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THE PRODUCTION OF INFORMATION

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NECESSARY AND SUFFICIENT CONDITIONS FOR ACHIEVING STOCKHOLDER UNANIMITY

OVER THE PRODUCTION OF INFORMATION

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ABSTRACT

The conditions under which stockholders will be unanimous in the choice of their firm's plan for the production of goods are well-known: no technological externalities, a competitive market in the production of goods, and spanning of the marginal returns of the production process by existing securities in the marketplace. However, not well-known are the conditions which give stockholder unanimity over the amount of information that the firm should produce. It is shown here that even if the conditions giving unanimity in the production of goods are satisfied, the additional conditions of homogeneous beliefs and risk neutral behavior are necessary to achieve unanimity in the production of information.

INTRODUCTION

The conditions under which stockholders will be unanimous in the choice of their firm's plan for the production of goods are well-known: no technological externalities, a competitive market in the production of goods, and spanning of the marginal returns of the production process by existing securities in the marketplace.[1] However, not well-known are the conditions which give stockholder unanimity over the amount of information that the firm should produce. Describing these necessary and sufficient conditions is the purpose of this paper.

It is shown that the conditions assuring unanimity over the firm's production plan for goods are not enough to guarantee stockholder unanimity in the production of information. In addition to these, the following conditions are necessary and sufficient: (a) homogeneous beliefs, (b) risk neutral behavior, and, if the information produced is also released, (c) an absence of external effects of the information on other firms' values. The first two of these very restrictive conditions arise, in large part, because, by its very nature, information causes agents' implicit prices for state contingent consumption to change; the assumption of competitive markets in the production of goods, crucial in achieving unanimity in the production of goods, cannot extend to the production of information.[2] These more restrictive conditions are necessary to assure that the changing prices do not differentially affect the agents. The last of these three conditions, analogous to the requirement of no technological externalities, is needed to assure that the actions of one firm do not effect the values of other firms.

Since the necessary and sufficient conditions are so stringent, rarely, if ever, will stockholders agree on the amount of information that the firm

should produce; unlike in the production of goods, management will not have a guideline as to how much to invest in information. This leaves the theory of the firm incomplete.

Previous work in the area of information production was conducted by Omberg [4] who examined the case where the information produced is also publicly revealed. He concluded that all stockholders will be unanimous that the information production decision should be made so as to maximize firm value. However, in obtaining his result he makes the simplifying assumption that beliefs are affected by the production of information in a way that, in essence, eliminates the differential impact of the information on agents' implicit prices. By assuming away the basic problem with information production, its effect on prices, Omberg is not able to properly distinguish between the necessary and sufficient conditions giving unanimity in the production of goods and the more restrictive ones which give unanimity in the production of information. This is in contrast to the approach in this paper.

The plan of this work is as follows. In Section I the economic setting is described. A review of the conditions giving unanimity in the production of goods is presented in Section II. This is followed in Section III by a derivation of the necessary and sufficient conditions for unanimity in the production of information. A summary and conclusions section ends the paper.

I.   THE ECONOMY

Consider a one-good two-date economy with the date 0 state known and with uncertainty as to which of S possible date 1 states will occur. Each agent maximizes his expected utility where the objects of choice are units of current consumption and shares in firms. There are enough firms so that there is a complete market. The standard perfect competition assumption is made so that

agents believe that shareholding decisions do not affect state contingent prices in the economy and that decisions on the production of goods do not affect either state contingent prices nor agents' implicit prices for state contingent consumption.

