Title

A Long-Run Cost Function For Rail Rapid Transit Properties

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A LONG-RUN COST FUNCTION FOR RAIL RAPID TRANSIT PROPERTIES

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A. BART Operating Costs and the Case for Inference from Other Properties

Calculating the incremental costs of providing service on BART is difficult because of the paucity of data and the fact that it is relevant to a period of the system's operation which may not be typical of future operation. As detailed in an earlier paper [12], there are many current expenses borne by the BART District which relate to the youth of the technology and its application in this system. Precisely where the District is operating on a technological or managerial "learning curve" is difficult to assess from direct observation of current activities. The most thorough analysis possible at this time, then, would be one which relied both on BART data and also data from the universe of the rapid transit industry as it exists elsewhere.

In the analysis and estimation below we ask the question: what will happen to long-run costs of BART with optimal adjustment of the fixed factor. No property we observe will necessarily have made that optimal adjustment,yet we shall devise procedures for inference about that question from their experience.

We also want to know if costs will come down as BART is expanded?

The data which was available at the time of the earlier analysis of BART costs was basically time-series observations on the expenses and output of the system over nine months of early operation. Were this data not permeated by the learning-curve problems described above, it might be interpreted as observations on a short-run cost curve; that is, the basic plant (defined by the rights of way and the associated track and structures) was fixed, and as the system increased its level of operation, different points on this short-run cost relationship were explored. Taking the interpretation literally, a short-run total cost function was estimated which related the system output (measured by vehicle-miles, VM) and the level of expenses incurred by the District. The relationship between output and expenses was explored statistically in several expense categories. Assuming that the relationship between output and cost was a linear one[1] allowed the following relationships to be established (see [9] for details):

Tabulation I Monthly Expense Functions

Maintenance of way and structures = $42,594 + .065 VM

Maintenance of support equipment = $141,534 + .172 VM

Maintenance of rolling stock = $148,145 + .169 VM

Other line-haul expenses = $32,848 + .077 VM

Additionally, vehicle attendant expenses were calculated to be roughly $.046 per vehicle-mile when trains are 10 cars in length. Other operating expenses, such as station expenses and the administration costs of the system were assumed to be invariant with respect to system output over the relevant range. The cost of additional vehicles[2] was calculated to be roughly $.25 per vehicle-mile.

In the range of output at which BART was operating at the time of analysis, an additional vehicle-mile increased costs in the categories described above by roughly $.78. On a per seat-mile basis, then, the incremental costs of BART service on the existing system was roughly 1.1¢. This calculation is detailed in Table I below.

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[1]The paucity of the data did not allow tests of alternative model formulations.

[2]The vehicle was assumed to have a price of $320,000, a life of 25 years, and annual utilization of 100,000 miles. An interest rate of 6% was used in the calculation. Each car has 72 seats.

Table I - Incremental Operating Expenses of the San Francisco Bay Area Rapid Transit District

Category of expense	Incremental cost per veh/mile	per seat/mile
Maintenance of way and structures	$ .065	
Maintenance of support equipment	.172	
Maintenance of rolling stock	.169	
Other line-haul expenses	.077	
Station expenses	0	
Administration	0	
Vehicle attendant expenses	.046	
Total incremental expenses excluding vehicle costs	.529	.0073
Interest on and depreciation of vehicles	.25	
Total incremental expenses including vehicle costs, but excluding capital costs of fixed system	.779	.0108

The size of incremental costs is important when decisions on pricing and subsidy are being considered. Economists claim that incremental or marginal cost pricing will encourage proper resource allocation under certain conditions, in spite of the fact that such prices may not yield sufficient revenue to cover all costs of operation. Extrapolating from the early experience of BART[3], at a level of output of 25,000,000 vehicle-miles annually, the annual total cost of BART service will be roughly as in Table II.

The average cost of providing a seat-mile of service on BART at an output rate of 25,000,000 vehicle-miles annually is thus estimated to be roughly 6.9¢, whereas the incremental cost is roughly 1.1¢ as outlined in Table I. Charging incremental cost would yield far less revenue than that required to finance the system. The property taxpayers of the Bay Area agreed in 1962 to subsidize the construction of the system by a special property tax levy, but required that BART meet operating expenses (defined as the first two categories in Table II) out of fare box revenues. These operating expenses, as can be seen from Table II, will average 2.18¢ per seat-mile at 25,000,000 vehicle-miles per year as compared again with the incremental costs of 1.1¢ per seat-mile. BART must legally charge fares, then, which will be on the average, twice as large as an economist might prescribe. An annual operating subsidy of about $19,000,000 would be necessary in order to charge the incremental cost fares.

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[3]The extrapolation assumed that the linear relationships between output and certain categories of expenses would continue to hold at this larger output level, and that other expenses, such as station expenses, administration, and other forms of overhead would not change.

Table II - Total Operating Expenses
at 25 million car-miles per year

Category of expense	Total annual cost	Average cost/seat-mi.
Operations	$ 32,990,000	$ .0183
Interest on and depreciation of vehicles	6,250,000	.0035
		.0218
Interest on and depreciation of all other capital	83,750,000	.0472
Total operating expense	$122,990,000	.0690

Notes:

The interest rate used was 6 percent

Vehicle life = 25 years

Structure life = 50 years

Land life infinite

Figures are in 1973 dollars.

These calculations rely rather heavily on two assumptions; one, that the early BART cost experience will be relevant at large outputs; and two, that the relationship between increased output and those costs that vary with output are the simple linear ones given in Tabulation 1. Both of these assumptions are somewhat cavalier, but the BART data itself is too sparse to test alternative hypotheses. Data is available, however, on the operating expenses and output of rapid transit systems elsewhere. A pooled time-series and cross-sectional analysis of the experience of these systems allows inference from more data and interpretation of the BART experience.

B. Definition of the Industry

We intend to investigate rail rapid transit systems operating in urban areas on exclusive (or nearly exclusive) rights-of-way. Rapid transit systems have average operating velocities of 20 miles per hour or more. They generally use lighter vehicles capable of higher acceleration than conventional railroad equipment.

