0.212401E-05	0.314276E-06	0.121914E-05	0.407014E-02	0.182796E-02
340.	280.	60.	0.213693E-05	0.326250E-06	0.123159E-05	0.393339E-02	0.180791E-02
340.	260.	80.	0.214105E-05	0.328895E-06	0.123497E-05	0.411776E-02	0.190121E-02
340.	240.	100.	0.215577E-05	0.331022E-06	0.124340E-05	0.326142E-02	0.150538E-02
340.	220.	120.	0.214437E-05	0.345059E-06	0.124471E-05	0.472418E-02	0.225621E-02
340.	200.	140.	0.216084E-05	0.338028E-06	0.124944E-05	0.540225E-02	0.252769E-02
340.	180.	160.	0.216890E-05	0.309452E-06	0.123918E-05	0.761007E-02	0.332625E-02
340.	160.	180.	0.220579E-05	0.318162E-06	0.126198E-05	0.665092E-02	0.293084E-02
360.	360.	0.	0.220929E-05	0.345476E-06	0.127738E-05	0.381852E-02	0.178618E-02
360.	340.	20.	0.214068E-05	0.322811E-06	0.123175E-05	0.430635E-02	0.196140E-02
360.	320.	40.	0.211572E-05	0.308770E-06	0.121224E-05	0.304577E-02	0.135397E-02

360.	300.	60.	0.211347E-05	0.312901E-06	0.121319E-05	0.326856E-02	0.146860E-02
360.	280.	80.	0.212714E-05	0.325452E-06	0.122630E-05	0.356715E-02	0.164215E-02
360.	260.	100.	0.213379E-05	0.330401E-06	0.123209E-05	0.432711E-02	0.200957E-02
360.	240.	120.	0.215106E-05	0.332934E-06	0.124200E-05	0.334371E-02	0.155238E-02
360.	220.	140.	0.214032E-05	0.346606E-06	0.124346E-05	0.504716E-02	0.242157E-02
360.	200.	160.	0.215738E-05	0.339152E-06	0.124827E-05	0.576523E-02	0.270722E-02
360.	180.	180.	0.216357E-05	0.310848E-06	0.123721E-05	0.705295E-02	0.309888E-02
380.	380.	0.	0.217800E-05	0.336342E-06	0.125717E-05	0.655776E-02	0.303952E-02
380.	360.	20.	0.214957E-05	0.325587E-06	0.123758E-05	0.440119E-02	0.201114E-02
380.	340.	40.	0.210726E-05	0.312370E-06	0.120982E-05	0.412187E-02	0.185371E-02
380.	320.	60.	0.208843E-05	0.296575E-06	0.119250E-05	0.261420E-02	0.113861E-02
380.	300.	80.	0.208799E-05	0.302417E-06	0.119520E-05	0.264441E-02	0.116891E-02
380.	280.	100.	0.210320E-05	0.317995E-06	0.121060E-05	0.367454E-02	0.167689E-02
380.	260.	120.	0.211231E-05	0.327125E-06	0.121972E-05	0.435151E-02	0.202112E-02
380.	240.	140.	0.213038E-05	0.331139E-06	0.123076E-05	0.285725E-02	0.133067E-02
380.	220.	160.	0.211794E-05	0.345953E-06	0.123194E-05	0.434132E-02	0.209589E-02
380.	200.	180.	0.213297E-05	0.339496E-06	0.123624E-05	0.484708E-02	0.229667E-02

Case k

flw	stdy	del	k1	k2	aver	gnmse	anmse
60.	60.	0.	0.230116E-05	0.650979E-06	0.147607E-05	0.518016E-01	0.356158E-01
120.	120.	0.	0.233019E-05	0.624992E-06	0.147759E-05	0.352093E-01	0.234863E-01
120.	60.	60.	0.217230E-05	0.498296E-06	0.133530E-05	0.191507E-01	0.116262E-01
180.	180.	0.	0.228854E-05	0.665369E-06	0.147695E-05	0.156318E-01	0.109118E-01
180.	60.	120.	0.222097E-05	0.488348E-06	0.135466E-05	0.208741E-01	0.123373E-01
180.	120.	60.	0.214818E-05	0.558372E-06	0.135328E-05	0.847905E-02	0.555352E-02
240.	240.	0.	0.242047E-05	0.639667E-06	0.153007E-05	0.723437E-02	0.478445E-02
240.	60.	180.	0.234139E-05	0.473410E-06	0.140740E-05	0.245384E-01	0.137316E-01
240.	120.	120.	0.224515E-05	0.538575E-06	0.139186E-05	0.739872E-02	0.461801E-02
240.	180.	60.	0.230103E-05	0.593766E-06	0.144740E-05	0.656446E-02	0.428115E-02
300.	300.	0.	0.238316E-05	0.645259E-06	0.151421E-05	0.370509E-02	0.248493E-02
300.	120.	180.	0.228275E-05	0.527555E-06	0.140515E-05	0.697043E-02	0.425146E-02
300.	180.	120.	0.232419E-05	0.582447E-06	0.145332E-05	0.551702E-02	0.353599E-02
300.	240.	60.	0.236824E-05	0.604883E-06	0.148656E-05	0.243827E-02	0.158057E-02
360.	360.	0.	0.239129E-05	0.616066E-06	0.150368E-05	0.937239E-03	0.610661E-03
360.	300.	60.	0.229909E-05	0.608663E-06	0.145388E-05	0.314438E-02	0.208168E-02
360.	240.	120.	0.234733E-05	0.595201E-06	0.147126E-05	0.227348E-02	0.146739E-02
360.	180.	180.	0.231078E-05	0.582472E-06	0.144663E-05	0.575776E-02	0.370318E-02
420.	420.	0.	0.230877E-05	0.628646E-06	0.146871E-05	0.242138E-02	0.162922E-02
420.	360.	60.	0.231526E-05	0.591790E-06	0.145352E-05	0.130261E-02	0.844769E-03
420.	300.	120.	0.225292E-05	0.602098E-06	0.142751E-05	0.198567E-02	0.132179E-02
420.	240.	180.	0.230590E-05	0.585791E-06	0.144584E-05	0.203667E-02	0.131601E-02
480.	480.	0.	0.227144E-05	0.657960E-06	0.146470E-05	0.114719E-02	0.799167E-03
480.	420.	60.	0.225339E-05	0.628534E-06	0.144096E-05	0.965613E-03	0.658663E-03
480.	360.	120.	0.227964E-05	0.601511E-06	0.144058E-05	0.113705E-02	0.751308E-03
480.	300.	180.	0.221055E-05	0.618112E-06	0.141433E-05	0.284548E-02	0.194366E-02
540.	540.	0.	0.230023E-05	0.667744E-06	0.148399E-05	0.178903E-02	0.124778E-02
540.	480.	60.	0.225546E-05	0.641830E-06	0.144864E-05	0.748650E-03	0.516428E-03
540.	420.	120.	0.224827E-05	0.620163E-06	0.143421E-05	0.647302E-03	0.438765E-03
540.	360.	180.	0.227554E-05	0.594100E-06	0.143482E-05	0.108985E-02	0.715676E-03
600.	600.	0.	0.232146E-05	0.648143E-06	0.148480E-05	0.231061E-02	0.157696E-02
600.	540.	60.	0.227087E-05	0.655894E-06	0.146338E-05	0.136905E-02	0.952202E-03
600.	480.	120.	0.224479E-05	0.639912E-06	0.144235E-05	0.620366E-03	0.428353E-03
600.	420.	180.	0.223957E-05	0.619642E-06	0.142960E-05	0.665047E-03	0.451570E-03

