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Quasi-Orthogonal Space-Frequency and Space-Time-Frequency Block Codes for MIMO OFDM Channels

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***Abstract*— In this paper, we propose a novel class of Space-Frequency and Space-Time-Frequency block codes based on Quasi-Orthogonal designs, over a frequency selective Rayleigh fading channel. The proposed Space-Frequency code is able to achieve rate-one and full space and multipath diversity gains available in the MIMO-OFDM channel. As simulation results demonstrate, the code outperforms the existing Space-Frequency block codes in terms of bit error rate performance. By coding across the three dimension of space, time and frequency, we propose a Quasi-Orthogonal Space-Time-Frequency code that is capable of achieving rate-one and exploiting all of the spatial, multipath and temporal diversity gains offered by the channel. In case of a channel which is quasi-static over adjacent OFDM symbol durations, we propose a Space-Time-Frequency code that benefits from a reduced maximum likelihood decoding complexity.**

***Index Terms*— MIMO-OFDM, space-frequency codes, quasi-orthogonal codes, wireless communication, space-time-frequency codes.**

I. Introduction

SPATIAL diversity is a popular diversity method for combating the effects of fading without the need to increase the bandwidth. Space diversity can be implemented in the form of transmit and/or receive diversity creating Multiple-Input Multiple-Output (MIMO) channels. Orthogonal Frequency Division Multiplexing (OFDM) is a technique used in broadband wireless systems. The idea is to split the high rate data stream into a number of lower rate streams and modulate them over a number of subcarriers. This technique creates frequency-flat subchannels within a frequency selective channel. Thus, a combination of MIMO and OFDM is a promising technique for high data rate broadband wireless systems. A frequency selective channel offers an additional degree of diversity known as *multipath* or *frequency* diversity. In a MIMO-OFDM system, it is desirable to achieve multipath as well as spatial diversity gains. Space-Frequency (SF) and Space-Time-Frequency (STF) codes have been designed to achieve some levels of space and multipath diversity. Space-Frequency codes use the two dimensions of space (antenna) and frequency tones (subcarriers) to code over. It is proved that a MIMO-OFDM system can achieve a maximum diversity gain equal to the product of the number of its transmit antennas, the number of its receive antennas and the number of multipaths present in the frequency selective channel as long as the channel correlation matrix is full rank [1]–[3]. Space-Time-Frequency codes use the three dimensions of space, frequency and time to code across, therefore STF codes are capable of achieving an additional temporal diversity advantage on top of space and multipath diversity gains offered by the MIMO-OFDM channel. Authors in [4] and [5] prove that the STF code can achieve a diversity order equal to the product of the number of its transmit antennas, the number of its receive antennas, the number of independent channel taps and the rank of the temporal correlation matrix of the channel.

Space-time coded OFDM was first introduced in [6] by using space-time trellis codes over frequency tones. Authors in [7] introduced a space-frequency-time coding method over MIMO-OFDM channels. They used trellis coding to code over space and frequency and Orthogonal Space-Time Block codes (OSTBC) [8] to code over OFDM blocks. It is noteworthy that in the case of more than two transmit antennas the OSTBC can provide a rate of at most $\frac{3}{4}$ and we are not able to have rate-one transmission. In [9], authors point out the analogy between antennas and frequency tones and based on capacity calculation, propose a grouping method that reduces the complexity of code design for MIMO-OFDM systems. The idea of subcarrier grouping is further pursued in [2] with precoding and in [10] with bit interleaving. Reference [11] proposes a repetition mapping technique to transform the existing space-time codes, designed for quasi-static flat fading channels, to full-diversity codes in frequency selective fading channels. Note that their proposed method provides a tradeoff between diversity and symbol rate. Later on, the authors proposed a rate-one, full-diversity space-frequency block code in [12]. Their proposed scheme can obtain a target diversity gain but the decoding complexity grows exponentially with the desired diversity. We use their design as a reference to compare our proposed structure in terms of performance and complexity.

Quasi-Orthogonal Space-Time Block Code (QOSTBC) structures for quasi-static channels were first introduced in [13] and [14]. Original QOSTBC designs provide rate-one codes and pairwise Maximum Likelihood (ML) decoding but fail to achieve full-diversity. Later on, improved quasi-orthogonal codes were proposed through constellation rotation

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[15]–[19]. It is worthwhile to mention that rotation-based constellations to gain diversity were first introduced in [20] and later on used for multi-antenna systems in [21]. A rotated QOSTBC provides full diversity, rate one and better performance compared to OSTBC. These benefits together with the simple decoding capabilities of rotated quasi-orthogonal codes, motivate us to design Space-Frequency codes based on quasi-orthogonal structures.

In this paper, we provide a systematic method of designing rate-one, full-diversity space-frequency and space-time-frequency codes for two transmit antennas, using QOSTBCs. We specifically construct sample SF and STF codes for a frequency selective channel with two channel taps. As the simulation results suggest, the proposed codes have a better performance and under certain conditions, provide reduced decoding complexity compared to the existing rate-one codes. Furthermore, assuming that the channel is quasi-static over two OFDM symbols, we show that the decoding complexity of the space-time-frequency code can be reduced. Both SF and STF code structures provide full symbol rate (one symbol per frequency tone per time slot) and achieve any desired multipath (frequency) diversity available in the frequency selective fading channel.

During the review process of this paper, it came to our attention that in an independent work, reference [22] has discussed the connection between QOSTBC codes and the STF scheme published in [2]. Therefore, it is not surprising that in some cases, our STF codes, which are based on QOSTBCs, match the STF scheme of [2]. Note that no SF schemes are presented in [2] or [22].

The rest of the paper is organized as follows. In Section II, we describe the MIMO-OFDM channel model and the general structure of a SF code. In Section III, we introduce a general class of quasi-orthogonal space-time block codes for quasi-static flat fading channels. We use this class of QOSTBC as an underlying structure to design rate-one full-diversity space-frequency codes in Section IV. This class of SF codes is referred to as Quasi-Orthogonal Space-Frequency (QOSF) code structure. We continue with the design of Quasi-Orthogonal Space-Time-Frequency (QOSTF) block codes in Section V. In Section VI, the decoding of QOSF and QOSTF codes is discussed. Simulation results are presented in Section VII and finally some concluding remarks are provided in Section VIII.

Notation: Throughout this paper we use bold letters to represent matrices and underlined letters to represent vectors. Superscripts ^{T}, * and ^{H} stand for transpose, conjugate and conjugate transpose, respectively; $A \circ B$ denotes the Hadamard product of the matrices A and B, while $A \otimes B$ denotes their Kronecker product and $||A||_F$ represents the Frobenius norm of the matrix A. Also, diag$(\underline{a}_1, \ldots, \underline{a}_n)$, where $\underline{a}_i$ is a row vector of size T, denotes a $n \times nT$ block diagonal matrix where the vectors $\underline{a}_1, \ldots, \underline{a}_n$ are the block diagonal elements. Note that $\mathcal{C}^{M \times N}$ is used to represent the set of $M \times N$ matrices over complex numbers.

