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# **Lateral Control of Tractor-Semitrailer Vehicles in Automated Highway Systems**

**Chieh Chen**

**Masayoshi Tomizuka**

*University of California, Berkeley*

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# Lateral Control of Tractor-Semitrailer Vehicles in Automated Highway Systems

Chieh Chen and Masayoshi Tomizuka

Department of Mechanical Engineering

University of California at Berkeley

Berkeley, California 94720

## **Abstract**

Linear quadratic optimal control with and without frequency shaping is utilized to design a steering controller for tractor-semitrailer vehicles in Automated Highway Systems (AHS). To enhance driving safety, we propose to use independent braking forces in the trailer as another control input. The combined steering and braking controller is designed by utilizing Input/Output linearization and backstepping design techniques.

## **Keywords**

Vehicle dynamics, Lateral Control, Steering Control, Braking control, AHS

## Executive Summary

This report summarizes the second year research on the lateral control of tractor-semitrailer vehicles in automated highway systems (AHS) conducted in the PATH project, MOU129, Steering and Braking Control of Heavy Duty Vehicles. We utilize a complex simulation model and two simplified control models for the tractor-trailer vehicles developed in the first year of the project (Chen and Tomizuka 1995). Two types of controllers are considered: the steering controller and the coordinated steering-braking controller. Linear quadratic optimal control with and without frequency shaping (LQ and FSLQ) is utilized to design control algorithms for the first type of controllers. The lateral acceleration of the vehicle is explicitly included in the performance index of the FSLQ formulation, and the FSLQ steering control algorithm will minimize the lateral acceleration so that the possibility of vehicle rollovers is reduced. Furthermore, an integral action (I-control) is included in the FSLQ approach. Simulations show that the FSLQ controller has better tracking performance than the LQ controller. The coordinated steering-braking controller is considered in order to enhance driving safety and avoid unstable yaw motion. In this controller, the braking force for each of the rear trailer wheels is controlled independently in addition to the steering angle. The combined steering and braking controller is designed by utilizing Input/Output linearization and backstepping design techniques. Simulations show that both the tractor and the trailer yaw errors under the Input/Output linearization control with independent braking force control are smaller than those without independent braking force control.

# 1 Introduction

Past research on advanced vehicle control systems (AVCS) for automated highway systems (AHS) has emphasized passenger vehicles (Fenton and Mayhan 1991, Shladover et. al. 1991, and Peng and Tomizuka 1993). The study of heavy vehicles for AHS applications has, however, gained interest only recently (Zimmermann, Fuchs, et al. 1994, Blosseville, Blondeel and Graton 1995, Favre 1995, Bishel 1993, Yanakiev and Kanellakopoulos 1995, Chen and Tomizuka 1995, and Kanellakopoulos and Tomizuka 1996). The study of heavy-duty vehicles for AHS is important for several reasons. In 1993, the share of the highway miles accounted for by the truck traffic was around 28% (Federal Highway Administration 1994). This is a significant percentage of the total highway miles by all the vehicles in US. According to Motor Vehicles Facts and Figures (American Automobile Manufacturers' Association 1993), California has one of the largest number of establishments manufacturing truck and truck-trailer combinations in the US. In 1991, the total number of registered trucks (light, commercial and truck-trailer combinations) formed approximately 10% of the national figures. In 1991, 34.3% of the highway taxes came from heavy vehicles and was above the national average of 30.9%. Also, due to several economic and policy issues, heavy vehicles have the potential of becoming the main beneficiaries of the automated guidance (Kanellakopoulos and Tomizuka 1996). The main reasons are:

- On average, a truck travels six times the miles as compared to a passenger vehicle. Possible reduction in the number of drivers will reduce the operating cost substantially.
- Relative equipment cost for automating the heavy vehicles is far less than the passenger vehicles.
- Automation of heavy vehicles will have a significant impact on the overall safety of the automated guidance system. Trucking is a tedious job and automation will contribute positively to reducing the driving stress and thereby contributing to the safety.

Thus Commercial trucks and buses will gain significant benefit from AVCS, and may actually become automated earlier than passenger vehicles due to economical considerations. Various organizations are thus getting increasingly involved with the automated guidance of trucks. Renault

and INRETS in France are involved in the study of AHS as applied to freight transport in France and associated issues at various levels (including socio-economic) (Blosseville, Blondeel and Gratton 1995). In U.S., Ervin (1992) tried to identify the domain of research on IVHS which might be of interest to the heavy vehicles manufacturers.

This report follows Chen and Tomizuka (1995) and is concerned with lateral lane following control of tractor-semitrailer vehicles in AHS. A complex nonlinear simulation model for the tractor semitrailer vehicle, developed in Chen and Tomizuka (1995), will be used to validate the effectiveness of the lane following control algorithms. This simulation model includes the roll, pitch and yaw motions of tractor-semitrailer vehicles.

The organization of this report is as follows. In section 2, linear quadratic optimal control with and without frequency shaping is utilized to design the steering controller for tractor-semitrailer vehicles. In section 3, a nonlinear model for steering and braking control is formulated. As safety is always of primary concern in AHS, we propose to use independent braking of the trailer to directly control the trailer yaw motion; consequently, jack-knifing may be avoided during autonomous driving operations of tractor-semitrailer vehicles. Conclusions are given in section 4.

## **2 Linear Controller**

Two linear control algorithms have been designed based on Linear Quadratic (LQ) optimal control (Anderson and Moore 1990) and Frequency-Shaped Linear Quadratic (FSLQ) optimal control (Gupta 1980). The linear quadratic optimal control algorithm makes a compromise between tracking performance and steering input. Perfect tracking is not attempted in these formulations. The frequency-shaped linear quadratic optimal control algorithm is developed to include the lateral acceleration of the vehicle in the performance index; thus the lateral acceleration is reduced, vehicle rolling over is avoided and driver's ride quality is enhanced. Moreover, since we can penalize high frequency components of the control input, the FSLQ control algorithm will not excite any unmodelled high frequency dynamics and is more robust than LQ optimal control.

## 2.1 Linear Quadratic Optimal Control

A linear control model for a tractor-semitrailer vehicle was developed in Chen and Tomizuka (1995) and has the following form:

$$M\ddot{q}_r + D\dot{q}_r + Kq_r = F\delta + E_1\dot{\epsilon}_d + E_2\ddot{\epsilon}_d \quad (1)$$

where  $q_r = [y_r, \epsilon_r, \epsilon_f]^T$  is the generalized coordinate vector, the inertial matrix  $M$  is

$$M = \begin{pmatrix} m_1 + m_2 & -m_2(d_1 + d_3) & -m_2d_3 \\ -m_2(d_1 + d_3) & I_{z1} + I_{z2} + m_2(d_1 + d_3)^2 & I_{z2} + m_2d_3^2 + m_2d_1d_3 \\ -m_2d_3 & I_{z2} + m_2d_3^2 + m_2d_1d_3 & I_{z2} + m_2d_3^2 \end{pmatrix} \quad (2)$$

the damping matrix  $D$  is

$$D = \frac{2}{V} \begin{pmatrix} C_{\alpha f} + C_{\alpha r} + C_{\alpha t} & l_1C_{\alpha f} - l_2C_{\alpha r} - (l_3 + d_1)C_{\alpha t} & -l_3C_{\alpha t} \\ l_1C_{\alpha f} - l_2C_{\alpha r} - (l_3 + d_1)C_{\alpha t} & l_1^2C_{\alpha f} + l_2^2C_{\alpha r} + (l_3 + d_1)^2C_{\alpha t} & l_3(l_3 + d_1)C_{\alpha t} \\ -l_3C_{\alpha t} & l_3(l_3 + d_1)C_{\alpha t} & l_3^2C_{\alpha t} \end{pmatrix} \quad (3)$$