Case 1

flw	stdy	del	k1	k2	aver	gnmse	anmse
20.	20.	0.	0.243242E-06	0.102756E-06	0.172999E-06	0.000000E+00	0.165478E+01
40.	40.	0.	0.329458E-06	0.216983E-06	0.273220E-06	0.000000E+00	0.310310E+00
40.	20.	20.	0.896328E-07	0.264122E-07	0.580225E-07	0.000000E+00	0.362669E+01
60.	60.	0.	0.259904E-06	0.236251E-06	0.248077E-06	0.000000E+00	0.501938E+00
60.	20.	40.	0.311010E-07	0.000000E+00	0.155505E-07	0.000000E+00	0.374419E+01
60.	40.	20.	0.113490E-06	0.606117E-07	0.870511E-07	0.000000E+00	0.160503E+01
80.	80.	0.	0.232152E-06	0.151809E-06	0.191980E-06	0.285201E+00	0.272714E+00
80.	20.	60.	0.205116E-07	0.000000E+00	0.102558E-07	0.000000E+00	0.356882E+01
80.	40.	40.	0.753812E-07	0.992807E-08	0.426546E-07	0.000000E+00	0.208418E+01
80.	60.	20.	0.801087E-07	0.547777E-07	0.674432E-07	0.000000E+00	0.106647E+01
100.	100.	0.	0.160161E-06	0.139660E-06	0.149910E-06	0.182268E+00	0.181416E+00
100.	20.	80.	0.217902E-07	0.000000E+00	0.108951E-07	0.000000E+00	0.369501E+01
100.	40.	60.	0.834947E-07	0.107404E-07	0.471176E-07	0.000000E+00	0.223424E+01
100.	60.	40.	0.658338E-07	0.483220E-07	0.570779E-07	0.000000E+00	0.115414E+01
100.	80.	20.	0.111633E-06	0.702059E-07	0.909195E-07	0.000000E+00	0.214778E+00
120.	120.	0.	0.188235E-06	0.153203E-06	0.170719E-06	0.000000E+00	0.270448E+00
120.	20.	100.	0.207128E-07	0.000000E+00	0.103564E-07	0.000000E+00	0.368741E+01
120.	40.	80.	0.810324E-07	0.124581E-07	0.467452E-07	0.000000E+00	0.213916E+01
120.	60.	60.	0.703902E-07	0.467351E-07	0.585626E-07	0.000000E+00	0.115021E+01
120.	80.	40.	0.966180E-07	0.525097E-07	0.745638E-07	0.000000E+00	0.237051E+00
120.	100.	20.	0.941921E-07	0.689657E-07	0.815789E-07	0.000000E+00	0.201929E+00
140.	140.	0.	0.166572E-06	0.123384E-06	0.144978E-06	0.109843E+00	0.107406E+00
140.	20.	120.	0.187689E-07	0.000000E+00	0.938447E-08	0.000000E+00	0.370338E+01
140.	40.	100.	0.719210E-07	0.141274E-07	0.430242E-07	0.000000E+00	0.214163E+01
140.	60.	80.	0.679212E-07	0.377564E-07	0.528388E-07	0.000000E+00	0.124070E+01
140.	80.	60.	0.779885E-07	0.374404E-07	0.577145E-07	0.000000E+00	0.454851E+00
140.	100.	40.	0.590883E-07	0.334896E-07	0.462890E-07	0.000000E+00	0.510525E+00
140.	120.	20.	0.823022E-07	0.559372E-07	0.691197E-07	0.000000E+00	0.387887E+00
160.	160.	0.	0.154225E-06	0.132357E-06	0.143291E-06	0.135603E+00	0.134814E+00
160.	20.	140.	0.183860E-07	0.000000E+00	0.919299E-08	0.000000E+00	0.372226E+01
160.	40.	120.	0.471767E-07	0.121978E-07	0.296873E-07	0.000000E+00	0.282898E+01
160.	60.	100.	0.539137E-07	0.302212E-07	0.420675E-07	0.000000E+00	0.159149E+01
160.	80.	80.	0.520649E-07	0.248208E-07	0.384428E-07	0.000000E+00	0.115601E+01
160.	100.	60.	0.369381E-07	0.224920E-07	0.297150E-07	0.000000E+00	0.126519E+01
160.	120.	40.	0.511902E-07	0.390776E-07	0.451339E-07	0.000000E+00	0.686855E+00
160.	140.	20.	0.856813E-07	0.734482E-07	0.795647E-07	0.208154E+00	0.206924E+00
180.	180.	0.	0.142878E-06	0.120097E-06	0.131487E-06	0.253606E+00	0.251703E+00
180.	20.	160.	0.122208E-07	0.761736E-09	0.649129E-08	0.000000E+00	0.580639E+01
180.	40.	140.	0.152304E-07	0.933360E-08	0.122820E-07	0.000000E+00	0.584194E+01
180.	60.	120.	0.374298E-07	0.245821E-07	0.310060E-07	0.000000E+00	0.233168E+01
180.	80.	100.	0.331551E-07	0.239403E-07	0.285477E-07	0.000000E+00	0.168275E+01
180.	100.	80.	0.232659E-07	0.208357E-07	0.220508E-07	0.000000E+00	0.200329E+01
180.	120.	60.	0.364933E-07	0.282983E-07	0.323958E-07	0.000000E+00	0.843517E+00
180.	140.	40.	0.514071E-07	0.469125E-07	0.491598E-07	0.000000E+00	0.469199E+00
180.	160.	20.	0.762985E-07	0.601724E-07	0.682354E-07	0.000000E+00	0.433725E+00
200.	200.	0.	0.108758E-06	0.783610E-07	0.935593E-07	0.209491E+00	0.203963E+00
200.	20.	180.	0.239633E-09	0.988549E-10	0.169244E-09	0.000000E+00	0.346932E+01
200.	40.	160.	0.818868E-09	0.000000E+00	0.409434E-09	0.000000E+00	0.367499E+01
200.	60.	140.	0.128876E-07	0.808183E-08	0.104847E-07	0.000000E+00	0.494364E+01
200.	80.	120.	0.176149E-07	0.962779E-08	0.136214E-07	0.000000E+00	0.250128E+01
200.	100.	100.	0.138774E-07	0.574265E-08	0.981003E-08	0.000000E+00	0.344870E+01
200.	120.	80.	0.302287E-07	0.123013E-07	0.212650E-07	0.000000E+00	0.112788E+01
200.	140.	60.	0.323690E-07	0.220313E-07	0.272002E-07	0.000000E+00	0.468558E+00
200.	160.	40.	0.444617E-07	0.299421E-07	0.372019E-07	0.000000E+00	0.454168E+00
200.	180.	20.	0.643141E-07	0.523211E-07	0.583176E-07	0.000000E+00	0.333158E+00
220.	220.	0.	0.919185E-07	0.783078E-07	0.851131E-07	0.317959E+00	0.315926E+00
220.	40.	180.	0.736585E-09	0.000000E+00	0.368292E-09	0.000000E+00	0.376917E+01
220.	60.	160.	0.997347E-08	0.842432E-08	0.919890E-08	0.000000E+00	0.491641E+01
220.	80.	140.	0.147150E-07	0.894454E-08	0.118298E-07	0.000000E+00	0.252453E+01
220.	100.	120.	0.105020E-07	0.616752E-08	0.833474E-08	0.000000E+00	0.337594E+01
220.	120.	100.	0.242627E-07	0.977385E-08	0.170183E-07	0.000000E+00	0.122270E+01
220.	140.	80.	0.260441E-07	0.138165E-07	0.199303E-07	0.000000E+00	0.535531E+00