There is some cause to suspect heterogeneity in our sample. We have included high-speed trolleys as well as the more conventional heavier cars used in the New York subways. These streetcar lines use President's Conference Committee (PCC) rolling stock. The President's Conference Committee was a committee made up of presidents of U.S. street railway companies which met in 1927 to decide on a mutually acceptable design for a streetcar which was then procured for all the cooperating properties.

Let us assume that urban rapid transit firms produce their services according to a Cobb-Douglas[4] production function

$$Q = A\, L^{\beta_1}\, E^{\beta_2}\, R^{\beta_3}\, T^{\beta_4}$$

Q is annual output measured in million vehicle-miles

L is labor input in annual hours

E is electricity in annual kilowatt-hours

R is rolling stock in vehicles

T is miles of single track

In the short run the properties choose rolling stock, electricity, and labor to minimize costs for a predetermined level of output. In the short run the amount of track they have is fixed and this can be seen as the fixed factor of economic theory[5]. Thus the problem is

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[4] There is probably a case for allowing electricity to be combined with vehicles with an elasticity of substitution other than unity, assumed by the Cobb-Douglas form. A constant-elasticity-of-substitution between vehicles and electricity form could be used. As a practical matter the price of vehicle services is assumed constant over the sample below so the importance of this problem disappears. Precedents for the use of the Cobb-Douglas form in rail industry studies include Klein [7], Borts [1], Griliches [3], Keeler [6]. If we chose a single technology we would observe fixed factor proportions. We have included heavy-car, light-rail and rubber-tired technologies in this study.

$$\min \quad C = wL + p_e E + p_r R + p_t T$$

subject to $T = \bar{T}$

and the production function,

where w, p_e, p_r, and p_t are the unit prices of labor, electricity, "rental" price of rolling stock and track respectively and C is total operating cost.

The La Grangean expression is

$$\ell = wL + p_e E + p_r R + p_t T + \lambda (T - \bar{T}) + \mu (Q - AL^{\beta_1} E^{\beta_2} R^{\beta_3} T^{\beta_4})$$

Differentiating with respect to the choice variables and setting the resulting expressions equal to zero we have:

$$\ell_L = w - \mu \beta_1 Q/L = 0$$

$$\ell_E = p_e - \mu \beta_2 Q/E = 0$$

$$\ell_R = p_r - \mu \beta_3 Q/R = 0$$

Solving the first-order conditions for an interior solution yields

$$L = C_1 Q^{1/\rho} (p_e/p_e)^{\beta_2/\rho} \left(\frac{p_r}{w}\right)^{\beta_3/\rho} T^{-\beta_4/\rho}$$

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[5]This treatment pursues the method of estimating a long-run cost function due to Keller [6]. It does not assume that all properties in the sample are in long-run equilibrium with optimal capital stock.

$$E = C_2 Q^{1/\rho} (w/p_e)^{\beta_1/\rho} (p_r/p_e)^{\beta_3/\rho} T^{-\beta_4/\rho}$$

$$R = C_3 Q^{1/\rho} (w/p_r)^{\beta_1/\rho} (p_e/p_r)^{\beta_2/\rho} T^{-\beta_4/\rho}$$

$$C_1 = [A (\beta_2/\beta_1)^{\beta_2} (\beta_3/\beta_1)^{\beta_3}]^{-1/\rho}$$

$$C_2 = [A (\beta_1/\beta_2)^{\beta_1} (\beta_3/\beta_2)^{\beta_3}]^{-1/\rho}$$

$$C_3 = [A (\beta_1/\beta_3)^{\beta_1} (\beta_2/\beta_3)^{\beta_2}]^{-1/\rho}$$

$$\rho = \sum_{j=1}^{3} \beta_j$$

There is a certain symmetry about these solutions. All factor demand equations are homogeneous of degree zero in factor prices. They all depend on output and the amount of the fixed factor available.

When all the choice variables are at their optimal or cost-minimizing levels the short-run operating cost (SROC) of the property is

$$SROC = p_t T + [C_1+C_2+C_3]\ Q^{1/\rho} w^{\beta_1/\rho} p_e^{\beta_2/\rho} p_r^{\beta_3/\rho} T^{-\beta_4/\rho} \qquad (1)$$

where SROC is in millions of dollars.

Now, we will drop the price of vehicles, p_r, as an independent variable because car prices are determined nationally and do not vary significantly over the cross-section[6]. Furthermore, trends over time are too confounded with qualitative change to measure the price of a constant service accurately and independently of trends in other variables.

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[6] Vehicle replacement costs do differ over time, however. We discuss below our efforts to use the price of vehicles as an independent variable in our equations.

Equation (1) states that there is a fixed component to annual operating cost proportional to the scale of the system, measured by our proxy, miles of track, as well as costs which vary with output. The variable p_t is an annualized price, user cost or "rental" price allowing for amortization of investments in way and structures as well as interest on capital invested and fixed components of operating expenses that vary with system scale[7].

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[7]As a method of estimation we shall first estimate only the maintenance of way and structures and later add on the fixed capital costs. This is necessitated by the poor quality of data on capital costs (track and structures "rental") for the properties involved. The use of book values would be misleading because most of the properties have track and structures on their books at acquisition cost which grossly understates replacement costs. As a practical matter, since these costs enter additively, we have the option of calculating the capital costs for the properties independent of the parameter estimation process.

C. The Data

The data used included observations on 11 North American transit properties. Observations on the price of transit labor in dollars per hour, the price of electricity in dollars per kilowatt-hour, annual operating expenses (less depreciation and interest) in millions of dollars, and output in millions of annual vehicle-miles were generally available for each of these properties' years of operation between 1960 and 1970.

Data was available from nine properties for eleven years in all but one case (Newark in 1960) and on Montreal and PATCO (Lindenwold) for a few recent years. This gave us a time-series of cross-sections consisting of 105 observations[8].

[8]The current names of the organizations which operate the properties are the Metropolitan Transportation Authority (New York), Chicago Transit Authority, Massachusetts Bay Transportation Authority, Toronto Transit Commission, Southeastern Pennsylvania Transportation Authority (SEPTA or Philadelphia), Montreal Urban Community Transit Commission, Port Authority of New York and New Jersey (Port Authority Trans-Hudson or PATH), Cleveland Transit System, Port Authority Transit Commission (Lindenwold or PATCO), the City of Shaker Heights Department of Transportation and Public Service Coordinated Transport in Newark.

