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### Authors

Weston, J. Fred

Tuan, Pham D.

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Changes in Credit Policy:  
Reconciliation and Extensions

by

J. Fred Weston

Pham D. Tuan

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CHANGES IN CREDIT POLICY: RECONCILIATION AND EXTENSIONS

J. Fred Weston and Pham D. Tuan  
Graduate School of Management  
University of California, Los Angeles

## CHANGES IN CREDIT POLICY: RECONCILIATION AND EXTENSIONS

Several articles on credit policy were published in Financial Management in 1976-79.<sup>(1)</sup> This paper seeks to reconcile the divergent views presented and to make some extensions to the material. The presentation will emphasize the general pattern of the relations.

The definition of symbols used and the initial conditions are set forth in Table 1. The symbols are used in the analysis of four major types of credit policies: (1) change credit standards, (2) change the credit terms, (3) change collection policy and (4) change the cash discount policy. The four major types of credit policy changes are illustrated by hypothetical examples in Table 2.

Changing credit standards refers to the quality control of credit customers. Credit standards are used to seek to control the collection behavior and loss experience on credit sales. Changing credit standards brings in new customers (or eliminates some of the old). A change in credit terms or collection policies increases (or decreases) sales among the existing customers. As a result the investment and profitability implications of a change in credit standards are different from the other changes in credit policy.

Finally, we focus on the question of optimal discount policy. From our general approach, we will derive formulas for the maximum profitable discount rate and the optimal discount rate and demonstrate that our formulation is essentially equivalent to that of Hill and Riener.

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1) The articles are: John S. Oh, "Opportunity Cost in the Evaluation of Investment in Accounts Receivable," Summer 1976; Edward A. Dyl, "Another Look at the Evaluation of Investments in Accounts Receivable," Winter 1977; Joseph C. Atkins and Yong H. Kim, "Comment and Correction: Opportunity Cost in the Evaluation of Investment in Accounts Receivable," Winter 1977; Tirlochan S. Walia, "Explicit and Implicit Cost of Changes in the Level of Accounts Receivable and the Credit Policy Decision of the Firm," Winter 1977; Ned C. Hill and Kenneth D. Riener, "Determining the Cash Discount in the Firm's Credit Policy," Spring, 1979.

Table 1

Symbols and Initial Conditions for

## ANALYSIS OF CREDIT POLICY

<u>Symbols</u>	<u>Initial Conditions</u>
$\Delta P$ = change in profit resulting from new policy	
$\Delta I$ = change in investment in receivables	
$S_0$ = present level of total credit sales	\$10,000,000
$S_1$ = expected level of total credit sales under new policy	
$= S_0 + \Delta S$	
$\Delta S$ = change in credit sales under new policy	
$C_0$ = average collection period for old total sales	60 days
$C_1$ = new average collection period for new total sales	
$C_n$ = average collection period for change in sales	
$B_0$ = percentage bad debt losses on old total sales	2%
$B_1$ = new percentage bad debt losses for new total sales	
$B_n$ = percentage bad debt losses on change in sales	
$V$ = percentage variable cost of sales = variable costs/sales	60%
$d_0$ = old discount rate for early payment	
$d_1$ = new discount rate for early payment	
$D_0$ = percentage of discounted sales under old policy	
$D_1$ = percentage of discounted sales under new policy	
$k$ = required rate of return on investment in receivables	10%
$\Delta E$ = change in collection expenses	

Table 2

Explanation of Different Types of Credit Policy Changes

I. Relaxing credit

- A. Lower credit standards--current ratio from 2/1 requirement to 1.5/1 requirement; debt ratio from 40 percent of total assets to 50 percent of total assets
- B. Lengthen credit terms--change credit policy from n/50 to n/60 and the average collection period goes from 60 to 70 days.
- C. Relax collection policy--the average collection period increases from 60 days to 70 days.
- D. Provide discounts--change from credit terms of net 60 days to 2%/15 days, net for payment in 60 days.

II. Tightening credit

- A. Raise credit standards--reversal of IA
- B. Shorten credit terms--change credit policy from n/50 to n/30 and the average collection period goes from 60 to 40 days.
- C. Relax collection policy--the average collection period decreases from 60 days to 40 days.
- D. Reduce discounts--change from credit terms of 2/15, n/60 to 2/10, n/60.

