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Contracting for Information (Preliminary and Incomplete)*

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Abstract

We study optimal contracts between a decision maker and an expert where the decision maker can commit to make transfers to the expert contingent on his advice but cannot commit to take an action contingent on advice. We show that optimal contracts never entail full revelation. Instead, the decision maker pays the expert to reveal for some states and doesn't compensate the expert at all in others.

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1 Introduction

In many situations of economic interest, those with the power to make decisions lack important information about the economic consequences of their choices. As a result, decision makers often seek advice from better informed parties—experts—prior to making decisions. Examples of such situations abound. CEOs consult investment bankers, strategic planners, and marketing specialists before making corporate decisions. Congressional representatives hold hearings and consult lobbyists to learn more about the impact of proposed legislation. Investors read reports of equity analysts and call up stock brokers for advice and tips before deciding on an investment strategy. Legislators vote on specialized legislation reported out of committees.

A feature common to all of these situations is that the expert being consulted may well have preferences that do not coincide with those of the decision maker. As a result, the expert may have the incentive to mislead or to withhold information. In such situations it is important that the decision maker be able to elicit as much information as possible from the expert. Indeed, the ability to do this is commonly thought of as the mark of an effective leader.

The strategic interaction between an uninformed decision-maker and an informed expert was first studied by Crawford and Sobel (1982) in a now classic paper.[1] In their model the expert, after learning the realization of the payoff relevant state of nature, sends a costless message to the decision maker, who then takes an action that has consequences for both parties. Interest in the problem arises, of course, from the assumption that the preferences of the two parties are not perfectly aligned. Crawford and Sobel (hereafter CS) obtain a complete characterization of the set of equilibria in their model and identify the Pareto dominant equilibrium. They show that preference divergence between the two parties inevitably leads to withholding of information by the expert; that is, full revelation is never an equilibrium outcome. Further, as the degree of preference divergence increases, the amount of information disclosed by the expert decreases. Once the preference divergence is sufficiently large, the expert can credibly disclose no information whatsoever.

Importantly, no contracts are available to the decision maker in CS model in soliciting advice from experts; however, in many situations, some form of contracting is possible. In this paper, we study the case where the decision maker can set a contract for non-negative transfers as a function of the advice offered by the expert, but cannot contract over actions after having received advice. That is, there is imperfect commitment on the part of the decision maker. The leading example of this situation is in legislative settings where a non-expert median floor legislator can informally contract with an expert, but biased, committee. While the floor median is held accountable by voters for its decisions regarding bills coming to the floor, the content of these bills might be influenced by transfers of resources from the floor to the committee as a function of the details of the bill reported out of committee.

[1] The Crawford and Sobel model (hereafter 'CS model') is discussed in more detail below.

We find that even this limited power of commitment is sufficient to enable the decision maker to get the expert to fully reveal his information in a quite general setting; however, it is *never* cost-effective for the decision maker to induce this outcome. That is, fully revealing contracts are never optimal contracts under fairly general conditions. When preferences follow the structure of the standard model of the informational theory of legislative rules (see Gilligan and Krehbiel, 1987)—the so-called uniform-quadratic case—we find that the fully revealing contract is always worse than having no contract whatsoever—regardless of the degree of preference misalignment between the two parties.

If fully revealing contracts are not optimal, then just what do optimal contracts look like? Here, we derive a number of properties of optimal contracts on our way to characterizing optimal contracts for the uniform-quadratic case. The most notable of these properties is that it is never cost-effective for the decision maker to "pay" for pooling. That is, unless the expert is fully disclosing his information, the decision maker should withhold compensation. With this property and several others, we identify the structure of optimal contracts: The optimal scheme induces full disclosure through compensation up to one-fourth of the time. Otherwise, the decision maker simply solicits the advice of the expert with no compensation whatsoever. Finally, we compare this result to the case where the decision maker has the power to commit to actions, but no transfers are possible. This roughly corresponds to the case of closed rules in legislative settings. We find that when the degree of preference misalignment is large, commitment to transfers is preferred to commitment to actions, but the situation is reversed when preferences are more closely aligned.

Related Literature

The nearest antecedent to this paper is the important paper by Baron (2000). Baron compares contracting under deference and under open rules in the Gilligan and Krehbiel uniform-quadratic setting. Under deference, the decision maker has full commitment power; whereas under the open rule, the setting is quite similar to ours—the decision maker can commit to transfers but not to actions. One key difference is that contracting in our paper consists solely of non-negative transfers whereas in Baron, transfers may be negative but the expert committee must receive sufficient incentives to "specialize" and obtain expertise about an issue. In the language of mechanism design, Baron considers the case where the agent, the expert, has an ex ante participation constraint that must be satisfied whereas out model is one where the agent has limited liability.

We extend the results of Baron in three directions: First, our results on full revelation hold for quite general preferences. Second, while optimal revealing contracts are characterized in Baron's paper, here we characterize the fully optimal contracts, which differ somewhat from those in the earlier work. Finally, we compare commitment solely to transfers with commitment solely to actions.

Along somewhat similar lines is Dewatripont and Tirole (1999) who examine optimal contracting of "advocates." The key distinction between that paper and ours

is that Dewatripont and Tirole are mainly concerned with solving the moral hazard problem of information gathering on the part of the agent or agents whereas our concerns center on using contracts for eliciting information from already informed experts.

Our paper is also related to the literature on the informational theory of legislative rules (see for instance Gilligan and Krehbiel, 1987, 1989; Krishna and Morgan, 2001b) as well as to the literature on strategic information transmission more generally (see, for instance, Crawford and Sobel, 1982; Austen-Smith, 1990, 1993; Krishna and Morgan, 2001a; Battaglini, 2002). However, none of these papers allow for contracts including transfers from the decision maker to the expert or experts.