II. Channel Model

In this section, we define the channel model we use throughout the paper. Consider a MIMO-OFDM system with M_T transmit and M_R receive antennas. We assume that the receiver has perfect channel knowledge while the transmitter does not know the channel. Throughout this work, we assume no spatial fading correlation exists in between antennas. Each channel between transmit antenna i and receive antenna j is assumed to have L independent channel taps and the channel impulse response vector in discrete-time is given as $[h_{ij}(0), \ldots, h_{ij}(L-1)] \in \mathcal{C}^{1 \times L}$. It is assumed that all channels have the same power-delay profile. Note that each $h_{ij}(l)$ is a zero mean complex Gaussian random variable with a variance of σ_l^2. For normalization purposes, we assume that $\sum_{l=0}^{L-1} \sigma_l^2 = 1$. Also, assume that we have N frequency subcarriers. A space-frequency codeword is represented by

$$\mathbf{C}_{SF} = \begin{bmatrix} c_1(0) & c_2(0) & \ldots & c_{M_T}(0) \\ c_1(1) & c_2(1) & \ldots & c_{M_T}(1) \\ \vdots & \vdots & \ddots & \vdots \\ c_1(N-1) & c_2(N-1) & \ldots & c_{M_T}(N-1) \end{bmatrix} \in \mathcal{C}^{N \times M_T}, \quad (1)$$

where $c_i(n)$ is the data transmitted by the i^{th} transmit antenna at the n^{th} frequency subcarrier. A space-time-frequency codeword has an additional dimension of time added to the above SF codeword. In general we can express a STF codeword transmitted during the t^{th} OFDM symbol by $\mathbf{C}_{STF}^t = [c_i^t(n)] \in \mathcal{C}^{N \times M_T}$. The OFDM transmitter performs an N-point IFFT over the frequency tones. In order to remove the Inter Symbol Interference (ISI) which is caused by the multipath delay of the channel, one needs to add a cyclic prefix to each OFDM symbol. The length of the cyclic prefix should be equal to or greater than the delay spread of the multipath channel. Note that the addition of cyclic prefix comes at the cost of reducing the spectral efficiency. After removing the cyclic prefix and applying FFT on frequency tones, the received signal at receive antenna j at the n^{th} subcarrier during the t^{th} OFDM symbol duration is given by

$$r_j^t(n) = \sum_{i=1}^{M_T} c_i^t(n) H_{ij}^t(n) + \mathcal{N}_j^t(n), \ \ n = 0, 1 \ldots, N-1, \quad (2)$$

where $H_{ij}^t(n)$ is the frequency response of the channel at the n^{th} frequency subcarrier within the t^{th} OFDM symbol duration given by

$$H_{ij}^t(n) = \sum_{l=0}^{L-1} h_{ij}^t(l) e^{-j2\pi l \frac{n}{N}}, \qquad n = 0, 1 \ldots, N-1. \quad (3)$$

Also, $\mathcal{N}_j^t(n)$ is a circularly symmetric zero-mean Gaussian noise term corresponding to the n^{th} frequency subcarrier and the t^{th} OFDM symbol duration.

In other words, for the t^{th} OFDM symbol, the receiver equation can be represented in matrix format by

$$\begin{bmatrix} \underline{r}^t(0) \\ \vdots \\ \underline{r}^t(N-1) \end{bmatrix} = \text{diag}\big(\underline{C}^t(0), \ldots, \underline{C}^t(N-1)\big) \begin{bmatrix} \mathbf{H}^t(0) \\ \vdots \\ \mathbf{H}^t(N-1) \end{bmatrix} + \begin{bmatrix} \mathcal{N}_1^t(0) & \ldots & \mathcal{N}_{M_R}^t(0) \\ \vdots & \ddots & \vdots \\ \mathcal{N}_1^t(N-1) & \ldots & \mathcal{N}_{M_R}^t(N-1) \end{bmatrix}, \quad (4)$$

where $\underline{r}^t(n) \in \mathcal{C}^{1\times M_R}$ is the vector of received signals, $\underline{C}^t(n) \in \mathcal{C}^{1\times M_T}$ is the vector of transmitted symbols and $\mathbf{H}^t(n) \in \mathcal{C}^{M_T\times M_R}$ is the matrix of channel coefficients in frequency domain during the corresponding OFDM symbol duration and subcarrier. For the channel model characterized in this section, the maximum achievable diversity by using a SF code, is equal to LM_TM_R [1]. In order to achieve such maximum diversity gains, the number of subcarriers, N, has to be larger than or equal to the number of independent delay paths, L. For a STF code the maximum achievable diversity level is $LM_TM_R\tau$, where τ is the rank of the channel temporal correlation matrix [4].

III. Generalized Block-Diagonal Quasi-Orthogonal Space-Time Block Codes

In this section, we introduce a class of space-time block codes based on quasi-orthogonal designs for any number of transmit antennas over a quasi-static flat fading channel model. This class of QOSTBC has a block-diagonal structure that will be useful later in Section IV to build space-frequency block codes. Let us denote the Alamouti scheme [23] for the two indeterminate variables x_1 and x_2 by

$$\mathbf{A}(x_1, x_2) = \begin{bmatrix} x_1 & x_2 \\ -x_2^* & x_1^* \end{bmatrix}. \quad (5)$$

The quasi-orthogonal space-time block code for four transmit antennas has a structure given by [13], [14],

$$\mathbf{C}_4 = \begin{bmatrix} \mathbf{A}(s_1, s_2) & \mathbf{A}(\tilde{s}_3, \tilde{s}_4) \\ \mathbf{A}(\tilde{s}_3, \tilde{s}_4) & \mathbf{A}(s_1, s_2) \end{bmatrix}, \quad (6)$$

where s_1 and s_2 belong to a constellation $\mathcal{A}$ and $\tilde{s}_3$ and $\tilde{s}_4$ belong to the rotated constellation $e^{j\theta}\mathcal{A}$. The code in (6) provides a full-diversity rate-one transmission scheme for four transmit antennas over quasi-static channels. Now consider the following code structure,

$$\mathbf{C}_4 = \begin{bmatrix} \mathbf{A}(s_1 + \tilde{s}_3, s_2 + \tilde{s}_4) & \mathbf{0} \\ \mathbf{0} & \mathbf{A}(s_1 - \tilde{s}_3, s_2 - \tilde{s}_4) \end{bmatrix}. \quad (7)$$