Overview of Effects of Credit Policy Changes

A summary view of the computation formulas is presented in Table 3.<sup>(1)</sup> The table reveals some consistent patterns and nice symmetries in the analysis. Changing credit standards is the only policy change which does not involve the existing credit sales ( $S_0$ ) or remaining credit sales ( $S_1$ ). Its formula is the only one that involves the change in sales ( $\Delta S$ ) only after credit policies have been changed. For the other three types of changes in credit policy in a given direction, the formulas for the change in investment in receivables are the same. But they change for relaxing vs tightening credit policy:

$$\text{Relax credit policy} \quad \Delta I = VC_1 \Delta S / 360 + (C_1 - C_0) S_0 / 360$$

$$\text{Tighten credit policy} \quad \Delta I = VC_0 \Delta S / 360 + (C_1 - C_0) S_1 / 360$$

When credit policy is relaxed, the new collection period applies to the change in sales. There is also additional investment in receivables due to the change in collection period on the existing sales. When credit policy is tightened, some sales are lost. Receivables will decline by the old collection period applied to the credit sales that are no longer made. In addition, the new level of sales,  $S_1$  is lower than  $S_0$  the old level of sales. Hence the investment in receivables will decline by the change in credit period applied to the remaining level of sales.

We next consider the pattern of the effect on profitability of the remaining three changes in credit policy. A nice logic obtains in the pattern of relations. The basic credit policy is a change in credit terms whose profitability is determined by the change in sales, the effects of the new bad debt loss experience, plus the opportunity cost of the change in

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<sup>1)</sup> Details of the analysis are presented in Appendix A available from the authors.

Table 3

## Summary Overview of Relations for a Firm with Excess Capacity

Change in Policy	Effect on Investment in Receivables ( $\Delta I$ )	Effect on Profit ( $\Delta P$ )
<b>I. Relaxing Credit</b>		
A. Lower Credit Standards	$VC_n \Delta S / 360$	$\Delta S(1-V) - B_n \Delta S - k\Delta I$
B. Lengthen Credit Terms	$VC_1 \Delta S / 360 + (C_1 - C_0) S_0 / 360$	$\Delta S(1-V) - (B_1 S_1 - B_0 S_0) - k\Delta I$
C. Relax Collection Policy	$VC_1 \Delta S / 360 + (C_1 - C_0) S_0 / 360$	$\Delta S(1-V) - (B_1 S_1 - B_0 S_0) - k\Delta I - S_1 \Delta E$
D. Provide Discounts	$VC_1 \Delta S / 360 + (C_1 - C_0) S_0 / 360$	$\Delta S(1-V) - (B_1 S_1 - B_0 S_0) - k\Delta I - (d_1^D S_1 - d_0^D S_0)$
<b>II. Tightening Credit</b>		
A. Raise Credit Standards	$VC_n \Delta S / 360$	$\Delta S(1-V) - B_n \Delta S - k\Delta I$
B. Shorten Credit Terms	$VC_0 \Delta S / 360 + (C_1 - C_0) S_1 / 360$	$\Delta S(1-V) - (B_1 S_1 - B_0 S_0) - k\Delta I$
C. Tighten Collection Policy	$VC_0 \Delta S / 360 + (C_1 - C_0) S_1 / 360$	$\Delta S(1-V) - (B_1 S_1 - B_0 S_0) - k\Delta I - S_1 \Delta E$
D. Reduce Discount	$VC_0 \Delta S / 360 + (C_1 - C_0) S_1 / 360$	$\Delta S(1-V) - (B_1 S_1 - B_0 S_0) - k\Delta I - (d_1^D S_1 - d_0^D S_0)$



the investment in receivables. If collection policy is changed, we simply add a term for the change in collections expenses applied to the new level of sales. If a change in credit terms is made, the cost, instead of collection expenses, is the change in the discounts taken under the new policy as compared with the old. In addition, the three formulas are unchanged between tightening credit and relaxing credit policy. Only the signs of the terms are affected by the change in the direction of credit policy.

The summary pattern of Table 3 reflects each of the major types of changes in credit policy. The next issue we consider is the practical significance of the general assumption of the existence of excess capacity.

Reconsideration of the Excess Capacity Assumption

The formulas generally employed treat the discount factor on an annualized basis, implying that the excess capacity will exist forever. But in a world of continuing growth, it is unlikely that a firm will have excess capacity more than a few years. We next consider the effect of elimination of excess capacity after some finite time period on the analysis.

We assume that there is no excess capacity after  $H$  years, where  $H=2$ . At that time total costs ( $T$ ) will be 90 percent of sales so that the profit margin ( $P$ ) will be 10 percent of sales. We will illustrate the effects on  $IB$ , lengthening credit terms. The equation to perform the analysis is:

$$\begin{aligned}
 NPV = & \sum_{i=1}^H [\Delta S(1-V) - (B_1 S_1 - B_0 S_0)](1+k)^{-i} & (1) \\
 & + \sum_{j=0}^{\infty} [\Delta S(1-T) - (B_1 S_1 - B_0 S_0)](1+k)^{-j} (1+k)^{-(H+1)} \\
 & - [VC_1 \Delta S + (C_1 - C_0) S_0] / 360 - [(T-V) C_1 \Delta S / 360] (1+k)^{-H}.
 \end{aligned}$$

The logic of equation (1) is to determine whether the PV of the increased cash flows exceeds the PV of the investment, that is, whether the NPV is positive.