the  $K$  matrix is

$$K = \begin{pmatrix} 0 & -2(C_{\alpha f} + C_{\alpha r} + C_{\alpha t}) & -2C_{\alpha t} \\ 0 & -2(l_1C_{\alpha f} - l_2C_{\alpha r} - (l_3 + d_1)C_{\alpha t}) & 2(l_3 + d_1)C_{\alpha t} \\ 0 & 2l_3C_{\alpha t} & 2l_3C_{\alpha t} \end{pmatrix} \quad (4)$$

and the vectors  $F$ ,  $E_1$  and  $E_2$  are

$$F = 2C_{\alpha f} \begin{pmatrix} 1 \\ l_1 \\ 0 \end{pmatrix} \quad (5)$$

$$E_1 = \begin{pmatrix} -(m_1 + m_2)V - \frac{2}{V}(l_1C_{\alpha f} - l_2C_{\alpha r} - (l_3 + d_1)C_{\alpha t}) \\ m_2(d_1 + d_3)V - \frac{2}{V}(l_1^2C_{\alpha f} + l_2^2C_{\alpha r} + (l_3 + d_1)^2C_{\alpha t}) \\ m_2d_3V - \frac{2}{V}l_3(l_3 + d_1)C_{\alpha t} \end{pmatrix} \quad (6)$$

and

$$E_2 = \begin{pmatrix} m_2(d_1 + d_3) \\ -(I_{z1} + I_{z2} + m_2(d_1 + d_3)^2) \\ -(I_{z2} + m_2d_3^2 + m_2d_1d_3) \end{pmatrix} \quad (7)$$

respectively. Notations of state variables and vehicle parameters are defined in Appendix 1. Note that Eq.(1) can be rewritten in state space form as

$$\frac{d}{dt}x = Ax + B\delta + H_1\dot{\epsilon}_d + H_2\ddot{\epsilon}_d \quad (8)$$

In this formulation we will regard the lane following problem as a disturbance rejection regulation problem; i.e., we will treat the  $\dot{\epsilon}_d$  and  $\ddot{\epsilon}_d$  terms in equation (8) as disturbances to the system. Then, for LQ formulation we have the state equation

$$\frac{d}{dt}x = Ax + B\delta \quad (9)$$

and the performance index

$$J = \int_{t=0}^{\infty} (x^T(t)Qx(t) + \delta(t)R\delta(t))dt \quad (10)$$

where  $Q \geq 0$ ,  $R > 0$  and  $(\cdot)^T$  denotes the transpose of  $(\cdot)$ .

The optimal control that minimizes the performance index is  $\delta = -Kx$ , where

$$K = R^{-1}B^T H \quad (11)$$

and  $H$  is the positive solution of the algebraic Riccati equation,

$$A^T H + HA - HBR^{-1}B^T H + Q = 0 \quad (12)$$

## 2.2 Frequency-Shaped Linear Quadratic Optimal Control

The frequency-shaped cost functional for vehicle lateral control is (Peng and Tomizuka 1990)

$$\begin{aligned} J = & \frac{1}{2\pi} \int_{-\infty}^{\infty} (\ddot{Y}_r^*(j\omega)q_{y_a}^*(j\omega)q_{y_a}(j\omega)\ddot{Y}_r(j\omega) + \ddot{\epsilon}_f^*(j\omega)q_{\epsilon_{f_a}}^*(j\omega)q_{\epsilon_{f_a}}(j\omega)\ddot{\epsilon}_f(j\omega) \\ & + Y_r^*(j\omega)q_i^*(j\omega)q_i(j\omega)Y_r(j\omega) + Y_r^*(j\omega)q_y^*(j\omega)q_y(j\omega)Y_r(j\omega) \\ & + \epsilon_r^*(j\omega)q_{\epsilon}^*(j\omega)q_{\epsilon}(j\omega)\epsilon_r(j\omega) + \epsilon_f^*(j\omega)q_{\epsilon_f}^*(j\omega)q_{\epsilon_f}(j\omega)\epsilon_f(j\omega))d\omega \end{aligned} \quad (13)$$

where  $\ddot{Y}_r$ ,  $\ddot{\epsilon}_f$ ,  $Y_r$ ,  $\epsilon_r$  and  $\epsilon_f$  are the Laplace transforms of  $\ddot{y}_r$ ,  $\ddot{\epsilon}_f$ ,  $y_r$ ,  $\epsilon_r$  and  $\epsilon_f$ ,  $y_r$  is as defined in Appendix 1,  $(\cdot)^*$  denotes the complex conjugate of  $(\cdot)$  and the frequency shaping filters are given as follows.

$$q_{y_a}(s) = \frac{k_{y_a}}{1 + \lambda_{y_a}s} \quad (14)$$

$$q_{\epsilon_f a}(s) = \frac{k_{\epsilon_f a}}{1 + \lambda_{\epsilon_f a} s} \quad (15)$$

$$q_i(s) = \frac{k_i}{s} \quad (16)$$

$$q_y(s) = \frac{k_y}{1 + \lambda_y s} \quad (17)$$

$$q_\epsilon(s) = \frac{k_\epsilon}{1 + \lambda_\epsilon s} \quad (18)$$

$$q_{\epsilon_f}(s) = \frac{k_{\epsilon_f}}{1 + \lambda_{\epsilon_f} s} \quad (19)$$

Notice that we penalize the lateral acceleration of the vehicle, angular acceleration of the articulation angle, lateral displacement, tractor relative yaw angle and semitrailer articulation angle. We add an integral term for the lateral displacement so that the steady state error of the lateral displacement goes to zero. Define the augmented state variables

$$z_1 = \frac{k_{y a}}{1 + \lambda_{y a} s} s^2 Y_r \quad (20)$$

$$z_2 = \frac{k_{\epsilon_f a}}{1 + \lambda_{\epsilon_f a} s} s^2 \epsilon_f \quad (21)$$

$$z_3 = \frac{k_i}{s} y_r \quad (22)$$

$$z_4 = \frac{k_y}{1 + \lambda_y s} y_r \quad (23)$$

$$z_5 = \frac{k_\epsilon}{1 + \lambda_\epsilon s} \epsilon_r \quad (24)$$

$$z_6 = \frac{k_{\epsilon_f}}{1 + \lambda_{\epsilon_f} s} \epsilon_f \quad (25)$$

From Parseval's equality, the frequency-weighted performance index can be transformed to

$$J = \int_{t=0}^{\infty} (z_1^2 + z_2^2 + z_3^2 + z_4^2 + z_5^2 + z_6^2 + R\delta^2) dt \quad (26)$$

Defining a new state vector  $x_T$  as

$$x_T = (x^T, z_1, z_2, z_3, z_4, z_5, z_6)^T, \quad (27)$$

the total augmented system can be written as

$$\frac{d}{dt} x_T = A_T x_T + B_T \delta \quad (28)$$

and the performance index as

$$J = \int_{t=0}^{\infty} (x_T^T(t) Q_T x_T(t) + R\delta(t)^2) dt \quad (29)$$

The original optimal control problem with the frequency shaped performance index (13) has been transformed to a standard LQ problem given by (28) and (29), and the solution is given as described in section 2.1



## 2.3 Simulation Results

The simulation scenario we used is shown in Fig. 1. The vehicle travels along a straight roadway with initial lateral displacement 15 cm and enters a curved section with radius of curvature 450 m at time  $t = 5$  sec. Fig. 2 and 3 show the simulation results of the LQ and FSLQ optimal controllers, respectively, at vehicle speed equal to 60 MPH. Fig. 4 shows the comparison of these two controllers. We see that the FSLQ controller has better tracking performance without increasing the control effort.