It is straightforward to show that the diversity conditions and the coding gain structure of the codes in Equations (6) and (7) are the same; therefore their Bit-Error-Rate (BER) vs. Signal-to-Noise-Ratio (SNR) behavior is the same although their transmission schemes are different [24, page 121]. We now extend the above structure to design a QOSTBC for $M_T = 2k$ transmit antennas where $k = 2^r$ for some positive integer r. For a block of $2k$ symbols, $\{s_1, \ldots, s_{2k}\}$, where s_i's are taken from a constellation $\mathcal{A}$, we define a new set of combined symbols, $\{\mathcal{S}_1, \ldots, \mathcal{S}_{2k}\}$, as follows

$$[\mathcal{S}_1\ \mathcal{S}_3\ \ldots\ \mathcal{S}_{2k-1}]^T = \mathbf{\Theta}[s_1\ s_3\ \ldots\ s_{2k-1}]^T, \quad (8a)$$

$$[\mathcal{S}_2\ \mathcal{S}_4\ \ldots\ \mathcal{S}_{2k}]^T = \mathbf{\Theta}[s_2\ s_4\ \ldots\ s_{2k}]^T. \quad (8b)$$

where $\mathbf{\Theta} = \mathbf{T} \times \text{diag}\{1, e^{j\theta_1}, \ldots, e^{j\theta_{k-1}}\}$ and $\mathbf{T} \in \mathcal{C}^{k\times k}$ is a Hadamard matrix[1]. Note that the above structure is not unique and one can use any invertible linear combination of the s_i's to construct the combined symbols $\mathcal{S}_i$'s. We now present a general class of QOSTBCs for the $M_T = 2k$ transmit antennas, over a quasi-static flat-fading channel as follows

$$\mathbf{C}_{2k} = \begin{bmatrix} \mathbf{A}(\mathcal{S}_1, \mathcal{S}_2) & \mathbf{0} & \ldots & \mathbf{0} \\ \mathbf{0} & \mathbf{A}(\mathcal{S}_3, \mathcal{S}_4) & \ldots & \mathbf{0} \\ \vdots & \vdots & \ddots & \vdots \\ \mathbf{0} & \mathbf{0} & \ldots & \mathbf{A}(\mathcal{S}_{2k-1}, \mathcal{S}_{2k}) \end{bmatrix} \in \mathcal{C}^{2k\times 2k}. \quad (9)$$

As will be discussed later, $\mathbf{C}_{2k}$ is capable of achieving full-diversity. Note that the block diagonal structure of the code in (9) is desirable in designing space-frequency codes in Section IV. The code in (9) is designed for $M_T = 2k = 2^{r+1}$ number of transmit antennas. By eliminating the proper rows and columns of the codeword matrix and the corresponding symbols from the set of combined symbols, we can design codes for an arbitrary number of transmit antennas. The resulting codes achieve full diversity and rate-one. As an example, suppose we want to design a generalized QOSTBC code for $M_T = 6$ transmit antennas. First, we select the code designed for $M_T = 8$ transmit antennas. To come up with a code for $M_T = 6$, we eliminate the last two rows and the last two columns of the codeword $\mathbf{C}_8$ and omit the symbols s_7 and s_8 from the combined symbols in (8a) and (8b) as well. To further design a code for $M_T = 5$ transmit antennas, we omit the symbol s_6 in the combined symbols and eliminate the last column of the codeword $\mathbf{C}_6$. These codes are still rate-one and achieve full-diversity.

A. Design Criteria

Let us denote two distinct sets of symbols by $\{s_1, s_2, \ldots, s_{2k}\}$ and $\{u_1, u_2, \ldots, u_{2k}\}$, where $s_i,\ u_i \in \mathcal{A},\ \forall i \in \{1, 2, \ldots, 2k\}$. We construct the sets of combined symbols, $\{\mathcal{S}_1, \mathcal{S}_2, \ldots, \mathcal{S}_{2k}\}$ and $\{\mathcal{U}_1, \mathcal{U}_2, \ldots, \mathcal{U}_{2k}\}$, corresponding to s_i's and u_i's respectively, by using the equations (8a) and (8b). Now let us define the set of differences (pairwise combined-symbol errors) $\{D_1, D_2, \ldots, D_{2k}\}$, where $D_i = \mathcal{S}_i - \mathcal{U}_i,\ \forall i \in \{1, 2, \ldots, 2k\}$. It is easily seen that,

$$\det\{(\mathbf{C}_{2k}^i - \mathbf{C}_{2k}^j)^H(\mathbf{C}_{2k}^i - \mathbf{C}_{2k}^j)\} = (|D_1|^2 + |D_2|^2)^2(|D_3|^2 + |D_4|^2)^2 \ldots (|D_{2k-1}|^2 + |D_{2k}|^2)^2. \quad (10)$$

The rotation angles $\{\theta_1, \theta_2, \ldots, \theta_{k-1}\}$ for the QOSTBC given by (9), are chosen such that for all distinct sets of $\{s_1, \ldots, s_{2k}\}$ and $\{u_1, \ldots, u_{2k}\}$, the following two conditions are satisfied:

1) Diversity: To guarantee full-diversity, it is necessary to ensure that the rotation angles are chosen such that for $d_i = s_i - u_i,\ \forall s_i,\ u_i \in \mathcal{A}$,

$$|D_1| \quad = \quad |d_1 + e^{j\theta_1}d_2 + \cdots + e^{j\theta_{k-1}}d_k| \neq 0,$$

If we switch s_i and u_i for any $i \in \{2, \ldots, k\}$, we get $D_j \neq 0, \forall j \in \{1, 3, \ldots, 2k-1\}$ as well.

2) Coding Gain: To maximize the coding gain, the following optimization problem needs to be solved,

$$\max_{\theta_1,\ldots,\theta_{k-1}} \ \min_{D_1, D_3 \ldots, D_{2k-1}} |D_1 D_3 \ldots D_{2k-1}|.$$

[1] An $n\times n$ Hadamard matrix is a matrix of +1's and -1's such that $HH^T = nI_n$.

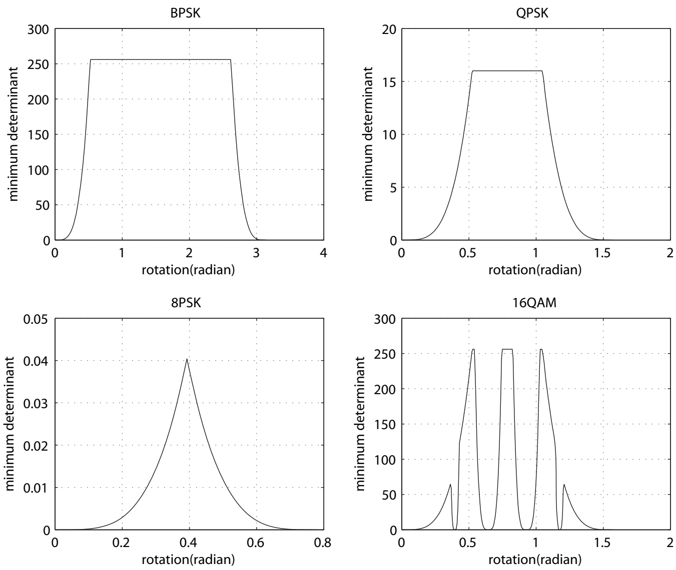

Fig. 1. Optimal rotation angles for two transmit antennas.

