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UNIVERSITY OF CALIFORNIA, SAN DIEGO

**Detecting gravitational lensing from the Cosmic Microwave
Background**

A dissertation submitted in partial satisfaction of the
requirements for the degree
Doctor of Philosophy

in

Physics

by

Chang Feng

Committee in charge:

Professor Brian Keating, Chair
Professor Samuel Buss
Professor Bruce Driver
Professor Aneesh Manohar
Professor Hans Paar

2014

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The Dissertation of Chang Feng is approved, and it is acceptable in quality and form for publication on microfilm and electronically:

Chair

University of California, San Diego

2014

EPIGRAPH

*I do not know what I may appear to the world, but to myself I seem to have been
only like a boy playing on the sea-shore, and diverting myself in now and then
finding a smoother pebble or a prettier shell than ordinary, whilst the great ocean
of truth lay all undiscovered before me. — Issac Newton*

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Chapter 4, in full, is a reprint of material as it appears in Physical Review D 86, 063519, 2012 (arXiv:1207.3326) [2], as written by the author of this dissertation. “Measuring gravitational lensing of the cosmic microwave background using cross correlation with large scale structure”, Chang Feng, Grigor Aslanyan, Aneesh V. Manohar, Brian Keating, Hans P. Paar, and Oliver Zahn.

Chapter 6, in full, is a reprint of material as it appears on Physical Review Letters 113, 021301 (2014) (arXiv:1312.6646) [3], as written by the author of this dissertation. “Measurement of the Cosmic Microwave Background Polarization Lensing Power Spectrum with the POLARBEAR experiment”, POLARBEAR Collaboration, P.A.R. Ade, Y. Akiba, A.E. Anthony, K. Arnold, M. Atlas, D. Barron, D. Boettger, J. Borrill, S. Chapman, Y. Chinone, M. Dobbs, T. Elleflot, J. Errard, G. Fabbian, C. Feng, D. Flanagan, A. Gilbert, W. Grainger, N.W. Halverson, M. Hasegawa, K. Hattori, M. Hazumi, W.L. Holzapfel, Y. Hori, J. Howard, P. Hyland, Y. Inoue, G.C. Jaehnig, A. Jaffe, B. Keating, Z. Kermish, R. Keskitalo, T. Kisner, M. Le Jeune, A.T. Lee, E. Linder, E. M. Leitch, M. Lungu, F. Matsuda, T. Matsumura, X. Meng, N.J. Miller, H. Morii, S. Moyerman, M.J. Myers, M. Navaroli, H. Nishino, H. Paar, J. Peloton, E. Quealy, G. Rebeiz, C.L. Reichardt, P.L. Richards, C. Ross, I. Schanning, D.E. Schenck, B. Sherwin, A. Shimizu, C. Shimmin, M. Shimon, P. Siritanasak, G. Smecher, H. Spieler, N. Stebor, B. Steinbach, R. Stompor, A. Suzuki, S. Takakura, T. Tomaru, B. Wilson, A. Yadav, O. Zahn.

VITA

- University of California, San Diego** 2014
Department of Physics
Degree: **Ph.D. in Physics**
Supervisor: Professor Brian Keating
- Fudan University** 2009
Department of Physics
Degree: **Master of Science**
Supervisor: Professor Rukeng Su, Vice President of High Energy Physics Society of China
- Guilin University of Electronic Technology** 2006
School of Electromechanical Engineering, Traffic Engineering
Degree: **Bachelor of Traffic Engineering**

PUBLICATIONS

A Measurement of the Cosmic Microwave Background B-Mode Polarization Power Spectrum at Sub-Degree Scales with POLARBEAR. (Contributing author). Accepted by the Astrophysical Journal. arXiv:1403.2369.

Measurement of the cosmic microwave background polarization lensing power spectrum with the POLARBEAR experiment. (**Corresponding author**). Phys. Rev. Lett. 113, 021301 (2014). arXiv:1312.6646. Highlighted as PRL Editors' Suggestion.

Evidence for Gravitational Lensing of the Cosmic Microwave Background Polarization from Cross-Correlation with the Cosmic Infrared Background. (Contributing author). Phys. Rev. Lett. 112, 131302. arXiv:1312.6645. Highlighted as PRL Editors' Suggestion.

Measuring gravitational lensing of the cosmic microwave background using cross correlation with large scale structure. **Chang Feng**, Grigor Aslanyan, Aneesh V. Manohar, Brian Keating, Hans P. Paar, Oliver Zahn. Phys. Rev. D 86, 063519 (2012). arXiv:1207.3326

Reconstruction of Gravitational Lensing Using WMAP 7-Year Data. **Chang Feng**, Brian Keating, Hans P. Paar, Oliver Zahn. Phys. Rev. D 85, 043513 (2012). arXiv:1111.2371

Observational constraints on the dark energy and dark matter mutual coupling. **Chang Feng**, Bin Wang, Elcio Abdalla, Rukeng Su. Physics Letters B, 665 (2008) 111-119. arXiv:0804.0110

Testing the viability of the interacting holographic dark energy model by using combined observational constraints. **Chang Feng**, Bin Wang, Yungui Gong and Ru-Keng Su. Journal of Cosmology and Astroparticle Physics, 09 (2007) 005. arXiv:0706.4033

ABSTRACT OF THE DISSERTATION

**Detecting gravitational lensing from the Cosmic Microwave
Background**

by

Chang Feng

Doctor of Philosophy in Physics

University of California, San Diego, 2014

Professor Brian Keating, Chair

Gravitational lensing of the Cosmic Microwave Background (CMB) measures all the matter content in the Universe. It can be used to constrain neutrino masses, calibrate biased tracers for large scale structure, and remove contamination of primordial B-modes. The theoretical framework, which includes simulations and reconstruction of gravitational lensing effects from CMB observations, has been established and applied through this dissertation. From observations of the CMB's temperature anisotropy, WMAP datasets are used to probe gravitational lensing effects. It is found that the lensing signal can not be directly detected from WMAP alone but can be indirectly detected at $> 3\sigma$ if WMAP's CMB observations are cross-correlated with galaxy surveys. Other than the CMB temperature,

the CMB polarization is of great importance because the CMB's polarization is more sensitive than its temperature to probing lensing effects. From the ground-based small-scale polarization experiment, POLARBEAR, we (for the first time) measure polarization lensing and lensing B-modes from different types of correlation functions. The B-mode power spectrum is measured, showing the evidence for lensing B-modes at the 2σ level. Lensing reconstruction with B-modes is also performed. From the auto-correlation of the lensing reconstruction with B-modes, the polarization lensing and lensing B-mode signal is measured at the 4.2σ level, including systematics. This signal measures dark matter fluctuations with 27% uncertainty. The matter structure seen in the lensing reconstruction is further validated by the cross-correlation with cosmic infrared background, which shows evidence for polarization lensing at 4σ . This state-of-the-art technique is capable of mapping all gravitating matter in the Universe, is sensitive to the sum of neutrino masses, and is essential for cleaning the lensing B-mode signal in searches for primordial gravitational waves.

Chapter 1

Introduction

1.1 The formation of the Cosmic Microwave Background

The Universe began with the Big Bang. After this beginning, it is believed that the Universe experienced an inflation stage during which quantum fluctuations, powered by the inflaton¹, were exponentially stretched and frozen at the superhorizon scale. After these fluctuations re-entered the horizon, large scale anisotropies were seeded. At the end of the inflation stage, the reheating process converted the energy of inflaton field into fundamental particles. The very early Universe was very hot and dense. Here photons Compton-scattered with electrons and the Universe was in a thermal equilibrium state, forming a blackbody radiation spectrum. As the Universe expanded, the energy density diluted and Thomson scattering between photons and electrons began. Initially the mean free path was still too small and photons could not escape. As the temperature of the Universe decreased, eventually protons were able to capture electrons and the interactions between electrons and photons were not very strong any more. From then on photons began free-streaming. Today if the observer is put at the origin of the rest frame, these photons appear to be coming from a thin surface which we call the last scattering surface (LSS) because after that process, photons didn't scatter too much. The photons which escaped from the last scattering surface

formed the Cosmic Microwave Background (CMB). The epoch when photons and electrons decoupled was associated with the electron-proton combination process so we also call it recombination epoch, which is around redshift $z \sim 1100$, roughly 380,000 years after the Big Bang.

The CMB temperature pattern is almost Gaussian and has an average temperature of 2.7 Kevin on the sky. If the mean is subtracted, the remaining temperature fluctuations shows an anisotropic structure which contains a lot of information about how the Universe evolved. The primary way to extract the information is to calculate a two-point correlation function of the CMB map, i.e., the angular power spectrum of the CMB, which is actually the variance of the Gaussian fluctuations at different angular scales. From the CMB power spectrum, we can extract information about the age, the geometry of the Universe, and even neutrino masses.

The CMB temperature anisotropy $T(\mathbf{n})$ can be decomposed into spherical harmonic coefficients we call temperature modes. T_{lm} is defined as

$$T(\mathbf{n}) = \sum_{lm} T_{lm} Y_{lm}(\mathbf{n}). \quad (1.1)$$

Here $Y_{lm}(\mathbf{n})$ are the spherical harmonics, and $\mathbf{n}$ is a direction on the sky.

The angular power spectrum is given by the two-point correlation function, i.e.,

$$\langle T_{lm} T_{l'm'}^* \rangle = C_l^{TT} \delta_{ll'} \delta_{mm'}. \quad (1.2)$$

Besides the CMB temperature anisotropy, the CMB is also polarized. At the recombination epoch, the CMB quadrupole was developed by scalar and tensor perturbations, giving rise to the CMB polarization pattern through Thomson scattering. The polarization patterns are described by Stokes parameters $Q(\mathbf{n})$ and $U(\mathbf{n})$. We usually decompose them into so-called electric (E-) and magnetic (B-) modes [4] because this decomposition disentangles their parity properties and does not require a specific coordinate system. The relationship between them is written as

$$[Q(\mathbf{n}) \pm iU(\mathbf{n})] = \sum_{lm} [E_{lm} \pm iB_{lm}]_{\pm 2} Y_{lm}(\mathbf{n}). \quad (1.3)$$

¹The quanta of a scalar field.

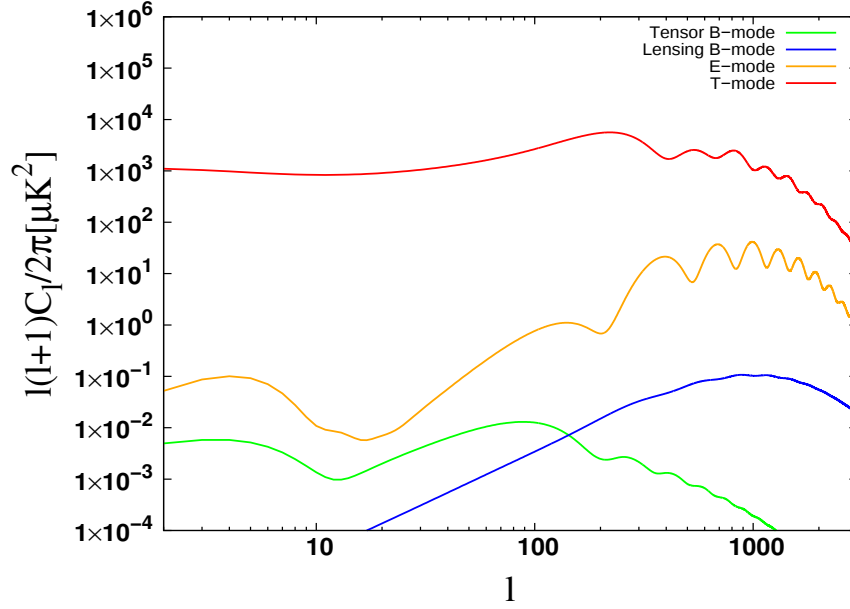


Figure 1.1: CMB power spectra showing T , E , B -power spectra for the scalar perturbations and only the B -mode power spectrum for the tensor perturbation.

In this equation, $_{\pm 2}Y_{lm}(\mathbf{n})$ is spin-2 harmonics. In analogy to the definition of the temperature power spectrum, we can also define E-mode and B-mode power spectra

$$\langle E_{lm} E_{l'm'}^* \rangle = C_l^{EE} \delta_{ll'} \delta_{mm'}, \quad (1.4)$$

and

$$\langle B_{lm} B_{l'm'}^* \rangle = C_l^{BB} \delta_{ll'} \delta_{mm'}. \quad (1.5)$$

In Figure 1.1, an example of all these power spectra is shown. In this plot, we show T , E , B -power spectra for the scalar perturbations, and only show the B -mode power spectrum for the tensor perturbation. These power spectra are calculated by the CAMB code [5].

As mentioned earlier, the CMB polarization was created due to the Thomson scattering at the recombination epoch. This type of anisotropy is a few orders of magnitude smaller than the temperature anisotropy as seen from Figure 1.1, so detecting these faint signals necessitates a very high-sensitivity CMB experiment. Scientists have been working hard to measure these signals because the CMB observation can help answer many complicated questions in cosmology. However, to

theoretically calculate the CMB power spectra, many theorists had done a tremendous amount of work to establish the theoretical CMB framework. The essence of the CMB is a photon radiative transport problem. In order to solve this problem, we need to know the perturbations of the metric as well as the density, velocity and pressure for each species. The core ingredients of the CMB theory are cold dark matter, baryonic matter, photons and neutrinos. The first two are modeled as perfect fluids and the continuity equation is used. For photons and neutrinos, Boltzmann equations are used to describe their deviations from the Bose-Einstein and Fermi-Dirac distributions during the very early stage of the Universe. All the energy-momentum perturbations determine the Universe's geometric distortions as indicated by Einstein's field equation [6]. Because of this, the photon radiative transport must be solved simultaneously with all other components which it is very complicated and time-consuming if one wants to directly solve all these coupled perturbation equations. Fortunately, a line-of-sight method [7] was developed to greatly speed up the calculations. The basic idea is to make an integral form of the photon transport solution, and the integrand has a sharp peak around the last scattering epoch. The physical implication is quite straightforward, i.e., most of the anisotropies were formed during the last scattering surface. Based on this technique, cosmologists are able to easily compute power spectra and compare them to measurements.

1.2 Milestones of CMB experiments

The CMB was first detected by Penzias and Wilson in 1965 [8]. In 1990, the Cosmic Background Explorer (COBE) satellite made a very precise measurement of the blackbody spectrum. After further analyzing the data, they found a structure in the CMB temperature anisotropy [9]. The quadrupole temperature was found to be $13 \pm 4 \mu\text{K}$, revealing the fact that the amount of anisotropy was roughly at the $O(10^{-5})$ level. In 1999, cosmologists were able to measure the acoustic oscillations using temperature anisotropy data. Another groundbreaking progress was soon made in 2002 by an experiment called Degree Angular Scale Interferometer (DASI).

DASI for the first time detected CMB E-mode polarization at the 4.9σ level [10]. For the CMB polarization, we know that E-mode polarization is about two orders of magnitude weaker than the temperature anisotropy, and B-mode polarization is about two orders of magnitude fainter than the E-mode polarization at sub-degree scales. As the development of low temperature detector technology advanced, in 2013 and 2014, the CMB field started to enter a new era, the B-mode era [11, 12, 3, 13, 14]. After decades of hard work, we are achieving an unprecedented sensitivity for the CMB polarization observations.

The CMB primary anisotropies are almost Gaussian and this is due to the nature of quantum fluctuations generated by inflation. These primary anisotropies can provide a great deal of cosmological information, however, there are also secondary anisotropies imprinted on the CMB which can also greatly enhance our understanding of the Universe. In some cases, the secondary effects are also important.

1.3 CMB secondary anisotropies

There are various CMB secondary anisotropies which are also of great interest. These secondary effects include weak gravitational lensing, the integrated Sachs-Wolfe (ISW) effect, and the Sunyaev-Zeldovich (SZ) effect.

CMB photons traverse the entire Universe, i.e., the cosmic web formed by dark matter. This structure distorts space and time according to what General Relativity (GR) predicts, and photon geodesics are deflected by this structure. The depth of the potential well at last scattering is at $O(10^{-5})$ level [15], although the GR predicted deflection is as twice as large as the Newtonian gravity prediction, the deflection of the CMB photon is still very small so this is within the weak-lensing regime. In the observer's rest frame, we can expect that the CMB photons at the sky direction $\mathbf{n}'$ actually came from an original direction $\mathbf{n}$ and the deflection imposed by the dark matter distribution determines how big the direction shift is according to the relation $\mathbf{n}' = \mathbf{n} + \mathbf{d}(\mathbf{n})$. Here $\mathbf{d}(\mathbf{n})$ denotes the deflection angle. CMB lensing measures all the matter content along the line of sight with no

free parameters. This is quite important for recovering neutrino masses because neutrinos decoupled from the hot plasma at very early time. Therefore neutrinos didn't interact with photons and simply were affected by the background evolution history. In this sense, neutrino signatures were weakly imprinted on primary CMB fluctuations. Like CMB photons, neutrinos also traversed the entire Universe. They became non-relativistic in the late Universe and fell into gravitational potential wells. In this way neutrinos altered the matter angular correlation function (or matter power spectrum) at small scales and CMB lensing is just the observable which measures this matter power spectrum most precisely. Thus CMB lensing is the most sensitive probe of neutrino masses. Neutrino masses are the final missing part in the standard model of particle physics. The experimental measurement of the neutrino masses will greatly improve our understanding of the standard model of particle physics. Given the confidence obtained with the increasing precision of recent CMB lensing measurements, we believe the neutrino mass problem will be solved in the near future.

The integrated Sachs-Wolfe effect consists of early ISW and late ISW. At recombination, CMB photons trapped in potential wells were released and their energies were redshifted, thus forming the early ISW. In the late Universe, CMB photons traversed gravitational potential wells which simultaneously decayed because dark energy was accelerating the expansion of the Universe. This mechanism generated the late ISW effect which largely affects the large angular scale anisotropies because on small scales CMB photons fell into and climbed out of the potential wells very quickly. Thus photon energy was almost exactly balanced.

On small scales, another important secondary effect is called SZ effect which is due to the inverse Compton scattering between electrons and photons. High energy electrons with high temperature can blueshift CMB photons, and this is known as thermal SZ effect (tSZ); on the other hand, electrons can gain high energy if their galaxy clusters have significant kinematic motions. These energetic electrons then excite CMB photons exactly as high temperature electrons do. This effect is called the kinematic SZ effect (kSZ).

1.4 Cosmological implications of CMB observations

The CMB illuminates the entire Universe. This makes CMB observations very useful to test theoretical predictions. The standard model in cosmology can be described only by six parameters such as baryon density fraction Ω_b , dark matter density fraction Ω_c , etc. This model can be extended to describe new physics such as sterile neutrino masses and the number of relativistic species.

The Wilkinson Microwave Anisotropy Probe (WMAP) made very precise measurements of CMB temperature anisotropy and some of the key questions in cosmology were answered. WMAP, for the first time, gave accurate numbers for the standard cosmological model, such as the age of the Universe and the fractions of individual matter components in the Universe [16]. Also, WMAP confirmed the predicted existence of correlation between temperature and E-mode polarization [17].

Recently Planck satellite improves those measurements. Planck found that the Hubble constant is much smaller and the matter density contrast is much higher than WMAP values. Due to the great sensitivity of Planck, gravitational lensing effects were precisely detected, and for the first time the ISW effect was measured at 2σ [18].

As the development of CMB experiments continues, interesting and important physics will be probed, and we believe the CMB will serve as a laboratory, teaching us more about the Universe in every respect.

Chapter 2

Gravitational Lensing

Gravitational lensing of the CMB by the intervening dark matter halos at redshift of a few encodes valuable information in the temperature anisotropy and polarization. We illustrate the inner-workings of CMB lensing by employing standard quadratic estimators, in both flat-sky and full-sky cases. We illustrate this by reconstructing fiducial deflection angle maps and their power spectra from lensed CMB simulations.

2.1 Introduction

According to standard cosmology, shortly after the big bang an inflationary phase in the expansion of the universe was set off by the slow rolling of the inflaton field. Quantum fluctuations in this field eventually lead to density perturbations as well as shear-like modes, the gravitational waves. Later on, these gravitational waves provide the prerequisite quadrupole moment in the CMB to form polarization upon scattering off free electrons in the post-recombination era. In particular, these generate curl-like B-mode polarization. Measuring this B-mode signal could in principle give us the energy scale of inflation, which is yet unknown. Recently, the BICEP team for the first time detected this primordial B-mode signal [14].

B-mode polarization can be also generated by the secondary gravitational lensing effect. The primordial CMB is kept Gaussian (or nearly so) until it is lensed by the intervening dark matter halos at redshifts of a few. Gravitational lensing of

the CMB re-maps the sky, aliasing power from one angular scale to another, and thereby generating non-Gaussianity which is characterized by non-trivial higher-order statistics of the CMB.

From this perspective, the B modes produced by gravitational lensing can be viewed as a contaminant that could potentially swamp the inflationary B modes. The delensed CMB will enable us to investigate further issues, such as primordial non-gaussianity. However, separating the lensing-generated B-mode from the primordial B-mode is of additional interest; the B-mode from lensing leads to a unique contribution to the B-mode power spectrum on multipoles $l \sim 1000$ (unlike the lensed temperature anisotropy or E-mode polarization which are only slightly blurred by lensing) which can be used to probe the evolution and clustering of dark matter halos, thereby providing us with valuable information about, e.g., neutrino masses, clustering of dark matter, etc. For these reasons, reliably measuring the inflation B-mode polarization, which peaks at horizon scales ($l \sim 100$) actually requires, in principle, an order of magnitude higher resolution observations, on the few arcminute scales ($l \lesssim 2000 - 3000$) to ensure that the lensing-induced signal is fully captured by the experiment and to allow an optimal separation of the two signals.

It can be shown that the connected part of the four-point correlation function of CMB temperature anisotropy and polarization contains the relevant information about the lensing potential power spectrum. The standard estimators for extracting gravitational lensing signal build on this fact [19, 20, 21]. The fact that the gravitational potential introduces off-diagonal elements into CMB's covariance provides us for ways of extracting lensing signal from observed CMB.

Quadratic estimator [19, 20, 21], has been exploited to extract gravitational lensing signal using CMB modes in harmonic space, incorporating both temperature and polarization data. They have obtained the expressions for the estimators of deflection angle and have comprehensively illustrated the workings of the framework of extracting lensing either from patchy or full sky CMB. This method adopted the first order approximation.

Configuration-space based approaches to the same problem have been pro-

posed as well. For example, maximum-likelihood method [22] analyzes the convergence field by using temperature anisotropy in real space.

We will only focus on reconstruction using the established formalism both on flat- and full-sky. The reconstructed lensing signal containing information on the evolution of perturbations up to, essentially, moderate redshifts [23], will ultimately well-constrain certain cosmological parameters, such as the sum of neutrino masses $\sum_i m_{\nu i}$ which may determine the hierarchy problem of neutrino, and correct the B-mode signal to recover the B-mode from gravitational waves generated by inflation.

2.2 Lensing effects in the flat-sky

The primordial temperature $\tilde{T}(\mathbf{n})$, polarization $\tilde{Q}(\mathbf{n})$ and $\tilde{U}(\mathbf{n})$ can be decomposed into plane waves as

$$\begin{aligned}\tilde{T}(\mathbf{n}) &= \int \frac{d^2l}{(2\pi)^2} \tilde{T}(\mathbf{l}) e^{i\mathbf{l} \cdot \mathbf{n}} \\ (\tilde{Q} + i\tilde{U})(\mathbf{n}) &= - \int \frac{d^2l}{(2\pi)^2} [\tilde{E}(\mathbf{l}) + i\tilde{B}(\mathbf{l})] e^{+2i\phi_1} e^{i\mathbf{l} \cdot \mathbf{n}} \\ (\tilde{Q} - i\tilde{U})(\mathbf{n}) &= - \int \frac{d^2l}{(2\pi)^2} [\tilde{E}(\mathbf{l}) - i\tilde{B}(\mathbf{l})] e^{-2i\phi_1} e^{i\mathbf{l} \cdot \mathbf{n}}.\end{aligned}\tag{2.1}$$

Here ϕ is the rotation angle in l space. We can now remap the unlensed maps and obtain a single realization of the lensed sky. The general remapping procedure reads

$$X(\mathbf{n}) = \tilde{X}(\mathbf{n} + \mathbf{d}(\mathbf{n}))\tag{2.2}$$

where X can be either the temperature anisotropy or linear polarization Stokes parameters, i.e., $X = \{T, Q, U\}$. Once the lensed maps are obtained in real space we can carry out 2D inverse Fourier transformation to obtain the corresponding lensed maps in l space following equations

$$\begin{aligned}T(\mathbf{l}) &= \int d^2n T(\mathbf{n}) e^{-i\mathbf{l} \cdot \mathbf{n}} \\ (Q + iU)(\mathbf{n}) &= - \int \frac{d^2l}{(2\pi)^2} [E(\mathbf{l}) + iB(\mathbf{l})] e^{+2i\phi_1} e^{i\mathbf{l} \cdot \mathbf{n}} \\ (Q - iU)(\mathbf{n}) &= - \int \frac{d^2l}{(2\pi)^2} [E(\mathbf{l}) - iB(\mathbf{l})] e^{-2i\phi_1} e^{i\mathbf{l} \cdot \mathbf{n}},\end{aligned}\tag{2.3}$$

where from the last two equations, we obtain the lensed E and B modes. Here we adopt the notation: $\mathcal{F} \rightarrow \int \frac{d^2 l}{(2\pi)^2} e^{i\mathbf{l} \cdot \mathbf{n}}$, $\mathcal{F}^{-1} \rightarrow \int \frac{d^2 l}{(2\pi)^2} e^{-i\mathbf{l} \cdot \mathbf{n}}$. Now we can analytically express E and B modes using the observed Stokes parameters as

$$\begin{aligned} E(\mathbf{l}) &= -\frac{1}{2} \{ e^{-2i\phi_1} \mathcal{F}^{-1}[(Q + iU)(\mathbf{n})] + e^{+2i\phi_1} \mathcal{F}^{-1}[(Q - iU)(\mathbf{n})] \} \\ B(\mathbf{l}) &= -\frac{1}{2i} \{ e^{-2i\phi_1} \mathcal{F}^{-1}[(Q + iU)(\mathbf{n})] - e^{+2i\phi_1} \mathcal{F}^{-1}[(Q - iU)(\mathbf{n})] \}. \end{aligned} \quad (2.4)$$

From these modes we can also get the E and B maps which are defined in the following equations:

$$\begin{aligned} E(\mathbf{n}) &= \int \frac{d^2 l}{(2\pi)^2} E(\mathbf{l}) e^{i\mathbf{l} \cdot \mathbf{n}} \\ B(\mathbf{n}) &= \int \frac{d^2 l}{(2\pi)^2} B(\mathbf{l}) e^{i\mathbf{l} \cdot \mathbf{n}}. \end{aligned} \quad (2.5)$$

Ultimately, the simulated lensed power spectrum is obtained. X can be T, E, B.

$$\langle X(\mathbf{l}) X'(\mathbf{l}') \rangle = (2\pi)^2 C_l^{xx'} \delta(\mathbf{l} + \mathbf{l}'). \quad (2.6)$$

Note that $X^*(\mathbf{l}) = X(-\mathbf{l})$. This equation could also be expressed as

$$\langle X^*(\mathbf{l}) X'(\mathbf{l}') \rangle = (2\pi)^2 C_l^{xx'} \delta(\mathbf{l} - \mathbf{l}'). \quad (2.7)$$

2.2.1 Two-point correlation function

Having described the lensing simulation procedure, we will next extract information from two-point correlation function. From the lensing re-mapping $X(\mathbf{n}) = \tilde{X}(\mathbf{n} + \mathbf{d})$, we expand it to first order in the deflection angle $\mathbf{d}$, as an example, the temperature expansion is

$$\delta T(\mathbf{l}) = - \int \frac{d^2 l'}{(2\pi)^2} \tilde{T}(\mathbf{l}') [\mathbf{l}' \cdot \mathbf{L}] \phi(\mathbf{L}) = \int \frac{d^2 l'}{(2\pi)^2} \tilde{T}(\mathbf{l}') W(\mathbf{l}', \mathbf{K}). \quad (2.8)$$

Note that $W(\mathbf{l}, \mathbf{K}) = -\mathbf{l} \cdot \mathbf{K} \phi(\mathbf{K})$, $\mathbf{K} = \mathbf{l} - \mathbf{l}'$, $\mathbf{L} = \mathbf{l} + \mathbf{l}'$, which yields

$$\begin{aligned} T(\mathbf{l}) T(\mathbf{l}') &= (\tilde{T}(\mathbf{l}) + \delta T)(\tilde{T}(\mathbf{l}') + \delta T) \\ &= \tilde{T}(\mathbf{l}) \tilde{T}(\mathbf{l}') + \tilde{T}(\mathbf{l}) \delta T(\mathbf{l}') + \tilde{T}(\mathbf{l}') \delta T(\mathbf{l}) + \dots \\ &= (2\pi)^2 \tilde{C}_l^{TT} \delta(\mathbf{L}) + [\tilde{C}_l^{TT} \mathbf{l} \cdot \mathbf{L} + \tilde{C}_{l'}^{TT} \mathbf{l}' \cdot \mathbf{L}] \phi(\mathbf{L}) + \dots \end{aligned} \quad (2.9)$$

The higher order terms are neglected. The ensemble average of the above equation is then calculated over many sky realizations. We can similarly obtain relations for TE, TB, EE, EB, as in the table of [20].

