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# Array Thinning with Periodic and Aperiodic Distributions in a Fabry-Perot Cavity for Gain Enhancement

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## Introduction

We explore the possibility of using periodic and aperiodic arrangements of sources inside a Fabry-Perot Cavity (FPC) made of a ground plane and a partially reflective surface (PRS), as in Fig. 1. The cavity is used to enhance the gain of each source [1]-[6]. The enhanced gain permits to design array antennas with large distances between the radiating elements without the occurrence of secondary lobes [7].

It is convenient to design and build arrays as a collection of sub-arrays, though this encounters the problem of the appearance of large secondary lobes. In this summary we show that a radiating element inside the FPC acts like a special sub-array with a single radiator. The aperiodic distribution of the sub-arrays yields a way to decrease the level of side lobes. Indeed, “aperiodic-tiling” arrays, intimately related to the concept of “quasi-crystals” in solid-state physics [8], have been recently shown to exhibit intrinsic thinning capabilities [9], which can be further enhanced via geometric optimization [10]. Here we analyze three different arrays: a Cartesian one, based on a square lattice, the Danzer one [8], and the spiral one.

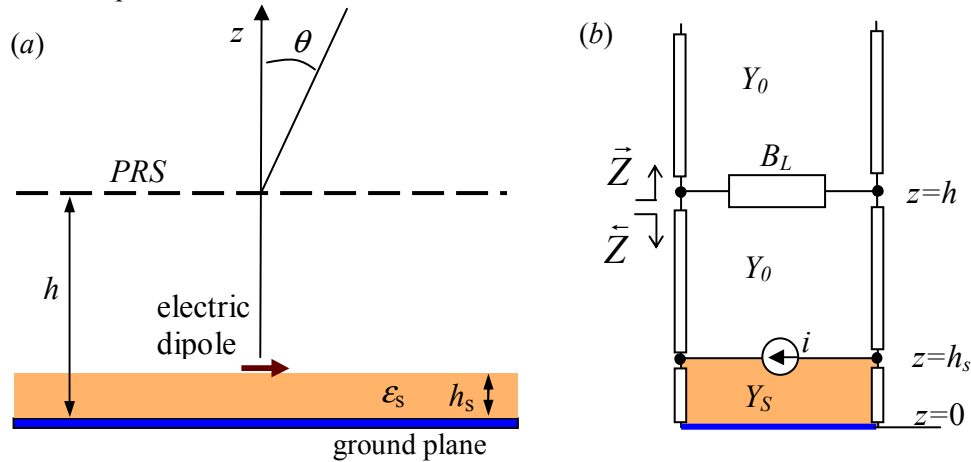


Fig. 1. Fabry-Perot Cavity resonator with electric dipole excitation. (b)  $z$ -transmission line to model the radiation by the electric dipole at  $z = h_s$ .

## The Fabry-Perot Cavity and the Resonant Condition

The FPC consists of a grounded dielectric layer where the source is located and a PRS. The FPC is modeled as a transmission line (TL), shown in Fig. 1(b), and the PRS, usually realized as a Frequency selective surface [1],[5],[6], is modeled as a shunt susceptance  $B_L$  as shown in [6]. The resonance condition for the FPC, that provides a good directivity design, is given by the following equation [7]

$$\Im m \{ \vec{Z}(h) + \vec{Z}(h) \} = 0, \quad (1)$$

where  $\bar{Z}(h)$  and  $\bar{Z}(h)$  are the impedances at  $z = h$  looking upward and downward, respectively, right above the PRS represented by the equivalent shunt load  $B_L$ . The resonance condition leads to the determination of the exact cavity thickness  $h$ . More details can be found in [7]. Here, we merely mention that if one has  $B_L = \infty$ , and  $\epsilon_s = 1$ , the resonant condition (1) exactly leads to  $h = \lambda_0/2$  as expected. The definition of accepted gain in the boresight direction ( $\theta = 0$ ) is

$$G_{acc} = \frac{S(\theta=0)4\pi r^2}{P_{acc}} = \frac{\text{EIRP}(\theta=0)}{P_{acc}} \quad (2)$$

In (2),  $S(\theta=0) = |E(r, \theta=0)|^2 / (2\eta_0)$  denotes the power density radiated by the antenna at boresight, at a far-field distance  $r$ , EIRP is the Effective Isotropic Radiated Power, and  $P_{acc} = (1 - |\Gamma|^2)P_{in}$  is the total power delivered to (*accepted* by) the single antenna (the case of multiple radiators is treated next) with  $P_{in}$  denoting the power of the *incident* wave at the antenna terminals and  $\Gamma$  the antenna input reflection coefficient. We look at the accepted gain because it is not affected by the antenna impedance mismatch, and is thus mostly related to the properties of the FPC. In this summary, we limit the analysis to ideal dipole radiators, and so we can assume  $P_{acc} = P_{in}$ . In the following we have designed a cavity with  $h = 12.26$  mm,  $B_L = 1/\eta_0$ , with a substrate of  $h_s = 30$  mils = 0.762 mm and  $\epsilon_s = 2.5$ , such that a single electric dipole provides a gain  $G = 11.20$  dB at  $f = 14$  GHz.

