Monograph No. 13

The Production and Cost of a Free Urban Public Service: Resource Allocation in the Oakland Public Library Branch System

Ronald K. H. Choy

August 1970

University of California at Berkeley

FOREWORD

The use of modern analytical methods to study the provision of urban public services and facilities is a relatively recent development in planning and urban studies. Urban services have long been taken for granted. However, as cities' budgetary and social problems have grown, the role of public services in creating and maintaining the quality of life has become evident even as their performance has been called into question. Being heavily labor intensive, their costs have grown faster than those in many other sectors of the economy. Rapidly unionizing municipal employees have exercised the power of their control over vital public services to maintain and in many cases improve their economic status in the face of inflation. Faced with progressively more serious budgetary pressures on services, cities are looking for ways to increase productivity and are reexamining the purpose of functions that seemed self-evidently desirable in easier times.

While the demand for analysis of services has grown, an adequate response faces difficult technical problems. Methods of operations research can answer limited questions such as how to locate and dispatch ambulances and fire trucks. But for larger planning issues, these methods do not provide an adequate analytic structure. For most urban public services, prices are undefined, production functions are difficult to specify, outputs are vague and/or multiple, and it is hard to distinguish supply from demand. This monograph deals with some of these problems through the application of production theory to non-priced public services, using public libraries as an example. Mr. Choy develops the equivalent of a production function for such public goods and uses it to test planning alternatives against an actual proposal using data from an operating public library. The work demonstrates the feasibility of this type of analysis and provides an important contribution toward resolution of the theoretical problems involved. It is a welcome addition to the small but developing literature on analysis of urban service systems.

Michael B. Teitz

THE PRODUCTION AND COST OF A FREE URBAN PUBLIC SERVICE: RESOURCE ALLOCATION IN THE OAKLAND PUBLIC LIBRARY BRANCH SYSTEM

ABSTRACT

RONALD K. H. CHOY

The problem of providing branch library services can be studied at two levels: the production activity of the branch library system as a whole, and the production activity of the individual branch libraries. Chapter 2 studies the former and Chapters 3 and 4 the latter. The chapters are two separate studies that are built around different behavioral assumptions, different models, and different estimation methods. They are similar in that both apply the same economic conditions, both examine the Oakland Public Library branch system, and both use the production function tool. However, the specific kinds of production functions are different. The branch library aggregate production function describes the activity of the system and says nothing about the individual branches. The branch library production function describes the activity of the individual branches and says nothing about the system as a whole.

The main concern of Chapter 2 is with an estimation method of obtaining an aggregate production function. The results are tentative and experimental, but the estimation method is feasible. Using the estimated aggregate production function as an analytical tool, an examination of the Oakland branch library system's resource allocation shows it is employing too many staff services and too few book services. A more detailed examination of staff services shows there are too many professionals and too few

nonprofessionals. One result of this imbalance is that costs are higher than necessary. Reallocating resources away from professional staff toward nonprofessional staff and books would generate some cost savings, which could be devoted to other uses.

The central concern of Chapter 3 is the optimal allocation of resources within the branch libraries in the Oakland Public Library branch system. The analysis of Oakland's branch library activity is done from the point of view of a branch library manager who is in charge of only the economic aspects of providing library services. The focus is on an economic definition of an optimum, the economic conditions that have to be satisfied, and the economic criteria for evaluating alternative allocations.

An optimizing model of the resource allocation process is developed and used to evaluate the current situation. Contrary to the conventional wisdom, branches with a high capacity do not produce relatively more output than branches with a lower capacity so that Oakland's larger branch libraries are not necessarily more efficient than its smaller ones. Furthermore, most branches are employing very nearly optimal input combinations. The combinations are optimal because they minimize cost for the existing capacity levels, but the capacity utilization measurements show that some branches have large excess capacity.

In Chapter 4, the model is used to prescribe and evaluate alternative resource allocations. While this exercise is illuminating, the alternatives have to be considered in light of the limited way in which they were generated. They consider only economic optimality, which is just one of the many aspects of providing branch library services. The analysis clearly indicates that economic optimality cannot solve all of the Oakland Public Library's problems.

ACKNOWLEDGEMENTS

To Professor Michael B. Teitz goes my greatest thanks. Through my years at Berkeley he has guided my intellectual development, kindled my interest in urban public services, introduced me to the Oakland Public Library, and advised and encouraged me on the dissertation research. Generous credit must also go to Professors William Alonso and Leonard A. Merewitz for their guidance, assistance, and suggestions on the technical details and on the broad policy questions of the dissertation.

The study would not have been possible without the generous assistance of Mr. William H. Brett, Oakland's Librarian, and his staff. A special note of thanks and debt of gratitude must go to Miss Lorna Blumen. Her patient and insightful answers to my many questions were invaluable and influential to the development of my ideas about providing library services.

My wife's contributions to all stages of the research are considerable. From the initial study design through the final proof reading Susan's support, assistance, criticism, and encouragement have been indispensable.

Many members of the Department of City and Regional Planning also contributed through discussion and exchange of ideas. In particular, Professor Douglass B. Lee, Jr., and Donald Miller devoted considerable time to reading early drafts and to making many helpful suggestions.

Professor Melvin M. Webber and the staff of the Center for Planning and Development Research generously allowed me to use their facilities. Support by the Department of City and Regional Planning enabled me to use the University of California Computer Center. Finally, thanks are due to Miss Norma Montgomery, who expertly typed the final draft.

TABLE OF CONTENTS

LIST OF TABLES

TABLE Page

LIST OF FIGURES

FIGURE | Page

Chapter 1

PROVIDING URBAN PUBLIC SERVICES: THE OAKLAND PUBLIC LIBRARY BRANCH SYSTEM

Providing Urban Public Services

Today's urbanite routinely consumes many public services without giving much thought to his actions. Nevertheless, his life and well being depend on these services, a number of which have been an integral part of urban life since the beginning of cities. With the growth of urban centers during the past century, the growth of public expenditures to provide the required urban public services has increased dramatically, but financial resources have always seemed to be inadequate to meet the ever increasing demand. Suppliers of the services are in desperate need of help. Extremely large quantities of resources flow through the urban public sector, but there is very little rigorously developed information on the demand for urban public services or their production and cost. Economists have recently taken an active interest in federal governmental expenditures, and have not yet turned comparable attention to local government expenditures. This study of the Oakland Public Library branch system is intended to be a beginning in the developing of this need information.

The main concern of beleagured suppliers is how to provide current services for less money or how to provide more services for the same amount of money. This is an economic question, and this study attempts to respond by using the tools of economic theory. While the economic aspect is only part of the total supply problem, it is a convenient starting point for a subject that has not yet been studied rigorously.* The basic analytical tool for such a study is the production function. Applied to the Oakland Public Library branch system it prescribes resource allocations that are economically optimal, which is of major interest to suppliers. In addition to this, the analysis reveals that a major problem confronting the Oakland Public Library is to determine what kind of services it should provide. Determining how to provide branch library services more cheaply is one thing. Determining what to provide is another. This second, and considerably more difficult, problem is beyond the scope of this study.

Production Function Analysis

The economic notion of a production function has well developed theoretical and empirical foundations. A production function relates inputs to a production process with outputs from that process. It is a rule that expresses the maximum amount of output obtainable from any given combination of inputs. The technology of the process is described by the particular form of the function and by the value of its parameters. While economists have long used production functions to analyze activities of many different sectors of the economy, it is a rare instance to find an application to public services let alone urban public services.** The

* The contribution of economics to military analysis is described by Hitch [7]. There are also examples of how economic analysis can be applied to such seemingly uneconomic problems as crime and punishment [Becker, 1] and freedom [Moore, 9].

**Two examples are a study of the United States Post Office [Merewitz, 8] and a study of British hospitals [Feldstein, 4].

many studies of urban public expenditures that have appeared in the literature in recent years* are informative, but they are only descriptive. They attempt to explain expenditure levels in terms of a whole host of both supply and demand variables. On the other hand, a production function analysis is prescriptive. It can indicate why costs are high or low and what action to take to improve the cost situation. A production function can do this because associated with it is a cost function, which is an integral part of and derived directly from the technological relationships of production. While costs and expenditures are similar, they are not the same.

The Oakland Public Library Branch System**

Oakland's library was founded in 1868 as a private subscription library. After some hard financial times, state enabling legislation allowed the City to accept the Library as a gift in 1878. During its century of history, the Oakland Library has served such notable literary figures as Ina Donna Coolbrith, the first poet laureate of California, Charles Warren Stoddard, Bret Harte,* Jack London, and Gertrude Stein. Since the Library became public, the branch system has grown from five reading rooms to the twenty-two branches listed in Table 1.1. The present West Oakland branch is one of the original reading rooms.

The Oakland Library is currently entering a period of change. A new city charter has just transferred control of the Library from an independent library commission to the city manager. Under the former 1911

---

* Hirsch [6] and Siegel [13] review this literature.

**It should be made clear at this point that this is a study of the branch system and does not include the main library, which is a distincly separate system.

* At one time, Coolbrith, Stoddard and Harte were known as the Golden Gate Trinity.

	Year Opened	Leased or Owned	Comment
Baymont	1964	Leased	Replaced Sheffield
Ina D. Coolbrith	1890	Owned	Carnegie building in 1918
Dimond	1915	Leased	Present site in 1950
Eastmont	1945	Leased	Present site in 1953
Elmhurst	1911	Owned	New building in 1949
Fruitvale	1909	Leased	Present site in 1956
Gibson	1925	Owned	New building in 1929
Glenview	1935	Owned	New building
Golden Gate	1899	Owned	Carnegie building in 1918
Charles S. Greene	1902	Owned	Carnegie bldg; was main until 1951
Lakeview	1930	Owned	New building in 1949
Laurel	1911	Leased	Moved in 1922, again in 1954
Lockwood	1916	Leased	Present site in 1929
Jack London	1887	Leased	New building in 1970
Melrose	1911	Owned	Carnegie building in 1916
Montclair	1930	Owned	New building
Park Boulevard	1878	Owned	New building in 1933
Piedmont	1912	Leased	Present site in 1932
Rockridge	1919	Leased	
Sequoyah-Chabot	1967	Leased	Was Grass Valley station
Temescal	1899	Owned	Carnegie building in 1918
West Oakland	1878	Leased	

Table 1.1 -- The Oakland Public Library Branch System

charter, the city manager controlled the Library's budget while the Librarian reported to the commission. Under the new arrangement, the commission has only an advisory role so that the Librarian is now clearly responsible only to the city manager. This clarification of responsibility and the appointment of a new Librarian have stimulated changes aimed at revitalizing the Oakland Public Library. One proposed change of particular interest is the restructuring of the branch system to provide "efficient service" [Oakland Public Library, 11]. While the notion of efficiency is ambiguous [Hall and Winsten, 5], it appears that lower costs is the object of this change. By reallocating resources within and among branches, the hope is that at least the same level of service can be provided at a lower level of cost.

The results of this study show that cost savings in the Oakland Library branch system are indeed possible. However, the study's narrow focus on only economic aspects tends to exaggerate the extent of these savings. A general pattern of allocating too many resources to staff services and too few resources to book services is found for the system as a whole and also for individual branches. A correction of this imbalance in the input mix would certainly result in some savings, but they would be relatively small because the input mix is already quite efficient.

Another potential source of cost savings is to reduce the excess output capacity of selected branches. Reducing capacity would certainly reduce the amount of excess capacity, but in terms of economic efficiency raising output to reduce the excess is equally justified. While this second alternative does not reduce cost, it is preferable because it increases output. If the Oakland Library branch system could stimulate the demand for its services, it would solve the major problem facing it today.

Changing Supply to Stimulate Demand

The Oakland librarians have known for some time that the demand for library services is lagging badly. This study verifies and quantifies this fact. Attempts to deal with this problem have generally yielded disappointing results, and the librarians are anxiously searching for a successful plan of attack. Their thinking appears to be that they need only change certain supply characteristics to bring about an increase in the quantity of library services demanded. The least difficult characteristics to change are quantities of staff and book service inputs. Other possibilities are input qualities such as the staff's attitude toward different users, the new book selections, and improving the condition and appearance of the buildings. However, these changes are marginal, and even if they do produce a change, the change also would be marginal. The magnitude of the present discrepancy between capacity and output levels at some branches suggests that marginal changes would not make a sizeable dent in the problem. More significant results might be produced by more drastic changes in basic supply characteristics. For example, changing the spatial pattern and the number of branches would affect the accessibility of branch library services. These changes would also affect the availability and quality of services. Even though such a course of action would be difficult to take because of the high setup costs required and the general inertia of the system, the Oakland Library staff is now thinking along these lines [Oakland Public Library, 11]. This kind of change may stimulate demand, but it is not at all certain to do so because there is evidence that the initial assumption that demand will significantly respond to changes in supply characteristics is incorrect.

An alternative approach would start with the assumption that demand is relatively autonomous of supply characteristics and would aim at directly stimulating demand by altering the structure of the users' demand functions. An initial necessary condition is that the Oakland librarians restructure their thinking on the nature of their product and the market they are competing in. After a close examination of the product, the reasons for lagging demand and the appropriate attack become obvious.

The Nature of Oakland's Branch Library System

The most fundamental characteristics of Oakland's library services are that they are public and free. A close examination of Oakland's branch library services does not reveal any economic support for having the public sector provide them. Ownership rights, consumption control, and price exclusion are all possible [Davis and Whinston, 2, and Demsetz, 3], which all indicate that there is no necessary economic reason for branch library services to be provided publicly. Furthermore, exhibiting no jointness in provision, they are not collective consumption goods but are private consumption goods.* They are free because merit

*Notice that Samuelson's [12] definition of a collective consumption good is not invoked as a means of designating branch library services as "public." Samuelson's model is not useful for classifying goods. Instead, the model supplies allocative norms for the provision of any good or service that happens to be provided publicly rather than privately. The conditions define norms for Pareto optimality that allow all goods to be treated as if they were public in a descriptively meaningful sense. Samuelson shows that the market pricing system, with a single unit price want [Musgrave, 10] attributes are imputed to them and public because it is a convenient institutional form for promoting the merit want aspects.

People tend to consume too little of the merit want branch library services because of imperfect knowledge. Because consumption of these services is good for them, people have to be encouraged to consume more. To provide this encouragement, librarians may change consumers' preferences or control the distribution of consumption benefits. The rationale for this kind of encouragement is that the librarians know what is good for the people and that these actions will ensure the people act in their own best interests. By altering the structure of demand functions, preference alteration can eliminate consumers' ignorance due to inadequate product information and correct consumers' irrationality due to incorrect preferences. However, preference alteration is contrary to the great

---

applicable to everyone, would not work because multiple pricing is necessary, and even if trying out multiple prices were possible potential consumers would not reveal their true preferences for the service so that there is no way of determining what to charge each person. This double failure occurs when the service exhibits jointness in provision; that is, when it is a collective consumption good. In the extreme, once an amount of a service is provided for one person, it is provided for all persons. Since different persons have different utility functions, each person will subjectively evaluate the same objective amount of the aspects of the service differently. In order to have efficient resource allocation in this situation, each person is required to pay a different relative price for the service. Thus, jointness in provision requires a multiple pricing system, which the market cannot provide.

Even if an idealized multiple pricing system could be put into effect by an all powerful central body, jointness in provision directly implies the impossibility, in the extreme, of exclusion, which is external economies narrowly defined. No matter what a person pays for the service, he always has the same amount to consume so he is not motivated to reveal his true valuation of the services. Since the central body cannot know each person's utility function, it cannot know what to charge each person.

It is clear that branch library services do not exhibit any more jointness in provision than do any other kinds of goods found in the market. The conclusion is that branch library services have only private goods characteristics.

importance that welfare economics places on consumers' sovereignty, and irrationality is hard to distinguish from true but odd preferences.

In practice, Oakland's librarians have not made concerted efforts to encourage consumption by changing consumers' preferences, but have made some efforts to encourage consumption by controlling the distribution of consumption benefits. Providing branch library services free removes the constraint of the consumers' limited budget. Also, controlling the availability [Teitz, 14] of branch library services to each segment of Oakland's population redistributes real income. A great deal of the apparent economic inefficiency found by this study can be attributed to this redistribution effort. This is not to suggest that the extra cost is unwarranted, only that the cost exists. However, the distributional efforts have not been successful at stimulating the lagging demand for branch library services. It is clear that the Oakland librarians must pay more attention to preference alteration as a means of encouraging the consumption of merit want branch library services.

Changing Demand to Stimulate Demand

The nature of Oakland's branch library services points to the conclusion that the Oakland branch library system must actively compete for consumers if it expects to sell its product. Being a private good is sufficient reason to require competition, and being publicly provided and free are advantages rather than hinderances. If this is not enough, competition is necessary for two more reasons. First, the library no longer has the monopoly on information, education, and recreation that it once had. Second, the printed page no longer has the monopoly in the field of communication that it once had. Professional librarians have already acknowledged the disappearance of these two monopolies, but they have been reluctant to accept the conclusion that libraries must now compete for consumers. It is not surprising that their attempts to adjust to the growing market of substitutable services and media have been largely unsuccessful.

A successful competitive attack must be based on a solid knowledge of the structure of the demand functions for branch library services. In spite of their professional training and expertise, there is a fair amount of evidence that Oakland's librarians do not know the nature of the demand functions they face. A successful competitive attack has to take full advantage of the merit want, private good nature of Oakland's branch library services. For any private good, one of the main ways of actively competing is advertising, which also requires a solid knowledge of the structure of the demand functions. The essence of advertising is preference alteration, which is also a major way of encouraging the consumption of merit wants. If the Oakland branch libraries were seriously committed to the merit want benefits of their product, advertising is the only feasible way of achieving widespread consumption.

Advertising is a proven way to induce people to consume services they would not consume otherwise, and there need not be any sinister connotations associated with preference alteration. Advertising and merit wants can correct preferences with adequate product information so that just publicizing the branch library services currently being provided would be an easy first step. Consumer responses would provide information for possible revisions of the kinds and mix of services. The object of advertising is to convert nonusers into users by giving them information they did not have before on the good chance that the branch library can satisfy a heretofore partially or completely unsatisfied demand.

Passively sitting by and waiting for people to come is not proving to be a good strategy. The results of this study show that some of the required resources to finance a modest advertising campaign are available. Eliminating some input mix inefficiency and reducing some excess capacity would release resources that could be devoted to increasing demand. Most of the conditions for positive results in terms of increasing use already exist in the system. The only missing crucial ingredient is the willingness to take this course of action.

Providing Branch Library Services

The problem of providing branch library services can be studied at two levels: the production activity of the branch library system as a whole, and the production activity of the individual branch libraries. Chapter 2 studies the former and Chapters 3 and 4 the latter. The chapters are two separate studies that are built around different behavioral assumptions, different models, and different estimation methods. They are similar in that both apply the same economic conditions, both examine the Oakland Public Library branch system, and both use the production function tool. However, the specific kinds of production functions are different. The branch library aggregate production function describes the activity of the system and says nothing about the individual branches. The branch library production function describes the activity of the individual branches and says nothing about the system as a whole. Because of their fundamental differences, the two studies should not be expected to yield exactly consistent results. However, because of their fundamental similarities, the two studies should be expected to complement and supplement each other.

A final word of explanation is that the two studies have different objectives. The main concern of Chapter 2 is with an estimation method for obtaining an aggregate production function. Chapter 3 concentrates on the problem of resource allocation. An optimizing model of the resource allocation process is developed and used to evaluate the current situation. In Chapter 4, the model is used to prescribe and evaluate alternative resource allocations. While this exercise is illuminating, the alternatives have to be considered in light of the limited way in which they were generated. They consider only economic optimality, which is just one of the many aspects of providing branch library services. The analysis clearly indicates that economic optimality cannot solve all of the Oakland Public Library's problems.

REFERENCES

1. G. S. Becker, "Crime and Punishment: An Economic Approach," Journal of Political Economy, 76.2 (1968) 169-217.

2. O. A. Davis and A. B. Whinston, "On the Distinction Between Public and Private Goods," American Economic Review, 57.2 (1967) 360-373.

3. H. Demsetz, "The Exchange and Enforcement of Property Rights," Journal of Law and Economics, 7 (1964) 11-26.

4. M. S. Feldstein, Economic Analysis For Health Service Efficiency: Econometric Studies of the British National Health Service, Amsterdam: North-Holland Publishing Company, 1967.

5. M. Hall and C. B. Winsten, "The Ambiguous Notion of Efficiency," Economic Journal, 69.273 (1959) 71-86.

6. W. Z. Hirsch, "The Supply of Urban Public Services," in H. S. Perloff and L. Wingo, Jr. (eds.), Issues in Urban Economics, Baltimore: Johns Hopkins Press, 1968.

7. C. Hitch, "Economics and Military Operations Research," Review of Economics and Statistics, 40.2 (1958) 199-209.

8. L. A. Merewitz, The Production Function in the Public Sector: Production of Postal Services in the United States Post Office, unpublished Ph.D. dissertation, University of California, Berkeley, June, 1969.

9. T. G. Moore, "An Economic Analysis of the Concept of Freedom," Journal of Political Economy, 77.4(1) (1969) 532-544.

10. R. A. Musgrave, The Theory of Public Finance: A Study in Public Economy, New York: McGraw-Hill, 1959.

11. Oakland Public Library, A Survey of the Oakland Public Library, 1968-1969, With Recommendations For a Long Range Master Plan, April, 1969.

12. P. A. Samuelson, "The Pure Theory of Public Expenditure," Review of Economics and Statistics, 36.4 (1954) 387-389.

13. B. N. Siegel, "On the Positive Theory of State and Local Expenditures," in P. L. Kleinsorge (ed.), Public Finance and Welfare: Essays in Honor of C. Ward Macy, Eugene: University of Oregon Books, 1966.

14. M. B. Teitz, "Toward a Theory of Urban Public Facility Location," Papers of the Regional Science Association, 21 (1968) 35-51.

Chapter 2
EXPLORATION OF AN ESTIMATION METHOD: THE BRANCH LIBRARY AGGREGATE PRODUCTION FUNCTION

Method and Problem

Some years ago, Houthakker [3] outlined an estimation method for obtaining an aggregate production function. Very few references have been made to this short article,* and none of these is of an empirical nature. This chapter explores the implementation of Houthakker's idea, and the primary concern is to demonstrate that the method is feasible. Because of the tentative and experimental character of the results, any use of the estimated production function as a decision making tool has to be interpreted in the same spirit.

Houthakker assumes that production within some aggregate is carried on in cells, which may be firms, establishments, machines, or even places. Production within each cell follows fixed factor proportions. Factors are divided into fixed and variable inputs. The variable inputs are available to each cell at common fixed prices, and the fixed inputs are peculiar to each cell. Fixed inputs may represent capital equipment, entrepreneurial ability, locational advantage, or anything, and they need never be aggregated. Although there are fixed proportions within each cell, different cells may have different requirements for the variable factors. At any array of prices for the output and the variable inputs

---

*Levhari [4] showed how the analysis could be reversed, and Solow [6] notes some possibilities of the untried method.

and under competitive assumptions, those cells that can earn nonnegative quasirents after paying the variable inputs will produce at capacity. Houthakker's innovation is to show how output and variable inputs for each cell can be aggregated to yield a production function for the set of cells as a whole.

Using the estimated aggregate production function as an analytical tool, an examination of the Oakland branch library system's resource allocation shows it is employing too many staff services and too few book services. A more detailed examination of staff services shows there are too many professionals and too few nonprofessionals. One result of this imbalance is that costs are higher than necessary. Reallocating resources away from professional staff toward nonprofessional staff and books would generate some cost savings, which could be devoted to other uses. The analysis tends to overstate the extent of the savings because of the inherent limitations of the model and the method of measuring staff services. However, these factors do not necessarily completely nullify the conclusions. The important point is that resource allocation could be improved, and some cost savings are possible.

The next section of the chapter develops a branch library, excess budget maximization version of Houthakker's theorem. Following this, the basic version is extended to facilitate the estimation procedure. The estimated branch library aggregate production function is then applied to the question of optimal resource allocation.

Houthakker's Aggregation Theorem: The Cost Minimization Analogy

Applying the Houthakker aggregate production function estimation method to the Oakland Public Library branch system requires a modification to fit the phenomenon. Houthakker assumes nonnegative profit maximization in the development of his theorem. Since the branch library system "sells" its output at a zero price, and does not try to make a profit, the appropriate analogous assumption is cost minimization. Going a step further, costs are assumed to be minimized subject to a budget limit so that the complete analogy is the assumption of nonnegative excess budget minimization. The analogy is complete in that the formal mathematics of the problem are exactly the same.

The branch library system is composed of a set of individual branches, each of which can be thought of as a production cell. In order to produce one or more outputs, $y_1, \ldots, y_k$, each branch needs two or more inputs, $x_1, \ldots, x_m$. Some of these inputs are variable, $x_1, \ldots, x_n$ $(n < m)$, and the others are fixed, $x_{n+1}, \ldots, x_m$. The set of outputs that each branch produces is determined by demand, which is assumed to be exogenous. The nature of a branch's production activity is determined by the nature of its fixed inputs because it is the fixed inputs that limit the number of production possibilities open to it. Each possibility consists of a combination of inputs that can produce the given outputs, and it is assumed that no output is possible without at least one variable input and one fixed input.

If the production possibilities that each branch actually uses are known, the outputs and variable inputs can be summed over all cells to obtain the total outputs and the total variable inputs for the entire branch system. Since the object is to derive the aggregate production function, the output and variable input totals are of interest. Fixed inputs do not have to be summed because it is the amount each branch has and the distribution among branches that are important. There is no particular reason for wanting the totals, and since fixed inputs are likely not to be measurable it may not even be possible to compute the totals. With fixed inputs limiting the production possibilities of each branch and hence the entire system, not every input vector X is attainable. Some of the vectors that are possible and attainable are more efficient than others.

Definition: An input vector $X = (X_1,...,X_n)$ for the branch library system is efficient if, given the set of outputs $Y_1,...,Y_k$ and the distribution of fixed inputs $x_{n+1},...,x_m$ among branches, X is attainable and there is no attainable $X' = (X'_1,...,X'_n)$ such that $X' \leq X$.

Introducing variable input wage rates, w_i, and the expenditure upper limit, E, the assumption that the excess budget reaches a maximum easily can be proved.

Theorem: If each branch utilizes that production possibility that maximizes its excess budget $E - \sum_{i=1}^{n} w_i x_i$, where E, $w_i > 0$, then the total input vector X is efficient.

Proof: X is attainable. In order to attain another X', some branches have to change their variable inputs by amounts Δx_i. This changes their excess budget to $E - \Sigma w_i(x_i + \Delta x_i) \leq E - \Sigma w_i x_i$. It is not possible that $E - \Sigma w_i(x_i + \Delta x_i) > E - \Sigma w_i x_i$ because this would mean that the budget excess before the change was not at a maximum. Total budget excess for the branch system will be $\sum E-\Sigma w_i(X_i + \Delta X_i) = \sum E-\Sigma w_i X_i' \leq \sum E-\Sigma w_i X_i$. Therefore, it is not possible for $X' \leq X$. At least one of the X_i' must be greater than the corresponding X_i, and, therefore, X is efficient.

Definition: A production function is a function $F(Y_1,\ldots Y_k;X_1,\ldots,X_n)$ that equals one if and only if the input vector is efficient.

For the branch system, the price mechanism can solve the following problem: $\sum E-\Sigma w_i X_i$ is a maximum if $F(Y_1,\ldots,Y_k;X_1,\ldots X_n) = 1$. To make use of this, the production possibilities have to be restricted to insure that the function F is continuous. The required assumptions are that production cells be small and numerous, that production possibilities among cells tend to be different, that indivisibilities arise from only the fixed factors, and that cases in which several production possibilities yield an equal excess budget are so rare that they can be neglected.

The considerable generality of the relationship between the aggregate production function, F, and the production possibilities of the individual branches can be made much more specific by assuming that there is only one output and by confining the discussion to the special case of fixed proportions between a branch's output and variable inputs. Each branch is assumed to follow fixed proportions production activity, but different branches will have different proportions. A key and unusual assumption is that these proportions are random variables from a generalized Pareto density function. It is this assumption, which enables the use of probability density function manipulations, that is the basis of the aggregation procedure that yields a Cobb-Douglas production function for F.

To simplify exposition, assume that there are two variable inputs x_1 and x_2, at least one fixed input, and one output y. For branch j, suppose that v_{1j} units of x_1 and v_{2j} units of x_2 are needed to produce one unit of y. Fixed inputs limit output to o_j units of y. With expenditure limit E_j and prices w_1 and w_2 for the variable inputs, positive unit excess budget $e - w_1v_{1j} - w_2v_{2j}$, $e = E_j/o_j$, implies production to capacity so that o_jv_{1j} units of x_1 and o_jv_{2j} units of x_2 are used. Figure 2.1 shows the values of v_{1j} and v_{2j} for which the excess budget is

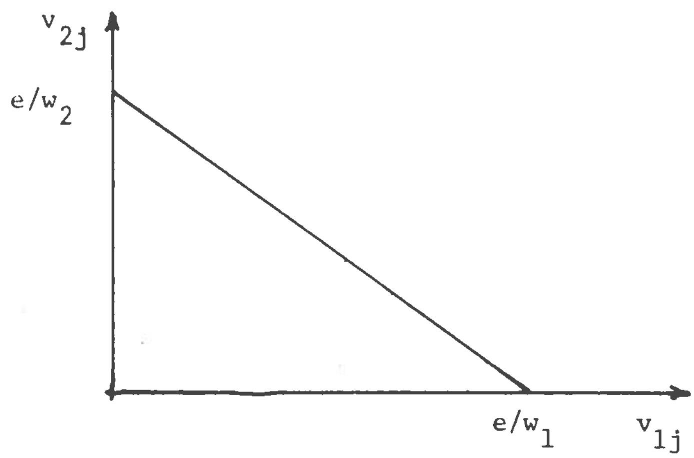

Figure 2.1 -- Positive Excess Budget

positve. At all points below the line $e - w_1v_{1j} - w_2v_{2j} = 0$ the excess budget is positive, and at all points above the line the excess budget is negative.

Assuming the smoothness and continuity conditions for production possibilities given above, suppose that there are $\phi(v_1,v_2)dv_1dv_2$ cells for which v_{1j} lies between v_1 and $v_1 + dv_1$ and for which v_{2j} lies between v_2 and $v_2 + dv_2$. Since each of these cells is producing one unit of output, together they produce a total of $\phi(v_1,v_2)dv_1dv_2$ units of output. The function ϕ is called the <u>input-output distribution</u> for the set of cells, and the explicit form of ϕ is a generalized Pareto type of density function:

$$\phi(v_1,v_2) = Rv_1^{r_1-1} v_2^{r_2-1} \qquad (1 \leqq r_1,r_2).$$

Redefining all prices relative to e and integrating ϕ over all values of v_1 and v_2 for which production permits a positive excess budget yields the aggregate output of the branch system:

$$Y = \int_0^{\frac{1}{w_1}} \int_0^{\frac{1-w_1v_1}{w_2}} \phi(v_1,v_2)dv_2dv_1 = \frac{(1+r_1+r_2)R\beta_{1+r_1,1+r_2}}{r_1\ r_2\ w_1^{r_1}\ w_2^{r_2}},$$

where $\beta_{a,b} = \int_0^1 t^a(1-t)^b dt$ is the beta function.

Similarly for the aggregate variable inputs:

$$X_1 = \int_0^{\frac{1}{w_1}} \int_0^{\frac{1-w_1v_1}{w_2}} v_1\ \phi(v_1,v_2)dv_2dv_1 = \frac{R\ \beta_{1+r_1,1+r_2}}{r_1w_1^{1+r_1}\ w_2^{r_2}},$$

$$X_2 = \int_0^{\frac{1}{w_1}} \int_0^{\frac{1-w_1v_1}{w_2}} v_2\ \phi(v_1,v_2)dv_2dv_1 = \frac{R\ \beta_{1+r_1,1+r_2}}{r_2w_1^{r_1}\ w_2^{1+r_2}}.$$

Solving for w_1 and w_2 from X_1 and X_2 and substituting into Y give a Cobb-Douglas aggregate production function:

$$Y = (1+r_1+r_2)\left[\frac{R\ \beta_{1+r_1,1+r_2}}{r_1^{1+r_1}\ r_2^{1+r_2}}\right]^{\frac{1}{1+r_1+r_2}} X_1^{\frac{r_1}{1+r_1+r_2}} X_2^{\frac{r_2}{1+r_1+r_2}}$$

$$= q\ X_1^{a_1}\ X_2^{a_2}\quad .$$

Extensions and Further Conditions

With three or more inputs, there are more complex constants to deal with. The general expressions for n variable inputs are:

$$X_k = \frac{D}{r_k\ w_k\ \Pi[w_i^{r_i}]} \qquad (k=1,\ldots,s)\ ,$$

$$Y = \frac{(1+\Sigma r)D}{\Pi[r_i\ w_i^{r_i}]}$$

$$= (1+\Sigma r)\ \{\frac{D}{\Pi[r_i^{1+r_i}]}\}^{1/(1+\Sigma r)}\ \Pi[X_k^{r_k/(1+\Sigma r)}]$$

$$= q\ \Pi[X_k^{a_k}]\ ,$$

where Π and Σ are products and sums over all n inputs, and

$$D = R \prod_{i=1}^{n-2} [1 + \sum_{k=i+1}^{n} r_k] \prod_{i=1}^{n-1} [\beta_{1+r_i,\ 1 + \sum_{k=i+1}^{n} r_k}].$$

Before proceeding to estimate the parameters of the Pareto density function, the nature of the function needs to be examined more closely. A complete statement of a single variable Pareto density function is:

$$f(x) = \begin{cases} cA^{-c}x^{c-1}, & 0 \leq x \leq A,\ 1 \leq c<\infty \\ 0, & x < 0,\ A < x. \end{cases}$$

The required condition $\int_0^A cA^{-c}x^{c-1}dx = 1$ easily can be shown to hold.

A further desirable condition is that the density function have a positive finite mean; that is, $m = \int_0^A cA^{-c}x^{c}dx = \frac{cA}{c+1} > 0.$

The condition that $1 \leq c < \infty$ insures $0 < m < \infty$, and the implied limits on A are $m < A \leq 2m$.

Returning to the input-output distribution, the above relationships imply that $R = r_1 R_1^{-r_1} r_2 R_2^{-r_2}$ and that $0 \leq v_1 \leq R_1$, $0 \leq v_2 \leq R_2$. In a probability sense, R_1 is the upper limit on v_1, and R_2 is the upper limit on v_2. In an economic sense, $1 - w_1v_1 - w_2v_2$ defines the upper limits on v_1 and v_2. There seems to be no necessary relationship between these two notions. Finally, it is a routine exercise to check that all of these relationships and conditions hold for any number of inputs.

Estimation Procedure and Sampling Properties

Two standard methods for estimating the parameters of a probability density function are the method of moments and maximum likelihood. Because of superior efficiency properties, maximum likelihood estimates are usually preferable. Assuming that a set of s branches utilizes n variable inputs, the maximum likelihood equation is:

$$L^* = \prod_{j=1}^{s} \phi(v_{1j},\ldots,v_{nj};\ R_1,\ldots,R_n;\ r_1,\ldots,r_n) = \prod_{i=1}^{n} [r_i^s\ R_i^{-sr_1} \prod_{j=1}^{s} v_{ij}^{r_i-1}]$$

Applying a logarithmic transformation to make L* linear,

$$L = \ln L^* = \sum_{i=1}^{n} [s \ln r_i - s\ r_i \ln R_i + (r_i - 1) \sum_{j=1}^{s} \ln v_{ij}] .$$

Maximizing L with respect to each r_i is equivalent to maximizing L* with respect to each r_i, but being linear, L is easier to manipulate than L*:

$$\frac{dL}{dr_i} = \frac{s}{r_i} - s \ln R_i + \sum_{j=1}^{s} \ln v_{ij} = 0 \quad (i = 1,\ldots,n) .$$

Solving for r_i,

$$r_i = \frac{s}{s \ln R_i - \sum \ln v_{ij}} \qquad (i = 1,\ldots,n) ,$$

which is the maximum likelihood equation for r_i. Unfortunately, the R_i are also unknown, and no maximum likelihood estimate exists for them since $dL/dR_i = 0$ implies that $R_i \to \infty$. A way out of this dilemma is to use the relationship between the mean, m_i, and the upper limit, R_i, and substitute $R_i = m_i(1+r_i)/r_i$ $(i = 1,\ldots,n)$ into the likelihood function:

$$L^* = \prod_{j=1}^{s} \phi(v_{1j},\ldots,v_{nj};\ m_1,\ldots,m_n;\ r_1,\ldots,r_n)$$

$$= \prod_{i=1}^{n} [r_i^{s(1+r_i)}\ (1+r_i)^{-sr_i}\ m_i^{-sr_i} \prod_{j=1}^{s} v_{ij}^{r_i-1}]$$

Again linearizing L* and maximizing with respect to each r_i,

$$L = \ln L^* = \sum_{i=1}^{n} [s(1+r_i)\ln r_i - sr_i \ln(1+r_i) - sr_i \ln m_i + (r_i-1) \sum_{j=1}^{s} \ln v_{ij}],$$

$$\frac{dL}{dr_i} = \frac{s}{r_i} + \frac{s}{1+r_i} + s\ln r_i - s\ln(1+r_i) - s\ln m_i + \sum_{j=1}^{s} \ln v_{ij} = 0 \quad (i=1,\ldots,n).$$

A value for r_i that will satisfy this nonlinear expression can be obtained by using a simple numerical analysis technique such as the Newton-Raphson method:* $r_{(n+1)} = r_{(n)} - F[r_{(n)}] / F'[r_{(n)}]$, where $r_{(n)}$ is the nth iteration estimate of r, and $F[r] = \frac{dL}{dr} - r$. Estimates of m_i are obtained from the method of moments because no maximum likelihood estimates exist for them since $dL/dm_i = 0$ implies that $m_i \to \infty$. In order to maximize L, the values of R_i $(i = 1,\ldots,n)$ must be as small as possible. Since $m_i < R_i \leqq 2m_i$, $R_i = m_i$ is the lower limit. Substituting in the corresponding maximum likelihood equation provides a starting value for the iterative solution of r_i:

$$r_{i(0)} = \frac{s}{s\ \ln m_i - \sum \ln v_{ij}} \quad .$$

The sampling properties of the estimates of r_i are not difficult to determine. Checking that further conditions on the existence of derivatives and expected values of the density function are satisfied [Fisz, 2, p. 487], as $s \to \infty$ the maximum likelihood estimate of r_i has an asymptotically normal distribution $N(r_i;\ r_i/\sqrt{s}\ )$, where

$$\frac{r_i}{\sqrt{s}} = \frac{1}{\sqrt{sE[(\frac{\partial \ln\phi}{\partial r_i})^2]}} \quad ,$$

*See any numerical analysis text such as McCracken and Dorn [5, pp. 133-135].

$$E[(\frac{\partial \ln \phi}{\partial r_i})^2] = \int_0^{R_1},\ldots,\int_0^{R_n} (\frac{\partial \ln \phi}{\partial r_i})^2 \quad \phi(v_1,\ldots,v_n)dv_n,\ldots,dv_1 = \frac{1}{r_i^2} .$$

Branch Library Output and Input Services

The first step in the empirical part of this exploration is to define a production cell in such a way that the assumption of fixed proportions production activity is valid. The notion of a branch library production cell can be made more definite by limiting the time period of operation to one month. Thus, a production cell is a branch library operating for one month. Applying this definition to the activities of nineteen Oakland Public Library branches during fiscal year 1968-1969 yields 222 cells. The aggregation will be over branches and over months so that the aggregate production function will describe the production activities of the branch system for one year.

Measuring the output of a free, public, multiservice facility is admittedly a speculative undertaking, and the public branch library is no exception. Converting the different services to a common unit is an even more speculative undertaking. The usual practice is to use output prices as conversion weights, but no explicit nonzero prices exist for the branch libraries' services. Neither do the branch libraries have an explicit set of self-determined weights that could serve as price surrogates. However, it should be possible to devise such a set of weights, which most critics would find not objectionable, so that output could be measured as a linear combination of the individual services.