Table III

TOTAL ANNUAL OPERATING EXPENSES (LESS INTEREST AND DEPRECIATION [$ x 10^6])

	1960	1961	1962	1963	1964	1965	1966	1967	1968	1969	1970
NEW YORK	215.09	222.39	229.37	239.81	252.33	261.08	267.88	302.04	337.80	362.02	447.78
CHICAGO	29.82	30.42	31.71	31.08	31.40	32.98	34.45	35.50	37.88	43.94	54.61
MBTA	22.45	22.35	22.27	22.40	23.59	23.36	24.59	26.45	29.54	35.05	41.80
TORONTO	4.34	4.54	4.65	5.58	5.93	6.17	11.22	12.66	14.83	17.02	18.55
SEPTA (PHILADELPHIA)	13.87	14.58	14.65	13.77	14.34	14.83	15.52	16.66	17.71	19.30	20.63
MONTREAL							1.40	13.05	13.96	14.51	15.40
PATH (N.Y., N.J.)	7.51	7.72	9.17	10.13	11.28	12.65	13.09	14.93	17.09	17.82	18.90
CLEVELAND	2.24	2.27	2.40	2.67	2.94	2.93	3.10	3.12	3.46	4.18	4.47
PATCO (LINDENWOLD)										3.65	4.34
SHAKER HEIGHTS	1.21	1.28	1.31	1.33	1.39	1.41	1.47	1.52	1.65	1.68	1.87
NEWARK		.693	.624	.626	.629	.675	.652	.660	.744	.859	.988

Source: Institute for Defense Analyses, Economic Characteristics of the Urban Public Transportation Industry (Washington, Department of Transportation and Government Printing Office, 1972). Table 6B2, p. 6-82

Table IV

Annual Vehicle Miles

(Passenger Car-Miles)

(x 10^6)

	1960	1961	1962	1963	1964	1965	1966	1967	1968	1969	1970
NEW YORK	305.15	300.82	304.14	306.09	314.30	314.69	302.03	319.73	339.79	344.57	359.82
CHICAGO	44.63	44.19	43.91	43.82	43.86	44.17	45.44	45.08	44.79	45.62	51.49
MBTA	15.80	15.01	14.74	14.56	14.92	14.63	14.64	14.54	14.29	13.83	13.65
TORONTO	7.1	7.0	7.0	9.0	9.5	9.3	17.8	16.4	20.5	22.7	22.7
SEPTA	17.52	16.77	16.28	14.92	15.38	14.78	14.80	14.78	14.62	14.57	14.59
MONTREAL							4.67	24.21	20.36	19.35	18.37
PATH	5.53	6.11	5.98	6.37	6.23	5.8	7.65	8.67	8.88	9.48	9.25
CLEVELAND	4.7	4.53	4.53	4.47	4.43	4.26	4.20	4.15	4.07	4.81	4.56
LINDENWOLD										2.93	3.67
SHAKER HTS.	1.27	1.27	1.25	1.23	1.25	1.25	1.24	1.23	1.24	1.22	1.23
NEWARK		.64	.63	.64	.61	.64	.61	.60	.59	.59	.60

Source: Same as Table III, Table 6B.5, p. 6 - 84

Table V

Total Miles of Single Track

	1960	1961	1962	1963	1964	1965	1966	1967	1968	1969	1970
NEW YORK	837	841	841	841	841	841	841	846	847	842	842
CHICAGO	204	203	202	202	211	211	211	211	209	243	243
MBTA	160	158	152	151	151	151	151	151	151	151	151
TORONTO	13	13	13	18	18	18	46	46	46	60	60
SEPTA	65	65	65	65	65	65	65	65	65	58	58
MONTREAL							27	33	33	33	33
PATH	21	21	21	21	21	21	21	34	34	35	35
CLEVELAND	34	34	34	34	34	34	34	34	34	43	43
LINDENWOLD										34	34
SHAKER HTS.	30	30	30	30	30	30	30	30	30	30	30
NEWARK	9	9	9	9	9	9	9	9	9	9	9

Source: Same as Table III, Table 6-B.7, p. 6 - 85

Table VI

Electric Power Rates (¢/kwh)

	1960	1961	1962	1963	1964	1965	1966	1967	1968	1969	1970
NEW YORK	2.22	2.24	2.25	2.31	2.29	2.30	2.29	2.30	2.30	2.32	2.39
CHICAGO	1.88	1.89	1.90	1.95	1.93	1.99	1.93	1.94	1.94	1.96	2.02
MBTA	1.83	1.85	1.86	1.91	1.89	1.90	1.89	1.90	1.90	1.92	1.98
TORONTO	.57	.58	.58	.60	.59	.59	.59	.59	.59	.60	.62
SEPTA	1.54	1.55	1.56	1.60	1.59	1.59	1.58	1.59	1.59	1.60	1.65
MONTREAL	.57	.58	.58	.60	.59	.59	.59	.59	.59	.60	.62
PATH	2.22	2.24	2.25	2.31	2.29	2.30	2.29	2.30	2.30	2.32	2.39
CLEVELAND	1.072	1.082	1.086	1.12	1.11	1.11	1.10	1.11	1.11	1.12	1.15
LINDENWOLD	1.54	1.55	1.56	1.60	1.59	1.59	1.58	1.59	1.59	1.60	1.65
SHAKER HTS.	1.072	1.08	1.08	1.11	1.13	1.13	1.12	1.13	1.13	1.14	1.12
NEWARK	2.22	2.24	2.25	2.31	2.29	2.29	2.28	2.30	2.30	2.32	2.39

Sources: Rates were assumed to be as in U.S. Federal Power Commission's Typical Electric Bills (1968), p. XXI, for the region. Unless other data was was available several data points for 1960 were available in Lang & Soberman, Urban Rail Transit, MIT Press, 1964, p. 76.

When interpolation or extrapolation was necessary, a series for wholesale electric prices from the Bureau of Labor Statistics was used. The Canadian rates were interpolated from the same source as Table III.

