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Quantum Information Processing with Trapped Electrons in Paul Traps

by

Qian Yu

A dissertation submitted in partial satisfaction of the

requirements for the degree of

Doctor of Philosophy

in

Applied Science and Technology

in the

Graduate Division

of the

University of California, Berkeley

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Professor Hartmut Häffner, Chair

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Fall 2025

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by

Qian Yu

Abstract

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University of California, Berkeley

Professor Hartmut Häffner, Chair

This work presents first experimental and theoretical steps towards establishing a novel architecture for quantum information processing (QIP) using trapped electrons in Paul traps. Electrons are attractive for quantum computing because they are extremely light and are natural two-level spin systems (qubit) with a large enough magnetic moment to be manipulated with well-established microwave technology and thermal reservoirs.

To that end, we investigate the feasibility for all-electric quantum computing with trapped electron spin qubits. We identify the necessary experimental steps to realize a trapped electron quantum processor through a concrete design proposal, including trapping, cooling, electronic detection, spin readout, and single- and multi-qubit gate operations. We numerically study the electron dynamics in linear Paul traps to find the threshold temperature to form two-electron Wigner crystals. We also derive the gate infidelities due to motional errors for trapped electrons and perform numerical simulations that indicate that two-qubit Bell-state fidelities of the order of 99.99% can be achieved.

We design a millimeter-sized PCB Paul trap driven at 1.6 GHz and trap electrons in a room-temperature ultrahigh vacuum setup. Cold electrons are introduced into the trap by photo-ionization of calcium atoms and stay confined for several tens of milliseconds with secular frequencies ranging from several tens to hundreds MHz. A fraction of these electrons remain trapped longer and show no measurable loss for measurement times up to a second. We estimate the temperature of trapped electrons through the width of the motional resonance.

To demonstrate cooling and electronic detection of the electron motion, we trap electrons in a 4 K cryostat as the first step and design a cooling and detection circuit. We study the effects of trap anharmonicity and trap voltage fluctuation on the detection signal to identify feasible operating parameters. In addition, we propose to cool the electron motion by bringing its micromotion sideband into resonance with a tank circuit and show that this should yield temperatures lower than what can be achieved with conventional resistive cooling.

To my family, and to Nemo.

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Chapter 1

Introduction

Ongoing efforts to build a quantum computer are based on various physical implementations. The spin state of an electron forms the archetypal two-level system and makes the electron a natural candidate for realizing a quantum bit. Electrons are used for quantum information processing in both solid-state and atomic systems, for instance confined in quantum dots or bound to donors in semiconductors [1, 2], bound as valence electrons in trapped atomic ions [3, 4], or manipulated via excitation of the outer-shell electron to Rydberg states in neutral atoms [5, 6]. In these cases, confinement to either the host solid-state environment or to a much heavier nucleus can limit the potential of the electron spin qubit. In condensed matter systems unwanted coupling of the charge and magnetic moment of the electron to the imperfect environment limits coherence times; for trapped ions, entanglement is typically mediated by the slow motion of the heavy ions in a shared trapping potential [7, 8], which limits the gate speed; and in neutral-atom platforms, the large atomic mass constrains the speed of atom transport in optical tweezers and lattices [9].

An approach that promises to remove these limitations is to confine individual free electrons in actual vacuum or a vacuum-like environment [10–12]. Trapping single electrons in vacuum has previously been achieved in Penning traps [13–16]. Electrons were confined in cryogenic Penning traps in the early 1970s [17] by combining a large magnetic field and a constant electric quadrupole field. There are also efforts underway to use electrons trapped on the surface of superfluid helium or solid neon for quantum information applications in milli-Kelvin environments [18–23], where single electrons have been detected and manipulated [21, 23].

The work done here studies electrons trapped within the type of traps used for the currently most advanced ion trap quantum computers, namely quadrupole Paul traps, where the confinement is provided by a rapidly oscillating quadrupole electric field in combination with a DC electric field. Ions trapped in Paul traps are among the most established implementations of quantum computing, where qubits are encoded in the internal states of the valence electrons and entangled using spin-dependent forces that couple the internal states of the ions to their collective motion [24]. Compared to commonly trapped ions, the charge-to-mass ratio of the electron is larger by a factor $10^4 - 10^5$, such that motion-based multi-qubit

gates and shuttling operations could be sped up. Further, the spin degree-of-freedom of an electron can be cooled, controlled, and read out using microwave-based electronics only. This removes most of the optical engineering challenges of trapped-ion control. At the same time, the trapped electron approach promises to retain the key advantages of trapped ions, i.e. long coherence times, as well as the ability to flexibly connect various regions of the quantum computer by shuttling the qubits through a microstructured trap array.

The remainder of this thesis is organized as follows. In Chapter 2 we introduce the important physical concepts and basic principles underlying trapped electrons in Paul traps. This is followed in Chapter 3 with a basic overview of using trapped electron spin states for quantum information processing. We discuss cooling, detection, spin readout, and quantum computation with electrons. In Chapter 4, we discuss the sources of decoherence for trapped electron qubits and carry out simulations of quantum gates to determine the dominant error sources. We then demonstrate electron trapping at room-temperature in Chapter 5, followed by a discussion of the loss mechanism. Later in Chapter 6, we describe efforts towards cooling and electronic detection of electron motion and propose micromotion sideband cooling. Chapter 7 summarizes our results and discusses prospects.

Chapter 2

Trapped-electron fundamentals

Charged particles have naturally strong interactions with electromagnetic fields and can therefore be confined through their charge.

A three-dimensional harmonic potential capable of confining a charged particle can be written as

$$\Phi = \frac{U}{2} (Ax^2 + By^2 + Cz^2), \quad (2.1)$$

where x, y, z denote three orthogonal spatial axes and A, B, C are the corresponding quadratic coefficients. For stable confinement, the curvature must have the same sign in all directions, requiring

$$\text{sgn}(A) = \text{sgn}(B) = \text{sgn}(C) \neq 0. \quad (2.2)$$

However, any static electric potential in free space must satisfy Laplace's equation,

$$\nabla^2 \Phi_{\text{static}} = 0, \quad (2.3)$$

which for the quadratic form above implies

$$A + B + C = 0. \quad (2.4)$$

This condition contradicts the requirement that all three curvature coefficients share the same sign, making it impossible to confine charged particles using only static electric fields.

Consequently, two alternative trapping approaches were developed and remain widely used today: the Penning trap [25], which employs a static electric field and a static magnetic field, and the Paul trap (radio-frequency, RF trap) [26, 27], which uses oscillating electric fields. This work focuses on the Paul trap.

In this chapter, we describe the physics of trapped electrons in Paul traps, which will be useful for later discussions throughout the thesis.

2.1 Paul traps

2.1.1 Operating principles

In 1953, Wolfgang Paul introduced the quadrupole Paul traps [27]. The idea is to generate 3-D confinement with rapidly oscillating electric fields. If the potential oscillates fast enough so that the corresponding time scale is faster than the time scale of the motion of the charged particle, the average effect will be a confining force in all dimensions.

The electric potential of Paul traps is approximately quadrupolar in the center of the trapping region, which can be decomposed into a time-dependent part that varies sinusoidally at a frequency of Ω_{rf} and a static DC part:

$$\begin{aligned}\Phi(x, y, z, t) &= \Phi_{\text{DC}} + \Phi_{\text{rf}} \\ &= \frac{U_0}{2}(\alpha x^2 + \beta y^2 + \gamma z^2) \\ &\quad + \frac{V_0}{2} \cos(\Omega_{\text{rf}} t)(\alpha' x^2 + \beta' y^2 + \gamma' z^2),\end{aligned}\tag{2.5}$$

where the coefficients satisfy $\alpha + \beta + \gamma = 0$, $\alpha' + \beta' + \gamma' = 0$ in order to fulfill the Laplace's equation $\nabla^2 \Phi = 0$ at every instant in time.

Linear Paul trap fulfills $-(\alpha + \beta) = \gamma$, and $\alpha' = -\beta'$ leading to a dynamical confinement in the $x - y$ plane (radial directions):

$$U_{\text{rf}}(t) = qV_0 \cos(\Omega_{\text{rf}} t) \left(\frac{x^2 - y^2}{2r_0^2} \right),\tag{2.6}$$

and static potential confinement in the z direction (axial direction):

$$U_{\text{dc}} = q\kappa U_0 \left(\frac{2z^2 - x^2 - y^2}{2r_0^2} \right).\tag{2.7}$$

Here κ , r_0 are the geometry-dependent trap parameters, q is the charge of the trapped particle, V_0 is the voltage amplitude of the RF potential, U_0 is the voltage amplitude of the DC potential.

2.1.2 Expansion of the potential in spherical harmonics

To obtain a general solution to Laplace's equation, we will expand the potential in terms of spherical harmonics. The general solution to the Laplace's equation in three dimensions can be expanded in terms of spherical harmonics,

$$\Phi(r, \theta, \varphi) = \sum_{l=0}^{\infty} \sum_{m=-l}^l (A_{lm} r^l + B_{lm} r^{-l-1}) Y_{lm}(\theta, \varphi).\tag{2.8}$$

Here $Y_{lm}(\theta, \varphi)$ are the spherical harmonics, A_{lm} and B_{lm} are the expansion coefficients determined by the boundary conditions. If we require the potential to be finite at the location of the electron, i.e. at $r = 0$, then B_{lm} must vanish to avoid divergence of the potential. The resulting solution reduces to

$$\Phi(r, \theta, \varphi) = \sum_{l=0}^{\infty} \sum_{m=-l}^l A_{lm} r^l Y_{lm}(\theta, \varphi). \quad (2.9)$$

We then express the spherical harmonics in cartesian coordinates through the standard coordinate transformation

$$\begin{aligned} x &= r \sin(\theta) \cos(\varphi), \\ y &= r \sin(\theta) \sin(\varphi), \\ z &= r \cos \theta. \end{aligned} \quad (2.10)$$

Further, we can perform a coordinate transformation to re-express the potential in Cartesian coordinate and use a one-index real basis M_j and Y_j to rewrite the coefficients A_{lm} and basis Y_{lm} in Eq. (2.9) following the convention of the multipole expansion [28]. The convention is shown in Tab. 2.1, up to $j = 9$.

j	l, m
1	0, 0
2	1, -1
3	1, 0
4	1, 1
5	2, -2
6	2, -1
7	2, 0
8	2, 1
9	2, 2

Table 2.1: Mapping between the two-index notation l, m and the single-index notation j .

With these substitution, the potential can be expressed as

$$\Phi(x, y, z) = \sum_{j=1}^{\infty} M_j Y_j(x, y, z). \quad (2.11)$$

Following this notation, we can expand the DC potential up to $j = 9$ (which corresponds to

$l = 2$ in the two-index basis) as

$$\begin{aligned}\Phi_{\text{DC}}(x, y, z) = & M_1^{\text{DC}} + M_2^{\text{DC}} \left(\frac{-y}{r_0} \right) + M_3^{\text{DC}} \left(\frac{z}{r_0} \right) + M_4^{\text{DC}} \left(\frac{-x}{r_0} \right) \\ & + M_5^{\text{DC}} \left(\frac{6xy}{r_0^2} \right) + M_6^{\text{DC}} \left(\frac{-3yz}{r_0^2} \right) + M_7^{\text{DC}} \left(\frac{2z^2 - x^2 - y^2}{2r_0^2} \right) \\ & + M_8^{\text{DC}} \left(\frac{-3xz}{r_0^2} \right) + M_9^{\text{DC}} \left(\frac{3x^2 - 3y^2}{r_0^2} \right) + O(r^3),\end{aligned}\quad (2.12)$$

and the RF potential will be

$$\begin{aligned}\Phi_{\text{rf}}(x, y, z, t) = & [V_{\text{bias}} + \cos(\Omega_{\text{rf}}t)] \left[M_1^{\text{rf}} + M_2^{\text{rf}} \left(\frac{-y}{r_0} \right) + M_3^{\text{rf}} \left(\frac{z}{r_0} \right) + M_4^{\text{rf}} \left(\frac{-x}{r_0} \right) \right. \\ & + M_5^{\text{rf}} \left(\frac{6xy}{r_0^2} \right) + M_6^{\text{rf}} \left(\frac{-3yz}{r_0^2} \right) + M_7^{\text{rf}} \left(\frac{2z^2 - x^2 - y^2}{2r_0^2} \right) \\ & \left. + M_8^{\text{rf}} \left(\frac{-3xz}{r_0^2} \right) + M_9^{\text{rf}} \left(\frac{3x^2 - 3y^2}{r_0^2} \right) + O(r^3) \right],\end{aligned}\quad (2.13)$$

where V_{bias} is the bias RF voltage, and a constant length r_0 is used to normalize the Y_j -basis to be dimensionless.

The higher-order expansion terms can be found in Tab. A.1 in Appendix. A. Note that the normalization constant r_0 is neglected in Tab. A.1, and the x, y, z variables shown are assumed to be already normalized to be dimensionless.

2.2 Electron motion

2.2.1 Classical equation of motion

Neglecting higher-order terms for now, the classical equations of motion for a single electron with charge $-e$ and mass m in a potential of the form given by Eq. (2.5) are decoupled in the spatial coordinates. We focus on the motion in the x direction here. The classical equation of motion can be written as:

$$\ddot{x} = \frac{e}{m} [U_0 \alpha + V_0 \cos(\Omega_{\text{rf}}t) \alpha'] x. \quad (2.14)$$

This can be transformed to the standard form of the Mathieu differential equation

$$\frac{d^2 x}{d\xi^2} + (a_x - 2q_x \cos(2\xi))x = 0, \quad (2.15)$$

upon making the substitutions

$$\begin{aligned}\xi &= \frac{\Omega_{\text{rf}} t}{2}, \\ a_x &= \frac{-4eU_0\alpha}{m\Omega_{\text{rf}}^2}, \\ q_x &= \frac{-2eV_0\alpha'}{m\Omega_{\text{rf}}^2}.\end{aligned}\tag{2.16}$$

Here, q_x and a_x are dimensionless parameters corresponding to the confinement strength provided by the RF and DC potential, respectively.

The general form of the stable solutions are [29]

$$\begin{aligned}x(\xi) &= Ae^{i\beta_x\xi} \sum_{n=-\infty}^{\infty} C_{2n}e^{i2n\xi} \\ &+ Be^{-i\beta_x\xi} \sum_{n=-\infty}^{\infty} C_{2n}e^{-i2n\xi},\end{aligned}\tag{2.17}$$

where A and B are arbitrary constants that can be used to satisfy boundary conditions or normalize a particular solution. The real-valued characteristic exponent β_x and the coefficients C_{2n} are functions of a_x and q_x only and do not depend on the initial conditions. By substituting Eq. (2.17) into Eq. (2.15) we obtain a recursion relation

$$\begin{aligned}C_{2n+2} - D_{2n}C_{2n} + C_{2n-2} &= 0, \\ D_{2n} &= [a_x - (2n + \beta_x)^2]/q_x,\end{aligned}\tag{2.18}$$

that connects the coefficients and β_x to a_x and q_x .

In the context of Paul traps, a_i and q_i ($i \in \{x, y, z\}$) are often known as the stability parameters of the Mathieu equation, which describe the stability of trajectories for a charged particle in a quadrupole trap, and have been studied theoretically and experimentally [29, 30]. The trajectory is stable when the characteristic exponent $0 < \beta_i < 1$, and the region of stability in the $a_i - q_i$ plane is bounded by pairs of (a_i, q_i) that lead to $\beta_x = 0$ or $\beta_x = 1$. The stable region that contains the points $(a_i, q_i) = (0, 0)$ is called the lowest stability region, which is the region we are interested in in this work.

When $(|a_x|, q_x^2) \ll 1$, the lowest-order approximation to the electron trajectory $x(t)$ can be found by assuming $C_{\pm 4} = 0$, and together with the initial condition $A = B$, we have [29]

$$x(t) \approx A_x \cos(\beta_x \frac{\Omega_{\text{rf}} t}{2}) [1 - \frac{q_x}{2} \cos(\Omega_{\text{rf}} t)],\tag{2.19}$$

where

$$\beta_x \approx \sqrt{a_x + q_x^2/2},\tag{2.20}$$

and A_x is the amplitude of the motion and depends on the initial conditions.

The trajectory consists of two parts: one harmonic oscillation at frequency

$$\omega_{\text{sec}} = \beta_x \Omega_{\text{rf}}/2 \ll \Omega_{\text{rf}} \quad (2.21)$$

namely, the secular motion, superposed with a faster oscillation at the RF drive frequency Ω_{rf} with a smaller amplitude reduced by a factor of $q_x/2$, the so called micromotion. Notice that this is identical to the solution found by the so called pseudopotential approximation that will be explained in the next section.

2.2.2 Pseudopotential approximation

To develop an intuitive understanding of the classical motion and the trapping potential, we introduce the pseudopotential approximation in this section. This approach provides an effective potential experienced by charged particles in a rapidly oscillating electric field. Consider a single electron with charge $-e$ in a potential composed of a static component and a time-dependent component that oscillates at frequency Ω_{rf} ,

$$\Phi(\mathbf{r}) = \Phi_{\text{DC}}(\mathbf{r}) + \Phi_{\text{rf}}(\mathbf{r}) \cos(\Omega_{\text{rf}} t). \quad (2.22)$$

We can write down the equation of motion using Newton's second law:

$$\begin{aligned} \mathbf{F} &= m\ddot{\mathbf{r}} \\ &= e\nabla\Phi_{\text{DC}}(\mathbf{r}) + e\nabla\Phi_{\text{rf}}(\mathbf{r}) \cos(\Omega_{\text{rf}} t) \\ &= \mathbf{F}_{\text{DC}}(\mathbf{r}) + \mathbf{F}_{\text{rf}}(\mathbf{r}) \cos(\Omega_{\text{rf}} t) \\ &= \mathbf{F}_{\text{DC}}(\mathbf{r}) - e\mathbf{E}_{\text{rf}}(\mathbf{r}) \cos(\Omega_{\text{rf}} t). \end{aligned} \quad (2.23)$$

Since the electric field is inhomogeneous, the time-averaged force $\langle \mathbf{F}_{\text{rf}}(\mathbf{r}(t)) \rangle$ depends on the spatial coordinates $\mathbf{r}$. When Ω_{rf} is sufficiently large, the motion caused by $\langle \mathbf{F}_{\text{rf}}(\mathbf{r}(t)) \rangle$ is temporally separated from Ω_{rf} , and Ω_{rf} only causes a small oscillation around the average position.

Based on the above rationale, we can write the total trajectory $\mathbf{r}(t)$ as a sum of a slow motion $\mathbf{r}_s(t)$ and a fast motion $\mathbf{r}_f(t)$:

$$\mathbf{r}(t) = \mathbf{r}_s(t) + \mathbf{r}_f(t). \quad (2.24)$$

We treat $\mathbf{r}_f(t)$ as a perturbation of $\mathbf{r}_s(t)$, i.e. $|\mathbf{r}_f(t)| \ll |\mathbf{r}_s(t)|$, and further expand the forces $\mathbf{F}_{\text{DC}}(\mathbf{r})$ and $\mathbf{F}_{\text{rf}}(\mathbf{r})$ in Taylor series:

$$\begin{aligned} \mathbf{F}_{\text{DC}}(\mathbf{r}_s + \mathbf{r}_f) &= \mathbf{F}_{\text{DC}}(\mathbf{r}_s) + (\mathbf{r}_f \cdot \nabla) \mathbf{F}_{\text{DC}}(\mathbf{r}_s) + \cdots \\ \mathbf{F}_{\text{rf}}(\mathbf{r}_s + \mathbf{r}_f) &= \mathbf{F}_{\text{rf}}(\mathbf{r}_s) + (\mathbf{r}_f \cdot \nabla) \mathbf{F}_{\text{rf}}(\mathbf{r}_s) + \cdots \end{aligned} \quad (2.25)$$

Plugging it into Eq. (2.23), we have:

$$m(\ddot{\mathbf{r}}_s + \ddot{\mathbf{r}}_f) = \mathbf{F}_{\text{DC}}(\mathbf{r}_s) + \mathbf{r}_f \cdot \nabla \mathbf{F}_{\text{DC}}(\mathbf{r}_s) + [\mathbf{F}_{\text{rf}}(\mathbf{r}_s) + (\mathbf{r}_f \cdot \nabla) \mathbf{F}_{\text{rf}}(\mathbf{r}_s)] \cos(\Omega_{\text{rf}} t). \quad (2.26)$$

Since the amplitudes of the oscillations $\mathbf{r}_f$ are much smaller than $\mathbf{r}_s$, we can neglect the terms containing $\mathbf{r}_f$ for the leading-order approximation. By averaging over a period of Ω_{rf} , the right-hand side of Eq. (2.26) is $\mathbf{F}_{DC}(\mathbf{r}_s)$. So, to the leading order

$$\ddot{\mathbf{r}}_s = \mathbf{F}_{DC}(\mathbf{r}_s), \quad (2.27)$$

and we have:

$$m\ddot{\mathbf{r}}_f = \mathbf{F}_{rf}(\mathbf{r}_s) \cos(\Omega_{rf}t). \quad (2.28)$$

Without loss of generality, we assume that the initial condition is $\mathbf{r}_f = 0$. Then we have the solution for $\mathbf{r}_f$ to the leading order by integrating twice:

$$\mathbf{r}_f = -\frac{\mathbf{F}_{rf}(\mathbf{r}_s)}{m\Omega_{rf}^2} \cos(\Omega_{rf}t). \quad (2.29)$$

To the next order, we look at the remaining terms in Eq. (2.26) after plugging Eq. (2.28) and Eq. (2.29) into it. Then, by averaging over a period of $2\pi/\Omega_{rf}$, we are left with an equation of motion only for $\mathbf{r}_s$:

$$\begin{aligned} m\ddot{\mathbf{r}}_s &= \mathbf{F}_{DC}(\mathbf{r}_s) - \frac{(\mathbf{F}_{rf}(\mathbf{r}_s) \cdot \nabla) \mathbf{F}_{rf}(\mathbf{r}_s)}{m\Omega_{rf}^2} \langle \cos^2(\Omega_{rf}t) \rangle \\ &= \mathbf{F}_{DC}(\mathbf{r}_s) - \frac{1}{2m\Omega_{rf}^2} (\mathbf{F}_{rf}(\mathbf{r}_s) \cdot \nabla) \mathbf{F}_{rf}(\mathbf{r}_s). \end{aligned} \quad (2.30)$$

Using the dot product rule of the ∇ operation:

$$\nabla(\mathbf{u} \cdot \mathbf{v}) = (\mathbf{u} \cdot \nabla)\mathbf{v} + (\mathbf{v} \cdot \nabla)\mathbf{u} + \mathbf{u} \times (\nabla \times \mathbf{v}) + \mathbf{v} \times (\nabla \times \mathbf{u}), \quad (2.31)$$

and notice that $\mathbf{F}_{rf}$ is curl-less since it is a conservative force, we have

$$\frac{1}{2}\nabla(\mathbf{F}_{rf} \cdot \mathbf{F}_{rf}) = (\mathbf{F}_{rf} \cdot \nabla)\mathbf{F}_{rf} + \mathbf{F}_{rf} \times (\nabla \times \mathbf{F}_{rf}) = (\mathbf{F}_{rf} \cdot \nabla)\mathbf{F}_{rf}. \quad (2.32)$$

Therefore, the equation of motion for $\mathbf{r}_s$ can be written as

$$m\ddot{\mathbf{r}}_s = \mathbf{F}_{DC}(\mathbf{r}_s) - \frac{1}{4m\Omega_{rf}^2} \nabla \|\mathbf{F}_{rf}(\mathbf{r}_s)\|^2. \quad (2.33)$$

We call this force the secular force

$$\mathbf{F}_{sec} = \mathbf{F}_{DC}(\mathbf{r}_s) - \frac{1}{4m\Omega_{rf}^2} \nabla \|\mathbf{F}_{rf}(\mathbf{r}_s)\|^2. \quad (2.34)$$

Now we can mimic the definition of the electric field and forces $\mathbf{E} = -\nabla\Phi$, $\mathbf{F} = Ze\mathbf{E}$ with $Z = -1$ for a single electron, and define the secular potential:

$$\begin{aligned} \Phi_{sec} &= \Phi_{DC} + \Phi_{ps} \\ &= \Phi_{DC} - \frac{e}{4m\Omega_{rf}^2} \|\mathbf{E}_{rf}\|^2. \end{aligned} \quad (2.35)$$

Here $\mathbf{E}_{\text{rf}}$ is the peak static electric field associated with the fast oscillating electric potential Φ_{rf} , as defined in Eq. (2.23). The second term

$$\Phi_{\text{ps}} = -\frac{e}{4m\Omega_{\text{rf}}^2} \|\mathbf{E}_{\text{rf}}\|^2 \quad (2.36)$$

is called the pseudopotential, also known as the ponderomotive potential. Consider an oscillating electric potential for a linear Paul trap as in Eq. (2.6),

$$\Phi_{\text{rf}}(t) = V_0 \cos(\Omega_{\text{rf}} t) \left(\frac{x^2 - y^2}{2r_0^2} \right). \quad (2.37)$$

The equivalent pseudopotential is

$$\Phi_{\text{ps}} = -\frac{eV_0^2}{4mr_0^4\Omega_{\text{rf}}^2} (x^2 + y^2). \quad (2.38)$$

When the electric potential is purely quadratic, the effective pseudopotential reduces to that of a harmonic oscillator. Neglecting the anti-confinement from the static electric component, we obtain the secular trap frequencies corresponding to Eq. (2.38):

$$\omega_x = \omega_y = \frac{eV_0}{\sqrt{2}mr_0^2\Omega_{\text{rf}}}. \quad (2.39)$$

From Eq. (2.16) and Eq. (2.36), the stability parameters a_x , q_x and the depth of a quadrupole trap scale as $(m\Omega_{\text{rf}}^2)^{-1}$. If we wish to maintain the same stability parameters and trap depth for electrons as for ions, the much smaller electron mass— 10^4 to 10^5 times lighter than typical ions—requires the drive frequency Ω_{rf} to be roughly two orders of magnitude higher. Since typical ion traps are driven at tens of MHz, an electron Paul trap must therefore operate in the GHz regime.

At the same time, from Eq. (2.39) the secular frequency scales as $(m\Omega_{\text{rf}})^{-1}$. Under the constraint that $m\Omega_{\text{rf}}^2$ is fixed, this implies that the secular frequency scales as $1/\sqrt{m}$. Consequently, the trap frequency for electrons is expected to be approximately two orders of magnitude higher than that for trapped ions.

The spatial extent of the ground state of a particle with secular frequency ω_{sec} is $x_0 = \sqrt{\hbar/(2m\omega_{\text{sec}})}$. Since the secular frequency scales as $1/\sqrt{m}$, the ground-state extent also scales as $1/\sqrt{m}$. For ions, this extent is extremely small – for example, a Ca^+ ion at $\omega_a = 2\pi \times 1 \text{ MHz}$ has a ground-state extension of only $\sim 11 \text{ nm}$. The effects of trap anharmonicities are therefore often negligible in ion traps. In contrast, trapped electrons explore a much larger volume and consequently experience stronger anharmonicities farther away from the trap center. This effect becomes even more pronounced when the particle is not cooled to its motional ground state, as the spatial extent increases as $\sqrt{n}$ with the thermal occupation number n . For these reasons, considering trap anharmonicities is essential for trapped electrons. We examine this in the next section.

2.2.3 Amplitude-dependent frequency shift

Anharmonicity arises because the Paul trap does not produce a pure quadrupole potential because of the asymmetry or fabrication imperfections of the RF and DC electrodes. It leads to amplitude-dependent shifts in the secular frequency and, for a particle at finite temperature, to a corresponding broadening of the motional spectrum [31]. The amount of anharmonicity can be altered by changing the voltages applied to the trap electrodes.

In this section, we introduce the amplitude-dependent frequency shift of trapped electrons arising from an anharmonic potential [31, 32]. For simplicity, we consider only a static anharmonic potential along the z -direction—the axial direction of a linear Paul trap. A complete treatment of dynamics in a time-dependent anharmonic potential requires solving a nonlinear Mathieu equation [33–37], which lies beyond the scope of this thesis.

Expanding the one-dimensional static potential around its minimum yields

$$\frac{U(z) - U(0)}{m} = \frac{1}{2}\omega_0^2 z^2 + \sum_{k=3}^{\infty} B_k z^k, \quad (2.40)$$

where ω_0 is the small-amplitude trap frequency and B_k quantify the anharmonicity of the trap. Differentiating Eq. (2.40) gives the equation of motion

$$\ddot{z} + \omega_0^2 z = - \sum_{k=3}^{\infty} k B_k z^{k-1}. \quad (2.41)$$

In a perfectly harmonic potential, the frequency of the electron oscillation is ω_0 and is independent of its displacement from the center of the trap. However, in the presence of anharmonicity, the oscillation frequency will have a shift which depends on the motion amplitude A . To calculate this dependence systematically, we use the Lindstedt–Poincaré method. The basic idea of the Lindstedt–Poincaré method is to absorb the frequency shift into a rescaled time variable. We write the physical frequency as

$$\omega(A) = \omega_0 \left[1 + \sum_{n=2}^{\infty} \varepsilon_n A^n \right], \quad (2.42)$$

where A is the oscillation amplitude and ε_n are dimensionless coefficients to be determined. Introducing a dimensionless time variable

$$\tau = \omega(A) t, \quad (2.43)$$

we have $\frac{d}{dt} = \omega(A) \frac{d}{d\tau}$ and hence $\ddot{z} = \omega(A)^2 z''$, where the primes denote derivatives with respect to τ . Also, denote $B_2 = \omega_0^2/2$ and $C_k = B_k/B_2$, and Eq. (2.41) becomes

$$\omega(A)^2 z'' + \omega_0^2 z + \frac{\omega_0^2}{2} \sum_{k=3}^{\infty} k C_k z^{k-1} = 0. \quad (2.44)$$

We now expand the solution itself in powers of the small amplitude A ,

$$z(\tau) = Az_1(\tau) + A^2z_2(\tau) + A^3z_3(\tau) + \cdots, \quad (2.45)$$

insert both Eq. (2.42) and Eq. (2.45) in Eq. (2.44). Using

$$\omega(A)^2 = \omega_0^2 [1 + 2\varepsilon_2 A^2 + 2\varepsilon_3 A^3 + (2\varepsilon_4 + \varepsilon_2^2) A^4 + \mathcal{O}(A^5)], \quad (2.46)$$

we expand Eq. (2.44) in powers of A and collect terms order by order.

In order A we obtain

$$z_1'' + z_1 = 0, \quad (2.47)$$

whose bounded solution we choose as

$$z_1(\tau) = \cos \tau. \quad (2.48)$$

In order A^2 there is still no contribution from the frequency correction, and the only nonlinear term is proportional to $C_3 z^2$. Using $z_1(\tau) = \cos \tau$,

$$z_2'' + z_2 = -\frac{3}{2}C_3 z_1^2 = -\frac{3}{2}C_3 \cos^2 \tau = -\frac{3C_3}{4} [1 + \cos 2\tau]. \quad (2.49)$$

The right-hand side contains only a constant and a second harmonic, so there is no resonant term proportional to $\cos \tau$. A particular solution can be chosen as

$$z_2(\tau) = -\frac{3}{4}C_3 + \frac{1}{4}C_3 \cos 2\tau.$$

In order A^3 both the quartic nonlinearity and the first frequency correction ε_2 enter. After inserting $z(\tau) = Az_1 + A^2z_2 + A^3z_3$ into Eq. (2.44) and collecting all terms proportional to A^3 , one finds an inhomogeneous equation of the form

$$z_3'' + z_3 = -2\varepsilon_2 z_1'' - 2C_4 z_1^3 - 3C_3 z_1 z_2, \quad (2.50)$$

Substituting the expression for z_1 and z_2 and using the trigonometric identities

$$\begin{aligned} \cos^3 \theta &= \frac{1}{4}(3 \cos \theta + \cos 3\theta), \\ \cos \theta \cos 2\theta &= \frac{1}{2} \cos \theta + \frac{1}{2} \cos 3\theta. \end{aligned} \quad (2.51)$$

The driving term on the right-hand side can be written as

$$F_3(\tau) = \left(2\varepsilon_2 - \frac{3}{2}C_4 + \frac{15}{8}C_3^2\right) \cos \tau + (\text{higher harmonics}). \quad (2.52)$$

The term proportional to $\cos \tau$ is resonant with the homogeneous solution of $z_3'' + z_3 = 0$ and would produce secular growth ($\propto \tau \sin \tau$) unless its coefficient vanishes. The Lindstedt–Poincaré prescription is precisely to choose ε_2 such that the resonant part of $F_3(\tau)$ is canceled. Imposing

$$2\varepsilon_2 - \frac{3}{2}C_4 + \frac{15}{8}C_3^2 = 0$$

gives the lowest-order frequency coefficient

$$\varepsilon_2 = -\frac{15}{16}C_3^2 + \frac{3}{4}C_4. \quad (2.53)$$

The procedure above generalizes systematically to all higher orders. In order A^n the equation of motion takes the generic form

$$z_n'' + z_n = F_n(\tau), \quad (2.54)$$

where the driving term $F_n(\tau)$ is the sum of harmonics $\cos(m\tau)$ with integer m , whose coefficients are polynomials in the anharmonic coefficients C_k , the lower-order functions $z_j(\tau)$ with $j < n$, and the already-determined frequency coefficients $\varepsilon_2, \dots, \varepsilon_{n-1}$. The condition for bounded motion is that the projection of $F_n(\tau)$ onto the resonant harmonic $\cos \tau$ vanishes:

$$[F_n(\tau)]_{\cos \tau} = 0. \quad (2.55)$$

This condition yields a linear equation for ε_n , which can be solved recursively.

Following this, we get the amplitude-dependent frequency shift for the first few orders

$$\begin{aligned} \varepsilon_2 &= -\frac{15}{16}C_3^2 + \frac{3}{4}C_4, \\ \varepsilon_3 &= -\frac{15}{16}C_3^3 + \frac{3}{4}C_3C_4, \\ \varepsilon_4 &= -\frac{2565}{1024}C_3^4 + \frac{645}{128}C_3^2C_4 - \frac{21}{64}C_4^2 - \frac{105}{32}C_3C_5 + \frac{15}{16}C_6. \end{aligned} \quad (2.56)$$

2.2.4 Quantum-mechanical equation of motion

In this section, we introduce the quantum-mechanical description of the motion of trapped electrons in Paul traps, which will be essential for understanding the micromotion sideband cooling scheme discussed in Sec. 6.4. Following the setup in Sec. 2.2.1, we focus on the one-dimensional case where the trapping potential is explicitly time dependent. Assuming a quadratic time-dependent potential, the motion of the electron along the x -axis is governed by the Hamiltonian

$$H_{\text{motion}} = \frac{\hat{p}^2}{2m} + \frac{mW(t)}{2}x^2. \quad (2.57)$$

Here, the function $W(t)$ defines the time-dependent potential felt by the electron

$$W(t) = \frac{\Omega_{\text{rf}}^2}{4}(a_x + 2q_x \cos(\Omega_{\text{rf}}t)), \quad (2.58)$$

and a_x and q_x are the dimensionless Mathieu parameters defined in Eq. (2.16) related to the static and RF trapping potential, respectively.

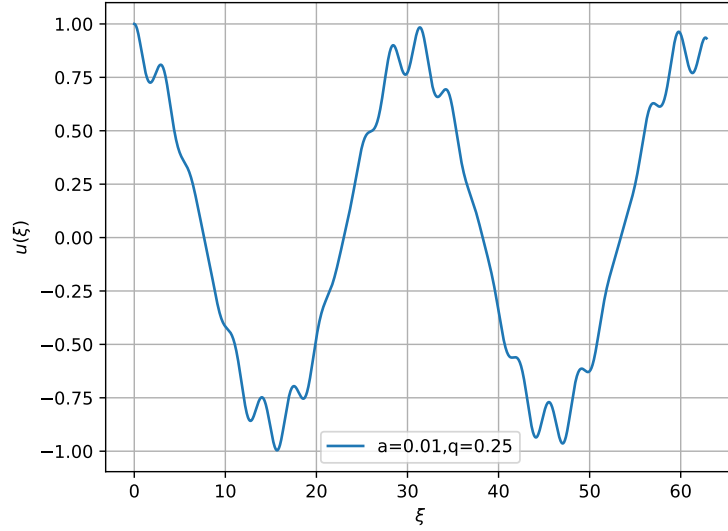


Figure 2.1: Solution of $u(\xi)$ with substitution $\xi = \Omega_{\text{rf}}t/2$. Initial condition is $u(0) = 1$, $\dot{u}(0) = i\nu$. Parameters of the Mathieu equation are set to be $a = 0.01, q = 0.25$, which corresponds to the characteristic component $\beta = 0.2$.

In the Heisenberg picture, the equations of motion are

$$\dot{\hat{x}}(t) = \hat{p}/m, \quad (2.59)$$

$$\dot{\hat{p}}(t) = -mW(t)\hat{x}, \quad (2.60)$$

or, in terms of $\hat{x}$ alone,

$$\ddot{\hat{x}}(t) + W(t)\hat{x} = 0. \quad (2.61)$$

This equation is equivalent to the classical Mathieu equation. To solve Eq. (2.61), we can use a special solution of the Mathieu equation subject to the boundary conditions $u(0) = 1$, $\dot{u}(0) = i\nu$ [29]

$$u(t) = e^{i\beta_x\Omega_{\text{rf}}t/2} \sum_{n=-\infty}^{\infty} C_{2n} e^{in\Omega_{\text{rf}}t}, \quad (2.62)$$

and

$$\nu = \Omega_{\text{rf}} \sum_{n=-\infty}^{\infty} C_{2n}(\beta_x/2 + n). \quad (2.63)$$

Here, the real-valued characteristic exponent β_x , the coefficients C_{2n} and ν are functions of a_x and q_x only. Figure 2.1 illustrates the numerical solution of $u(t)$ at $a = 0.01, q = 0.25$, which corresponds to a characteristic component of $\beta = 0.2$.

The solutions to the equations of motion can be expressed in terms of $u(t)$ and the operators of a reference oscillator

$$\begin{aligned}\hat{x}(t) &= x_0(\hat{a} u^*(t) + \hat{a}^\dagger u(t)), \\ \hat{p}(t) &= \frac{p_0}{\nu}(\hat{a} \dot{u}^*(t) + \hat{a}^\dagger \dot{u}(t)),\end{aligned}\tag{2.64}$$

where $x_0 = \sqrt{\hbar/2m\nu}$, $p_0 = \sqrt{\hbar m\nu/2}$. The operator $\hat{a}$ is the annihilation operator of a static potential harmonic oscillator of mass m and frequency ν and fulfills $[\hat{a}, \hat{a}^\dagger] = 1$, which is called the *reference oscillator*. [29]

We also note that, when $|a_x|, q_x^2 \ll 1$, to the lowest-order approximation by assuming $C_{\pm 4} = 0$,

$$u(t) \approx e^{i\nu t} \frac{1 + \frac{q_x}{2} \cos(\Omega_{\text{rf}} t)}{1 + \frac{q_x}{2}},\tag{2.65}$$

with $\nu \approx \beta_x \Omega_{\text{rf}}/2 = \omega_x$, where ω_x is the radial secular frequency, and $\beta_x \approx \sqrt{a_x + q_x^2/2}$.

