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Infinite Dimensional State-space  
as a Pricing Tool for Subsidized Energy Systems

A thesis submitted in partial satisfaction  
of the requirements for the degree Master of Science  
in Chemical Engineering

by

Abdullah Tariq A. Alabdulhadi

2017



## ABSTRACT OF THE THESIS

Infinite DimEnsional State-space  
as a Pricing Tool for Subsidized Energy Systems

by

Abdullah Tariq A. Alabudhadi

Master of Science in Chemical Engineering

University of California, Los Angeles, 2017

Professor Vasilios I. Manousiouthakis, Chair

In this work, the Infinite-Dimensional State-space (IDEAS) conceptual framework is utilized as a systematic tool for determining the dynamic subsidies and flowrates for an overall system consisting of various subsystems interacting with each other via monetary and physical flows. The study's goal is to identify significant improvements over traditional practices involving fixed monetary subsidies and constant material flows. The IDEAS framework is able to identify intensified overall system designs, and to quantify the process intensification potential (PIP) of the technologies employed in the overall system. The resulting mathematical formulation is a mixed integer linear program (MILP). A case study is utilized to illustrate the proposed methodology, focusing on an overall system consisting of refineries, petrochemical plants, and power plants that are provided with liquid and gas hydrocarbon feedstock. It is demonstrated that the overall system's PIP is 11%.

The thesis of Abdullah Tariq A. Alabdulhadi is approved.

Vijay K. Dhir

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Vasilios I. Manousiouthakis, Committee Chair

University of California, Los Angeles

2017

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# 1. Introduction

Process Intensification (PI) aims to achieve significant enhancement to existing processes, by redesigning them, so that they are significantly improved in regard to a variety of metrics, such as energy efficiency, capital and operation cost, environmental footprint and many others. Typically, PI is often achieved by combining several unit operations into a single unit. A commonly referred to PI paradigm is the reactive distillation process for methyl acetate production patented by Eastman Chemical in the 1970s<sup>1</sup>. This intensified process integrates five process steps into one, thus reducing energy consumption by 80%, and large reduction in capital cost<sup>2-4</sup>.

The Infinite Dimensional State-Space approach (IDEAS), is a novel framework that was first introduced in 2000<sup>5</sup>. The IDEAS framework formulates the process network synthesis problem as an infinite linear program, thus guaranteeing identification of the global optimum. In addition, IDEAS can achieve truly ground-breaking designs, since it doesn't employ any a-priori assumptions regarding process network structure, thus avoiding designer preconceptions<sup>4</sup>. As a result, IDEAS can serve as a methodical process intensification tool, that can identify rigorous performance limits for a technology under consideration. In this regard, IDEAS has the capability of identifying the "Process Intensification Potential" (PIP) for various process technologies. A technology's "*Process Intensification Potential*" (PIP) is defined as "*a metric's maximum level of improvement, over a technology's current state of the art, that can be attained by any conceivable design employing the same technology*". Indeed, each technology's state of the art implementation holds room for improvement. It is thus desirable to identify rigorous limits on a technology's possible improvement. If these limits are close to the performance of the current state



of the art designs, process intensification is not feasible for the considered technology, and alternative technologies must be considered. If, on the other hand, IDEAS can identify designs whose performance represents a significant improvement over known designs, then IDEAS is validated as a systematic process intensification tool.

The sharply fluctuating energy prices constitute a substantial headwind for much of the energy value chain, especially for energy systems where the upstream, downstream, and power sectors are synergized with the aid of governmental subsidies. In such cases, the economic viability of the downstream and power sectors depends heavily on the technology costs and the government regulation on fuel prices, which are often fixed. In the current decade, energy systems with an abundant supply of oil were able to achieve sizable margins in the downstream sector within the high oil price environment. However, the uncertainty of future oil prices makes it difficult to put forward strategies for the energy and petrochemical sectors<sup>6</sup>. An energy model that takes into account fossil fuels in a dynamic selection of hydrocarbon prices, and a flexible downstream sector that is capable of liquid or gas feedstock inlets can lessen the burden that oil price fluctuations have.

As mentioned earlier IDEAS has been employed as a PI tool, to increase energy efficiency, reduce equipment size, and other typical chemical processes optimization activities. In this work, the IDEAS framework is employed to optimize subsidy levels for a variety of resources within a multi-industrial complex.

## 2. IDEAS mathematical Formulation

The IDEAS framework consists of two network blocks. The first one is called Operator Network (OP); in which, all unit operations (reactors, heat exchangers, etc.) are incorporated. The second network block is the Distribution Network (DN); in which, resources are distributed throughout the overall system. These two network blocks are interconnected to each other. The overall system's inlet and outlet streams enter and exit the DN, while the OP inlets and outlets are also DN outlets and inlets respectively as shown in Figure 1. The globally optimal synthesis of several process networks has been successfully pursued using the IDEAS framework. Examples include synthesis of mass-exchange networks<sup>5</sup>, separation network synthesis<sup>7</sup>, synthesis of power cycles<sup>8</sup>, synthesis of complex distillation networks<sup>9-10</sup>, synthesis of reactor network<sup>11-12</sup>, synthesis of reactive distillation network<sup>13</sup>, construction of attainable region<sup>14</sup>, and construction of batch attainable region<sup>15</sup>.

