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On the Possibility of Second-Order Phase Transitions in Spontaneously Broken Gauge Theories

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In the "Type-II" regime, $m_{\text{Higgs}} \gtrsim m_{\text{gauge}}$, the finite-temperature phase transition in spontaneously-broken gauge theories (including the standard model) must be studied using a renormalization group treatment. Previous studies within the $(4 - \epsilon)$ -expansion suggest a 1st-order transition in this regime. We use analogies with experimentally accessible phase transitions in liquid crystals, and theoretical investigations of superconductor phase transitions to argue that, in this range, the critical behavior of a large class of gauge-Higgs-fermion systems changes from 1st to 2nd-order as a function of Higgs mass. We identify a set of models which, within the $(2 + \epsilon)$ -expansion, possess fixed points that can describe this 2nd-order behavior. As usual, a definitive demonstration that the claimed critical behavior occurs (and a reliable estimate of m_{Higgs} at the tricritical point) will probably require numerical simulations.

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There has been much recent interest in the physics of the finite-temperature electroweak phase transition (EWPT) [1]. The motivation behind this considerable body of work, is the realization that the observed baryon-number asymmetry of the universe might be generated at time of the EWPT, through the anomaly in the conservation laws for the baryon and lepton number currents. In relation to this, it is a pleasant fact that classic studies of the nature of the phase transition in coupled gauge-scalar systems indicate that they are 1st-order [2,3,4]. Therefore one of the primary conditions for a net generation of baryon number, namely that thermal equilibrium is not maintained, arises naturally in the case of the standard model due to the supercooling of the false-vacuum phase and its later decay. In this letter we will reconsider the question of the order of the phase transition in a large class of coupled gauge-scalar-fermion systems (including a version of the two-doublet standard model) in the “Type-II” regime (roughly speaking $m_{\text{Higgs}} \gtrsim m_{\text{gauge}}$). We will show that for large enough scalar masses there is reason to believe that the transition changes over from 1st to 2nd-order, passing through a tricritical point at some value of the ratio of Higgs to gauge boson masses.

For Higgs masses in the range we consider our conclusions probably do not directly impinge on the question of weak scale baryogenesis, since (at least in many simple extensions of the standard model) even if the transition were 1st-order, the value of the scalar expectation value just after the completion of the phase transition is such that the baryon number asymmetry is “washed out” [1]. Nevertheless, the mass of the Higgs in our world might well turn out to be in the Type-II regime, and it is an interesting theoretical question as to the nature of the transition in that case. The analysis might also have some practical importance in non-minimal extensions of the standard model and for other phase transitions, although we caution the reader that the critical scalar mass is both model dependent and difficult to estimate.

We start with a review of the current state of knowledge concerning the order of the phase transition in gauge-Higgs systems from the perspective of Refs. [2] and [4]. Consider an abelian gauge field B_μ coupled to a complex scalar field ϕ ,

with action

$$S = \int d^4r \left(-\frac{1}{4}F^2 + |D_\mu\phi|^2 - m^2|\phi|^2 - \frac{\lambda}{4}|\phi|^4 \right), \quad (1)$$

where $D_\mu = \partial_\mu - ieB_\mu$. The study of the finite-temperature critical behaviour of this theory proceeds by considering the action in Euclidean space with periodized time of length $\beta = 1/T$. The fourier modes of the (periodic) fields ϕ and B are then labelled by a continuous three-dimensional momentum \mathbf{k} and an integer $n \in Z$, where $\omega_n = 2\pi n/\beta$ now appears in place of k^0 . For a weakly coupled theory (the only case considered in this letter), and near the transition temperature, $1/\beta \sim \mu/g$ (or $\mu/\sqrt{\lambda}$ if it is smaller) is then parametrically large compared to the typical mass scale μ of the theory, and may be used as an expansion parameter to isolate the $n = 0$ mode [4]. This is achieved by integrating out the $n \neq 0$ modes to obtain an effective three-dimensional theory of the $n = 0$ mode alone. Even though near the transition effective masses are vanishing, no IR problems arise in this procedure since the $n \neq 0$ propagators are cutoff by effective “masses” $\omega_n = 2\pi n/\beta \neq 0$.*