Note that the minimum coding gain is achieved when one of the following sets $\{D_1, D_3, \ldots, D_{2k-1}\}$ or $\{D_2, D_4, \ldots, D_{2k}\}$ are zero. Without loss of generality, we have assumed that $\{D_2, D_4, \ldots, D_{2k}\}$ is the zero set.

The decoding of the quasi-orthogonal STBC structure in (9) is done for k symbols at a time. Thus the decoding complexity grows exponentially with k. The above class of QOSTBCs provides rate-one, full-diversity block codes for any number of transmit antennas at the expense of higher decoding complexity compared to the orthogonal space-time block codes. For the case of $M_T = 2$ transmit antennas or equivalently $k = 1$, the code in (9) reduces to the well-known Alamouti code. For the case of $M_T = 4$ transmit antennas and consequently $k = 2$, we obtain the quasi-orthogonal code given by (7).

B. Optimal Rotation Angles

The optimum rotation angles, θ_i's, are determined such that the coding gain is maximized while the code is full-diversity. The code in Equation (7) has the same performance as the rotated quasi-orthogonal codes for four transmit antennas discussed in [15]–[17]. It is full-rate and achieves full-diversity and has a pairwise maximum likelihood decoding. The minimum coding gain structure of the code in (7) is also the same as the minimum coding gain of the existing quasi-orthogonal codes. Therefore, the optimum rotation angles for this code, for MPSK constellation is π/M (for M even) and $\pi/2M$ (for M odd) and for QAM is $\pi/4$ [18], [24]–[26]. Notice that the optimal rotation angles are not unique. Fig. 1 depicts the values for the minimum determinant vs. the rotation angles for $M_T = 4$ transmit antennas. Table I lists some of the optimal rotation angles for BPSK and QPSK constellations for 4, 6 and 8 number of antennas. The results are obtained through exhaustive search.

TABLE I
OPTIMAL ROTATION ANGLES

	$M_T = 4$	$M_T = 6$	$M_T = 8$
constellation	θ_1	θ_1, θ_2	$\theta_1, \theta_2, \theta_3$
BPSK	$\pi/2$	$\pi/4, 3\pi/4$	$\pi/4, \pi/2, 3\pi/4$
QPSK	$\pi/4$	0.4638,0.9275	$\pi/8, \pi/4, 3\pi/8$

IV. Quasi-Orthogonal Space-Frequency Code Structure

It has been shown in [1] that by applying the existing orthogonal space-time block codes to frequency domain, it is not guaranteed that we achieve the multipath diversity gain of a frequency selective fading channel. In this section we provide a guideline for constructing space-frequency block codes based on quasi-orthogonal designs, that is guaranteed to exploit any desired level of multipath diversity.

Consider the MIMO-OFDM system described in Section II where $M_T = 2$ and $L \leq N$. In order to design a SF block code that exploits full spatial diversity and a multipath diversity of L, we use the QOSTBC designed for $2L$ transmit antennas in quasi-static channel model given by Equation (9). A general SF codeword, based on the aforementioned quasi-orthogonal design, is expressed as

$$\mathbf{C}_{SF} = \begin{bmatrix} \mathbf{G}^{1T} & \mathbf{G}^{2T} & \ldots & \mathbf{G}^{mT} & \ldots \end{bmatrix}^T \in \mathcal{C}^{N\times 2}, \quad (11)$$

where,

$$\mathbf{G}^m = \begin{bmatrix} \mathbf{A}(\mathcal{S}_1^m, \mathcal{S}_2^m) \\ \mathbf{A}(\mathcal{S}_3^m, \mathcal{S}_4^m) \\ \vdots \\ \mathbf{A}(\mathcal{S}_{2L-1}^m, \mathcal{S}_{2L}^m) \end{bmatrix} \in \mathcal{C}^{2L\times 2}, \quad (12)$$

and the superscript $m \in \{1, \ldots, \lfloor \frac{N}{2L} \rfloor\}$ denotes the block number. Note that if N, the number of subcarriers, is not an integer multiple of $2L$, we need to pad the space-frequency codeword with zeros. For simplicity, let us assume from now on that $N = 2Lp$, for some integer p. The proof of full-diversity for the QOSF code of (11) is presented in the Appendix A of the paper.

As an example, consider a multipath channel with $L = 2$ and $M_T = 2$. We construct the QOSF code as follows,

$$\mathbf{C}_{SF} = \begin{bmatrix} s_1^1 + \tilde{s}_3^1 & s_2^1 + \tilde{s}_4^1 \\ -s_2^{1*} - \tilde{s}_4^{1*} & s_1^{1*} + \tilde{s}_3^{1*} \\ s_1^1 - \tilde{s}_3^1 & s_2^1 - \tilde{s}_4^1 \\ -s_2^{1*} + \tilde{s}_4^{1*} & s_1^{1*} - \tilde{s}_3^{1*} \\ s_1^2 + \tilde{s}_3^2 & s_2^2 + \tilde{s}_4^2 \\ -s_2^{2*} - \tilde{s}_4^{2*} & s_1^{2*} + \tilde{s}_3^{2*} \\ s_1^2 - \tilde{s}_3^2 & s_2^2 - \tilde{s}_4^2 \\ -s_2^{2*} + \tilde{s}_4^{2*} & s_1^{2*} - \tilde{s}_3^{2*} \\ \vdots & \vdots \end{bmatrix} \quad (13)$$

Note that in general, there is a tradeoff between the amount of multipath diversity one can achieve and the corresponding decoding complexity. Also, note that although we designed the codes for two transmit antennas, the design can be generalized to more than two transmit antennas by simply using the

QOSTBC designed for $M_T L$ antennas in (9) and applying it to frequency domain following the same guidelines provided in this section.

V. Quasi-Orthogonal Space-Time-Frequency Code Structure

Consider a multipath channel described in Section II where $M_T = 2$ transmit antennas. Assume a temporal diversity of τ is desired, therefore we spread our codeword across τ OFDM symbol durations. We choose a generalized QOSTBC code, given by (9), corresponding to $2L\tau$ transmit antennas to build our QOSTF code. The codeword transmitted during the t'th OFDM symbol duration is given by

$$\mathbf{C}_{STF}^t = \begin{bmatrix} \mathbf{G}_t^{1^T} & \mathbf{G}_t^{2^T} & \ldots & \mathbf{G}_t^{m^T} & \ldots \end{bmatrix}^T \in \mathcal{C}^{N\times 2}, \quad (14)$$

where $t \in \{1,\ldots,\tau\}$ and for a block index of $m \in \{1,\ldots,\lfloor \frac{N}{2L} \rfloor\}$,

$$\mathbf{G}_t^m = \begin{bmatrix} \mathbf{A}(\mathcal{S}_{2L(t-1)+1}^m, \mathcal{S}_{2L(t-1)+2}^m) \\ \mathbf{A}(\mathcal{S}_{2L(t-1)+3}^m, \mathcal{S}_{2L(t-1)+4}^m) \\ \vdots \\ \mathbf{A}(\mathcal{S}_{2Lt-1}^m, \mathcal{S}_{2Lt}^m) \end{bmatrix} \in \mathcal{C}^{2L\times 2}. \quad (15)$$