The general form is

$$\langle X(\mathbf{l})X'(\mathbf{l}') \rangle = (2\pi)^2 \tilde{C}_l^{XX'} \delta(\mathbf{L}) + f_\alpha(\mathbf{l}, \mathbf{l}') \phi(\mathbf{L}) \quad (2.10)$$

and the coefficients in front of the potential is listed in [20]. Equation 2.10 tells us that modes satisfying $\mathbf{L} = \mathbf{l}_1 + \mathbf{l}_2 = 0$, contribute to the power spectra, and from arbitrary modes ($\mathbf{L} \neq 0$), they contribute to the potential of the intervening mass.

2.2.2 Four-point correlation function

We can parametrize an estimator of the deflection field as [20]

$$d(\mathbf{L}) = \frac{A(L)}{L} \int \frac{d^2 l}{(2\pi)^2} x(\mathbf{l}) x'(\mathbf{l}') F(\mathbf{l}, \mathbf{l}'). \quad (2.11)$$

For different combinations, we have different estimators with different filters F which are functions of the coefficients derived in the last section. In the following, we use index α or β to denote them. Then we calculate the two-point correlation of this estimator

$$\begin{aligned} d_\alpha(\mathbf{L}) d_\beta(\mathbf{L}') &= \frac{A_\alpha(L)}{L} \int \frac{d^2 l_1}{(2\pi)^2} x_\alpha(\mathbf{l}_1) x'_\alpha(\mathbf{l}_2) F_\alpha(\mathbf{l}_1, \mathbf{l}_2) \frac{A_\beta(L')}{L'} \\ &\times \int \frac{d^2 l'_1}{(2\pi)^2} x_\beta(\mathbf{l}'_1) x'_\beta(\mathbf{l}'_2) F_\beta(\mathbf{l}'_1, \mathbf{l}'_2) \\ &= \frac{A_\alpha(L)}{L} \frac{A_\beta(L')}{L'} \int \frac{d^2 l_1}{(2\pi)^2} \frac{d^2 l'_1}{(2\pi)^2} x_\alpha(\mathbf{l}_1) x'_\alpha(\mathbf{l}_2) x_\beta(\mathbf{l}'_1) x'_\beta(\mathbf{l}'_2) \\ &\times F_\alpha(\mathbf{l}_1, \mathbf{l}_2) F_\beta(\mathbf{l}'_1, \mathbf{l}'_2). \end{aligned} \quad (2.12)$$

For a Gaussian field, a four-point correlation can be broken down into the summation of the product of two-point correlation functions (Wick's theorem)(Eq(2) in [24])

$$\langle x(\mathbf{l}_1) x(\mathbf{l}_2) x(\mathbf{l}_3) x(\mathbf{l}_4) \rangle = \sum \langle x(\mathbf{l}_1) x(\mathbf{l}_2) \rangle \langle x(\mathbf{l}_3) x(\mathbf{l}_4) \rangle + \text{perms.} \quad (2.13)$$

We can apply this relation to the four-point correlation function above and get the result

$$\begin{aligned} \langle d_\alpha(\mathbf{L})d_\beta^*(\mathbf{L}') \rangle &= (2\pi)^2 \delta(\mathbf{L} - \mathbf{L}') (C_L^{dd} + N_{\alpha\beta}^{(0)}(L) \\ &\quad + \text{higher-order terms}), \end{aligned} \quad (2.14)$$

where the second term is called reconstruction noise given in [20].

2.2.3 Estimator average

As we showed in the previous section, the simple quadratic estimator proposed by [20] is shown in equation 2.11, where $F(\mathbf{l}, \mathbf{l}')$ is a filter that needs to be determined by minimizing the covariance of the estimator. The parameter $A(L)$ is given from the following procedure. Taking the average of this estimator over many samples,

$$\begin{aligned} \langle d(\mathbf{L}) \rangle &= \frac{A(L)}{L} \int \frac{d^2 l}{(2\pi)^2} \langle x(\mathbf{l})x'(\mathbf{l}') \rangle F(\mathbf{l}, \mathbf{l}') \\ &= \frac{A(L)}{L} \int \frac{d^2 l}{(2\pi)^2} f(\mathbf{l}, \mathbf{l}') \phi(\mathbf{L}) F(\mathbf{l}, \mathbf{l}') \\ &= \frac{A(L)}{L} \left[\int \frac{d^2 l}{(2\pi)^2} f(\mathbf{l}, \mathbf{l}') F(\mathbf{l}, \mathbf{l}') \right] \phi(\mathbf{L}) \\ &= L \phi(\mathbf{L}) \end{aligned} \quad (2.15)$$

From this constraint we determine the parameter form as

$$A(L) = L^2 \left[\int \frac{d^2 l}{(2\pi)^2} f(\mathbf{l}, \mathbf{l}') F(\mathbf{l}, \mathbf{l}') \right]^{-1}. \quad (2.16)$$

This is exactly the same as the result in [20]. Generally, we can calculate the estimator of $d(\mathbf{L})$ from the definition. However, it's a 2D integration problem. The time complexity is about N^2 . N denotes the number of 1D grid point. When N is very large, it lacks efficiency. We should resort to alternative ways which can speed up our calculations. The idea is to use the Fast Fourier Transformation (FFT) which reduces the time complexity to $N \log N$.

2.2.4 Real-space estimators

With the derived parameters in the previous sections, we can write down the TT estimator which is

$$d(\mathbf{L}) = \frac{A(L)}{L} \int \frac{d^2 l}{(2\pi)^2} T(\mathbf{l}) T(\mathbf{l}') \frac{\tilde{C}_l^{TT} \mathbf{L} \cdot \mathbf{l} + \tilde{C}_{l'}^{TT} \mathbf{L} \cdot \mathbf{l}'}{2C_l^{TT} C_{l'}^{TT}}. \quad (2.17)$$

From this equation, we integrate it over $\mathbf{L}$, and define a scalar map in real space,

$$G(\mathbf{n}) = \int \frac{d^2 L}{(2\pi)^2} e^{i\mathbf{L} \cdot \mathbf{n}} \int \frac{d^2 l}{(2\pi)^2} T(\mathbf{l}) T(\mathbf{l}') \frac{\tilde{C}_l^{TT} \mathbf{L} \cdot \mathbf{l} + \tilde{C}_{l'}^{TT} \mathbf{L} \cdot \mathbf{l}'}{2C_l^{TT} C_{l'}^{TT}}. \quad (2.18)$$

Then we define another map in real space by the relation $\mathbf{G}(\mathbf{n}) = \nabla D(\mathbf{n})$. This gives the map form as

$$\mathbf{D}(\mathbf{n}) = \int \frac{d^2 L}{(2\pi)^2} e^{i\mathbf{L} \cdot \mathbf{n}} \int \frac{d^2 l}{(2\pi)^2} T(\mathbf{l}) T(\mathbf{l}') \frac{\tilde{C}_l^{TT} \mathbf{l} + \tilde{C}_{l'}^{TT} \mathbf{l}'}{2C_l^{TT} C_{l'}^{TT}}. \quad (2.19)$$

This map can be separated into two maps: $\mathbf{D}(\mathbf{n}) = \mathbf{A}(\mathbf{n})B(\mathbf{n})$ with $\mathbf{A}(\mathbf{n}) = \int \frac{d^2 l}{(2\pi)^2} [T(\mathbf{l}) \frac{\tilde{C}_l^{TT}}{C_l^{TT}} e^{i\mathbf{l} \cdot \mathbf{n}}]$ and $B(\mathbf{n}) = \int \frac{d^2 l'}{(2\pi)^2} [T(\mathbf{l}') \frac{1}{C_{l'}^{TT}} e^{i\mathbf{l}' \cdot \mathbf{n}}]$. This method was originally proposed in [19], and [25] used this method to do the temperature reconstruction. The TT estimator now can be re-written as

$$d_{TT}(\mathbf{L}) = \frac{A(L)}{L} \mathbf{L} \mathcal{F}^{-1}[\mathbf{A}(\mathbf{n})B(\mathbf{n})]. \quad (2.20)$$

For the other estimators, the real-space derivations are very similar to the temperature case.

2.3 Lensing effects in the full-sky

For the full-sky case, to account for the curvature of the sphere, we use spherical harmonics to expand any quantity. The full-sky temperature is defined by

$$T(\mathbf{n}) = \sum_{lm} {}_0a_{lm}^{TT} {}_0Y_{lm}, \quad (2.21)$$

the full-sky polarization is

$$(Q + iU)(\mathbf{n}) = \sum_{lm} {}_2a_{lm} {}_2Y_{lm}, \quad (2.22)$$

and the full-sky lensing potential is

$$\phi(\mathbf{n}) = \sum_{lm} {}_0a_{lm}^{\phi\phi} {}_0Y_{lm}. \quad (2.23)$$

The deflection field in the full-sky is just the gradient of the lensing potential,

$$\begin{aligned} \mathbf{d}(\mathbf{n}) &= \nabla \phi(\mathbf{n}) \\ &= \nabla \sum_{lm} {}_0a_{lm}^{\phi\phi} {}_0Y_{lm} \\ &= \sum_{lm} {}_0a_{lm}^{\phi\phi} \nabla {}_0Y_{lm} \\ &= \sum_{lm} {}_0a_{lm}^{\phi\phi} \alpha_{l0} {}_{+1}Y_{lm} \bar{\mathbf{m}} + \sum_{lm} {}_0a_{lm}^{\phi\phi} \beta_{l0} {}_{-1}Y_{lm} \mathbf{m}. \end{aligned} \quad (2.24)$$

This equation is also shown in [26, 27]. In this equation, the orthogonal basis $\mathbf{m}$ and $\bar{\mathbf{m}}$ in the full-sky can be chosen as $\bar{m} = (e_1 - ie_2)/\sqrt{2}$ and $m = (e_1 + ie_2)/\sqrt{2}$, where (e_1, e_2) refers to (e_θ, e_ϕ) . The coefficients in this equation are defined as

$$\alpha_{ls} = -\sqrt{\frac{(l-s)(l+s+1)}{2}} \quad (2.25)$$

and

$$\beta_{ls} = \sqrt{\frac{(l+s)(l-s+1)}{2}}. \quad (2.26)$$

The lensing process is to shift the pixel to another site with rotation if it's polarization. The method has been studied by [28, 27]. Here we just review the procedure described in the appendix of [28]. What the lensing does in the full-sky is symbolically expressed as

$${}_sP(\mathbf{n}') = e^{isR} {}_s\tilde{P}(\mathbf{n}) \quad (2.27)$$

for a spin- s field P . To incorporate the lensing effects, the field P should not only be shifted but also be rotated by an angle R . Temperature field is spin 0, so the rotation is not required. The deflection field has two components and can be described as $\mathbf{d} = \nabla \phi = (d \cos \alpha, d \sin \alpha)$. From this decomposition, we solve the unknowns d , α and R for the new position and the rotation angle. Specifically, on the sphere, the new position $\mathbf{n}' = (\theta', \phi + \Delta\phi)$ is given by

$$\begin{aligned} \cos \theta' &= \cos d \cos \theta - \sin d \sin \theta \cos \alpha \\ \sin \Delta\phi &= \frac{\sin \alpha \sin d}{\sin \theta'}. \end{aligned} \quad (2.28)$$

The entire procedure is given by [28].

2.3.1 Spherical Harmonic Transformation

The spin-weighted harmonic function contains two components

$${}_sY_{lm} = {}_s\lambda_{lm}e^{im\phi}. \quad (2.29)$$

${}_s\lambda_{lm}$ is a real function so it satisfies the relation ${}_s\lambda_{lm}^* = {}_s\lambda_{lm}$. The negative m part is related to the positive part by ${}_s\lambda_{l-m} = (-1)^{m+s}{}_s\lambda_{lm}$. And the south hemisphere is related to north sphere by ${}_s\lambda_{lm}(\pi - \theta) = (-1)^{l+m}{}_s\lambda_{lm}(\theta)$. With these properties, the spin-weighted spherical harmonics possess the properties: ${}_sY_{lm}^* = (-1)^{m+s}{}_sY_{l-m}$ and ${}_sY_{lm}(\pi - \theta, \phi + \pi) = (-1)^l{}_sY_{lm}(\theta, \phi)$. For the full-sky analysis, we can define the polarization modes as [28]

$$\begin{aligned} {}_sa_{lm} &= {}_sE_{lm} + i{}_sB_{lm} \\ {}_{-s}a_{lm} &= (-1)^s[{}_sE_{lm} - i{}_sB_{lm}]. \end{aligned} \quad (2.30)$$

Since we know the complex conjugate relations for ${}_sE_{lm}^* = (-1)^m{}_sE_{l-m}$ and ${}_sB_{lm}^* = (-1)^m{}_sB_{l-m}$, we can obtain the similar relations for the polarization modes as well. They are ${}_sa_{lm}^* = (-1)^m(-1)^s{}_s a_{l-m}$, ${}_sa_{l-m}^* = (-1)^{m+s}{}_s a_{lm}$, and ${}_{-s}a_{l-m}^* = (-1)^{m+s}{}_s a_{lm}$.

From the full-sky definition in Eq. (2.30), the E and B modes can be expressed as

$$\begin{aligned} {}_sE_{lm} &= \frac{1}{2}[{}_sa_{lm} + (-1)^s{}_s a_{lm}] \\ i{}_sB_{lm} &= \frac{1}{2}[{}_sa_{lm} - (-1)^s{}_s a_{lm}]. \end{aligned} \quad (2.31)$$

For the full-sky analysis, the spherical harmonics transformation (SHT) is the basis for many calculations. The backward transformation is

$$\begin{aligned} {}_sP(\theta, \phi) &= \sum_{lm} {}_sa_{lm} {}_s\lambda_{lm}(\cos \theta)e^{im\phi} \\ &= \sum_{m=-l_{max}}^{l_{max}} \left[\sum_l {}_sa_{lm} {}_s\lambda_{lm}(\cos \theta) \right] e^{im\phi}. \end{aligned} \quad (2.32)$$

We can use the symmetry properties derived earlier to make the above calculation more efficient. The idea is to break the summation into four parts like the ones shown in the equation

$$\begin{aligned}
{}_sP(\theta, \phi) &= \left[\sum_{m=0}^{l_{max}} \sum_l {}_s a_{lm} {}_s \lambda_{lm}(\cos \theta) \right. \\
&+ \sum_{m=0}^{l_{max}} \sum_l (-1)^{m+l} {}_s a_{lm} {}_{-s} \lambda_{lm}(\cos \theta) \\
&+ \sum_{m=0}^{l_{max}} \sum_l (-1)^{m+s} {}_s a_{l-m} {}_{-s} \lambda_{lm}(\cos \theta) \\
&\left. + \sum_{m=0}^{l_{max}} \sum_l (-1)^{l+s} {}_s a_{l-m} {}_s \lambda_{lm}(\cos \theta) \right] e^{im\phi}.
\end{aligned} \tag{2.33}$$

The forward transformation is

$$\begin{aligned}
{}_s a_{lm} &= \int \sin \theta d\theta d\phi {}_s P(\theta, \phi) {}_s \lambda_{lm}^*(\cos \theta) e^{-im\phi} \\
&= \int d\theta \sin \theta {}_s \lambda_{lm}^*(\cos \theta) \left[\int d\phi {}_s P(\theta, \phi) e^{-im\phi} \right] \\
&= \int d\theta \sin \theta {}_s \lambda_{lm}^*(\cos \theta) \nabla_m(\theta) \\
&= \sum_{i=0}^{4N_{side}} \Delta\theta_i \sin \theta_i {}_s \lambda_{lm}^*(\cos \theta_i) \nabla_m(\theta_i) \\
&= \sum_{i=0}^{4N_{side}} \Delta\theta_i \sin \theta_i {}_s \lambda_{lm}(\cos \theta_i) \nabla_m(\theta_i) \\
&= (2\pi) \sum_{i=0}^{4N_{side}} \Delta z_i {}_s \lambda_{lm}(z_i) \nabla_m(z_i).
\end{aligned} \tag{2.34}$$

In Eqs. (2.33, 2.34), the quantity ${}_s \lambda_{lm}$ should satisfy the orthogonality relation given by

$$\frac{1}{2\pi} \delta_{ll'} = \int_{-1}^{+1} dz {}_s \lambda_{lm}(z) {}_s \lambda_{l'm'}(z), \tag{2.35}$$

and this quantity in principle can be directly calculated from its definition, however, the brute-force of this calculation is not computationally efficient. Rather than

doing this directly, we use recursive method [29] to compute this coefficient. In [30], there is another implementation of the SHT.

Chapter 3

Reconstruction of Gravitational Lensing Using WMAP 7-Year Data

Chapter origin: **Chang Feng**, Brian Keating, Hans P. Paar, Oliver Zahn.
Phys. Rev. D **85**, 043513 (2012), arXiv:1111.2371 [1].

Gravitational lensing by large scale structure introduces non-Gaussianity into the Cosmic Microwave Background and imprints a new observable, which can be used as a cosmological probe. We apply a four-point estimator to the Wilkinson Microwave Anisotropy Probe (WMAP) 7-year coadded temperature maps alone to reconstruct the gravitational lensing signal. The Gaussian bias is simulated and subtracted, and the higher order bias is investigated. We measure a gravitational lensing signal with a statistical amplitude of $\mathcal{C} = 1.27 \pm 0.98$ using all the correlations of the W- and V-band Differencing Assemblies (DAs). We therefore conclude that WMAP 7-year data alone, can not detect lensing.

3.1 Introduction

Gravitational lensing of the Cosmic Microwave Background (CMB) provides information on the mass distribution between the surface of last scattering

and the observer, thus potentially providing information, for example, on dark energy and neutrino masses. In addition, gravitational lensing causes E -modes to be converted into large angular scale B -modes, thereby potentially contaminating B -mode signature of inflationary gravitational waves [31]. Because lensing deflects CMB photons by approximately $3'$, a perturbative treatment to first order is generally valid. An estimator for the deflection angle has been devised by Hu [26, 19].

The first attempt to detect lensing by Hirata et al. [32] used the cross-correlation between the WMAP 1-year data and selected luminous red galaxies (LRGs) from the Sloan Digital Sky Survey (SDSS). No statistically significant signal was found. The first detection of lensing was performed by Smith et al. [33] who used the cross-correlation between the NRAO VLA Sky Survey (NVSS) of radio galaxies with a higher mean redshift than the Sloan LRGs and a fully-optimal lensing estimator on the statistically more powerful WMAP 3-year data. Evidence for lensing was found at the 3.4σ level. Using a similar estimator as in [32], Hirata et al. [34] obtained results consistent with, though at slightly lower significance than [33], using WMAP 3-year data, LRGs and quasars from the SDSS data, as well as data from the NVSS. Recently, Smidt et al. [35] used an estimator based upon the kurtosis of the CMB temperature four-point correlation function to estimate lensing from WMAP 7-year data only and claimed evidence for lensing at the 2σ level. Recently, the Atacama Cosmology Telescope (ACT) collaboration successfully detected gravitational lensing [36] at the 4σ level. The South Pole Telescope (SPT) detected the effects of gravitational lensing on the angular power spectrum [25].

In this paper we present a search for gravitational lensing using the WMAP 7-year data alone and the standard optimal quadratic estimator [26, 19] which differs from the kurtosis estimator of [35]. We apply the quadratic estimator to WMAP-7 temperature maps alone for the first time in the hopes that our analysis might serve as a touchstone allowing for consistent comparison between different lensing extraction techniques. We review the notation for full-sky reconstruction of gravitational lensing in Section 3.2. We discuss the sky-cut used in our analysis

in Section 3.3. Then we introduce our modified estimator in Section 3.4 making use of the optimal quadratic estimator of [26]. We introduce the WMAP 7-year data in Section 3.5, and describe the details of the calculations, including the noise model, and analysis in Section 3.6. Results of a null test are shown in Section 3.7, and we discuss the conclusions of our work in Section 3.8.

3.2 Gravitational Lensing

The effect of lensing on the CMB's primordial temperature $\tilde{T}$ in direction $\mathbf{n}$ can be represented by

$$T(\mathbf{n}) = \tilde{T}(\mathbf{n} + \mathbf{d}(\mathbf{n})), \quad (3.1)$$

where T is the lensed temperature and $\mathbf{d}(\mathbf{n}) = \nabla\phi$, with ϕ being the lensing potential. The two-point correlation function of the temperature field following [21], is:

$$\begin{aligned} \langle T_{lm} T_{l'm'} \rangle &= \tilde{C}_l^{TT} \delta_{ll'} \delta_{m-m'} (-1)^m + \\ &\sum_{LM} (-1)^M \begin{pmatrix} l & l' & L \\ m & m' & -M \end{pmatrix} f_{lLl'}^{TT} \phi_{LM}, \end{aligned} \quad (3.2)$$

where the second term encodes the effects of lensing with the weighting factor $f_{lLl'}^{TT}$ given by

$$f_{lLl'}^{TT} = \tilde{C}_l^{TT} {}_0F_{l'L} + \tilde{C}_{l'}^{TT} {}_0F_{lLl'}. \quad (3.3)$$

Here $\tilde{C}_l^{TT}$ are the unlensed temperature power spectra. and

$$\begin{aligned} {}_0F_{lLl'} &= \sqrt{\frac{(2l+1)(2l'+1)(2L+1)}{4\pi}} \times \\ &\frac{1}{2} [L(L+1) + l'(l'+1) - l(l+1)] \begin{pmatrix} l & L & l' \\ 0 & 0 & 0 \end{pmatrix}. \end{aligned} \quad (3.4)$$

The lensing estimator is constructed from an average over a pair of two-

point correlations [26, 19] and has the form

$$d_{LM}^{TT} = \frac{A_L^{TT}}{\sqrt{L(L+1)}} \times \sum_{l'l'mm'} (-1)^M g_{l'l}^{TT}(L) \begin{pmatrix} l' & l & L \\ m' & m & -M \end{pmatrix} T_{l'm'} T_{lm}. \quad (3.5)$$

The requirement that the estimator in Eq. (3.5) is unbiased and has minimal variance results in

$$A_L^{TT} = L(L+1)(2L+1) \left[\sum g_{ll'}^{TT}(L) f_{lLl'}^{TT} \right]^{-1} \quad (3.6)$$

and

$$g_{ll'}^{TT}(L) = \frac{f_{lLl'}^{TT}}{2C_l^{\text{tot}} C_{l'}^{\text{tot}}}, \quad (3.7)$$

with $C_l^{\text{tot}} = C_l^{TT} + N_l^{TT}$, where C_l^{TT} are the lensed power spectra and N_l^{TT} is the instrumental noise. In the following, the summations are from l and $l' = 0$ to 750 and $|m| \leq l$, $|m'| \leq l'$. The WMAP 7-year data do not contain additional information at higher multipoles.

To reduce computation time we follow [21] and define three maps for the TT estimator:

$${}_0A^T(\mathbf{n}) = \sum_{lm} \frac{1}{C_l^{\text{tot}}} T_{lm} {}_0Y_{lm}(\mathbf{n}), \quad (3.8)$$

$$X(\mathbf{n}) = \sum_{lm} \frac{\tilde{C}_l^{TT}}{C_l^{\text{tot}}} T_{lm} \alpha_{l0+1} Y_{lm}(\mathbf{n}), \quad (3.9)$$

$$Y(\mathbf{n}) = \sum_{lm} \frac{\tilde{C}_l^{TT}}{C_l^{\text{tot}}} T_{lm} \beta_{l0-1} Y_{lm}(\mathbf{n}), \quad (3.10)$$

and take the inverse Spherical Harmonic Transform (SHT) of ${}_0A^T X$ and ${}_0A^T Y$ to get

$$\Upsilon_{LM}^{(1)} = \beta_{L0} \int d\mathbf{n} {}_{+1}Y_{LM}^* {}_0A^T X \quad (3.11)$$

$$\Upsilon_{LM}^{(2)} = \alpha_{L0} \int d\mathbf{n} {}_{-1}Y_{LM}^* {}_0A^T Y \quad (3.12)$$

with

$$\alpha_{ls} = -\sqrt{\frac{(l-s)(l+s+1)}{2}} \quad (3.13)$$

$$\beta_{ls} = \sqrt{\frac{(l+s)(l-s+1)}{2}}. \quad (3.14)$$

Using Eqs. (3.8), (3.9) and (3.10) the expression for d_{LM}^{TT} in Eq. (3.5) becomes

$$d_{LM}^{TT} = \frac{A_L^{TT}}{\sqrt{L(L+1)}} [\Upsilon_{LM}^{(1)} + \Upsilon_{LM}^{(2)}]. \quad (3.15)$$

A similar procedure is followed for the efficient calculation of A_L^{TT} in Eq. (3.6). The resulting expression is given in [37] (originally proposed in [38]):

$$\begin{aligned} A_l^{TT} = & \int_{+1}^{-1} \left[\left[\xi_{00}^T(\theta) \xi_{11}^T(\theta) - \xi_{01}^T(\theta) \xi_{01}^T(\theta) \right] d_{-1-1}^l(\theta) \right. \\ & \left. + \left[\xi_{00}^T(\theta) \xi_{1-1}^T(\theta) - \xi_{01}^T(\theta) \xi_{0-1}^T(\theta) \right] d_{1-1}^l(\theta) \right] d(\cos \theta) \end{aligned} \quad (3.16)$$

with the ξ^T given by

$$\xi_{00}^T(\theta) = \sum_l \frac{2l+1}{4\pi} \frac{1}{C_l^{TT} + N_l^{TT}} d_{00}^l(\theta), \quad (3.17)$$

$$\xi_{0\pm 1}^T(\theta) = \sum_l \frac{2l+1}{4\pi} \sqrt{l(l+1)} \frac{\tilde{C}_l^{TT}}{C_l^{TT} + N_l^{TT}} d_{0\pm 1}^l(\theta), \quad (3.18)$$

$$\xi_{1\pm 1}^T(\theta) = \sum_l \frac{2l+1}{4\pi} l(l+1) \frac{(\tilde{C}_l^{TT})^2}{C_l^{TT} + N_l^{TT}} d_{1\pm 1}^l(\theta), \quad (3.19)$$

here $d_{ss'}^l(\theta)$ are Wigner d-functions.

3.3 Sky Cut

In order to eliminate contaminated data, regions such as the galactic plane and bright point sources in the full-sky map must be removed using a mask, thereby introducing a sky-cut. For example, in [34], the Kp2 mask was used to make 84.7% of the sky uncontaminated. In [35], the more conservative KQ75 mask was used to clean artifacts around the galactic plane and point sources.

The sky-cut can be removed as a separate component to get a full-sky map before we process the data. One such technique is the ‘‘inpainting’’ method in

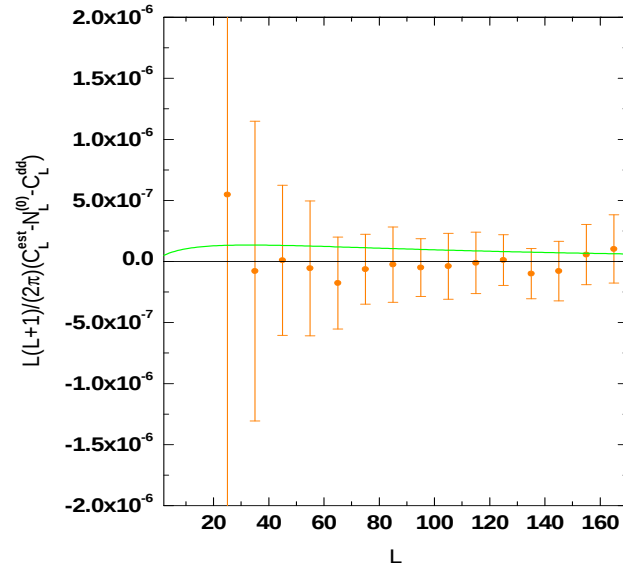


Figure 3.1: The higher order bias calculated from $(C_L^{est} - N_L^{(0)}) - C_L^{dd}$ for all correlations of the WMAP's W- and V-band DAs. The simulated higher order bias from averaging 700 (to be discussed in Figure 3.10) realizations is shown in orange. For comparison, the simulated lensing signal is shown in green.

which the estimated values of pixels in the map are substituted for those removed by the mask. Perotto et al. have simulated the full sky reconstruction for PLANCK [39]. The full-sky map recovered in this way will bias the lensing reconstruction slightly.

Another method proposed by A. Benoit-Levy [40] apodizes the masked regions of the map and inpaints the masked regions of the map by constrained Gaussian random values of the unlensed temperature. In this way, the sky-cut-induced coupling approximately reduces to a unit matrix. However, for WMAP, we have to remove a big portion of the sky, reducing f_{sky} dramatically to 0.3. The unbiased estimator could be scaled up by a factor of $1/f_{\text{sky}}$, but the signal-to-noise ratio would be reduced significantly. This means the uncertainty of the reconstructed signal would be larger.

