LARGE ANGLE pp SCATTERING AS AN EXPERIMENTAL TEST OF THE MINIMAL INTERACTION HYPOTHESIS

Authors
Chiu, Charles B.
Harte, John
Tan, Chiang-1.

Publication Date
1967-06-15

University of California

Ernest O. Lawrence Radiation Laboratory

LARGE ANGLE pp SCATTERING AS AN EXPERIMENTAL TEST OF THE MINIMAL INTERACTION HYPOTHESIS

Charles B. Chiu, John Harte, and Chung-I Tan

June 15, 1967

To be submitted to Nuovo Cimento

UCRL-17553
Preprint

UNIVERSITY OF CALIFORNIA
Lawrence Radiation Laboratory
Berkeley, California

AEC Contract No. W-7407-eng-48

LARGE ANGLE pp SCATTERING AS AN EXPERIMENTAL TEST OF THE MINIMAL INTERACTION HYPOTHESIS

Charles B. Chiu, John Harte, and Chung-I Tan

June 15, 1967

UCRL-17553

LARGE ANGLE pp SCATTERING AS AN EXPERIMENTAL TEST OF THE MINIMAL INTERACTION HYPOTHESIS*

Charles B. Chiu, John Harte† and Chung-I Tan

Lawrence Radiation Laboratory
University of California
Berkeley, California

June 15, 1967

ABSTRACT

Recently a lower bound of the general form $\sim\exp[Cs^{\gamma}\ln s]$ has been suggested for the pp scattering amplitude at fixed angle, where $\gamma \geq 1/2$. We discuss in this note the characteristic features in the pp differential cross section should the minimal interaction hypothesis be valid. We compare this form with the experimental data for the cases $\gamma = 1/2$ and $\gamma = 1$, and find that the present data beyond the recently observed "break" in the 90 deg. data are compatible with both cases although $\gamma = 1/2$ gives a slightly better fit to the data. Should the lower bound behavior indeed saturate the amplitude in the large momentum transfer region, then the break can be attributed to the transition between the region where the lower bound contribution dominates and the smaller $|t|$ region where specific t-channel exchange mechanisms are dominating.

I. INTRODUCTION

Several years ago, Cerulus and Martin derived[1] a lower bound for the pp scattering amplitude at high energy. This bound is given by

$$f(s, z_s) \geq \exp[-(N - 1)\, C(z_s)\, s^{\frac{1}{2}} \ln s] \qquad (1)$$

The symbol z_s refers to the cosine of the scattering angle in the s channel c.m. system. The factor N is a constant which is associated with their assumption of the polynomial bound s^N for the amplitude, and $C(z_s)$ is a known non-negative function of z_s. Subsequently, Kinoshita pointed out[2] that the observed pp differential cross section (dcs) at large angles is actually quite close to this lower bound. This led him to postulate the principle of minimal interaction which states that the scattering amplitude in the large angle region at high energies takes the minimum value consistent with the general requirements of analyticity and unitarity.

The term $s^{\frac{1}{2}} \ln s$ in Eq. (1) is closely related to the specific upper bound assumed for the scattering amplitude in the t plane. In a recent paper[3] two of us showed that if one alters this assumption, the energy dependence of the lower bound could be altered. More specifically, if one assumes that the scattering amplitude is bounded in the region to the left of the lines $\xi = r \exp i(\pi \pm \Theta_{max}) \equiv t - t_o$ where $t_o = (2\mu)^2$, and μ is the pion mass, then the lower bound becomes

$$f(s, z_s) \gtrsim \exp[-(N - 1)\, C_\gamma(z_s)\, s^\gamma \ln s] \qquad (2)$$

where

$$\gamma = \frac{\pi}{2\Theta_{max}} \qquad (3)$$

and $C_\gamma(z_s)$ is a known function of z_s and also depends on the value of γ. The assumption made by Cerulus and Martin essentially corresponds to $\gamma = 1/2$, whereas for the case of a linearly rising Regge trajectory, $\gamma = 1$. In Cerulus and Martin's work, the factor $C(z_s)$ in Eq. (1) approaches infinity as z_s approaches 0, so their lower bound vanishes at 90 deg. Fortunately, however, in the modified lower bound of Eq. (2), $C_\gamma(z_s)$ is everywhere finite for all γ.

