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FESR AND FIXED POLES FOR K^+p ELASTIC SCATTERING*

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ABSTRACT

Assuming that exchange degeneracy is exact, the residue of the Pomeranchukon trajectory for K^+p elastic scattering is calculated from FESR's and the low-energy phase shifts. It is shown that in order to be consistent with some recent results on s-channel helicity conservation for Pomeranchukon dominated processes, at least one fixed pole must be included in the B amplitude, and in particular, this consistency is achieved by introducing a fixed pole at $J = 0$.

I. INTRODUCTION

It has been suggested by Gilman et al.[1] that s-channel helicity conservation might be a general property of all Pomeranchukon exchange amplitudes.

Furthermore, some recent work by Langacker[2] shows that s-channel helicity conservation is very likely to hold approximately at high s for elastic processes, as a simple constraint of parity conservation and unitarity. If one assumes that exchange degeneracy is exact, one has only Pomeranchukon exchange in K^+p elastic scattering, and therefore this process seems to be ideally suited for studying s-channel helicity conservation. We will assume throughout that the Pomeranchukon is a simple Regge pole, and that the trajectory is given by

$$\alpha_P(t) = 1 . \tag{1}$$

The whole analysis has also been carried out using a slope of 0.3 for the Pomeranchukon trajectory;[3] however, the final results are practically identical to those obtained by using Eq. (1). This implies that even if the Pomeranchukon cut is included, our conclusion remains unchanged.

II. ANALYSIS AND RESULTS

A. Amplitudes and FESR

We employ integer moment finite-energy sum rules (FESR) of the standard form.[4] The notation for the kinematics of the reaction $K^+p \to K^+p$ is shown in Fig. 1. The amplitude can be written in terms of the invariant amplitudes A,B as:

$$M(s,t,u) = \bar{u}(p')\{-A(s,t,u) + \frac{i}{2}\gamma\cdot(q+q')B(s,t,u)\}u(p) .$$

Here the differential cross section is given by

$$\frac{d\sigma}{dt} = \frac{1}{64\pi s p^2}|M|^2 .$$

It is a matter of straightforward calculation to obtain the s-channel helicity nonflip and flip amplitudes

$$g_{++} = -\cos\left(\frac{\theta_s}{2}\right)\{2mA + [2E(s)^{\frac{1}{2}} - 2m^2]B\} ,$$

$$g_{+-} = -\sin\left(\frac{\theta_s}{2}\right)\{2EA + 2m\omega B\} , \quad (2)$$

as well as the t-channel helicity nonflip and flip amplitudes

$$f_{++} = \frac{A'}{(t-4m^2)^{\frac{1}{2}}} ,$$

$$f_{+-} = \frac{\phi^{\frac{1}{2}}}{(t-4m^2)^{\frac{1}{2}}} B , \quad (3)$$

where E and ω are the C.M. energies of the nucleon and the kaon respectively and $\phi = 4tp_t^2 q_t^2 \sin^2\theta_t$ is the Kibble function. We have introduced the definition[5]

$$A' = (4m^2 - t)A + m(s-u)B . \quad (4)$$

It is convenient to use the variables $\nu \equiv s-u/4m$ and t. Amplitudes with even or odd behavior under $\nu \to -\nu$ may be formed in an obvious way:

$$A'^{(\pm)}(\nu,t) = \frac{1}{2}[A'(\nu,t) \pm A'(-\nu,t)]$$

$$B^{(\pm)}(\nu,t) = \frac{1}{2}[B(\nu,t) \mp B(-\nu,t)]$$

and one can derive the following sum rules for these amplitudes,[6]

$$\frac{1}{\nu_1^{2n+2}}\int_0^{\nu_1} \nu^{2n+1}\,\mathrm{Im}\,A'^{(+)}(\nu,t)d\nu = \sum_i \frac{a_i^+(t)}{[\alpha_i^+(t)+2n+2]}\left(\frac{\nu_1}{\nu_0}\right)^{\alpha_i^+(t)} \quad (5.a)$$

where the $\alpha_i^+(t)$ are all the Regge trajectories contributing to the amplitude $A'^{(+)}$, and where ν_0 is just a scale factor. Analogously,

$$\frac{1}{\nu_1^{2n+1}}\int_0^{\nu_1} \nu^{2n}\,\mathrm{Im}\,A'^{(-)}(\nu,t)d\nu = \sum_i \frac{a_i^-(t)}{[\alpha_i^-(t)+2n+1]}\left(\frac{\nu_1}{\nu_0}\right)^{\alpha_i^-(t)} \quad (5.b)$$

$$\frac{1}{\nu_1^{2n}}\int_0^{\nu_1} \nu^{2n}\,\mathrm{Im}\,B^{(+)}(\nu,t)d\nu = \sum_i \frac{b_i^+(t)}{[\alpha_i^+(t)+2n]}\left(\frac{\nu_1}{\nu_0}\right)^{\alpha_i^+(t)} \quad (5.c)$$

and

$$\frac{1}{\nu_1^{2n+1}} \int_0^{\nu_1} \nu^{2n+1} \, \mathrm{Im}\, B^{(-)}(\nu,t) d\nu = \sum_i \frac{b_i^-(t)}{[\alpha_i^-(t) + 2n + 1]} \left(\frac{\nu_1}{\nu_0}\right)^{\alpha_i^-(t)} . \qquad (5.d)$$