Finally, our paper is somewhat related to questions addressed in Bester and Strausz (2002). That paper seeks to extend the revelation principle to settings where the principal is unable to commit to one or more dimensions of the contracting space—as in the question we consider. They offer a characterization of incentive feasible direct mechanisms.[2]

The remainder of the paper proceeds as follows: In section 2 we sketch the model. Section 3 presents results on fully revealing contracts and shows that these contracts are never optimal. Section 4 derives various structural properties of optimal contracts and then uses these to characterize in closed form the optimal contract in this setting. Section 5 compares the value of commitment to transfers with commitment to actions. Finally, section 6 concludes.

2 Preliminaries

In this section we sketch a simple model of decision making between a single decision maker and a single expert when the decision maker has some power to contract with the expert but where commitment by the decision maker to a particular action is impossible. We do not model any of the examples mentioned in the introduction explicitly. Rather our model is a stylized representation of the interaction between a decision maker and an expert across a broad range of institutional settings. The overall structure extends the model of CS to a setting where some contracting is possible but where the commitment power of the decision maker is limited.

Consider a *decision maker* who takes an action $y \in \mathbf{R}$, the payoff from which depends on some underlying state of nature $\theta \in [0,1]$. The state of nature θ is distributed according to the density function $f(\cdot)$. The decision maker has no information about θ, but this information is available to an *expert* who observes θ.

The expert then offers "advice" to the decision maker by sending a message $m \in [0,1]$ after observing the state. The action taken by the decision maker is not in any way bound by the advice of the experts. Instead, she is free to interpret the messages

[2]For technical reasons, their result does not apply in the CS model.

however she likes as well as to choose any action.[3]

The payoff functions of the agents are of the form $U(y, \theta, b_i)$ where b_i is a parameter which differs across agents. For the decision maker, agent 0, b_0 is normalized to be 0. For the experts, agent 1, $b_1 = b > 0$. We write $U(y, \theta) \equiv U(y, \theta, 0)$ as the decision maker's payoff function. We suppose that U is twice continuously differentiable and satisfies $U_{11} < 0$, $U_{12} > 0$, $U_{13} > 0$. Since $U_{13} > 0$ the parameter b measures how closely the expert's interests are aligned with those of the decision maker and it is useful to think of b as a measure of how *biased* the expert is, relative to the decision maker. We also assume that for each i, $U(y, \theta, b_i)$ attains a maximum at some y. Since $U_{11} < 0$, the maximizing action is unique. The biases are commonly known.

These assumptions are satisfied by "quadratic loss functions." In this case, the decision maker's payoff function is

$$U(y, \theta) = -(y - \theta)^2 \tag{1}$$

and the expert's payoff function is

$$U(y, \theta, b_i) = -(y - (\theta + b))^2 \tag{2}$$

where $b > 0$. An important case, first introduced by CS and used extensively in the political science literature, combines quadratic loss functions with the assumption that the state θ is uniformly distributed on $[0, 1]$. We will refer to this as the "uniform-quadratic" case.

Define $y^*(\theta) = \arg\max_y U(y, \theta)$ to be the *ideal* action for the decision maker when the state is θ. Similarly, define $y^*(\theta, b) = \arg\max_y U(y, \theta, b)$ be the ideal action for the expert. Since $U_{13} > 0$, $b > 0$ implies that $y^*(\theta, b) > y^*(\theta)$

Notice that with quadratic loss functions, the ideal action for the decision maker is to choose an action that matches the true state exactly: for all θ, $y^*(\theta) = \theta$. The ideal action for an expert with bias b is $y^*(\theta, b) = \theta + b$.

Prior to offering any advice, the decision maker is free to contract with the expert as follows: The decision maker can specify a schedule of monetary transfers, $t(m)$ which can depend on the exact advice offered by the expert. We assume that these transfers are required to be non-negative and that the preferences of the two parties are additively separable and risk neutral in the amount of the transfer. That is, when such a contract is offered by the decision maker and the expert offers advice m, the payoffs to the decision maker taking an action y in state θ are:

$$V(y, \theta, m) = U(y, \theta) - t(m)$$

while the payoffs to the expert are

$$V(y, \theta, m, b) = U(y, \theta, b) + t(m).$$

[3] In the political science literature, this is referred to as the "open rule" (Gilligan and Krehbiel (1989)).

The structure of the contracting environment is one where the decision maker has the power to commit to a transfer, but cannot commit to what action to undertake as a function of the advice offered by the expert. The timing of the game is as follows:

$\vdash$ —	— — — + — — —	— — — + — — —	— — —	— — $\dashv$
Decision Maker proposes $t(m)$	Expert learns θ	Expert sends message m		DM chooses action y

A strategy for the expert μ specifies the message $m = \mu(\theta)$ that he sends in any state θ. A strategy for the decision maker y specifies the action $y(m)$ that she takes following any message m by the expert as well as a transfer $t(m)$ to be sent to the expert. Let $P(\cdot|m)$ denote the cumulative distribution function that specifies posterior beliefs about the state held by the decision maker after the message m.

In a *perfect Bayesian equilibrium* (1) for all messages m, the decision maker's action $y(m)$ maximizes her expected payoff given her posterior beliefs $P(\cdot|m)$; (2) the beliefs $P(\cdot|m)$ are formed using the expert's strategy μ by applying Bayes' rule wherever possible; (3) given the decision maker's strategy y and the contract $t(\cdot)$ for all states θ, $\mu(\theta)$ maximizes the expert's payoff. Finally, the transfer schedule $t(\cdot)$ is chosen by the decision maker to maximize her expected payoff anticipating $\mu(\theta)$ and $y(m)$.

The following notation, employed by CS, proves useful: If the expert's advice, m_1, reveals only that the state lies in the interval $[a_0, a_1]$, then we shall say that the expert has *pooled* over this interval. Let $y[a_0, a_1]$ denote the decision maker's optimal action under these circumstances. Specifically, $y[a_0, a_1] \in \arg\max_y \int U(y, \theta)\, dP(\theta|m_1)$.