The most important output of branch libraries is circulation. More specialized services such as answering reference questions that require extensive research and a collection of items on some unique subject are provided by the main library and not the branches. Work with children is another set of branch library outputs. It includes story hours, visits by school classes, and visits to school classes. Finally, just being an open, free, public building with tables, chairs, and a collection of books and magazines generates outputs. The branch library is a place where indoor activities such as studying and browsing can go on. In this respect, it is much like a free public park where corresponding outdoor activities can go on because the park is open and available for use.

Expert professional librarian judgment is not opposed to fixing circulation at about eighty per cent of total branch library output. The remaining twenty per cent is accounted for by the remaining two output sets. Any given branch can expect its output mix to vary from month to month, and the monthly average mix can be expected to vary from branch to branch, but the system wide total output is split at about eighty to twenty. Since circulation is so dominant, it would be most convenient to convert the other outputs to circulation equivalents by applying suitable weights, which are derived by again resorting to professional judgment. Each child attending a story hour is treated as being equivalent to two books circulated, and each class visit and class visited is treated as being equivalent to twenty-five books circulated. Finally, each hour a branch is open is treated as being equivalent to three books circulated. To summarize, the single measure of branch library output is circulation equivalents, and it is a linear combination of circulation, attendance at story hours, class visits, classes visited, and hours open:

circulation equivalents = circulation + 2 x attendance at story hours + 25 x class visits + 25 x classes visited + 3 x hours open.

Circulation equivalents measure quantity of output, but not all of the quantity. Circulation only counts the books borrowed and not the length of time each book is out. Hours open is only a very rough proxy for other consumption activities, and no allowances are made for length of activity time and body count variations. The output measure also does not differentiate among users. One person taking out ten books and ten persons taking out one book each would both count as ten. A more serious deficiency is that no allowances are made for quality variations. The output measure counts only undifferentiated successes and no failures. The times a user fails to find what he wants, finds a substitute, or finds a complement are not treated as separate outputs. The reason for ignoring these quality factors is the lack of adequate data and not the lack of adequate measures.

One of the major variable inputs into the branch library production process is staff services. The branches use some seven different staff classifications, and these can be conveniently grouped into three categories. Supervising and senior librarians are senior professionals; junior librarians, educational assistants, and library assistants are professionals; and intermediate typist clerks and pages are nonprofessionals. The level of staff services that a branch employs is determined by dividing that branch's total wages and salaries for each category by the corresponding average wage rate. This measures the quantity-quality of equivalent man-months for each category of staff services, and it assumes that a staff member's wage rate accurately reflects his productivity.

The other major variable input into the branch library production process is book services. Each branch has a stock of books that provides a flow of book services. The flow at any branch is determined by the amount of time the stock is available for use, which is equal to the amount of time the branch is open. The possession of a book does not by itself input a unit of book services into the production process. The book must be available, which means that the branch at least must be open. The flow of book services is measured by the utilization rate of the book stock, and the utilization rate is measured by the actual number of hours the branch is open as a proportion of the reasonable maximum number of hours it could be open. This reasonable maximum is 325 hours per month, which is based on twelve hours a day six days a week and four hours on Sunday. Thus, the unit of book services is book-months.

A minor variable input into the branch library production process is furniture services. The stock-flow characteristics of this input permit it to be treated in a manner similar to book services. However, additions to and deletions from the stock are not made on a regular period to period basis so that the stock levels are actually fixed. This is not an inherent characteristic, but it does make for an uninteresting variable input. Since no current resources are allocated to it and since it is so similar to book services, confusion and unnecessary detail are lessened by leaving this unimportant input out of the analysis.

The major fixed input that limits branch library output capacity is the building. Its location, physical size, condition, and appearance are difficult to significantly alter except in the very long-run. For the

Oakland Public Library, it could be argued that the stock of librarians is really a fixed input because they are civil service positions and it is difficult to alter the stock in the short-run.* While this is certainly true for the entire library system as a whole, it is not true for a particular branch library. The stock of librarians at any branch can and does vary in response to changes in output levels so that it is definitely a variable rather than fixed input.

Summarizing the branch library output and input services,

Q = circulation equivalents of output,

S = equivalent man-months of staff services,

SP = equivalent man-months of senior professional staff services,

P = equivalent man-months of professional staff services,

N = equivalent man-months of nonprofessional staff services,

B = book-months of book services,

U = unaggregated fixed inputs.

(See Appendix 1 for a listing of the basic data, Appendix 2 for a listing of the random variables, and Appendix 5 for a glossary of variable names.

Two Specifications

The measurement of staff services suggests two specifications for the production process. Each category of staff services can be treated as a separate variable input, or all staff services can be aggregated into one input, which is measured by dividing each branch's total salaries

---

*Eads, Nerlove, and Raduchel [1] similarly argue that a local service airline firm's stock of pilots and copilots is a fixed factor in the short-run.

by an average wage rate. The single measure implies that each category of staff services is a perfect substitute for any other category. This is certainly not absolutely true, but it is instructive to consider this very simple specification. For the most part, attention is focused on staff and book services because these inputs figure prominently in current expenditures. Furniture stocks, being long-lived and very durable, are not replenished regularly and are less important as far as the allocation of regular expenditures is concerned. This secondary treatment does not mean that these inputs are not important, but that not too many useful insights are gained by devoting much attention to them during this exploratory attempt at unraveling the operations of the branch system. With this in mind, the two specifications of output as a function of staff and book services will only differ in the treatment of staff services.

The initial basic computations are shown in Table 2.1. The Newton-

Input i	$m_i = \sum_j v_{ij}/s$	$\sum_j \ln v_{ij}$
Senior Professional	2.44×10^{-4}	-1954.6
Professional	2.94×10^{-4}	-1917.5
Nonprofessional	1.16×10^{-4}	-2057.7
Staff Services	6.68×10^{-4}	-1648.0
Book Services	2.04	138.5

Table 2.1 -- Initial Maximum Likelihood Computations For Estimating The Pareto Density Function Parameters

Raphson solutions for the r_i in Table 2.2 take about six iterations before the values change only beyond the twelfth decimal place so that conversion to a stable value is fairly rapid. (See Appendix 3 for complete iteration results.) Finally in Table 2.3, the parameters of the

Input	Initial Values		Final Values		Standard Error
i	R_i	r_i	R_i	r_i	$\sqrt{r_i}/\sqrt{222}$
Senior Professional	2.44×10^{-4}	2.05	3.88×10^{-4}	1.70	.34
Professional	2.94×10^{-4}	1.98	4.73×10^{-4}	1.64	.33
Nonprofessional	1.16×10^{-4}	4.78	1.45×10^{-4}	3.99	.42
Staff Services	6.68×10^{-4}	8.91	7.69×10^{-4}	6.60	.67
Book Services	2.04	11.14	2.31	7.61	.72

Table 2.2 -- Newton-Raphson, Maximum Likelihood Estimates of the Pareto Density Function Parameters.

two aggregate production functions are easily computed from the density function parameters. The sum of the variable input elasticities is less than one. Since the aggregate production function must be homogeneous of degree one, the residual elasticity must be allocable to the unaggregated fixed factor that limits output. Inserting the estimated elasticities and constant and the actual aggregate variable input levels into the aggregate production function expression, the computed value for aggregate output is less than the actual amount. In other words, variable inputs account for only some of the output. The "missing" factor accounts for the remaining output. Houthakker [3, p. 31] notes that it is not strictly correct to include a term for this limiting input in the aggregate production function because its distribution among cells is an

	Name	Single Staff Specification	Three Staff Specification
Elasticities			
Senior Professional	a_{SP}	-	.107
Professional	a_P	-	.103
Nonprofessional	a_N	-	.250
Staff Services	a	.434	-
Book Services	b	.500	.477
Fixed Inputs	u	.066	.063
Constant	q	37.194	86.216
Proportion of Output Accounted For By Variable Inputs	-	.761	.827

Table 2.3 -- The Branch Library Aggregate Production Function Parameters

essential element of the problem, and it is not legitimate to aggregate fixed inputs.

Branch Library System Resource Allocation

The aggregate production function provides a means of examining the Oakland Library branch system's resource allocation among inputs. In order to do this, the function's role in the excess budget maximization model has to be made more explicit. Recasting the problem into a simple Lagrange function simplifies the exposition. The problem is to maximize the excess budget subject to the condition that output capacity is sufficient to meet demand. Since the budget excess is maximized if and only if costs, C, are minimized,* the problem can be restated as a constrained cost minimization one. The Lagrange functions, G, are

$$G(S,B;g) = C + g[Q - F(S,B)], \text{ where } F = qS^{a}B^{b} ;$$

$$\text{or } G(SP,P,N,B;g) = C + g[Q - F(SP, P, N, B)], \text{ where } F = qSP^{a_{SP}}P^{a_P}N^{a_N}B^{b} ;$$

where g is the Lagrange multiplier. The interpretation of g is the extra cost incurred by a one unit increase in output, which is marginal cost.

The first order conditions for a saddle point minimum of the Lagrange functions are**

$$G_S = w - gF_S \geqq 0,$$

$$G_B = p - gF_B \geqq 0,$$

$$G_g = Q - F \leqq 0,$$

$$SG_S + BG_B + gG_g = 0;$$

or

$$G_{SP} = w_{SP} - gF_{SP} \geqq 0,$$

$$G_P = w_P - gF_P \geqq 0,$$

$$G_N = w_N - gF_N \geqq 0,$$

$$G_B = p - SF_B \geqq 0,$$

$$G_g = Q - F \leqq 0,$$

$$SPG_{SP} + PG_P + NG_N + BG_B + gG_g = 0.$$

---

*$C = wS + pB$ or $C = w_{SP}SP + w_PP + w_NN + pB$.

**Partial derivatives of functions are denoted by the name of the function subscripted by the argument with respect to which the differentiation is being taken.

An extra data requirement for this examination is the prices of the variable inputs shown in Table 2.4. Wage rates for the different staff inputs are readily available and present no problem. The book service price is borrowed from Chapter 3, where its derivation is fully discussed.

Input	Name	Wage/Month
Senior Professional	w_{SP}	800.28
Professional	w_P	613.30
Nonprofessional	w_N	356.18
Staff Services	w	624.47
Book Services	p	.06

Table 2.4 -- Variable Input Prices*

The optimal relative amounts of each input can be determined by comparing actual marginal products with corresponding price ratios. Considering first the simple specification, the condition for an optimal input mix is $p/w = F_B/F_S = bS/aB$. Inserting the appropriate values,

$$.953 \times 10^{-4} = .06/624.47 \overset{?}{=} .500 \times 670/.434 \times 2194055 = 3.52 \times 10^{-4} .$$

The condition clearly is not satisfied. The conclusion is that staff services are relatively too high or book services are relatively too low. Disaggregation of staff services reveals a slightly different pattern. In Table 2.5, the lower triangle contains the ratios of the row variable price to the column variable price. The upper triangle contains the ratios of the column variable marginal product to the row variable marginal

*See Appendix 4 for computations.

Price Ratio \ Marginal Product Ratio	Senior Professional	Professional	Nonpro- fessional	Book Services
Senior Professional	--	.81	4.09	4.91×10^{-4}
Professional	.77	--	5.06	6.07×10^{-4}
Nonprofessional	.44	.58	--	1.20×10^{-4}
Book Services	$.74 \times 10^{-4}$	$.97 \times 10^{-4}$	1.67×10^{-4}	--

Table 2.5 -- Relative Input Levels Are Not Optimal

product. The conclusions, summarized in Table 2.6, show that the level of senior professional staff services is uniformly relatively too high, and the level of nonprofessional staff services is uniformly relatively too low. The

	Senior Professional	Professional	Nonprofes- sional	Book Services
Senior Professional	---	SP too high	SP too high	SP too high
Professional	P too low	---	P too high	P too high
Nonprofessional	N too low	N too low	---	N too low
Book Services	B too low	B too low	B too high	---

Table 2.6 -- Relative Input Optimal Changes

level of professional staff services is relatively too high, and the level of book services is relatively too low.

While this examination of the input mix is informative, it can only deal with relative amounts of inputs and not absolute amounts. The optimum levels of each input can be determined by solving the model for the reduced form, which expresses each endogenous variable as a function of only exogenous variables. However, this is not strictly possible with this aggregate production function because it is not completely specified. The unaggregated fixed inputs enter into the function in some unknown manner, which also makes the complete production function unknown. However, if fixed inputs are assumed to enter the production function in a manner similar to the variable inputs, they can be treated as a constant that is just "carried" around in the same way as the constant q. Taking this approach makes the derivation of the reduced form a simple matter.

$$D^S(Q,w,p) = \left[\frac{Q}{q^*} \left(\frac{w}{a}\right)^{a-s} \left(\frac{p}{b}\right)^b \right]^{1/s},$$

$$D^B(Q,w,p) = \left[\frac{Q}{q^*} \left(\frac{w}{a}\right)^{a} \left(\frac{p}{b}\right)^{b-s} \right]^{1/s},$$

$$H(Q,w,p) = s\left[\frac{Q}{q^*} \left(\frac{w}{a}\right)^{a} \left(\frac{p}{b}\right)^{b} \right]^{1/s},$$

$$M(Q,w,p) = \left[\frac{Q^{1-s}}{q^*} \left(\frac{w}{a}\right)^{a} \left(\frac{p}{b}\right)^{b} \right]^{1/s},$$

where $s = a + b$, and $q^* = qU^u$; or

$$D^{SP}(Q,w_{SP},w_P,w_N,P) = \left[\frac{Q}{q^*} \left(\frac{w_{SP}}{a_{SP}}\right)^{a_{SP}-s} \left(\frac{w_P}{a_P}\right)^{a_P} \left(\frac{w_N}{a_N}\right)^{a_N} \left(\frac{P}{b}\right)^b \right]^{1/s},$$

$$D^{P}(Q,w_{SP},w_P,w_N,p) = \left[\frac{Q}{q^*} \left(\frac{w_{SP}}{a_{SP}}\right)^{a_{SP}} \left(\frac{w_P}{a_P}\right)^{a_P-s} \left(\frac{w_N}{a_N}\right)^{a_N} \left(\frac{P}{b}\right)^b \right]^{1/s},$$

$$D^{N}(Q,w_{SP},w_P,w_N,p) = \left[\frac{Q}{q^*} \left(\frac{w_{SP}}{a_{SP}}\right)^{a_{SP}} \left(\frac{w_P}{a_P}\right)^{a_P} \left(\frac{w_N}{a_N}\right)^{a_N-s} \left(\frac{P}{b}\right)^b \right]^{1/s},$$

$$D^B(Q,w_{SP},w_P,w_N,p) = \left[\frac{Q}{q^*} \left(\frac{w_{SP}}{a_{SP}}\right)^{a_{SP}} \left(\frac{w_P}{a_P}\right)^{a_P} \left(\frac{w_N}{a_N}\right)^{a_N} \left(\frac{p}{b}\right)^{b-s} \right]^{1/s},$$

$$H(Q,w_{SP},w_P,w_N,p) = s\left[\frac{Q}{q^*} \left(\frac{w_{SP}}{a_{SP}}\right)^{a_{SP}} \left(\frac{w_P}{a_P}\right)^{a_P} \left(\frac{w_N}{a_N}\right)^{a_N} \left(\frac{p}{b}\right)^{b} \right]^{1/s},$$

$$M(Q,w_{SP},w_P,w_N,p) = \left[\frac{Q^{1-s}}{q^*} \left(\frac{w_{SP}}{a_{SP}}\right)^{a_{SP}} \left(\frac{w_P}{a_P}\right)^{a_P} \left(\frac{w_N}{a_N}\right)^{a_N} \left(\frac{p}{b}\right)^{b} \right]^{1/s},$$

where $s = a_{SP} + a_P + a_N + b$ and $q^* = qU^u$.

The reduced form is composed of the derived demand functions for each aggregate variable input, the aggregate cost function, and the Lagrange multiplier function. Given the variable input prices, the cost function expresses the minimum cost required to produce a Q level of output. The variable input demand functions express the optimal levels of each input required to produce the same Q. These input levels are optimal because they are the least cost combinations: $wD^S + pD^B = H$, or $w_{SP}D^{SP} + w_PD^P + w_ND^N + pD^B = H$. The Lagrange multiplier function expresses the optimal marginal cost, which is the interpretation of the multiplier: $H_Q = M$.

Inserting the proper values into the reduced form functions, Table 2.7 shows the optimal level of each endogenous variable. If the

Variable	Actual Level	Optimal Level	Optimal/Actual
Staff	670	333	.49
Book	2194055	4027075	1.84
Cost	548902	447265	.81
Marginal Cost	.48	.39	.81

Table 2.7a -- Optimal Input and Cost Levels

Variable	Actual Level	Optimal Level	Optimal/Actual
Senior Professional	241	51	.21
Professional	287	64	.22
Nonprofessional	138	267	1.93
Book Services	2194055	3047160	1.39
Cost	548902	355905	.65
Marginal Cost	.48	.31	.65

Table 2.7b -- Optimal Input and Cost Levels

branch system were employing staff and book services in the amounts given by the derived demand functions, then its costs would be minimized and its input mix would be optimal for the given input prices and output level. Suppose that maximum output instead of minimum cost were the objective, then the dual relationship between the cost function and the production function can be put to use. A reduced form output function can be derived by inverting the cost function:

$$R(C,w,P) = q*(\frac{C}{s})^{s} \; (\frac{a}{w})^{a} \; (\frac{b}{p})^{b} \;, \quad \text{and}$$

$$R(C,w_{SP},w_{P},w_{N},p) = q* \; (\frac{C}{s})^{s} \; (\frac{a_{SP}}{w_{SP}})^{a_{SP}} \; (\frac{a_{P}}{w_{P}})^{a_{P}} \; (\frac{a_{N}}{w_{N}})^{a_{N}} \; (\frac{b}{p})^{b} \; .$$

Given the input prices and the cost level, Table 2.8 shows that this reduced form output function expresses the maximum possible output that the branch system could produce.

Actual Output	Maximum Output	Maximum/Actual	Specification
1224389	1482531	1.21	single staff
1224389	1837728	1.50	three staff

Table 2.8 -- Maximum Output Level

Evaluating the Method and the Results

The main objective of this chapter is to show that Houthakker's aggregation theorem is a feasible method for estimating an aggregate production function. Some strong assumptions are necessary, but this is not unusual, and the assumptions are no stronger or weaker than those required for the usual regression methods used by economists. In spite of the shortcomings of the data set, the objective has been fulfilled: the method is feasible.

How does Houthakker's method compare with the usual regression method for obtaining an estimated aggregate production function? The usual practice is to assume an explicit form a priori for the production function, and Levhari [4] shows that there is a one-to-one correspondence between the density function and the derived production function. In this respect, Houthakker's method is not really different and not necessarily superior.

Another aspect to consider is the data requirements. One common regression method requires a number of observations on the inputs and output of the particular aggregate. The only suitable type of data for this case is time series, which introduces time related extraneous factors. An alternative regression method requires a number of observations on similar aggregates. Here the data must be cross sectional, which introduces cross sectionally related extraneous factors. In contrast, Houthakker's method requires a number of observations on different cells of the same aggregate. While the data is cross sectional, it is easier to control for unwanted variations among cells of the same aggregate than among different aggregates. If cells are defined with care, time does not introduce extraneous factors, and cross sectional variations among cells can be explicitly allowed for by the fixed input assumptions. With respect to this data characteristic, Houthakker's method seems to be superior to regression.

The significant contribution of Houthakker's method is that the aggregate production function is constructed by actually summing up the inputs and output of the individual production units that make up the aggregate. However, a drawback is that the absence of fixed inputs from the aggregate production function results in an incomplete specification. The theorem acknowledges that fixed inputs exist and are important in determining the set of efficient production possibilities, but then it completely ignores them in the derivation of the aggregate function. The not surprising result is that the derived production function yields almost no information about the limiting, unaggregated, fixed inputs. In the end, there are still too many unresolved questions about how fixed inputs fit into production functions.

Some of the particular steps taken along the estimation path may not be the most ideal, but for this exploratory study they are sufficient to demonstrate the feasibility of the method. Other numerical methods can be applied to estimate the density function parameters. Especially unsatisfactory is the method of moments estimate of the means of the various random variables. The importance of the means in determining the allowable upper limits on the random variables is enough reason to search for a better estimating method. The method of moments usually yields inefficient estimates so that other methods are usually preferrable even if more difficult computations are necessary. A further unsatisfactory feature is the estimation of each density function parameter independently of all the others. While this is certainly a computational convenience, it seems intuitively plausible that the mutual dependence of all the random variables, through their economic relationships, somehow should be taken into account in the estimation procedure. This possible bias of the estimates needs to be examined more closely.

Turning to the behavioral assumptions, the two most important ones concern fixed proportions production functions for each cell and cost minimization as the objective. Since a production cell is defined as one branch library operating for one month, the fixed proportions cell production function is easy to accept and more likely than not is a valid assumption. The objective of minimizing cost is a bit more difficult to accept as valid even if it is an appropriate objective. Checking the number of production coefficients that exceed their allowable limit (Appendix 2) reveals that about forty per cent of the cells are not minimizing costs and are inefficient. According to the aggregation theorem, if a cell is efficiently minimizing its production costs, the probability of observing a production coefficient greater than its limit is zero. Three possible explanations of this discrepancy are immediately obvious. First, the Pareto density function assumption is wrong. The production coefficients are not Pareto random variables. Second, the estimate of the limit is too low. Given its dependence on the estimate of the mean, the importance of the method of estimating the mean becomes more apparent. Third, the implied managerial model requires high mobility of and control over the inputs, but mobility and control may be low so that rigidities cause inefficient production coefficients to be used. Each of these possibilities is supported by some evidence so that finding so many inefficient coefficients is not at all surprising. Invalidating the cost minimizing assumption casts doubt on the validity of applying the method to the Oakland Library branch system. However, this is not to say that the theorem has no merit, rather, the phenomenon has to be selected with more care toward validation of the crucial assumptions.

Moving on to the computed results themselves, the production function elasticities look reasonable, although it is difficult to say how reasonable. The requirement that the aggregate production function be homogeneous of degree one in all its inputs severely limits its ability to shed some light on the question of returns to scale for the set of cells. About all that can be gleaned is returns to scale from the variable inputs individually and collectively. An interesting finding is that the staff and book elasticities are about the same size. Being a Cobb-Douglas function, this means that the aggregate production activity is equally intensive in book and staff services, which means that the two inputs are equally important.

The results of the application of the production function show that too many resources are being allocated to professional staff positions, which leaves too few resources for the other purchased input services. The drastic cutback of professional staff implied by the model has to be considered in the light of the model's inherent limitations. Especially important to keep in mind is what is not included in the model, which is just as vital to evaluating the branch system's performance as what is included.

The measure of professional staff services assumes that all effort contributes directly to output. Administrative, overhead type activities are counted in with direct output related activities so that the measure of staff services is greater than the actual level of inputs. Another shortcoming of the staff services measure is that a person's wage is assumed to accurately reflect his productivity. The extent to which this overestimates professionals' direct output productivity and underestimates nonprofessionals' direct output productivity is reflected in the differences between the actual and optimal input levels.

Cost minimization is a convenient economic objective, but the branch system has other important noneconomic and even anti-economic objectives that also effect the input mix and that have to be given their due consideration when making resource allocation decisions. However, even allowing for a reasonable amount of slack does not reverse the conclusion that fewer professional staff services and more nonprofessional and book services would improve resource allocation optimality and reduce cost.

Appendix 1 -- Branch Library Outputs and Inputs For Nineteen Branches For Each Month, 1 July 1968 through 30 June 1969*

(1) Circulation

(2) Attendance at Story Hours

(3) Class Visits and Classes Visited

(4) Hours Open

(5) Circulation Equivalents

(6) Senior Professional Staff Salaries

(7) Professional Staff Salaries

(8) Nonprofessional Staff Salaries

(9) Staff Salaries

(10) Volumes in the Book Stock

(11) Book-months of Book Services

(12) Utilization Rate

Source: The records of the Oakland Public Library

---

*1) The Fruitvale branch is not included in the sample because it houses the federally funded Latin American project, which causes it to have a different budget constraint.

2) The Charles S. Greene branch is not included in the sample because it houses some main library activities.

3) The Lockwood branch was closed for the first six months of 1969.

4) No stations are included in the sample.

APPENDIX 1 (CONTINUED) -- BRANCH LIBRARY OUTPUTS AND INPUTS

BRANCH AND MONTH	(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)	(9)	(10)	(11)	(12)
BAYMONT												
7-68	5728	0	0	262	6514	766	633	429	1828	5951	4797	.806
8-68	5115	0	0	267	5916	766	693	439	1898	5984	4916	.822
9-68	4078	0	0	229	4765	766	693	432	1891	5969	4206	.705
10-68	4339	25	2	265	5234	786	693	161	1640	5939	4843	.815
11-68	4648	0	1	234	5375	786	693	142	1621	5967	4296	.720
12-68	3440	0	1	239	4182	786	693	172	1651	6002	4414	.735
1-69	4401	0	0	208	5025	787	710	146	1642	6083	3893	.640
2-69	3695	0	1	138	4134	564	710	124	1398	6171	2620	.425
3-69	4241	0	0	200	4841	824	710	157	1690	6198	3814	.615
4-69	4279	0	0	208	4903	824	710	151	1685	6213	3976	.640
5-69	4140	0	1	208	4789	824	710	160	1694	6251	4001	.640
6-69	4632	0	0	200	5232	824	710	167	1701	6283	3866	.615
INA D. COOLBRITH												
7-68	2804	8	0	200	3420	804	627	85	1516	12041	7410	.615
8-68	1995	0	0	200	2595	804	583	96	1483	12084	7436	.615
9-68	2108	0	0	184	2660	804	583	113	1500	12106	6854	.566
10-68	3691	25	35	216	5264	804	583	96	1483	12061	8016	.665
11-68	3646	50	27	192	4997	804	583	90	1477	12028	7106	.591
12-68	3522	25	24	200	4772	804	583	112	1499	12040	7409	.615
1-69	3700	25	30	208	5124	824	384	84	1292	12041	7706	.640
2-69	3043	0	21	114	3910	824	394	131	1349	12077	4236	.351
3-69	3823	0	29	200	5148	824	72	157	1054	12125	7462	.615
4-69	3935	25	24	208	5209	824	65	154	1042	12150	7776	.640
5-69	3616	0	24	208	4840	824	72	161	1057	12210	7814	.640
6-69	3119	0	10	200	3969	824	96	177	1097	12280	7557	.615

APPENDIX 1 (CONTINUED) -- BRANCH LIBRARY OUTPUTS AND INPUTS

BRANCH AND MONTH	(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)	(9)	(10)	(11)	(12)
DIMOND												
7-68	8519	21	2	262	9397	866	1186	314	2366	19131	15423	.806
8-68	7887	0	0	267	8688	866	1186	92	2144	19195	15769	.822
9-68	6036	74	0	229	6871	866	1186	145	2197	19236	13554	.705
10-68	7141	88	3	265	8187	866	1068	168	2102	19270	15712	.815
11-68	7135	56	10	234	8199	866	1213	156	2235	19306	13900	.720
12-68	6341	24	0	245	7124	866	1213	173	2252	19356	14591	.754
1-69	7449	42	0	259	8310	888	1243	167	2298	19412	15470	.797
2-69	6922	48	0	221	7681	888	1243	275	2406	19471	13240	.680
3-69	7367	44	0	248	8199	888	1243	315	2446	19530	14903	.763
4-69	7537	56	3	258	8498	888	1256	322	2466	19589	15551	.794
5-69	7749	44	8	259	8814	888	1243	315	2446	19698	15698	.797
6-69	7615	0	6	251	8518	888	1243	315	2446	19832	15316	.772
EASTMONT												
7-68	3859	0	0	200	4459	729	996	124	1849	10428	6417	.615
8-68	3541	0	0	216	4189	804	693	129	1626	10468	6957	.665
9-68	2815	0	0	184	3367	804	693	110	1607	10492	5940	.566
10-68	3381	0	2	208	4055	804	693	133	1630	10522	6734	.640
11-68	3788	0	17	192	4789	804	693	122	1619	10573	6246	.591
12-68	3425	0	8	200	4225	804	693	150	1647	10633	6543	.615
1-69	3743	0	0	208	4367	824	0	122	946	10722	6862	.640
2-69	3079	0	0	146	3517	824	0	98	922	10833	4867	.449
3-69	3214	0	0	200	3814	824	427	116	1368	10906	6711	.615
4-69	3317	0	0	208	3941	824	610	139	1574	10957	7012	.640
5-69	3485	0	0	208	4109	824	598	136	1558	11023	7055	.640
6-69	3294	0	0	200	3894	824	598	20	1442	11092	6826	.615

APPENDIX 1 (CONTINUED) -- BRANCH LIBRARY OUTPUTS AND INPUTS

BRANCH AND MONTH	(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)	(9)	(10)	(11)	(12)
ELMHURST												
7-68	4888	120	0	262	5914	804	693	209	1706	15433	12441	.806
8-68	4906	120	2	267	5997	804	1326	243	2373	15495	12730	.822
9-68	4116	25	1	229	4878	804	1326	234	2364	15554	10960	.705
10-68	4869	30	3	265	5799	804	1338	343	2485	15618	12735	.815
11-68	4928	0	1	220	5613	804	1338	340	2482	15718	10640	.677
12-68	4081	28	0	239	4854	804	1338	290	2432	15823	11636	.735
1-69	5656	0	0	259	6433	1671	1371	280	3322	15996	12748	.797
2-69	5141	22	0	221	5848	1671	710	222	2603	16202	11017	.680
3-69	5193	64	1	248	6090	1671	710	288	2669	16323	12456	.763
4-69	5202	30	2	258	6086	1671	710	297	2678	16431	13044	.794
5-69	5700	23	4	259	6623	1671	710	288	2669	16577	13211	.797
6-69	5257	118	1	251	6271	1671	710	249	2630	16716	12910	.772
GLENVIEW												
7-68	4810	53	0	273	5735	804	860	135	1798	14375	12075	.840
8-68	4142	0	0	267	4943	804	1255	152	2212	14395	11826	.822
9-68	3550	0	0	229	4237	804	958	115	1877	14409	10153	.705
10-68	4017	0	0	265	4812	804	908	143	1855	14407	11747	.815
11-68	3891	0	5	234	4718	804	908	147	1859	14428	10388	.720
12-68	3482	0	0	245	4217	804	908	172	1884	14463	10903	.754
1-69	3990	0	0	251	4743	824	930	142	1896	14442	11154	.772
2-69	2885	0	0	158	3359	824	930	144	1898	14371	6987	.486
3-69	3640	0	0	200	4240	824	930	157	1910	14269	8781	.615
4-69	3455	0	0	208	4079	824	930	153	1907	14197	9086	.640
5-69	4012	0	0	208	4636	824	930	157	1910	14211	9095	.640
6-69	4310	0	0	200	4910	824	930	177	1931	14236	8761	.615

APPENDIX 1 (CONTINUED -- BRANCH LIBRARY OUTPUTS AND INPUTS

BRANCH AND MONTH	(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)	(9)	(10)	(11)	(12)
GOLDEN GATE												
7-68	1819	50	1	208	2568	729	693	124	1546	11639	7449	.640
8-68	1936	0	0	216	2584	729	693	129	1551	11604	7712	.665
9-68	1744	0	0	184	2296	729	693	64	1486	11497	6509	.566
10-68	1869	0	0	216	2517	729	693	107	1529	11374	7559	.665
11-68	1884	0	2	192	2510	729	693	94	1516	11318	6686	.591
12-68	1533	0	2	200	2183	729	693	49	1471	11242	6918	.615
1-69	1521	0	0	200	2121	747	710	65	1522	10967	6749	.615
2-69	1043	0	1	114	1410	747	710	214	1671	10660	3739	.351
3-69	1278	0	2	184	1880	747	710	154	1611	10517	5954	.566
4-69	1441	5	1	208	2100	747	710	63	1520	10476	6705	.640
5-69	1328	0	2	192	1954	747	710	40	1497	10493	6199	.591
6-69	1261	40	0	200	1941	0	710	158	868	10521	6474	.615
LAKEVIEW												
7-68	11275	36	0	278	12181	806	669	638	2113	19096	16334	.855
8-68	9731	0	0	283	10580	1550	669	686	2905	19157	16681	.871
9-68	8917	0	0	249	9664	1550	669	719	2938	19208	14716	.766
10-68	9569	41	4	281	10594	1585	714	733	3032	19248	16642	.865
11-68	9336	42	2	250	10220	1585	681	744	3010	19310	14854	.769
12-68	7990	41	2	268	8926	1585	681	799	3065	19385	15985	.825
1-69	8459	28	3	275	9415	1584	698	755	3037	19436	16446	.846
2-69	7276	0	3	251	8104	1584	698	681	2963	19523	15078	.772
3-69	8239	61	4	268	9265	1584	777	727	3088	19629	16186	.825
4-69	7960	24	2	274	8880	1584	785	770	3139	19718	16624	.843
5-69	8420	90	4	274	9522	1584	905	812	3300	19853	16738	.843
6-69	9000	54	0	272	9924	1584	770	817	3171	20008	16745	.837

APPENDIX 1 (CONTINUED) -- BRANCH LIBRARY OUTPUTS AND INPUTS

BRANCH AND MONTH	(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)	(9)	(10)	(11)	(12)
LAUREL												
7-68	8314	0	0	262	9100	729	1252	329	2310	16551	13343	.806
8-68	8339	0	0	267	9140	712	1252	329	2294	16578	13619	.822
9-68	6364	0	0	229	7051	729	1252	322	2303	16622	11712	.705
10-68	6732	0	1	265	7552	744	1264	337	2345	16661	13585	.815
11-68	6487	0	22	234	7739	744	1264	337	2345	16773	12077	.720
12-68	5538	0	13	251	6616	744	1204	337	2285	16893	13047	.772
1-69	7283	0	3	259	8135	763	1296	345	2404	17016	13560	.797
2-69	6415	0	6	221	7228	763	1296	350	2409	17168	11674	.680
3-69	7078	0	6	248	7972	763	1296	345	2404	17255	13167	.763
4-69	7637	0	10	248	8631	763	1296	345	2404	17325	13220	.763
5-69	7460	0	8	259	8437	763	1296	345	2404	17409	13874	.797
6-69	8215	0	1	251	8993	763	1296	345	2404	17516	13528	.772
LOCKWOOD												
7-68	1896	38	0	208	2596	729	693	103	1525	10331	6612	.640
8-68	1907	0	0	216	2555	729	693	99	1521	10362	6887	.665
9-68	1570	0	0	184	2122	729	830	110	1668	10390	5882	.566
10-68	1808	0	0	183	2356	729	693	92	1514	10407	5860	.563
11-68	790	0	0	192	1366	688	693	38	1420	10487	6195	.591
12-68	1131	0	0	200	1731	731	693	111	1535	10571	6505	.615

APPENDIX 1 (CONTINUED) -- BRANCH LIBRARY OUTPUTS AND INPUTS

BRANCH AND MONTH	(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)	(9)	(10)	(11)	(12)
JACK LONDON												
7-68	1081	40	2	208	1835	729	686	92	1507	11610	7430	.640
8-68	1135	0	0	216	1783	729	657	106	1492	11650	7743	.665
9-68	800	0	1	184	1377	729	657	99	1485	11693	6620	.566
10-68	1017	76	0	208	1793	729	669	110	1508	11719	7500	.640
11-68	856	48	0	177	1483	729	669	65	1463	11730	6388	.545
12-68	893	0	1	168	1422	729	669	4	1402	11759	6078	.517
1-69	1138	0	0	184	1690	747	686	31	1464	11786	6673	.566
2-69	816	6	1	136	1261	747	686	33	1466	11823	4947	.418
3-69	1023	11	3	200	1720	747	686	89	1522	11868	7303	.615
4-69	909	22	1	200	1578	747	686	0	1433	11908	7328	.615
5-69	1111	260	12	208	2555	747	686	47	1480	11970	7661	.640
6-69	1073	0	0	200	1673	747	686	65	1498	12033	7405	.615
MELROSE												
7-68	5760	63	6	273	6855	866	760	492	2118	23461	19707	.840
8-68	4906	0	4	267	5807	866	1228	412	2506	23572	19365	.822
9-68	3873	0	1	229	4585	866	1228	423	2517	23672	16680	.705
10-68	4759	0	2	265	5604	866	657	412	1935	23764	19377	.815
11-68	5154	0	3	218	5883	866	657	412	1935	24038	16124	.671
12-68	4208	0	2	248	5002	866	657	412	1935	24292	18537	.763
1-69	5406	0	0	259	6183	888	673	438	1999	24505	19529	.797
2-69	5093	0	1	221	5781	888	1040	444	2372	24737	16821	.680
3-69	5279	0	0	248	6023	888	1209	466	2563	24777	18907	.763
4-69	5132	0	0	262	5918	888	1209	433	2530	24789	19984	.806
5-69	5618	0	1	259	6420	888	1209	422	2519	24889	19835	.797
6-69	5071	0	4	248	5915	888	1209	444	2541	25031	19101	.763

APPENDIX 1 (CONTINUED) -- BRANCH LIBRARY OUTPUTS AND INPUTS

BRANCH AND MONTH	(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)	(9)	(10)	(11)	(12)
MONTCLAIR												
7-68	12047	10	0	289	12934	843	1637	321	2801	14883	13234	.889
8-68	10374	0	0	283	11223	862	1591	337	2790	14941	13010	.871
9-68	8926	47	0	249	9767	823	1697	319	2840	15006	11497	.766
10-68	10066	143	1	285	11232	862	1120	358	2340	15072	13217	.877
11-68	9682	33	0	261	10531	844	1134	406	2384	15120	12143	.803
12-68	8524	20	0	265	9359	864	1087	393	2344	15211	12403	.815
1-69	10951	38	0	275	11852	865	1114	393	2372	15299	12945	.846
2-69	9960	72	0	237	10815	867	1114	395	2375	15381	11216	.729
3-69	10619	0	0	268	11423	1643	1194	401	3239	15522	12800	.825
4-69	11019	0	2	278	11903	1643	1114	354	3111	15656	13392	.855
5-69	10058	0	2	275	10933	1622	1142	404	3168	15775	13348	.846
6-69	11184	0	0	271	11997	1602	1217	306	3125	15878	13240	.834
PARK BOULEVARD												
7-68	4888	0	0	262	5674	804	693	115	1612	12857	10365	.806
8-68	4626	11	0	267	5449	804	693	135	1632	12879	10581	.822
9-68	3278	0	0	229	3965	804	693	128	1625	12895	9086	.705
10-68	3475	0	0	265	4270	804	693	156	1653	12911	10527	.815
11-68	3357	0	0	234	4059	804	693	136	1633	12935	9313	.720
12-68	2959	0	0	245	3694	804	693	113	1610	12965	9774	.754
1-69	3539	0	1	250	4314	824	710	115	1649	13007	10005	.769
2-69	2994	0	1	158	3493	824	748	126	1697	13069	6354	.486
3-69	3640	0	0	224	4312	824	710	131	1665	13123	9045	.689
4-69	3882	0	1	258	4681	824	710	140	1674	13159	10446	.794
5-69	3950	16	0	256	4750	824	736	134	1695	13212	10407	.788
6-69	4237	15	0	239	4984	824	723	88	1635	13270	9759	.735