2.3 Equivalent electrical representation

In this section, we describe the induced image currents and the equivalent electrical representation of trapped electrons which will be useful for understanding the dynamical behavior including cooling and image current detection.

2.3.1 Induced image currents

Image currents arise because a charge moving in the vicinity of conductors continuously rearranges the surface charge on the electrodes in such a way that the boundary conditions of the electrostatic problem are satisfied. The time dependence of this induced surface charge is what we measure as a current in the external detection circuit [38]. In this section, we derive the standard expression for the induced image charge and the corresponding image current in an arbitrary electrode geometry [39]. The result is later used to motivate the equivalent circuit model for trapped electrons in Paul traps.

Consider a point charge with charge Q located at position $\mathbf{r}_p$. According to Maxwell's equation, the electric potential $\Phi(\mathbf{r})$ obeys Poisson's equation

$$\nabla^2 \Phi(\mathbf{r}) = -\frac{\rho(\mathbf{r})}{\epsilon_0},\tag{2.66}$$

with charge density

$$\rho(\mathbf{r}) = Q \delta(\mathbf{r} - \mathbf{r}_p).\tag{2.67}$$

Here, ϵ_0 denotes the vacuum permittivity.

When the charged particle is surrounded by different segments of conducting electrodes, Poisson's equation with the specified boundary conditions has a unique solution according

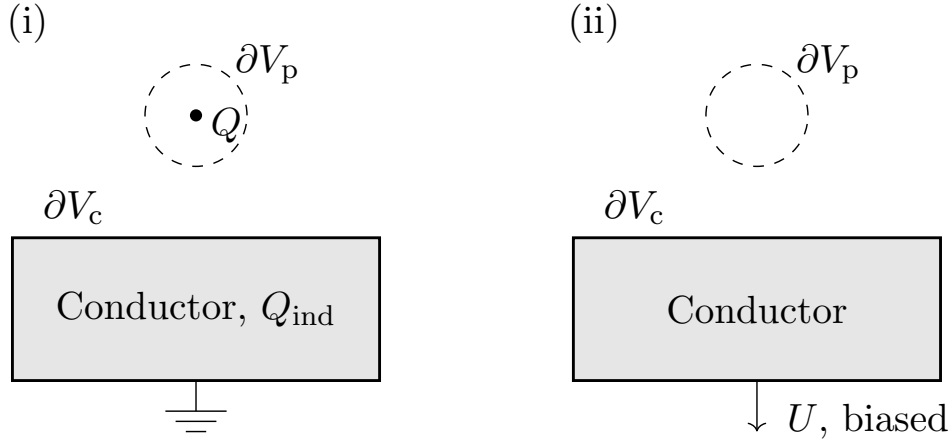


Figure 2.2: (i) A charged particle with charge Q at position $\mathbf{r}_p$ induces an image charge Q_{ind} on a grounded conductor. (ii) The charged particle is absent and the conductor is biased at potential U .

to the uniqueness theorem. Moreover, the linearity of the equation allows us to construct this solution by superposition. In practice, the total electrostatic potential is obtained by considering the contribution from the charged particle together with the responses of each conductor and then summing these components. Therefore, it is sufficient to derive the image charge induced on a particular conductor, where we denote the induced charge by Q_{ind} .

To compute Q_{ind} , we consider two scenarios that share the same spatial region and the same electrode geometry, as shown in Fig. 2.2:

- (i): All electrodes are grounded ($\Phi_1 = 0$ on all conducting surfaces), and the point charge Q is present at $\mathbf{r}_p$. The resulting potential is $\Phi_1(\mathbf{r})$. The induced charge on the conductor in this configuration is the desired Q_{ind} .
- (ii): The point charge is removed and only the conductor is held at a potential U (while all other conductors remain grounded). The resulting potential is denoted $\Phi_2(\mathbf{r})$.

We define an infinitesimal spherical surface ∂V_p centered on the charge position $\mathbf{r}_p$ and a surface ∂V_c that coincides with the outer surface of the conductor. The volume between the sphere and all conductors is denoted by V_s . Everywhere inside V_s the space is charge-free, so both potentials obey Laplace's equation,

$$\nabla^2 \Phi_1(\mathbf{r}) = 0, \quad \nabla^2 \Phi_2(\mathbf{r}) = 0, \quad \mathbf{r} \in V_s. \quad (2.68)$$

The presence of charge is encoded via Gauss's law on the bounding surfaces. For case (i), the small sphere ∂V_p encloses the point charge Q , and the conductor surface ∂V_c encloses

the induced charge Q_{ind} :

$$-\oint_{\partial V_p} \nabla \Phi_1(\mathbf{r}) \cdot d\mathbf{S} = \frac{Q}{\varepsilon_0}, \quad (2.69)$$

$$-\oint_{\partial V_c} \nabla \Phi_1(\mathbf{r}) \cdot d\mathbf{S} = \frac{Q_{\text{ind}}}{\varepsilon_0}. \quad (2.70)$$

For case (ii), there are no free charges in space, so any Gaussian surface enclosing only vacuum carries no net charge. In particular, a sphere ∂V_p around the point $\mathbf{r}_p$ and the conductor surface ∂V_c both enclose zero charge:

$$-\oint_{\partial V_p} \nabla \Phi_2(\mathbf{r}) \cdot d\mathbf{S} = 0, \quad (2.71)$$

$$-\oint_{\partial V_c} \nabla \Phi_2(\mathbf{r}) \cdot d\mathbf{S} = 0. \quad (2.72)$$

We now apply Green's second identity to the pair of harmonic functions Φ_1 and Φ_2 in the charge-free region V_s :

$$\int_{V_s} [\Phi_1(\mathbf{r}) \nabla^2 \Phi_2(\mathbf{r}) - \Phi_2(\mathbf{r}) \nabla^2 \Phi_1(\mathbf{r})] dV = \oint_{\partial V_s} [\Phi_1(\mathbf{r}) \nabla \Phi_2(\mathbf{r}) - \Phi_2(\mathbf{r}) \nabla \Phi_1(\mathbf{r})] \cdot d\mathbf{S}. \quad (2.73)$$

Since both Φ_1 and Φ_2 satisfy Eq. (2.68), the volume integral on the left-hand side vanishes, leaving

$$\oint_{\partial V_s} [\Phi_1(\mathbf{r}) \nabla \Phi_2(\mathbf{r}) - \Phi_2(\mathbf{r}) \nabla \Phi_1(\mathbf{r})] \cdot d\mathbf{S} = 0. \quad (2.74)$$

The boundary ∂V_s is composed of the small sphere ∂V_p , the conductor surface ∂V_c , and any additional boundaries (such as other electrodes or a surface at infinity). On all boundaries other than ∂V_p and ∂V_c , at least one of the potentials vanishes by construction (either grounded conductors or vanishing potential at infinity), so their contribution to Eq. (2.74) is zero. Hence, we can write

$$\oint_{\partial V_p} [\Phi_1(\mathbf{r}) \nabla \Phi_2(\mathbf{r}) - \Phi_2(\mathbf{r}) \nabla \Phi_1(\mathbf{r})] \cdot d\mathbf{S} + \oint_{\partial V_c} [\Phi_1(\mathbf{r}) \nabla \Phi_2(\mathbf{r}) - \Phi_2(\mathbf{r}) \nabla \Phi_1(\mathbf{r})] \cdot d\mathbf{S} = 0. \quad (2.75)$$

Next we use the fact that the small sphere ∂V_p is infinitesimal: the potentials vary negligibly over its surface and can be replaced by their values at the center, $\Phi_1(\mathbf{r}_p)$ and $\Phi_2(\mathbf{r}_p)$. Then

$$\begin{aligned} & \oint_{\partial V_p} [\Phi_1(\mathbf{r}) \nabla \Phi_2(\mathbf{r}) - \Phi_2(\mathbf{r}) \nabla \Phi_1(\mathbf{r})] \cdot d\mathbf{S} \\ & \approx \Phi_1(\mathbf{r}_p) \oint_{\partial V_p} \nabla \Phi_2(\mathbf{r}) \cdot d\mathbf{S} - \Phi_2(\mathbf{r}_p) \oint_{\partial V_p} \nabla \Phi_1(\mathbf{r}) \cdot d\mathbf{S}. \end{aligned} \quad (2.76)$$

The potentials are constant on the surface of the conductor in each problem:

$$\begin{aligned}\Phi_1(\mathbf{r}) &= 0 \quad \text{on } \partial V_c \quad (\text{grounded in (i)}), \\ \Phi_2(\mathbf{r}) &= U \quad \text{on } \partial V_c \quad (\text{biased in (ii)}),\end{aligned}\tag{2.77}$$

so the contribution from ∂V_c becomes

$$\oint_{\partial V_c} [\Phi_1(\mathbf{r}) \nabla \Phi_2(\mathbf{r}) - \Phi_2(\mathbf{r}) \nabla \Phi_1(\mathbf{r})] \cdot d\mathbf{S} = -U \oint_{\partial V_c} \nabla \Phi_1(\mathbf{r}) \cdot d\mathbf{S}.\tag{2.78}$$

Substituting Eqs. (2.76) and (2.78) into Eq. (2.75), and then using Gauss's law relations Eqs. (2.69)–(2.72), we obtain

$$\begin{aligned}0 &= \Phi_1(\mathbf{r}_p) \oint_{\partial V_p} \nabla \Phi_2(\mathbf{r}) \cdot d\mathbf{S} - \Phi_2(\mathbf{r}_p) \oint_{\partial V_p} \nabla \Phi_1(\mathbf{r}) \cdot d\mathbf{S} - U \oint_{\partial V_c} \nabla \Phi_1(\mathbf{r}) \cdot d\mathbf{S} \\ &= \Phi_1(\mathbf{r}_p) \cdot 0 - \Phi_2(\mathbf{r}_p) \left(-\frac{Q}{\varepsilon_0} \right) - U \left(-\frac{Q_{\text{ind}}}{\varepsilon_0} \right) \\ &= \frac{Q}{\varepsilon_0} \Phi_2(\mathbf{r}_p) + \frac{U}{\varepsilon_0} Q_{\text{ind}}.\end{aligned}\tag{2.79}$$

Multiplying by ε_0 gives

$$Q \Phi_2(\mathbf{r}_p) + U Q_{\text{ind}} = 0,\tag{2.80}$$

and hence

$$Q_{\text{ind}} = -Q \frac{\Phi_2(\mathbf{r}_p)}{U}.\tag{2.81}$$

The induced charge equals the negative of the particle charge multiplied by the potential that the biased conductor alone would generate at the particle's position, normalized by the applied voltage.

By charge conservation, the induced current I_{ind} is

$$I_{\text{ind}}(t) = \frac{dQ_{\text{ind}}(t)}{dt}.\tag{2.82}$$

Differentiating Eq. (2.81):

$$\begin{aligned}I_{\text{ind}}(t) &= \frac{dQ_{\text{ind}}}{dt} \\ &= -Q \frac{d}{dt} \left[\frac{\Phi_2(\mathbf{r}_p(t))}{U} \right] \\ &= -Q \frac{1}{U} \nabla_{\mathbf{r}_p} \Phi_2(\mathbf{r}_p) \cdot \dot{\mathbf{r}}_p(t).\end{aligned}\tag{2.83}$$

Since the field produced by the biased electrode in case (ii) is

$$\mathbf{E}_U(\mathbf{r}) = -\nabla \Phi_2(\mathbf{r}),\tag{2.84}$$

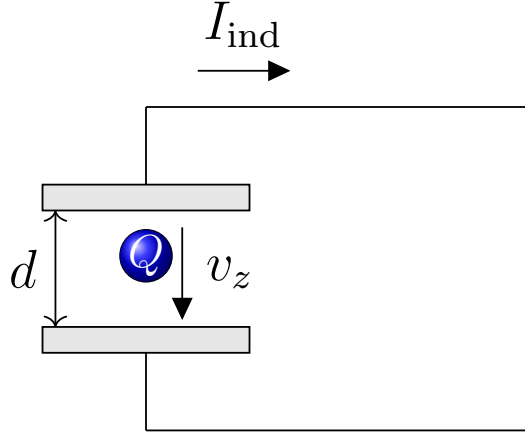


Figure 2.3: A charged particle moving between the plates of a capacitor induces an image current. The direction of I_{ind} shown corresponds to the case $Q < 0$.

we obtain the compact and widely used expression

$$I_{\text{ind}}(t) = Q \frac{\dot{\mathbf{r}}_{\text{p}}(t) \cdot \mathbf{E}_U(\mathbf{r}_{\text{p}})}{U}. \quad (2.85)$$

The normalized field $\mathbf{E}_U/U$ is known as the weighting field of the electrode and depends solely on the geometry of the conducting electrode. Equation (2.85) states that the induced current is equal to the projection of the particle velocity onto the weighting field, multiplied by its charge. For a trapped electron, $Q = -e$, the image current has the opposite sign to the electron velocity.

The general result in Eq. (2.85) becomes particularly transparent for simple geometries. Consider a parallel-plate capacitor with plate separation d , biased with a voltage U as in the schematic example in Fig. 2.3. The electric field between the plates is homogeneous and is given by

$$\mathbf{E}_U = \frac{U}{d} \hat{\mathbf{z}}, \quad (2.86)$$

assuming that the plates are normal to the z -axis. If the particle moves along z with velocity $v_z = \dot{z}_{\text{p}}$, Eq. (2.85) simplifies to

$$I_{\text{ind}}(t) = Q \frac{v_z(t)}{d}. \quad (2.87)$$

The induced current is proportional to the velocity component perpendicular to the plates, with a constant proportionality Q/d . For more complicated three-dimensional electrode geometries, it is convenient to define an *effective distance* d_{eff} such that the image current still looks like Eq. (2.87). For a given direction of motion $\hat{\mathbf{v}} = \dot{\mathbf{r}}_{\text{p}}/|\dot{\mathbf{r}}_{\text{p}}|$, we define

$$\frac{1}{d_{\text{eff}}} = \hat{\mathbf{v}} \cdot \frac{\mathbf{E}_U(\mathbf{r}_{\text{p}})}{U}. \quad (2.88)$$

Then Eq. (2.85) can be written as

$$I_{\text{ind}}(t) = Q \frac{v(t)}{d_{\text{eff}}}, \quad (2.89)$$

where $v = |\dot{\mathbf{r}}_{\text{p}}|$ is the scalar speed along $\hat{\mathbf{v}}$. This definition incorporates all geometric information about the conducting electrode into a single parameter d_{eff} , analogous to the spacing of the plate in the parallel-plate example.

In practice, d_{eff} can be obtained from electrostatic simulations of the weighting field and directly determines the coupling strength between the motion of a trapped electron and the readout electrode. A smaller d_{eff} corresponds to a larger induced current for a given electron velocity and therefore to stronger electrical coupling.

For a cloud of N electrons whose center-of-mass (COM) coordinate is $\mathbf{R}_{\text{cm}}$ and COM velocity is $\dot{\mathbf{R}}_{\text{cm}}$, the total induced current is obtained by summing Eq. (2.85) over all particles. If the electrons experience the same weighting field and move coherently in phase, their contributions add constructively, and one finds

$$I_{\text{ind}}^{(N)}(t) = NQ \frac{\dot{\mathbf{R}}_{\text{cm}}(t) \cdot \mathbf{E}_U(\mathbf{R}_{\text{cm}})}{U}. \quad (2.90)$$

This collective enhancement of the image current is the basis for the sensitive detection of electron clouds, whereas single-electron detection requires minimizing noise and optimizing d_{eff} to maximize the signal from an individual trapped electron.

2.3.2 Trapped electrons as an equivalent circuit

To see how we can treat the trapped electron as an equivalent lumped circuit [40–42], it is useful to consider the simple scenario shown in Fig. 2.4. Suppose that a single electron is bounded between the plates of a parallel-plate capacitor with the spring constant of the trapping potential k . The electron induces image charges on the capacitor plates and as the electron moves, it causes these image charges to flow in the external circuit connected to the capacitor plates. If the capacitor plate is large compared to the separation between them, the end effects can be neglected, and as in Eq. (2.87) the induced current can be written as:

$$I_{\text{ind}} = \frac{qv_z}{d}, \quad (2.91)$$

where v_z is the electron velocity component perpendicular to the capacitor plates, d is the separation between the two capacitor plates, and $q < 0$ is the electron charge so that

$$q = -e. \quad (2.92)$$

The equation of motion for the electron in the z -direction can be written as:

$$\begin{aligned} m\ddot{z} &= -kz - m\gamma_e \dot{z} + F_e \\ &= -m\omega_e^2 z - m\gamma_e \dot{z} + F_e. \end{aligned} \quad (2.93)$$

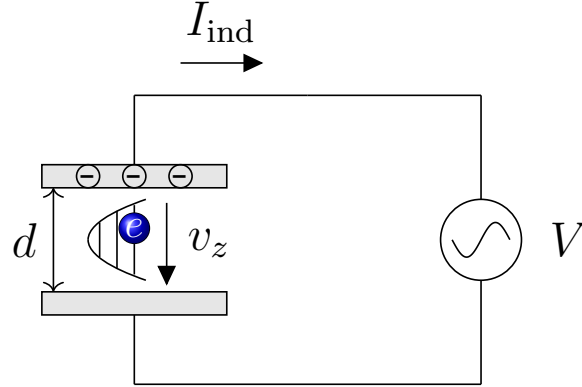


Figure 2.4: A trapped electron inside a parallel-plate capacitor with plate separation d , capacitance C , and effective spring constant k induces an image current I_{ind} in the nearby trap electrode.

Here, m is the electron mass, ω_e is the corresponding secular frequency, and γ_e is the damping rate of the electron motion. This damping can arise from cooling mechanisms that transfer motional energy to an external reservoir—for example, resistive cooling (Sec. 2.4.3) or micromotion sideband cooling (Sec. 6.4)—as well as from coupling to other motional modes. In the absence of any external circuit or active cooling and considering only motion along the z -direction, we can neglect such effects and treat $\gamma_e = 0$. F_e is the external force on the electron:

$$F_e = \frac{qV}{d} + F_{\text{ind}}, \quad (2.94)$$

where V/d is the electric field between the capacitor plates obtained when no charge is present. F_{ind} is the additional force on the electron due to the attraction of the induced charges. For a single electron,

$$F_{\text{ind}} \approx \frac{8q^2}{d^2} z, \quad (2.95)$$

which will lead to an oscillation frequency shift of

$$\frac{\Delta\omega_e}{\omega_{e0}} = -\frac{4q^2}{m\omega_{e0}^2 d^3}. \quad (2.96)$$

In the case of n electrons, the center-of-mass (COM) motion will experience a larger shift by a factor of n . For a single electron, this effect is small $\approx 10^{-9}$ and we can neglect it. Substituting Eq. (2.91) and Eq. (2.94) into Eq. (2.93) we get:

$$L_e \frac{dI}{dt} + R_e I + \frac{1}{C_e} \int I dt = V. \quad (2.97)$$

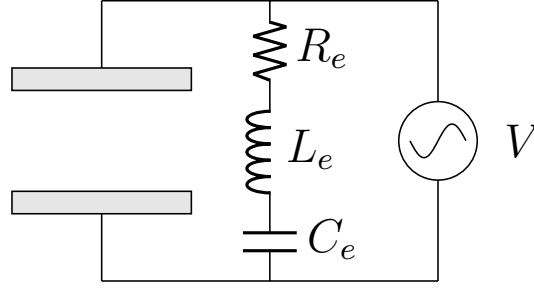


Figure 2.5: Electrical equivalent circuit representing a trapped electron inside a capacitor, corresponding to the setup in Fig. 2.4.

Here

$$L_e = \frac{md^2}{q^2}, \quad (2.98)$$

$$R_e = \gamma_e L_e, \quad (2.99)$$

$$C_e = \frac{1}{\omega_e^2 L_e}. \quad (2.100)$$

Equation (2.97) relates the current in a series RLC circuit to the voltage across it, and therefore we may replace the harmonically bound electron between the capacitor plate by an equivalent series RLC circuit with a resonance frequency of the oscillation of a single trapped electron (see Fig. 2.5).

Next, we consider the case of n electrons with the same spring constant k . For simplicity, we only include their mutual interactions in the z direction. Then, the equation of motion for the j -th electron is

$$m\ddot{z}_j = -kz_j - m\gamma_e\dot{z}_j + F'_{ej} + \sum_{k \neq j}^n F_{jk}. \quad (2.101)$$

Here, F_{jk} is the direct Coulomb force on the j -th electron from the k -th electron. Generally for trapped electrons the damping rate γ_e includes the contribution of resistive cooling of the COM motion, and the coupling of the COM motion to the internal mode motion which depends on the anharmonicity of the trapping field. With a small number of particles, it is dominated by the resistive cooling effect, which will be explained later. We also have

$$F'_{ej} = \frac{qV}{d} + F'_{\text{ind}j}, \quad (2.102)$$

$$F'_{\text{ind}j} = \sum_k^n F_{\text{ind}}(j, k), \quad (2.103)$$

where $F_{\text{ind}}(j, k)$ is the force on the j -th electron due to the image charges of the k -th electron. The same as before, we neglect the effect from $F'_{\text{ind}j}$.

Adding Eq. (2.101) for all electrons, we have

$$m\ddot{Z} = -kZ - m\gamma_e\dot{Z} + \frac{qV}{d}, \quad (2.104)$$

where $Z = \sum_k^n z_k/n$ is the COM coordinate for all electrons. Notice that the sum of $\sum_j^n \sum_{k \neq j}^n F_{jk}$ vanishes because of Newton's third law $F_{jk} = -F_{kj}$.

Comparing Eq. (2.104) with Eq. (2.93), we can see that the COM motion of the electron cloud obeys the same equation as a single electron. Generalizing Eq. (2.91) for n electrons, we have

$$I_{\text{ind}} = n \frac{q\dot{Z}}{d}. \quad (2.105)$$

Therefore, analogously to Eq. (2.97), for n electrons, we have

$$L_{ne} = \frac{L_e}{n}, \quad (2.106)$$

$$R_{ne} = \frac{R_e}{n}, \quad (2.107)$$

$$C_{ne} = nC_e. \quad (2.108)$$

Note that we have linearized the problem and neglected higher-order effects in the electric field which give rise to induced currents at the harmonics of ω_e . In this linear approximation ($z/d \ll 1$) we can observe only the COM motion at ω_e which is described by the series RLC circuit.

2.4 Interaction with an external circuit

The equivalent circuit derived in the previous section describes how the motion of a single trapped electron (or the COM motion of an electron cloud) can be represented by a lumped series resonator with parameters (L_e, R_e, C_e) (or (L_{ne}, R_{ne}, C_{ne})). In practice, this circuit is not isolated. In this section, we study the coupling of trapped electrons to an external RLC circuit, also known as a *tank circuit*, which will be used for resistive cooling and image current detection [40–42].

2.4.1 External circuitry

The COM motion of n trapped electrons is represented by the series RLC circuit with parameters

$$L_{ne} = \frac{L_e}{n}, \quad R_{ne} = \frac{R_e}{n}, \quad C_{ne} = nC_e, \quad (2.109)$$

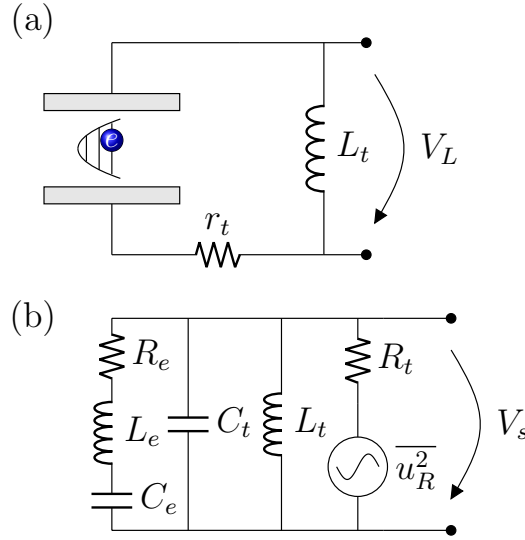


Figure 2.6: (a) A trapped electron inside the pickup electrodes is coupled to an external tank circuit. L_t denotes the inductance of the tank circuit, r_t represents RF losses, and V_L is the voltage across the inductor. (b) Electrical equivalent circuit of (a), where L_t is the tank circuit inductance, C_t is the parasitic capacitance of the trap and circuit, R_t is the effective resistance of the tank circuit, u_R is the Johnson noise of the resistor, and V_s is the detected signal voltage.

and the resonance frequency of this branch is the secular frequency ω_e

$$\omega_e = \frac{1}{\sqrt{L_{ne}C_{ne}}} = \frac{1}{\sqrt{L_e C_e}}. \quad (2.110)$$

The damping rate of the COM motion can be written as

$$\gamma_e = \frac{R_{ne}}{L_{ne}} = \frac{R_e}{L_e}, \quad (2.111)$$

this is also the linewidth of the motion for a single electron

$$\gamma_e = \Delta\omega_e. \quad (2.112)$$

The corresponding quality factor is

$$\begin{aligned} Q_e &= \frac{\omega_e}{\Delta\omega_e} \\ &= \frac{1}{\omega_e R_e C_e}. \end{aligned} \quad (2.113)$$

Suppose that an inductor is connected between the two capacitor plates, also known as the pickup electrodes, as sketched in Fig. 2.6 (a). It forms a series LCR resonator that contains an inductor L_t that can tune away the trap capacitance C_t at the desired resonance frequency to achieve a pure resistive impedance. This network is analogous to a damped harmonic oscillator. In the absence of trapped electrons, the LCR circuit has a resonance frequency of

$$\omega_t = \frac{1}{\sqrt{L_t C_t}}, \quad (2.114)$$

and we use Q_t , the resonator quality factor, as a measure of resistive loss. The value is

$$Q_t = \frac{\omega_t}{\Delta\omega_t}. \quad (2.115)$$

where $\Delta\omega_t$ is the full width at half maximum (FWHM) of the noise signal in power, this also equals the damping rate γ_t of the oscillating current in the resonator

$$\begin{aligned} \Delta\omega_t &= \gamma_t \\ &= \frac{r_t}{L_t}. \end{aligned} \quad (2.116)$$

The measured mean noise power of the voltage across the inductor $V_L = L_t \dot{I}(t) = L_t \ddot{Q}(t)$ is

$$\overline{V_L^2} = 4k_B T R(\omega) \Delta\nu, \quad (2.117)$$

where k_B is the Boltzmann constant, $\Delta\nu$ is the observation width, and T is the temperature of the circuit. This is the familiar result for Johnson noise (also known as thermal noise, Johnson-Nyquist noise), with the effective resistance $R(\omega)$ given by

$$R(\omega) = \left(\frac{L_t}{r_t C_t} \right) \frac{(\frac{1}{2}\gamma_t)^2}{(\omega_t - \omega)^2 + (\frac{1}{2}\gamma_t)^2}. \quad (2.118)$$

Therefore, in the absence of trapped electrons, the measured “noise resonance” is a Lorentzian lineshape with a resonance frequency of ω_t and a FWHM of γ_t .

When $\omega \approx 1/\sqrt{L_t C_t}$, it is often more convenient to use the parallel LCR representation and Fig. 2.6 (b) shows the corresponding effective circuit. Here L_t is the inductance of the tank circuit, and the capacitance C_t includes the effective distributed capacitance and the contribution of the trap electrodes. The resistive loss in the LC circuit is inversely proportional to an effective resistance R_t in parallel with the LC at the resonance frequency, u_R denotes the Johnson noise of the resistor and V_s is the input voltage signal detected by a preamplifier. We do not include the Johnson noise of R_e because it is typically much smaller than R_t . The Johnson noise associated with a resistor with resistance R held at temperature T is given by

$$\overline{u_R^2} = 4k_B T R \Delta\nu, \quad (2.119)$$

where k_B is the Boltzmann constant and $\Delta\nu$ is the spectral width.

When $\omega_e = \omega_t$, at the resonance frequency, the capacitive reactance of the trap electrodes is canceled by the inductive reactance of the inductor L_t and the tank circuit is effectively a resistor R_t . The effective parallel resistance is given by

$$R_t = \frac{L_t}{r_t C_t}. \quad (2.120)$$

We often find it convenient to use this simplification as shown in Fig. 2.6 (b). Therefore, the relations of Q_t with tank circuit parameters can be expressed as

$$Q_t = \omega_t R_t C_t = R_t / \omega_t L_t. \quad (2.121)$$

The higher Q_t of the circuit, the higher the parallel impedance R_t at the resonance frequency.

2.4.2 Coupling to a tank circuit

An electron confined in the trap interacts with the tank circuit, as shown in Fig. 2.6. The motion of the electron $z(t)$ induces an oscillating current

$$I_{\text{ind}}(t) = \dot{Q}(t) \quad (2.122)$$

in the tank circuit, which in turn, produces an electric field that acts back on the electron.

The coupling between the two oscillators is described by the interaction potential

$$\begin{aligned} V_{\text{int}} &= qU(z)V_{\text{ind}} \\ &= \frac{q}{d_{\text{eff}} C_t} z(t) Q(t). \end{aligned} \quad (2.123)$$

where $U(z)$ is the potential at the electron position when applying 1 V on the coupling electrode, $q = -e$ is the electron charge and $Q(t)$ is the charge on the capacitance C_t (equivalently the charge that flows in the series LCr loop). Including the Johnson noise generated by the finite loss of the tank circuit and assuming that the electron has no intrinsic damping, the coupled equations for the tank circuit and the electron are [40]

$$L_t \left(\frac{d^2}{dt^2} + \gamma_t \frac{d}{dt} + \omega_t^2 \right) Q(t) + \frac{q}{d_{\text{eff}} C_t} z(t) = V(t), \quad (2.124)$$

$$m \left(\frac{d^2}{dt^2} + \omega_e^2 \right) z(t) + \frac{q}{d_{\text{eff}} C_t} Q(t) = 0. \quad (2.125)$$

In a time-domain Langevin description, the dissipative element r_t in the series LCr loop produces a fluctuating voltage source $V(t)$ in series with the resonator. The stochastic voltage $V(t)$ has zero mean $\langle V(t) \rangle = 0$, and is taken to be white on the scale of the narrow

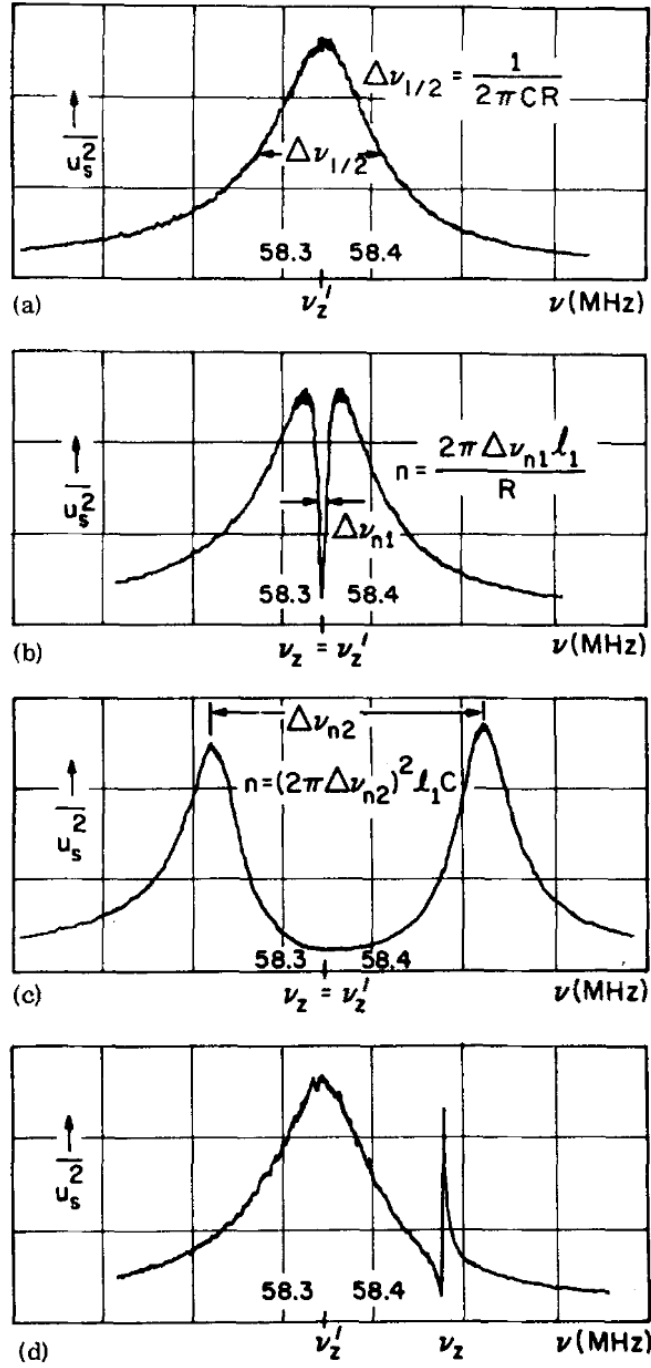


Figure 2.7: Noise spectra corresponding to the circuit in Fig. 2.6. (a) Spectrum with no electrons in the trap. (b) Spectrum for approximately 5×10^3 electrons on resonance ($\omega_e = \omega_t$). (c) Spectrum for approximately 3.4×10^5 electrons on resonance. (d) Spectrum for approximately 5×10^3 electrons when the electron frequency is detuned by 130 kHz from the tank resonance. Figure adapted from Ref. [40].

measurement bandwidth around the tank circuit resonance. Its correlation function is fixed by the fluctuation–dissipation theorem,

$$\langle V(t)V(t') \rangle = 2k_B T_t (L_t \gamma_t) \delta(t - t') = 2k_B T_t r_t \delta(t - t'). \quad (2.126)$$

Next we derive the resulting “noise dip” used for electron-number detection using the series LCr representation [40]. The complex impedance of the tank circuit is

$$Z_t(\omega) = r_t + i\omega L_t + \frac{1}{i\omega C_t}. \quad (2.127)$$

The resistive loss produces Johnson noise. It is convenient to represent this noise at the pickup node using a Norton equivalent: a current noise source $I_N(t)$ in parallel with the noiseless impedance $Z_t(\omega)$. In the absence of trapped electrons, this current drives the node voltage according to

$$V_s^{(0)}(\omega) = Z_t(\omega) I_N(\omega). \quad (2.128)$$

The (one-sided) current-noise power spectral density (PSD) is related to the real part of the admittance,

$$S_{I_N}(\omega) = 4k_B T_t \Re\{Y_t(\omega)\}, \quad Y_t(\omega) \equiv \frac{1}{Z_t(\omega)}. \quad (2.129)$$

Consequently, the electron-free node-voltage noise is

$$S_{V_s}^{(0)}(\omega) = |Z_t(\omega)|^2 S_{I_N}(\omega) = 4k_B T_t |Z_t(\omega)|^2 \Re\left\{\frac{1}{Z_t(\omega)}\right\} = 4k_B T_t \Re\{Z_t(\omega)\}, \quad (2.130)$$

which is the usual Johnson–Nyquist result written in impedance form.¹

The pickup voltage drives the motion of the electron through force $F = qV_s/d_{\text{eff}}$. In the frequency domain ($z \propto e^{-i\omega t}$) the driven harmonic oscillator obeys

$$m(\omega_e^2 - \omega^2) z(\omega) = \frac{q}{d_{\text{eff}}} V_s(\omega), \quad (2.131)$$

so that

$$z(\omega) = \frac{q}{md_{\text{eff}}} \frac{V_s(\omega)}{\omega_e^2 - \omega^2}. \quad (2.132)$$

The image current induced at the pickup node is proportional to the electron velocity,

$$I_{\text{ind}}(\omega) = i\omega \frac{q}{d_{\text{eff}}} z(\omega), \quad (2.133)$$

¹For an ideal series $L_t C_t r_t$, $\Re\{Z_t\} = r_t$ is frequency-independent. The *measured* “noise resonance” discussed in Sec. 2.4.1 arises when one measures a particular circuit voltage (e.g. across L_t or at the amplifier input) whose transfer function weights the Johnson source by the resonator response, leading to the effective Lorentzian resistance $R(\omega)$ used there.

which, after inserting Eq. (2.132), gives a linear relation between induced current and node voltage:

$$I_{\text{ind}}(\omega) = Y_e(\omega) V_s(\omega), \quad Y_e(\omega) = i\omega \frac{q^2}{md_{\text{eff}}^2} \frac{1}{\omega_e^2 - \omega^2}. \quad (2.134)$$

For n electrons in the COM mode, the coupling scales as $q^2/m \rightarrow nq^2/m$, so that

$$Y_e(\omega) = i\omega \frac{nq^2}{md_{\text{eff}}^2} \frac{1}{\omega_e^2 - \omega^2}. \quad (2.135)$$

Thus, the electron COM motion loads the pickup node as a frequency-dependent admittance $Y_e(\omega)$.

With electrons present, Kirchhoff's current law at the node states that the noisy Norton current I_N splits between the tank circuit branch and the electron-induced current:

$$I_N(\omega) = \frac{V_s(\omega)}{Z_t(\omega)} + I_{\text{ind}}(\omega) = \left[\frac{1}{Z_t(\omega)} + Y_e(\omega) \right] V_s(\omega). \quad (2.136)$$

Solving for the node voltage,

$$V_s(\omega) = \frac{I_N(\omega)}{Z_t(\omega)^{-1} + Y_e(\omega)} = \frac{I_N(\omega)Z_t(\omega)}{1 + Z_t(\omega)Y_e(\omega)}. \quad (2.137)$$

Dividing by the electron-free response $V_s^{(0)}(\omega) = I_N(\omega)Z_t(\omega)$ from Eq. (2.128) gives the compact transfer relation [40]

$$\frac{V_s(\omega)}{V_s^{(0)}(\omega)} = \frac{1}{1 + Z_t(\omega)Y_e(\omega)} = \frac{\omega_e^2 - \omega^2}{\omega_e^2 - \omega^2 + i\omega \frac{nq^2}{md_{\text{eff}}^2} Z_t(\omega)}, \quad (2.138)$$

where we used Eq. (2.135) in the last step. The measured noise PSD follows immediately because the system is linear and the only stochastic drive is the tank circuit Johnson noise:

$$S_{V_s}(\omega) = S_{V_s}^{(0)}(\omega) \left| \frac{\omega_e^2 - \omega^2}{\omega_e^2 - \omega^2 + i\omega \frac{nq^2}{md_{\text{eff}}^2} Z_t(\omega)} \right|^2. \quad (2.139)$$

In writing $Z_t(\omega) = \Re\{Z_t(\omega)\} + i\Im\{Z_t(\omega)\}$, the denominator of Eq. (2.139) can be arranged as

$$\omega_e^2 - \omega^2 + i\omega \frac{nq^2}{md_{\text{eff}}^2} Z_t(\omega) = \left(\omega_e^2 - \omega^2 - \omega \frac{nq^2}{md_{\text{eff}}^2} \Im\{Z_t\} \right) + i \left(\omega \frac{nq^2}{md_{\text{eff}}^2} \Re\{Z_t\} \right). \quad (2.140)$$

This motivates defining a circuit-induced frequency shift and damping rate,

$$\tilde{\omega}_e^2(\omega) \equiv \omega_e^2 - \omega \frac{nq^2}{md_{\text{eff}}^2} \Im\{Z_t(\omega)\}, \quad (2.141)$$

$$\gamma_z(\omega) \equiv \frac{nq^2}{md_{\text{eff}}^2} \Re\{Z_t(\omega)\}. \quad (2.142)$$

Substituting these into Eq. (2.139) yields the standard “noise-dip” expression [40]:

$$S_{V_s}(\omega) = S_{V_s}^{(0)}(\omega) \frac{(\omega_e^2 - \omega^2)^2}{(\tilde{\omega}_e^2 - \omega^2)^2 + (\omega\gamma_z)^2}. \quad (2.143)$$

Equation (2.143) explicitly shows that the dissipative part $\Re\{Z_t\}$ produces an effective damping γ_z of the electron COM motion (see further discussion in Sec. 2.4.3), while the reactive part $\Im\{Z_t\}$ shifts the resonance. As a result, the electron COM motion partially shorts the tank circuit Johnson noise near $\omega \simeq \omega_e$, producing a dip in the observed noise resonance.