As opposed to a separate-component solution, we obtain an all-inclusive lensing reconstruction pipeline, using the built-in filter of the estimator to treat the data without pre-conditioning it. The optimal estimator for the potential based on the maximum likelihood is derived by Hirata [22]. The full inverse variance $(\mathbf{C} + \mathbf{N})^{-1}$, instead of $(C_L^{TT} + N_L^{TT})^{-1}$, was used by [33] because it is an optimal filter when there are sky-cuts and inhomogeneous noise. The sky-cut generates artifacts in harmonic space, as does lensing. $(\mathbf{C} + \mathbf{N})^{-1}$ can be used to filter those modes affected by the sky-cut. However, we do not use this filter because the inversion of $(\mathbf{C} + \mathbf{N})$ is computationally challenging [33]; instead we use the estimator Eq. (3.5) which is identical to the one of [34], and it is an excellent approximation to the maximum likelihood estimator. We note that, while $(C_L^{TT} + N_L^{TT})^{-1}$ will be suboptimal to a full $(\mathbf{C} + \mathbf{N})^{-1}$ filter, it preserves the simplicity and efficiency of the lensing reconstruction procedure.

3.4 The lensing estimator

For WMAP, we modify the estimator slightly to deal with the instruments' anisotropic temperature noise.

The observed lensed temperature map $\mathbb{T}$ is given by

$$\mathbb{T}(\mathbf{n}) = M(\mathbf{n}) \left[\int d\mathbf{n}' T(\mathbf{n}') B(\mathbf{n}, \mathbf{n}') + N(\mathbf{n}) \right] \quad (3.20)$$

and likewise the “observed” unlensed temperature map $\tilde{\mathbb{T}}$ is

$$\tilde{\mathbb{T}}(\mathbf{n}) = M(\mathbf{n}) \left[\int d\mathbf{n}' \tilde{T}(\mathbf{n}') B(\mathbf{n}, \mathbf{n}') + N(\mathbf{n}) \right] \quad (3.21)$$

Here $M(\mathbf{n})$ represents the mask, $B(\mathbf{n}, \mathbf{n}')$ the beam, and $N(\mathbf{n})$ the noise.

For a pair of maps α and β , “ $TT(\alpha \times \beta)$ ” denotes the cross-correlation between these two temperature maps. A harmonic mode of the reconstruction including noise is estimated as

$$\begin{aligned} \mathbb{d}_{LM}^{TT(\alpha \times \beta)} &= \frac{A_L^{TT(\alpha \times \beta)}}{\sqrt{L(L+1)}} \sum_{l'm'm'} (-1)^M f_{lLl'}^{TT} \begin{pmatrix} l' & l & L \\ m' & m & -M \end{pmatrix} \\ &\times \frac{\mathbb{T}_{l'm'}^{(\alpha)} \mathbb{T}_{lm}^{(\beta)}}{\mathbb{C}_{l'}^{(\alpha)} \mathbb{C}_l^{(\beta)}} \end{aligned} \quad (3.22)$$

following Eq. (3.5), and a harmonic mode of the Gaussian bias is estimated as

$$\begin{aligned} \mathbb{N}_{LM}^{TT(\alpha \times \beta)} &= \frac{A_L^{TT(\alpha \times \beta)}}{\sqrt{L(L+1)}} \sum_{l'm'm'} (-1)^M f_{lLl'}^{TT} \begin{pmatrix} l' & l & L \\ m' & m & -M \end{pmatrix} \\ &\times \frac{\tilde{\mathbb{T}}_{l'm'}^{(\alpha)} \tilde{\mathbb{T}}_{lm}^{(\beta)}}{\mathbb{C}_{l'}^{(\alpha)} \mathbb{C}_l^{(\beta)}}. \end{aligned} \quad (3.23)$$

Here $\mathbb{C}$ are the power spectra of the observed lensed temperature, determined from $\langle \mathbb{T}_{lm} \mathbb{T}_{l'm'} \rangle$. As was done in [36] and [41] we use the same power spectra in Eq. (3.22) and Eq. (3.23). In order to deal with the non-uniform noise distribution in the WMAP data, we symmetrize $\mathbb{d}_{LM}^{TT(\alpha \times \beta)}$ as in [34], denoting the symmetrized cross-correlation “ $TT(\alpha \bullet \beta)$ ” between these two temperature maps,

$$\mathbb{d}_{LM}^{TT(\alpha \bullet \beta)} = \frac{\mathbb{d}_{LM}^{TT(\alpha \times \beta)} + \mathbb{d}_{LM}^{TT(\beta \times \alpha)}}{2} \quad (3.24)$$

and

$$\mathbb{N}_{LM}^{TT(\alpha \bullet \beta)} = \frac{\mathbb{N}_{LM}^{TT(\alpha \times \beta)} + \mathbb{N}_{LM}^{TT(\beta \times \alpha)}}{2}. \quad (3.25)$$

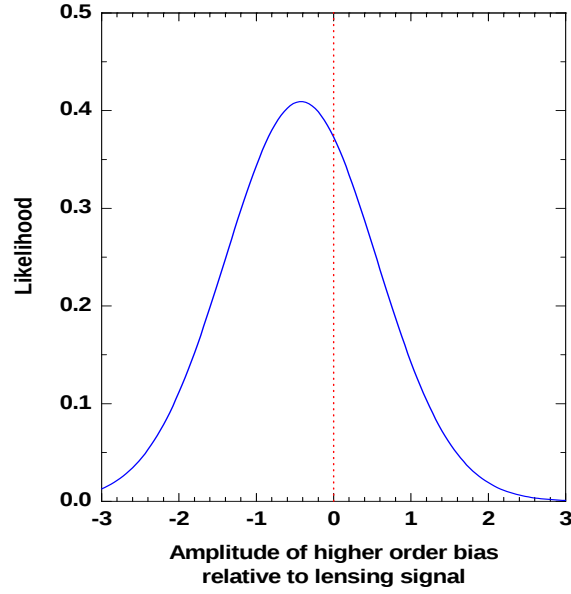


Figure 3.2: The normalized likelihood of the amplitude of the higher order bias limited to the region $20 < L < 170$, to the simulated lensing signal. This confirms that the higher order bias is consistent with zero and negligible.

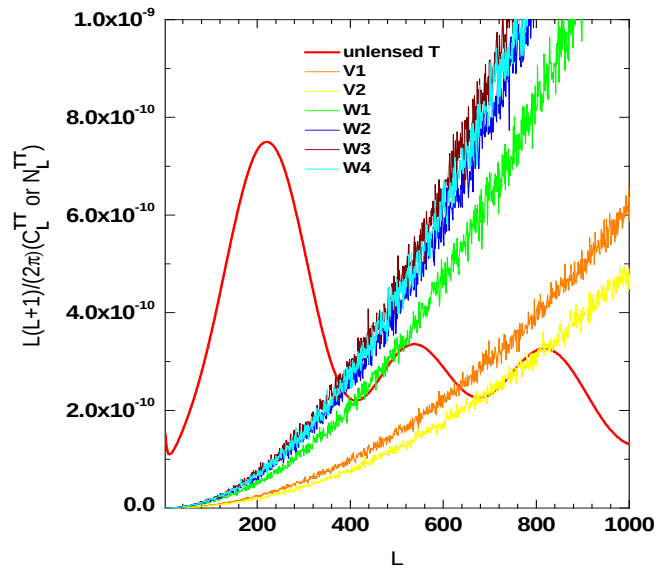


Figure 3.3: WMAP noise for each DA and the TT power spectrum as a function of L .

We refer to $C_L^{est} = \langle d_{LM}^* d_{LM} \rangle$ as the reconstruction including noise, and $N_L^{(0)} = \langle N_{LM}^* N_{LM} \rangle$ as the Gaussian bias, with the superscript “ $TT(\alpha \bullet \beta)$ ” omitted. Thus we obtain

$$\begin{aligned} \mathbb{d}_{LM}^{TT(\alpha \bullet \beta)} &= \frac{1}{2} \left\{ \frac{A_L^{TT(\alpha \times \beta)}}{\sqrt{L(L+1)}} [\beta_{L0} \int d\mathbf{n}_{+1} Y_{LM\ 0}^* A^{T(\alpha)} X^{(\beta)} \right. \\ &+ \alpha_{L0} \int d\mathbf{n}_{-1} Y_{LM\ 0}^* A^{T(\alpha)} Y^{(\beta)}] \\ &+ \frac{A_L^{TT(\beta \times \alpha)}}{\sqrt{L(L+1)}} [\beta_{L0} \int d\mathbf{n}_{+1} Y_{LM\ 0}^* A^{T(\beta)} X^{(\alpha)} \\ &+ \alpha_{L0} \int d\mathbf{n}_{-1} Y_{LM\ 0}^* A^{T(\beta)} Y^{(\alpha)}] \Big\}, \end{aligned} \quad (3.26)$$

$$\begin{aligned} \mathbb{A}_L^{TT(\alpha \times \beta)} &= \int_{+1}^{-1} d(\cos \theta) [(\xi_{00}^{T(\alpha)}(\theta) \xi_{11}^{T(\beta)}(\theta) - \xi_{01}^{T(\alpha)}(\theta) \xi_{01}^{T(\beta)}(\theta)) d_{-1-1}^L(\theta) \\ &+ (\xi_{00}^{T(\alpha)}(\theta) \xi_{1-1}^{T(\beta)}(\theta) - \xi_{01}^{T(\alpha)}(\theta) \xi_{0-1}^{T(\beta)}(\theta)) d_{1-1}^L(\theta)], \end{aligned} \quad (3.27)$$

following a reasoning similar to the one near the end of Section 3.2.

The two-point correlation of the Gaussian bias estimator is essentially a four-point correlation function of the primordial temperature modes. It should be carefully subtracted since, for a noise-dominated experiment such as WMAP, the Gaussian four-point bias is several orders of magnitude larger than the lensing power spectra. In [36] phase-randomized data maps are used to simulate this Gaussian bias. However, this approach does not work for the present lensing reconstruction since WMAP’s noise is not isotropic. Evidence for this can be seen from the normalization factor $\mathbb{A}_L^{TT(\alpha \times \beta)}$ which is not equal to $N_L^{(0)TT(\alpha \bullet \beta)}$ whereas they should be equal for isotropic noise [39]. The normalization factor Eq. (3.27) only contains the partial contribution coming from the non-isotropic noise while the Gaussian bias squared from Eq. (3.25) consists of all the correlations generated by the non-isotropic noise, see [42] and [43]. If the phases of the WMAP temperature maps are randomized in order to remove the lensing-induced coupling between modes, it will also remove the strong correlation of the noise. The Gaussian bias calculated in this way will be significantly lower than that from the standard

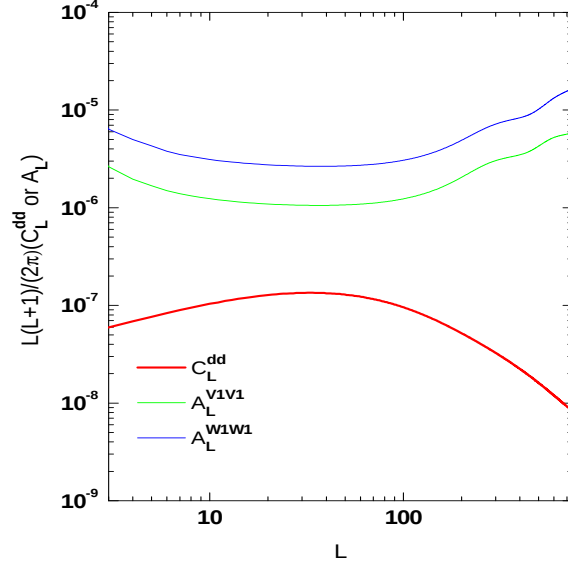


Figure 3.4: Comparison of $\mathbb{A}_L$ (Eq. (3.27)) and the expected lensing signal as function of L . The estimator noise is about two orders of magnitude higher than the signal C_L^{dd} , indicating the difficulty of detecting lensing from WMAP-7 data alone.

approach [44]. So we have to perform simulations which use the simulated WMAP noise and temperature maps, rather than the randomized WMAP data to get the Gaussian bias term.

The deflection power spectrum is

$$C_L^{dd} = \langle [\mathbb{d}_{LM}^{TT(\alpha\bullet\beta)}]^* \mathbb{d}_{LM}^{TT(\alpha\bullet\beta)} - [\mathbb{N}_{LM}^{TT(\alpha\bullet\beta)}]^* \mathbb{N}_{LM}^{TT(\alpha\bullet\beta)} \rangle. \quad (3.28)$$

This estimator is essentially the same as in [36] except that here it is the full-sky version and the noise $\mathbb{N}_{LM}$ is not obtained from the phase-randomized data. We subtract the Gaussian bias for each realization of the estimator, and all the estimated power spectra are averaged to get the binned power spectra $\langle C_b^{dd} \rangle$ for the b -th bin [41]. The averaged power spectrum in a range of L labeled by the index b is

$$C_b^{dd} = \sum_{L \in b} \frac{L(L+1)}{b(b+1)} C_L^{dd}. \quad (3.29)$$

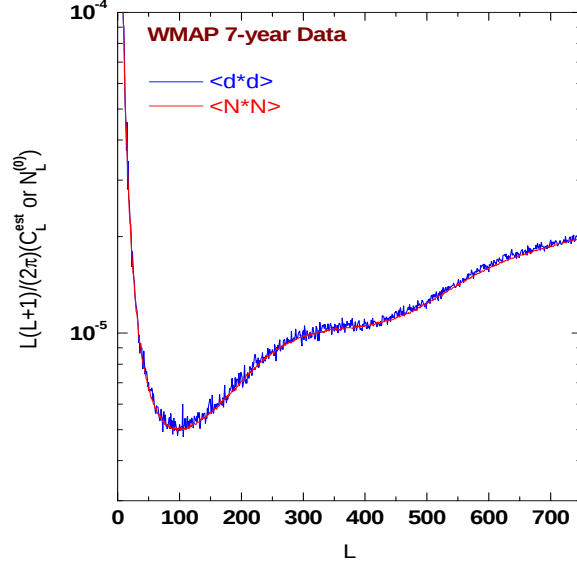


Figure 3.5: The averaged reconstruction including noise (C_L^{est}) (blue) of *WMAP data* and the Gaussian bias $N_L^{(0)}$ (red) from 700 realizations. Since lensing is approximately 100 times smaller than C_L^{est} , the two curves are almost indistinguishable; however, this confirms the precision of the noise model.

The statistical uncertainty is given by $\sigma_b = [\langle (C_b - \bar{C}_b)^2 \rangle]^{1/2}$. After the subtraction of the Gaussian bias, there remains the higher order biases, see [36] (where it was called “null bias”), [41], and [45].

We expand T in harmonic space as

$$T_{LM} = \tilde{T}_{LM} + \delta T_{LM} + \delta^2 T_{LM} + \delta^3 T_{LM} + \dots, \quad (3.30)$$

see [46]. Here the power n in δ^n denotes the order in ϕ^n . We expand the noise bias as

$$N_L = N_L^{(0)} + N_L^{(1)} + N_L^{(2)} + \dots, \quad (3.31)$$

where the index n in $N^{(n)}$ denotes the order of its dependence upon $[\phi^2]^n$, excluding terms that contribute to the lensed power spectrum. The four-point function $\langle \mathfrak{d}_{LM}^* \mathfrak{d}_{LM} \rangle$ contains terms of different order in $\delta^n T$. A term of the type $\langle \delta T \delta T \tilde{T} \tilde{T} \rangle$ contributes to C_L^{dd} and the first order noise $N_L^{(1)}$ while terms of the

type $\langle \delta T \delta T \delta T \delta T \rangle$, $\langle \delta^2 T \delta^2 T \tilde{T} \tilde{T} \rangle$, $\langle \delta^2 T \delta T \delta T \tilde{T} \rangle$, and $\langle \delta^3 T \delta T \tilde{T} \tilde{T} \rangle$ generate the second order noise $N_L^{(2)}$. Following [46], the higher order bias term is calculated as the difference between the estimated power spectrum and the sum of its prediction and the lowest order noise (i.e., Gaussian bias): $C_L^{est} - (C_L^{dd} + N_L^{(0)})$, using Monte Carlo simulations.

We study the statistical significance of the detection as follows. Following [34], the reconstructed power spectra $C^{(obs)}$ are compared with their theoretical prior $C^{(th)}$ by minimizing a χ^2 defined as

$$\chi^2(\mathcal{C}) = \sum_{AB} (C_A^{(obs)} - \mathcal{C} C_A^{(th)}) \mathbf{C}_{AB}^{-1} (C_B^{(obs)} - \mathcal{C} C_B^{(th)}) \quad (3.32)$$

and varying $\mathcal{C}$. Here A or B label the range in L , and C_A or C_B is the band-power. The covariance matrix $\mathbf{C}$ is calculated from the Monte Carlo simulation as $\mathbf{C}_{AB} = \langle (C_A^{(sim)} - \bar{C}_A^{(sim)})(C_B^{(sim)} - \bar{C}_B^{(sim)}) \rangle$. The best fit $\mathcal{C}$ is obtained by setting the derivative of χ^2 to zero:

$$\mathcal{C} = \frac{\sum_{AB} C_A^{(th)} \mathbf{C}_{AB}^{-1} C_B^{(obs)}}{\sum_{AB} C_A^{(th)} \mathbf{C}_{AB}^{-1} C_B^{(th)}}. \quad (3.33)$$

A non-zero value of $\mathcal{C}$ indicates the presence of lensing. The significance of a non-zero value can be judged if its variance is known. The variance of $\mathcal{C}$ is given by

$$(\Delta \mathcal{C})^2 = \frac{1}{\sum_{AB} C_A^{(th)} \mathbf{C}_{AB}^{-1} C_B^{(th)}} \quad (3.34)$$

and the significance of the detection of lensing is $\mathcal{C}/\Delta \mathcal{C}$.

We show the higher order bias in Figure 3.1. The higher order bias $N_L^{(1)} + N_L^{(2)} + \dots$ is seen to be negative for $L < 20$ and positive for $L > 170$ and consistent with zero for $20 < L < 170$ where the amplitude is -0.42 ± 0.98 (0.43σ), compared to the simulated lensing signal C_L^{dd} by using 15 bins with $\Delta L = 10$ starting from $L = 20$. In Figure 3.2, the likelihood of the amplitude of the higher order bias limited to the region $20 < L < 170$ confirms that the bias is consistent with zero. Thus subtraction of the higher order bias is not required as long as we limit L to this region.

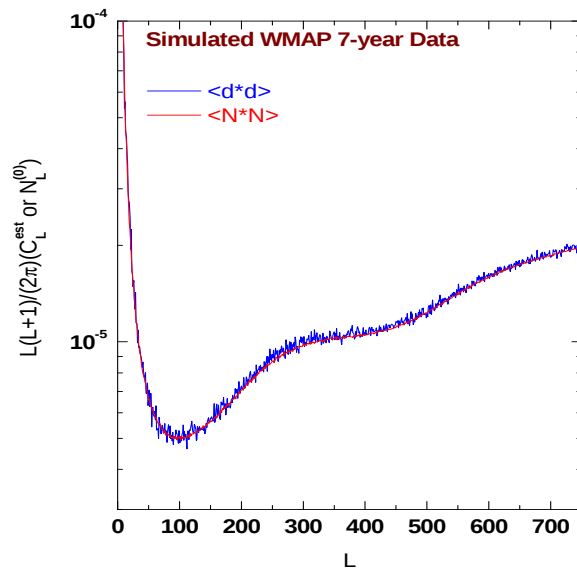


Figure 3.6: The averaged reconstruction including noise (C_L^{est}) (blue) of *simulated WMAP data* and the Gaussian bias $N_L^{(0)}$ (red) from 700 realizations. Since lensing is approximately 100 times smaller than C_L^{est} , the two curves are almost indistinguishable; however, this confirms the precision of the noise model.

Table 3.1: Measurements of lensing $\mathcal{C}$ and its significance $\mathcal{C}/\Delta\mathcal{C}$.

Data set	$\mathcal{C}$	$\mathcal{C}/\Delta\mathcal{C}$
WMAP-7 ALL ¹	1.27 ± 0.98	1.30σ
WMAP-7 V+W ²	0.97 ± 0.47	2.06σ
WMAP-1 ALL $\times$ LRGs ³	1.0 ± 1.1	0.91σ
WMAP-3 ALL $\times$ (LRGs+QSOs+NVSS) ⁴	1.06 ± 0.42	2.52σ
WMAP-3 (Q+V+W) $\times$ NVSS ⁵	1.15 ± 0.34	3.38σ
ACT ⁶	1.16 ± 0.29	4.00σ
SPT ⁷	-	$\sim 4.90\sigma$

3.5 WMAP 7-year Data

The lensing reconstruction depends most sensitively on the high- L modes which are supplied by WMAP’s DAs in the V (2 DAs) and W (4 DAs) frequency bands. Thus we use WMAP’s coadded temperature maps with r9 resolution (Healpix’s $n_{side} = 512$) using all possible distinct pairings: three auto-correlations for the two V-band DAs, ten auto-correlations for the four W-band DAs, and eight cross-correlations between the W- and V-band DAs for a total of 21 correlations (labeled “ALL”). Smith et al. [33], used the Q-band DAs in addition to the W- and V-band DAs of WMAP 3-year temperature maps. Hirata et al. [34], used 153 one-year DAs from the WMAP 3-year data in the W- and V-bands. Recently, Smidt et al. [35] used the W- and V-frequency bands of the WMAP 7-year data. This work adopts six DAs of WMAP’s 7-year temperature map, making the data selection slightly different from other work, although the same signal-to-noise is expected. The WMAP temperature maps contain very high levels of noise as shown in Figure 3.3. The normalization factor $\mathbb{A}_L$ shown in Figure 3.4 is about two orders of magnitude higher than the signal C_L^{dd} ; indicative of the difficulty of extracting the lensing from the noisy data. We calculate the noise in each band from WMAP’s data instead of using an analytical form as [32] and [34] do. The noise is simulated according to the prescription in [44], and the beam transfer functions are supplied by WMAP.

Table 3.2: Summary of $\mathcal{C}$ and its significance $\mathcal{C}/\Delta\mathcal{C}$ for this work.

Type	$\mathcal{C}$	$\mathcal{C}/\Delta\mathcal{C}$
higher order bias	-0.42 ± 0.98	0.43σ
curl null test	0.38 ± 0.79	0.47σ
reconstructed lensing	1.27 ± 0.98	1.30σ

3.6 Simulation and Analysis

We use the CAMB code [5] to obtain the power spectra $\tilde{C}_l^{TT}$, and $C_l^{\phi\phi}$ using a six parameter Λ CDM model with $\mathbf{P} = \omega_b h^2, \omega_c h^2, h, \tau, A_s, n_s = 0.0226, 0.112, 0.70, 0.09, 2.1 \times 10^{-9}, 0.96$. These are input into a pipeline that has elements as follows.

Gaussian maps of the deflection angle field $\mathbf{d}(\mathbf{n})$ and unlensed temperature $\tilde{T}(\mathbf{n})$ are generated using their respective power spectra. We use $\tilde{C}_l^{TT}$, and $C_l^{\phi\phi}$ to create one realization of the simulated deflection field and lensed temperature maps $T(\mathbf{n})$ are generated using Eq. (4.1) with the $\tilde{T}(\mathbf{n})$ and $\mathbf{d}(\mathbf{n})$ found above.

Using the inverse SHT, the temperature maps are converted into harmonic modes a_{lm} which are convolved with the beam transfer functions b_l . Using SHT, these are transformed into configuration space and WMAP-based noise is added. We mask the galactic plane and point sources using WMAP’s KQ75 mask. Using inverse SHT, the resulting maps are transformed back into harmonic space where new a_{lm} are kept up to $l_{max} = 750$ and $|m_{max}| = 750$.

The noise simulation is crucial to this work because the $N_L^{(0)}$ is one hundred times larger than C_L^{dd} . The six V- and W-band DAs, labeled by $\alpha = V1, V2, W1, W2, W3, W4$, have different noise variances, different beam transfer functions, and different relative phases. To mimic the WMAP DAs, we simulate the Gaussian

¹All 21 correlations of WMAP-7’s W- and V-band DAs in this work.

²[35] WMAP-7 V and W bands.

³[32] WMAP-1 W- and V-band DAs, LRGs.

⁴[34] WMAP-3 W- and V-band DAs, LRGs, QSOs and NVSS.

⁵[47] WMAP-3 Q-, V-, W-band DAs, NVSS.

⁶[36] ACT temperature maps.

⁷[25] SPT temperature maps.

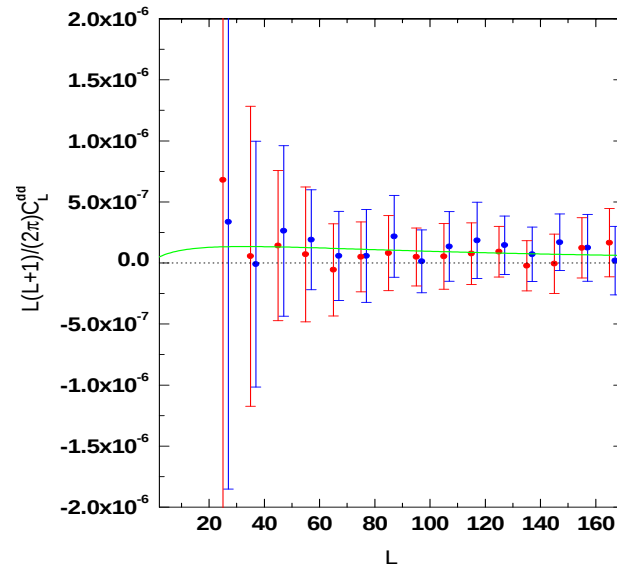


Figure 3.7: The reconstructed power spectra (C_L^{dd}) of the deflection angle field from all correlations of WMAP's W- and V-band DAs. The green curve is the simulated lensing signal, and the data points are the reconstructed lensing signal from simulations (red), and the reconstructed lensing signal from data (blue). The red and blue data points show the consistency between the simulated and real WMAP data for the lensing reconstruction.

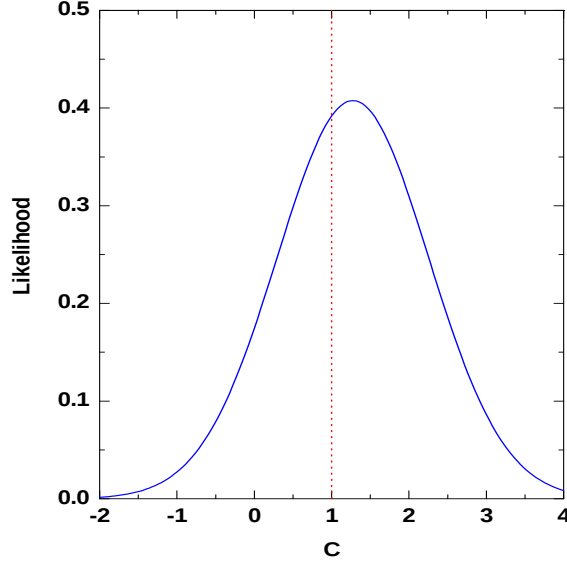


Figure 3.8: The normalized likelihood distribution for $\mathcal{C}$ for all 21 correlations of WMAP’s W- and V-band DAs.

bias as follows. Using

$$\begin{aligned} \tilde{\mathbb{T}}^{(i)\alpha}(\mathbf{n}) &= M(\mathbf{n}) \left[\int d\mathbf{n}' \tilde{T}^{(i)}(\mathbf{n}') B^{\alpha}(\mathbf{n}, \mathbf{n}') \right. \\ &\quad \left. + N^{(i)\alpha}(\mathbf{n}) \right], \end{aligned} \quad (3.35)$$

we set the index i (an arbitrary running index) for Eq. (3.35) and generate an unlensed temperature map $\tilde{T}^{(i)}(\mathbf{n})$, and six noise maps $N^{(i)\alpha}(\mathbf{n})$, $\alpha = V1, V2, W1, W2, W3, W4$. Then we make an observed map $\tilde{\mathbb{T}}^{(i)\alpha}(\mathbf{n})$ using Eq. (3.35), and repeat this procedure to make another observed map $\tilde{\mathbb{T}}^{(i)\beta}(\mathbf{n})$. Subsequently, we calculate the Gaussian bias $N_L^{(0)}$ using Eq. (3.25) for the pair $(\alpha \bullet \beta)$. In the same way, we generate 21 realizations for all the correlations. Finally we increase the index i , and repeat the whole procedure until the ensemble $\{N_L^{(0)}\}$ has 700 elements.