Notice that the degenerate case where, for all m, $t(m) = 0$ corresponds exactly to the model of CS. They show that every equilibrium of the single expert game has the following structure.[4] There are a finite number of equilibrium actions $y_1, y_2, ..., y_N$. The equilibrium breaks the state space into N disjoint intervals $[0, a_1)$, $[a_1, a_2)$, ... , $[a_{n-1}, a_n)$, ... , $[a_{N-1}, 1]$ with action y_n resulting in any state $\theta \in [a_{n-1}, a_n)$. The equilibrium actions are monotonically increasing in the state, that is, $y_{n-1} < y_n$. Finally, at every "break point" a_n the following "no arbitrage" condition

$$U(y_n, a_n, b) = U(y_{n+1}, a_n, b) \tag{3}$$

is satisfied. In other words, in state a_n the expert is indifferent between the actions y_n and y_{n+1}. Since $U_{12} > 0$, for all $\theta < a_n$, the expert strictly prefers y_n to y_{n+1} and for all $\theta > a_n$, the reverse is true. Thus (3) serves as an incentive (or self-selection) constraint absent any transfers.

[4] CS actually characterize the set of *Bayesian* equilibrium outcomes. In the single expert game this is the same as the set of *perfect Bayesian* equilibrium outcomes.

3 Full Revelation

CS have shown that absent any commitment power on the part of the decision maker, it is impossible for her to induce the expert to fully reveal his private information. However, the addition of the limited power to commit to non-negative transfers, this is no longer the case. Indeed, full revelation is always implementable when the decision maker has the power to make non-negative transfers to the expert contingent on her advice. The main result in this section is to show that, despite the fact that such a scheme always leads to the decision maker obtaining his first-best action, it is never cost effective to induce such an outcome. That is, full revelation is *never* an optimal contract.

First, we show that a fully revealing scheme is always feasible. To see this, first notice that under such a scheme the expert offers truthful advice; that is, $\mu(\theta) = \theta$. Further, the decision maker anticipates that this will be the case; hence $y(m) = m$. For truthful revelation on the part of the expert to be incentive compatible requires that in every state θ the expert prefers to tell the truth rather than to lie and suggest that the state is some $\theta' \neq \theta$. That is for all θ, θ'

$$U(y^*(\theta), \theta, b) + t(\theta) \geq U(y^*(\theta'), \theta, b) + t(\theta').$$

Suppose that $t^*(\cdot)$ is a scheme that induces full revelation. This requires that the expert's utility from reporting an arbitrary state m when the true state is θ is no greater than reporting the truth, that is,

$$U(y^*(\theta), \theta, b) + t^*(\theta) \geq U(y^*(m), \theta, b) + t^*(m)$$

The first-order condition for the expert's maximization problem is

$$U_1(y^*(\theta), \theta, b) y^{*\prime}(\theta) + t^{*\prime}(\theta) = 0$$

which results in the differential equation

$$t^{*\prime}(\theta) = -U_1(y^*(\theta), \theta, b) y^{*\prime}(\theta)$$

Since $U_1(y^*(\theta), \theta, b) > 0$ and $y^{*\prime}(\theta) > 0$, a scheme that induces full revelation is downward sloping. Thus among all schemes that induce full revelation the one that is best for the decision maker satisfies $t^*(1) = 0$. This scheme is

$$t^*(\theta) = \int_\theta^1 U_1(y^*(\alpha), \alpha, b) y^{*\prime}(\alpha) d\alpha \tag{4}$$

To see that this scheme indeed induces full revelation, notice that we can write, for all θ and m,

$$\begin{aligned} U(y^*(\theta), \theta, b) &= U(y^*(m), \theta, b) + \int_m^\theta U_1(y^*(\alpha), \theta, b) y^{*\prime}(\alpha) d\alpha \\ &\geq U(y^*(m), \theta, b) + \int_m^\theta U_1(y^*(\alpha), \alpha, b) y^{*\prime}(\alpha) d\alpha \\ &= U(y^*(m), \theta, b) + t^*(m) - t^*(\theta) \end{aligned}$$

where the inequality follows from the fact that $U_{12} > 0$.

To summarize:

Proposition 1 *The power to contract using non-negative transfers without the ability to commit to actions is sufficient for a decision maker to induce full revelation. The optimal fully revealing contract entails transfers going to zero in the highest states.*

Now we show that such a scheme is never cost-effective from the perspective of the decision maker.

To see this, consider an alternative contract $t(\cdot)$ that induces the following behavior: the expert reveals any state $\theta \in [0, z]$ where $z < 1$ and pools thereafter. No payment is made if the reported state $m > z$. At $\theta = z$, the expert must be indifferent between reporting that the state is z and reporting that it is above z. If we denote by t_z the payment in state z, then we must have

$$U(y^*(z), z, b) + t_z = U(y([z, 1]), z, b). \tag{5}$$

Since for z close to 1, $U(y^*(z), z, b) < U(y([z, 1]), z, b)$, it follows that $t_z > 0$.

It is routine to verify that

$$\left.\frac{dt_z}{dz}\right|_{z=1} = U_1(y^*(1), 1, b)\left(\left.\frac{d}{dz}y[z, 1]\right|_{z=1} - y^{*\prime}(1)\right).$$

Incentive compatibility over the interval $[0, z]$ requires that

$$t(\theta) = t_z + \int_\theta^z U_1(y^*(\alpha), \alpha, b)\, y^{*\prime}(\alpha)\, d\alpha.$$

which is again always greater than zero, so this alternative scheme is feasible.