As shown in Fig. 2.7 from Ref. [40], the number of electrons can be counted from the noise spectrum of V_s . When the tank circuit is tuned to the electron COM frequency ($\omega_e = \omega_t$), the impedance of the tank circuit is approximately real in a narrow window around ω_e , $Z_t(\omega) \approx R_t$. If the electron-induced feature is narrow compared to the tank linewidth, $\gamma_z(\omega_e) \ll \gamma_t$, the baseline $S_{V_s}^{(0)}(\omega)$ can be treated as constant across the dip and Eq. (2.143) reduces to an inverted Lorentzian [40]

$$S_{V_s}(\omega) \simeq 4k_B T_t R(\omega) \left[1 - \frac{(\Gamma_0/2)^2}{(\omega_e - \omega)^2 + (\Gamma_0/2)^2} \right], \quad (2.144)$$

with $\Gamma_0 \equiv \gamma_z(\omega_e) \propto n$. In this regime, the dip FWHM is $\Delta\omega_{ne} \simeq \Gamma_0$, giving the electron number as

$$n \simeq \frac{\Delta\omega_{ne}}{\Delta\omega_e}, \quad (2.145)$$

where $\Delta\omega_e$ denotes the single-electron linewidth due to dissipation in the external circuitry.

For sufficiently large n the electron-induced loading is no longer a perturbation. When $\Gamma_0 \gtrsim \gamma_t$ the single noise resonance develops into two maxima (Fig. 2.7 (c)) and the position of the two peaks corresponds to the parallel resonances formed when the reactive impedance of the electrons is equal to and opposite to the reactive impedance of the external tank circuit. In this regime, the characteristic peak-to-peak separation scales as

$$\Delta\omega_{ne} \sim \sqrt{\Gamma_0 \Delta\omega_t} \sim \sqrt{n \Delta\omega_e \Delta\omega_t}, \quad (2.146)$$

so that the electron number can be inferred from the measured splitting as

$$n \simeq \frac{(\Delta\omega_{ne})^2}{\Delta\omega_e \Delta\omega_t}, \quad (2.147)$$

where $\Delta\omega_{ne}$ denotes the peak-to-peak separation and $\Delta\omega_t = \gamma_t$ is the tank circuit linewidth in the absence of trapped electrons.

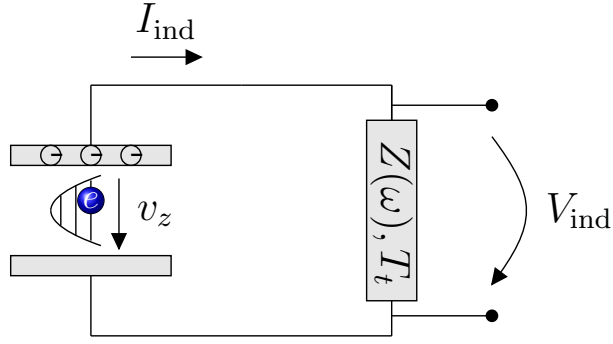


Figure 2.8: Schematic of resistive cooling: the motional energy of the trapped electron is dissipated through the induced image current flowing into an effective resistor held at temperature T_t .

2.4.3 Resistive cooling

Resistive cooling comes from the coupling between the electron motion and the tank circuit. The thermal current noise in the resistor R_t drives the COM motion, while the motion in turn dissipates energy back into R_t through the induced image current. In thermal equilibrium, the COM mode acquires a mean energy $k_B T_t$ set by the temperature of the tank circuit. To understand the damping, let us first consider the situation with a single electron without external drive and without noise voltage, as shown in Fig. 2.8. Assuming that the tank circuit has impedance $Z(\omega)$, the voltage drop across the tank circuit has the form

$$V_{\text{ind}} = \Re\{Z(\omega)\} I_{\text{ind}}. \quad (2.148)$$

Near the center of the trap, this potential produces an electric field of the form

$$E_{\text{ind}} = \frac{V_{\text{ind}}}{d_{\text{eff}}}, \quad (2.149)$$

where the effective distance d_{eff} is defined as Eq. (2.88). Therefore, the force exerted on a single electron with charge $q = -e$ can be expressed as

$$\begin{aligned} F_{\text{damp}} &= qE_{\text{ind}} \\ &= \frac{qV_{\text{ind}}}{d_{\text{eff}}} \\ &= -\left(\frac{q^2 \Re\{Z(\omega)\}}{d_{\text{eff}}^2}\right) \dot{z}. \end{aligned} \quad (2.150)$$

This force opposes the motion of the electron, in the form of a dissipative force $F_{\text{damp}} = -m\gamma\dot{z}$. The damping coefficient is given by

$$\gamma = \frac{q^2}{d_{\text{eff}}^2} \frac{\Re\{Z(\omega)\}}{m}. \quad (2.151)$$

When $\omega_e = \omega_t$, at the resonance frequency, the capacitive reactance of the trap electrodes is canceled by the inductive reactance of the inductor L_t and the tank circuit is effectively a resistor R_t . The on-resonance impedance of the circuit is, therefore,

$$\begin{aligned} Z(\omega_t) &= R_t \\ &= Q_t \omega_t L_t \\ &= Q_t \sqrt{\frac{L_t}{C_t}}. \end{aligned} \tag{2.152}$$

Note that the damping coefficient is inversely proportional to the mass. Therefore, a heavier particle, like a proton or an ion, is much less damped compared to an electron. The cooling time constant is just the inverse of the damping constant

$$\tau = \frac{m}{R_t} \frac{d_{\text{eff}}^2}{q^2}. \tag{2.153}$$

Another way of seeing this is that the energy of the electron oscillation is dissipated via the effective resistance

$$I_{\text{ind}}^2 R_t = -F_{\text{damp}} \dot{z}, \tag{2.154}$$

thus

$$F_{\text{damp}} = -\frac{q^2}{d_{\text{eff}}^2} R_t \dot{z}, \tag{2.155}$$

and we get the same result.

For the COM mode of n trapped electrons, the damping coefficient scales linearly with the particle number

$$\gamma_n = n\gamma. \tag{2.156}$$

Energy stored in internal (non-COM) degrees of freedom must first couple to the COM motion before it can be dissipated.

2.4.4 Electronic detection

Electronic detection uses the image current induced by the motion of the electrons to generate a measurable voltage across an effective resistance. In practice, the trap capacitance is tuned by an inductor into a narrowband resonator; within the detection bandwidth near $\omega \simeq \omega_t$, the network can be modeled by an effective impedance $Z_t(\omega)$, whose dissipative part $\Re\{Z_t\}$ produces resistive damping and whose Johnson noise drives the motion. When the detected signal is small, it is often necessary to drive the electron motion amplitude to a level where it can be detected above the noise.

Following Ref. [42], to understand electronic detection let us now consider the motion of the electron in the z -direction and include the effect of drive and noise, as shown in

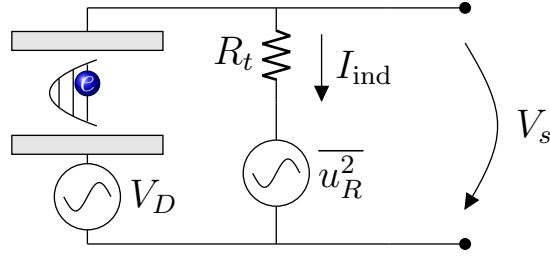


Figure 2.9: External circuit used to damp and detect the electron motion. The motion is driven by the oscillatory potential V_D and by the Johnson noise from the effective resistor u_R , while the resulting signal voltage V_s is detected at the output.

Fig. 2.9. The equation of motion with damping, restoring force, and nondissipative force can be written as

$$m \left(\frac{d^2}{dt^2} + \gamma_e \frac{d}{dt} + \omega_e^2 \right) z(t) = F(t), \quad (2.157)$$

where

$$\gamma_e = \frac{q^2 R_t}{m d_{\text{eff}}^2} \quad (2.158)$$

is the resistive damping rate due to the external circuit. The nondissipative force is given by

$$F = \frac{q}{d_{\text{eff}}} (V_D - u_R), \quad (2.159)$$

where V_D is the drive voltage, u_R is the voltage of the Johnson noise from the resistor, and $q = -e$ is the electron charge. The two voltages appear with opposite signs because they are usually applied to opposing electrodes. The detected signal voltage V_S is the sum of the noise response and the driven response and is thus given by

$$V_S = u_R + V_{\text{ind}} = u_R + \frac{q}{d_{\text{eff}}} \dot{z} R_t. \quad (2.160)$$

Fourier-transforming the equation of motion in Eq. (2.157) with $z \propto e^{-i\omega t}$ yields

$$z(\omega) = \left(\frac{q}{m d_{\text{eff}}} \right) \frac{V_D(\omega) - u_R(\omega)}{\omega_e^2 - \omega^2 - i\omega\gamma_e}. \quad (2.161)$$

Substituting Eq. (2.158) we have the axial resonance [42]

$$\begin{aligned} V_S(\omega) &= u_R(\omega) - i\omega \frac{q R_t}{d_{\text{eff}}} z(\omega) \\ &= u_R(\omega) - i\omega \gamma_e \frac{V_D(\omega) - u_R(\omega)}{\omega_e^2 - \omega^2 - i\omega\gamma_e} \\ &= \frac{(\omega_e^2 - \omega^2) u_R(\omega) - i\omega \gamma_e V_D(\omega)}{\omega_e^2 - \omega^2 - i\omega\gamma_e}. \end{aligned} \quad (2.162)$$

This is a Lorentzian response with a full width half maximum of γ_e . Since $u_R(\omega)$ scales with $\sqrt{R_t}$ and γ_e scales linearly with R_t , the signal-to-noise ratio (SNR) is improved by increasing the effective resistance R_t .

To suppress broadband noise, the signal is commonly measured with phase-sensitive detection [42]: $V_s(t)$ is mixed with a local oscillator at the drive frequency ω with phase φ and low-pass filtered to a narrow bandwidth $\Delta\omega$. The resulting demodulated output can be written as

$$S = \int dt F(t) V_s(t), \quad (2.163)$$

where

$$\begin{aligned} F(t) &= F(\omega, \Delta\omega, \varphi; t) \\ &= 2 \int_{-\Delta\omega/2}^{\Delta\omega/2} \frac{d\omega'}{2\pi} \cos[(\omega + \omega')t - \varphi]. \end{aligned} \quad (2.164)$$

We can write the signal as the sum of terms corresponding to the external drive and the noise,

$$S = S_D + S_N. \quad (2.165)$$

Near resonance, the coherent drive produces a dispersive/absorptive Lorentzian in the demodulated signal [42]

$$S_D = V_d \frac{(\gamma_e^2/4) \cos \varphi + (\gamma_e/2)(\omega - \omega_e) \sin \varphi}{(\omega - \omega_e)^2 + \gamma_e^2/4}, \quad (2.166)$$

so that $\varphi = 0$ gives the familiar absorptive profile and $\varphi = -\pi/2$ gives the dispersive profile. Since $\langle u_R(t) \rangle = 0$, the average value of the signal after performing the experiment a large number of times is due entirely to the coherent drive $\langle S \rangle = S_D$. The magnitude of the noise fluctuations is

$$\Delta S^2 = \langle (S - \langle S \rangle)^2 \rangle = \langle S_N^2 \rangle. \quad (2.167)$$

We have [42]

$$\Delta S^2 = 4k_B T R_t \Delta\nu \frac{(\omega_e^2 - \omega^2)^2}{(\omega_e^2 - \omega^2)^2 + (\omega\gamma_e)^2}, \quad (2.168)$$

which explicitly displays the “noise-dip” on resonance discussed in Sec. 2.4.2. Far away from resonance, the Johnson noise is

$$\Delta S^2 = 4k_B T R_t \Delta\nu. \quad (2.169)$$

The Johnson noise sets a practical floor for how small a coherent excitation can be and still be observed. In the absence of a coherent drive, the electron comes into thermal equilibrium with a typical energy of the order $k_B T$. A drive near ω_e increases the energy by an additional coherent amount E . A simple detectability criterion is that the peak demodulated signal exceeds the rms noise in the measurement bandwidth $S_D \gtrsim \Delta S$.

Using the absorptive quadrature ($\varphi = 0$) on resonance ($\omega = \omega_e$), Eq. (2.166) gives a peak signal $S_D = V_d$. In the same bandwidth $\Delta\nu$, the off-resonant Johnson fluctuations are approximately

$$\Delta S^2 \simeq 4k_B T R_t \Delta\nu. \quad (2.170)$$

Relating the drive amplitude to the coherent axial energy via the steady-state result

$$E = \frac{V_d^2}{2\gamma_e R_t}, \quad (2.171)$$

one obtains the rough threshold [42]

$$E \gtrsim k_B T \left(\frac{2\Delta\nu}{\gamma_e} \right), \quad (2.172)$$

i.e., the narrower detection bandwidth allows coherent excitations below $k_B T$ to be resolved.

2.4.5 Quantum-mechanical description of the coupling

To further develop intuition for the interaction between trapped electrons and external circuits, in this section we establish a basic quantum-mechanical framework describing the coupling between a trapped electron and an electrical resonator. Specifically, we consider a single electron confined in a Paul trap and coupled to a cryogenic tank circuit, with both systems treated fully quantum mechanically. The total Hamiltonian is

$$\hat{H} = \hat{H}_e + \hat{H}_t + \hat{H}_{\text{int}}, \quad (2.173)$$

where $\hat{H}_e$ describes the motion of the electron in the coupling direction, $\hat{H}_t$ is the Hamiltonian of the quantized tank circuit, and $\hat{H}_{\text{int}}$ captures the electron–circuit interaction.

Electron motion

In the axial direction, the electron behaves as a simple harmonic oscillator with annihilation operator a_z ,

$$H_e^{(z)} = \hbar\omega_z \left(a_z^\dagger a_z + \frac{1}{2} \right). \quad (2.174)$$

In the radial direction, the electron experiences a time-dependent quadrupole potential driven at the rf frequency Ω_{rf} . The effective quantum Hamiltonian for motion along x can be written as

$$H_e^{(x)} = \frac{\hat{p}^2}{2m} + \frac{m}{2} \left[\omega_x^2 + 2q_x \Omega_{\text{rf}} \cos(\Omega_{\text{rf}} t) \right] \hat{x}^2, \quad (2.175)$$

which corresponds to the quantized version of the Mathieu equation. As discussed in Sec. 2.2.4, the Heisenberg-picture solution can be expressed using the special solution $u(t)$,

$$\hat{x}(t) = x_0 \left(\hat{a} u^*(t) + \hat{a}^\dagger u(t) \right), \quad (2.176)$$

where $x_0 = \sqrt{\hbar/(2m\omega_x)}$. Note that $u(t)$ captures all the time dependence of $x(t)$.

Tank circuit

A quantized LC resonator with inductance L_t and capacitance C_t has Hamiltonian [43]

$$H_t = \frac{\hat{\Phi}^2}{2L_t} + \frac{\hat{Q}^2}{2C_t}, \quad (2.177)$$

with conjugate operators obeying $[\hat{\Phi}, \hat{Q}] = i\hbar$. In the Heisenberg picture, the charge and flux operators can be written in terms of ladder operators a_ϕ and $a_\phi^\dagger$,

$$\begin{aligned} \hat{Q}(t) &= iQ_0 \left(a_\phi^\dagger e^{i\omega_t t} - a_\phi e^{-i\omega_t t} \right), \\ \hat{\Phi}(t) &= \Phi_0 \left(a_\phi^\dagger e^{i\omega_t t} + a_\phi e^{-i\omega_t t} \right), \end{aligned} \quad (2.178)$$

where $\omega_t = 1/\sqrt{L_t C_t}$, and

$$Q_0 = \sqrt{\frac{\hbar \omega_t C_t}{2}}, \quad \Phi_0 = \sqrt{\frac{\hbar}{2\omega_t C_t}}. \quad (2.179)$$

The Hamiltonian thus becomes the familiar harmonic-oscillator form,

$$H_t = \hbar \omega_t \left(a_\phi^\dagger a_\phi + \frac{1}{2} \right). \quad (2.180)$$

Electron–circuit interaction

The voltage across the tank circuit is coupled to the electron with charge $q = -e$ through the potential $U(x)$ created by the coupling electrodes. The interaction energy is [10]

$$H_{\text{int}} = q U(x) V, \quad (2.181)$$

where $V = \hat{Q}/C_t$ is the electrode voltage, and $U(x)$ is the potential at the position of the electron when 1 V is applied to the coupling electrode. Expanding the potential around $x = 0$,

$$U(x) = U(0) + \frac{x}{d_{\text{eff}}} + \frac{x^2}{d_2^2} + O(x^3), \quad (2.182)$$

where d_{eff} and d_2 are the expansion coefficients. The linear term produces the dominant resonant interaction, whereas the quadratic term gives us the parametric coupling.

Keeping only the linear term,

$$H_{\text{int}} = q \frac{\hat{x}}{d_{\text{eff}}} \frac{\hat{Q}}{C_t}. \quad (2.183)$$

Using this framework, we can rederive resistive cooling in Sec. 2.4.3. If we consider the coupling between the axial motion of the electron and the tank circuit using $x_0^{(z)} = \sqrt{\hbar/(2m\omega_z)}$, the interaction Hamiltonian becomes

$$\hat{H}_{\text{int}} = q \left[\frac{x_0^{(z)}}{d_{\text{eff}}} (a_z + a_z^\dagger) \right] \left[i \frac{Q_0}{C_t} (a_\phi^\dagger - a_\phi) \right]. \quad (2.184)$$

The total Hamiltonian is therefore

$$\begin{aligned}\hat{H} = & \hbar\omega_z\left(a_z^\dagger a_z + \frac{1}{2}\right) + \hbar\omega_t\left(a_\phi^\dagger a_\phi + \frac{1}{2}\right) \\ & + i\hbar g(a_\phi^\dagger - a_\phi)(a_z + a_z^\dagger),\end{aligned}\quad (2.185)$$

with coupling strength

$$g = \frac{qx_0^{(z)}Q_0}{\hbar d_{\text{eff}}C_t} = \frac{q}{2d_{\text{eff}}}\sqrt{\frac{\omega_t}{m\omega_z C_t}}. \quad (2.186)$$

Assuming the resonant condition $\omega_z = \omega_t$ and applying the rotating-wave approximation, the interaction-picture Hamiltonian reduces to

$$H_I = i\hbar g (a_\phi^\dagger a_z - a_\phi a_z^\dagger). \quad (2.187)$$

The Heisenberg equations of motion are then

$$\begin{aligned}\dot{a}_z &= -\frac{\gamma_e}{2}a_z - g a_\phi, \\ \dot{a}_\phi &= -\frac{\gamma_t}{2}a_\phi + g a_z,\end{aligned}\quad (2.188)$$

where γ_e is the intrinsic damping rate of the electron motion and γ_t the linewidth of the tank circuit.

When $\gamma_e \ll \gamma_t$, the tank circuit can be adiabatically eliminated:

$$a_\phi \approx \frac{2g}{\gamma_t}a_z. \quad (2.189)$$

Substituting back, we obtain the effective damping of the electron mode,

$$\frac{d}{dt}a_z = -\left(\frac{\gamma_e}{2} + \frac{2g^2}{\gamma_t}\right)a_z. \quad (2.190)$$

Thus the total energy dissipation rate is

$$\gamma = \gamma_e + \frac{4g^2}{\gamma_t}, \quad (2.191)$$

which is equivalent to Eq. (2.151) when $\gamma_e = 0$, and is directly analogous to the Purcell effect in cavity QED.

Chapter 3

Quantum computing with trapped electrons

In this chapter, we discuss the basic ingredients for quantum computing with trapped electrons including trapping, cooling, motion detection, spin readout, initialization, and single- and multi-qubit gates as shown in Fig. 3.1. We study the anticipated challenges and potential roadblocks in the context of a electron trap blueprint. Furthermore, we discuss the threshold temperature to form the Wigner crystal for quantum computation and also the scalability of the system. [11, 12, 44, 45]

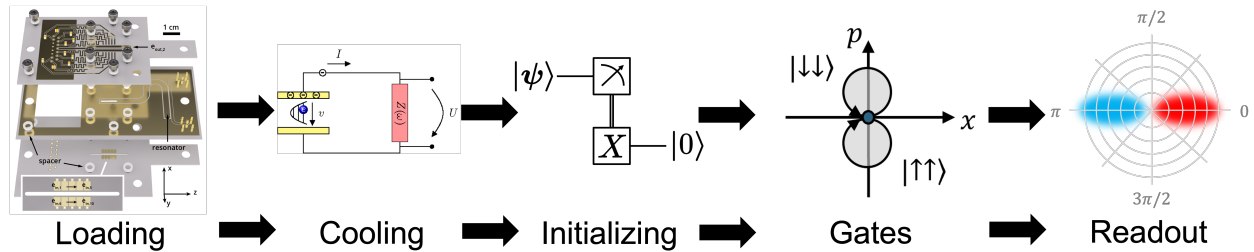


Figure 3.1: Basic principle of a trapped-electron platform. Quantum information is stored in superpositions of the electron spin states. Electrons are loaded into a Paul trap, their motion is cooled and electronically detected using a cryogenic tank circuit, initialization is performed via spin readout followed by a conditional flip, quantum gates are driven by magnetic fields and magnetic-field gradients, and spin readout is achieved through a spin-dependent force generated by the magnetic-field gradient.

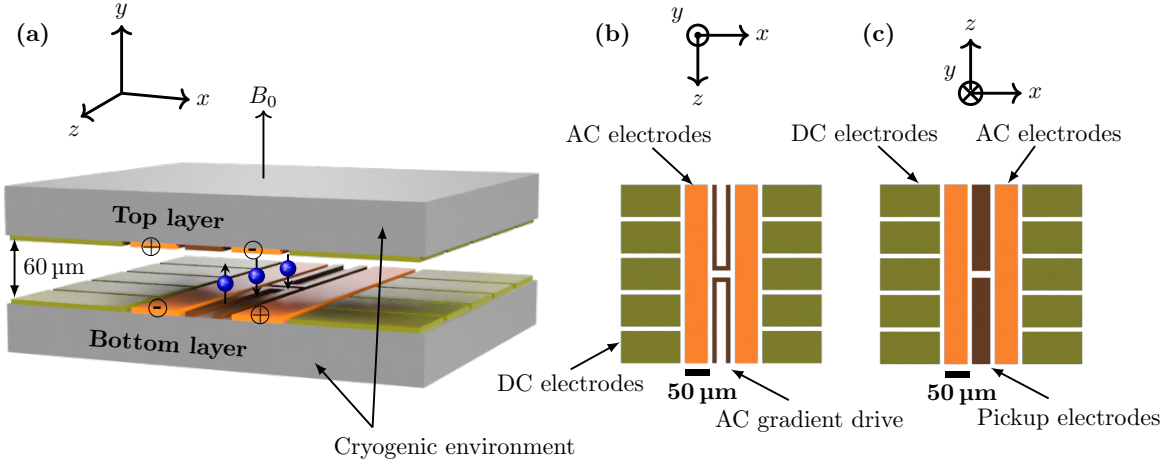


Figure 3.2: (a) A two-layer trap design blueprint used to analyze the feasibility of trapped-electron quantum computing. Two substrates separated by $60\text{ }\mu\text{m}$ support four out-of-phase AC electrodes (AC electrodes). Driving currents through the center electrodes of the bottom layer (AC gradient drive) generates either a magnetic-field gradient or a homogeneous magnetic field along the z -axis, depending on the relative phase. The center electrodes of the top layer (pickup electrodes) detect the image current generated by the electron motion. The plus and minus signs indicate the relative phases of the AC drive. (b) Top view of the bottom substrate. (c) Bottom view of the top substrate. Figure reproduced from Ref. [44].

3.1 Creating and trapping electrons

In this chapter, we base our discussion on the device blueprint shown in Fig. 3.2, which is assumed to operate in a cryogenic environment near 4 K. This section describes the trap design and how electrons are created in situ and subsequently confined in the quadrupolar trapping potential of this device.

3.1.1 Electron trap blueprint

The electron trap consists of two electrode layers separated by $60\text{ }\mu\text{m}$. Since there are very few constraints on optical access, we introduce a top plate from a conventional surface trap. The addition of the top layer has two advantages: First, the AC ground of the top plate significantly increases trap depth. Second, it allows for separating the cooling and detection circuit in the top layer from high-power electrical signals (order 1 W) in the bottom layer, such as the AC voltage needed to trap the electrons, as well as the wires through which the magnetic field gradient will be applied. This opens up the possibility to reduce the temperature of the cooling circuit significantly below the temperature of bottom substrate receiving the largest heat-load.

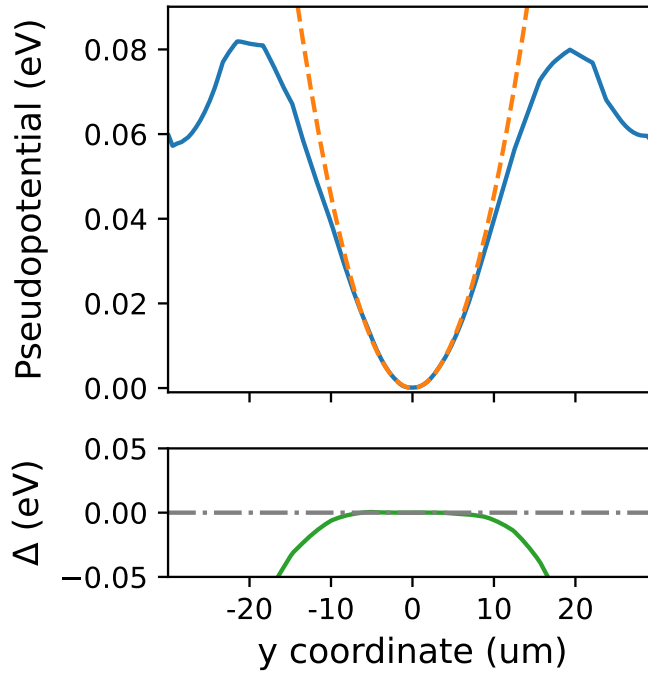


Figure 3.3: Top: pseudopotential along the y -axis through the trap center (blue continuous curve) of the electron trap blueprint shown in Fig. 3.2, compared to an ideal harmonic potential (dashed orange). The blue continuous curve is obtained using Ansys finite-element simulations, and the dashed line is a harmonic fit corresponding to a trap frequency of 2 GHz. Bottom: deviation Δ of the pseudopotential from a harmonic potential.

As mentioned in Sec. 2.2.1, the vastly different charge-to-mass ratio for bare electrons requires a higher AC drive frequency as opposed to that used for commonly trapped ions. This can be achieved by incorporating capacitively-coupled resonators to introduce microwaves to the surface-electrode trap. With this blueprint trap geometry, we assume that the two AC electrodes in each plane are driven out of phase with each other and opposite to the other plane with an amplitude of $V_0 = 14$ V with a frequency of

$$\Omega_{\text{rf}} = 2\pi \times 10.6 \text{ GHz.}$$

Due to the geometry's symmetry, the minimum trapping potential lies midway between the layers and is centered laterally along the x - z axes. Using the pseudopotential approximation, the resulting trap depth is approximately 80 meV from Ansys FEM simulation, as shown in Fig. 3.3. This yields a transverse secular frequency of

$$\omega_t = 2\pi \times 2 \text{ GHz.}$$

Further, we assume an axial trap frequency of

$$\omega_a = 2\pi \times 300 \text{ MHz},$$

produced by static voltages applied to the DC electrodes shown in Fig. 3.2.

The numerical simulation of electron trajectories shows substantial losses for electrons with an initial energy above 22 meV as shown in Fig. 3.4. The stability of the electron trajectory is analyzed by numerically integrating the 2-D motion of the electron along the radial directions. Because the surplus ionization energy is small compared to the potential energy in the trapping potential, we assume the initial kinetic energy of the electrons to be zero. The simulation variables are the phase ϕ of the AC drive at ionization time ($t = 0$) assuming a time dependence of $\cos(\omega_{ac}t + \phi)$ and the initial energy of electrons in the AC trapping potential, which is determined by the ionization distance from the trap center. Figure 3.4 shows the simulation result, which is a map of the simulated electron storage time up to 100 μ s. From the simulation, electron trajectories are not lost during the simulation duration for initial energies less than $k_B \times 250$ K, which corresponds to an ionization distance of about 8 μ m from the trap center. For higher energies, the stability of the electron trajectory depends strongly on the phase of the AC drive, and electron trajectories are most stable when the phase of the AC electric field is zero when the electron is separated from its parent atom.

3.1.2 Electron source

Previous experiments involving trapping electrons in Penning traps, or guiding electrons in a linear quadrupole potential employed electron guns, either as the primary electron source [46], or to create secondary electrons through collision ionization of background gas [40, 47]. Such approaches typically produce electrons with energies comparable to or exceeding the trap depth unless active cooling is present. However, without a cooling method, the created electrons must have low energies to be confined by the trapping potential. For this we consider generating the electrons using two-photon photoionization (PI) of neutral atoms from a thermal atomic beam, which is a technique widely used for ion-trap loading [48]. Neutral atoms are supplied by a resistively heated oven, and the neutral flux is tuned by adjusting the oven current. Ionization occurs when the atoms intersect the PI beams near the trap center, allowing the localized generation of electrons directly within the trapping region. This loading method enables both the creation of very cold electrons by tuning the laser frequencies close to the ionization threshold, and preferential in-situ ionization in the trapping region by optical alignment.

By tuning the ionizing laser close to threshold, the imparted kinetic energy scales as

$$E_e \approx \frac{m_e}{m_{Ca}} E_{Ca} \approx 1 \text{ } \mu\text{eV}, \quad (3.1)$$

far below the 80 meV trap depth. Here, m_e and m_{Ca} denote the electron and calcium masses, respectively, and E_{Ca} is the kinetic energy of calcium atoms emitted from the heated oven,

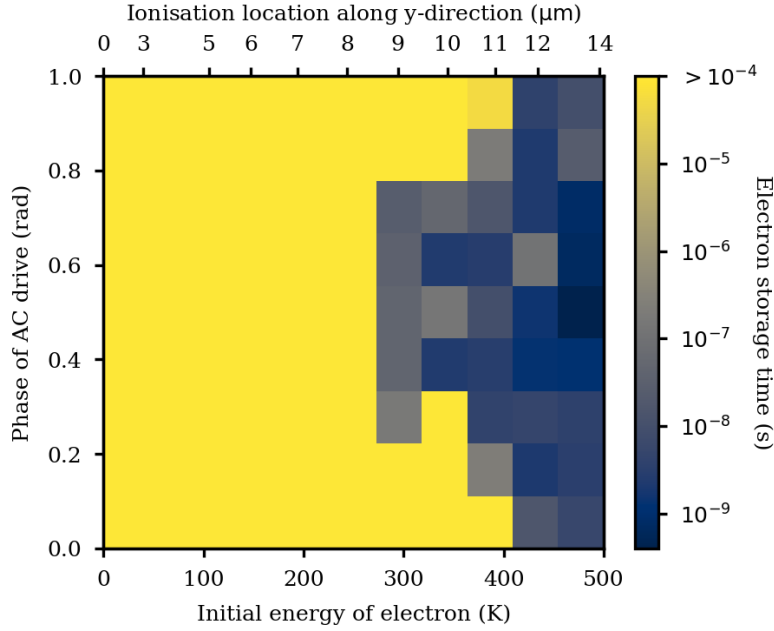


Figure 3.4: Electron trajectory simulation of the two-dimensional secular motion. Electron trajectories are stable irrespective of the drive phase when the electron separates from the atom for initial energies less than 250 K. Figure reproduced from Ref. [44].

assumed to be at 1,000 K. Since fewer charged particles are introduced around the trap using this method, we also reduce accidental charging of the trap, which would modify the trapping potential.

3.2 Damping the electron motion

Damping the electron motion to minimize the average extent of their wave function is essential for long-term confinement, high-fidelity state detection, and quantum control. In Chapter 2, we developed the theoretical framework of resistive cooling and image current detection. Here we apply these concepts to the blueprint device introduced in Sec. 3.1 and quantify the achievable cooling rates, steady-state temperatures, and thermal requirements.

In the blueprint trap, damping is achieved by coupling the induced image current to a high-impedance tank circuit anchored at $T_t = 0.4$ K. As discussed in Sec. 2.4.3, this coupling produces a voltage that opposes the electron's motion and dissipates motional energy until the induced current is of similar magnitude as the thermal (Johnson) current fluctuations in the circuit. For an electron thermalized at 0.4 K, the average micromotion amplitude of the

transverse secular motion is

$$x_{MM} = \frac{q}{2}x_t \approx 80 \text{ nm},$$

where the Mathieu stability parameter $q = 0.53$, and $x_t = \sqrt{2k_B T / (m_e \omega_t^2)} \approx 0.3 \mu\text{m}$ is the average amplitude of the transverse secular motion, with $\omega_t = 2\pi \times 2 \text{ GHz}$.

The image current induced by the motion along the direction i is

$$I_{\text{ind}} = \frac{qv_i}{d_{\text{eff}}^{(i)}},$$

where v_i is the electron velocity projected on a certain axis ($i \in (x, y, z)$) and $d_{\text{eff}}^{(i)}$ is the effective distance between the electron and the pickup electrode along the direction i . The effective distance can be calculated from the electrostatic simulation, with

$$d_{\text{eff}}^{(i)} = \frac{1}{E_{1V}^{(i)}}, \quad (3.2)$$

where $E_{1V}^{(i)}$ is the i -th component of the electric field at the target electron location when the pickup electrode is held at 1 V. For the blueprint trap, we find

$$d_{\text{eff}}^{(y)} = 138 \mu\text{m}, \quad d_{\text{eff}}^{(z)} = 254 \mu\text{m}.$$

where for the y -direction the cooling circuit is attached to the two center pickup electrodes on the top layer, a reduction of about a factor of two compared to the geometry of the ideal plate capacitor, and for the axial (z) direction the cooling circuit is attached to one of the top center pickup electrodes.

From Sec. 2.4.3, the cooling time constant is

$$\tau = \frac{m_e}{q^2} \frac{d_{\text{eff}}^2}{R_t}, \quad (3.3)$$

where $R_t = Q_t \sqrt{L_t / C_t}$ is the on-resonance impedance of the tank circuit and $q = -e$ is the electron charge.

Assuming a moderate quality factor of $Q_t = 1,000$, a capacitance of $C_t = 1 \text{ pF}$, an inductance of $L_t = 6 \text{ nH}$ at a resonance frequency of $2\pi \times 2 \text{ GHz}$ results in an on-resonance impedance of $R_t = 80 \text{ k}\Omega$ and

$$\tau_y \approx 8 \mu\text{s}.$$

For the axial mode at $2\pi \times 300 \text{ MHz}$ ($L_t = 250 \text{ nH}$),

$$\tau_z \approx 4 \mu\text{s}.$$

At equilibrium, the average energy of the cooled mode will be that of the temperature of the tank circuit

$$\langle E \rangle = \hbar\omega(\bar{n} + \frac{1}{2}) = k_B T_t,$$

assuming $\omega_t = 2\pi \times 2$ GHz for one of the transverse modes and $T_t = 0.4$ K, we obtain the average motional excitation $\bar{n} \approx 4$.

Since the vertical transverse mode can be cooled following this protocol, the horizontal transverse mode and the axial mode can also be cooled by coupling them to the vertical transverse mode. Such parametric coupling is common practice for electrons and ions in Penning traps [42] and has been demonstrated for ions in a Paul trap [49]. To achieve this, our electrode geometry allows us to generate a quadrupole field with projection overlap on a pair of modes that oscillates at the difference frequency between the two modes. This leads to an interaction that causes an energy exchange between the two motional modes. Assuming that we hold the high-frequency vertical transverse mode at 0.4 K, the low-frequency axial mode will be cooled to a temperature reduced by the frequency ratio,

$$T_a = T_t \cdot \frac{\omega_a}{\omega_t} \approx 60 \text{ mK}. \quad (3.4)$$

Scaling results from Ref. [49], we expect coupling times between the two motional modes to be below 1 μ s.

Since resistive cooling acts only on the motion of the electron and does not affect the spin state of the qubit, the electrons can be cooled while keeping the qubits coherent, and no sympathetic coolant-particles are needed to be able to perform high fidelity operations on the electron qubits. In fact, the cooling does not need to be turned off for most of the time: only during multi-qubit gates where a common mode of motion is used as a bus, as well as during the spin-motion conversion process for detection, must the cooling be switched off. We also note that efficient cooling takes place only if the electron motion is in resonance with the resonance frequency of the tank circuit. Thus, we can adjust the cooling time constant by changing the trapping potential.

3.2.1 Thermal load considerations

Since we need the tank circuit to be physically cold to damp the electron motion, a realistic cryogenic trapped-electron experiment must account for the heat generated by

1. current-carrying cables entering the cryostat
2. Joule dissipation inside the on-chip trap electrodes
3. thermal conduction between different cryogenic stages

External black-body radiation can be effectively shielded and almost completely suppressed because the operation of the electron trap requires only very minimal optical access. In the following, we give the relevant formulas and then evaluate the thermal load for the parameters of the blueprint device.

Joule heating from current-carrying wires (30 K $\rightarrow$ 4 K). To determine the heat load on the 4 K stage, we assume that the current-carrying wires that go from a 30 K stage to the 4 K stage are made of niobium. For a wire with resistivity ρ , length L , and cross-sectional area $A = \pi d^2/4$ with d the diameter of the wire, the DC resistance is

$$R = \rho \frac{L}{A}, \quad (3.5)$$

and the Joule heating at current I is

$$P = I^2 R. \quad (3.6)$$

Since the AC electrodes are driven at MHz–GHz frequencies, we need to consider the frequency-dependent skin depth δ , the AC skin-depth-limited resistance is

$$R_{ac} = \rho \frac{L}{A_{eff}}, \quad (3.7)$$

with

$$A_{eff} = \text{perimeter} \times \delta. \quad (3.8)$$

For niobium wires of

$$L = 10 \text{ cm}, \quad d = 1 \text{ mm},$$

with an electrical resistivity at 30 K of

$$\rho_{Nb}(30 \text{ K}) = 1.5 \text{ n}\Omega \text{ m},$$

and skin depth of

$$\delta(300 \text{ MHz}) = 1 \text{ }\mu\text{m},$$

$$\delta(10 \text{ GHz}) = 0.2 \text{ }\mu\text{m},$$

For $I = 1 \text{ A}$ of current at 300 MHz with a duty cycle of $\approx 10\%$, we have

$$P = 0.1 \times I^2 R_{ac}(300 \text{ MHz}) = 4.5 \text{ mW}.$$

For $I = 1 \text{ A}$ of current at 10 GHz,

$$P = I^2 R_{ac}(10 \text{ GHz}) = 250 \text{ mW}.$$

Joule heating inside the on-chip hairpin AC electrodes. More important will be the heat load from Joule dissipation in the AC-electrodes themselves, which is directly transferred to the substrate. Assume AC gradient electrodes with a cross section of $10 \times 0.5 \text{ }\mu\text{m}^2$ and a total length of 5 mm operating with a duty cycle of $\approx 10\%$ for spin-readout and gates.