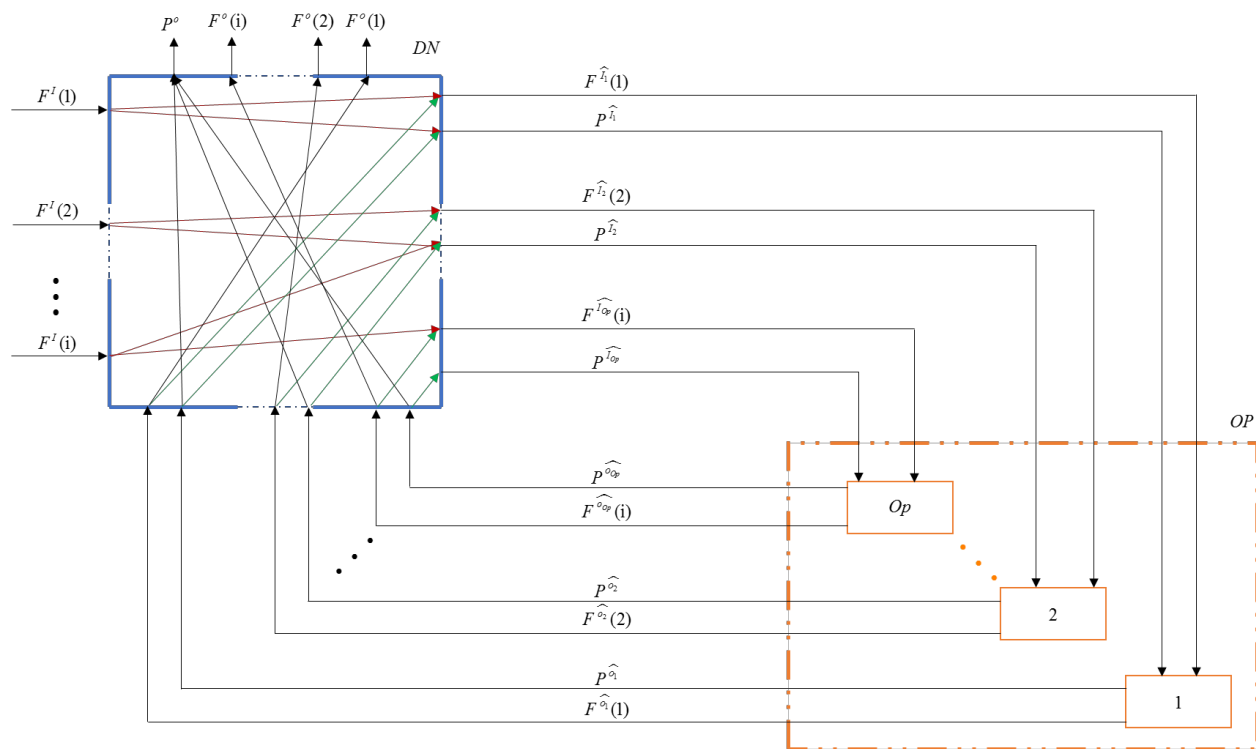


Figure 1: IDEAS representation of an integrated hydrocarbon and energy system

### 3. Background on Subsidized Energy Systems

Energy systems, where upstream, downstream, and power industries are interconnected in a complex manner, involve a variety of raw materials and end products. These typically involve crude oil and light-end hydrocarbon gases as feeds, and refined products and polymers, as shown in Figure 2, which illustrates the inlet and outlet nature of these components within the IDEAS framework's Distribution Network (DN).

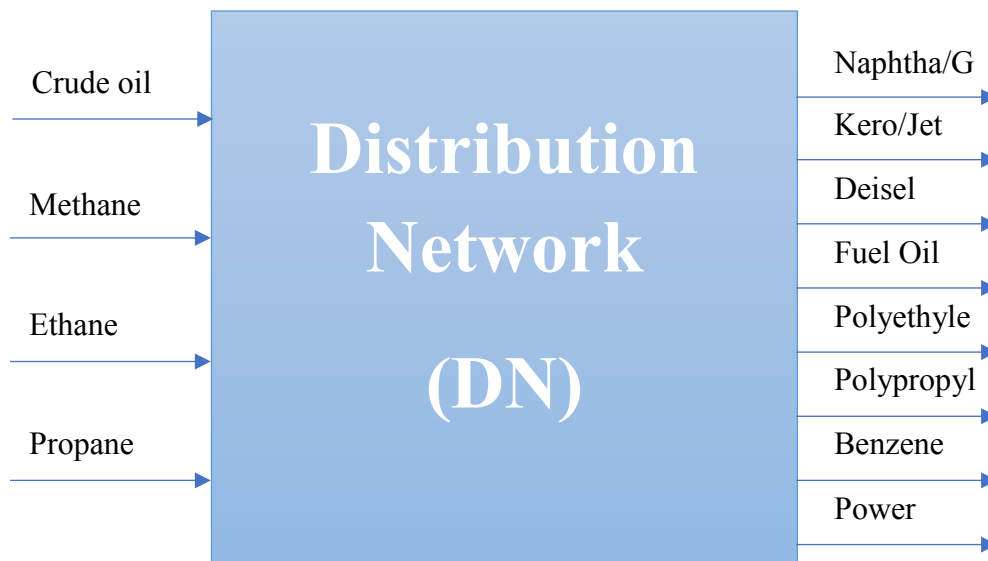


Figure 2: Hydrocarbon and energy distribution network (DN)

Correspondingly, Figures 3, 4, and 5 illustrate the inlet and outlet nature of these components for particular subsystems considered within the IDEAS framework's Process Operator (OP).

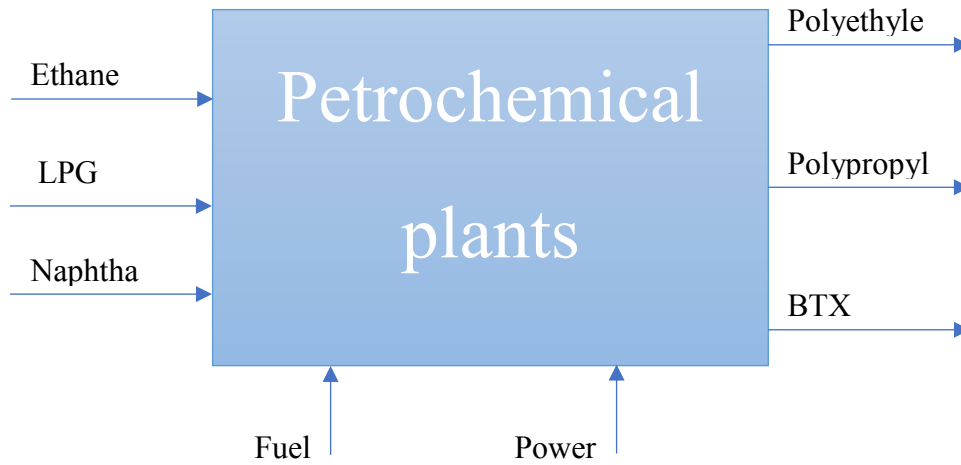


Figure 3: Refineries operation process (OP)