In this paper we discuss the characteristic features expected in the dcs, should the amplitude coincide with the lower bound of Eq. (2). A simple and clear cut method is suggested for distinguishing between this lower bound behavior and other proposed forms for the large angle data, such as the Orear fit.[4]

Since the assumption of minimal interaction should be correct, if at all, only for large momentum transfer, we first look for the lower bound behavior in the pp dcs data at 90 deg.[5] There is an apparent break in the slope in this dcs data, when plotted as a function of t, which occurs near $|t| = 7\ (\text{GeV/c})^2$. This implies that at 90 deg, the lower bound cannot possibly be saturating the dcs below $|t| \cong 7\ (\text{GeV/c})^2$. For the region $|t| \gtrsim 7\ (\text{GeV/c})^2$, we found that their data together with all other presently available data are not sufficient to determine conclusively whether or not the scattering amplitude should indeed follow the form of Eq. (2) either for the case $\gamma = 1/2$ or for $\gamma = 1$. For the case $\gamma = 1/2$, the data is compatible with the lower bound behavior. For the case $\gamma = 1$, the fit is adequate for $|t| \gtrsim 8\ (\text{GeV/c})^2$. It starts to deviate noticeably from the data near $|t| = 7\ (\text{GeV/c})^2$. We have constructed a model to fit the large angle scattering data for $|t| > 2(\text{GeV/c})^2$. In the fit, the dcs in the large $|t|$ region is dominated by the lower bound form with $\gamma = 1$, whereas in the smaller $|t|$ region this amplitude is dominated by the contribution with Regge-like dependence, $(-z_t)^{\alpha}$.

Although the predictions for these two cases $\gamma = 1/2$ and $\gamma = 1$ investigated in the large $|t|$ region are both compatible with the present data, they show considerable difference even within the present accessible energy region where no accurate measurements are available; this indicates that further accurate large angle measurements will be very significant in testing the minimal interaction hypothesis and in providing some indirect evidence for the asymptotic behavior of the Regge trajectories.

In Sec. II we discuss the characteristic features associated with the lower bound behavior and compare with some other forms which have been suggested heretofore in fitting large angle pp scattering data. Sec. III contains our comparison between the experimental data and the lower bound behavior for the cases $\gamma = 1/2$ and $\gamma = 1$. We present our concluding remarks in Sec. IV.

II. THE LOWER BOUND BEHAVIOR AND A COMPARISON WITH OTHER FORMS

The scattering amplitude which is obtained from the minimal interaction hypothesis is given by the right hand side of Eq. (2). We have checked numerically[6] for $0 \leqslant z_s \leqslant 1$ and have found that $C_1(z_s) = a t_o^{-1} \sin^2 \frac{\theta_s}{2}$ where $a \cong 1.803$, $z_s = \cos \theta_s$, and $t_o = 4\mu^2$. Therefore, denoting by $F(s, z_s)$ this lower bound amplitude, we have

$$F(s,z_s) = D_o \ \exp[-a(N - 1) \left(\frac{-t}{t_o}\right)^{\gamma} (\ln s - D_1)] \tag{4}$$

where, unfortunately at this stage, γ, N, D_o and D_1 are unknown constants. We assume that the triplet amplitude is asymptotically equal to the singlet amplitude. So

$$\frac{d\sigma}{dt} = \frac{1}{16\pi k_s^2 s} \left[|F(s,z_s)|^2 + |F(s, -1+z_s)|^2 - \mathrm{Re}F^*(s,z_s)F(s, -1+z_s) \right] \tag{5}$$

where k_s refers to the momentum in the c.m. system. For any assumed value of γ, the parameters D_o, N, and D_1 have to be determined experimentally. As mentioned in the introduction, γ is related to the boundedness property of the scattering amplitude in the t plane, and N appears in Eq. (4) because of the assumption of a power bound s^N in the derivation of the lower bound. In the case that the Pomeranchuk trajectory is exactly linear in t, and it dominates the asymptotic behavior at $t = 0$ and $t = t_o$, it turns out that $N = \mathrm{Re}\, \alpha(t_o)$ and $(N - 1)/t_o$ is the slope of the trajectory.

The parameter D_1 in this case is related to the value of the residue function at $t=0$ and to its maximum value at $\mathrm{Re}\, t = t_o$. For $\gamma \neq 1$, the correspondence is not clear.

Let us examine some characteristic features of the experimental data[7] which may allow a comparison with Eq. (4). Referring to Fig. (1), one observes that the fixed angle data and the fixed s data fall on intersecting curves with the former considerably more steep than the latter. We define $r_z = \left.\frac{\partial F(s,z_s)}{\partial |t|}\right|_{z_s}$ and $r_s = \left.\frac{\partial F(s,z_s)}{\partial |t|}\right|_s$ where the derivatives are taken at fixed z_s and s respectively. Note these derivatives are defined to be related to the experimental dcs through Eq. (5). Once the slopes r_z and r_s and the value of the dcs are known at a given point, then for a given value of γ, one can solve algebraically for the parameters D_o, D_1, and N.