220.	160.	60.	0.322684E-07	0.148317E-07	0.235500E-07	0.000000E+00	0.370419E+00
220.	180.	40.	0.480671E-07	0.275268E-07	0.377970E-07	0.000000E+00	0.206338E+00
220.	200.	20.	0.515238E-07	0.416089E-07	0.465663E-07	0.000000E+00	0.304820E+00
240.	240.	0.	0.111352E-06	0.849341E-07	0.981430E-07	0.175655E+00	0.172473E+00
240.	100.	140.	0.895156E-08	0.732948E-08	0.814052E-08	0.000000E+00	0.331182E+01
240.	120.	120.	0.238741E-07	0.106154E-07	0.172448E-07	0.000000E+00	0.123232E+01
240.	140.	100.	0.269076E-07	0.145315E-07	0.207196E-07	0.000000E+00	0.558622E+00
240.	160.	80.	0.324520E-07	0.147290E-07	0.235905E-07	0.000000E+00	0.424367E+00
240.	180.	60.	0.467319E-07	0.242483E-07	0.354901E-07	0.000000E+00	0.199124E+00
240.	200.	40.	0.438979E-07	0.304411E-07	0.371695E-07	0.000000E+00	0.271642E+00
240.	220.	20.	0.573823E-07	0.409815E-07	0.491819E-07	0.000000E+00	0.366401E+00
260.	260.	0.	0.102563E-06	0.807684E-07	0.916657E-07	0.245851E+00	0.242377E+00
260.	80.	180.	0.166003E-07	0.745112E-08	0.120257E-07	0.000000E+00	0.239814E+01
260.	100.	160.	0.830340E-08	0.696916E-08	0.763628E-08	0.000000E+00	0.321029E+01
260.	120.	140.	0.236723E-07	0.106634E-07	0.171679E-07	0.000000E+00	0.123574E+01
260.	140.	120.	0.255526E-07	0.137446E-07	0.196486E-07	0.000000E+00	0.585183E+00
260.	160.	100.	0.300498E-07	0.142244E-07	0.221371E-07	0.000000E+00	0.462226E+00
260.	180.	80.	0.423304E-07	0.231006E-07	0.327155E-07	0.282300E+00	0.257916E+00
260.	200.	60.	0.393378E-07	0.284165E-07	0.338772E-07	0.000000E+00	0.338277E+00
260.	220.	40.	0.495843E-07	0.377196E-07	0.436520E-07	0.000000E+00	0.360763E+00
260.	240.	20.	0.630981E-07	0.506437E-07	0.568709E-07	0.215973E+00	0.213383E+00
280.	280.	0.	0.999700E-07	0.940058E-07	0.969879E-07	0.197387E+00	0.197201E+00
280.	100.	180.	0.729536E-08	0.650286E-08	0.689911E-08	0.000000E+00	0.514968E+01
280.	120.	160.	0.174910E-07	0.884409E-08	0.131675E-07	0.000000E+00	0.160933E+01
280.	140.	140.	0.246223E-07	0.129408E-07	0.187816E-07	0.000000E+00	0.556146E+00
280.	160.	120.	0.252391E-07	0.130851E-07	0.191621E-07	0.000000E+00	0.646994E+00
280.	180.	100.	0.327632E-07	0.187439E-07	0.257535E-07	0.000000E+00	0.568719E+00
280.	200.	80.	0.277051E-07	0.199790E-07	0.238421E-07	0.000000E+00	0.644079E+00
280.	220.	60.	0.317971E-07	0.262318E-07	0.290144E-07	0.000000E+00	0.676490E+00
280.	240.	40.	0.458892E-07	0.341993E-07	0.400443E-07	0.335167E+00	0.328027E+00
280.	260.	20.	0.679510E-07	0.502934E-07	0.591222E-07	0.241966E+00	0.236570E+00
300.	300.	0.	0.100340E-06	0.814786E-07	0.909091E-07	0.171422E+00	0.169577E+00
300.	120.	180.	0.868833E-08	0.711955E-08	0.790394E-08	0.000000E+00	0.288863E+01
300.	140.	160.	0.171787E-07	0.118683E-07	0.145235E-07	0.000000E+00	0.956467E+00
300.	160.	140.	0.186383E-07	0.123656E-07	0.155019E-07	0.000000E+00	0.111940E+01
300.	180.	120.	0.270421E-07	0.182427E-07	0.226424E-07	0.000000E+00	0.915546E+00
300.	200.	100.	0.257980E-07	0.180196E-07	0.219088E-07	0.000000E+00	0.835241E+00
300.	220.	80.	0.301599E-07	0.229244E-07	0.265422E-07	0.000000E+00	0.803815E+00
300.	240.	60.	0.403811E-07	0.286268E-07	0.345039E-07	0.000000E+00	0.515685E+00
300.	260.	40.	0.563591E-07	0.392109E-07	0.477850E-07	0.000000E+00	0.330472E+00
300.	280.	20.	0.702439E-07	0.539740E-07	0.621089E-07	0.000000E+00	0.302127E+00
320.	320.	0.	0.103928E-06	0.924490E-07	0.981886E-07	0.975955E-01	0.972620E-01
320.	140.	180.	0.150620E-07	0.943772E-08	0.122499E-07	0.000000E+00	0.121283E+01
320.	160.	160.	0.171704E-07	0.109251E-07	0.140478E-07	0.000000E+00	0.132753E+01
320.	180.	140.	0.236919E-07	0.160722E-07	0.198821E-07	0.000000E+00	0.109295E+01
320.	200.	120.	0.225213E-07	0.158246E-07	0.191730E-07	0.000000E+00	0.966190E+00
320.	220.	100.	0.258636E-07	0.198625E-07	0.228631E-07	0.000000E+00	0.780767E+00
320.	240.	80.	0.332456E-07	0.231947E-07	0.282201E-07	0.000000E+00	0.523185E+00
320.	260.	60.	0.471179E-07	0.340236E-07	0.405707E-07	0.000000E+00	0.277543E+00
320.	280.	40.	0.539297E-07	0.437427E-07	0.488362E-07	0.000000E+00	0.239464E+00
320.	300.	20.	0.696248E-07	0.549368E-07	0.622808E-07	0.146292E+00	0.144258E+00
340.	340.	0.	0.909647E-07	0.843672E-07	0.876660E-07	0.000000E+00	0.200666E+00
340.	160.	180.	0.148141E-07	0.808937E-08	0.114517E-07	0.000000E+00	0.158328E+01
340.	180.	160.	0.212524E-07	0.137218E-07	0.174871E-07	0.000000E+00	0.124159E+01
340.	200.	140.	0.202607E-07	0.135921E-07	0.169264E-07	0.000000E+00	0.109539E+01
340.	220.	120.	0.252928E-07	0.168371E-07	0.210650E-07	0.000000E+00	0.887832E+00
340.	240.	100.	0.318662E-07	0.188571E-07	0.253617E-07	0.000000E+00	0.657103E+00
340.	260.	80.	0.417501E-07	0.288145E-07	0.352823E-07	0.000000E+00	0.416380E+00
340.	280.	60.	0.444301E-07	0.356377E-07	0.400339E-07	0.000000E+00	0.376413E+00
340.	300.	40.	0.508523E-07	0.445320E-07	0.476922E-07	0.319865E+00	0.318461E+00
340.	320.	20.	0.668904E-07	0.577815E-07	0.623359E-07	0.000000E+00	0.299933E+00
360.	360.	0.	0.861364E-07	0.764036E-07	0.812700E-07	0.128635E+00	0.128174E+00
360.	180.	180.	0.177665E-07	0.142597E-07	0.160131E-07	0.000000E+00	0.155928E+01
360.	200.	160.	0.170140E-07	0.142260E-07	0.156200E-07	0.000000E+00	0.136601E+01
360.	220.	140.	0.226038E-07	0.174595E-07	0.200317E-07	0.000000E+00	0.100440E+01
360.	240.	120.	0.276135E-07	0.197690E-07	0.236913E-07	0.000000E+00	0.843546E+00

360.	260.	100.	0.371040E-07	0.286445E-07	0.328742E-07	0.000000E+00	0.492869E+00
360.	280.	80.	0.379745E-07	0.332654E-07	0.356200E-07	0.000000E+00	0.414267E+00
360.	300.	60.	0.416209E-07	0.378124E-07	0.397167E-07	0.000000E+00	0.319557E+00
360.	320.	40.	0.497390E-07	0.419519E-07	0.458455E-07	0.000000E+00	0.266650E+00
360.	340.	20.	0.554727E-07	0.492802E-07	0.523765E-07	0.242510E+00	0.241662E+00
380.	380.	0.	0.857643E-07	0.774909E-07	0.816276E-07	0.185291E+00	0.184816E+00
380.	200.	180.	0.132775E-07	0.117960E-07	0.125368E-07	0.000000E+00	0.222824E+01
380.	220.	160.	0.184116E-07	0.147330E-07	0.165723E-07	0.000000E+00	0.152524E+01
380.	240.	140.	0.240728E-07	0.173715E-07	0.207221E-07	0.000000E+00	0.116207E+01
380.	260.	120.	0.331732E-07	0.256965E-07	0.294349E-07	0.000000E+00	0.647461E+00
380.	280.	100.	0.338712E-07	0.299879E-07	0.319295E-07	0.000000E+00	0.516626E+00
380.	300.	80.	0.384683E-07	0.329532E-07	0.357107E-07	0.000000E+00	0.417151E+00
380.	320.	60.	0.421239E-07	0.335014E-07	0.378126E-07	0.000000E+00	0.369682E+00
380.	340.	40.	0.457162E-07	0.385613E-07	0.421388E-07	0.314322E+00	0.312057E+00
380.	360.	20.	0.596833E-07	0.501423E-07	0.549128E-07	0.184783E+00	0.183388E+00

Case m

flw	stdy	del	k1	k2	aver	gnmse	anmse
60.	60.	0.	0.410890E-06	0.374839E-06	0.392864E-06	0.000000E+00	0.196932E+00
120.	120.	0.	0.576451E-06	0.496058E-06	0.536255E-06	0.103027E+00	0.102448E+00
120.	60.	60.	0.347782E-06	0.294068E-06	0.320925E-06	0.206719E+00	0.205272E+00
180.	180.	0.	0.549707E-06	0.488969E-06	0.519338E-06	0.393155E-01	0.391811E-01
180.	120.	60.	0.481858E-06	0.415464E-06	0.448661E-06	0.494754E-01	0.492046E-01
180.	60.	120.	0.353651E-06	0.332267E-06	0.342959E-06	0.216717E+00	0.216507E+00
240.	240.	0.	0.566077E-06	0.509722E-06	0.537900E-06	0.140082E-01	0.139698E-01
240.	180.	60.	0.497439E-06	0.436112E-06	0.466775E-06	0.352495E-01	0.350974E-01
240.	120.	120.	0.455213E-06	0.405695E-06	0.430454E-06	0.361473E-01	0.360277E-01
240.	60.	180.	0.370513E-06	0.344096E-06	0.357305E-06	0.231772E+00	0.231455E+00
300.	300.	0.	0.605696E-06	0.524579E-06	0.565137E-06	0.104734E-01	0.104195E-01
300.	240.	60.	0.520085E-06	0.496762E-06	0.508424E-06	0.100039E-01	0.999861E-02
300.	180.	120.	0.501654E-06	0.446158E-06	0.473906E-06	0.307201E-01	0.306148E-01
300.	120.	180.	0.453791E-06	0.426803E-06	0.440297E-06	0.339188E-01	0.338869E-01
360.	360.	0.	0.578583E-06	0.540180E-06	0.559382E-06	0.457959E-02	0.457420E-02
360.	300.	60.	0.556992E-06	0.501279E-06	0.529135E-06	0.598491E-02	0.596832E-02
360.	240.	120.	0.528393E-06	0.492471E-06	0.510432E-06	0.652484E-02	0.651676E-02
360.	180.	180.	0.506891E-06	0.454229E-06	0.480560E-06	0.258785E-01	0.258008E-01
420.	420.	0.	0.562922E-06	0.503129E-06	0.533025E-06	0.777420E-02	0.774974E-02
420.	360.	60.	0.548721E-06	0.489101E-06	0.518911E-06	0.336289E-02	0.335179E-02
420.	300.	120.	0.550099E-06	0.483395E-06	0.516747E-06	0.547701E-02	0.545419E-02
420.	240.	180.	0.533559E-06	0.472782E-06	0.503171E-06	0.617933E-02	0.615679E-02
480.	480.	0.	0.549274E-06	0.513560E-06	0.531417E-06	0.606330E-02	0.605646E-02
480.	420.	60.	0.510955E-06	0.478297E-06	0.494626E-06	0.539396E-02	0.538808E-02
480.	360.	120.	0.530904E-06	0.471136E-06	0.501020E-06	0.319472E-02	0.318335E-02
480.	300.	180.	0.541618E-06	0.466280E-06	0.503949E-06	0.481875E-02	0.479182E-02
540.	540.	0.	0.549125E-06	0.493875E-06	0.521500E-06	0.376711E-02	0.375653E-02
540.	480.	60.	0.519423E-06	0.466786E-06	0.493105E-06	0.505878E-02	0.504437E-02
540.	420.	120.	0.506041E-06	0.463786E-06	0.484913E-06	0.567930E-02	0.566852E-02
540.	360.	180.	0.524258E-06	0.464729E-06	0.494494E-06	0.328231E-02	0.327042E-02
600.	600.	0.	0.556896E-06	0.495493E-06	0.526195E-06	0.225110E-02	0.224344E-02
600.	540.	60.	0.526469E-06	0.466875E-06	0.496672E-06	0.161540E-02	0.160959E-02
600.	480.	120.	0.517504E-06	0.457048E-06	0.487276E-06	0.359302E-02	0.357919E-02
600.	420.	180.	0.508333E-06	0.458980E-06	0.483657E-06	0.488686E-02	0.487414E-02