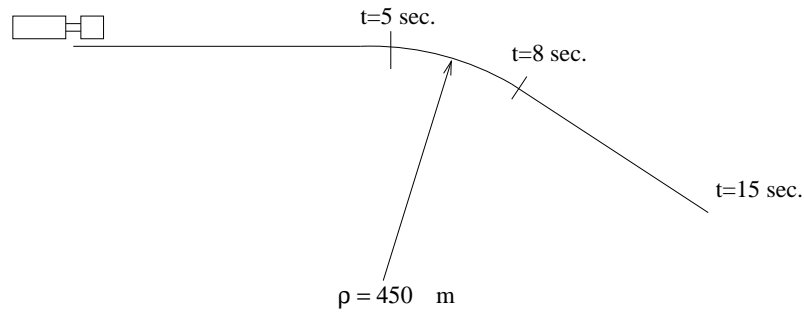


Figure 1: Simulation Scenarios

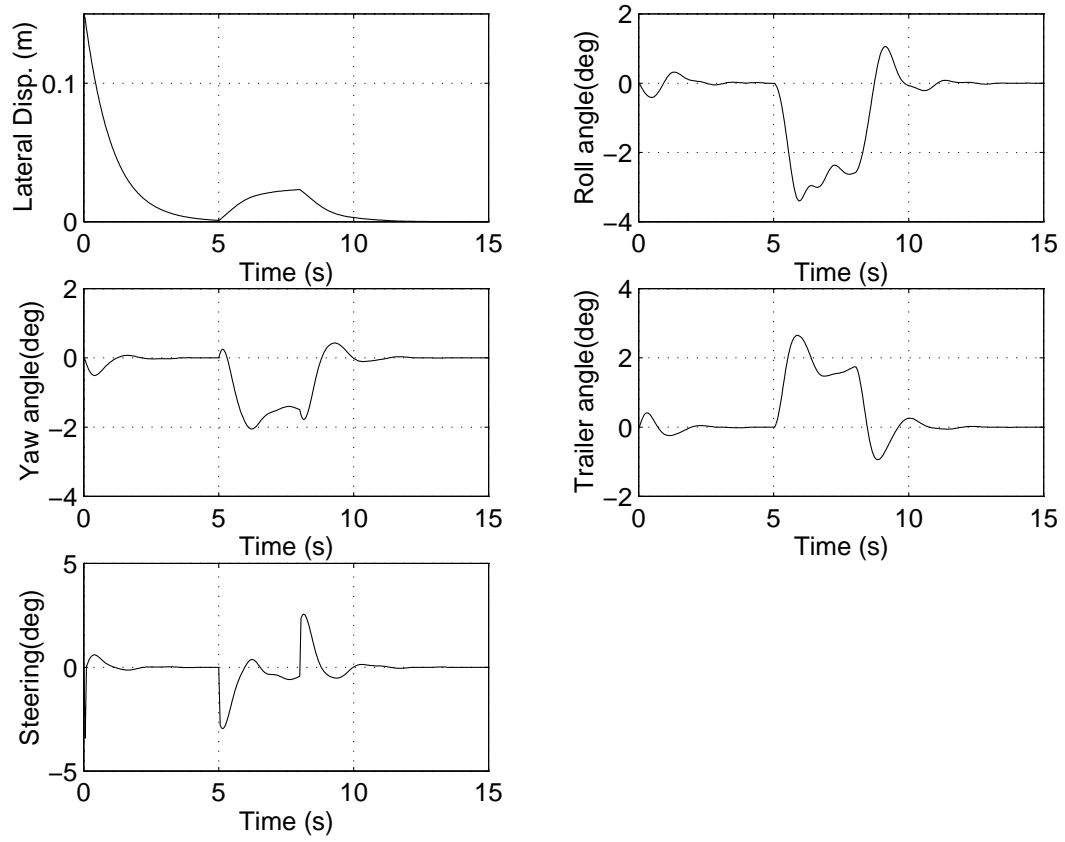


Figure 2: Linear Quadratic Optimal Control

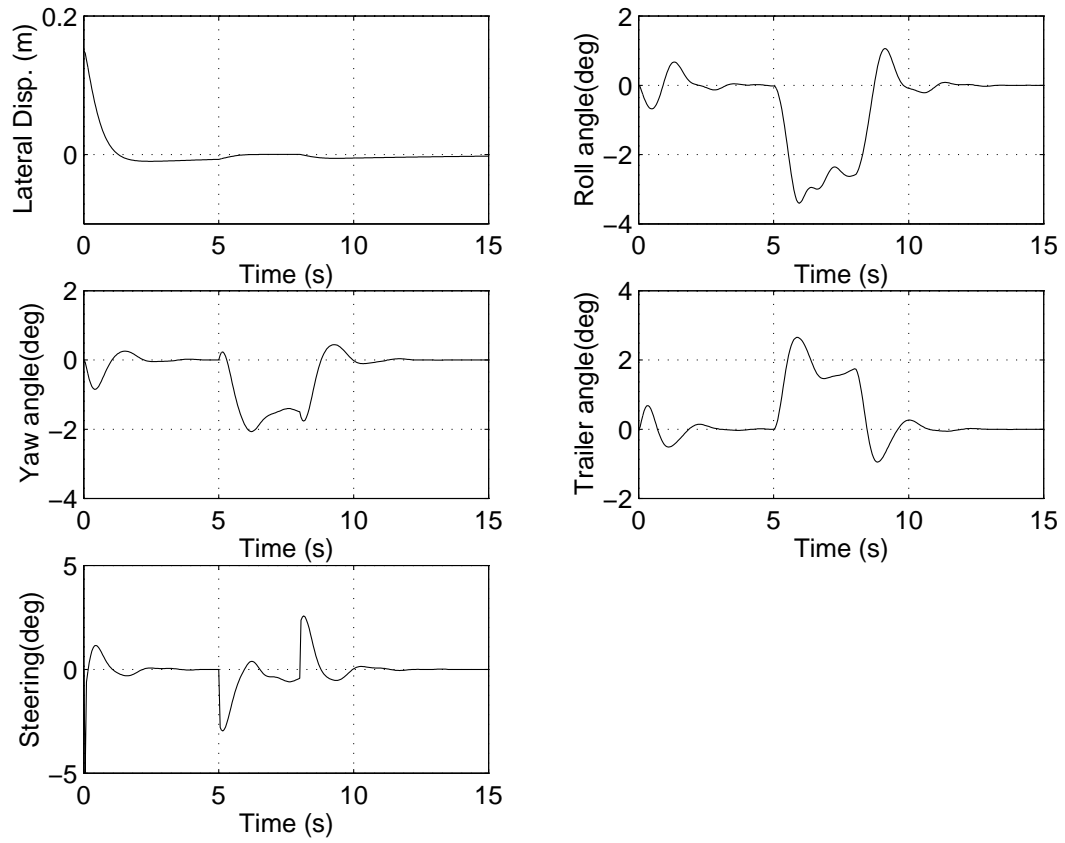


Figure 3: Frequency Shaped Linear Quadratic Optimal Control

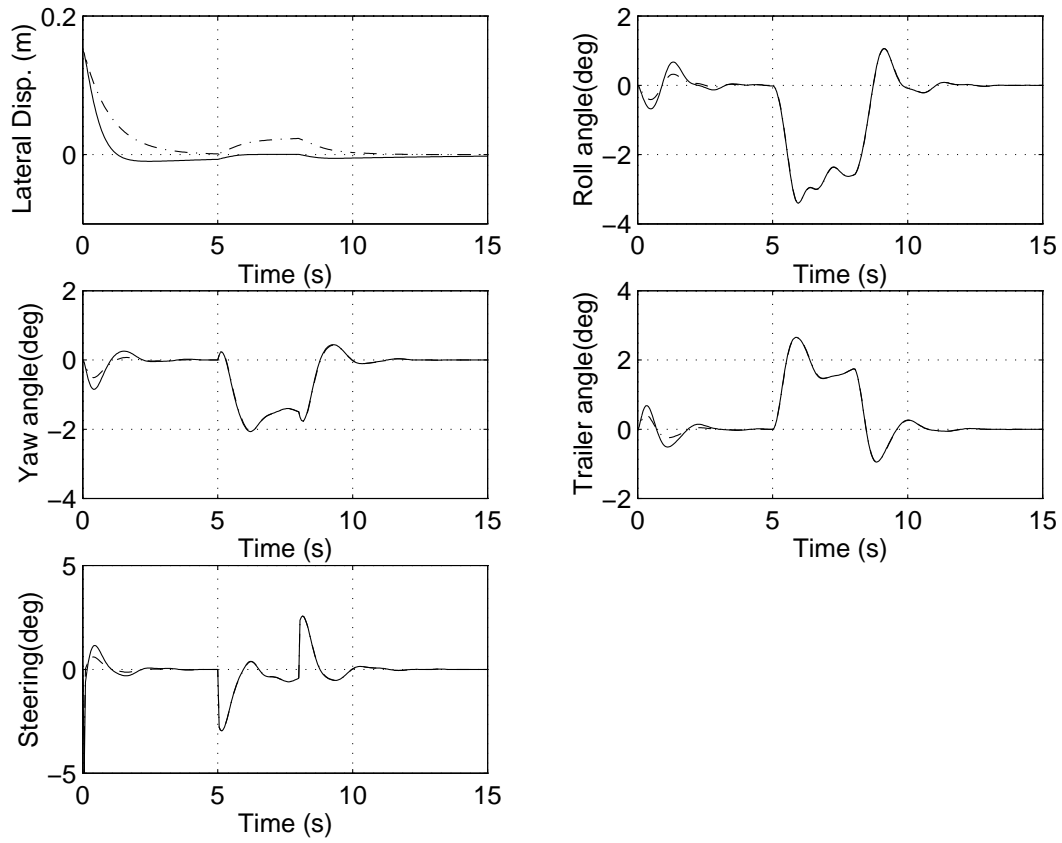


Figure 4: Comparison of LQ (dashdot line) and FSLQ (solid line) Optimal Control

### 3 Nonlinear Controller

Although linear control algorithms designed in section 2 show satisfactory results, they are sensitive to the vehicle's longitudinal speed. One way to solve this problem is to utilize gain scheduling techniques with respect to the longitudinal speed. Another way is to apply nonlinear control techniques. In this section, a nonlinear control model is formulated. Motivated by Matsumoto and Tomizuka (1992), we propose to use not only the tractor's front wheel steering input but also the trailer's unilateral tire braking to provide the differential torque for directly controlling the trailer yaw motion.

#### 3.1 Nonlinear Steering and Braking Control Model

We relax the assumption in Chen and Tomizuka (1995) that the longitudinal speed is constant, and that tractor relative yaw motion and trailer's articulation angle are small. Furthermore, we include the trailer's left and right braking forces into the control model as additional control inputs. The control model can be represented as

$$M(q_r)\ddot{q}_r + C(\dot{q}_r, \dot{\epsilon}_d, \dot{\epsilon}_d) = H \cdot U \quad (30)$$

where

$$M(q_r) =$$

