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Review: Tatsuoka, *Multivariate Analysis*

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REVIEWS

Maurice M. Tatsuoka *Multivariate Analysis: Techniques for Educational and Psychological Research.* New York: John Wiley and Sons, Inc., 1971. Pp. xiii + 310. $10.95

As is stated in the introductory chapter, this text is designed for graduate-level, research-oriented students in psychology, education and related fields. These students want to be sophisticated users of multivariate analytical techniques, but are not mathematical statisticians. The text serves this audience far better than the more mathematically advanced presentations of Anderson [1958] or Morrison [1967] and the theoretically parallel texts such as Cooley and Lohnes [1962 and 1971]. Yet it is sophisticated and complete enough to be a good first text for students of psychometrics and allied disciplines.

The overall presentation is clear and concise. The notational systems readily followed, predominantly consistent in that they are changed only for pedogogic efficacy. It is a text that is constructed with teaching in mind. Throughout the text there are examples to illustrate all major analyses, as well as exercises to test the students' understanding of major points. New ideas and techniques are developed by analogy to previously learned techniques, so that students are gently drawn into a fuller understanding of a whole body of analytical methods.

Chapter 2 contains the elementary topics of matrix algebra needed to begin the study of multivariate analysis. While the discussion of addition, subtraction, transposition, multiplication by scalars and matrices, and matrix inversion theoretically covers all needed elements, I have found it necessary to augment this material with a more formal presentation on these topics. It seems that at this juncture students without a background in matrix algebra approach the material as a new technique to be learned, rather than a convenient and useful, simplifying tool to aid their study of multivariate analysis. This chapter also contains a matrix formulation of multiple regression which well illustrates the simplified representation of such problems when presented in matrix form rather than algebraic form. It is assumed that students have studied multiple regression in prior courses. For students without such a background this causes some difficulties, since this material was not designed to provide a sufficient basis for really studying the technique.

Chapter 3 deals with analysis of covariance. The logic of the method is developed for the single covariate case and then extended to multiple covariates. The mathematics are developed with remarkable clarity and completeness. Tatsuoka chooses to develop the single covariate case with notation specific to this case and then switch to a more general notation in the development of the multiple covariate analysis of covariance. The specific notation seems to help the students grasp what the analysis does to the data, and the switch to the more general notation is easily mastered. In his presentation no mention is made of expected mean squares. While this somewhat simplifies the presentation it creates a difficulty in that no explanation is given as to why the residual sum of squares which one calculates are not independently distributed; and why the modified residual sums of square are independent.

The presentation could be considerably strengthened by inclusion of material which discusses the difficulties encountered in making inferences from the analysis of covariance model [c.f. Lord, 1967; Werts and Linn, 1971a, b, and c; Hall, 1971]. Even without added material on the use and misuse of ANCOVA this chapter provides a good discussion of the basic logic and mathematics, and a good transition into the truly multivariate techniques which follow.

In Chapter 4 Tatsuoka develops the mathematics and geometry of the multivariate normal distribution, the sampling distribution of sample centroids, Hotellings T^2, generalized variance, Wilk's Λ, the statistical approximations of Rao and Bartlett as well as Schatzoff's exact distribution for Λ. Rather than mathematically deriving T^2 and Λ, Tatsuoka develops them by generalization from the univariate t and F statistics. This approach, which is used very effectively throughout the text, avoids the time and tedium involved in formal derivations which are not really needed for students who are not mathematical statisticians, and yet provides a solid basis for conceptual understanding of the statistics and their relation to the original data.

Chapter 5 begins with additional needed topics in matrix algebra such as linear transformation, orthogonal rotation, eigenvalues and eigenvectors, rank of a matrix, and grammian matrices. Delaying the introduction of these topics until just prior to their use has a pedogically desirable result. Students, by this time, have seen the utility of matrix representation and seem to absorb this material more readily than the elementary matrix algebra in Chapter 2. Additional material is included on the application of eigenvalues and eigenvectors in principal components analysis and factor analysis, on principal components as a preliminary to other multivariate techniques, and a theoretical supplement on generalized inverses. The material on generalized inverses could be made more practical by showing the application of generalized inverses in finding least squares solutions to consistent systems of linear equations. The use of principal components as a preliminary analysis could also be made more practical by reference to, if not explanation of, reduced rank regression models c.f.; Tucker, 1957; Burket, 1964], and by reference to corresponding techniques in the later discussion of discriminant analysis and canonical correlation.