Note that $\{\mathcal{S}_1^m,\ \mathcal{S}_2^m, \ldots,\ \mathcal{S}_{2L\tau}^m\}$ are defined in (8a) and (8b). The proof that our proposed QOSTF code achieves full space, frequency and time diversity over independently changing channels, is provided in Appendix B of the paper. In general, for larger temporal diversity advantage τ, one can spread the codewords across an arbitrary number of OFDM blocks but there is a delay of τ OFDM symbols associated with the decoding process. If the channel is quasi-static over the adjacent OFDM blocks, i.e. the channel stays constant for B time slots, STF coding cannot provide additional temporal diversity advantage. However, in such a scenario we propose a QOSTF code that provides reduced ML decoding complexity. For instance, if $B = 2$, we select an underlying generalized QOSTBC that provides a diversity advantage of $2L$ and spread the codeword across the $B = 2$ adjacent OFDM symbols as follows,

$$\mathbf{C}^1 = \begin{bmatrix} \mathcal{S}_1^1 & \mathcal{S}_3^1 & \ldots & \mathcal{S}_{2L-1}^1 & \mathcal{S}_1^2 & \ldots \\ \mathcal{S}_2^1 & \mathcal{S}_4^1 & \ldots & \mathcal{S}_{2L}^1 & \mathcal{S}_2^2 & \ldots \end{bmatrix}^T \in \mathcal{C}^{N\times 2},$$
$$\mathbf{C}^2 = \begin{bmatrix} -\mathcal{S}_2^{1*} & -\mathcal{S}_4^{1*} & \ldots & -\mathcal{S}_{2L}^{1*} & -\mathcal{S}_2^{2*} & \ldots \\ \mathcal{S}_1^{1*} & \mathcal{S}_3^{1*} & \ldots & \mathcal{S}_{2L-1}^{1*} & \mathcal{S}_1^{2*} & \ldots \end{bmatrix}^T \in \mathcal{C}^{N\times 2}. \quad (16)$$

Now, as an example, consider a channel with $L = 2$ that is quasi-static over $B = 2$ adjacent OFDM symbols (consequently $\tau = 1$). The proposed QOSTF code is given as,

$$\mathbf{C}^1 = \begin{bmatrix} s_1^1 + \tilde{s}_3^1 & s_2^1 + \tilde{s}_4^1 \\ s_1^1 - \tilde{s}_3^1 & s_2^1 - \tilde{s}_4^1 \\ s_1^2 + \tilde{s}_3^2 & s_2^2 + \tilde{s}_4^2 \\ s_1^2 - \tilde{s}_3^2 & s_2^2 - \tilde{s}_4^2 \\ \vdots & \vdots \end{bmatrix} \in \mathcal{C}^{N\times 2},$$
$$\mathbf{C}^2 = \begin{bmatrix} -s_2^{1*} - \tilde{s}_4^{1*} & s_1^{1*} + \tilde{s}_3^{1*} \\ -s_3^{1*} + \tilde{s}_4^{1*} & s_1^{1*} - \tilde{s}_2^{1*} \\ -s_2^{2*} - \tilde{s}_4^{2*} & s_1^{2*} + \tilde{s}_3^{2*} \\ -s_2^{2*} + \tilde{s}_4^{2*} & s_1^{2*} - \tilde{s}_3^{2*} \\ \vdots & \vdots \end{bmatrix} \in \mathcal{C}^{N\times 2}. \quad (17)$$

The code in (17) has a pairwise ML decoding which is simplified compared to the existing codes and also the QOSF code discussed in Section IV. Details of decoding are provided in Section VI.

VI. Decoding of QOSF and QOSTF Block Codes

Assume we have $M_R = 1$ receive antenna and $H_i(n)$ represents the one tap channel gain between transmit antenna i and the single-antenna receiver at carrier frequency n. Let $\underline{H}(n) = \begin{bmatrix} H_1(n) & H_2(n) \end{bmatrix}^T$. Assume that the channel is quasi-static over adjacent OFDM symbols and consider the QOSTF code in (16). Due to the independence of different blocks of data corresponding to different values of m, the Maximum-Likelihood (ML) decoding is reduced into independent ML decoding per block. Assuming perfect channel information at the receiver, the ML decision rule for the m'th block is given by

$$\arg \min_{\{\mathcal{S}_1^m, \mathcal{S}_2^m, \ldots, \mathcal{S}_{2L}^m\}} \sum_{n=0}^{L-1} ||\underline{y}(n+(m-1)L) - \mathbf{A}(\mathcal{S}_{2n+1}^m, \mathcal{S}_{2n+2}^m)\underline{H}(n+(m-1)L)||_F^2 \quad (18)$$

where $\underline{y}(n) = [r^1(n)\ r^2(n)]^T$ represents a vector containing the received signals at two consecutive OFDM symbols over the n'th subcarrier. Furthermore, the Alamouti structure of the subblocks enables independent decoding per sets of $\{\mathcal{S}_1^m, \ldots, \mathcal{S}_{2L-1}^m\}$ and $\{\mathcal{S}_2^m, \ldots, \mathcal{S}_{2L}^m\}$. Therefore, the QOSTF code in (17) has a pairwise ML decoding. Note that there is a delay of two OFDM symbols associated with the decoding of the STF code. Based on the discussion above, for the same level of diversity, our proposed STF code has a decoding complexity which is a power of 1/2 of the decoding complexity of the code in Ref. [12].

In most urban communication channel models, the root-mean-square value of the delay spread is smaller than 2.5μsec. For a typical sampling frequency of 1 MHz one can assume that if the number of OFDM tones is larger than 500, two adjacent frequency tones undergo the same fading [27] and a block fading model can be adopted. Therefore, one can separate the decoding formulas for the sample QOSF code in (13) into two independent functions each containing a pair of the symbols. Under these assumptions, the decoding complexity of the QOSF code is considerably reduced and is similar to the decoding complexity of the QOSTF code. Note that in general sphere decoding methods can be used to reduce the decoding complexity for both QOSF and QOSTF codes.

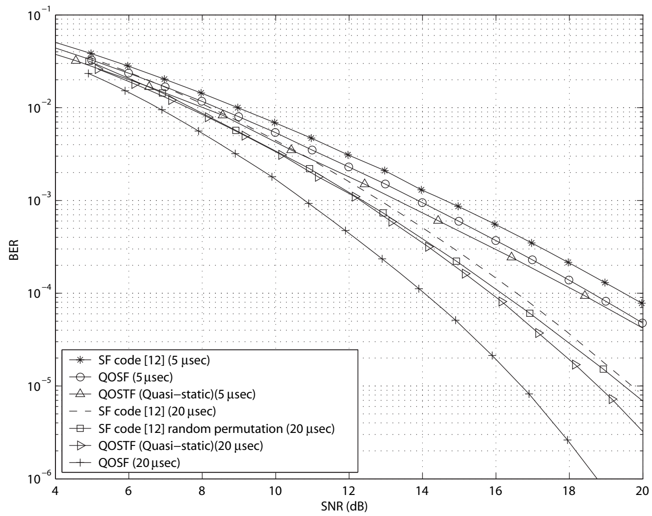

Fig. 2. BER vs. SNR for a 2-ray channel with delay spreads of 5 μsec and 20 μsec respectively; 1 bit/sec/Hz using BPSK.