$$\begin{pmatrix} m_1 + m_2 & -m_2(d_1 + d_3 \cos \epsilon_f) & -m_2 d_3 \cos \epsilon_f \\ -m_2(d_1 + d_3 \cos \epsilon_f) & I_{z1} + I_{z2} + m_2 d_1^2 + m_2 d_3^2 + 2m_2 d_1 d_3 \cos \epsilon_f & I_{z2} + m_2 d_3^2 + m_2 d_1 d_3 \cos \epsilon_f \\ -m_2 d_3 \cos \epsilon_f & I_{z2} + m_2 d_3^2 + m_2 d_1 d_3 \cos \epsilon_f & I_{z2} + m_2 d_3^2 \end{pmatrix} \quad (31)$$

$$C(\dot{q}_r, \dot{\epsilon}_d, \dot{\epsilon}_d)(1) =$$

$$\begin{pmatrix} \frac{2}{x}((C_{\alpha f} + C_{\alpha r} + C_{\alpha t})(\dot{y}_r - \dot{x}\epsilon_r) + (l_1 C_{\alpha f} - l_2 C_{\alpha r} - (l_3 + d_1)C_{\alpha t})(\dot{\epsilon}_r + \dot{\epsilon}_d) - l_3 C_{\alpha t} \dot{\epsilon}_f) \\ -2C_{\alpha t} \dot{\epsilon}_f + m_2 d_3 \sin \epsilon_f (\dot{\epsilon}_r + \dot{\epsilon}_d + \dot{\epsilon}_f)^2 + (m_1 + m_2)\dot{x}\dot{\epsilon}_d - m_2(d_1 + d_3 \cos \epsilon_f)\ddot{\epsilon}_d \end{pmatrix} \quad (32)$$

$$C(\dot{q}_r, \dot{\epsilon}_d, \ddot{\epsilon}_d)(2) =$$

$$\left( \begin{array}{l} \frac{2}{x}((l_1 C_{\alpha f} - l_2 C_{\alpha r} - (l_3 + d_1) C_{\alpha t})(\dot{y}_r - \dot{x}\epsilon_r) + (l_1^2 C_{\alpha f} + l_2^2 C_{\alpha r} + (l_3 + d_1)^2 C_{\alpha t})(\dot{\epsilon}_r + \dot{\epsilon}_d) \\ + l_3(l_3 + d_1) C_{\alpha t} \dot{\epsilon}_f) + 2(l_3 + d_1) C_{\alpha t} \epsilon_f - m_2 d_3 \sin \epsilon_f (\dot{y}_r - \dot{x}\epsilon_r)(\dot{\epsilon}_r + \dot{\epsilon}_d) \\ - 2m_2 d_1 d_3 \sin \epsilon_f (\dot{\epsilon}_r + \dot{\epsilon}_d) \dot{\epsilon}_f - m_2 d_1 d_3 \sin \epsilon_f \dot{\epsilon}_f^2 - m_2 (d_1 + d_3 \cos \epsilon_f) \dot{x} \dot{\epsilon}_d \\ + (I_{z1} + I_{z2} + m_2 d_1^2 + m_2 d_3^2 + 2m_2 d_1 d_3 \cos \epsilon_f) \ddot{\epsilon}_d \end{array} \right) \quad (33)$$