APPENDIX 1 (CONTINUED) -- BRANCH LIBRARY OUTPUTS AND INPUTS

BRANCH AND MONTH	(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)	(9)	(10)	(11)	(12)
PIEDMONT												
7-68	6513	0	2	262	7349	804	787	120	1711	13124	10580	.806
8-68	6182	0	2	267	7033	804	633	125	1562	13123	10781	.822
9-68	5117	0	0	229	5804	804	333	115	1252	13134	9254	.705
10-68	5887	0	0	265	6682	804	44	109	957	13152	10724	.815
11-68	5400	0	0	234	6102	804	124	0	928	13174	9485	.720
12-68	4807	0	0	245	5542	804	174	131	1109	13211	9959	.754
1-69	5791	0	2	259	6618	824	587	122	1533	13198	10518	.797
2-69	5145	0	2	221	5858	824	561	116	1502	13171	8956	.680
3-69	5370	0	10	264	6412	824	536	120	1480	13152	10683	.812
4-69	5974	0	2	258	6798	824	536	136	1496	13116	10412	.794
5-69	5839	0	1	259	6641	824	536	138	1498	13111	10448	.797
6-69	5459	0	0	227	6140	824	536	151	1511	13121	9165	.698
ROCKRIDGE												
7-68	5655	76	0	262	6593	729	1247	111	2087	15939	12849	.806
8-68	5365	0	0	267	6166	729	1247	135	2111	15958	13110	.822
9-68	4370	0	0	229	5057	729	1270	143	2143	15973	11255	.705
10-68	5212	0	0	265	6007	744	601	158	1503	15985	13034	.815
11-68	4713	0	0	234	5415	744	692	142	1579	15996	11517	.720
12-68	4155	0	0	248	4899	744	554	152	1450	16010	12217	.763
1-69	4991	0	0	259	5768	763	568	246	1577	16022	12768	.797
2-69	3511	0	0	136	3919	763	568	127	1458	16039	6712	.418
3-69	4404	2	0	200	5008	763	568	153	1484	16051	9878	.615
4-69	4584	7	0	208	5222	763	568	144	1475	16046	10269	.640
5-69	4584	0	0	208	5208	763	607	138	1509	16040	10266	.640
6-69	4294	0	0	200	4894	763	0	165	928	15969	9827	.615

APPENDIX 1 (CONTINUED) -- BRANCH LIBRARY OUTPUTS AND INPUTS

BRANCH AND MONTH	(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)	(9)	(10)	(11)	(12)
SEQUOYAH-CHABOT												
7-68	1442	0	0	164	1934	0	1246	142	1388	7369	3719	.505
8-68	1044	0	0	168	1548	729	693	165	158	7387	3819	.517
9-68	2842	0	32	160	4122	729	0	43	772	7408	3647	.492
10-68	2832	51	52	213	4873	744	455	95	1293	7430	4870	.655
11-68	2855	110	50	184	4877	744	523	85	1352	7457	4222	.566
12-68	2290	75	50	61	3873	744	453	71	1268	7481	1404	.188
1-69	3749	83	55	200	5890	0	1246	71	1317	7506	4619	.615
2-69	3001	79	20	200	4259	0	710	136	846	7533	4636	.615
3-69	3430	48	20	200	4626	0	710	146	856	7550	4646	.615
4-69	3268	46	20	200	4460	0	710	146	856	7586	4668	.615
5-69	3614	45	20	200	4804	0	710	164	874	7649	4707	.615
6-69	1898	71	0	200	2640	0	710	164	874	7694	4735	.615
TEMESCAL												
7-68	7488	120	0	262	8514	826	508	195	1529	12721	10255	.806
8-68	7139	78	5	267	8221	826	964	244	2034	12784	10503	.822
9-68	5832	11	2	229	6591	826	1086	207	2119	12795	9016	.705
10-68	6324	124	99	245	9782	846	1288	231	2365	12844	9682	.754
11-68	5442	51	8	234	6446	846	1180	231	2257	13032	9383	.720
12-68	5706	65	8	243	6765	846	1180	256	2282	13222	9886	.748
1-69	6686	52	7	259	7742	1630	673	125	2428	13312	10609	.797
2-69	6157	128	5	221	7201	1630	673	132	2435	13364	9088	.680
3-69	6792	55	3	248	7721	1630	673	251	2554	13322	10166	.763
4-69	7815	72	4	258	8833	1630	673	276	2579	13253	10521	.794
5-69	7053	82	7	259	8169	1630	673	276	2579	13293	10593	.797
6-69	7781	190	8	251	9114	1630	673	301	2604	13370	10326	.772

APPENDIX 1 (CONTINUED) -- BRANCH LIBRARY OUTPUTS AND INPUTS

BRANCH AND MONTH	(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)	(9)	(10)	(11)	(12)
WEST OAKLAND												
7-68	1483	50	0	208	2207	804	645	142	1591	8531	5460	.640
8-68	1368	0	0	214	2010	804	645	152	1601	8569	5642	.658
9-68	1211	45	0	184	1853	804	645	145	1594	8603	4871	.566
10-68	1430	0	0	208	2054	804	657	168	1629	8620	5517	.640
11-68	1233	10	0	184	1805	804	657	145	1606	8884	5030	.566
12-68	1033	20	3	192	1724	804	657	175	1636	9159	5411	.591
1-69	1321	20	1	200	1986	824	673	135	1632	9326	5739	.615
2-69	1031	15	1	152	1542	824	673	149	1646	9503	4444	.468
3-69	1452	30	3	200	2187	824	673	153	1650	9533	5866	.615
4-69	1639	45	2	208	2403	824	673	160	1657	9555	6115	.640
5-69	1565	30	0	208	2249	824	673	157	1653	9607	6148	.640
6-69	1535	30	6	200	2345	824	673	177	1674	9660	5945	.615

Appendix 2 -- Branch Library Input-Output Proportions: The Random Variables

SP/Q = Senior Professional Man-months/Circulation Equivalents

P/Q = Professional Man-months/Circulation Equivalents

N/Q = Nonprofessional Man-months/Circulation Equivalents

B/Q = Book-months/Circulation Equivalents

APPENDIX 2 (CONTINUED) -- INPUT-OUTPUT PROPORTIONS

BRANCH AND MONTH	[illegible]/V	E/V	N/P	S/V	[illegible]/V
BAYMONT					
7-68	.000147	.000159	.000185	.000444	.74
8-68	.000162	.000181	.000208	.000514	.[illegible]
9-68	.000211	.000237	.000255	.000534	.[illegible]
10-68	.000180	.000216	.000095	.000502	.[illegible]
11-68	.000183	.000210	.000074	.000433	.50
12-68	.000235	.000270	.000115	.000632	1.06
1-69	.000196	.000230	.000091	.000523	.77
2-69	.000170	.000200	.000084	.000541	.63
3-69	.000213	.000255	.000091	.000559	.79
4-69	.000210	.000235	.000087	.000550	.81
5-69	.000215	.000242	.000094	.000547	.84
6-69	.000197	.000221	.000093	.000521	.74
INA B. COULSON					
7-68	.000264	.000249	.000070	.000710	2.17
8-68	.000347	.000366	.000103	.000915	2.87
9-68	.000378	.000337	.000120	.000903	2.55
10-68	.000191	.000181	.000051	.000451	1.52
11-68	.000201	.000191	.000051	.000473	1.42
12-68	.000211	.000199	.000056	.000503	1.55
1-69	.000221	.000182	.000045	.000404	1.59
2-69	.000263	.000164	.000084	.000553	1.98
3-69	[illegible]	.000023	[illegible]	.000328	1.45
4-69	.000138	.000020	.000083	.000320	1.49
5-69	.000213	.000024	.000093	.000350	1.61
6-69	.000255	.00004[illegible]	.000125	.000443	1.90
SIMONE					
7-68	.000115	.000206	.000064	.000413	1.54
8-68	.000125	.000223	.000030	.000395	1.82
9-68	.000157	.000241	.000050	.000512	1.97
10-68	.000132	.000215	.000053	.000411	1.92
11-68	.000132	.000241	.000053	.000436	1.70
12-68	.000152	.000276	.000068	.000506	2.05
1-69	.000134	.000244	.000057	.000443	1.86
2-69	.000144	.000264	.000101	.000502	1.72
3-69	.000135	.000247	.000108	.000478	1.32
4-69	.000131	.000241	.000106	.000455	1.83
5-69	.000126	.000230	.000100	.000444	1.74
6-69	.000130	.000238	.000104	.000460	1.80

APPENDIX 2 (CONTINUED) -- INPUT-OUTPUT PROPORTIONS

[illegible] AND MONTH	SO/Q	F/Q	N/Q	S/Q	[illegible]/Q
EASTMENT					
7-68	.000284	.000364	.000078	.000654	1.44
8-68	.000240	.00027[illegible]	.000087	.000622	1.56
9-68	.000298	.000336	.000[illegible]42	.000764	1.76
10-68	.000243	.000279	.000[illegible]2	.000644	1.66
11-68	.000210	.000236	.000072	.000541	1.50
12-68	.000238	.000267	.00012[illegible]	.000624	1.55
1-69	.000236	.000[illegible]	.000[illegible]78	.000347	1.57
2-69	.000253	.000[illegible]	.000[illegible]78	.000420	1.38
3-69	.000270	.000143	.000[illegible]65	.000574	1.76
4-69	.000261	.000253	.000[illegible]	.000[illegible]	1.78
5-69	.000251	.000237	.000[illegible]3	.000[illegible]07	1.72
6-69	.000264	.00025[illegible]	.000[illegible]14	.0005[illegible]	1.75
ELMHURST					
7-68	.000170	.000191	.000099	.000462	2.10
8-68	.000168	.000361	.000114	.000634	2.12
9-68	.000206	.000443	.000134	.000776	2.25
10-68	.000173	.000376	.000165	.000685	2.20
11-68	.000179	.000389	.000170	.000708	1.90
12-68	.000207	.000449	.000168	.000802	2.40
1-69	.000325	.000347	.000122	.000827	1.98
2-69	.000357	.000198	.000106	.000713	1.88
3-69	.000343	.000190	.000133	.000732	2.05
4-69	.000343	.000190	.000137	.000705	2.14
5-69	.000315	.000175	.000122	.000645	1.99
6-69	.000333	.000185	.000111	.000671	2.06
GLENVIEW					
7-68	.000175	.000245	.000064	.000502	2.11
8-68	.000203	.000414	.000085	.000715	2.39
9-68	.000237	.000369	.000076	.000709	2.40
10-68	.000209	.000308	.000084	.000617	2.44
11-68	.000213	.000314	.000087	.000631	2.20
12-68	.000238	.000351	.000114	.000715	2.59
1-69	.000217	.000320	.000084	.000649	2.35
2-69	.000307	.000451	.000120	.000905	2.08
3-69	.000243	.000358	.000104	.000722	2.07
4-69	.000252	.000372	.000105	.000749	2.23
5-69	.000222	.000327	.000095	.000660	1.96
6-69	.000210	.000309	.000101	.000630	1.78

APPENDIX 2 (CONTINUED) -- INPUT-OUTPUT PROPORTIONS

BRANCH AND MONTH	SP/A	F/A	N/P	S/P	P/A
GOLDEN GATE					
7-68	.000355	.000440	.001135	.000954	2.9[illegible]
8-68	.000353	.000437	.001140	.000961	2.[illegible]
9-68	.000357	.000422	.000978	.001036	2.[illegible]
10-68	.000362	.000449	.001117	.000973	3.0[illegible]
11-68	.000373	.000455	.001015	.000957	2.[illegible]
12-68	.000417	.000518	.001003	.001074	3.17
1-69	.000440	.000546	.001[illegible]	.001140	3.1[illegible]
2-69	.000462	.000421	.000488	.001394	2.[illegible]
3-69	.000467	.000616	.001230	.001372	3.17
4-69	.000444	.000551	.000984	.001159	3.19
5-69	.000473	.000532	.001054	.001227	3.17
6-69	.000000	.000576	.000229	.000716	3.34
LAKEVIEW					
7-68	.000083	.00009[illegible]	.000147	.000278	1.34
8-68	.000183	.000105	.000182	.000440	1.56
9-68	.000200	.000113	.000209	.000437	1.52
10-68	.000187	.000110	.000194	.000455	1.57
11-68	.000194	.000109	.000204	.000472	1.45
12-68	.000222	.000124	.000251	.000550	1.79
1-69	.000210	.000121	.000225	.000516	1.75
2-69	.000244	.000140	.000236	.000586	1.[illegible]
3-69	.000214	.000137	.000220	.000534	1.75
4-69	.000223	.000144	.000244	.000566	1.57
5-69	.000208	.000155	.000239	.000555	1.76
6-69	.000199	.000127	.000231	.000512	1.69
LAUREL					
7-68	.000100	.000224	.000132	.000437	1.47
8-68	.000097	.000223	.000101	.000432	1.49
9-68	.000129	.00029[illegible]	.000124	.000523	1.6[illegible]
10-68	.000123	.000273	.000125	.000497	1.80
11-68	.000127	.000266	.000122	.000485	1.55
12-68	.000141	.000297	.000143	.000553	1.97
1-69	.000117	.000263	.000115	.000473	1.57
2-69	.000132	.000282	.000136	.000534	1.62
3-69	.000120	.000265	.000122	.000483	1.65
4-69	.000110	.000245	.000112	.000445	1.53
5-69	.000113	.000250	.000115	.000456	1.64
6-69	.000108	.000235	.000108	.000428	1.59

APPENDIX [illegible] (CONTINUED) -- [illegible]

[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]
[illegible]					
7-68	[illegible]	[illegible]	.000111	[illegible]	[illegible]
8-68	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]
9-68	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]
10-68	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]
11-68	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]
12-68	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]
[illegible]					
7-68	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]
8-68	.000511	[illegible]	[illegible]	[illegible]	4.24
9-68	[illegible]	.000778	[illegible]	.001727	4.81
10-68	[illegible]	[illegible]	[illegible]	.001347	[illegible]
11-68	.000614	[illegible]	.000124	[illegible]	4.51
12-68	.000541	[illegible]	[illegible]	[illegible]	[illegible]
1-69	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]
2-69	[illegible]	[illegible]	[illegible]	.001861	[illegible]
3-69	[illegible]	[illegible]	[illegible]	.001417	4.25
4-69	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]
5-69	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]
6-69	[illegible]	[illegible]	[illegible]	.001433	4.43
MELROSE					
7-68	.000158	[illegible]	[illegible]	.000495	2.87
8-68	.000166	.000345	[illegible]	.000691	3.33
9-68	.000236	.000437	[illegible]	.000879	3.64
10-68	.000193	.000191	.000206	.000553	3.46
11-68	.000184	.000182	[illegible]	.000527	2.74
12-68	.000216	.000214	.000231	[illegible]	3.71
1-69	.000179	.000177	[illegible]	[illegible]	3.15
2-69	.000192	[illegible]	.000216	.000657	2.91
3-69	.000184	.000327	.000217	.000681	3.14
4-69	.000187	[illegible]	[illegible]	.000685	[illegible]
5-69	.000173	[illegible]	.000185	[illegible]	[illegible]
6-69	[illegible]	[illegible]	.000211	[illegible]	3.23

APPENDIX 2 (CONTINUED) -- INPUT-OUTPUT PROPORTIONS

BRANCH AND MONTH	SP/L	F/Q	N/P	S/Q	S/Q
MONTCLAIR					
7-68	.000081	.000206	.000070	.000347	1.92
8-68	.000096	.000231	.000086	.000398	1.16
9-68	.000105	.000243	.000072	.000464	1.14
10-68	.000096	.000163	.000086	.000334	1.18
11-68	.000100	.000176	.000108	.000362	1.19
12-68	.000115	.000189	.000118	.000401	1.34
1-69	.000091	.000153	.000093	.000320	1.03
2-69	.000100	.000148	.000103	.000352	1.04
3-69	.000190	.000170	.000098	.000454	1.12
4-69	.000172	.000153	.000084	.000419	1.12
5-69	.000185	.000170	.000104	.000464	1.22
6-69	.000167	.000158	.000072	.000417	1.10
PARK BOULEVARD					
7-68	.000177	.000189	.000057	.000455	1.83
8-68	.000184	.000227	.000073	.000483	1.94
9-68	.000253	.000285	.000091	.000656	2.29
10-68	.000235	.000265	.000102	.000620	2.47
11-68	.000248	.000278	.000094	.000644	2.25
12-68	.000272	.000305	.000086	.000692	2.65
1-69	.000239	.000268	.000075	.000512	2.32
2-69	.000245	.000349	.000101	.000778	1.62
3-69	.000233	.000268	.000085	.000618	2.13
4-69	.000220	.000247	.000084	.000573	2.23
5-69	.000217	.000253	.000079	.000571	2.19
6-69	.000207	.000237	.000050	.000525	1.96
PIEDMONT					
7-68	.000137	.000175	.000046	.000373	1.44
8-68	.000143	.000147	.000060	.000356	1.53
9-68	.000173	.000094	.000056	.000345	1.59
10-68	.000150	.000011	.000046	.000229	1.60
11-68	.000165	.000033	.000000	.000244	1.55
12-68	.000181	.000051	.000066	.000320	1.80
1-69	.000156	.000145	.000052	.000371	1.59
2-69	.000176	.000156	.000056	.000411	1.53
3-69	.000161	.000136	.000053	.000370	1.67
4-69	.000151	.000129	.000055	.000353	1.53
5-69	.000135	.000132	.000058	.000361	1.57
6-69	.000163	.000142	.000069	.000394	1.49

APPENDIX [illegible] (CONTINUED) -- INPUT-OUTPUT [illegible]

[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]
[illegible]					
7-68	.000134	.000308	.000047	.000507	1.9[illegible]
8-68	.000148	.000350	[illegible]	[illegible]	2.1[illegible]
9-68	.000[illegible]	.00041[illegible]	.000[illegible]	.000[illegible]	2.2[illegible]
10-68	.000155	.000154	.000074	.000401	2.17
11-68	.000172	.000209	.000074	.000457	2.13
12-68	.000193	.000184	.000067	.000474	2.4[illegible]
1-69	.000165	.000141	.000127	.000438	2.21
2-69	.000243	.000234	.000091	.000576	1.71
3-69	.000197	.000185	.000066	.000474	1.[illegible]7
4-69	.000183	.000177	.000077	.000453	1.47
5-69	.000193	.000192	.000075	.000444	1.97
6-69	.000195	.000200	.000045	.000306	2.[illegible]1
[illegible]					
7-68	.000000	.001051	[illegible]	.001164	1.9[illegible]
8-68	.000588	.000743	.000797	.001641	2.47
9-68	.000221	.000036	.000038	.000838	.38
10-68	.000191	.000152	.000035	.000425	1.03
11-68	.000151	.000175	.000044	.000446	.[illegible]7
12-68	.000241	.000191	.000051	.000526	.36
1-69	.000000	.000345	.000034	.000358	.78
2-69	.000000	.000272	.000090	.000316	1.09
3-69	.000000	.000250	.000085	.000246	1.06
4-69	.000000	.000260	.000092	.000507	1.05
5-69	.000000	.000241	.000056	.000291	.93
6-69	.000000	.000439	.000174	.000539	1.7[illegible]
TEMESCAL					
7-68	.000121	.000097	.000044	.000288	1.2[illegible]
8-68	.000126	.000191	.000045	.000396	1.2[illegible]
9-68	.000157	.000215	[illegible]	.000515	1.37
10-68	.000108	.000215	.000066	.000397	.9[illegible]
11-68	.000164	.000208	.000101	.000561	1.4[illegible]
12-68	.000156	.000184	.000106	.000492	1.46
1-69	.000263	.000142	.000046	.000532	1.37
2-69	.000283	.000152	.000051	.000541	1.26
3-69	.000264	.000142	.000091	.000530	1.32
4-69	.000231	.000134	.000088	.000458	1.1[illegible]
5-69	.000240	.000134	.000095	.000506	1.39
6-69	.000223	.000126	.000043	.000459	1.13

APPENDIX 2 (CONTINUED) -- INPUT-OUTPUT PROPORTIONS

BRANCH AND MONTH	SH/S	I/S	O/R	N/S	[illegible]/S
WEST [illegible]					
7-68	.000455	.000477	.000190	.001154	2.47
8-68	.000500	.000593	.000213	.001274	2.81
9-68	.000542	.000659	.000222	.001378	2.6[illegible]
10-68	.000499	.000522	.000230	.001273	2.[illegible]
11-68	.000557	.000595	.000226	.001426	2.7[illegible]
12-68	.000485	.000621	.000225	.00152[illegible]	3.14
1-69	.000518	.000553	.000190	.001314	2.[illegible]
2-69	.000608	.000712	.000272	.001719	2.[illegible]
3-69	.000473	.000542	.000197	.001209	2.[illegible]
4-69	.000428	.000457	.000187	.001104	2.54
5-69	.000458	.000443	.000195	.001177	2.73
6-69	.000439	.000466	.000211	.001143	2.54

Appendix 3 -- Newton-Raphson Iteration Results

Iteration	Parameter	Senior Professional	Professional	Nonprofessional	Staff	Book
	R	.000244	.000294	.000116	.000668	2.04
0	r	2.05	1.98	4.78	8.91	11.14
	R	.000363	.000442	.000141	.000743	2.23
1	r	1.64	1.58	3.86	6.05	6.64
	R	.000393	.000480	.000146	.000778	2.35
2	r	1.70	1.64	3.99	6.57	7.51
	R	.000388	.000473	.000145	.000770	2.31
3	r	1.70	1.64	3.99	6.60	7.61
	R	.000388	.000473	.000145	.000769	2.31
4	r	1.70	1.64	3.99	6.60	7.61
	r(3) - r(4)	-3.2×10^{-6}	-3.6×10^{-6}	-2.9×10^{-6}	-1.6×10^{-6}	-9.2×10^{-4}
5	r(4) - r(5)	-5.8×10^{-12}	-7.29×10^{-12}	-1.9×10^{-12}	-3.0×10^{-9}	-8.2×10^{-8}
6	r(5) - r(6)	-1.4×10^{-14}	-1.6×10^{-14}	-5.9×10^{-14}	-6.5×10^{-13}	-2.3×10^{-13}

Appendix 4 -- Computation of Staff Wages

Staff Category	Total Salaries	Total Man-Months	Monthly Wage
Senior Professional	193,747.70	242.10	800.28
Professional	178,980.29	291.83	613.30
Nonprofessional	52,126.44	146.42	356.18
Staff	424,854.43	680.35	624.47

Appendix 5 -- Glossary of Names

a	Output - staff elasticity
b	Output - book elasticity
B	Book services
C	Cost
D	Input derived demand function
F	Aggregate production unction
g	Lagrange multiplier
G	Lagrange function
H	Cost function
m_i	The mean of the v_{ij}
M	Lagrange multiplier function
N	Nonprofessional staff services
p	Book service price
P	Professional staff services
q	Production function constant
Q	Circulation equivalents of output
r_i	Pareto density function parameter
R	Output function
R_i	Upper limit on the random variables v_{ij}
s	$= a + b$ or $a_{SP} + a_P + a_N + b$
S	Staff services
SP	Senior professional staff services
u	Output - fixed input elasticity
U	Unaggregated fixed nputs
v_{ij}	The amount of variable input i that branch j needs to provide one unit of output; the Pareto random variables
w	Staff service price

REFERENCES

1. G. Eads, M. Nerlove, and W. Raduchel, "A Long-Run Cost Function for the Local Service Airline Industry: An Experiment in Non-Linear Estimation," Review of Economics and Statistics, 51.3 (1969) 258-270.

2. M. Fisz, Probability Theory and Mathematical Statistics, Third Edition, New York: John Wiley & Sons, 1963.

3. H. S. Houthakker, "The Pareto Distribution and the Cobb-Douglas Production Function in Activity Analysis," Review of Economic Studies, 23(1)60 (1955-1956) 27-31.

4. D. Levhari, "A Note on Houthakker's Aggregate Production Function in a Multifirm Industry," Econometrica, 36.1 (1968) 151-154.

5. D. D. McCracken and W. S. Dorn, Numerical Methods and FORTRAN Programming, New York: John Wiley & Sons, 1964.

6. R. M. Solow, "Some Recent Developments in the Theory of Production," in M. Brown (ed.), The Theory and Empirical Analysis of Production, New York: Columbia University Press, 1967.

Chapter 3
RESOURCE ALLOCATION OPTIMALITY:
THE BRANCH LIBRARY PROVISION FUNCTION

Problem, Method, and Hypotheses

The central concern of this chapter is the optimal allocation of resources within twenty of the branch libraries in the Oakland Public Library branch system. The analysis of Oakland's branch library activity is done from the point of view of a branch library manager who is in charge of only the economic aspects of providing library services. The focus is on an economic definition of an optimum, the economic conditions that have to be satisfied, and the economic criteria for evaluating alternative allocations. This narrow and limited focus on such a broad problem should always be kept in mind when drawing conclusions about any of the analytical results. What is not included is just as important as what is included.

This entire chapter is built around the programming framework. It is used to examine the resource allocation problem, to build an analytical model of the allocation process, to estimate the parameters of the model, and to measure the optimality of actual allocations.

The basic analytical tool used to examine branch library resource allocation activities is the economic notion of a production function. In its most idealized and general form, the neoclassical production function expresses the maximum possible output technologically obtainable by transforming a given set of inputs by some designated process. The previous chapter dealt with the aggregate production function, which describes the activity of the branch library system as a whole and says nothing about any individual branch's production function. This chapter deals with the branch library production function, which describes the activities of any individual branch and says nothing about the system as a whole. This branch library production function contains a wealth of information that the Oakland branch library mangers would find helpful for answering resource allocation questions such as the feasible capacity level of library services, the optimal levels of input services, and the expected minimum cost.

One way of answering these questions is to set the resource allocation problem in a programming framework. The basic programming approach looks at allocation activities as a program that has objectives and that has constraints on achieving those objectives. Among all feasible ways of satisfying all the constraints, the one that best achieves the objectives is the optimal solution to the program. An analytical model of resource allocation activities can be built around this framework, and the model provides a theory of measuring resource allocation optimality.

The first step in measuring the optimality is to estimate the parameters of the branch library production function. The production function definition given above is exactly the statement of a programming problem, which suggests linear programming as a possible estimation method.* It is preferable to regression methods because the most

---

*Aigner and Chu [1] discuss in more detail the relationships, interpretations, and applications of mathematical programming and production functions.

sophisticated regression techniques can estimate a production function that expresses only the average output and not the maximum output. Figure 3.1 illustrates this difference. Given a set of input and output data points, a regression production function goes through the middle while a linear programming production function goes over the top

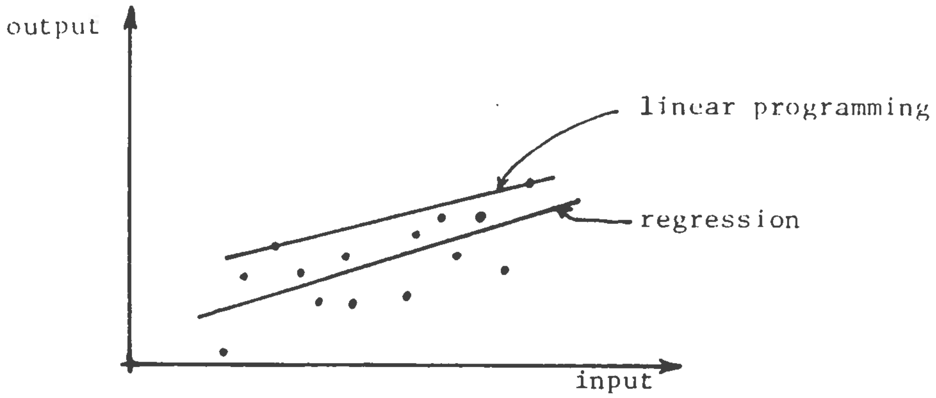

Figure 3.1 -- The Difference Between Linear Programming and Regression Production Function Estimation

so that it is a least upper bound or production frontier concept. Branches that actually achieve this frontier have lower technical efficiency. While the average is of interest, the frontier concept is clearly more useful for determining resource allocation optimality. The linearity feature of linear programming is no more of a problem than the linearity feature of regression, and the computation procedures are no more or no less difficult. From a theoretical standpoint, linear programming is definitely superior. From a computational standpoint, the two estimation methods are of equal difficulty. From an implementation standpoint, neither method is necessarily better. As Table 3.1 shows, the choice depends on the kind and magnitude of error in the estimation sample data set. Manager's error is the primary error that the branch library production function is designed to measure. Small manager's error would nullify the selective powers of linear programming. Relatively

Table 3.1 -- Which Estimation Method to Use If an Error is Large or Small

Error	Large	Small
Manager's Error	Linear Programming	Regression
Random Error	Regression	Linear Programming
Measurement Error	Regression	Linear Programming

large random and measurement errors introduce deviations that are not the fault of the managers. Linear programming would penalize or reward managers for appearing to be worse or better than they really are. Overall, if the sample data are tightly clustered in a line so that there are no clearly superior observations, regression would be preferable. If the data are more loosely scattered, linear programming is preferable. In the case of the sample data for this study, measurement error is nearly zero, and manager's error clearly overshadows random error so that linear programming is the more suitable estimation method.

The second step in measuring optimality is to compare the maximum output a branch library could be producing with the output it is actually producing. A branch's potential maximum output, or capacity, from its set of inputs is given by the estimated production function. The ratio of the actual output level to the capacity output level is the per cent of capacity utilized and is a measure of inefficiency due to excess capacity. The cost of this inefficiency is the resources allocated to hiring the extra inputs that generate the excess capacity. This cost reduces resource allocation optimality.

The third step in measuring optimality is to compare marginal product ratios with corresponding input price ratios. Assuming continuous input substitution, the actual capacity level could be achieved by hiring different combinations of inputs. There is one combination that costs the least. The ratio of this minimum cost to actual cost is a measure of inefficiency due to employing the wrong input mix. The difference between the actual and minimum costs is the wasted resources allocated to hiring the wrong relative amounts of inputs. This cost reduces resource allocation optimality.

At each step, it is possible to set up and test hypotheses about the nature of branch library activities in Oakland. The hypotheses at the estimation step are:

H1: A regional branch has the same capacity as a regular branch;

H2: The branch library production function exhibits constant returns to scale.

The evidence of the data shows no reason to accept H1 and no strong reason to reject H2. Regional branches have a lower capacity than other regular branches with the same inputs, and the branch library production function exhibits only slightly increasing returns to scale.

The hypothesis at the capacity utilization step is:

H3: The size of a branch library has no effect on its capacity utilization.

The evidence of the data shows no strong reason to reject H3. Contrary to the conventional wisdom, branches with a high capacity do not produce relatively more output than branches with a lower capacity. The general conclusion from testing H1, H2, and H3 is that Oakland's larger branch libraries are not necessarily more efficient than its smaller ones.

The hypotheses at the input mix efficiency step are:

H4: The book service price is not zero;

H5: The long run average deletion rate is about five per cent;

H6: The appropriate discount rate is zero;

H7: Input mix efficiency is low.

Input prices are required data for measuring input mix efficiency. The staff service price, \$624.47/man-month, presents no empirical problem, but the book service price does. As owner of the book stock from which book services flow, the library does not "buy" book services from itself at an explicit price. However, it is possible to devise an implicit book service price that the library could use to facilitate and improve resource allocation. This price depends on the deletion and utilization rates of the book stock, the discount rate opportunity cost of capital, the inflation rate of the price of new books, and the price of new books. The evidence of the data shows no reasons to reject H4 and H5 and no strong reasons to accept H6 and H7. Given the data on utilization rates, the inflation rate, and the new book price and assuming deletion and discount rates to be five per cent, the implicit book service prices range from \$ .66 to \$1.27 and average \$ .71/book-year. The resulting input mix efficiency measurements show that all branches employ relatively too many staff services or relatively too few book services. However, the lowest input mix measurement is only .84, the next seven are between .90 and .95, and the remaining twelve are between .96 and 1.00. These high measurements show that most branches are employing very nearly optimal input combinations. The combinations are optimal because they minimize cost for the existing capacity levels, but the capacity utilization measurements show that some branches have large excess capacity. Is demand too low or capacity too high? The model's limitations prevent it from answering this question.

The Resource Allocation Problem

To a library user, a branch library is a bundle of physical items: building, books, furniture, and librarians. He knows that when he goes to a branch he can engage in certain activities such as borrowing, reading, browsing, research, studying, and conversation. These consumption activities are available to him because of the particular composition of the bundle of input items that makes up that branch library. These inputs provide consumption opportunities, and these opportunities are the branch library outputs.* To fit this situation, the notion of a production function must be redefined. The usual definition implies the cause and effect relationship of inputs actively making outputs. However, libraries do not work that way. Librarians and books working together do not make anything, but they do provide opportunities for people to consume library services. As a reminder of this distinction, the term provision function is used in place of the term production function.

The objective of the idealistic librarian is to be able to provide

---

*These ideas are developed further by Lancaster [12], Baumol [5], and Quandt and Baumol [14].

all the library services demanded by all users. Since providing library services is not a free activity, since providing more services usually requires higher expenditures, and since monetary resources are not unlimited, this objective cannot be fully achieved. Librarians must content themselves with providing a more reasonable lower level of services. Given the level of services to be provided, the branch manager is charged with providing this level with minimum expenditures. Since inputs are not free, he is faced with the resource allocation problem: what combination of inputs will produce this level of services at minimum cost?

Many different inputs are used to provide library services. They can be conveniently grouped as staff services, book services, building services, furniture services, and other equipment services.* Of these, the analysis is concerned only with staff and book services. A number of factors lead to this decision. First, staff services and additions to the book stock are purchased regularly and comprise more than ninety-five per cent of a branch's operating expenditures. Utilities take up most of the remaining expenditures. Second, the owned buildings, furniture, and equipment are one-shot special expenditures. These stocks last a long time and are purchased neither regularly nor frequently. Third, because the Oakland Public Library owns these stocks, no explicit expenditure is required to employ their services. Fourth, because the Library pays no taxes, it has little incentive to and

---

*Notice that inputs are defined as the services provided by physical objects because inputs are flows while the source of these flows are stocks. The importance of this distinction becomes clear during a later discussion of the role of the book collection.

therefore in fact does not keep track of depreciation costs, interest costs, and capital loss costs. Fifth, the role that these stocks play in the provision process is to limit capacity. Since they are fixed in the short run they are relatively unimportant to a single period resource allocation problem. Finally, a model of the provision process that utilizes staff and book services inputs is sophisticated enough for testing many hypotheses about the optimality of resource allocations. Including more inputs would only make the analysis unnecessarily confusing. Moreover, any conclusions reached about the book stock can be extended to apply to the other stocks.

The market for books is such that a manager can purchase as many books as he wants without affecting the price of books. The prices are set by the publishers so that all buyers must accept them as given parameters. Managers must also accept as given parameters the prices of staff services because they are set by the Board of Library Directors and are subject to civil service regulations. Except for institutional constraints, a manager theoretically could purchase all the staff services he wants without affecting the prices. His resource allocation problem can be reduced to a decision on how many staff services and books to purchase.

An economically rational manager bases his decision on many signals. One of these is the price of the input services. In the case of staff services, the signal is explicit and unambiguous. A wage rate is assumed to reflect both the quantity and quality of staff services it can purchase. In the case of book services, the signal is implicit and very fuzzy. The implicit character stems from the fact that as owner of the book stock, which provides the flow of book services, the library charges itself a zero price even though the cost to the library of making the stocks available is positive. Three factors give rise to this positive cost. They can be better understood by pretending that library managers are also in the business of renting out their book stock. The rental charge would have to cover the cost of replacing worn out and obsolete items, the cost of the capital required to purchase the stock, and the cost of the loss of capital as the real value of money declines. These are the costs of depreciation, interest, and inflation all evaluated at the current price of new items. It is obvious that none of these costs is necessarily zero so that if the library managers charged a zero rent for the use of their book stock they clearly would be irrational if not foolish. Even if they rented their book stock to only themselves so that all payments and receipts would cancel, completely ignoring the existence of these costs could easily lead to a nonoptimal mix of staff and book services. If this implicit book service price's signal were clear, an optimal resource allocation would be much simpler to devise. Unfortunately, as mentioned earlier, the notions of depreciation, interest, and inflation are so totally alien to the library provision process that they can be given only very simplistic roles in the model, and how they might affect resource allocation evaluations can only be speculated.

The discussion of the price of book services introduces the factor of time, and its role in the model should be made clear. All variables are functions of time. Output provided, input services employed, and items purchased are all flows over time. Prices and stocks change over time, and the various parameters used in the model are rates of change over time. The passage from one time period to the next makes itself felt through the identity of the closing and opening inventories of book stock so that the stock provides an intertemporal link. Since the closing inventory is what a manager must start the next time period with, he has an interest in maintaining the quality and quantity of the book stock. This added complication to devising an optimal resource allocation arises because the process of providing services wears out the stock and the passage of time makes it more obsolete so that additions have to be purchased regularly.

The optimal book stock net change rate depends on the prices of the inputs, the demand for library services, and the size of the library building. Output capacity must be sufficient to provide for demand, and capacity is partially determined by the size of the stock. A change in the relative prices of the inputs affects the optimal amounts of required inputs, which in turn affects the optimal size of the book stock. The size of the building enters in because branch library services are provided from a building that encloses a finite amount of space. Since the book stock requires space, the time will eventually come when additions must be at least matched by deletions. Given the optimal book stock net change rate and the deletion rate, the manager must purchase at least enough additions to insure that the stock is maintained.

A Resource Allocation Model

The task now is to assemble the parts of the resource allocation problem into a model of the programming mold. At the end of each one year time period the manager must calculate the budget he requires for the coming year. His list of information includes the closing book stock inventory, the prices staff and book services are selling at, the demand for library services, the optimal book stock net change rate, the discount and inflation rates, the book stock deletion and utilization rates, and expert estimates of the available technology and the productivity of the staff and book services he can purchase. On the basis of this information list, he must determine how many staff services inputs and book stock additions he will need to purchase. To emphasize the importance of the time factor, it is assumed that the manager orders and receives items throughout the year, but he does not pay his bills until the end of the year. The amount of monetary resources he needs at the beginning of the year is the discounted present value of the estimated expenditures he will actually make at the end of the year.

The manager's resource allocation problem can be formalized into the general programming problem of finding the amount of staff services and book stock additions that will minimize the discounted present value of expenditures subject to the constraints that provision capacity is sufficient to provide for all the services demanded and that the book stock is maintained sufficiently into the following year.* The problem is formalized further by assuming mathematical functions for each of the parts. Given the above information list and some

*The formal structure of this problem is based on the capital theory work of Jorgenson [10].

further conditions on the functions, it is possible to calculate the optimal resource allocation for each branch library, which can then be used as a standard to evaluate the actual allocations selected by a manager and to indicate how the allocation might be improved.

The resource allocation programming problem can be solved with a Lagrange function, a standard tool of economists with a well developed theory behind it. Assuming that all functions are twice differentiable, the problem is to minimize a quasi-convex function subject to quasi-convex constraints. The Arrow-Enthoven [3] extension of the Kuhn-Tucker theorem applies so it is not necessary to check the second order conditions to insure that there is a saddle point minimum. The remainder of this section discusses the objective function and the two constraints, sets up the appropriate Lagrange function, solves it for the structural form of the model, and finally derives the reduced form.*

For each branch, the objective is to minimize the discounted present value of expenditures. Total expenditures, E, is the sum of expenditures for staff services, S, and for book stock additions, A: $E(t) \equiv w(t)S(t) + x(t)A(t)$, where w is the price of one equivalent man-month of staff services, and x is the price of one book. Let r be the annual discount rate. The discounted present value of total expenditures t years hence is

---

*To economize on notation when no confusion will arise, the argument t, which stands for time, is left out since all variables are functions of time. Partial derivatives of functions are denoted by the name of the function subscripted by the argument with respect to which the differentiation is being taken. Derivatives with respect to time are denoted by a dot above the variable name. Appendix 7 contains a glossary of variable names.

$$(\frac{1}{1+r})^t \ [wS + xA] \ .$$

The case where $t = 1$ is the only one of interest here, but the generalization helps show how t figures in the derivations that follow.

Given this objective, there are two constraints. The first is that each branch library must have at least enough capacity, Q*, to provide the services demanded, Q. Capacity is a notion that implies the fullest use of inputs so that when a demand is made for services, the library is able to provide them. Capacity depends upon the levels of staff and book services, B, and the manner in which they work together. This is all summarized in the provision function; $Q^*(t) \equiv F[S(t),B(t)] = qS^aB^b \geqq Q(t)$, where q, a, and b are parameters to be estimated. The explicit form of the provision function embodies the technology of the provision process. The form popularly known as Cobb-Douglas [7] is chosen because it is general and manageable enough for empirical work, and sophisticated enough for testing hypotheses. A more sophisticated expression is not warranted by the experimental character of the assumptions and data.

The second constraint is that each branch is required to maintain its book stock into the next year. The difference between additions and deletions must not be less than the required net change, N, of the book stock, V: $N(t) \leqq A(t) - y\, V(t)$, where y is the book stock annual deletion rate. The level of book services provided by the stock is proportional to the size of the stock, where the proportionality factor is the annual utilization rate, u, of the stock: $B(t) = u\, V(t)$.