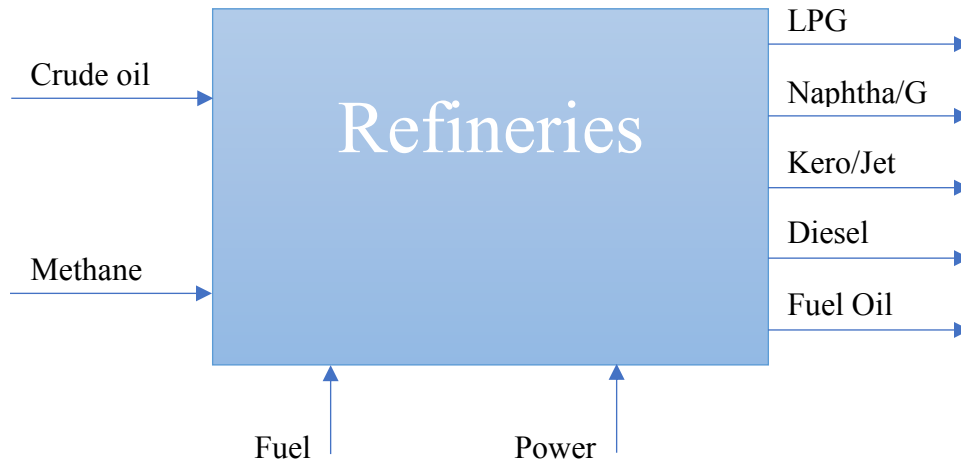


Figure 4: Petrochemicals operation process (OP)

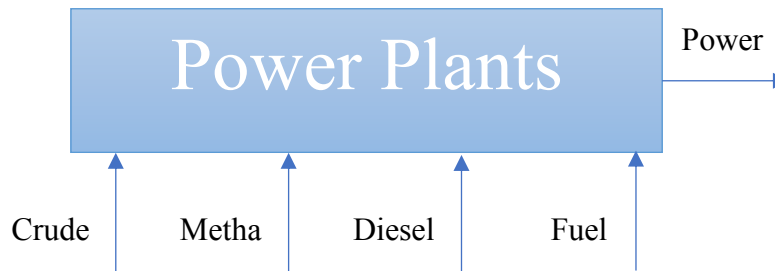


Figure 5: Power plants operation process (OP)

## 4. Mathematical Formulation

### Objective Function

The goal of the considered mathematical formulation is to maximize the DN's net profit, while maintaining desirable profit margins for all sub-systems.

The objective function (eq. 1) consists of three triple sums. Each term in the first triple sum represents the DN entity's revenue obtained from the OP subsystems for OP inlet related purchases, which consists of two components: raw material cost and external power acquisition cost for each OP subsystem. Each term in the second triple sum represents the DN entity's cost associated with the acquisition of OPs' outlets, which again consists of two components: product cost and external power acquisition cost from each OP subsystem. Finally, each term in the third triple sum represents the revenues from DN entity's outlets leaving the overall system, which also consists of two components: product revenue and external power sale revenue for the overall system

$$\left\{ \begin{array}{l} \sum_{j=1}^{Op} \sum_{i=1}^n \sum_{kv=1}^m \left[ \left( F^{\hat{I}_{jkv}}(i) * C_{kv}(i) \right) + \left( P^{\hat{I}_{jkv}} * C_{kv}^{Pw} \right) \right] \\ - \sum_{j=1}^{Op} \sum_{i=1}^n \sum_{kv=1}^m \left[ \left( F^{\hat{O}_{kv}}(i) * C_{kv}(i) \right) + \left( P^{\hat{O}_{kv}} * C_{kv}^{Pw} \right) \right] \\ + \sum_{j=1}^{Op} \sum_{i=1}^n \sum_{kv=1}^m \left[ \left( F^O(i) * C_{int}(i) \right) + \left( P^O * C_{int}^{Pw} \right) \right] \end{array} \right\} \quad (1)$$

where  $T \in \{I_j, O, O\}$  with  $I_j, O, O$  denoting OP inlet, OP outlet, and overall system outlet

respectively; and  $F^{T_{kv}}(i)$  and  $P^{T_{kv}}$  are the molar flowrate of component  $i$  and the power supply at the corresponding prices  $C_{kv}$ ,  $C_{\text{int}}(i)$ ,  $C_{kv}^{pw}$  or  $C_{\text{int}}^{pw}$ .

To enable the global solution of the optimization problem, each physical stream carrying any component ( $i$ ) from the DN to the OP, and vice versa, is represented by a total of  $kv = 1, m$  substreams. Each of these substreams is made available at the fixed price,  $C_{kv}(i)$ , ranging from 0 (100% subsidy) to  $C_{\text{int}}(i)$  (international price = no subsidy) in increments of  $\frac{C_{\text{int}}(i)}{m}$ . For each component ( $i$ ), only one of these  $kv = 1, m$  fixed price substreams can have a non-zero molar flowrate. This is enforced by the optimization program, through binary flag variables as shown in equations (2) and (3), thus, enabling the identification of the optimal subsidy price for each component. In addition, depending on the system design and the particular case study characteristics, some components ( $i$ ) may not appear in certain subsystem operators.

$$\left. \begin{aligned} \sum_{kv=1}^m \lambda_{kv}(i) &= 1 \quad \forall i = 1, n; \quad \lambda_{kv}(i) \in \{0, 1\} \quad \forall i = 1, n, \quad \forall kv = 1, m & (2) \\ \sum_{kv=1}^m \lambda_{kv}^{pw} &= 1; \quad \lambda_{kv}^{pw} \in \{0, 1\} \quad \forall kv = 1, m & (3) \end{aligned} \right\}$$

## DN Mass Balances

The DN total mass mixing and splitting balances can be written as follows:

$$\left\{ \begin{array}{l} F^I(i) = F^{Ol}(i,i) + \sum_{kv=1}^m \sum_{j=1}^{Op} F^{\hat{I}_{jkv}I}(i,i) \quad \forall i=1,n \quad (4) \\ F^{\hat{I}_{jkv}}(i) = \sum_{kv=1}^m F^{\hat{I}_{jkv}O_{\bar{kv}}}(i,i) + F^{\hat{I}_{jkv}I}(i,i) \quad \forall i=1,n \quad \forall j=1,Op \quad \forall kv=1,m \quad (5) \\ F^O(i) = F^{Ol}(i,i) + \sum_{kv=1}^m F^{OO_{\bar{kv}}}(i,i) \quad \forall i=1,n \quad (6) \\ F^{\widehat{O_{\bar{kv}}}}(i) = \sum_{kv=1}^m \sum_{j=1}^{Op} F^{\hat{I}_{jkv}\widehat{O_{\bar{kv}}}}(i,i) + F^{OO_{\bar{kv}}}(i,i) \quad \forall i=1,n \quad \forall \bar{kv}=1,m \quad (7) \\ F^{\widehat{O_{\bar{kv}}}}(i) = \sum_{j=1}^{Op} F^{\widehat{O_{j\bar{kv}}}}(i) \quad \forall i=1,n \quad \forall \bar{kv}=1,m \quad (8) \end{array} \right.$$

where  $\hat{I}$ ,  $O$ ,  $O_{\bar{kv}}$ ,  $I$  denoting OP inlet, OP outlet, overall system outlet and overall system inlet respectively.

