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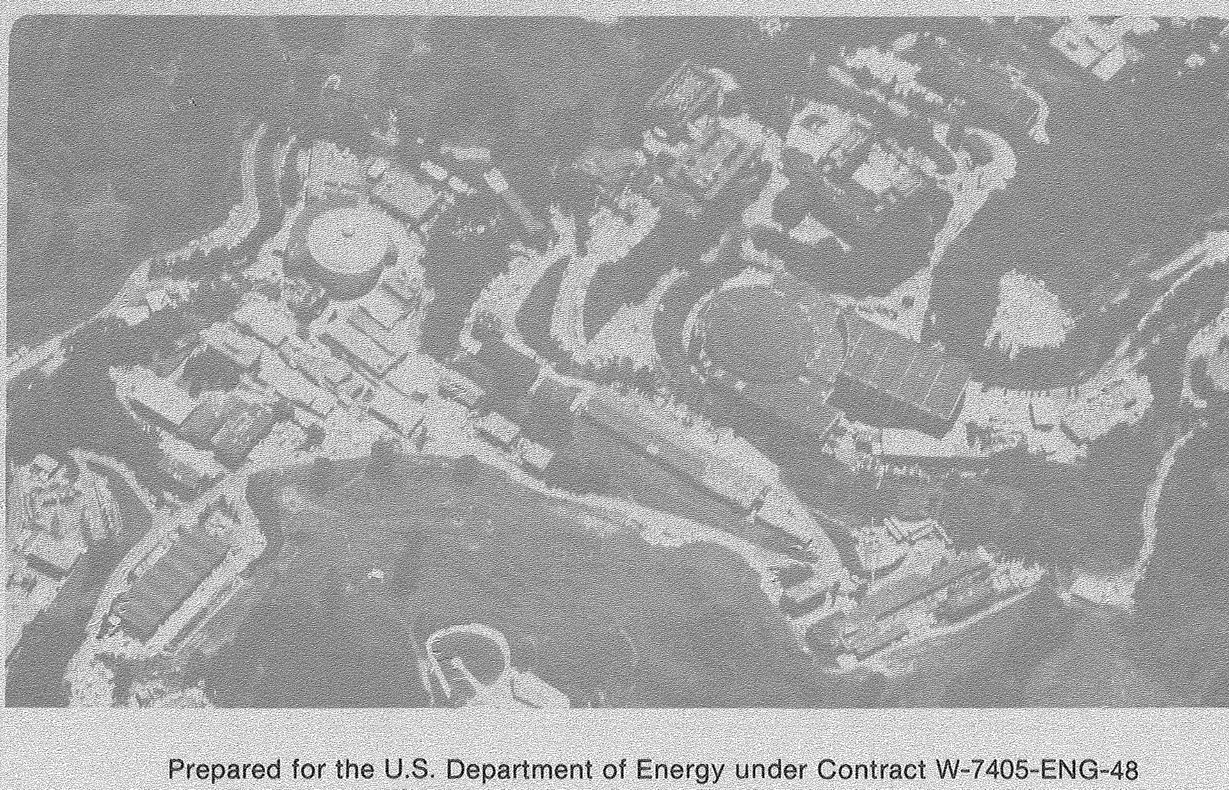

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QUARKONIUM PRODUCTION VIA THE ONE GLUON MECHANISM

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ABSTRACT

It is determined that quarkonium production from a single virtual gluon is suppressed due to the lack of colored resonances. The branching ratio to charmonium in $\Upsilon(9.4)$ decay is estimated to be between 10^{-2} and 10^{-3} times smaller than previous estimates.

Heavy quarkonium states offer an interesting laboratory in which to test ideas developed on possible theories of the strong interactions. For QCD, it is supposed that a heavy quarkonium state ($J^{PC} = 1^{--}$) decays into three gluons which materialize by fragmenting into hadrons. We expect heavy flavors to be produced in this decay process only through a virtual timelike gluon of large invariant mass. This process is illustrated by figure 1.

The total rate for producing heavy flavors in the final state for this process has been estimated.[1] The estimate is based upon an analogue to electromagnetic heavy flavor production in e^+e^- annihilation. It is well known that in e^+e^- annihilation the cross-section for producing a specific flavor is roughly equal to the cross-section for producing a free pair of quarks, if one averages over resonance bumps.[2] This same idea of duality was applied to heavy flavor production in quarkonium decay by calculating the rate for production of a pair of free heavy quarks, as in figure 1. The authors of reference 1 thus determine the branching ration of Υ to charm + anti-charm + two gluons. The branching ratio is given as a function of the dimensionless parameter ξ, which is defined as Q^2/M_Υ^2, where Q^2 is the invariant mass squared of the charm—anti-charm pair produced. Their results are summarized in figure 2. The total branching ratio to charmed particles is obtained by integrating over the entire kinematic range of Q^2. It is assumed that the contribution for $Q^2 < (2M_D)^2$ manifests itself in final state charmonium (η_c, χ, ψ), and for $Q^2 > (2M_D)^2$ becomes a final state $D\bar{D}$ pair. This gives an estimate of

2.7% for $D\bar{D}$ production, and 1.1% for charmonium production in Υ decay.