The amplitude A' is given by $A' = A'^{(+)} + A'^{(-)}$, so that one is tempted to simply add (5.a) and (5.b) to get a sum rule for the full amplitude A'; however, one immediately notices that whereas (5.a) involves an odd moment sum rule, (5.b) involves an even moment sum rule. This problem can be overcome by taking a wrong moment sum rule for one of the amplitudes [say, $A'^{(-)}$] and allowing for the possibility of nonsense-wrong signature fixed poles.[7] We would like to remind the reader that a fixed pole in the partial-wave amplitude at a nonsense value of J with the wrong signature has no effect on the asymptotic behavior of the physical amplitude.

When we write the sum rule for A', we will have contributions from both α_i^+ and α_i^- as well as contributions from the fixed poles, and due to exchange degeneracy the contribution coming from an α_i^+ will be cancelled by the contribution of an exchange degenerate partner α_i^-, except for the Pomeranchukon contribution which is the only one that survives. The sum rule is therefore simplified and it reads

$$S_n \equiv \frac{1}{\nu_1^{2n+2}} \int_0^{\nu_1} \nu^{2n+1} \, \mathrm{Im}\, A'(\nu,t) d\nu = \frac{a_P(t)}{\alpha_P(t) + 2n + 2} \left(\frac{\nu_1}{\nu_0}\right)^{\alpha_P(t)} + \mathrm{F.P.} \qquad (6.a)$$

Here F.P. represents the fixed poles contribution, which we have not written explicitly since we will discuss it later. Similar considerations lead to [wrong moment sum rule for $B^{(-)}$]

$$S_n' \equiv \frac{1}{\nu_1^{2n}} \int_0^{\nu_1} \nu^{2n} \, \mathrm{Im}\, B(\nu,t) d\nu = \frac{b_P(t)}{\alpha_P(t) + 2n} \left(\frac{\nu_1}{\nu_0}\right)^{\alpha_P(t)} + \mathrm{F.P.} \qquad (6.b)$$

It is worth mentioning at this point that the possibility of fixed J-plane poles not associated with the third double-spectral function at right signature points has been proposed by Finkler.[8] These poles do contribute to the asymptotic behavior of the scattering amplitude, and in particular to the real part of the physical amplitude.[9-13] Unfortunately, from our FESR formulas [for instance (6.b)] it is impossible to tell whether F.P. is generated by a wrong signature pole in $B^{(-)}$ or by a right signature pole in $B^{(+)}$.

B. Input

We use as input the low-energy phase-shift analysis of S. Kato et al.[14] (hereafter referred to as "Yokosawa's solution") which seems to be the most recent and accurate at present. In particular, we use their solution I which seems to be favored with respect to solution II.[15] In this solution the $S_{\frac{1}{2}}$ phase shifts are repulsive.

Since attractive $S_{\frac{1}{2}}$ phase shifts have also been proposed in the past, we also performed our analysis using solution II of Ayed et al.[16] (hereafter referred to as "Bareyre's solution"), which is a typical solution of this kind, and has been used by Meyers and Salin in their work on K^+p scattering.[17] This solution, however, does not show the correct threshold behavior, and furthermore is incompatible with forward dispersion relations.[18]

In the case of Yokosawa's solution, a linear interpolation for the phase shifts has been performed between $\delta_\ell = 0$ at threshold and δ_ℓ at $p_{lab} = 0.52$ GeV/c which is the lowest value of the momentum in Yokosawa's analysis. This interpolation is consistent with s-wave dominance and $k^{2\ell+1}$ behavior of phase shifts at low energies, and as a matter of fact it turns out that the contribution from the low-energy part of the integrals in the FESR's is quite small, and therefore the results are insensitive to this interpolation.

The cutoff values used for the upper limits of the integrals in the FESR's are those values of ν corresponding to:

(a) $p_{lab} = 2.53$ GeV/c in the case of Bareyre's solution, and

(b) $p_{lab} = 1.89$ GeV/c in the case of Yokosawa's phase shifts.

C. Results Neglecting the Contribution of Fixed Poles

Throughout this section, we will assume that the fixed poles contribution is negligible in all our sum rules, and we will do our calculations using S_0 and S_0' in (6). To leading order in s, the condition for s-channel helicity conservation is that the amplitude A vanish, or from Eq. (4), that the energy independent dimensionless ratio

$$R \equiv \frac{m(s-u)B}{A'} = 1 \quad . \tag{7}$$

In the case of Bareyre's solution, and for small t, we obtain $R \approx 1$. A plot of R for different values of t is shown in Fig. 2 for Yokosawa's solution. The values of R for Yokosawa's solution are clearly inconsistent with s-channel helicity conservation. As we will show later however, different moment FESR's are not compatible with each other unless one introduces at least one additional J-plane singularity.