Case n

flw	stdy	del	k1	k2	aver	gnmse	anmse
60.	60.	0.	0.117200E-05	0.739904E-06	0.955954E-06	0.607178E-01	0.576165E-01
120.	120.	0.	0.941072E-06	0.709290E-06	0.825181E-06	0.312838E-01	0.306668E-01
120.	60.	60.	0.115012E-05	0.587688E-06	0.868903E-06	0.349620E-01	0.312999E-01
180.	180.	0.	0.898964E-06	0.774604E-06	0.836784E-06	0.128834E-01	0.128123E-01
180.	120.	60.	0.906195E-06	0.705618E-06	0.805907E-06	0.114771E-01	0.112994E-01
180.	60.	120.	0.115311E-05	0.622926E-06	0.888017E-06	0.260974E-01	0.237717E-01
240.	240.	0.	0.859274E-06	0.809453E-06	0.834363E-06	0.110354E-01	0.110255E-01
240.	180.	60.	0.851267E-06	0.790209E-06	0.820738E-06	0.924129E-02	0.922850E-02
240.	120.	120.	0.893162E-06	0.730867E-06	0.812014E-06	0.937558E-02	0.928194E-02
240.	60.	180.	0.115618E-05	0.639765E-06	0.897970E-06	0.286730E-01	0.263023E-01
300.	300.	0.	0.805950E-06	0.740857E-06	0.773404E-06	0.867932E-02	0.866395E-02
300.	240.	60.	0.780122E-06	0.742736E-06	0.761429E-06	0.104030E-01	0.103968E-01
300.	180.	120.	0.798554E-06	0.767036E-06	0.782795E-06	0.915077E-02	0.914706E-02
300.	120.	180.	0.837562E-06	0.713190E-06	0.775376E-06	0.668294E-02	0.663995E-02
360.	360.	0.	0.800612E-06	0.705098E-06	0.752855E-06	0.117784E-01	0.117310E-01
360.	300.	60.	0.776135E-06	0.651433E-06	0.713784E-06	0.723172E-02	0.717654E-02
360.	240.	120.	0.765227E-06	0.646337E-06	0.705782E-06	0.948369E-02	0.941641E-02
360.	180.	180.	0.773393E-06	0.690401E-06	0.731897E-06	0.845173E-02	0.842457E-02
420.	420.	0.	0.829245E-06	0.693490E-06	0.761367E-06	0.504010E-02	0.500004E-02
420.	360.	60.	0.797170E-06	0.703990E-06	0.750580E-06	0.493393E-02	0.491492E-02
420.	300.	120.	0.766405E-06	0.651070E-06	0.708738E-06	0.122740E-01	0.121927E-01
420.	240.	180.	0.774060E-06	0.609110E-06	0.691585E-06	0.152099E-01	0.149936E-01
480.	480.	0.	0.757854E-06	0.700389E-06	0.729121E-06	0.680786E-02	0.679729E-02
480.	420.	60.	0.741186E-06	0.658410E-06	0.699798E-06	0.631958E-02	0.629747E-02
480.	360.	120.	0.750403E-06	0.651416E-06	0.700910E-06	0.676058E-02	0.672688E-02
480.	300.	180.	0.756969E-06	0.610716E-06	0.683842E-06	0.540531E-02	0.534350E-02
540.	540.	0.	0.756018E-06	0.692376E-06	0.724197E-06	0.441330E-02	0.440478E-02
540.	480.	60.	0.731249E-06	0.675101E-06	0.703175E-06	0.444318E-02	0.443610E-02
540.	420.	120.	0.722697E-06	0.660099E-06	0.691398E-06	0.552411E-02	0.551279E-02
540.	360.	180.	0.734701E-06	0.655926E-06	0.695314E-06	0.500406E-02	0.498800E-02
600.	600.	0.	0.741767E-06	0.708975E-06	0.725371E-06	0.233363E-02	0.233244E-02
600.	540.	60.	0.735269E-06	0.688629E-06	0.711949E-06	0.247655E-02	0.247389E-02
600.	480.	120.	0.722905E-06	0.675708E-06	0.699307E-06	0.277348E-02	0.277033E-02
600.	420.	180.	0.714608E-06	0.663352E-06	0.688980E-06	0.424659E-02	0.424072E-02

Case o

flw	stdy	del	k1	k2	aver	gnmse	anmse
20.	20.	0.	0.143307E-05	0.338917E-06	0.885994E-06	0.000000E+00	0.151238E+00
40.	40.	0.	0.137958E-05	0.743086E-06	0.106133E-05	0.366427E-01	0.333481E-01
40.	20.	20.	0.140038E-05	0.373781E-06	0.887083E-06	0.000000E+00	0.207978E+00
60.	60.	0.	0.117357E-05	0.938937E-06	0.105625E-05	0.120783E-01	0.119293E-01
60.	20.	40.	0.141182E-05	0.397410E-06	0.904617E-06	0.000000E+00	0.209384E+00
60.	40.	20.	0.134729E-05	0.763279E-06	0.105528E-05	0.276926E-01	0.255723E-01
80.	80.	0.	0.102509E-05	0.945194E-06	0.985143E-06	0.918220E-02	0.916710E-02
80.	20.	60.	0.138416E-05	0.421691E-06	0.902926E-06	0.000000E+00	0.214061E+00
80.	40.	40.	0.130015E-05	0.763897E-06	0.103202E-05	0.288142E-01	0.268693E-01
80.	60.	20.	0.111635E-05	0.917995E-06	0.101717E-05	0.112282E-01	0.111215E-01
100.	100.	0.	0.961785E-06	0.916284E-06	0.939035E-06	0.164753E-01	0.164656E-01
100.	20.	80.	0.139813E-05	0.429980E-06	0.914056E-06	0.000000E+00	0.225522E+00
100.	40.	60.	0.131844E-05	0.750583E-06	0.103451E-05	0.290668E-01	0.268773E-01
100.	60.	40.	0.111290E-05	0.912747E-06	0.101282E-05	0.112307E-01	0.111210E-01
100.	80.	20.	0.102067E-05	0.920263E-06	0.970468E-06	0.704688E-02	0.702802E-02
120.	120.	0.	0.936756E-06	0.861624E-06	0.899190E-06	0.144784E-01	0.144531E-01
120.	20.	100.	0.140552E-05	0.434896E-06	0.920209E-06	0.000000E+00	0.228609E+00
120.	40.	80.	0.133214E-05	0.750341E-06	0.104124E-05	0.303835E-01	0.280120E-01
120.	60.	60.	0.111993E-05	0.911654E-06	0.101579E-05	0.113811E-01	0.112615E-01
120.	80.	40.	0.102568E-05	0.918507E-06	0.972093E-06	0.566073E-02	0.564353E-02
120.	100.	20.	0.957226E-06	0.908003E-06	0.932615E-06	0.124113E-01	0.124027E-01
140.	140.	0.	0.920619E-06	0.831495E-06	0.876057E-06	0.131243E-01	0.130903E-01
140.	20.	120.	0.141002E-05	0.427890E-06	0.918955E-06	0.000000E+00	0.225091E+00
140.	40.	100.	0.132477E-05	0.745731E-06	0.103525E-05	0.301535E-01	0.277952E-01
140.	60.	80.	0.111140E-05	0.909567E-06	0.101048E-05	0.123251E-01	0.122021E-01
140.	80.	60.	0.100546E-05	0.919819E-06	0.962637E-06	0.587045E-02	0.585883E-02
140.	100.	40.	0.936407E-06	0.904031E-06	0.920219E-06	0.131696E-01	0.131655E-01
140.	120.	20.	0.924091E-06	0.850949E-06	0.887520E-06	0.151243E-01	0.150986E-01
160.	160.	0.	0.915092E-06	0.848071E-06	0.881581E-06	0.171181E-01	0.170933E-01
160.	20.	140.	0.142706E-05	0.425837E-06	0.926451E-06	0.000000E+00	0.220862E+00
160.	40.	120.	0.132903E-05	0.746146E-06	0.103759E-05	0.314687E-01	0.289859E-01
160.	60.	100.	0.111515E-05	0.907897E-06	0.101153E-05	0.124704E-01	0.123395E-01
160.	80.	80.	0.100548E-05	0.916956E-06	0.961218E-06	0.584394E-02	0.583154E-02
160.	100.	60.	0.934425E-06	0.897717E-06	0.916071E-06	0.120044E-01	0.119995E-01
160.	120.	40.	0.918161E-06	0.847378E-06	0.882769E-06	0.147723E-01	0.147485E-01
160.	140.	20.	0.894100E-06	0.824657E-06	0.859378E-06	0.138575E-01	0.138349E-01
180.	180.	0.	0.938559E-06	0.841028E-06	0.889793E-06	0.968069E-02	0.965162E-02
180.	20.	160.	0.144439E-05	0.421068E-06	0.932730E-06	0.000000E+00	0.218242E+00
180.	40.	140.	0.133678E-05	0.742320E-06	0.103955E-05	0.327289E-01	0.300533E-01
180.	60.	120.	0.112420E-05	0.902939E-06	0.101357E-05	0.133432E-01	0.131842E-01
180.	80.	100.	0.101514E-05	0.907372E-06	0.961258E-06	0.599576E-02	0.597691E-02
180.	100.	80.	0.947506E-06	0.882116E-06	0.914811E-06	0.114412E-01	0.114266E-01
180.	120.	60.	0.920140E-06	0.838749E-06	0.879444E-06	0.144331E-01	0.144022E-01
180.	140.	40.	0.897095E-06	0.814156E-06	0.855625E-06	0.136877E-01	0.136555E-01
180.	160.	20.	0.898680E-06	0.834776E-06	0.866728E-06	0.155915E-01	0.155703E-01
200.	200.	0.	0.937444E-06	0.839350E-06	0.888397E-06	0.109293E-01	0.108959E-01
200.	20.	180.	0.146912E-05	0.415209E-06	0.942166E-06	0.000000E+00	0.218162E+00
200.	40.	160.	0.135488E-05	0.739180E-06	0.104703E-05	0.342495E-01	0.312886E-01
200.	60.	140.	0.114060E-05	0.903301E-06	0.102195E-05	0.144463E-01	0.142515E-01
200.	80.	120.	0.103040E-05	0.905280E-06	0.967839E-06	0.671343E-02	0.668539E-02
200.	100.	100.	0.964235E-06	0.869781E-06	0.917008E-06	0.124245E-01	0.123915E-01
200.	120.	80.	0.928569E-06	0.833125E-06	0.880847E-06	0.142091E-01	0.141674E-01
200.	140.	60.	0.904368E-06	0.811698E-06	0.858033E-06	0.130757E-01	0.130376E-01
200.	160.	40.	0.899972E-06	0.833458E-06	0.866715E-06	0.151982E-01	0.151759E-01
200.	180.	20.	0.921324E-06	0.838415E-06	0.879870E-06	0.932695E-02	0.930625E-02
220.	220.	0.	0.948562E-06	0.875564E-06	0.912063E-06	0.658326E-02	0.657271E-02
220.	40.	180.	0.137578E-05	0.736495E-06	0.105614E-05	0.349823E-01	0.317779E-01
220.	60.	160.	0.116089E-05	0.899633E-06	0.103026E-05	0.135151E-01	0.132979E-01
220.	80.	140.	0.104993E-05	0.899466E-06	0.974696E-06	0.647780E-02	0.643921E-02
220.	100.	120.	0.984752E-06	0.861629E-06	0.923190E-06	0.123825E-01	0.123274E-01
220.	120.	100.	0.943157E-06	0.827928E-06	0.885542E-06	0.137107E-01	0.136526E-01