For the simple case of Eq. (1) this leads to an effective three-dimensional theory for the $n = 0$ modes of ϕ and B of the general form

$$S_{\text{eff}} = \int d^3r \left(\frac{1}{4}\mathbf{F}^2 + |\mathbf{D}\phi|^2 + a|\phi|^2 + \frac{b}{4}|\phi|^4 + \dots \right), \quad (2)$$

where in general all coupling constants have suppressed temperature dependencies. There is one temperature dependence, however, that must be kept – that of the effective mass term $a = a'(T - T_0)$ since it’s vanishing at the temperature T_0 drives (in mean field or Landau theory, and ignoring the gauge field) a second order transition and leads to IR divergences once we take into account fluctuations. (As we see in a moment this is only the “transition temperature” for a second order phase transition – we will have to modify our statements slightly in the first order case.)

* It is simple to include fermions in this discussion. Their antiperiodicity in the time direction implies that $\omega_n = (2n+1)\pi/\beta$ and thus fermions do not possess zero modes which participate in the three-dimensional effective action (although they can, of course, affect the numerical values of its effective coupling constants).

The effect of the gauge field on the transition can be qualitatively understood by formally integrating out the gauge field to define a free energy (or finite temperature effective action) $F(\phi, T)$ depending only on ϕ . Since the three dimensional action Eq. (2) is quadratic in B we can evaluate $\langle B^2 \rangle_\phi$ near T_0 by the equipartition theorem, leading to

$$\langle B^2 \rangle_\phi \propto T_0 \int d^3k \frac{1}{(k^2 + M_B^2(\phi))}, \quad (3)$$

where $M_B(\phi) \propto |\phi|$ is the ϕ dependent mass of the gauge boson. Regulating and renormalizing the integral in Eq. (3) we end up with the finite contribution $\langle B^2 \rangle_\phi \propto -T_0|\phi|$. We can then substitute this back into the free energy (2) to discover that a term proportional to $|\phi|^3$ with a negative sign has been generated in $F(\phi, T)$ [2]. This, within the framework of mean field theory, inevitably leads to a 1st-order transition. However, when fluctuations in ϕ are also considered complications arise.

Define T_c to be the temperature at which the symmetric “false” phase at $\langle \phi \rangle = 0$ is degenerate with the unsymmetric “true” phase at $\langle \phi \rangle \neq 0$. Below T_c the false phase is at best metastable. Define $T_* < T_c$ to be the temperature at which the false phase first becomes mechanically *unstable* (rather than metastable).[†] In other words T_* is defined by $d^2V(\langle \phi \rangle = 0, T_*)/d\langle \phi \rangle^2 = 0$ – the point of vanishing of the scalar mass around the false phase. To leading order T_* is equal to T_0 above. Because of the IR divergences that occur in the loop expansion evaluation of the partition function at T_* there is (for a given size of the effective scalar couplings in $F(\phi, T)$, and in less than four spatial dimensions) a range of temperatures around T_* for which the loop expansion *inevitably fails*. This range temperatures is known as the Ginzburg region [6].

[†] T_* is the “spinodal point” at which the false phase can first evolve into the true phase by *small amplitude, long wavelength* fluctuations rather than by the better known (to particle physicists) process of critical bubble nucleation. Parenthetically, the correctly defined effective potential for the false phase should possess an imaginary part in perturbation theory for low momentum fluctuations corresponding to this decay process [5]. This seems not to be widely appreciated.

This is *not* just an academic concern in the case of first order transitions if T_c is within the Ginzburg region surrounding T_* . Generally speaking, this is the case in the “Type-II” parameter range roughly given by $m_{\text{scalar}} \gtrsim m_{\text{gauge}}$. For instance, an application of the Ginzburg criterion to the analogue of $F(\phi, T)$ for the standard model shows that T_c is within the Ginzburg region for $m_{\text{Higgs}} \gtrsim 100$ GeV (fairly independent of the top quark mass).[‡] In this situation the standard effective potential formalism fails to give any reliable information, and the only known way to proceed in the study of the critical behavior is the Wilson-Fisher renormalization group (RG) [7, 8].