Table VII

OPERATOR'S WAGES

	1960	1961	1962	1963	1964	1965	1966	1967	1968	1969	1970
NEW YORK	2.63	2.79	2.79	3.15	3.45	3.55	3.78	4.01	4.21	4.52	4.82
CHICAGO	2.44	2.55	2.67	2.80	2.92	3.06	3.20	3.35	3.66	3.92	4.17
MBTA	2.48	2.60	2.73	2.87	3.01	3.17	3.32	3.49	3.83	3.97	4.11
TORONTO*	2.10	2.21	2.33	2.45	2.57	2.79	2.85	2.97	3.00	3.27	3.53
SEPTA	2.23	2.33	2.44	2.55	2.67	2.80	2.92	3.06	3.26	3.54	3.81
MONTREAL*	1.88	2.00	2.13	2.27	2.41	2.57	2.74	2.91	3.10	3.28	3.45
PATH	2.63	2.79	2.97	3.15	3.35	3.55	3.78	4.01	4.21	4.52	4.82
CLEVELAND	2.23	2.34	2.46	2.59	2.72	2.86	3.01	3.66	3.28	3.37	3.46
LINDENWOLD	2.23	2.33	2.44	2.55	2.67	2.80	2.92	3.06	3.26	3.54	3.81
SHAKER HTS.	2.23	2.34	2.46	2.59	3.72	2.86	3.01	3.16	3.28	3.37	3.46
NEWARK	2.30	2.41	2.52	2.64	2.76	2.89	3.03	3.17	3.66	3.83	4.00

Sources: *Canadian wages are from Institute for Defense Analyses, Economic Characteristics of the Urban Public Transportation Industry. (Linearly interpolated for missing years).

U.S. Wage rates are the U.S. Department of Labor Bulletin #1620; Union Wages and Hours: Local Transit Operating Employees: Missing years were calculated using a time-series index from the same bulletin.

D. Estimation

Two estimation techniques were employed: a Guass-Newton non-linear process and a log-linear form after subtraction. Equation (1) is non-linear. It would be linear in logarithms were there no $p_t T$ term. The Gauss-Newton method employs a Taylor's series approximation about initial values of the parameters to be estimated and seeks to minimize the sum of squared errors through a process of successive approximations to the parameters which sometimes converges in the time alloted and sometimes does not.

The second method of dealing with the nonlinear form subtracted $p_t T$ from both sides of equation (1). Then the right hand side is linear in logarithms and can be estimated by ordinary least-squares methods, although the implicit error structure differs from the Gauss method[9]. Such OLS procedures were iterated for different values of p_t until a minimum sum of squared errors resulted. This is shown in Model 5 in Table VIII.

Results of several estimation techniques and models are given in Table VIII. Those for a Gauss-Newton estimation of equation (1), called Model 1, are given in the first column of that table. All the parameter estimates are reasonable. It is reassuring that the more miles of track a property is endowed with the lower are its nontrack-related costs. The regression coefficient is -.938 and it is significantly different from zero. All other estimates are of the expected signs.

The New York system produces over 8 times the output of the next larger system. We felt that it was necessary to test the reasonableness of

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[9]The estimation of p_t by iteration and seeking a minimum sum of squared residuals in the estimated equation can yield a maximum likelihood estimate of p_t and the other parameters. Such a process is often recommended in the literature of non-linear estimation [11, p. 342]. However, the error structure assumptions inherent in the log-linear form are somewhat debatable. In order to take the logarithms of both sides of

Table VIII: Estimated Cost Functions for Temporal Cross-Sections of Urban Rapid Transit Properties in North America 1960 - 1970

	Model 1	Model 2	Model 3	Model 5
	Regression Coefficients (Standard Errors)			
Track coefficient	.1604 (.0127)	.0493 (.0189)	.0995 (.0138)	.110
Gamma	5.318 (13.33)	8.143 (4.062)	25.89 (18.66)	7.885 (2.166)
Wage exponent	1.456 (.1551)	1.091 (.1176)	1.264 (.1099)	1.343 (.2703)
Electricity price exponent	.7969 (.4374)	.5840 (.0806)	.7940 (.1180)	.6524 (.1180)
Vehicle-miles exponent	1.861 (.313)	1.1520 (.140)	1.574 (.185)	1.042 (.066)
Track exponent	-.9381 (.366)	-.3661 (.158)	-.8433 (.208)	-.3675 (.092)
R^2	.966	.951	.983	.915
SSE	3571	1.468		22.56

pooling such a radically different system with the other properties in the sample. We tried fitting the model by the Gauss-Newton method on all 105 observations, and then fitting them on all observations save the eleven on New York. The sum of squared errors without N.Y. was 795, that with N.Y. was 3571. We estimated six parameters in each equation.
The quantity

$$\frac{(3571 - 795)/11}{795/99}$$

is distributed as F with 11 and 99 degrees of freedom. The calculated value is 31.4 which exceeds the critical F of 1.95 so that we can reject the hypothesis that the 11 additional observations on New York fit the same relationship[10].

In addition to problems of heterogeneity of the sample, we have problems of heteroscedasticity. The average residual from estimated Model 1 for N.Y. was 7.0 while that for the small properties was 0.68. We tried dividing through both sides of the equation by T, miles of track and a proxy for size. We raised this to several candidate powers on the theory that there is an optimal correction for heteroscedasticity. Keeler [6] suggests comparing the absolute values of the residuals for a number of the smallest firms and a number of the largest firms. His criterion for a minimum of heteroscedasticity is that the two groups differ least significantly (by a Mann-Whitney U-Test). Griliches [3] had suggested as a criterion that the estimated standard èrror of the coefficient of prime interest be at a minimum. However, estimated standard errors in a model with heteroscedasticity are biased downward so that a low standard error is not conclusive. [5, p. 216]

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the equation, and have an additive error in the resultant equation the error must be multiplicative in the following fashion

$$C = p_t T + \gamma p_e^{\delta} w^{\beta} Q^{\rho} T^{\lambda} e^{\varepsilon}$$

Then if we take logs we have an additive, ε, in the form to estimate:

$$\log [C - p_t T] = \log \gamma + \delta \log p_e + \ldots + \varepsilon$$

We may take ε to be normally distributed. Note that if γ was equal to zero in the true model, this form implies that $p_t T$ is known without error, The best adjustment for heteroscedasticity was Model 2 where we divided both sides of the equation by $T^{.75}$. The estimated equation is then[11]

$$SROC/T^{.75} = .049\ T^{.25} + 7{,}143\ w^{1.09} p_e^{.58}\ Q^{1.15}\ T^{-1.12}$$

Model 3 divided both sides of the equation by $T^{.50}$, which assumes that the variance of the error terms are proportional to the property's miles of track.