For copper at 4 K, 300 MHz, the resistivity $\rho = 1 \text{ n}\Omega \text{ m}$, and the skin depth $\delta \approx 0.9 \text{ }\mu\text{m}$ that is larger than the thickness of the electrode, we have

$$A_{eff} = 10 \text{ }\mu\text{m} \times 0.5 \text{ }\mu\text{m} = 5 \text{ }\mu\text{m}^2.$$

The AC resistance is thus

$$R_{ac} = \rho \frac{L}{A_{\text{eff}}} = (1 \times 10^{-9}) \frac{5 \times 10^{-3}}{5 \times 10^{-12}} = 1 \Omega.$$

Thus for $I = 1$ A of current,

$$P = I^2 R_{ac} = 1 \text{ W}.$$

Including a 10% duty cycle yields

$$P = 100 \text{ mW}.$$

Joule heating inside the on-chip RF trapping electrodes. Regarding the AC-trapping electrodes, assuming a drive voltage of 14 V at 10 GHz, and a capacitance of 1 pF, this will lead to a current of 1 A. Suppose further that the current flows through the entire AC copper electrode of length 5 mm and cross section $50 \times 0.5 \mu\text{m}^2$. The skin depth of copper at 10 GHz is $\approx 0.16 \mu\text{m}$, which is less than the electrode thickness $0.5 \mu\text{m}$. So, the perimeter is $2(w + t) = 101 \mu\text{m}$, and

$$A_{\text{eff}} = 101 \mu\text{m} \times 0.16 \mu\text{m} = 16.2 \mu\text{m}^2.$$

Then

$$R_{ac} = 1 \times 10^{-9} \frac{5 \times 10^{-3}}{1.62 \times 10^{-11}} \approx 0.31 \Omega.$$

For 1 A of current,

$$P = 0.31 \text{ W}.$$

Two such electrodes form a pair, so the pair dissipates

$$P_{\text{pair}} \approx 0.6 \text{ W},$$

and the total four RF trapping electrodes dissipate ≈ 1.2 W.

This heat load can be mitigated by providing a sufficiently large heat sink to a typical cryo pulse-tube cooler with 1.8 W of cooling power at 4 K by carefully choosing the trap substrate with high heat conductivity (e.g. sapphire) and trap mounting materials.

Thermal conduction between the 4 K and 0.4 K stage. To reach 0.4 K in the motion of the trapped electron, the resonant circuit must be cryogenically cooled to the same temperature. This can be done by thermally anchoring the resonant circuit at 0.4 K and connecting it to the pickup electrodes at 4 K with a Nb (Niobium) wire. The wire needs to be a good electrical conductor and a good thermal insulator to ensure that the heat load of the trap on the resonant circuit is lower than the maximum cooling power of the 0.4 K stage, which is ≈ 1.4 mW.

The thermal conduction between stages is

$$P_{\text{th}} = \kappa \frac{A}{L} \Delta T, \tag{3.9}$$

where κ is the thermal conductivity and ΔT is the temperature difference.

The thermal conductivity of niobium at 4 K is $\kappa \approx 10 \text{ W/(m K)}$. For a Nb wire of diameter $100 \mu\text{m}$ and length $L = 1 \text{ cm}$, the heat load on the resonant circuit is:

$$P_{\text{th}} = 10 \times \frac{7.85 \times 10^{-9}}{0.01} \times 3.6 \approx 2.8 \times 10^{-4} \text{ W} = 280 \mu\text{W}.$$

which is within the cooling capacity of a commercial dilution refrigerator with a maximum cooling power of $\approx 1.4 \text{ mW}$.

3.3 Detecting electrons

Detection of trapped ions is typically performed via fluorescence imaging. However, for trapped electrons, we need a different mechanism to detect without fluorescence detection. Different types of destructive and non-destructive techniques are used in various experiments for the detection of electrons. The destructive methods usually use the discharge of electrons onto a charge-collector assembly or phosphor screen, allowing for the acquisition of spatiotemporal profiles of the charged particles. On the other hand, the non-destructive methods usually rely on resonant detection of the image current induced by electron motion. In this section, we outline possible detection methods for trapped electrons and quantify the requirements on the readout circuitry in the presence of the large-amplitude RF trap drive.

3.3.1 Destructive detection

Electrons may be detected destructively by extracting them from the trap and directing them to various electron detectors such as micro-channel plate detector (MCP), channeltron, or superconducting micro-strip single-electron detector (SSED) [50].

In our room-temperature experiment later described in Chap. 5, a voltage pulse is applied to selected DC electrodes to distort the confining potential and extract the trapped electrons. A positively biased metal mesh (grid) located between the trap and the detector guides and accelerates the electrons onto a two-stage MCP detector. In our cryogenic apparatus, later discussed in Chap. 6, the same extraction principle is used with a channeltron detector mounted inside the room-temperature environment.

3.3.2 Non-destructive detection

For quantum information processing, a non-destructive readout method is required so that electrons remain trapped after detection. The general physical principle discussed in Sec. 2.4.4 has also already been shown in Penning traps [42] and Paul traps [51]: the motion of the electrons induces an image current in the trap electrodes, which can be detected by measuring the voltage drop over the attached tank circuit. Although there are more advanced (and sensitive) schemes [42, 52, 53], here we focus first on the simplest where the electron is

exited above its equilibrium temperature T_{eq} with a brief (sub- μs) pulse. While the electron is cooled, a Johnson-noise-limited amplifier picks up the voltage drop over the cooling tank circuit. This signal is then sent through another amplifier outside the cryostat and recorded at room temperature. The actual signal is then extracted via a Fourier transform of the voltage.

We propose to detect the axial motion at 300 MHz because (i) its frequency lies far from the 10.6 GHz trap drive, allowing much more effective filtering of the drive signal, and (ii) the axial confinement is generated by static (DC) voltages, which offer better stability of the trap frequency and allow for more precise control of anharmonicities.

For a single trapped electron, the detection circuit sets both the thermal background and the linewidth of any motional feature, as discussed in Sec. 2.4.2. If the electron is coherently excited above thermal equilibrium by an energy E_{el} , the driven axial motion produces a narrow spectral feature centered at the secular frequency ω_e (see Sec. 2.4.4) and the linewidth of this feature is set by the resistive damping rate, $\Delta\omega \simeq \gamma$.

A final complication, we mention, is that the amplifier can increase the electronic temperature of the tank circuit, i.e., the kinetic energy of the electrons inside the circuit. This effect is well-known for Penning trap experiments and can be avoided by choosing the right amplifier types dissipating only very little power, such as a high electron mobility transistors (HEMT) or a Josephson traveling wave parametric amplifier (JTWPA) [54, 55]. Alternatively, we may employ one circuit for cooling and another for detection.

3.3.3 Strong drive in motion detection

Although this electronic detection technique has been standard in Penning traps for almost 50 years [17], a challenge specific to Paul traps is the presence of a strong RF trap drive of ~ 100 V at ~ 10 times the trap frequency. The danger is that the strong drive may saturate the amplifiers detecting the signals. Therefore, we need an effective filtering of the trap drive.

To estimate the degree of filtering necessary to successfully detect an electron in the presence of the 10.6 GHz drive, we model the electron signal as a Johnson-noise source of bandwidth $\Delta f \sim 1$ MHz at 0.4 K, where the bandwidth is determined by the coupling of the electron to the tank circuit $\gamma = 1/\tau$. The noise power available in a matched load is

$$P_J = k_B T \Delta f \approx 5.5 \times 10^{-18} \text{ W}. \quad (3.10)$$

Expressing this in dBm,

$$P_J(\text{dBm}) = 10 \log_{10} \left(\frac{P_J}{1 \text{ mW}} \right) = 10 \log_{10}(5.5 \times 10^{-15}) \approx -142 \text{ dBm}. \quad (3.11)$$

Assuming the amplifier has a dynamic range of

$$\text{DR} = 70 \text{ dB}, \quad (3.12)$$

the largest tolerable pickup of the RF drive is

$$P_{\max} = P_J + \text{DR} = -142 \text{ dBm} + 70 \text{ dB} = -72 \text{ dBm}. \quad (3.13)$$

Driving the trap typically requires power on the level of

$$P_{\text{drive}} \approx 1 \text{ W} = 30 \text{ dBm}. \quad (3.14)$$

Thus, the required total suppression (filtering) of the trap-drive signal is

$$\text{Suppression} = P_{\text{drive}} - P_{\max} = 30 \text{ dBm} - (-72 \text{ dBm}) = 102 \text{ dB}. \quad (3.15)$$

Several mechanisms provide this suppression:

1. The detection resonant circuit that has a narrow linewidth at

$$\omega_a = 2\pi \times 300 \text{ MHz},$$

itself serves as a filter. For a simple RLC resonance, the voltage response at a detuned angular frequency ω scales as

$$|H(\omega)| \simeq \frac{1}{\sqrt{1 + Q^2 \left(\frac{\omega - \omega_a}{\omega_a} \right)^2}}.$$

where Q is the loaded quality factor. Assuming $\Omega_{\text{rf}} = 2\pi \times 10.6 \text{ GHz}$ and a conservative $Q = 300$

$$|H(\omega_{\text{rf}})| \sim 10^{-4},$$

corresponding to about 80 dB suppression of the 10.6 GHz drive at the resonator output. For a somewhat higher loaded quality factor, $Q \approx 500$, the off-resonant suppression increases to $\sim 85 \text{ dB}$.

2. The amplifier will be optimized to amplify the 300 MHz signal rather than the high frequency drive.
3. We can further filter the signal via low-pass filters or with another resonant circuit if necessary.

Together, these mechanisms can help achieve the required suppression of the 10.6 GHz trapping drive.

3.4 Spin readout and initialization

Quantum information will be stored in superpositions of the spin states of the electron, denoted as $|\uparrow\rangle$ and $|\downarrow\rangle$. An externally applied homogeneous magnetic field defines the quantization axis and generates a well-isolated two-level system suitable for quantum computation. In this section, we describe the principles of spin readout and qubit initialization of trapped electrons in the context of this blueprint device.

3.4.1 Electron spin as qubits

Electron spin states provide a naturally clean and well-characterized qubit. The two spin states, denoted as $|\uparrow\rangle$ and $|\downarrow\rangle$, form an ideal two-level system without nearby leakage transitions, and the energy split between the two spin states is given by the Zeeman splitting

$$\hbar\omega_{\text{qubit}} = g_e\mu_B B, \quad (3.16)$$

where g_e is the electron g -factor, μ_B the Bohr magneton, and B the magnetic field strength.

In the blueprint device considered in this work (see Fig. 3.5), we assume that a homogeneous magnetic field of $B_0 = 36$ G is applied along the z -axis, which splits the degeneracy of the two spin states and produces a qubit frequency

$$\omega_{\text{qubit}} = 2\pi \times 100 \text{ MHz}.$$

Note that the coordinate axes in Fig. 3.5 are redefined relative to those in Fig. 3.2. In all subsequent discussions of spin readout and quantum gate operations, we adopt the convention shown in Fig. 3.5.

3.4.2 Spin-dependent drive

As discussed in Ref. [11], the spin direction can be read-out by first coupling the spin to the electron motion and then measuring the phase of the motion. In order to couple the spin direction to the motion, one can use a spin-dependent drive of an oscillating magnetic field gradient at the axial frequency ω_a . Since the direction of the force due to the magnetic field gradient depends on the spin direction, as demonstrated by the Stern-Gerlach experiment, this creates a state-dependent displacement acting on the initial thermal state.

The electron qubits will be manipulated with the help of the two “hairpin” electrodes between the AC electrodes of the bottom layer, labeled “AC gradient drive” in Fig. 3.2. As shown in Fig. 3.5, an oscillating magnetic field gradient $B'_y(y)$ is created by currents of opposite direction in the middle segments of the hairpin electrodes and can also be used to drive multi-qubit gates. Reversing the current direction in one of the hairpin electrodes produces an oscillating homogeneous magnetic field B_x that is perpendicular to the quantization axis and can drive single-qubit gates.

In the presence of an oscillating linear magnetic-field gradient, the axial Hamiltonian can be written as

$$H = \frac{\hat{p}_y^2}{2m} + \frac{1}{2}m\omega_a^2\hat{y}^2 + \sum_{n=3}^{\infty} V_n\hat{y}^n - \mu_B B'_y(y)\hat{\sigma}_y\hat{y}e^{i\omega_a t}, \quad (3.17)$$

where $\hat{y}$ and $\hat{p}_y$ are the position and momentum operators along the axial direction, m is the mass of the electron, ω_a is the axial trap frequency, $\hat{\sigma}_y$ is the Pauli operator along the axial y -axis, and V_n is the n th-order expansion coefficient of the potential, accounting for anharmonicity which will be discussed later. The last term corresponds to a resonant spin-dependent driving force whose sign depends on the spin state, where $B'_y(y)$ is the derivative

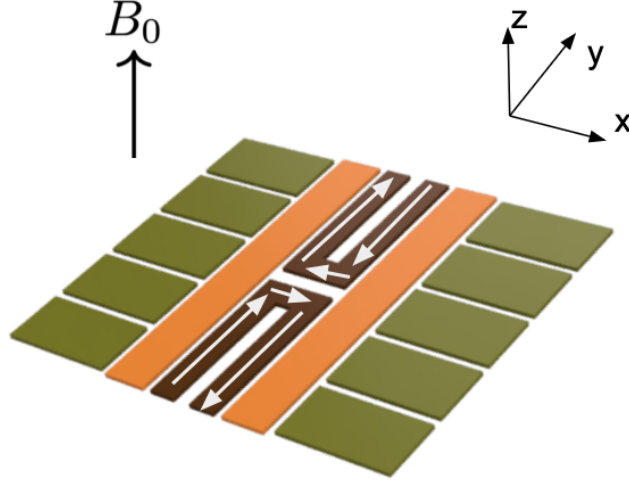


Figure 3.5: Magnetic-field control using hairpin electrodes. Oscillating currents of opposite direction in the central segments of the hairpin electrodes generate an oscillating magnetic-field gradient $B'_y(y)$, enabling multi-qubit gates (white arrows). Reversing the current direction in one hairpin electrode produces an oscillating homogeneous magnetic field B_x perpendicular to the quantization axis for single-qubit gates. The coordinate axes are redefined relative to Fig. 3.2 and are used in Sec. 3.4 and Sec. 3.5.

of the magnetic field along y with respect to y . Since the spin state is projected onto an eigenstate of $\hat{\sigma}_y$ after measurement, the operator can be replaced by a scalar $\sigma_y = \pm 1$.

Expressing $\hat{y}$ in terms of ladder operators

$$\hat{y} = y_0(\hat{a} + \hat{a}^\dagger), \quad y_0 = \sqrt{\frac{\hbar}{2m\omega_a}},$$

and moving to the frame rotating with ω_a as well as using the rotating-wave approximation, Eq. (3.17) becomes

$$H = \sum_{n=3}^{\infty} nV_n y_0^n (\hat{a} + \hat{a}^\dagger)^n - \mu_B B'_y(y) y_0 \sigma_y (\hat{a} + \hat{a}^\dagger). \quad (3.18)$$

The Heisenberg equation of motion then gives

$$i\hbar \frac{d\hat{a}}{dt} = - \sum_{n=3}^{\infty} nV_n y_0^n (\hat{a} + \hat{a}^\dagger)^{n-1} + \mu_B y_0 B'_y(y) \sigma_y, \quad (3.19)$$

Neglecting the anharmonicity of the trapping potential for now, the equation of motion becomes

$$i\hbar \frac{d\hat{a}}{dt} = \mu_B z_0 B'_y(y) \sigma_y, \quad (3.20)$$

which shows that the drive produces a coherent oscillation at a rate proportional to the spin.

It is convenient to decompose the coherent response into components in-phase (A_0) and in-quadrature ($A_{\pi/2}$). The real part of $\langle \hat{a} \rangle$, which corresponds to A_0 , grows linearly at the rate

$$\frac{dA_0}{dt} = \frac{\mu_B B'_y(y)}{2m\omega_a} \sigma_y. \quad (3.21)$$

The sign of A_0 directly reveals the measurement result, i.e., spin up or down, and the modulus of A_0 determines the size of the signal.

For the spin-dependent motion to be resolvable above the thermal spread of the axial motion, one requires

$$\frac{|A_0(\tau)|}{\sigma_{\text{std}}} = \frac{B'_y(y) \mu_B \tau}{2m\omega_a \sigma_{\text{std}}} > 1, \quad (3.22)$$

where σ_{std} is the width of the electron wave packet

$$\sigma_{\text{std}} = \sqrt{n_{\text{th}} z_0}, \quad (3.23)$$

with

$$n_{\text{th}} = \frac{1}{1 - e^{-1/k_B T}}. \quad (3.24)$$

The measurement result is encoded in the phase of the motion, and thus the critical requirement is to maintain the coherence of the motion. Therefore, it is important to create the displaced state faster than the motional dephasing time. Thus, one of the limiting factors is the strength of the magnetic field gradient.

In the prototype two-layer trap, the spin-dependent magnetic-field gradient is generated by the two “hairpin” electrodes on the bottom substrate. When designing the trap, we optimized the geometries, especially the distance between the hairpin electrodes to suppress the residual magnetic field to avoid the resulting AC Zeeman shift. Figure 3.6 shows the magnetic field when driving a current of 1 A through these conductors. This corresponds to a magnetic field gradient of approximately

$$B'_y(y) \approx 120 \text{ T/m}.$$

Once the displacement of the motion exceeds that of the random background excitation due to the finite temperature of the electron, a parametric amplification scheme can be used to increase the displacements to easily measurable sizes [11]. This can be done by modulating the stiffness of the trap potential at twice the motion frequency of the electron [56, 57].

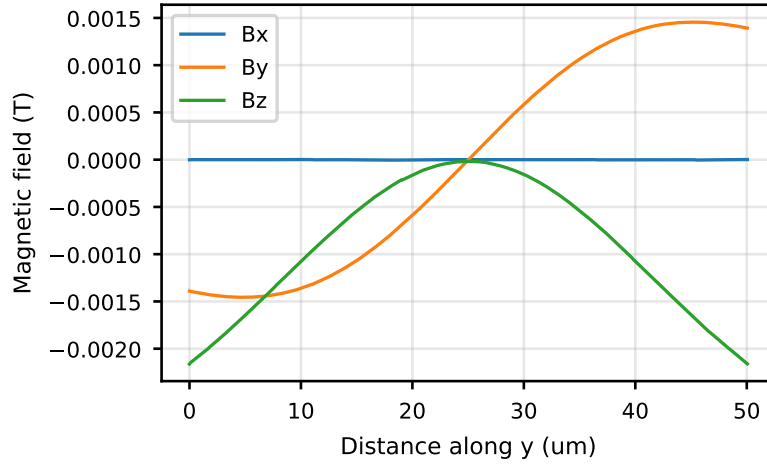


Figure 3.6: Magnetic-field amplitude generated by applying a 1 A current in opposite directions through the central segments of the hairpin electrodes. The point $y = 25 \mu\text{m}$ corresponds to the trap center. The spacing between the electrodes is optimized to suppress the residual B_z component and to avoid unwanted AC Zeeman shifts.

3.4.3 Effect of anharmonicity

Since it is critical to conserve phase coherence, it is necessary to consider the harmonicity of the axial trapping potential created by the DC electrodes in Fig. 3.2. Experimentally, potentials applied to the twenty DC electrodes can be used to minimize anharmonicities in the axial potential.

The optimized axial trapping potential can be expanded as

$$V(y) = V(0) (1 + c_2 y^2 + c_4 y^4 + c_6 y^6), \quad (3.25)$$

with $c_2 = 1 (\mu\text{m})^{-2}$. We find coefficients $c_4 = 10^{-7} (\mu\text{m})^{-4}$ and $c_6 = -2 \times 10^{-9} (\mu\text{m})^{-6}$, while odd and higher even order terms are negligible, assuming a typical 16-bit DAC voltage uncertainty of $200 \mu\text{V}$ for each trapping voltage.

From Sec. 2.2.3, the resulting fractional frequency shift is

$$\frac{\Delta\omega_a}{\omega_a} \approx \frac{3A^2 c_4}{4c_2} + \frac{15A^4 c_6}{16c_2}, \quad (3.26)$$

where A is the amplitude of the electron motion.

To excite the motion above the Johnson noise at 0.4 K, the amplitude must exceed $A > 1.3 \mu\text{m}$. At this amplitude, the relative frequency shift due to trap anharmonicity would be only

$$\frac{\Delta\omega_a}{\omega_a} \approx 1.2 \times 10^{-7},$$

corresponding to an absolute shift of the resonance frequency of 36 Hz, which is negligible on the considered timescale of 10 μ s.

3.4.4 Qubit initialization

With the state read-out in place, we can initialize the electron spin by first measuring it and then flipping it conditioned on the measurement result. If the electron is found in the state in which we would like to initialize it, nothing is done. If it is in the other state, we perform a π -pulse using a microwave field.

3.5 Quantum gates with trapped electrons

3.5.1 Single-qubit gates

Single-qubit gates can be performed using a homogeneous transverse magnetic field created by the hairpin electrodes drive near the Zeeman splitting, similar to how error rates of 10^{-6} have been achieved for ions [58].

For an electron with $g_e = 2$, the magnetic moment operator is

$$\hat{\mu} = -\mu_B \hat{\sigma}, \quad V = -\hat{\mu} \cdot \mathbf{B}, \quad (3.27)$$

where μ_B is the Bohr magneton and $\hat{\sigma}$ is the vector of Pauli operators.

Assume a static magnetic field along the z direction,

$$\mathbf{B}_0 = B_0 \hat{z}, \quad (3.28)$$

which defines the quantization axis $\hat{z}$. The Zeeman Hamiltonian is then

$$\hat{H}_{\text{spin}} = \frac{1}{2} \hbar \omega_0 \hat{\sigma}_z, \quad \omega_0 = \frac{e B_0}{m}, \quad (3.29)$$

where m is the electron mass.

The axial motional Hamiltonian is

$$\hat{H}_a = \frac{\hat{p}_y^2}{2m} + \frac{1}{2} m \omega_y^2 \hat{y}^2 + \sum_{n=3}^{\infty} V_n \hat{y}^n = \hbar \omega_y a^\dagger a + \sum_{n=3}^{\infty} V_n y_0^n (a + a^\dagger)^n, \quad (3.30)$$

where $y_0 = \sqrt{\hbar/(2m\omega_a)}$ is the zero-point motion and a ($a^\dagger$) are the annihilation (creation) operators of the axial mode.

We apply a driving (RF) field that oscillates along the x axis

$$V_{\text{drive}} = \mu_B \hat{\sigma}_x B_{1x} e^{i\omega t}. \quad (3.31)$$

Neglecting the trap field anharmonicities, in the frame rotating with the drive frequency ω and after performing the rotating-wave approximation (RWA), the effective single-qubit Hamiltonian is

$$\hat{H} = -\frac{1}{2}\hbar\delta\hat{\sigma}_z + \frac{1}{2}\hbar\Omega\hat{\sigma}_x, \quad (3.32)$$

where

$$\delta = \omega - \omega_0, \quad \Omega = \frac{2\mu_B B_{1x}}{\hbar} = \frac{eB_{1x}}{m}. \quad (3.33)$$

In resonance ($\delta = 0$), the spin-flip probability is

$$P_{\uparrow \rightarrow \downarrow}(t) = \sin^2\left(\frac{\Omega t}{2}\right). \quad (3.34)$$

In our blueprint trap design, applying 0.1 A of current to the hairpin electrodes at frequency ω_0 corresponds to a single-qubit gate Rabi frequency of $\Omega \approx 2\pi \times 10$ MHz.

3.5.2 Two-qubit gates

For performing two-qubit gates, the Mølmer-Sørensen gate [24, 59] and its controlled phase gate ($\sigma_z \otimes \sigma_z$) variant [8] are considered. While typically performed using optical-frequency radiation, it can also be implemented using static or oscillating magnetic field gradients [60–65]. The general idea is that a force that oscillates nearly resonant with a mode frequency of the two-electron crystal excites the motion if the force on the individual electrons has the correct symmetry.

An oscillating magnetic field couples the internal states and the motion of the electrons

$$\mathbf{B}_{\text{drive}} = \cos(\omega t + \phi) \left[\mathbf{B}^0 + \vec{\delta}^i \mathbf{B}' \right], \quad (3.35)$$

where $\vec{\delta}^i$ is the displacement of the electron i from its equilibrium position, $\mathbf{B}^0$ is the magnetic field strength at the equilibrium position of the electron and $\mathbf{B}' = \partial \mathbf{B} / \partial \hat{\delta}$ is the magnetic field gradient.

We can write the displacement $\vec{\delta}^i$ for the electron i in terms of normal modes [66]

$$\vec{\delta}^i = \hat{q} \sum_{j=1}^n b_{jn} q_j, \quad (3.36)$$

where q_j is the amplitude of the j -th normal mode of the electron chain and b_{jn} is its eigenvector. For the COM mode of the two-electron chains, $b_j = 1/\sqrt{2}$ for both electrons. We quantize the motion using the normal-mode creation and annihilation operators

$$q_j = q_j^0 (a_j + a_j^\dagger), \quad (3.37)$$

where $q_j^0 = \sqrt{\hbar/(2m\omega_j)}$ is the size of the ground-state wave packet, ω_j is the frequency for the j -th normal mode, and m is the mass of each electron.

Taking into account only the axial COM mode $a(a^\dagger)$, the interaction energy between the electron i and the magnetic field is

$$\begin{aligned}
\hat{H}_i &= -\hat{\boldsymbol{\mu}} \cdot \mathbf{B}_{\text{drive}} \\
&= \mu_B \hat{\boldsymbol{\sigma}} \cdot \mathbf{B}_{\text{drive}} \\
&= \mu_B \cos(\omega t + \phi) \left[\hat{\sigma}_{i,x} (\mathbf{B}_x^0 + \mathbf{B}'_x(y) \frac{y_0}{\sqrt{2}} (a + a^\dagger)) \right. \\
&\quad + \hat{\sigma}_{i,y} (\mathbf{B}_y^0 + \mathbf{B}'_y(y) \frac{y_0}{\sqrt{2}} (a + a^\dagger)) \\
&\quad \left. + \hat{\sigma}_{i,z} (\mathbf{B}_z^0 + \mathbf{B}'_z(y) \frac{y_0}{\sqrt{2}} (a + a^\dagger)) \right].
\end{aligned} \tag{3.38}$$

We consider a two-electron chain aligned along the axial direction. The interaction-free Hamiltonian for the two-electron system is

$$\hat{H}_0 = \sum_{i=1}^2 \frac{1}{2} \hbar \omega_0 \hat{\sigma}_{i,z} + \hbar \omega_a a^\dagger a, \tag{3.39}$$

where $\hat{\sigma}_{i,z}$ is the Pauli z -operator for electron i and $a(a^\dagger)$ is the creation and annihilation operator of the axial COM model.

$\sigma_z \sigma_z$ gate

For the phase gate variant, a gradient oscillating at $\omega_a \pm \delta$ with $\delta \ll \omega_a$ excites the center-of-mass motion of the electron crystal along the axial direction if the electrons are in the same spin-state, i.e. $|\uparrow\uparrow\rangle$ or $|\downarrow\downarrow\rangle$. If the electrons are in the $|\uparrow\downarrow\rangle$ or $|\downarrow\uparrow\rangle$ state, the net force vanishes and the electron-COM (Center-Of-Mass) motion is not excited.

Consider only the term $B'_z(y)$ in Eq. 3.38, the corresponding Hamiltonian in the interaction picture with respect to H_0 is given by [8, 59]

$$\begin{aligned}
\hat{H}_I &= \mu_B \frac{\partial B_z}{\partial y} \frac{y_0}{\sqrt{2}} \frac{e^{i(\omega t + \phi)} + e^{-i(\omega t + \phi)}}{2} (\hat{\sigma}_{1,z} + \hat{\sigma}_{2,z}) (a e^{-i\omega_a t} + a^\dagger e^{i\omega_a t}) \\
&= \mu_B \frac{\partial B_z}{\partial y} \frac{y_0}{2\sqrt{2}} (I \otimes \hat{\sigma}_z + \hat{\sigma}_z \otimes I) (\hat{a} e^{i(\delta t + \phi)} + \hat{a}^\dagger e^{-i(\delta t + \phi)}) \\
&= \hbar \Omega_R (I \otimes \hat{\sigma}_z + \hat{\sigma}_z \otimes I) (\hat{a} e^{i(\delta t + \phi)} + \hat{a}^\dagger e^{-i(\delta t + \phi)}),
\end{aligned} \tag{3.40}$$

where $\delta = \omega - \omega_a$ is the detuning of the drive, Ω_R is the Rabi frequency of the two-qubit gate.

$$\Omega_R = \mu_B \frac{\partial B_z}{\partial y} \frac{y_0}{2\sqrt{2}\hbar}. \tag{3.41}$$

Using the Magnus expansion [67],

$$U(t) = e^{\Omega(t)}$$

$$\Omega(t) = \sum_k \Omega_k(t) = \int_0^t A(\tau) d\tau + \frac{1}{2} \int_0^t d\tau \int_0^\tau d\sigma [A(\tau), A(\sigma)] + \dots \quad (3.42)$$

with

$$A(t) = -\frac{i}{\hbar} \hat{H} = \dot{\gamma}(t)a - \dot{\gamma}^*(t)a^\dagger, \quad (3.43)$$

and, in our case,

$$\dot{\gamma}(t) = (-i\Omega_R)e^{i(\delta t + \phi)}(I \otimes \hat{\sigma}_z + \hat{\sigma}_z \otimes I), \quad (3.44)$$

one finds the first-order term

$$\Omega_1(t) = (I \otimes \hat{\sigma}_z + \hat{\sigma}_z \otimes I)(\alpha \hat{a}^\dagger - \alpha^* \hat{a}), \quad (3.45)$$

with

$$\alpha = (-i\Omega_R) \frac{2}{\delta} \sin\left(\frac{\delta t}{2}\right) e^{-i\phi} e^{-i\delta t/2}. \quad (3.46)$$

At gate time

$$T = \frac{2\pi}{\delta}, \quad (3.47)$$

the oscillator returns to its original phase space point for all four logical eigenstates, enclosing an area in phase space. For our parameters, the anticipated gate time for a two-qubit phase gate is about 2 μ s. The propagator reduces to a pure spin operation,

$$U_{zz}(T) = \exp\left[\frac{2\pi i}{\delta^2} \sum_n \Omega_{j,n}^z \hat{\sigma}_z^{(n)}\right], \quad (3.48)$$

where $\Omega_{j,n}^z = (\pm)\Omega_R$.

Choosing

$$\Omega_R = \frac{\delta}{4} \quad (3.49)$$

will implement a $\sigma_z \sigma_z$ phase gate. For example,

$$|\uparrow\downarrow\rangle \rightarrow i|\uparrow\downarrow\rangle, \quad |\uparrow\uparrow\rangle \rightarrow |\uparrow\uparrow\rangle. \quad (3.50)$$

An entangled state can be generated using

$$R_x\left(\frac{\pi}{2}\right) U_{\text{gate}} R_x\left(\frac{\pi}{2}\right) |\downarrow\downarrow\rangle = \frac{1}{\sqrt{2}} (|\downarrow\downarrow\rangle - i|\uparrow\uparrow\rangle), \quad (3.51)$$

The advantage of the $\sigma_z \otimes \sigma_z$ gate scheme is that static qubit frequency shifts commute with the gate Hamiltonian and thus can be canceled with a spin-echo sequence. However,

since the drive is near the motional frequency, this might cause unwanted excitation via residual electric fields. The $\sigma_z \otimes \sigma_z$ gate can be implemented with all fields far detuned from the motional frequencies [68]. Such variants have been implemented using ions [60] with Bell state fidelities $F = \text{Tr}(\rho_{\text{exp}}\rho_{\text{Bell}})$ reaching 99.7% [69], limited by motional heating and trap frequency instabilities, and 99.9% [68], limited mainly by motional dephasing, where ρ_{exp} , ρ_{Bell} are the density matrix operators of the experimentally prepared state and the ideal Bell-state intended to prepare respectively.

$\sigma_\phi\sigma_\phi$ gate

Now consider the Mølmer-Sørensen type gate where we simultaneously apply two equal-amplitude microwave fields detuned from the first blue and red sidebands with the frequencies (assuming $\omega_0 > \omega_a$, which can be achieved by increasing the static magnetic field) [24, 59]

$$\omega_b = \omega_0 + \omega_a - \delta, \quad (3.52)$$

$$\omega_r = \omega_0 - \omega_a + \delta, \quad (3.53)$$

with $|\mathbf{B}_b| = |\mathbf{B}_r|$ and phases ϕ_b and ϕ_r .

As discussed above, driving the hairpin electrodes with currents in the opposite direction in the middle segments will create an oscillating magnetic field gradient $B'_y(y)$. Consider only the $B'_y(y)$ term in Eq. 3.38, the interaction Hamiltonian is given by

$$\begin{aligned} \hat{H}_I = & \mu_B \frac{\partial B_{b,y}}{\partial y} \frac{y_0}{\sqrt{2}} \frac{e^{i(\omega_b t + \phi_b)} + e^{-i(\omega_b t + \phi_b)}}{2} (\hat{\sigma}_{1,y} + \hat{\sigma}_{2,y})(a + a^\dagger) \\ & + \mu_B \frac{\partial B_{r,y}}{\partial y} \frac{y_0}{\sqrt{2}} \frac{e^{i(\omega_r t + \phi_r)} + e^{-i(\omega_r t + \phi_r)}}{2} (\hat{\sigma}_{1,y} + \hat{\sigma}_{2,y})(a + a^\dagger). \end{aligned} \quad (3.54)$$

The corresponding Hamiltonian in the interaction picture with respect to $\hat{H}_0$ is given by

$$\begin{aligned} \hat{H}_I = & \mu_B \frac{\partial B_{b,y}}{\partial y} \frac{y_0}{\sqrt{2}} \frac{e^{i(\omega_b t + \phi_b)} + e^{-i(\omega_b t + \phi_b)}}{2} (-i\hat{S}_+ e^{i\omega_0 t} + i\hat{S}_- e^{-i\omega_0 t})(ae^{-i\omega_a t} + a^\dagger e^{i\omega_a t}) \\ & + \mu_B \frac{\partial B_{r,y}}{\partial y} \frac{y_0}{\sqrt{2}} \frac{e^{i(\omega_r t + \phi_r)} + e^{-i(\omega_r t + \phi_r)}}{2} (-i\hat{S}_+ e^{i\omega_0 t} + i\hat{S}_- e^{-i\omega_0 t})(ae^{-i\omega_a t} + a^\dagger e^{i\omega_a t}), \end{aligned} \quad (3.55)$$

where $\hat{S}_\pm = \hat{\sigma}_{1,\pm} + \hat{\sigma}_{2,\pm}$ is the raising (lowering) operator.

Keeping only near-resonant terms, the interaction Hamiltonian can be written as

$$\hat{H}_I = i\hbar\Omega_R(I \otimes \hat{\sigma}_- + \hat{\sigma}_- \otimes I) (ae^{-i(\delta t - \phi_b)} + a^\dagger e^{i(\delta t + \phi_r)}) + \text{h.c.}, \quad (3.56)$$

where

$$\Omega_R = \frac{\mu_B}{\hbar} \frac{\partial B_y}{\partial y} \frac{y_0}{2\sqrt{2}}. \quad (3.57)$$

For simplicity, take $\phi_b = \phi_r = 0$. Then an equivalent form for a $\sigma_\phi \sigma_\phi$ gate is

$$\hat{H}_I = \hbar \Omega_R (I \otimes \hat{\sigma}_y + \hat{\sigma}_y \otimes I) (ae^{-i\delta t} + a^\dagger e^{i\delta t}). \quad (3.58)$$

Instead, if we consider the $B'_x(y)$ term in Eq. 3.38, the gate Hamiltonian will be

$$\hat{H}_I = \hbar \Omega_R (I \otimes \hat{\sigma}_x + \hat{\sigma}_x \otimes I) (ae^{-i\delta t} + a^\dagger e^{i\delta t}), \quad (3.59)$$

when $\phi_b = \phi_r = 0$. For the $\sigma_\phi \otimes \sigma_\phi$ gate, an additional dynamical decoupling field is needed to suppress errors due to fluctuations of the qubit frequency, which is the main source of error from other microwave-based trapped-ion experiments.

In our configuration, using the axial COM mode with frequency $\omega_a = 2\pi \times 300$ MHz and a bichromatic magnetic field gradient of 120 T/m (corresponds to 1 A of current) leads to a two-qubit gate Rabi frequency of $\Omega_R = 2\pi \times 105$ kHz, which corresponds to a single-loop gate time of ~ 2.4 μ s.

3.6 Wigner crystal

To perform two-qubit operations on a trapped electron quantum processor, it is essential that the electrons form a Wigner crystal inside the trap [45]. When the kinetic energy of the electrons is large, they will form a cloud: the oscillation amplitude exceeds their mutual separation, enabling the electrons to exchange positions. In this regime, the eigenmodes of the system are not well defined, as the normal modes become strongly coupled and effectively scrambled by reordering events. Furthermore, radial secular motion becomes coupled with RF-driven micromotion, allowing energy to flow from micromotion to secular modes through a process commonly known as RF heating [70].

When the electrons are sufficiently cooled so that the potential energy associated with their mutual Coulomb repulsion dominates the kinetic energy, their motion becomes stable and well described by normal modes. In this low-energy regime, the electrons form a Wigner crystal, analogous to the Coulomb crystals widely studied in trapped-ion systems [71]. A stable Wigner crystal is therefore essential for implementing high-fidelity two-qubit gates that rely on shared motional modes when the participating electrons are confined within the same trap. In this work, we define the electrons as in the cloud state if they undergo positional swapping along any spatial direction.