## OP Total and Component Mass Balances

$$F^{\hat{I}_{jkv}}(i) = F^{O_{j\bar{kv}}}(i) \quad \forall i=1,n \quad \forall \bar{kv}=1,m \quad \forall kv = \bar{kv} \quad \forall j = j1,Op \quad (9)$$

Equation 9 enforces the process operator total mass balance for each component, subsystem, and price considered. In addition, since all components in the overall system strictly consist of a combination of carbon and hydrogen atoms, the component mass balances in the OP take the following form:



Carbon and Hydrogen Mass Balances:

$$\left\{ \sum_{i=1}^n \sum_{kv=1}^m \left( Ca(i) * F^{\hat{I}_{jkv}}(i) \right) - \sum_{i=1}^n \sum_{kv=1}^m \left( Ca(i) * F^{O_{j\bar{kv}}}(i) \right) = 0 \right\}, \forall j = j1, Op \quad (10)$$

$$\left\{ \sum_{i=1}^n \sum_{kv=1}^m \left( Hy(i) * F^{\hat{I}_{jkv}}(i) \right) - \sum_{i=1}^n \sum_{kv=1}^m \left( Hy(i) * F^{O_{j\bar{kv}}}(i) \right) = 0 \right\}, \forall j = j1, Op \quad (11)$$

where  $Ca(i)$  and  $Hy(i)$  represent the number of carbon and hydrogen atoms per mole of component  $i$  respectively. Equations (10) and (11) enforce that the total carbon and hydrogen atom count in all incoming streams to each subsystem  $j$  is equal to the corresponding counts in all outgoing streams.

### Energy Balances

Equation (12) enforces energy conservation in the OP.  $H_f(i)$  represents the heat of formation of component  $i$ ,  $Q_j^{in-eff}$  represents the heat inlet to subsystem  $j$ , and  $W_j^{in-eff} \geq 0$ ,  $W_j^{out} \geq 0$  represent the work consumed or generated respectively by facility  $j$ .

$$\left\{ \begin{array}{l} \sum_{i=1}^n \sum_{kv=1}^m \left( H_f(i) * F^{\hat{I}_{jkv}}(i) \right) + Q_j^{in-eff} + W_j^{in-eff} \\ \sum_{i=1}^n \sum_{kv=1}^m \left( H_f(i) * F^{\hat{O}_{j\bar{kv}}}(i) \right) + W_j^{out} \end{array} = \right\} \forall j = j1, Op \quad (12)$$

Equation (13) defines the aforementioned  $Q_j^{in-eff}$ , in terms of the heat of combustion ( $\Delta H_c$ ) of component  $i$ , its flowrate  $F_j^{i,jkv}$ , and the efficiency  $\eta_j^{Q^{in}}$  of the combustion.

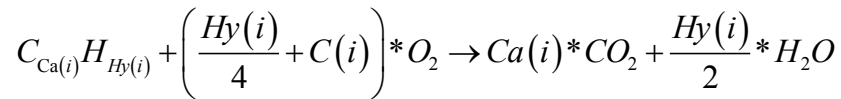
$$\left\{ Q_j^{in-eff} = \sum_{i=1}^i \sum_{kv=1}^m \left( \eta_j^{Q^{in}}(i) * \underbrace{\Delta H_c(i) * F_j^{i,jkv}(i)}_{Q_j^{in}} \right) \right\}, \forall j = j1, Op \quad (13)$$

Similarly, equation (14) defines the aforementioned  $W_j^{in-eff}$ , in terms of  $Q_j^{in-eff}$ , and the correlation factor  $\frac{D_j(i)}{B_j(i)}$  to the plant capacity of component ( $i$ ).

$$\left\{ W_j^{in-eff} = Q_j^{in-eff} * \sum_{i=1}^n \frac{D_j(i)}{B_j(i)} \right\}, \forall j = 1, Op \quad \text{and} \quad \left[ \begin{array}{l} B_j(i) : \text{Correlation between } i \text{ and } Q_j^{in} \\ D_j(i) : \text{Correlation between } i \text{ and } W_j \end{array} \right] \quad (14)$$

### Enforcing subsystem CO<sub>2</sub> and H<sub>2</sub>O emission specifications

Considering the combustion reaction,



the amount of CO<sub>2</sub> and H<sub>2</sub>O emissions ( $F_j^{CO_2}$  and  $F_j^{H_2O}$  respectively) for subsystem  $j$  are bounded above as follows:

$$\left\{ F_j^{CO_2} = \sum_{i=1}^n \sum_{kv=1}^m \left( Ca(i) * F_j^{i,jkv}(i) \right) \leq ULCO_2(j) \right\}, \forall j = j1, Op \quad (15)$$

$$\left\{ F_j^{H_2O} = \sum_{i=1}^n \sum_{kv=1}^m \left( \frac{Hy(i)}{2} * F^{i_{jkv}}(i) \right) \leq ULH_2 O(j) \right\}, \forall j = j1, Op \quad (16)$$

### Subsystem profit requirement

As stated earlier, the problem formulation ensures that each subsystem maintains a margin of profitability. This requirement is quantified by equation (17).

$$\left\{ \sum_{i=1}^n \sum_{kv=1}^m \left( F^{O_{kv}}(i) * C_{kv}(i) \right) \geq (1 + PM_j) * \left( \sum_{i=1}^n \sum_{kv=1}^m \left( F^{i_{jkv}}(i) * C_{kv}(i) \right) + \sum_{kv=1}^m \left( W_{jkv}^{in} * C_{kv}^{Pw} \right) \right) + OH_j \right\}, \forall j = j1, Op \quad (17)$$

where  $(OH_j)$  and  $(PM_j)$  represent the fixed overhead cost and the minimum profit margin for subsystem  $j$ .

The resulting optimization problem, can be concisely stated as follows

$$v = \left\{ \begin{array}{l} \min(1) \\ s.t. (2) - (17) \end{array} \right\} \quad (18)$$

## 5. Case Study

The proposed problem formulation is illustrated in a case study involving the components listed in Table 1 and the subsystems listed in Table 2.