The various parts of the model can now be assembled into the programming problem of minimizing the appropriate Lagrange function:

$$L(S,B,N,A;m,n;t) = (\frac{1}{1+r})^t \; [\; wS + xA \;] + m \;[\; Q - F(S,B) \;] + n \;[N - A + yB/u] \;,$$

where m(t) and n(t) are Lagrange multipliers. The multiplier m may be interpreted as the extra expenditure required to provide for sufficient capacity if the quantity of services demanded increases by one unit. The multiplier n may be interpreted as the extra expenditure required to meet a one unit increase in the book stock net change. Assuming that all of the expressions are twice differentiable the necessary conditions for a saddle point minimum of the Lagrange function are:

$$L_S = (\frac{1}{1+r})^t w - m\, F_S \geqq 0,$$

$$L_B - \dot{L}_N = -m\, F_B + n\, y/u - \dot{n} \geqq 0,$$

$$L_A = (\frac{1}{1+r})^t x - n \geqq 0,$$

$$L_m = Q - F(S,B) \leqq 0,$$

$$L_n = N - A + yB/u \leqq 0,$$

$$SL_S + BL_B + N\dot{L}_N + AL_A + mL_m + nL_n = 0.$$

These expressions constitute the structural form of the model. They set out the explicit relationships among the variables, both endogenous and exogenous, and summarize the theory behind the model. Using simple algebra, it is possible to derive the reduced form of the model where the endogenous variables, S, B, A, m, n, are expressed as functions of only the exogenous variables, w, x, Q, N, y, u, r, and the parameters q, a, b.

Assuming that the first three conditions hold with strict equality and that t = 1, n can be set equal to $(\frac{1}{1+r})x$ so that $\dot{n} = (\frac{1}{1+r})\dot{x} - (\frac{1}{1+r})x \ln(1+r)$. Substituting, $(\frac{1}{1+r})x(y/u + \ln(1+r) - \dot{x}/x) = m\, F_B$.

Combining the two marginal productivity conditions gives the familiar equilibrium condition that the marginal rate of technical substitution of book for staff services, MRS_{BS}, must equal the ratio of their prices:

$$MRS_{BS} \equiv F_B/F_S = \frac{x[y/u + \ln(1+r) - \dot{x}/x]}{w}$$

The price of staff services is already known to be w. It is now seen that the price of book services is the expression $x(y/u + \ln(1+r) - \dot{x}/x)$, which is hereafter referred to as p. This price is the implicit shadow price mentioned earlier. It is the cost to the library of employing one unit of book services for one year.

The marginal products of the provision function are $F_B = bQ^*/B$ and $F_S = aQ^*/S$ so that $\frac{bS}{aB} = \frac{p}{w}$. Substituting back into the provision function yields the derived demand functions, $D^S(Q,w,p)$ and $D^B(Q,w,p)$, for the to input services:

$$D^S(Q^*,w,p) = [\frac{Q^*}{q} (\frac{ap}{bw})^b]^{1/s} ,$$

$$D^B(Q^*,w,p) = [\frac{Q^*}{q} (\frac{bw}{ap})^a]^{1/s} ,$$

where s = a + b, which is the degree of returns to scale of the provision function. Solving for m and A yields the marginal expenditure function, M(Q,w,p), and the derived demand function for new books, G(N,Q,w,p):

$$M(Q^*,w,p) = [\frac{Q^{*1-s}}{q} (\frac{w}{a})^a (\frac{p}{b})^b]^{1/s} ,$$

$$G(N,Q^*,w,p) = N + \frac{y}{u} [\frac{Q^*}{q} (\frac{bw}{ap})^a]^{1/s} .$$

The definition of the cost, C, of input services is introduced here: $C \equiv wS + pB$. While it is not too different, cost should not be confused with expenditure because the cost of book services, pB, is not the same as the expenditure for book stock additions, xA. The notion of cost will prove useful later on when evaluating resource allocations. A cost function, H(Q,w,p), which is the dual of the provision function [Shepherd, 15, and Uzawa, 16], can be derived by substituting the derived demand functions D^S and D^B for S and B in the cost equation so that

$$H(Q^*,w,p) = s\ [\frac{Q^*}{q}\ (\frac{w}{a})^a\ (\frac{p}{b})^b\ ]^{1/s} .$$

There are several conditions the provision function should satisfy. The first condition is the intuitively reasonable one that zero amounts of inputs result in zero provision capacity: $F(0,0) = 0$. The second condition is that as the amount of one input increases, the amount of the other input remaining the same, the rate of change of provision capacity is nonnegative: $F_S \geqq 0$, $F_B \geqq 0$. In economic terms, this is the condition that marginal products be nonnegative. With positive levels of inputs, this condition can be met if and only if q, a, b $\geqq 0$. The third condition imposes the additional constraint on the marginal products that they must increase at a decreasing rate: $aQ^*(a-1)/S^2 = F_{SS} \leqq 0$, $bQ^*(b-1)/B^2 = F_{BB} \leqq 0$. This is the familiar law of diminishing marginal returns, and it is satisfied if and only if a,b $\leqq 1$. Together, the second and third conditions require $0 \leqq a,b \leqq 1$.

The parameter associated with each input is the elasticity of provision capacity with respect to the input, which is defined as the per cent change in capacity from a one per cent change in the input. Being a ratio of per cents, a provision elasticity is a unitless number that indicates an input's relative importance in influencing capacity changes. The value of the elasticities are of further interest because the provision function is homogeneous. Multiplying both inputs by some positive number, h, increases the value of the function by h^s, where $s = a+b$ is the degree of homogeneity. This sum is of interest because the provision function is said to have increasing, constant, or decreasing returns to scale as $s \gtreqless 1$. The returns to scale of the activity is a bit of useful information because costs decrease, are constant, or increase as $s \gtreqless 1$ [Bassett, 4]. This means that any given uniform increase of inputs results in corresponding responses of cost and provision capacity that are proportionately greater, the same, or less.

<u>The Behavioral Assumptions</u>

Having assembled some mathematical functions into a resource allocation model, the behavioral assumptions that are behind the functions need to be made explicit. The definition of total expenditures is straightforward and needs no elaboration, but the objective function involves the discount rate, r, which calls for some explanation. The discount rate is specified as a parameter independent of time. If the situation required a multiperiod expenditure minimization objective, the correct specification would be a different discount rate for each time period. However, the Oakland branch library situation requires only a one period objective so that using a single valued r is not incorrect. Another curious feature of the discount rate is its value.

While r can be thought of as an ordinary interest rate, it could also be a municipal bond current yield rate, a market current yield on privately invested capital, the opportunity cost of public rather than private expenditures for library services, or some even more complex synthetic social discount rate. An argument for or against any particular definition or value of r is beyond the scope of this study. The point to be made is that the discount rate has a role in the resource allocation problem.

The provision function indicates the maximum amount of services that can be provided by employing a given amount of input services: $Q^* \equiv F(S,B)$. By definition, this maximum amount is at least as great as the actual amount provided: $Q^* \geqq Q$. The difference between these two figures is a measure of a branch's inefficiency in providing library services. It is a measure of inefficiency in the sense that with the inputs being employed more services than the actual amount could have been provided.

Four possible causes of this inefficiency suggest themselves. First, the conventional wisdom says that larger branch libraries provide more and better services than smaller ones. These supposedly inherent scale qualities imply that larger branches utilize relatively more of their provision capacity so that just being larger will result in higher capacity utilization.

Second, the library manager may be over employing inputs even when allowing for contingency provision ability to meet unexpectedly heavy demand. Assuming that the estimate of expected demand is reasonably accurate, the manager underestimates the technological possibilities and the productivity of his inputs so that he hires too much inputs, which causes Q* to rise above Q. Productivity depends on both the quality and quantity of an input, and it is more likely that errors would be made in quality rather than quantity estimates. Being in charge of running the provision process, the manager should be an expert in evaluating the technology and the productivity of his inputs so that this cause of underutilized capacity can only be attributable to poor management skills.

Third, the demand for branch library services is overestimated to be at the Q* level. Being ready with what the manager thinks is the correct capacity, the expected users do not all show up so that Q falls below Q*, and capacity is underutilized. Estimating demand requires knowing the demand functions of all potential users. By their own admission, professional librarians do not know enough about this subject so that it would not be surprising to find inaccurate demand estimates.

Fourth, the effect known as Wagner's "law of increasing expansion of public, and particularly state, activities" [Wagner, 17] causes provision capacity to be set independently of demand. The law justifies expenditures for providing library services in terms of objective criteria, like population, rather than in terms of a theory of value. Many of the American Library Association standards [2] are worded in terms of so many books per so many population, so many branches per so many thousand population, and so on. Given that these standards are being followed, a Wagner's law effect would explain the insensitivity of capacity to output levels. In other words, the factors that determine capacity are not the factors that determine output. Moreover, there is an upward bias to capacity and a resistance to decline so that it would not be surprising to find large gaps between capacity and output.

Each of these possible causes can be supported by evidence found in the Oakland branch library system. However, there is no strong evidence that any single cause dominates the others. The resource allocation model is not capable of shedding too much light on the causes of low or high capacity utilization. It can only measure the amount of resources wasted due to this inefficiency.

Treating the deletion rate as a parameter is a strong assumption that implies a constant rate of decrease of the book stock. One interpretation is that each volume is treated uniformly, and all are deleted at the same rate. This is certainly not a true description of the deletion process. An alternative interpretation is that only the obsolete and worn out volumes are deleted while the newer ones remain fully untouched in the stock. The factors that influence the deletion rate are the size of the stock relative to the holding capacity of the building, the physical condition of individual books, and the "productivity" of individual books. Notice that only the first factor relates to the stock as a whole, while the other two relate to individual books. This means that unless the holding capacity is being reached, during any particular time period there is no cause for deletions just for the sake of deletions. If the deletion decision is made individually for each book, there is no assurance that the total number of deletions will add up to any predetermined figure. This kind of possibility casts doubt on the way deletions have been specified in the model. However, the fact that the book stock has a productive life-time that is greater than one period makes the deletion process a long-run continuous activity. While any annual deletion rate time series may show fluctuations, the average over time will tend to be constant. This long-run average is the correct interpretation of the parametric deletion rate.

If additions less deletions keep filling the library, the book holding capacity of the building will eventually be reached. At this overflowing point, it is obvious that the book stock net change, N, will hover close to zero. This limiting situation requires a mutual adjustment between the number of additions and the deletion rate to maintain the required book stock zero net change. The obvious question here is when is a library filled to the brim with books? The book density appears to be a functionally determined constant arrived at by trial and error. It is obvious to any library user whether a branch has too few books or too many, but at what point the number becomes too few or too many is not obvious. As a crude attempt to come up with a number, Table 3.2 shows the books per square feet of floor space for twenty Oakland branch libraries. In only two cases, Dimond and Lockwood, is the density over ten books per square foot, and these branches both have the appearance of being cramped for space. On the other hand, Laurel, Park, Piedmont, and Rockridge have densities between nine and ten and they appear to be filled, but not overcrowded. The rule of thumb for determining the limiting book stock size is ten times the square feet of floor space.

When a branch library's book stock is far from its holding capacity, it is not at all obvious what the optimal net change is. However, since the book stock is the source of the flow of book services, B = uV, and

TABLE 3.2 -- BOOK STOCK CHANGES AND HOLDING CAPACITY

	Book Stock Net Change N	Additions A	Deletions vV	Book Stock V	Square Feet of Floor Area f	Books/Square Foot V/f
BAYMONT	346	666	320	6112	1662	3.7
INA D. COOLBRITH	278	670	392	12159	2624	4.6
DIMOND	799	1147	348	19491	1692	11.5
EASTMONT	721	844	123	10760	1757	6.1
ELMHURST	1367	1591	224	16088	3228	5.0
GIBSON	-1061	135	1196	7351	1080	6.8
GLENVIEW	-127	635	762	14300	2011	7.1
GOLDEN GATE	-1106	356	1462	11084	2184	5.1
LAKEVIEW	1013	1255	242	19566	2680	7.3
LAUREL	1029	1311	282	17052	1834	9.3
LOCKWOOD	391	577	186	10512	775	13.6
JACK LONDON	467	621	154	11821	3250	3.6
MELROSE	1685	1956	271	24251	2826	8.6
MONTCLAIR	1070	1375	305	15379	3906	3.9
PARK BOULEVARD	449	562	113	13071	1449	9.0
PIEDMONT	1	520	519	13120	1471	8.9
ROCKRIDGE	-30	292	322	15913	1656	9.6
SEQUOYAH-CHABOT	345	399	54	7531	908	8.3
TEMESCAL	727	1763	1036	13028	2021	6.4
WEST OAKLAND	1166	1404	238	9099	1443	6.3

the derived demand function for book services, $D^B(Q,w,p)$, expresses the optimal level of book services, $D^V = D^B/u$ should express the optimal book stock so that $N \equiv \dot{D}^V$, where $\dot{D}^V = D^V[\frac{a}{s}(\frac{\dot{w}}{w} - \frac{\dot{p}}{p}) + \frac{\dot{Q}}{sQ} - \frac{\dot{u}}{u}]$ and

$$\dot{p} = \dot{x}\ [y/u + \ln(1+r) - \ddot{x}/\dot{x}\ ] + x\ [\ \frac{u\dot{y} - y\dot{u}}{u^2} + \frac{\dot{r}}{1+r}\ ]\ .$$

The optimal value for N depends on changes in several factors, the most important of which are the demand for library services, the prices of purchased items, and the size of the stock. If $\ddot{x}$, $\dot{y}$, $\dot{u}$, and $\dot{r}$ are assumed to be insignificantly different from zero, the function is not so complex after all. The biggest unknown is the rate of change of demand, which is not necessarily more difficult to estimate than the level of demand itself.

Borrowing results from ahead and taking $\dot{w}/w = -.05$ and $\dot{p}/p = -.04$ (see Appendix 6), $\dot{D}^V = D^V\ [-.007 + \dot{Q}/Q]/1.05$ so that unless $\dot{Q}/Q > .007$, the optimal book stock net change would be negative. This conclusion is very tenuous and many factors have been assumed to be zero. Before this notion can be useful, this whole subject has to be restructured to allow the complex factors to be examined in greater detail. The present model is not designed for this purpose, and the simplicity of the results attest to this.

The reduced form of the model expresses each endogenous variable as a function of only exogenous variables. Taking the cost function $\overline{C} \equiv H(Q^*,w,p)$, how does $\overline{C}$ compare with $C = wS + pB$? Given the prices w and p, $\overline{C}$ is the minimum cost required to provide the Q^* capacity level. The feature of $\overline{S} \equiv D^S(Q^*,w,p)$ and $\overline{B} \equiv D^B(Q^*,w,p)$ is that they satisfy the equilibrium condition $MRS_{BS} = b\bar{S}/a\bar{B} = p/w$. $[\bar{B},\bar{S}]$ is the optimal input combination that provides Q* level of capacity because it is the least cost one: $w\bar{S} + p\bar{B} = \bar{C}$. Similarly, $\bar{A} \equiv G(N,Q^*,w,p)$ is the optimal level of additions to the stock because $w\bar{S} + x\bar{A} = \bar{E}$ is the minimum expenditure required to maintain book stock net change N and capacity level Q*. The differences $S-\bar{S}$ and $A-\bar{A}$ could be translated into the corresponding expenditure difference $E-\bar{E}$, which could be a measure of wasted resources. However, the corresponding measure $C-\bar{C}$ gives more insights into the possible causes of waste and inefficiency.

The duality of the cost and provision functions enables the cost functions to be inverted to obtain a reduced form capacity function:

$$Q' \equiv R(C,w,p) = q\left(\frac{C}{s}\right)^s \left(\frac{a}{w}\right)^a \left(\frac{b}{p}\right)^b .$$

Given w and p, the capacity function expresses the maximum level of library services that could be provided for a given level of cost. This relationship between the cost and provision functions will be useful when developing the theory of measuring resource allocation optimality.

One final note is that an alternative definition of m is marginal cost, H_Q, since $H_Q = M(Q,w,p)$, which again points to the similarity of cost and expenditure. It was mentioned earlier that returns to scale affected the relative changes of cost and provision. This can clearly be shown by observing that changes in marginal cost are positive, constant, or negative as a + b = s is less than, equal to, or greater than one:

$$H_{QQ} = M_Q = \frac{1-s}{s}\ \frac{m}{Q} \gtreqless 0 \quad \text{as } s \lesseqgtr 1.$$

Costs will correspondingly rise faster than, the same as, or slower than a rise in provision capacity.

The Theory of Measuring Resource Allocation Optimality

A branch library's resource allocation optimality can be measured by some very simple economic indexes, which are based on the provision function as the technological standard of the maximum output obtainable with a given set of inputs.* The indexes for each branch can be compared with those of other branches to evaluate relative performance. The indexes indicate where and how much inefficiency exists, but they cannot suggest by themselves what steps need to be taken to remedy the situation. The very simplicity of these economic indicators prohibits them from revealing all the causes of any given inefficiency. This limit on the information from these indicators must be kept in mind when examining the measurements.

The provision function is graphed in Figure 3.2 as a surface in the three dimensional space defined by book, staff, and library services.** All branch libraries are located somewhere in the space on or below the frontier surface, and the coordinates of a branch's position are its levels of B, S, and Q. From its position at [B,S,Q], straight "above" it on the frontier is the position [B,S,Q*], and the distance between the two positions is excess capacity, Q* - Q. In Figure 3.3, cutting the

---

*This analysis is in the spirit of Farrell [8] and Nerlove [13]. Feldstein [9] develops similar average indexes to measure the efficiency of British hospitals.

**The function is drawn as having constant returns to scale.

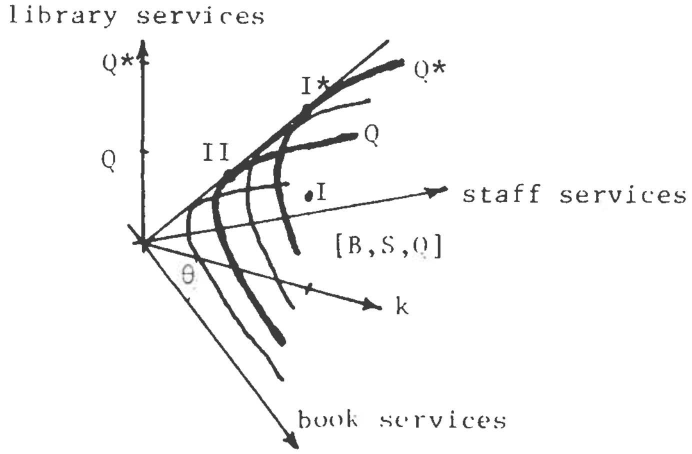

Figure 3.2 -- Provision Frontier

frontier with a plane parallel to the book-staff plane at level Q produces the familiar isoquant. In Figure 3.4, cutting the frontier by a plane through the origin and perpendicular to the B-S plane produces the provision frontier for all levels of inputs that maintain the ratio S/B = k, where k is the tangent of the angle the cutting plane makes with the B axis. In each diagram, a branch is at position I, and the optimality of position I is measured relative to other positions such as II or I*.

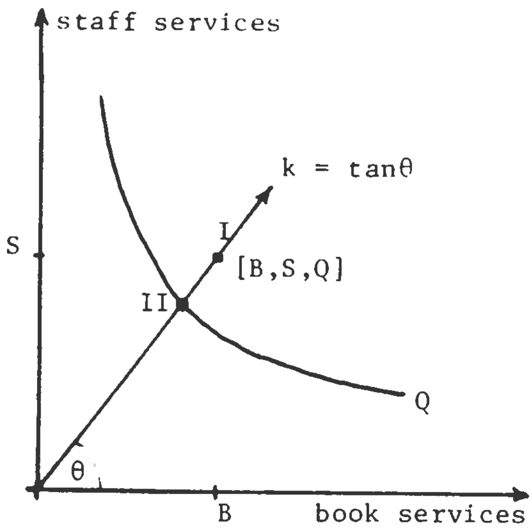

Figure 3.3 -- Isoquant

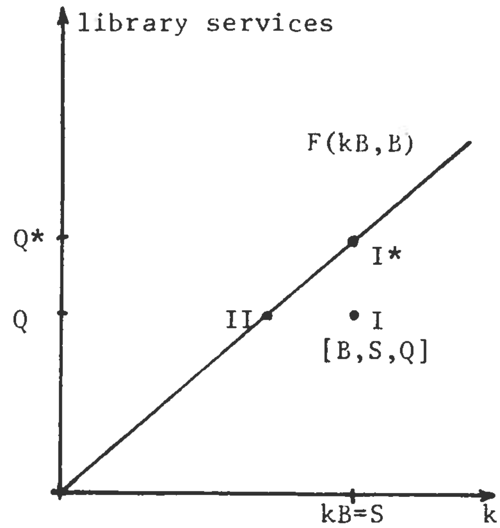

Figure 3.4 -- Provision Function

A key relationship in the theory is the isoquant and isocost tangency. A necessary condition for an optimal resource allocation is that the marginal rate of technical substitution of book for staff services must equal their price ratio: $MRS_{BS} \equiv F_B/F_S = p/w$. The negative of the slope of an isoquant, $\bar{Q}$, is also equal to the MRS_{BS}:

$$d\bar{Q} = F_S dS + F_B dB = 0 \ ,$$

$$- \left.\frac{dS}{dB}\right|_{\bar{Q}} = \left.\frac{F_B}{F_S}\right|_{\bar{Q}} \equiv MRS_{BS}\Big|_{\bar{Q}} \ ,$$

so that for optimality $\left.\frac{F_B}{F_S}\right|_{\bar{Q}} = \frac{p}{w} = - \left.\frac{dS}{dB}\right|_{\bar{Q}}$.

The slope of an isocost, $\bar{C}$, is the negative of the ratio of the input prices: $- \left.\frac{dS}{dB}\right|_{\bar{C}} = \frac{p}{w}$. In Figure 3.5, whether the aim is to minimize costs for a given level of capacity, $\bar{Q}$, or maximize capacity for a given level of costs, $\bar{C}$, the condition to satisfy is the mutual tangency of an isoquant and an isocost.

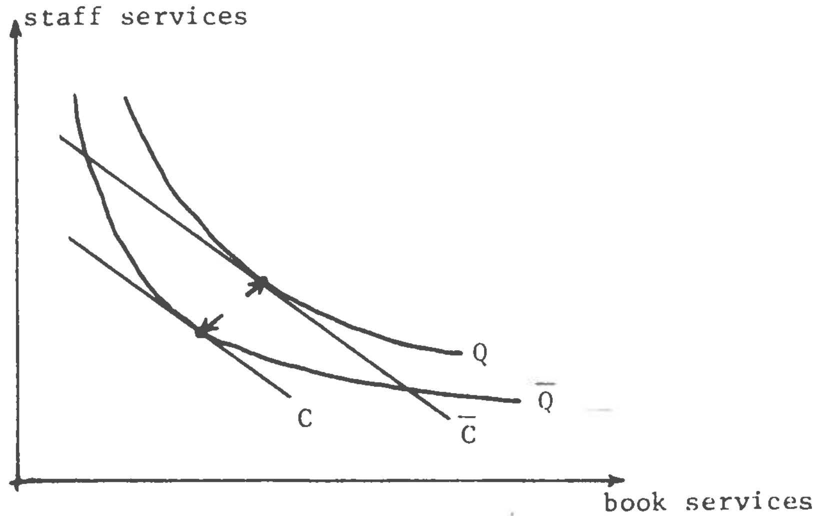

Figure 3.5 -- Isoquant and Isocost Tangent

The most obvious efficiency index is the ratio of actual services provided, Q, to the capacity level, Q*. Q/Q* measures the per cent of capacity utilization and indicates the relative amount of excess capacity at a branch. A low capacity utilization index can most easily be raised by bringing Q* down to Q, which is a shift from I to Q in Figure 3.3. A more desirable and more difficult way to eliminate excess capacity is to boost Q up to Q*, which is a vertical shift from I to Q* in Figure 3.4.

An alternative measure of capacity utilization efficiency is the ratio of the costs corresponding to Q and Q*. At position I, cost is C = wS + pB. One way of reducing Q* to Q is to reduce all inputs by the same proportions. In Figure 3.6, this is a shift along the ray k to position II, which would reduce cost to C* = wS* + pB*. Using the homogeneity property of the provision function, inputs and cost have to be reduced by the factor $(Q/Q^*)^{1/s} = h$.* The alternative capacity utilization measure is h = C*/C, which measures resources wasted on excess capacity.

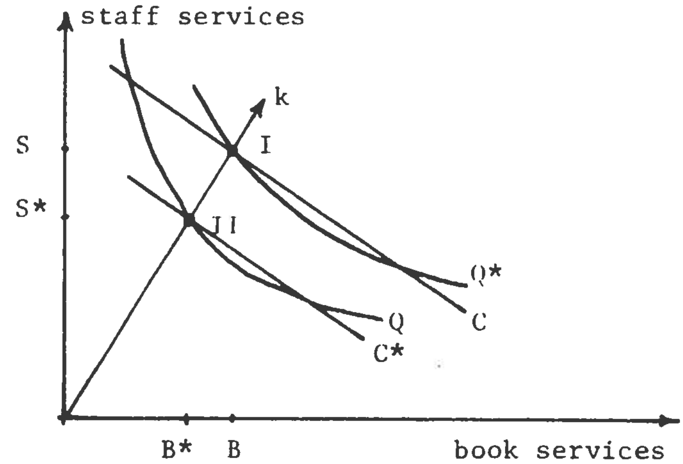

Figure 3.6 -- Capacity Utilization

Let S = hS and B* = hB so that C* = wS* + pB* = whS + phB = hC. If $Q^* = qS^aB^b$, then $Q = qS^{*a}B^{*b} = q(hS)^a(hB)^b = h^sQ^*$ so that $h = (Q/Q^*)^{1/s}$.

Capacity level Q* is obtained by hiring inputs S and B, which costs C = wS + pB. Because of input substitution possibilities, Q* can also be obtained by hiring any one of many other input combinations, and one of these is the optimal combination because it costs the least. The efficiency index of the actual combination [B,S] is the ratio of the minimum cost, $\bar{C}$, to the actual cost, C. $\bar{C}/C$ measures the input mix optimality. The resources C - $\bar{C}$ are wasted by hiring a nonoptimal input mix. In Figure 3.7, this waste can be eliminated by shifting from I* to III along Q*.

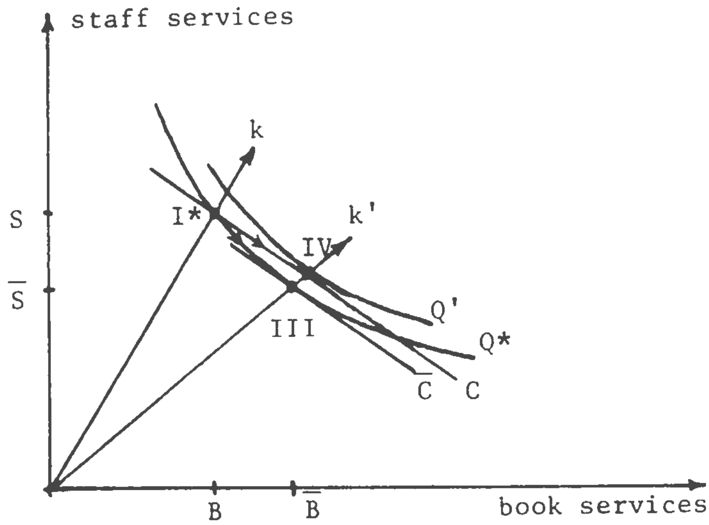

Figure 3.7 -- Input Mix

Again, an alternative measure of input mix efficiency is the ratio of the capacities corresponding to $\bar{C}$ and C. At position I* in Figure 3.7, cost is C = wS + pB. A nonmaximum capacity due to hiring a nonoptimal input combination can be boosted by substituting book for staff services until the combination shifts from ray k = S/B to ray k' = $\bar{S}/\bar{B}$. Using the reduced form capacity function, a shift from position I* to position IV along C will boost capacity to the maximum possible level, Q'. The alternative input mix measure is Q*/Q'.

A measure of <u>overall</u> provision efficiency is the ratio of the actual output level to the maximum possible output level, Q/Q'. This overall efficiency index takes into account both capacity utilization and input mix and is the simple product of the two indexes, Q/Q* x Q*/Q' = Q/Q'. The corresponding measure of overall costliness is the ratio C'/C = C'/C* x C*/C, where position V in Figure 3.8 shows that C' is the minimum cost required to provide Q level of output and is analogous to $\overline{C}$.

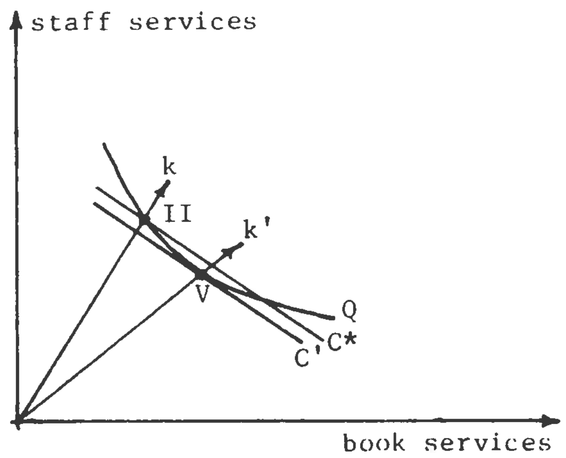

Figure 3.8 -- Minimum Cost

The relationship between the two sets of indexes is shown in Figure 3.9, which graphs the cost function for a given set of input prices.

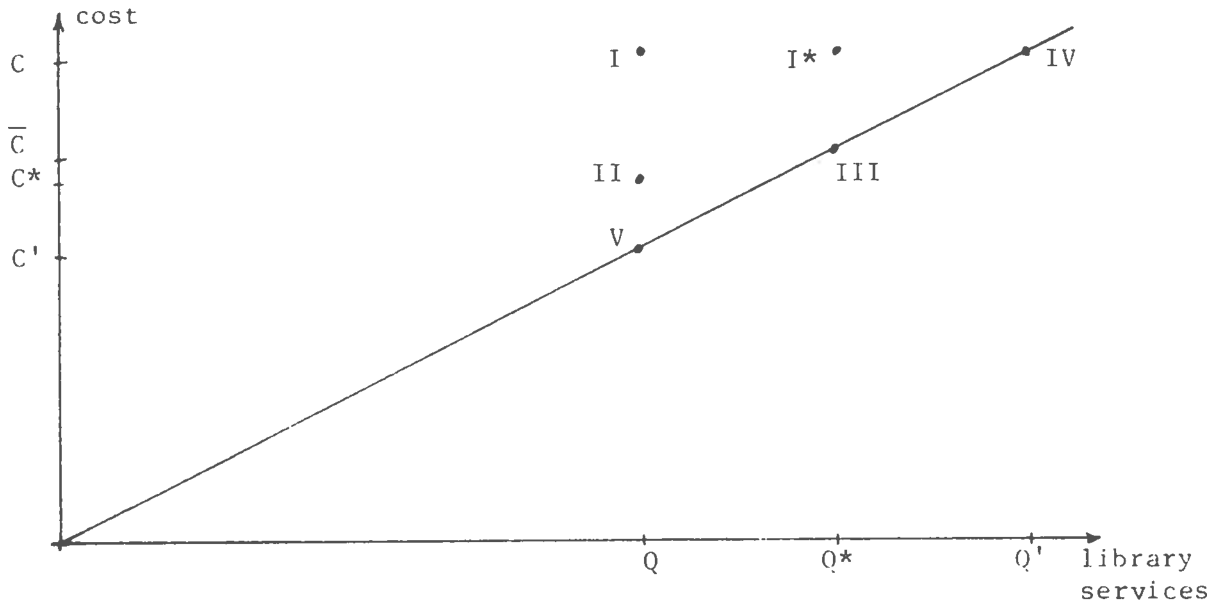

Figure 3.9 -- Cost Function

It has already been shown that $C^*/C = (Q/Q^*)^{1/s}$. Similarly, $C'/C^* = \overline{C}/C = (Q^*/Q')^{1/s}$ and $C'/C = (Q/Q')^{1/s}$. Since s denotes the returns to scale, the two sets of indexes do not have the same values unless s=1. However, no matter what the value of s, there is a direct correspondence between the two sets of indexes so knowing one set is as good as knowing the other.

A Linear Programming Estimation of the Provision Function

The first step in the empirical investigation of resource allocation optimality at twenty branches of the Oakland Public Library branch system is the estimation of the provision function parameters q, a, and b. The theory developed in the previous section and the definition of the provision function suggest an estimation procedure. The provision function defines the best that a manager of library j can do with his input combination $[B_j,S_j]$. From this follows the operational objective of minimizing the difference $Q^*_j - Q_j$. In other words, the aim is to be on the frontier. As always, the manager must contend with the constraint that $Q^*_j \geqq Q_j$. For the branch system as a whole the operational objective is to minimize the sum, over all branches, of the differences between actual and capacity levels of library services: minimize $\Sigma[Q^*_j - Q_j]$. The constraints on this objective are that $Q^*_j \geqq Q_j$ for each branch. This is a general programming problem, and for estimation purposes it can be transformed into a linear programming problem. Linear programming is the simplest kind of programming, and it is used here because more sophisticated nonlinear methods go far beyond the crude level of this particular model. The transformation is accomplished by taking the logarithm of the provision function, which yields the required linear functions: $\ln q + a\ln S + b\ln B = \ln Q^* \geqq \ln Q$. The objective function is the sum over all twenty branches of the differences $\ln Q^*_j - \ln Q_j$. The constraints are that the inequality $\ln q + a\ln S_j + b\ln B_j \geqq \ln Q_j$ must hold for every branch and that $a,b \geqq 1$. The sum a + b is left unconstrained so that the data will indicate what the returns to scale are. This all can be summarized as a standard minimum program to find $\ln q$, a, $b \geqq 0$ such that

$$\sum_{j=1}^{20} [\ln q + a\ln S_j + b\ln B_j - \ln Q_j]$$

is a minimum subject to the constraints

$$\ln q + a\ln S_j + b\ln B_j \geqq \ln Q_j, \text{ for all } j=1,\ldots,20,$$

$$-a \geqq -1,$$

$$-b \geqq -1.$$

The objective function can be rewritten as $20\ln q + a\Sigma\ln S_j + b\Sigma\ln B_j - \Sigma\ln Q_j$. Since $\Sigma\ln Q_j$ is a constant that does not involve any of the unknowns, it can be left out of the objective function without affecting the solution.

The basic estimation problem must be modified slightly to allow for the special character of the five regional branch libraries in the sample. In the Oakland system, regional branches have higher level professional staff and more specialized and larger book collections. These additional inputs increase cost and capacity, but do not necessarily affect efficiency. To avoid penalizing or rewarding regional branches for their extra capacity, a dummy variable is used to account for their special situation. The effect of the dummy variable is to alter the provision capacity by a multiplicative factor that is absorbed into the provision function constant so that the actual estimated provision function is $qe^{zZ}S^aB^b = Q^* \geqq Q$. The dummy variable Z takes on the value 1 if a branch is a regional one and 0 otherwise, and z is another parameter to be estimated. In the linear programming formulation of the problem, the term z must be added to each regional branch constraint, and a 5z must be added to the objective function.

One further modification must be made to the estimation program. It is possible for q and z to be negative. This minor computational point can be handled by adding the terms $-20\ln q^*$ and $-5z^*$ to the objective function, adding the term $-\ln q^*$ to all the constraints, and adding the term $-z^*$ to the regional branch constraints. The simplex algorithm is left free to select the most appropriate values for the provision function constant and the regional branch constant. In matrix notation, the final estimation program is to find

$[\ln q \;\; \ln q^* \;\; z \;\; z^* \;\; a \;\; b] \geqq 0$ such that $[20 \;\; -20 \;\; 5 \;\; -5 \;\; \Sigma \ln S_j \;\; \Sigma \ln B_j] \begin{bmatrix} \ln q \\ \ln q^* \\ z \\ z^* \\ a \\ b \end{bmatrix}$

is a minimum subject to $\begin{bmatrix} 1 & -1 & 0 & 0 & \ln S_1 & \ln B_1 \\ . & . & . & . & . & . \\ . & . & . & . & . & . \\ . & . & . & . & . & . \\ . & . & 1 & -1 & . & . \\ . & . & . & . & . & . \\ . & . & . & . & . & . \\ . & . & . & . & . & . \\ 1 & -1 & 0 & 0 & \ln S_{20} & \ln B_{20} \\ 0 & 0 & 0 & 0 & -1 & 0 \\ 0 & 0 & 0 & 0 & 0 & -1 \end{bmatrix} \begin{bmatrix} \ln q \\ \ln q^* \\ z \\ z^* \\ a \\ b \end{bmatrix} \geqq \begin{bmatrix} \ln Q_1 \\ . \\ . \\ . \\ . \\ . \\ . \\ . \\ \ln Q_{20} \\ -1 \\ -1 \end{bmatrix}$.

Every programming problem has a dual, and this estimation problem is no exception. The dual problem is to find the vector $[v_1 \;\; v_2 \;\; \ldots \;\; v_{20} \;\; v_{21} \;\; v_{22}] \geqq 0$ such that $[\ln Q_1 \;\; \ln Q_2 \;\; \ldots \;\; \ln Q_{20} \;\; -1 \;\; -1] \begin{bmatrix} v_1 \\ v_2 \\ . \\ . \\ . \\ v_{20} \\ v_{21} \\ v_{22} \end{bmatrix}$ is a minimum subject to

$$\begin{bmatrix} 1 & 1 \; . \; . \; . & 1 & 0 & 0 \\ -1 & -1 \; . \; . \; . & -1 & 0 & 0 \\ 0 \ldots & 1 \; . \; . \; . & 0 & 0 & 0 \\ 0 \ldots & -1 \; . \; . \; . & 0 & 0 & 0 \\ \ln S_1 & \ln S_2 \ldots & \ln S_{20} & -1 & 0 \\ \ln B_1 & \ln B_2 \ldots & \ln B_{20} & 0 & -1 \end{bmatrix} \begin{bmatrix} v_1 \\ v_2 \\ . \\ . \\ . \\ v_{20} \\ v_{21} \\ v_{22} \end{bmatrix} \leqq \begin{bmatrix} 20 \\ -20 \\ 5 \\ -5 \\ \ln S_j \\ \ln B_j \end{bmatrix} .$$

The interpretation of the dual is to maximize output subject to the constraints that the total number of branches and regionals not increase and that the total amounts of inputs used by the branch system not exceed the total amounts available. The dual variables, $v_1, \ldots, v_{20}, v_{21}, v_{22}$, are weights that redistribute the inputs among the branches. That is, they are the level at which each branch should be operating in order to achieve the optimal solution to the dual problem. By the complementary slackness theorem, at most four of the dual variables will be nonzero, and these will correspond to the most "efficient" branches that are on the estimated provision frontier. In linear programming terms, these branches constitute the basis of the optimal solution.

The estimation sample is based on the activities of twenty of Oakland's branch libraries during the year July, 1968, through June, 1969. Shown in Table 3.3, it is the same data discussed and used in Chapter 2, and the derivations of the output and input measures are exactly the same (see Appendix 1). Library services are measured by circulation equivalents: circulation equivalents = circulation + 2 x attendance at story hours + 25 x class visits + 25 x classes visited + 3 x hours open. Equivalent man-months of staff services is equal to total wages and salaries for the year divided by the average monthly wage rate, \$624.47. The flow of book-years of book services depends on the book stock and the utilization rate. Over the course of a year, the reasonable maximum number of hours a branch could be open is 3900 hours, and the per cent of this amount that a branch is actually open is the utilization rate.