The estimates produced by iterated least-squares on a log-linear model are given as Model 5 in Table VIII. The estimates are not very different from those of the Gauss-Newton method which is reassuring. Generally the standard errors of the regression coefficients were lower.

The economic interpretation of the estimate of p_t is that it costs roughly $110,000 per mile of single track annually to operate a rapid transit system without any traffic. These are all maintenance and other operating costs and do not include any capital interest, or depreciation costs. The coefficient of output is the elasticity of short-run variable costs with respect to increases in output.

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a strong a priori statement. If the true γ is greater than zero the model states that the error is related to the dimension of the independent variables, a credible assumption.
The Gauss-Newton estimation process, on the other hand, assumes an additive error:

$$C = p_t T + \gamma p_e^{\delta}\ w_{11}^{\beta} Q^{\rho}\ T^{\lambda} + \varepsilon.$$

The selection between these models rests on one's belief in the true form of the underlying error structure.

[10] Gregory C. Chow, "Tests of Equality Between Sets of Coefficients in Two Linear Regressions", Econometrica XXVIII (July, 1960) 691-605

[11] The estimated standard error of the exponent of output (the parameter of interest) was smallest in this model. However, the best correction for heteroscedasticity in more complicated models did not have the smallest standard errors.

Table IX

Cost Functions for More Complicated Models

Model	NY	SH	MED	γ	w	p_e	p_v	Q	T	Converge? R^2	Date	Deflated?
6	.17	-.13	-.05	4.95	1.71	.57		1.27	-.60	yes	14May74	yes
	(.02)	(.02)	(.02)	(2.6)	(.20)	(.08)		(.20)	(.16)	.97		
7	.24	-.17	-.09	.61	3.59	.45		1.81	-1.28	no	30Jul74	no
	(.006)	(.02)	(.007)	(.83)	(.30)	(.20)		(.21)	(.24)	.99+		
8	.30	-.15	-.45	.34	1.85	.60	.21	1.34	-.77	no	5Aug74	no
	(.15)	(.25)	(.13)	(13.8)	(4.10)	(2.00)	(4.02)	(1.71)	(1.93)	.88		
9	------	.17T	--------	19.9	.93	1.29	.38	2.66	-2.09	yes	5Aug74	no
		(.008)		(65)	(.42)	(.21)	(.36)	(.22)	(.24)	.998		

(Standard errors)

Our estimate implies nearly constant returns and we cannot reject the hypothesis of constant returns. There are economics of density, however. The fraction of costs variable is less than one.

Analysis of residuals revealed that actual cost is always larger for New York than estimated cost. This suggests a mis-specification. We should allow for New York to be different from the rest of the sample. In rapid transit it has more rolling stock than the rest of the country combined.

There are ways to test objectively for the likeness of the cost behavior of properties in the sample. First, we allow the fixed cost per mile of track alone to vary among properties. This is done by defining three groups: New York, the small properties: Shaker Heights, Cleveland, Newark, PATCO[12] and the medium properties: Chicago, MBTA, Toronto, SEPTA, Montreal, PATH. "Dummy" variables represented the different groups.

It is also possible to allow the short-run elasticities of output with respect to inputs to vary among the train and streetcar systems. Unfortunately, the statistical method is already too strained to make this extensive test. The $(X'X)^{-1}$ matrix became nearly singular indicating a problem of identification. (See below at p.)

Model 6 allows for differences in fixed costs among the properties and corrects for heteroscedasticity. New York has significantly more fixed costs than other properties and the medium group has significantly less fixed costs. The small group has even less fixed costs. The signs and magnitudes of these estimates agree with intuition. Parameter estimates for Model 6 are given in Table IX. The equations resulting from Model 6 are:

$$SROC_S = .04T + 4.95\, w^{1.71} p_e^{.57} Q^{1.27} T^{-.595}$$

$$SROC_M = .12T + 4.95\, w^{1.71} p_e^{.57} Q^{1.27} T^{-.595}$$

$$SROC_{NY} = .165T + 4.95\, w^{1.71} p_e^{.57} Q^{1.27} T^{-.595}$$

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[12] An alternative typology considered only Shaker Heights and Newark as small. Such a model had a larger sum of squared residuals than the one reported.

We examined five corrections for heteroscedasticity: deflating by T^g where g took on the values 0 to 1.0 in steps of .25. In all cases[13] residuals for New York, the largest property, and Newark, the smallest, were compared. We had 10 observations on Newark and 11 on New York. The Mann-Whitney U statistic serves as an index of the equality of the variance of residuals. If the statistic is less than the tabulated value we can reject the hypothesis of homoscedasticity. For a two-tailed test at $\alpha = .02$, the critical value is 22 comparing a group of 10 and 11. If U is smaller than that there is evidence that one vector of errors is stochastically larger than the other.

Table X Results of Alternative Adjustments for Heteroscedasticity

g	U (Mann-Whitney)
0	2
.25	1
.50	13
.75	59
1.00	23.5

It appears, that of those corrections tried, dividing by $T^{.75}$ is the best correction for heteroscedasticity. Qualitatively, the errors are homoscedastic with deflation by either $T^{.75}$ or T.

Short-run marginal costs for each type of property is the same in this model.

$$\frac{\partial SROC}{\partial Q} = 6.287\, P_1^{1.71} P_e^{.57} Q^{.27} T^{-.595}$$

It remains to add in fixed costs due to interest and depreciation and then we can proceed to calculate the long-run cost function for fixed rail rapid transit properties in North America.

The objective ought to be to estimate replacement cost. We can

[13]Only models with g = 0 and g = .75 are reported here.

then use a fixed interest rate of 6% and the annuity form of depreciation assuming a 25-year equipment life, a 50-year life for track and structures, and an infinite life for rights-of-way.

Data on the replacement costs of transit equipment is difficult to develop because of the wide variety of equipment types in existence and because much of the cost data is out of date. Using data in the estimated cost to completion of the BART system, the annualized capital cost is as displayed in Table XI below[14].