We now denote by Δ the difference of the two slopes

$$\Delta = r_z - r_s \tag{6}$$

For the lower bound amplitude, we have

$$\Delta_\gamma = -a(N-1)\,\frac{(-t)^{\gamma-1}}{t_o^{\gamma}}\left[1 - \frac{4M^2}{s}\right] \xrightarrow[s\to\infty]{} -a(N-1)\,\frac{|t|^{\gamma-1}}{t_o^{\gamma}} \tag{7}$$

Here both a and t_o are known contstants. For $s \gg 4M^2$, Δ_γ is constant at fixed t, and the t dependence is determined solely from the value of γ. For $\gamma > 1$, $|\Delta_\gamma|$ increases with the increase of t; for $\gamma = 1$ it is independent of t; while, for $\gamma < 1$, it decreases with the increase of t. Identical statements can also be made for the dependence of $|\Delta_\gamma|$ on s at fixed z_s. The determination of the quantity Δ at several t values will be extremely useful in testing the minimal interaction hypothesis. We note here, that since the amplitude has to be properly symmetrized to calculate the cross section (see Eq. (5)), it is necessary to fit the dcs data using this form to extract the experimental value of Δ. On the other hand, in the region where the cross section falls off rapidly, one can approximate Eq. (5) by the first term, i.e. the $F(s,z_s)$ contribution alone. For this case, the Δ value can be obtained directly from the data.

The quantity Δ can be conveniently used to distinguish various forms which have been proposed to fit the large angle scattering data. For example, for

$$f_I \sim \beta(t)(-z_t)^{\alpha(t)}, \quad \Delta_I = \frac{4\alpha}{4M^2(1 - z_s)-(3 + z_s)t} \xrightarrow[s \to \infty]{} \frac{4\alpha}{(3 + z_s)|t|} \tag{8}$$

and for

$$f_{II} \sim \exp\,(-C_1\, t^{\frac{1}{2}}), \qquad \Delta_{II} = 0 \tag{9}$$

It can be shown that the data require that Δ be negative near the break region. Therefore the form f_I requires $\alpha(t)$ to be negative there. The form f_{II} is ruled out. In the model proposed by Huang, Jones and Teplitz,[8] it was assumed that the Pomeranchuk trajectory and the Regge cut are dominating. Both of these contributions have the form $(-z_t)^\alpha$. At the break, in their model, $\alpha_P = -1$

and $\alpha_c = 0$. The contribution of the Pomeranchuk trajectory vanishes there, and the cut is the only contribution, which in turn gives $\Delta = 0$. We have checked explicitly that their solution gives the wrong fixed-s angular dependence near the break region. Also, the model of Sakmar and Wojtaszek[9] which attempts to explain the break with a positive Pomeranchuk trajectory, should not give the correct fixed-energy angular dependence in the break region. In the larger $|t|$ region, the Δ value is experimentally well determined at one point, $P_L \sim 15$ GeV/c and $t = -10.4$ $(\mathrm{GeV/c})^2$, by the data of Ref. 5 and Ref. 7a. We estimated $\Delta = -0.18 \pm 0.02$ $(\mathrm{GeV/c})^{-2}$. For the form f_I, this would imply $\alpha = -1.35$ at $t = -10.4$ $(\mathrm{GeV/c})^2$.

To take another example, for the form

$$f_{III} \sim \exp[-C_1 \sin^2\theta\, s] \tag{10}$$

we obtain

$$\Delta_{III} \sim -2\, C_1\, (z_s + 1) \tag{11}$$

which may be distinguished from Δ_γ by the lack of z_s dependence in latter at fixed t. Sometimes it is more convenient to define the fixed s and fixed z_s slopes by taking derivatives in $|t|^{\frac{1}{2}}$; that is we let

$$\bar{\Delta} = \left.\frac{\partial F(s,z_s)}{\partial |t|^{\frac{1}{2}}}\right|_{z_s} - \left.\frac{\partial F(s,z_s)}{\partial |t|^{\frac{1}{2}}}\right|_{s} \tag{12}$$

For the lower bound amplitude with $\gamma = 1/2$, we obtain

$$\bar{\Delta}_{\frac{1}{2}} = -2a(N - 1)\, t_o^{-\frac{1}{2}} , \qquad (13)$$

while for the Orear formula[4] we have

$$f_{IV} \sim \exp\,(-C_1 \sin\Theta_s\, s^{\frac{1}{2}}), \quad \bar{\Delta}_{IV} \sim -\, 2^{\frac{1}{2}} C_1 \frac{1 - z_s}{(1 + z_s)^{\frac{1}{2}}} \qquad (14)$$

Both $\bar{\Delta}_{\frac{1}{2}}$ and $\bar{\Delta}_{IV}$ are independent of t, but f_{IV} will be characterized by a shrinkage of $\bar{\Delta}_{IV}$ as the angle is decreased from 90 deg, whereas $\bar{\Delta}_{\frac{1}{2}}$ is a constant.

III. THE COMPARISON OF THE MINIMAL INTERACTION BEHAVIOR WITH THE DATA

A sample of pp dcs data[7] is shown in Fig. 1, where $d\sigma/dt$ is plotted as a function of t. For a given energy, the t value is ranging from 0 to $-2k_s^2$ (the latter is the t value at 90 deg). So as the energy is increased, correspondingly the t-interval is enlarged. Since it is plausible that the lower bound behavior is more likely to describe the large angle scattering region where no specific cross channel exchange mechanism is dominating, our comparison of the data with the lower bound behavior is made by first looking at the 90 deg data. Note the slope in the dcs below $7(\mathrm{GeV}/c)^2$ is steeper than the slope above. This implies that the data below $7(\mathrm{GeV}/c)^2$ cannot be saturated by the lower bound. Thus we shall concentrate first on the region with $|t| \gtrsim 7(\mathrm{GeV}/c)^2$, and see whether the data in this region are compatible with the lower bound behavior or not. For definiteness we consider the two cases $\gamma = 1/2$ and $\gamma = 1$ below.