220.	140.	80.	0.916749E-06	0.810253E-06	0.863501E-06	0.115716E-01	0.115276E-01
220.	160.	60.	0.910596E-06	0.833980E-06	0.872288E-06	0.138978E-01	0.138710E-01
220.	180.	40.	0.923740E-06	0.842224E-06	0.882982E-06	0.871574E-02	0.869717E-02
220.	200.	20.	0.937671E-06	0.840636E-06	0.889153E-06	0.105054E-01	0.104742E-01
240.	240.	0.	0.964849E-06	0.848882E-06	0.906866E-06	0.623184E-02	0.620636E-02
240.	60.	180.	0.117634E-05	0.908006E-06	0.104217E-05	0.133374E-01	0.131164E-01
240.	80.	160.	0.106531E-05	0.906342E-06	0.985827E-06	0.622134E-02	0.618090E-02
240.	100.	140.	0.100176E-05	0.864750E-06	0.933257E-06	0.118569E-01	0.117930E-01
240.	120.	120.	0.957664E-06	0.829547E-06	0.893605E-06	0.123918E-01	0.123281E-01
240.	140.	100.	0.929107E-06	0.813771E-06	0.871439E-06	0.992148E-02	0.987803E-02
240.	160.	80.	0.920214E-06	0.837741E-06	0.878977E-06	0.120473E-01	0.120208E-01
240.	180.	60.	0.930082E-06	0.849042E-06	0.889562E-06	0.787420E-02	0.785786E-02
240.	200.	40.	0.941031E-06	0.847828E-06	0.894429E-06	0.926201E-02	0.923687E-02
240.	220.	20.	0.946801E-06	0.873958E-06	0.910379E-06	0.605592E-02	0.604623E-02
260.	260.	0.	0.984268E-06	0.833850E-06	0.909059E-06	0.748698E-02	0.743574E-02
260.	80.	180.	0.107538E-05	0.914809E-06	0.995094E-06	0.615254E-02	0.611249E-02
260.	100.	160.	0.101369E-05	0.869249E-06	0.941472E-06	0.114564E-01	0.113890E-01
260.	120.	140.	0.968764E-06	0.832065E-06	0.900414E-06	0.114514E-01	0.113854E-01
260.	140.	120.	0.938372E-06	0.817915E-06	0.878144E-06	0.884994E-02	0.880831E-02
260.	160.	100.	0.926694E-06	0.842658E-06	0.884676E-06	0.106126E-01	0.105886E-01
260.	180.	80.	0.934208E-06	0.854999E-06	0.894603E-06	0.728910E-02	0.727481E-02
260.	200.	60.	0.942357E-06	0.852832E-06	0.897594E-06	0.824607E-02	0.822556E-02
260.	220.	40.	0.944275E-06	0.875552E-06	0.909913E-06	0.546657E-02	0.545877E-02
260.	240.	20.	0.955636E-06	0.847593E-06	0.901614E-06	0.550058E-02	0.548083E-02
280.	280.	0.	0.967661E-06	0.845004E-06	0.906332E-06	0.867894E-02	0.863920E-02
280.	100.	180.	0.102629E-05	0.871793E-06	0.949043E-06	0.110289E-01	0.109558E-01
280.	120.	160.	0.980548E-06	0.833895E-06	0.907221E-06	0.108732E-01	0.108022E-01
280.	140.	140.	0.949142E-06	0.821289E-06	0.885215E-06	0.843886E-02	0.839485E-02
280.	160.	120.	0.936558E-06	0.846764E-06	0.891661E-06	0.976229E-02	0.973754E-02
280.	180.	100.	0.942790E-06	0.860166E-06	0.901478E-06	0.712090E-02	0.710594E-02
280.	200.	80.	0.950458E-06	0.857559E-06	0.904009E-06	0.797859E-02	0.795753E-02
280.	220.	60.	0.948220E-06	0.878646E-06	0.913433E-06	0.523902E-02	0.523142E-02
280.	240.	40.	0.957875E-06	0.852813E-06	0.905344E-06	0.502305E-02	0.500614E-02
280.	260.	20.	0.981501E-06	0.836027E-06	0.908764E-06	0.640442E-02	0.636339E-02
300.	300.	0.	0.946166E-06	0.846207E-06	0.896187E-06	0.833827E-02	0.831233E-02
300.	120.	180.	0.991467E-06	0.830360E-06	0.910914E-06	0.105741E-01	0.104914E-01
300.	140.	160.	0.959349E-06	0.818498E-06	0.888923E-06	0.828402E-02	0.823203E-02
300.	160.	140.	0.945986E-06	0.844203E-06	0.895095E-06	0.926024E-02	0.923031E-02
300.	180.	120.	0.950262E-06	0.858844E-06	0.904553E-06	0.699692E-02	0.697905E-02
300.	200.	100.	0.956879E-06	0.854942E-06	0.905911E-06	0.769295E-02	0.766860E-02
300.	220.	80.	0.952647E-06	0.875396E-06	0.914021E-06	0.489001E-02	0.488128E-02
300.	240.	60.	0.959611E-06	0.851792E-06	0.905702E-06	0.440642E-02	0.439081E-02
300.	260.	40.	0.977464E-06	0.834088E-06	0.905776E-06	0.518422E-02	0.515174E-02
300.	280.	20.	0.960393E-06	0.834438E-06	0.897415E-06	0.675689E-02	0.672361E-02
320.	320.	0.	0.940762E-06	0.842023E-06	0.891393E-06	0.753646E-02	0.751334E-02
320.	140.	180.	0.969570E-06	0.811197E-06	0.890383E-06	0.831958E-02	0.825377E-02
320.	160.	160.	0.955140E-06	0.837568E-06	0.896354E-06	0.902820E-02	0.898936E-02
320.	180.	140.	0.958861E-06	0.852748E-06	0.905804E-06	0.684336E-02	0.681989E-02
320.	200.	120.	0.964837E-06	0.848695E-06	0.906766E-06	0.759481E-02	0.756366E-02
320.	220.	100.	0.959506E-06	0.868935E-06	0.914221E-06	0.471466E-02	0.470309E-02
320.	240.	80.	0.964739E-06	0.846193E-06	0.905466E-06	0.420899E-02	0.419095E-02
320.	260.	60.	0.978183E-06	0.828744E-06	0.903463E-06	0.435576E-02	0.432596E-02
320.	280.	40.	0.962338E-06	0.823320E-06	0.892829E-06	0.602034E-02	0.598385E-02
320.	300.	20.	0.945600E-06	0.830989E-06	0.888294E-06	0.771163E-02	0.767954E-02
340.	340.	0.	0.946416E-06	0.851410E-06	0.898913E-06	0.807133E-02	0.804879E-02
340.	160.	180.	0.961731E-06	0.832452E-06	0.897092E-06	0.877003E-02	0.872449E-02
340.	180.	160.	0.964544E-06	0.848285E-06	0.906414E-06	0.652127E-02	0.649445E-02
340.	200.	140.	0.969565E-06	0.844476E-06	0.907020E-06	0.734209E-02	0.730718E-02
340.	220.	120.	0.963692E-06	0.865285E-06	0.914489E-06	0.454568E-02	0.453252E-02
340.	240.	100.	0.967774E-06	0.843631E-06	0.905702E-06	0.404593E-02	0.402693E-02
340.	260.	80.	0.979478E-06	0.826020E-06	0.902749E-06	0.408909E-02	0.405955E-02
340.	280.	60.	0.963571E-06	0.819629E-06	0.891600E-06	0.568072E-02	0.564371E-02
340.	300.	40.	0.946639E-06	0.825597E-06	0.886118E-06	0.731053E-02	0.727643E-02
340.	320.	20.	0.940526E-06	0.832977E-06	0.886752E-06	0.759262E-02	0.756470E-02
360.	360.	0.	0.951662E-06	0.889187E-06	0.920424E-06	0.527559E-02	0.526951E-02
360.	180.	180.	0.967820E-06	0.846727E-06	0.907273E-06	0.641611E-02	0.638754E-02