Table XI: Annual Interest and
Amortization Costs of BARTD Capital

Type of Capital	Capital Cost	Life	Annual Cost @ 6%
Track & structures[1]	$1,249,649,000	50 yr.	$79,277,000
Right-of-way	95,000,000	infinite	5,700,000
Vehicles	79,860,000	25 yr.	6,258,000
Totals	1,424,509,000		91,235,000
(Per mile of single track)	9,496,727		608,233

---

[1]Includes cost of station construction, electrification, train control, utility relocation, engineering and charges, contingencies, and pre-operating expense.

[2]BARTD acquired 3700 parcels to August, 1974 for approximately $95 million. Not all of this land is chargeable to the provision of transit services. Land for parking lots, buildings, yards and shops clearly is for the land under elevated sections the answer is not clear. Linear park services are provided as a joint product. Generally, for its cut and cover tunnel work BARTD purchased only subterranean easements. In other cases, however, land was purchased, structures were razed and once the tunnel was covered the land was saleable. According to BARTD, 188 parcels valued at $5.2 million are excess.

[14]The source of this data is the July 1, 1972 Comparative Data Report of the Bay Area Rapid Transit District.

The total annual capital costs for BART thus reduce to roughly \$608,000 per mile of single track. Thus to include capital costs in Equation (1), .608 should be added to the estimated p_t assuming that BART capital costs are "typical" of the modern standard of transit construction. Equation (1) now becomes a short-run total cost function (SRTC) and using the parameters of Model 5 it may be written:

$$(2)\quad SRTC = (.110 + .608)T + 7.895w^{1.343}p_e^{.652}Q^{1.042}T^{-.367}$$

To derive the long-run total cost (LRTC) function, it is necessary to optimize in the choise of the fixed factor, T. The LRTC function can be found by differentiating (2) with respect to T, solving for the optimum T and substituting the result back into (2). In so doing, the LRTC function can be shown to be

$$(3)\quad LRTC = 7.421\, w^{.982}p_e^{.477}Q^{.762}$$

Long run marginal costs LRMC can be determined by differentiating (3) with respect to Q:

$$(4)\quad \frac{\partial LRTC}{\partial Q} = LRMC = 5.655w^{.982}p_e^{.477}Q^{-.238}$$

The sign of the exponent of output Q in Equation (4) indicates that long-run marginal costs decline with output. This is symptomatic of long-run economies of scale in the provision of fixed-rail rapid transit service and is a basic conclusion of our study.

E. Use of estimated Equations to Predict BARTD Operating Costs.

We will now apply our cost functions derived from Model 5 to the values of the independent variables obtaining an estimate of operating cost at an output of 25 million vehicle-miles in 1973 dollars. These data are given in Table XII. The estimate of short-run BARTD operating costs SROC (exclusing interest and depreciation) is about $35.5 million (Table XII). This is not far from the $33 million in 1973 dollars projected from simple extrapolation of early BART experience as presented in Table II. It also indicates that in spite of the high level of vehicle maintenance expenses and other costs which BART is currently experiencing because of start-up problems, the operating costs are very similar to those other transit systems in North America would incur were they facing the wage rates and other predetermined variable values that BARTD faces.

If there are eventually savings in the expense categories in which BART is currently experiencing very high costs as learning-by-doing takes place, BART may be more economical than other comparable systems in the area of operating costs.

Table XII BARTD Operating Parameters 1973

w = \$5.34/hr. base wage of train attendants

p_e = \$.012/kwh

T = 150 miles of single track

Q = 25 million vehicle-miles

Source: R.G. Snyder to W.C. Hein, Interoffice Memo, SF BARTD, March 13, 1973

Table XIII BARTD Operating Costs Under Two Models

A: Short-run costs Model 5	Annual Costs \$ million[1]
$SROC = .110T + 7.895\ w^{1.343} p_e^{.652} Q^{1.042} T^{-.367}$	35.5
$SRTC = .718T + 7.895\ w^{1.343} p_e^{.652} Q^{.042} T^{-.467}$	126.8
$SRMC = 8.227\ w^{1.343} p_e^{.652} Q^{.042} T^{-.367}$	\$.79/vm

B: Long-run costs Model 5	
$LRTC = 7.421\ w^{.982} p_e^{.477} Q^{.762}$	54.2
$LRMC = 5.655\ w^{.982} p_e^{.477} Q^{-.238}$	\$1.65/vm

C: Short-run costs Model 6 (Assuming BART a medium property)	
$SROC = .112T + 4.95\ w^{1.71} p_e^{.57} Q^{1.266} T^{-.595}$	37.6
$SRTC = .72T + 4.95\ w^{1.71} p_e^{.57} Q^{1.266} T^{-595}$	128.8
$SRMC = 6.267\ w^{1.71} p_e^{.57} Q^{.27} T^{-.595}$	\$1.08/vm

D: Long-run costs Model 6	
$LRTC = 4.668\ w^{1.072} p_e^{.357} Q^{.794}$	74.7
$LRMC = 3.706\ w^{1.072} p_e^{.357} Q^{-.206}$	\$2.37/vm

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[1] Unless otherwise specified.

Table XIV Economies of Density and

Economies of Scale in Transportation

	Short Run		Long Run	
	$\eta C_{srv},Q$	Concl. on Density	$\eta C_{er},Q$	Conclusion
Eads, Nerlove and Raduchel, Local Service Airlines	2.125	diseconomies	.77	CRTS[1]
Keeler, Railroads	1.261 (.433)	economies	1.007	CRTS
Merewitz and Pozdena Fixed-Rail Transit	1.27	economies	.80	IRTS

[1]Eads, Nerlove and Raduchel [2] called this point estimate of IRTS, CRTS for reasons explained in the text.