Tatsuoka's discussion of factor analysis is very abbreviated. In his introduction he acknowledges this—properly stating that this topic is so large that it is better dealt with in separate courses and texts. However, as a teacher I have always succumbed to the temptation to augment this material to give students at least an introductory understanding of factor analysis. The major error is that such a discussion sould not be undertaken at this point in a course. Factor analysis is primarily an exploratory method. A good deal of time has been spent readying students for the study of comfirmatory techniques such as discriminant analysis, canonical correlation and MANOVA. Students find the shift in logic from confirmatory methods to exploratory methods rather diffcult to handle at this juncture. If factor analysis is introduced at this point, the result is a serious loss of continuity in the course. This is, of course, in no way a criticism of Tatsuoka's efforts. Rather it is a warning to instructors that if they wish to augment the content of the text with some further discussion of factor analysis that they would be well advised to delay it until much later in the course.

In Chapter 6, Tatsuoka develops discriminant analysis and canonical correlation. In the discussion of discriminant analysis the criterion again generalized from the single variable criterion for measuring group-mean differences. The need to determine a linear combination of the original variables which maximizes the corresponding multivariate criterion flows easily from this. Tatsuoka develops the discriminate analysis on two groups as a special case of multiple regression, but emphasizes that this holds only for the two group situation. With K groups, discriminant analysis is a special case of canonical correlation. He uses this notion to introduce canonical correlation first as a special approach to discriminant analysis, before treating the general model for canonical correlation. In addition to a very complete derivation of the mathematical basis of canonical correlation and significance tests, a section on interpreting canonical variates is included. This section is widely appreciated since people seem to find canonical correlation the most difficult to interpret of all the multivariate techniques.

Chapter 7 provides a brief, but thorough, discussion of MANOVA for factorial designs. The development is again easily achieved by generalization from ANOVA, with the significance tests primarily coming out of previous work with Wilks' Λ. Equation 7.9 (p. 199) contains the only important typographical error I have discovered in the entire text. The second line of the equation should be either

$$V = [\nu_e + \nu_h - (p + \nu_h + 1)/2] \ln \prod_{i=1}^{t} (1 + \lambda_i)$$

or

$$V = [\nu_e + \nu_h - (p + \nu_h + 1)/2] \sum_{i=1}^{t} \ln (1 + \lambda_i)$$

This chapter also includes a good discussion of discriminant analysis as a follow-up to MANOVA; a set of guidelines for extending MANOVA to nonorthogonal and other, more complex designs; as well as a brief discussion and comparison of Wilks' Λ with Hotelling's trace criterion and Rao's largest root criterion.

Chapter 8 contains a discussion of procedures for classifying people into well defined, pre-existing groups, with or without prior probabilities using Bayesian procedure, and using discriminant analysis to reduce the dimensionality of the classification space. Procedures are also developed for using this information, such as predicted success, in the classification. Although this chapter reads well and seems very completely developed, with several interesting examples; I have little information on student reaction to the contents, since it has been predominantly ignored in order to have time for discussion of factor analysis.

Appendices are provided on determinants, the pivotal condensation method of matrix inversion, symbolic differentiation by vectors or matrices, Hotelling's iterative procedure for solving eigenvalue problems, statistical and numerical tables, and answers to exercises. The placement of methods in appendices allows for their inclusion in or exclusion from discussion without loss of continuity. A special credit should be given for the inclusion of the appendix dealing with symbolic matrix differentiation. Although inclusion of some of the summary tables from Dwyer and MacPhail [1948] and Dwyer [1967] on matrix differentiation would have even made it better, this material greatly enhances the student's ability to follow the mathematical development of multivariate techniques.

My high esteem for this text is supported by the very positive student reaction ot it. They feel that it is a text that is readable and understandable with almost no additional explanation. They feel that lectures which iterate the text material are almost completely unnecessary. Throughout the text students are made to see the similarities in the mathematical bases of many multivariate techniques. Slowly students become aware that the major differences in techniques are substantive rather than mathematical. The ability of this text to be read and understood frees the instructor to spend more lecture time on the substantive aspects of the multivariate research process. The result is a learning experience which more fully prepares students to handle the difficult problems encountered in all aspects of multivariate research.