360.	200.	160.	0.972257E-06	0.842919E-06	0.907588E-06	0.741726E-02	0.737960E-02
360.	220.	140.	0.965674E-06	0.864090E-06	0.914882E-06	0.450482E-02	0.449093E-02
360.	240.	120.	0.968866E-06	0.842981E-06	0.905924E-06	0.403228E-02	0.401282E-02
360.	260.	100.	0.979653E-06	0.825527E-06	0.902590E-06	0.394838E-02	0.391960E-02
360.	280.	80.	0.963348E-06	0.819603E-06	0.891475E-06	0.540675E-02	0.537160E-02
360.	300.	60.	0.944006E-06	0.825134E-06	0.884570E-06	0.670350E-02	0.667323E-02
360.	320.	40.	0.936586E-06	0.832031E-06	0.884308E-06	0.705046E-02	0.702582E-02
360.	340.	20.	0.939746E-06	0.848929E-06	0.894338E-06	0.730944E-02	0.729060E-02
380.	380.	0.	0.940756E-06	0.897138E-06	0.918947E-06	0.513494E-02	0.513205E-02
380.	200.	180.	0.976291E-06	0.841545E-06	0.908918E-06	0.735764E-02	0.731721E-02
380.	220.	160.	0.969348E-06	0.862318E-06	0.915833E-06	0.446373E-02	0.444849E-02
380.	240.	140.	0.972568E-06	0.840587E-06	0.906578E-06	0.409477E-02	0.407307E-02
380.	260.	120.	0.982887E-06	0.822630E-06	0.902758E-06	0.405408E-02	0.402214E-02
380.	280.	100.	0.966132E-06	0.816693E-06	0.891413E-06	0.545312E-02	0.541481E-02
380.	300.	80.	0.946288E-06	0.821138E-06	0.883713E-06	0.644673E-02	0.641440E-02
380.	320.	60.	0.938326E-06	0.828020E-06	0.883173E-06	0.672473E-02	0.669850E-02
380.	340.	40.	0.939046E-06	0.845200E-06	0.892123E-06	0.672837E-02	0.670976E-02
380.	360.	20.	0.104358E-05	0.979586E-06	0.101158E-05	0.483082E-02	0.482599E-02

Case p

flw	stdy	del	k1	k2	aver	gnmse	anmse
60.	60.	0.	0.121896E-05	0.780119E-06	0.999541E-06	0.248404E-01	0.236434E-01
120.	120.	0.	0.128687E-05	0.100631E-05	0.114659E-05	0.125961E-01	0.124075E-01
180.	180.	0.	0.116661E-05	0.105811E-05	0.111236E-05	0.202962E-01	0.202479E-01
240.	240.	0.	0.109878E-05	0.103398E-05	0.106638E-05	0.693656E-02	0.693016E-02
240.	60.	180.	0.112511E-05	0.737542E-06	0.931326E-06	0.327484E-01	0.313306E-01
300.	300.	0.	0.111635E-05	0.108485E-05	0.110060E-05	0.914815E-02	0.914628E-02
300.	120.	180.	0.106921E-05	0.989813E-06	0.102951E-05	0.112290E-01	0.112123E-01
360.	360.	0.	0.113995E-05	0.106979E-05	0.110487E-05	0.570136E-02	0.569561E-02
360.	180.	180.	0.110574E-05	0.100146E-05	0.105360E-05	0.472541E-02	0.471384E-02
420.	420.	0.	0.116080E-05	0.107616E-05	0.111848E-05	0.498910E-02	0.498196E-02
420.	240.	180.	0.106562E-05	0.992040E-06	0.102883E-05	0.286607E-02	0.286240E-02
420.	60.	360.	0.116815E-05	0.742500E-06	0.955323E-06	0.343227E-01	0.326193E-01
480.	480.	0.	0.117919E-05	0.112643E-05	0.115281E-05	0.443095E-02	0.442863E-02
480.	300.	180.	0.109138E-05	0.102613E-05	0.105876E-05	0.443441E-02	0.443020E-02
480.	120.	360.	0.109060E-05	0.100411E-05	0.104736E-05	0.107055E-01	0.106872E-01
540.	540.	0.	0.119857E-05	0.116438E-05	0.118148E-05	0.228670E-02	0.228622E-02
540.	360.	180.	0.112406E-05	0.102497E-05	0.107451E-05	0.445298E-02	0.444351E-02
540.	180.	360.	0.111044E-05	0.102696E-05	0.106870E-05	0.401680E-02	0.401067E-02
600.	600.	0.	0.119021E-05	0.117982E-05	0.118502E-05	0.229654E-02	0.229650E-02
600.	240.	360.	0.106180E-05	0.103523E-05	0.104852E-05	0.209830E-02	0.209797E-02
600.	420.	180.	0.115885E-05	0.108394E-05	0.112139E-05	0.244020E-02	0.243748E-02

Case q

flw	stdy	del	k1	k2	aver	gnmse	anmse
60.	60.	0.	0.196391E-05	0.183012E-05	0.189702E-05	0.168666E-01	0.168456E-01
120.	120.	0.	0.187256E-05	0.177654E-05	0.182455E-05	0.990271E-02	0.989585E-02
180.	180.	0.	0.184117E-05	0.172971E-05	0.178544E-05	0.105075E-01	0.104973E-01
240.	240.	0.	0.184505E-05	0.175208E-05	0.179857E-05	0.374884E-02	0.374634E-02
240.	60.	180.	0.187801E-05	0.161849E-05	0.174825E-05	0.316514E-02	0.314770E-02
300.	300.	0.	0.182374E-05	0.173187E-05	0.177780E-05	0.238861E-02	0.238701E-02
300.	120.	180.	0.185423E-05	0.160322E-05	0.172873E-05	0.958829E-02	0.953775E-02
360.	360.	0.	0.178386E-05	0.170555E-05	0.174470E-05	0.181890E-02	0.181799E-02
360.	180.	180.	0.180060E-05	0.161287E-05	0.170674E-05	0.626857E-02	0.624961E-02
420.	420.	0.	0.172917E-05	0.165837E-05	0.169377E-05	0.160848E-02	0.160778E-02
420.	240.	180.	0.176594E-05	0.166111E-05	0.171353E-05	0.153642E-02	0.153498E-02
420.	60.	360.	0.180225E-05	0.157598E-05	0.168911E-05	0.332438E-02	0.330947E-02
480.	480.	0.	0.171661E-05	0.164496E-05	0.168079E-05	0.124866E-02	0.124809E-02
480.	300.	180.	0.170596E-05	0.168010E-05	0.169303E-05	0.203867E-02	0.203855E-02
480.	120.	360.	0.175511E-05	0.156273E-05	0.165892E-05	0.743532E-02	0.741032E-02
540.	540.	0.	0.169245E-05	0.162435E-05	0.165840E-05	0.180982E-02	0.180906E-02
540.	360.	180.	0.172029E-05	0.164571E-05	0.168300E-05	0.149423E-02	0.149350E-02
540.	180.	360.	0.173712E-05	0.158058E-05	0.165885E-05	0.610305E-02	0.608946E-02
600.	600.	0.	0.163912E-05	0.159628E-05	0.161770E-05	0.143869E-02	0.143844E-02
600.	240.	360.	0.171144E-05	0.164567E-05	0.167855E-05	0.243933E-02	0.243839E-02
600.	420.	180.	0.170040E-05	0.160028E-05	0.165034E-05	0.116715E-02	0.116607E-02