It is useful to note that:

$$\frac{dt(\theta)}{dz} = \frac{dt_z}{dz} + U_1(y^*(z), z, b)\, y^{*\prime}(z).$$

Observe that on the interval $[0, z]$, the new scheme t is parallel to the fully revealing scheme t^*. Indeed, for all $\theta \leq z$ we have,

$$t(\theta) - t^*(\theta) = t_z - t^*(z)$$

The expected utility resulting from the new scheme is

$$V = \int_0^z (U(y^*(\theta), \theta) - t(\theta))\, f(\theta)\, d\theta + \int_z^1 U(y[z, 1], \theta)\, f(\theta)\, d\theta$$

Differentiating with respect to z, we obtain

$$\begin{aligned}\frac{dV}{dz} &= \left(U\left(y^*\left(z\right),z\right)-t_z\right)f\left(z\right)-U\left(y\left[z,1\right],z\right)f\left(z\right)\\ &\quad -\int_0^z\left(\frac{dt\left(\theta\right)}{dz}\right)f\left(\theta\right)d\theta\\ &= \left(U\left(y^*\left(z\right),z\right)-t_z\right)f\left(z\right)-U\left(y\left[z,1\right],z\right)f\left(z\right)\\ &\quad -\int_0^z\left(\frac{dt_z}{dz}+U_1\left(y^*\left(1\right),1,b\right)y^{*\prime}\left(1\right)\right)f\left(\theta\right)d\theta\end{aligned}$$

When $z=1$, we have

$$\begin{aligned}\left.\frac{dV}{dz}\right|_{z=1} &= -\left.\frac{dt_z}{dz}\right|_{z=1}-U_1\left(y^*\left(1\right),1,b\right)y^{*\prime}\left(1\right)\\ &= -\left(U_1\left(y^*\left(1\right),1,b\right)\left(\left.\frac{d}{dz}y\left[z,1\right]\right|_{z=1}-y^{*\prime}\left(1\right)\right)\right)-U_1\left(y^*\left(1\right),1,b\right)y^{*\prime}\left(1\right)\\ &= -U_1\left(y^*\left(1\right),1,b\right)\left.\frac{d}{dz}y\left[z,1\right]\right|_{z=1}\\ &< 0\end{aligned}$$

Thus we have shown that for z close enough to 1, the alternative scheme $t\left(\cdot\right)$ yields a higher expected utility for the decision maker than the fully revealing scheme $t^*\left(\cdot\right)$.

Thus we have established the main result for this section:

Proposition 2 *A fully revealing contract is* never *optimal.*

The economic trade-off captured in this result is the following: For states near the highest possible state, the direct contracting costs of inducing truth-telling are relatively inexpensive ($t^*\left(\theta\right)$ is close to zero when θ is close to 1); however the indirect effect of obtaining this revelation is to increase the information extraction costs for *all* of the lower states. The alternative contract shows that the informational benefits of additional revelation in the high states never justifies these increased costs. The decision maker can *locally* give up a small amount of information by inducing pooling for the highest states, but more than recovers this in the *global* reduction in the costs of information extraction for lower states.

3.1 The Uniform-Quadratic Case

In the uniform-quadratic case, a sharper comparison between revealing schemes and alternatives is possible. Specifically, we show that a fully revealing contract is *always* inferior to no contract whatsoever.

Using equation (4) ,the optimal fully revealing contract in the uniform-quadratic case, we obtain:

$$t^*(\theta) = 2b(1-\theta)$$

and the expected payoff to the decision maker under this scheme is simply $-b$.

With no contracting whatsoever, the situation is exactly that in CS. They have shown that the highest expected payoff to the decision maker is

$$V = -\frac{1}{12N(b)^2} - \frac{b^2\left(N(b)^2 - 1\right)}{3},$$

where

$$N(b) = \left\lceil -\frac{1}{2} + \frac{1}{2}\sqrt{1 + \frac{2}{b}} \right\rceil$$

is the number of partition elements in the most informative equilibrium.

We now show that the optimal fully revealing contract is always worse than no contract. First, consider the sequence of biases $b_N = \frac{1}{2N(N+1)}$ for $N = 1, 2, ...$These are the levels of bias where the maximum number of partition elements change. When $b = b_N$,

$$\frac{1}{12N^2} + \frac{b_N^2(N^2-1)}{3} = \frac{1}{6N(N+1)} = \frac{1}{3}b_N,$$

which implies that for any $b = b_N$ in this sequence, the optimal fully revealing contract is worse than no contract.

Next, notice that, for a fixed N, the payoffs under both schemes are decreasing monotonically in b. This implies that, for all $b \leq \frac{1}{4}$, the optimal fully revealing contract is worse than no contract. Finally, notice that for $b > \frac{1}{4}$, $N = 1$, hence $V = -\frac{1}{12} > b$,which completes the argument.

To summarize:

Proposition 3 *In the uniform-quadratic case, the optimal fully revealing contract is worse than no contract whatsoever.*

4 The Structure of Optimal Contracts

Having established that fully revealing contracts are not optimal, the remainder of the paper is devoted to characterizing optimal contracts. To obtain a precise characterization, we restrict attention to the uniform-quadratic case. In the first subsection, we identify key structural characteristics of any optimal contract. In the second, we offer a closed-form solution to the optimal contracting problem.

4.1 Properties of Optimal Contracts

4.1.1 Monotonicity of Equilibrium Actions

The first property holds in general: Any incentive compatible contract must lead to (weakly) higher actions in higher states. This simplifies the analysis of optimal contracts considerably. Lemma 4 below implies that each action occurs either on an interval (pooling) or at a point (revealing).