To determine the threshold temperature below which Wigner crystallization is maintained, we numerically simulate the full three-dimensional dynamics of two electrons in a linear Paul trap, as reported in Ref. [45]. The trajectory is obtained by numerically integrating the equations of motion using the Velocity Verlet algorithm,

$$m\ddot{\mathbf{r}} = \mathbf{F}_{\text{trap}}(\mathbf{r}, t) + \mathbf{F}_i^{\text{Coul}}, \quad (3.60)$$

where $\mathbf{F}_{\text{trap}}(\mathbf{r}, t)$ is the trapping force corresponding to the trapping potential discussed in Sec. 2.1

$$\mathbf{F}_{\text{trap}}(\mathbf{r}, t) = -q\nabla[\Phi_{\text{dc}}(\mathbf{r}) + \Phi_{\text{rf}}(\mathbf{r}, t)], \quad (3.61)$$

and $\mathbf{F}_i^{\text{Coul}}$ ($i \in (1, 2)$) is the Coulomb force

$$\mathbf{F}_1^{\text{Coul}} = \frac{q^2}{4\pi\epsilon_0} \frac{\mathbf{r}_1 - \mathbf{r}_2}{|\mathbf{r}_1 - \mathbf{r}_2|^3}, \quad (3.62)$$

$$\mathbf{F}_2^{\text{Coul}} = \frac{q^2}{4\pi\epsilon_0} \frac{\mathbf{r}_2 - \mathbf{r}_1}{|\mathbf{r}_1 - \mathbf{r}_2|^3}, \quad (3.63)$$

corresponding to the Coulomb potential between electrons at positions $\mathbf{r}_1$ and $\mathbf{r}_2$

$$U_{\text{Coul}}(\mathbf{r}_1, \mathbf{r}_2) = \frac{q^2}{4\pi\epsilon_0} \frac{1}{|\mathbf{r}_1 - \mathbf{r}_2|} = \frac{q^2}{4\pi\epsilon_0} \frac{1}{\sqrt{\Delta x^2 + \Delta y^2 + \Delta z^2}}. \quad (3.64)$$

We simulate the full 3-D motion in which two electrons oscillate in both the radial and the axial directions. The two-electron system decomposes into center-of-mass (COM) and relative (stretch) modes along each axis, yielding six normal modes in total, four of which are non-degenerate (assuming the two radial modes share the same frequency). Because only the relative motion changes the inter-electron spacing, it dominates the crystal-to-cloud transition. Consequently, we define the *threshold energy* as the maximum initial kinetic energy in the relevant relative coordinate (axial or radial) for which the Wigner-crystal lifetime exceeds 1 ms. The axial and radial threshold energies are obtained separately by varying the initial kinetic energy in the respective direction while keeping the remaining modes thermally populated at

$$E_{\text{kin}} = \frac{1}{2} k_B T, \quad T = 0.4 \text{ K},$$

consistent with the proposed cryogenic operating temperature.

The result obtained is shown in Fig. 3.7, where the electron reordering rate f_{reorder} is plotted with respect to the temperature T . The reordering rate is defined as the inverse of the lifetime of the Wigner crystal. $\bar{f}_{\text{reorder}}$ is the mean reordering rate assuming that the thermal energies of trapped electrons follow a Boltzmann distribution

$$P(E, T) \propto \exp \{-E/(k_B T)\}.$$

The mean rate is calculated by weighting the simulated reordering rate with the Boltzmann distribution, of the form

$$\bar{f}_{\text{reorder}}(T) = \int_0^\infty P(E, T) f_{\text{reorder}}(E) dE.$$

The resulting curve is plotted in red, and we found that for a mean reordering rate below 10^3 , the axial and radial temperatures must be below 1.99 K and 7.82 K, respectively. The rate threshold of 10^3 was chosen as a compromise between the desired long lifetimes of the Wigner crystal of more than 1 ms and the computational cost of simulating for such long times. A double exponential fit of the form $f_{\text{reorder}}(E) = \exp \{A [1 - \exp \{(E_0 - E)/\tau\}] + f_0\}$ is performed to extrapolate the reordering rate beyond the 1 ms limit, with fitting parameters A , E_0 , τ , f_0 . Finally, assuming a representative gate time of 2 μs , achieving error rates at the level 10^{-4} requires crystal lifetimes of 20 ms, which corresponds to axial and radial temperatures below 1.52 K and 5.91 K, respectively.

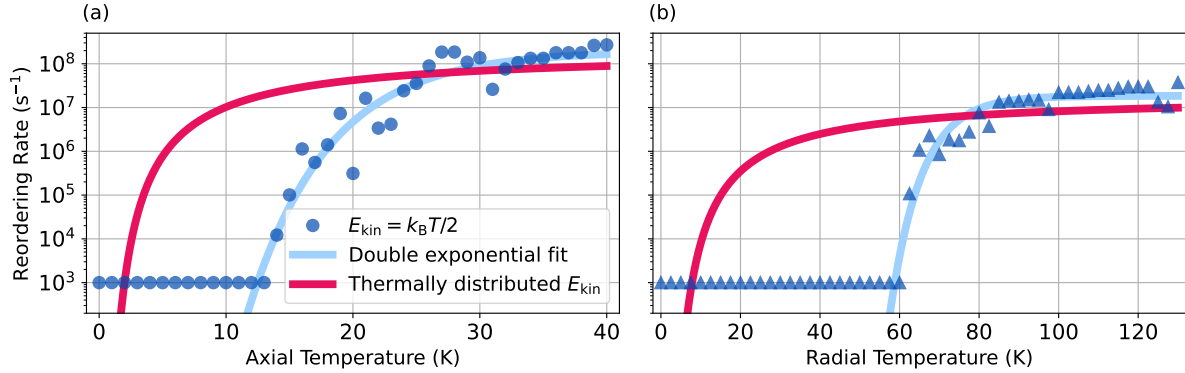


Figure 3.7: Reordering rate of a two-electron chain as a function of the initial (a) axial and (b) radial energies. Blue points show numerical simulations at fixed energies, with light-blue curves representing double-exponential fits. The mean reordering rate $\bar{f}_{\text{reorder}}$, obtained by averaging over a Boltzmann distribution, is plotted in red. The horizontal axes are expressed in Kelvin for comparison with the cryogenic environment. Simulations use an RF drive at $\Omega_{\text{rf}}/2\pi = 10.6$ GHz, radial frequency $\omega_r/2\pi = 2$ GHz, and axial frequency $\omega_a/2\pi = 300$ MHz. Figure reproduced from Ref. [45]. The simulations were performed by Andris Huang and Edith Hausten.

3.7 Scalability

Fault-tolerant quantum computation requires a large number of physical qubits. In this section, we outline the most natural path towards scalability in our platform, drawing inspiration from the quantum charge-coupled device (QCCD) model developed for trapped ions [72–74], while highlighting key advantages offered by the much higher trap frequencies attainable for electrons.

3.7.1 Modularization

A modular approach is preferred for constructing a large-scale quantum computer, and the most promising strategy for scaling trapped electron systems is to adapt the quantum charge-coupled device (QCCD) architecture from trapped ions [72–74], as shown in Fig. 3.8. Since the trap frequencies for electrons we consider are approximately two orders of magnitude larger than typical values for trapped ions, we expect that electron transport and operations such as splitting and merging of the electron crystals can be carried out substantially faster than on equivalent trapped ions QCCD architecture.

Combining two-qubit gates with shuttling operations, we imagine entangling two electrons at specific sites in the QCCD device. Shuttling and splitting operations on electron crystals can be used to isolate a selected pair of electrons in a region where a localized mag-

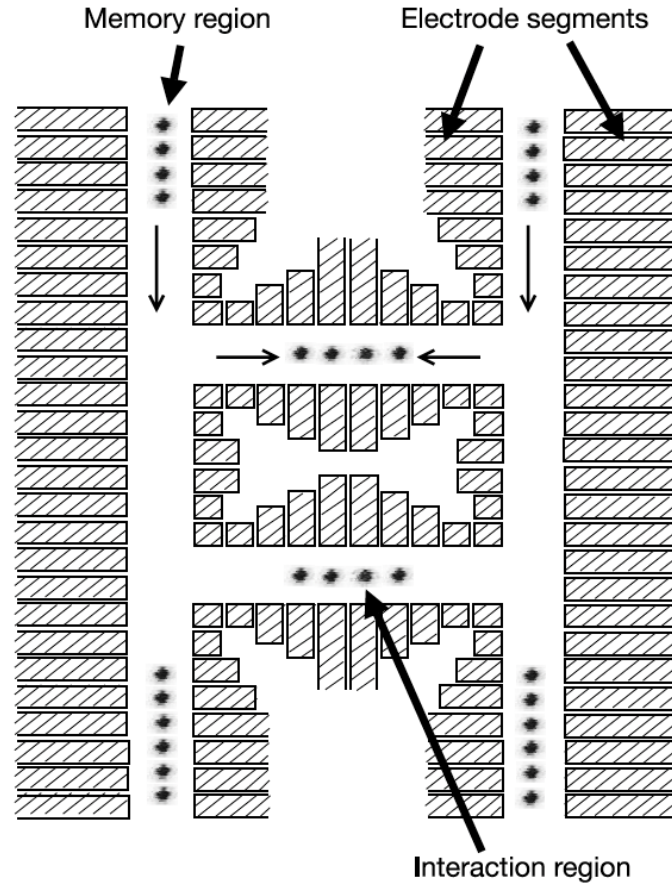


Figure 3.8: Conceptual diagram of a QCCD (quantum charge-coupled device) architecture. Electrons are stored in memory zones and transported to interaction zones for logic operations. Thin arrows indicate transport and confinement along the local trap axis. Figure adapted from Ref. [72].

netic field gradient can be applied to perform gates. These processing sites, with current-carrying wires providing the necessary magnetic-field gradient, would be on the order of $200\text{ }\mu\text{m}$ long to allow for high-fidelity single-qubit addressing. In addition, the trap frequency would serve as an important discriminating element. Thus, we expect that crosstalk for multi-qubit gates can be even lower than that for single-qubit gates. The electrons could be more densely packed in dedicated storage regions and can be shuttled between these two types of region in a conveyor-belt fashion, and junctions connect different processing and storage units with each other on a 2D-grid to create large entangled states.

In this context, it is worthwhile to mention that the cooling mechanism through image currents and parametric coupling does not impact the internal state of the electrons. Thus, we can cool the electrons while keeping the quantum information intact, making the QCCD architecture even more attractive. This is to be contrasted to trapped ions, where laser cooling on an ion destroys the quantum information stored in the ion thereby requiring another ion species for recooling during the computation. Together with fast cooling times on the order of microseconds, we expect very little overhead from cooling.

A popular concept for many optically active qubits, including trapped ions, is that a distributed network of qubit nodes can be created using remote entanglement through photon interference. Electrons do not offer this modality, but to build a modular electron quantum computer, coupling of remote electron qubits could be engineered with a high-impedance co-planar waveguide that distributes image currents between distant sites. In Ref. [10], the authors estimate such a coupling between two electrons to be on the order of 100 kHz , which is a significant improvement over current implementations of remote entanglement in trapped ion systems [75, 76].

3.7.2 Individual addressing

As with any qubit platform controlled by microwave radiation, selective manipulation of individual qubits is of concern, especially if they are packed densely. However, several mechanisms enable individual addressing of trapped electrons. A straightforward approach is to apply a magnetic-field gradient, which shifts the local Zeeman splitting of each electron and thereby separates their spin-resonance frequencies. This enables frequency-selective single-qubit rotations. Care must be taken that the quadratic (second-order) component of the magnetic field remains sufficiently small, as excessive curvature can introduce dephasing and reduce the coherence time T_φ . A further concern with this approach is that the expected spacing is only on the order of $5\text{ }\mu\text{m}$ which, in turn, would require quite strong magnetic field gradients. These constraints can be relaxed by physically separating electrons during computation using the QCCD architecture paradigm discussed above in the context of scaling to isolate the individual qubits by approximately $100\text{ }\mu\text{m}$ or more.

Another approach is to shape the excitation field to avoid excitation of one of the electrons. For instance, we can position the two-electron crystal so that one electron is in the microwave-node while the other is not. Therefore, we can change the spin-state of one electron, while the other electron will not experience the microwave excitation. A similar method

has already been shown for ions in Ref. [77]. We note that both methods can be combined, substantially reducing the demands. Furthermore, even if residual off-resonant excitation occurs, single-qubit addressing errors are generally easy to characterize and correct: since single-qubit errors are unitary and local, they can be undone with calibrated rotations. This is in contrast to entangling gates, where unwanted crosstalk or addressing errors are non-local and significantly harder to compensate.

3.7.3 Participation control for multi-qubit operations

Similarly to single-qubit gates, we need to determine which electrons will participate in the gates. With the anticipated shuttling strategy focusing on small electron crystals of up to two electrons, we can use localized magnetic field gradients to determine to which ion-pairs the gate will be applied. Additionally, we can utilize the trap frequency as an additional discriminating element. Even detuning the trap frequency by a few tens of MHz will shut off the gate mechanism almost entirely. Thus, we anticipate no difficulties in controlling to which ions the multi-qubit gates are applied.

3.8 DiVincenzo's criteria

In the following, we summarize how the trapped-electron technology proposed in this chapter meets the DiVincenzo's criteria for the realization of a fault-tolerant quantum computer [78]:

1. A scalable physical system with well-characterized qubits:

The qubit is encoded in the spin states of a single electron confined in a Paul trap in a static magnetic field. Scalability is provided by adopting a QCCD-style architecture in which electrons are stored in densely packed memory regions and shuttled to dedicated processing zones, in close analogy to advanced trapped-ion devices.

2. The ability to initialize the qubits:

Initialization of the electron spin can be achieved by projective measurement followed by a conditional spin flip. Once the spin has been projected into either $|\uparrow\rangle$ or $|\downarrow\rangle$, a single-qubit π pulse is applied if necessary to prepare a chosen fiducial state (e.g. $|\downarrow\rangle$).

3. Coherence times that are much longer than the characteristic gate times:

The electron spin resides in an ultra-high vacuum, cryogenic environment. Magnetic-field noise can be strongly suppressed by shielding, and the coherence times are expected to be comparable to those achieved in state-of-the-art trapped-ion experiments, where T_2 values exceeding 1 s have been demonstrated [79]. In contrast, as discussed previously, the characteristic gate time is on the order of 1 μ s.

4. A universal set of quantum gates:

A universal set of quantum gates requires arbitrary single-qubit rotations together with at least one entangling two-qubit gate. In our architecture, single-qubit rotations are implemented by resonant microwave magnetic fields transverse to the quantization axis, generated by currents in the on-chip hairpin electrodes. Entangling gates are realized by oscillating magnetic-field gradients that couple the electron spin to the motional modes, enabling Mølmer-Sørensen-type interactions between pairs of electrons. Together, these operations form a universal gate set.

5. The ability to perform qubit-specific measurements.

Finally, qubit-specific measurement is achieved through the spin-dependent displacement and image-current detection scheme. A localized oscillating magnetic-field gradient is used to couple the spin of a selected electron to a chosen motional mode, while other electrons are either spectator electrons at different trap frequencies or physically separated in the QCCD layout. The resulting spin-dependent motional state is read out via the induced voltage across a cryogenic tank circuit.

In summary, the trapped electron in Paul trap platform satisfies all five DiVincenzo's criteria, and we believe it constitutes a viable route toward scalable, fault-tolerant quantum computation.

Chapter 4

Sources of decoherence of trapped electrons

High-fidelity gate operations are crucial to achieving fault-tolerant quantum computing. In this chapter, we analyze the common error sources for quantum gates between trapped electrons including motional heating, motional dephasing, trap frequency drift, magnetic field gradient inhomogeneity, trapping potential anharmonicity, and spin decoherence. We further carry out numerical simulations to determine dominant error sources and estimate the obtainable control over the trapped electron quantum computer.

We also note that in Ref. [80], we derive analytic expressions for gate infidelities of one and two-qubits arising from motional errors in trapped electrons and trapped ions. These results incorporate the impact of driving-field inhomogeneities on one- and two-qubit gates, as well as static motional frequency shifts, trap anharmonicities, heating, and motional dephasing on two-qubit gates. The treatments of static frequency shifts, anharmonicities, heating, and dephasing are applicable to both laser-based and laser-free gates, while the analysis of driving field inhomogeneities specifically applies to laser-free implementations.

4.1 Model and theory

In the following, we use the Bell-state fidelity as a proxy for operation fidelities. The Bell-state fidelity is defined as

$$F = \text{Tr}(\rho_{\text{exp}} |\Psi_{\text{Bell}}\rangle \langle \Psi_{\text{Bell}}|), \quad (4.1)$$

where $|\Psi_{\text{Bell}}\rangle$ is the state vector of the ideal targeted Bell-state and ρ_{exp} is the reduced density matrix of the total system tracing out the motional degree of freedom. The Bell-state infidelity is calculated as $1 - F$.

In this chapter, we consider the two-qubit gate Hamiltonian as described in Eq. (3.59):

$$\hat{H}_g = \hbar\Omega_R(\hat{I} \otimes \hat{\sigma}_x + \hat{\sigma}_x \otimes \hat{I})(\hat{a}e^{-i\delta t} + \hat{a}^\dagger e^{i\delta t}), \quad (4.2)$$

where Ω_R is the Rabi frequency of the two-qubit gate. When the qubit is initialized as $|\downarrow\downarrow\rangle$, the target state is

$$\begin{aligned} |\Psi_{\text{Bell}}\rangle &= U_g(T) |\downarrow\downarrow\rangle \\ &= \frac{1}{\sqrt{2}} (|\downarrow\downarrow\rangle + i |\uparrow\uparrow\rangle), \end{aligned} \quad (4.3)$$

for a gate time $T = 2\pi/\delta$ and Rabi frequency $\Omega_R = \delta/4$.

To analyze the effect of different error sources, we model two electron spins coupling to one motional mode and truncate the motional space to $n = 500$. In all simulations, the system begins in a product state between the two electron-spin qubits and the axial motional mode. Specifically, both qubits are initialized in the spin-down state $|\downarrow\rangle$, while the motional mode is prepared in a thermal state with mean occupation $\bar{n}$.

$$\rho(0) = |\downarrow\downarrow\rangle\langle\downarrow\downarrow| \otimes \rho_{\text{th}}(\bar{n}), \quad (4.4)$$

where

$$\rho_{\text{th}}(\bar{n}) = \sum_{n=0}^{\infty} \frac{\bar{n}^n}{(\bar{n} + 1)^{n+1}} |n\rangle\langle n| \quad (4.5)$$

is the thermal state of the harmonic oscillator with the mean number of phonons $\bar{n}$. Cooling the high-frequency radial mode at $\omega_t = 2\pi \times 2 \text{ GHz}$ to $T = 0.4 \text{ K}$ corresponds to a mean phonon occupation of $\bar{n} = 4$, as discussed in Sec. 3.2. Using parametric coupling between the radial and axial motions, the axial mode can be cooled to the same phonon number. For gate operations, we focus on the axial center-of-mass (COM) mode at $\omega_a = 2\pi \times 300 \text{ MHz}$, since the stretch mode would suffer substantially from cross-Kerr coupling between radial and axial motions. With a magnetic field gradient of 120 T/m , the axial COM mode yields a Rabi frequency of $\Omega_R = 2\pi \times 105 \text{ kHz}$, corresponding to a single-loop gate time of $2.4 \mu\text{s}$.

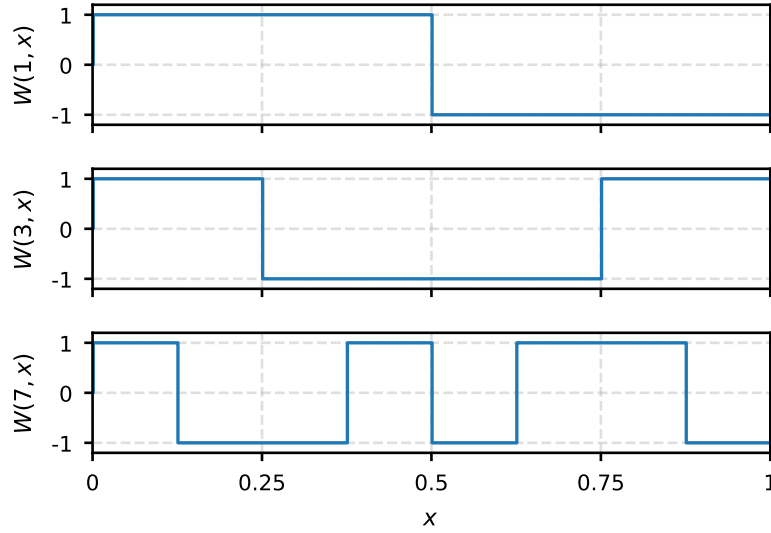
We numerically integrate the master equation in the Lindblad form using QuTip [81]:

$$\dot{\rho}(t) = -\frac{i}{\hbar} [\hat{H}, \rho] + \sum_n \left[\hat{L}_n \rho \hat{L}_n^\dagger - \frac{1}{2} \rho \hat{L}_n^\dagger \hat{L}_n - \frac{1}{2} \hat{L}_n^\dagger \hat{L}_n \rho \right], \quad (4.6)$$

where $\hat{L}_n$ is a Lindblad operator, $\hat{H} = \hat{H}_g + \hat{H}_e$ is the total Hamiltonian consisting of the gate dynamics $\hat{H}_g$ and the possible error sources described by $\hat{H}_e$.

4.1.1 Walsh modulation

Many of the error sources change slowly compared to the gate time and can be taken as quasi-static. This opens up the possibility of using dynamical decoupling sequences (for example, Walsh modulation) as used in state-of-the-art trapped-ion experiments [64, 68, 69, 82–84] with a modest overhead in gate duration. Using Walsh modulation [85], one can suppress errors from slow frequency noise, which prevents the motional trajectory from closing exactly. The resulting residual entanglement between spin and motion would cause errors that grow with the temperature of the motional mode.

Figure 4.1: Walsh functions $W(1, x)$, $W(3, x)$ and $W(7, x)$.

Walsh modulation suppresses such errors by periodically flipping the sign of the spin-dependent force. In the MS interaction, these sign flips alter the direction of motion in phase space, causing the trajectory to form a multi-loop geometric figure whose end point is guaranteed to return to the origin to higher order in the frequency error. This is implemented by applying controlled phase shifts to the red and blue sideband fields,

$$\phi_r \rightarrow \phi_r + \pi, \quad \phi_b \rightarrow \phi_b + \pi, \quad (4.7)$$

which reverses the sign of the Hamiltonian,

$$\hat{H} \rightarrow -\hat{H},$$

exactly at a set of times determined by a Walsh function $W(k, x)$ of index k . A Walsh function is a piecewise-constant sequence that alternates between ± 1 in a pattern designed to cancel successive orders of timing or detuning error, as shown in Fig. 4.1. Notice that $W(3, x)$ can be constructed as two sequential $W(1, x)$ functions with a phase flip on the second pulse. Similarly, $W(7, x)$ can be constructed as sequential $W(3, x)$ pulses with a phase flip on the second pulse.

Ref. [85] shows that by choosing a Walsh function with index $k = 2^n - 1$, where n is an integer and a detuning $\delta = 2^{n+1}\pi/t_g$, where t_g denotes the gate time, phase flips only occur at integer multiples of $T = 2\pi/\delta$ and the resulting multi-loop trajectory guarantees that the leading n orders of detuning error cancel in the displacement operator $\mathcal{D}(\alpha)$.

4.2 Motional heating

Instabilities of the trap frequency are caused by voltage noise, finite temperature of the electrons in combination with anharmonic potentials, and surface electric field noise. Trap frequency instabilities lead to a fluctuating detuning of the drive frequency from the motional sidebands at ω_a , and thus to errors.

One of the dominant sources of error can be motional heating due to surface electric field noise. Since the spin state is entangled with the motional mode during the gate, any heating of the motion will lead to an error. Following Ref. [86], we model motional heating as a coupling to an infinite temperature bath. And this leads to the Lindblad operators

$$\begin{aligned} L_- &= \sqrt{\gamma} \hat{a}, \\ L_+ &= \sqrt{\gamma} \hat{a}^\dagger, \end{aligned} \quad (4.8)$$

where $\gamma = \dot{n}_e$ is the motional heating rate, $\hat{a}^\dagger$ and $\hat{a}$ are the creation and annihilation operator of the gate motional mode, respectively.

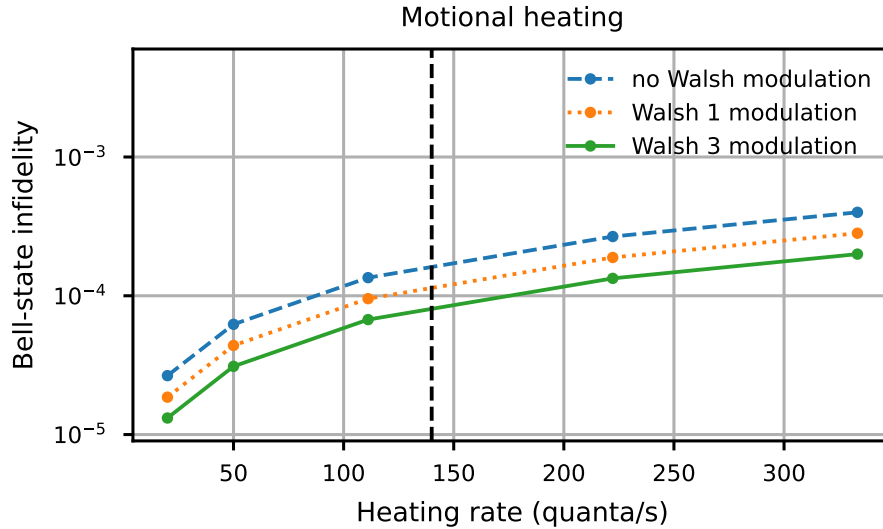


Figure 4.2: Numerical simulation of gate error due to motional heating for an electron two-qubit gate with $\bar{n} = 40$ and $\Omega_R = 2\pi \times 105$ kHz. Blue dashed, orange dotted, and green solid curves correspond to no modulation, Walsh-1 modulation, and Walsh-3 modulation, respectively. The black dashed line indicates the anticipated heating rate of $\dot{n} = 140$ quanta/s.

The motional heating rate is proportional to the electric-field spectral noise density $S(\omega_a)$ [87, 88]

$$\dot{n} = \frac{e^2}{4m_e \hbar \omega_a} S(\omega_a). \quad (4.9)$$

For a 30- μm ion-electrode distance, $S(\omega_a)$ near 1 MHz is expected to be approximately $10^{-12} \text{ V}^2\text{m}^2/\text{Hz}$ at cryogenic temperatures [87]. This corresponds to a heating rate of $\dot{n}_{Ca} = 100$ quanta/s at $\omega_a^{Ca} = 2\pi \times 1 \text{ MHz}$ for Ca^+ ions.

In order to extrapolate to GHz frequencies, we assume that the spectral noise density follows the power law

$$S(\omega_a) \propto 1/\omega_a^\gamma.$$

For ion traps, values of γ ranging from 1 to 1.5 have been measured [88–91]. Taking into account the light mass of electrons and an axial frequency of $\omega_a = 2\pi \times 300 \text{ MHz}$, the expected motional heating rate for electrons is

$$\dot{n}_e = \frac{m_{Ca}}{m_e} \left(\frac{\omega_a^{Ca}}{\omega_a} \right)^{1+\gamma} \dot{n}_{Ca}. \quad (4.10)$$

For niobium traps at cryogenic temperatures, an exponent $\gamma = 1.3$ has been found [90], leading to an estimate of the heating rate of 14 quanta/s, which corresponds to a limit of the motional coherence time of 70 ms. Due to the uncertainties of extrapolation over two orders of magnitude in frequency, we conservatively assume a heating rate of 140 quanta/s. For transverse modes near 2 GHz, motional heating should be reduced by approximately two orders of magnitude to order 1 quanta/s. The simulation result is shown in Fig. 4.2 assuming an initial $\bar{n} = 40$ which corresponds to a temperature of 4 K to be more conservative.

4.3 Trap frequency instabilities

Formally, we distinguish between fast noise (dephasing) and slow noise (trap frequency fluctuations) compared to the gate duration. Slow fluctuations can be effectively mitigated using modified phase space trajectories [64, 84, 85, 92, 93]. Fast noise, on the other hand, cannot be easily mitigated, other than by speeding up the quantum gates or by performing more loops in phase space.

Trap frequency fluctuation (Motional frequency drift)

Trap frequency fluctuations are modeled with the Hamiltonian

$$\hat{H}_e = \hbar \Delta \hat{a}^\dagger \hat{a}, \quad (4.11)$$

where Δ is the motional frequency detuning from ω_a .

Trap frequency fluctuations due to noise of the voltages applied to the electrodes are expected to be slow if low-pass filtered. Typical voltage sources provide fractional voltage stabilities on the order of 10^{-5} , corresponding to a motional frequency fluctuation $\Delta = 2\pi \times 3 \text{ kHz}$. Using Walsh modulation one can suppress errors from slow frequency noise [85]. The simulation result is shown in Fig. 4.3 assuming an initial $\bar{n} = 40$ which corresponds to a temperature of 4 K to be more conservative.

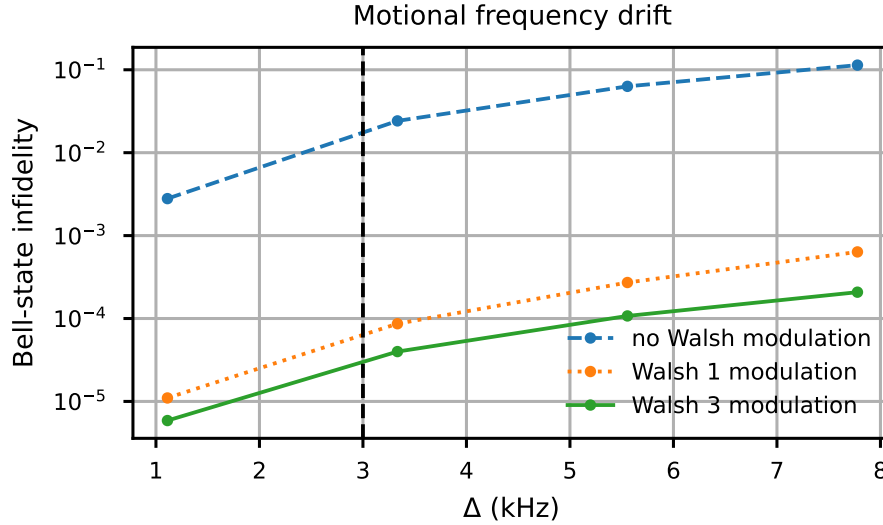


Figure 4.3: Numerical simulation of gate error due to motional frequency drift for an electron two-qubit gate with $\bar{n} = 40$ and $\Omega_R = 2\pi \times 105$ kHz. Blue dashed, orange dotted, and green solid curves correspond to no modulation, Walsh-1 modulation, and Walsh-3 modulation, respectively. The black dashed line corresponds to an expected frequency fluctuation of $\Delta = 2\pi \times 3$ kHz.

Motional dephasing

Motional dephasing is modeled with the Lindblad operator

$$L = \sqrt{\Gamma} \hat{a}^\dagger \hat{a},$$

where $\Gamma = 2/\tau_a$ is the dephasing rate of the electron motion and τ_a is the motional coherence time.

In order to estimate the size of the fast noise, we draw on the insights of a recent ion trap experiment where for a trapping height of $30 \mu\text{m}$ its impact has been estimated to contribute 6×10^{-4} to the Bell-state infidelity [57, 68]. The gate time in these experiments was several hundred microseconds, more than two orders of magnitude slower than what we expect for electrons. Thus, we expect that a significant amount of the fast noise observed in the ion trap experiments is actually slow in the context of electron quantum computing and can be removed with Walsh modulations. Assuming a $1/f$ characteristic, the remainder of the noise leads to a dephasing rate of $\Gamma = 2\pi \times 1.8 \times 10^{-3}$ Hz. The simulation result is shown in Fig. 4.4 assuming an initial $\bar{n} = 40$ which corresponds to a temperature of 4 K to be more conservative.

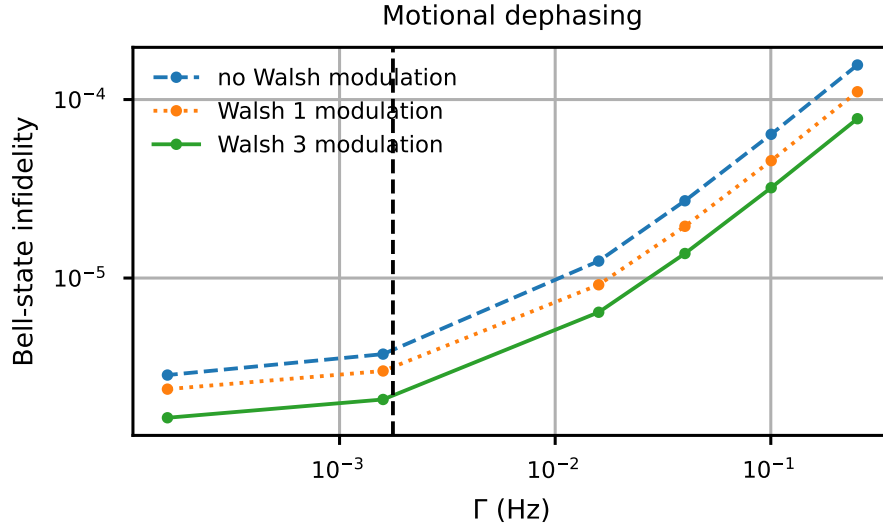


Figure 4.4: Numerical simulation of gate error due to motional dephasing for an electron two-qubit gate with $\bar{n} = 40$ and $\Omega_R = 2\pi \times 105$ kHz. Blue dashed, orange dotted, and green solid curves correspond to no modulation, Walsh-1 modulation, and Walsh-3 modulation, respectively. The black dashed line indicates the estimated dephasing rate of $\Gamma = 2\pi \times 1.8 \times 10^{-3}$ Hz.

4.4 Inhomogeneity of the magnetic field gradient

Because of a much lighter mass, the extent of the wave function for trapped electrons is much larger than that for trapped ions. For instance, the ground state extension of a trapped electron with a trap frequency of $\omega_a = 2\pi \times 300$ MHz is 175 nm, whereas for a Ca^+ ion at $\omega_a = 2\pi \times 1$ MHz it is about 11 nm. As a result, error sources usually not considered for trapped ions may become relevant. In particular, the electron experiences a variation of the force over the extent of its wavefunction, due to any inhomogeneity of the magnetic field gradient. Thus, for electron qubits, the homogeneity of the applied microwave radiation in combination with the finite electron temperature is relevant. The size of this effect depends critically on the temperature of the electrons and the relative strength of the magnetic field gradient inhomogeneity.

To estimate the lowest order effect of magnetic field inhomogeneity, we do a Taylor expansion of the magnetic field at the center-of-mass of the two-electron chain along the axial direction

$$B = B_0 + B_1 y + B_2 y^2 + B_3 y^3,$$

where B_j denotes the j^{th} order expansion coefficient of the magnetic field.

Due to symmetry, the contribution of the second order term $B_2 = 0$ (corresponding to

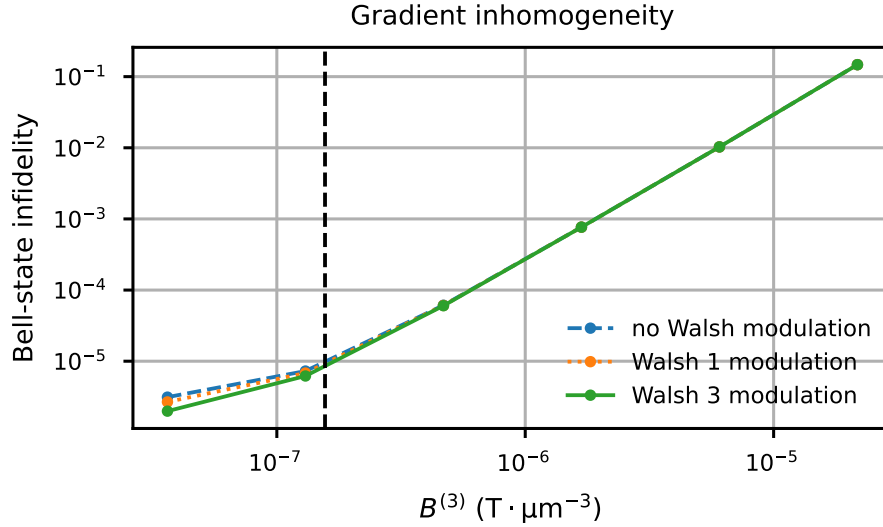


Figure 4.5: Numerical simulation of gate error due to magnetic-field inhomogeneity for an electron two-qubit gate with $\bar{n} = 40$ and $\Omega_R = 2\pi \times 105$ kHz. Blue dashed, orange dotted, and green solid curves correspond to no modulation, Walsh-1 modulation, and Walsh-3 modulation, respectively. The black dashed line indicates an estimated gradient-inhomogeneity coefficient of $B_3 = 1.5 \times 10^{-7} \text{ T}/\mu\text{m}^3$.

the first order term in magnetic field gradient (∇B) cancels over the extent of the electron motion. The third order term B_3 (corresponding to the second order term in ∇B), on the other hand, introduces a loss of fidelity due to a non-canceling force.

Keeping only the slowly oscillating terms rotating $\propto \delta$, the magnetic field gradient inhomogeneity error is modeled with the Hamiltonian [80]:

$$\hat{H}_e = 3\hbar\Omega_{\text{inhomo}}(\hat{I} \otimes \hat{\sigma}_x + \hat{\sigma}_x \otimes \hat{I})(aa^\dagger ae^{-i\delta t} + a^\dagger aa^\dagger e^{i\delta t}), \quad (4.12)$$

where

$$\Omega_{\text{inhomo}} = \Omega_R \cdot \frac{3y_0^2 B_3}{B_1}. \quad (4.13)$$

Using the hairpin design shown in Fig. 3.2, we find the third order expansion coefficient of the magnetic field to be $B_3 = 1.5 \times 10^{-7} \text{ T}/\mu\text{m}^3$. The simulation result is shown in Fig. 4.5 assuming an initial $\bar{n} = 40$ which corresponds to a temperature of 4 K to be more conservative.

Finally, we note that this effect can be reduced by reducing the temperature of the electrons. We also anticipate that by optimizing the geometry of the hairpin design and/or adding additional wires, the homogeneity can be substantially improved.

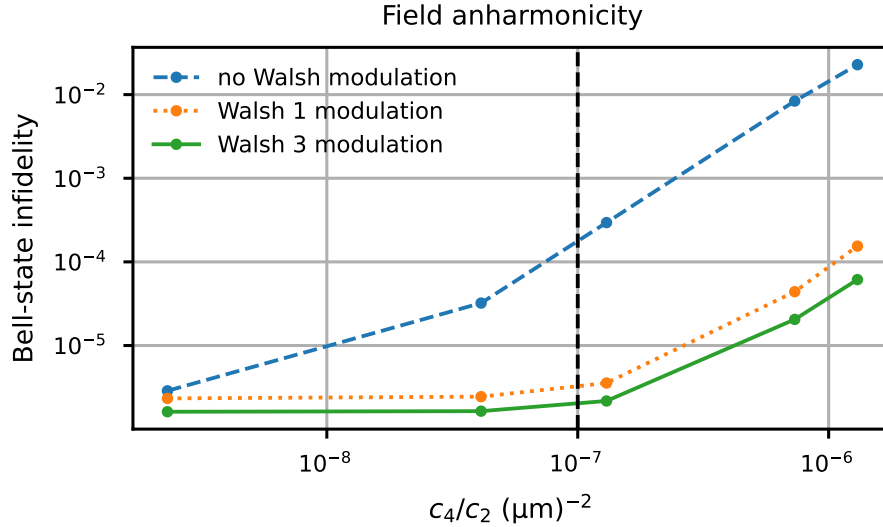


Figure 4.6: Numerical simulation of gate error due to anharmonicity in the trapping potential for an electron two-qubit gate with $\bar{n} = 40$ and $\Omega_R = 2\pi \times 105$ kHz. Blue dashed, orange dotted, and green solid curves correspond to no modulation, Walsh-1 modulation, and Walsh-3 modulation, respectively. The black dashed line indicates an estimated anharmonic coefficient ratio of $c_4/c_2 = 10^{-7}(\mu\text{m})^{-2}$.

