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TRANSFER ELECTRON-PHOTON SCATTERING

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POSSIBLE ENHANCEMENT OF TWO-PHOTON EXCHANGE EFFECTS IN LARGE MOMENTUM
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ABSTRACT

An estimate is made of the two-photon exchange contribution to elastic electron-proton scattering. The dependence of the $NN\gamma$ vertex function on the invariant momentum-squared of each leg is assumed to be given by a definite form recently derived in a crossing symmetric bootstrap model. The two-photon effects turn out to be surprisingly large at large momentum transfer; recent form factor measurements are consistent with the results of this calculation, but a decisive test of the theory awaits e^+p scattering at $t \approx -10(\text{BeV}/c)^2$.

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† AEC Postdoctoral Fellow under contract no. AT(11-1)-34.

We have recently investigated¹ an off-shell, crossing symmetric bootstrap equation for the vertex function coupling three composite hadrons. This non-linear equation treats the constituents of a composite particle as, themselves, composite, and bootstraps the entire vertex function rather than just the coupling constant. We obtained the general result that a vertex function connecting three legs with momenta p_1 , p_2 , p_3 is given (up to an undetermined polynomial) by

$$\Gamma(p_1^2, p_2^2, p_3^2) \sim e^{-\alpha\sqrt{p_1^2 p_2^2 p_3^2}} \quad (1)$$

in the asymptotic limit in which one or more of the invariants p_1^2 , p_2^2 and p_3^2 approach infinity.

The prediction that this model makes, therefore, for the nucleon electromagnetic form factor is apparently

$$F(t) \sim e^{-b\sqrt{-t}} \quad (2)$$

where $b = am_N^2$. On the other hand, recent measurements² of $F(t)$ indicate that Eq. (2) is not an adequate description of the form factor. In fact, if one plots (see Fig. 1) $G_M(t)$ against $(-t)^{1/2}$, one notices that the data out to $t = -4(\text{BeV}/c)^2$ is well described by an exponential,³ but then decreases less rapidly. In this note, we shall suggest an explanation for this behavior.

Let us look at a particular two-photon exchange graph, shown in Fig. 2, in which the two photon-nucleon vertices are connected by a one-particle intermediate state. While the contribution of this graph to the scattering amplitude is smaller by an approximate factor of the fine structure constant, α , we will show that Eq. (1) causes an enhancement of this graph relative to the one-photon exchange graph at large momentum transfers. In order to see

how this comes about, we write the contribution of this two-photon exchange graph, A_2 , as

$$A_2((k+p)^2, q^2) \sim \alpha^2 \int d^4 q' [\text{propagators and spin factors}] \times \\ \times e^{-am_N [\sqrt{-q'^2(p-q')^2} + \sqrt{-(q-q')^2(p-q')^2}]} \quad (3)$$

where the momentum variables are defined in Fig. 2. We have used Eq. (1) to describe the two hadronic vertices in this graph. The explicit propagators and spin factors have not been written down since the vertex functions are undetermined up to polynomials and we can only make an order of magnitude estimate of A_2 .

The crucial point is that in the integration over q' , when the virtual mass of the one-particle intermediate state, with momentum $p-q'$, vanishes, then the exponent in the integrand also vanishes. By the method of steepest descent, this implies that the integral around the closed loop will be dominated, in the limit $|q^2| \rightarrow \infty$, by the integration over the hyperboloid $(p-q')^2 = 0$, and the leading term in the asymptotic limit of Eq. (3) will decrease like a power, rather than an exponential, in $\sqrt{-t}$.⁴ A similar result holds for the graph with crossed photons.

We therefore obtain the approximate, large momentum transfer electron proton scattering amplitude (neglecting s dependence)

$$A_{ep}(t) = \alpha P_1(t) e^{-\sqrt{-t}} + \alpha^2 P_2(t) \quad (4)$$

where $P_1(t)$ and $P_2(t)$ are undetermined polynomials or inverses of polynomials. It is clear from Eq. (4) that the two-photon exchange contribution must eventually dominate that from one-photon exchange. Let us attempt to compare Eq. (4) with experiment. Since $d\sigma_{ep}/d\Omega$ and the square of the form factor which one extracts from $d\sigma_{ep}/d\Omega$ by making the assumption of one-photon