Essential to the referenced calculation is the assumption of duality. However, this does not seem a reasonable assumption since the produced $c\bar{c}$ pairs are not produced in a color singlet state. Since colored resonances have not been observed, one should not expect dynamical enhancement of $c\bar{c}$ production through the one gluon mechanism at discrete energies, and thus not have an operative version of duality working. Instead, one must calculate the rate for the specific color singlet final state desired. This means that the process "g" $\rightarrow$ $(c\bar{c})$ + g must be calculated to obtain a reliable estimate of the branching ratio to charm in Υ decay.

The lowest order diagrams that one must calculate to obtain a gauge invariant result are those of figure 3. In words, they correspond to heavy quark-antiquark production via a virtual gluon, followed by propagation of the system in a color octet state, with the subsequent emission of an on-shell gluon which is coupled either directly to a quark line (fig. 3a,b) or to a virtual gluon exchanged between the quark lines (fig. 3c). The remaining quark system is then projected onto the desired color singlet state. The justification for doing a perturbative treatment of soft gluon emission is well founded. It rests upon a "quasi-dipole" type coupling of a long wavelength probe (gluon) to a small source (quarkonium). This conspires with limited phase space to suppress higher order soft gluon emission, although the coupling constant is not necessarily small. For this same reason, we cannot calculate soft gluon emission from the $(Q\bar{q})(\bar{Q}q)$ final state

(where q denotes a light quark), and must restrict ourselves to quarkonium production.

In order to illustrate the mechanics of heavy quarkonia production and to specifically estimate the branching ratio of Υ to charmonium, we will assume the $Q\bar{Q}$ potential is reasonably approximated by the one-gluon-exchange coulomb potential. This approximation should yield an order of magnitude estimate for charmonium, and will become increasingly more accurate as one deals with heavier quark systems.

The calculation of the relevent S-matrix element begins by adopting a modified interaction picture where the Hamiltonian is divided into an external perturbative part (H_E) which describes the coupling of the $Q\bar{Q}$ system to external gluons, and a part treated "non-perturbatively" which describes the internal interactions of the $Q\bar{Q}$ system. The internal Hamiltonian for the $Q\bar{Q}$ system in the attractive (repulsive) color singlet (octet) state is $H_1 (H_8)$. The lowest order S-matrix element for the process "g" $\rightarrow \Phi$ + g, where Φ generically refers to a $Q\bar{Q}$ color singlet bound state, is

$$S = -2\pi\delta(E_f - E_i) < g\Phi| \int_0^\infty dt \, H_E(t) \, e^{-i(H_8+\epsilon_1)t} \, H_E(0)|"g" > \quad (1)$$

with ϵ_1 the binding energy of the state Φ. Inserting a complete set of intermediate color octet $Q\bar{Q}$ states and rotating to Euclidean space yields

$$S = 2\pi i\delta(E_f - E_i) < g\Phi| \int_0^\infty d\tau \, H_E(\tau) \, e^{-(H_8+\epsilon_1)\tau} \, |Q\bar{Q}>_8$$

$$* \frac{V d^3 p_Q}{(2\pi)^3} \frac{V d^3 p_{\bar{Q}}}{(2\pi)^3} \, {}_8< Q\bar{Q}|H_E(0)|"g" > \quad (2)$$

To evaluate the $(Q\bar{Q})_8 \to \Phi g$ matrix element we closely follow the techniques used by Peskin in deriving his operator product expansion for heavy quark systems.[3] Due to the specific matrix element we are calculating, we shall see that the only diagrams we must calculate are those of figure 4. Neglecting terms of order p_Q^2/m^2, and restricting the external gluon to time-like polarization, figure 4a + 4b can be reduced to

$$(4a + 4b,\ \text{timelike}) = - \frac{i(2\pi)^3\delta^3(P_f-P_i)}{V^{3/2}(2k_0)^{1/2}} \frac{(igT_b)_{\alpha\beta}}{\sqrt{3}} \delta_{s_1s_1'}\delta_{s_2s_2'}$$

$$* \langle\Phi|\{A_0^b(\tau,R+\frac{r}{2}) - A_0^b(\tau,R-\frac{r}{2})\}\, e^{-(H_8+\varepsilon_1)\tau} d\tau\, |Q\bar{Q}\rangle_8 \tag{3}$$