The first term is the cash flow with excess capacity; the second term is the cash flow when there is no excess capacity; the third term is the investment when excess capacity exists and the last term is the additional investment at the end of the second year to reflect the full costs of the investment in receivables. With  $\Delta S = \$500,000$ ,  $V = 60\%$ ,  $B_1 = 3\%$ ,  $B_0 = 2\%$ ,  $S_0 = \$10,000,000$ ,  $k = 10\%$ ,  $T = 90\%$ ,  $C_1 = 70$  days,  $C_0 = 60$  days, the NPV of the policy change is clearly negative.

$$\begin{aligned} \text{NPV} &= \$85,000(1.7355) + [(-\$65,000)/(.1)](.826) - \$336,111 - (\$29,167)(.826) \\ &= \$147,518 - \$536,900 - \$336,111 - \$24,092 \\ &= -\$749,585 \end{aligned} \tag{1.1}$$

When we assume that the bad debt loss ratio rises to only 2.2 percent instead of 3 percent, the calculations are as shown in equation (1.2):

$$\begin{aligned} \text{NPV} &= \$169,000(1.7355) + [(\$19,000)/(.1)](.826) - \$336,111 - (\$29,167)(.826) \\ &= \$293,300 + \$156,940 - \$336,111 - \$24,092 \\ &= \$90,037 \end{aligned} \tag{1.2}$$

The NPV of the investment resulting from the credit policy change is now positive. Perhaps the firm would be prudent to experiment with a change from n/60 to n/55. The additional investment in receivables would be smaller. The new bad debt loss ratio might not increase appreciably. Such factors might make the change in credit policy more attractive.

Nevertheless, the point is that as the firm's sales grow, the assumption of excess capacity is not likely to be a realistic one for an infinite time horizon. The analysis of the change in the credit policy decision should take into account the time at which the investment in receivables goes in at full cost and the cash flow from the change in sales also reflects the total costs of production. The effects on the credit policy decision are likely to be

substantial. In any event, the assumption of excess capacity for an infinite time horizon is not realistic in a world in which the economy grows in real terms and industries and firms grow as well.

Optimizing Credit Policy

The Hill and Riener (HR) (1979) paper presents the analytics for computing a maximum profitable discount and an optimal discount rate under a set of assumptions with respect to the effects of discounts on the timing of payments on credit sales, changes in sales volume, fraction of credit sales which take discounts, and changes in the bad debt loss ratio. Their approach is to find the present value of the discount decision using the Atkins and Kim (1977) methodology.

In formulating the decision rule for calculating the maximum discount, HR first present Case 1 in which only a change in the timing of payments is affected. The maximum profitable discount rate derived is:

$$d_{1m} = 1 - (1+i)^{M-N'} \left[ 1 - \frac{1}{D_1} + \frac{(1+i)^{N'-N}}{D_1} \right]$$

where:

$d_{1m}$  = maximum profitable discount rate

$i$  = opportunity cost per day of the firm's funds = (.10/365)

$N$  = ACP under old discount policy = 90 days

$M$  = ACP when discounts are taken under new discount policy = 10 days

$N'$  = ACP when discounts are not taken under new discount policy = 120 days

$D_1$  = percentage of discounted sales under new discount policy = .5

Using the data of their numerical example, we obtain  $d_{1m}$

$$d_{1m} = 1 - (1.00027)^{10-120} \left[ 1 - \frac{1}{.5} + \frac{(1.00027)^{120-90}}{.5} \right] = 1.37\%$$

The same result is obtained using the methodology proposed by Oh (1976) and Dyl (1977). This is our Case ID for which

$$\Delta I = VC_1 \Delta S / 360 + (C_1 - C_0) S_0 / 365$$

$$\Delta P = \Delta S(1-V) - (d_1 D_1 S_1 - d_0 D_0 S_0) - k \Delta I$$

Using the HR data, we have

$$C_0 = N = 90 \text{ days}$$

$$C_1 = D_1 M + (1 - D_1) N' = .5(10) + .5(120) = 65 \text{ days}$$

Since  $k = .10$  and  $\Delta S = 0$ , it follows that

$$\Delta I = 0 + (65 - 90) S_0 / 365$$

$$\begin{aligned} \Delta P &= 0 - d_{1m} (.50) S_0 - .10 \Delta I \\ &= d_{1m} (.50) S_0 + (.1) \frac{25}{365} S_0 \end{aligned}$$

At the maximum profitable discount rate,  $d_{1m}$ , the value of  $\Delta P$  would be zero because  $P$  would be at a maximum. Setting  $\Delta P = 0$  and solving for  $d_{1m}$ , we obtain

$$d_{1m} = \frac{2.5}{365} \frac{1}{.5} = .0137 = 1.37\%$$

The same result is obtained, illustrating that the two approaches are equivalent. They both take the timing of cash flows into account by different, but equivalent, formulations.