$$C(\dot{q}_r, \dot{\epsilon}_d, \ddot{\epsilon}_d)(3) =$$

$$\left( \begin{array}{l} \frac{2}{x}(-l_3 C_{\alpha t}(\dot{y}_r - \dot{x}\epsilon_r) + l_3(l_3 + d_1) C_{\alpha t}(\dot{\epsilon}_r + \dot{\epsilon}_d) + l_3^2 C_{\alpha t} \dot{\epsilon}_f) + m_2 d_3 \sin \epsilon_f (\dot{\epsilon}_r + \dot{\epsilon}_d)^2 \\ + 2l_3 C_{\alpha t} \epsilon_f - m_2 d_3 \sin \epsilon_f (\dot{y}_r - \dot{x}\epsilon_r)(\dot{\epsilon}_r + \dot{\epsilon}_d) - m_2 d_3 \cos \epsilon_f \dot{x} \dot{\epsilon}_d + (I_{z2} + m_2 d_3^2 + m_2 d_1 d_3 \cos \epsilon_f) \ddot{\epsilon}_d \end{array} \right) \quad (34)$$

with  $C(\dot{q}, \dot{\epsilon}_d, \ddot{\epsilon}_d)(i), i = 1 \dots 3$ , representing the  $i$ -th element of the column vector  $C(\dot{q}, \dot{\epsilon}_d, \ddot{\epsilon}_d)$ ,

$$H = \begin{pmatrix} 2C_{\alpha f} & 0 \\ 2l_1 C_{\alpha f} & \frac{T_{w3}}{2} \\ 0 & \frac{T_{w3}}{2} \end{pmatrix} \quad (35)$$

and

$$U = \begin{pmatrix} \delta \\ F_2 - F_1 \end{pmatrix} \equiv \begin{pmatrix} \delta \\ T \end{pmatrix} \quad (36)$$

Notice that  $F_1$  and  $F_2$  stand for the longitudinal forces at the left and right wheels of the trailer. Thus  $T$  is the differential force acting on the trailer. Notice also that  $F_i < 0$  when it is a braking force,  $F_i > 0$  when it is a traction force. In fact, the control inputs  $F_1$  and  $F_2$  at the wheels of the trailer can only be negative; i.e., the braking force is the only input. This would be a big constraint on the control input. However, the differential force  $T$  can be both positive and negative. Furthermore, the braking force  $F_i, i = 1, 2$ , is determined by the tire force model and is a function of the tire slip ratio. Specifically, as shown in Fig. 5, the wheel dynamics are

$$I_w \dot{\omega}_i = -F_i r + \tau_i \quad (37)$$

where  $\omega_i$  is the angular velocity of the wheel,  $F_i$  is the braking force generated at the tire/ground interface, and  $\tau_i$  is the braking torque applied at the braking disk of the wheel. The tire slip ratio is defined as

$$\lambda_i = \frac{\omega_i r - V}{V} \quad (38)$$

and the braking force is

$$F_i = C_l \lambda_i \quad (39)$$

Eq. (30) as well as Eqs. (37) (38) and (39) will be used to design the coordinated steering and braking control algorithm in section 3.2.

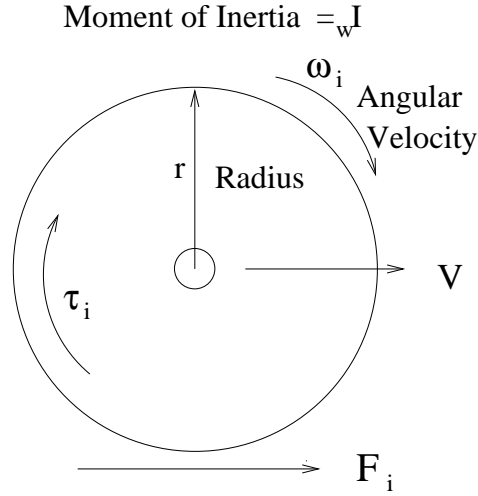


Figure 5: Wheel Dynamics

### 3.2 Coordinated Steering and Independent Braking Control

In this subsection, a coordinated steering and braking control algorithm will be designed. We use the tractor's steering input to achieve lane following and the trailer's differential braking to enhance the stability of the trailer. The control algorithm will be designed in two steps. In the first step, we assume the braking forces  $F_1$  and  $F_2$  are control inputs. Then the desired steering command  $\delta_d$  and the desired differential braking force  $T_d$  are determined by input/output linearization scheme. If  $T_d > 0$ , we have  $F_{1d} = -T_d$  and  $F_{2d} = 0$ . On the other hand, if  $T_d < 0$ , we have  $F_{1d} = 0$  and  $F_{2d} = T_d$ . In the second step the required braking torques  $\tau_1$  and  $\tau_2$  are determined to generate the desired braking forces  $F_{1d}$  and  $F_{2d}$  by utilizing backstepping design methodologies (Kanellakopoulos, Kokotovic and Morse 1991) (Kokotovic 1992).

#### Step 1

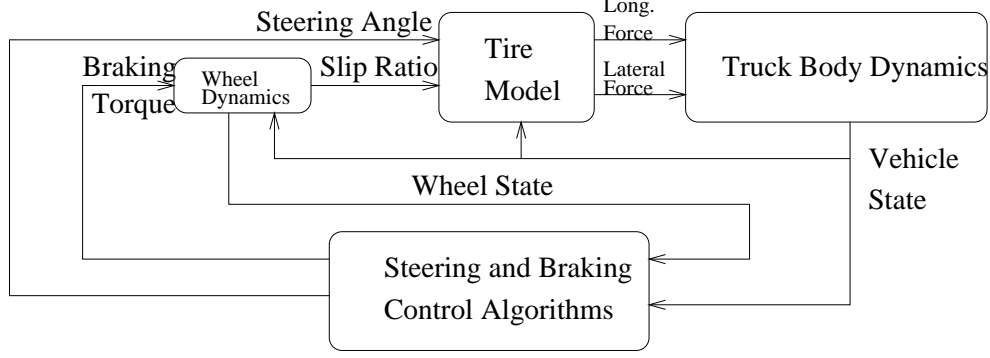


Figure 6: Vehicle system including wheel dynamics

First, we define the first system output  $e_1$  as the lateral tracking error

$$e_1 = y_r \quad (40)$$

and the second output  $e_2$  as the articulation angle between the tractor and the trailer

$$e_2 = \epsilon_f \quad (41)$$

Differentiating  $e_1$  and  $e_2$  twice, we obtain

$$\begin{pmatrix} \ddot{e}_1 \\ \ddot{e}_2 \end{pmatrix} = \begin{pmatrix} M^{-1}(1) \\ M^{-1}(3) \end{pmatrix} C(\dot{q}, \dot{\epsilon}_d, \ddot{\epsilon}_d) + \begin{pmatrix} M^{-1}(1) \\ M^{-1}(3) \end{pmatrix} HU \quad (42)$$

The number  $i$  in the parenthesis  $M^{-1}(i)$  denotes the  $i$ -th row of the  $M^{-1}$  matrix. For notational simplicity, we define

$$J = \begin{pmatrix} M^{-1}(1) \\ M^{-1}(3) \end{pmatrix} H \quad (43)$$

If the matrix  $J$  is nonsingular, we can choose the control input  $U$  as

$$U = -J^{-1} \begin{pmatrix} M^{-1}(1) \\ M^{-1}(3) \end{pmatrix} C(\dot{q}, \dot{\epsilon}_d, \ddot{\epsilon}_d) - J^{-1} \left( K_D \begin{pmatrix} \dot{e}_1 \\ \dot{e}_2 \end{pmatrix} + K_P \begin{pmatrix} e_1 \\ e_2 \end{pmatrix} \right) \quad (44)$$