exchange, will differ only by a polynomial, we are justified in comparing the experimental form factor to the absolute value of the amplitude in Eq. (4). Let us set $P_1(t)$ equal to a constant, c , for simplicity, and take the parameters b and c directly from the exponential fit in Fig. (1). Then we expect the form factor to behave like

$$G_M(t) \sim |1.5e^{-2.12\sqrt{-t}} + \alpha P_2(t)| \quad (5)$$

Calculating the discrepancy between the experimental data for $G_M(t)$ and the exponential term, alone, we find that the maximum value of the last term in Eq. (5) must be $\approx 5 \times 10^{-3}$ which is indeed of order α . For large t , $P_2(t) \approx |t|^{-1}$ is consistent with the data, but we have not attempted a detailed fit due to the ambiguities in Eq. (4).

The prediction that two-photon exchange effects may actually dominate can be dramatically tested in large momentum transfer e^+p scattering, provided that the second term in Eq. (4) has an appreciable real part. The momentum transfer $t \approx -10(\text{BeV}/c)^2$ should be a good one to look for a large interference effect since at this value the two terms in Eq. (5) are roughly comparable. If the second term in Eq. (5) is predominantly real, then we are witnessing constructive interference in the e^-p data and therefore would expect the e^+p data to actually dip below the exponential term near $t \approx -10(\text{BeV}/c)^2$. We note that since a term with exponential dependence in $\sqrt{-t}$ can describe the experimental data out to $|t| \cong 4(\text{BeV}/c)^2$ to within a few percent, Eq. (4) is consistent with the fact that no significant interference effects have been observed at small momentum transfers.

Finally we wish to point out that a test of this prediction would provide a strong argument in favor of the bootstrap model of Ref. 1, since the

effects described here arise because of the particular combination of momenta that appear in the exponent of Eq. (1). Had our vertex function depended only on the momentum of the off-shell photons, or had the exponent in Eq. (1) been, e.g., $(p_1^2 + p_2^2 + p_3^2)^{1/2}$, then we would have obtained the result that the α^2 term in the amplitude had an asymptotic dependence on t similar to that of the lowest order term, and therefore would be negligible.

FOOTNOTES AND REFERENCES

1. J. Harte, University of California, Berkeley, Preprint, February, 1968, "The Asymptotic Vertex Function in a Crossing Symmetric Bootstrap Model of the Hadrons." See also: J. Harte, "Crossing Symmetric Bootstrap and Exponentially Falling Form Factors" to be published, Phys. Rev. January, 1968, and J. Stack, Phys. Rev. 164, 1904 (1967).
2. D. H. Coward, et al., Phys. Rev. Letters, to be published.
3. Evidence to support the idea that the asymptotic behavior has already set in at momentum transfers of order $2-4(\text{BeV})^2$ is given in Ref. 1. There we give an argument which suggests that the constant a , in Eq. (1), should be a universal constant, independent of the types of particles coupling at the vertex, if the bootstrap is truly reciprocal. We also show that a comparison of the measured $NN\gamma$ and $N^*N\gamma$ form factors supports this conclusion.
4. Had the electron also been composite, then the electron-photon vertex would also be described by Eq. (1), and the leading asymptotic term in the scattering amplitude would again be an exponential in t . This can be easily understood by considering the Wick-rotated form of Eq. (3), since in a Euclidean space, the two conditions, $(p-q')^2 = 0$ and $(k+q')^2 = 0$, cannot be simultaneously satisfied for $s \neq 0$. We have also had to assume (but have not yet proven) the validity of the Wick-rotation in the crossing symmetric Bethe-Salpeter equation in order to derive Eq. (1).

FIGURE CAPTIONS

- Fig. 1. Data points are the experimental values of $G_{MP}(t)/\mu$ as measured by D. H. Coward et al., Ref. 2. The solid curve is an exponential fit (first term in Eq. (5)).
- Fig. 2. Possible two-photon exchange graph contributing to e^-p scattering. The dashed, wavy, and solid lines represent electrons, photons, and protons respectively. The shaded circle represents a composite particle vertex, described by Eq. (1).