Table 1: components list

<i>i</i>	<b>Component Name</b>	<b>Component Number</b>
	Crude Oil	1
	Methane	2
	Methane (fuel)	3
	Ethane	4
	LPG	5
	Naphtha	6
	Kero/Jet Fuel	7
	Diesel	8
	Fuel Oil	9
	Polyethylene	10
	Polypropylene	11
	Benzene	12

Table 2: Subsystems list

<i>j</i>	<b>OP Name</b>	<b>OP Number</b>
	Refining Sector	<i>j</i> 1
	Petrochemical Sector	<i>j</i> 2
	Power Sector	<i>j</i> 3

The objective function of the resulting IDEAS maximization formulation is:

$$\left\{ \begin{aligned} & \sum_{i=6}^{12} \sum_{kv=1}^m \left( F^{OO_{kv}^-}(i,i) * C_{\text{int}}(i) \right) + \sum_{i=1}^3 \sum_{kv=1}^m \left( F^{\hat{I}_{j1kv}^I}(i,i) * C_{kv}(i) \right) \\ & + \sum_{i=3}^5 \sum_{kv=1}^m \left( F^{\hat{I}_{j2kv}^I}(i,i) * C_{kv}(i) \right) + \sum_{i=5}^6 \sum_{kv=1}^m \left( F^{\hat{I}_{j2kv}\hat{O}_{kv}^-}(i,i) * C_{kv}(i) \right) \\ & + \sum_{kv=1}^m \left( F^{\hat{I}_{j3kv}^I}(1,1) * C_{kv}(1) \right) + \sum_{kv=1}^m \left( F^{\hat{I}_{j3kv}^I}(3,3) * C_{kv}(3) \right) \\ & + \sum_{kv=1}^m \left( F^{\hat{I}_{j3kv}\hat{O}_{kv}^-}(8,8) * C_{kv}(8) \right) + \sum_{kv=1}^m \left( F^{\hat{I}_{j3kv}\hat{O}_{kv}^-}(9,9) * C_{kv}(9) \right) \\ & - \sum_{i=6}^{12} \sum_{kv=1}^m \left( F^{OO_{kv}^-}(i,i) * C_{kv}(i) \right) + \sum_{kv=1}^m \left( P^{OO_{kv}^-} * (C_{\text{int}}^{pw} - C_{kv}^{pw}) \right) \end{aligned} \right\} \quad (*)$$

The first double sum represents overall system product sales at international prices for components 6-12. In the first operator ( $j1$ ), components 1-3 are received as feedstock, and thus the second double sum represents the raw material cost for this subsystem. In the second operator ( $j2$ ), petrochemical plants, components 3-5 are received from the overall system inlet, and thus the third double sum represents raw material cost for this subsystem. Subsystem ( $j2$ ) also receives raw material from the bottom side of the DN, which is represented by the fourth double sum. In the third operator ( $j3$ ), power plants, components 1, 3 are received from the overall system inlet, while components 8, 9 are received from the bottom side of the DN. Subsystem product purchases by the DN entity at subsidized prices, are represented by the fifth double sum. The final single sum represents the power sale revenue for the overall system, which is based on the difference between international and subsidized prices.

The constraints of the resulting IDEAS maximization formulation are:

$$\left\{ \begin{array}{l}
 \left( \begin{array}{l}
 \sum_{i=1}^2 \sum_{kv=1}^m \left( Ca(i) * F^{\hat{I}_{j1kv}^I} (i,i) \right) - \sum_{i=5}^6 \sum_{kv=1}^m \left( Ca(i) * F^{\hat{I}_{j2kv}^{\hat{O}_{kv}}} (i,i) \right) \\
 - \sum_{i=6}^9 \sum_{kv=1}^m \left( Ca(i) * F^{OO_{kv}} (i,i) \right) - \sum_{i=8}^9 \sum_{kv=1}^m \left( Ca(i) * F^{\hat{I}_{j3kv}^{\hat{O}_{kv}}} (i,i) \right)
 \end{array} \right) = 0 \\
 \left( \begin{array}{l}
 \sum_{i=1}^2 \sum_{kv=1}^m \left( Hy(i) * F^{\hat{I}_{j1kv}^I} (i,i) \right) - \sum_{i=5}^6 \sum_{kv=1}^m \left( Hy(i) * F^{\hat{I}_{j2kv}^{\hat{O}_{kv}}} (i,i) \right) \\
 - \sum_{i=6}^9 \sum_{kv=1}^m \left( Hy(i) * F^{OO_{kv}} (i,i) \right) - \sum_{i=8}^9 \sum_{kv=1}^m \left( Hy(i) * F^{\hat{I}_{j3kv}^{\hat{O}_{kv}}} (i,i) \right)
 \end{array} \right) = 0 \\
 \left( \begin{array}{l}
 \sum_{i=4}^5 \sum_{kv=1}^m \left( Ca(i) * F^{\hat{I}_{j2kv}^I} (i,i) \right) + \sum_{i=5}^6 \sum_{kv=1}^m \left( Ca(i) * F^{\hat{I}_{j2kv}^{\hat{O}_{kv}}} (i,i) \right) \\
 - \sum_{i=10}^{12} \sum_{kv=1}^m \left( Ca(i) * F^{OO_{kv}} (i,i) \right)
 \end{array} \right) = 0 \\
 \left( \begin{array}{l}
 \sum_{i=4}^5 \sum_{kv=1}^m \left( Hy(i) * F^{\hat{I}_{j2kv}^I} (i,i) \right) + \sum_{i=5}^6 \sum_{kv=1}^m \left( Hy(i) * F^{\hat{I}_{j2kv}^{\hat{O}_{kv}}} (i,i) \right) \\
 - \sum_{i=10}^{12} \sum_{kv=1}^m \left( Hy(i) * F^{OO_{kv}} (i,i) \right)
 \end{array} \right) = 0 \\
 Ca(3) * \sum_{kv=1}^m \left( F^{\hat{I}_{j1kv}^I} (3,3) \right) \leq X \\
 Ca(3) * \sum_{kv=1}^m \left( F^{\hat{I}_{j2kv}^I} (3,3) \right) \leq Y \\
 \left( \begin{array}{l}
 Ca(1) * \sum_{kv=1}^m \left( F^{\hat{I}_{j3kv}^I} (3,3) \right) + Ca(3) * \sum_{kv=1}^m \left( F^{\hat{I}_{j3kv}^I} (3,3) \right) \\
 + Ca(8) * \sum_{kv=1}^m \left( F^{\hat{I}_{j3kv}^{\hat{O}_{kv}}} (8,8) \right) + Ca(9) * \sum_{kv=1}^m \left( F^{\hat{I}_{j3kv}^{\hat{O}_{kv}}} (9,9) \right)
 \end{array} \right) \leq Z
 \end{array} \right.$$