If you are in need of a teaching text for multivariate analysis, I highly recommend this one. It reaches a pragmatically necessary balance between breadth and depth of presentation. More depth could be included, notably a discussion of the general linear hypothesis formulation of many of these techniques, but would, I think, make the material more difficult to understand. Of course more techniques could be discussed, such as a formal discussion of factor analysis. However, it is doubtful that any more techniques could adequately be handled in the time allotted to a semester or quarter course. More space could be devoted to the substantive aspects of the research process. But multivariate research is, in many ways, a highly personalized process. The insights of one researcher-teacher may differ markedly from another. Although this material is highly valuable, it may be better to learn it in the context of a classroom where it can be kept in proper perspective. While this is not ideal text, it does seem to me to be the optimum text which meets the constraints of our current educational systems.

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Fred N. Kerlinger. *Foundations of Behavioral Research* (Second edition). New York: Holt, Rinehart, & Winston, 1973. Pp. xx + 741. $13.95.

This effort is a revision of the 1964 book; the revision involves a longer and more comprehensive treatment of research design, including new chapters on multivariate analysis. An important addition for instructors who will use the book is the inclusion of study suggestions at the end of almost every chapter.

The author's intention is "to help students understand the fundamental nature of the scientific approach to problem solution." To achieve this goal, Kerlinger provides a book which consists of 37 chapters, plus appendices. It is organized into 10 parts. Parts 1 to 5 provide the conceptual and mathematical foundations of behavioral research; the last five parts are concerned with statistical, experimental, and technical details of research design. The book ends with chapters on multivariate analysis, including one on factor analysis. Throughout the book the emphasis is on education and psychology problems; however, these aspects can be generalized easily to other behavioral sciences.

The author's effort is an ambitious one and it appears that he has achieved his goal. He is to be commended for having gathered under one cover such a comprehensive coverage of foundation, experimental, statistical, and technical aspects of research design. It appears that he has provided at least an introductory treatment of all topics which would seem to be important in behavioral research design. The book seems to be well written and clear in style. The approach is simple, systematic, and logical as the author moves from one topic to the next one; topics are integrated in an excellent fashion.

Now for a few comments on specific details. It is refreshing to see that Kerlinger treats the important topic of estimating magnitude of experimental effects in a knowledgeable fashion. Many psychological statisticians writing on this topic in recent years seem to think that the analysis of variance approaches to this problem are a recent solution; they show no awareness of Haggard's excellent book published in 1958 tracing the history of the intraclass correlation procedures. Kerlinger is very much aware of Haggard's contribution, as is indicated in his discussion.

One inappropriate implication by Kerlinger (p. 288) is that interval scale data are required for using Anova procedures (although in a footnote on the previous page discussing the assumptions of normal distribution and homogeneity of variance, he cites a number of articles which indicate that scale properties do not enter into Anova requirements). The interval scale assumption was suggested by Stevens and continued by Siegel, Senders, and a few others. Although many individuals have discussed this aspect in journal articles and rejected it as a definite requirement (*e.g.*, Anderson, Boneau, Gaito, Lord), some individuals still persist in thinking otherwise. It should be quite clear that scale properties do not enter into Anova assumptions. The mathematical or structural model for each design shows a statement of "*NID* (0, σ_e^2)" which *explicitly* indicates that "errors are normally, independently distributed with a mean of zero and one variance, σ_e^2," showing the assumptions of normality, independence, and homogeneity of errors; *an assumption of interval scale is nowhere to be found.*

The misconception of interval scale requirement has occurred because of a confusion between measurement-empirical aspects and those of statistical nature. For rejection or nonrejection of null hypotheses in any Anova problem, only the statistical assumptions *explicitly* indicated in the mathematical model are relevant.

In actual fact, the assumptions of normality and homogeneity of errors are weak assumptions, but the requirement of independence is a strong one. There is little effect on probability levels by violations of the first two assumptions, but drastic deviations may occur with lack of independence. Kerlinger discusses adequately the normality and homogeneity aspects, but pays no attention to the more important independence requirement.

Because of its simplicity and comprehensiveness, this book should be of great value to students; it would seem to be an excellent textbook, or reference book, for graduate students in the behavioral sciences, especially in Psychology and in Education. Also behavioral science researchers probably will find this book useful in the planning and execution of their research efforts.

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