There is a manager for each firm, executing the production plan chosen by stockholders. The manager can also invest some of the firm's resources in information collection at date 0, before consumption and production decisions have been made. Investing I_j in information by firm j results in a signal that state r will occur, where prob(state r is signalled/true state is s) $=\lambda^j_{rs}(I_j)$, with $\lambda^{j'}_{rs}(I_j) > 0$, $\lambda^{j''}_{rs}(I_j) < 0$ for r=s and $\lambda^{j'}_{rs}(I_j) < 0$, $\lambda^{j''}_{rs}(I_j) > 0$ for r≠s. Given this information, each agent i appropriately revises his prior subjective probability for the occurrence of each state s, π^i_s, according to Bayes' rule.[3]

II. A REVIEW OF THE CONDITIONS FOR UNANIMITY IN THE PRODUCTION OF GOODS

In order to understand why the conditions producing unanimity over information production are so restrictive, it is useful to review why the assumptions of no technological externalities, competitive markets, and spanning are all that are necessary for unanimity in the production of goods. Consider a specific firm j. Assume that the manager of firm j has already decided to invest I_j in information.[4] If state r is signalled and publicly released, agent i will solve the problem:

maximize $\quad E(U_{ir}) = w_i(c_{or}^i) + \sum_s \pi_{s/r}^i y_i(c_{sr}^i)$ $\qquad\qquad$ (1)

$\{c_{or}^i, t_{ikr}\}$

subject to: $\quad c_{or}^i + \sum_k t_{ikr}v_{kr} = \bar{c}_0^i + \sum_{k \neq j}\bar{t}_{ikr}(v_{kr}-q_{okr})+\bar{t}_{ijr}(v_{jr}-q_{ojr}-I_j)$ $\quad$ (2)

$\qquad\qquad c_{sr}^i = \sum_k t_{ikr}q_{skr}$ $\qquad\qquad\qquad\qquad\qquad$ ∀s $\qquad$ (3)

where:

$\pi_{s/r}^i$ = agent i's subjective probability for the occurrence of state s given signal r;

c_{or}^i = initial consumption of agent i given signal r;

$\bar{c}_0^i$ = initial consumption endowment of agent i;

c_{sr}^i = consumption of agent i in state s given signal r;

t_{ikr} = fractional shareholding of agent i in firm k given signal r;

$\bar{t}_{ik}$ = fractional shareholding endowment of agent i in firm k;

q_{okr} = input into firm k's production process given signal r;

q_{skr} = output of firm k in state s given signal r;

v_{kr} = price of firm k given signal r;

$w_i(c_{or}^i)$ and $y_i(c_{sr}^i)$ are assumed to be increasing, concave functions of their respective arguments.

$\qquad$ Substituting (2) and (3) into the objective function (1) gives:

maximize $\quad E(U_{ir}) = w_i(\bar{c}_0^i + \sum_{k \neq j}\bar{t}_{ik}(v_{kr}-q_{okr}) + \bar{t}_{ij}(v_{jr}-q_{ojr}-I_j) - \sum_k t_{ikr}v_{kr})$

$\{t_{ikr}\}$

$\qquad\qquad\qquad + \sum_s \pi_{s/r}^i y_i(\sum_k t_{ikr}q_{skr})$ $\qquad\qquad\qquad$ (4)

This yields the first order condition for shares in firm j which can be written as:

$$v_{jr} = \sum_s \pi^i_{s/r} \frac{\partial y_i(c^i_{sr})/\partial c^i_{sr}}{\partial w_i(c^i_{or})/\partial c^i_{or}} q_{sjr} \qquad (5)$$

That is, in equilibrium each agent purchases shares up until the point where his marginal valuation of the firm's outputs is equal to the price of the firm

$(\pi^i_{s/r} \dfrac{\partial y_i(c^i_{sr})/\partial c^i_{sr}}{\partial w_i(c^i_{or})/\partial c^i_{or}}$ is agent i's implicit price for state s comsumption, that

is, his marginal rate of substitution of state s for data 0 consumption.)