where R is the center of mass coordinate of the Φ, r is the relative quark spacing, and the A-field has been made dimensionless. Furthermore, the approximation can be made that

$$\{A_0^b(\tau,R+\frac{r}{2}) - A_0^b(\tau,R-\frac{r}{2})\} = \underset{\sim}{r}\cdot\underset{\sim}{\partial}A_b^0(\tau,R) + O\left(\frac{k_0}{m}\right)^2$$

$$\cong \underset{\sim}{r}\cdot\underset{\sim}{\partial} \sum_{n=0}^{\infty} \frac{1}{n!}\tau^n \left[\left(\frac{d}{dt}\right)^n A_b^0(t,R)\right]_{t=0} \tag{4}$$

which neglects terms of order $(k_0/m)^2$ and keeps all terms of order (k_0/ε_1). Doing the now trivial τ-integration yields

$$(4a + 4b,\ \text{timelike}) = - \frac{i(2\pi)^3\delta^3(P_f-P_i)}{V^{3/2}(2k_0)^{1/2}} \frac{(igT_b)_{\alpha\beta}}{\sqrt{3}} \delta_{s_1s_1'}\delta_{s_2s_2'}$$

$$* \sum_{n=0}^{\infty} \langle\Phi|r_i \frac{1}{(H_8+\varepsilon_1)^{n+1}} |Q\bar{Q}\rangle_8 (\partial_0)^n(\partial^i A_0^b) \quad . \tag{5}$$

Evaluating figures 4a + 4b for external gluons of spacelike polarization yields two terms of different spin structures. The spin singlet term corresponds to the spacelike gluon coupling to the quark color convection current, and yields after manipulations similar to those used deriving equation 5

$$(4a + 4b,\ \text{spacelike}) = - \frac{(2\pi)^3\delta^3(P_f-P_i)}{V^{3/2}(2k_0)^{1/2}} \frac{(igT_b)_{\alpha\beta}}{\sqrt{3}} \delta_{s_1s_1'}\delta_{s_2s_2'}$$

$$\times \sum_{n=0}^{\infty} \langle\Phi| \frac{2p_Q^i}{m} \frac{1}{(H_8+\varepsilon_1)^{n+1}} |Q\bar{Q}\rangle_8 (\partial_0)^n A_i^b \quad . \tag{6}$$

The spin flip term corresponds to the spacelike gluon coupling to the quark spin current, and yields the gauge invariant expression

$$(4a + 4b,\ \text{spacelike}) = \frac{i(2\pi)^3\delta^3(P_f-P_i)}{V^{3/2}(2k_0)^{1/2}} \frac{(igT_b)_{\alpha\beta}}{\sqrt{3}} \sum_{n=0}^{\infty}$$

$$\Bigg[\chi_{s_1'}^\dagger\sigma_j\chi_{s_1}\delta_{s_2s_2'} \langle\Phi| \frac{1}{(H_8+\varepsilon_1)^{n+1}} |Q\bar{Q}\rangle_8 (\partial_0)^n$$

$$\times \frac{(\underset{\sim}{k} \times \underset{\sim}{A}^b(R+\frac{r}{2}))^j}{2m} + \delta_{s_1s_1'}\chi_{s_2}^\dagger\sigma_j\chi_{s_2'}$$

$$\times \langle\Phi| \frac{1}{(H_8+\varepsilon_1)^{n+1}} |Q\bar{Q}\rangle_8 (\partial_0)^n \frac{(\underset{\sim}{k} \times \underset{\sim}{A}^b(R-\frac{r}{2}))^j}{2m} \Bigg] \tag{7}$$

where χ is the non-relativistic quark two-component spinor. In order to evaluate figure 4c, we first isolate the effective interaction induced by the three-gluon-vertex. This is done by calculating figure 5 in the limit of $(x - y) \to 0$. Also note that the legs which will ultimately connect to quark lines have timelike polarization to lowest order in (p/m). Therefore, in Feynman gauge, with $\lambda = j$

$$(\text{fig. } 5) = g t_{bac}\, A_b^j(x) \int d^4w\, (\partial_x^j - \partial_y^j)\, \frac{1}{4\pi^2(x-w)^2}\, \frac{1}{4\pi^2(y-w)^2}$$

which reduces to

$$(\text{fig. } 5) = -\frac{g t_{bac} A_b^j(x)\,(y-x)_j}{4\pi^2(y-x)^2} \quad . \qquad (8)$$