D. Results Including One Fixed Pole

If one assumes that there is only one fixed pole in a certain amplitude, one can in principle calculate its position and residue by using different moment sum rules; however, in practice this method is highly unreliable because one has to calculate certain ratios that are very sensitive to small errors in the sum rules.[19] Therefore, we will not attempt to calculate the position and residue of the fixed pole in this fashion.

On the other hand, we can still estimate very roughly the magnitude of the relative contribution of the fixed pole to the sum rule as follows:[20] Assume that the fixed pole has a trajectory ϵ and a residue γ. Let us also assume that $\epsilon \neq 0$. Then, using the fact that $\alpha_P(t) = 1$, from (6) we have

$$S_0' = b^+(t)\frac{\nu_1}{\nu_0} + \frac{\gamma}{\epsilon}\left(\frac{\nu_1}{\nu_0}\right)^\epsilon ,$$

$$S_1' = b^+(t)\frac{\nu_1}{3\nu_0} + \frac{\gamma}{\epsilon+2}\left(\frac{\nu_1}{\nu_0}\right)^\epsilon ,$$

so that the quantity

$$R.C. \equiv \frac{3S_1' - S_0'}{3S_1' + S_0'}$$

should give us a good idea of the relative contribution of the fixed pole to the B amplitude. The same argument holds in the case $\epsilon = 0$, for which

$$S_0' = b_P(t)\left(\frac{\nu_1}{\nu_0}\right) + \gamma , \tag{8}$$

$$S_1' = b_P(t)\,\frac{\nu_1}{3\nu_0} . \qquad (9)$$

By the way, from the above formulas we see that if there is a fixed pole at $J = 0$ in B, one should use the sum rule S_1' and not S_0' in computing b_P. It is now obvious that for A' one has a relative contribution given by

$$R.C. = \frac{3S_0 - 5S_1}{5S_1 + 3S_0} .$$

The results are shown in Fig. 3; and it is evident that there is a negligible relative contribution to A', whereas there is a very sizeable contribution to B, for both sets of phase shifts.

One might interpret this contribution as coming from broken exchange degeneracy; however, the very flat K^+p total cross section indicates that exchange degeneracy holds very accurately for this process (at least at $t = 0$).

An alternative interpretation (which by the way happens to be consistent with Finkler's predictions) is that the additional contribution comes from a fixed pole at $J = 0$ in the B amplitude sum rule; i.e., we use (9) to calculate $b_P(t)$, and combine this with (8) to obtain γ. Once these quantities are known, one can easily calculate R [see Eq. (7)] by using the asymptotic behavior of A' and B in Regge theory. Furthermore, one can calculate the ratio [see Eq. (2)]

$$R' = \frac{g_{+-}/s^{\frac{1}{2}} \sin(\theta_s/2)}{g_{++}/\cos(\theta_s/2)} \times m$$

for any large value of s. We show our results in Figs. 4 - 6. All

the results are shown at the cutoff value of the FESR integrals, and in Fig. 6 we also include a plot of R' for $s = 10\ \mathrm{GeV}^2$. Bareyre's solution is clearly incompatible with s-channel helicity conservation. For Yokosawa's solution, R is seen to be very close to 1, and R' is very small and decreasing as s increases, in agreement with some earlier work on KN scattering.[21] Our results are in sharp contradiction with those of Meyers and Salin after we introduce the fixed pole at $J = 0$. It is now only fair to ask what happens if instead of using (6), one uses a wrong moment sum rule for $A'^{(+)}$ and a wrong moment sum rule for $B^{(+)}$. We find that if one does the calculation in this manner, there is no evidence of a fixed pole at $J = -1$, and furthermore, all our previous results are essentially unchanged.

III. CONCLUSIONS

As we have mentioned before, the repulsive $S_{\frac{1}{2}}$ wave solution seems to be favored at the present time, and therefore we will draw our conclusions from the results given by Yokosawa's solution. We would like to point out that Yokosawa's solution is a typical solution of this kind, and very similar solutions have previously been proposed in the literature.[17] Therefore, we expect our qualitative results to hold not only for Yokosawa's phase shifts, but also for the other similar $S_{\frac{1}{2}}$ repulsive phase-shift solutions.

The FESR's indicate that in order to have s-channel helicity conservation for K^+p elastic scattering, at least one fixed pole must be included in the B amplitude, and in particular, s-channel helicity conservation is achieved by introducing a fixed pole at $J = 0$. Furthermore, there is no evidence of fixed poles in A'. Unfortunately, we cannot test Finkler's prediction since we have no way of determining whether our pole has right or wrong signature; and for that matter, we may even have a combination of both types of poles contributing to our FESR's.

A final remark is in order here. Yokosawa's solution seems to indicate some evidence for an exotic baryon resonance with strangeness +1. If this is the case, and if duality holds for this type of resonance, the fixed pole introduced here may be reinterpreted as one or more very low-lying ordinary Regge trajectories with fairly large residues.

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FOOTNOTES AND REFERENCES

* This work was supported by the U.S. Atomic Energy Commission.

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   Since a detailed discussion of the properties of right-signature fixed poles is outside the scope of this paper, we refer the

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FIGURE CAPTIONS

Fig. 1. Kinematics of the reaction. μ = kaon mass; m = proton mass.