Given that firm j has a production function of the form $f_j(q_{1j},\ldots,$ $q_{sj}) = q_{oj}$ and that agent i optimally chooses his shareholdings given the firm's production plan, the effect of an increase in q_{sjr} (keeping q_{njr}, $n \neq s$, fixed) on his utility is:

$$\frac{\partial E(U_{ir})}{\partial q_{sjr}} = \frac{\partial w_i(c^i_{or})}{\partial c^i_{or}} [(\bar{t}_{ij}-t_{ijr}) \frac{\partial v_{jr}}{\partial q_{sjr}} - \bar{t}_{ij} \frac{\partial q_{ojr}}{\partial q_{sjr}} + t_{ijr} \pi^i_{s/r} \frac{\partial y_i(c^i_{sr})/\partial c^i_{sr}}{\partial w_i(c^i_{or})/\partial c^i_{or}}]$$

$$(6)$$

assuming that $\dfrac{\partial v_{kr}}{\partial q_{sjr}} = 0$, $k \neq j$, that is, that there are no technological externalities. Given perfect competition, $\dfrac{\partial v_{jr}}{\partial q_{sjr}}$ can be written as (using equation (5)):

$$\frac{\partial v_{jr}}{\partial q_{sjr}} = \pi^i_{s/r} \frac{\partial y_i(c^i_{sr})/\partial c^i_{sr}}{\partial w_i(c^i_{or})/\partial c^i_{or}} \qquad \text{for each i} \qquad (7)$$

The change in the value of the firm comes solely from an increase in the level of its outputs; implicit prices remain fixed. Further, from the assumption of spanning, (7) is identical for all agents and is equal to the market price of state s contingent consumption. Using (7), (6) becomes:

$$\frac{\partial E(U_{ir})}{\partial q_{sjr}} = \frac{\partial w_i(c^i_{or})}{\partial c^i_{or}} \bar{t}_{ij} [\frac{\partial v_{jr}}{\partial q_{sjr}} - \frac{\partial q_{ojr}}{\partial q_{sjr}}] \qquad (8)$$

The terms involving t_{ijr} cancel out. The effect of an increase in output on the cost to the agent of buying the t_{ijr} shares, $-t_{ijr} \frac{\partial v_{jr}}{\partial q_{sjr}}$, (which, in equilibrium, is equal to the agent's marginal valuation of his share of the firm's additional output) is just offset by the marginal value to the agent of the additional future consumption now provided by the shares, $t_{ijr} \; \pi^i_{s/r} \frac{\partial y_i(c^i_{sr})/\partial c^i_{sr}}{\partial w_i(c^i_{or})/\partial c^i_{or}}$.

The effect on the agent's utility of an increase in output therefore comes solely from its effect on the value of the agent's endowment in shares, or, equivalently, on the value of the agent's initial wealth. All agents who hold a positive endowment are unanimous that q_{sjr} should be increased as long as initial wealth is increased as a result; that is, as long as $\frac{\partial v_{jr}}{\partial q_{sjr}} - \frac{\partial q_{okr}}{\partial q_{sjr}} > 0$. All agree that the objective function of the firm should be the maximization of its net market value, $v_{jr} - q_{ojr}$.

This is just the familiar Fisher separation theorem under uncertainty. With state contingent consumption prices remaining fixed (because of the perfect competition assumption), each agent benefits, through an expansion of his opportunity set, from any action by the firm which increases his initial wealth.

There is one case where the assumption of competition in the production of goods is not needed for unanimity -- when each agent's endowment of shares in the firm is equal to his optimal holdings in the firm, that is, when $\bar{t}_{ij} = t_{ijr}$. The exact form of $\frac{\partial v_{jr}}{\partial q_{jr}}$, the change in the value of the firm with respect to an increase in one output, can then be left unspecified. As seen from (6) the term involving $\frac{\partial v_{jr}}{\partial q_{sjr}}$ drops out from the derivative of utility; what remains is:

$$\frac{\partial E(U_{ir})}{\partial q_{sjr}} = \frac{\partial w_i(c^i_{or})}{\partial c^i_{or}} \; [\bar{t}_{ij}(\pi^i_{s/r} \frac{\partial y_i(c^i_{sr})/\partial c^i_{sr}}{\partial w_i(c^i_{or})/\partial c^i_{or}} - \frac{\partial q_{ojr}}{\partial q_{sjr}})] \qquad (9)$$

By the spanning assumption (9) is of the same sign for all shareholders with positive endowments. There is again unanimity. However, in this case it

is a local unanimity. All agree on the optimal direction of change for q_{sjr}. The result does not hold for large movements from the starting production plan since the optimal shareholding level will also move away from the endowment level. Utility will then again be sensitive to the form of $\dfrac{\partial v_{jr}}{\partial q_{sjr}}$.