In the Type-II regime consider the coupled RG equations (within a $(4 - \epsilon)$ -expansion) for ϕ and B in a slight generalization of the model (1) – specifically the case of N complex scalar fields. The RG equations are:

$$\frac{de^2}{ds} = \beta_{e^2}(e^2, \epsilon) = \epsilon e^2 - \frac{N}{24\pi^2} e^4, \quad (4)$$

for the abelian gauge coupling, and

$$\frac{d\lambda}{ds} = \beta_\lambda(e^2, \lambda, \epsilon) = \epsilon \lambda - \frac{N+4}{4\pi^2} \lambda^2 - \frac{3}{8\pi^2} e^4 + \frac{3}{4\pi^2} e^2 \lambda, \quad (5)$$

for the quartic scalar self-coupling. For convenience we have taken s to increase in the IR. We are interested, of course, in the case $\epsilon = 1$ if we are to describe the critical behaviour of our model in the physical number of spatial dimensions. Recall that the RG equations derived in the ϵ -expansion are only rigorously true in the limit $\epsilon \rightarrow 0$. Nevertheless, two decades of experience have shown that the stable fixed points identified within the expansion lead to a surprisingly accurate description of many second-order transitions at $d = 3$.

[‡] There are many equivalent ways of expressing the “Ginzburg criterion.” Probably the simplest in the field theory context is a direct comparison between the tree and 1-loop three and four-point functions calculated from the free energy $F(\phi, T)$.

For $N \geq 183$ the equations (4) and (5) possess a stable fixed point with real couplings, leading to a prediction of a 2nd-order transition. For a lesser number of scalar fields the only physically accessible fixed points (i.e. with non-complex values of the couplings) are the Gaussian and Heisenberg ones, both of which are unstable with respect to the charge. This lack of a stable fixed point and the associated runaway of the coupling e was interpreted in Ref. [2] as the sign of a (weakly) 1st-order transition even in the Type-II regime. Note that the size of the 1st-order transition was predicted to be far too small to be experimentally detected. (The critical region in the high- T_c materials, $|T - T_c|/T_c \sim 10^{-2}$, is large enough that experiments measuring the non-mean-field critical properties are feasible.)

In an excellent paper, Ginsparg [4] pointed out that the critical behavior of a finite-temperature four-dimensional field theory could be described by an effective three-dimensional theory, as in Eq. (2). He also performed an extension of the work of Ref. [2] to a large class of non-abelian gauge-Higgs theories. The result is that, within the $(4 - \epsilon)$ -expansion, theories with an asymptotically free *gauge* coupling constant (not necessarily the entire theory) have no stable fixed point of the coupled RG equations. This was interpreted as implying that these theories also all underwent 1st-order transitions in the Type-II regime. The physical intuition behind this result is that small, amplitude fluctuations of the scalar field give the gauge field a mass, in turn suppressing the gauge field fluctuations which had tended to disorder the scalar field. Therefore the system is unstable to a sudden jump in the amplitude of the scalar field.

Unfortunately, the conclusions of Ref. [2] are known to be incorrect in this Type-II regime. It is possible to argue (again within the $(4 - \epsilon)$ -expansion) that the critical properties of the smectic-A to nematic (SAN) phase transition in liquid crystals are isomorphic to that of the superconductor [9].*

* Note that the free-energy describing this transition is of essentially the same form as the superconductor – the director field in the smectic-A phase playing the role of the vector potential. There are differences though – especially the spatially anisotropic nature of the smectic-A phase which makes comparison with critical theories delicate.

is that this phase transition should similarly be weakly 1st-order. The liquid-crystal case differs from that of the superconductor in that the size of the latent heat was such that it could easily be detected. Experimentally, however, the SAN transition is observed to be second order with approximately XY exponents [10]. (The situation is complicated by the evidence of “anisotropic scaling.”)