The estimated provision function is $e^{5.64-.14Z} S^{.68} B^{.37} = Q^* \geq Q$. The four most efficient branches on this estimated provision frontier are Montclair, Piedmont, Sequoyah-Chabot, and Temescal, which is on the

TABLE 3.3 -- OUTPUTS AND INPUTS

	Circulation	Attendance at Story Hours	Class Visits + Classes Visited	Hours Open	Circulation Equivalents	Staff Services	Book Services	Book Stock	Utilization Rate
BAYMONT	52736	25	6	2658	60910	32.6	4166	6112	.682
INA D. COOLBRITH	39002	158	224	2330	51908	25.4	7264	12159	.597
DIMOND	87698	497	32	2998	98486	44.5	14983	19491	.769
EASTMONT	40941	0	27	2370	48726	28.5	6539	10760	.608
ELMHURST	59937	580	15	2978	70406	48.7	12285	16088	.764
GIBSON	12587	304	0	908	15919	7.0	1712	7351	.233
GLENVIEW	46184	53	5	2738	54629	36.7	10040	14300	.702
GOLDEN GATE	18657	95	11	2314	26064	28.5	6577	11084	.593
LAKEVIEW	106172	417	24	3223	117275	57.3	16170	19566	.826
LAUREL	85862	0	70	2994	96594	45.3	13091	17052	.768
LOCKWOOD	9102	38	0	1216	13226	14.7	3278	10512	.312
JACK LONDON	11852	463	21	2289	20170	28.4	6938	11821	.587
MELROSE	60259	63	24	2997	69976	44.0	18636	24251	.768
MONTCLAIR	123410	363	5	3236	133969	52.7	12761	15379	.830
PARK BOULEVARD	44825	42	3	2887	53645	31.7	9676	13071	.740
PIEDMONT	67484	0	21	2990	76979	26.5	10059	13120	.767
ROCKRIDGE	55838	85	0	2716	64156	30.9	11082	15913	.696
SEQUOYAH-CHABOT	32265	608	319	2150	47906	21.3	4152	7531	.551
TEMESCAL	80215	1028	156	2976	95099	44.5	9942	13028	.763
WEST OAKLAND	16301	295	16	2358	24365	31.3	5501	9099	.605

frontier because it is the most efficient regional branch. All other branches are below the frontier so that it is a least upper bound on the actual provision levels of the entire set of branch libraries.

Hypothesis 1: A regional branch has the same capacity as a regular branch. The negative value of the regional branch constant is the evidence against accepting H1. Regional branches have a lower capacity than other regular branches with the same inputs.

Hypothesis 2: The provision function exhibits constant returns to scale. The sum of the two provision elasticities is 1.05, which is not very strong evidence for rejecting H2. The provision function exhibits only very slightly increasing returns to scale.

Having obtained estimates for the provision function parameters, the quality of these estimates has to be examined. As with any estimation method, the estimated values of lnq, z, a, and b depend upon the particular sample of observations. Changing the values of some of the observed variables could change the estimates. Sensitivity analysis of the programming problem provides some indications of exactly how the estimates would react to certain changes. Since the estimates depend on which branches are in the basis, the focus of sensitivity analysis is on changing variables that would lead to a branch entering or leaving the basis. (See Appendix 2 for essential computations.)

How much more demand is required at any nonbasic branch before it can enter the basis? The values of the slack variables shown in Table 3.4 answer this question. Considering all the approximations built into the model, Gibson and Baymont could easily be in the basis and possibly Dimond too. For Jack London, West Oakland, Golden Gate, and Lockwood,

	lnQ	lnQ*-lnQ	Per Cent of lnQ
Baymont	11.02	.05	.5
Ina D. Coolbrith	10.86	.25	2.3
Dimond	11.50	.12	1.0
Eastmont	10.79	.35	3.2
Elmhurst	11.16	.44	3.9
Gibson	9.68	.02	.2
Glenview	10.91	.56	5.2
Golden Gate	10.17	.98	9.6
Lakeview	11.67	.14	1.2
Laurel	11.48	.24	2.0
Lockwood	9.49	.95	10.0
Jack London	9.91	1.25	12.6
Melrose	11.16	.53	4.8
Montclair	11.81	0	0
Park Boulevard	10.89	.47	4.3
Piedmont	11.25	0	0
Rockridge	11.07	.32	2.9
Sequoyah-Chabot	10.78	0	0
Temescal	11.46	0	0
West Oakland	10.10	1.04	10.3

Table 3.4 -- The Demand Increase Required Before a Branch Can Enter The Basis

	lnS	lnQ-lnQ* / a	Per Cent of lnS	lnB	lnQ-lnQ* / b	Per Cent of lnB
Baymont	3.48	- .07	- 2.1	8.34	- .14	- 1.6
Ina D. Coolbrith	3.23	- .36	-11.2	8.89	- .67	- 7.5
Dimond	3.80	- .17	- 4.5	9.62	- .32	- 3.3
Eastmont	3.35	- .51	-15.3	8.79	- .95	-10.8
Elmhurst	3.89	- .65	-16.7	9.42	-1.20	-12.7
Gibson	1.95	- .03	- 1.6	7.45	- .06	- .8
Glenview	3.60	- .83	-23.1	9.21	-1.53	-16.6
Golden Gate	3.35	-1.44	-42.9	8.79	-2.66	-30.2
Lakeview	4.05	- .21	- 5.1	9.69	- .38	- 4.0
Laurel	3.81	- .35	- 9.1	9.48	- .64	- 6.7
Lockwood	2.69	-1.40	-52.0	8.10	-2.58	-31.9
Jack London	3.35	-1.84	-55.0	8.85	-3.40	-38.4
Melrose	3.78	- .78	-20.7	9.83	-1.44	-14.7
Montclair	3.96	0	0	9.45	0	0
Park Boulevard	3.46	- .69	-19.9	9.18	-1.27	-13.9
Piedmont	3.28	0	0	9.22	0	0
Rockridge	3.43	- .48	-13.9	9.31	- .88	- 9.4
Sequoyah-Chabot	3.06	0	0	8.33	0	0
Temescal	3.80	0	0	9.20	0	0
West Oakland	3.45	-1.54	-44.6	8.61	-2.84	-32.9

Table 3.5 -- The Input Decrease Required Before a Branch Can Enter the Basis

	$\frac{\ln Q - \ln Q^*}{a+b}$	Per Cent of lnS	Per Cent of lnB
Baymont	- .05	- 1.4	- 5.8
Ina D. Coolbrith	- .24	- 7.3	- 2.7
Dimond	- .11	- 3.0	- 1.2
Eastmont	- .33	- 9.9	- 3.8
Elmhurst	- .42	-10.8	- 4.5
Gibson	- .02	- 1.1	- .3
Glenview	- .54	-15.0	- 5.8
Golden Gate	- .93	-27.9	-10.6
Lakeview	- .14	- 3.3	- 1.4
Laurel	- .23	- 5.9	- 2.4
Lockwood	- .91	-33.8	-11.2
Jack London	-1.20	-35.7	-13.5
Melrose	- .51	-13.4	- 5.2
Montclair	0	0	0
Park Boulevard	- .45	-12.9	- 4.9
Piedmont	0	0	0
Rockridge	- .31	- 8.9	- 3.3
Sequoyah-Chabot	0	0	0
Temescal	0	0	0
West Oakland	-1.00	-28.9	-11.6

Table 3.6 -- The Uniform Input Decrease Required Before a Branch Can Enter The Basis

the provision frontier is a long way off. An alternative way of reaching the frontier is to reduce the level of inputs to meet the demand. Holding one input fixed and reducing the other is one way of doing this. Table 3.5 shows the results. Another way is to reduce both inputs according to some rule. One convenient rule, which takes advantage of the homogeneity property of the provision function, is to reduce both inputs by the same proportions. Table 3.6 shows the results.

Turning to the branches in the basis, how much can demand vary before a branch is forced to leave the basis? Changing the demand of a basic branch will directly affect the value of the estimated parameters. For a branch to remain in the basis, such changes are limited by the requirement that the parameters be nonnegative. Looking at the results in Table 3.7, all parameters seem to be equally sensitive to the changes. The required ranges of variation are moderate for Montclair, Piedmont, and Sequoyah-Chabot, while the range for Temescal results from its special regional character. Taking into account the low value of

Basic Branch	Minimum Change	Critical Parameter	Maximum Change	Critical Parameter	Per Cent Variation	Per Cent Effective Variation
Montclair	-.427	z	.164	a	5	2
Piedmont	-.226	a	.336	b	5	3
Sequoyah-Chabot	-.564	lnq	.298	b	8	3
Temescal	$-\infty$	-	.136	z	∞	1

Table 3.7 -- Changing Basic Branch Demand

Gibson's slack variable reduces the limit on decreasing demand because decreasing it beyond -.022 automatically brings Gibson into the basis. Thus, the effective range of variation reduces to a fairly small per cent.

A final set of variables that can affect the basis is the aggregate supplies of inputs and the number of branches and regionals. This is exactly the same as the previous change, but involves working with the dual instead. The results in Table 3.8 indicate that Piedmont is the

Aggregate Supply	Negative Change	Leaving Branch	Positive Change	Leaving Branch	Per Cent Variation
Constant	- .586	Sequoyah-Chabot	.857	Piedmont	7
Regional	- 3.357	Montclair	5.000	Temescal	167
Staff	- 1.748	Montclair	3.145	Piedmont	7
Book	- 3.895	Piedmont	4.739	Sequoyah-Chabot	5

Table 3.8 -- Changing the Aggregate Supplies

most sensitive branch although Montclair and Sequoyah-Chabot are not that much less sensitive. Again, the special character of the regional branches contribute to the large variation range for the regional dummy variable sum, which has no special significance. The other ranges are moderate.

The general conclusion of the sensitivity analysis is that the estimated provision function could very easily be different. Small changes in a few variables would bring about a change in the set of basic branches. Gibson and Baymont seem like the most likely candidates for inclusion into the select group.

Measurements of Inefficiency

There are many possible reasons why a particular branch may have a high or low efficiency index. To analyze each branch individually would attribute to the indexes interpretative powers they do not possess. However, the indexes can reveal some insights into general patterns of inefficiency in the system. Since the capacity and cost sets of indexes are equivalent, all the relevant points can be made by referring only to the capacity set. However, this does not mean that costs are unimportant.

The values of the three different measures of output and capacity come from different sources. The actual level of output, Q, is data. The capacity level, Q*, is computed from the estimated provision function: $Q^* \equiv F(S,B) = e^{5.64-.14Z}\, S^{.68}\, B^{.37}$ for actual levels of staff and book services, S and B. The maximum capacity level, Q', is computed from the estimated reduced form capacity function: $Q' \equiv R(C,w,p) = e^{5.64-.14Z}$ $(C/1.05)^{1.05}\ (.68/w)^{.68}\ (.37/p)^{.37}$, where $C = wS + pB$ for actual levels of S and B, w = 624.47 (Appendix 3), and p has to be determined.

Hypothesis 3: The size of a branch library has no effect on its capacity utilization. What might cause excess capacity? The conventional wisdom is that larger libraries provide more and better services and, therefore, are more efficient than smaller ones. The result of these supposedly inherent scale qualities is that larger libraries would end up satisfying relatively more demand, which should show up in a relatively high capacity utilization index. Looking at the results shown in Table 3.9, there seems to be no strong relationship between efficiency and size. Measuring size by either actual level of services provided or capacity, large branches are not necessarily more efficient than small ones. Also, regional branches are not necessarily more efficient than the others. While as a group Oakland's large branches do better than the small ones as a group, there are enough outstanding exceptions to cast doubt on any strong generalization about efficiency and size. More evidence in favor of H3 is that the coefficient of correlation between Q/Q*

TABLE 5.5 -- CAPACITY UTILIZATION EFFICIENCY

	Output Q	Capacity Q*	Capacity Utilization Index Q/Q*
BAYMONT	50912	54055	.95
INA D. COOLBRITH	51969	6[illegible]746	.7[illegible]
DIMOND	96450	110687	.87
EASTMONT	48726	68628	.71
ELMHURST	70405	109352	.64
GIBSON	15919	16265	.98
GLENVIEW	54629	96042	.57
GOLDEN GATE	26064	69173	.38
LAKEVIEW	117275	135044	.87
LAUREL	96394	122152	.79
LOCKWOOD	13326	34182	.39
JACK LONDON	20170	70367	.29
MELROSE	65570	118948	.55
MONTCLAIR	137462	133069	1.03
PARK BOULEVARD	53665	85541	.63
PIEDMONT	75576	70978	1.06
ROCKRIDGE	64155	84598	.76
S[illegible]	47976	47906	1.00
TEMESCAL	65664	65093	1.01
WEST OAKLAND	24365	69119	.35

and Q is .61, and between Q/Q^* and Q^* it is .15*. This does not lend much support to the conventional wisdom that larger branches are better than smaller ones. There seems to be more to efficiency than mere size so there is no strong reason to reject H3.

Turning now to measuring input mix efficiency, the optimal input mix condition is that the marginal rate of technical substitution of book for staff services equals their price ratio: $MRS_{BS} = bS/aB = p/w$. Inputs' marginal productivities are a technological bit of information that managers should be able to gather on their own. Managers accept input prices as economic information determined elsewhere in the system. Only an incorrect book service price, p, can cause an incorrect price ratio because the staff service price, w, is explicit and common knowledge information. Before measuring input mix efficiency, the correct value for p has to be determined.

Hypothesis 4: The book service price is not zero. Since the book service price is implicit, a reasonable possibility is that the branch library managers naively assume it is zero. Being free inputs, there would be no budgetary constraint on the amount of book services a branch could hire. Every branch library would be expected to be filled to its limit with books. Since this conclusion does not conform to the facts of the situation, there is no reason to reject H4. The book service price is not zero.

The resource allocation model generates a theoretical expression for the implicit book service price: $p = x(y/u + \ln(1+r) - \dot{x}/x)$.

---

*The 95% confidence intervals are .04 to .83 and -.32 to .55 respectively.

The new book price, x, the utilization rate, u, and the inflation rate, $\dot{x}/x$, are items for which data is readily available. While data is also available for the deletion rate y, the one point observation is inadequate, but a reasonable value can be inferred. In the absence of any data, a similar tactic is employed to obtain a value for the discount rate r.

Hypothesis 5: The long run average deletion rate is about five per cent. The deletion rate enters into the optimal book stock net change constraint. The assumption that deletions are made at some constant rate turns out to be important for determining the implicit book service price. The argument for the parameteric assumption is that the deletion process is a long-run, continuous activity that averages out over many years to some given constant rate. This is especially true if the flow of additions does not fluctuate too wildly over the years so that the wearing out and obsolescence rates are stable. Assuming that this kind of stability exists, what is the appropriate long-run average deletion rate? The American Library Association standards explicitly sets five per cent as the lower bound, but then implies that a ten per cent rate would better reflect obsolescence.* Of the twenty branches in the sample, Table 3.10 shows that only Gibson and Golden Gate deleted more than ten per cent of their stock during the observation year. Baymont, Glenview, and Temescal deleted between five and ten per cent, and all others deleted less than five per cent, with most of them less than two per cent. If the ALA standard is the correct standard, then Oakland's branch libraries did not

*Standard 36i [ALA, 2].

delete enough books. While the cross sectional, one point observations of the actual deletion figures are interesting and informative, the long-run rate is what should go into the determination of the book service price. For this reason, the ALA five per cent figure has the most intuitive appeal, but the ten per cent figure cannot be rejected as outrageous.

Baymont	5.2	Lockwood	1.8
Ina D. Coolbrith	3.2	Jack London	1.3
Dimond	1.8	Melrose	1.1
Eastmont	1.1	Montclair	2.0
Elmhurst	1.4	Park Boulevard	.9
Gibson	16.3	Piedmont	4.0
Glenview	5.3	Rockridge	2.0
Golden Gate	13.2	Sequoyah-Chabot	.7
Lakeview	1.2	Temescal	8.0
Laurel	1.7	West Oakland	2.6

Table 3.10 -- Deletion Rate

Hypothesis 6: The appropriate discount rate is zero. The proper discount rate, r, is a much more slippery notion because it can have so many different empirical twists to the basic criterion of measuring opportunity cost.* Depending on the risk involved, the sector of the economy from which the funds are taken, the private investment that may have been displaced, the workings of the money market, the lifetime of

*See Kaimen [11] and Baumol [6] for a laymen's discussion of the issues.

the flow of services from the stock, the desired amount of intergenerational transfers, and any kind of social discount rate that may be attached to public provision of library services, the appropriate discount rate could vary from, say, three to twelve per cent. Since the Oakland Public Library does not explicitly use a discount rate, it would be unfair to evaluate it against an arbitrarily chosen value for r, and this study is not concerned with determining the correct rate. The important point is that resources would be more optimally allocated if the discount rate were made explicit and were not equal to zero. With this in mind, discount rates of five per cent and ten per cent are used to compute alternative values for the book service price.

Hypothesis 7: Input mix efficiency is low. Assuming a new book price of \$3.91 (Appendix 4), inflation rate of six per cent (Appendix 5), discount rates of five or ten per cent, and deletion rates of the actual values, five per cent, or ten per cent, several alternative book service prices can easily be computed. Each price determines a different maximum capacity level, Q', and therefore a different input mix efficiency measurement, Q*/Q'. Table 3.11 presents the evidence against H7. Input mix efficiency is uniformly high and increases as p increases. This relationship indicates that staff services are relatively too high or book services are relatively too low. In Table 3.12, comparing the actual input ratio S/B to the various corresponding optimal ratios verifies this high staff bias. Further evidence that staff services receive too much resources is to look at the division of cost between book and staff services. Selecting $p = 3.91(.05/u + \ln 1.05 + .06)$ as the most intuitively appealing explicit value for the implicit book service price, Table 3.13 shows that the staff services' actual

TABLE 3.11 -- INPUT MIX EFFICIENCY

	Q*/Q' For p=3.91(y/u+1n1.05+.06)	Q*/Q' For p=3.91(y/u+1n1.10+.06)	Q*/Q' For p=3.91(.05/u+1n1.05+.06)	Q*/Q' For p=3.91(.05/u+1n1.10+.06)	Q*/Q' For p=3.91(.10/u+1n1.05+.06)	Q*/Q' For p=3.91(.10/u+1n1.10+.06)
BAYMONT	.80	.84	[illegible]	[illegible]	.86	.80
INA G. COOLBRITH	.75	.77	.74	[illegible]	.76	.75
DIMOND	.45	[illegible]	.47	[illegible]	.59	.49
EASTMONT	.63	.64	.67	.60	.70	.70
ELMHURST	.55	.61	.61	[illegible]	[illegible]	[illegible]
GIBSON	.90	[illegible]	[illegible]	[illegible]	.90	.95
GLENVIEW	.55	.55	[illegible]	.56	.57	.57
GOLDEN GATE	.38	.38	.36	.37	.37	[illegible]
LAKEVIEW	.80	[illegible]	.84	[illegible]	.85	.87
LAUREL	.73	.73	.77	.78	.79	.79
LOCKWOOD	.36	[illegible]	[illegible]	[illegible]	.39	[illegible]
JACK LONDON	.26	.27	[illegible]	[illegible]	[illegible]	.29
MELROSE	.57	[illegible]	[illegible]	.58	[illegible]	.58
MONTCLAIR	[illegible]	[illegible]	[illegible]	.17	[illegible]	.20
PARK BOULEVARD	[illegible]	[illegible]	.61	[illegible]	[illegible]	.63
PIEDMONT	[illegible]	1.[illegible]	[illegible]	1.[illegible]	1.[illegible]	[illegible]
ROCKRIDGE	.72	.72	[illegible]	.72	.72	.72
SHELLEY [illegible]	[illegible]	.91	[illegible]	[illegible]	[illegible]	.90
TEMESCAL	[illegible]	[illegible]	[illegible]	[illegible]	.37	[illegible]
WEST OAKLAND	.31	[illegible]	[illegible]	.31	.34	.35

TABLE 3.12 -- OPTIMAL INPUT RATIOS

	Actual S/B	S'/B' For p=3.91(v/u+1n1.05+.06)	S'/B' For p=3.91(v/u+1n1.10+.06)	S'/B' For p=3.91(.05/u+1n1.05+.06)	S'/B' For p=3.91(.05/u+1n1.10+.06)	S'/B' For p=3.91(.10/u+1n1.05+.06)	S'/B' For p=3.91(.10/u+1n1.10+.06)
BAYMONT	7.8	2.1	2.7	2.1	2.6	3.0	3.5
INA D. COOLBRITH	3.5	1.9	2.4	2.2	2.8	3.2	3.7
DIMOND	3.0	1.5	2.1	2.0	2.5	2.8	3.3
EASTMONT	4.4	1.5	2.0	2.2	2.7	3.2	3.7
ELMHURST	4.0	1.5	2.0	2.0	2.6	2.8	3.3
GIBSON	4.1	9.3	9.9	3.7	4.3	6.2	6.8
GLENVIEW	3.7	2.1	2.7	2.1	2.6	2.9	3.4
GOLDEN GATE	4.3	3.8	4.4	2.2	2.8	3.2	3.7
LAKEVIEW	3.5	1.4	2.0	2.0	2.5	2.7	3.2
LAUREL	3.5	1.5	2.0	2.0	2.5	2.8	3.3
LOCKWOOD	4.5	1.9	2.5	3.1	3.7	5.0	5.5
JACK LONDON	4.1	1.5	2.1	2.2	2.8	3.2	3.8
MELROSE	2.4	1.4	2.0	2.0	2.5	2.8	3.3
MONTCLAIR	4.1	1.5	2.1	2.0	2.5	2.7	3.2
PARK BOULEVARD	3.3	1.4	1.9	2.0	2.6	2.8	3.4
PIEDMONT	2.6	1.9	2.4	2.0	2.6	2.8	3.3
ROCKRIDGE	2.8	1.6	2.1	2.1	2.6	2.9	3.5
SEQUOYAH-CHABOT	5.1	1.4	1.9	2.3	2.8	3.4	3.9
TEMESCAL	4.5	2.5	3.0	2.0	2.6	2.8	3.3
WEST OAKLAND	5.7	1.8	2.3	2.2	2.8	3.2	3.7

	p = 3.91 (.05/u + ln 1.05 - .06)	wS/C
Baymont	$ .71	.87
Ina D. Coolbrith	.75	.74
Dimond	.68	.73
Eastmont	.75	.78
Elmhurst	.68	.78
Gibson	1.27	.67
Glenview	.70	.76
Golden Gate	.75	.78
Lakeview	.66	.77
Laurel	.68	.76
Lockwood	1.05	.73
Jack London	.76	.77
Melrose	.68	.68
Montclair	.66	.80
Park Boulevard	.69	.75
Piedmont	.68	.71
Rockridge	.71	.71
Sequoyah-Chabot	.78	.80
Temescal	.68	.80
West Oakland	.75	.83
Average	$ .71	.76

Table 3.13 -- Book Service Price and Staff's Share of Cost

shares of cost are between 67% and 87% and average about 76%. By a simple extension of Euler's theorem, staff services' share should be about 65%.*

The high input mix efficiency is a very surprising finding, or is it? Consider that high input mix efficiency means minimum cost and that librarians are concerned about minimizing expenditures. If expenditures are being minimized, it is reasonable that costs are also being minimized: given that $N \leq A - yV = A - By/u$, the relationship between expenditure and cost is $E = wS + xA \geq wS + xN + xBy/u = C + xN - [x\ln(1+r) - \dot{x}]B$. All things considered, the Oakland branch library managers are doing a sterling job at minimizing expenditures for the <u>existing capacity</u> levels.

However, the capacity utilization indexes show that some capacity levels are too high relative to actual output. It is easy to explain Golden Gate, Lockwood, Jack London, and West Oakland because they are located in neighborhoods where demand is known to be low. Elmhurst, Glenview, Melrose, and Park Boulevard do not have this disadvantage, but their capacity utilization indexes range between .57 and .64. Is demand lagging or capacity sprinting? A more detailed analysis of the situation facing each branch library is needed before this question can be answered with any factual basis. The limits of the resource allocation model have been reached.

---

*By Euler's theorem, $SF_S + BF_B = sF(S,B)$ since $F(S,B)$ is homogeneous of degree s. Dividing through by $sF(S,B)$ and substituting for F_S and F_B, $a/s + b/s \equiv 1$. Multiplying by cost, C, shows the division of cost between the two inputs:

$$\frac{a}{s}C + \frac{b}{s}C = C.$$

Since a = .68 and b = .37, staff's share of cost is a/s = .68/1.05 = .65.

Appendix 1 -- The Sample

The Sample consists of twenty of the twenty-two branches. Fruitvale is not included because it houses the Latin-American project, which is federally funded. Charles S. Greene is not included because it houses some Main Library activities. The data are merely twelve month sums of the data listed in Chapter 2, Appendix 1. The five regional branches are Dimond, Elmhurst, Lakeview, Melrose, and Temescal.

Appendix 2 -- Linear Programming Computations

The linear programming estimation problem can be summarized by the initial tableau:

$$\begin{bmatrix} T & -I & u \\ r' & 0 & 0 \end{bmatrix},$$

where T is the matrix of inputs,

r is the vector of total inputs,

u is the vector of outputs,

I is the identity matrix,

0 is the vector of zeros.

The solution is summarized by the final inverse tableau:

$$\begin{bmatrix} \overline{T} & -R^{-1} & \begin{matrix} x \\ s \end{matrix} \\ \overline{r}' & r'_R R^{-1} & v \end{bmatrix},$$

where R is the final basis,

$\overline{T} = R^{-1}T$,

$\overline{r} = r - r'_R R^{-1}T$ is the vector of dual slack variables,

$\begin{bmatrix} x \\ s \end{bmatrix} = R^{-1}u$,

x is the vector of primal variables,

s is the vector of primal slack variables,

$r'_R R^{-1}$ is the vector of dual variables,

$v = r'_R R^{-1}u$ is the value of the objective function.

	Constant	Regional Dummy Z*	Staff lnS	Book lnB	Slack - Baymont	Slack - Ina D. Coolbrith	Slack - Dimond	Slack - Eastmont	Slack - Elmhurst	Slack - Gibson	Slack - Glenview	Slack - Golden Gate	Slack - Lakeview	Slack - Laurel	Slack - Lockwood	Slack - Jack London	Slack - Melrose	Slack - Montclair	Slack - Park Boulevard	Slack - Piedmont	Slack - Rockridge	Slack - Sequoyah-Chabot	Slack - Temescal	Slack - West Oakland	Output lnQ
Baymont	1	0	3.483	8.335	-1																				11.017
Ina D. Coolbrith	1	0	3.234	8.891		-1																			10.857
Dimond	1	-1	3.796	9.615			-1																		11.498
Eastmont	1	0	3.349	8.786				-1																	10.794
Elmhurst	1	-1	3.886	9.416					-1																11.162
Gibson	1	0	1.946	7.445						-1															9.675
Glenview	1	0	3.604	9.214							-1														10.908
Golden Gate	1	0	3.349	8.791								-1													10.168
Lakeview	1	-1	4.048	9.691									-1												11.672
Laurel	1	0	3.814	9.480										-1											11.478
Lockwood	1	0	2.688	8.095											-1										9.490
Jack London	1	0	3.346	8.845												-1									9.912
Melrose	1	-1	3.784	9.833													-1								11.156
Montclair	1	0	3.964	9.454														-1							11.805
Park Boulevard	1	0	3.456	9.177															-1						10.890
Piedmont	1	0	3.227	9.216																-1					11.251
Rockridge	1	0	3.431	9.313																	-1				11.069
Sequoyah-Chabot	1	0	3.057	8.331																		-1			10.777
Temescal	1	-1	3.795	9.204																			-1		11.463
West Oakland	1	0	3.445	8.613																				-1	10.101
	20	-5	68.751	179.745	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0

Appendix 2 -- The Initial Tableau

	Constant	Regional Dummy	Staff	Book	Baymont	...	Melrose	Montclair	Park Boul-evard	Piedmont	Rockridge	Sequoyah-Chabot	Temescal	West Oak-land	
q	1		1.332	- .017		...		-1.578		-7.413		9.991			5.638
z*		1	.013	0		...		.829		- .065		.236	-1		.136
a			- .638	- .001		...		1.592		-2.020		.428			.679
b			- .293	-21.594		...		- .395		1.631		-1.236			.368
Slack - Baymont			-3.330	- 8.334	-1	...		.677		- .855		1.178			.0504
Slack - Ina D. Coolbrith			-3.334	- 8.891		...		.061		.555		.384			.245
Slack - Dimond			-3.916	-12.640		...		- .160		.666		- .506	1		.117
Slack - Eastmont			-3.377	- 8.786		...		.286		.151		.563			.348
Slack - Elmhurst			-3.911	- 9.407		...		.061		.161		- .223			.440
Slack - Gibson			-2.088	- 7.445		...		-1.419		.799		1.620			.0215
Slack - Glenview			-3.664	- 9.215		...		.521		.337		.142			.564
Slack - Golden Gate			-3.375	- 8.782		...		.283		.160		.556			.976
Slack - Lakeview			-4.099	- 9.688		...		.211		.282		- .493	1		.141
Slack - Laurel			-3.873	- 9.470		...		.751		.344		- .096			.235
Slack - Lockwood			-2.753	- 8.095		...		- .494		.360		1.134			.950
Slack - Jack London			-3.391	- 8.844		...		.256		.255		.489			1.250
Slack - Melrose			-3.972	- 9.832		...	-1	- .265		1.046		- .781	1		.531
Slack - Park Boulevard			-3.558	- 9.176		...		.300	-1	.575		.125			.468
Slack - Rockridge			-3.584	- 9.313		...		.208		.846	-1	- .054			.323
Slack - West Oakland			-3.387	- 8.613		...		.506		- .324		.818		-1	1.043
	0	0	0	0	0	...	0	-2.783	0	-6.360	0	-5.857	-5	0	224.847

Appendix 2 -- The Final Inverse Tableau

Appendix 3 -- Computation of the Staff Wage

$$w = \frac{\text{Total salaries}}{\text{Total man-months}} = \frac{\$424,854.43}{680.35} = \$624.47$$

Appendix 4 -- Computation of the New Book Price

$$x = \frac{\text{Total amount spent for new books}}{\text{Total books catalogued}} = \frac{\$90,324.45}{23,100} = \$3.91$$

Appendix 5 -- Computation of the New Book Price Inflation Rate

Average Cost per Book for the Oakland Public Library, 1963-69

1963-1964	\$3.70
1964-1965	4.07
1965-1966	3.97
1966-1967	4.44
1967-1968	4.51
1968-1969	5.03

$$\frac{\dot{x}}{x} = \frac{3.70 - 5.03}{6 \times 3.70} = -.06$$

Since the money value of a book is rising over time, real value of the dollar is falling. $\dot{x}/x$ measures the rate of loss of real value due to rising prices.

Appendix 6 -- Computation of Staff and Book Service Prices Inflation Rates.

From the Personnel Classification and Pay Plan, Resolutions 9706-G, 19 June 1968, and 9711-D, 21 November 1968, of the Board of Library Directors of the City of Oakland,

$$\frac{\dot{w}}{w} = -.05 \ .$$

$$\frac{\dot{p}}{p} \quad \frac{\dot{x}\ [y/u + \ln(1+r)]}{x\ [y/u + \ln(1+r) - \dot{x}/x]} = -.06 \left[\frac{.05/.68 + .0488}{.05/.68 + .0488 + .06}\right]$$

$$= -.06\ (.12/.18) = -.04 \ .$$

Appendix 7 -- Glossary of Names

a	Capacity - staff elasticity
A	Additions to the book stock
b	Capacity - book elasticity
B	Book services
C	Cost
D	Input derived demand function
e	Base of the natural logarithm
E	Expenditure
F	Branch library provision function
G	New books derived demand function
H	Cost function
ln	Natural logarithm
L	Lagrange function
m	Lagrange multiplier
M	Lagrange multiplier function
n	Lagrange multiplier
N	Book stock net change
p	Book service price
q	Provision function constant
Q	Output of library services
Q*	Output capacity
r	Discount rate
R	Capacity function
s	= a + b
S	Staff services
t	Time
u	Utilization rate
v	Dual variables of the linear programming estimation problem
V	Book stock
w	Staff service price
x	Price of new additions
y	Deletion rate
z	Regional branch constant
Z	Regional branch dummy variable

REFERENCES

1. D. J. Aigner and S. F. Chu, "On Estimating the Industry Production Function," American Economic Review, 58.4 (1968) 826-839.

2. American Library Association, Minimum Standards For Public Library Systems, 1966, Chicago, 1967.

3. K. J. Arrow and A. C. Enthoven, "Quasi-Concave Programming," Econometrica, 29.4 (1961) 779-800.

4. L. Bassett, "Returns to Scale and Cost Curves," Southern Economic Journal, 36.2 (1969) 189-190.

5. W. J. Baumol, "Calculation of Optimal Product and Retailer Characteristics: The Abstract Product Approach," Journal of Political Economy, 75.5 (1967) 674-685.

6. W. J. Baumol, "On the Appropriate Discount Rate for Evaluation of Public Projects," in United States Congress Joint Economic Committee, The Planning-Programing - Budgeting System: Progress and Potential, Ninetieth Congress, First Session, 1967.

7. C. W. Cobb and P. H. Douglas, "A Theory of Production," American Economic Review, 18.1 (1928) 139-165.

8. M. J. Farrell, "The Measurement of Productive Efficiency," Journal of the Royal Statistical Society, Series A, 120.3 (1957) 253-281.

9. M. S. Feldstein, Economic Analysis for Health Service Efficiency: Econometric Studies of the British National Health Service, Amsterdam: North-Holland Publishing Co., 1967.

10. D. W. Jorgenson, "The Theory of Investment Behavior," in R. Ferber (ed.), Determinants of Investment Behavior, New York: Columbia University Press, 1967.

11. M. I. Kaimen, "Interest Rate Guidelines For Federal Decision Making," in United States Congress Joint Economic Committee, The Planning-Programing-Budgeting System: Progress and Potential, Ninetieth Congress, First Session, 1967.

12. K. J. Lancaster, "A New Approach to Consumer Theory," Journal of Political Economy, 74.2 (1966) 132-157.

13. M. Nerlove, Estimation and Identification of Cobb-Douglas Production Functions, Chicago: Rand McNally & Co., 1965.

14. R. E. Quandt and W. J. Baumol, "The Demand for Abstract Transport Modes: Theory and Measurement," Journal of Regional Science, 6.2 (1966) 13-26.

15. R. W. Shephard, Cost and Production Functions, Princeton: Princeton University Press, 1953.

16. H. Uzawa, "Duality Principles in the Theory of Cost and Production," International Economic Review, 5.2 (1964) 216-220.

17. A. Wagner, "The Nature of the Fiscal Economy," in R. A. Musgrave and A. T. Peacock (eds.), Classics in the Theory of Public Finance, London: MacMillan, 1958; Originally in Finanzwissenschaft, Part I, Third Edition, Leipzig, 1883, 4-16, 69-76.

Chapter 4

ALTERNATIVE ALLOCATIONS

The Resource Allocation Model as a Decision Making Tool

The model developed in the previous chapter is a useful tool for making resource allocation decisions. Used properly, it can quickly generate alternative allocations and evaluate their economic optimality. However, the limitations of its evaluation powers should be kept in mind. Economic optimality is only one of the many characteristics of any particular allocation, and it is not necessarily the most important one. Other legitimate, and sometimes more important, objectives cannot be evaluated by the model. For this reason, no particular alternative is necessarily the best. The best alternative can be decided on only after considering a larger and broader set of branch library objectives.

Although the model can generate alternatives, it cannot distinguish reasonable allocations from unreasonable ones. A few simple, fundamental criteria can help to warn about possibly unreasonable proposals. The first criterion is the minimum staffing rule * that at least two persons be on duty whenever a branch is open. The second criterion is derived from the rule of thumb that the book stock holding capacity of a branch building is ten times the square feet of its floor area. The third criterion is a minimum book stock of five thousand volumes.** The fourth criterion is

*A minimum staff consists of a library assistant, who is a professional, and a page, who is a nonprofessional. This rule is made for security reasons.

**This figure is determined by the professional judgment of Oakland's librarians.

that provision capacity be at least sufficient to handle current demand levels. The fifth and final criterion is that cost be no greater than current levels since there are already excess costs. All five criteria warn against alternatives by placing boundaries, shown in Figure 4.1 and Table 4.1, on the selection of feasible input combinations. In the

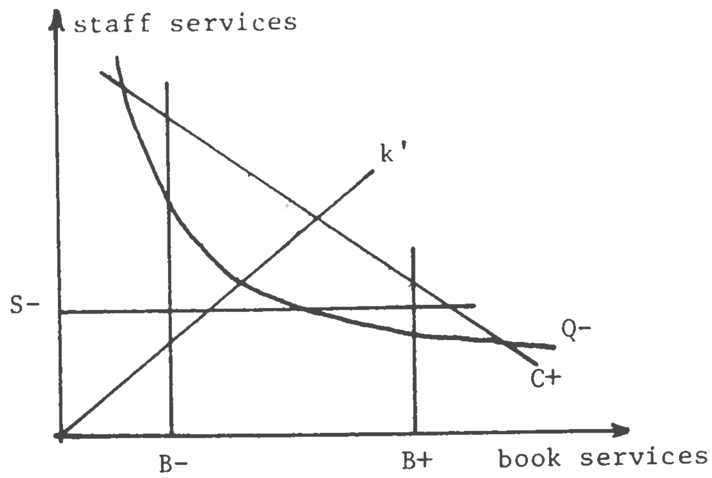

Figure 4.1 -- Reasonable Boundaries

input space, the boundaries form a reasonable domain. A combination falling outside this domain is a warning that it violates at least one fundamental limit. Applying these rules warns against about sixty per cent of the choices. The remaining choices are not necessarily automatically acceptable just because they fall inside the boundaries. They are only not too unreasonable, and other factors have to be considered before an intelligent selection can be made. One such factor that the model does handle is the optimality of the input mix. Any combination closer to the optimal mix ray, k', is better than any combination farther from it. However, this is about the limit of the model's discriminating powers, and the search for other factors has to go beyond the scope of the

	Minimum Staff[1]	Minimum Book[2]	Maximum Book[3]	Demand	Cost[4]
	S-	B-	B+	Q-	C+
Baymont	20.0	3408	11327	60910	23306
Ina D. Coolbrith	17.6	2987	15676	51908	21317
Dimond	22.6	3844	13006	98486	37989
Eastmont	17.9	3039	10677	48726	22672
Elmhurst	22.4	3818	24649	70406	38783
Gibson	6.8	1164	2514	15919	6538
Glenview	20.6	3511	14119	54629	30003
Golden Gate	17.4	2967	12958	26064	22752
Lakeview	24.3	4132	22148	117275	46465
Laurel	22.6	3839	14080	96594	37213
Lockwood	9.2	1559	2416	13226	12632
Jack London	17.3	2935	19074	20170	22982
Melrose	22.6	3843	21718	69976	40138
Montclair	22.4	4149	32408	133969	41322
Park Boulevard	21.8	3707	10741	53645	26453
Piedmont	22.5	3834	11278	76979	23385
Rockridge	20.5	3482	11532	64156	27128
Sequoyah-Chabot	16.2	2757	5006	47906	16521
Temescal	22.4	3816	15422	95099	34540
West Oakland	17.8	3023	8724	24365	23688

Table 4.1 -- Reasonable Boundaries

[1] S- = 29.4u equivalent man-months; u is the utilization rate.

[2] B- = 5000u book-years.

[3] B+ = square feet x 10u book-years.

[4] p = 3.91(.05/u + ln 1.05 + .06).

model itself.

Alternative Zero: The Existing Input Levels

The current input levels constitute a feasible alternative that cannot be ignored. The results of the previous chapter show that the outlook, if existing input levels are continued, is not all that dismal. It could be better, but it could also be a lot worse. The input mix efficiency is generally high, although a number of branches could benefit from improved mixes. As the two previous chapters showed, improving input mix efficiency would be easier if the implicit book service price were made explicit. Managers would then have a clear and correct signal to follow in their resource allocation decisions. A more serious problem with the current situation is capacity underutilization. However, it would be better to solve this by stimulating demand rather than by lowering input levels.

Other alternatives have to be evaluated against the current situation to determine whether or not change is desirable because there is no justification for change unless another alternative is shown to be better. To determine if a better choice exists, alternative combinations are generated and compared to the current one. Each alternative assumes that the levels of certain variables are fixed and that others can be varied by the managers. These different combinations of flexibility and rigidity yield different resource allocations, which may be better or worse in some sense than the current one. Each alternative is accompanied by a table that allows comparison of the relative status of each branch.