III. THE INFORMATION PRODUCTION DECISION

Consider now the decision of the manager of firm j as to how much information to produce. Denote by $E(U^{*}_{ir}(I_j))$ the maximum expected utility of agent i given that I_j is invested in information, signal r is obtained and publicly released and all agents act optimally. Taking expectations over r gives:

$$E_r[E(U^{*}_{ir}(I_j))] = \sum_r p^i_r \; E(U^{*}_{ir}(I_j)) \tag{10}$$

where p^i_r is agent i's subjective probability that state r will be signalled.[5] $E_r[E(U^{*}_{ir}(I_j))]$ is the agent's maximum expected utility given that I_j has been invested in information.

Agents will be unanimous in their choice of I_j only if (10) is maximized at the same value of I_j for each agent. The following proposition characterizes the conditions assuring this:

Proposition: For arbitrary shareholding endowments, $\{\bar{t}_{ik}\}$, and information production function, $\lambda^j_{rs}(I_j)$, necessary and sufficient conditions for information production unanimity are that (a) all agents have homogeneous beliefs, (b) all agents be risk neutral with identical rates of time preference, and (c) the production of information not affect the value of any firm other than j.

Proof: See the appendix.

To understand the need for these conditions, consider the effect on agent i's utility of a small increase in I_j:

$$\frac{\partial E_r[E(U^*_{ir})]}{\partial I_j} = \sum_r \frac{\partial p^i_r}{\partial I_j} E(U^*_{ir}) + \sum_r p^i_r [\frac{\partial w_i(c^{i*}_{or})}{\partial c^{i*}_{or}} [\sum_k (\bar{t}_{ik} - t^*_{ikr})\frac{\partial v^*_{kr}}{\partial I_j} - \bar{t}_{ij}] + \sum_s \frac{\partial \pi^i_{s/r}}{\partial I_j} y_i (\sum_k t^*_{ikr} q^*_{skr})]$$

(11)

where an asterisk represents the optimal value of the variable given that state r has been signalled.[6] (The functional notation, indicating the variables' dependence on I_j, has been suppressed.)

Increased information production affects the agent's utility in two ways: first, it changes the probability of occurrence of each signal r and thereby the probability of attaining the associated utility level $E(U^*_{ir})$; and second, given signal r, it changes the agent's perception of the probability that each state s will occur and, through this, changes the market price of each firm. Consider this last effect. By changing each agent's subjective probabilities an increase in the amount of information produced affects implicit prices; the assumption of perfect competition does not hold with respect to the production of information. This brings in an additional complication not faced in the production of goods, making it more difficult to achieve unanimity. Mathematically the result of these changing prices on utility, given signal r, can be written as:

$$\frac{\partial w_i(c^{i*}_{or})}{\partial c^{i*}_{or}} [\sum_k (\bar{t}_{ik} - t^*_{ikr})\frac{\partial v^*_{kr}}{\partial I_j}] + \sum_s \frac{\partial \pi^i_{s/r}}{\partial I_j} y_i (\sum_k t^*_{ik} q^*_{skr})$$

(12)

There are two related effects. First, the change in all agents' implicit prices causes a change in the price of each firm (or, equivalently, a change in agents' marginal valuation of the firm's outputs) and thereby a change in each agent's cost of purchasing his shares in the firm. This effect on utility is represented by the first term in (12). Second, the change in implicit prices causes each agent to revise the valuation of his entire (as opposed to marginal) share of each firm's output. This effect is represented by the

second term in (12). Unless the effect on price of the change in all agents' marginal valuation of each firm's outputs is exactly offset by the change in each agent i's average valuation of his share of each firm's output, the changing prices will differentially impact agents; this would lead to a lack of unanimity over the amount of information to produce. These two effects will offset each other if there are homogeneous beliefs (so that the change in all agents' marginal valuation of each firm's outputs is equal to agent i's change in marginal valuation alone) and if agent i is risk neutral (so that marginal valuation of output equals average valuation of output). Further, as a technical point, the rate of time preference embodied in the utility functions must be the same for all agents; otherwise equilibrium will fail to exist with those having low rates of time preference buying infinite amounts of a risk free security and those having high rates of time preference selling short infinite amounts.