In HR's Case 2, the cash discount rate affects sales volume as well as the timing of payments. Their derivation of  $d_{1m}$  gives:

$$d_{1m} = 1 - (1+i)^{M-N'} \left[ 1 - \frac{1}{D_1} + \frac{(1+i)^{N'-N} + Vg(1+i)^{N'-Q}}{D_1(1+g)} \right],$$

where the new symbols are:

$$g = (S_1/S_0) - 1$$

Q = average payment date of variable costs.

In their numerical example,  $N=N'=M=0$ ;  $Q=0$ ;  $g=.1$ ;  $V=.8$ , hence,

$$d_{1m} = 1 - \left[ 1 - \frac{1}{.5} + \frac{1 + (.8)(.1)(1.0)}{5(1.1)} \right] = .0364 = 3.64\%$$

Now we use the HR data in our Case ID formulation.

$$C_0 = 0$$

$$C_1 = .5(0) + .5(0) = 0$$

$$\Delta S = .1S_0$$

$$\Delta I = 0$$

$$\Delta P = .1S_0(1-.8) - d_{1m}(.5)(1.1S_0)$$

As before, setting  $\Delta P=0$  and solving for  $d_{1m}$ , we have

$$d_{1m} = \frac{.02S_0}{(.5)(1.1)S_0} = .03636 = 3.64\%$$

Again we obtain the HR results.

HR develop the optimal discount rate when the timing of payments only is affected by discount policy. Their result is:

$$d_1^* = \frac{1 - \left(1 + \frac{k}{365}\right)^{M-N'}}{2}$$

Using our notation for ID again, we have

$$\Delta I = VC_1 \Delta S / 360 + (C_1 - C_0) S_0 / 360$$

$$\Delta P = \Delta S(1-V) - (B_1 S_1 - B_0 S_0) - (d_1 D_1 S_1 - d_0 D_0 S_0) - k \Delta I$$

To relate to the HR symbols and assumptions,

$$C_0 = N$$

$$\Delta S = 0; S_1 = S_0; B_1 = B_0$$

$$C_1 = D_1 M + (1-D_1)N'$$

$$= f(d_1)M + [1-f(d_1)]N'$$

We now can rewrite  $\Delta I$  and  $\Delta P$ ,

$$\Delta I = 0 + \{f(d_1)M + [1-f(d_1)]N' - N\} S_0 / 365$$

$$\Delta P = 0 - 0 - [d_1 f(d_1) S_0 - 0] - k \Delta I$$

The optimal discount is chosen to maximize  $\Delta P$ . This occurs when

$$\frac{\partial \Delta P}{\partial d_1} = 0$$

So we insert the right-hand side of  $\Delta I$  into  $\Delta P$  and differentiate with respect to  $d_1$  to obtain

$$- f(d_1) S_0 - d_1 f'(d_1) S_0 - k [f'(d_1)M - f'(d_1)N'] S_0 / 360 = 0$$

Simplifying

$$f(d_1) + f'(d_1) \left\{ d_1 + \frac{k(M-N')}{360} \right\} = 0$$

HR assume  $f(d_1) = d_1$  so that  $f'(d_1) = 1$ . All that is required is a relationship between  $D_1$  and  $d_1$ . We will assume a relationship such that  $f(d_1) = 20d_1$  so that  $f'(d_1) = 20$ .

We now have

$$20d_1 + 20 \left[ d_1 + \frac{k(M-N')}{365} \right] = 0$$

and  $d_1^* = \frac{k(N'-M)}{2(365)}$ .

In HR's numerical example,  $M = 10$  days and  $N' = N = 90$  days. These are inserted in their formula for  $d_1^*$ .

$$d_1^* = \frac{1 - \left(1 + \frac{.10}{365}\right)^{10-90}}{2} = .0108382 = 1.084\%$$

Using our formula, we have

$$d_1^* = \frac{k(N'-M)}{2(365)} = \frac{.1(90-10)}{2(365)} = \frac{8.0}{730} = .010959 = 1.096\%$$

The difference would not affect the practical result which would be to the nearest integer discount rate. <sup>(1)</sup>

#### Summary

Based on the assumption of a firm with excess capacity for an infinite period of time, we first illustrated the effects of changes of four major credit policies on both the investment in receivables and the firm's profitability. These four major credit policy changes used are either relaxing or tightening the credit standards, the credit terms, the collection period, and the cash discount policy.