This control law cancels the system nonlinearities and inserts the desired error dynamics. Thus the closed loop system becomes

$$\begin{pmatrix} \ddot{e}_1 \\ \ddot{e}_2 \end{pmatrix} + K_D \begin{pmatrix} \dot{e}_1 \\ \dot{e}_2 \end{pmatrix} + K_P \begin{pmatrix} e_1 \\ e_2 \end{pmatrix} = 0. \quad (45)$$



Step 2

In *Step 1* we regard  $T$  as a real control input; then the desired steering command and the desired differential braking forces  $T_d$  are set in (44). In this step we will 'backstep' to determine the braking torques  $\tau_1$  and  $\tau_2$  on the trailer's left and right wheels. Recall that the wheel dynamics is

$$I_w \dot{\omega}_i = -F_i r + \tau_i \quad (46)$$

and the tire force is

$$F_i = C_{lt} \lambda_i \quad (47)$$

where the slip ratio  $\lambda_i$  is defined as

$$\lambda_i = \frac{\omega_i r - V}{V} \quad (48)$$

Combining equations (46), (47) and (48), we obtain

$$\begin{aligned} \dot{F}_i &= C_{lt} \dot{\lambda}_i \\ &= C_{lt} \left( \frac{\partial \lambda_i}{\partial V} \dot{V} + \frac{\partial \lambda_i}{\partial \omega_i} \dot{\omega}_i \right) \\ &= C_{lt} \left( -\frac{\omega_i r}{V^2} \dot{V} + \frac{r}{I_w V} (-C_{lt} \lambda_i r + \tau_i) \right) \end{aligned} \quad (49)$$

Thus the equations governing the vehicle dynamics and wheel dynamics are

$$\begin{pmatrix} \ddot{e}_1 \\ \ddot{e}_2 \end{pmatrix} = \begin{pmatrix} M^{-1}(1) \\ M^{-1}(3) \end{pmatrix} C(\dot{q}, \dot{e}_d, \ddot{e}_d) + JU \quad (50)$$

and

$$\dot{F}_i = C_{lt} \left( -\frac{\omega_i r}{V^2} \dot{V} + \frac{r}{I_w V} (-C_{lt} \lambda_i r + \tau_i) \right) \quad (51)$$

Recall that  $T = F_2 - F_1$  and both  $F_1$  and  $F_2$  are negative. Thus if  $T_d$  determined in (44) is positive, we have  $F_{1d} = -T_d$  and  $F_{2d} = 0$ ; i.e., the braking controller will apply the brake torque on the trailer's left wheel. On the other hand, if  $T_d$  is negative, we have  $F_{1d} = 0$  and  $F_{2d} = T_d$ ; i.e., the braking controller will apply the brake torque on the trailer's right wheel. From Eq. (44) in *step 1*, we can choose

$$\delta = \delta_d(q_r, \dot{q}_r, \dot{e}_d) \quad (52)$$

and

$$T = T_d(q_r, \dot{q}_r, \dot{e}_d) \quad (53)$$

so that equation (50) becomes

$$\ddot{e}_1 + k_{d1}\dot{e}_1 + k_{p1}e_1 = 0 \quad (54)$$

and

$$\ddot{e}_2 + k_{d2}\dot{e}_2 + k_{p2}e_2 = 0. \quad (55)$$

Note that  $T$  is determined by the braking force  $F_i$ , and that braking force  $F_i$  can be adjusted only through equation (51); i.e., the braking torque  $\tau_i$  is the actual control input. Therefore,  $T$  cannot be simply set to  $T_d$  all the time, and  $\tau_i$  must be adjusted so that the difference between  $T_d$  and  $T$  is brought to zero. This is the main idea in the backstepping procedure. We define two new variables  $\eta_1$  and  $\eta_2$  as

$$\eta_1 = F_1 - F_{1d} \quad (56)$$

and

$$\eta_2 = F_2 - F_{2d} \quad (57)$$

respectively. Then we have

$$\begin{aligned} T &= F_2 - F_1 \\ &= F_{2d} + \eta_2 - F_{1d} - \eta_1 \\ &= T_d + \eta_2 - \eta_1 \end{aligned} \quad (58)$$

Since

$$\begin{aligned} \dot{\eta}_1 &= \dot{F}_1 - \dot{F}_{1d} \\ &= C_{1t}(-\frac{\omega_1 r}{V^2}\dot{V} + \frac{r}{I_w V}(-C_{1t}\lambda_1 r + \tau_1)) - \dot{F}_{1d} \end{aligned} \quad (59)$$

and

$$\begin{aligned} \dot{\eta}_2 &= \dot{F}_2 - \dot{F}_{2d} \\ &= C_{2t}(-\frac{\omega_2 r}{V^2}\dot{V} + \frac{r}{I_w V}(-C_{2t}\lambda_2 r + \tau_2)) - \dot{F}_{2d} \end{aligned} \quad (60)$$

we choose

$$\tau_1 = C_{lt}\lambda_1 r + \frac{L_w V}{r} \left( -\frac{\omega_1 r}{V^2} \dot{V} + \frac{1}{C_{lt}} (\dot{F}_{1d} - k_1 \eta_1) \right) \quad (61)$$

and

$$\tau_2 = C_{lt}\lambda_2 r + \frac{L_w V}{r} \left( -\frac{\omega_2 r}{V^2} \dot{V} + \frac{1}{C_{lt}} (\dot{F}_{2d} - k_2 \eta_2) \right) \quad (62)$$

we obtain

$$\ddot{e}_1 + k_{d1} \dot{e}_1 + k_{p1} e_1 + J_{12}(\eta_2 - \eta_1) = 0 \quad (63)$$

$$\ddot{e}_2 + k_{d2} \dot{e}_2 + k_{p2} e_2 + J_{22}(\eta_2 - \eta_1) = 0 \quad (64)$$

$$\dot{\eta}_1 + k_1 \eta_1 = 0 \quad (65)$$

$$\dot{\eta}_2 + k_2 \eta_2 = 0 \quad (66)$$

where  $J_{12}$  and  $J_{22}$  are the (1,2) and (2,2) elements of the matrix  $J$ . Defining the state vector  $(x_1, x_2, x_3, x_4)^T$  as  $(e_1, \dot{e}_1, e_2, \dot{e}_2)^T$  and transforming equations (63) and (64) to state space form,

we have

$$\frac{d}{dt} \begin{pmatrix} x_1 \\ x_2 \\ x_3 \\ x_4 \end{pmatrix} = \begin{pmatrix} 0 & 1 & 0 & 0 \\ k_{p1} & k_{d1} & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & k_{p2} & k_{d2} \end{pmatrix} \begin{pmatrix} x_1 \\ x_2 \\ x_3 \\ x_4 \end{pmatrix} + \begin{pmatrix} 0 & 0 \\ J_{12} & -J_{12} \\ 0 & 0 \\ J_{22} & -J_{22} \end{pmatrix} \begin{pmatrix} \eta_1 \\ \eta_2 \end{pmatrix} \quad (67)$$

Then the overall system can be rewritten as

$$\frac{d}{dt} \begin{pmatrix} x_1 \\ x_2 \\ x_3 \\ x_4 \\ - \\ \eta_1 \\ \eta_2 \end{pmatrix} = \begin{pmatrix} 0 & 1 & 0 & 0 & | & 0 & 0 \\ -k_{p1} & -k_{d1} & 0 & 0 & | & J_{12} & -J_{12} \\ 0 & 0 & 0 & 1 & | & 0 & 0 \\ 0 & 0 & -k_{p2} & -k_{d2} & | & J_{22} & -J_{22} \\ - & - & - & - & - & - & - \\ 0 & 0 & 0 & 0 & | & -k_1 & 0 \\ 0 & 0 & 0 & 0 & | & 0 & -k_2 \end{pmatrix} \begin{pmatrix} x_1 \\ x_2 \\ x_3 \\ x_4 \\ - \\ \eta_1 \\ \eta_2 \end{pmatrix} \quad (68)$$

We see that the overall system matrix can be divided by four blocks and the lower off-diagonal block is identically zero. Thus the eigenvalues of the overall system are the union of those of the block diagonal matrices. Since each block diagonal matrix is asymptotically stable, the overall system is asymptotically stable.