Stimulated by these findings, the Type-II superconductor transition in three-dimensions has been reconsidered [11]. By starting with a lattice model, and applying duality arguments, the partition function of the superconductor can be mapped onto that of a set of directed strings with repulsive contact interactions. With the aid of monte carlo simulations, this system was then studied and shown to lead to a 2nd-order transition with XY-model exponents (but with inverted asymmetry of the amplitudes of the singular terms with respect to temperature). Therefore a new fixed point of the RG equations must exist at $\epsilon = 1$ *which we cannot see by analytically continuing away from $d = 4$.*

Indeed, as we will argue in detail below, fluctuations which drive the critical behaviour of a system 2nd-order get stronger as we approach two dimensions. As we approach four dimensions 1st-order behaviour is favored. Thus, roughly speaking, the most reliable predictions of expansions in $(4 - \epsilon)$ -dimensions are that of 2nd-order transitions. The obvious consequence for non-abelian theories is that care must be taken in identifying the lack of a stable fixed-point in the expansion away from four-dimensions with a 1st-order transition.

If the theories that we are considering possess a 2nd-order transition then there must exist a fixed point of the RG equations. If the $(4 - \epsilon)$ expansion fails to find this fixed point, then how are we to proceed?

To be specific, what can we say about the critical properties of the following three-dimensional Lagrangian describing N complex p -vectors coupled to an $SU(p) \times U(1)$ gauge-theory?

$$L = |\mathbf{D}\phi^a|^2 + \frac{1}{4}\mathbf{G}^2 + \frac{1}{4}\mathbf{F}^2 + \lambda(\bar{\phi}^a \cdot \phi^a)(\bar{\phi}^b \cdot \phi^b) + \gamma(\bar{\phi}^a \cdot \phi^b)(\bar{\phi}^b \cdot \phi^a) + \text{mass terms}, \quad (6)$$

where $D_\mu = \partial_\mu - ieB_\mu - igA_\mu$, with A and B the $SU(p)$ and $U(1)$ gauge fields respectively (G and F are the associated field strengths). Here $a, b = 1, \dots, N$, repeated indices are summed, and $\bar{\phi} \cdot \phi$ denotes the inner product among p -vectors. An extra $SU(N)$ global symmetry has been imposed on the scalar sector so that tractable RG equations result. This is an obvious generalization of the standard-model.[†]

Consider the apparently unrelated gauged non-linear σ -model in two dimensions [13], defined by the action

$$S_\sigma = \frac{1}{t} \int d^2x \left(|\partial_\mu \phi_i^a|^2 + \frac{1}{2} \left((\bar{\phi}_i^a \partial_\mu \phi_j^a)(\bar{\phi}_j^b \partial_\mu \phi_i^b) + h.c. \right) \right), \quad (7)$$

with the additional constraint $\bar{\phi}_i^a \phi_j^a = \delta_{ij}$. Here $i, j = 1, \dots, p$ and $a, b = 1, \dots, N$, so that this σ -model is that of the complex grassmann manifold $U(N)/U(p) \times U(N-p)$. Note that in the action Eq. (7), we have already integrated out a $U(p)$ gauge field that originally appeared in covariant derivatives of ϕ . This leads to the second term in Eq. (7). (The elimination of the gauge field is exact, since, by definition, it possesses no kinetic term.) The beta-function for the single coupling t of this model (t is a dimensionless parameter proportional to the temperature) up to three-loops and in $(2 + \epsilon)$ -dimensions is [13]:

$$\beta_t(t) = -\epsilon t + Nt^2 + 2(p(N-p) + 1)t^3 + \mathcal{O}(t^4). \quad (8)$$

This theory is, of course, asymptotically free in the coupling t at $d = 2$ (we have again defined s to increase in the IR). In the language of statistical-mechanics, this translates in the statement that we have an IR-unstable fixed point at a critical

[†] In the $SU(2) \times U(1)$ two-doublet case ($p = 2, N = 2$), Eq. (6) corresponds to $\lambda + \gamma = \lambda_3$, $\gamma = -\lambda_4/2$, $\lambda_1 = \lambda_2 = \lambda_5 = \lambda_6 = 0$ and $4\lambda_3 v^2 = m^2$ with $v_1^2 = v_2^2 = v^2$, in the notation of [12] (before finite- T effects are taken into account). Here the zero-temperature mass term is $-m^2 \bar{\phi}^a \cdot \phi^a$.