If these conditions exist, then, the net effect on utility of the price changes, given signal r, is equal to $\sum_k \bar{t}_{ik} \frac{\partial v^*_{kr}}{\partial I_j}$. The additional requirement that $\frac{\partial v^*_{kr}}{\partial I_j} = 0$, $k \neq j$, ensures that this sum will be of the same sign for all of firm j's shareholders. It eliminates the externality effect of the information production activity of firm j, which, if present, could cause a breakdown in unanimity similar to that caused by technological externalities in the production of goods.

Given these conditions (11) simplifies to:

$$\frac{\partial E_r[E(U^*_{ir})]}{\partial I_j} = \bar{t}_{ij}[\sum_r \frac{\partial p^i_r}{\partial I_j} (v^*_{jr} - q^*_{ojr} - I_j) + \sum_r p^i_r (\frac{\partial v^*_{jr}}{\partial I_j} - 1)] \qquad (13)$$

$$= \bar{t}_{ij}[\frac{\partial E_r(v^*_{jr} - q^*_{ojr} - I_j)}{\partial I_j}] \qquad (14)$$

where $E_r(v^*_{jr} - q^*_{ojr} - I_j)$ is the expected net market value of the firm, after information costs. All stockholders gain utility from an increase in I_j as long as the change in the net market value of the firm is positive; in other words, as long as their expected net wealth (after information costs) increases. This is an intuitive result since all stockholders are risk neutral.

Unlike in the production of goods, the necessary and sufficient conditions for information production unanimity do not change if it is assumed, a priori, that shareholding endowments equal final shareholdings. Without any prior restrictions on endowments, it has been shown that risk neutrality, homogeneous beliefs, and no externalities are necessary and sufficient to produce unanimity. But these conditions imply that endowments and final shareholdings will be equal (since there will be no trading in the market-place). Assuming, a priori, that they are equal therefore cannot change the necessary and sufficient conditions.

These results were derived under the assumption that any information produced is also released. It will now be demonstrated that the conditions under which there is unanimity remain very restrictive even if it is assumed that the information is not publicly released. Note, first, that in order for the choice of production plan by the manager of firm j not to reveal his information, the manager must not announce, and stockholders must not be able to discern, the plan. For this to be possible, the manager must raise the same amount of funds, q_{oj}, for input regardless of the signal he receives. Given signal r he will then invest q_{ojr} in the production process, with the remaining $q_{oj} - q_{ojr}$ going into a riskless investment. In this context, then, agent i solves the problem:

maximize $\quad E_r[E(U_{ir})] = w_i(c_0^i) + \sum_r p_r^i \sum_s \pi_{s/r}^i \, y_i(c_{sr}^i)$ (15)

$c_0^i, \{t_{ik}\}$

subject to: $\quad c_0^i + \sum_k t_{ik} v_k = \bar{c}_0^i + \sum_{k \neq j} \bar{t}_{ik}(v_k - q_{ok}) + \bar{t}_{ij}(v_j - q_{oj} - I_j)$ (16)

$$c_{sr}^i = \sum_{k \neq j} t_{ik} q_{skr} + t_{ij}[q_{sjr} + R(q_{oj} - q_{ojr})] \qquad \forall r \qquad (17)$$

where R = 1 + the riskfree rate of interest.

The difference between this problem and that represented by equations (1) - (3) is that the agent chooses the same initial consumption and shareholding level for each signal that the manager receives and that the value of each firm is constant across signals.