Fig. 2. R as a function of t for Yokosawa's solution I, without including fixed poles.

Fig. 3a. R.C. for B as a function of t.

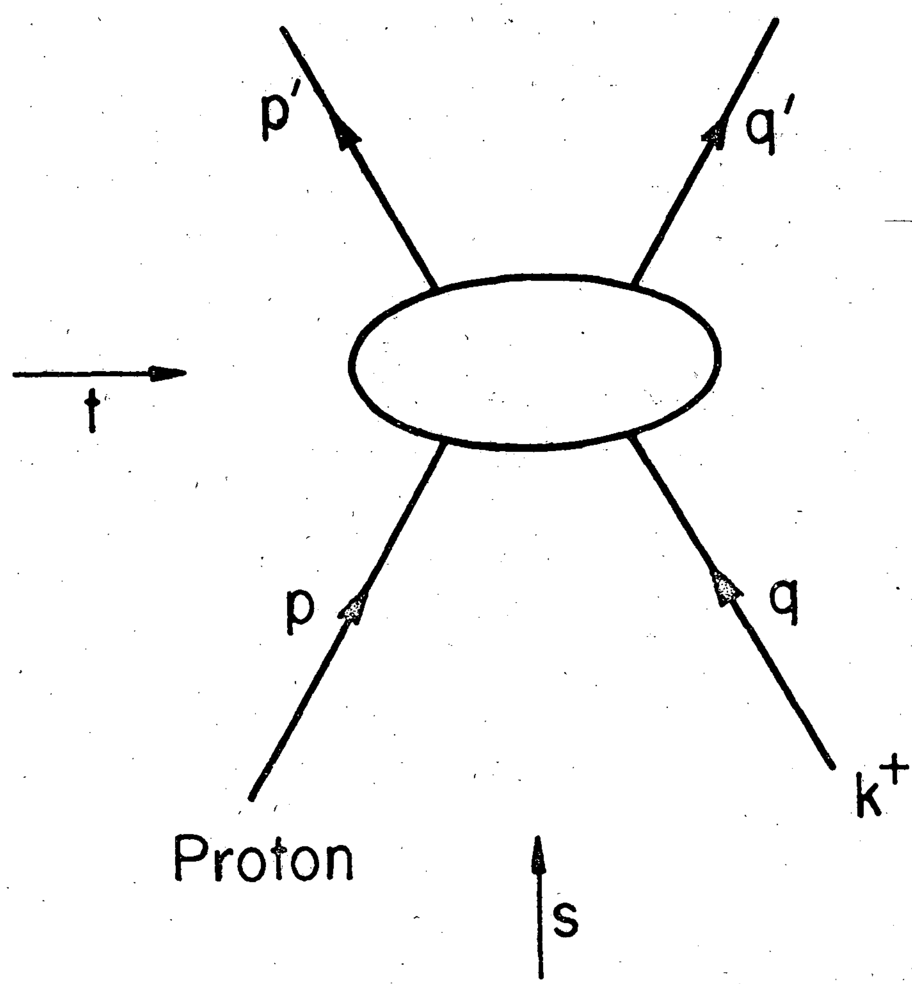
Fig. 3b. R.C. for A' as a function of t.

Fig. 4a. Residue of fixed pole for Bareyres solution.

Fig. 4b. Residue of fixed pole for Yokosawa's solution.

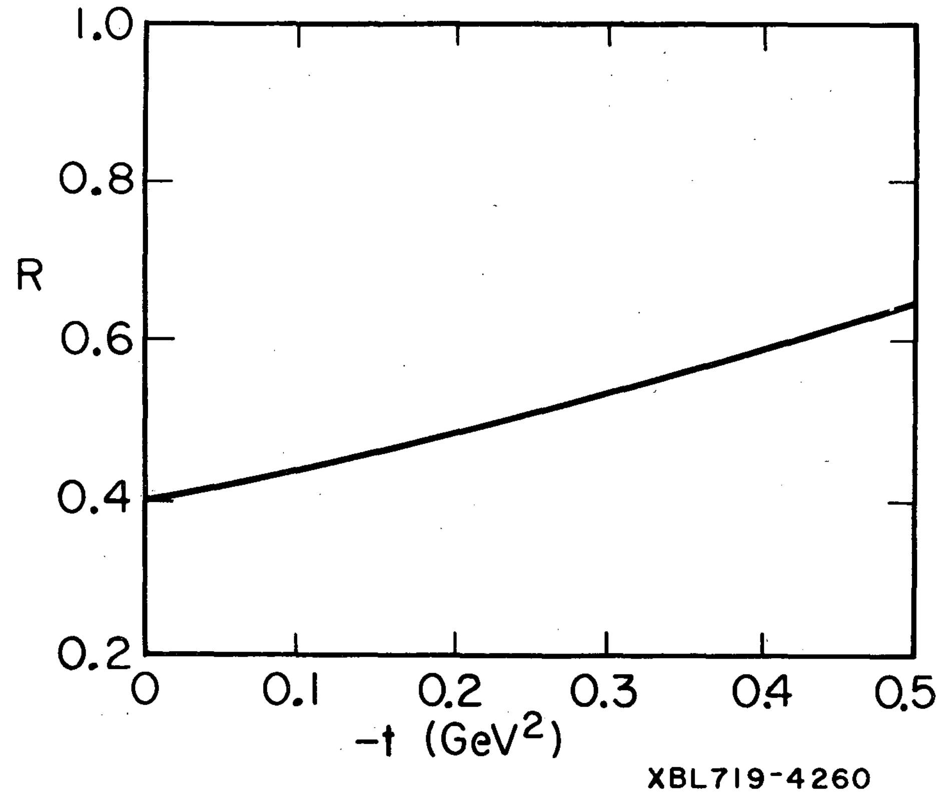
Fig. 5. R as a function of t, including one fixed pole at J = 0.