TABLE 4.2 -- INPUTS, OUTPUT, AND COST FOR ALTERNATIVE 0
THE EXISTING INPUT LEVELS

BRANCH	OUTPUT	STAFF	BOOK	CAPACITY	COST	CAPACITY UTILIZ.	INPUT MIX
BAYMONT	60910	32.6	4166	64056	23306	.95	.84
INA D. COOLBRITH	51908	25.4	7264	66346	21317	.78	.98
DIMOND	98486	44.5	14983	110687	37989	.89	.98
EASTMONT	48726	28.5	6539	69028	22672	.71	.95
ELMHURST	70406	48.7	12285	109352	38783	.64	.95
GIBSON	15919	7.0	1712	16265	6538	.98	1.00
GLENVIEW	54629	36.7	10040	96042	30003	.57	.96
GOLDEN GATE	26064	28.5	6577	69173	22752	.38	.95
LAKEVIEW	117275	57.3	16170	135044	46465	.87	.96
LAUREL	96594	45.3	13091	122152	37213	.79	.97
LOCKWOOD	13226	14.7	3278	34182	12632	.39	.98
JACK LONDON	20170	28.4	6938	70367	22982	.29	.96
MELROSE	69976	44.0	18636	118948	40138	.59	1.00
MONTCLAIR	133969	52.7	12761	133969	41322	1.00	.94
PARK BOULEVARD	53645	31.7	9676	85691	26453	.63	.98
PIEDMONT	76979	26.5	10059	76979	23385	1.00	.99
ROCKRIDGE	64156	30.9	11082	88585	27128	.72	.99
SEQUOYAH-CHABOT	47906	21.3	4152	47906	16521	1.00	.93
TEMESCAL	95099	44.5	9942	95099	34540	1.00	.93
WEST OAKLAND	24365	31.3	5501	69119	23688	.35	.91

Alternative One: The Recommendations For a Long Range Master Plan

During 1968-1969, the staff of the Oakland Public Library undertook a study of their library system in order to develop recommendations for restructuring it [Oakland Public Library, A Survey of the Oakland Public Library, 1968-69, With Recommendations for a Long Range Master Plan, 1969]. The Library administration is currently in the process of implementing the recommendations so that this represents a real alternative.

The recommendations for reorganizing the branch system stem from a concern with the steadily rising average cost of branch circulation. This dilemma is caused not so much by rising costs as by declining circulation. In the aggregate, Oakland residents are not using the library as much as in the past. The reason for this is quite obvious. Oakland's population is becoming more predominantly made up of minority groups who are not as avid library users as the people they are replacing. To attack the rising average cost, the recommendations mainly call for increasing output by reallocating resources within and among branches in order to alter the kinds and amounts of library services provided by the branches.

The plan calls for a hierarchy of branches, as shown in Table 4.3. At the top are three regional branch libraries, which are much larger than the others and which provide a fairly wide range of specialized services. In other words, they are smaller versions of the main library. The three regional branches are Temescal, which will be greatly enlarged, and Dimond and Eastmont, which will be new buildings. The classification of branches into A, B, or C is based on current output levels. Table 4.3

shows that Class A branches remain unchanged and Classes B and C are reduced to reading centers. The distinguishing characteristic of the reading centers is that they offer virtually no specialized reference services.

Class	Hours/ Week	Staff/ Year	Basic Minimum Book Stock	Expected Circulation/ Year
Regional	70	179.1	200,000	192,000
A	60	52.1-57.0	15,000-20,000	84,000
B	38-48	17.0-26.4	5,000-10,000	60,000
C	30-36	13.0-20.8	5,000	36,000

Table 4.3 -- The Plan's Branch Hierarchy

Since the new regional branch is so different from the existing ones, the resource allocation model is not capable of evaluating it. The activities, the scale of operation, and the technology employed are so completely different that the model is not even an approximate representation. However, the reading centers, listed in Table 4.4, are not very different from the current branches so evaluating them with the model is acceptable.

Class A Branches	Class B Reading Centers	Class C Reading Centers
Dimond	Baymont	Ina D. Coolbrith
Elmhurst	Eastmont	Gibson
Lakeview	Glenview	Golden Gate
Laurel	Park Boulevard	Lockwood
Melrose	Rockridge	Jack London
Montclair		Sequoyah-Chabot
Piedmont		West Oakland
Temescal		

Table 4.4 -- Branches and Reading Centers

The input levels in the plan are given as ranges so there are several combinations of input levels, each of which is supposed to provide enough capacity to meet the expected demand. The basic minimum book stock is a branch's or reading center's permanent collection. This basic collection is augmented by more current selections from the large circuit collection that belongs to the branch system as a whole. The output figures are expressed only in terms of circulation, but they can be converted to the model's circulation equivalents output measure with an appropriate weight. The weights, shown in Table 4.5, are averages of output that is not circulation as a per cent of circulation and are derived from the corresponding branches of each class. To evaluate input mix efficiency, the book

Class	Weight*
A	.10
B	.12
C	.33

Table 4.5 -- Circulation Conversion Weights

service price uses a five per cent deletion rate for Class A branches and a ten per cent deletion rate for Class B and C reading centers, a utilization rate based on seventy-six hours per week as the feasible maximum number of open hours, a five per cent discount rate, and a six per cent inflation rate. Ten per cent for deletions seems more reasonable than five per cent because reading centers have items that are more current

---

*Weight = (Output - Circulation)/Circulation = w

(w+1)expected circulation = expected output

(See Appendix for computations.)

on the average so that they tend to become obsolete more rapidly, which implies the higher rate.

One way of evaluating the plan's proposals is to assume that the staff input figures are fixed and then determine the required optimal book stock. This is similar to Alternative Four. Table 4.6 shows that the Class A branches have about twice the capacity needed to provide for the expected output. For the shorter open hours, the Class B and C reading centers have insufficient output capacity, but for the longer hours, the reading centers have about the right capacity levels. Another way of evaluating the proposals is to determine the optimal input levels required to provide just enough capacity to handle the expected output. This is similar to Alternative Seven. Table 4.7 shows the results.

Comparing the two sets of results, two conclusions are clear. The shorter hours leave the reading centers with too little capacity, and the Class A branches have too much capacity. The first conclusion has no serious consequences because the alternative longer hours is a feasible proposal. However, why are the Class A proposals so far off the mark? An indication of where the problem may lie is to note that Class A branches are allocated about one hundred to two hundred per cent more staff than Class B reading centers, but the expected circulation is only twenty-five per cent greater. The corresponding figures comparing Class C reading centers to Class A branches are two hundred to three hundred per cent more staff for only one hundred and fifty per cent more circulation. Has capacity or output been incorrectly estimated? The model cannot answer this question.

Book Service Price	Class	Hours/ Week	Expected Output	Staff	Optimal Book	Capacity	Cost	Stock	Capacity Utiliza- tion
p = 3.91(.05/u + ln1.05 + .06)	A	60	92333	57.0	28626	190199	54845	36259	.485
		60	92333	52.1	26193	173315	50184	33177	.533
p = 3.91(.10/u + ln1.05 + .06)	B	48	67116	26.4	8535	72238	25379	13514	.929
		38	67116	17.0	4774	43388	16407	9547	1.547
	C	36	47999	20.8	5616	52678	19996	11855	.911
		30	47999	13.0	3111	30888	12541	7883	1.554

Table 4.6 -- Alternative One: The Optimal Plan With Staff Held Fixed, Input Mix Efficiency = 1.00

Book Service Price	Class	Hours/ Week	Expected Output	Optimal Staff	Optimal Book	Cost	Stock	Circuit Stock
p = 3.91(.05/u + ln1.05 + .06)	A	60	92333	28.6	14351	27496	18178	-1822-3178
p = 3.91(.10/u + ln1.05 + .06)	B	38	67116	25.9	7242	24892	14842	4842-9842
		48	67116	24.6	7956	23656	12597	2597-7597
	C	30	47999	19.9	4741	19110	12011	7011
		36	47999	19.0	5138	18295	10847	5847

Table 4.7 -- Alternative One: The Optimal Plan, Capacity Utilization = 1.00 and Input Mix Efficiency = 1.00

Alternatives Two, Three, Four, and Five: A Fixed Input

In each one of this set of alternatives, one input is held fixed and the other is required to take up all of the slack. A2 and A3 have the additional constraint that all excess capacity must be eliminated by bringing capacity down to demand. A4 and A5 have the constraint that the input mix must be optimal; that is, the isocost and isoquant must be tangent.

Figures 4.2 and 4.3 illustrate the alternatives. A2, fixed staff services, and A3, fixed book services, are represented by shifts from the point [B,S] to the Q level isoquant. A4, fixed staff services, and A5, fixed book services, require shifts to points on the k' ray, the optimal input mix expansion path.

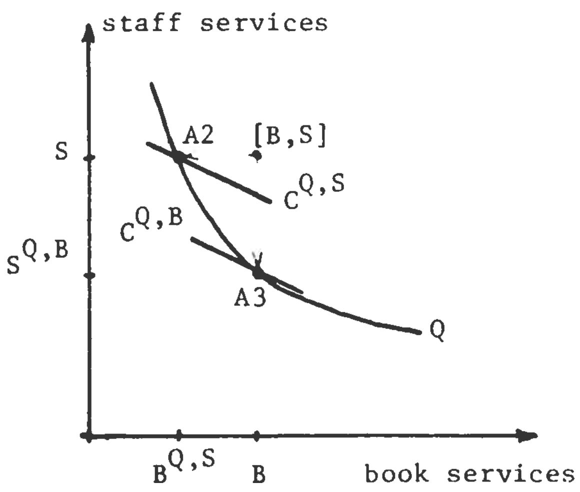

Figure 4.2 -- Alternatives Two and Three: Fix An Input and Eliminate Excess Capacity

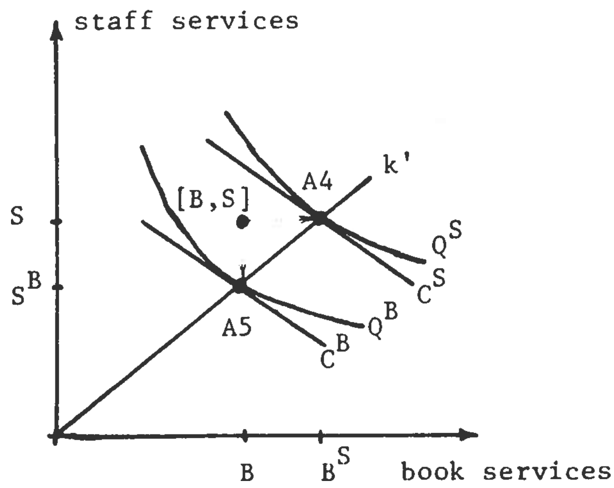

Figure 4.3 -- Alternatives Four and Five: Optimal Mix With A Fixed Input

TABLE 4.[illegible] -- INPUTS, OUTPUT, AND COST FOR ALTERNATIVE 2
FIX STAFF AND ELIMINATE EXCESS CAPACITY

BRANCH	STAFF	BOOK	CAPACITY	COST
[illegible]AYMENT	32.6	2632	60913	27826
IRA D. COOLBRITH	25.4	3726	51908	18654
DIMOND	44.5	10905	99484	35217
EASTMONT	23.5	2536	48724	1[illegible]4[illegible]1
ELMHURST	46.7	3759	70406	32940
GIBSON	7.0	1614	15919	6415
GLENVIEW	2[illegible].7	2164	34629	24459
GOLDEN GATE	23.5	462	26064	18136
LAKEVIEW	57.3	11017	117275	43954
LAUREL	45.3	6913	46594	33012
LOCKWOOD	14.7	245	13226	9443
JACK LONDON	22.4	232	20170	1784[illegible]
MELROSE	44.0	4402	59976	35462
MONTCLAIR	52.7	12761	133969	41322
PARK BOULEVARD	31.7	2707	53645	21643
PIEDMONT	26.5	10055	76979	23335
ROCKRIDGE	30.9	4608	64154	22554
SEQUOYAH-CHABOT	21.[illegible]	4152	47906	16521
TEMESCAL	44.5	9542	950[illegible]	3454[illegible]
WEST OAKLAND	21.3	223	243[illegible]	19311

TABLE 4.9 -- INPUTS, OUTPUT, AND COST FOR ALTERNATIVE 2
FIX BOOK AND ELIMINATE EXCESS CAPACITY

BRANCH	STAFF	BOOK	CAPACITY	COST
BAYMONT	30.2	4166	[illegible]	21952
INA C. COOLBRITH	17.7	7264	[illegible]	16599
DIMOND	37.5	14983	[illegible]	35595
EASTMONT	17.1	6539	44726	15535
ELMHURST	25.5	12295	[illegible]	24272
GIBSON	6.8	1712	[illegible]	5401
GLENVIEW	14.0	10040	54624	17054
GOLDEN GATE	6.8	6577	26764	9189
LAKEVIEW	44.5	16170	117275	39756
LAUREL	37.1	13091	56594	25934
LOCKWOOD	3.6	3278	13226	5717
JACK LONDON	4.5	6538	20171	9076
MELROSE	25.1	18636	69976	25243
MONTCLAIR	52.7	12761	133969	41322
PARK BOULEVARD	13.9	9676	53645	16595
PIEDMONT	26.5	10059	75979	23385
ROCKRIDGE	19.2	11082	64154	19826
SEQUOYAH-CHABOT	21.3	4152	47906	16521
TEMESCAL	44.5	9942	95095	34549
WEST OAKLAND	6.7	5501	24345	9332

How reasonable is an alternative with a fixed input? The answer is not very. Keeping one input fixed requires large changes in the other. The book stock, from which book services flow, can be changed incrementally, but large shifts are not really feasible in the short run. Staff positions are subject to civil service regulations and rigid budgeting procedures, which make large changes infeasible in the short run.

Large changes may be made at a few selected branches by shifting stock among branches, but they cannot be made at all branches because the aggregate stocks of both books and staff are relatively fixed. Given this situation, no alternative in this set is particularly feasible as a general policy, but application to selected branches is feasible.

Another point against these alternatives is that the implied levels of staff and book services are not necessarily desirable. For some branches, the low staff services, $S^{Q,B}$ and S^{B}, fall well below the required staffing minimum of two persons on duty at all times. Moreover, the high book services, B^{S}, requires a book stock that is well above holding capacity of some branches, while the low book services, $B^{Q,S}$, goes below the minimum book stock for other branches. Further disadvantages are that the cost of maintaining A4 appears to be prohibitively high for some branches, and the capacity resulting from A3 and A5 is lower than current demand for other branches. The general conclusion is that none of the alternatives is a completely desirable alternative to A0. Much more flexibility in setting input levels is needed in order to design a better alternative.

Alternatives Six and Seven: Minimize Cost

Figure 4.4 illustrates two ways of minimizing cost. A6 minimizes cost for the existing capacity while A7 minimizes cost and eliminates excess

TABLE 4.10 -- INPUTS, OUTPUT, AND COST FOR ALTERNATIVE OPTIMAL INPUT MIX WITH STAFF FIXED

BRANCH	STAFF	BOOK	CAPACITY	COST
[illegible]AYMONT	32.[illegible]	1546[illegible]	1[illegible]3747	31[illegible]5[illegible]
INA D. COOLBRITH	[illegible]5.4	11403	7[illegible]	24432
DIMOND	44.5	22150	127744	42[illegible]5[illegible]
EASTMONT	24.5	12892	[illegible]859[illegible]	2741[illegible]
ELMHURST	48.7	24167	14[illegible]234	4[illegible]
GIBSON	7.0	1871	1[illegible]	6740
GLENVIEW	36.7	17646	115149	35357
GOLDEN GATE	25.5	12759	8[illegible]257	27418
LAKEVIEW	57.3	25253	1[illegible]793[illegible]	55125
LAUREL	45.3	22542	149167	4364[illegible]
LOCKWOOD	14.7	4725	3910[illegible]	14155
JACK LONDON	23.4	12650	67754	27315
MELROSE	44.0	21981	126178	42344
MONTCLAIR	52.7	26941	176327	50695
PARK BOULEVARD	31.7	15535	101983	30493
PIEDMONT	25.5	13164	84982	25498
ROCKRIDGE	30.9	14802	98532	29754
SEQUOYAH-CHABOT	21.3	9221	64236	20474
TEMESCAL	44.5	22057	127471	42798
WEST OAKLAND	31.3	14153	97627	30166

TABLE 4.11 -- INPUTS, OUTPUT, AND COST FOR ALTERNATIVE [illegible]
(OPTIMAL INPUT MIX WITH [illegible] FIXED)

BRANCH	STAFF	BOOK	CAPACITY	COST
[illegible]	[illegible]	4166	26291	6446
INA D. COOLBRITH	16.2	7264	44847	15564
DIMOND	30.1	14583	[illegible]	[illegible]
EASTMONT	14.4	6525	43529	13097
ELMHURST	24.0	12265	60074	23431
GIBSON	6.4	1712	15332	6144
GLENVIEW	20.9	10040	55489	20117
GOLDEN GATE	14.7	4577	44155	14133
LAKEVIEW	31.7	16170	[illegible]	[illegible]
LAUREL	26.2	13051	84455	25341
LOCKWOOD	10.2	3278	76667	9826
JACK LONDON	15.6	6558	46802	14931
MELROSE	47.5	18526	106667	35565
MONTCLAIR	29.5	12761	60552	24012
PARK BOULEVARD	19.7	9674	62134	18993
PIEDMONT	20.2	10055	64127	19483
ROCKRIDGE	23.1	11082	72779	22276
SEQUOYAH-CHABOT	5.6	4152	27845	8220
TEMESCAL	20.0	9942	55259	19290
WEST OAKLAND	12.2	5501	[illegible]	11726

capacity. In these alternatives, the extra cost resulting from a non-optimal input mix is eliminated. However, both alternatives have drawbacks. Choosing to keep capacity at present levels, Q*, is not particularly desirable because it assumes that the current excess capacity is acceptable. However, the alternative of cutting out all excess capacity may be too severe especially in light of the limited kinds of objectives the model can handle. This pure efficiency type of solution has the possibility of being broader if some intermediate capacity level, Q°, where $Q < Q° < Q*$, were selected so that the cost savings from minimizing costs would yield enough savings to make it possible to bring Q up to Q° as a first step towards reaching even higher levels. The appropriate value of Q° depends on the marginal value of having another unit of excess capacity against the marginal cost of that extra unit. This kind of tradeoff should be explicitly considered by library managers because it can facilitate making resource allocation decisions.

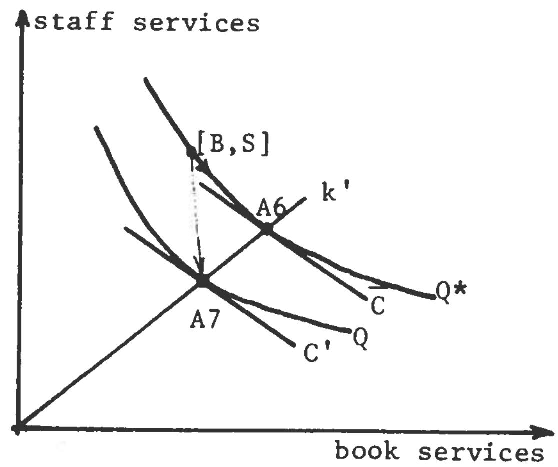

Figure 4.4 -- Alternatives Six and Seven: Minimize Cost For Existing Capacity or No Excess Capacity

TABLE 4.[illegible] -- INPUTS, OUTPUT, AND COST FOR ALTERNATIVE 4
MINIMIZE COST FOR EXISTING CAPACITY

BRANCH	STAFF	BOOK	CAPACITY	COST
BAYMONT	[illegible].5	5754	64[illegible]	1977[illegible]
INA C. COOLBRITH	21.7	573[illegible]	[illegible]	20953
DIMOND	[illegible]	1520[illegible]	11[illegible]97	37361
EASTMONT	22.4	10157	59[illegible]2[illegible]	21[illegible]02
ELMHURST	[illegible].4	19055	1[illegible]552	36963
GIBSON	5.8	1814	10265	4532
GLENVIEW	[illegible].1	14475	96042	29[illegible]93
GOLDEN GATE	22.6	1010[illegible]	60172	21724
LAKEVIEW	47.5	23754	135[illegible]	447[illegible]2
LAUREL	27.5	19625	12215[illegible]	34056
LOCKWOOD	12.5	4155	341[illegible]	17449
JACK LONDON	[illegible].0	10244	70[illegible]7	22119
MELROSE	41.6	20651	1159[illegible]	40025
MONTCLAIR	42.5	20721	135[illegible]	38391
PARK BOULEVARD	26.8	13155	556[illegible]1	25821
PIEDMONT	24.1	11977	76675	22198
ROCKRIDGE	27.9	12371	88585	26478
SEQUOYAH-CHABOT	16.1	6967	4793[illegible]	15470
TEMESCAL	33.6	16672	95099	32348
WEST OAKLAND	22.5	10155	69119	21645

TABLE 4.13 -- INPUTS, OUTPUT, AND COST FOR ALTERNATIVE 7
ELIMINATE EXCESS CAPACITY AND MINIMIZE COST

BRANCH	STAFF	BOOK	CAPACITY	COST
BAYMONT	19.6	9296	60910	18849
INA D. COOLBRITH	17.1	7699	51908	16494
DIMOND	34.7	17270	98486	33416
EASTMONT	16.1	7282	48726	15487
ELMHURST	25.2	12511	70406	24270
GIBSON	6.6	1777	15919	6399
GLENVIEW	17.6	8443	54629	16917
GOLDEN GATE	8.9	3978	26064	8549
LAKEVIEW	40.6	20758	117275	39117
LAUREL	29.9	14883	96594	28812
LOCKWOOD	5.2	1677	13226	5025
JACK LONDON	7.0	3104	20170	6703
MELROSE	25.0	12458	69976	24108
MONTCLAIR	40.5	20721	133969	38991
PARK BOULEVARD	17.1	8409	53645	16505
PIEDMONT	24.1	11977	76979	23198
ROCKRIDGE	20.5	9824	64156	19747
SEQUOYAH-CHABOT	16.1	6967	47906	15470
TEMESCAL	33.6	16672	95099	32348
WEST OAKLAND	8.3	3750	24365	7993

Alternative Eight: Eliminate Excess Capacity

If the existing input mix efficiency is deemed to be adequate, but excess capacity is not to be tolerated, uniformly decreasing both inputs until capacity is down to the demand level would be an appealing alternative. In many cases, substantial cost savings would result. There is nothing wrong with this, but Figure 4.5 shows that with a little more adjustment, additional savings can be made by improving the input mix. However, this results in a situation identical to A7. Thus, while A8 is reasonable, it can be regarded as an intermediate step to A7, which is even more efficient. Again, however, an intermediate shift might well be more desirable.

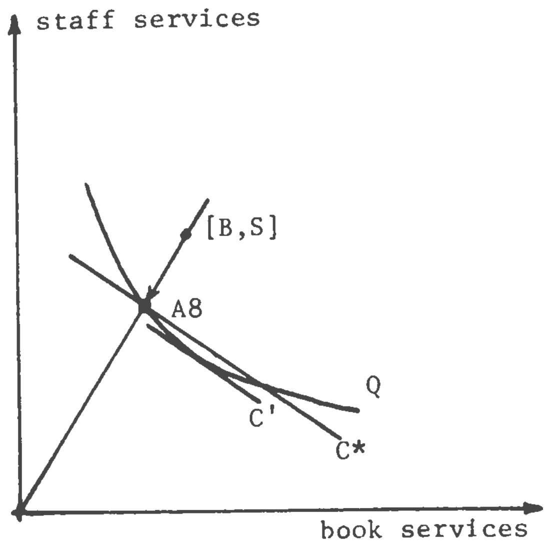

Figure 4.5 -- Alternative Eight: Uniform Reduction of Inputs to Eliminate Excess Capacity

TABLE 4.14 -- INPUTS, OUTPUT, AND COST FOR ALTERNATIVE 8
ELIMINATE EXCESS CAPACITY

BRANCH	STAFF	BOOK	CAPACITY	COST
BAYMONT	31.0	3970	60910	22211
INA D. COOLBRITH	20.1	5746	51908	16861
DIMOND	39.8	13401	98486	33977
EASTMONT	20.4	4688	48726	16254
ELMHURST	32.0	8066	70406	25465
GIBSON	6.9	1677	15919	6405
GLENVIEW	21.4	5856	54629	17500
GOLDEN GATE	11.2	2588	26064	8953
LAKEVIEW	50.0	14131	117275	40605
LAUREL	36.2	10461	96594	29736
LOCKWOOD	5.9	1323	13226	5099
JACK LONDON	8.6	2103	20170	6965
MELROSE	26.5	11226	69976	24177
MONTCLAIR	52.7	12761	133969	41322
PARK BOULEVARD	20.2	6185	53645	16910
PIEDMONT	26.5	10059	76979	23385
ROCKRIDGE	22.7	8142	64156	19931
SEQUOYAH-CHABOT	21.3	4152	47906	16521
TEMESCAL	44.5	9942	95099	34540
WEST OAKLAND	11.6	2031	24365	8747

Alternative Nine: Maximize Capacity

Assuming that Wagner's Law holds and that a desire to improve input mix efficiency exists, capacity can be maximized at current cost levels. Figure 4.6 shows that the only difference between this and A6 and A7 is that excess capacity is ignored. Since no branch manager in good conscience could ignore excess capacity, this alternative must be termed undesirable.

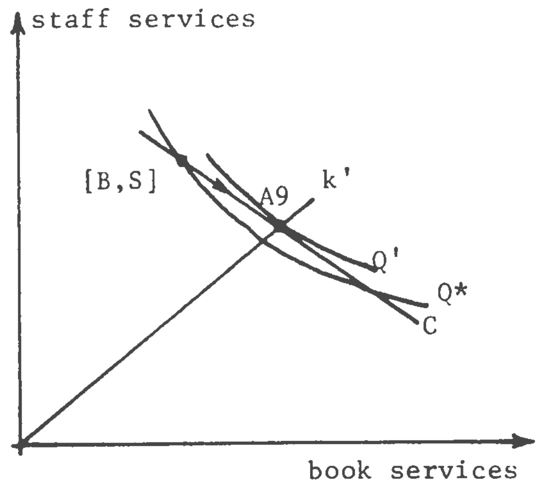

Figure 4.6 -- Alternative Nine: Maximize Capacity

Alternative Ten: Maximize Output

The previous alternatives are concerned with resource allocation within a branch, and give only superficial attention to alternative uses of the possible resource savings. Alternative Ten considers reallocation among the branches so that the system as a whole can provide more library

TABLE 4.15 -- INPUTS, OUTPUT, AND COST FOR ALTERNATIVE 3
MAXIMIZE CAPACITY

BRANCH	STAFF	BOOK	CAPACITY	COST
BEYMONT	24.2	11499	75063	23306
INA C. COOLBRITH	22.1	9949	67692	21317
DIMOND	39.5	15633	112699	[illegible]
EASTMONT	23.8	10660	72613	22673
ELMHURST	[illegible]	19993	114996	[illegible]
GIBSON	6.8	1915	16231	6534
GLENVIEW	31.2	14974	90512	40093
GOLDEN GATE	23.6	10587	72651	22752
LAKEVIEW	49.2	24657	140425	46405
LAUREL	35.7	19222	126257	37213
LOCKWOOD	13.1	4216	34709	12632
JACK LONDON	22.9	10644	73244	22932
MELROSE	41.7	20741	119308	40136
MONTCLAIR	42.9	21960	142263	41322
PARK BOULEVARD	27.5	13477	87885	26453
PIEDMONT	24.3	12073	77627	23335
ROCKRIDGE	28.2	13495	89447	27128
SEQUOYAH-CHABOT	17.2	7440	51317	16521
TEMESCAL	35.9	17802	101254	34540
WEST OAKLAND	24.5	11114	75661	23583

services. Given that some branches have shiftable excess inputs, the branches where more inputs would provide the most services are the ones on the provision frontier, that is, the ones with zero excess capacity. If zero excess capacity is actually limiting their output, increasing capacity would allow these branches to provide more library services. The cost of this additional output is zero since the added resources, which make the extra capacity available, were idle before the shift. This something-for-nothing result should be attractive to the library managers.

The four branches on the provision frontier are the most likely candidates for more inputs since they are operating at the lowest levels of excess capacity.* The dual to the linear programming estimation problem provides alternative approaches to this problem. The dual variables can be interpreted as the multiple by which the corresponding branch's inputs, transformed to logarithms, should be increased so that the output of library services for the branch system as a whole will be at a maximum. Since the dual constraints are aggregate input supply limitations, what the dual does is to close down all underutilized branches and reallocate all inputs to overutilized ones. The simplicity and extremeness of this solution result from the narrowness of the objective function only to maximize output. This alternative becomes unquestionably infeasible when realistic operational considerations and other branch

---

*Since the estimated provision frontier is only a least upper bound, the actual excess capacity for these four branches is not necessarily zero.

library objectives are also considered.

Although the possibility of implementing the dual solution in its pure form must be rejected, a more moderate and experimental approach suggests itself. If the dual variables are interpreted as relative weights, which indicate the added output each branch could provide from a given increase in capacity, reallocation can be done incrementally. Branches with large excess capacity would lose inputs to branches with low excess capacity, and the biggest output increase would be at the branch with the highest value dual variable. However, as the sensitivity analysis in Chapter 3 demonstrates, incremental changes of the basic constraint values can easily lead to changes in the branch system provision function. In other words, after each small reallocation, the provision function and allocation weights should be reestimated.

As the reallocation proceeds, different branches enter and leave the set of branches that define the provision function and that are candidates for more resources. Over the course of many shifts, this experimental-incremental approach can improve the capacity-output efficiency and output level of the branch system as a whole.

The feasibility of this alternative depends on the mobility of inputs among branches, the fine tuning ability of the branch managers, and the presence of unsatisfied demand. A high degree of control over and mobility of inputs may be somewhat unrealistic, but equally unrealistic is the assumption of no control and complete immobility. This alternative is certainly not impossible. The presence of unsatisfied demand at some branches is obvious, and this alternative can do something about it.

Summary of Alternatives

Tables 4.16 - 4.19 summarize all alternatives for each branch, and Table 4.20 summarizes the reasonable and unreasonable proposals. Isoquant graphs, Figures 4.8 - 4.27, also summarize all alternatives for each branch. As in Figure 4.7, each isoquant graph is marked with existing input levels, the actual input ratio ray, and the optimal input ratio ray.

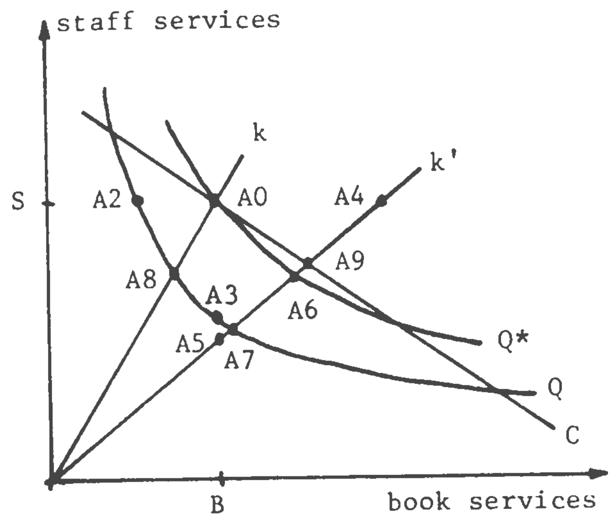

Figure 4.7 -- Typical Isoquant Graph Summarizing All Alternatives For A Branch

The possible cost savings that correspond to the capacity utilization and input mix indexes are shown in Table 4.21. While the savings from eliminating all excess capacity are quite substantial, one last warning is that this pure efficiency solution is as narrow and limited as the model that generates it.

TABLE 4.16 -- STAFF SERVICES FOR ALL ALTERNATIVES

BRANCH	ACTUAL	A3	A5	A6	A7	A8	A9
BAYMONT	22.6	30.2	[illegible]	2[illegible].5	19.6	31.0	24.2
INA D. COOLBRITH	25.4	17.7	15.[illegible]	21.7	17.1	26.1	22.1
DIMOND	44.5	37.5	30.1	28.9	34.7	39.9	39.5
EASTMONT	23.5	17.1	14.4	22.4	16.1	20.4	23.6
ELMHURST	48.7	25.5	24.8	39.4	25.2	32.0	40.3
GIBSON	7.0	6.3	[illegible].4	6.8	6.6	6.0	6.8
GLENVIEW	36.7	16.0	[illegible]	30.1	17.6	21.4	31.2
GOLDEN GATE	28.5	6.9	11.7	22.6	8.9	11.2	23.6
LAKEVIEW	57.3	46.5	31.7	46.5	40.6	50.0	44.3
LAUREL	45.3	32.1	[illegible].3	37.5	29.9	36.2	38.7
LOCKWOOD	14.7	3.6	1[illegible]	12.9	5.2	5.9	13.1
JACK LONDON	23.4	4.6	15.6	23.0	7.0	8.6	23.9
MELROSE	44.0	20.1	37.6	41.6	25.[illegible]	26.5	41.7
MONTCLAIR	52.7	52.7	24.9	40.6	4[illegible].5	52.7	42.9
PARK BOULEVARD	31.7	18.9	18.7	2[illegible].8	17.1	20.2	27.5
PIEDMONT	26.5	26.5	20.2	24.1	24.1	26.5	24.3
ROCKRIDGE	30.9	19.2	2[illegible].1	27.9	20.5	22.7	29.2
SEQUOYAH-CHABOT	21.3	21.3	[illegible].6	16.1	16.1	21.3	17.2
TEMESCAL	44.5	44.5	[illegible]	33.[illegible]	33.6	44.5	35.9
WEST OAKLAND	21.3	6.7	1[illegible].2	2[illegible]	[illegible]	11.6	24.6

TABLE 4.17 -- BOOK SERVICES FOR ALL ALTERNATIVES

BRANCH	ACTUAL	A2	A4	A6	A7	A8	A9
BAYMONT	4166	3632	15463	9754	[illegible]	3970	11495
INA D. COOLBRITH	7264	3726	11483	9733	7690	5745	9949
DIMOND	14983	10905	22159	19303	17270	13401	19633
EASTMONT	6535	2536	12492	10157	7232	4588	10660
ELMHURST	12285	3709	24167	19055	12511	8066	19993
GIBSON	1712	1414	1471	1514	1777	1677	1315
GLENVIEW	10040	2164	17545	14475	3443	5656	10587
GOLDEN GATE	6577	462	12750	10110	3074	2588	24557
LAKEVIEW	15170	11017	29253	23754	20758	14131	19222
LAUREL	13091	5913	22542	18525	14883	10451	4216
LOCKWOOD	3278	248	4725	4155	1677	1323	10644
JACK LONDON	6938	332	12650	10244	3104	2103	20741
MELROSE	18636	4402	21581	20631	12458	11226	21950
MONTCLAIR	12761	12761	26941	20721	20721	12751	13477
PARK BOULEVARD	9576	2707	15535	13155	8406	6195	12073
PIEDMONT	13058	13058	13164	11477	11477	10059	13495
ROCKRIDGE	11092	[illegible]	[illegible]	13371	9324	5142	7440
SEQUOYAH-CHABOT	4152	4152	[illegible]	[illegible]	6067	4152	17302
TEMESCAL	6942	5643	22557	16072	15572	9842	11114
WEST OAKLAND	5501	[illegible]	14153	10156	3750	2031	[illegible]

TABLE 4.18 -- OUTPUT AND CAPACITY FOR ALL ALTERNATIVES

BRANCH	ACTUAL	CAPACITY	A4	A5	A8
BAYMONT	60910	64066	1[illegible]3747	26291	76[illegible]
INA D. COOLBRITH	51908	66246	73505	48747	67692
DIMOND	32486	110687	127794	44985	112635
EASTMONT	49726	65035	[illegible]4595	44[illegible]	70613
ELMHURST	70450	109352	149936	59774	114976
GIBSON	15919	15265	1[illegible]	15308	16231
GLENVIEW	34025	35642	111168	[illegible]	99512
GOLDEN GATE	25064	69173	[illegible]57	441[illegible]	72501
LAKEVIEW	117375	135044	167730	90225	140425
LAUREL	66564	122152	141167	94458	126257
LOCKWOOD	12226	34182	301[illegible]	26[illegible]7	34709
JACK LONDON	20170	70367	97754	45802	73244
MELROSE	65976	118548	124178	110667	119338
MONTCLAIR	131469	133969	176367	60650	142363
PARK BOULEVARD	33045	85691	11[illegible]3	52134	97388
PIEDMONT	74070	76579	84[illegible]2	44117	77627
ROCKRIDGE	64156	33586	[illegible]	72770	99447
SEQUOYAH-CHABOT	47906	47906	94216	27559	51317
TEMESCAL	65099	65099	127471	55350	101984
WEST OAKLAND	24365	45119	97427	36[illegible]	75941

TABLE 4.19 -- COST FOR ALL ALTERNATIVES

BRANCH	ACTUAL	A2	A3	A4	A5	A6	A7	A8
BAYMONT	23306	22926	21992	31352	3446	16778	13849	22211
INA D. COOLBRITH	21317	17554	16506	24432	15544	20653	16494	16661
DIMOND	37989	35217	33595	42846	23982	37361	33416	33977
EASTMONT	22672	13691	15535	27410	13907	21602	15487	16254
ELMHURST	38783	32540	24272	46850	23341	35963	24270	25465
GIBSON	6539	6415	6401	6740	6164	6532	6399	6405
GLENVIEW	30003	24460	17758	35357	20117	24003	16917	17500
GOLDEN GATE	22752	18136	9185	27413	14132	21724	8549	8953
LAKEVIEW	46465	42054	39756	58125	30471	44742	39117	40505
LAUREL	37213	33012	22030	43440	25343	36056	28312	29736
LOCKWOOD	12632	5443	5717	14155	9820	12449	5025	5099
JACK LONDON	22982	17966	8076	27315	14931	22119	6703	6965
MELROSE	40133	30452	25243	42344	30045	40023	24108	24177
MONTCLAIR	41322	41322	41322	51695	24012	33091	38991	41322
PARK BOULEVARD	26452	21648	16563	30492	18962	25921	15505	16910
PIEDMONT	23365	23365	23315	25604	17483	23198	23198	23385
ROCKRIDGE	27128	22555	17426	20754	22275	16378	19747	19931
SEQUOYAH-CHABOT	16521	16521	16521	20474	9220	15470	15470	16521
TEMESCAL	34640	34640	34547	42708	19242	33348	33348	34540
WEST OAKLAND	21685	14611	8572	30160	11724	21645	7993	8747

Branch	Existing Input Levels: A0	Fix S and Q* → Q: A2	Fix C and Q*→ Q: A3	Optimal Mix With S Fixed: A4	Optimal Mix With B Fixed: A5	Minimize Cost for Q*: A6	Minimize Cost for Q* → Q: A7	Q* → Q With Same Input Mix: A8	Maximize Capacity for C: A9	Reasonable Number
Baymont				B+,C+	Q-,S-		S-		B+	5
Ina D. Coolbrith			S-	C+	Q-,S-		S-			5
Dimond	B+			B+,C+	Q-	B+	B+	B+	B+	2
Eastmont		B-	S-	B+,C+	Q-,S-		S-			4
Elmhurst		B-		C+	Q-					6
Gibson				C+	Q-,S-		S-			6
Glenview		B-	S-	B+,C+		B+	S-		B+	3
Golden Gate		B-	S-	C+	S-		S-	S-,B-		3
Lakeview				B+,C+	Q-	B+			B+	5
Laurel				B+,C+	Q-	B+	B+		B+	4
Lockwood	B+	B-	S-	B+,C+		B+	S-	S-,B-	B+	1
Jack London		B-	S-	C+	S-		S-	S-,B-		2
Melrose			S-	B+,C+						7
Montclair				C+	Q-					7
Park Boulevard		B-	S-	B+,C+	S-	B+	S-	S-	B+	1
Piedmont				B+,C+	Q-,S-	B+	B+		B+	4
Rockridge			S-	B+,C+		B+			B+	5
Sequoyah-Chabot				B+,C+	Q-,S-	B+,S-	B+,S-		B+	4
Temescal				B+,C+	Q-,S-	B+	B+		B+	4
West Oakland		B-	S-	B+,C+	S-	B+	S-	S-,B-	B+	1
Reasonable Number	18	12	10	0	4	9	4	14	8	79