(1) $\gamma = 1/2$: With the form of Eq. (4) we first fit the data in the region near 90 deg and then enlarge the region with decreasing angle. We found all the data for $|t| \geq 7(\mathrm{GeV}/c)^2$ shown in Fig. 1 (or in Fig. 2), which includes some points with scattering angle down to 50 deg, can be fitted with the form of Eq. (4). The parameters obtained for the fit are

$$D_0 = 1.30 \text{ mb}^{\frac{1}{2}} \text{ GeV},$$

$$N - 1 = 0.47 t_0^{\frac{1}{2}},$$

$$D_1 = 2.56.$$

The fit to these data points are illustrated in Fig. (2). Although this fit is quite reasonable, since there are three parameters involved, the data are only enough to determine the values for these parameters but not sufficient to determine conclusively whether the observed amplitude should indeed follow the lower bound behavior or not. We illustrate the extension of the lower bound contributions to the lower $|t|$ region in Fig. 2. In the smaller $|t|$ region, the extension of the lower bound contribution deviates substantially from the data. Note that this deviation for fixed s is more noticeable than that for fixed z_s.

(2) $\gamma = 1$: The same data points as for the case $\gamma = 1/2$ are also fitted for $\gamma = 1$. The fit to the $|t| > 7(\text{GeV/c})^2$ scattered points are comparable to that for the case of $\gamma = 1/2$. Fits to the data at 16.9 GeV/c data and the 90 deg data are less satisfactory. They are illustrated by the dotted lines shown in Fig. 2. It turns out that the fit at 16.9 GeV/c is strongly coupled to the 90 deg fit near the break region. As the fit for the 16.9 GeV/c data is improved, the fit at 90 deg deviates further from the data. These dotted lines shown are a typical compromise of the situation. The corresponding parameters obtained for this case are:

$$D_o = 0.36 \text{ mb}^{\frac{1}{2}} \text{ GeV},$$

$$N - 1 = 0.15 t_o ,$$

$$D_1 = 3.06 .$$

Finally we consider a third possibility.

(3) Phenomenological model for large angle pp scattering for $|t| > 2(\text{GeV/c})^2$:

We feel that the less satisfactory fit for the case $\gamma = 1$ at this stage should not necessarily be taken to imply that the data prefer $\gamma = 1/2$ rather than $\gamma = 1$. It is conceivable, that the lower bound behavior could start to dominate at a slightly larger $|t|$ value. More explicitly, we construct the following phenomenological model for the large angle scattering amplitude. In the large $|t|$ region, it is dominated by the lower bound behavior for $\gamma = 1$, while in the region where the lower bound behavior is not dominating, we assume the scattering amplitude is characterized by the Regge-like behavior, $(-z_t)^{\alpha}$. Lacking knowledge of how the transition is made from one form to the other, we write

$$\frac{d\sigma}{dt} = \frac{d\sigma}{dt}\bigg|_{\ell.b.} + \frac{d\bar{\sigma}}{dt} \tag{15}$$

where $\frac{d\sigma}{dt}\Big|_{\ell.b.}$ is given by Eq. (4) and Eq.(5) and the added term $\frac{d\bar{\sigma}}{dt}$ is pressumably associated with the contribution of the exchanged Regge trajectories. This term dominates below the break region (the break is at $P_L \sim 8.4$ BeV/c or $|t| \sim 7$ (GeV/c)^2), and it falls off more rapidly than the lower bound amplitude in the larger $|t|$ region. For pp scattering, the dominating Regge trajectories in the small $|t|$ region are the Pomeranchuk trajectory and the secondary trajectories, such as P' and ω. From the present knowledge of Regge analysis on pp scattering,[11] for $|t| < 1$ (GeV/c)^2 the P contribution in the energy region is falling faster than the contribution of P' plus ω. So for $|t| > 2$ (GeV/c)^2, in the moderate energy region between

3-8 GeV/c region (we will come back to this point later on), the dcs might be mainly dominated by secondary trajectory contributions or even lower lying trajectories. We write,[11]

$$\frac{d\bar{\sigma}}{dt} = \frac{1}{16\pi s k_s^2 \left(1 - \frac{t}{4M^2}\right)^2} \left| \beta(t)\,(-z_t)^{\alpha(t)} \right|^2 \tag{16}$$

and parameterize

$$\alpha(t) = \frac{1}{2} + \alpha_1 t$$

$$\beta(t) = \bar{D}_o \exp \bar{D}_1 t$$

We note here that the $\alpha(t)$ should be taken as an effective power which might be associated with the resultant contribution of Regge poles. The zero intercept is not crucial in our fit; for definiteness we take it to be 1/2, the nominal value for the secondary trajectories. Here $\bar{D}_o$, $\bar{D}_1$ and α_1 are parameters left to be determined from the experiment. Since we do not include the Pomeranchuk contribution (the Pomeranchuk is rather flat whereas it will be seen that our effective trajectory is rather steep) in this model, we do not attempt to fit the data in the region where Pomeranchuk is important.