When the credit standards are changed, the sales will be affected either by losing those customers with less desired credit standings or by bringing in new customers, whereas in the latter case an assumption of no effects on existing accounts and sales has been made. When the credit terms are altered, the average collection period would be either shortened or lengthened. When a collection policy is changed, different collection outlays and efforts are expected. When a new discount policy is implemented, due to the changing incentives for early payments, the payment pattern and the collection period should all be affected. Based on these cause-effect relationships, the equations needed for calculating the effects on investment in receivables and profitability are developed.

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<sup>1)</sup>This equivalence is developed more generally in Appendix B available from the authors.

Second, we questioned the validity of the general assumption of excess capacity in a world of continuing growth. Therefore, we developed an alternative equation to analyze the effects of credit policy changes for a firm with excess capacity only for a limited period of time. In this revised equation, a concept of full cost has been employed for the subsequent investment in receivables whenever the excess capacity has been used up after some finite time period.

Finally, we focused on the computation of an optimal discount rate. We demonstrated that the HR results are readily duplicated using annualized discount factors. Thus there is no difference between approaches presented by their authors as distinct alternatives.

We have made clear the logic behind the four major credit policy changes. But it is not possible to predict how these changes will affect the profitability of investment in receivables. The results depend on the interrelationships among the parameters of change in the factors affected. The framework set forth in this paper, assisted by empirical analysis can be used to move toward the development of optimal credit policies.



## Appendix A

Detail of Computations for Changes in Credit PolicyIA. Lower Credit StandardsRelations

$$\begin{aligned} \Delta I &= (\text{percentage variable cost})(\text{collection period on change in sales}) \\ &\quad (\text{change in sales})/360 \\ &= V \cdot C_n \cdot \Delta S / 360 \end{aligned}$$

$$\begin{aligned} \Delta P &= (\text{profit on change in sales}) - (\text{bad debt losses on change in sales}) - \\ &\quad (\text{opportunity cost of } \Delta I) \\ &= \Delta S(1-V) - B_n \cdot \Delta S - k \cdot \Delta I \end{aligned}$$

New conditions

$$C_n = 90 \text{ days}, \Delta S = \$500,000, B_n = 4\%$$

Calculations

$$\Delta I = (.60)(90)(\$500,000)/360 = \$75,000$$

$$\begin{aligned} \Delta P &= (\$500,000)(1-.60) - (.04)(\$500,000) - (.10)(\$75,000) \\ &= \$200,000 - \$20,000 - \$7,500 = \$172,500 \end{aligned}$$

The logic of the formula is that by lowering credit standards, a new group of customers is brought in, with no effects on existing accounts and sales. In the investment formula, we would expect the credit sales to the new customers with lower financial strength to experience a longer collection period than the old accounts and to experience a higher bad debt loss ratio. This is reflected in the new collection period in the investment formula and in the new bad debt loss ratio in the profitability formula. For the data of the example, the lowering of credit standards results in an increase in profitability.

When credit standards are raised, the formulas used are exactly the same. But now sales are decreased because some customers are now no longer granted credit sales. Since the change in sales,  $\Delta S$ , reflected in each term of the formulas, is negative, the signs of each term are simply reversed, as shown in IIA.

#### II.A. Raise Credit Standards

$$\begin{aligned}\Delta I &= (\text{percentage variable cost})(\text{collection period on change in sales}) \\ &\quad (\text{change in sales})/360 \\ &= V \cdot C_n \cdot \Delta S/360\end{aligned}$$

$$\begin{aligned}\Delta P &= (\text{profit on change in sales}) - (\text{bad debt losses on change in sales}) \\ &\quad - (\text{opportunity cost of } \Delta I) \\ &= \Delta S(1-V) - B_n \cdot \Delta S - k \cdot \Delta I\end{aligned}$$

#### New conditions

$$C_n = 80 \text{ days}, \Delta S = -\$500,000, B_n = 3\%$$

#### Calculations

$$\Delta I = (.60)(80)(-\$500,000)/360 = -\$66,667$$

$$\begin{aligned}\Delta P &= (-\$500,000)(.4) - (.03)(-\$500,000) - (.1)(-\$66,667) \\ &= -\$200,000 + \$15,000 + \$6,667 = -\$178,333\end{aligned}$$

Here the loss in sales due to the tighten credit standards offsets the other benefits. Hence under the data assumed, the tighten credit standards result in decreased profits.

The remaining three changes in credit policy all affect existing customers as well as the new credit sales. The relations between the next three changes in credit policies discussed are:

Change credit terms--average collection period changes because terms are altered

Change collection policy--average collection period changes because collection outlays and other efforts are altered

Change discount terms--average collection period changes because the reward for early payment is altered

The subsequent formulas will therefore reflect the effect of the change. The effect on the investment in receivables in each case will reflect the new collection experience on new sales as well as the change in collection period on the existing or remaining accounts. The formulas for calculating the change in profitability will reflect the change in sales, the change in investment in receivables and new bad debt loss experience. These relations are illustrated for a lengthening of credit terms.