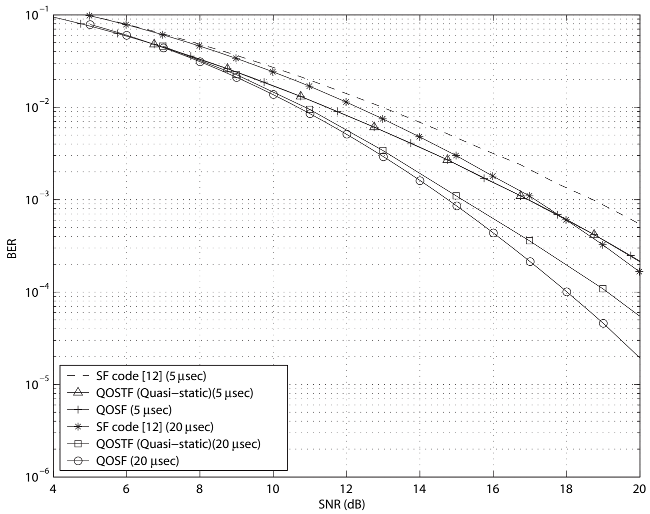

Fig. 3. BER vs. SNR for a 2-ray channel with delay spreads of 5 μsec and 20 μsec respectively; 2 bits/sec/Hz using QPSK.

VII. Simulation Results

The MIMO-OFDM system we use in our simulation model consists of $M_T = 2$ transmit antennas, $M_R = 1$ receive antenna and $N = 128$ subcarriers. We assume that the receiver has perfect channel state information. Assume that the average symbol power per transmit antenna is $E_s = \frac{1}{M_T}$ and the noise variance is $\frac{1}{SNR}$. We carry out the simulations for two different channel models. First we use a 2-ray equal power channel model with delay spreads of 5 μsec and 20 μsec. Then we use an exponential decay power profile model where the root mean square (rms) delay spread of the channel is 5 μsec and the maximum delay spread is set to be 10 times the rms delay spread. The length of the cyclic prefix is set to be 20 μsec in all cases. For our QOSF and QOSTF schemes, the rotation angles are $\theta = \frac{\pi}{2}$ and $\theta = \frac{\pi}{4}$ for BPSK and QPSK constellations, respectively. We compare our proposed scheme to the space-frequency block codes presented by Su et al. in [12], which are the best existing SF block codes in the literature. Note that the decoding for the code in [12] is performed for four symbols at a time.

Fig. 2 depicts the bit error rate performance of our QOSF and QOSTF codes given by (13) and (17) compared to the SF code in [12]. Note that for the code in (17) the channel is considered to be quasi-static over two adjacent OFDM symbols. The symbols are chosen from a BPSK constellation, therefore ignoring the cyclic prefix, we have a spectral efficiency of 1 bit/sec/Hz. It is evident from the figures that both QOSTF and QOSF schemes outperform the code in [12]. As seen in the figure, in the case of 20μsec delay spread, even when random subcarrier permutation (interleaving) is applied to [12] to improve its performance, still the QOSF and QOSTF schemes are superior. As the delay spread of the channel increases, the QOSF code dominates the other curves and outperforms both the code in [12] and the QOSTF over quasi-static channel. Nevertheless, we have to restate that in case of the QOSTF code over quasi-static channel, we benefit from a reduced decoding complexity. In Fig. 2, at a bit error rate of 10^{-5} and a delay spread of 20 μsec, QOSF code outperforms the code in [12] by almost 3 dB. Fig. 3 depicts the bit error rate vs. signal to noise ratio for the code in [12] and the QOSF and QOSTF codes at a spectral efficiency of 2 bits/sec/Hz. The superiority of our proposed QOSF scheme over that of [12] is evident from the figure. In Fig. 3, at a bit error rate of 10^{-4} and a delay spread of 20 μsec, again we observe a performance advantage of about 3 dB over the scheme in [12]. In Fig. 4 we study the performance of the QOSTF code of (17) over the following channel scenarios:

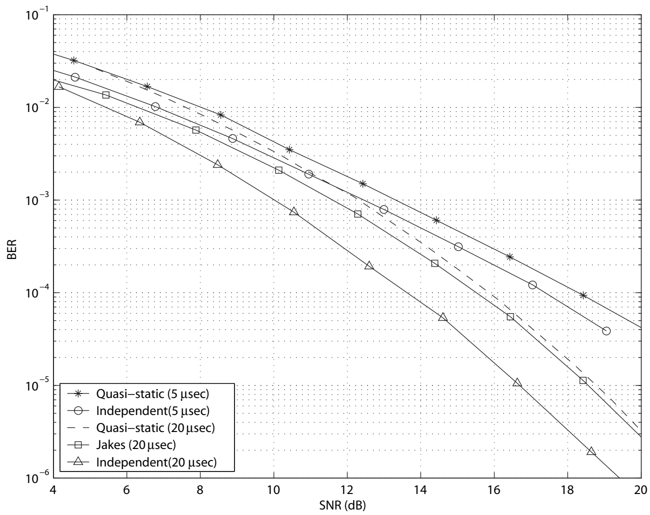

Fig. 4. BER vs. SNR of QOSTF code for a 2-ray channel with delay spreads of 5 μsec and 20 μsec; 1 bit/sec/Hz using BPSK.

1) The channel is quasi-static over two OFDM symbol durations,
2) The channel changes from one OFDM symbol to the next in a correlated manner following a Jakes model [28] with $f_D T = 0.0025$,
3) The channel changes independently from one OFDM symbol to the next.

We observe that the QOSTF code over independent channel realizations offers the best bit-error-rate performance. While

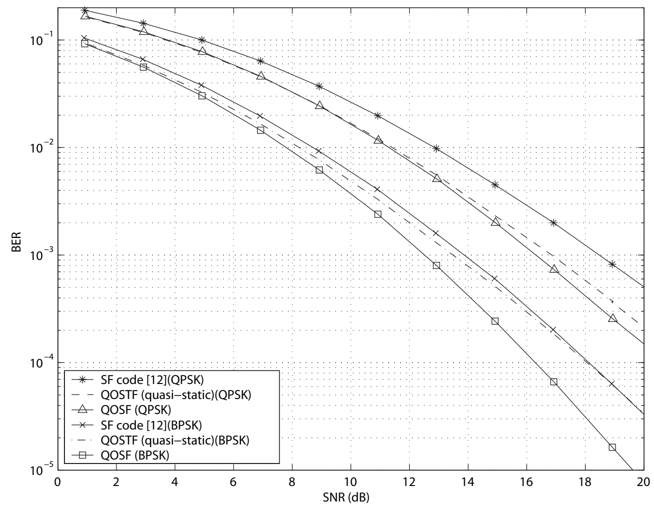

Fig. 5. BER vs. SNR for an exponential decay power delay profile; 1 bit/sec/Hz using BPSK and 2 bits/sec/Hz using QPSK.

the QOSTF scheme over Jakes channel model falls in between the quasi-static and independent scenarios. We have to mention that throughout our simulations, we used the simplified decoding for QOSTF in the case of quasi-static fading scenario.