We proceed in a similar manner simulating the reconstruction including

noise, except setting $T^{(i)}(\mathbf{n}) = T(\mathbf{n})$ and $N^{(i)\alpha}(\mathbf{n}) = N^\alpha(\mathbf{n})$. Using

$$\begin{aligned} \mathbb{T}^{(i)\alpha}(\mathbf{n}) &= M(\mathbf{n}) \left[\int d\mathbf{n}' T^{(i)}(\mathbf{n}') B^\alpha(\mathbf{n}, \mathbf{n}') \right. \\ &\quad \left. + N^{(i)\alpha}(\mathbf{n}) \right], \end{aligned} \quad (3.36)$$

we set the index i for Eq. (3.36), and generate a lensed temperature map $T^{(i)}(\mathbf{n})$, and six noise maps $N^{(i)\alpha}(\mathbf{n})$, $\alpha = V1, V2, W1, W2, W3, W4$. Then we make an observed map $\mathbb{T}^{(i)\alpha}(\mathbf{n})$ using Eq. (3.36) and repeat this procedure to make another observed map $\mathbb{T}^{(i)\beta}(\mathbf{n})$. Subsequently we calculate the reconstruction including noise C_L^{est} using Eq. (3.24) for the pair $(\alpha \bullet \beta)$. In the same way, we generate 21 realizations for all the correlations. Finally we increase the index i , and repeat the whole procedure until the ensemble $\{C_L^{est}\}$ has 700 elements. Eq. (3.28) is then used to obtain the deflection power spectrum C_L^{dd} .

We show the reconstruction including noise C_L^{est} and the Gaussian bias $N_L^{(0)}$ in Figures 3.5, and 3.6 for the real and the simulated WMAP data, respectively. The simulation is consistent with the data, and we confirm that the two terms in Eq. (3.28) nearly have the same magnitude, and the lensing induced difference is not visible because the lensing signal C_L^{dd} is one hundred times smaller than the Gaussian bias $N_L^{(0)}$. We use Eq. (3.28) to calculate the reconstructed lensing power spectra in Figure 3.7.

The likelihood distribution of $\mathcal{C}$ is shown in Figure 3.8, where it is seen lensing is detected at only 1.30σ confidence level.

3.7 Curl Null Test

To check for systematic effects, we employ the ‘‘curl null test’’. The deflection angle field can be written as the sum of a gradient and a curl term [48]:

$$D_i(\mathbf{n}) = d_i(\mathbf{n}) + \epsilon_{ij} \nabla^j \delta(\mathbf{n}). \quad (3.37)$$

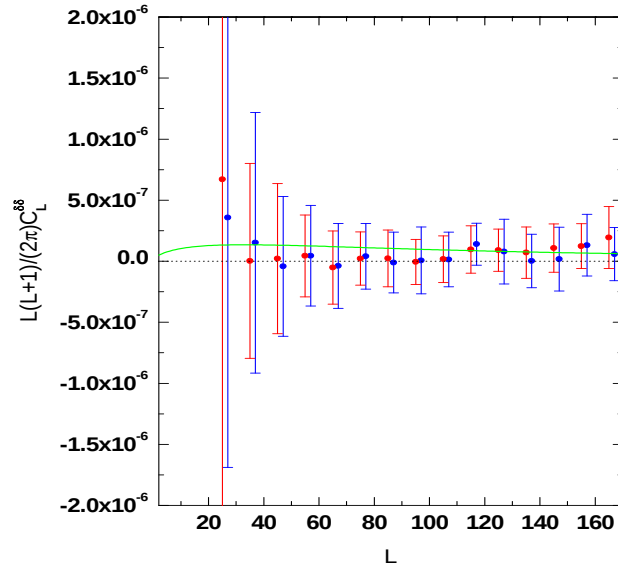


Figure 3.9: Curl null test for all correlations of WMAP's W- and V-band DAs: $C_L^{\delta\delta}$ from the simulated WMAP data (red), and $C_L^{\delta\delta}$ from the real WMAP data (blue), for comparison, the simulated lensing signal C_L^{dd} (solid green). The red and blue data points show the consistency between the simulated and the real WMAP data for the curl null test.

The first term leads to the Hu estimator [26, 19]

$$d_{LM}^{TT} = \frac{A_L^{TT}}{\sqrt{L(L+1)}} \int d\mathbf{n} Y_{LM}^* \nabla^i [{}_0A^T(\mathbf{n}) \nabla_i {}_0B^T(\mathbf{n})] \quad (3.38)$$

whose efficient form is given in Eq. (3.26), here ${}_0A^T(\mathbf{n})$ is given by Eq. (3.8) and

$${}_0B^T(\mathbf{n}) = \sum_{lm} \frac{\tilde{C}_l^{TT}}{C_l^{tot}} T_{lm} {}_0Y_{lm}(\mathbf{n}). \quad (3.39)$$

The estimator for the curl part in Eq. (3.37) is

$$\delta_{LM}^{TT} = \sum_{ij} \epsilon^{ij} \frac{A_L^{TT}}{\sqrt{L(L+1)}} \int d\mathbf{n} Y_{LM}^* \nabla_i [{}_0A^T(\mathbf{n}) \nabla_j {}_0B^T(\mathbf{n})] \quad (3.40)$$

and the corresponding efficient form is

$$\begin{aligned} \delta_{LM}^{TT(\alpha\bullet\beta)} &= \frac{1}{2} \left\{ \frac{A_L^{TT(\alpha\times\beta)}}{\sqrt{L(L+1)}} \left[\beta_{L0} \int d\mathbf{n}_{+1} Y_{LM}^* {}_0A^{T(\alpha)} X^{(\beta)} \right. \right. \\ &\quad \left. \left. - \alpha_{L0} \int d\mathbf{n}_{-1} Y_{LM}^* {}_0A^{T(\alpha)} Y^{(\beta)} \right] \right. \\ &\quad \left. + \frac{A_L^{TT(\beta\times\alpha)}}{\sqrt{L(L+1)}} \left[\beta_{L0} \int d\mathbf{n}_{+1} Y_{LM}^* {}_0A^{T(\beta)} X^{(\alpha)} \right. \right. \\ &\quad \left. \left. - \alpha_{L0} \int d\mathbf{n}_{-1} Y_{LM}^* {}_0A^{T(\beta)} Y^{(\alpha)} \right] \right\}, \end{aligned} \quad (3.41)$$

which can be compared with Eq. (3.26). We show the resulting power spectra $C_L^{\delta\delta}$, averaged from 700 realizations from the real and the simulated WMAP data separately in Figure 3.9. The averaged curl component amplitude is 0.38 ± 0.79 consistent with zero as expected, compared to the simulated C_L^{dd} .

3.8 Results and Discussion

In this work, we have applied the optimal quadratic estimator to WMAP-7 temperature maps alone for the first time.

We have monitored the convergence behavior for the mean value $\mathcal{C}$, the error $\Delta\mathcal{C}$, and the detection significance $\mathcal{C}/\Delta\mathcal{C}$ of the reconstructed lensing signal C_L^{dd} . We find that all these quantities converge after producing 700 realizations of

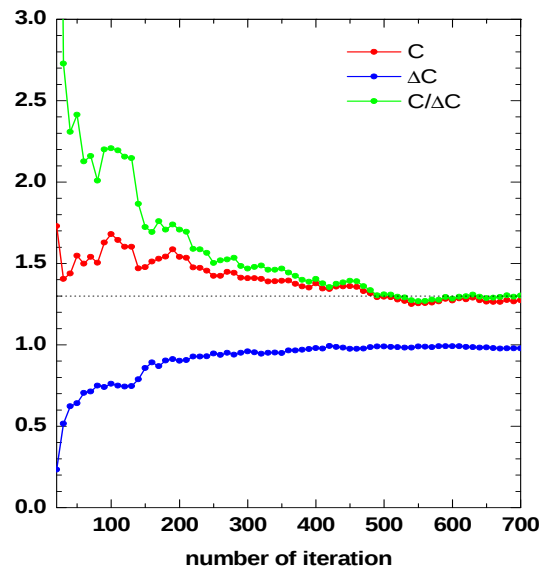


Figure 3.10: The convergence behavior. The values of mean amplitude $\mathcal{C}$ (red), the error $\Delta\mathcal{C}$ (blue), and the detection significance $\mathcal{C}/\Delta\mathcal{C}$ (green) of the reconstructed lensing signal C_L^{dd} are plotted for every 10 realizations. It is seen that convergence is reached after 700 realizations.

the reconstructed lensing signal, see Figure 3.10. We determine the significance of the lensing detection and find $\mathcal{C} = 1.27 \pm 0.98$ (1.30σ), while Smidt et al. found $\mathcal{C} = 0.97 \pm 0.47$ (2.06σ). The result is shown in Table 3.5 as well as a comparison with [32, 34, 47, 36, 35, 25]. All our results have been corrected by the sky fraction. We find evidence for lensing only at 1.30σ , using all correlations of WMAP-7's W- and V-band DAs. The resulting constraint on the lensing amplitude differs from [35] and this can be explicated from several aspects. In terms of the estimator, we use the optimal estimator derived from minimum variance principle [26], rather than the kurtosis estimator in [35]. We adopt the individual beam transfer function associated with each DA, not the averaged one for each frequency. We have taken into account the impact of the higher order bias, afterwards restricting the reconstruction in a proper multiple range that marginally overlaps with [35]. In terms of the noise model, we estimate the noise in a way which mimics WMAP, not simply generating random underlying skies and associated noises with independent phases. All these factors may jointly contribute to the difference between us and Smidt et al. A summary of various tests in this work is shown in Table 3.2. We do not observe a significant lensing signal from the WMAP 7-year temperature data.

We did not apply a correction for higher order bias terms $N_L^{(1)}$, $N_L^{(2)}$, ..., because they are expected to be small owing to the fact that we limited the region of L to $20 < L < 170$, where the higher order bias is consistent with zero. The higher order bias can be obtained via an iterative solution[43] but it is computationally demanding and not warranted in the present case because we do not obtain a significant signal.

We applied the curl null test to all the correlations of W- and V-band DAs as a systematic check, since we observe a small amount of power from the reconstructed gravitational lensing signal (Figure 3.7). The reconstruction procedure passes the curl null test.

The effects of beam systematics and the galactic and foreground contaminations are quite small compared to the statistical error. We do not correct the statistical result for the presence of point sources because they introduce negligible systematics [44].

We have demonstrated, using a nearly optimal estimator, that WMAP-7 data does not have the power to detect gravitational lensing, which is unfortunate since WMAP data is the only publicly available data set with sufficient angular resolution to detect lensing. However, WMAP-7 does have value as a publicly available tool to assess the efficacy of lensing algorithms and to test for systematic biases.

We would like to acknowledge helpful discussions with Joseph Smidt, Meir Shimon, Aneesh V. Manohar, Grigor Aslanyan, Alexander van Engelen and Edward Wollack. We acknowledge the use of CAMB, Healpix software packages.

Chapter 3, in full, is a reprint of material as it appears in Physical Review D 85, 043513, 2012 (arXiv:1111.2371) [1], as written by the author of this dissertation. “Reconstruction of gravitational lensing using WMAP 7-year data”, Chang Feng, Brian Keating, Hans P. Paar, and Oliver Zahn.

Chapter 4

Measuring gravitational lensing of the cosmic microwave background using cross correlation with large scale structure

Chapter origin: **Chang Feng**, Grigor Aslanyan, Aneesh V. Manohar, Brian Keating, Hans P. Paar, Oliver Zahn. **Phys. Rev. D** **86**, 063519 (2012), arXiv:1207.3326 [2].

We cross correlate the gravitational lensing map extracted from cosmic microwave background measurements by the Wilkinson Microwave Anisotropy Probe (WMAP) with the radio galaxy distribution from the NRAO VLA Sky Survey (NVSS) by using a quadratic estimator technique. We use the full covariance matrix to filter the data, and calculate the cross-power spectra for the lensing-galaxy correlation. We explore the impact of changing the values of cosmological parameters on the lensing reconstruction, and obtain statistical detection significances at $> 3\sigma$. The results of all cross correlations pass the curl null test as well as a complementary diagnostic test using the NVSS data in equatorial coordinates. We forecast the potential for Planck and NVSS to constrain the lensing-galaxy cross

correlation as well as the galaxy bias. The lensing-galaxy cross-power spectra are found to be Gaussian distributed.

4.1 Introduction

The cosmic microwave background (CMB) temperature anisotropy contains a wealth of cosmological information and has played a pivotal role in our understanding of the Universe. Besides the primordial fluctuations, various secondary anisotropies, e.g. gravitational lensing, the thermal Sunyaev-Zel'dovich effect, the kinetic Sunyaev-Zel'dovich effect, as well as the integrated Sachs-Wolfe effect, are playing an increasingly important role in constraining cosmological constituents and dynamics.

Among the secondary effects imprinted on the CMB gravitational lensing is of great importance. The projected gravitational lensing potential is a line-of-sight probe which contains information about the geometric distance traversed by CMB photons and time-dependent gravitational potentials. As such it is very sensitive to late universe parameters, such as the sum of neutrino masses, the dark energy equation of state and spatial curvature. Since the projected gravitational lensing potential contains both geometric and structure growth information, it effectively breaks the angular diameter distance degeneracy [23]. Gravitational lensing measurements can also be used to de-lens the B -mode polarization of the CMB [37], enabling us to learn about primordial gravitational waves [4] and the energy scale of inflation.

Tentative CMB weak lensing searches have been done with WMAP-7 year data sets [35, 1] using non-Gaussian statistics. However, WMAP-7 alone cannot detect weak lensing of the CMB because WMAP temperature maps have insufficient sensitivity [1]. Recently, the Atacama Cosmology Telescope [36] and South Pole Telescope (SPT) [25] have performed the first internal lensing reconstruction detections using non-Gaussianity. In addition, Atacama Cosmology Telescope and SPT also measured the gravitational lensing signal from the smoothing effects of the acoustic peaks on the CMB temperature power spectrum [49, 50, 51]. As the

experimental sensitivity improves, internal measurements, either from the power spectrum or the trispectrum, will become more precise in the near future.

The correlation between lensing and large scale structure arises from large scale structure, which deflects CMB photons in the late universe. The signal-to-noise ratio of lensing measurements can be enhanced if the CMB maps are cross correlated with highly sensitive large scale structure tracers, such as luminous red galaxies (LRGs) (which cover the redshift range $0.2 < z < 0.7$), quasars (which covers the redshift range $z < 2.7$) from the Sloan Digital Sky Survey (SDSS), or the NVSS of radio galaxies which has a higher mean redshift ($z \sim 1$) than the LRGs and quasars. Hirata *et al.* [32] used the cross correlation between WMAP-1 and LRGs and quasars from SDSS imaging and found no statistically significant signal. Then Smith *et al.* [33] used the cross correlation between WMAP-3 and NVSS, and found a 3.4σ signal, including systematics. Using a slightly less optimal estimator than Ref. [33], Hirata *et al.* [52] obtained results consistent with, though at slightly lower significance than, Ref. [33] for WMAP-3 with LRGs (0.95σ), WMAP-3 with quasars (1.64σ), and WMAP-3 with NVSS (2.13σ) respectively. Recently, SPT found a greater than 4σ cross correlation between the SPT convergence field and the galaxy survey from the Blanco Cosmology Survey, the Wide-field Infrared Survey Explorer, and Spitzer [53]. In this work, we use WMAP data released in years 1, 3, 5, 7, with NVSS to probe the lensing-galaxy correlation. We follow the methods developed in Smith *et al.* [33] and Hirata *et al.* [52] using all of WMAP's datasets and compare our results to these earlier analyses.

The structure of the paper is as follows. We introduce the data sets in Sec. 4.2. Gravitational lensing effects on the CMB as well as the lensing extraction technique are reviewed in Sec. 4.3. We describe the cross correlation estimators in Sec. 4.4, and the forecast for Planck in Sec. 4.5. We discuss our results in Sec. 4.6.

4.2 Data Sets: WMAP and NVSS

The CMB data we use are from WMAP's Q-, V-, and W-band raw differencing assemblies (DAs). All of these DAs are masked by the Kp0 mask (Fig. 4.1)

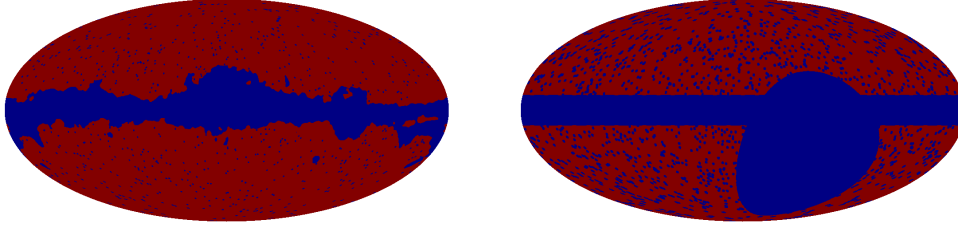


Figure 4.1: WMAP Kp0 mask (left) with $f_{\text{sky}} = 0.77$ and NVSS mask (right) with $f_{\text{sky}} = 0.573$.

to remove bright sources and the galactic plane leaving 77% of the sky.

The input for the galaxy distribution is the NVSS of radio galaxies. The NVSS [54] team provides the software “NVSSlist” to convert its raw catalog to a deconvolved one which is corrected for known biases and systematic errors. We use the deconvolved catalog to extract the galaxy count map. We use this software here, without specifying either a minimum or maximum flux cut. The NVSS map is pixelized with a HEALPIX pixelization scheme with $N_{\text{side}} = 256$. We remove the galactic plane ($|b| < 10^\circ$) and the part of the sky unobserved by the survey ($\delta < -36.87^\circ$). We also carefully remove bright sources with flux > 1 Jy and mask out a disk of radius 1° around them, forming the NVSS mask shown in Fig. 4.1. The resulting galaxy count map has 1,224,990 sources, a sky fraction $f_{\text{sky}} = 0.573$, a mean number of sources per pixel $\bar{n} = 2.72$, and a surface density of 170,249 galaxies per steradian. This agrees well with previous studies [55].

4.3 Gravitational lensing of the CMB

The effect of lensing on the CMB’s primordial temperature $\tilde{T}$ in direction $\mathbf{n}$ can be represented by

$$T(\mathbf{n}) = \tilde{T}(\mathbf{n} + \mathbf{d}(\mathbf{n})), \quad (4.1)$$

where T is the lensed temperature and the deflection angle field $\mathbf{d}(\mathbf{n}) = \nabla\phi$, with ϕ being the lensing potential. The operator ∇ is the covariance derivative on the sphere with respect to the angular position $\mathbf{n}$. We use Gaussian natural units with $\hbar = c = 1$ throughout this paper.

The two-point correlation function of the temperature field is [56]:

$$\begin{aligned} \langle T_{lm} T_{l'm'} \rangle &= \tilde{C}_l^{TT} \delta_{ll'} \delta_{m-m'} (-1)^m + \\ &\sum_{LM} (-1)^M \begin{pmatrix} l & l' & L \\ m & m' & -M \end{pmatrix} f_{lLl'}^{TT} \phi_{LM}, \end{aligned} \quad (4.2)$$

where the second term encodes the effects of lensing with the weighting factor $f_{lLl'}^{TT}$ given by

$$f_{lLl'}^{TT} = \tilde{C}_l^{TT} {}_0F_{l'Ll} + \tilde{C}_{l'}^{TT} {}_0F_{lLl'}. \quad (4.3)$$

We use the standard spherical harmonic decomposition

$$T(\mathbf{n}) = \sum_{lm} T_{lm} Y_{lm}(\mathbf{n}), \quad (4.4)$$

which defines the temperature modes T_{lm} . We use a similar notation for all other quantities defined on a sphere, e.g. ϕ_{LM} are the modes of the lensing potential, etc.

Here $\tilde{C}_l^{TT}$ is the unlensed temperature power spectrum, and

$$\begin{aligned} {}_0F_{lLl'} &= \sqrt{\frac{(2l+1)(2l'+1)(2L+1)}{4\pi}} \times \\ &\frac{1}{2} [L(L+1) + l'(l'+1) - l(l+1)] \begin{pmatrix} l & L & l' \\ 0 & 0 & 0 \end{pmatrix} \end{aligned} \quad (4.5)$$

are proportional to the Wigner $3j$ symbols. Equation (4.2) provides a way to extract ϕ_{LM} from the TT correlations.

In the late universe, the Poisson equation relates the lensing potential $\Phi(\mathbf{k})$ to the density contrast $\delta(\mathbf{k})$,

$$k^2 \Phi(\mathbf{k}) = \frac{3H_0^2 \Omega_m}{2a} \delta(\mathbf{k}), \quad (4.6)$$

where Ω_m is the matter fraction, a is the scale factor and H_0 is the Hubble constant. Using the definition $D(\chi) = -2(1/\chi - 1/\chi_*)$, the projected lensing potential can be expressed as an integral along the line-of-sight,

$$\phi(\mathbf{n}) = \int_0^{\chi_*} d\chi \Phi(\chi \mathbf{n}) D(\chi), \quad (4.7)$$

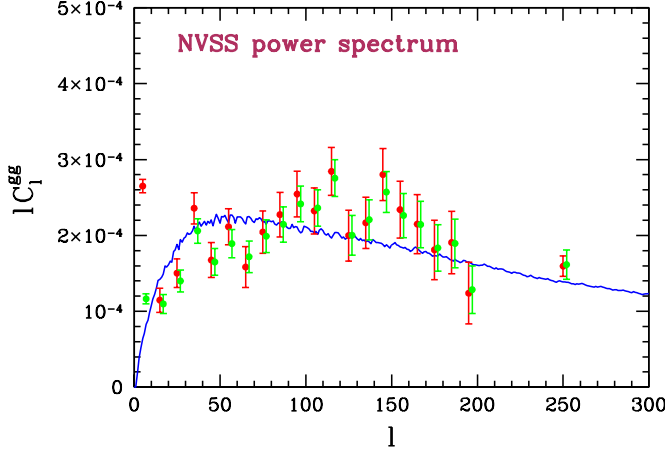


Figure 4.2: The NVSS galaxy auto-power spectrum. The 1σ error bars are from 1000 Monte Carlo simulations of the NVSS galaxy map with galaxy bias $b_g = 1$. For both sets of data points, the red points are from the pseudo- C_l method [57], and the green are from the spherical harmonic estimation [60]. The theoretical galaxy auto-power spectrum is fit to the red data points derived from the pseudo- C_l method. Both methods show a consistent galaxy auto-power spectrum from the NVSS data. The first bin of the red data points largely deviates from the theoretical curve due to systematic effects.

and it is integrated from 0 to the comoving distance at the last scattering surface χ_* . Here $\chi(z)$ is the comoving distance at redshift z . The galaxy overdensity is also given by a line-of-sight integration as

$$g(\mathbf{n}) = \frac{\int d\chi b_g \mathcal{N}(\chi) \delta(\chi \mathbf{n})}{\int d\chi \mathcal{N}(\chi)}. \quad (4.8)$$

To better understand the projected galaxy overdensity, $\mathcal{N}(\chi) = dN/d\chi$ is the comoving distance distribution of the galaxies. For NVSS galaxies, there is a lack of accurate photometric redshifts so approximations to the redshift distributions are made [57, 58, 59]. Following [33], we use a Gaussian distribution

$$\frac{dN}{dz} \propto e^{-\frac{(z-z_0)^2}{2\sigma^2}} \quad (4.9)$$

where $\sigma = 0.8$ for $z < z_0 (= 1.1)$, and $\sigma = 0.3$ for $z > z_0$, and the comoving distribution $dN/d\chi$ is easily derived from Eq. (4.9).

The parameter $b_g(z)$ is the redshift-dependent galaxy bias. To keep the model as simple as possible we treat the galaxy bias as a constant which can be

determined from a fit to the data shown in Fig. 4.2. This is different from [52, 58] which used the cross correlation of NVSS with the SDSS and with sources from the 2-Micron All Sky Survey to determine the galaxy bias. The galaxy bias is of great importance because the cross correlation can be directly translated into primordial non-Gaussianity [61] and may enable general relativity to be tested on cosmological scales [62].

Equations (4.7) and (4.8) give the general definitions for the lensing potential and galaxy overdensity which are both Gaussian fields, characterized by their variances, (i.e. the power spectra) $C_l^{\phi\phi}$ and C_l^{gg} . From Eq. (4.7) and (4.8), one sees that the lensing-galaxy cross correlation is built on the relation described by Eq. (7.6). Using the Limber approximation $k \sim l/\chi$ we calculate the theoretical galaxy auto-power spectrum and cross-power spectrum in Eqs. (4.10) and (7.1).

The galaxy-galaxy power spectrum ($\langle g_{lm} g_{lm}^* \rangle$) is

$$C_l^{gg} \simeq \left(\frac{1}{\int d\chi \mathcal{N}(\chi)} \right)^2 \int d\chi b_g^2 \frac{\mathcal{N}^2(\chi)}{\chi^2} P_\delta \left(\frac{l}{\chi} \right) \quad (4.10)$$

and it will be used later for determining the galaxy bias and for simulating galaxy maps. This power spectrum shows the galaxy clustering strength on different angular scales. We calculate the power spectrum of the NVSS overdensity map using two independent methods: a pseudo- C_l method and a spherical harmonic estimation, as described in Refs. [57, 60] respectively. We find that both methods agree very well (Fig. 4.2). As a final check, we have computed the NVSS galaxy power spectrum using the NVSS galaxy map in both equatorial and galactic coordinates and find a negligible difference, as expected, since the galaxy clustering is an intrinsic property of the Universe and does not depend on the choice of coordinate system. The galaxy power spectrum we use, obtained using the pixelized map in equatorial coordinates, is plotted in Fig. 4.2.

The lensing-galaxy cross-power spectrum is

$$\begin{aligned} C_l^{\phi g} &\simeq \frac{3H_0^2 \Omega_m}{2} \frac{1}{\int d\chi \mathcal{N}(\chi)} \\ &\times \int d\chi b_g (1+z) D(\chi) \frac{\mathcal{N}(\chi)}{k^2 \chi^2} P_\delta \left(\frac{l}{\chi} \right) \end{aligned} \quad (4.11)$$

Table 4.1: The 6-parameter Λ CDM model used for the simulations of the temperature, galaxy and lensing potential. The derived parameter σ_8 , based on the 6-parameter model, is shown in column eight. Using these parameters 1000 galaxy simulations with $b_g = 1$ were performed to get the reconstructed galaxy biases as well as the 1σ error bars. From column nine, we see that all the reconstructed galaxy biases are consistent with the input value $b_g = 1$. Furthermore, the galaxy biases of the real data are calculated based on the simulations and shown in column ten. Two independent methods were used to calculate the galaxy auto-power spectra, as specified in the footnotes.

Data set	$\Omega_b h^2$	$\Omega_{\text{CDM}} h^2$	H_0	A_s	n_s	τ	σ_8	b_g^{sim}	b_g^{data}
WMAP-1 ³	0.0226	0.1104	72	2.212×10^{-9}	0.96	0.117	0.76	0.98 ± 0.12^1	2.00 ± 0.12^1
								0.96 ± 0.10^2	1.95 ± 0.10^2
WMAP-3 ⁴	0.02186	0.1105	70.4	2.393×10^{-9}	0.947	0.073	0.77	0.98 ± 0.11^1	1.97 ± 0.11^1
								0.96 ± 0.09^2	1.92 ± 0.09^2
WMAP-5 ⁵	0.02305	0.1182	69.7	2.484×10^{-9}	0.969	0.094	0.85	0.98 ± 0.10^1	1.85 ± 0.10^1
								0.96 ± 0.08^2	1.81 ± 0.08^2
WMAP-7 ⁶	0.02258	0.1109	71	2.43×10^{-9}	0.963	0.088	0.80	0.98 ± 0.11^1	1.91 ± 0.11^1
								0.96 ± 0.09^2	1.86 ± 0.09^2

which shows the mutual influence between the gravitational potential and the galaxy clustering in the late universe on different angular scales. $P_\delta(k)$ is the matter power spectrum defined using the same convention as Ref. [63]. The primordial scalar curvature perturbations are evaluated at the pivot scale $k_0 = 0.002 \text{ Mpc}^{-1}$. The cross-power spectrum $C_l^{\phi g}$ will be used to simulate the correlated galaxy maps and will also be fit to data to determine the detection significance.

4.4 Cross Correlation Estimation

Monte Carlo simulations are used to estimate the cross correlation between the CMB and the galaxy distribution. The variances $\tilde{C}_l^{TT}$, C_l^{TT} , $C_l^{\phi\phi}$ are computed using CAMB [64] with the cosmological parameters listed in Table 4.1. In addition to these, we derive C_l^{gg} and $C_l^{\phi g}$ from Eq. (4.10) and Eq. (7.1) with the parameters listed in Table 4.1. Simulated CMB temperature modes, $\tilde{a}_{lm}$, are drawn from Gaussian distributions with zero means and variances $\tilde{C}_l^{TT}$. In this work, we will use two sets of cosmological parameters because we want to check the consistency of our results with a previous study [33] and also because we want to explore the

impact of using the newest parameters from WMAP-7 on the lensing-galaxy cross correlations.

We convert these $\tilde{a}_{lm}$ to an unlensed temperature map, $\tilde{T}(\mathbf{n})$, on which we do a cubic interpolation to precisely implement Eq. (4.1). This produces a lensed temperature map $T(\mathbf{n})$ that is converted back to harmonic space to give the lensed modes a_{lm} . Then each DA's beam and pixel window transfer function (the pixel window transfer function has negligible effects on the cross-power spectra) from WMAP are multiplied by these modes which are subsequently transformed into a temperature map containing the lensing signal.