With this parameterization, we made a least squares fit to all the data illustrated in Fig. 1 for $P_L > 5$ GeV/c and $|t| > 2\ (\text{GeV/c})^2$. The results are also illustrated in Fig. 1. Note the data near the 90 deg. region, including the break region, can be adequately fitted.

The 90 deg. data for $|t| < 7(\mathrm{GeV/c})^2$ together with the off 90° data at 5 and 7 GeV/c essentially determines the energy dependence and the angular dependence of the Regge term. We note that the extrapolation of the fit down to 3 GeV/c at 90 deg. and the extrapolations at 5 and 7 GeV/c down to $|t| \gtrsim 1(\mathrm{GeV/c})^2$ are in good agreement. Above 7 GeV/c the energy dependence of our simple Regge term begin to show noticeable shrinkage effect; we attribute this to be an indication that the Pomeranchuk contribution begins to set in. As the energy increases, eventually the Pomeranchuk contribution will take over for fixed t; then the data will show little shrinkage effect. The parameters for the fits are

$$D_o = 0.30 \text{ mb}^{\frac{1}{2}} \text{ GeV},$$
$$N - 1 = 0.14 t_o ,$$
$$D_1 = 3.09 ,$$
$$\overline{D}_o = 28.2 \text{ mb}^{\frac{1}{2}} \text{ GeV},$$
$$\overline{D}_1 = -0.11 \text{ GeV}^{-2}$$
$$\alpha_1 = 0.70 \text{ GeV}^{-2} .$$

Note the lower bound amplitude parameters are quite similar to those which we have obtained above. Also the fit for the data beyond $|t| = 7\ (\mathrm{GeV/c})^2$ are comparable to that for the case $\gamma = 1/2$ as illustrated in Fig. 2.

To illustrate the relative importance of the two terms in Eq. (15), we also show in Fig. 1 the extensions of the Regge and the lower bound contribution at 90 deg. near the break region

$|t| = 7\ (\mathrm{GeV/c})^2$ and at 60 deg. near $|t| = 5\ (\mathrm{GeV/c})^2$. The extension of the Regge contribution at 9 GeV/c is also illustrated in Fig. 1.

The break occurs when the contributions of the Regge amplitude and the lower bound amplitude are comparable. The break for $z_s = 0$ occurs at $t \cong -7.1\ (\mathrm{GeV/c})^2$. The loci of this break as a function of z_s is illustrated in Fig. 1. Since the dcs is relatively flat near 90 deg., the break moves inward in $|t|$ nearly horizontally as the angle decreases. As the angle further decreases, both the $|t|$ value and $\frac{d\sigma}{dt}$ at the break decrease. This result is in contrast to the prediction of Huang, Jones and Teplitz[9]. In their model the break occurs at the point where the leading Regge trajectory passes through -1 and hence is found at a fixed value of t as the scattering angle is varied.

We illustrate in Fig. 2 also the prediction of the fit with Eq. (15) for this case at $P_L = 25$ GeV/c and at 60 and 90 degrees in the large $|t|$ region. Comparing with the corresponding curves for case (1), one sees that at 25 GeV/c the difference could already be quite noticeable although their values are comparable at 16.9 GeV/c.

Finally we note here that the value of N obtained from these fits is reasonably well determined by the 16.9 GeV/c data and the 90 deg. data. As mentioned in the previous section, for $\gamma = 1$ one could associate the quantity $(N - 1)/t_0$ with the slope of the Pomeranchuk trajectory, which is equal to $0.15\ \mathrm{GeV}^{-2}$, this value is compatible with the present knowledge of the slope of the Pomeranchuk trajectory.[11] For the case (1) the value of N is found to be 1.13.

IV. SUMMARY

We have shown that the extended lower bound, which has a fixed-t power behavior, can be readily compared with the data through Eq. (4). It is rather encouraging that the lower bound behavior is compatible with the existing data for $|t| \gtrsim 7 (GeV/c)^2$ (or for $\gamma = 1$, $|t| \gtrsim 8 (GeV/c)^2$) which include some data with the scattering angle down to 50 deg. At this stage, the experimental constraints are not sufficient to determine conclusively whether or not the lower bound behavior should coincide with the physical amplitude. The data can be fitted either for $\gamma = 1/2$ or $\gamma = 1$. Should the minimal interaction hypothesis be valid, one expects that the quantity $|\Delta|$ will decrease, stay constant or increase as $|t|$ increases depending on whether the corresponding value of γ is less than, equal to or greater than one. Future experimental measurements, even in the present accelerator energy region, will afford a significant test of the minimal interaction hypothesis. In the process of testing for this hypothesis, one will also be able to learn about the value of γ, and hence about the boundedness property of the scattering amplitude in the t-plane, which is of great theoretical interest.