This problem yields the first order condition for shares which can be written as follows:

$$v_j = \sum_r \sum_s p_r^i \, \pi_{s/r}^i \, \frac{\partial y_i(c_{sr}^i)/\partial c_{sr}^i}{\partial w_i(c_0^i)/\partial c_0^i}[q_{sjr} + R(q_{oj} - q_{ojr})] \qquad (18)$$

Given that the manager of the firm now holds inside information useful for the production decision, choice of the optimal plan for the production of goods becomes more difficult. Only under certain circumstances will stockholders be unanimous on how the manager should use his inside information in choosing the production plan, given that they do not have access to that information when making their consumption-investment decisions.[7] Fortunately, the necessary and sufficient conditions for achieving unanimity in information production in this context, homogeneous beliefs and risk neutral behavior (see the discussion below), are also sufficient to guarantee unanimity in the production of goods. All stockholders will agree that it is optimal for the manager to choose q_{ojr}, $\{q_{sjr}\}$ so as to maximize $\frac{E_{s/r}(q_{sjr})}{R} - q_{ojr}$, where $E_{s/r}(q_{sjr})$ is the expected value of the output given signal r.

Given that shareholdings and the plan for the production of goods are chosen optimally, the effect on the agent's maximum utility of an increase in I_j is given by:

$$\frac{\partial E_r[E(U_{ir}^*)]}{\partial I_j} = \sum_r \frac{\partial p_r^i}{\partial I_j} \sum_s \pi_{s/r}^i \, y_i [\sum_{k \neq j} t_{ik}^* q_{skr}^* + t_{ij}^* (q_{sjr}^* + R(q_{oj} - q_{ojr}^*))]$$

$$+ \sum_r p_r^i [\frac{\partial w_i(c_o^{i*})}{\partial c_o^{i*}} [\sum_k (\bar{t}_{ik} - t_{ik}^*) \frac{\partial v_k^*}{\partial I_j} - \bar{t}_{ij}]$$

$$+ \sum_s \frac{\partial \pi_{s/r}^i}{\partial I_j} \, y_i [\sum_{k \neq j} t_{ik}^* q_{skr}^* + t_{ij}^* (q_{sjr}^* + R(q_{oj} - q_{ojr}^*))]]^8 \qquad (19)$$

(19) is almost identical to (11), the major difference being that $w_i(c_o^{i*})$ and $\{v_k^*\}$ do not depend on r. As can be verified, the necessary and sufficient conditions for information production unanimity in this setting remain very similar to those presented before, specifically, homogeneous beliefs and risk neutral behavior, with the same rate of time preference, exhibited by all agents.

The conditions for unanimity remain restrictive, even though the information is not released, because perfect competition with respect to information production is still lacking. Agents continue to change their implicit prices conditional on a given signal as the amount of information produced increases. Even though the information is not revealed, each agent knows what his revised probabilities would be given each signal and given the amount invested in information. As the amount of information produced increases, these probabilities change, and consequently so do the agents' implicit prices.

It is no longer necessary, however, to explicitly assume that information production by firm j not affect the values of the other firms. Because the information is not being revealed, the managers of the other firms will not be able to use it to revise their firms' production plans. $\{q_{skr}\}$ and q_{okr} will be constant over r for each firm $k \neq j$. The values of these firms will remain constant with respect to the amount of information produced by firm j.

IV. SUMMARY AND CONCLUSIONS

Whether or not the information is released, the conditions under which there is unanimity over the amount of information that the firm should produce are very restrictive compared to those under which there is unanimity in the production of goods. The difference arises because the basic nature of information is such that it causes investors' valuation of firm output, and therefore the equilibrium value of the firm, to change. This violates the assumption of perfect competition, necessary to assure unanimity in the production of goods. Without restrictive conditions, these price changes differentially affect agents, resulting in a lack of unanimity.

Because the conditions guaranteeing unanimity ar so restrictive, they would rarely, if ever, be expected to hold in the economy. Unless sidepayments are made among stockholders,[9] there will not be agreement on the optimal amount of investment in information. There will be no guideline for the manager to follow in choosing the investment level. If sidepayments were possible, however, the manager would continue to produce more information as long as those stockholders gaining utility would be willing to adequately compensate those stockholders losing utility. Whether mechanisms for implementing such a payment system are feasible and of sufficiently low cost to be justified is an open question.