Fig. 6. R' as a function of t and energy in Yokosawa's solution, including one fixed pole.



XBL719-4258

Fig. 1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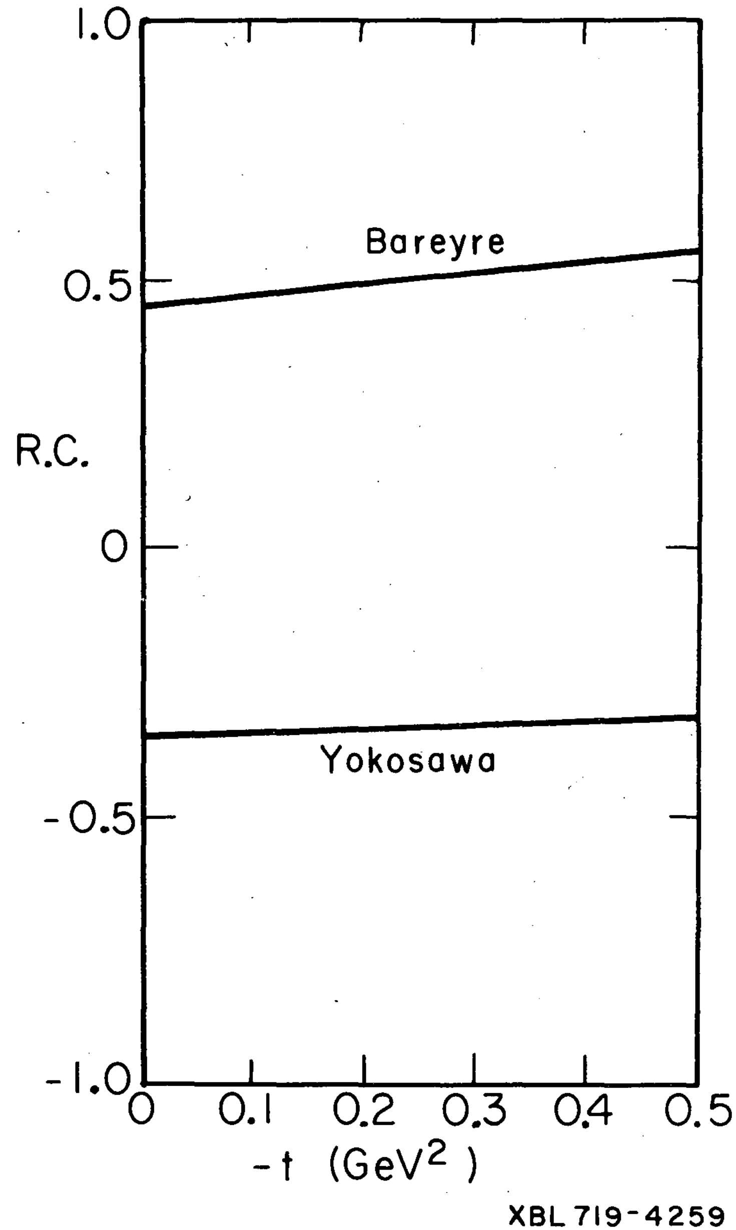
0.8
0.6
0.4
0.2
R
0
0.1
0.2
0.3
0.4
0.5
−t (GeV2)
XBL719-4260