We can now make the insertion of figure 5 onto the quark lines which yields figure 4c. After manipulations similar to those used deriving equation 5, we find

$$(\text{fig. } 4c) = \frac{i(2\pi)^3\delta^3(P_f-P_i)}{V^{3/2}(2k_0)^{1/2}}\, \frac{3g^2}{8\pi}\, \frac{(igT_b)_{\alpha\beta}}{\sqrt{3}}\, \delta_{s_1 s_1'}\delta_{s_2 s_2'}$$

$$\sum_{n=0}^{\infty} \langle\Phi|\, \frac{r_i}{r}\, \frac{1}{(H_8+\varepsilon_1)^{n+1}}\, |Q\bar{Q}\rangle_8\, (\partial_0)^n A_b^i \quad . \qquad (9)$$

However, this term can be rewritten in a way which makes manifest the fact that it is effectively the same order in g^2 as the terms from figures 4a and 4b. This is done by using the relation $H_8 - H_1 = 3g^2/8\pi r$, valid for QCD in coulomb approximation, and the commutation relations of r and H_8, which yields

$$(\text{fig. } 4c) = \frac{i(2\pi)^3\delta^3(P_f-P_i)}{V^{3/2}(2k_0)^{1/2}}\, \frac{(igT_b)_{\alpha\beta}}{\sqrt{3}}\, \delta_{s_1 s_1'}\delta_{s_2 s_2'} \sum_{n=0}^{\infty}$$

$$\Bigg[\langle\Phi|\, r_i\, \frac{1}{(H_8+\varepsilon_1)^n}\, |Q\bar{Q}\rangle_8\, (\partial_0)^n A_b^i + \langle\Phi|\, -\frac{2ip_Q^i}{m}\, \frac{1}{(H_8+\varepsilon_1)^{n+1}}\, |Q\bar{Q}\rangle_8\, (\partial_0)^n A_i^b\Bigg] \quad . \qquad (10)$$

Note that the first n=0 term in equation (10), which is a potentially gauge non-invariant contribution, is zero because of the specific process computed. i.e.

$$\langle\Phi|\, \underset{\sim}{r}\cdot \underset{\sim}{A}^b\, |Q\bar{Q}\rangle_8 \;\; d^3p_Q d^3p_{\bar{Q}}\; \delta^3(p_Q+p_{\bar{Q}}) \sim \int d^3r\, \Phi(r)\, \underset{\sim}{r}\cdot \underset{\sim}{A}_b\, \delta^3(r) \sim 0$$

Since $\Phi(r)$ is finite as $r \to 0$. Now adding together equations (5), (6), and (10) yields for the non-spin-flip part of the sum of diagrams 4a, b, and c, the gauge invariant expression

$$(\text{fig. } 4) = -\frac{i(2\pi)^3\delta^3(P_f-P_i)}{V^{3/2}(2k_0)^{1/2}}\, \frac{(igT_b)_{\alpha\beta}}{\sqrt{3}}\, \delta_{s_1 s_1'}\delta_{s_2 s_2'}$$

$$\sum_{n=0}^{\infty} \langle\Phi|\, r_i\, \frac{1}{(H_8+\varepsilon_1)^{n+1}}\, |Q\bar{Q}\rangle_8\, \{(\partial_0)^n(\partial^i A_b^0 - \partial^0 A_b^i)\} \quad , \qquad (11)$$

while equation (7) is the gauge invariant expression for the spin-flip part. Now that we have the expressions for the $(Q\bar{Q})_8 \to \Phi + g$ matrix elements, we can go back and evaluate the S-matrix elements of equation (3) for specific final states. For $\eta\,(^1S_0)$ production, only the contribution of equation (7) is relevant due to the spin structure. Doing the spin = 0 projection, the sum over all n, and substituting $\frac{p_Q^2}{m} + \varepsilon_8$ for H_8 yields

$$S\big(\eta(^1S_0)\big) = -\frac{i(2\pi)^4\delta^4(Q-P-k)\, g^2\, \underset{\sim}{\varepsilon}^a\cdot(\underset{\sim}{k} \times \underset{\sim}{\varepsilon}_b)}{(2Q)^{1/2}(2k_0)^{1/2} V\sqrt{6}}\, \frac{d^3p_Q}{(2\pi)^3}$$

$$\times \langle ^1S_0|\, \frac{1}{m(\varepsilon_T-k_0)+p_Q^2-i\varepsilon}\, |Q\bar{Q}\rangle_8 \quad , \qquad (12)$$

where $\underset{\sim}{\varepsilon}^a$ and $\underset{\sim}{\varepsilon}^b$ are the polarization vectors of the incident and final state gluons and $\varepsilon_T = \varepsilon_1 + \varepsilon_8$. Note that the amplitude has the expected physical behavior of developing a finite absorptive part when $k_0 > \varepsilon_T$, i.e. when $Q > 2m + \varepsilon_8$, which is the threshold for a "physical" intermediate state. Using the coulombic 1S bound state wavefunction for the matrix element, squaring the amplitude and summing over final states, yields for the rate