### Sparse Array of Radiators

As shown in [7], a FPC can be used for array thinning. The directivity of the FPC allows for an array inter-element distance larger than a wavelength. Here, we generalize and try to systematize this concept for arbitrary arrays inside a FPC and we want to explore the periodic and aperiodic arrangements of radiators. The resonant condition is not affected by the distribution of radiators. We consider  $N$  electric sources all polarized along the same direction, say  $y$ ,  $\mathbf{J}(\mathbf{r}) = J_0 \hat{\mathbf{y}} \sum_{n=1}^N \delta(\mathbf{r} - \mathbf{D}_n)$ , and in phase, so as to provide a boresight radiation. Here,  $\mathbf{D}_n$  is the displacement in the  $x$ - $y$  plane of the  $n$ -th source. We define the array factor as

$$AF(\mathbf{k}_t) = \sum_{n=1}^N e^{j\mathbf{k}_t \cdot \mathbf{D}_n} \quad (3)$$

where  $\mathbf{k}_t = k(\hat{\mathbf{x}}u + \hat{\mathbf{y}}v)$ , with  $u = \sin\theta \cos\phi$  and  $v = \sin\theta \sin\phi$ , is the transverse wavevector related to a  $\theta, \phi$  direction of observation. The formulas for EIRP and power for single radiators in [7] can still be used if one multiplies the TL TE and TM current generators (see Fig. 1(b), and [7]) by the array factor  $AF(\mathbf{k}_t)$ . The EIRP( $\theta=0$ ) for  $N$  sources is exactly equal to  $N^2$  times the EIRP( $\theta=0$ ) of a single source. The field radiated by the array in a  $\theta, \phi$  direction is equal to that radiated by a single source, multiplied by the array factor  $AF(\mathbf{k}_t)$ . Then, mutual distances  $(\mathbf{D}_n - \mathbf{D}_m) = D_{nm}[\hat{\mathbf{x}} \cos\phi_{nm} + \hat{\mathbf{y}} \sin\phi_{nm}]$  are represented in terms of polar coordinates

$D_{nm}$  and  $\phi_{nm}$ . Generalizing what was done in [7] with the use of  $|AF(\mathbf{k}_t)|^2 = \sum_{n=1}^N \sum_{m=1}^N e^{jk_t D_{nm} \cos(\phi - \phi_{nm})}$ , leads to the integral representation for the accepted power representation for  $N$  radiators

$$P_{acc} = |J_0|^2 \sum_{n=1}^N \sum_{m=1}^N \Re e \left\{ \frac{1}{8\pi} \int_{-\infty}^{+\infty} dk_t \mathfrak{S}_{nm}(k_t) k_t \right\}, \quad (4)$$

with

$$\mathfrak{S}_{nm}(k_t) = [\tilde{Z}' + \tilde{Z}'' ] J_0(k_t D_{nm}) + [\tilde{Z}' - \tilde{Z}'' ] J_2(k_t D_{nm}) \cos(2\phi_{nm}),$$

where  $\tilde{Z}' = \tilde{Z}' + \tilde{Z}''$  and  $\tilde{Z}'' = \tilde{Z}'' + \tilde{Z}'$  are the TM and TE total impedances evaluated at  $z = h_s$ , and  $J_0$  and  $J_2$  are the Bessel functions of 0<sup>th</sup> and 2<sup>nd</sup> order.