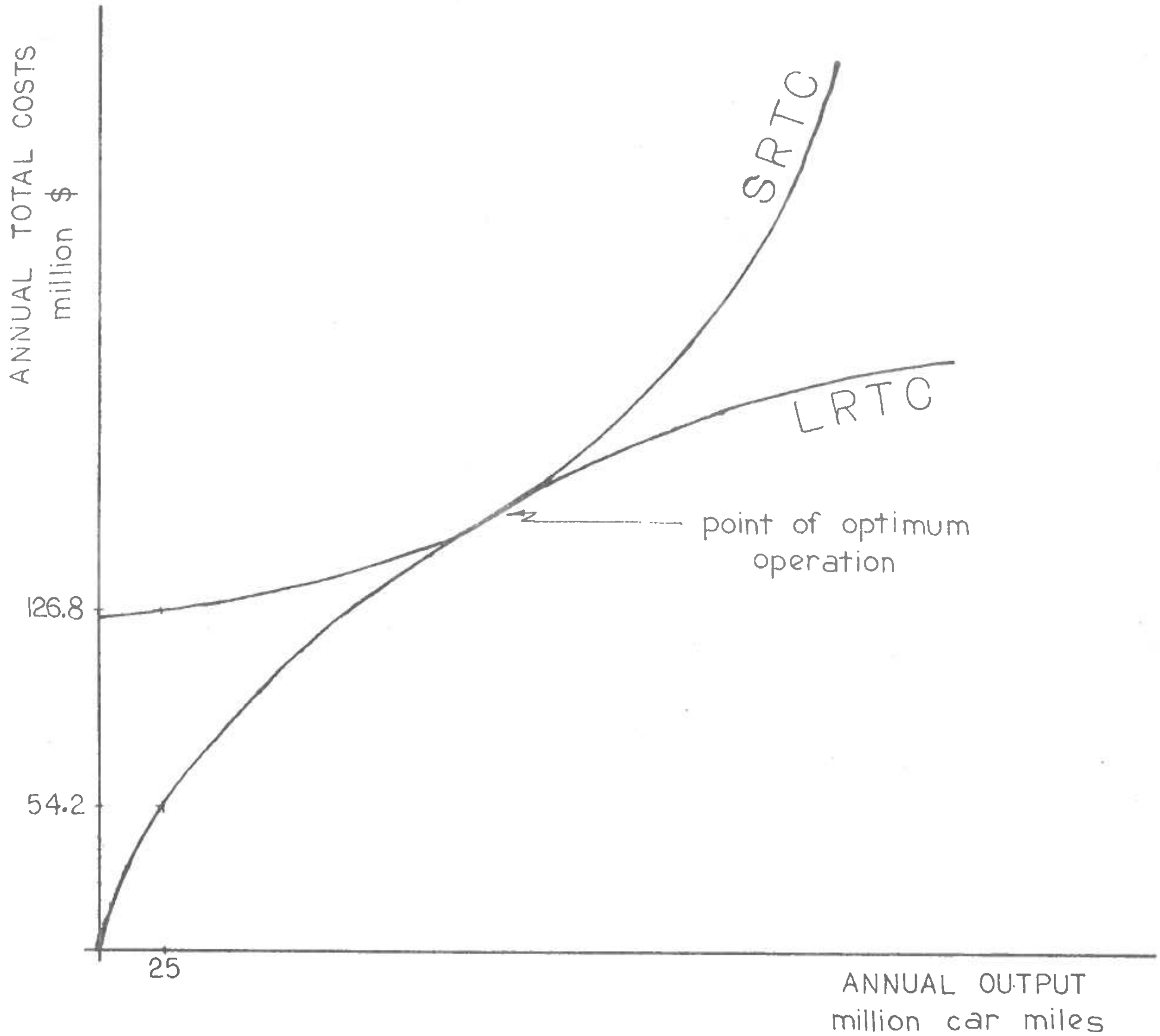

FIG. 1

Long run and Short run total costs of BART under MODEL 5

In Table XIII we have used Model 6 for the same prediction with similar results.

The long-run cost functions indicate that BART is operating at less than optimum long-run capacity at 25 million car-miles annually. One would expect excess capacity at the outset. The relative shape of the short-run and long-run total cost functions may be something like that illustrated in Figure 1 below. The SRTC function has a positive slope which increases with output; LRTC has a positive slope which decreases with output. Although a point of optimum output for BART's endowment of track is labelled it is unlikely that any property would choose to operate there. That level of output would allow trains to be running at all times of day and, incidentally, at infinite length.

However, although BART operating expenses are within six per cent of what the mean property would experience (by Model 5), if we were to add capital expenses at the proper concept of user costs, BART would appear more costly. The logic of BART was to substitute capital for labor, a popular cost-saving strategy of the 'sixties, but BART has used lavish capital to achieve operating cost economies of about 7% (2.5/35.5) less than those of other North American properties, or 12% less (4.5/37.6) according to Model 6. These calculations assume that BART expenses will grow linearly. If that assumption is not true, our best estimate of BART costs are given by predictions from the estimated equations.

The estimated function may also be used to derive incremental costs of a system with BART's parameters. Differentiating the estimate of Model 5 with respect to output yields an incremental cost function of the form.

$$\text{Incremental cost/vehicle-mile} = 8.23w^{1.34}p_e^{.652}Q^{.042}T^{-.367}$$

At a BART output of 25 million vehicle-miles, the incremental cost per vehicle mile is $.79. (See Table 13, Part A) The cost per seat-mile is $.011 without interest on and depreciation of the vehicles. Comparing this with the same number calculated from actual BART experience reported in Table I (roughly $.53 per vehicle-mile or $.0073 per seat-mile), it can be seen that the two estimates are fairly close. This gives support to the conclusion that a reasonable estimate of the incremental cost of

BART service at a level of 25 million vehicle-miles is roughly 1.1 to 1.4¢ per seat-mile when vehicle interest and depreciation costs are included.

F. Explorations with More Complicated Models

In our early estimation we adjudged deflation by $T^{.75}$ to be the best adjustment for heteroscedasticity. We encountered problems other than heteroscedasticity, however. There was a misspecification and in estimating a single undifferentiated model for all properties without distinguishing among small, medium and large properties. Thus, we decided to work with a model with dummy variables representing these different sizes. Formal tests for optimal correction for heteroscedasticity were deferred until we could examine the errors from these properly specified models.