temperature $t_c = \epsilon - 2\epsilon^2(p(N-p) + 1)/N + \mathcal{O}(\epsilon^3)$, resulting in a 2nd-order transition.[‡] The correlation length exponent ν (defined by $\xi(t) \sim |t - t_c|^{-\nu}$) is simply related to β_t ;

$$\nu^{-1} = -d\beta_t/dt|_{t_c} = \epsilon + 2(p(N-p) + 1)\epsilon^2/N^2 + \mathcal{O}(\epsilon^3), \quad (9)$$

which in the large N limit reduces to

$$\nu^{-1} = \epsilon + 2p\epsilon^2/N. \quad (10)$$

(It is also simple to calculate other exponents after applying the well known scaling laws [8] to ν and the anomalous dimension of the operator $\bar{\phi}^a \cdot \phi^b - (p/N)\delta_{ab}$ [13].)

Returning to the action Eq. (6) we may also consider its beta-functions, but in $(4 - \epsilon)$ -dimensions. Standard calculations show that they are Eq. (4) with the replacement $N \rightarrow pN$ supplemented by $\beta_g = \epsilon g^2 + \frac{g^4}{8\pi^2} \left(\frac{11p}{3} - \frac{N}{6} \right)$, and some rather complicated expressions for β_λ and β_γ [14]. These equations have a stable fixed point in the large N limit (at finite p) with an associated set of critical exponents. For instance the exponent ν is given by

$$\nu^{-1} = 2 - \epsilon + 48p\epsilon/N + \mathcal{O}(1/N^2). \quad (11)$$

We may also study the RG equations for the action (6) directly within the $1/N$ expansion [8] and for arbitrary dimension $2 < d < 4$. For instance we now find [15]

$$\nu = \frac{1}{d-2} \left(1 + \frac{2(d^2 - d) \sin(d\pi/2) \Gamma(d-1)p}{\pi N \Gamma^2(d/2)} \right). \quad (12)$$

Along with the other exponents this can be expanded near *both* four and two dimensions: for $\epsilon = (4 - d)$ we recover Eq. (11), while for $\epsilon = (d - 2)$ we obtain Eq. (10)

[‡] Recall that with respect to temperature a fixed point describing a 2nd-order transition is *unstable*, since dilation of the coordinates decreases the effective correlation length, and therefore, increases the reduced temperature $|T - T_c|/T_c$.

[15]. Similar relations hold between the values of the other critical exponents calculated (where they are well defined), in the large N limit, for, respectively, the σ -model near two dimensions, and the original $SU(p) \times U(1)$ gauge theory near four dimensions. The interpretation of these relations we wish to emphasize is that, in the large- N limit, both the $(4 - \epsilon)$ and $(2 + \epsilon)$ -expansions are in fact describing the *same* fixed point, albeit from differing perspectives.

What about the nature of the phase transition in the system (6) for small N ? Define the continuous function $d(N)$ as the number of (spatial) dimensions at which, for a given N , the transition switches over from 1st to 2nd-order ($d(N)$ is clearly model-dependent). The curve $d(N)$ in (N, d) -space is then a line of singularities through which it is impossible to analytically continue (see Fig. 1). The results of the $(4 - \epsilon)$ -expansion tell us that at some critical value N_c , $d(N)$ drops infinitesimally below four dimensions (considering N as a continuous parameter which we decrease away from large values). The important point is then the following: If we assume the existence of a fixed point at $d = 3$, even below N_c , then as we smoothly change N from just above, to just below N_c , we expect, by continuity, that the properties of the fixed point at $d = 3$ do not greatly alter. So if we have a description of the fixed point that is smooth through N_c , then this should give an at least qualitatively correct description of its properties into the region $N < N_c$. But the RG analysis of the system Eq. (7) within the $(2 + \epsilon)$ -expansion is exactly such a description!