### 3.3 Simulation Results

We use the same scenario as in subsection 2.3. Vehicle longitudinal speed is 26.4 m/s (60 MPH). Fig. 7 shows the simulation results of the input/output linearization scheme without independent braking control. Figs. 8 and 9 show the simulation results of the input/output linearization scheme with trailer independent braking. The results of these two scheme are shown in Fig. 10, from which we see that the independent braking helps reducing both the tractor and trailer yaw errors.

Compared with the LQ and FSLQ controller, the Input/Output linearization controller has better tracking performance because it utilizes the information of the desired tracking signal  $\dot{\epsilon}_d$  to compute the control input. Thus the I/O linearization control scheme is a feedback plus feedforward controller. On the other hand, the LQ and FSLQ control scheme, a pure feedback control law, treats the desired tracking signal  $\dot{\epsilon}_d$  as a disturbance.

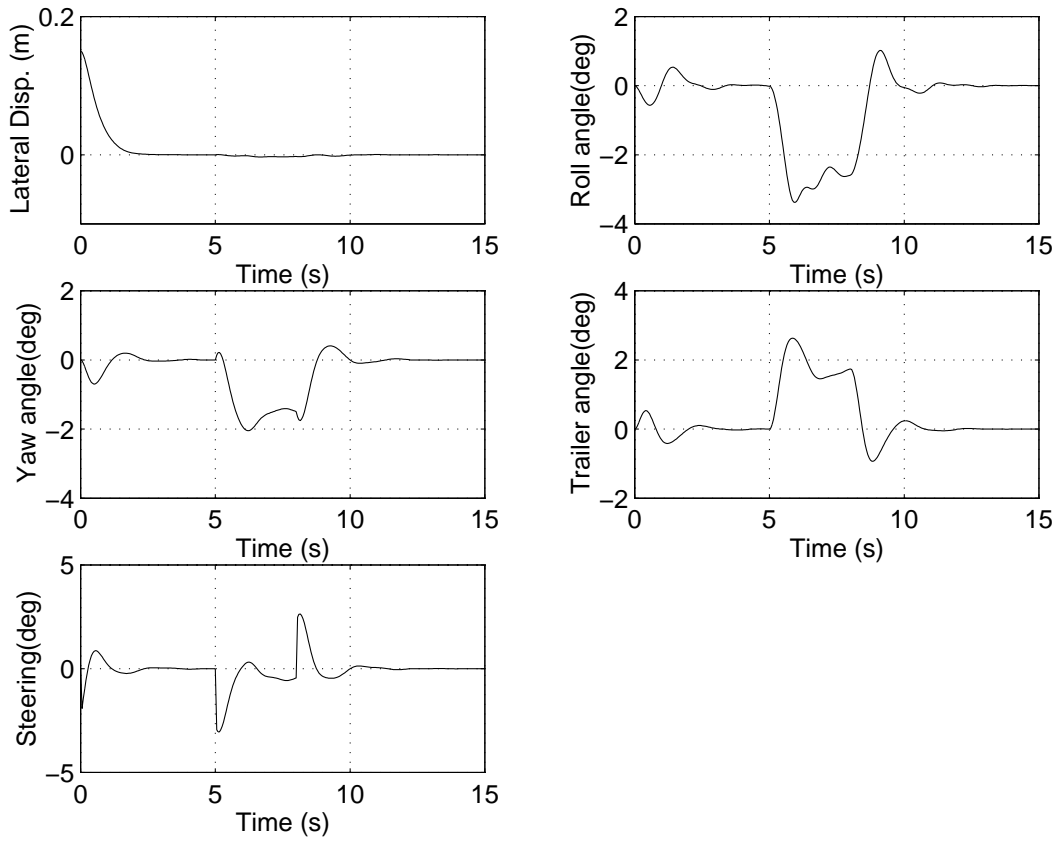


Figure 7: Input/Output Linearization Control

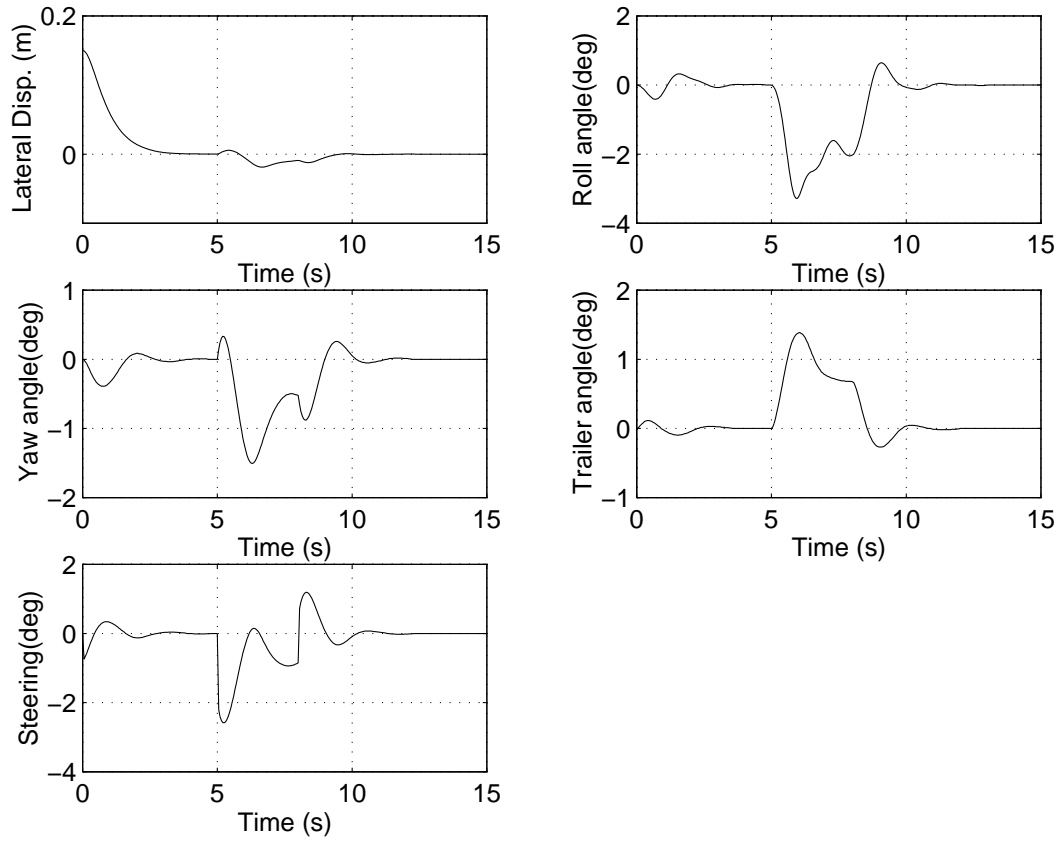


Figure 8: Input/Output Linearization Control with Trailer Independent Braking

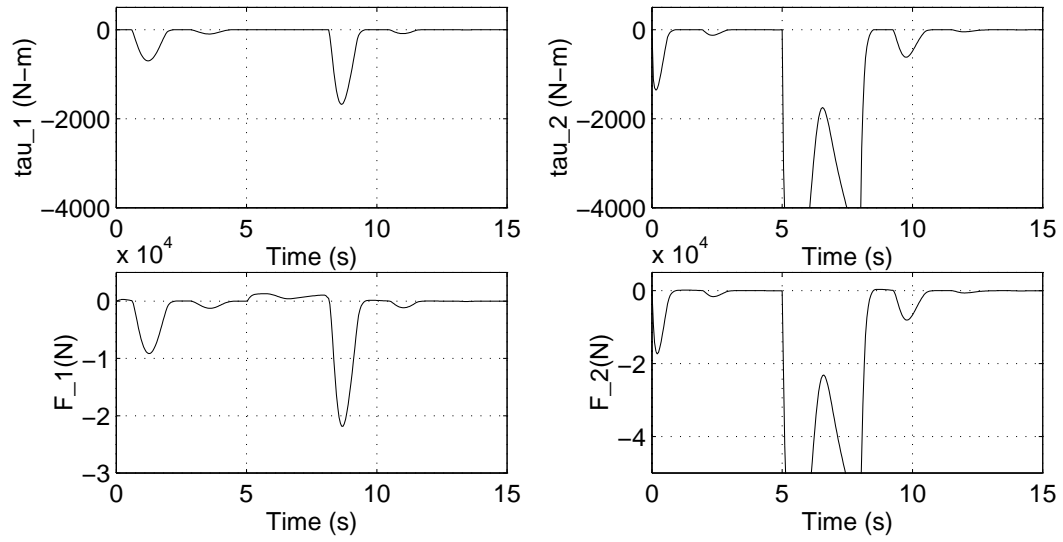


Figure 9: Input/Output Linearization Control with Trailer Independent Braking

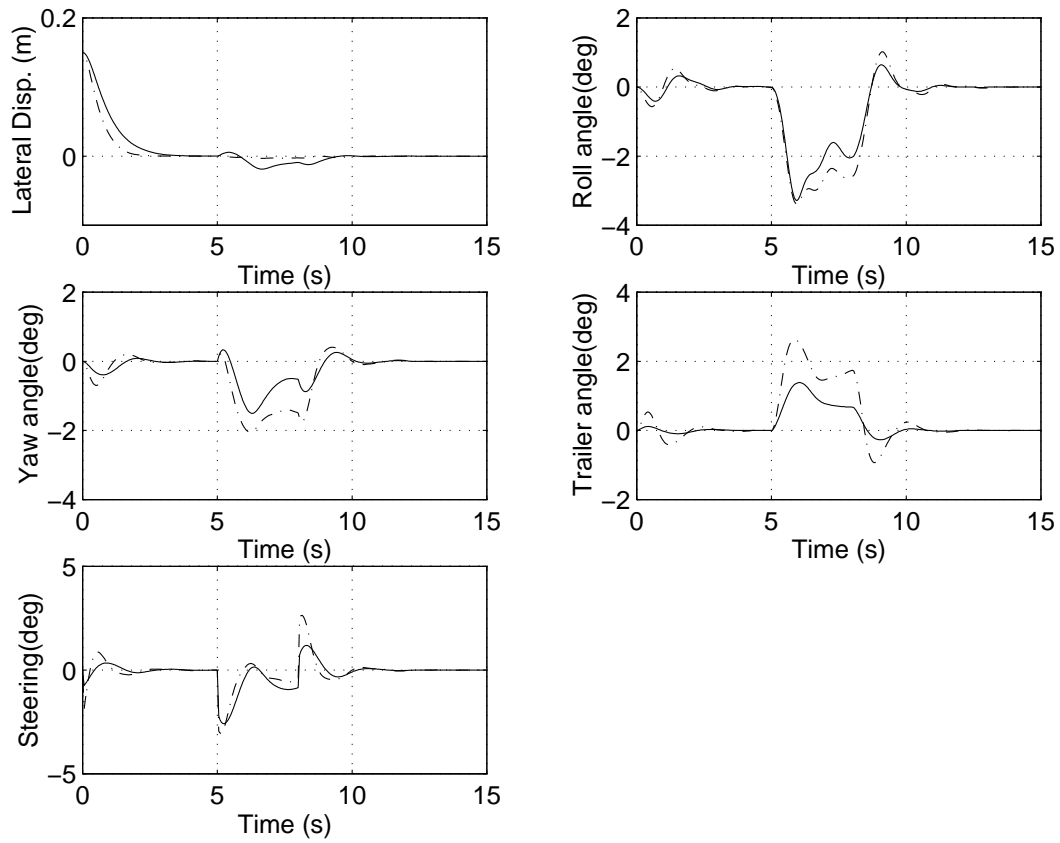


Figure 10: Comparison of Input/Output Linearization Control with (solid line) and without (dashdot line) Trailer Independent Braking