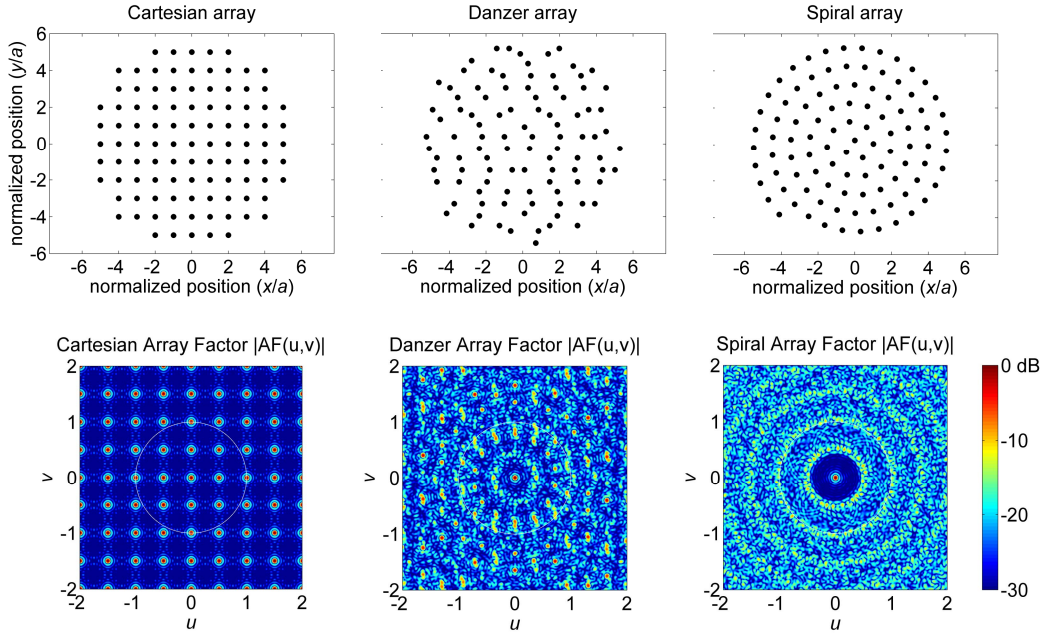


Fig. 2. Array of dipole sources distributed as a square lattice (Cartesian), as a Danzer tiling [8]-[10], and as a spiral. The Array Factor (AF) in (3) is shown for the three cases, assuming a distance parameter  $a = 2\lambda_0$ .

In Fig. 2 we report the geometry of three different array distributions, and the AF in (3): the first is on a periodic square lattice (Cartesian), the second follows a Danzer-type aperiodic-tiling distribution [8] which was found in [9],[10] to exhibit good thinning capabilities in free space, and the third is along the spiral  $\rho = \Delta_\rho \alpha / (2\pi)$ . The three arrays have the same *source density*  $s$ , defined as the number of sources per  $\lambda_0^2$ , and are parameterized by using a distance parameter  $a$  defined through  $s = \lambda_0^2 / a^2$ . For the periodic case,  $a$  corresponds to the period. For the spiral,  $\Delta_\rho = a$ , and the contour was sampled with period  $a$ .

Fig. 2 shows that for  $a = 2\lambda_0$  the Cartesian array has a few strong side lobes in free space, that are present on the principal planes. The other two arrays have the same power radiated in side lobes, as for the Cartesian case, which however is distributed azimuthally, and thus lower sidelobes. In Fig. 3 we plot the gain of the three arrays inside the FPC. Note that the arrays exhibit increasing gain for increasing distance parameter  $a$ , with comparable trends and values. As shown in [7], for the Cartesian array we can easily have interelement distances of the order of  $a = 1.5\lambda_0$ . However, here in Fig. 3(b) we show that the spiral and Danzer arrays in the FPC exhibit rather low side lobes also for distance parameters as  $a = 2\lambda_0$ . Even larger distances could be used.

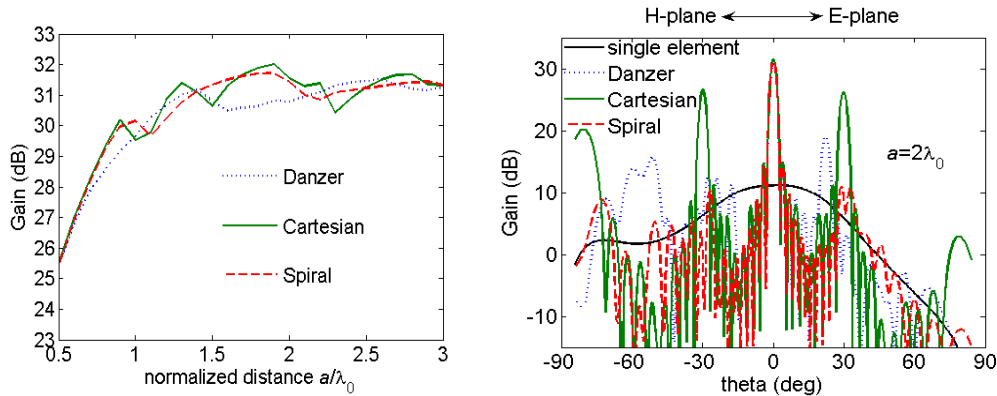


Fig. 3. Gain of the Danzer, Cartesian (periodic), and spiral arrays inside the FPC. The FPC is designed so as to provide a gain of 11.20dB for a single dipole. The Gain of the array inside the FPC is plotted versus the distance parameter  $a$ . For  $a = 2\lambda_0$  we plot the radiation patterns along the E and H plane. Note the large side lobes of the Cartesian distribution

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