We now proceed with the result. Let $Y(\theta)$ be the equilibrium action chosen in state θ with an incentive compatible compensation scheme in place. Then,

Lemma 4 *In any incentive compatible compensation scheme, $Y(\cdot)$ is nondecreasing.*

Proof. Let θ_1 and θ_2 be such that $\theta_1 < \theta_2$ and suppose that $Y(\theta_1) > Y(\theta_2)$. Let t_1 and t_2 be the transfers made in the two states, respectively. In state θ_1,incentive compatibility requires that

$$\begin{aligned} U(Y(\theta_1), \theta_1, b) + t_1 &\geq U(Y(\theta_2), \theta_1, b) + t_2 \\ U(Y(\theta_1), \theta_1, b) - U(Y(\theta_2), \theta_1, b) &\geq t_2 - t_1. \end{aligned}$$

Similarly, in state θ_2

$$\begin{aligned} U(Y(\theta_1), \theta_2, b) + t_1 &\leq U(Y(\theta_2), \theta_2, b) + t_2 \\ U(Y(\theta_1), \theta_2, b) - U(Y(\theta_2), \theta_2, b) &\leq t_2 - t_1. \end{aligned}$$

and so we conclude that

$$U(Y(\theta_1), \theta_1, b) - U(Y(\theta_2), \theta_1, b) \geq U(Y(\theta_1), \theta_2, b) - U(Y(\theta_2), \theta_2, b)$$

On the other hand, since $\theta_1 < \theta_2$ but $Y(\theta_1) > Y(\theta_2)$, $U_{12} > 0$ implies that

$$U(Y(\theta_1), \theta_1, b) - U(Y(\theta_2), \theta_1, b) < U(Y(\theta_1), \theta_2, b) - U(Y(\theta_2), \theta_2, b)$$

which is a contradiction. ■

4.1.2 Never Pay for Pooling

Having now shown that optimal contracts have distinct actions that occur either in an interval of states (pooling) or for a unique state (separating), we are in a position to first examine the compensation for actions that occur in a pooling interval. The finding in Lemma 5 below is that it is *never* cost-effective for the decision maker to make a positive transfer in a pooling interval. Specifically,

Lemma 5 *If $t(\cdot)$ is an optimal scheme, and $[x_{i-1}, x_i]$ is some pooling interval then $t(\theta) = 0$ for all $\theta \in (x_{i-1}, x_i)$.*

Proof. Suppose not, so that there is some positive transfer $t_i > 0$ in the interval (x_{i-1}, x_i). We show that alternative contracting schemes, which we construct below, improve the expected payoff of the decision maker.

The alternative contract we construct is one in which there is full separation in the interval (x_{i-1}, w) and pooling in the interval $[w, x_i]$. We will construct an alternative transfer scheme $\bar{t}$ so that the incentives for the rest of the contract are unchanged. In fact, for all $\theta \notin (x_{i-1}, x_i)$, $\bar{t}(\theta) = t(\theta)$.

To ensure that transfers for states greater than x_i are unaffected, choose $\bar{t}_i$,the transfer in the interval $[w, x_i]$, top solve

$$-\left(\frac{x_i + w}{2} - (x_i + b)\right)^2 + \bar{t}_i = -\left(\frac{x_i + x_{i-1}}{2} - (x_i + b)\right)^2 + t_i$$

or equivalently,

$$\bar{t}_i = \left(\frac{x_i + w}{2} - (x_i + b)\right)^2 - \left(\frac{x_i + x_{i-1}}{2} - (x_i + b)\right)^2 + t_i.$$

Since the payoff to the expert in state x_i is identical in the alternative scheme to his payoff in the original scheme, incentives are unaffected for $\theta > x_i$.

Next, we need to show that such a $\bar{t}_i$ is non-negative. To see this, notice that $\bar{t}_i < t_i$ since $w > x$; however, when $w = x_{i-1}$, $\bar{t}_i = t_i > 0$ so (by continuity) if w is close enough to x_{i-1}, then $\bar{t}_i > 0$.

The reduction in the transfer over the interval $[w, x_i]$ is characterized by:

$$\frac{d\bar{t}_i}{dw} = \left(\frac{w - x_i}{2} - b\right).$$

Next, we have to choose transfers so that truthful revelation is incentive compatible over the $[x_{i-1}, w]$ interval. At $\theta = w$, this means that $\bar{t}(w)$ solves

$$-b^2 + \bar{t}(w) = -\left(\frac{x_i + w}{2} - (w + b)\right)^2 + \bar{t}_i$$

or, equivalently,

$$\begin{aligned} \bar{t}(w) &= \bar{t}_i - \left(\frac{x_i - w}{2} - b\right)^2 + b^2 \\ &= \bar{t}_i + (x_i - w)\left(b - \frac{1}{4}(x_i - w)\right). \end{aligned}$$

It is useful to note that

$$\frac{d\bar{t}(w)}{dw} = -2b.$$

It is straightforward to compute the remainder of the contract: For $\theta < w$,

$$\bar{t}(\theta) = 2b(w - \theta) + \bar{t}(w).$$

Again, it is useful to note that, for $\theta < w$

$$\frac{d\bar{t}(\theta)}{dw} = 2b - 2b = 0.$$

Next, notice that the payoffs for an expert in state x_{i-1} are identical under the alternative scheme compared to the old scheme. That is, under the alternative scheme, the expert earns:

$$\begin{aligned} & -b^2 + t(x_{i-1}) \\ = \; & 2b(x_i - x_{i-1}) - \left(\frac{x_i + x_{i-1}}{2} - (x_i + b)\right)^2 + t_i \\ = \; & -\left(\frac{x_i + x_{i-1}}{2} - (x_{i-1} + b)\right)^2 + t_i \end{aligned}$$

which is exactly what he earned under the old scheme. Therefore, incentives for $\theta < x_{i-1}$ are unaffected.