Since the dcs is relatively flat near 90 deg., the break should move in horizontally in the $d\sigma/dt$ vs $|t|$ plot. If our assumptions are correct we find that the location of the break should decrease both in magnitude and in $|t|$ as the angle further decreases. Experimental verification of this conjecture would be very interesting.

ACKNOWLEDGMENT

The authors express their appreciation to Prof. Geoffrey Chew and Dr. Jerome Finklestein for discussions and criticism.

FOOTNOTES AND REFERENCES

* This work was performed under the auspices of the U. S. Atomic Energy Commission.

+ AEC postdoctoral fellow under contract No. AT (11 - 1) -34.

1. F. Cerulus and A. Martin, Physics Letters 8, 80 (1964), see also A. Martin, Nouvo Cimento 37, 671 (1965).

2. T. Kinoshita, Phys. Rev. Letters 12, 257 (1964).

3. C. Chiu and C.I. Tan, Power Behavior of the Scattering Amplitude at Fixed-Momentum Transfer and a Reconsideration of the Cerulus-Martin Lower Bound, UCRL-17552, in preparation.

4. J. Orear, Phys. Rev. Letters 12, 112 (1964).

5. C. W. Akerlof, R. H. Hieber, A. D. Krisch, K. W. Edwards, L. G. Ratner and K. Ruddick, Phys. Rev. Letters 17, 1105 (.966).

6. We checked that the values of $C_1(z)/C_1(0)$ and $(1 - z)$ agree better than 10^{-9} for $z \leq 0.55$. They start to deviate slightly from each other as z is increased. At $z \sim 0.85$, this deviation is maximum. The maximum difference is still less than 0.01. Once one sets $C_1(z) = C_1(0)(1 - z)$, then the formula of Eq. (5) follows readily.

7. In Fig. (1) we include the data of Ref. 5 and data points,
   (a) at 16.9 GeV/c by J. V. Allaby, G. Bellettini, G. Cocconi, A. N. Diddens, M. L. Good, G. Matthiae, E. J. Sacharidis, A. Silverman and A. M. Wetherell, Physics Letters 23, 389 (1966),
   (b) between 3 to 7 BeV/c by A. Clyde et al., see A. R. Clyde, Ph.D. Thesis, University of California, Berkeley, 1966 (unpublished),

(c) at 19.6 BeV/c by K. J. Foley, S. J. Lindenbaum, W. A. Love, S. Ozaki, J. J. Russell and L. C. L. Yuan, Phys. Rev. Letters 10, 376 (1963),

(d) energies between 11.1 to 31.8 GeV/c by G. Cocconi, V. T. Cocconi, A. D. Krisch, J. Orear, R. Rubinstein, D. B. Scarl and B. T. Ulrich, W. F. Baker, E. W. Jenkins and A. L. Read, Phys. Rev. 138, B165 (1965).

8. K. Huang, C. E. Jones, and V. L. Teplitz, Phys, Rev. Letters 18, 146 (1967).

9. I. A. Sakmar and Joseph H. Wojtaszek, Single Regge Pole Fit to 90° p-p Scattering, University of Miami preprint, 1967. In this paper the authors actually used a $P_{\alpha}(z_t)$ dependence. The trajectory they used is nearly zero at $t = -7$ $(\text{GeV})^2$ and for small α, $P_{\alpha}(z_t) \approx z_t^{\alpha}$.

10. See for example, W. Rarita, R. J. Riddell, Jr., C. B. Chiu and R. J. N. Phillips, Regge Pole Model for πp, pp and $\bar{p}p$ Scattering, University of California, in preparation.

11. It turns out that the form $\beta(t)(-z_t)^{\alpha(t)}$ for the present case is relatively flat near 90° region and does not decrease monotonically beyond 90°. So we have not symmetrized the expression here. Later on we will see that this approximate form is adequate in fitting the data. Our effective residue function $\beta(t)$ is parametrized to be a smooth function as required by the data. We argue that the zeros associated with the individual trajectory passing through the unphysical signature point are either smoothed out due to the contribution of several trajectories or that the effect of the

fixed-J singularity discussed by Mandelstam and Wang (see S. Mandelstam and L. L. Wang, Gribov-Pomeranchuk Poles in Scattering Amplitudes, University of California preprint, 1967) is important here.