## 4 Conclusions

Two types of controllers were considered for lateral guidance of tractor-semitrailer vehicles in AHS. The first type of controllers manipulated the vehicle front wheel steering angles and were designed utilizing LQ optimal control and FSLQ optimal control theory. In the design of the second type of controllers, we utilized tractor front wheel steering angles and trailer independent braking forces to control the tractor and the trailer motion. Input/Output linearization and backstepping design methodologies were utilized to determine the coordinated steering angle and braking torques on the trailer wheels.

# Appendix 1: Nomenclature

- $\delta$  : Tractor front wheel steering angle.
- $\dot{x}$ : Longitudinal velocity of the vehicle.
- $y_r$ : Lateral displacement of tractor relative to the road centerline.
- $\dot{\epsilon}_d, \ddot{\epsilon}_d$ : Desired yaw rate and desired yaw acceleration of the vehicle at curved section.
- $\epsilon_r, \dot{\epsilon}_r$ : Yaw angle and yaw rate of the tractor relative to the road centerline.
- $\epsilon_f, \dot{\epsilon}_f$ : articulation angle and rate.
- $\alpha_i$ : Lateral slip angle at  $i$  -  $th$  wheel.

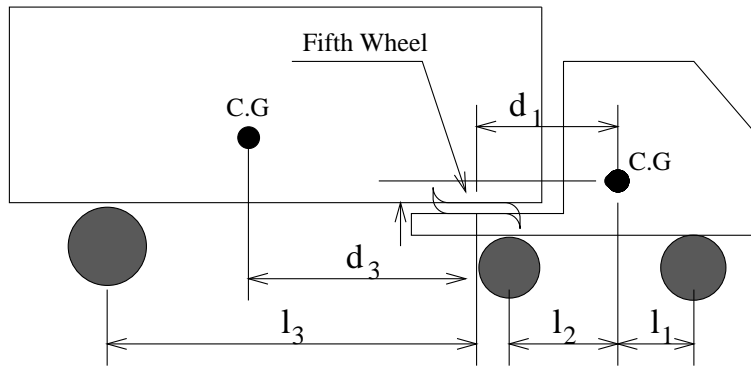


Figure 11: Vehicle Parameters

## Vehicle parameters :

- $m_1$  : tractor mass.
- $I_{z1}$ : tractor moment of inertia.
- $m_2$ : semitrailer mass.
- $I_{z2}$ : semitrailer moment of inertia.
- $Tw_3$ : semitrailer rear axle track width.

- $l_1, l_2, l_3, d_1, d_3$  : see Fig. 11.

**Tire parameters :**

- $C_{\alpha f}, C_{\alpha r}, C_{\alpha t}$ : cornering stiffness.

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