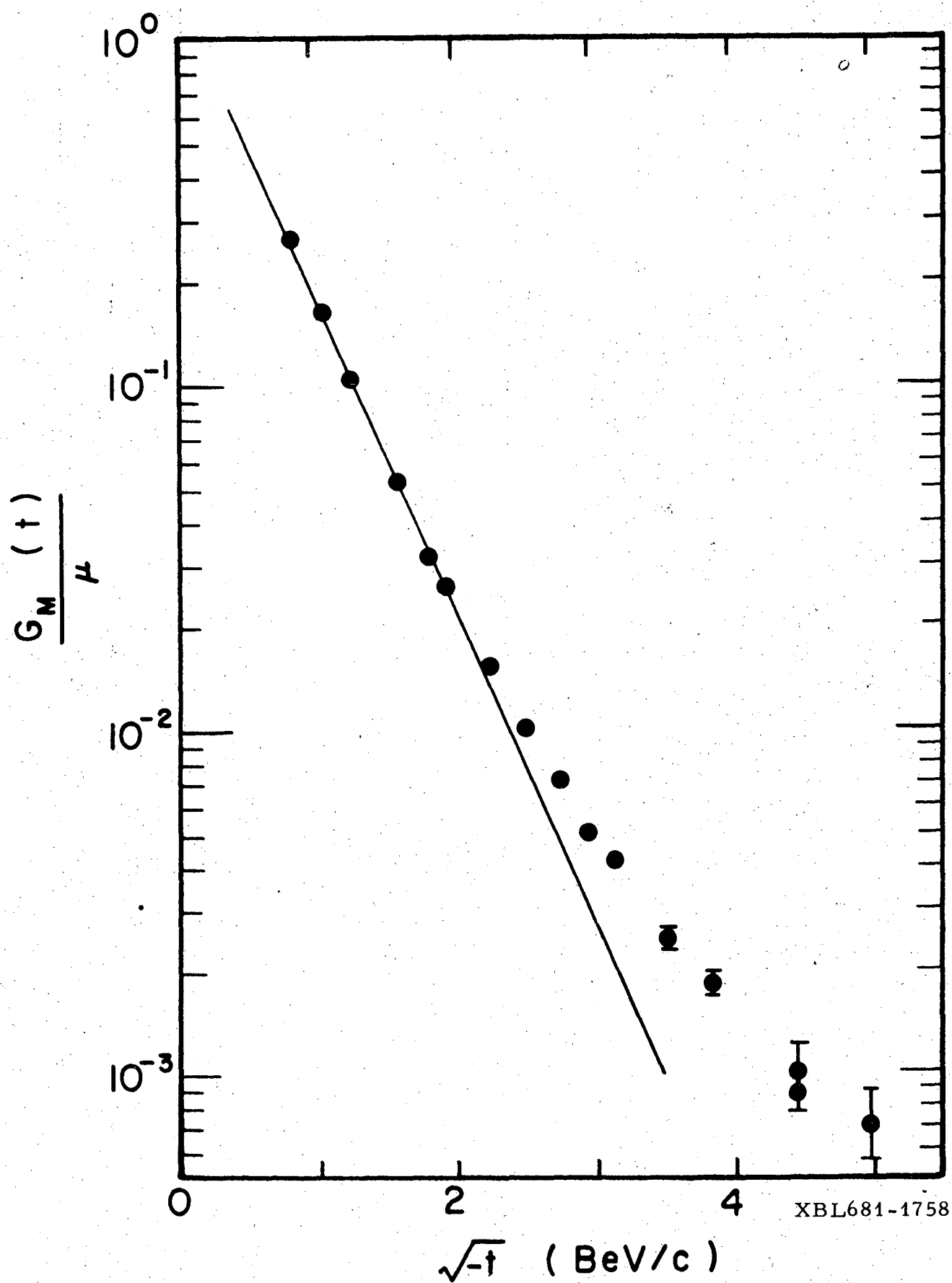
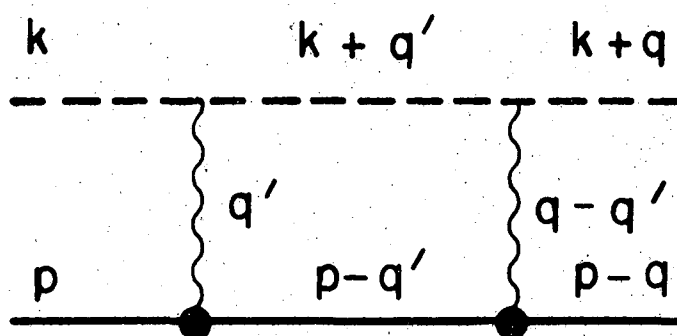


FIG. 1

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FIG. 2

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