We therefore propose that the IR behavior of the $SU(p) \times U(1)$ gauge model in three dimensions and for $N < N_c$ is described by the gauged non-linear σ -model of Eq. (7). In particular the correlation exponent ν of the $SU(p) \times U(1)$ gauge theory in three spatial dimensions is approximated by the expression Eq. (9) at $\epsilon = 1$. Furthermore, using the Josephson scaling law $\alpha = 2 - d\nu$ [8] we may extract a prediction for the specific heat exponent α . For the interesting case of $N = p = 2$ this gives $\nu = 2/3$ and $\alpha = 0$. It is also possible to extract other exponents from the formulae of Ref. [13], although we will not do so here. We note that in cases where both $(4 - \epsilon)$ and $(2 + \epsilon)$ expansions predict 2nd-order phase transitions, the

expansion away from two dimensions is generally less quantitatively reliable, and quite sophisticated techniques are required to extract accurate exponent values at $d = 3$. Thus Eq. (9) and the associated value of α should be taken as a rough guide only. We also caution the reader that we have assumed $d(N)$ is a monotonically decreasing function of N (as N is decreased), and that $d(N) \geq 3$ for values of (N, p) as low as $(2, 2)$. Although we have the analogies with the superconducting and liquid-crystal systems to support these assumptions it is possible to be misled, so that numerical simulations are really required to settle the issue definitively. These simulations would also enable us to find the value of the ratio of Higgs to gauge boson masses at the tricritical point, at which the system switches over from 1st to second order behavior. For the interesting case of small N , this ratio is likely to be hard to extract analytically.

To better understand the physics behind the arguments of the preceding paragraphs it is useful to consider a simpler, purely scalar, system [18,19]. Let ϕ be an n -component complex scalar field with three-dimensional free-energy given by

$$F = \int d^3x (|\nabla\phi|^2 - a|\phi|^2 + b|\phi|^4 + \dots). \quad (13)$$

Consider the limit in which $b \rightarrow \infty$ with $a/2b$ fixed. In this limit the value of the field ϕ becomes very well localized around its vacuum expectation value $\langle |\phi|^2 \rangle = a/2b$. However, the $(n - 1)$ Goldstone modes are free to fluctuate, and dominate the IR behavior. Indeed, formally integrating out the “amplitude fluctuations” in this limit we arrive at the free-energy of the non-linear $O(n)$ sigma-model,

$$F = \int d^3x (\nabla\phi)^2, \quad (14)$$

with the constraint $|\phi|^2 = a/2b$. In this free-energy only the “phase fluctuations” appear, and the associated system is known to undergo a 2nd-order transition near $d = 2$ [17]. Note that *for a fixed amplitude* of ϕ , the addition of a cubic term to (13) (so that a 1st-order transition is naively predicted for this model), has, almost

by definition, no effect – we still arrive at Eq. (14), with its 2nd-order transition. Of course, for any finite b , we cannot just throw away the amplitude fluctuations – the correct procedure is to consider the RG flow of the couplings of the (higher-dimension) operators that now appear in the action (14) when the amplitude of ϕ is allowed to fluctuate. A very important result of this procedure for our discussion is that, near two-dimensions, all these higher-dimension operators are *irrelevant* (in the Wilson-Fisher sense) [18,19] and so do not affect the order of the transition. (To our knowledge, no explicit proof of the analogous statement for gauged non-linear sigma-models has been presented. Nevertheless we expect that similar statements continue to hold [16].) One way to think about such phase-fluctuation-driven phase transitions is that near $d = 2$ the true transition temperature is driven well below its mean-field value $T_c^{(\text{mf})}$ (recall that $T_c \propto \epsilon$ [17]), so that the effective mass term of the amplitude fluctuations, $a \propto (T_c^{(\text{mf})} - T)$, is actually quite large at the transition. Further note that generally speaking, the effect of these phase-fluctuations *increases* as we move to more complex non-abelian systems. This gives us some reason to hope that the analogies to superconductor and liquid-crystal systems (which are, of course, abelian) may not be misleading.

We can heuristically include gauge fields A in this discussion by adding to Eq. (13) a gauge kinetic term $g^{-2}F^2$, and changing derivatives to covariant derivatives. Now, if in addition to considering fixed amplitude ϕ -fields, we also take the formal limit $g \rightarrow \infty$, then we can integrate out A leading to an action of the form Eq. (7). Despite the brutal nature of these manipulations, the large- N expansion and analogies presented above, indicate that the resulting theory may give a good description of the (static) far-IR behavior of the original theory in three dimensions. This procedure also generalizes to models other than the $SU(p) \times U(1)$ gauge system considered above.