Fig. 2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0.5
0
-0.5
-1.0
R.C.
Bareyre
Yokosawa
0
0.1
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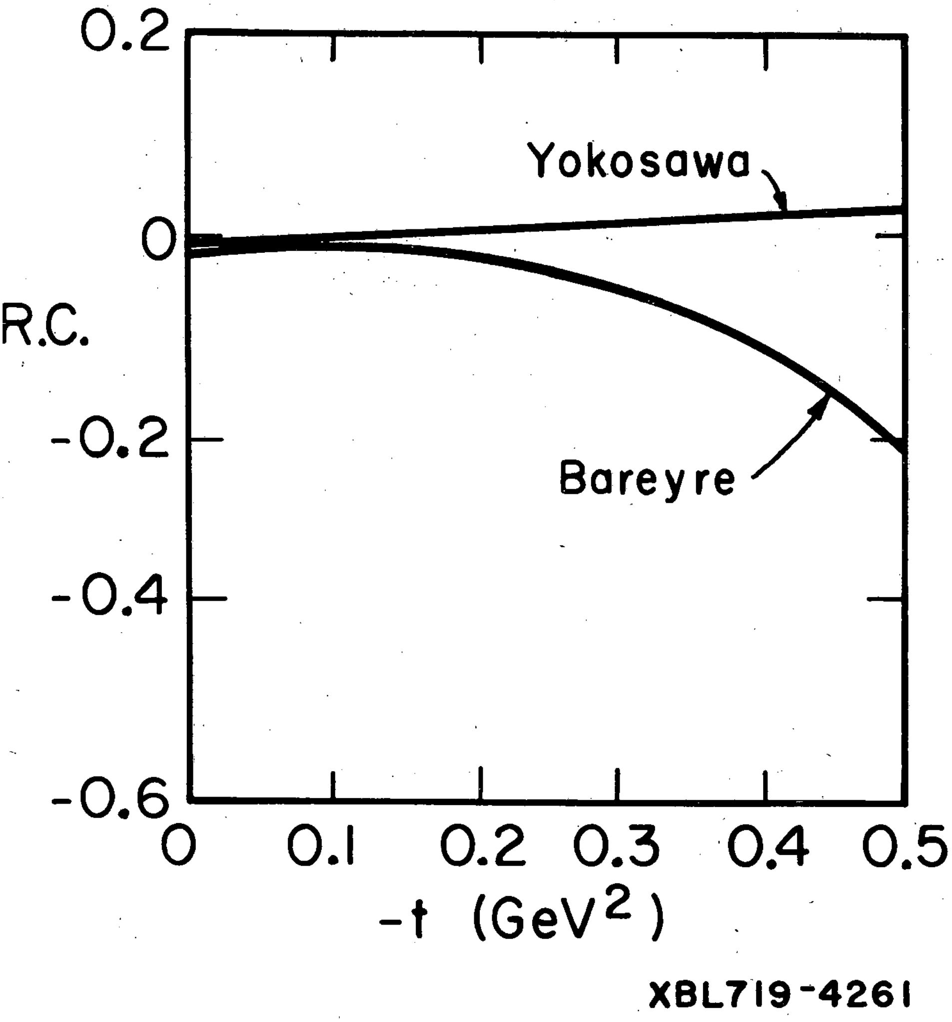
0.3
0.4
0.5
-t (GeV2)
XBL 719-4259

Fig. 3a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-0.4
-0.6
R.C.
Yokosawa
Bareyre
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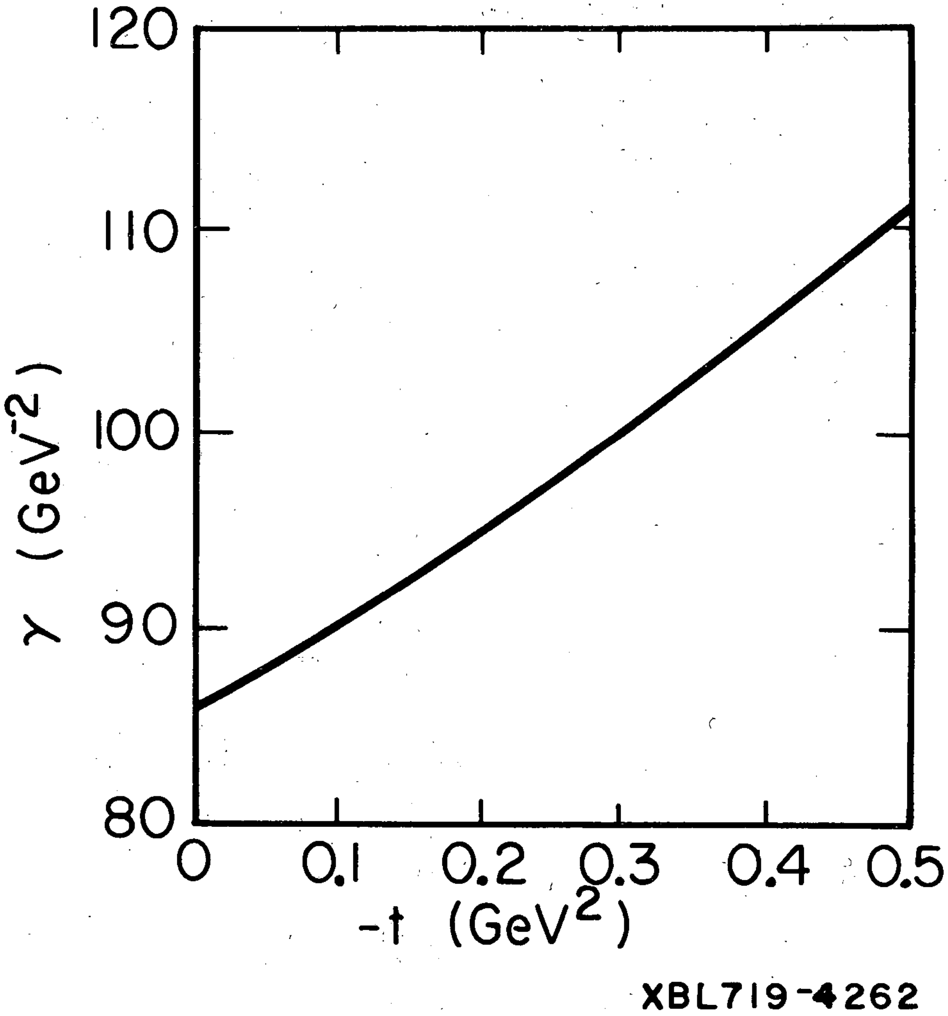
0.3
0.4
0.5
-t (GeV²)
XBL719-4261