#### I.B. Lengthen Credit Terms

For example, credit terms are changed from n/50 to n/60 and the new ACP becomes 70 days.

$$\begin{aligned}\Delta I &= (\text{percentage variable cost})(\text{new collection period})(\text{change in sales})/360 \\ &\quad + (\text{new collection period} - \text{old collection period})(\text{old total sales})/360 \\ &= VC_1 \Delta S / 360 + (C_1 - C_0) S_0 / 360\end{aligned}$$

$$\begin{aligned}\Delta P &= (\text{profit on change in sales}) - (\text{change in bad debt losses on all} \\ &\quad \text{sales}) - (\text{opportunity cost of } \Delta I) \\ &= \Delta S \cdot (1 - V) - (B_1 S_1 - B_0 S_0) - k \Delta I\end{aligned}$$

#### New conditions

$$C_1 = 70 \text{ days}, \Delta S = \$500,000, B_1 = 3\%$$

#### Calculations

$$\begin{aligned}\Delta I &= (.60)(70)(\$500,000)/360 + (70-60)(\$10,000,000)/360 \\ &= \$58,333 + \$277,778 = \$336,111\end{aligned}$$

$$\begin{aligned}\Delta P &= (\$500,000)(1-.60) - [(.03)(\$10,500,000) - (.02)(\$10,000,000)] - \\ &\quad (.10)(\$336,111) \\ &= \$200,000 - \$115,000 - \$33,611 \\ &= \$51,389\end{aligned}$$

Analysis of the effects of shortening credit terms employs the same formulas. Since the change in sales is negative, the sign of each term is changed.

### II.B. Shorten Credit Terms

For example, change from n/50 to n/30 and the new ACP becomes 40 days.

$$\begin{aligned}\Delta I &= (\text{percentage variable cost})(\text{old collection period})(\text{change in sales})/360 \\ &\quad + (\text{new collection period} - \text{old collection period})(\text{new total sales})/360 \\ &= VC_o \Delta S/360 + (C_1 - C_o) S_1/360\end{aligned}$$

$$\begin{aligned}\Delta P &= (\text{profit on change in sales}) - (\text{change in bad debt losses on} \\ &\quad \text{all sales}) - (\text{opportunity cost of } \Delta I) \\ &= \Delta S(1-V) - (B_1 S_1 - B_o S_o) - k \Delta I\end{aligned}$$

#### New conditions

$$C_1 = 40 \text{ days, } \Delta S = -\$500,000, B_1 = 1\%$$

#### Calculations

$$\begin{aligned}\Delta I &= (.60)(60)(-\$500,000)/360 + (40-60)(\$9,500,000)/360 \\ &= -\$50,000 - \$527,778 = -\$577,778\end{aligned}$$

$$\begin{aligned}\Delta P &= (-\$500,000)(1-.60) - [(.01)(\$9,500,000) - (.02)(\$10,000,000)] - \\ &\quad (.10)(-\$577,778) \\ &= -\$200,000 + \$105,000 + \$57,778 \\ &= -\$37,222\end{aligned}$$

Under the facts assumed, the shorter credit terms causes sales to decline by a magnitude that causes a reduction in profits.

Relaxing the collection period probably involves a reduction in collection efforts and expenses. It causes the average collection period to become longer and involves an increase in the bad debt loss ratio. This is illustrated below.

### I.C. Relax Collection Policy (lengthen ACP)

For example, ACP goes from 60 days to 70 days.

$$\begin{aligned}\Delta I &= (\text{percentage variable cost})(\text{new collection period for all sales}) \\ &\quad (\text{change in sales})/360 + (\text{new collection period} - \text{old collection} \\ &\quad \text{period})(\text{old total sales})/360 \\ &= V \cdot C_1 \cdot \Delta S/360 + (C_1 - C_0) \cdot S_0/360\end{aligned}$$

$$\begin{aligned}\Delta P &= (\text{profit on change in sales}) - (\text{change in bad debt losses on new total} \\ &\quad \text{sales}) - (\text{change in collection expenses}) - (\text{opportunity cost of } \Delta I) \\ &= \Delta S(1-V) - (B_1 S_1 - B_0 S_0) - S_1 \Delta E - k \Delta I\end{aligned}$$