4.5 Anharmonicity of the trapping potentials

Anharmonicities in potentials lead to the so-called cross-mode coupling discovered in Ref. [94] and theoretically analyzed in Ref. [95]. Briefly, the anharmonicity of the Coulomb-interaction between the electrons leads to frequency fluctuations of all but the center-of-mass modes. To counteract this, one typically cools all modes near the motion ground state. To avoid the complications of ground state cooling, one can use the center-of-mass mode instead, which is immune to this effect. Further, this noise is slow and would be mitigated by our dynamical decoupling methods.

The effect of the temperature on the COM mode frequency is given solely by the anharmonicity of the trapping potential itself. Expanding the axial trapping potential to only the fourth order

$$V(y) = V(0)(1 + c_2 y^2 + c_4 y^4).$$

The trapping potential anharmonicity can be modeled with the Hamiltonian

$$\hat{H}_e = V_4 \hat{y}^4 = V(0) c_4 y_0^4 (a + a^\dagger)^4. \quad (4.14)$$

As discussed in Sec. 3.4.3, with the optimized trap potential up to the fourth order $V(z) = V(0)(1 + c_2 z^2 + c_4 z^4)$, we find $c_4/c_2 = 10^{-7}(\mu\text{m})^{-2}$. The simulation result is shown

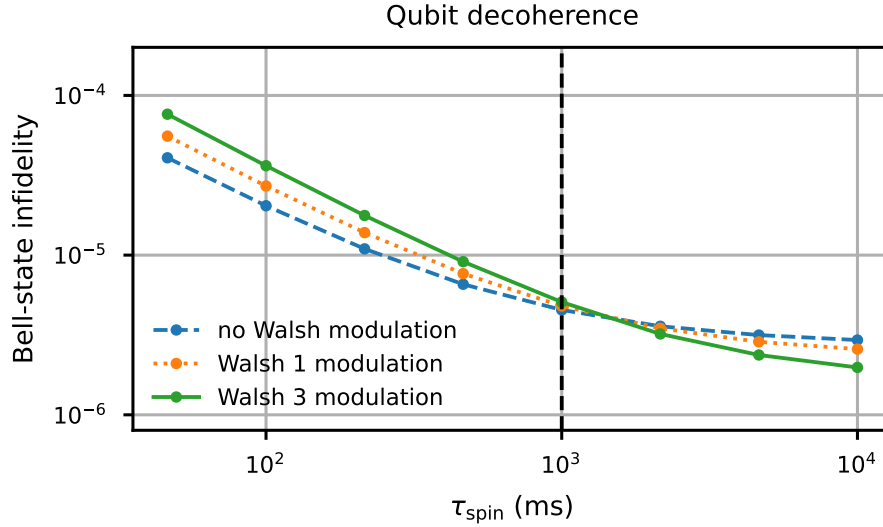


Figure 4.7: Numerical simulation of gate error due to qubit decoherence for an electron two-qubit gate with $\bar{n} = 40$ and $\Omega_R = 2\pi \times 105$ kHz. Blue dashed, orange dotted, and green solid curves correspond to no modulation, Walsh-1 modulation, and Walsh-3 modulation, respectively. The black dashed line indicates an assumed spin coherence time of $\tau_{\text{spin}} = 1$ s.

in Fig. 4.6 assuming an initial $\bar{n} = 40$ which corresponds to a temperature of 4 K to be more conservative.

4.6 Qubit decoherence

Qubit decoherence (spin dephasing) is modeled with the Lindblad operator

$$L = \sqrt{\frac{1}{2\tau_{\text{spin}}}} \hat{\sigma}_z,$$

where τ_{spin} is the spin coherence time.

The decoherence of the electron spin is expected to be dominated by magnetic field noise. The magnetic moment of the electron-qubit is nearly identical to that spanned by the $S_{1/2}$ ground state of alkaline earth metal ions such as Ca^+ and Sr^+ . Using magnetic shielding, coherence times of 300 ms (2 s with a spin-echo) have been observed for Zeeman qubits in ions [96]. The greatly reduced optical overhead requirements for electron qubits as compared to the experiment in Ref. [96] will allow for more efficient magnetic shielding, leading to expected spin-coherence times in excess of 1 s without spin-echo. In addition, we may use an additional layer of oxygen-free high-conductivity copper inside the cryostat, or even

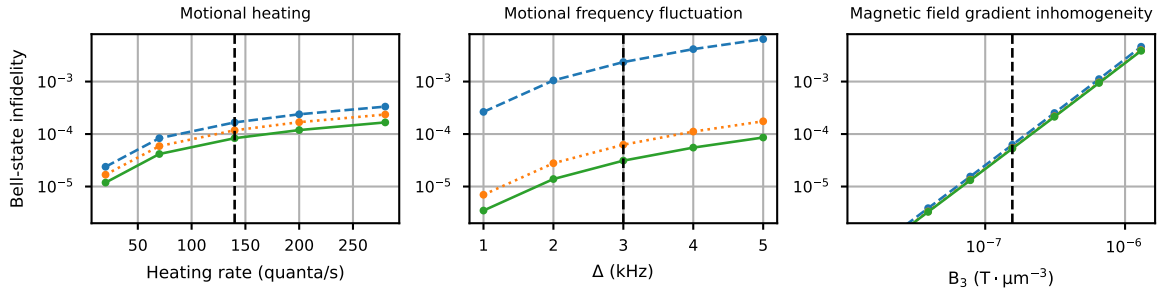


Figure 4.8: Numerical simulations of the three dominant error sources for an electron two-qubit gate with $\bar{n} = 4$ and $\Omega_R = 2\pi \times 105$ kHz. Blue dashed, orange dotted, and green solid curves correspond to no modulation, Walsh-1 modulation, and Walsh-3 modulation, respectively. For the magnetic-field-gradient inhomogeneity case, the three modulation traces overlap. Black dashed lines indicate the anticipated magnitude of each error mechanism. From these estimates, the total contribution to the Bell-state infidelity from the error sources considered here can be constrained to below 10^{-4} . Figure reproduced from Ref. [44].

a superconducting solenoid placed around the trap, both of which have been demonstrated to shield magnetic fields very effectively [97, 98]. This corresponds to only small contributions to Bell-state infidelities on the order of 3×10^{-6} . The simulation result is shown in Fig. 4.7 assuming an initial $\bar{n} = 40$ which corresponds to a temperature of 4 K to be more conservative.

In laser-free trapped-ion entangling gates, the qubit frequency shift error can be suppressed effectively in a dynamically decoupled Mølmer-Sørensen gate [69] and $\sigma_z \otimes \sigma_z$ gate with intrinsic dynamical decoupling [68]. We expect a similar performance for electron qubits.

4.7 Summary

Although the analysis in the previous sections used a conservative estimate of $\bar{n} = 40$ (corresponding to a radial temperature of 4 K), we now adopt $\bar{n} = 4$ to reflect the expected radial temperature of 0.4 K in the cryogenic setup discussed in Sec. 3.2. Figure 4.8 summarizes the Bell-state infidelities ($1 - F$) arising from the most relevant decoherence mechanisms, assuming a two-qubit Rabi frequency of $\Omega_R = 2\pi \times 105$ kHz and an initial $\bar{n} = 4$. Many of these error sources vary slowly compared to the gate duration and may therefore be treated as quasi-static. This enables the use of dynamical decoupling sequences (for example, Walsh modulation) as used in state-of-the-art trapped-ion experiments [64, 68, 69, 82–84], with only a modest overhead in gate duration. For instance, if the detuning and the strength of the magnetic field gradient are kept the same, Walsh-3 modulation increases the total

gate time by a factor of 4 relative to the unmodulated single-loop gate, resulting in a gate duration of $9.6 \mu\text{s}$. In practice, this overhead can be reduced by increasing the detuning or the strength of the magnetic field gradient.

	Motional heating	Trap frequency fluctuation	Motional dephasing	Magnetic field gradient inhomogeneity	Trapping potential anharmonicity	Qubit decoherence
Magnitude	140 quanta/s	3 kHz	$1.8 \times 10^{-3} \text{ Hz}$	$1.5 \times 10^{-7} \text{ T}/\mu\text{m}^3$	$10^{-7} \mu\text{m}^{-2}$	1 s
Infidelity	8×10^{-5}	3×10^{-5}	5×10^{-8}	5×10^{-5}	2×10^{-7}	3×10^{-6}

Table 4.1: Estimated contributions of the dominant error sources for an electron two-qubit gate using Walsh-3 modulation, with $\bar{n} = 4$ and $\Omega_R = 2\pi \times 105 \text{ kHz}$. Listed are the characteristic magnitudes of each error mechanism and the corresponding estimated infidelity contributions. Table reproduced from Ref. [44].

In summary, we find that the largest source of infidelities will likely be magnetic field gradient inhomogeneities and motional heating, both expected to lead to errors smaller than 10^{-4} as shown in Table 4.1. Although these infidelities meet some commonly assumed error correction thresholds, they can be further improved. For example, surface treatments can reduce motional heating [90] and thus suppress one of the largest error sources. Increasing the electron-electrode distance could also reduce motional heating. However, this will compromise achievable magnetic field gradients and motional frequencies. Another way to reduce these errors is to cool the motional modes of the electrons substantially below 0.4 K. As derived in Ref. [80], single-qubit gate errors due to field inhomogeneities, and prominent error rates for two-qubit gates such as motional heating, motional dephasing, motional frequency fluctuation, trap anharmonicity, and field inhomogeneities increase with the motional occupation number $\bar{n}$, and therefore with the temperature of the electron motion. For the specific case of two-qubit errors due to field inhomogeneities, Eq. (47) of Ref. [80] shows that the infidelity contains a contribution term proportional to $4\bar{n}^2$ and additional terms linear in $\bar{n}$. Evaluating the full expression for a single-loop gate and an initial state $|\downarrow\downarrow\rangle$ shows that cooling the motion from $\bar{n} = 4$ to $\bar{n} = 0.4$ reduces this gate error by approximately a factor of 8.

Chapter 5

Trap electrons in a room-temperature Paul trap

Although potential applications to quantum computing with trapped electrons will require cryogenic environments [10–12], here we concentrate on demonstrating electron trapping in a proof-of-principle experiment at room temperature. This chapter presents our room-temperature electron trapping experiment, which has been published in Ref. [99]. We discuss the design of the three-layer printed circuit board (PCB) Paul trap, the experimental setup, the measurement results, and the loss mechanisms.

Compared with conventional ion-trap experiments, there are three main challenges in moving from ion to electron trapping. First, because of the smaller mass of electrons, the trapping RF field must be at higher frequencies. The Mathieu stability parameters and the pseudopotential trap depth scale as $(m\Omega_{\text{rf}}^2)^{-1}$, making the required drive frequency Ω_{rf} roughly two orders of magnitude larger than in typical ion traps. Second, because we do not initially apply any cooling mechanism, electrons must be created with kinetic energies low enough to remain confined from the moment of ionization. This requires both a sufficiently deep trapping potential and a source of cold electrons injected directly into the trap center. Third, in the absence of conventional fluorescence detection, we need an alternative detection method to demonstrate trapping and characterize the trapping dynamics. We will address all of this in the following sections.

5.1 Three-layer PCB trap

5.1.1 Trap design

The electron trap used in this experiment is shown in an exploded view in Fig. 5.1. It consists of three double-sided printed circuit boards (PCBs) separated by ceramic spacers and the spacing between layers is 1.27 mm. The substrate is chosen as Rogers RO 4003C for low RF loss compared to FR-4.

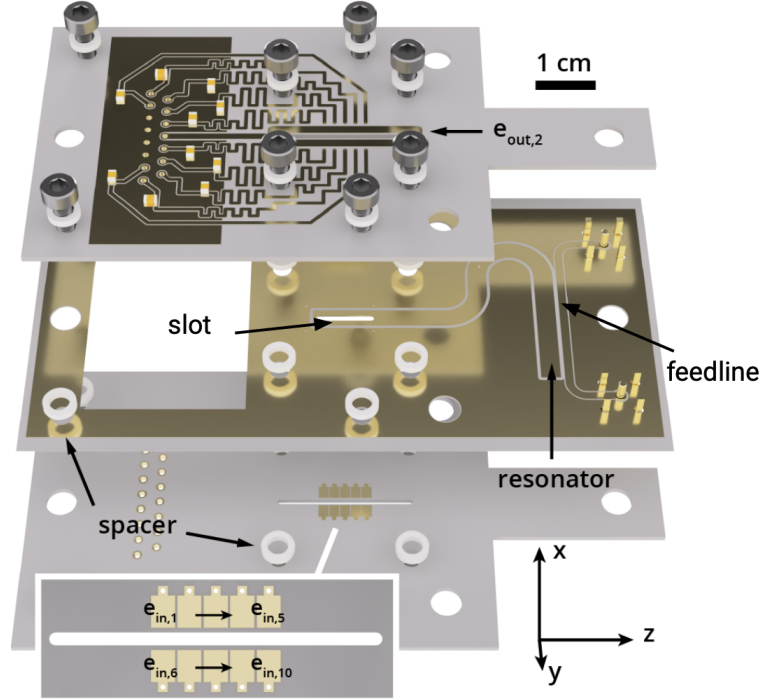


Figure 5.1: Exploded view of the three-layer PCB electron trap. Driving the halfwave coplanar-waveguide resonator (central board) generates a quadrupole microwave trap inside the slot at the resonator end. The two outer boards are identical and contain electrodes $e_{\text{in},1-10}$ ($e_{\text{in},11-20}$) on the bottom (top) surfaces to provide static confinement along the axial z -direction (see bottom inset for electrode labels). The boards are separated by 1.27 mm alumina spacers and have a footprint of approximately 5×10 cm. Figure reproduced from Ref. [99].

The two outer boards, mirroring each other about the central board, each feature ten rectangular electrodes along the slot on the inside board surface, labeled $e_{\text{in},j}$ with $j = 1-20$. Electrodes $e_{\text{in},1-10}$ are visible on the lower board and magnified in the inset, while electrodes $e_{\text{in},11-20}$ are on the hidden side of the upper board. The traces delivering voltages to the electrodes are on the outside surfaces, visible for the top board, and linked to a ground electrode via 10 pF NP0 capacitors for filtering. Both boards also feature a single electrode that surrounds the slot on the outside surface and is labeled $e_{\text{out},1}$ for the bottom board and $e_{\text{out},2}$ for the top board. Electrodes $e_{\text{in},1-20}$ are used to apply a static quadrupole field, confining electrons in the z (axial) direction.

The central board hosts a halfwave coplanar waveguide resonator whose end serves as the RF electrode. Figure 5.2 shows a closer view of the central region of the trap. The slot

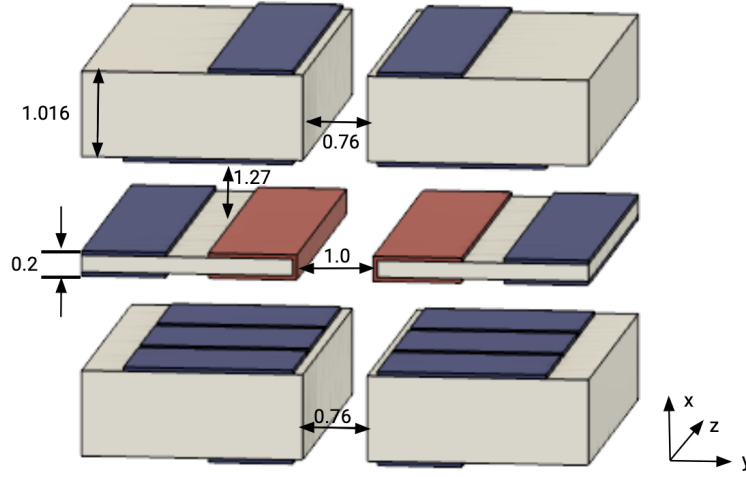


Figure 5.2: Central view of the three-layer PCB electron trap. The red electrode denotes the RF electrode, the blue electrode on the central board is ground, and the blue electrodes on the outer boards are DC electrodes. Dimensions (in millimeters) are labeled in the diagram.

is wider on the two outer boards compared to the central board to provide better shielding of the Ca flux to mitigate charging.

5.1.2 Trap RF drive

In order to generate the required high-voltage drive in the GHz regime, we integrate a coplanar waveguide resonator into the trap design. The $\lambda/2$ waveguide resonator is capacitively coupled to a microwave feedline and the coupling can be tuned by adjusting the gap between the feedline and the resonator. The end of the resonator in the center of the board contains a slot and functions as the trap's RF electrode, providing an AC quadrupole field which confines electrons inside the slot in the radial (x and y) directions. The microwave voltage is applied through SMA connectors to the central board. The resonator is held at DC ground potential via a tap in its center which connects it to the grounded top surface of the board. Future cryogenic experiments can take advantage of previous work on waveguide resonators in the context of superconducting qubits [100, 101].

Comparing the simulated microwave resonance with measurements yields good agreement, as shown in Fig. 5.3. The solid blue line shows the Ansys HFSS simulation of the RF resonance. The quality factor of the resonator is about 40. When fully assembled, we measure a resonance frequency of about $2\pi \times 1.60$ GHz with a loaded Q of roughly 35, as shown in the orange line in Fig. 5.3. For the experiment, we kept the trap drive in continuous-wave mode. For ease of presentation, we denote P_{rf} as the output power of the Marconi RF signal

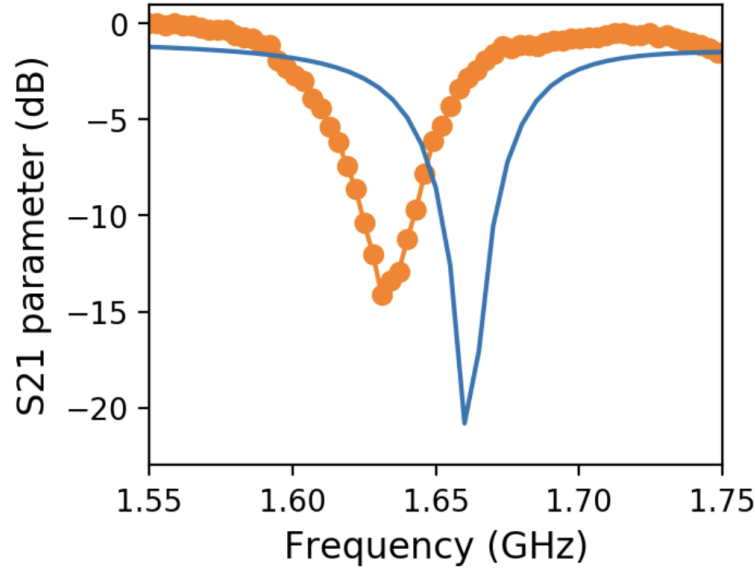


Figure 5.3: Comparison of Ansys HFSS simulations (blue) and experimental measurements (orange) of the RF resonance of the assembled three-layer trap. The observed resonance has a quality factor of approximately 40.

generator used as our RF source. The RF output passes through a 15 dB attenuator, a 5 W amplifier (Mini-Circuits ZHL-5W-202-S+) and a circulator before entering the chamber. We find that we can reach about 100 V on the microwave resonator with an input power of 5 W.

5.1.3 Trapping field

Pseudopotential

The effective trapping potential in the radial directions for trapped electrons can be described by the pseudopotential approximation as discussed in Sec. 2.2.2.

$$\Phi_{\text{ps}} = -\frac{e}{4m\Omega_{\text{rf}}^2} \|\mathbf{E}_{\text{rf}}\|^2.$$

$\mathbf{E}_{\text{rf}}$ is retrieved from electrostatic simulation using the finite element method (FEM) with Ansys Maxwell. Figure 5.4 shows the simulation result for a drive amplitude of 90 V and with all DC electrodes grounded. In calculating the pseudopotential, Ω_{rf} is set to $2\pi \times 1.6$ GHz. Figure 5.4 (a) displays a map of the pseudopotential for a cut through the trap in the xy -plane in the center of the slot, highlighting the trap substrate and electrodes as hashed gray and yellow areas, respectively. Figure 5.4 (b) and (c) show the trap depth based on the

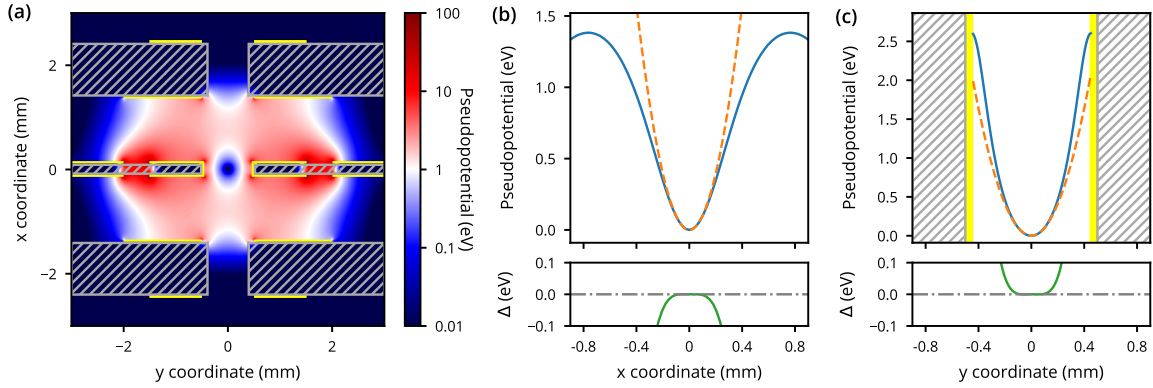


Figure 5.4: Electron trapping potential. (a) Contour plot of the trapping potential based on the pseudopotential approximation in the xy -plane in the center of the slot for 90 V amplitude at 1.6 GHz frequency on the resonator electrode. The circuit board substrate is indicated by the hashed areas, metal electrodes are highlighted in yellow. (b) Top: pseudopotential along the x -axis through the trap center (blue continuous curve), compared to an ideal harmonic potential (dashed orange). Bottom: deviation Δ of pseudopotential from harmonic potential. (c) Top: pseudopotential along the y -axis through the trap center (blue continuous curve), compared to an ideal harmonic potential (dashed orange). Location of trap substrate (electrodes) shown as hashed grey (solid yellow) area. Bottom: deviation Δ of pseudopotential from harmonic potential. Figure reproduced from Ref. [99].

pseudopotential approximation (continuous blue curves), which is about 1.3 eV (or 15,000 K), limited by the weaker confinement along the x -direction.

Near the trap center, the pseudopotential is well approximated by a harmonic potential. Comparing the numerically obtained pseudopotential with an ideal harmonic potential (see the orange dashed curve in (b) and (c)) shows close agreement out to about 100 μm from the trap center, as exemplified by the green curves in the bottom panels, which show their difference Δ . Within 200 μm of the trap center, the pseudopotential deviates by less than 2% from the ideal harmonic potential. The corresponding radial secular frequency for an electron moving in this potential is about $2\pi \times 300$ MHz.

DC field

To generate the desired static potential, we control the coefficients of all multipole terms up to second order by controlling the DC voltages for all individual DC electrodes $e_{\text{in}, 1-10}$ and

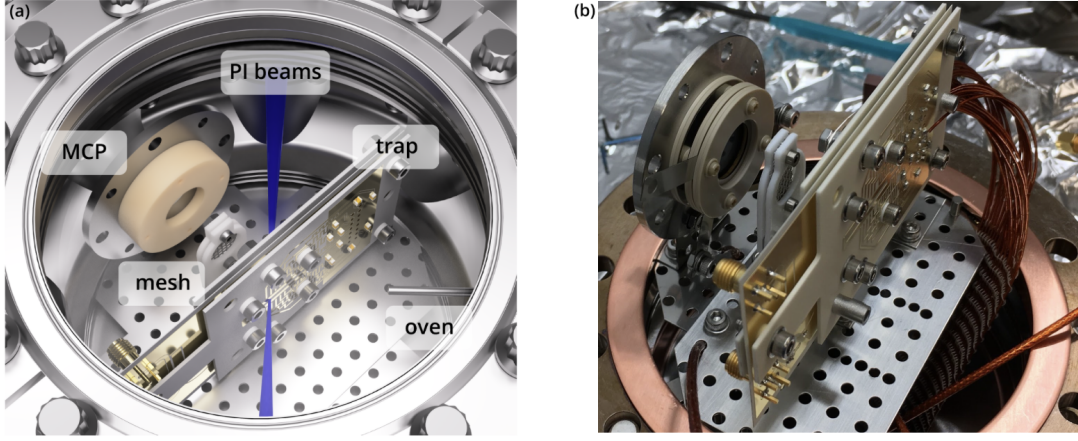


Figure 5.5: Experimental setup. (a) Simplified rendering of the apparatus inside the UHV chamber, showing the PCB electron trap, microchannel-plate (MCP) detector, mesh, photoionization (PI) laser beams, and a calcium oven supplying atoms for photoionization. (b) Photograph of the assembled chamber. Figure (a) reproduced from Ref. [99].

$e_{\text{in}, 11-20}$. We express the static trapping potential in terms of multipole coefficients

$$\begin{aligned}
 \Phi(x, y, z) = & C + E_x(-x) + E_y(-y) + E_z(-z) \\
 & + U_1 \left(\frac{x^2 - y^2}{2} \right) + U_2 \left(\frac{2z^2 - x^2 - y^2}{2} \right) \\
 & + U_3 \left(\frac{xy}{2} \right) + U_4 \left(\frac{yz}{2} \right) + U_5 \left(\frac{xz}{2} \right) \\
 & + O(r^3).
 \end{aligned} \tag{5.1}$$

During the trapping experiment, we tune U_2 to change the axial confinement strength with a typical $U_2 = -0.5 \text{ V/mm}^2$. E_x , E_y and E_z can be varied to achieve micromotion compensation. Ref. [102] has a detailed explanation of the multipole control method. The wires soldered to the outer boards supply DC voltages in the $\pm 32 \text{ V}$ range from a 16 bit digital-to-analog-converter (DAC), which generates a static quadrupole field that confines electrons in the axial direction along the slot.

5.2 Experimental setup

The main components of our experimental setup and their alignment in the UHV chamber are shown in Fig. 5.5 (a), omitting electric leads for simplicity. A picture of the component

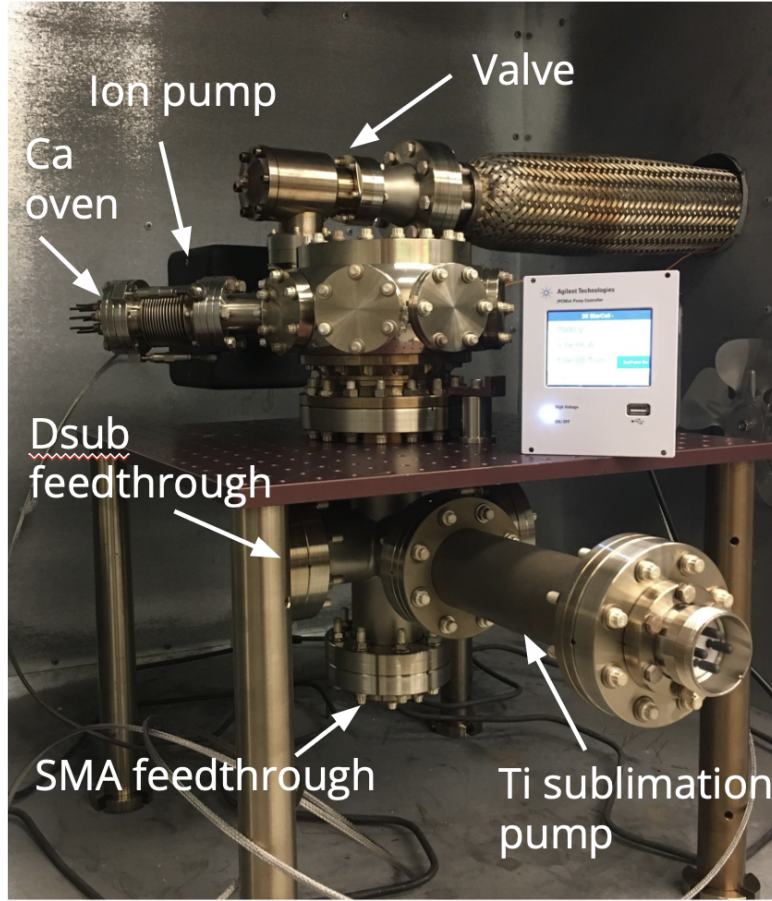


Figure 5.6: Photograph of the UHV chamber assembly used in the experiment.

inside the chamber is shown in Fig. 5.5 (b). The PCB trap is located in the center of the chamber. A resistive oven on the side of the chamber is aligned to direct an atomic calcium beam through the trap slots when heated (steel tube labeled ‘oven’). A steel mesh (grid) and a two-stage MCP are used to direct and detect electrons extracted from the trap, respectively. The two photoionization (PI) laser beams are overlapped and traverse the chamber at right angles to the Ca oven for electron creation.

5.2.1 Vacuum System

The entire UHV chamber used for the experiment is shown in Fig. 5.6. The Kimball octagon is connected to an ion pump and a valve via a 2.75-inch tee. The bottom of the octagon is attached to a 6-inch close coupler that is connected to a 6-way 4.5-inch cross (Nor-Cal) via a

6-inch to 4.5-inch zero-length reducer. The 6-inch close coupler is for extending the chamber space and mounting the assembly with groove grabbers. The 6-way cross has a titanium sublimation pump, a D-sub feedthrough and an SMA feedthrough attached.

In Paul trap experiment, it is crucial to maintain a good vacuum since particles are subject to loss due to collisions with the background gas. Before assembly, the experiment chamber was pre-baked at 200 C for a week. After assembly, the chamber is baked at 150 C for 12 days and the baking temperature is limited by the requirement of the MCP. Before ramping down the temperature, the pressure of the chamber is 1.4×10^{-7} Torr. After cooling down, turning on the ion pump and firing the Ti-sublimation pump, the chamber reaches a base pressure below 4×10^{-10} Torr at room-temperature.

5.2.2 Electron source

The initial experiment lacks a cooling method; therefore, the electrons must be created near the trap center and have enough low energies to stay confined. To solve this problem, we consider the generation of electrons using two-photon photoionization (PI) of neutral atoms from a thermal atomic beam. This is a technique widely used for ion-trap loading [48]. Neutral calcium atoms are supplied by a resistively heated oven, and the neutral flux is tuned by adjusting the oven current. During the experiment, we supply a constant current to the calcium oven I_{oven} typically between 4 ~ 6 A. Ionization occurs when the atoms intersect the PI beams near the trap center, allowing the localized generation of electrons directly within the trapping region. This loading method enables both the creation of very cold electrons by tuning the laser frequencies close to the ionization threshold, and preferential in-situ ionization in the trapping region by optical alignment.

For PI, we use laser light at 422 nm and at 390 nm. The 422 nm single-mode laser is tuned to be on resonance with the neutral calcium 4^1S_0 - 4^1P_1 transition, while the free-running multi-mode 390 nm laser diode (NDU4316) is tuned by temperature and current to maximize the electron ionization rate. Although the 4^1P_1 -continuum ionization threshold is approximately 389.8 nm, we find the ionization rate to peak when the diode center wavelength is approximately 390.3 ± 0.2 nm, suggesting that ionization occurs through Rydberg states [48]. As such, we expect electrons to inherit only minimal kinetic energy from the ionization process, and their energy in the trapping potential is rather determined by their ionization location and the phase of the microwave field. The experiment typically uses about 0.5 mW of 422 nm laser light and approximately 2 mW of 390 nm light. In the default alignment configuration, the two PI laser beams overlap and traverse the chamber at right angles to the calcium oven, focusing near the trapping region with a beam waist of about 30 μm . Since they are co-propagating, calcium ionization conditions are met over an extended volume, which follows the laser beam path, and the majority of electrons are created outside of the trap. In the experiment, the 390 nm laser is pulsed with an AOM to control the timing of the PI process and the 422 nm laser is running CW.

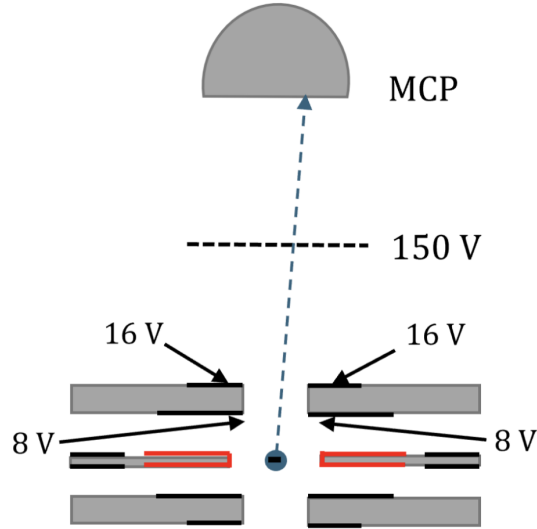


Figure 5.7: Schematic of electron extraction and detection. A 20 ns extraction pulse is applied to three electrodes: $e_{\text{out},1}$ at 16 V and two inner DC electrodes $e_{\text{in},3}$ and $e_{\text{in},8}$ at 8 V. These pulses eject trapped electrons from the trap. A two-stage MCP detector records the electrons, while a metal mesh (grid) biased at 150 V directs them toward the detector.

5.2.3 Electron extraction and detection

To evidence trapping, we detect electrons destructively by applying an extraction pulse to extract electrons out of the trap and detect them with a two-stage microchannel plate detector (MCP) (Hamamatsu F4655-11). We hold the front stage of the MCP at positive bias to attract negatively charged electrons. The first stage, second stage, and MCP anode are kept at 200 V, 2200 V and 2500 V with high-voltage power supplies (SRS PS350), respectively. A steel mesh between the trap and the MCP is used to direct the electrons towards the detector. The MCP signal is shaped and amplified before sending to a time-to-digital converter (TDC), which we often call a "time tagger" (Swabian instruments time tagger 20), as the stop signal for recording. The mesh is typically at a potential of 150 V. When applying the DC voltages for the static trapping field, we compensate for the multipoles generated by the mesh.

As shown in Fig. 5.7, to eject trapped electrons from the trap, an extraction pulse is applied to three electrodes, $e_{\text{out},1}$ (outer DC electrode) with 16 V amplitude and $e_{\text{in},3,8}$ (two inner central DC electrodes) with 8 V amplitude. The duration of the extraction pulse is about 20 ns. The extraction pulses are added to the static voltages for DC confinement via a bias tee, which originate from a digital-to-analog converter (DAC).

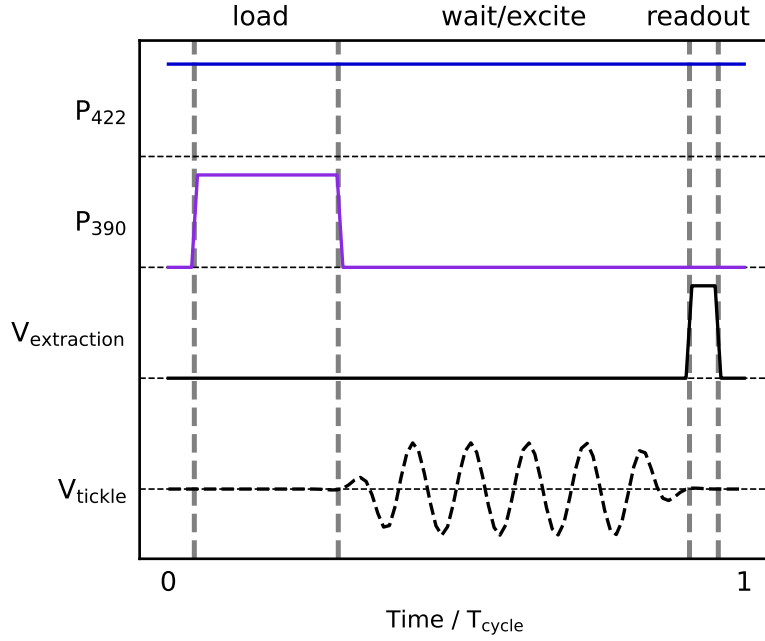


Figure 5.8: Illustration of one cycle of the experimental protocol. Free electrons are created during the loading phase when the 390-nm laser is switched on. An RF tone is applied to one DC electrode in some experiments during the “wait/excite” phase. Voltage pulses to three DC electrodes in the “readout” phase eject electrons in the direction of the mesh and MCP. Figure reproduced from Ref. [99].

5.2.4 Pulse sequence

The pulse sequence and timing for an experimental cycle are illustrated in Fig. 5.8. It starts with a loading phase of variable duration t_{load} where the 390 nm PI laser is pulsed. The 422 nm PI laser is on during the full cycle. The loading is followed by a variable time t_{wait} , where we either keep all settings constant or apply an RF tone at frequency ω_{tickle} to the electrode $e_{\text{in}, 17}$ to excite the electron motion. Finally, the extraction pulse is applied to eject trapped electrons.