A similar situation occurs for the q -state Potts model for $q = 3, 4$ [20]. For $q \geq 3$ such models possess a cubic invariant and Landau theory predicts a 1st-order phase transition. Recall that the lower-critical dimension for systems with a discrete symmetry is $d = 1$, so that as we approach $d = 1$ from above, fluctuations drive

the transition temperature to zero. Studies of the RG in $(1 + \epsilon)$ -dimensions show that all Potts models possess a 2nd-order transition near $d = 1$. The interesting point is that it is rigorously known that the three-state Potts model is 2nd-order upto $d = 2$ (the four-state Potts model also has a continuous transition at $d = 2$, although the situation is more complicated due to the merging of the critical and tricritical fixed points). This is despite the fact that a $(6 - \epsilon)$ -expansion (analogous to the $(4 - \epsilon)$ -expansion we considered previously) finds no stable fixed point. The interpretation is again that “phase” fluctuations have suppressed amplitude fluctuations. Our proposal for the critical behaviour of the gauged systems is that they are similarly “phase-fluctuation-driven” 2nd-order transitions, but now in three dimensions.

We wish to emphasize that many models other than the $SU(p) \times U(1)$ example of Eq. (6) may be analyzed in the framework described above, and that this analysis leads to similar results. (Of particular interest are the $O(p)$ gauge theories – and related non-linear sigma models – discussed in Refs. [13] and [15].) Namely, within a $(2 + \epsilon)$ -dimensional renormalization-group analysis, and in the Type-II regime, *2nd-order* transitions are predicted. Work is in progress concerning these theories, as well as predictions for the additional critical exponents, and an analysis of the RG flows of the higher-dimension operators, of the $SU(p) \times U(1)$ systems [16].

Finally, we mention that it might be possible to extend our predictions of the critical behavior of the $SU(p) \times U(1)$ systems all the way down to $p = 2$ and $N = 1$ – the minimal standard model. Although, for these values, the action Eq. (7) no longer makes sense, the critical exponents (considered as analytic functions of N and p) might still be qualitatively correct. This is similar to considerations made in the study of Anderson localization [21], and 0-component spins applied to random walks [8]. It is also possible that our results have some relevance to the difficulties encountered in the study of the EWPT in the Type-I regime [1]. For instance, the proximity of a 2nd-order phase transition could result in an anomalously weak 1st-order transition in this regime.

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FIGURE CAPTIONS

- 1) The character of the critical behavior of the $SU(p) \times U(1)$ gauge theory coupled to N p -vectors, in d -dimensions, changes from 1st to 2nd-order as we cross the curve $d(N)$. Expansions in $\epsilon = (4 - d)$ inevitably break down as $d(N)$ is approached. The 2nd-order transitions occurring along the hatched line at $d = 3$, are only accessible via $(2 + \epsilon)$ -expansions away from the gauged non-linear sigma-model formulated in two-dimensions.

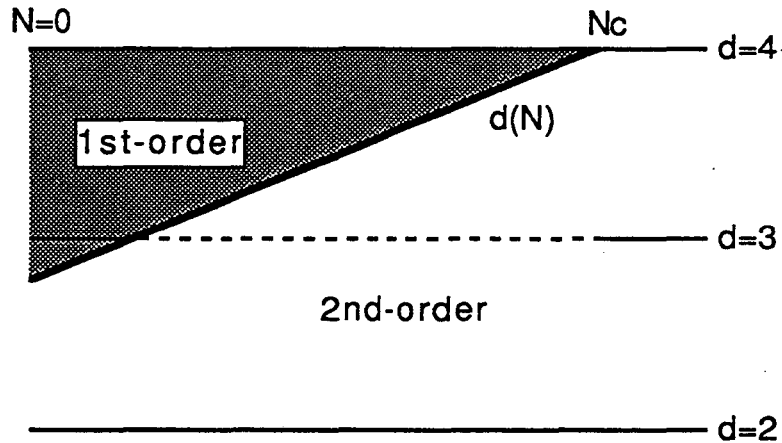


Figure 1