$$R(\text{"g"} \to \eta + g) = \frac{g^4(Q-m_\eta)^3}{36\pi^2 Q a^3 \left| \left(\frac{1}{a} + \sqrt{m(\varepsilon_T + m_\eta - Q)} \right)^2 \right|^2} , \qquad (13)$$

where "a" is the Bohr radius. For $\chi(^3P_J)$ production, spin structure demands that only equation (11) contributes to the amplitude. We find the ^3P_J states produced with their statistical weight $R^{J=0}: R^{J=1}: R^{J=2}$ equal to 1:3:5 and

$$R\left(\text{"g"} \to \chi(^3P_J) + g\right) = \frac{g^4(Q-m_\chi)^3 m^2(2J+1)}{432\pi^2 Q \left| \left(\sqrt{m(\varepsilon_T' + m_\chi - Q)} + \frac{1}{2a} \right)^2 \right|^4 a^5} , \qquad (14)$$

where $\varepsilon_T' = \frac{\varepsilon_1}{4} + \varepsilon_8$ and m_χ is the χ-state mass.

To make the connection to the branching ratio of Υ to charmonium, one can make use of Fritzsch's results by dividing out the rate of "g" $\to c\bar{c}$ and multiplying by the rate of "g" $\to \Phi + g$ where Φ denotes either η_c or $\chi_c(^3P_J)$. These scaling factors which must be applied to the results of figure 2 are, for η_c production

$$\frac{R(\text{"g"} \to \eta_c + g)}{R(\text{"g"} \to c\bar{c})} = \frac{4g^2(Q-m_{\eta_c})^3}{9\pi a^3 \left| \left(\frac{1}{a} + \sqrt{m(\varepsilon_T + m_{\eta_c} - Q)} \right)^2 \right|^2 \sqrt{Q^2 - m_{\eta_c}^2}\; Q} \qquad (15)$$

and for χ-production

$$\frac{R(\text{"g"} \to \chi_c + g)}{R(\text{"g"} \to c\bar{c})} = \frac{g^2 m^2 (Q-m_{\chi_c})^3}{3\pi a^5 Q \left| \left(\frac{1}{2a} + \sqrt{m(\varepsilon_T' + m_{\chi_c} - Q)} \right)^2 \right|^4 \sqrt{Q^2 - m_{\eta_c}^2}} \qquad (16)$$

To apply these results to the charmonium system, the values of the bound state parameters can be determined from fitting m_ψ and $m_{\psi'}$ to a coulomb spectrum. (Find $m_c = 1.9$ GeV, $a = .81$ GeV^{-1}.) The numerical evaluation of (15) and (16) is found in tables 1 and 2. They need only be evaluated up to an incident energy of 3.75 GeV, as above this $D\bar{D}$ production dominates.

If our "scaling factors" in tables 1 and 2 are folded with the previous results of figure 2, one obtains for the branching ratio of Υ to η_c + anything

$$BR(\Upsilon \to \eta_c + X) \cong 3 \times 10^{-5} , \qquad (17)$$

and for Υ to χ_c + anything

$$BR(\Upsilon \to \chi_c + X) \cong 3 \times 10^{-6} . \qquad (18)$$

Note that this branching ratio is between 10^{-2} and 10^{-3} times smaller than that predicted using the assumption of duality, and the associated implicit assumption of dynamical resonance enhancement. (A branching ratio of 10^{-2} would be just observable in an upcoming CESR experiment.)[4]

Thus, the lack of colored resonances allows heavy quarkonium production with soft gluon emission to be suppressed by limited phase space and a "dipole-type" coupling. This calculation assumed a coulombic potential for the $Q\bar{Q}$ interaction, which nicely illustrated the behavior of a system with no colored resonances. It should also give a reasonable order-of-magnitude estimate for the above branching ratios, and will become even more reliable for higher mass quarkonia.

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4. Kirk Shinsky - private communication.

FIGURE CAPTIONS

Figure 1: Υ decay into charm + anticharm + hadrons

Figure 2: Branching ratio of Υ to charm + anticharm as a function $\xi = Q^2/m_\Upsilon^2$

Figure 3: Gauge invariant set of diagrams for "g" → quarkonium + g Q, k, and P are external 4-momenta.

Figure 4: Lowest order gluon emission from $Q\bar{Q}$ system. α, β, α', β', b are color indices and s_1, s_1', s_2, s_2' are spin states.

Figure 5: Tri-gluon insertion in coordinate space. w, x, y, z are coordinates, a, b, c are color indices, and λ, μ, ν are polarization indices.