V.   APPENDIX

Proof of Proposition:

Differentiating $E_r[E(U^*_{ir}(I_j))]$ with respect to I_j gives:

$$\frac{\partial E_r[E(U^*_{ir})]}{\partial I_j} = \sum_r \frac{\partial p^i_r}{\partial I_j}\ [w_i(\bar{c}^i_0 + \sum_{k\neq j}\bar{t}_{ik}(v^*_{kr}- q^*_{okr}) + \bar{t}_{ij}(v^*_{jr}-q^*_{ojr}-I_j) - \sum_k t^*_{ikr}v^*_{kr})$$

$$+ \sum_s \pi^i_{s/r}\ y_i(\sum_k t^*_{ikr}q^*_{skr})]$$

$$+ \sum_r p^i_r[\frac{\partial w_i(c^{i*}_{or})}{\partial c^{i*}_{or}}\ [\sum_k(\bar{t}_{ik}-t^*_{ikr})\frac{\partial v^*_{kr}}{\partial I_j} - \bar{t}_{ij}] + \sum_s \frac{\partial \pi^i_{s/r}}{\partial I_j}\ y_i(\sum_k t^*_{ikr}q^*_{skr})]$$

$$(A1)$$

where an asterisk represents the optimal value of the variable given that state r has been signalled. (The functional notation, indicating the variables' dependence on I_j, has been suppressed.)

To prove sufficiency note first that with homogeneous beliefs $\frac{\partial v^*_{kr}}{\partial I_j}$ can be written as:

$$\frac{\partial v^*_{kr}}{\partial I_j} = \frac{\partial \pi^i_{s/r}}{\partial I_j}\ \frac{\partial y_i(c^{i*}_{sr})/\partial c^{i*}_{sr}}{\partial w_i(c^{i*}_{or})/\partial c^{i*}_{or}}\ q^*_{skr} \qquad \text{for each } i \qquad (A2)$$

This follows because, first, $\frac{\partial \pi^i_{s/r}}{\partial I_j}$ is equal for all agents, and second, combined with the assumption of a complete market (so that $\pi^i_{s/r}\ \frac{\partial y_i(c^{i*}_{sr})/\partial c^{i*}_{sr}}{\partial w_i(c^{i*}_{or})/\partial c^{i*}_{or}}$ is the same for all agents), $\frac{\partial y_i(c^{i*}_{sr})/\partial c^{i*}_{sr}}{\partial w_i(c^{i*}_{or})/\partial c^{i*}_{or}}$ is equal for each agent.

With risk neutrality agent i's utility function can be written as:

$$E(U_{ir}) = c_{or}^i + a_i \sum_s \pi_{s/r}^i c_{sr}^i \tag{A3}$$

(a_i, the agent's subjective rate of time preference, must be identical over all agents, or equilibrium will fail to exist.)

Using (A2) and (A3), (A1) simplifies to:

$$\frac{\partial E_r[E(U_{ir}^*)]}{\partial I_j} = \sum_r \frac{\partial p_r^i}{\partial I_j} [\sum_{k \neq j} \bar{t}_{ik}(v_{kr}^* - q_{okr}^*) + \bar{t}_{ij}(v_{jr}^* - q_{ojr}^* - I_j)] + \sum_r p_r^i (\sum_k \bar{t}_{ik} \frac{\partial v_{kr}^*}{\partial I_j} - \bar{t}_{ij}) \tag{A4}$$

Finally, with the value of each firm $k \neq j$ being unaffected by the information production of firm j (so that v_{kr}^* and q_{okr}^* are independent of r) (A4) becomes:

$$\frac{\partial E_r[E(U_{ir}^*)]}{\partial I_j} = \sum_r \frac{\partial p_r^i}{\partial I_j} \bar{t}_{ij}(v_{jr}^* - q_{ojr}^* - I_j) + \sum_r p_r^i \bar{t}_{ij}(\frac{\partial v_{jr}^*}{\partial I_j} - 1) \tag{A5}$$

This is of the same sign for all agents long in the risky security. All agents will then agree on the sign of the effect on utility of an increase in I_j and therefore will agree on the optimal I_j. This completes the sufficiency part of the proof.