Thus, the expected utility of the alternative contract is

$$E\bar{U} = k - \int_{x_{i-1}}^{w} \bar{t}(\theta)\, d\theta - \int_{w}^{x_i} \left(\left(\frac{w + x_i}{2} - \theta\right)^2 + \bar{t}_i\right) d\theta,$$

where k is identical to what what it was under EU.

$$\begin{aligned} \frac{dE\bar{U}}{dw} &= -\bar{t}(w) - \int_{x_{i-1}}^{w} \frac{d\bar{t}(\theta)}{dw} d\theta + \left(\frac{w + x_i}{2} - w\right)^2 + \bar{t}_i - (x_i - w)\frac{d\bar{t}_i}{dw} \\ &= -\bar{t}(w) + \left(\frac{w + x_i}{2} - w\right)^2 + \bar{t}_i - (x_i - w)\left(\frac{w - x_i}{2} - b\right) \\ &= \left(\frac{x_i + w}{2} - (w + b)\right)^2 - b^2 + \left(\frac{x_i - w}{2}\right)^2 - (x_i - w)\left(\frac{w - x_i}{2} - b\right) \\ &= (x_i - w)^2. \end{aligned}$$

When $w = x_{i-1}$,

$$\begin{aligned} \frac{dE\bar{U}}{dw}|_{w=x_{i-1}} &= (x_i - x_{i-1})^2 \\ &> 0. \end{aligned}$$

Therefore, the alternative contract yields higher payoffs than the putative optimal contract, which is a contradiction. ■

4.1.3 No separation to the right of pooling

The previous property establishes that the only transfers in optimal contracts occur under full separation. The final property we establish is that inducing such separation is only cost-effective for the lowest states. That is, once a contract calls for a pooling interval over a set of states, it never pays to induce separation for higher states. Specifically,

Lemma 6 *There do not exist* $0 \leq w < x < z \leq 1$ *such that the optimal contract results in pooling on the interval* $[w, x]$ *and separation on the interval* $[x, z]$.

Proof. If there is pooling on $[w, x]$ followed by separation then at x we must have

$$-\left(\frac{w+x}{2} - (x+b)\right)^2 + t = -b^2 + t\,(x)$$

At the point x, both the actions $\frac{w+x}{2}$ and x are too low for the expert and x is better. Thus we must have $t > 0$. But this is impossible by the previous lemma. ■

Taken together, these properties imply that optimal contracts consist of separation for some set of low states followed by a set of pooling intervals where no transfers are paid. In the next subsection, we construct such optimal contracts.

4.2 Optimal contracts

From the preceding subsection, we know that under the optimal contract the expert is induced to reveal up to some state s and not compensated thereafter. Suppose that for the interval $[s, 1]$, the largest number of feasible partition elements in any equilibrium is K. We first show that such a contract is feasible. Further, we show that inducing fewer an equilibrium that entails fewer than K partition elements is infeasible owing to non-negativity constraints.

Lemma 7 *Suppose that the optimal contract calls for revelation on* $[0, s]$ *and pooling thereafter. Such a contract is feasible if and only if the equilibrium containing the maximum number of partition elements is played over* $[s, 1]$.

Proof. First, suppose that a size K partition of $[s, 1]$ is possible, then the break-points of the partition are

$$x_j = \frac{j}{K} + \frac{K-j}{K}s - 2bj\,(K-j)$$

for $j = 1, 2, ..., K$.

For a size K partition to be feasible, we need that $x_1 > s$. Or, substituting for x_1

$$b < \frac{1-s}{2K\,(K-1)}.$$

For a size $K+1$ partition to be impossible requires (using identical reasoning)

$$b \geq \frac{1-s}{2K\left(K+1\right)}$$

Thus, we know that $\frac{1-s}{2K(K+1)} \leq b < \frac{1-s}{2K(K-1)}$.

At s, the indifference condition is

$$\begin{aligned} -b^2 + t\left(s\right) &= -\left(\frac{s+x_1}{2} - \left(s+b\right)\right)^2 \\ t\left(s\right) &= b^2 - \left(\frac{s+x_1}{2} - \left(s+b\right)\right)^2. \end{aligned}$$

Substituting for x_1 yields

$$t\left(s\right) = \frac{1}{4}\frac{4b^2K^2 - 1 + 2s + 4bK^2 - s^2 - 4bsK^2 - 4b^2K^4}{K^2}.$$

After rearranging, we can rewrite the numerator as $-\left(s - 1 + 2bK^2 - 2Kb\right)\left(s - 1 + 2bK^2 + 2Kb\right)$. Since $t\left(s\right) \geq 0$ in any feasible contract, this implies

$$\frac{1-s}{2K\left(K+1\right)} \leq b < \frac{1-s}{2K\left(K-1\right)}$$

which is exactly the condition that there be at most K partition elements in the interval $\left[s,1\right]$. ■

It now remains to determine the optimal value of s. From the last section for fixed s, the only K that is feasible is

$$K = \left\lceil -\frac{1}{2} + \frac{1}{2}\sqrt{1 + \frac{2\left(1-s\right)}{b}} \right\rceil,$$

which is the most informative CS equilibrium on $\left[s,1\right]$. Next, notice that we can rewrite the expected payoff to the decision maker from revealing until s and playing the most informative CS equilibrium in $[s,1]$ as

$$\begin{aligned} EV &= -\int_0^s \left(2b\left(s-\theta\right) + t\left(s\right)\right) d\theta - \sum_{k=1}^{K} \int_{x_{k-1}}^{x_k} \left(y_k - \theta\right)^2 d\theta \\ EV &= \frac{1}{12}\left(16sK^2b^2 - 4b^2K^2 + \frac{6s - 9s^2 + 4s^3 - 1}{K^2} - 16b^2s + 4b^2 - 12bs\right). \end{aligned}$$

Keeping K fixed and differentiating with respect to s yields the first-order condition for a local optimum

$$\frac{\partial EV}{\partial s} = 2bK^2\left(4bK^2 - 4b - 3\right) + 3\left(2s-1\right)\left(s-1\right) = 0.$$

Therefore, the conjectured optimal solution is

$$s^* = \frac{3}{4} - \frac{1}{4}\sqrt{4 + \frac{1}{3}\left((8bK)^2 - (8bK^2 - 3)^2\right)}$$

Of course, this derivation is only heuristic. One can easily verify by direct calculation that in the interval $[s^*, 1]$ there can be at most K partition elements in any equilibrium. It is also interesting to notice that, for all b, $0 \leq s^* \leq \frac{1}{4}$ (and in a nontrivial way). That is, under the conjectured optimum, it never pays to induce revelation more than one-fourth of the time.

It remains to verify that no global deviation from s^* is payoff improving.

4.2.1 Proof of optimality of s^*

Claim: For all b, the payoff to the decision maker from choosing $s > s^*$ is worse than her payoff from choosing $s = s^*$.