$$\left. \begin{aligned}
& \left( \sum_{i=5}^6 \sum_{kv=1}^m \left( F^{\hat{I}_{j2kv} \hat{O}_{\bar{kv}}} (i, i) * C_{kv} (i) \right) + \sum_{i=6}^9 \sum_{kv=1}^m \left( \left( F^{O \hat{O}_{\bar{kv}}} (i, i) \right) * C_{kv} (i) \right) \right. \\
& \left. + \sum_{i=8}^9 \sum_{kv=1}^m \left( \left( F^{\hat{I}_{j3kv} \hat{O}_{\bar{kv}}} (i, i) \right) * C_{kv} (i) \right) \right) \geq 0 \\
& - (1 + PM) * \left( \sum_{i=1}^3 \sum_{kv=1}^m \left( F^{\hat{I}_{j1kv} I} (i, i) * C_{kv} (i) \right) + \sum_{kv=1}^m \left( P^{\hat{I}_{j1kv} \hat{O}_{\bar{kv}}} * C_{kv}^{Pw} \right) + OH_1 \right) \\
& \left( \sum_{i=10}^{12} \sum_{kv=1}^m \left( \left( F^{O \hat{O}_{\bar{kv}}} (i, i) \right) * C_{kv} (i) \right) \right. \\
& \left. - (1 + PM) * \left( \sum_{i=3}^5 \sum_{kv=1}^m \left( F^{\hat{I}_{j2kv} I} (i) C_{kv} (i) \right) + \sum_{i=5}^6 \sum_{kv=1}^m \left( \left( F^{\hat{I}_{j2kv} \hat{O}_{\bar{kv}}} (i, i) \right) * C_{kv} (i) \right) \right) \right) \geq 0 \\
& \left. + \sum_{kv=1}^m \left( W_{j2kv}^{in} * C_{kv}^{Pw} \right) + OH_2 \right) \\
& \left( \sum_{kv=1}^m \left( \left( P^{\hat{O}_{\bar{kv}}} + P^{\hat{I}_{j2kv} \hat{O}_{\bar{kv}}} + P^{\hat{I}_{j2kv} \hat{O}_{\bar{kv}}} \right) * C_{kv}^{Pw} \right) \right. \\
& \left. - (1 + PM) * \left( \sum_{kv=1}^m \left( F^{\hat{I}_{j3kv} I} (1, 1) * C_{kv} (1) \right) + \sum_{kv=1}^m \left( F^{\hat{I}_{j3kv} I} (3, 3) * C_{kv} (3) \right) \right) \right) \geq 0 \\
& \left. + \sum_{kv=1}^m \left( F^{\hat{I}_{j3kv} \hat{O}_{\bar{kv}}} (8, 8) * C_{kv} (8) \right) + \sum_{kv=1}^m \left( F^{\hat{I}_{j3kv} \hat{O}_{\bar{kv}}} (9, 9) * C_{kv} (9) \right) + OH_3 \right) \\
& \sum_{kv=1}^m \lambda_{kv} (i) = 1, \forall i = 1, 12 \\
& \sum_{kv=1}^m \lambda_{kv}^{pw} = 1 \\
& \text{all } \lambda \in \{0, 1\}
\end{aligned} \right\}$$

$$\left\{ \begin{array}{l}
0 \leq P^{\hat{I}_{jkv} \hat{O}_{kv}} \leq \lambda_{kv}^{pw} * W_j^{inMax} \quad \forall kv = 1, m \text{ and } j = 1, 2 \\
0 \leq P^{O\hat{O}_{kv}} + \sum_{j=1}^2 P^{\hat{I}_{jkv} \hat{O}_{kv}} \leq \lambda_{kv}^{pw} * W_{j3}^{outMax} \quad \forall kv = 1, m \\
0 \leq F^{\hat{I}_{j1kv}^I} (1,1) \leq \lambda_{kv} (1) * F^{Feed} (1), \quad \forall kv = 1, m \\
0 \leq F^{\hat{I}_{j1kv}^I} (2,2) \leq \lambda_{kv} (2) * F^{Feed} (2), \quad \forall kv = 1, m \text{ and } j = \{1, 3\} \\
0 \leq \sum_{j=1}^3 F^{\hat{I}_{j1kv}^I} (3,3) \leq \lambda_{kv} (3) * F^{Feed} (3), \quad \forall kv = 1, m \\
0 \leq F^{\hat{I}_{j2kv}^I} (4,4) \leq \lambda_{kv} (4) * F^{Feed} (4), \quad \forall kv = 1, m \\
0 \leq F^{\hat{I}_{j2kv}^I} (5,5) \leq \lambda_{kv} (5) * F^{Feed} (5), \quad \forall kv = 1, m \\
\lambda_{kv} (5) * F^{\min} (5) \leq F^{\hat{I}_{j2kv}^I O_{kv}} (5,5) \leq \lambda_{kv} (5) * F^{\max} (5), \quad \forall kv = 1, m \\
\lambda_{kv} (6) * F^{\min} (6) \leq F^{O\hat{O}_{kv}} (6,6) + F^{\hat{I}_{j2kv}^I O_{kv}} (6,6) \leq \lambda_{kv} (6) * F^{\max} (6), \quad \forall kv = 1, m \\
\lambda_{kv} (7) * F^{\min} (7) \leq F^{O\hat{O}_{kv}} (7,7) \leq \lambda_{kv} (7) * F^{\max} (7), \quad \forall kv = 1, m \\
\lambda_{kv} (8) * F^{\min} (8) \leq F^{O\hat{O}_{kv}} (8,8) + F^{\hat{I}_{j2kv}^I O_{kv}} (8,8) \leq \lambda_{kv} (8) * F^{\max} (8), \quad \forall kv = 1, m \\
\lambda_{kv} (9) * F^{\min} (9) \leq F^{O\hat{O}_{kv}} (9,9) + F^{\hat{I}_{j2kv}^I O_{kv}} (9,9) \leq \lambda_{kv} (9) * F^{\max} (9), \quad \forall kv = 1, m \\
\lambda_{kv} (10) * F^{\min} (10) \leq F^{O\hat{O}_{kv}} (10,10) \leq \lambda_{kv} (10) * F^{\max} (10), \quad \forall kv = 1, m \\
\lambda_{kv} (11) * F^{\min} (11) \leq F^{O\hat{O}_{kv}} (11,11) \leq \lambda_{kv} (11) * F^{\max} (11), \quad \forall kv = 1, m \\
\lambda_{kv} (12) * F^{\min} (12) \leq F^{O\hat{O}_{kv}} (12,12) \leq \lambda_{kv} (12) * F^{\max} (12), \quad \forall kv = 1, m \\
F^{\hat{I}_{j1kv}^I} (1,1) \geq 0, \quad \forall kv = 1, m \text{ and } i = 1, 3 \\
F^{\hat{I}_{j2kv}^I} (3,3) \geq 0, \quad \forall kv = 1, m \text{ and } i = 3, 5 \\
F^{\hat{I}_{j3kv}^I} (1,1) \geq 0, \quad \forall kv = 1, m \\
F^{\hat{I}_{j3kv}^I} (3,3) \geq 0, \quad \forall kv = 1, m \\
F^{\hat{I}_{j2kv}^I O_{kv}} (5,5) \geq 0, \quad \forall kv = 1, m \text{ and } i = 5, 6 \\
F^{\hat{I}_{j3kv}^I O_{kv}} (8,8) \geq 0, \quad \forall kv = 1, m \text{ and } i = 8, 9 \\
F^{O\hat{O}_{kv}} (6,6) \geq 0, \quad \forall kv = 1, m \text{ and } i = 6, 10 \\
F^{O\hat{O}_{kv}} (11,11) \geq 0, \quad \forall kv = 1, m \text{ and } i = 11, 12
\end{array} \right.$$