Table 4.20 -- Reasonable and Unreasonable Proposals

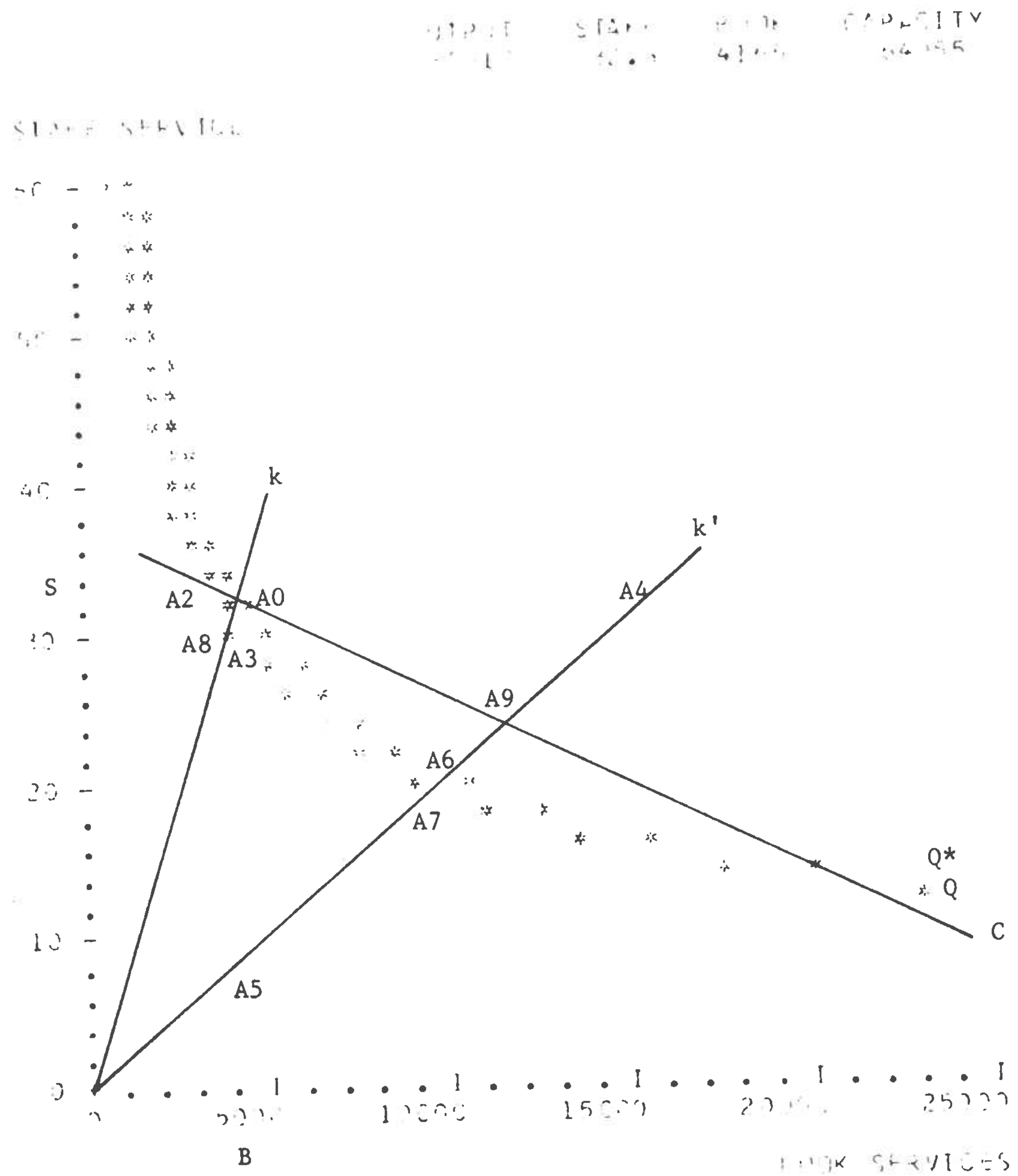

FIGURE 4.5 -- SUMMARY OF ALL ALTERNATIVES - RAYMONT

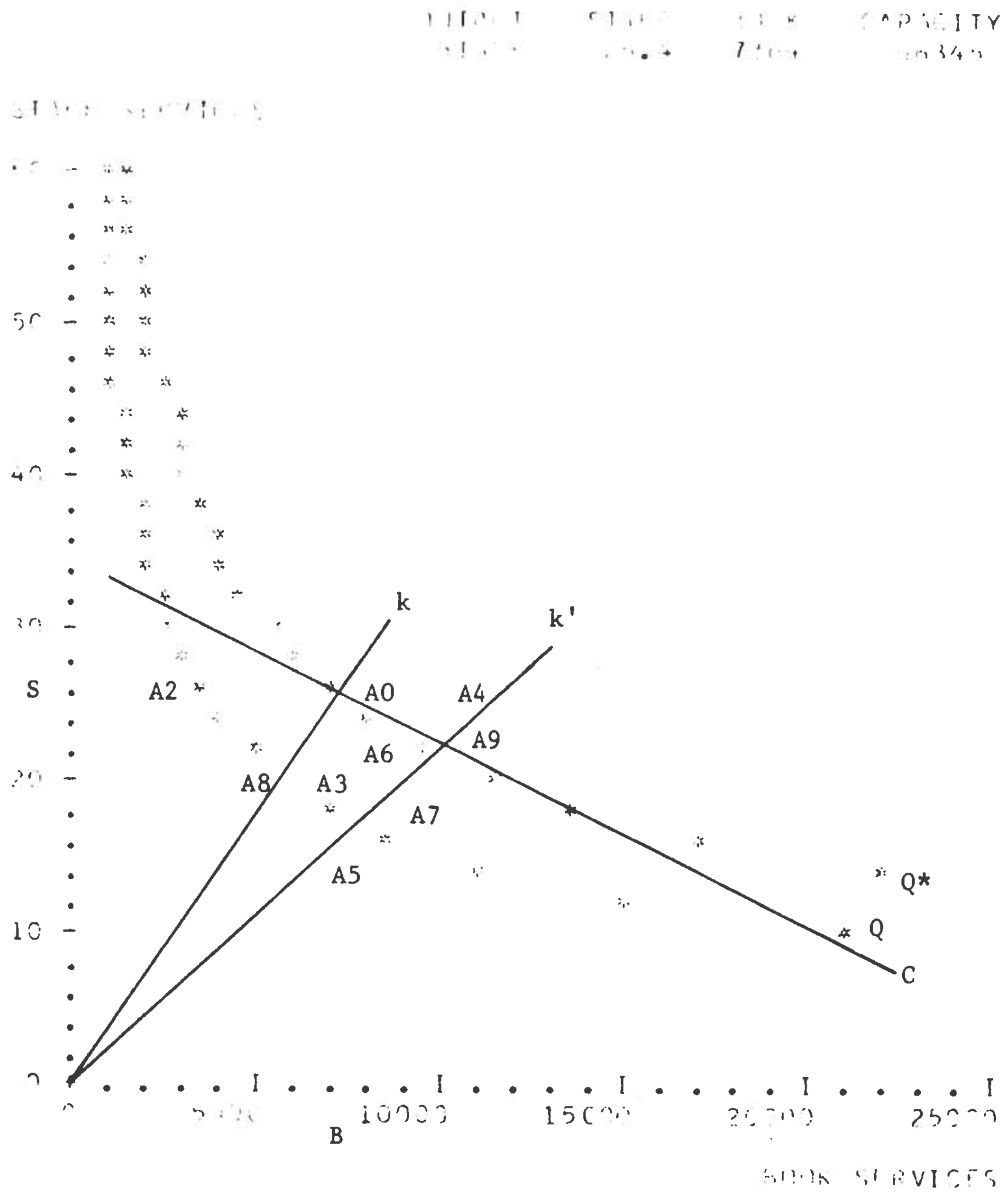

FIGURE 4.9 -- SUMMARY OF ALL ALTERNATIVES - INA D. COOLBRITH

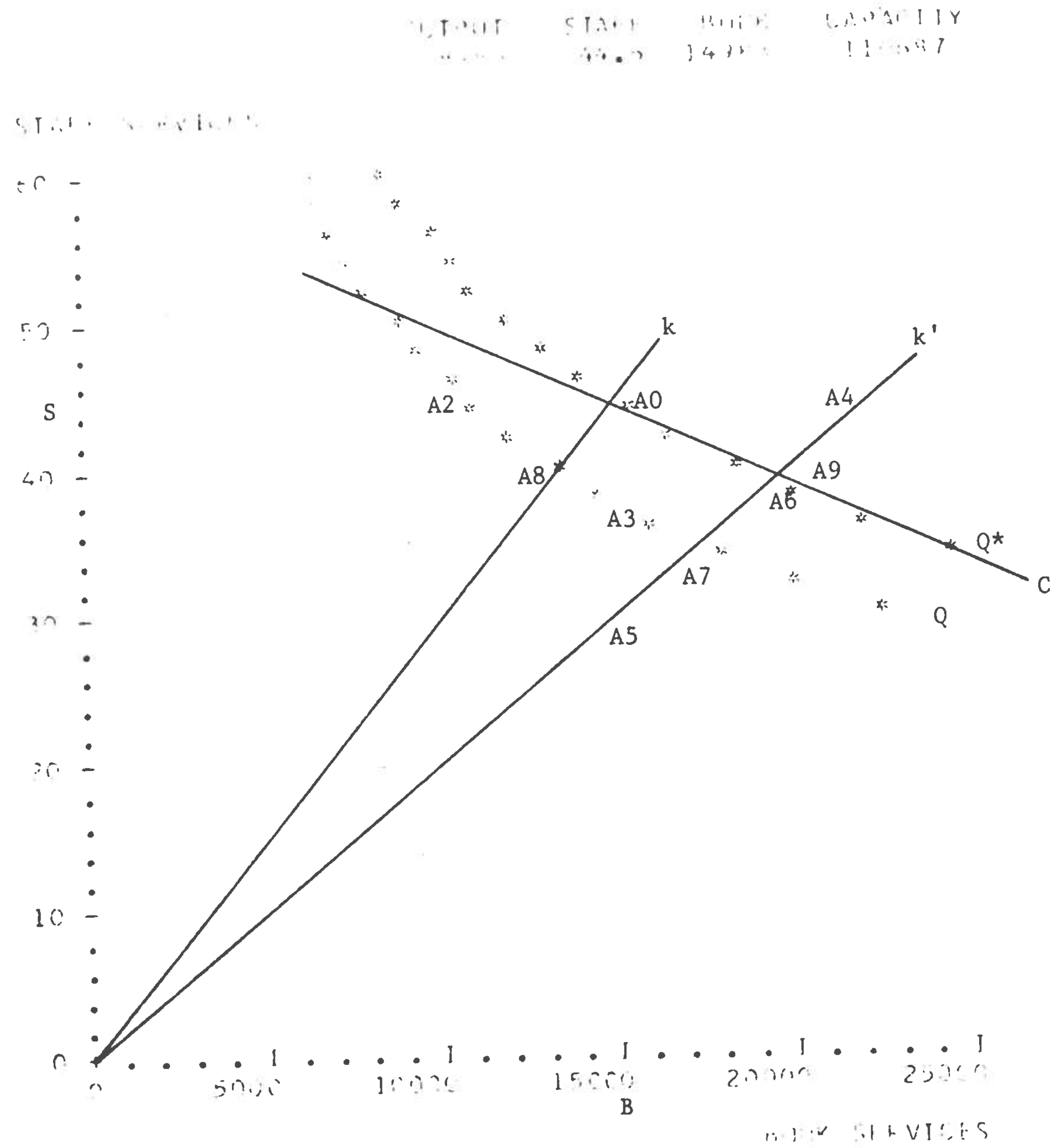

FIGURE 4.10 -- SUMMARY OF ALL ALTERNATIVES - DIMOND

OUTPUT	STAFF	BOOK	CAPACITY
418726	28.5	6539	69028

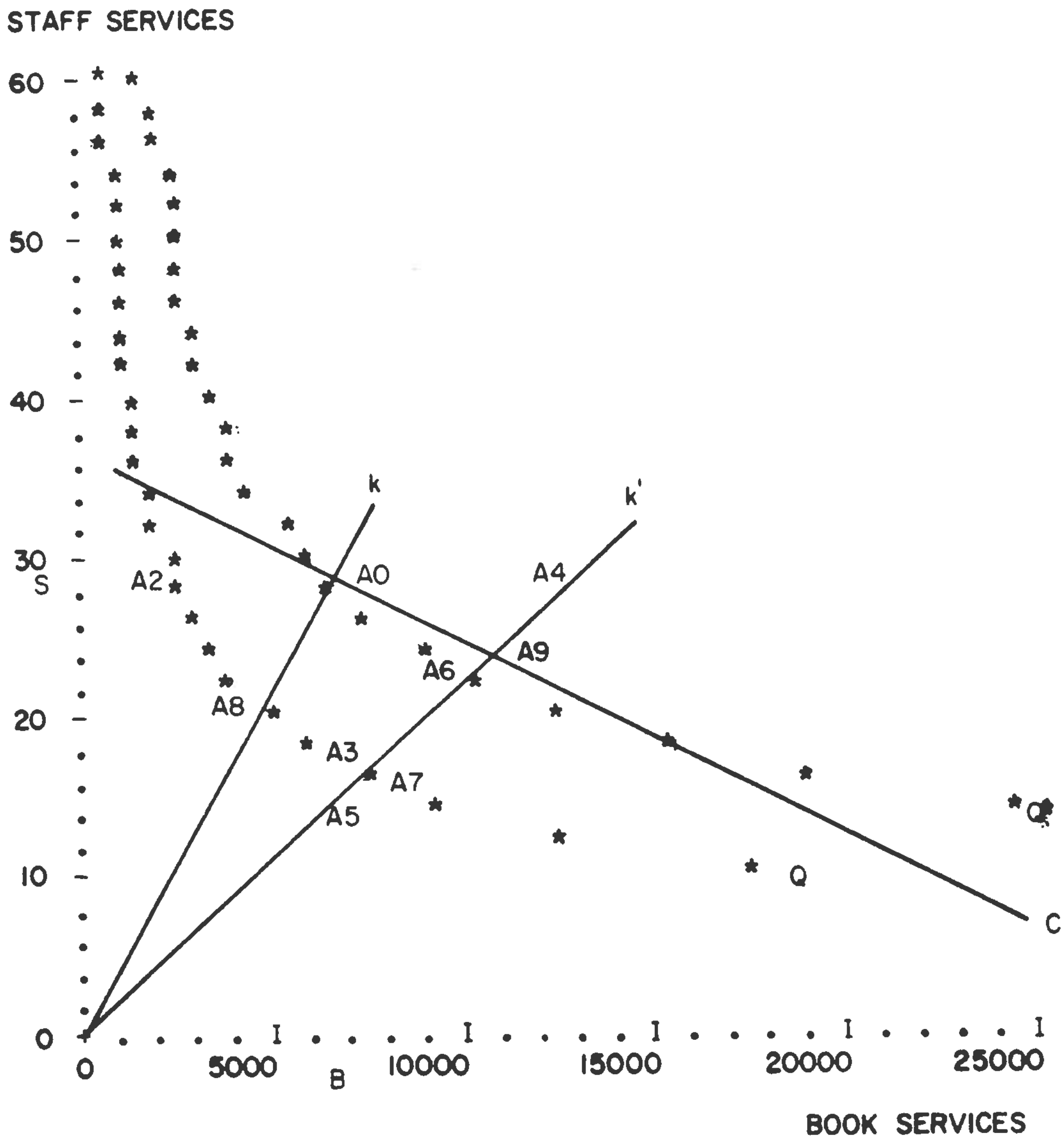

Figure 4.11 -- SUMMARY of ALL ALTERNATIVES-EASTMONT

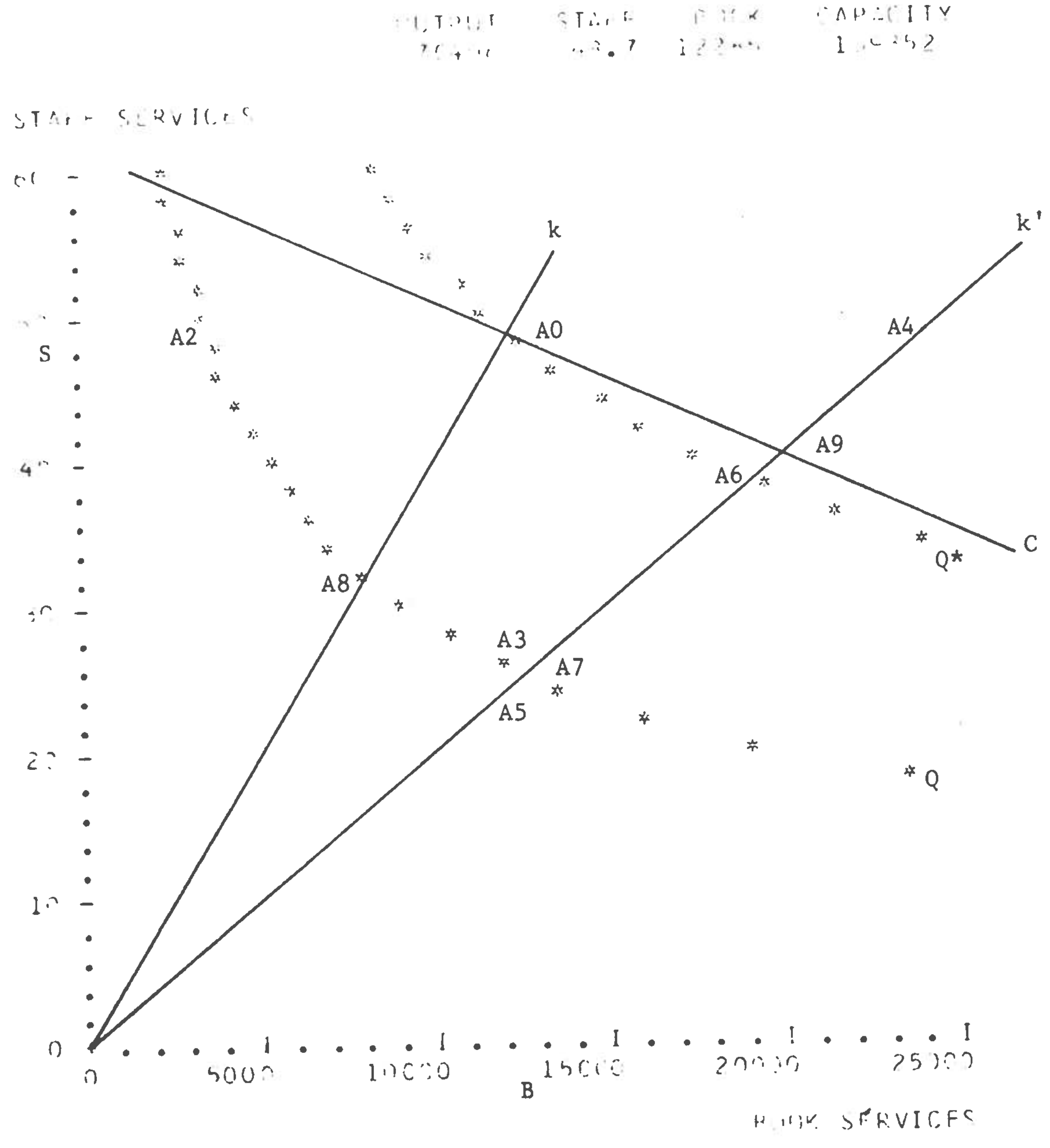

FIGURE 4.12 -- SUMMARY OF ALL ALTERNATIVES - ELMHURST

OUTPUT	STAFF	BOOK	CAPACITY
15[illegible]	7.[illegible]	1711	16255

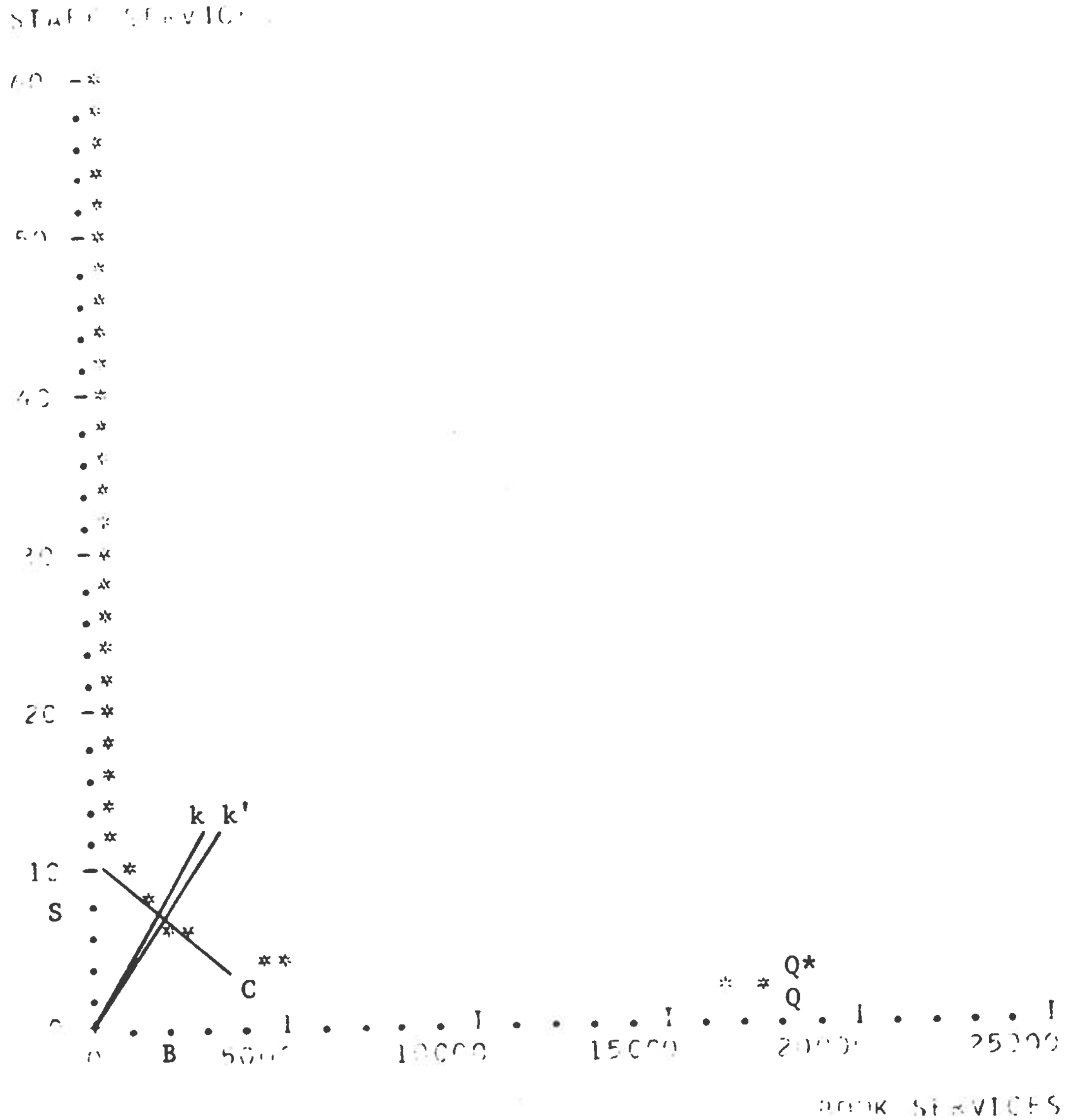

FIGURE 4.13 -- SUMMARY OF ALL ALTERNATIVES - GIBSON

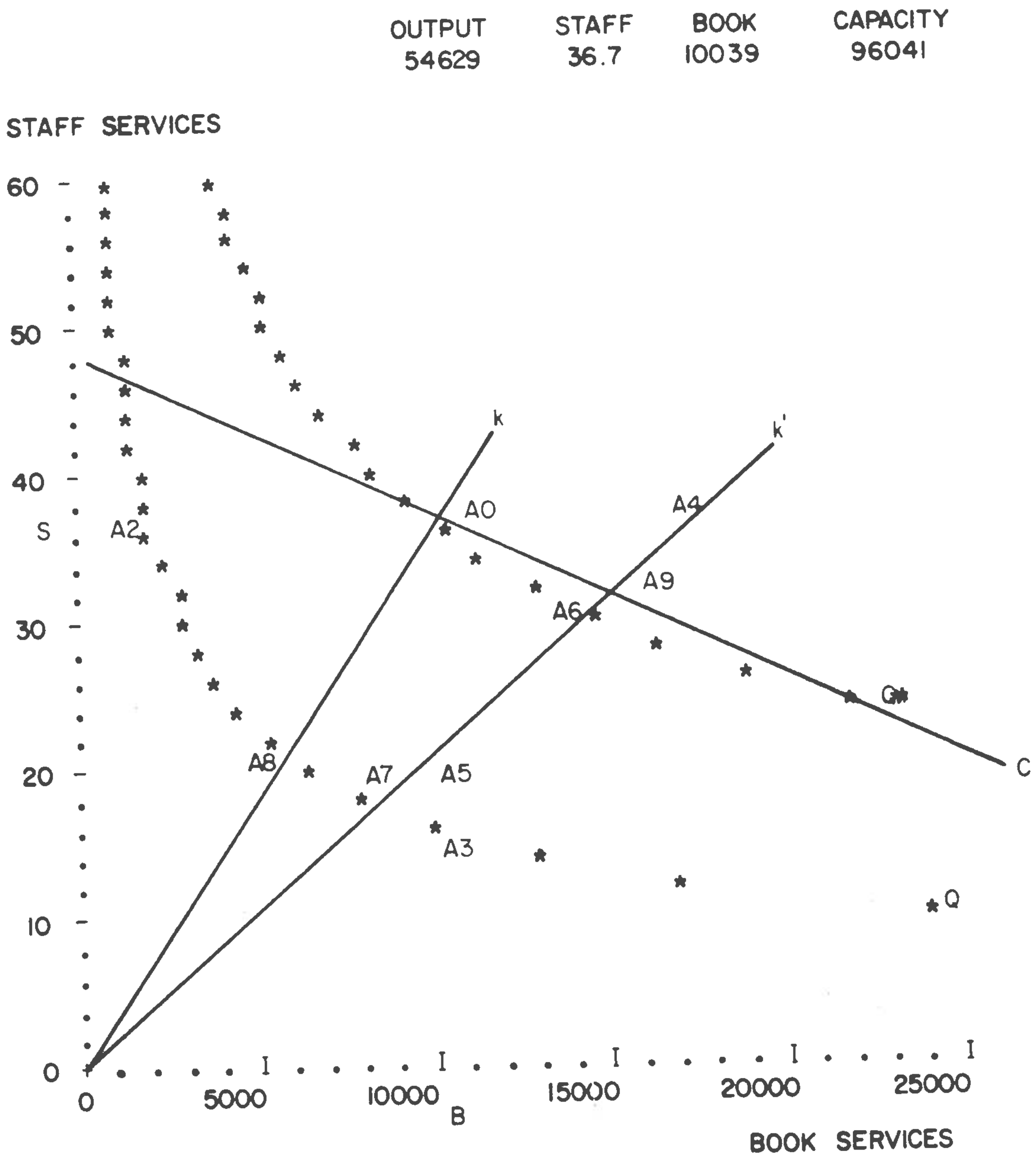

Figure 4.14 -- SUMMARY of ALL ALTERNATIVES - GLENVIEW

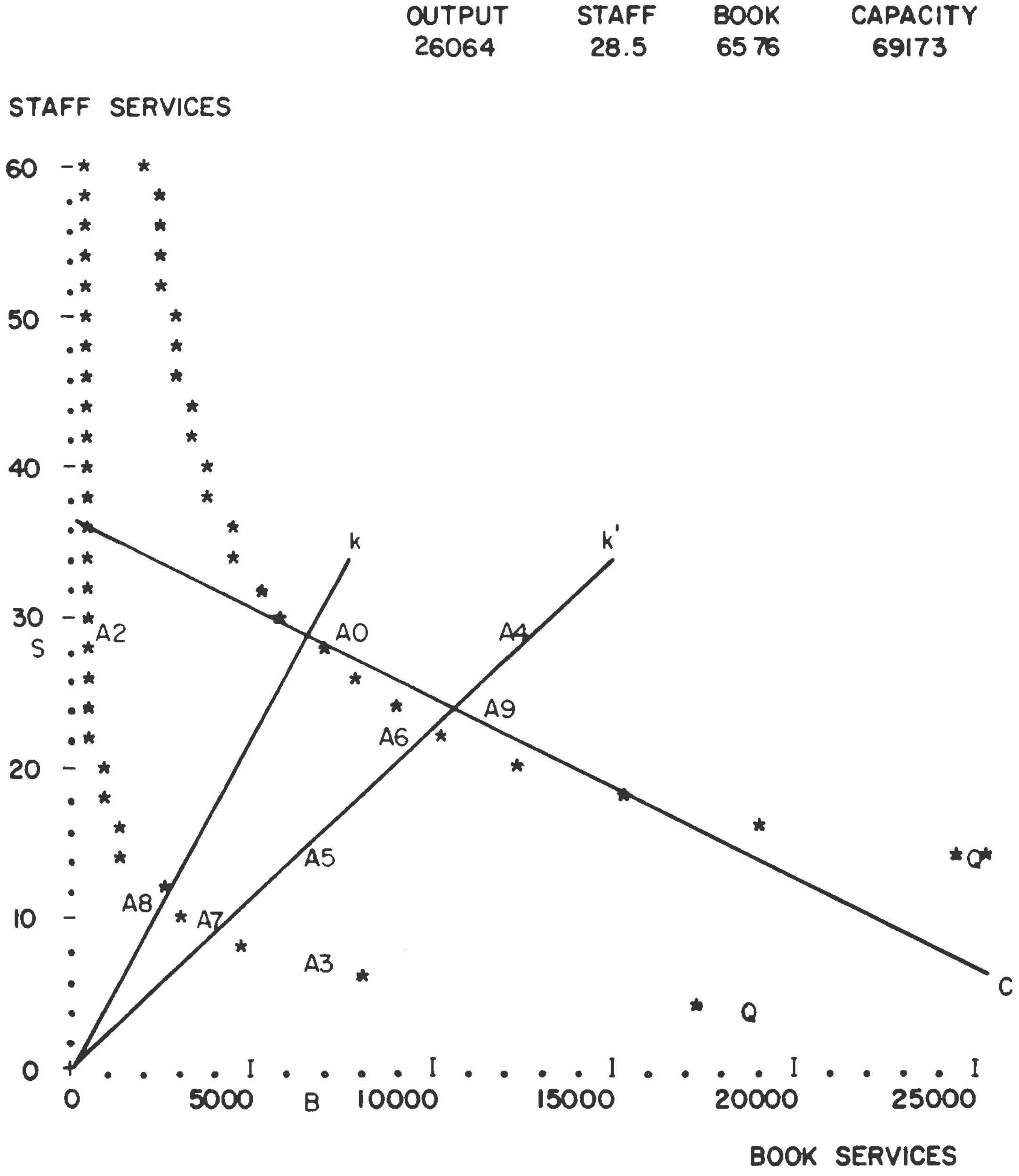

Figure 4.15 -- SUMMARY of ALL ALTERNATIVES - GOLDEN GATE

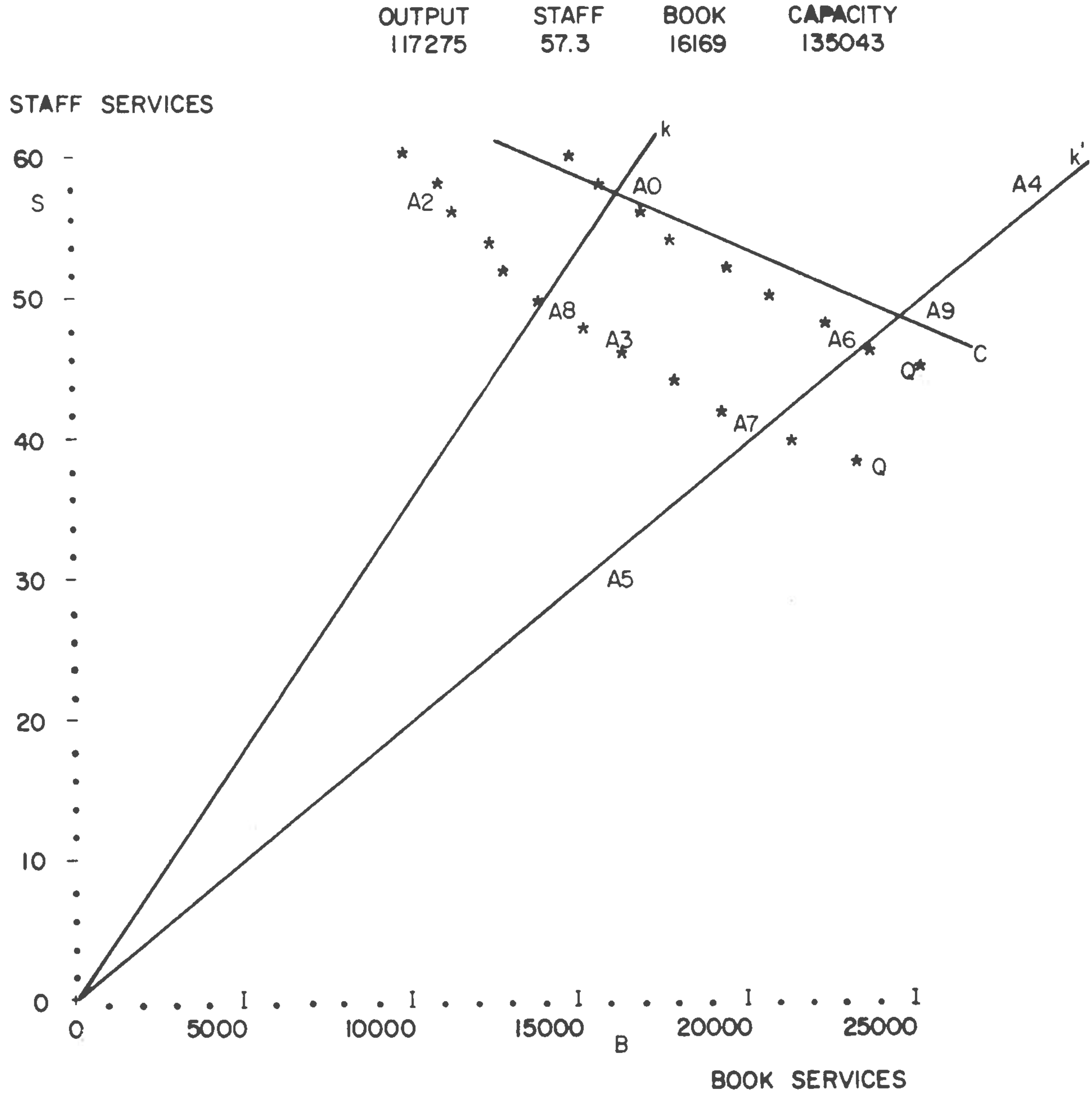

Figure 4.16 -- SUMMARY of ALL ALTERNATIVES - LAKEVIEW

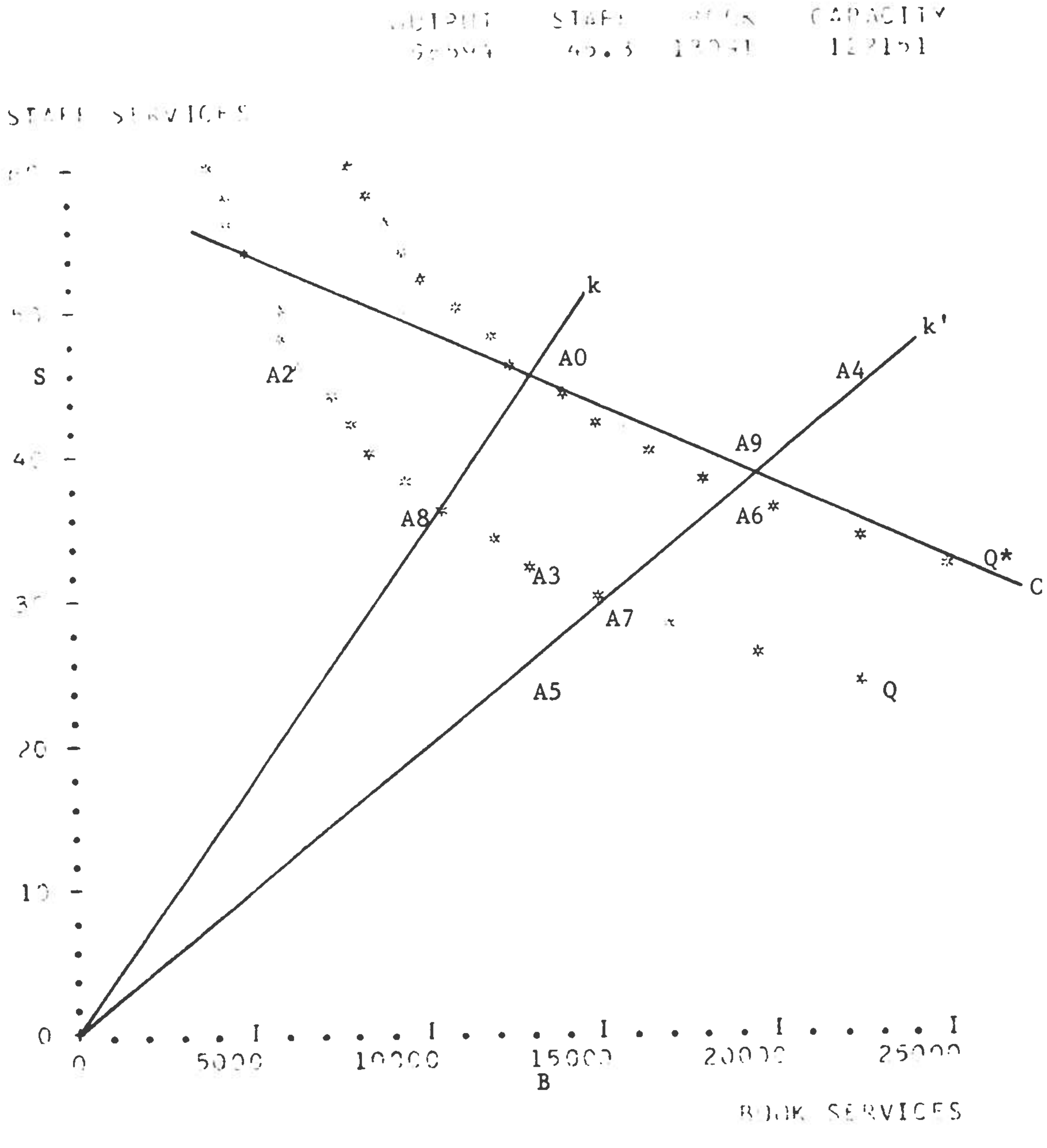

FIGURE 4.17 -- SUMMARY OF ALL ALTERNATIVES - LAUREL

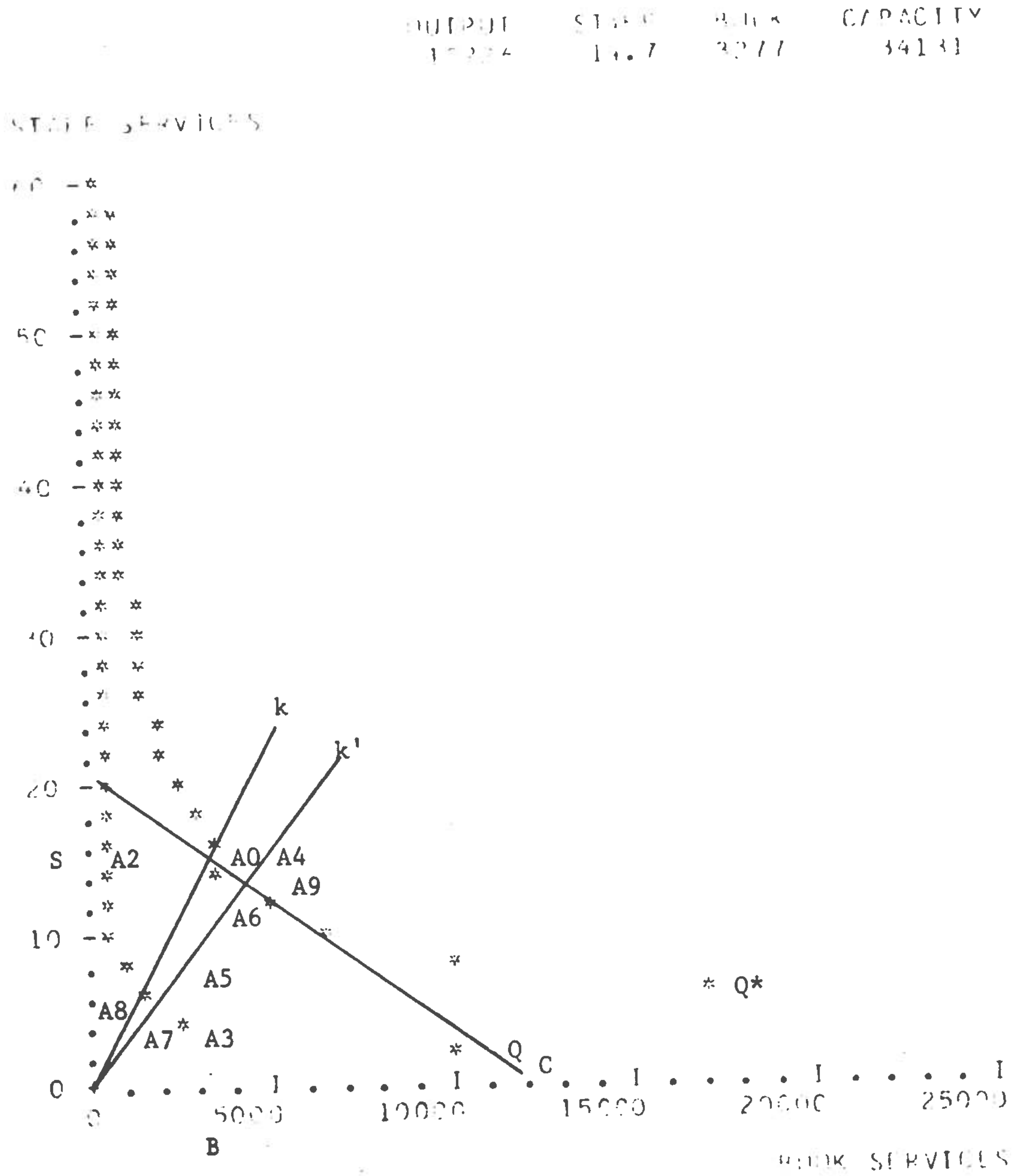

FIGURE 4.18 -- SUMMARY OF ALL ALTERNATIVES - LOCKWOOD

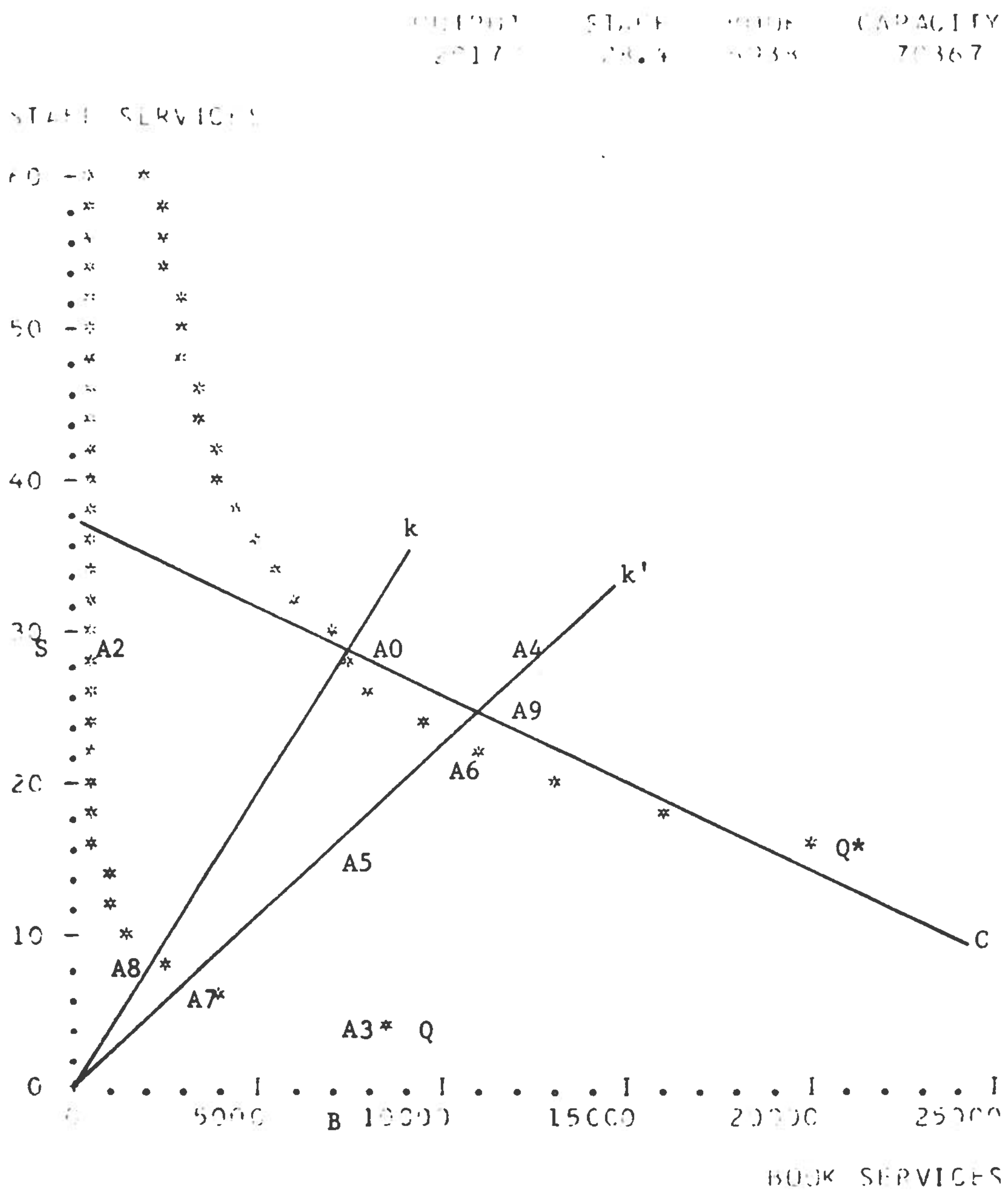

FIGURE 4.19 -- SUMMARY OF ALL ALTERNATIVES - JACK LONDON

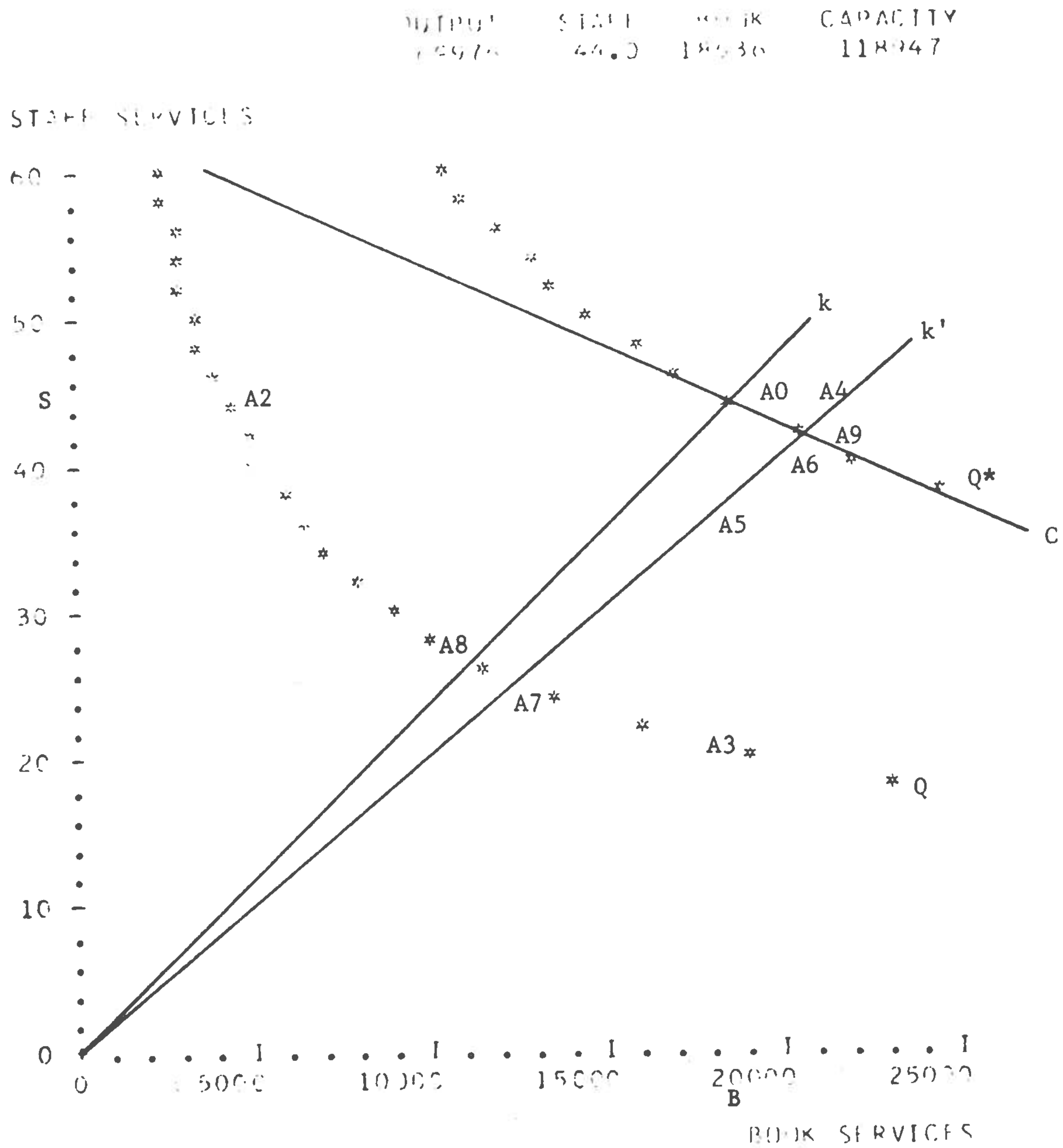

FIGURE 4.20 -- SUMMARY OF ALL ALTERNATIVES - MELROSE

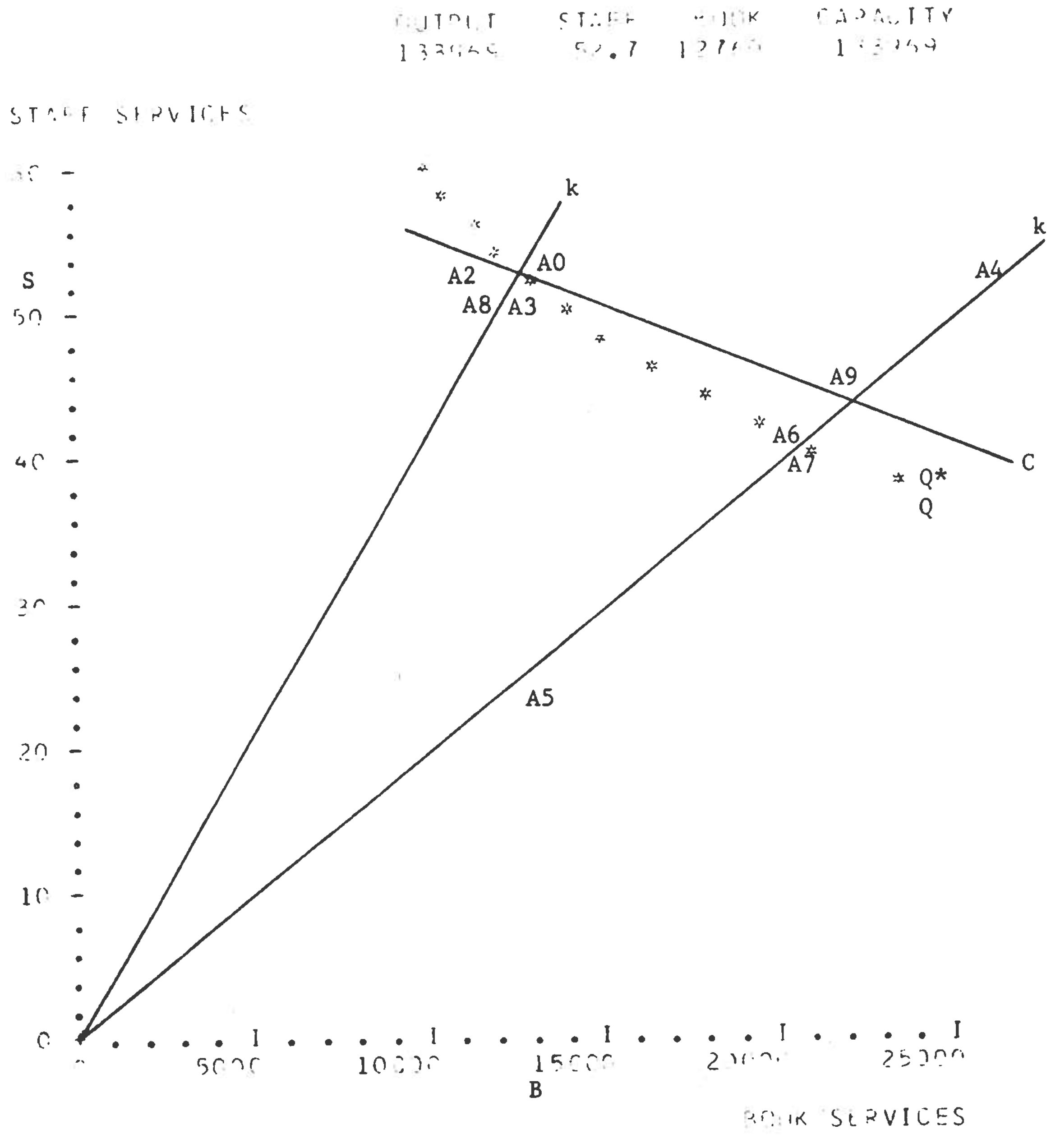

FIGURE 4.21 -- SUMMARY OF ALL ALTERNATIVES - MONTCLAIR

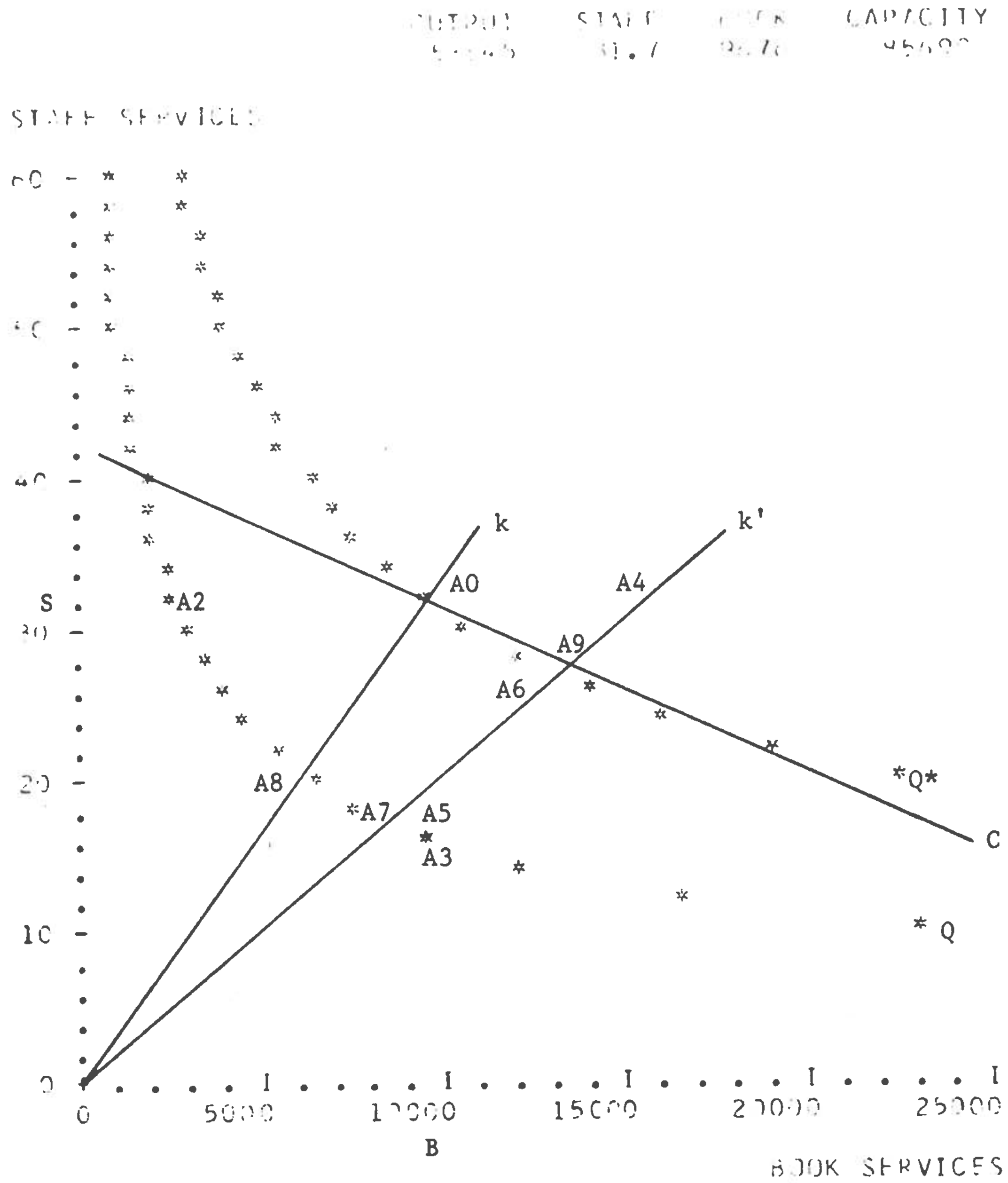

FIGURE 4.22 -- SUMMARY OF ALL ALTERNATIVES - PARK BOULEVARD

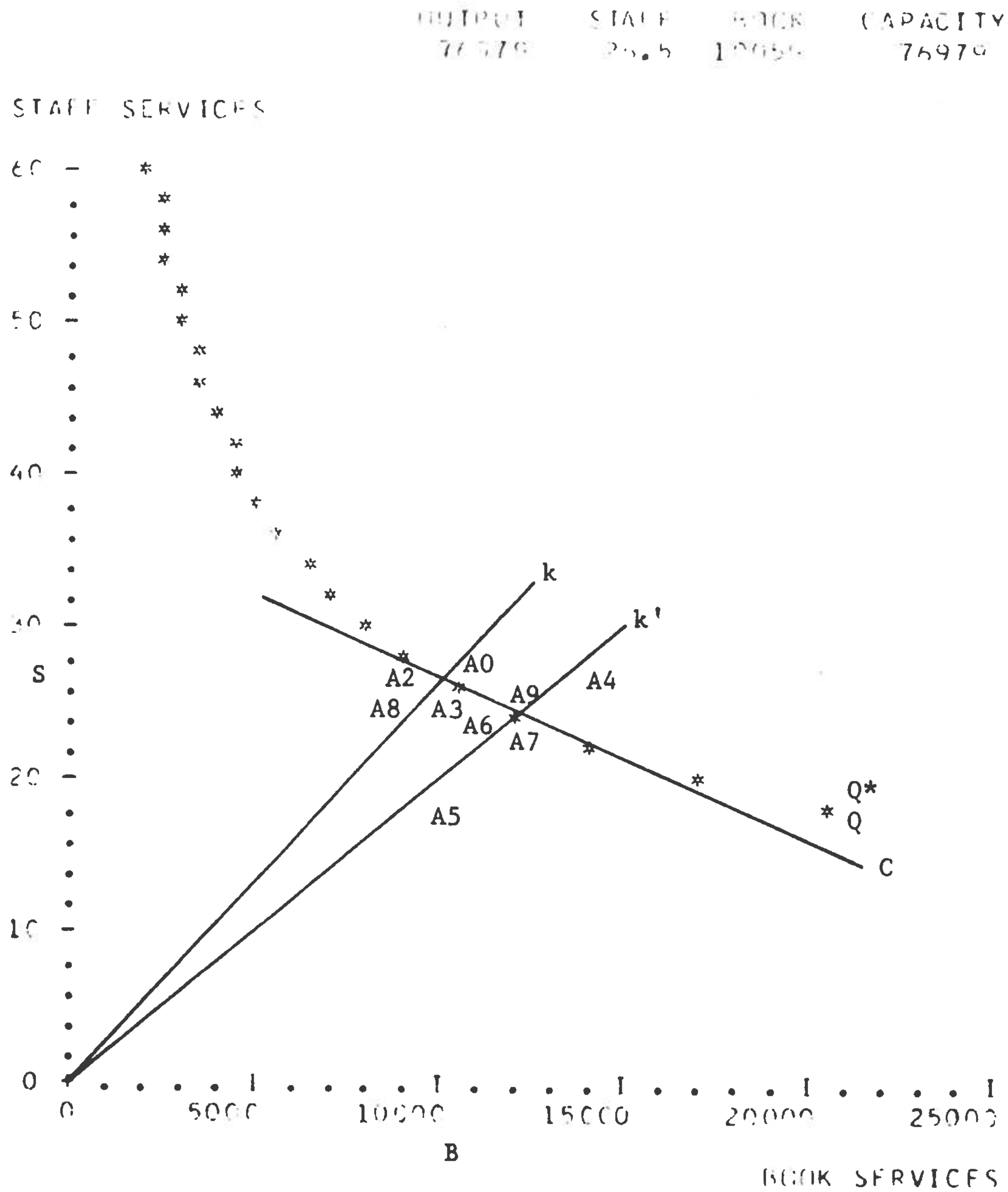

FIGURE 4.23 -- SUMMARY OF ALL ALTERNATIVES - PIEDMONT

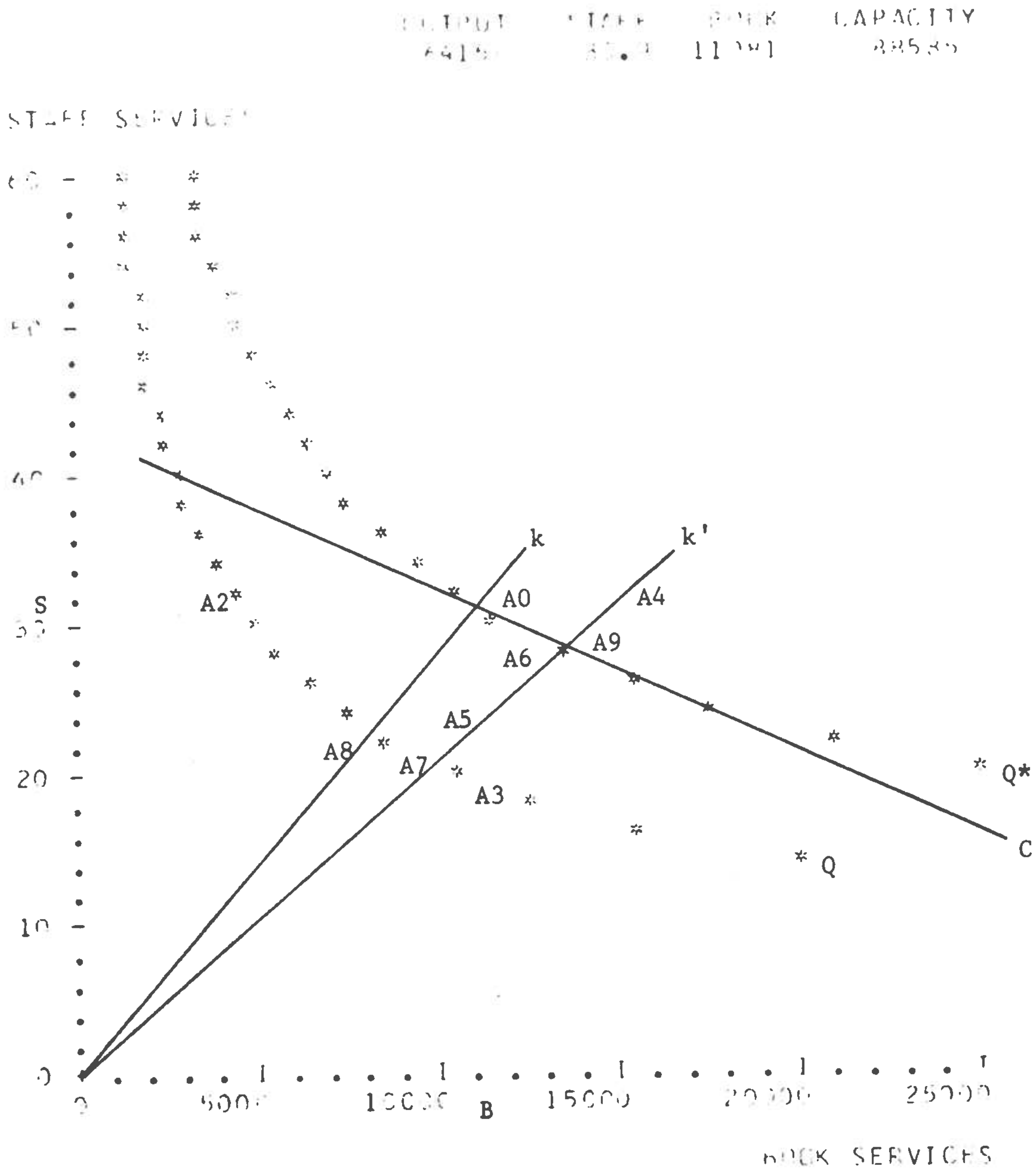

FIGURE 4.24 -- SUMMARY OF ALL ALTERNATIVES - ROCKRIDGE

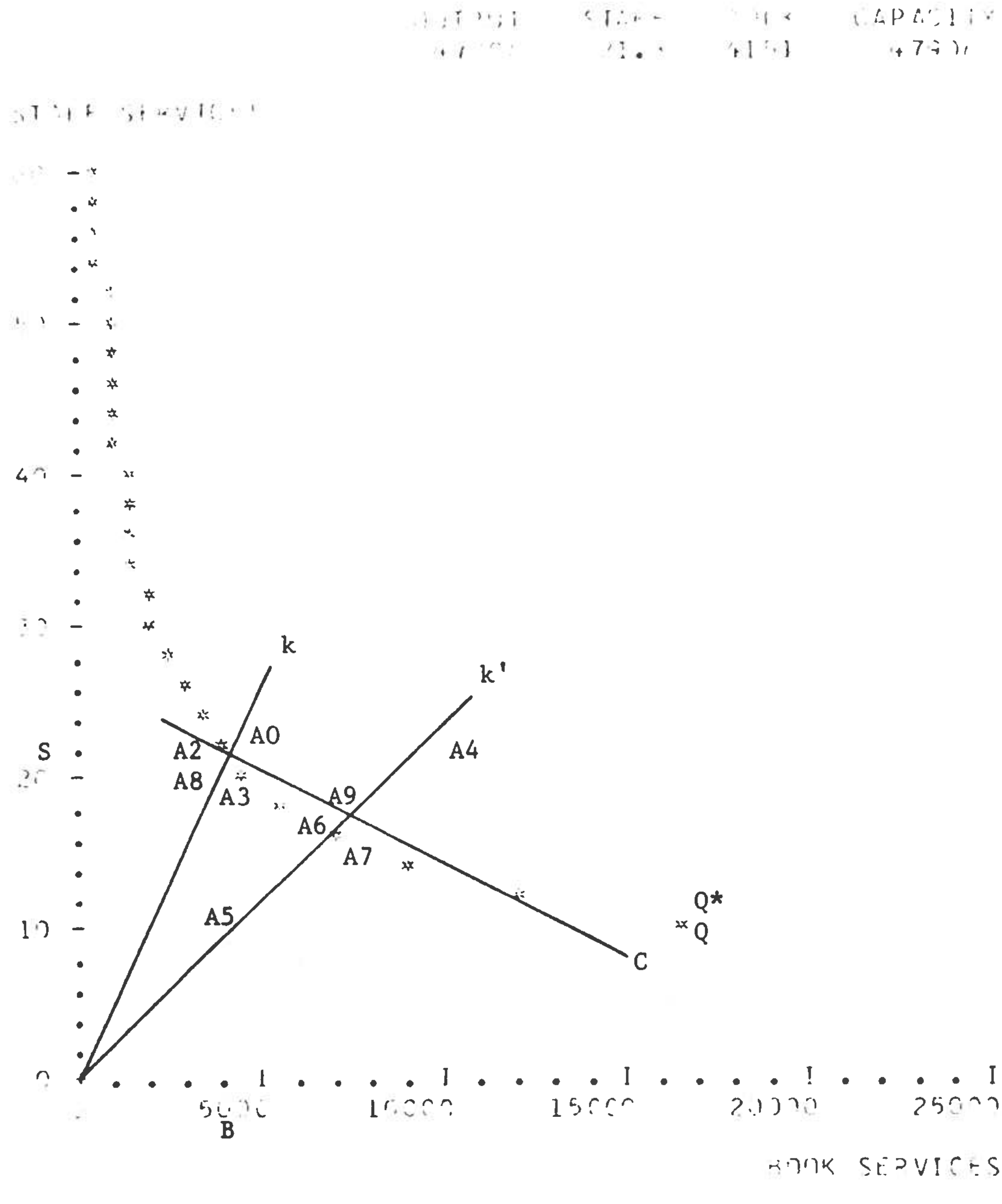

FIGURE 4.25 -- SUMMARY OF ALL ALTERNATIVES - SEQUOYAH-CHABOT

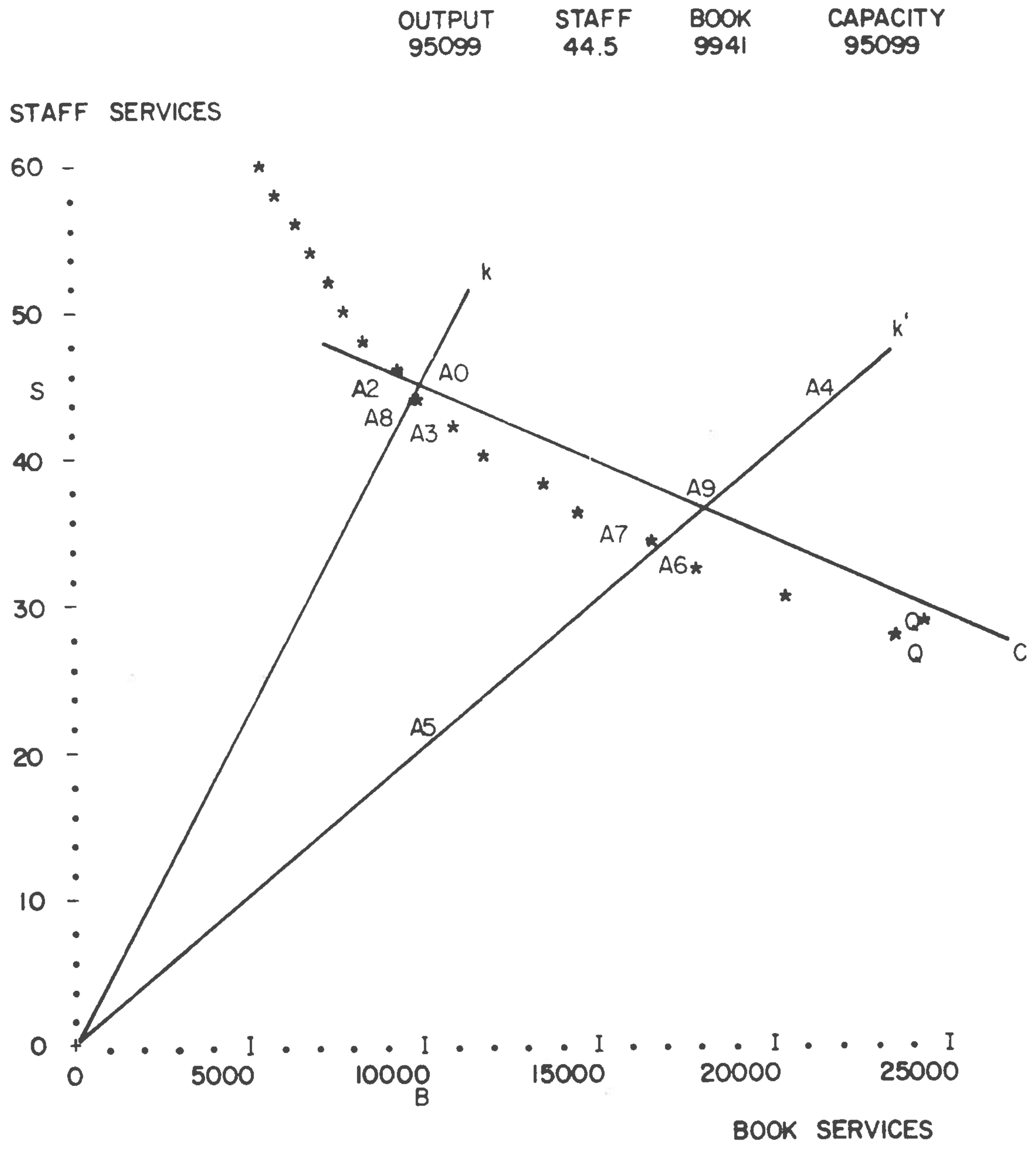

Figure 4.26 -- SUMMARY of ALL ALTERNATIVES - TEMESCAL