It is interesting to note that although QOSF, QOSTF and the code in [12] achieve full spatial and multipath diversity, the slopes of the performance curves are not equal. We conjecture that as the concept of diversity of a code is based on asymptotical analysis for large SNR, the slopes of the BER-SNR curves will be equal for larger values of SNR not captured in our simulations.

It is also worthwhile to mention that decreasing the FFT length while keeping the bandwidth constant, improves the performance of the QOSF scheme by decreasing the correlation between adjacent subcarriers.

In Fig. 5, the performances of the QOSF and QOSTF codes are compared with the code of [12] for an exponential decay channel model for 1 bit/sec/Hz and 2 bits/sec/Hz using BPSK and QPSK respectively. As the simulation result suggests, even in more practical channel models, the proposed QOSF code demonstrates a superior performance over that of [12], which to the best of our knowledge, is the best available SF block code in the literature.

VIII. Conclusion

In this paper, we introduced a class of space-time block codes for an arbitrary number of transmit antennas based on generalizing the quasi-orthogonal space-time block codes. We then proposed a class of space-frequency block codes that is capable of achieving rate one and full spatial and multipath diversity in a frequency selective MIMO-OFDM channel structure. In general, the decoding complexity of our SF scheme grows exponentially with the desired diversity level although sphere decoding can be utilized to reduce the complexity. We discussed the conditions under which the ML decoding complexity is reduced.

We also designed a class of quasi-orthogonal space-time-frequency block codes. Our proposed STF codes, in addition to the space and frequency diversity gains, are able to exploit the temporal diversity gains of the channel as well, thus achieving the maximum possible diversity level. If the channel is quasi-static over B OFDM symbol durations, there are no temporal diversity gains offered by the channel. In this case, we proposed to use the STF structure to reduce the decoding complexity. Note that in general there is a delay of B OFDM symbols associated with the decoding of the STF codes while the SF code does not produce any decoding delays.

Appendix
A

In this appendix, we prove that the SF code given by Equations (11) and (12) provides a diversity of $2L$ over any two-antenna frequency selective channel with L independent channel taps.

Proof: Assuming that $N > 2L$, the diversity order of a space-frequency code, for any two distinct codewords $\mathbf{C}$ and $\mathbf{E}$, is determined by the minimum rank of the matrix $\mathbf{F}(\mathbf{C},\mathbf{E}) \in \mathcal{C}^{N\times 2L}$ given by [1],

$$\mathbf{F}(\mathbf{C},\mathbf{E}) = \left[\ (\mathbf{C}-\mathbf{E}) \quad \mathbf{\Psi}(\mathbf{C}-\mathbf{E}) \ \ldots \ \mathbf{\Psi}^{L-1}(\mathbf{C}-\mathbf{E})\ \right],$$

where $\mathbf{\Psi} = \mathrm{diag}\{w^k\}_{k=0}^{N-1}$ and $w = e^{-j\frac{2\pi}{N}}$. For a block index $m \in \{1,\ldots,\lfloor \frac{N}{2L} \rfloor\}$, let us denote the difference between the two symbols s_i^m and u_i^m to be $d_i^m = s_i^m - u_i^m$. Assume that,

$$\exists\ m_0 \text{ such that } \{d_1^{m_0},\ldots,d_L^{m_0},d_{L+1}^{m_0},\ldots,d_{2L}^{m_0}\} \neq 0.$$

To achieve the minimum rank, we further assume that $\forall m \neq m_0$, $\{d_1^m,\ldots,d_{2L}^m\} = 0$; because the rank of $\mathbf{F}(\mathbf{C},\mathbf{E})$ can not decrease further if for some $m_1 \neq m_0$, $\{d_1^{m_1},\ldots,d_{2L}^{m_1}\} \neq 0$. Moreover, it is obtained numerically, that for the practical constellations BPSK, QPSK and 16QAM, the minimum coding gain is achieved when one of the sets $\{d_1^{m_0},\ldots,d_L^{m_0}\}$ or $\{d_{L+1}^{m_0},\ldots,d_{2L}^{m_0}\}$ is zero. Without loss of generality, let us assume $\{d_1^1,\ldots,d_L^1\}$ is the non-zero set. Thus only the first $2L$ rows of $\mathbf{F}(\mathbf{C},\mathbf{E})$ have non-zero elements. Let us denote the non-zero part of $\mathbf{F}(\mathbf{C},\mathbf{E})$ by $\tilde{\mathbf{F}}(\mathbf{C},\mathbf{E}) \in \mathcal{C}^{2L\times 2L}$ given as,

$$\begin{bmatrix} D_1 & 0 & \ldots & D_1 & 0 \\ 0 & D_1^* & \ldots & 0 & w^{L-1}D_1^* \\ D_3 & 0 & \ldots & w^{2(L-1)}D_3 & 0 \\ \vdots & \vdots & \ddots & \vdots & \vdots \\ D_{2L-1} & 0 & \ldots & w^{2(L-1)(L-1)}D_{2L-1} & 0 \\ 0 & D_{2L-1}^* & \ldots & 0 & w^{(L-1)(2L-1)}D_{2L-1}^* \end{bmatrix}. \tag{19}$$

Next, we prove that the columns of the above matrix are linearly independent, resulting in a full-rank $\tilde{\mathbf{F}}(\mathbf{C},\mathbf{E})$. Due to the full-diversity criteria of the generalized QOSTBC we already know that $D_1 = |d_1 + e^{j\theta_1}d_2 + \cdots + e^{j\theta_{k-1}}d_L| \neq 0$, where $d_i = s_i - u_i,\ \forall\ s_i,\ u_i \in \mathcal{A}$. Note that if we switch s_j and u_j for any $j \in \{2,\ldots,L\}$, we get $D_j \neq 0$ as well. Therefore any even and any odd column of $\tilde{\mathbf{F}}(\mathbf{C},\mathbf{E})$ are already independent. Let us denote the matrix constructed by the odd rows and odd columns of $\tilde{\mathbf{F}}(\mathbf{C},\mathbf{E})$ by $\tilde{\mathbf{F}}_{odd}(\mathbf{C},\mathbf{E}) \in \mathcal{C}^{L\times L}$ and similarly the matrix constructed by even rows and even columns of $\tilde{\mathbf{F}}(\mathbf{C},\mathbf{E})$ by $\tilde{\mathbf{F}}_{even}(\mathbf{C},\mathbf{E}) \in \mathcal{C}^{L\times L}$. One can easily show that,

$$\det(\tilde{\mathbf{F}}(\mathbf{C},\mathbf{E})) = \det(\tilde{\mathbf{F}}_{odd}(\mathbf{C},\mathbf{E}))\det(\tilde{\mathbf{F}}_{even}(\mathbf{C},\mathbf{E})) \tag{20}$$