5.2.5 Experiment synchronization

Figure 5.9 shows a schematic of the electronics setup to synchronize the experiment. In the initial experiment setup as shown in Fig. 5.9 (a), an arbitrary waveform generator (AWG) functions as the experiment clock (Rigol DG4062). It provides the trap extraction pulse and the start signal triggering a time tagger. The extraction pulses are added to the static

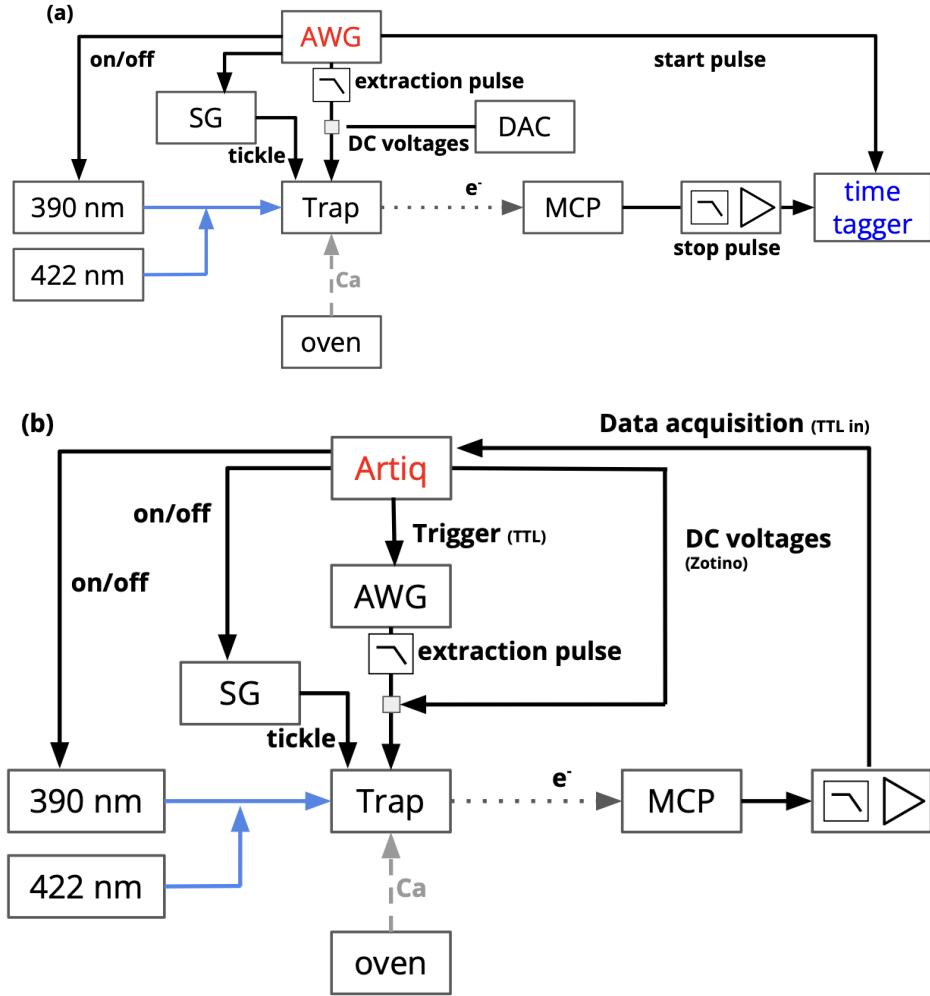


Figure 5.9: Control schematic and key electronics for synchronizing the experiment. (a) An arbitrary waveform generator (AWG) provides the extraction pulses to the trap, and the start signal for a time tagger. It controls the timing for pulsing on and off the PI loading beams and a signal generator (SG), which excites the electron motion ('tickle'). Pulses from the MCP provide the stop Signals to the TDC. Low-pass filters prevent electronic pickup of the microwave trap drive and the extraction pulses by the MCP detection circuit which contains pulse shaping and amplification elements. The Ca oven and a digital-to-analog converter (DAC) for the DC trap voltages are operated with constant settings during an experiment. (b) Control schematic using ARTIQ. The ARTIQ sends trigger TTLS to the AOM for pulsing 390 nm laser, to an AWG which sends the extraction pulse, and to a signal generator (SG) used to apply the tickle pulse. The DC voltages are provided by ARTIQ Zotino channel. The electron detection signal is picked off from the MCP anode supply voltage with a high-pass filter, shaped and amplified to be acquired by the ARTIQ TTL input channel.

voltages for DC confinement which originate from a digital-to-analog converter (DAC). The AWG further controls the timing for switching on and off both the 390 nm PI laser for loading and a signal generator (SG) used to apply an RF tone (labeled ‘tickle’) to the trap. The electron detection signal is picked from the MCP anode supply voltage with a high-pass filter, shaped and amplified to be used as the TDC stop signal. Low-pass filtering of the extraction pulses ($2\pi \times 50$ MHz cut-off) prevents electronic pickup at the MCP and a further low-pass filter ($2\pi \times 200$ MHz cut-off) in front of the shaping and amplification circuit removes pickup at the frequency of the microwave drive.

Later, the control and synchronization electronics of the experiment underwent an upgrade, and we started to use ARTIQ for experimental control, as shown in Fig. 5.9 (b). The ARTIQ system sends trigger TTLs to the AOM for pulsing the 390 nm laser light, to an AWG which sends the extraction pulse, and to a signal generator (SG) used to apply the tickle pulse. The DC voltages are provided by ARTIQ Zotino channel. The electron detection signal is picked up from the MCP anode supply voltage with a high-pass filter, shaped and amplified to be acquired by the ARTIQ TTL input channel.

5.3 Experimental results

In Fig. 5.10 we show a histogram of MCP detection events of a typical trapping experiment, where loading and waiting times are $t_{\text{load}} = t_{\text{wait}} \approx 10 \mu\text{s}$. Data are displayed as probability to record an event during a 1 ns time bin, and we acquire data for 10^7 experimental cycles. During the loading period, we observe a small constant signal mirroring the shape of the 390 nm laser pulse, likely from just created but not trapped electrons. Application of the extraction pulse at the end of the experimental cycle results in a large and sharply localized signal from the MCP with an appropriate DC and RF trapping field, demonstrating that the electrons remain in trap $10 \mu\text{s}$ after the end of the loading pulse. The inset displays a close-up of the readout signal, which peaks with a full-width at a half-maximum of about 2 ns. Note that the inset uses a linear scale for the readout signal, while the full cycle is displayed using a semi-logarithmic scale to show the background during loading as well.

5.3.1 Loading efficiency

To investigate electron loading, we use the protocol introduced in Fig. 5.8 and vary t_{load} for a fixed waiting time $t_{\text{wait}} = 10 \mu\text{s}$. The oven current was kept at $I_{\text{oven}} = 4.25$ A for this study. Increasing the oven current will increase the loading efficiency but will also make the vacuum worse and also make the charging issue more severe, so that we need to adjust for micromotion compensation field more often. For ease of presentation, we sum the detections in a 50 ns wide window around the readout signal. The result of the electron loading measurement is shown in Fig. 5.11(a). The left ordinate displays the fraction of cycles with at least one detection pulse from the MCP, we abbreviate it as detection probability p_e . p_e approaches unity with a time constant $\tau_{1/e} \approx 80 \mu\text{s}$ as t_{load} increases. We employ a

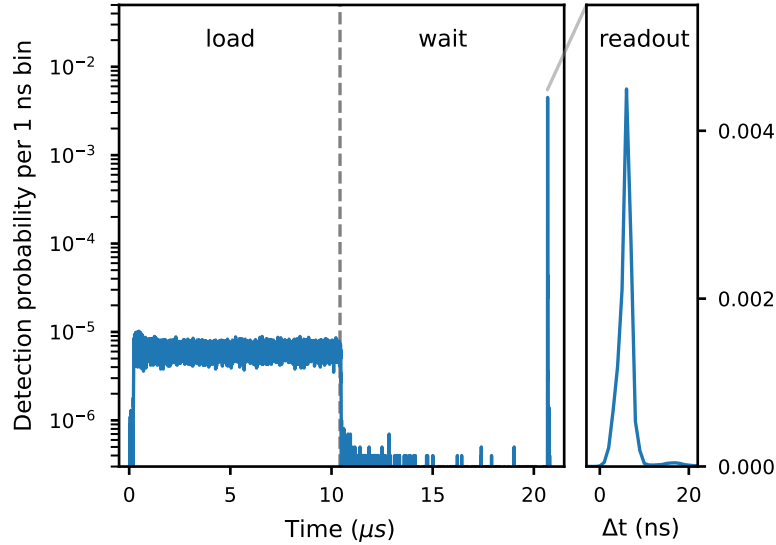


Figure 5.10: Typical MCP detection histogram for an experiment with loading and wait times of $t_{\text{load}} = t_{\text{wait}} \approx 10 \mu\text{s}$. During loading, untrapped electrons are accelerated into the MCP, reproducing the temporal profile of the 390 nm laser pulse. The extraction pulse empties the trap, producing a sharp, localized signal. Inset: magnified view of the histogram during the readout phase. Figure reproduced from Ref. [99].

simple threshold method to detect MCP pulses and set a dead time of 60 ns following each detection to prevent double counting of some events due to voltage ringing, which sets a natural limit of one detection event per cycle.

Electron number estimation

To estimate the number of trapped electrons, we convert the measured detection probability to electron number assuming that electron detection events at the MCP follow Poisson statistics. Concretely, we find the mean λ for a Poisson distribution $P \sim \text{Poisson}(k, \lambda)$

$$P(k, \lambda) = \frac{\lambda^k e^{-\lambda}}{k!},$$

such that $P(0, \lambda) = e^{-\lambda}$ matches the measured probability of not registering events during readout

$$P(0, \lambda) = P(\text{No detection}) = 1 - p_e,$$

where p_e is the probability of detection. The product of λ and the electron loss (inverse of the detection efficiency) provides an estimate of the average number of electrons N_e in the

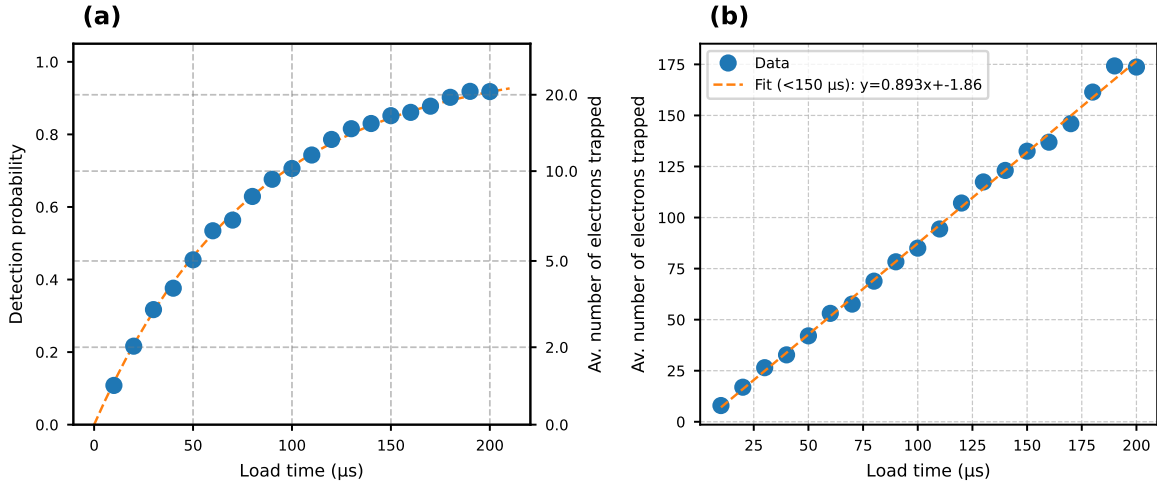


Figure 5.11: Trapped electron loading efficiency for $t_{\text{wait}} = 10 \mu\text{s}$, $U_2 = -0.5 \text{ V/mm}^2$, $P_{\text{rf}} = 2 \text{ dBm}$, and $I_{\text{oven}} = 4.25 \text{ A}$. (a) Electron trapping and detection probability as function of loading time. The dashed curve shows an exponential fit to $1 - \exp(-t/\tau_{1/e})$ with $\tau_{1/e} = 80.3 \pm 0.5 \mu\text{s}$. Error bars due to statistical uncertainty are too small to be visible. (b) Estimated average number of trapped electrons as a function of loading time. The dashed line is a linear fit to data points with $t_{\text{load}} < 150 \mu\text{s}$, corresponding to an average loading rate of one electron per $10 \mu\text{s}$. Figure (a) reproduced from Ref. [99].

trap at the point when the extraction sequence is applied

$$\bar{N}_e = \frac{1}{E_{\text{det}}} \lambda = -\frac{1}{E_{\text{det}}} \ln(1 - p_e), \quad (5.2)$$

where E_{det} is the detection efficiency of the experiment.

Detection efficiency

To be registered in our readout scheme, electrons have to escape the trap, pass through the mesh, impinge on the MCP, and trigger the electron multiplication process. From electron trajectory simulations, we estimate the extraction process to be close to lossless, and hence the open area of the mesh and the non-unity MCP efficiency are the main loss factors for detection.

The open area of the mesh is ≈ 0.5 . The MCP detection efficiency depends on the energy of incoming electrons, which is determined by the potential difference between trap and the first MCP stage, and the voltages we apply to the second stage and the anode of the MCP. For sufficiently high MCP voltages and electron energies of a few hundred

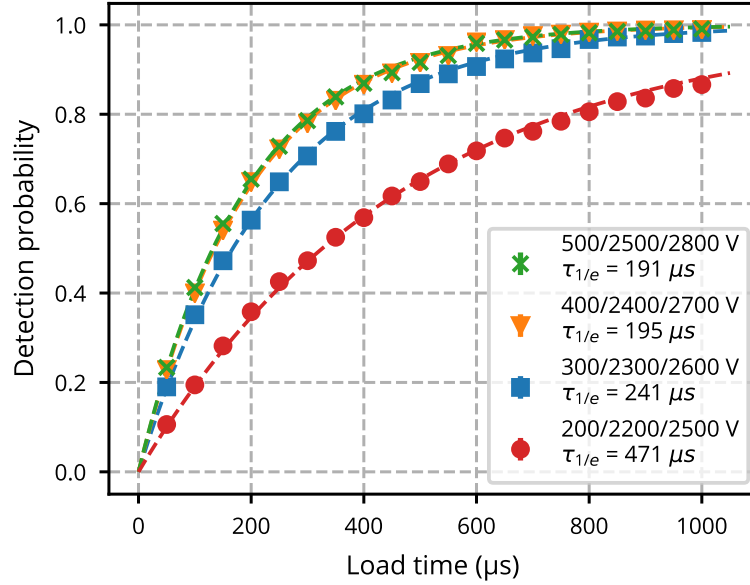


Figure 5.12: Calibration of relative MCP detection efficiency. Electron loading rates (compare Fig. 5.11(a)) are measured for various MCP voltage settings. For typical operating conditions used in this work (200/2200/2500 V), the MCP efficiency is approximately 40% of its maximum. Figure reproduced from Ref. [99].

eV, the efficiency should saturate approximately at the MCP open area of about 0.6, such that the upper bound of the electron detection efficiency would be about $1/3$. We performed electron loading measurements with a range of MCP voltages to calibrate the MCP detection efficiency for the operating conditions used in Fig. 5.11, where the front stage, second stage, and the anode are at 200 V, 2200 V, 2500 V, respectively.

Figure 5.12 shows the data (symbols) and fits to $1 - \exp(-t/\tau_{1/e})$ (dashed curves). The legend displays the MCP voltages and the fit parameter $\tau_{1/e}$. We note that the $\tau_{1/e}$ values are different from the data in Fig. 5.11 (a), as the intensity of the photoionization beams was lower here. We observe a clear saturation of the detection efficiency as the voltages approach 500 V, 2500 V, 2800 V for the first stage, second stage, and anode of the MCP, respectively. For these voltages, the MCP efficiency should correspond to the open area of 0.6 and it follows that for the typical operating conditions used here, the efficiency is reduced by a factor of about 2.5. A caveat in this estimate is that we are not accounting for possible degradation of the MCP which would lower the efficiency somewhat.

Overall, we then estimate that 12% of the electrons leaving the trap in the extraction

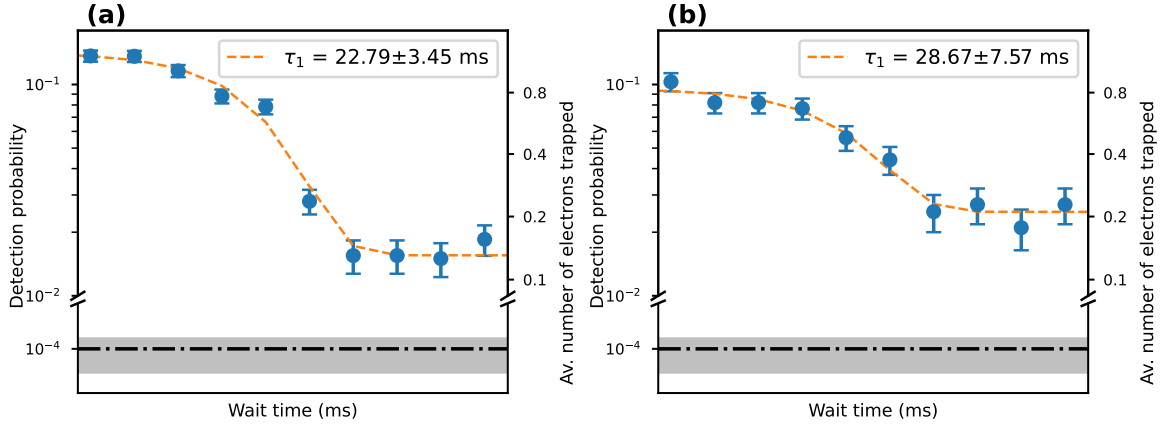


Figure 5.13: Trapped electron lifetime for $t_{\text{load}} = 10 \mu\text{s}$, $U_2 = -0.5 \text{ V/mm}^2$, $P_{\text{rf}} = 2 \text{ dBm}$, and $I_{\text{oven}} = 4.25 \text{ A}$. (a) Storage-time measurement with co-propagating PI beams. A double-exponential fit (orange dashed) to the electron number $\bar{N}_e$ shows that $\sim 90\%$ of electrons decay with $1/e$ constant $\tau_1 = 23 \pm 3 \text{ ms}$, while the remaining 10% show no measurable decay. The background detection level is indicated by the horizontal dash-dotted line. Error bars and the gray band indicate one standard deviation. (b) Same measurement with PI beams intersecting at a 15° angle. A double-exponential fit yields $\tau_1 = 29 \pm 8 \text{ ms}$ for $\sim 75\%$ of electrons, while the remainder show no measurable decay.

process generate a detectable signal from the MCP

$$E_{\text{det}} = 0.12.$$

Taking into account the loss during the readout process and the fraction of detections per cycle, we can estimate the average number of electrons in the trap for each measurement setting. The right ordinate in Fig. 5.11(a) shows the estimated number of trapped electrons. Figure 5.11(b) plots the same data with number estimation on the y axis, it can be seen that the number of electrons is proportional to the loading time: it takes on average $10 \mu\text{s}$ to load one electron, and we trap on average about 20 electrons for a loading time of $200 \mu\text{s}$.

5.3.2 Electron lifetime

To measure the electron storage time in the trap, we set the loading time such that the trap rarely contains more than a single electron, and record the readout signal as a function of the wait time t_{wait} , see Fig. 5.13 for the data. Here, each data point contains $\approx 10^3$ repetitions. During measurement, the order of different t_{wait} is randomized to account for parameter drifts that affect the loading probability. Both measurements show a bimodal lifetime distribution with two distinct regimes, where a portion of the electrons are lost

within 100 ms (exponential decay), while the remaining electrons, which we call long-lived electrons, show no detectable loss after 1 s. The dark horizontal dashed-dotted line displays the background detection level independently measured to be about 1×10^{-4} detections per cycle.

To understand the cause of this bimodal distribution, we tuned the alignment of the two PI beams. In both experiment, the focus of 390 nm laser is about $30 \mu\text{m}$, and the focus of 422 nm laser is about $40 \mu\text{m}$. The fraction of these two electron populations appears to depend on the precise alignment of the PI lasers. In the default configuration, where the two beams are co-propagating, the fraction of the long-lived electrons is about 10%, as shown in Fig. 5.13 (a). We further separate the optical path of the two PI beams and still let them enter the chamber through the same viewport. The focuses of the two beams are overlapped using a beam profiler. Limited by the optical access we have on the apparatus, the maximum angle formed between the two PI beams is about 15 degrees, where the fraction of long-lived electrons increases to 25%, as shown in Fig. 5.13 (b). When the PI beams are moved along the axial direction while keeping two beams co-propagating, we find that the storage time of short-lived electrons will change, but the fraction of the long-lived electrons remains roughly the same. Another observation is that larger PI beams in the trapping region are correlated with fewer long-lived electrons.

From the above observations, we speculate that trap anharmonicity is the driving force behind electron loss in our trap. Electrons created near the trap center stay in the trap for much longer time while electrons created further away from the trap center are lost, for instance, due to coupling between their secular motion and the microwave drive of the trap. We expect such a coupling from anharmonicities of the trap potential. Only the trajectories for single electrons confined within the harmonic trapping region are stable, so with the ionization volume dictated by the alignment of the PI beams, we expect that the majority of initially trapped electrons (which are created outside of the central $100 \mu\text{m}$) will experience a sufficiently anharmonic potential to be driven out of the trap eventually. The tail of long-lived electrons in Fig. 5.13 is then attributed to the single electrons that remain in a stable trajectory after all other particles have been heated out of the trap. A more detailed discussion about this loss mechanism can be found below in Sec. 5.4. An interesting experiment that can further confirm this hypothesis and can also be used to investigate the trapping volume of this electron trap would be to let the two PI beams intersect only at the trap center. In this way, we can control the location of the created electrons more precisely. We tried to pass two PI beams from different viewports so that they form a 90 degree angle and only intersect in the trap center, but the signal was too small to be resolved from the background.

Beyond the regime of trapping single electrons, we see a slight dependence of the short-lived electron lifetime on the number of trapped electrons, see Fig. 5.14 for the data. The offset exponential fit shows different decay constants τ_1 for different t_{load} . In general, a larger electron density in the trap leads to a slightly shorter lifetime.

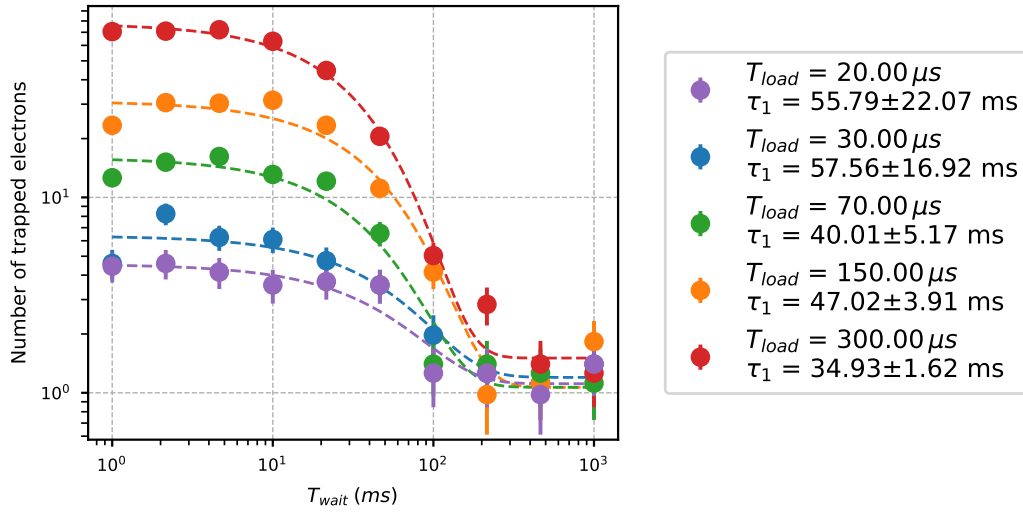


Figure 5.14: Dependence of trapped-electron lifetime on the number of loaded electrons for $U_2 = -0.5 \text{ V/mm}^2$, $P_{\text{rf}} = 2 \text{ dBm}$, and $I_{\text{oven}} = 4.25 \text{ A}$. Each data point corresponds to 500 repetitions. The orange dashed curve is an exponential fit with an offset showing that the $1/e$ decay constant τ_1 varies with t_{load} (and thus the number of trapped electrons). Larger electron numbers generally lead to shorter storage times.

5.3.3 Trap frequencies

The frequencies of the electron motion in the trap can be measured with an excitation pulse (‘tickle’ pulse) following the experimental protocol from Fig. 5.8. For this measurement, we load about ten electrons on average and set the wait time to $t_{\text{wait}} = 2 \text{ ms}$. We apply the tickle pulse to one of the DC electrodes $e_{\text{in}, 17}$ on top of the DC confinement voltage, which has a projection in all three directions. During the waiting time, we try to excite the motion of trapped electrons with an RF tone at a frequency ω_{tickle} . We step ω_{tickle} in increments of 1 MHz from 20 to 350 MHz and monitor electron loss, which is indicative of motional resonance.

The top panel in Fig. 5.15 shows the electron loss spectrum for a tickle power of about -12 dBm applied (5 mV measured excitation voltage amplitude) and features two prominent dips. We can identify the resonances based on their response to DC and microwave voltages, revealing the dip at about $2\pi \times 40 \text{ MHz}$ as the axial mode, while the resonance at $2\pi \times 300 \text{ MHz}$ is due to one of the radial modes of motion. We find no evidence of multi-electron Wigner crystal modes, which is consistent with having weakly or non-interacting electrons with a range of energies in the trap. For low excitation tickle powers, only the fundamental resonances are visible. Exciting the system more strongly with a tickle power of about -6 dBm (20 mV measured excitation voltage amplitude) reveals a series of harmonics of the

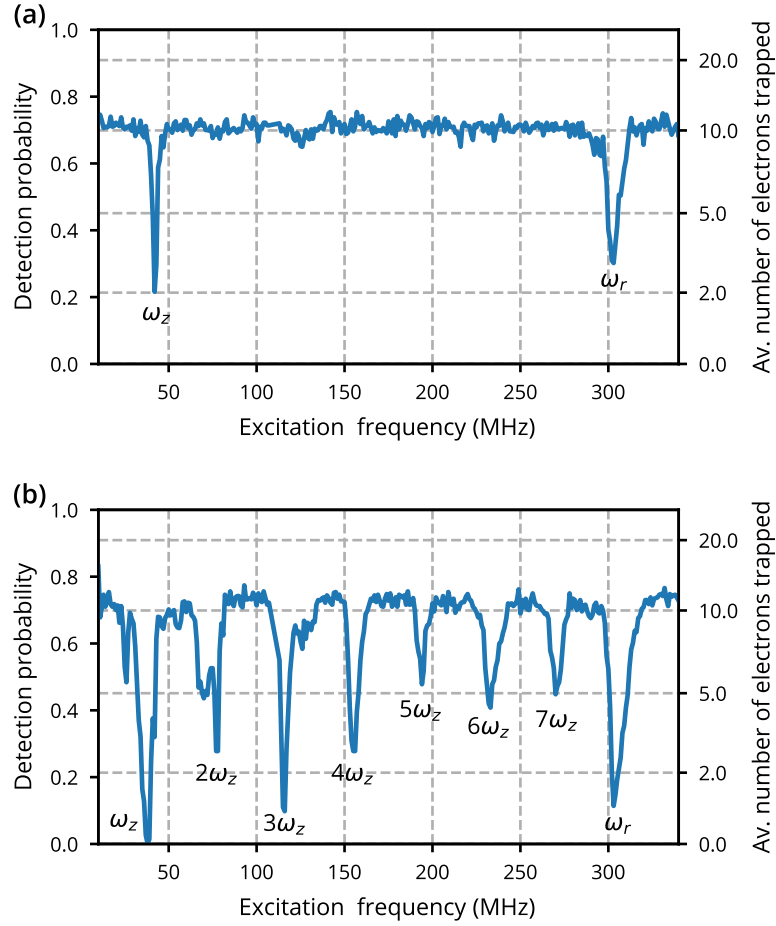


Figure 5.15: Measured axial and radial trap frequencies for $U_2 = -0.5 \text{ V/mm}^2$, $P_{\text{rf}} = 2 \text{ dBm}$, $t_{\text{load}} = 200 \text{ } \mu\text{s}$, and $I_{\text{oven}} = 4.25 \text{ A}$. Each data point corresponds to 500 repetitions. Measurements are taken with a wait time $t_{\text{wait}} = 2 \text{ ms}$ during which an RF tickle pulse is applied to one of the DC electrodes $e_{\text{in}, 17}$. The axial and radial resonances are labeled ω_z and ω_r . (a) -12 dBm tickle excitation, (b) -6 dBm tickle excitation. Figure reproduced from Ref. [99].

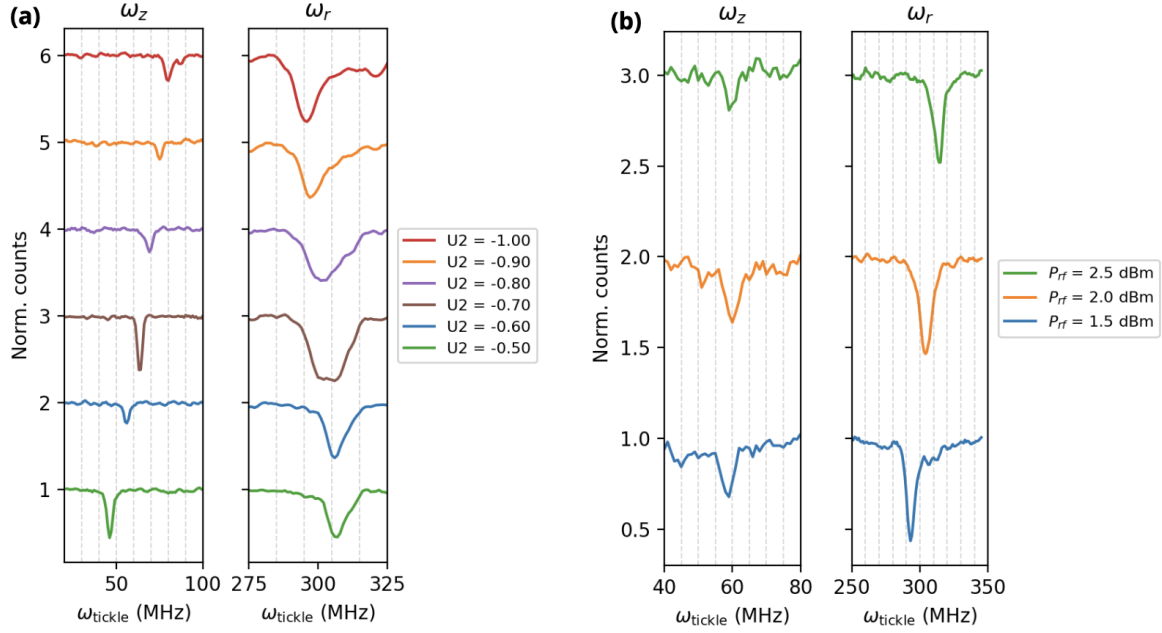


Figure 5.16: Trap-frequency measurements under varying DC and RF confinement strength. Each data point corresponds to 500 repetitions. Measurements use $t_{\text{load}} = 200 \mu\text{s}$, $I_{\text{oven}} = 4.25 \text{ A}$, and a wait time of $t_{\text{wait}} = 2 \text{ ms}$ during which an RF tickle pulse is applied to $e_{\text{in}, 17}$. Axial and radial resonances are denoted ω_z and ω_r . (a) DC confinement dependence: axial and radial frequencies for various static fields at $P_{\text{rf}} = 2 \text{ dBm}$ with -12 dBm tickle excitation. (b) RF confinement dependence: axial and radial frequencies for various RF powers at $U_2 = -0.7 \text{ V/mm}^2$ with -18 dBm tickle excitation.

axial mode, as shown in Fig. 5.15 (b).

Changing the DC and microwave voltages, we can tune the axial mode frequency between 30 and 100 MHz and the radial mode between 200 and 380 MHz, limited by the voltage sources used in the experiment. See Fig. 5.16 for the axial and radial resonance responses to different DC and RF confinement strengths.

As can be seen from this experiment, even with a millimeter-sized trap, the achieved trap frequency of trapped electrons is already almost two orders of magnitude higher than that for typical ion trap experiments. This is one of the main motivations of our work, as the secular frequency sets a natural speed for ion trap quantum information processors.

Electron temperature estimation

The width of the resonances observed in the trap frequency measurements can be linked to the possible range of electron motional amplitudes, and therefore the energy or temperature of trapped electrons, through trap anharmonicities. The general idea is that hot electrons

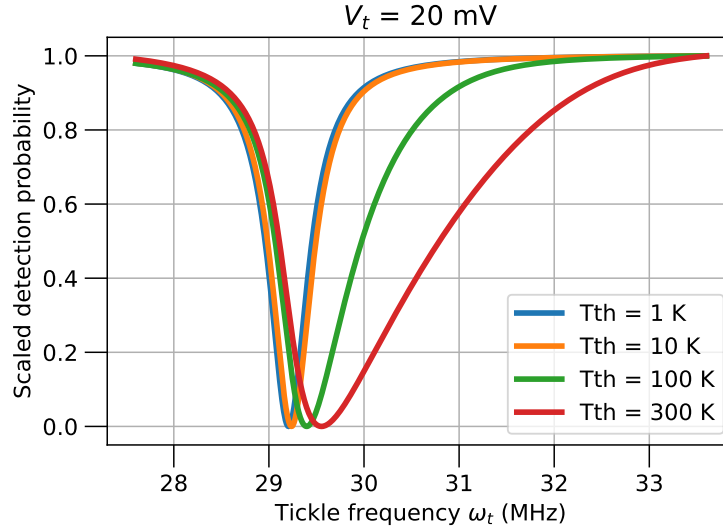


Figure 5.17: Axial trap frequency signal width for a tickle excitation of 20 mV at different radial temperatures. To predict the response of the electron in frequency space, we carried out simulations of the electron dynamics at fixed energies after being excited by a tickle pulse at a certain frequency and fitted a Lorentzian to the simulated response spectrum. We also simulated the temperature-dependent frequency shifts of the trapping potential of the three-layer PCB trap. Assuming a Boltzmann distribution for the radial temperature, we convolved the two results and calculated the predicted shape of our trap frequency measurement.

will experience large detectable frequency shifts due to the anharmonicity of the trapping potential, as discussed in Sec. 2.2.3. Figure 5.17 shows a numerical simulation of the axial trap frequency spectrum probed by a tickle pulse at various radial temperatures. From these spectra, the radial temperature can be inferred from the frequency broadening of the axial response. To model the behavior of the electron in the frequency domain, we simulate the electron dynamics at fixed energies (and thus fixed trap frequencies) after applying a tickle pulse at a given excitation frequency. A Lorentzian fit to the simulated response yielded a FWHM of 0.5 MHz for a single motional frequency component. We additionally performed full electron trajectory simulations to determine the temperature-dependent frequency shifts induced by the trapping potential of the three-layer PCB trap, using the potential obtained from an Ansys FEM simulation. These simulations show an axial trap frequency shift of approximately $\Delta\omega_z \approx 0.5 \text{ MHz}$ for a radial temperature of $T_r = 100 \text{ K}$. Assuming a Boltzmann distribution of radial energies, we convolved the resulting temperature-dependent frequency shifts with the intrinsic Lorentzian response to obtain the predicted axial frequency lineshape. We note that the experimentally observed spectra already display the expected asymmetric broadening, which allows us to place an upper bound of approximately $T_r \lesssim 1000 \text{ K}$ on the radial temperature of the trapped electrons, corresponding to an axial trap frequency width

of about 3 MHz.

5.4 Loss mechanism

Long storage times are essential if trapped electrons are to be used as qubits. Therefore, it is important to understand the loss mechanism of trapped electrons. In this section, we discuss possible mechanisms for electron loss in our room-temperature Paul trap experiment, focusing mainly on loss due to collisions with background gas particles and loss due to trap anharmonicities.

5.4.1 Collisions with the background gas

The dominant loss mechanism for laser-cooled ions is collisions with the background gas. Collisions, in particular with heavier atoms and molecules, can impart sufficient energy to kick an ion out of its trapping potential or may lead to the formation of molecules. Given the light mass of the electron, one might expect collisional loss and electron capture to play a major role in our trap as well.

In order to quantify this loss channel, we change the pressure in the vacuum chamber by more than one order of magnitude by turning the ion pump on and off. We measure the decay constant at chamber pressures of about $\sim 5 \times 10^{-10}$ mbar and $\sim 2 \times 10^{-8}$ mbar. Although operating the MCP with the ion pump on substantially increases the background detection rate, we corrected for this effect by independently measuring the background count rate with no electrons present and subtracting this rate from the detected signal. After correcting for this effect, we find that the extracted storage times for short-lived electrons are essentially unchanged within our measurement uncertainty. This rules out collision with the background gas as the primary loss channel for short-lived electrons.

In more detail, an ion pump with line of sight to the main vacuum chamber is used to keep the pressure below the 1×10^{-10} mbar level. We found that operating the MCP while the ion pump is on leads to background detections in excess of 10 kilo-counts per second. When the Ca oven is hot and the trap microwave drive is on, the pressure in the chamber increases to $\sim 5 \times 10^{-10}$ mbar and the electron detections due to the ion pump increase by 1-2 orders of magnitude, lowering the signal to noise ratio of our measurements. Hence, we typically operate with the ion pump switched off during measurements. The pressure in the chamber then increases above 1×10^{-8} mbar. Figure 5.18 shows two measurements of electron storage time. The data shown as circles are taken under typical conditions, where the ion pump is turned off and the chamber pressure is $\sim 2 \times 10^{-8}$ mbar. Upon turning on the ion pump and repeating the measurement, we record the data shown as triangles. To account for the detections caused by the ion pump, the background level is measured separately and subtracted from this data set. As the storage times agree within the measurement uncertainties we rule out collisions with the background gas as the dominant loss mechanism for short-lived electrons.

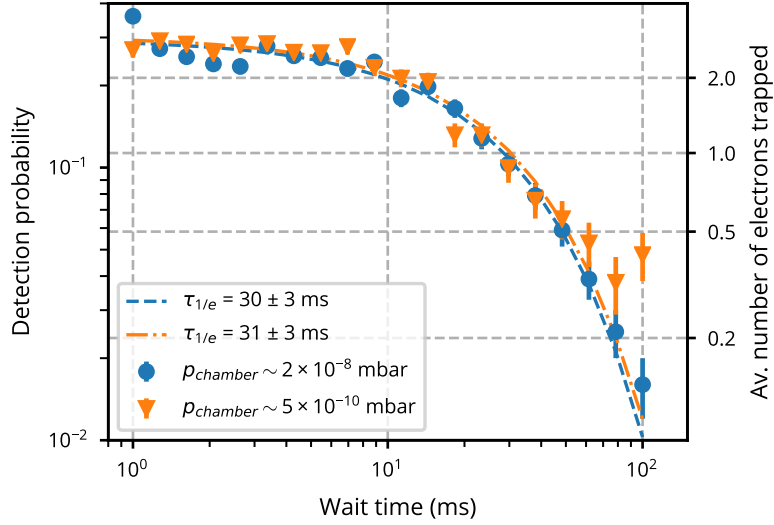


Figure 5.18: Electron storage-time measurements for $U_2 = -1.5 \text{ V/mm}^2$ and $P_{\text{rf}} = 5 \text{ dBm}$. Each data point corresponds to 1000 repetitions. Filled circles (dashed fit) correspond to typical operating conditions with the ion pump switched off. Triangles (dash-dotted fit) correspond to reduced background pressure achieved by operating the ion pump. Figure reproduced from Ref. [99].

We also note here that although collisions with the background gas are not the dominant loss mechanism for our room-temperature setup, they do become a primary limitation in our cryogenic trapping experiments. At room temperature, the chamber pressure is approximately $2 \times 10^{-7} \text{ Torr}$, and we typically observe a substantial increase in electron lifetime once the cryostat is cooled, consistent with the enhanced cryopumping and corresponding reduction in the density of the residual gas.