Case r

flw	stdy	del	k1	k2	aver	gnmse	anmse
60.	60.	0.	0.182034E-05	0.124756E-05	0.153395E-05	0.383722E-01	0.370346E-01
120.	120.	0.	0.144770E-05	0.134208E-05	0.139489E-05	0.949658E-02	0.948297E-02
120.	60.	60.	0.169519E-05	0.111780E-05	0.140649E-05	0.237778E-01	0.227760E-01
180.	180.	0.	0.144223E-05	0.130673E-05	0.137448E-05	0.103257E-01	0.103006E-01
180.	60.	120.	0.155070E-05	0.114730E-05	0.134900E-05	0.273386E-01	0.267275E-01
180.	120.	60.	0.130703E-05	0.124223E-05	0.127463E-05	0.606588E-02	0.606196E-02
240.	240.	0.	0.140917E-05	0.135285E-05	0.138101E-05	0.765004E-02	0.764686E-02
240.	60.	180.	0.162138E-05	0.110886E-05	0.136512E-05	0.300997E-01	0.290390E-01
240.	120.	120.	0.128956E-05	0.124836E-05	0.126896E-05	0.587902E-02	0.587747E-02
240.	180.	60.	0.134264E-05	0.127331E-05	0.130798E-05	0.718237E-02	0.717733E-02
300.	300.	0.	0.144195E-05	0.136339E-05	0.140267E-05	0.488799E-02	0.488416E-02
300.	120.	180.	0.133316E-05	0.124880E-05	0.129098E-05	0.570439E-02	0.569830E-02
300.	180.	120.	0.135174E-05	0.128754E-05	0.131964E-05	0.462948E-02	0.462674E-02
300.	240.	60.	0.140485E-05	0.131995E-05	0.136240E-05	0.379232E-02	0.378864E-02
360.	360.	0.	0.144113E-05	0.138534E-05	0.141323E-05	0.493130E-02	0.492938E-02
360.	180.	180.	0.137368E-05	0.126740E-05	0.132054E-05	0.510731E-02	0.509904E-02
360.	240.	120.	0.140142E-05	0.131501E-05	0.135822E-05	0.385463E-02	0.385073E-02
360.	300.	60.	0.141219E-05	0.131830E-05	0.136524E-05	0.333874E-02	0.333479E-02
420.	420.	0.	0.145051E-05	0.142833E-05	0.143942E-05	0.239282E-02	0.239268E-02
420.	360.	60.	0.140719E-05	0.134517E-05	0.137618E-05	0.325025E-02	0.324860E-02
420.	300.	120.	0.141222E-05	0.131005E-05	0.136114E-05	0.263001E-02	0.262630E-02
420.	240.	180.	0.140403E-05	0.131767E-05	0.136085E-05	0.374591E-02	0.374213E-02
480.	480.	0.	0.143892E-05	0.139088E-05	0.141490E-05	0.150188E-02	0.150145E-02
480.	420.	60.	0.142564E-05	0.138711E-05	0.140638E-05	0.123047E-02	0.123024E-02
480.	360.	120.	0.141023E-05	0.134316E-05	0.137670E-05	0.248791E-02	0.248644E-02
480.	300.	180.	0.141702E-05	0.131625E-05	0.136663E-05	0.209837E-02	0.209552E-02
540.	540.	0.	0.142774E-05	0.139185E-05	0.140980E-05	0.153791E-02	0.153766E-02
540.	480.	60.	0.142471E-05	0.137165E-05	0.139818E-05	0.123158E-02	0.123113E-02
540.	420.	120.	0.143320E-05	0.138765E-05	0.141043E-05	0.989882E-03	0.989624E-03
540.	360.	180.	0.142504E-05	0.134221E-05	0.138363E-05	0.235328E-02	0.235117E-02
600.	600.	0.	0.140105E-05	0.135510E-05	0.137808E-05	0.344240E-03	0.344144E-03
600.	540.	60.	0.144375E-05	0.133840E-05	0.139107E-05	0.108986E-02	0.108830E-02
600.	480.	120.	0.148545E-05	0.130510E-05	0.139527E-05	0.195847E-02	0.195029E-02
600.	420.	180.	0.147800E-05	0.134980E-05	0.141390E-05	0.127625E-02	0.127363E-02

Case s

flw	stdy	del	k1	k2	aver	gnmse	anmse
60.	60.	0.	0.178378E-05	0.127823E-05	0.153101E-05	0.970620E-02	0.944162E-02
120.	120.	0.	0.189468E-05	0.143714E-05	0.166591E-05	0.156289E-01	0.153341E-01
120.	60.	60.	0.180938E-05	0.108717E-05	0.144828E-05	0.143597E-01	0.134669E-01
180.	180.	0.	0.176217E-05	0.159247E-05	0.167732E-05	0.994480E-02	0.991935E-02
180.	120.	60.	0.181769E-05	0.146577E-05	0.164173E-05	0.966764E-02	0.955658E-02
180.	60.	120.	0.189783E-05	0.113031E-05	0.151407E-05	0.135725E-01	0.127006E-01
240.	240.	0.	0.157895E-05	0.152551E-05	0.155223E-05	0.502570E-02	0.502421E-02
240.	180.	60.	0.161693E-05	0.156257E-05	0.158975E-05	0.601580E-02	0.601404E-02
240.	120.	120.	0.165286E-05	0.152173E-05	0.158730E-05	0.585180E-02	0.584181E-02
240.	60.	180.	0.173666E-05	0.117344E-05	0.145505E-05	0.122874E-01	0.118271E-01
300.	300.	0.	0.156320E-05	0.148302E-05	0.152311E-05	0.325302E-02	0.325076E-02
300.	120.	180.	0.161672E-05	0.147949E-05	0.154811E-05	0.534533E-02	0.533483E-02
360.	360.	0.	0.152607E-05	0.144769E-05	0.148688E-05	0.148392E-02	0.148289E-02
360.	300.	60.	0.149617E-05	0.143774E-05	0.146696E-05	0.151247E-02	0.151187E-02
360.	240.	120.	0.146072E-05	0.142907E-05	0.144490E-05	0.301450E-02	0.301414E-02
360.	180.	180.	0.154914E-05	0.148920E-05	0.151917E-05	0.343860E-02	0.343726E-02
420.	420.	0.	0.156989E-05	0.151030E-05	0.154009E-05	0.188841E-02	0.188770E-02
420.	360.	60.	0.151512E-05	0.144174E-05	0.147843E-05	0.718686E-03	0.718243E-03
420.	300.	120.	0.148949E-05	0.143589E-05	0.146269E-05	0.938963E-03	0.938648E-03
420.	240.	180.	0.144869E-05	0.144000E-05	0.144435E-05	0.298500E-02	0.298497E-02
480.	480.	0.	0.157168E-05	0.151103E-05	0.154136E-05	0.193845E-02	0.193770E-02
480.	420.	60.	0.156506E-05	0.147908E-05	0.152207E-05	0.146882E-02	0.146765E-02
480.	360.	120.	0.152517E-05	0.142571E-05	0.147544E-05	0.853164E-03	0.852195E-03
480.	300.	180.	0.150077E-05	0.142574E-05	0.146325E-05	0.960818E-03	0.960187E-03
540.	540.	0.	0.156043E-05	0.152300E-05	0.154172E-05	0.136865E-02	0.136845E-02
540.	480.	60.	0.154621E-05	0.149534E-05	0.152078E-05	0.942834E-03	0.942570E-03
540.	420.	120.	0.156348E-05	0.148325E-05	0.152337E-05	0.917751E-03	0.917115E-03
540.	360.	180.	0.152771E-05	0.143401E-05	0.148086E-05	0.820727E-03	0.819905E-03