Fig. 3b


120
110
100
90
80
γ (GeV⁻²)
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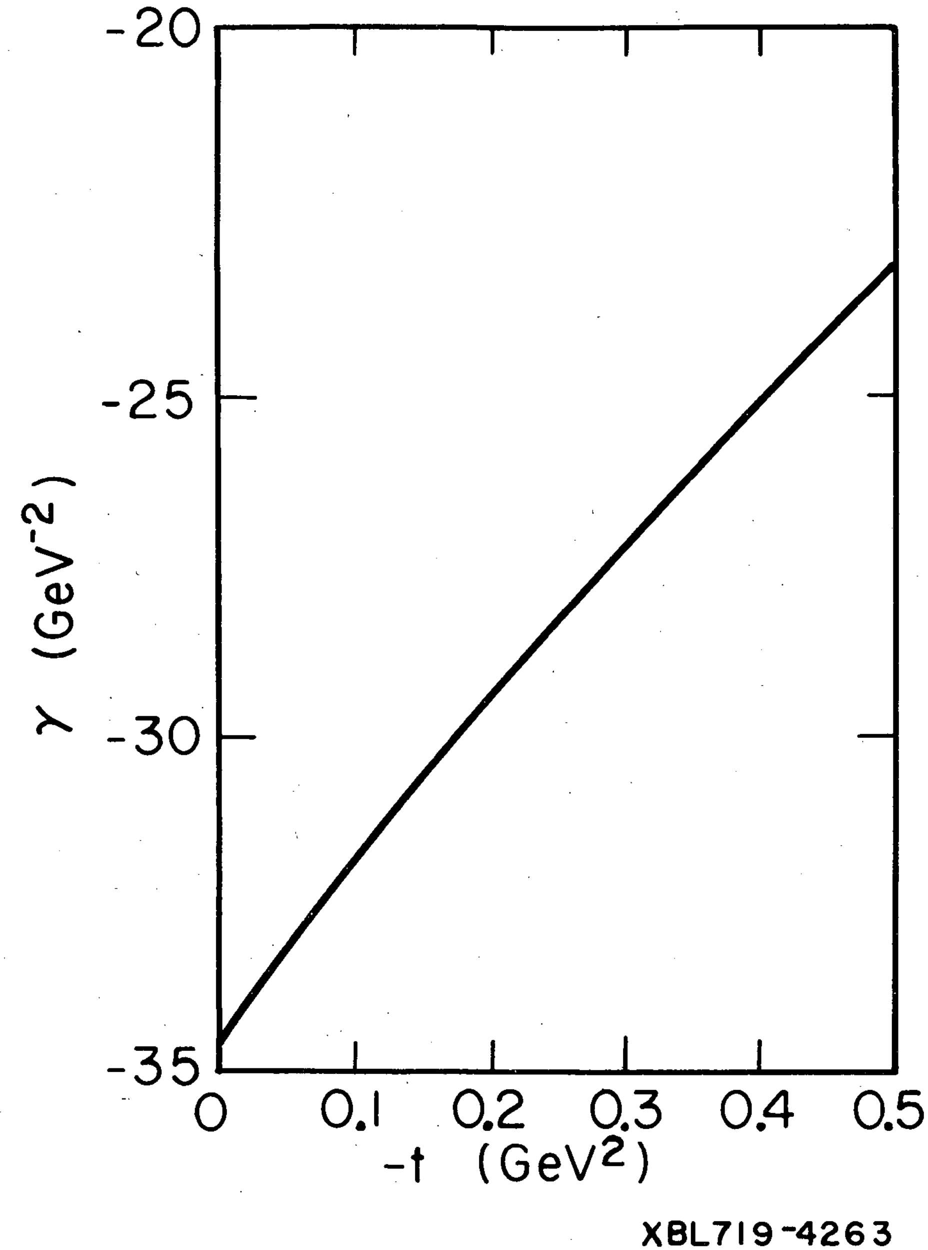
0.2
0.3
0.4
0.5
-t (GeV²)
XBL719-4262