At $s = \frac{1}{4}$, the most informative partition has K elements where

$$\frac{3}{8K(K+1)} \leq b < \frac{3}{8K(K-1)}$$

For any $s > s^*$

$$\begin{aligned}
\frac{\partial EV}{\partial s} &= \frac{1}{6}\frac{8b^2K^4 - 8b^2K^2 - 6bK^2 + 3 - 9s + 6s^2}{K^2} \\
&= \frac{1}{48}\frac{64b^2K^4 - 48bK^2 + 24(1-2s)(1-s) - 64b^2K^2}{K^2} \\
&< \frac{1}{48}\frac{64b^2K^4 - 48bK^2 - 64b^2K^2 + 24(1-2s^*)(1-s^*)}{K^2} \\
&= 0
\end{aligned}$$

This shows that all $s > s^*$ are suboptimal since for $s > s^*$ the most informative partition of $[s, 1]$ can have at most K elements. In particular, $\frac{dU}{ds} < 0$ at $s = \frac{1}{4}$.

Claim: For all b, the payoff to the decision maker from choosing $s < s^*$ is worse than her payoff from choosing $s = s^*$.

For $s < s^*$ and fixed K,

$$\frac{\partial EV}{\partial s} > 0$$

The only thing left to verify is that for $s < s^*$, the utility is lower than at s^* even if the number of elements in the most informative partition of $[s, 1]$ is greater than K.

Suppose that when $s = 0$, the maximal size of the partition of $[s, 1]$ is N (as in CS).

For $L = N-1, N-2, ...K+1, K$ define s_L to be the smallest s for which it is not possible to make a size $L+1$ partition. That is,

$$\begin{aligned} -\frac{1}{2} + \frac{1}{2}\sqrt{1 + \frac{2(1-s_L)}{b}} &= L \\ s_L &= 1 - 2bL(L+1). \end{aligned}$$

The decision maker's expected payoff function is not differentiable at the points s_L since there is a "regime" change from $L+1$ to L element partitions. We can however, find the right and left derivatives of EV at s_L and s_{L-1}, respectively.

The right derivative of EV at $s = s_L = 1 - 2bL(L+1)$ is,

$$\begin{aligned} \left.\frac{\partial EV}{\partial s}\right|^{+}_{s=s_L} &= \frac{1}{48}\frac{64b^2L^4 - 48bL^2 + 24(1-2s)(1-s) - 64b^2L^2}{L^2} \\ &= \frac{1}{3}8b(2L+1)(L+1)\left(b - \frac{3}{8L(L+1)}\right) \end{aligned}$$

But since for all $s \in [s_L, s_{L-1})$, there does *not* exist a partition of $[s,1]$ with $L+1$ elements and $s < \frac{1}{4}$

$$b \geq \frac{(1-s)}{2L(L+1)} > \frac{3}{8L(L+1)},$$

and so

$$\left.\frac{\partial EV}{\partial s}\right|^{+}_{s=s_L} > 0.$$

Similarly, the left derivative of U at $s = s_{L-1} = 1 - 2bL(L-1)$

$$\begin{aligned} \left.\frac{\partial EV}{\partial s}\right|^{-}_{s=s_{L-1}} &= \frac{1}{48}\frac{64b^2L^4 - 48bL^2 + 24(1-2s)(1-s) - 64b^2L^2}{L^2} \\ &= \frac{1}{3}8b(2L-1)(L-1)\left(b - \frac{3}{8L(L-1)}\right). \end{aligned}$$

But since at s_{L-1}, there does not exist a partition of $[s_{L-1}, 1]$ with L elements and $s_{L-1} < \frac{1}{4}$

$$b \geq \frac{(1-s_L)}{2L(L-1)} > \frac{3}{8L(L-1)},$$

and so we also have

$$\left.\frac{\partial EV}{\partial s}\right|^{-}_{s=s_{L-1}} > 0.$$

Finally, note that when $L = K$, we have

$$\left.\frac{\partial EV}{\partial s}\right|^{+}_{s=s_K} > 0,$$

but

$$\left.\frac{\partial EV}{\partial s}\right|_{s=s_{K-1}}^{-} < 0.$$

This completes the proof.

We now state the main result of the paper:

Proposition 8 *In the uniform-quadratic case, the optimal contract is as follows:*

Transfers to the expert:

$$t(m) = \begin{cases} 2b(s^* - m) + t(s^*) & m \in [0, s^*] \\ 0 & otherwise \end{cases},$$

where

$$s^* = \frac{3}{4} - \frac{1}{4}\sqrt{4 + \frac{1}{3}\left((8bK)^2 - (8bK^2 - 3)^2\right)},$$

K is such that

$$\frac{3}{8K(K+1)} \leq b < \frac{3}{8K(K-1)},$$

and

$$t(s^*) = \frac{1}{4}\frac{4b^2K^2 - 1 + 2s^* + 4bK^2 - (s^*)^2 - 4bs^*K^2 - 4b^2K^4}{K^2}$$

The expert reports truthfully on $[0, s^*]$ and according to a size K CS partition on $[s^*, 1]$.

The decision maker chooses her ideal action for reports from $[0, s^*]$ sent by the expert and selects action $y([a_{i-1}, a_i])$ for reports corresponding to partition element $[a_{i-1}, a_i]$.

Discussion The structure of the optimal contract has the somewhat unusual property that the interval in which the decision maker finds it optimal to compensate the expert in exchange truth-telling is non-monotonic in the bias of that expert. In figure 1, shown below, we see that for extremely biased experts ($b > .25$) the revealing interval over which the expert is compensated increases from 0 up to $\frac{1}{4}$.