We then simulate Gaussian noise in map space where the pixel noise is assumed to be uncorrelated and Gaussian distributed with zero mean and pixel-independent variance determined from $\sigma_0/\sqrt{N_{\text{obs}}}$. Here, both σ_0 and N_{obs} are supplied by the WMAP team for different DAs. We add this noise map to the signal map and apply the Kp0 mask to get a simulated WMAP DA made in the same way as the real WMAP maps were produced. The entire procedure can be summarized by Eq. (4.12) in which a_{lm} is the lensed CMB mode, $n(\mathbf{n})$ the white noise, $M^{\text{WMAP}}(\mathbf{n})$ the mask map, ν the index of the DA channel, p_l the pixel window transfer function, b_l the beam transfer function

$$T^{(\nu)}(\mathbf{n}) = M^{\text{WMAP}}(\mathbf{n}) \left[\sum_{lm} p_l b_l^{(\nu)} a_{lm} Y_{lm}(\mathbf{n}) + \left(\frac{\sigma_0}{\sqrt{N_{\text{obs}}(\mathbf{n})}} \right)^{(\nu)} n(\mathbf{n}) \right]. \quad (4.12)$$

To maximize the signal-to-noise ratio, we compute a single harmonic mode $\hat{a}_{lm}$ from eight Q, V, W-band DAs. This reduction step is expressed as [33]

$$\begin{aligned} \hat{\mathbf{a}} &= (\mathbf{S} + \mathbf{N})^{-1} \mathbf{a} \\ &= \mathbf{S}^{-1/2} \mathbf{A}^{-1} \mathbf{S}^{1/2} \mathbf{N}^{-1} \mathbf{a}. \end{aligned} \quad (4.13)$$

Here $\mathbf{a}$ is the vector of a_{lms} , $\mathbf{S}$ the signal covariance matrix, $\mathbf{N}$ the noise covariance matrix, and $\mathbf{A} = \mathbf{I} + \mathbf{S}^{1/2} \mathbf{N}^{-1} \mathbf{S}^{1/2}$. We use the second form of Eq. (4.13) and filter the raw CMB modes using a multigrid-preconditioned-complex conjugate gradient method. The master equation that has to be solved is

$$\mathbf{A} \mathbf{x} = \mathbf{y}, \quad (4.14)$$

where $\mathbf{x} = \mathbf{S}^{1/2}\hat{\mathbf{a}}$, and $\mathbf{y} = \mathbf{S}^{1/2}\mathbf{N}^{-1}\mathbf{a}$. Equation (4.14) is better for numerical computations because $\mathbf{A}$ is close to the unit matrix. Appendix 4.7 gives details of the numerical calculation of Eq. (4.14). We solve Eq. (4.14) with $\hat{a}_{lm}$ for both the temperature ($\hat{T}_{lm}$) and the galaxy ($\hat{g}_{lm}$).

We use the standard quadratic estimator to reconstruct a noisy lensing potential map $\hat{\phi}_{lm}$ in harmonic space [26, 19],

$$\sum_{lm} \hat{\phi}_{lm} Y_{lm}(\mathbf{n}) = \nabla^i ({}_0A^T(\mathbf{n}) \nabla_i {}_0B^T(\mathbf{n})), \quad (4.15)$$

where

$${}_0A^T(\mathbf{n}) = \sum_{lm} \hat{T}_{lm} Y_{lm}(\mathbf{n}) \quad (4.16)$$

and

$${}_0B^T(\mathbf{n}) = \sum_{lm} \tilde{C}_l^{TT} \hat{T}_{lm} Y_{lm}(\mathbf{n}). \quad (4.17)$$

In the above equations, ∇_i is the gradient operator on a sphere and $\nabla^i = g^{ij} \nabla_j$. Here g_{ij} is the metric of a sphere.

We also use Monte Carlo simulations for the NVSS galaxy maps. The simulated galaxy modes g_{lm} are drawn from a Gaussian distribution and transformed into a galaxy overdensity map $g(\mathbf{n})$ at HEALPIX resolution $N_{\text{side}} = 1024$. The galaxy modes must satisfy the correct galaxy-galaxy auto-correlation and lensing-galaxy cross correlation. From these two constraints the simulated galaxy mode must be

$$g_{lm} = \frac{C_l^{\phi g}}{C_l^{\phi\phi}} \phi_{lm} + \sqrt{C_l^{gg} - \frac{(C_l^{\phi g})^2}{C_l^{\phi\phi}}} G_{lm}, \quad (4.18)$$

where G_{lm} is a complex Gaussian random variable, and ϕ_{lm} is inherited from the deflection field in Eq. (4.1). From this equation we see that the lensing-galaxy correlation is encoded in the first term. A NVSS map is generated where the galaxy number count in each pixel is drawn from a Poisson distribution with mean

$$\lambda(\mathbf{n}) = \bar{n}(1 + g(\mathbf{n})). \quad (4.19)$$

The galaxy count map $\lambda(\mathbf{n})$ is used to generate a simulated galaxy overdensity map $g^{(\text{sim})}(\mathbf{n})$,

$$g^{(\text{sim})}(\mathbf{n}) = M^{\text{NVSS}}(\mathbf{n}) \left[\frac{\lambda(\mathbf{n})}{\bar{n}} - 1 \right], \quad (4.20)$$

Table 4.2: The two sets of cosmological parameters used in this work: we choose two sets of parameters (labeled “**Set-3**” and “**Set-7**”) to do the cross correlation calculations in this work. In order to compare our results with those from the previous studies [33], we use the parameters they used **Set-3** from WMAP-3’s cosmological parameters (row “WMAP-3” in Table 4.1) combined with the corresponding galaxy bias of Smith *et al.* [33]. Based on the newest cosmological parameters from WMAP-7 (row “WMAP-7” in Table 4.1) we construct a new parameter set, **Set-7** with the corresponding galaxy bias shown in Table 4.1.

Data set	$\Omega_b h^2$	$\Omega_{\text{CDM}} h^2$	H_0	A_s	n_s	τ	σ_8	b_g
Set-3	0.02186	0.1105	70.4	2.393×10^{-9}	0.947	0.073	0.77	1.70
Set-7	0.02258	0.1109	71	2.43×10^{-9}	0.963	0.088	0.80	1.91

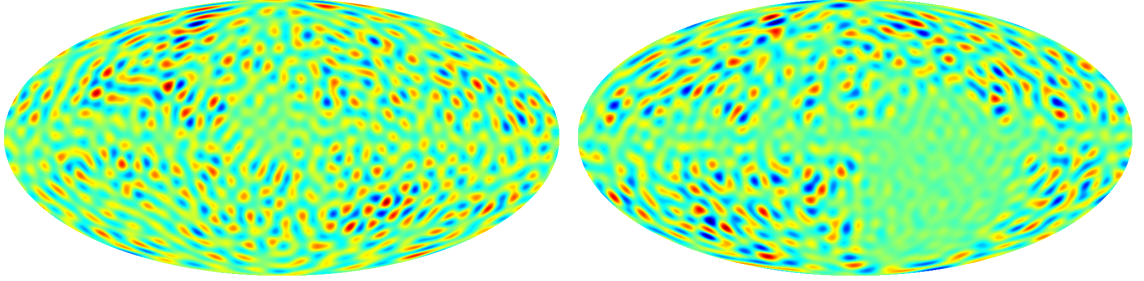


Figure 4.3: The noisy reconstruction of the lensing potential map (Eq.(4.15) from WMAP-7) band-pass filtered from $20 \leq l \leq 40$ (left). The analogous map from NVSS galaxy data [Eq.(4.21)] within the band $20 \leq l \leq 40$ (right).

where M^{NVSS} is the NVSS mask shown in Fig. 4.1. $g^{(\text{sim})}(\mathbf{n})$ automatically contains the shot-noise with the variance $N_l^{gg} = 1/\bar{n}$ for the galaxy overdensity map. We degrade this map to resolution $N_{\text{side}} = 256$ i.e. the same as the real NVSS data. The harmonic mode $g_{lm}^{(\text{sim})}$, which contains the shot-noise, is obtained from $g^{(\text{sim})}(\mathbf{n})$ and is further filtered using the same procedure as in Eq. (4.13),

$$\hat{g}_{lm} = (\mathbf{S} + \mathbf{N})^{-1} g_{lm}^{(\text{sim})}. \quad (4.21)$$

Here $\mathbf{S}$ represents the primordial galaxy covariance and $\mathbf{N}$ is the shot-noise covariance.

We show the noisy reconstruction of the potential maps and the filtered galaxy map in Fig. 4.3, using the measured WMAP and NVSS data.

The lensing-galaxy cross-power spectrum is the observable which will be

Table 4.3: Measure of lensing-galaxy cross correlation $\mathcal{C}$ and its significance $\mathcal{C}/\Delta\mathcal{C}$ using **Set-3**. For five columns of this table: the second column shows the simulation results, the third column is the case without gradient stripes removed, the fourth column is the case with gradient stripes removed (this column shows the statistical results of the lensing-galaxy cross correlations). The fifth column is the case by setting the NVSS map in equatorial coordinates as a complementary diagnostic test.

Data set	$\mathcal{C}^{\text{sim}}$	$\mathcal{C}/\Delta\mathcal{C}$	$\mathcal{C}^7$	$\mathcal{C}/\Delta\mathcal{C}$	$\mathcal{C}^8$	$\mathcal{C}/\Delta\mathcal{C}$	$\mathcal{C}^9$
WMAP-1×NVSS	1.00 ± 0.47	2.13σ	1.25 ± 0.47	2.66σ	1.24 ± 0.47	2.64σ	0.26
WMAP-3×NVSS	1.00 ± 0.35	2.86σ	1.20 ± 0.35	3.43σ	1.26 ± 0.35	3.60σ	0.17
WMAP-5×NVSS	1.00 ± 0.31	3.23σ	1.24 ± 0.31	4.00σ	1.27 ± 0.31	4.10σ	0.23
WMAP-7×NVSS	1.00 ± 0.30	3.33σ	1.14 ± 0.30	3.80σ	1.16 ± 0.30	3.87σ	0.15

Table 4.4: Measure of lensing-galaxy cross correlation $\mathcal{C}$ and its significance $\mathcal{C}/\Delta\mathcal{C}$ using **Set-7**. The format of this table is the same as Table 4.3.

Data set	$\mathcal{C}^{\text{sim}}$	$\mathcal{C}/\Delta\mathcal{C}$	$\mathcal{C}^7$	$\mathcal{C}/\Delta\mathcal{C}$	$\mathcal{C}^8$	$\mathcal{C}/\Delta\mathcal{C}$	$\mathcal{C}^9$
WMAP-1×NVSS	1.00 ± 0.41	2.44σ	1.01 ± 0.41	2.46σ	1.00 ± 0.41	2.44σ	0.20
WMAP-3×NVSS	1.00 ± 0.31	3.23σ	0.96 ± 0.31	3.10σ	1.01 ± 0.31	3.26σ	0.13
WMAP-5×NVSS	1.00 ± 0.28	3.57σ	0.98 ± 0.28	3.50σ	1.01 ± 0.28	3.61σ	0.18
WMAP-7×NVSS	1.00 ± 0.26	3.85σ	0.92 ± 0.26	3.54σ	0.93 ± 0.26	3.58σ	0.11

compared with the counterpart from data. The estimator of the lensing-galaxy cross correlation is expressed as

$$C_b^{\phi g} = \frac{1}{\mathcal{F}_b} \sum_{\substack{l \in b \\ -l \leq m \leq l}} (\hat{\phi}_{lm} - \langle \hat{\phi}_{lm} \rangle)^* \hat{g}_{lm}, \quad (4.22)$$

where $\mathcal{F}_b$ is the normalization factor at band b . It is shown in Ref. [65] that the normalization factor can be calculated by either a direct or a fast method for the full-sky coverage and that the fast method converges in $O(10^2)$ simulations. When there is a sky-cut these methods account for the sky-cut effect very well and a constant f_{sky} is often used. The factor f_{sky} is actually a function of l [66] so a simple constant approximation may potentially bias the cross-spectra reconstruction. Therefore an end-to-end simulation [33] is the best way to get the exact normalization accounting for the sky-cut and that is done here.

As a systematic check we note that the lensing signal consists of a gradient

Table 4.5: Fisher matrix analysis for WMAP×NVSS cross correlation. The 1σ error bars are determined from Eq. (4.28). We calculate two sets of the optimal bounds for this work, based on two sets of parameters: **Set-3** (column two); **Set-7** (column three).

Data set	$\mathcal{C}^{\text{optimal 10}}$	$\mathcal{C}/\Delta\mathcal{C}$	$\mathcal{C}^{\text{optimal 11}}$	$\mathcal{C}/\Delta\mathcal{C}$
WMAP-1×NVSS	1 ± 0.46	2.17σ	1 ± 0.39	2.56σ
WMAP-3×NVSS	1 ± 0.29	3.45σ	1 ± 0.25	4.00σ
WMAP-5×NVSS	1 ± 0.25	4.00σ	1 ± 0.21	4.76σ
WMAP-7×NVSS	1 ± 0.22	4.55σ	1 ± 0.19	5.26σ

and a curl component [67]. The curl component estimator ψ_{lm} is defined by

$$\sum_{lm} \psi_{lm} Y_{lm}(\mathbf{n}) = \sum_{ij} \epsilon^{ij} \nabla_i ({}_0A^T(\mathbf{n}) \nabla_j {}_0B^T(\mathbf{n})) \quad (4.23)$$

and should vanish because lensing does not generate vorticity. Similar to Eq. (4.22), the curl-galaxy cross correlation diagnostic is calculated by

$$C_b^{\psi g} = \frac{1}{\mathcal{F}_b} \sum_{\substack{l \in b \\ -l \leq m \leq l}} (\psi_{lm} - \langle \psi_{lm} \rangle)^* \hat{g}_{lm} \quad (4.24)$$

which should also vanish.

The amplitude of the cross correlation is determined using

$$\mathcal{C} = \frac{\sum_{AB} C_A^{(\text{th})} \mathbf{C}_{AB}^{-1} C_B^{(\text{obs})}}{\sum_{AB} C_A^{(\text{th})} \mathbf{C}_{AB}^{-1} C_B^{(\text{th})}}. \quad (4.25)$$

$\mathbf{C}_{AB}$ is the covariance matrix for the band powers and A and B stand for the band power index. We find that the off-diagonal correlations of $\mathbf{C}_{AB}$ are negligible, and the covariance matrix elements can be simply replaced by the band power variance σ_A^2 , i.e. $\mathbf{C}_{AB} = \sigma_A^2 \delta_{AB}$.

¹Pseudo- C_l method [57],

²Spherical harmonic estimation [60]

³WMAP1+CBI+ACBAR+2dFGRS+Ly α , [16]

⁴WMAP3 ALL, <http://lambda.gsfc.nasa.gov/product/map/dr2/parameters.cfm>

⁵WMAP5+BAO+SNALL+Ly α POST, <http://lambda.gsfc.nasa.gov/product/map/dr3/parameters.cfm>

⁶WMAP7, <http://lambda.gsfc.nasa.gov/product/map/dr4/parameters.cfm>

⁷Without gradient stripes removed.

⁸With gradient stripes removed.

⁹Galaxy map in equatorial coordinate.

¹⁰WMAP-3 year cosmological parameters and $b_g = 1.70$.

¹¹WMAP-7 year cosmological parameters and $b_g = 1.91$.

Table 4.6: Results of the curl null tests for WMAP $\times$ NVSS cross correlation. The curl null tests are performed based on two sets of parameters: **Set-3** (column two); **Set-7** (column three).

Data set	$\mathcal{C}^{10}$	$\mathcal{C}/\Delta\mathcal{C}$	$\mathcal{C}^{11}$	$\mathcal{C}/\Delta\mathcal{C}$
WMAP-1 $\times$ NVSS	-0.11 ± 0.47	-0.23σ	-0.03 ± 0.41	-0.07σ
WMAP-3 $\times$ NVSS	0.00 ± 0.35	0.00σ	0.04 ± 0.31	0.13σ
WMAP-5 $\times$ NVSS	0.05 ± 0.31	0.16σ	0.07 ± 0.28	0.25σ
WMAP-7 $\times$ NVSS	-0.05 ± 0.30	0.17σ	-0.03 ± 0.26	-0.12σ

Table 4.7: Gaussianity diagnostics for the probability distribution of $\{\mathcal{C}\}$ which is constructed from 1000 Monte Carlo simulations. The second column is the Kolmogorov-Smirnov test, and the critical value is 0.04 at 5% confidence level. The Kolmogorov-Smirnov test requires the maximum deviation be < 0.04 to validate the distribution is Gaussian. The third column is the skewness of $\{\mathcal{C}\}$, and the fourth column is the kurtosis of $\{\mathcal{C}\}$. The upper values in the cells are the results for **Set-3**, the lower values for **Set-7**. For a Gaussian distribution, the skewness should be 0 and the kurtosis should be 3. As can be seen, all the probability distribution functions pass the Kolmogorov-Smirnov test and are consistent with being Gaussian-distributed.

Data set	maximum distance[<0.04]	skewness[~ 0]	kurtosis[~ 3]
WMAP-1 $\times$ NVSS	0.02	0.02	2.89
	0.02	-0.05	2.77
WMAP-3 $\times$ NVSS	0.02	-0.14	2.62
	0.03	-0.17	2.58
WMAP-5 $\times$ NVSS	0.03	-0.17	2.53
	0.03	-0.23	2.43
WMAP-7 $\times$ NVSS	0.03	-0.21	2.44
	0.03	-0.21	2.36

We have described the procedures used to perform analysis on simulated or measured data. Now we summarize the analysis of the real WMAP and NVSS data. We fit the theoretical galaxy auto-power spectrum to the NVSS data in Fig. 4.2 and determine the galaxy biases (Table 4.1) using two methods. The error bars are determined from 1000 simulated galaxy maps with galaxy bias $b_g = 1$. Then we choose two sets of parameters (labeled “**Set-3**” and “**Set-7**”) in Table 4.2 to do the cross correlation calculations in this work. In order to compare our results with those from the previous studies [33], we use the parameters they used **Set-3** from WMAP-3’s cosmological parameters (row “WMAP-3” in Table 4.1) combined with the corresponding galaxy bias of Smith *et al.* [33]. Based on the newest cosmological parameters from WMAP-7 (row “WMAP-7” in Table 4.1) we construct a new parameter set **Set-7** with the corresponding galaxy bias shown in Table 4.1. For each of the parameter sets we calculate four lensing-galaxy cross correlations from WMAP-1 to WMAP-7.

We carefully treat the known systematics of NVSS, i.e. the gradient stripes which are generated by the declination-dependence of the galaxy overdensity field due to the low-flux calibration issue [54]. We first make a gradient stripe map only using $m = 0$ modes and then subtract it from the galaxy map. We calculate the lensing-galaxy cross correlations for two cases: without the gradient stripes removed and with the gradient stripes removed. We find that this systematic effect does not affect the lensing-galaxy cross correlations as seen from column $\mathcal{C}^a$ and column $\mathcal{C}^b$ in Table 4.3 and Table 4.4. The statistical results are those with the gradient stripes removed which are shown in Figs. 4.4 and 4.5 for the Kp0+NVSS mask combination. From the two figures, we find that the lensing-galaxy cross-power spectra are consistent with the theoretical predictions and the uncertainty of the cross-power spectrum is decreasing as the year of WMAP increases. All the error bars are calculated from 1000 Monte Carlo simulations, which we confirmed to be sufficient for convergences. As a complementary diagnostic test, we keep the NVSS galaxy overdensity map in equatorial coordinates and calculate the cross-power spectra and all the amplitudes are shown in column $\mathcal{C}^c$ in Table 4.3 and Table 4.4. As can be seen, they are negligible. All the cross correlation amplitudes

are summarized in Table 4.3 and Table 4.4. From the results of WMAP-3×NVSS in Table 4.3: for the statistical results, we get lensing detection significance level of 3.60σ and [33] got 4.02σ . Both analyses agree quite well. We find the cross-power spectra from WMAP-5 and WMAP-7 clearly and firmly show the lensing-galaxy correlation at $> 3\sigma$ level for both cases. All the results are within the optimal bounds shown in Table 4.5.

Assuming there is no cosmological parity violation the curl-galaxy correlation should be consistent with zero. We show the results of the curl null tests in Figs. 4.6 and 4.7. As expected, all the correlations are consistent with zero. The amplitude as well as the significance are given in Table 4.6.

We pixelized the NVSS catalog with different HEALPIX resolutions (e.g. $N_{\text{side}} = 512, 1024$) to probe the possible pixel artifacts that could afflict the cross-power spectra and because we want to examine the impact of possible long range spatial correlations. However, we find that different NVSS pixelization resolutions do not affect the cross correlation.

We also use the diagonal elements of the covariance matrix to do the analysis to check the consistency with previous studies. In this case, the estimator has a larger variance as pointed out by Smith *et al.* [68]. This contributes to the difference in significance levels between 4σ [33] and 2σ [52].

4.5 Forecast for Future Experiments

The revealed cross correlation between WMAP and NVSS hints that the detection significance would be further enhanced if the precision of the CMB data were improved. The upcoming Planck data will improve upon WMAP, so we expect that the cross correlation between Planck and NVSS will be more significant. To predict the optimal bound on the detection signal-to-noise ratio for lensing-galaxy cross correlation we first calculate the equivalent noise $N_l^{\phi g}$ from the following equation

$$N_l^{\phi g} = \left[N_l^{\phi\phi} N_l^{gg} \right]^{1/2}. \quad (4.26)$$

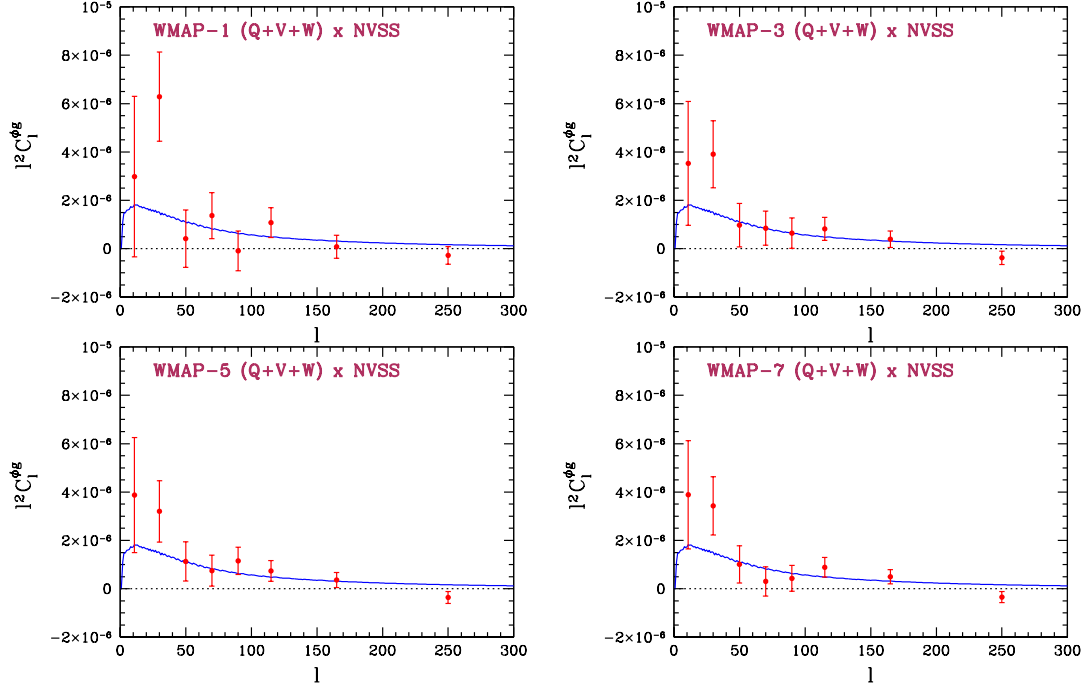


Figure 4.4: (Set-3) The lensing-galaxy cross-power spectra for WMAP $\times$ NVSS are calculated from Eq. (4.22). The Kp0 mask is used to remove the contaminated regions of the WMAP data. WMAP's data are provided from two Q bands, two V bands and four W bands. The NVSS mask is applied to the galaxy map to remove bright sources and unobserved regions. The theoretical cross-power spectra are shown in blue solid lines, and they are the same for all of the four panels. The real data are shown in the red scattered points. The statistical amplitude for WMAP-1 $\times$ NVSS is 1.24 ± 0.47 , for WMAP-3 $\times$ NVSS is 1.26 ± 0.35 , for WMAP-5 $\times$ NVSS is 1.27 ± 0.31 , for WMAP-7 $\times$ NVSS is 1.16 ± 0.30 . All the error bars are determined from 1000 Monte Carlo simulations. We find that the lensing-galaxy cross-power spectra are consistent with the theoretical predictions and the uncertainty of the cross-power spectrum is decreasing as the year of WMAP increases.

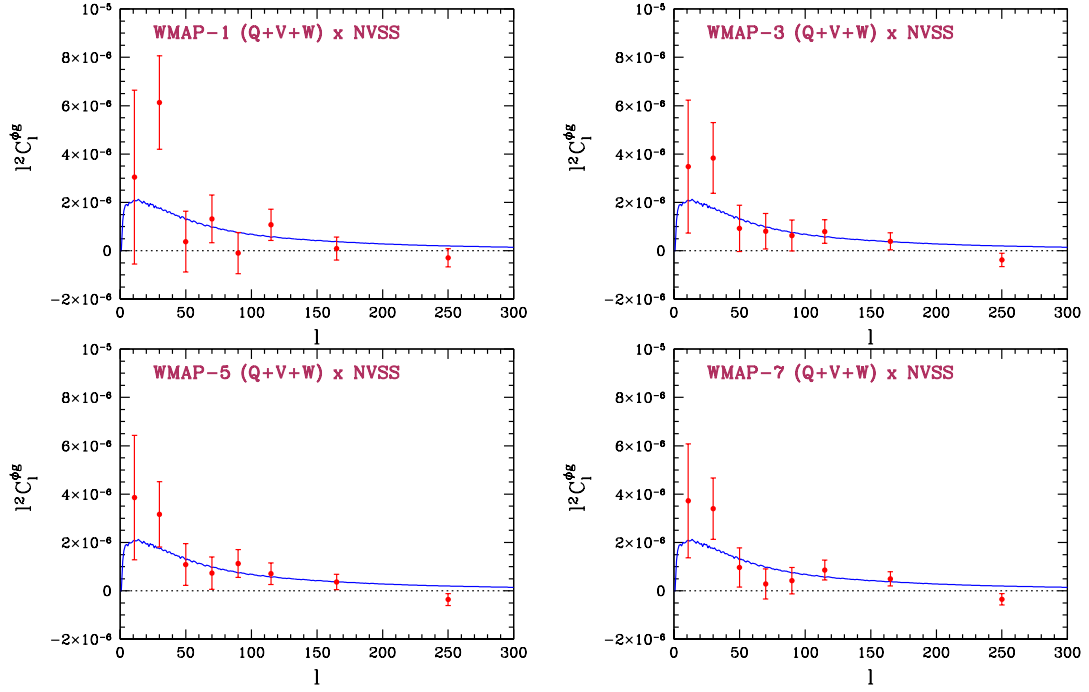


Figure 4.5: (Set-7) The lensing-galaxy cross-power spectra for WMAP $\times$ NVSS are calculated from Eq. (4.22). See Fig. 4.4 for detailed descriptions. The statistical amplitude for WMAP-1 $\times$ NVSS is 1.00 ± 0.41 , for WMAP-3 $\times$ NVSS is 1.01 ± 0.31 , for WMAP-5 $\times$ NVSS is 1.01 ± 0.28 , for WMAP-7 $\times$ NVSS is 0.93 ± 0.26 .