Fig. 4a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-35
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0.4
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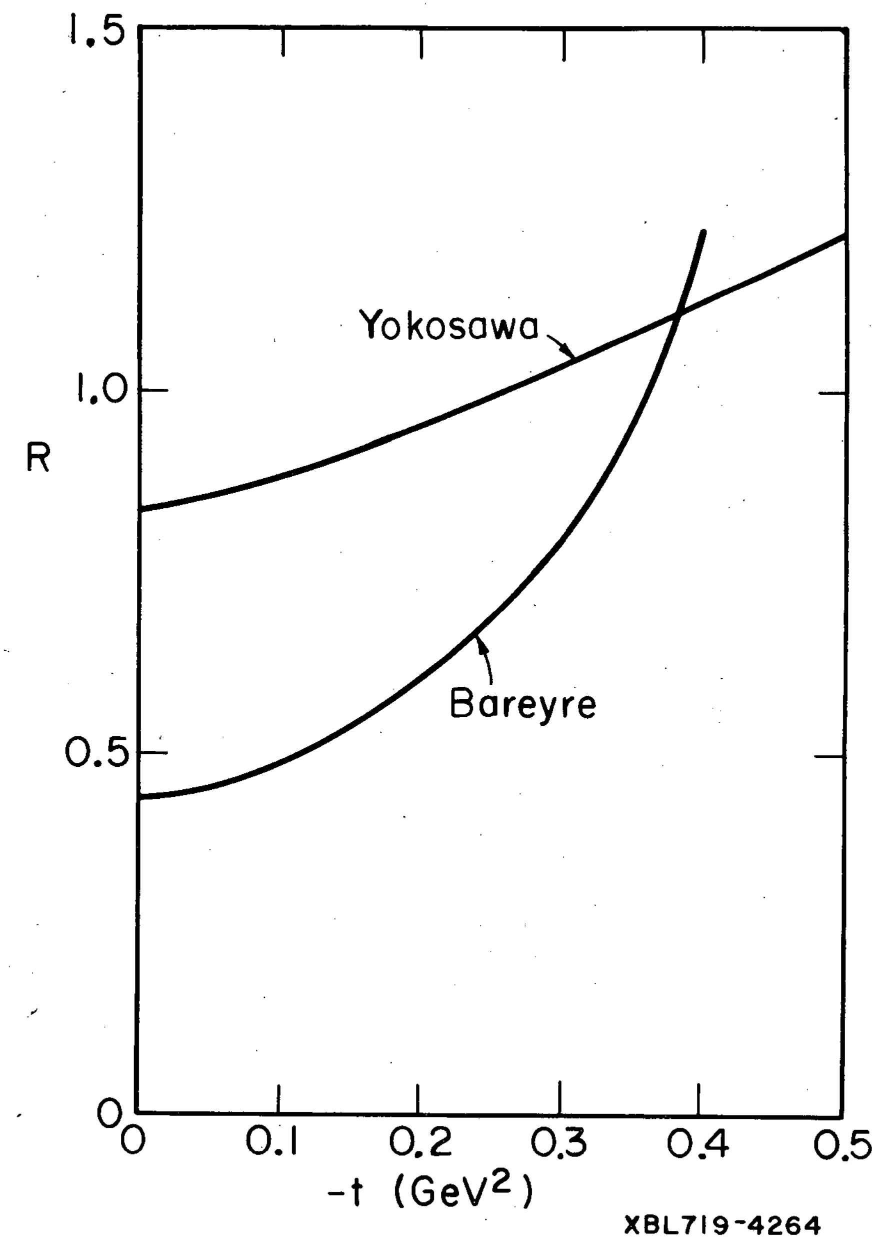
-t (GeV^2)
XBL719-4263

Fig. 4b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1.0
0.5
0
R
Yokosawa
Bareyre
0
0.1
0.2
0.3
0.4
0.5
-t (GeV^2)
XBL719-4264

Fig. 5

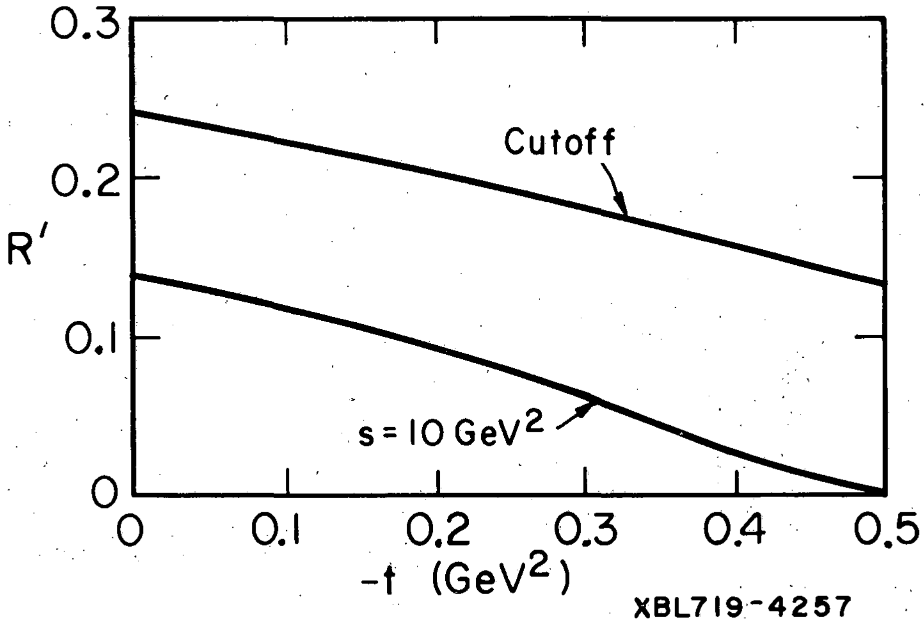

Fig. 6

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