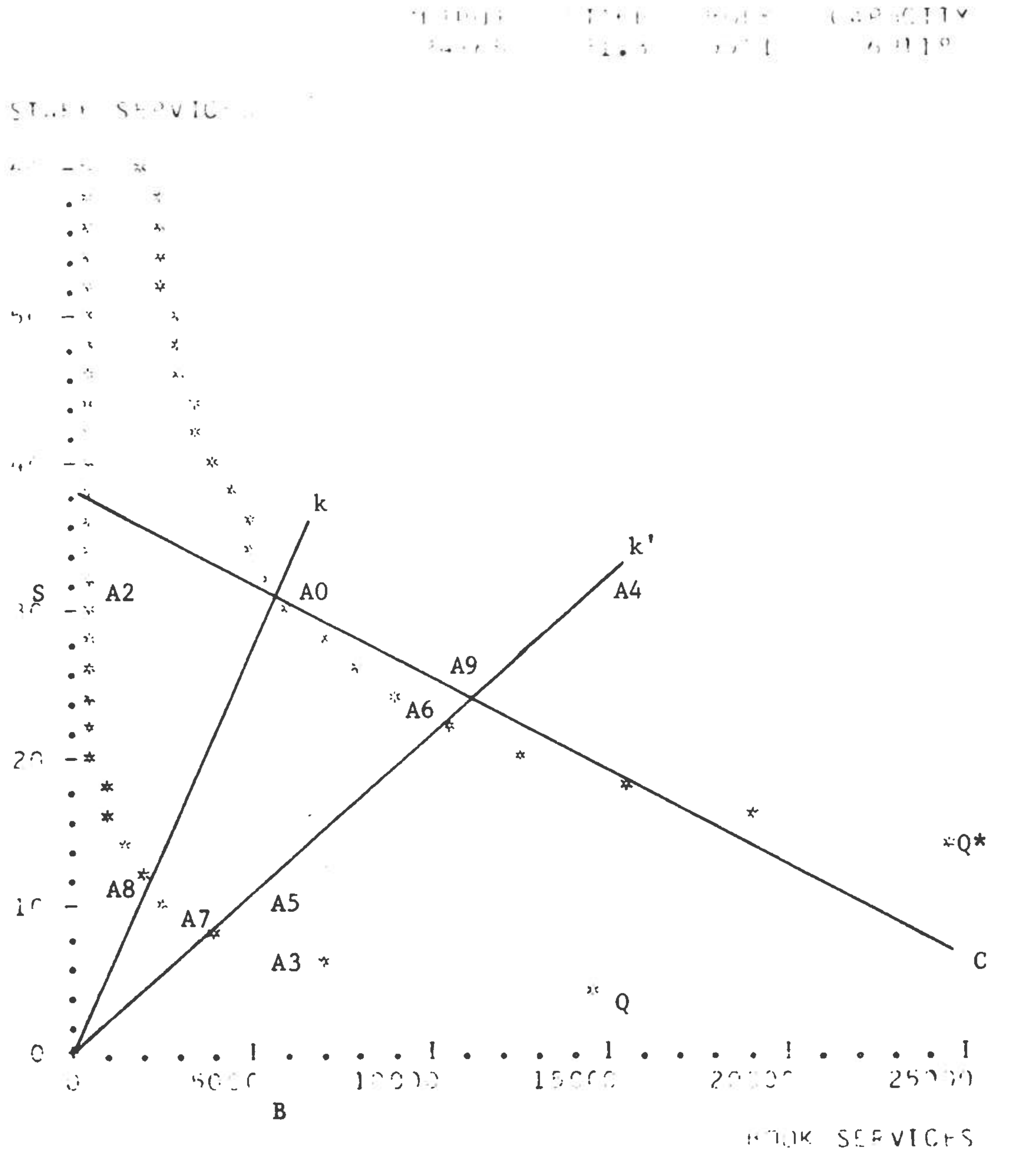

FIGURE 4.27 -- SUMMARY OF ALL ALTERNATIVES - WEST OAKLAND

TABLE 4.21 -- COST SAVINGS, p = 3.91(.05/u + ln1.05 + .06)

	Actual Cost C	Minimum Cost For Existing Capacity $\overline{C}$	No Excess Capacity Cost C^*	Minimum Cost For Existing Output C'	Input Mix Cost Savings $C - \overline{C}$	Capacity Cost Savings $C - C^*$	Input Mix Cost Savings C^*-C'	Overall Cost Savings $C - C'$
RAYMONT	23306	19778	22211	13840	3528	1095	3362	4457
INA C. COOLBRITH	21317	20953	16361	16494	464	4456	357	4823
DIMOND	37389	37361	33377	32416	628	4011	562	4573
EASTMONT	22672	21602	16254	15487	1071	5418	768	7186
ELMHURST	38783	36963	25465	24273	1821	13318	1195	14514
GIBSON	6538	6532	6405	6399	6	133	6	139
GLENVIEW	30003	29003	17500	16917	1001	12503	584	13087
GOLDEN GATE	22752	21724	[illegible]	8544	1027	13703	404	14202
LAKEVIEW	46460	44754	[illegible]	39117	1703	5359	1488	7347
LAUREL	37213	36056	29736	28811	1157	7477	925	8401
LOCKWOOD	12632	12449	5099	5025	183	7533	74	7607
JACK LONDON	22982	22119	6955	6703	863	16018	252	16279
MELROSE	40138	40023	24177	24107	115	15951	70	16030
MONTCLAIR	41322	38991	41322	38991	2331	0	2331	2331
PARK BOULEVARD	26453	25821	16910	16506	632	9544	404	9948
PIEDMONT	23355	23198	23355	23198	137	0	157	187
ROCKRIDGE	27128	26878	19931	19747	250	7197	184	7380
SEQUOYAH-CHABOT	16521	15470	16521	15470	1051	0	1051	1051
TEMESCAL	34540	32348	34540	32348	2192	0	2192	2192
WEST OAKLAND	23489	21545	8747	7983	1944	14941	764	15696

Appendix -- Circulation Conversion Weights

Weight = (Output - Circulation)/Circulation

Class A		Class B		Class C	
Dimond	.0888	Baymont	.1046	Ina D. Coolbrith	.2712
Elmhurst	.1250	Eastmont	.1323	Gibson	.1926
Lakeview	.0742	Glenview	.1236	Golden Gate	.2730
Laurel	.0901	Park Boulevard	.1324	Lockwood	.3195
Melrose	.1115	Rockridge	.1003	Jack London	.5087
Montclair	.0593			Sequoyah-Chabot	.4181
Piedmont	.0964			West Oakland	.3500
Temescal	.1485				
Total	.7938	Total	.5932	Total	2.3331
Weight	.0992	Weight	.1186	Weight	.3333