#### New conditions

$$C_1 = 70 \text{ days, } \Delta S = \$100,000$$

$$B_1 = 3\%, \Delta E = -1\%$$

#### Calculations

$$\begin{aligned}\Delta I &= (.60)(70)(\$100,000)/360 + (70-60)(\$10,000,000)/360 \\ &= \$11,667 + \$277,778 = \$289,445\end{aligned}$$

$$\begin{aligned}\Delta P &= (\$100,000)(1-.60) - [(.03)(\$10,100,000) - (.02)(\$10,000,000)] \\ &\quad - (\$10,100,000)(-.01) - (.10)(\$289,445) \\ &= \$40,000 - \$103,000 + \$101,000 - \$28,945 \\ &= \$9,055\end{aligned}$$

Analysis of the effects of tightening collection policy will involve an increase in collection efforts and expenses. It causes the average collection period to become shorter and results in a decrease in the bad debt loss ratio. The formulas are exactly the same as for relaxing collection efforts, but since the change in sales is negative, the sign of each term is changed.

### II.C. Tighten Collection Policy (shorten ACP)

$$\begin{aligned}\Delta I &= (\text{percentage variable cost})(\text{old collection period})(\text{change in sales})/360 \\ &\quad + (\text{new collection period} - \text{old collection period})(\text{new total sales})/360 \\ &= VC_0 \Delta S/360 + (C_1 - C_0) S_1/360\end{aligned}$$

$\Delta P = (\text{profit on change in sales}) - (\text{change in bad debt losses on new total sales}) - (\text{incremental collection expenses}) - (\text{opportunity cost of } \Delta I).$

$$= \Delta S(1-V) - (B_1 S_1 - B_0 S_0) - S_1 \Delta E - k \Delta I$$

New conditions

$$C_1 = 40 \text{ days, } \Delta S = -\$100,000, B_1 = 1\%, \Delta E = 1\%$$

Calculations

$$\begin{aligned} \Delta I &= (.60)(60)(-\$100,000)/360 + (40-60)(\$9,900,000)/360 \\ &= -\$10,000 - \$550,000 = -\$560,000 \end{aligned}$$

$$\begin{aligned} \Delta P &= (-\$100,000)(1-.60) - [(.01)(\$9,900,000) - (.02)(\$10,000,000)] \\ &\quad - (\$9,900,000)(.01) - (.10)(-\$560,000) \\ &= -\$40,000 + \$101,000 - \$99,000 + \$56,000 \\ &= \$18,000 \end{aligned}$$

For the particular relations postulated, profit is increased for both lengthening and shortening the ACP.

In analyzing the effects of discount policy, we will assume a change from net 60 to 2/15, n/60. We assume further that the discount will be taken on 60 percent of credit sales and that the remaining 40 percent takes 60 days to be collected. The average collection period under the new policy will be:

$$C_1 = (.60)(15) + (.40)(60) = 33 \text{ days}$$

In addition, the formulas will reflect the costs of the discounts and the change in the bad debt loss ratio because of the zero losses on the accounts which take the cash discounts.

I.D. Provide Discounts

$$\begin{aligned} \Delta I &= (\text{percentage variable cost})(\text{collection period on new sales})(\text{change in sales})/360 + (\text{new collection period} - \text{old collection period}) \\ &\quad (\text{old total credit sales})/360 \end{aligned}$$

$$= VC_1 \Delta S / 360 + (C_1 - C_0) S_0 / 360$$

$$\begin{aligned} \Delta P &= (\text{profit on change in sales}) - (\text{change in bad debt losses on all sales}) \\ &\quad - (\text{change in revenue due to discount}) - (\text{opportunity cost of } \Delta I) \\ &= \Delta S(1-V) - (B_1 S_1 - B_0 S_0) - (d_1 D_1 S_1 - d_0 D_0 S_0) - k \Delta I \end{aligned}$$

#### New conditions

$$C_1 = 33 \text{ days}, \Delta S = \$500,000, B_1 = 1\%, d_0 = 0, d_1 = .02, D_0 = 0, D_1 = .60$$

#### Calculations

$$\begin{aligned} \Delta I &= (.60)(33)(\$500,000)/360 + (33-60)(\$10,000,000)/360 \\ &= \$27,500 - \$750,000 = -\$722,500 \end{aligned}$$

$$\begin{aligned} \Delta P &= (\$500,000)(1-.60) - [(.01)(\$10,500,000) - (.02)(\$10,000,000)] \\ &\quad - [(.02)(.60)(\$10,500,000) - 0] - (.10)(-\$722,500) \\ &= \$200,000 + \$95,000 - \$126,000 + \$72,250 = \$241,250 \end{aligned}$$

To illustrate the reduction in discount terms, we will relate to the situation in ID after credit terms of 2/15, n/60 have been offered. Now we assume that the terms have been reduced to 2/10, n/60. Under the new terms, 40 percent of the accounts are paid with 10 days and 60 percent are paid within 60 days. Hence the new collection period rises from 33 days to 40 days.