Case t

flw	stdy	del	k1	k2	aver	gnmse	anmse
60.	60.	0.	0.895023E-06	0.609214E-06	0.752119E-06	0.460769E-01	0.444134E-01
120.	120.	0.	0.503227E-06	0.427948E-06	0.465587E-06	0.413000E-01	0.410300E-01
120.	60.	60.	0.294532E-06	0.209751E-06	0.252141E-06	0.207556E+00	0.201689E+00
180.	180.	0.	0.463999E-06	0.425078E-06	0.444538E-06	0.343470E-01	0.342811E-01
180.	60.	120.	0.270468E-06	0.236413E-06	0.253441E-06	0.220647E+00	0.219651E+00
180.	120.	60.	0.350198E-06	0.275089E-06	0.312643E-06	0.587143E-01	0.578671E-01
240.	240.	0.	0.445614E-06	0.399954E-06	0.422784E-06	0.259224E-01	0.258468E-01
240.	60.	180.	0.265771E-06	0.233761E-06	0.249766E-06	0.230900E+00	0.229952E+00
240.	120.	120.	0.358336E-06	0.242036E-06	0.300186E-06	0.495627E-01	0.477028E-01
240.	180.	60.	0.353447E-06	0.295083E-06	0.324265E-06	0.408578E-01	0.405269E-01
300.	300.	0.	0.434488E-06	0.380736E-06	0.407612E-06	0.228440E-01	0.227447E-01
300.	120.	180.	0.351378E-06	0.233060E-06	0.292219E-06	0.350450E-01	0.336087E-01
300.	180.	120.	0.340895E-06	0.265173E-06	0.303034E-06	0.248209E-01	0.244334E-01
300.	240.	60.	0.354293E-06	0.310203E-06	0.332248E-06	0.984964E-02	0.980627E-02
360.	360.	0.	0.385618E-06	0.371561E-06	0.378589E-06	0.145097E-01	0.145047E-01
360.	180.	180.	0.312882E-06	0.252703E-06	0.282792E-06	0.243581E-01	0.240823E-01
360.	240.	120.	0.331231E-06	0.309292E-06	0.320261E-06	0.139946E-01	0.139782E-01
360.	300.	60.	0.354498E-06	0.317675E-06	0.336086E-06	0.120784E-01	0.120421E-01
420.	420.	0.	0.348087E-06	0.316887E-06	0.332487E-06	0.208551E-01	0.208092E-01
420.	240.	180.	0.327059E-06	0.291979E-06	0.309519E-06	0.165342E-01	0.164811E-01
420.	300.	120.	0.335920E-06	0.289005E-06	0.312462E-06	0.883452E-02	0.878473E-02
420.	360.	60.	0.309603E-06	0.292958E-06	0.301280E-06	0.160797E-01	0.160674E-01
480.	480.	0.	0.334465E-06	0.313710E-06	0.324088E-06	0.150538E-01	0.150384E-01
480.	300.	180.	0.315755E-06	0.295242E-06	0.305498E-06	0.881383E-02	0.880390E-02
480.	360.	120.	0.295606E-06	0.286845E-06	0.291226E-06	0.124876E-01	0.124847E-01
480.	420.	60.	0.287334E-06	0.281090E-06	0.284212E-06	0.128242E-01	0.128226E-01
540.	540.	0.	0.313026E-06	0.302115E-06	0.307570E-06	0.119615E-01	0.119578E-01
540.	360.	180.	0.294876E-06	0.267096E-06	0.280986E-06	0.119315E-01	0.119023E-01
540.	420.	120.	0.282027E-06	0.257541E-06	0.269784E-06	0.120112E-01	0.119865E-01
540.	480.	60.	0.286882E-06	0.266032E-06	0.276457E-06	0.105039E-01	0.104890E-01
600.	600.	0.	0.337090E-06	0.311837E-06	0.324464E-06	0.115065E-01	0.114891E-01
600.	420.	180.	0.277294E-06	0.257762E-06	0.267528E-06	0.903901E-02	0.902697E-02
600.	480.	120.	0.280997E-06	0.262474E-06	0.271736E-06	0.851052E-02	0.850064E-02
600.	540.	60.	0.287462E-06	0.266953E-06	0.277208E-06	0.102022E-01	0.101882E-01
660.	660.	0.	0.323209E-06	0.308830E-06	0.316020E-06	0.117169E-01	0.117109E-01
660.	480.	180.	0.278594E-06	0.261246E-06	0.269920E-06	0.759537E-02	0.758752E-02
660.	540.	120.	0.279872E-06	0.263917E-06	0.271895E-06	0.874602E-02	0.873849E-02
660.	600.	60.	0.286378E-06	0.272047E-06	0.279212E-06	0.853194E-02	0.852632E-02

Case u

flw	stdy	del	k1	k2	aver	gnmse	anmse
60.	60.	0.	0.738934E-06	0.605068E-06	0.672001E-06	0.811610E-01	0.803558E-01
120.	120.	0.	0.438859E-06	0.338844E-06	0.388851E-06	0.899918E-01	0.885034E-01
120.	60.	60.	0.373607E-06	0.220502E-06	0.297055E-06	0.569338E-01	0.531527E-01
180.	180.	0.	0.299019E-06	0.263153E-06	0.281086E-06	0.655667E-01	0.652999E-01
180.	60.	120.	0.367671E-06	0.117784E-06	0.242727E-06	0.130858E+00	0.961851E-01
180.	120.	60.	0.230947E-06	0.150742E-06	0.190845E-06	0.131856E+00	0.126034E+00
240.	240.	0.	0.250988E-06	0.228844E-06	0.239916E-06	0.410540E-01	0.409665E-01
240.	60.	180.	0.282353E-06	0.123855E-06	0.203104E-06	0.113209E+00	0.959730E-01
240.	120.	120.	0.163247E-06	0.106277E-06	0.134762E-06	0.198287E+00	0.189428E+00
240.	180.	60.	0.148004E-06	0.109894E-06	0.128949E-06	0.149685E+00	0.146417E+00
300.	300.	0.	0.252343E-06	0.242276E-06	0.247309E-06	0.522458E-01	0.522241E-01
300.	120.	180.	0.150104E-06	0.913291E-07	0.120716E-06	0.163508E+00	0.153818E+00
300.	180.	120.	0.135619E-06	0.879988E-07	0.111809E-06	0.112820E+00	0.107704E+00
300.	240.	60.	0.176746E-06	0.148475E-06	0.162610E-06	0.271146E-01	0.269097E-01
360.	360.	0.	0.233846E-06	0.210117E-06	0.221981E-06	0.278322E-01	0.277527E-01
360.	180.	180.	0.132773E-06	0.881456E-07	0.110459E-06	0.108100E+00	0.103689E+00
360.	240.	120.	0.164962E-06	0.148873E-06	0.156917E-06	0.316040E-01	0.315209E-01
360.	300.	60.	0.171491E-06	0.153594E-06	0.162543E-06	0.183286E-01	0.182730E-01
420.	420.	0.	0.222118E-06	0.196957E-06	0.209537E-06	0.405824E-01	0.404361E-01
420.	240.	180.	0.167111E-06	0.134250E-06	0.150680E-06	0.311652E-01	0.307947E-01
420.	300.	120.	0.165002E-06	0.129396E-06	0.147199E-06	0.101639E-01	0.100152E-01
420.	360.	60.	0.178731E-06	0.139647E-06	0.159189E-06	0.269395E-01	0.265335E-01
480.	480.	0.	0.256082E-06	0.123381E-06	0.189731E-06	0.186970E-01	0.164104E-01
480.	300.	180.	0.187160E-06	0.863272E-07	0.136744E-06	0.333834E-01	0.288454E-01
480.	360.	120.	0.212934E-06	0.891112E-07	0.151023E-06	0.424801E-01	0.353410E-01
480.	420.	60.	0.204423E-06	0.103865E-06	0.154144E-06	0.347394E-01	0.310433E-01

Case v

flw	stdy	del	k1	k2	aver	gnmse	anmse
60.	60.	0.	0.674761E-06	0.542198E-06	0.608480E-06	0.835261E-01	0.825350E-01
120.	120.	0.	0.370062E-06	0.325785E-06	0.347924E-06	0.177210E+00	0.176493E+00
120.	60.	60.	0.189814E-06	0.880330E-07	0.138923E-06	0.000000E+00	0.317312E+00
180.	180.	0.	0.266229E-06	0.187655E-06	0.226942E-06	0.552961E-01	0.536389E-01
180.	60.	120.	0.166343E-06	0.624341E-07	0.114389E-06	0.000000E+00	0.259909E+00
180.	120.	60.	0.161128E-06	0.684067E-07	0.114767E-06	0.238305E+00	0.199419E+00
240.	240.	0.	0.225279E-06	0.188858E-06	0.207068E-06	0.608814E-01	0.604105E-01
240.	60.	180.	0.191123E-06	0.473748E-07	0.119249E-06	0.000000E+00	0.216671E+00
240.	120.	120.	0.141358E-06	0.573623E-07	0.993599E-07	0.302039E+00	0.248077E+00
240.	180.	60.	0.138546E-06	0.760818E-07	0.107314E-06	0.104241E+00	0.954118E-01
300.	300.	0.	0.177958E-06	0.145885E-06	0.161922E-06	0.865308E-01	0.856821E-01
300.	120.	180.	0.129760E-06	0.171141E-07	0.734371E-07	0.512642E+00	0.211095E+00
300.	180.	120.	0.105171E-06	0.324142E-07	0.687924E-07	0.109627E+00	0.789706E-01
300.	240.	60.	0.123652E-06	0.736073E-07	0.986296E-07	0.122734E+00	0.114835E+00
360.	360.	0.	0.160821E-06	0.131384E-06	0.146102E-06	0.546952E-01	0.541401E-01
360.	180.	180.	0.104676E-06	0.283548E-07	0.665153E-07	0.153935E+00	0.103268E+00
360.	240.	120.	0.125142E-06	0.658486E-07	0.954952E-07	0.106244E+00	0.960046E-01
360.	300.	60.	0.117312E-06	0.722045E-07	0.947581E-07	0.114965E+00	0.108453E+00
420.	420.	0.	0.141003E-06	0.129536E-06	0.135270E-06	0.559516E-01	0.558511E-01
420.	240.	180.	0.115084E-06	0.669279E-07	0.910062E-07	0.799899E-01	0.743905E-01
420.	300.	120.	0.100723E-06	0.697159E-07	0.852197E-07	0.850352E-01	0.822207E-01
420.	360.	60.	0.964220E-07	0.809382E-07	0.886801E-07	0.499665E-01	0.495857E-01
480.	480.	0.	0.128743E-06	0.121877E-06	0.125310E-06	0.761073E-01	0.760502E-01
480.	300.	180.	0.891378E-07	0.622496E-07	0.756937E-07	0.573483E-01	0.555392E-01
480.	360.	120.	0.839084E-07	0.728714E-07	0.783899E-07	0.384061E-01	0.382158E-01
480.	420.	60.	0.856443E-07	0.736452E-07	0.796447E-07	0.649801E-01	0.646114E-01

LAWRENCE BERKELEY LABORATORY
UNIVERSITY OF CALIFORNIA
INFORMATION RESOURCES DEPARTMENT
BERKELEY, CALIFORNIA 94720