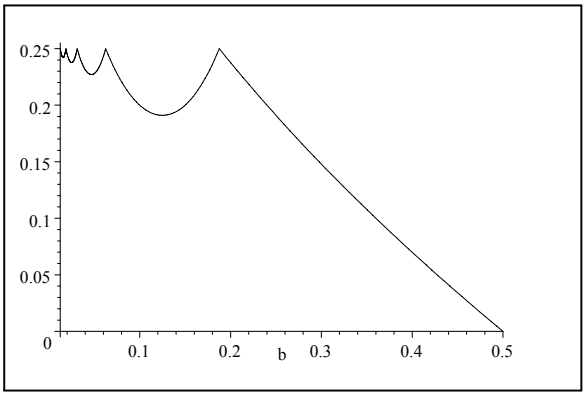

Figure 1: Optimal s^*

Beyond that, the interval starts to decrease again. Why is this? The key trade-off is that, by reducing the size of the revelatory interval, the decision maker can induce more information transmission for higher states. Obtaining this information for higher states is cost-effective for the expert in two respects. First, the more precise the information immediately to the right of s^*, the less expensive is the compensation scheme to induce revelation since this creates a parallel shift downward in the transfer schedule. At the same time, there is a direct benefit of obtaining more information in higher states at no cost whatsoever. As this trade-off becomes more or less favorable with changes in the bias, the optimal contract adjusts the length of the revealing interval. Interestingly, the net benefits from increased information to the right always outweigh the upside from increasing the revealing interval for states above $\theta = \frac{1}{4}$. That is, it is *never* in the decision maker's interest to compensate the expert more than one-fourth of the time.

5 What Type of Commitment?

In this section, we compare the expected payoff of the decision maker when she can commit to transfer schemes but not to actions as compared to where commitment to actions is possible, but transfers are not feasible. In an important paper, Dessein (2002) has shown that the optimal scheme when the decision maker can commit to actions is simply to delegate the choice of action to the expert but with the restriction that the space of feasible actions is $[0,1]$. In that case, the expected payoff to the decision maker is

$$\begin{aligned} EV' &= \int_0^{1-b} -b^2 d\theta + \int_{1-b}^{1} -(1-\theta)^2 d\theta \\ &= b^2\left(\frac{2}{3}b - 1\right) \end{aligned}$$

However, for b sufficiently large, it is optimal not to delegate at all. In these cases, the expert will convey no information whatsoever and the decision maker will choose the optimal action given her prior beliefs. The expected payoff to the decision maker in these cases is simply $-\frac{1}{12}$. Hence,

$$EV' = \max\left(b^2\left(\frac{2}{3}b - 1\right), -\frac{1}{12}\right)$$

In the case where transfers are feasible, but commitment to actions is impossible, the decision maker's expected payoffs under the optimal scheme are

$$\begin{aligned} EV &= \int_0^{s^*} -t(\theta)\, d\theta + \sum_{k=1}^{K} \int_{x_{k-1}}^{x_k} -\left(y([x_{k-1}, x_k]) - \theta\right)^2 d\theta \\ &= \frac{1}{12}\left(16 s^* K^2 b^2 - 4b^2K^2 + \frac{6s^* - 9(s^*)^2 + 4(s^*)^3 - 1}{K^2} - 16b^2 s^* + 4b^2 - 12bs^*\right), \end{aligned}$$

where s^* and K are defined in Proposition 8.

In figure 2 below, the two expected payoffs are plotted with the thicker line representing her expected payoffs when she can commit to actions and the thinner line indicating the decision maker's expected payoffs under the optimal contract identified in Proposition 8.

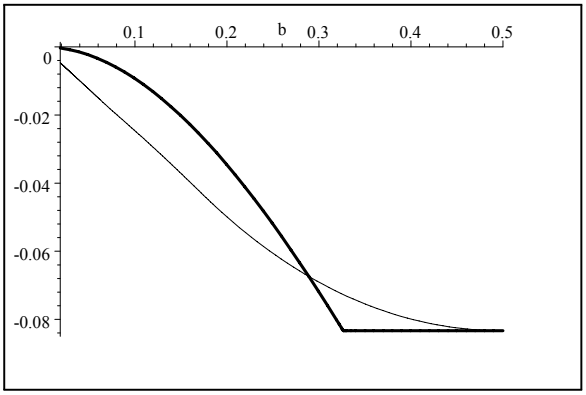

Figure 2: Actions (thick) v. Transfers (thin)

As the figure shows, when the bias of the expert is extreme, $b > .29$,then the ability to commit to transfers is more valuable than the ability to commit to actions; however, when the expert's incentives are less misaligned, the ability to commit to actions is more valuable.

6 Conclusions

In this paper, we study how decision maker's should optimally contract with experts for information. In our setting, decision makers lack the ability to commit to actions to be undertaken on the basis of the advice offered by the expert, but do have the ability to offer schedules of transfers to influence the incentives of the expert to report accurately. In such an environment, we show that it is always possible for the decision maker to design an incentive scheme leading the expert to fully reveal his information; however, such an incentive scheme is generally too expensive to be cost-effective for the decision maker to employ. Indeed, in the leading example of Crawford and Sobel, the uniform-quadratic case, fully revealing contracts are always worse for the decision maker than offering no contract at all.

We then turn to the question of the structure of optimal contracts. Here, we identify several key properties, the most important of which is that the only time it pays for the decision maker to contract for information is when that information completely discloses the knowledge of the expert. That is, it is never in the decision maker's interest to pay for imprecise information. The optimal contract we characterize has the feature that the decision maker creates incentives for the expert to fully reveal less than one-fourth of the time. Otherwise, the decision maker relies on the

degree of alignment between his incentives and the expert's incentives to acquire some imprecise information, but at no direct cost. Interestingly, the amount of revelation that the decision maker contracts for is not directly proportional to the bias of the expert. The key trade-off here is that by *not* incentivizing the expert to fully reveal, the decision maker is able to gain a direct benefit by receiving more precise information from the expert in circumstances where no payments are made as well as an indirect benefit—more precise information from experts decreases the cost of creating incentives to fully reveal. Finally, we show that this trade-off is non-monotonic in the bias of the expert.

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