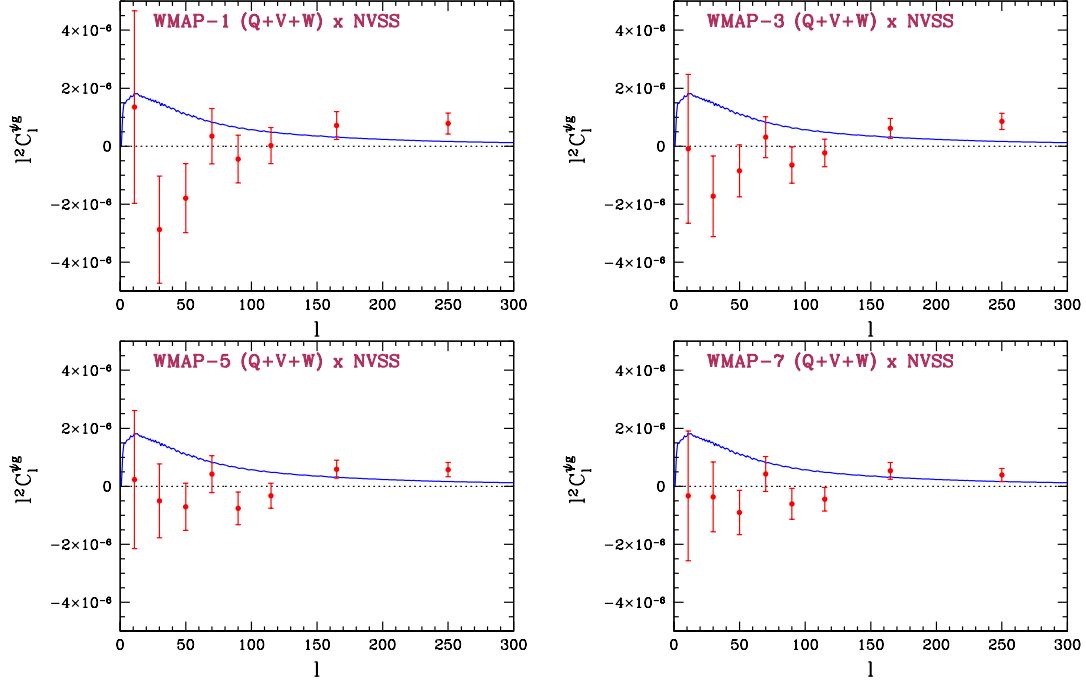


Figure 4.6: (Set-3) The curl null tests for WMAP $\times$ NVSS are calculated from Eq. (4.24). The Kp0 mask is used to remove the contaminated regions of the WMAP data. WMAP data are provided from two Q bands, two V bands and four W bands. The NVSS mask is applied to the galaxy map to remove bright sources and unobserved regions. The theoretical lensing-galaxy cross-power spectra with both WMAP and NVSS in galactic coordinates are shown in blue solid lines for comparison, and they are the same for all of the four panels. The curl amplitude for WMAP-1 $\times$ NVSS is -0.11 ± 0.47 , for WMAP-3 $\times$ NVSS is 0.00 ± 0.35 , for WMAP-5 $\times$ NVSS is 0.05 ± 0.31 , for WMAP-7 $\times$ NVSS is -0.05 ± 0.30 . As can be seen, all cross-power spectra for the curl null test are consistent with zero (black dotted line).

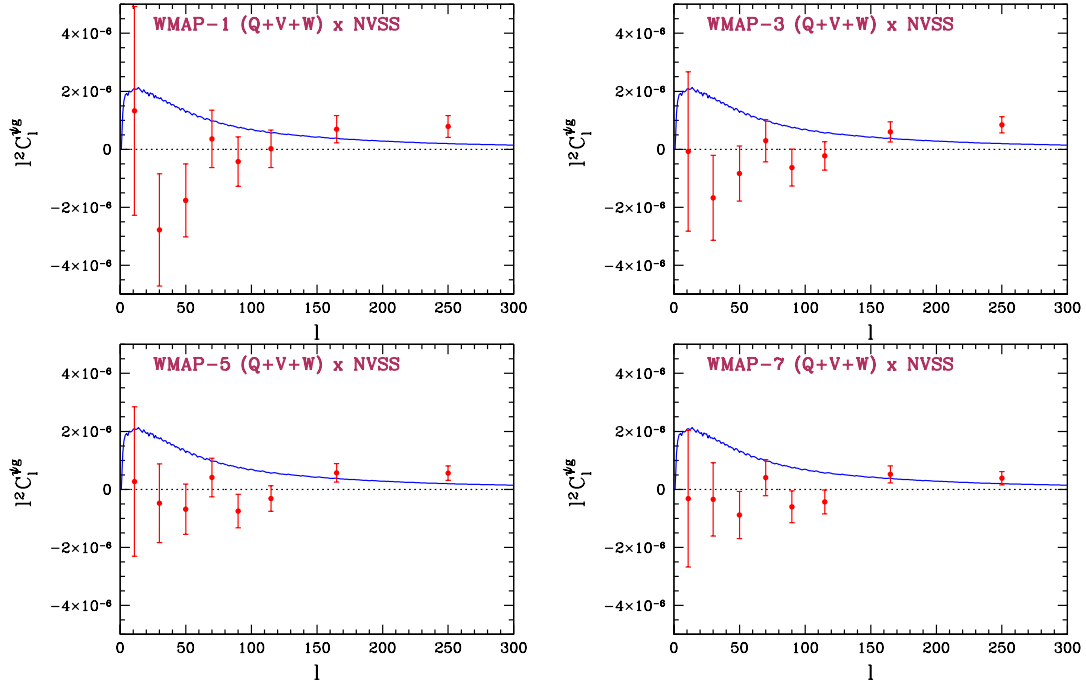


Figure 4.7: (Set-7) The curl null tests for WMAP $\times$ NVSS are calculated from Eq. (4.24). See Fig. 4.6 for detailed descriptions. The curl amplitude for WMAP-1 $\times$ NVSS is -0.03 ± 0.41 , for WMAP-3 $\times$ NVSS is 0.04 ± 0.31 , for WMAP-5 $\times$ NVSS is 0.07 ± 0.28 , for WMAP-7 $\times$ NVSS is -0.03 ± 0.26 . As can be seen, all cross-power spectra for the curl null test are consistent with zero (black dotted line).

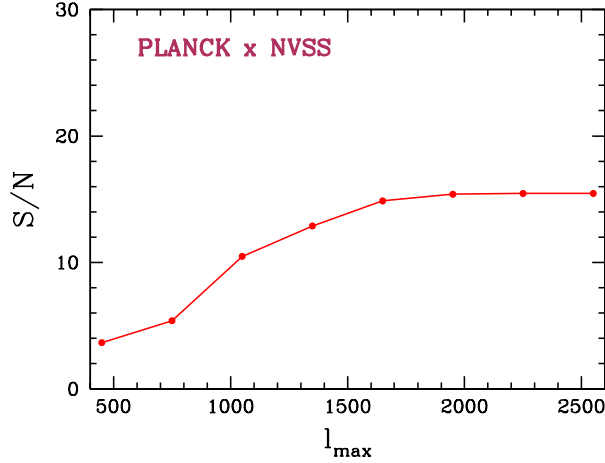


Figure 4.8: The signal-to-noise ratio for the lensing-galaxy cross correlation between Planck and NVSS as a function of the maximum multipole used in the analysis.

where $N_l^{\phi\phi}$ is the lensing reconstruction noise [56] and N_l^{gg} is the galaxy shot-noise. The efficient algorithm for calculating $N_l^{\phi\phi}$ is given in Refs. [37, 1]. This reconstruction noise can be minimized by combining different CMB channels and the minimum noise is

$$N_l^{\text{min},\phi\phi} = \frac{1}{\sum_{\nu} \left[N_l^{\nu,\phi\phi} \right]^{-1}}, \quad (4.27)$$

Both of the noise spectra effectively propagate the uncertainty $\Delta C_l^{\phi g}$ into the cross-power spectrum $C_l^{\phi g}$. Specifically, we express it as

$$\Delta C_l^{\phi g} = \sqrt{\frac{2}{(2l+1)f_{\text{sky}}}} (C_l^{\phi g} + N_l^{\phi g}). \quad (4.28)$$

The optimal bound is then determined from

$$\left[\sum_l \left(\frac{C_l^{\phi g}}{\Delta C_l^{\phi g}} \right)^2 \right]^{1/2}.$$

The redshift distribution Eq. (4.9) was used and the galaxy bias was set equal to $b_g = 1$. The instrumental properties for Planck are given in Refs. [69, 70]. We show the signal-to-noise ratio for Planck with NVSS as a function of l_{max} in Fig. 4.8. We find that the highest signal-to-noise ratio, i.e. 15σ , saturates at $l_{\text{max}} = 2000$. Since the lensing-galaxy cross-power spectrum scales as $C_l^{\phi g} \propto b_g$ as illustrated by Eq. (7.1), the amplitude of this cross-power spectra is degenerate with the galaxy bias and the signal-to-noise for the cross-power spectrum can also serve as a prediction of the detection significance for the galaxy bias. Thus, we see Planck can detect b_g with high precision which will lead to a better understanding of the correlation between the baryonic matter distribution and the dark matter distribution.

4.6 Conclusion

We have calculated the lensing-galaxy cross-power spectra using WMAP and NVSS and the full covariance matrix to filter the data sets. Specifically, we performed a thorough analysis of WMAP-1, WMAP-3, WMAP-5 and WMAP-7

raw DAs. The cross correlations between WMAP-5, -7's 8 DAs (2Q-bands+2V-bands+4W-bands) with NVSS clearly and firmly show signals at $> 3\sigma$ level. We took the effects of gradient stripes into account for the NVSS data, and determined the significance without and with gradient stripes removed. The major effects caused by the stripes can be seen from the first bin of either the galaxy auto-power spectrum or the lensing-galaxy cross-power spectrum; the first bin decreases if the gradient stripes are marginalized over. However, gradient stripes do not affect the lensing-galaxy correlation (compare Refs. [59, 71, 72]). We have explicitly shown all these results in Tables 4.3 and 4.4. In these two tables, column $\mathcal{C}^a$ are the results without the gradient stripes removed and column $\mathcal{C}^b$ are our main results with the gradient stripes removed. In order to validate the lensing-galaxy cross correlations, we produced a NVSS galaxy map in equatorial coordinates directly from the NVSS catalog and cross correlated it with the WMAP DA which is in galactic coordinates, we find that all the lensing-galaxy cross correlation amplitudes are negligible.

We investigated the impact of different NVSS pixelization resolutions and found no effect. We compared the sensitivities of the estimators both with the full and diagonal covariance matrix and found that the former more effectively reduces the variance, which is mainly caused by the sky-cut and the inhomogeneous instrumental noise. However, the former scheme involves the inversion of a large matrix which is computationally challenging.

We predicted the detection significance for the lensing-galaxy cross correlation or the galaxy bias for the upcoming Planck data with NVSS and found the detection significance will be improved by a factor of 5.

The minimum variance of the estimator assumes that the CMB and galaxy overdensity modes are Gaussian. However, if the CMB contains gravitational lensing, the bispectrum $\langle TTg \rangle$ is not zero; it induces an additional variance as indicated in Eq. (4.50). We analytically and numerically confirm that this variance is actually not noticeable for WMAP and NVSS as pointed out in Ref. [73]. Furthermore, being aware of the potential non-Gaussian shape of the probability distribution function (PDF) [74], we specifically investigate the PDF of the cross-power spectrum amplitude $\mathcal{C}$ in terms of Kolmogorov-Smirnov test, the skewness

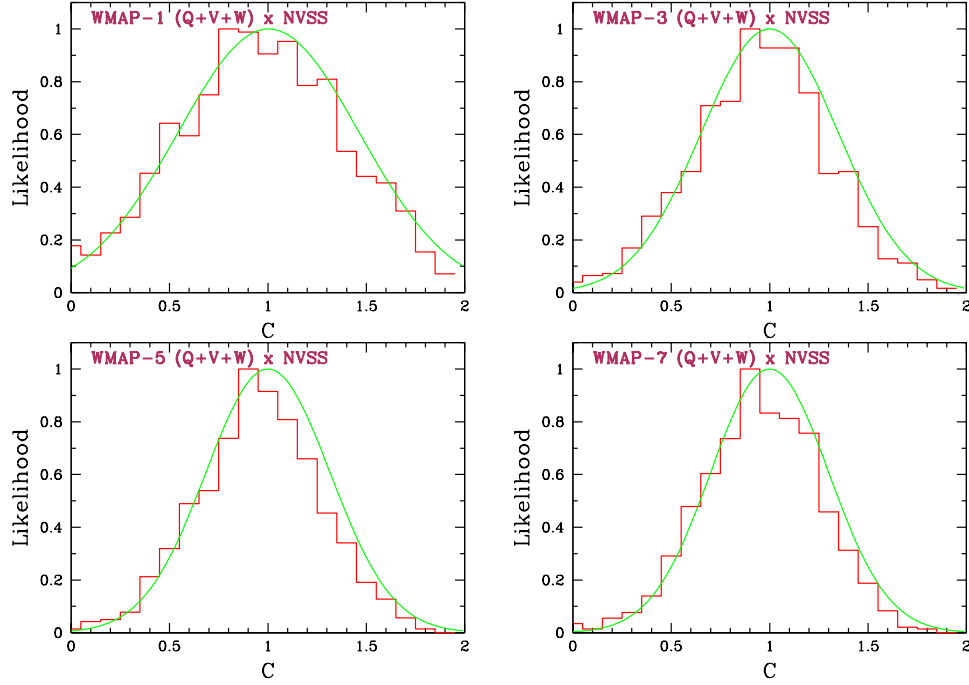


Figure 4.9: (Set-3) Probability distribution function for the lensing-galaxy cross correlation. The likelihood functions are normalized to 1. From 1000 simulations, a set of $\{C\}$ is generated for each one of the subfigures, then by counting the frequency of C within a bin, a step-like function (red) is plotted. For comparison, Gaussian likelihood (green) is plotted using the mean and the variance of the set $\{C\}$.

and the kurtosis. The diagnostic tests are shown in Table 4.7. All the PDFs pass the Kolmogorov-Smirnov tests. All the PDFs are consistent with being Gaussian-distributed (Figs. 4.9 and 4.10).

The lensing-galaxy cross correlations effectively link the early universe to the late universe and the CMB is served as a back light casting the dark cosmic web (which is formed by the dark matter) throughout the major expansion history of the universe. The gravitational lensing is a powerful tool to decode the information of dark matter distribution from the CMB and the lensing-galaxy cross-correlations further unveil the relationship between baryonic matter and dark matter.

We would like to acknowledge helpful discussions with Sudeep Das, Duncan Hanson, Christian Reichardt, Meir Shimon, and Amit Yadav. We acknowledge the use of CAMB, HEALPIX¹, and LAPACK software packages and the LAMBDA

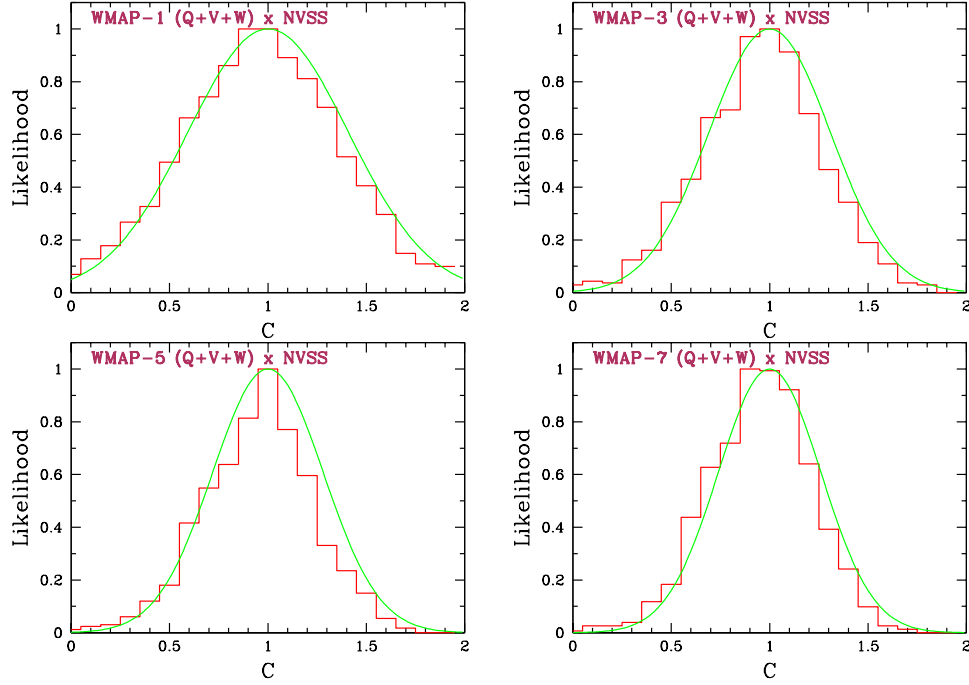


Figure 4.10: (Set-7) Probability distribution function for the lensing-galaxy cross correlation. See Fig. 4.9 for detailed descriptions.

archive. The computational resources required for this work were accessed via the GlideinWMS [75] on the Open Science Grid [76]. We are indebted to Frank Wuerthwein, Igor Sfiligoi, Terrence Martin, and Robert Konecny for their insight and support.

Chapter 4, in full, is a reprint of material as it appears in Physical Review D 86, 063519, 2012 (arXiv:1207.3326) [2], as written by the author of this dissertation. “Measuring gravitational lensing of the cosmic microwave background using cross correlation with large scale structure”, Chang Feng, Grigor Aslanyan, Aneesh V. Manohar, Brian Keating, Hans P. Paar, and Oliver Zahn.

¹<http://healpix.jpl.nasa.gov/>

4.7 APPENDIX A: Multigrid-Preconditioned Complex Conjugate Gradient Inversion

Given the signal covariance matrix $\mathbf{S}$ and the noise covariance matrix $\mathbf{N}$, and an array of the CMB modes $\mathbf{a}$, we define another covariance matrix $\mathbf{A} = \mathbf{I} + \mathbf{S}^{1/2}\mathbf{N}^{-1}\mathbf{S}^{1/2}$, and two vectors $\mathbf{x} = \mathbf{S}^{1/2}\mathbf{a}$, and $\mathbf{y} = \mathbf{S}^{1/2}\mathbf{N}^{-1}\mathbf{a}$. For the problem $\mathbf{A}\mathbf{x} = \mathbf{y}$, we write down the equations for constructing the matrix $\mathbf{A}$ and the vector $\mathbf{y}$,

$$N_{lm'l'm'}^{-1} = \sum_{\nu} p_l b_l^{(\nu)} p_{l'} b_{l'}^{(\nu)} \times \int d\mathbf{n} Y_{lm}^*(\mathbf{n}) Y_{l'm'}(\mathbf{n}) \left[\frac{M(\mathbf{n})}{\sigma^2} \right]^{(\nu)}, \quad (4.29)$$

$$[N^{-1}a]_{lm} = \sum_{\nu} p_l b_l^{(\nu)} \times \int d\mathbf{n} Y_{lm}^*(\mathbf{n}) \left[\frac{M(\mathbf{n})H(\mathbf{n})}{\sigma^2} \right]^{(\nu)}, \quad (4.30)$$

$$w_{lm}^{(\nu)} = \int d\mathbf{n} Y_{lm}(\mathbf{n}) \left[\frac{M(\mathbf{n})}{\sigma^2} \right]^{(\nu)}. \quad (4.31)$$

In the above equations, p_l is the window transfer function, $b_l^{(\nu)}$ is the specific beam transfer function corresponding to the DA of WMAP, and $M(\mathbf{n})$ is the mask map. For WMAP, $\nu = Q_1, Q_2, V_1, V_2, W_1, W_2, W_3, W_4$, $H(\mathbf{n}) = T(\mathbf{n})$ and $M(\mathbf{n})$ is the Kp0 mask. For NVSS, $\nu = 1$ and $H(\mathbf{n}) = g(\mathbf{n})$. Since NVSS has 45 arc-second FWHM resolution [54], we set $b_l^{(1)} = 1$ as NVSS's beam transfer function. The correspondence between the continuum and discrete forms of integration on the sphere is,

$$\int d\mathbf{n} \rightarrow \frac{4\pi}{N_{\text{pix}}} \sum_j, \quad (4.32)$$

where j denotes the pixel index according to the HEALPIX pixelization scheme and N_{pix} is the total number of pixels.

For comparison, we also use a suboptimal estimator which only takes the diagonal elements of the inverse noise matrix $N_{lm'l'm'}^{-1}$ shown in Eq. (4.29).

The filtering using the covariance matrix requires us to solve the linear equation $\mathbf{Ax} = \mathbf{y}$. We use the preconditioned conjugate gradient iteration to solve it, and the initial condition is chosen to be

$$\begin{aligned}\mathbf{x}^{(0)} &= 0, \\ \mathbf{r}^{(0)} &= \mathbf{y}, \\ \mathbf{p}^{(1)} &= \tilde{\mathbf{A}}^{-1}\mathbf{y},\end{aligned}\tag{4.33}$$

with the preconditioner defined as

$$\tilde{\mathbf{A}}^{-1} = \begin{pmatrix} \mathbf{A}_0^{-1} & \mathbf{0} \\ \mathbf{0} & \mathbf{A}_{\Delta}^{-1} \end{pmatrix},\tag{4.34}$$

here $\mathbf{A}_{\Delta}$ is the diagonal element of the matrix $\mathbf{A}$.

The iteration procedure is [32]

$$\begin{aligned}\mathbf{x}^{(i)} &= \mathbf{x}^{(i-1)} + \frac{\mathbf{r}^{(i-1)}\tilde{\mathbf{A}}^{-1}\mathbf{r}^{(i-1)}}{\mathbf{p}^{(i)}\mathbf{A}\mathbf{p}^{(i)}}\mathbf{p}^{(i)}, \\ \mathbf{r}^{(i)} &= \mathbf{y} - \mathbf{A}\mathbf{x}^{(i)}, \\ \mathbf{p}^{(i)} &= \tilde{\mathbf{A}}^{-1}\mathbf{r}^{(i-1)} + \frac{\mathbf{r}^{(i-1)}\tilde{\mathbf{A}}^{-1}\mathbf{r}^{(i-1)}}{\mathbf{r}^{(i-2)}\tilde{\mathbf{A}}^{-1}\mathbf{r}^{(i-2)}}\mathbf{p}^{(i-1)}.\end{aligned}\tag{4.35}$$

From Eq. (4.35), we find that two operations $\tilde{\mathbf{A}}^{-1}\mathbf{r}$ and $\mathbf{A}\mathbf{p}$ are computationally demanding if we evaluate them directly because $\mathbf{A}$ and $\mathbf{A}_0$ are $10^6 \times 10^6$ matrix.

In order to achieve the necessary efficiency, we recursively precondition the matrix $\mathbf{A}$ on a much coarser grid. The preconditioner is

$$\tilde{\mathbf{A}}^{-1} = \begin{pmatrix} \tilde{\mathbf{A}}_0^{-1} & \mathbf{0} \\ \mathbf{0} & \mathbf{A}_{\Delta}^{-1} \end{pmatrix},\tag{4.36}$$

and on the coarser grid the preconditioner is $\tilde{\mathbf{A}}_0^{-1}$. This multigrid strategy enables us to directly store the matrix $\tilde{\mathbf{A}}_0$ on the coarsest grid and we can further

analytically express the smallest inversion problem as follows²

$$\begin{aligned}
N_{12}^{-1} &= \sum_{\nu} \int p_1 b_1^{(\nu)} Y_1^* p_2 b_2^{(\nu)} Y_2 \sum_3 w_3^{(\nu)} Y_3 \\
&= \sum_{\nu} \sum_3 w_3^{(\nu)} \sqrt{\frac{(2l_1+1)(2l_2+1)(2l_3+1)}{4\pi}} \\
&\times (-1)^{m_1} \begin{pmatrix} l_1 & l_2 & l_3 \\ 0 & 0 & 0 \end{pmatrix} \begin{pmatrix} l_1 & l_2 & l_3 \\ -m_1 & m_2 & m_3 \end{pmatrix} \\
&\quad p_1 b_1^{(\nu)} p_2 b_2^{(\nu)}, \tag{4.37}
\end{aligned}$$

then the problem of preconditioning $\mathbf{A}$ with $\tilde{\mathbf{A}}_0$ on the finer grid can be iteratively solved by using Eq. (4.35). For this work we use three levels of the grids: (1) $N_{\text{side}} = 512, l_{\text{max}} = 1000$, (2) $N_{\text{side}} = 256, l_{\text{max}} = 400$, (3) $N_{\text{side}} = 128, l_{\text{max}} = 200$. We split the covariance matrix on the third grid at $l_{\text{split}} = 30$ to construct the minimum inversion problem.

For the coarsest grid, we explicitly calculate the inverse noise matrix N_{12}^{-1} [Eq. (4.37)] using LAPACK. The iteration process also needs the multiplication for $\mathbf{A}\lambda$, and this can be computed efficiently by doing spherical harmonic transformations:

$$\begin{aligned}
\mathbf{A}\lambda &= \sum_4 (I + S^{1/2} N^{-1} S^{1/2})_{14} \lambda_4 \\
&= \lambda_1 + \sum_{\nu} p_1 b_1^{(\nu)} S_1^{1/2} \left[\int d\mathbf{n} Y_1^* \left[\frac{\mathbf{M}(\mathbf{n})}{\sigma^2} \right]^{(\nu)} \right. \\
&\quad \left. \times \left(\sum_4 p_4 b_4^{(\nu)} S_4^{1/2} \lambda_4 Y_4 \right) \right]. \tag{4.38}
\end{aligned}$$

4.8 APPENDIX B: Non-Gaussianity

There are several possible non-Gaussian effects generated by using a nonzero bispectrum in the simulation. These can potentially bias our results. We analytically calculate this non-Gaussian bias in this appendix.

²In the following, we denote subscripts l_i or l_i, m_i by i for simplicity, so that $p_{l_i} \rightarrow p_i$, $Y_{l_i m_i} \rightarrow Y_i$, $N_{l_1 m_1 l_2 m_2}^{-1} \rightarrow N_{12}^{-1}$ etc.

We define the bispectrum by

$$\langle a_1 a_2 g_3 \rangle = b_{123} G(123), \quad (4.39)$$

where $b_{123} = (f_{123} C_2^{\text{TT}} + f_{213} C_1^{\text{TT}}) C_{l_3}^{\phi g}$, (see footnote 2 for notation) and

$$G(123) = \sqrt{\frac{(2l_1 + 1)(2l_2 + 1)(2l_3 + 1)}{4\pi}} \begin{pmatrix} l_1 & l_2 & l_3 \\ 0 & 0 & 0 \end{pmatrix} \begin{pmatrix} l_1 & l_2 & l_3 \\ m_1 & m_2 & m_3 \end{pmatrix}. \quad (4.40)$$

The estimator is

$$\hat{C} = \frac{1}{\mathcal{F}} (\hat{C}_A - \hat{C}_B) \quad (4.41)$$

where

$$\hat{C}_A = \frac{1}{2} \sum_{123} b_{123} G(123) \tilde{a}_1 \tilde{a}_2 \tilde{g}_3 \quad (4.42)$$

and

$$\hat{C}_B = \frac{1}{2} \sum_{123} b_{123} G(123) [C^{\text{TT}}]_{12}^{-1} \tilde{g}_3. \quad (4.43)$$

We define

$$\begin{aligned} \tilde{a} &= C^{-1} a, \\ f_k &= \frac{1}{2} \sum b_{12k} G(12k) [C^{\text{TT}}]_{12}^{-1}, \\ \langle \tilde{a}_1 \tilde{a}_2 \rangle &= [C^{\text{TT}}]_{12}^{-1}, \\ \langle \tilde{g}_1 \tilde{g}_2 \rangle &= [C^{\text{gg}}]_{12}^{-1}. \end{aligned}$$

The summation index i denotes a sum over $l_i m_i$.