We now need to show that both $\tilde{\mathbf{F}}_{odd}(\mathbf{C},\mathbf{E})$ and $\tilde{\mathbf{F}}_{even}(\mathbf{C},\mathbf{E})$ are full-rank. $\tilde{\mathbf{F}}_{odd}(\mathbf{C},\mathbf{E}) \in \mathcal{C}^{L\times L}$ can be represented by,

$$\begin{bmatrix} D_1 & D_1 & \dots & D_1 \\ D_3 & w^2 D_3 & \dots & w^{2(L-1)} D_3 \\ D_5 & w^4 D_5 & \dots & w^{4(L-1)} D_5 \\ \vdots & \vdots & \ddots & \vdots \\ D_{2L-1} & w^{2(L-1)} D_{2L-1} & \dots & w^{2(L-1)(L-1)} D_{2L-1} \end{bmatrix}.$$

To show that $\tilde{\mathbf{F}}_{odd}(\mathbf{C},\mathbf{E})$ is full-rank we prove that its determinant is non-zero. Using basic determinant properties, one can write,

$$\det(\tilde{\mathbf{F}}_{odd}(\mathbf{C},\mathbf{E})) = D_1 D_3 D_5 \dots D_{2L-1} \det(\mathbf{W}), \quad (21)$$

where,

$$\mathbf{W} = \begin{bmatrix} 1 & 1 & 1 & \dots & 1 \\ 1 & w^2 & w^4 & \dots & w^{2(L-1)} \\ 1 & w^4 & w^8 & \dots & w^{4(L-1)} \\ \vdots & \vdots & \vdots & \ddots & \vdots \\ 1 & w^{2(L-1)} & w^{4(L-1)} & \dots & w^{2(L-1)(L-1)} \end{bmatrix}. \quad (22)$$

Noting that $\mathbf{W}$ is a Vandermonde matrix [29], one can rewrite the determinant of $\tilde{\mathbf{F}}_{odd}(\mathbf{C},\mathbf{E})$ as follows,

$$\det(\tilde{\mathbf{F}}_{odd}(\mathbf{C},\mathbf{E})) = \Big(\prod_{\substack{i=1\\ i:odd}}^{2L-1} D_i\Big)\det(\mathbf{W}) = \prod_{\substack{i=1\\ i:odd}}^{2L-1} D_i \prod_{m=0}^{L-2}\prod_{n=m+1}^{L-1}(w^{2n}-w^{2m}). \quad (23)$$

The first term in $\det(\tilde{\mathbf{F}}_{odd}(\mathbf{C},\mathbf{E}))$, which is the product of D_i's for odd values of i, is non-zero because of the full-diversity characteristic of the underlying generalized QOSTBC. The second term is also non-zero because we have assumed that $N > 2L$, therefore $e^{-j\frac{2\pi}{N}l} < 1$, $\forall l \in \{1,\dots,2(L-1)\}$ and $w^i \neq w^j$, $\forall i \neq j$. Thus $\tilde{\mathbf{F}}_{odd}(\mathbf{C},\mathbf{E})$ is full-rank. In a similar manner it can be shown that $\tilde{\mathbf{F}}_{even}(\mathbf{C},\mathbf{E})$ is full-rank as well,

$$\det(\tilde{\mathbf{F}}_{even}(\mathbf{C},\mathbf{E})) = \prod_{\substack{i=1\\ i:odd}}^{2L-1} D_i^* \prod_{m=0}^{L-2}\prod_{n=m+1}^{L-1} w(w^{2n}-w^{2m}). \quad (24)$$

Consequently, $\mathbf{F}(\mathbf{C},\mathbf{E})$ has a minimum rank of $2L$. Thus we have proved that the code in Equation (11) achieves a diversity of $2L$, where two levels of diversity are due to transmit diversity and L levels are due to multipath/frequency diversity ∎

APPENDIX B

In this appendix, we prove that the STF code, given by Equations (14) and (15), provides a diversity of $2L\tau$ over any two-antenna frequency selective channel with L independent taps over τ independent OFDM symbols.

Proof: Assuming the channel taps are independent and no spatial correlation between antennas exists, also assuming that the second order statistics of the time correlation is the same for all transmit and receive antenna pairs and all paths, the diversity criterion is given by [4],

$$\text{diversity} = \min_{\mathbf{C},\mathbf{E}} \text{rank}(\boldsymbol{\Delta} \circ \mathbf{R}), \quad (25)$$

where,

$$\boldsymbol{\Delta} = \begin{bmatrix} \mathbf{D}^1_{STF} \\ \mathbf{D}^2_{STF} \\ \vdots \\ \mathbf{D}^{\tau}_{STF} \end{bmatrix} \begin{bmatrix} \mathbf{D}^1_{STF} \\ \mathbf{D}^2_{STF} \\ \vdots \\ \mathbf{D}^{\tau}_{STF} \end{bmatrix}^H, \quad (26)$$

$$\mathbf{D}^i_{STF} = \mathbf{C}^i_{STF} - \mathbf{E}^i_{STF} \quad \forall i \in \{1,\dots,\tau\},$$

and $\mathbf{R} = \mathbf{R}_\tau \otimes \mathbf{R}_f$ where $\mathbf{R}_f \in \mathcal{C}^{N\times N}$ is the frequency correlation matrix of the channel and $\mathbf{R}_\tau \in \mathcal{C}^{\tau\times\tau}$ is the temporal correlation matrix. For the sake of simplification, we further assume that the channel changes independently in time over adjacent OFDM symbols, therefore, $\mathbf{R}_\tau = \mathbf{I}_{\tau\times\tau}$. In this case, we can write the diversity criterion as follows,

$$\min_{\mathbf{C},\mathbf{E}} \sum_{i=1}^{\tau} \text{rank}(\mathbf{D}^i_{STF}{\mathbf{D}^i_{STF}}^H \circ \mathbf{R}_f), \quad (27)$$

Therefore, to achieve full space, time and frequency diversity gains of $2L\tau$, one needs to show that each of the elements $(\mathbf{D}^i_{STF}{\mathbf{D}^i_{STF}}^H) \circ \mathbf{R}_f$ is of rank $2L$ which is equivalent to the proof of full-diversity for SF codes provided in Appendix A. Therefore, under independent temporal correlation condition, the QOSTF in (14) provides full-space, time and frequency diversity of $2L\tau$ ∎

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