Table I

Q(in GeV)	$R_{\eta_c}/R_{c\bar{c}}$
3.0	0
3.1	3.7×10^{-5}
3.2	2.3×10^{-4}
3.3	7.1×10^{-4}
3.4	1.7×10^{-3}
3.5	3.4×10^{-3}
3.6	6.5×10^{-3}
3.7	1.2×10^{-2}
3.8	2.6×10^{-2}

Table II

Q(in GeV)	$R_\chi / R_{c\bar{c}}$
3.5	0
3.6	1.3×10^{-4}
3.7	2.3×10^{-3}
3.8	2.5×10^{-2}

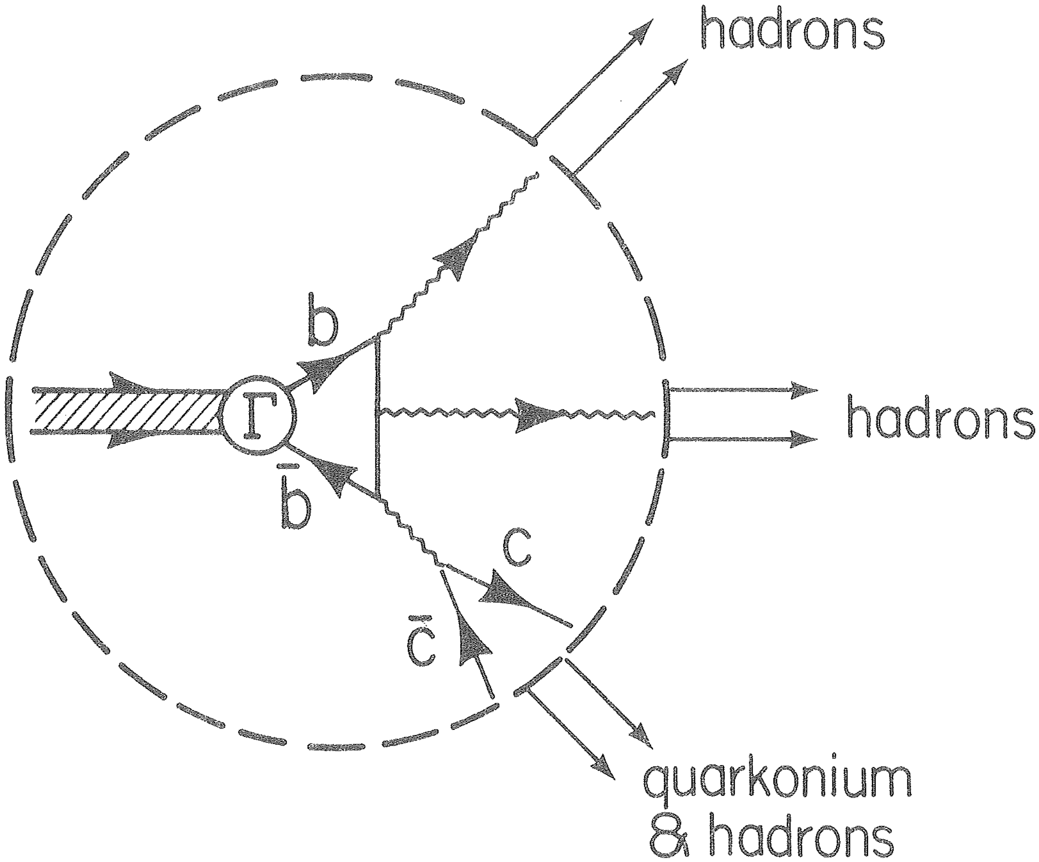

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Fig. 1

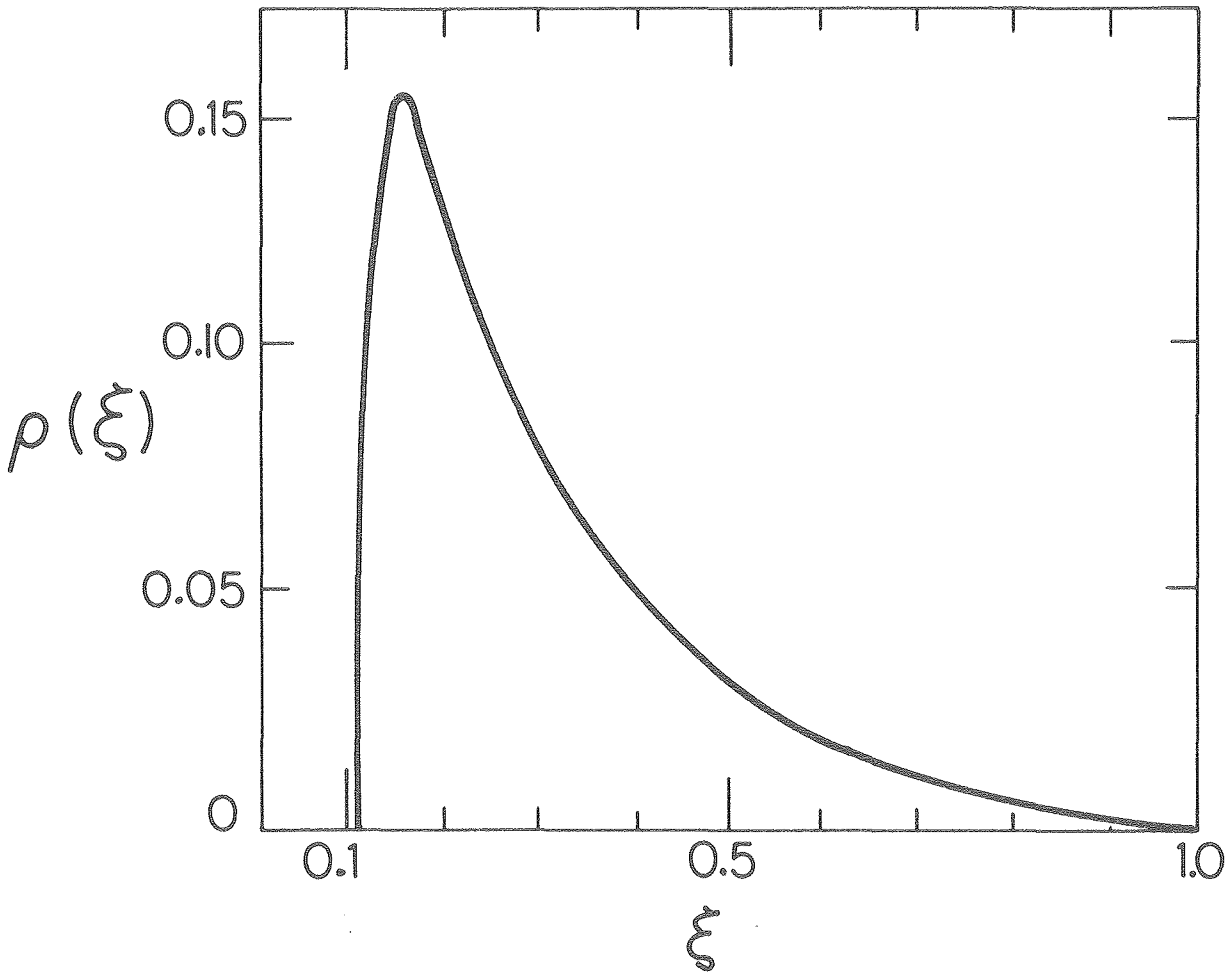

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Fig. 2