$$\left\{ \begin{array}{l}
F^{O_{kv}}(5) = F^{I_{j2kv} O_{kv}}(5,5), \forall kv = 1, m \\
F^{O_{kv}}(6) = F^{OO_{kv}}(6,6) + F^{I_{j2kv} O_{kv}}(6,6), \forall kv = 1, m \\
F^{O_{kv}}(7) = F^{OO_{kv}}(7,7), \forall kv = 1, m \\
F^{O_{kv}}(8) = F^{OO_{kv}}(8,8) + F^{I_{j3kv} O_{kv}}(8,8), \forall kv = 1, m \\
F^{O_{kv}}(9) = F^{OO_{kv}}(9,9) + F^{I_{j3kv} O_{kv}}(9,9), \forall kv = 1, m \\
F^{O_{kv}}(10) = F^{OO_{kv}}(10,10), \forall kv = 1, m \\
F^{O_{kv}}(11) = F^{OO_{kv}}(11,11), \forall kv = 1, m \\
F^{O_{kv}}(12) = F^{OO_{kv}}(12,12), \forall kv = 1, m \\
F^{I_{j1kv}}(2) = F^{I_{j1kv} I}(2,2), \forall kv = 1, m \\
F^{I_{j1kv}}(3) = F^{I_{j1kv} I}(3,3), \forall kv = 1, m \\
F^{I_{j2kv}}(3) = F^{I_{j2kv} I}(3,3), \forall kv = 1, m \\
F^{I_{j2kv}}(4) = F^{I_{j2kv} I}(4,4), \forall kv = 1, m \\
F^{I_{j2kv}}(5) = F^{I_{j2kv} I}(5,5) + F^{I_{j2kv} O_{kv}}(5,5), \forall kv = 1, m \\
F^{I_{j2kv}}(6) = F^{I_{j2kv} O_{kv}}(6,6), \forall kv = 1, m \\
F^{I_{j3kv}}(1) = F^{I_{j3kv} I}(1,1), \forall kv = 1, m \\
F^{I_{j3kv}}(3) = F^{I_{j3kv} I}(3,3), \forall kv = 1, m \\
F^{I_{j3kv}}(8) = F^{I_{j3kv} O_{kv}}(8,8), \forall kv = 1, m \\
F^{I_{j3kv}}(9) = F^{I_{j3kv} O_{kv}}(9,9), \forall kv = 1, m \\
F^O(6) = F^{OO_{kv}}(6,6), \forall kv = 1, m \\
F^O(7) = F^{OO_{kv}}(7,7), \forall kv = 1, m \\
F^O(8) = F^{OO_{kv}}(8,8), \forall kv = 1, m \\
F^O(9) = F^{OO_{kv}}(9,9), \forall kv = 1, m \\
F^O(10) = F^{OO_{kv}}(10,10), \forall kv = 1, m \\
F^O(11) = F^{OO_{kv}}(11,11), \forall kv = 1, m \\
F^O(12) = F^{OO_{kv}}(12,12), \forall kv = 1, m
\end{array} \right.$$

The resulting MILP formulation was solved repeatedly, using IBM CPLEX solver in Julia<sup>16</sup>, for an ever increasing level of discretization of subsidy price. This iterative procedure is used to establish that the infimum of the IDEAS ILP is adequately approximated. The corresponding IDEAS convergence plot is shown in Figure 6 establishing the adequacy of the considered level of discretization. The relevant case study parameter values are shown in Table 3 below:

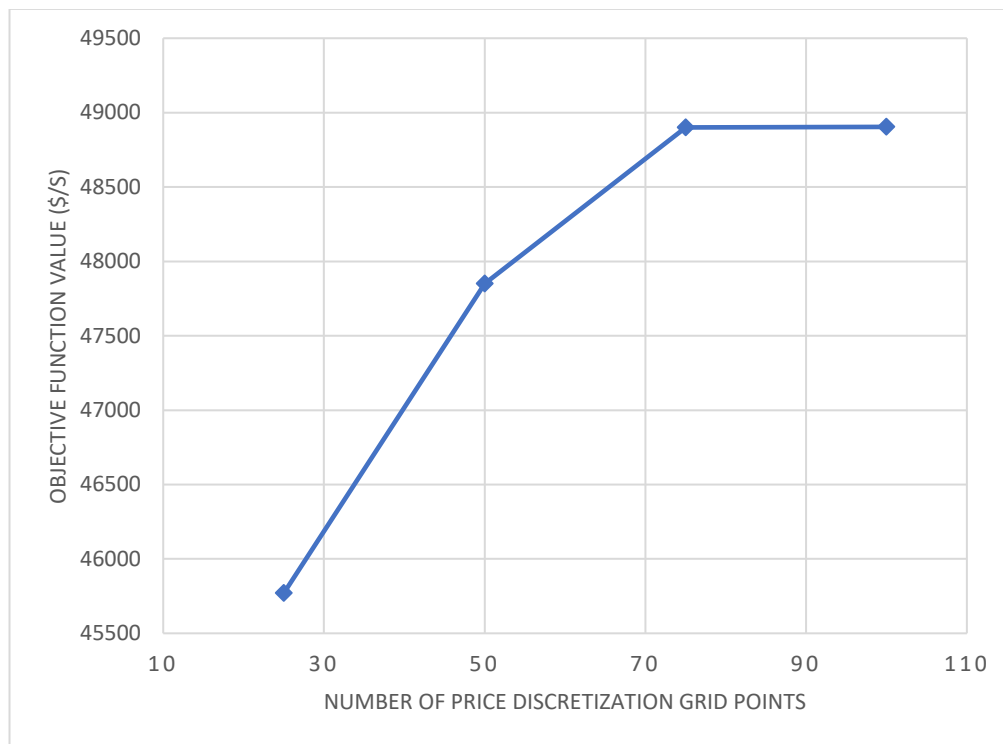


Figure 4: IDEAS convergence with the number of price discretization grid points

Table 3: Case study parameter values

Parameter	Value	Parameter	Value
$Ca(1)$	60	$Ca(2)$	1
$Ca(3)$	1	$Ca(4)$	2
$Ca(5)$	3	$Ca(6)$	8
$Ca(7)$	10	$Ca(8)$	14
$Ca(9)$	25	$Ca(10)$	2000
$Ca(11)$	3000	$Ca(12)$	6
$Hy(1)$	122	$Hy(2)$	4
$Hy(3)$	4	$Hy(4)$	6
$Hy(5)$	8	$Hy(6)$	18
$Hy(7)$	22	$Hy(8)$	30
$Hy(9)$	52	$Hy(10)$	4000
$Hy(11)$	6000	$Hy(12)$	6
$\Delta H_f(1)$	-1281.7 kJ/mol	$\Delta H_f(2)$	-74.87 kJ/mol
$\Delta H_f(3)$	-74.87 kJ/mol	$\Delta H_f(4)$	-84.0 kJ/mol
$\Delta H_f(5)$	-104.7 kJ/mol	$\Delta H_f(6)$	-250.3 kJ/mol
$\Delta H_f(7)$	-301.0 kJ/mol	$\Delta H_f(8)$	-403.3 kJ/mol
$\Delta H_f(9)$	-559.3 kJ/mol	$\Delta H_f(10)$	-107.1 kJ/mol

$\Delta H_f(11)$	-104.1 kJ/mol	$\Delta H_f(12)$	-49.0 kJ/mol
$\Delta H_c(1)$	38410 kJ/mol	$\Delta H_c(3)$	730 kJ/mol
$\Delta H_c(8)$	9033 kJ/mol	$\Delta H_c(9)$	16058 kJ/mol
$F^I(1)$	7 mol/s	$F^I(2)$	66000 mol/s
$F^I(3)$	66000 mol/s	$F^I(4)$	28000 mol/s
$F^I(5)$	19 mol/s	$C_{int}(1)$	0.3 \$/mol
$C_{int}(2)$	0.002 \$/mol	$C_{int}(3)$	0.002 \$/mol
$C_{int}(4)$	0.005 \$/mol	$C_{int}(5)$	0.13 \$/mol
$C_{int}(6)$	0.06 \$/mol	$C_{int}(7)$	0.06 \$/mol
$C_{int}(8)$	0.09 \$/mol	$C_{int}(9)$	0.12 \$/mol
$C_{int}(10)$	25 \$/mol	$C_{int}(11)$	37.8 \$/mol
$C_{int}(12)$	0.057 \$/mol	$C_{int}^{pw}$	$3.3 \cdot 10^{-5}$ \$/kwatt
PM	0.08		

Table 4 and 5 below list the optimum levels of subsidy, over international prices, for each of the considered components, and the optimum product flowrates respectively.

Table 4: Component subsidy levels generated by IDEAS

Component	subsidy
1	32%
2	58%
3	58%
4	1%
5	1%
6	89%
7	91%
8	99%
9	99%
10	92%
11	97%
12	98%
Pw	85%

Table 5: Overall system product flowrates generated by IDEAS

Component	Flow rate
6	1000.00 mol/s
7	300.00 mol/s
8	0.00 mol/s
9	0.00 mol/s
10	15000.00 mol/s
11	3000.00 mol/s
12	776.54 mol/s
Pw	2.22E7 kwatt

The optimum objective function value, representing the DN entity's net profit, is 48902.853 \$/s. Current standard practice, calls for fixed subsidy levels at 30% for liquids and 50% for gases. This subsidy policy results in a net profit for the DN entity of 43994.864 \$/s. Thus, the IDEAS identified policy represents a profit improvement of 11% over the current subsidy policy.

## 6. Conclusion

The IDEAS conceptual framework has been employed to generate a linear programming based formulation that identifies price subsidy policies for energy systems that includes refineries, petrochemical plants and power plants. The identified pricing policies maximize net revenue for the overall system, while maintaining desirable profit margins for all sub-systems. In this sense, IDEAS is employed as an intensification tool, for current state of the art fixed price subsidy systems. The proposed approach is demonstrated in a case study involving an energy system consisting of refineries, petrochemicals, and power plants that are fed with liquid and gas hydrocarbons. Some of the sub-system products are recycled within the overall system, while others exit the overall system as exports. Optimized discounts for all products are identified, as well as optimal sub-system and final product flowrates. The identified optimum improves net revenue by 11%, when compared to the fixed discount case.

## 7. Nomenclature

$OP$	Process Operator
$DN$	Distribution Network
$i$	Component number
$j$	Facility number
$F^{I_j}(i)$	Flowrate of component $i$ entering subsystem $j$
$F^I(i)$	Flowrate of component $i$ entering the overall system
$F^{O_j}(i)$	Flowrate of component $i$ exiting subsystem $j$
$F^O(i)$	Flowrate of component $i$ exiting the overall system
$P$	Power
$Ca(i)$	Carbon count of component $i$
$Hy(i)$	Hydrogen count of component $i$
$B_j(i)$	Correlation between component $i$ in $j$ and $Q_j^in$
$D_j(i)$	Correlation between component $i$ in $j$ and $W_j$
$C_{int}(i)$	International price of component $i$
$C_{kv}(i)$	Subsidized price of component $i$



$C_{\text{int}}^{pw}$	International price of power
$C_{kv}^{Pw}$	Subsidized price of power
$Q_j^{in-eff}$	Effective heat entering subsystem $j$
$Q_j^{in}$	heat entering subsystem $j$
$W_j^{in-eff}$	Effective Work entering subsystem $j$
$W_j^{in}$	Work entering subsystem $j$
$W_j^{out}$	Work generated from subsystem $j$
$PM_j$	Profit Margin at subsystem $j$
$\eta_j(i)$	Efficiency of Work production by fuel $i$ in $j$
$\Delta H_f(i)$	Heat of formation for component $i$
$\Delta H_c(i)$	Heat of reaction (combustion) for component $i$
$G_j(i)$	Correlation between component $i$ in $j$ and $Q_j^{loss}$
$OH_j$	Fixed overhead cost at subsystem $j$
$\lambda$	Binary flag variable

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