We define the normalization as

$$\begin{aligned} \mathcal{F} &= \frac{1}{2} \sum_{123456} b_{123} b_{456} G(123) G(456) \\ &\times [C^{\text{TT}}]_{14}^{-1} [C^{\text{TT}}]_{25}^{-1} [C^{\text{gg}}]_{36}^{-1}. \end{aligned} \quad (4.44)$$

The variance of the estimator $\hat{C}$ is $\sigma^2(\hat{C})$ which has contributions from three parts,

$$\sigma^2(\hat{C}) = \sigma^2(\hat{C}_A) + \sigma^2(\hat{C}_B) - 2\sigma^2(\hat{C}_A \hat{C}_B). \quad (4.45)$$

Now we explicitly determine the three variances. For the second term, we have the relation

$$\langle \tilde{a}_1 \tilde{a}_2 \tilde{g}_3 \rangle = \sum_{1'2'3'} [C^{\text{TT}}]_{11'}^{-1} [C^{\text{TT}}]_{22'}^{-1} [C^{\text{gg}}]_{33'}^{-1} b_{1'2'3'} G(1'2'3'), \quad (4.46)$$

so the last two variance terms can be easily expressed as

$$\sigma^2(\hat{C}_B) = \sigma^2(\hat{C}_A \hat{C}_B) = f^T [C^{\text{gg}}]^{-1} f. \quad (4.47)$$

For the first term, it is

$$\begin{aligned} \sigma^2(\hat{C}_A) &= \frac{1}{4} \sum_{123456} b_{123} b_{456} G(123) G(456) \langle \tilde{a}_1 \tilde{a}_2 \tilde{g}_3 \tilde{a}_4 \tilde{a}_5 \tilde{g}_6 \rangle \\ &\quad - \frac{1}{4} \sum_{123456} b_{123} b_{456} G(123) G(456) \langle \tilde{a}_1 \tilde{a}_2 \tilde{g}_3 \rangle \langle \tilde{a}_4 \tilde{a}_5 \tilde{g}_6 \rangle, \end{aligned} \quad (4.48)$$

and can be expanded as

$$\begin{aligned} &\frac{1}{4} \sum_{123456} b_{123} b_{456} G(123) G(456) \langle \tilde{a}_1 \tilde{a}_2 \tilde{g}_3 \tilde{a}_4 \tilde{a}_5 \tilde{g}_6 \rangle \\ &= \frac{1}{4} \sum_{123456} b_{123} b_{456} G(123) G(456) \left\{ \underbrace{\left[\langle \tilde{a}_1 \tilde{a}_2 \tilde{g}_3 \rangle \langle \tilde{a}_4 \tilde{a}_5 \tilde{g}_6 \rangle + \langle \tilde{a}_4 \tilde{a}_5 \tilde{g}_3 \rangle \langle \tilde{a}_1 \tilde{a}_2 \tilde{g}_6 \rangle \right]}_{\text{second term}} \right. \\ &\quad + \langle \tilde{a}_1 \tilde{a}_4 \tilde{g}_3 \rangle \langle \tilde{a}_2 \tilde{a}_5 \tilde{g}_6 \rangle + \langle \tilde{a}_2 \tilde{a}_5 \tilde{g}_3 \rangle \langle \tilde{a}_1 \tilde{a}_4 \tilde{g}_6 \rangle \\ &\quad + \left. \left[\langle \tilde{a}_1 \tilde{a}_5 \tilde{g}_3 \rangle \langle \tilde{a}_2 \tilde{a}_4 \tilde{g}_6 \rangle + \langle \tilde{a}_2 \tilde{a}_4 \tilde{g}_3 \rangle \langle \tilde{a}_1 \tilde{a}_5 \tilde{g}_6 \rangle \right]_{3 \cdot 3} \right. \\ &\quad + \underbrace{\left[\langle \tilde{a}_1 \tilde{a}_5 \rangle \langle \tilde{a}_2 \tilde{a}_4 \rangle \langle \tilde{g}_3 \tilde{g}_6 \rangle + \langle \tilde{a}_1 \tilde{a}_4 \rangle \langle \tilde{a}_2 \tilde{a}_5 \rangle \langle \tilde{g}_3 \tilde{g}_6 \rangle \right]}_{\text{normalization}} \\ &\quad + \left. \left[\langle \tilde{a}_1 \tilde{a}_2 \rangle \langle \tilde{a}_4 \tilde{a}_5 \rangle \langle \tilde{g}_3 \tilde{g}_6 \rangle \right]_{2 \cdot 2 \cdot 2} \right\}. \end{aligned} \quad (4.49)$$

$$\begin{aligned} \sigma^2(\hat{C}_A - \hat{C}_B) &= \mathcal{F} + \left\{ \frac{1}{4} \sum_{123456} b_{123} b_{456} G(123) G(456) \left[\langle \tilde{a}_4 \tilde{a}_5 \tilde{g}_3 \rangle \langle \tilde{a}_1 \tilde{a}_2 \tilde{g}_6 \rangle \right. \right. \\ &\quad + \langle \tilde{a}_1 \tilde{a}_4 \tilde{g}_3 \rangle \langle \tilde{a}_2 \tilde{a}_5 \tilde{g}_6 \rangle + \langle \tilde{a}_2 \tilde{a}_5 \tilde{g}_3 \rangle \langle \tilde{a}_1 \tilde{a}_4 \tilde{g}_6 \rangle \\ &\quad + \left. \left. \langle \tilde{a}_1 \tilde{a}_5 \tilde{g}_3 \rangle \langle \tilde{a}_2 \tilde{a}_4 \tilde{g}_6 \rangle + \langle \tilde{a}_2 \tilde{a}_4 \tilde{g}_3 \rangle \langle \tilde{a}_1 \tilde{a}_5 \tilde{g}_6 \rangle \right] \right\}_{\text{nonzero bispectrum}} \\ &= O(b^2) + O(b^4) \end{aligned} \quad (4.50)$$

When all the pieces are put together, we find that the nonvanishing bispectrum induces an extra term which is $O(b^4)$. We conclude that for WMAP, this contribution is very small and we numerically verified that this is indeed the case.

Chapter 5

Power Spectrum Estimation from Cosmic Microwave Background Maps

The CMB power spectra contain a lot of cosmological information. The core task of CMB data analysis is to compute the power spectra. In this chapter, the method of CMB map-making and power spectrum calculation is discussed.

5.1 Maximum likelihood map-making

The signal received by the detector has several components as seen from the following equation

$$d_t = n_t + A_{tp}s_p. \quad (5.1)$$

Here s is the primordial signal which we are measuring, n is the noise, A is the pointing matrix and d is the detected signal which contains everything. “t” is the index of the time ordered data (TOD), and “p” the index of the pixelised map. From this, it is easy to see that the pointing matrix A_{tp} is not invertible because it is not a square matrix. This is why we resort to the following algorithm. Given everything described above, the unknown map s is obtained from minimizing the

following likelihood function

$$\mathcal{L} = \frac{1}{(2\pi)^{D/2} \sqrt{\det N}} e^{-\frac{n^T N^{-1} n}{2}}. \quad (5.2)$$

In this equation, $N = \langle n^T n \rangle$ is the noise covariance matrix and D is dimension of the sampled time stream of the noise. We take the logarithm of this likelihood function and get

$$\begin{aligned} \ln \mathcal{L} &= -\frac{1}{2} n^T N^{-1} n - \frac{1}{2} \ln \det N \\ &= -\frac{1}{2} (d - As)^T N^{-1} (d - As) - \frac{1}{2} \ln \det N, \end{aligned} \quad (5.3)$$

then the variance-minimized map s is constrained from

$$\frac{\partial \ln \mathcal{L}}{\partial s} = 0. \quad (5.4)$$

The detailed expression for the left hand side is

$$\frac{\partial \ln \mathcal{L}}{\partial s} = \frac{1}{2} A^T N^{-1} (d - As) + [\frac{1}{2} A^T N^{-1} (d - As)]^T = 0. \quad (5.5)$$

The second part is the same as the first, so the above condition implies that $A^T N^{-1} (d - As) = 0$. From this we get the solution to the map-making

$$\hat{s} = [A^T N^{-1} A]^{-1} A^T N^{-1} d. \quad (5.6)$$

Note that the matrix $A^T N^{-1} A$ is $N_{pix} \times N_{pix}$ and invertible.

5.2 Power spectrum calculation

The observed CMB map is constructed from a few components: window $M(\mathbf{n})$, beam profile $B(\mathbf{n}', \mathbf{n})$, signal $\tilde{T}(\mathbf{n})$ and noise $N(\mathbf{n})$. The data structure is

$$T(\mathbf{n}) = M(\mathbf{n}) \int d^2 \mathbf{n}' \mathcal{F}[\tilde{T}(\mathbf{n}') B(\mathbf{n}', \mathbf{n})] + N(\mathbf{n}). \quad (5.7)$$

In the above equation the operator $\mathcal{F}$ means making a pixelised map from the TOD with polynomial filtering in time domain. We use F_l to account for this

operation in harmonic space. We first evaluate the power spectrum after filtering and beam smoothing. If we have a map just with these effects as

$$T^s(\mathbf{n}) = \int d^2n' \mathcal{F}[\tilde{T}(\mathbf{n}')B(\mathbf{n}', \mathbf{n})], \quad (5.8)$$

and this gives

$$T_{lm}^s = \sqrt{F_l} B_l \tilde{T}_{lm} \quad (5.9)$$

under the spherical harmonic transformation (SHT). The power spectrum is $C_l^{T^s T^s} = F_l B_l^2 \tilde{C}_l$. Now we add instrumental noise on the map and mask the map, the power spectrum becomes

$$C_l = \text{Power}[M(\mathbf{n})T^s(\mathbf{n})] + N_l. \quad (5.10)$$

We need to solve the power spectrum for $M(\mathbf{n})T^s(\mathbf{n})$ in order to get the final result for the data structure define in Eq. (5.7).

5.3 Flat-Sky mode-mode coupling matrix

For a small patch of the sky, the window $M(\mathbf{n})$ is used to remove poorly observed regions and this masking procedure is described as

$$T(\mathbf{n}) = \tilde{T}(\mathbf{n})M(\mathbf{n}). \quad (5.11)$$

We expand the mask and the underlying sky using plane waves and calculate the observed temperature mode (note $l = 2\pi k$)

$$T(\mathbf{k}) = \int d^2k' \tilde{T}(\mathbf{k}') K_{\mathbf{k}'\mathbf{k}} \quad (5.12)$$

with the coupling matrix being

$$K_{\mathbf{k}'\mathbf{k}} = \int d^2k'' M(\mathbf{k}'') \delta(\mathbf{k}' + \mathbf{k}'' - \mathbf{k}). \quad (5.13)$$

In these equations, we use $\mathbf{k}$ instead of $\mathbf{l}$ to be consistent with the derivations in [77].

The power spectrum of $T(\mathbf{k})$ is [77]

$$C_k^{TT} = \int dk' k' C_{k'}^{\tilde{T}\tilde{T}} W_{k'k} \quad (5.14)$$

with the mode-mode coupling matrix defined as

$$W_{k'k} = 2\pi \int dk'' k'' M_{k''} J(k', k, k''). \quad (5.15)$$

In this equation, the function $J(k', k, k'')$ is given in [77]. Due to the masking, uncorrelated CMB modes now are coupled. Fortunately once the mode-coupling matrix is obtained, we can easily decouple the modes.

5.4 Full-Sky mode-mode coupling matrix

Now we work in harmonic space. Again we define the masking effect like

$$T(\mathbf{n}) = M(\mathbf{n})\tilde{T}(\mathbf{n}) \quad (5.16)$$

on a sphere.

We expand the mask and the underlying sky into spherical harmonics and do the inverse spherical harmonic transformation to get the observed CMB mode

$$\begin{aligned} T_{lm} &= \int d^2n Y_{lm}^*(\mathbf{n}) M(\mathbf{n}) \tilde{T}(\mathbf{n}) \\ &= \int d^2n Y_{lm}^*(\mathbf{n}) \sum_{l''m''} M_{l''m''} Y_{l''m''}(\mathbf{n}) \sum_{l'm'} \tilde{T}_{l'm'} Y_{l'm'}(\mathbf{n}) \\ &= \sum_{l'm'} \tilde{T}_{l'm'} \int d^2n Y_{lm}^*(\mathbf{n}) \sum_{l''m''} M_{l''m''} Y_{l''m''}(\mathbf{n}) Y_{l'm'}(\mathbf{n}) \\ &= \sum_{l'm'} \tilde{T}_{l'm'} K_{l'm',lm}. \end{aligned} \quad (5.17)$$

The above equation defines the coupling matrix

$$\begin{aligned} K_{l'm',lm} &= \sum_{l''m''} M_{l''m''} \int d^2n Y_{l-m}(\mathbf{n}) (-1)^m Y_{l''m''}(\mathbf{n}) Y_{l'm'}(\mathbf{n}) \\ &= \sum_{l''m''} M_{l''m''} h_{ll''} \begin{pmatrix} l & l'' & l' \\ 0 & 0 & 0 \end{pmatrix} \begin{pmatrix} l & l'' & l' \\ -m & m'' & m' \end{pmatrix} (-1)^m. \end{aligned} \quad (5.18)$$

For this derivation, we use the identity

$$\int d^2n Y_{lm}(\mathbf{n}) Y_{l'm'}(\mathbf{n}) Y_{l''m''}(\mathbf{n}) = h_{ll''} \begin{pmatrix} l & l'' & l' \\ 0 & 0 & 0 \end{pmatrix} \begin{pmatrix} l & l'' & l' \\ m & m'' & m' \end{pmatrix} \quad (5.19)$$

and coefficients

$$h_{ll''} = \sqrt{\frac{(2l+1)(2l'+1)(2l''+1)}{4\pi}}. \quad (5.20)$$

Using Eq 5.17, we calculate the observed power spectrum which is [77]

$$C_l^{TT} = \sum_{l'} C_{l'}^{\tilde{T}\tilde{T}} W_{l'l} \quad (5.21)$$

with the mode-mode coupling matrix [78, 77]

$$W_{l'l} = \frac{2l'+1}{4\pi} \sum_{l''} M_{l''}(2l''+1) \begin{pmatrix} l & l'' & l' \\ 0 & 0 & 0 \end{pmatrix}^2. \quad (5.22)$$

This is the full-sky mode mixing matrix compared to Eq.5.13 which is the flat-sky counterpart of equation 5.22.

5.5 Discussions

For POLARBEAR experiment, we make the beam profile from Jupiter observations and the window from the inverse-noise-variance map. We calculate the mode-coupling matrix both numerically and analytically. For the numerical calculation of the matrix of $W_{bb'}$ (here b is the band index and it corresponds to multipole l_b), we first make a step function in C_b so the power spectrum we take is essentially $C_b \delta_{bb'}$ and create maps from this step function. Then we create maps from this step function and apply window to them. We calculate the power spectra for all of these maps. Finally the average ratio between the power spectra after masking and the input step function is the value of the matrix element $W_{b'b}$. Repeating this procedure for all the bands b , we get the entire matrix $W_{bb'}$. To account for the TOD filtering, we generate signal-only simulations and convert them into simulated TODs, incorporating the telescope scanning information. Then we apply the same polynomial filter as we do for the observed TODs and convert them back to maps. Finally we compare the averaged power spectra of these maps to the input power spectrum to make the filter transfer function. Going through all these procedures, we can recover the underlying power spectra TT, TE, TB, EE, EB, BB . For polarization data, it is slightly complicated. It is not trivial to separate E and B modes

and a few algorithms have been developed, such as the pure- B estimator [79] and the Matrix Based Map Purification method [80].

Chapter 6

Measurement of the Cosmic Microwave Background Polarization Lensing Power Spectrum with the POLARBEAR Experiment

Chapter origin: POLARBEAR Collaboration, P.A.R. Ade, Y. Akiba, A.E. Anthony, K. Arnold, M. Atlas, D. Barron, D. Boettger, J. Borrill, S. Chapman, Y. Chinone, M. Dobbs, T. Elleflot, J. Errard, G. Fabbian, C. Feng (**corresponding author**), D. Flanigan, A. Gilbert, W. Grainger, N.W. Halverson, M. Hasegawa, K. Hattori, M. Hazumi, W.L. Holzapfel, Y. Hori, J. Howard, P. Hyland, Y. Inoue, G.C. Jaehnig, A. Jaffe, B. Keating, Z. Kermish, R. Keskitalo, T. Kisner, M. Le Jeune, A.T. Lee, E. Linder, E. M. Leitch, M. Lungu, F. Matsuda, T. Matsumura, X. Meng, N.J. Miller, H. Morii, S. Moyerman, M.J. Myers, M. Navaroli, H. Nishino, H. Paar, J. Peloton, E. Quealy, G. Rebeiz, C.L. Reichardt, P.L. Richards, C. Ross, I. Schanning, D.E. Schenck, B. Sherwin, A. Shimizu, C. Shimmin, M. Shimon, P. Siritanasak, G. Smecher, H. Spieler, N. Stebor, B. Steinbach, R. Stompor, A. Suzuki, S. Takakura, T. Tomaru, B. Wilson, A. Yadav, O. Zahn. **Physical Re-**

view Letters 113, 021301 (2014) (arXiv:1312.6646) [3].

Gravitational lensing due to the large-scale distribution of matter in the cosmos distorts the primordial Cosmic Microwave Background (CMB) and thereby induces new, small-scale B -mode polarization. This signal carries detailed information about the distribution of all the gravitating matter between the observer and CMB last scattering surface. We report the first direct evidence for polarization lensing based on purely CMB information, from using the four-point correlations of even- and odd-parity E - and B -mode polarization mapped over ~ 30 square degrees of the sky measured by the POLARBEAR experiment. These data were analyzed using a blind analysis framework and checked for spurious systematic contamination using null tests and simulations. Evidence for the signal of polarization lensing and lensing B -modes is found at 4.2σ (stat.+sys.) significance. The amplitude of matter fluctuations is measured with a precision of 27%, and is found to be consistent with the Lambda Cold Dark Matter (Λ CDM) cosmological model. This measurement demonstrates a new technique, capable of mapping all gravitating matter in the Universe, sensitive to the sum of neutrino masses, and essential for cleaning the lensing B -mode signal in searches for primordial gravitational waves.

6.1 Introduction

As Cosmic Microwave Background (CMB) photons traverse the Universe, their paths are gravitationally deflected by large-scale structures. By measuring the resulting changes in the statistical properties of the CMB anisotropies, maps of this gravitational lensing deflection, which traces large-scale structure, can be reconstructed. Gravitational lensing of the CMB has been detected in the CMB temperature anisotropy in several ways: in the smoothing of the acoustic peaks of the temperature power spectrum [49, 50, 51], in cross-correlations with tracers of the large-scale matter distribution [32, 33, 52, 53, 2, 81, 82], and in the four-point correlation function of CMB temperature maps [36, 1, 25, 18].

The South Pole Telescope (SPT) collaboration recently reported a detection of lensed polarization using the cross-correlation between maps of CMB polarization and sub-mm maps of galaxies from *Herschel*/SPIRE [11]. A companion paper to this one has also shown the evidence of the CMB lensing-Cosmic Infrared Background cross-correlation results using POLARBEAR data [12], finding good agreement with the SPT measurements. This cross-correlation is immune to several instrumental systematic effects but the cosmological interpretation of this measurement requires assumptions about the relation of sub-mm galaxies to the underlying mass distribution [83].

In this Letter, we present the first direct evidence for gravitational lensing of the polarized CMB using data from the POLARBEAR experiment. We present power spectra of the lensing deflection field for two four-point estimators using only CMB polarization data, and tests for spurious systematic contamination of these estimators. We combine the two estimators to increase the signal-to-noise of the lensing detection.

6.2 CMB lensing

Gravitational lensing affects CMB polarization by deflecting photon trajectories from a direction on the sky $\mathbf{n} + \mathbf{d}(\mathbf{n})$ to a new direction $\mathbf{n}$. In the flat-sky approximation, this implies that the lensed and unlensed Stokes parameters are related by

$$(Q \pm iU)(\mathbf{n}) = (\tilde{Q} \pm i\tilde{U})(\mathbf{n} + \mathbf{d}(\mathbf{n})), \quad (6.1)$$

where $\tilde{Q}$ or $\tilde{U}$ denotes a primordial Gaussian CMB polarization map, Q and U are the observed Stokes parameters, and $\mathbf{d}(\mathbf{n})$ is the deflection angle. The CMB polarization fields defined in Eq. (6.1) are rotation-invariant under the transformation $e^{\pm 2i\phi}$ and can be decomposed into electric- (E -) and magnetic-like (B -) modes [4].

Taylor expanding Eq. (6.1) to first order in the deflection angle reveals that the off-diagonal elements of the two-point correlation functions of E - and B -modes are proportional to the lensing deflection field, $d(\mathbf{n})$. Quadratic estimators take advantage of this feature to measure CMB lensing [19, 26, 20]. The two lensing

quadratic estimators for CMB polarization are:

$$d_{EE}(\mathbf{L}) = \frac{A_{EE}(L)}{L} \int \frac{d^2\mathbf{l}}{(2\pi)^2} E(\mathbf{l})E(\mathbf{l}') \frac{C_l^{EE} \mathbf{L} \cdot \mathbf{l}}{\hat{C}_l^{EE} \hat{C}_{l'}^{EE}} \cos 2\phi_{\mathbf{l}\mathbf{l}'}, \quad (6.2)$$

and

$$d_{EB}(\mathbf{L}) = \frac{A_{EB}(L)}{L} \int \frac{d^2\mathbf{l}}{(2\pi)^2} E(\mathbf{l})B(\mathbf{l}') \frac{C_l^{EE} \mathbf{L} \cdot \mathbf{l}}{\hat{C}_l^{EE} \hat{C}_{l'}^{BB}} \sin 2\phi_{\mathbf{l}\mathbf{l}'}. \quad (6.3)$$

In Eqs. (6.2, 6.3), $\mathbf{l}$, $\mathbf{l}'$, and $\mathbf{L}$ are coordinates in Fourier space with $\mathbf{L} = \mathbf{l} + \mathbf{l}'$. The angular separation between $\mathbf{l}$ and $\mathbf{l}'$ is $\phi_{\mathbf{l}\mathbf{l}'}$, C_l^{EE} is the theoretical lensed power spectrum, $\hat{C}_l^{EE}$ and $\hat{C}_l^{BB}$ are lensed power spectra with experimental noise. The estimators are normalized by $A_{EE}(L)$ and $A_{EB}(L)$ so that they recover the input deflection power spectrum [20].

The power spectrum of these estimators is:

$$\begin{aligned} \langle d_\alpha(\mathbf{L}) d_\beta^*(\mathbf{L}') \rangle &= (2\pi)^2 \delta(\mathbf{L} - \mathbf{L}') (C_L^{dd} + N_{\alpha\beta}^{(0)}(L)) \\ &+ \text{higher-order terms).} \end{aligned} \quad (6.4)$$

Here, C_L^{dd} is the deflection power spectrum and $N_{\alpha\beta}^{(0)}$ is the lensing reconstruction noise, α and β are chosen from $\{EE, EB\}$, however we do not use $\alpha = \beta = EE$ as our focus is on the direct probe of CMB lensing represented by the conversion of E -to- B patterns. The BB estimator also probes B -modes, but it does not make a substantial contribution to the deflection power spectrum [20], so it is not used in this work. The four-point correlation function takes advantage of the fact that gravitational lensing converts Gaussian primary anisotropy to a non-Gaussian lensed anisotropy. When calculating this non-Gaussian signal, however, there is a ‘‘Gaussian bias’’ term $N^{(0)}$ which is the disconnected part in the four-point correlation that has to be subtracted. The Gaussian bias is zero when $\alpha \neq \beta$ (i.e., $\langle d_{EE}(\mathbf{L}) d_{EB}^*(\mathbf{L}') \rangle$) because $\langle E(\mathbf{l})B(\mathbf{l}') \rangle = 0$ under the assumption of parity invariance. However, the Gaussian bias is much larger than the lensing power spectrum in the $\alpha = \beta$ case. The Gaussian bias can be estimated, and removed, in several ways [36, 25, 18]; the methodology employed in this Letter is described in the Data Analysis section.

6.3 Data Analysis

The POLARBEAR experiment [84] is located at the James Ax Observatory in Northern Chile on Cerro Toco at West longitude $67^{\circ}47'10.4''$, South latitude $22^{\circ}57'29.0''$, elevation 5.20 km. The 1,274 polarization-sensitive transition-edge sensor bolometers are sensitive to a spectral band centered at 148 GHz with 26% fractional bandwidth [85]. The 3.5 meter aperture of the telescope primary mirror produces a beam with a $3.5'$ full width at half maximum (FWHM). Three approximately $3^{\circ} \times 3^{\circ}$ fields centered at right ascension and declination (23h02m, -32.8°), (11h53m, -0.5°), (4h40.2m, -45.0°), referred to as “RA23”, “RA12”, and “RA4.5”, were observed between May 2012 and June 2013. The patch locations are chosen to optimize a combination of low dust contrast, availability throughout the day, and overlap with other observations for cross-correlation studies.

The time-ordered data are filtered and binned into sky maps with $2'$ pixels. Observations of the same pixel are combined using their inverse-noise-variance weight estimated from the time-ordered data. All power spectra are calculated following the MASTER method [77]. We construct an apodization window from a smoothed inverse variance weight map. Pixels with an apodization window value below 1% of the peak value are set to zero, as are pixels within $3'$ of sources in the Australia Telescope 20 GHz Survey [86]. Q and U maps are transformed to E and B maps using the pure-B transform [79].

We reconstruct the lensing deflection field by applying the two estimators in Eqs. (6.2, 6.3) to the sky maps for $l, l' \in \{500, 2700\}$. In these estimators, C_l^{EE}, C_l^{BB} are calculated using CAMB [5] for the WMAP-9 best-fit cosmological model. The theoretical deflection power spectrum, which is used in simulations, is estimated with CAMB as well. We calculate power spectra for these reconstructions with the requirement that B -mode information is included, thus there are two estimates of the lensing power spectrum: $\langle d_{EE} d_{EB}^* \rangle$ and $\langle d_{EB} d_{EB}^* \rangle$, hereafter referred to as $\langle EEEB \rangle$ and $\langle EBEB \rangle$ respectively. Intuitively, these two four-point correlation functions can be split into a product of two two-point correlations, EE or EB , each of which is proportional to a deflection field (dark matter distribution) on the sky. So these four-point correlation functions estimate the squared

deflection field which is proportional to the deflection power spectrum. The first estimator $\langle EEEB \rangle$, which we will refer to as the cross-lensing estimator, is nearly free of Gaussian bias. The second estimator, $\langle EBEB \rangle$, requires calculation and removal of the large Gaussian bias [1, 36, 25, 18]. The unbiased, reconstructed lensing power spectrum is calculated as follows:

$$C_L^{dd} = (\langle d(\mathbf{L})d^*(\mathbf{L}) \rangle - N_L^{(0)})/T_L, \quad (6.5)$$

where both the Gaussian bias $N_L^{(0)}$ and the transfer function T_L are calculated using simulations. The mean estimated deflection is subtracted from the reconstructions and the realization-dependent Gaussian bias is subtracted for our final results.

We create 500 simulated lensed and unlensed maps to estimate the Gaussian bias and establish the lensing transfer function. The lensed and unlensed simulations are used in calculations to estimate the lensing amplitude and to test the null hypothesis of no lensing, respectively. In the following context, “lensed” or “unlensed” refers to the case with or without lensing sample variance. We create map realizations of the theoretical spectra calculated by CAMB. In the lensed case, map pixels are displaced following Eq. (6.1) to obtain lensed maps. We convolve each realization by the measured beam profile and filter transfer function, and add noise based on the observed noise levels in the polarization maps.

We estimate the Gaussian bias by estimating the lensing power spectrum from a suite of unlensed simulated maps. The finite area of the POLARBEAR fields results in a window function that couples to large-scale modes, biasing them at $l < 300$. This low- l bias has also been seen in temperature lensing reconstructions [87, 88]. After verifying with simulations that it is proportional to the lensing power spectrum, we correct this bias by calculating a transfer function derived from the ratio of the average simulated reconstructed lensing spectrum to the known input spectrum for $l < 300$. This transfer function produces only 0.2σ difference in the overall significance of the two lensing estimators $\langle EEEB \rangle$ and $\langle EBEB \rangle$. We validate the lensing reconstruction by correlating the estimated deflection fields from lensed map realizations with the known input deflection field. All the spectra for all patches and estimators agree with the input lensing power spectra.

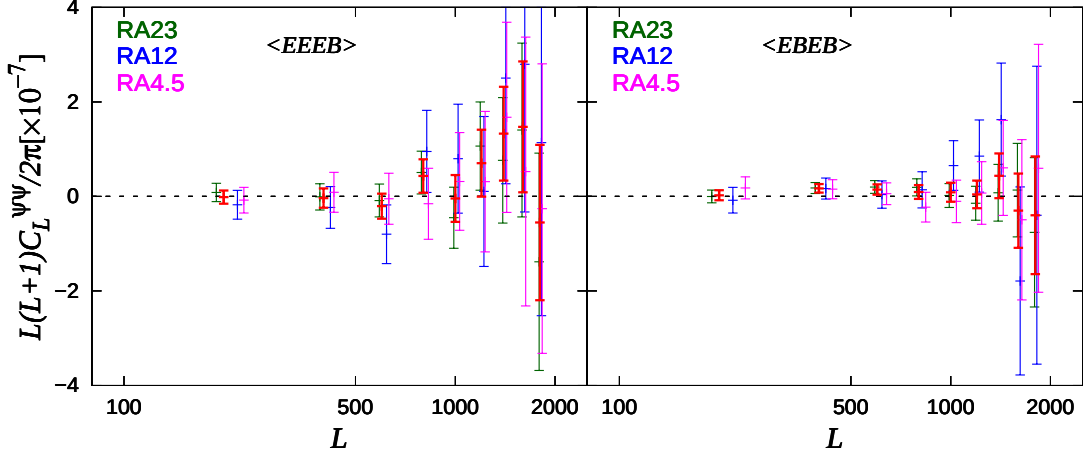


Figure 6.1: Curl null power spectra for each of the three patches for the $\langle EEEB \rangle$ and $\langle EBEB \rangle$ estimators. The patch-combined curl null power spectra are shown in red for the two lensing estimators. All the curl null power spectra are consistent with zero.

6.4 Correlations between lensing estimators

Assuming CMB polarization is lensed, the two lensing estimators $\langle EEEB \rangle$ and $\langle EBEB \rangle$ make a correlated measurement of the lensing power spectrum. Monte Carlo simulations can precisely estimate these correlations [89]. We produce 500 simulated lensing reconstructions for each lensing estimator, for each patch, and this correlation information is used to combine the two lensing estimators.

The covariance matrix between two band-powers is defined as

$$\mathbf{C}_{AB} = \langle (C_A^{\text{sim}} - \bar{C}_A^{\text{sim}})(C_B^{\text{sim}} - \bar{C}_B^{\text{sim}}) \rangle; \quad (6.6)$$

here the combined band-power is $C_A = (C_{\text{channel } 1}, C_{\text{channel } 2}, \dots)$ and each $C_{\text{channel } X}$ is co-added from simulations of all patches, with channel X either being $\langle EEEB \rangle$ or $\langle EBEB \rangle$ and A or B being the index of the band-power. The lensing amplitude A is constructed as

$$A = \frac{\sum_{AB} C_A^{(\text{th})} \mathbf{C}_{AB}^{-1} C_B^{(\text{obs})}}{\sum_{AB} C_A^{(\text{th})} \mathbf{C}_{AB}^{-1} C_B^{(\text{th})}} \quad (6.7)$$

using POLARBEAR data (obs) and the WMAP-9 best-fit Λ CDM model (th). The variance of A is

$$(\Delta A)^2 = \frac{1}{\sum_{AB} C_A^{(\text{th})} \mathbf{C}_{AB}^{-1} C_B^{(\text{th})}}, \quad (6.8)$$

and the significance of the lensing detection is $A/\Delta A$.