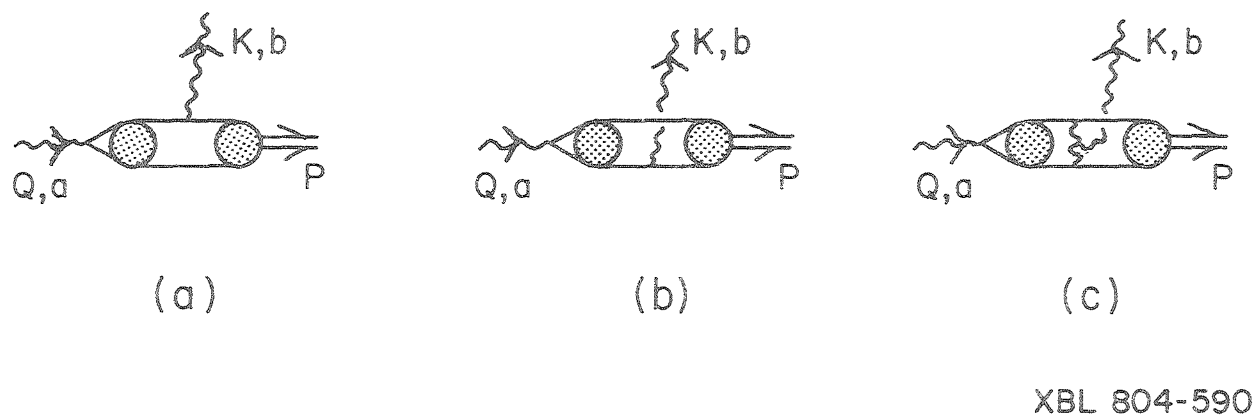

Fig. 3

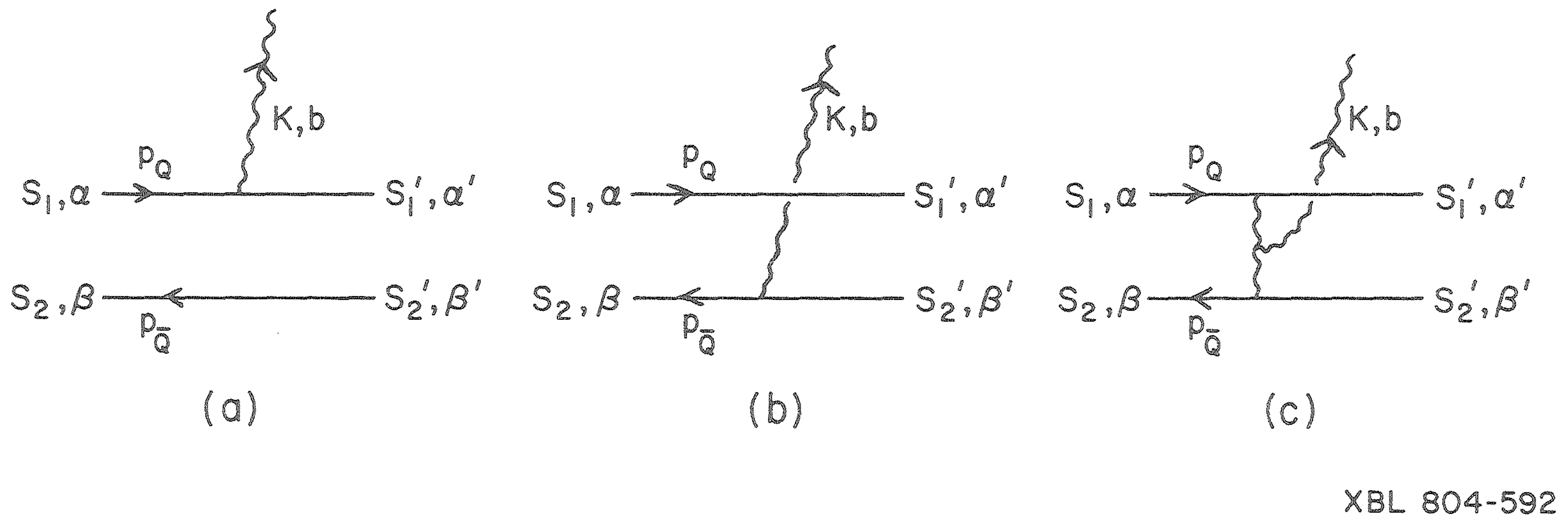

Fig. 4

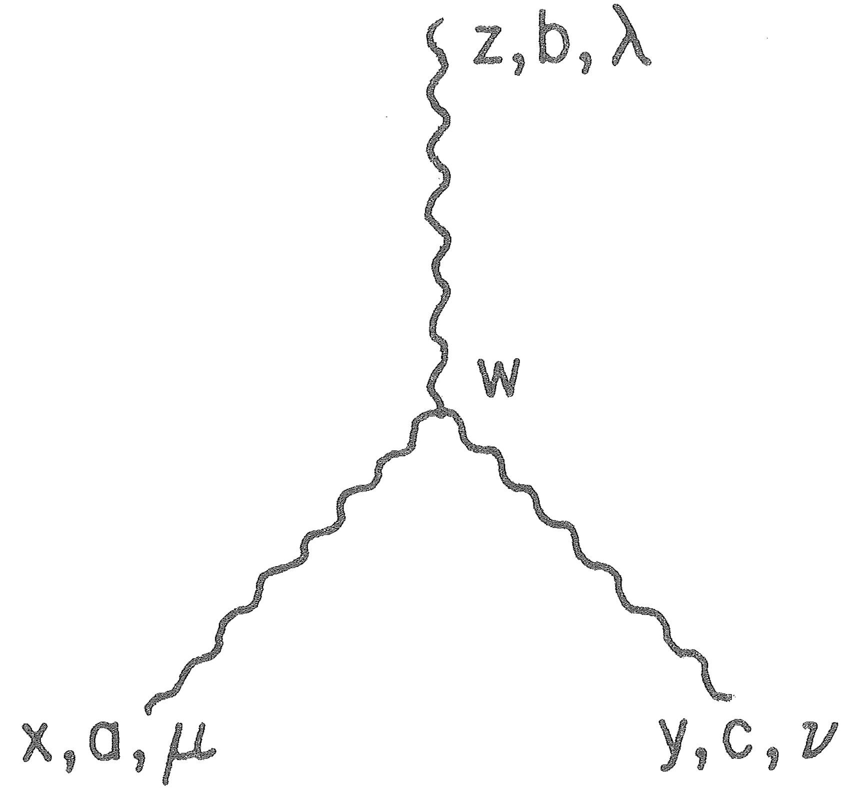

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Fig. 5