FIGURE CAPTIONS

Fig. 1. $d\sigma/dt$ vs t for pp-scattering. Data points: ● at 90 deg. by Akerlof et al. [5]; ■ at 16.9 GeV/c by Allaby et al. [7a]; o at 5 GeV/c, ▽ at 7 GeV/c and △ at 7.08 GeV/c by Clyde et al., [7b]; ♠ by Cocconi et al., [7d]. The solid lines for $|t| < 1\ (\mathrm{GeV/c})^2$ indicate the experimental curves for the dcs at 5, 7 GeV/c (from [7b]) and at 19.6 GeV/c (from [7d]). The solid lines are the fitted pp dcs at $\Theta = 60, 90$ deg. and at $P_L = 5, 7, 9$ and 16.9 GeV/c for case (3). The big open circles are fitted values for the associated data point. The dashed lines are the extension of the fit to the lower $|t|$ region. The dash-dotted line indicates the motion of the break as a function of z_s. The light solid lines indicate the contribution of I - $d\sigma/dt\Big|_{\ell.b.}$ and II - $d\bar{\sigma}/dt$, at 90 deg. near $|t| = 7(\mathrm{GeV/c})^2$ and at 60 deg. near $|t| = 5\ (\mathrm{GeV/c})^2$ and II - $d\bar{\sigma}/dt$ at $P_L = 9$ GeV/c near $|t| = 4(\mathrm{GeV/c})^2$.

Fig. 2. $d\sigma/dt$ vs t for pp-scattering, for $|t| > 3(\mathrm{GeV/c})^2$. Data points: △ at 7 GeV/c by Clyde et al. [7b], ● at 90 deg. by Akerlof et al. [5], ■ at 16.9 GeV/c by Allaby et al. [7a], ♠ by Cocconi et al. [7d]. The solid lines extended by the dashed lines are the fitted curves and the extrapolation to the smaller $|t|$ region for case (1) at 90, 60 and 50 deg. and at $P_L = 25$ GeV/c. The open circles indicate the fitted values for the associated data points for this case. The dotted lines indicate the fit for case (2) at 16.9 GeV/c and at the 90 deg. near the break region. The dash-dotted curves indicate the dcs obtained at 90 and 60 deg. and $P_L = 25$ GeV/c for case (3).