6.5 Estimation of systematic uncertainties

Systematic effects can generate spurious signals which could mimic the ones we want to probe. The statistical uncertainty of our measurements, which are $\Delta A = 0.30(0.47)$ for the unlensed (lensed) results, would overestimate the significance of our measurement if these systematic effects are neglected. We simulate the effect of measured instrument non-idealities and check the data for internal consistency and evidence of systematic instrumental errors using null tests. Leakage from temperature to polarization is constrained to be less than 0.5% by correlating temperature maps with polarization maps. A 0.5% leakage from temperature-to-polarization in maps was simulated and found to introduce an error of $\Delta A = \pm 0.10(\pm 0.13)$ into the unlensed (lensed) simulations. Polarized foregrounds are estimated based on models from the South Pole Telescope [90] assuming 5% polarization fraction and constant polarization angle [91]. This contamination was simulated and found not to bias the lensing estimators but it does increase the variance by an amount of $\Delta A = \pm 0.08(\pm 0.14)$ in unlensed (lensed) simulations.

We analyzed calibration and beam model uncertainty using lensed simulations. The beam model uncertainty is estimated from uncertainty in the point-source-derived beam-smoothing correction, and the variation in that correction across each field. We used the 1σ -bounds of the beam model as a simulated beam error and found that this created a change $\Delta A = {}^{+0.19}_{-0.16}$. Absolute calibration error exists due to sample variance in the calibration to Λ CDM (4% including beam uncertainties), uncertainty in the pixel polarization efficiency (4% upper bound), and uncertainty in the analysis transfer function (4% upper bound), where all uncertainties are quoted in terms of their effect on C_ℓ^{BB} since these are conservative limits for error on C_ℓ^{EB} and C_ℓ^{EE} [13]. We take 10% as a bound on the calibration uncertainty, this corresponds to a calibration uncertainty of $\Delta A = {}^{+0.22}_{-0.18}$ in A . The

total systematic error is $\Delta A = \pm 0.13^{+0.35}_{-0.31}$ for unlensed (lensed) simulations.

Null tests specific to the four-point lensing estimators are also examined. Deflection fields for different patches should be uncorrelated and this is used to test the lensing signals for potential contamination. We define a “swap-patch” lensing power spectra $C_L^{dd,\text{null}} = \langle d_{\text{patch } 1}(\mathbf{L}) d_{\text{patch } 2}^*(\mathbf{L}) \rangle$ to test for contamination common to different patches [36]. The deflection vector field can be decomposed into both gradient and curl components, of which only the gradient component is sourced by gravitational lensing (to leading order). The curl power spectrum $C_L^{\psi\psi}$ ’s consistency with zero is thus another check of data robustness [92]. While instrumental systematics could, in principle, mimic a lensing-like remapping of the CMB, such effects are generically expected to produce both gradient and curl-like deflections. A measurement of $C_L^{\psi\psi}$ is thus a sensitive test for instrumental systematics. Curl estimators are constructed by replacing $\mathbf{L} \cdot \mathbf{l}$ by $\mathbf{L} \times \mathbf{l}$ in Eqs. (6.2, 6.3). For each of the null power spectra tests, a χ^2 statistic is calculated assuming a null (zero signal) model. The probabilities to exceed the observed χ^2 values are consistent with a uniform distribution from zero to one; the lowest PTE out of 15 tests (which include 9 swap-patch null and 6 curl null tests) is 8%. For the curl null tests, the results are shown in Fig. 6.1.

As a further systematic check, parallel work shows that the mass distribution information seen from the lensing reconstructions in this work is strongly correlated with cosmic infrared background maps from the *Herschel* satellite [12].

In this work, for deflection power spectrum calculations, we adopted a blind analysis framework, whereby deflection power spectra were not viewed until the data selection and the analysis pipeline were established using realistic instrumental noise properties.

6.6 Results

We present the polarization lensing power spectrum measurements for each of the three POLARBEAR patches and the two *B*-mode estimators $\langle EEEB \rangle$ and $\langle EBEB \rangle$ in Fig. 6.2. The uncertainties in these band-powers do not include sample

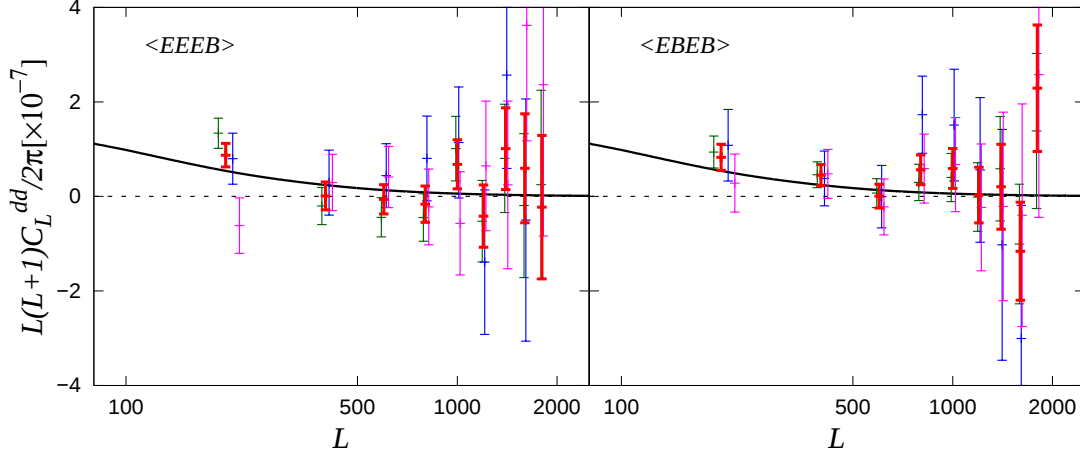


Figure 6.2: Measured polarization lensing power spectra for each of POLAR-BEAR’s three patches, for both lensing estimators $\langle EEEB \rangle$ (left) and $\langle EBEB \rangle$ (right). The lensing signal predicted by the Λ CDM model is shown as the solid black curve. The measured lensing power spectra are shown for each patch in dark green (RA23), blue (RA12) and magenta (RA4.5), respectively and are offset in L slightly for clarity. The patch-combined lensing power spectrum is shown in red.

variance, that is, they represent the no lensing case. Fig. 6.3 shows the patches co-added, and the estimators $\langle EEEB \rangle$ and $\langle EBEB \rangle$ combined. The left panel does not assume the existence of lensing, and we measure a lensing amplitude of $1.37 \pm 0.30 \pm 0.13$, where the errors are statistical and systematic, respectively (this amplitude is normalized to the expected WMAP-9 Λ CDM value). The rejection of the null hypothesis has a significance of 4.6σ statistically and 4.2σ combining statistical and systematic errors in quadrature. Without using EE reconstruction to aid in the measurement of E -to- B conversion, the lensing signal is detected at 3.2σ significance statistically.

The right panel of Fig. 6.3 assumes the predicted amount of gravitational lensing in the Λ CDM model. In this case, the $\langle EEEB \rangle$ and $\langle EBEB \rangle$ estimators are correlated, which changes the optimal linear combination of the two, and requires that lensing sample variance be included in the band-power uncertainties. Under this assumption, the amplitude of the polarization lensing power spectrum is measured to be $A = 1.06 \pm 0.47^{+0.35}_{-0.31}$. The last term gives an estimate of systematic error. Since A is a measure of power and depends quadratically on the amplitude of the matter fluctuations, we measure the amplitude with 27% error.

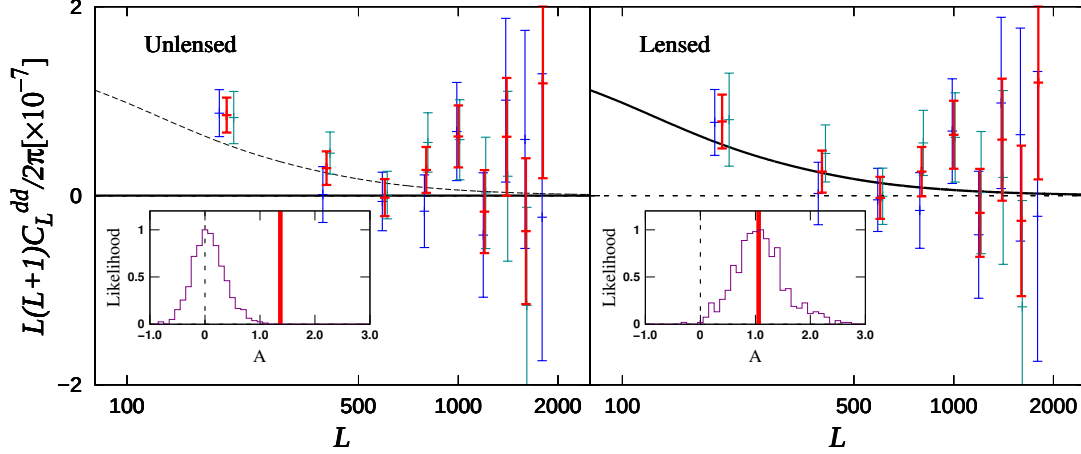


Figure 6.3: Polarization lensing power spectra co-added from the three patches and two estimators are shown in red. The lensing signal predicted by the Λ CDM model is shown as the dashed black curve in the left panel and the solid black curve in the right panel, respectively. The polarization lensing power spectrum $\langle EEEB \rangle$ is in blue and $\langle EBEB \rangle$ dark green. *Left:* A 4.2σ rejection of the null hypothesis of no lensing. These data indicate a lensing amplitude $A = 1.37 \pm 0.30 \pm 0.13$ normalized to the fiducial Λ CDM value. *Right:* The same data, assuming the existence of gravitational lensing to calculate error bars, including sample variance and including the covariance between $\langle EEEB \rangle$ and $\langle EBEB \rangle$. In this case, the lensing amplitude is measured as $A = 1.06 \pm 0.47^{+0.35}_{-0.31}$, corresponding to 54% uncertainty on the C_L^{dd} power spectrum (27% uncertainty on the amplitude of matter fluctuations). The histograms of the amplitudes A from 500 unlensed and lensed simulations are shown in the inset boxes.

The measured signal traces all the B -modes at sub-degree scales. This signal is presumably due to gravitational lensing of CMB, because other possible sources, such as gravitational waves, polarization cosmic rotation [93] and patchy reionization are expected to be small at these scales.

6.7 Conclusions

We report the evidence for gravitational lensing, including the presence of lensing B -modes, directly from CMB polarization measurements. These measurements reject the absence of polarization lensing at a significance of 4.2σ . We have performed null tests and have simulated systematics errors using the measured properties of our instrument, and we find no significant contamination. Our measurements are in good agreement with predictions based on the combination of the Λ CDM model and basic gravitational physics. This work represents an early step in the characterization of CMB polarization lensing after the precise temperature lensing measurement from Planck. The novel technique of polarization lensing will allow future experiments to go beyond Planck in signal-to-noise and scientific returns. Future measurements will exploit this powerful cosmological probe to constrain neutrino masses [83] and de-lens CMB observations in order to more precisely probe B -modes from primordial gravitational waves.

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Chapter 6, in full, is a reprint of material as it appears on Physical Review Letters 113, 021301 (2014) (arXiv:1312.6646) [3], as written by the author of this dissertation. “Measurement of the Cosmic Microwave Background Polarization Lensing Power Spectrum with the POLARBEAR experiment”, POLARBEAR Collaboration, P.A.R. Ade, Y. Akiba, A.E. Anthony, K. Arnold, M. Atlas, D. Barron, D. Boettger, J. Borrill, S. Chapman, Y. Chinone, M. Dobbs, T. Elleflot, J. Errard, G. Fabbian, C. Feng, D. Flanigan, A. Gilbert, W. Grainger, N.W. Halverson, M. Hasegawa, K. Hattori, M. Hazumi, W.L. Holzapfel, Y. Hori, J. Howard, P. Hyland, Y. Inoue, G.C. Jaehnig, A. Jaffe, B. Keating, Z. Kermish, R. Keskitalo, T. Kisner, M. Le Jeune, A.T. Lee, E. Linder, E. M. Leitch, M. Lungu, F. Matsuda, T. Matsumura, X. Meng, N.J. Miller, H. Morii, S. Moyerman, M.J. Myers, M. Navaroli, H. Nishino, H. Paar, J. Peloton, E. Quealy, G. Rebeiz, C.L. Reichardt, P.L. Richards, C. Ross, I. Schanning, D.E. Schenck, B. Sherwin, A. Shimizu, C.

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Chapter 7

Cross-correlation calculation

The cross correlation between CMB lensing and large-scale structure can be used to constrain astrophysical models. This type of correlation is not only systematics free, but it is a good observable to constrain cosmological models, such as galaxy bias and redshift distribution. In this chapter we discuss the detailed theoretical calculation of this correlation.

7.1 CMB lensing-large scale structure cross correlation

We will derive the exact formula for the brute-force calculations. The initial conditions are given by inflation. The correlation function of the scalar perturbation $A_s(\mathbf{k})$ is given by

$$\langle A_s(\mathbf{k})A_s(\mathbf{k}') \rangle = (2\pi)^3 P(k) \delta(\mathbf{k} + \mathbf{k}'). \quad (7.1)$$

In this equation, the primordial power spectra is $P(k)$, which is parametrized as $\ln \frac{P(k)}{P(k_0)} = (n-1) \ln \frac{k}{k_0} + \alpha/2 (\ln \frac{k}{k_0})^2 + \dots$ according to the CAMB [5] code, and in this equation, k_0 is the pivot scale. If the running index α is neglected, then the power spectrum is reduced to the ordinary one

$$P_{si}(k) = A \left[\frac{k}{k_0} \right]^{n-1}, \quad (7.2)$$

where $n \sim 1$ and $A \sim 2.4 \times 10^{-9}$ which is related to the power $P(k_0)$. A plane wave can be expanded in harmonic space as

$$e^{i\mathbf{k} \cdot \chi \mathbf{n}} = \sum_{lm} 4\pi i^l j_l(k\chi) Y_{lm}^*(\hat{k}) Y_{lm}(\mathbf{n}), \quad (7.3)$$

where $j_l(k)$ is the spherical Bessel function. The relationship between the projected gravitational potential (Φ_{lm}) and three-dimensional ($\Phi(\mathbf{k})$) modes is

$$\begin{aligned} \Phi(\mathbf{n}) &= \sum_{lm} \Phi_{lm} Y_{lm}(\mathbf{n}) \\ &= \int \frac{d^3k}{(2\pi)^3} \Phi(\mathbf{k}) \sum_{lm} 4\pi i^l j_l(k) Y_{lm}^*(\hat{k}) Y_{lm}(\mathbf{n}), \end{aligned} \quad (7.4)$$

After simplifying the above equation, we get the specific form for the relationship between Φ_{lm} and $\Phi(\mathbf{k})$

$$\Phi_{lm} = \int \frac{d^3k}{(2\pi)^3} \Phi(\mathbf{k}) 4\pi i^l j_l(k) Y_{lm}^*(\mathbf{k}). \quad (7.5)$$

This equation will be used to derive the cross-correlation power spectrum. The Poisson equation relates the gravitational potential and the density contrast via

$$k^2 \Phi(\mathbf{k}) = \frac{3H_0^2 \Omega}{2a} \delta(\mathbf{k}), \quad (7.6)$$

In this equation, Ω is the matter density fraction, H_0 Hubble constant and a the scale factor. The late time evolution of the lensing potential and the density are

$$\Phi(\mathbf{k}, z) = \frac{3H_0^2 \Omega}{2ak^2} A_s(\mathbf{k}) \delta^{\text{MT}}(\mathbf{k}, z) \quad (7.7)$$

and the initial condition is included in the density contrast as

$$\delta(\mathbf{k}, z) = A_s(\mathbf{k}) \delta^{\text{MT}}(\mathbf{k}, z). \quad (7.8)$$

The power spectra of the time-dependent potential and density contrast are

$$\langle \Phi(\mathbf{k}, z) \Phi(\mathbf{k}', z) \rangle = (2\pi)^3 P_\Phi(k, z) \delta(\mathbf{k} + \mathbf{k}') \quad (7.9)$$

and

$$\langle \delta(\mathbf{k}, z) \delta(\mathbf{k}', z) \rangle = (2\pi)^3 P_\delta(k, z) \delta(\mathbf{k} + \mathbf{k}'). \quad (7.10)$$

The CAMB code can calculate radial transfer functions $\Delta(k, z)$ for cold dark matter, baryon, neutrino and photons. So we define two variables $S_\delta(k, z) = \Delta(k, z)$ and $S_\psi(k, z) = \frac{3}{2}H_0^2\Omega_m a^{-1}k^{-2}\Delta(k, z)$. With these definitions, the power spectra of potential and density can be expressed in terms of the transfer function. They are

$$\begin{aligned} P_\delta &= \frac{2\pi^2}{k^3} P_{si}(k) \Delta(k, z)^2 \\ &= \frac{2\pi^2}{k^3} P_{si}(k) S_\delta^2(k, z) \end{aligned} \quad (7.11)$$

and

$$\begin{aligned} P_\Phi &= \frac{2\pi^2}{k^3} P_{si}(k) \left(\frac{3}{2}H_0^2\Omega_m\right)^2 a^{-2} k^{-4} \Delta(k, z)^2 \\ &= \frac{2\pi^2}{k^3} P_{si}(k) S_\psi^2(k, z). \end{aligned} \quad (7.12)$$

These power spectra will be used for the derivation of the cross-correlation.

Define $D(\chi) = -2(1/\chi - 1/\chi_{\text{CMB}})$. χ is the comoving distance. The definition of the gravitational lensing potential is [94]

$$\psi(\mathbf{n}) = -2 \int d\chi \Phi(\chi \mathbf{n}) \left(\frac{1}{\chi} - \frac{1}{\chi_{\text{CMB}}} \right) = \int d\chi \Phi(\chi \mathbf{n}) D(\chi). \quad (7.13)$$

Large-scale structure map, e.g., galaxies, can be defined as [32, 33, 52]

$$g(\mathbf{n}) = \frac{\int d\chi b_g \mathcal{N}(\chi) \delta(\chi \mathbf{n})}{\int d\chi \mathcal{N}(\chi)}, \quad (7.14)$$

here $\mathcal{N}(\chi)$ is the redshift distribution of this large-scale structure and b_g is the galaxy bias which relates the baryonic matter density to the dark matter density. It in general changes with respect to different scales and redshifts, but to first order, it is always assumed to be a constant. The most precise form of galaxy bias usually resorts to N-body simulations. By using the projection Eq. (7.5), we can do the spherical harmonic transformation for the gravitational lensing potential and large-scale structure. The lensing potential modes are given by

$$\psi_{lm} = (4\pi) i^l \int d\chi \int \frac{d^3k}{(2\pi)^3} D(\chi) \Phi(\mathbf{k}) j_l(k\chi) Y_{lm}^*(\hat{k}), \quad (7.15)$$

and the large-scale structure mode is

$$g_{lm} = \frac{(4\pi)i^l}{\int d\chi \mathcal{N}(\chi)} \int d\chi \int \frac{d^3k}{(2\pi)^3} b_g \mathcal{N}(\chi) \delta(\mathbf{k}) j_l(k\chi) Y_{lm}^*(\hat{k}). \quad (7.16)$$

We then define two functions as

$$\Delta_l^\psi(k) = \int d\chi D(\chi) S_\psi(k, z) j_l(k\chi), \quad (7.17)$$

and

$$\Delta_l^\mathcal{N}(k) = \frac{1}{\int d\chi \mathcal{N}(\chi)} \int d\chi b_g \mathcal{N}(\chi) S_\delta(k, z) j_l(k\chi). \quad (7.18)$$

Finally we calculate the power spectrum of the cross-correlation from definition and get

$$\begin{aligned} C_l^{\psi g} &= \langle \psi_{lm}^* g_{lm} \rangle \\ &= \frac{1}{\int d\chi \mathcal{N}(\chi)} \frac{2}{\pi} \int d\chi \int d\chi' \int k^2 dk b_g \frac{3H_0^2 \Omega}{2ak^2} D(\chi) \mathcal{N}(\chi') P_\delta(k) j_l(k\chi) j_l(k\chi') \\ &= 4\pi \int d \ln k P_{si} \Delta_l^\psi(k) \Delta_l^\mathcal{N}(k). \end{aligned} \quad (7.19)$$

This power spectrum can be directly calculated. However, the brute-force method is very time-consuming because the spherical Bessel function computation in equations 7.17 and 7.18) costs a lot of computing time. Fortunately, the Limber approximation, $k \sim l/\chi$ can be applied then the orthogonality of the Bessel functions greatly reduces the power spectrum to

$$C_l^{\psi g} \simeq \frac{3H_0^2 \Omega}{2} \frac{1}{\int d\chi \mathcal{N}(\chi)} \int d\chi b_g (1+z) D(\chi) \mathcal{N}(\chi) \frac{1}{k^2} \frac{1}{\chi^2} P_\delta\left(\frac{l}{\chi}\right). \quad (7.20)$$

This form of the power spectrum can be very easily calculated. We numerically validated that the Limber approximation gives a solution which is very close the exact one.

As we have seen from previous chapter, the cross-correlation between CMB lensing and other large-scale structure tracer could boost the signal to noise significantly. For POLARBEAR, we also applied this technique to our first season data. We chose Herschel 500 μm cosmic infrared background data to do the cross-correlation because at this wavelength, more structure is captured and the signal to noise is much better than the other Herschel bands, such as 250 μm and 350 μm .

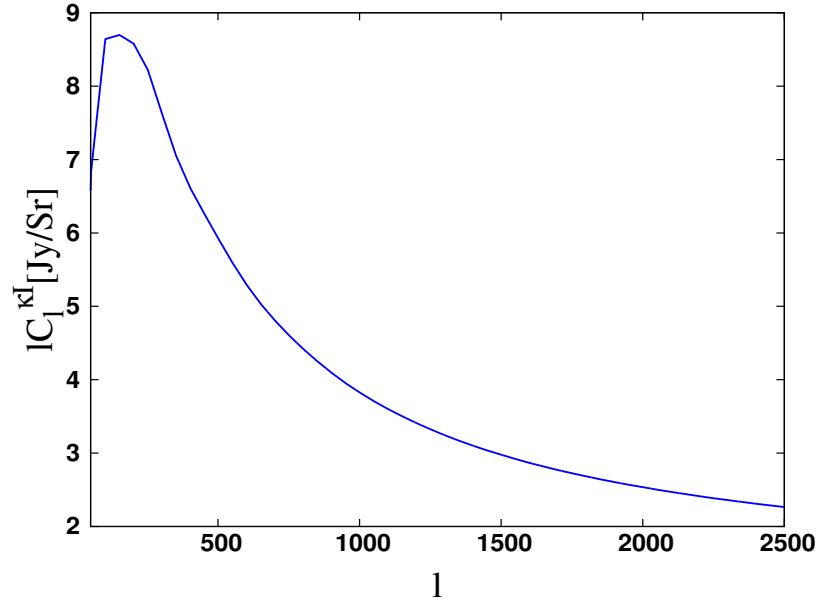


Figure 7.1: The cross correlation between the lensing convergence and CIB field map. The convergence field is related the lensing potential by $\kappa(\mathbf{n}) = -\frac{1}{2}\nabla^2\psi(\mathbf{n})$.

The non-linear correction from halo-fit is also made to $P_\delta(k)$. The primordial scalar curvature perturbations are evaluated at the pivot scale $k_0 = 0.002 \text{ Mpc}^{-1}$. The redshift distribution $\mathcal{N}(\chi)$ is given in [95]. As an example, the power spectrum of this cross-correlation is shown in Figure 7.1.

Chapter 8

Conclusion and Outlook

Since the first application to WMAP data in 2004 [32], CMB lensing has undergone tremendous developments during the last a few years. In 2011, the Atacama Cosmology Telescope (ACT) made the first CMB-only detection of CMB lensing from its temperature data at 4σ significance [36]. In 2013, South Pole Telescope Polarimeter (SPTPol) detected lensing B-modes at 7.7σ significance by cross-correlating its polarization lensing reconstruction with Herschel cosmic infrared background data [11]. In 2014, the POLARBEAR experiment, for the first time measured, the B-mode power spectrum at the 2σ significance level [13] and for the first time showed the evidence of both polarization lensing and lensing B-modes at 4.2σ using the four-point correlation functions of CMB polarization alone [3]. Notably, the signal measured from the four-point correlation function established a new technique for future polarization lensing studies and for the first time verified that the amount of gravitational deflection is consistent with what is predicted by General Relativity on cosmological scales. This signal traces all the matter content in the Universe so it has the potential to ultimately constrain neutrino masses. The POLARBEAR experiment also confirmed the cross-correlation signal detected by SPTPol at the 4σ confidence level [12].

In the near future, more detectors and more observing time will be available. This will greatly improve the precision of CMB lensing measurements. We expect a near full-sky high resolution lensing map will be achieved and questions about the nature of dark matter, and the masses of neutrino will be better answered.

Based on accurate lensing templates, we also expect the primordial B-model floor will be reached and physics at 10^{16} GeV energy will be tested. On the other hand, astrophysics can benefit from CMB lensing as well. An array of cross correlations can be performed and improved, such as CMB lensing with cosmic shear [96], thermal Sunyaev-Zel'dovich effect [97], and the Lyman- α forest [98], etc. We believe CMB lensing will greatly enrich not only the content of cosmology, but of physics as a whole.

In the near future, in addition to CMB lensing, other secondary effects will also be important. Cosmic birefringence (CB) is such an example. A pseudo-Nambu-Goldstone field (a scalar field) may be coupled with the electromagnetic field, that is, a coupling term of the form $\phi F_{\mu\nu} \tilde{F}^{\mu\nu}$ [99]. A prediction from this assumption is that the CMB's polarization would be rotated following the last scattering epoch. This can be mathematically written as

$$(Q(\mathbf{n}) \pm iU(\mathbf{n})) = (\tilde{Q}(\mathbf{n}) \pm i\tilde{U}(\mathbf{n}))e^{\pm 2i\alpha(\mathbf{n})}, \quad (8.1)$$

where $\alpha(\mathbf{n})$ is the spatial rotation projected on the sky, $Q(\mathbf{n})$ and $U(\mathbf{n})$ are Stokes parameters, and $\mathbf{n}$ is a direction on the sky. Through the CB mechanism, B-modes are also generated similar to gravitational lensing. So this source of B-modes makes primordial B-mode detection even more complicated. Thanks to ground-based small-scale CMB polarization experiments, we are on the right track towards exploring this exotic physics.

To study this effect, we need to estimate the spatial rotation based on Eq. (8.1), however, this equation is not directly invertible because we do not know the underlying polarization $\tilde{Q}(\mathbf{n}) \pm i\tilde{U}(\mathbf{n})$, thus, it is impossible to directly solve this equation. Fortunately, some previous works have already developed the algorithm to deal with this problem [100, 101, 102], and this is the so-called ‘‘quadratic estimator technique’’. For ground-based CMB experiments, the EB correlation will eventually give the best estimate of the CB rotation, so we will restrict the discussion to EB estimator. We use the EB estimator to form

$$\hat{\alpha}(L) \propto \int \frac{d^2\mathbf{l}}{(2\pi)^2} E(\mathbf{l}) B(\mathbf{l}') F_{EB}(\mathbf{l}, \mathbf{l}'). \quad (8.2)$$

In this equation, the particular form of the filter $F(\mathbf{l}, \mathbf{l}')$ is given in [100].

Currently the sensitivity to spatial rotation still hasn't reached the required noise level to make a detection so we don't know exactly what the amplitude of its power spectrum, instead we use this rotation estimation as a systematic test and potentially de-rotate the polarization observations [103].

Patchy reionization is another example of physics which can be probed by small scale CMB polarimeter such as POLARBEAR. It affects the optical depth anisotropically, so the optical depth becomes direction-dependent, i.e., $\tau(\mathbf{n})$ instead of a single number τ . The direct impact on the CMB polarization is an inhomogeneous suppression at different directions

$$(Q(\mathbf{n}) \pm iU(\mathbf{n})) = (\tilde{Q}(\mathbf{n}) \pm i\tilde{U}(\mathbf{n}))e^{-\delta\tau(\mathbf{n})}. \quad (8.3)$$

Mathematically this equation is very similar to Eq. 8.1, so we can also use the quadratic estimator to study $\tau(\mathbf{n})$. The calculation shows that the filter of $\tau(\mathbf{n})$ is not orthogonal to the lensing EB filter, so the reconstruction of $\tau(\mathbf{n})$ is biased. In [104], both lensing and patchy reionization effects are included in the simulations and it is found that even a small portion of the lensing signal can significantly bias the patchy reionization signal. Although the separation of the lensing-patchy reionization could be technically done, it greatly reduces the minimum signal-to-noise ratio for a detection. However, so far we only limit the discussion with the CMB regime, in the future, the noisy patchy reionization reconstruction from CMB polarization would be cross-correlated with other observations, such as 21 cm [105], and a detection of the patchy reionization is promising.

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