We examined the absolute values of the residuals from Model 6 with the $T^{.75}$ deflation and Model 7 with no correction for heteroscedasticity. We examined the residuals of Newark, the smallest property and New York, the largest. In looking at Model 7 we concluded that the variance of New York errors was stochastically higher than that of Newark. The Mann-Whitney U was equal to 2, less than the critical value of 25 for a two-tailed test at $\alpha = .02$ for 11 numbers in each group [15, Table K, p. 275]. Therefore there is a heteroscedasticity problem in Model 7. In Model 6, the corrected model, we made the same comparison of absolute values of residuals for New York and Newark. The U statistic was equal to 59, greater than the tabulated value, so we cannot reject the null hypothesis of equal variances. Model 6 errors appear to be homoscedastic. We do not yet know, however, whether this is an <u>optimal</u> correction for heteroscedasticity.

We developed a time series on vehicle prices. The gross influences on the acquisition cost of vehicles appeared to be the year in which they were produced and the number of seats they contained. The following cost estimating relationship was derived:

$$p_v = 9032 + 8528\,(Y-1950) + 1079\,(S-41)$$
$$(10{,}220)\quad(896)\qquad\qquad(426)\qquad\qquad R^2 = .75$$

where

p_v = the simulated replacement cost of a rapid transit vehicle in a particular year with a stated number of seats

Y = the calendar year for which capital costs are being calculated

S = the number of seats in the vehicle

Thus vehicle prices increased about $8500 per year and larger vehicles cost about $1100 more per extra seat. Thses prices were applied to each property's inventory of rolling stock year by year to calculate interest and depreciation. Interest was calculated at 6 per cent and depreciation was assumed to take place according to a declining balance model where a constant fraction of the capital's value expires every year. We estimated the average age of each property's fleet in each year. Annual vehicle capital cost is given by Equation (7)

$$C_k = N\,p_v\,e^{-da}\,(d+r) \tag{7}$$

where

p_v = replacement cost of the vehicle in the current year

a = average age of fleet

d = rate of depreciation, taken as 0.1

r = interest rate on invested capital, taken as .06

These inputed vehicle capital costs were added to short-run operating costs to yield an intermediate concept of costs but still excludes capital costs due to right-of-way and track and structures.

Model 8 was estimated with dependent variable as described above and vehicle replacement cost as an additional independent variable. Results were unsatisfactory because the Guass-Newton algorithm did not converge. We are straining our estimation method in endeavoring to estimate nine parameters with observations on only eight independent variables. (Track is used twice) Eight parameters seems to be the

the practical limit of what this method can estimate with these data.

The equation we are estimating could have identification problems because the number of estimated parameters is greater than the number of variables. Because of its non-linear nature all that is required for identification is a wide range in the data to limit the probability of a family of parameter values' providing the same sum of squared residuals. As we increase the number of parameters to be estimated we tax the scarce variation in the data which helps in identification. The increase from seven parameters to eight seems to be significant. There is another possible explanation of the method's failure to converge. That is that we are inadvertently finding a local minimum sum of squared errors rather than a global one. Our intuition (and many failures with the method when we tried to estimate eight or more parameters) suggests that the first explanation is more likely.

Thus we tried in Model 9 to economize parameters. Separate estimates for different size properties are no longer sought. This model has a high R^2 and the coefficient estimates are precise. The vehicle price is still not significant. Clearly, more work is needed on the proper concept of capital cost. Not only vehicle prices but rates of interest on bonds and contributions by subsidy from UMTA and other sources (e.g. the State of New York to New York City's MTA) are relevant to the cost of capital to these properties.

It would seem reasonable that the fixed cost per mile of track would not be constant for a given property over the years but would increase with time. Given the limitations of our estimation procedure, it is too much to ask to estimate such parameters which vary with time.

G. Conclusion

This effort to estimate a marginal cost function exploited Keeler's technique to derive the long-run cost function of economic theory from current observations on operating cost. In the short run managements of fixed-rail rapid transit properties cannot vary the length of track they have. Rather than assume that all properties in a cross-section are at a long-run equilibrium and represent observations on a long-run cost function we estimated a cost function over a time-series of cross sections. We next took the first partial derivative with respect to track. Setting this expression equal to zero gives an expression for the optimal level of track as a function of the exogenous variables the property faces (output, wages and electricity price). This expression for the optimal level of fixed-factor, when substituted in the short-run cost function gives the unconstrained least-cost locus from which management can choose in response to output and factor price constellations. This unconstrained locus is the long-run cost function. The short-run (or unconstrained) cost function must be above or equal to the long-run cost function.

Track was recognized as fixed in the short-run for fixed-rail rapid transit. This leads to the result that the enterprise will experience higher total operating costs when it is not able to vary the fixed factor.

Several studies now have derived numerical estimates for long-run and short-run cost functions. [3, 6] All three obtain estimates of the elasticity of short-run variable costs with respect to output which are greater than one. Such costs are strongly increasing in the local-service airline industry (the value is 2.125) and modestly increasing in railroading (1.261, standard error .433) and in urban rapid transit (1.27, standard error .20). The desideratum for economies of density is per cent variable [15] or fraction of short-run costs variable. There appear to be diseconomies of traffic density in local-service airlines which would indicate a "shortage" of the fixed factor, pilots and copilots. In railroads and subways, by contrast, Keeler finds per cent variable figures ranging from 7.2 to 78.7 [6, p. 206] and we observe a range of .42 to .88. The general finding is one of short-run costs less than one hundred per cent variable or economies of density. In the case of trains on track, there seems usually to be an excess of the fixed factor, track. Since we have not differentiated output by time of day our results seem to say we could run more trains on the given track at 3 a.m. in the morning, for example, to achieve more output with short-run cost increasing less than _pari passu_. While Keeler can discuss the attractive alternative of abandoning underutilized track, our industry admits of no such salubrious prescription.

In the long run Eads, Nerlove and Raduchel claim to perceive constant returns to scale [16] as did Keeler. We find increasing returns to scale in the fixed-rail urban rapid transit industry. Our best estimate (Model 6) of long-run marginal cost per vehicle-mile is $2.37. This is to be contrasted to long-run average cost of $5.62. [12]

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[15] This term, frequently used in transportation cost analysis would be called the elasticity of (total) short-run costs with respect to output by an economist.

[16] They make this judgment despite a point estimate of returns to scale of 1.3. They claim the estimate is biased toward IRTS if simultaneous-equations problems made output endogenous or jointly determined with input levels.

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