$$C_1 = (.40)(10) + (.60)(60) = 40 \text{ days}$$

Analysis of the effects of reduction in discount terms involves exactly the same formulas again, but the signs of the terms are changed, as shown:

#### II.D. Reduce Discounts

$$\begin{aligned} \Delta I &= (\text{percentage variable cost})(\text{old collection period})(\text{change in sales})/360 \\ &\quad + (\text{new collection period} - \text{old collection period})(\text{expected total sales})/360 \\ &= VC_0 \Delta S / 360 + (C_1 - C_0) S_1 / 360 \end{aligned}$$

$$\begin{aligned} \Delta P &= (\text{profit on change in sales}) - (\text{change in bad debt losses on all sales}) \\ &\quad - (\text{change in revenue due to lower discount}) - (\text{opportunity cost of } \Delta I) \end{aligned}$$

$$= \Delta S(1-V) - (B_1 S_1 - B_0 S_0) - (d_1 D_1 S_1 - d_0 D_0 S_0) - k \Delta I$$

Conditions

$$C_0 = 33 \text{ days}, C_1 = 40 \text{ days}, \Delta S = -\$200,000, B_1 = 1.5\%$$

$$d_0 = .02, d_1 = .02, D_0 = .60, D_1 = .40, S_0 = \$10,500,000, B_0 = 1\%$$

Calculations

$$\Delta I = (.60)(33)(-\$200,000)/360 + (40-33)(\$10,300,000)/360$$

$$= -\$11,000 + \$200,278 = \$189,278$$

$$\Delta P = (-\$200,000)(1-.6) - [(.015)(\$10,300,000) - (.01)(\$10,500,000)]$$

$$- [(.02)(.40)(\$10,300,000) - (.02)(.60)(\$10,500,000)] - (.10)(\$189,278)$$

$$= -\$80,000 - \$49,500 + \$43,600 - \$18,928 = -\$104,828$$

We have now illustrated the logic and numerical computations for relaxing and tightening the four major categories of changes in credit policies.



## Appendix B

A Formal Proof of the Equivalence Between Two Discount Methods

In this appendix we prove more generally the equivalence between discounting on a daily basis and using an annualized discount factor. In the HR equation for the optimal discount rate,

$\left(1 + \frac{k}{365}\right)^{M-N'}$  can be expanded into a binomial series. The general expression is

$$(a+b)^n = a^n + na^{n-1}b + \frac{n(n-1)}{2} a^{n-2}b^2 + \dots + b^n$$

In our analysis,

$$a = 1; b = \frac{k}{365} = \frac{.10}{365} = .000274; n = M-N'; M = 10; N' = 90$$

Hence using the first three terms of the expansion, we find

$$\begin{aligned} \left(1 + \frac{k}{365}\right)^{M-N'} &= 1 + (10-90) \frac{.10}{365} + \frac{(-80)(-81)}{2} (.000274)^2 \\ &= 1 - 80(.000274) + .00024 \\ &= 1 - .022 + .00024 \\ &= .978 + .00024 \\ &= .97824 \end{aligned}$$

Thus only the first two terms of the binomial expansion affect the resulting value to three decimal places. In fact, for small values of  $k$ , we can write approximately

$$\left(1 + \frac{k}{365}\right)^{M-N'} = 1 + \frac{k(M-N')}{365}$$

so that an equivalence is established between the approach which calculates the present value of benefits on a daily basis and the method which uses an annualized discount factor in taking the timing of cash flows into account.

Using the same type of approximation we may also show the equivalence of the two expressions for the maximum profitable discount rate. Indeed, HR's formulation may be written as

$$d_{1m} = 1 - (1+i)^{M-N'} \left(1 - \frac{1}{D_1}\right) - \frac{(1+i)^{M-N}}{D_1}$$

Since  $(1+i)^{M-N'} \approx 1 + (M-N')i$  and  $(1+i)^{M-N} \approx 1 + (M-N)i$ ,

for small  $i$ , the above equation becomes

$$\begin{aligned} d_{1m} &= 1 - [1+(M-N')i] \left(1 - \frac{1}{D_1}\right) - \frac{1 + (M-N)i}{D_1} \\ &= i \left[ N' - M + \frac{N-N'}{D_1} \right] \\ &= \frac{k}{365} \left[ N' - M + \frac{N-N'}{D_1} \right]. \end{aligned}$$

On the other hand, using our approach, the maximum profitable discount rate is the solution of

$$\Delta P = -d_{1m} D_1 S_0 + k [D_1 M + (1-D_1) N' - N] \frac{S_0}{365} = 0,$$

that is,

$$d_{1m} = \frac{k}{365} \left[ N' - M + \frac{N-N'}{D_1} \right],$$

which is an approximation to the discount rate given in HR.