To prove necessity for arbitrary $\{\bar{t}_{ik}\}$ and $\lambda_{rs}^j (I_j)$ note first that to achieve unanimity (A1) must equal zero for all agents at the same level of I_j. This implies that the terms involving $\bar{t}_{ik}$, for each k, must separately equal zero at that level and that those not involving any $\bar{t}_{ik}$ must be zero for all levels of I_j. Otherwise the $\{\bar{t}_{ik}\}$ could be chosen (keeping initial wealth and thereby all other variables unchanged) so as to ensure that (A1) was not zero

for all agents at the same level of I_j. Further, unless there are homogeneous beliefs, $\lambda_{rs}^{j'}(I_j)$ can be varied, changing $\frac{\partial v_{kr}^*}{\partial I_j}$, $\frac{\partial \pi_{s/r}^i}{\partial I_j}$ $\forall i,s$ and $\frac{\partial p_r^i}{\partial I_j}$ $\forall i$, and $\{\bar{t}_{ik}\}$ can be chosen, keeping everything else the same, to have (A1) not be zero for all agents at the same level of I_j.

Solving for the utility function satisfying the above requirements gives:

$$E(U_{ir}) = c_{or}^i + a_i \sum_s \pi_{s/r}^i c_{sr}^i \qquad (A6)$$

representing risk neutrality. (Again, a_i must be equal over all agents to assure that equilibrium will exist.) Given this, (A1) becomes:

$$\frac{\partial E_r[E(U_{ir}^*)]}{\partial I_j} = \sum_r \frac{\partial p_r^i}{\partial I_j} [\sum_{k \neq j} \bar{t}_{ik} (v_{kr}^* - q_{okr}^*) - \bar{t}_{ij} (v_{jr}^* - q_{ojr}^* - I_j)]$$

$$+ \sum_r p_r^i [\sum_{k \neq j} \bar{t}_{ik} \frac{\partial v_{kr}^*}{\partial I_j} + \bar{t}_{ij} (\frac{\partial v_{jr}^*}{\partial I_j} - 1)] \qquad (A7)$$

Finally, given arbitrary $\{\bar{t}_{ik}\}$ (A7) can only be guaranteed to be zero for all agents at the same I_j if the value of each firm $k \neq j$ is unaffected by the information production of firm j (so that v_{kr}^* and q_{okr}^* are independent of r). This completes the necessity part of the proof.

FOOTNOTES

1. These are the conditions for ex-ante unanimity, where each agent's final shareholding level is allowed to differ from his endowment shareholding level. See Leland (2).

2. Making the assumption that implicit prices remain fixed with respect to the production of information would trivialize information production. In particular it would make it useless for the production of goods; with implicit prices fixed, each firm's optimal production plan for goods would remain unaffected by the production of information.

3. Specifically:

$$\pi^i_{s/r} = \frac{\lambda^j_{rs}(I_j)\pi^i_s}{\sum_s \lambda^j_{rs}(I_j)\pi^i_s}$$

where: $\pi^i_{s/r}$ = agent i's posterior subjective probability for the occurrence of state s given that state r has been signalled.

4. For simplicity it is assumed that only firm j invests in information.

5. $p^i_r = \sum_s \lambda^j_{rs}(I_j)\pi^i_s$

6. Note that no terms involving derivatives of decisions variables, $\{t_{ikr}, q_{skr}\}$, appear in (11). This is because, for each value of I_j, they are chosen optimally. By the Implicit Function Theorem, the effect on utility of a change in these variables, as a result of an infinitesimal change in the parameter I_j, is zero.

7. For further discussion of the use of inside information for the production decision, see Leland [3] and Trueman [5].

8. This equation is derived by substituting (16) and (17) into (15), differentiating, and remembering that all terms involving derivatives of decision variables are zero by the Implicit Function Theorem.

9. These would be similar to those suggested by Grossman and Hart (1) to resolve the problem of a lack of unanimity in the production of goods.

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