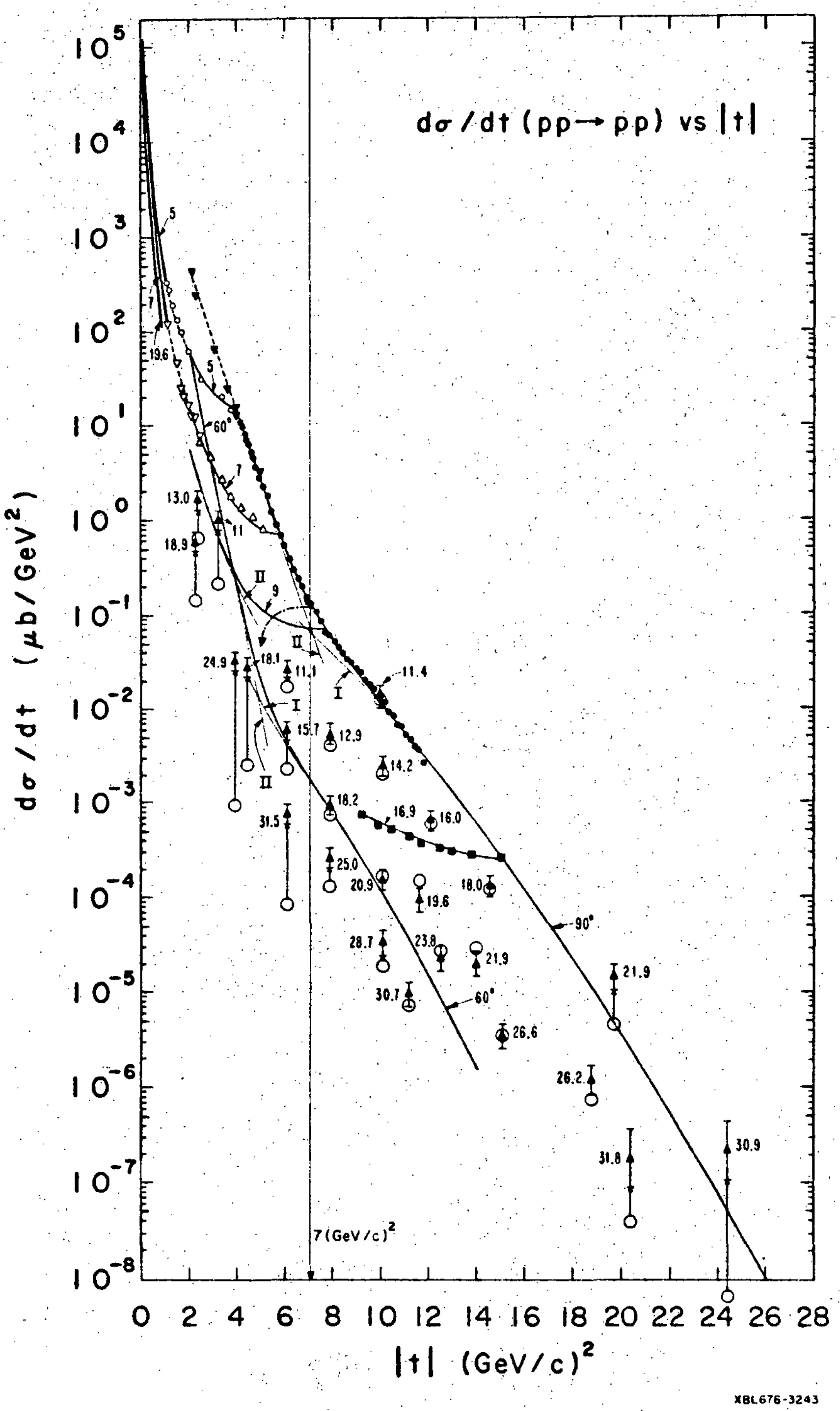

Fig. 1

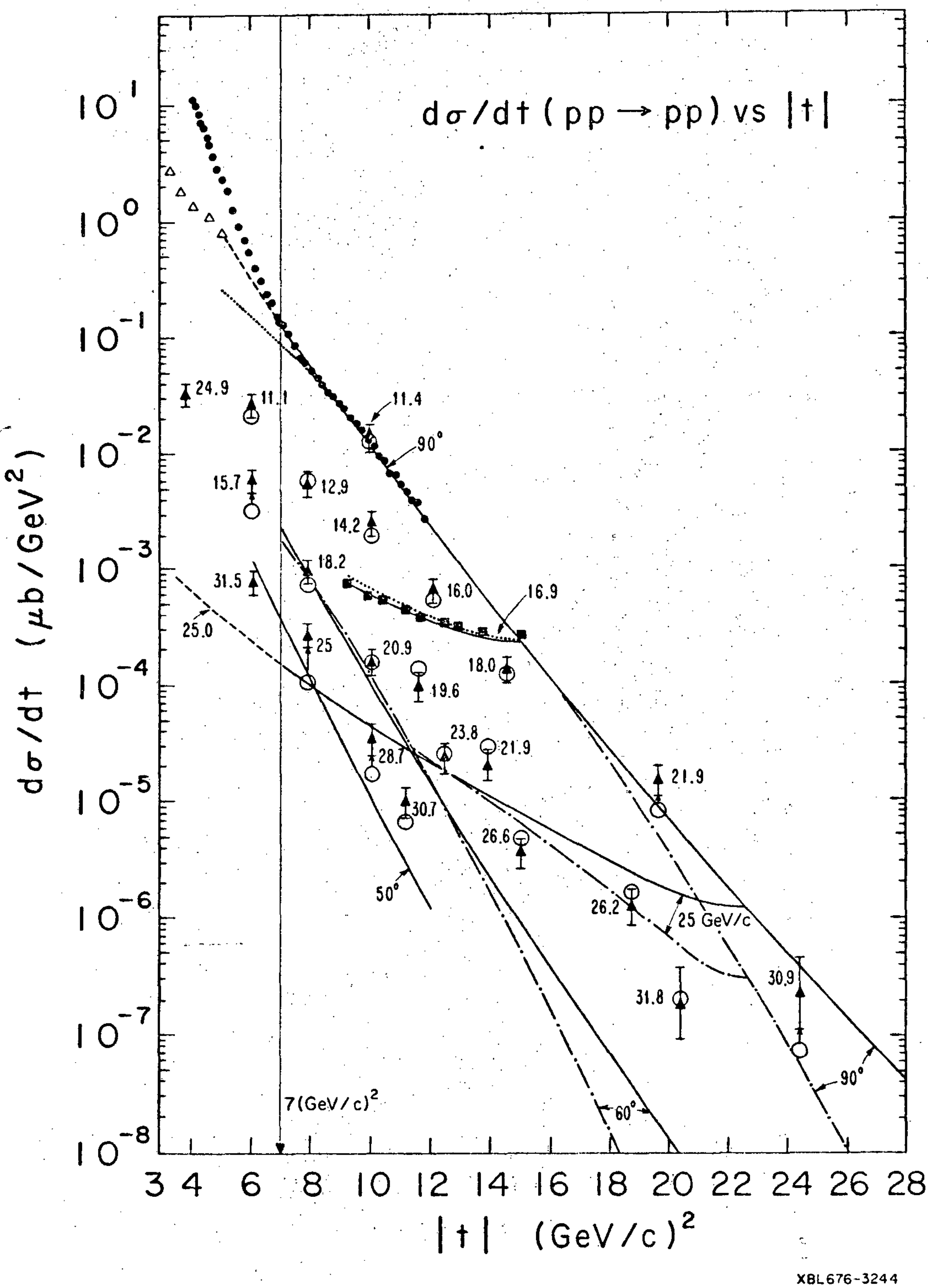
dσ/dt (pp → pp) vs |t|
dσ/dt (μb/GeV²)
|t| (GeV/c)²
7(GeV/c)²
11.4
90°
16.9
25 GeV/c
60°
50°
90°
24.9
11.1
15.7
12.9
14.2
18.2
31.5
25.0
16.0
25
20.9
19.6
18.0
28.7
23.8
21.9
30.7
21.9
26.6
26.2
31.8
30.9
XBL676-3244

Fig. 2