5.4.2 Trap anharmonicities

The behavior of trapped electrons in our experiment differs from that of laser-cooled ions in two important ways. First, electrons are much lighter, increasing their secular frequencies and ground state extension. Second, we did not apply any active cooling mechanism which damps the motion and concentrates particles in the trap center during the experiment. Combining these two, electrons in our trap can sample a much larger volume and hence experience anharmonicities in the trapping potential further away from the trap center. Although the general motion of a charged particle in an anharmonic AC potential is non-trivial [36], we can make two general distinctions to the harmonic case. First, normal modes of motion

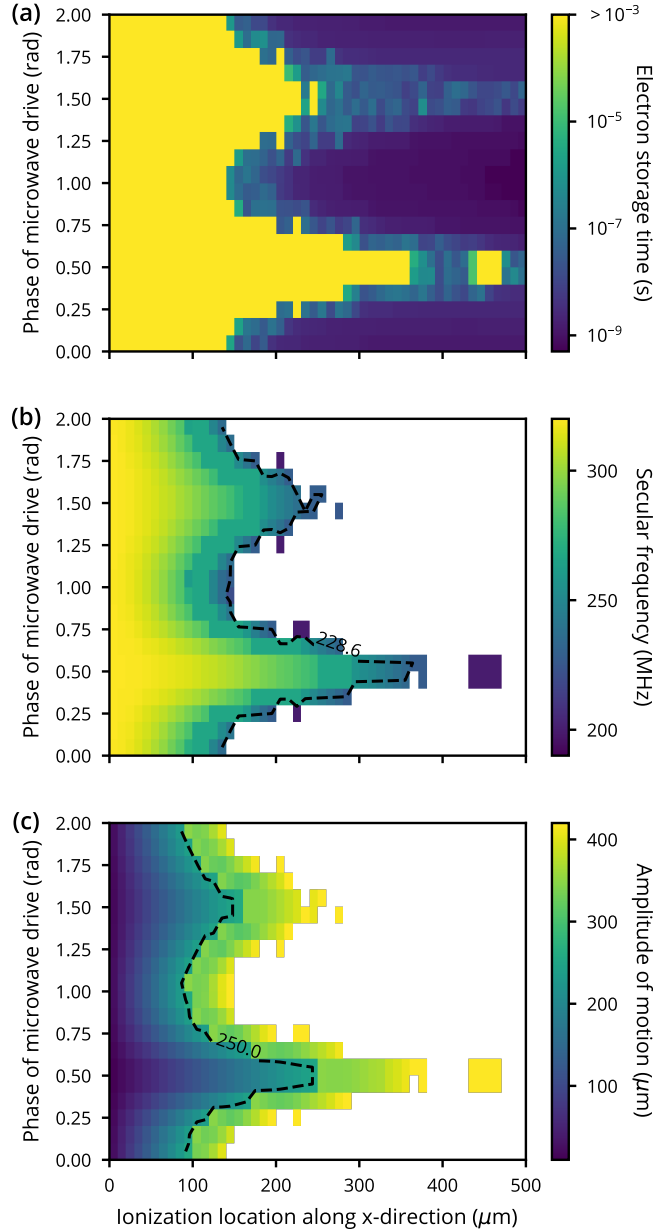


Figure 5.19: Electron motion simulations. (a) Storage times for one-dimensional motion through the trap along the x -axis and the trap center (see Fig. 5.4 (a), (b)). Storage times are calculated for a maximum time of 1 ms. The plot axes indicate the ionization location and the microwave phase at the time of ionization. The electron is assumed to have zero kinetic energy at the time of ionization. (b) Secular frequency of motion. The dashed curve highlights the frequency contour for the 7th subharmonic of the microwave drive frequency. (c) Corresponding motional amplitudes. The dashed contour line for an amplitude of 250 μm highlights where the motion frequency coincides with the 6th subharmonic of the drive frequency and the motional amplitude increases abruptly. White areas in (b) and (c) indicate storage times below 1 μs . Figure reproduced from Ref. [99].

become coupled and second, nonlinear resonances [103, 104] enable the transfer of energy from RF driven micromotion to secular motion, leading to heating and particle loss (RF heating) [70].

We gain insight into this process from numerical simulations of single-electron motion in the full time-dependent RF field, restricted to one spatial dimension along the x -axis for simplicity and computation efficiency (see Fig. 5.19). The path of the ionization lasers follows the x -axis when projected on the xy -plane of the pseudopotential and, based on neutral calcium fluorescence measurements we know electrons are created up to several millimeters away from the trap center. Hence, we choose to simulate the motion of a single charge in the time-dependent field from the microwave electrode along the x -axis. As discussed earlier, the energy, and consequentially the trajectory, of a single electron is determined by the surplus ionization energy, the ionization location, and the phase of the microwave field at ionization. Therefore, the initial conditions are parameterized by the ionization distance from the trap center and the RF phase at the moment of ionization, while the initial velocity is set to zero to reflect the small excess energy from the PI process. Figure 5.19 (a) shows the simulated storage time as a function of these two parameters capped at 1 ms for computational efficiency. We hypothesize that the trajectories that live up to this time are likely stable indefinitely in our simulation. We find that electrons created within roughly 120 μm from the trap center are stable on the simulation timescale for all RF phases, whereas those created further out exhibit strongly phase-dependent lifetimes ranging from sub-ns to ms. The transition from stable to unstable trajectories is very pronounced for all microwave phases.

Panels (b) and (c) of Fig. 5.19 display the corresponding secular frequencies and motional amplitudes for trajectories that survive longer than 1 μs . The transition to unstable trajectories in Fig. 5.19 (b) occurs at the same secular frequency for all parameter combinations (see dashed contour line) and we can identify this frequency as the 7th subharmonic of the microwave drive frequency: $1.6 \text{ GHz}/7 \approx 229 \text{ MHz}$. The 6th subharmonic around 267 MHz is also visible as a uni-colored band in parameter space where the secular frequency is locked to the subharmonic. At these subharmonic frequencies, energy is pumped from the driven micromotion into the secular motion, which heats up the motion. We can observe the heating effect on the map of motional amplitudes in Fig. 5.19 (c): the motional amplitude first increases smoothly as a function of the ionization distance from the trap center, until an amplitude of about 250 μm is reached (see labeled contour). Here, the motional amplitude suddenly increases. This jump coincides with the secular frequency hitting the 6th subharmonic. However, the trap anharmonicity is not sufficient to drive electrons out of the trap. Electron loss occurs for motional amplitudes just above 400 μm , when the 6th subharmonic is reached.

The analysis of one-dimensional electron motion illustrates the relevance of nonlinear resonances in anharmonic trapping potentials. Qualitatively, this behavior extends to three dimensions: because all motional modes are coupled, the density of nonlinear resonances in frequency space increases substantially [104]. Energy can be pumped from the driving field into the electron secular motion over a broad range of frequencies. Although many

electron trajectories appear stable in simulations—such as those near the sixth subharmonic discussed above—they are not robust to perturbations in the absence of a cooling mechanism at the same time. Perturbations to the motion, for instance, due to space charge effects of nearby untrapped or trapped electrons, or charges on the trap itself, or fluctuations in the power of the microwave drive, can eventually push an electron into a nonlinear resonance that heats it out of the trap. We believe that this mechanism underlies the millisecond-scale electron storage times observed in our measurements.

Chapter 6

Towards cooling and electronic detection

One of the central requirements for realizing a trapped-electron-based quantum computing platform is the ability to trap electrons at cryogenic temperatures and cool the electron motion to minimize the extent of their wave function, thus enabling high-fidelity operations.

In this chapter, we describe the cryogenic electron trapping experiment in a 4 K cryostat, present the experimental steps and considerations towards cooling and detecting electrons, and discuss a path towards an improved cooling scheme utilizing micromotion, in which the cooling tank circuit is tuned to the micromotion blue sideband and allows cooling of the electron secular motion to lower temperatures than standard resistive cooling.

6.1 Cryogenic experiment

6.1.1 The cryostat

The cryostat used in the experiment is a revived closed-cycle cryostat (ARS DE-210AF) acquired in 2008. The system operates on the Gifford-McMahon refrigeration cycle and consists of a compressor, a two-stage cold head (40 K and 4 K) and two helium exchange lines between them. The nominal cooling power of the cryostat at 4 K is about 1 W. The compressor provides high- and low-pressure helium gas (99.999% ultra-high purity), while a rotary valve driven by an external motor periodically connects the cold head to the two pressure reservoirs. Its position is synchronized with the motion of the displacer. A regenerative heat exchanger maximizes the refrigeration. To isolate the vacuum chamber from vibrations generated by the cold-head motor, the expander is connected to the trap chamber via a gas-filled bellow that separates the expander from the vacuum chamber. The rubber bellow is filled with helium gas at a pressure slightly above ambient pressure and is cooled by the heat exchanger. The helium gas, in turn, cools the sample in the vacuum chamber and eventually liquefies to provide a cooling buffer. To restore proper cryostat performance

after it had been exposed to atmosphere for several years, we replaced the adsorber in the gas filter and baked the displacer assembly to remove residual moisture.

Managing thermal loads

Reaching the lowest possible temperature requires minimizing the heat flow into the cold head. Heat enters through three primary mechanisms: conduction, convection, and thermal radiation.

Heat conduction is the transfer of energy from one molecule to another by direct contact. It arises from mechanical supports, wiring, and any unintended thermal shorts between stages. To reduce conductive loads, we eliminate the thermal short between the 40 K shield and the 300 K vacuum chamber by carefully leveling the cryostat to avoid direct contact, and improve the thermal contact by applying thermal grease, Stycast, GE varnish, and indium. Convection is the movement of heat as a result of the flow of fluid medium such as water or air. Residual gas conduction is suppressed by purging and sealing the cryostat with high-purity helium and leak-checking all feedthroughs. During the experiment, an activated charcoal cryo-sorption pump was added to the cold stage, reducing the base temperature by approximately 2 K. Radiation is the transfer of heat by electromagnetic waves. Unlike conduction or convection, heat transfer by radiation does not need any matter to help with the transfer. Thermal radiation is reduced by installing both 40 K and 4 K radiation shields, covering unused optical ports, and lining the aluminum shields with nickel-plated highly reflective surfaces. Multilayer insulation (superinsulation) further decreases the radiative load.

Base temperature and cooling power

Although the manufacturer specifications state a base temperature of 2.9 K without applied load, our system exhibits a higher base temperature after thermal management. In the experiment, the cryostat has a base temperature of about 6 K with the trap wired-up without any heat loads and 14 K when operating with 1 W of input power, similar to the input power required of the trap RF drive. This discrepancy can be attributed to the degraded heat exchangers used by the system to reduce vibration because the cryostat was exposed to the atmosphere for a few years before we revived the system.

As discussed in Sec. 3.6, for trapped electrons to form a Wigner crystal, thus enabling two-qubit gates, we need to cool the axial motion of the trapped electrons below 2 K. Also in Chap. 4 we mentioned that dominant error sources for trapped electron qubits scale inversely with the temperature of the motion. Ultimately, temperatures of 400 mK will be required for good performance, i.e., gate fidelities in excess of 0.9999.

Even though the current cryostat does not meet the temperature requirements, we carry out the first cryogenic electron trap experiment in this setup as a first step towards cooling and electronic detection. Meanwhile, we also acquired a cryogen-free closed-loop cooling

system from Bluefors that will be installed in early 2026. The system has a base temperature of 0.45 K with Helium-3 and a cooling power of more than 5 mW at 0.6 K and 1.35 W at 4 K.

6.1.2 Cryogenic electron trapping

In comparison with room-temperature operation, the trapping of electrons in a cryogenic environment introduces several additional technical challenges. Beyond the complexities of the cryostat itself, we must re-evaluate how to create electrons, how to extract and detect electrons, and how to operate electron detectors in a low-temperature, high-vacuum setting.

In the room-temperature setup, cold electrons are produced via two-photon photoionization of calcium atoms emitted from a resistively heated oven. However, a conventional oven typically dissipates more than 1 W of power and requires currents of a few Amps — both of which are incompatible with the limited cooling power available at 4 K. For cryogenic operation, two strategies may be considered. The first is to place the oven entirely outside the 4 K region, in the 300 K environment, and deliver calcium atoms through a narrow aperture into the trap region. The second is to adopt a laser-heated oven, in which heating power is delivered optically rather than electrically. This approach avoids running high-current leading to the cold stage and allows the oven assembly to be thermally isolated far more effectively.

Before implementing electronic detection of image currents, we still need to rely on destructive detection to evidence electron trapping. The microchannel plate detector (MCP) used at room temperature cannot operate at cryogenic temperatures and therefore must remain at 300 K. Extracting electrons through the nested 40 K and 4 K radiation shields requires openings in each shield, which inevitably introduce radiative heat load. In addition, because the MCP is located further from the trap compared to the room-temperature setup, the extraction fields must be carefully shaped to ensure efficient collection. To this end, we mount metallic grids on the windows of both 40 K and 4 K shields. These grids serve to shape and guide the electric field lines so that the extracted electrons are reliably directed toward the detector. Because MCPs are delicate, susceptible to damage under elevated pressures, and cannot tolerate operating pressures above 10^{-7} Torr, we used a channeltron (Photonis CEM 5901) as the electron detector for the cryogenic experiment.

6.1.3 Experimental setup

The cryogenic trapping apparatus is shown in Fig. 6.1. The electron trap, which is the same design as the PCB trap used in the room-temperature setup (see Sec. 5.1), is mounted on the 4 K base plate inside a nested 40 K/4 K radiation-shield assembly. The overall design follows the room temperature experimental geometry, but incorporates several additional constraints required for stable operation at cryogenic temperatures: thermal anchoring of all heat-dissipating components, minimization of radiative loading, access to neutral calcium atoms for electron creation, and reliable extraction of electrons toward a detector located at room temperature.

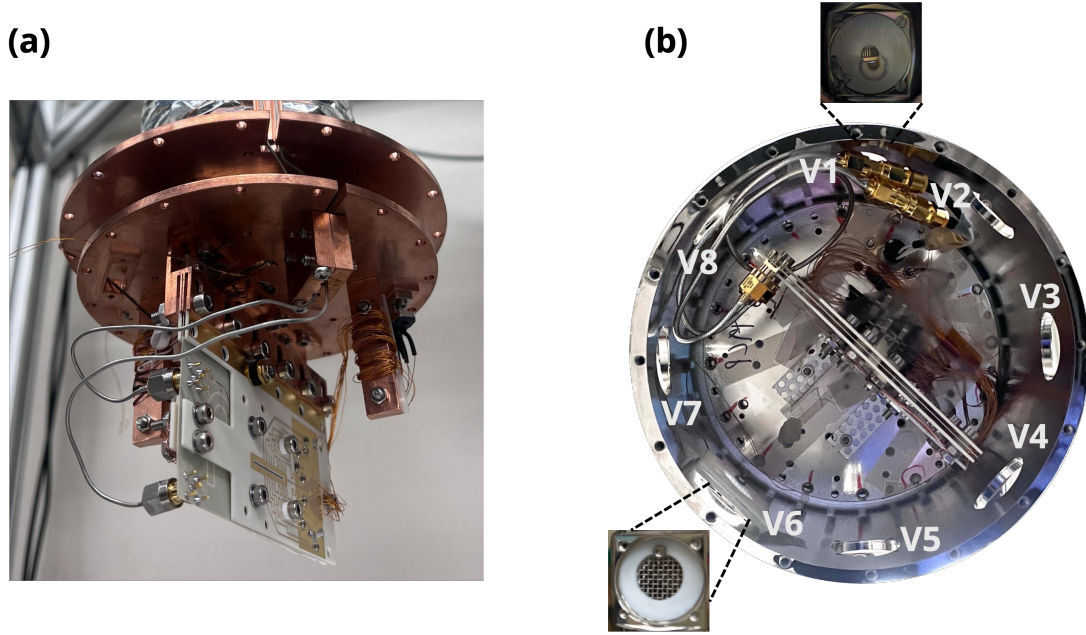


Figure 6.1: Cryogenic experimental setup. (a) The three-layer PCB electron trap mounted on the 4 K base plate. OFHC copper structures provide thermal anchoring for the PCB trap, the RF feedline, and the DC wiring. (b) Bottom view of the 4 K base plate showing viewports V1–V8. A small aperture on V1 (and also on 40 K shield) allows calcium atoms from the resistively heated oven on the 300 K octagon to reach the trap center. The two photoionization beams pass through window V3 and V7. A metal grid mounted on V6 (and another metal grid mounted on the 40 K shield) shapes the electric field so that electrons exit towards a channeltron detector located on the 300 K octagon. All other viewports are blanked to minimize radiative heat load onto the 4 K stage.

Figure 6.1 (a) shows the three-layer PCB electron trap mounted directly to the 4 K base plate using copper supports. These OFHC copper structures provide well-defined thermal connections for the trap body, RF cables, and all DC wiring. The trap is positioned at the geometric center of the 4 K shield, ensuring uniform optical access. RF and DC signals required for trapping are routed through the 40 K and 4 K stages with intermediate thermal anchoring to intercept heat before it reaches the base plate.

A bottom view of the 4 K base plate is shown in Fig. 6.1 (b), where eight viewports (V1–V8) provide optical and atomic access. To generate electrons, calcium atoms produced by a resistively heated oven located on the 300 K octagon enter the cryogenic region through a narrow aperture on viewport V1. Two photoionization (PI) laser beams, overlapped and aligned perpendicular to the atomic beam, enter the 4 K region through V3 and V7 and intersect with the calcium beams at the trap center. This geometry reproduces the room-temperature loading scheme while minimizing the optical access required at 4 K.

Efficient electron extraction toward the detector requires precise shaping of the electric field inside the radiation shields. For this purpose, the metal grids are mounted on viewport V6 of the 4 K shield and on the corresponding opening of the 40 K shield. These grids define an electrostatic funnel that guides electrons outward through both radiation shields to the channeltron detector located on the 300 K octagon. All remaining viewports are blanked with polished aluminum plates to suppress blackbody radiation from the 300 K environment and reduce the steady-state heat load on the cold stage.

6.2 Experiment results

6.2.1 Trapping

We successfully trapped electrons in the cryogenic setup using a protocol similar to that introduced in Chap. 5. As expected, the loading efficiency is reduced compared with that of the room-temperature experiment, since the atomic oven is located farther from the trap center in the cryogenic configuration.

In trapped-ion experiments, cryogenic operation typically yields longer storage times as a result of improved vacuum from cryopumping. In our system, however, the cryopumping efficiency is substantially diminished and the trap does not reach ultra-high vacuum conditions because of small openings in both the 40 K and 4 K shields for electron creation, extraction, and detection.

At room temperature, the chamber pressure is measured as 2×10^{-7} Torr. In lifetime measurements at cryogenic temperatures, we observed only short-lived electron signals and no evidence of a long-lived component. Moreover, the lifetime of the short-lived electrons is strongly temperature dependent: it increases from roughly 200 μ s at 300 K to about 2.3 ms at 120 K and to about 26 ms at 11 K. This clear correlation with temperature (and thus pressure) indicates that the dominant loss mechanism in our cryogenic setup is background-gas collisions from the exposed holes in the 40 K and 4 K shields.

6.2.2 Charging

A pronounced difference between the cryogenic and room-temperature experiments is the presence of much stronger charging effects at low temperature. Figure 6.2 shows two trap-frequency scans taken under identical operating conditions but with opposite scan directions. Each measurement takes about ~ 10 minutes. Between the two measurements the first order multipole terms E_x, E_y, E_z were adjusted to optimize for the trapping signal, and we find that changes in the field component perpendicular to the PCB surface (x -direction in Fig. 5.1) are particularly sensitive. This compensation field, together with the noticeable baseline shift between the two traces, indicates that strong charging effects induce measurable drifts in the trapping potential on timescales of only a few minutes.

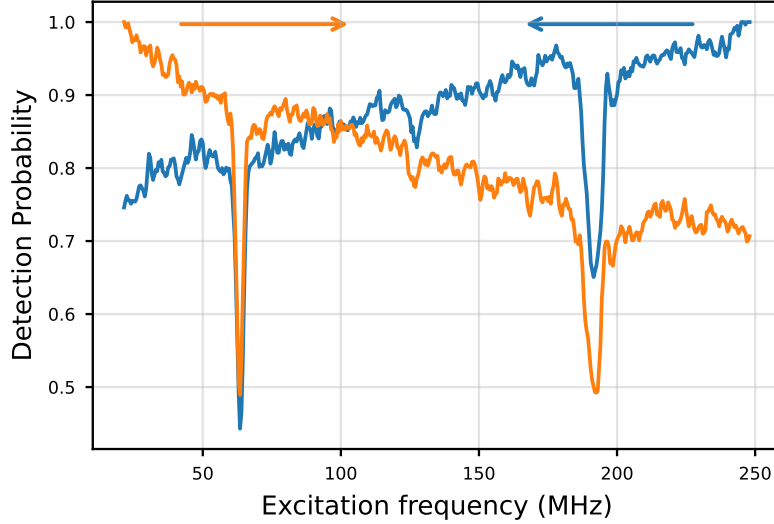


Figure 6.2: Cryogenic electron trap frequency measurements taken with $U_2 = -0.7 \text{ V/mm}^2$, $P_{\text{rf}} = 2.5 \text{ dBm}$, $t_{\text{load}} = 2 \text{ ms}$, $I_{\text{oven}} = 6.5 \text{ A}$, and a -21 dBm tickle excitation. Each data point represents 10^3 repetitions. The measurements were performed with a wait time $t_{\text{wait}} = 1 \text{ ms}$ during which an RF tickle pulse was applied to the DC electrode $e_{\text{in},17}$. The blue and orange traces correspond to frequency scans taken in opposite directions; each scan required approximately 10 minutes. Between the two scans, the first-order multipole terms E_x, E_y, E_z were adjusted to compensate for accumulated charging. The shift in the baseline secular frequency demonstrates that significant charging effects develop on timescales of only a few minutes.

We attribute this behavior to two main factors: (i) the strong calcium flux present during electron loading, and (ii) the openings in the 40 K and 4 K radiation shields that allow particles from the poor room-temperature vacuum to reach cold surfaces of the electron trap, creating a dielectric layer at the trap. The dielectric layer at the trap may lead to charging that then produces electric stray fields. To mitigate these effects, we installed a copper shield with a narrow 0.5 mm slit directly in front of the outer PCB board to reduce the flux of the background atoms, and to prevent calcium deposition on the electrodes. In future experiments that implement electronic detection, we will be able to eliminate the large openings required for destructive detection. Further improvements are possible by exploring alternative electron-loading methods. In particular, we plan to adopt a laser-heated oven placed inside the 4 K stage, where the heating power is delivered optically rather than electrically. This approach eliminates the need for openings in the radiation shields and is expected to reduce charging and radiative heat load.

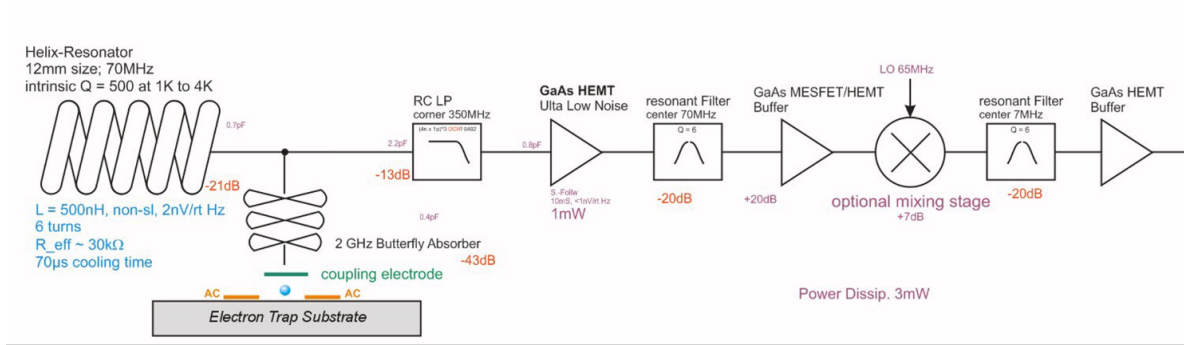


Figure 6.3: Schematic of the electronic detection circuit designed by Stahl-Electronics for a trap drive frequency of $\Omega_{\text{rf}}/2\pi = 2 \text{ GHz}$ and an axial motional frequency of $\omega_z/2\pi = 70 \text{ MHz}$. A helical resonator tuned to the axial mode frequency is used to detect and cool the electron motion. Suppression of RF pickup from the trap drive is achieved using a coplanar butterfly absorber and multiple filters. The signal is then amplified by several stages of low-noise amplifiers and can optionally be down-mixed with a local oscillator for heterodyne detection.

6.3 Considerations for electronic detection

Before attempting spin readout, we must first establish a reliable method to detect the presence of trapped electrons without removing them from the trap. The underlying physical mechanism and experimental considerations were introduced in Sec. 2.4.4 and Sec. 3.3: the electron’s motion induces an image current in the trap electrodes, which produces a measurable voltage drop across an attached resonant tank circuit.

Figure 6.3 shows the schematic of a single-electron detection circuit developed by Stahl-Electronics. The system incorporates an electron trap, a butterfly absorber, and multiple RF filters to suppress the pickup of the trap drive, a helical resonator for resistive cooling, and several stages of low-noise amplification to boost the signal to detectable levels.

If an electron is present, the measured power spectral density exhibits a characteristic feature: the Johnson noise of the tank circuit is “shorted out” at the electron’s motional frequency [42]. Figure 6.4 shows the expected spectra after two amplification stages. In the ideal case, the noise dip at the electron frequency would vanish completely. In practice, amplifier noise and frequency broadening mechanisms—such as fluctuations in trap strength and trap anharmonicities reduce the dip. Achieving high detection fidelity therefore requires a sufficiently large signal-to-noise ratio (SNR).

A crucial assumption in estimating the electron signal relative to the Johnson noise is that the electron oscillates at a well-defined frequency. If the electron frequency fluctuates, its induced signal spreads across frequency space and can become buried in the noise background. Thus, the achievable SNR depends not only on thermal and amplifier noise but also on the stability of the electron’s secular frequency.

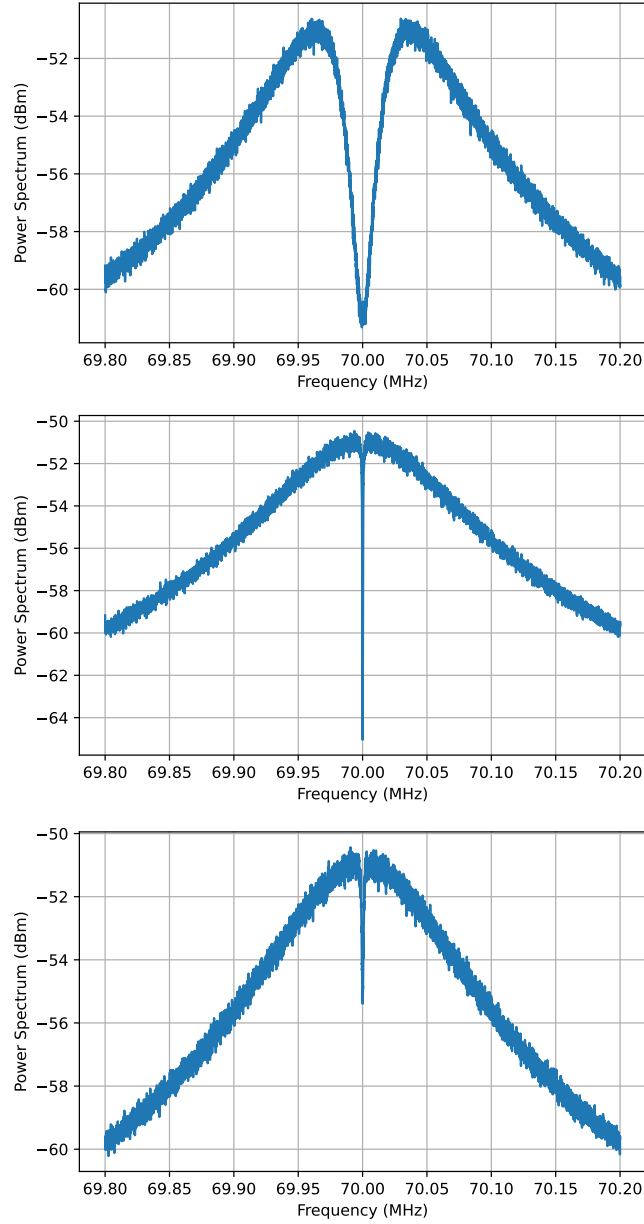


Figure 6.4: Expected spectra of the single-electron detection signal after two amplification stages, corresponding to the circuit in Fig. 6.3. The signal consists of a broad Lorentzian peak arising from the Johnson noise of the tank circuit, with a narrow dip at the electron’s motional frequency where the trapped electron “shorts out” this noise [42]. For these calculations we assume a tank circuit with quality factor $Q_t = 500$ and effective impedance of $300\ \Omega$, and an amplifier chain providing 20 dB gain and $1\text{ nV}/\sqrt{\text{Hz}}$ equivalent input noise. The spectrum analyzer noise floor is taken to be -135 dBm with a resolution bandwidth of 100 Hz. Signals shown here are averaged over 32 traces for clarity. The top panel uses $d_{\text{eff}} = 138\ \mu\text{m}$ and relative frequency fluctuations $\Delta f/f_0 = 10^{-4}$; the bottom two use a conservative value $d_{\text{eff}} = 800\ \mu\text{m}$ with $\Delta f/f_0 = 10^{-6}$ (middle) and $\Delta f/f_0 = 10^{-5}$ (bottom).

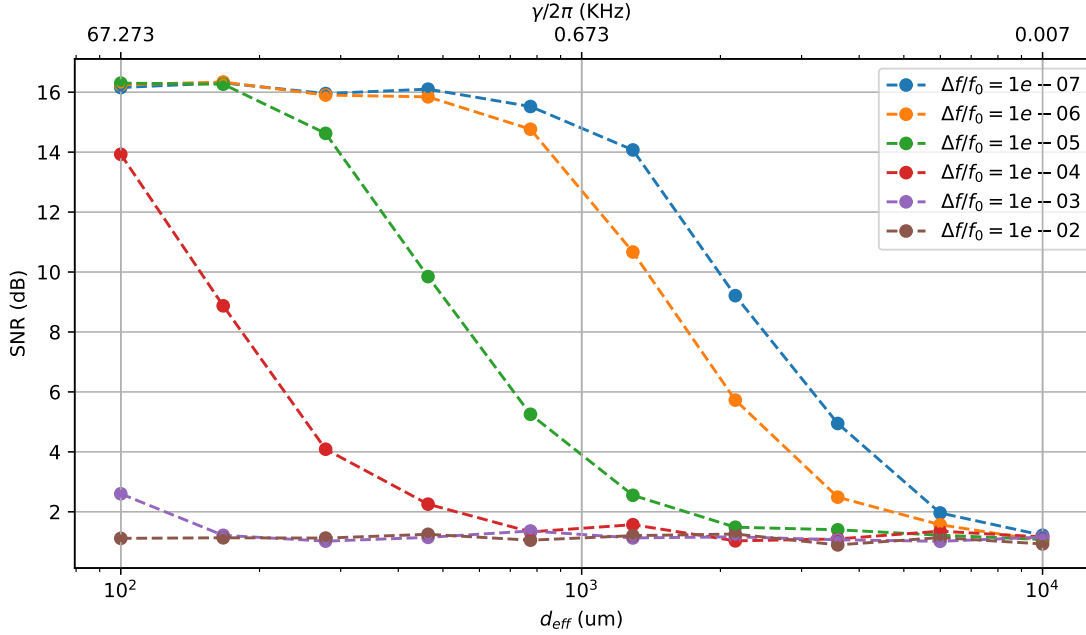


Figure 6.5: Estimated SNR for single-electron electronic detection at 4 K as a function of coupling strength d_{eff} . Different curves correspond to different assumed motional frequency broadening around an axial frequency of $2\pi \times 70$ MHz. The tank circuit is assumed to have an on-resonance effective resistance of $150 \text{ k}\Omega$. Simulations include both Johnson noise and amplifier noise, and were performed jointly with Andris Huang.

To estimate the expected frequency stability, we consider both trap voltage fluctuations and trap anharmonicities. DC voltages can typically be stabilized at the 10^{-6} level. For an axial frequency of $\omega_a = 2\pi \times 70$ MHz, this corresponds to a broadening of approximately 70 Hz. Trap anharmonicities, by contrast, generate amplitude-dependent frequency shifts that can dominate frequency broadening when the electron motion is warm.

Assuming relative frequency fluctuations of 10^{-7} at 4 K and 10^{-6} at 14 K due to anharmonicities, and taking an effective electron-electrode distance of $d_{\text{eff}} = 800 \text{ }\mu\text{m}$, we simulate the expected electron signals and plot the corresponding SNRs in Fig. 6.5. For a frequency broadening of $\Delta f/f_0 = 10^{-6}$, the SNR is approximately 14 dB (see Fig. 6.4, middle), while $\Delta f/f_0 = 10^{-5}$ would still yield an acceptable SNR of 5 dB.

For comparison, the prototype trap considered in Chap. 3 has a much smaller effective distance, $d_{\text{eff}} = 138 \text{ }\mu\text{m}$, resulting in a larger intrinsic linewidth of approximately $1/(2\pi \times 4.7 \text{ }\mu\text{s}) \approx 34 \text{ kHz}$. Under these conditions, even relatively large frequency fluctuations of $\Delta f/f_0 = 10^{-4}$ still result in an SNR primarily limited by amplifier noise (see Fig. 6.4, top).

These results emphasize the importance of minimizing trap anharmonicities, stabiliz-

ing trap voltages, and reducing electron–electrode separation to achieve robust, high-SNR electronic detection of single trapped electrons.

6.4 Micromotion sideband cooling

In Sec. 2.4.5, we showed that coupling the electron axial motion to a damped tank circuit leads to resistive (Purcell-enhanced) cooling of the secular modes. In that setting, the resonator acts as an engineered thermal reservoir at its resonance frequency, and the electron asymptotically acquires the corresponding thermal occupation. In this section, we introduce a new cooling configuration—micromotion sideband cooling, which builds on the same basic mechanism but exploits the periodic micromotion of the trapped electron to couple its secular motion to a reservoir at a higher effective frequency. This enables substantially lower final motional occupations than is possible with carrier (resistive) cooling alone, at the cost of longer cooling times.

6.4.1 Cooling mechanism

We now analyze micromotion sideband cooling within the same framework as Sec. 2.4.5. In the last part of Sec. 2.4.5, we considered the coupling between the axial motional mode and the tank circuit. Here we focus instead on the radial secular motion. For the radial mode of the electron along x , the equations of motion are

$$\begin{aligned}\dot{\hat{x}}(t) &= \frac{\hat{p}(t)}{m}, \\ \dot{\hat{p}}(t) &= -mW(t)\hat{x}(t) - \frac{e}{C_t d_{\text{eff}}} \hat{Q}(t).\end{aligned}\tag{6.1}$$

For the tank circuit, we have

$$\begin{aligned}\dot{\hat{Q}}(t) &= -\frac{\hat{\Phi}(t)}{L_t}, \\ \dot{\hat{\Phi}}(t) &= \frac{\hat{Q}(t)}{C_t} + \frac{e}{C_t d_{\text{eff}}} \hat{x}(t),\end{aligned}\tag{6.2}$$

where the last term arises from the interaction Hamiltonian

$$\hat{H}_{\text{int}} = e \frac{\hat{x}}{d_{\text{eff}}} \frac{\hat{Q}}{C_t}.\tag{6.3}$$

It is convenient to express these equations in terms of ladder operators $\hat{a}_x(\hat{a}_x^\dagger)$ and $\hat{a}_\phi(\hat{a}_\phi^\dagger)$ as in Eq. (2.64) and Eq. (2.178). In contrast to Sec. 2.4.5, the operators $\hat{a}$ and $\hat{a}_\phi$ are now explicitly time-dependent and our goal is to determine their dynamics.

Substituting Eq. (2.64) into Eq. (6.1) and using the fact that $u(t)$ solves the homogeneous Mathieu equation, we obtain a linear system (recall Sec. 2.2.4)

$$\begin{aligned}\dot{\hat{a}} u^*(t) + \hat{a}^\dagger u(t) &= 0, \\ \dot{\hat{a}} u^*(t) + \hat{a}^\dagger \dot{u}(t) &= -2i\nu \alpha (\hat{a}_\phi^\dagger e^{i\omega_t t} - \hat{a}_\phi e^{-i\omega_t t}),\end{aligned}\tag{6.4}$$

and

$$\alpha = \frac{ex_0 Q_0}{\hbar C_t d_{\text{eff}}} = \frac{e}{2d_{\text{eff}}} \sqrt{\frac{\omega_t}{m\omega_x C_t}}.\tag{6.5}$$

Using the Wronskian identity $u^*(t)\dot{u}(t) - u(t)\dot{u}^*(t) = 2i\nu$ [29], we can invert this system to obtain

$$\dot{\hat{a}}(t) = \alpha u(t) (\hat{a}_\phi^\dagger e^{i\omega_t t} - \hat{a}_\phi e^{-i\omega_t t}),\tag{6.6}$$

and its Hermitian conjugate.

Similarly, for the circuit we find

$$\dot{\hat{a}}_\phi(t) = \alpha (\hat{a} u^*(t) + \hat{a}^\dagger u(t)) e^{i\omega_t t} - \frac{\gamma_t}{2} \hat{a}_\phi(t) + \sqrt{\gamma_t} \hat{b}(t),\tag{6.7}$$

where we have included a dissipative term $-\gamma_t \hat{a}_\phi/2$ and a noise term $\sqrt{\gamma_t} \hat{b}(t)$ describing the coupling of the tank circuit to a thermal reservoir at temperature T_t [105].

We assume $\hat{b}(t)$ has a short correlation time, so that, in the frequency domain,

$$\langle \hat{b}[\omega]^\dagger \hat{b}[\omega'] \rangle = \bar{n}(\omega_t) \delta(\omega - \omega'),\tag{6.8}$$

where

$$\bar{n}(\omega_t) = \frac{1}{e^{\hbar\omega_t/k_B T_t} - 1}\tag{6.9}$$

is the Bose–Einstein occupation number of the bath at frequency ω_t . This noise self-correlation assumes that the tank circuit has a high quality factor and that we are only interested in the behavior of the circuit close to its resonance frequency.

We are interested in comparing two regimes:

1. *Carrier cooling (resistive cooling)*: $\omega_t \approx \omega_x$, the tank circuit is resonant with the radial secular motion, which is the case in resistive cooling as discussed in Sec. 2.4.3 and Sec. 2.4.5.
2. *Micromotion sideband cooling*: $\omega_t \approx \omega_x + \Omega_{\text{rf}}$, the tank circuit is tuned to the micromotion blue sideband of the secular radial motion.

We now make use of the rotating wave approximation, near $\omega_t = \omega_x$ and $\omega_t = \omega_x + \Omega_{\text{rf}}$. Dropping the fast-oscillating terms, in both cases we arrive at the coupled system of equations

$$\frac{d}{dt} \begin{bmatrix} \hat{a} \\ \hat{a}_\phi \end{bmatrix} = \underbrace{\begin{bmatrix} 0 & -g \\ g & -\gamma_t/2 \end{bmatrix}}_{\Gamma} \underbrace{\begin{bmatrix} \hat{a} \\ \hat{a}_\phi \end{bmatrix}}_v + \underbrace{\begin{bmatrix} 0 \\ \sqrt{\gamma_t} \hat{b} \end{bmatrix}}_{\beta}\tag{6.10}$$

or, in short-hand notation,

$$\dot{v} = \Gamma v + \beta. \quad (6.11)$$

Here, the constant g depends on the circuit frequency ω_t ,

$$\begin{aligned} g^{(\omega_x)} &= C_0 \alpha = \frac{eC_0}{2d_{\text{eff}}} \sqrt{\frac{1}{mC_t}}, \\ g^{(\omega_x + \Omega_{\text{rf}})} &= C_2 \alpha = \frac{eC_2}{2d_{\text{eff}}} \sqrt{\frac{\omega_x + \Omega_{\text{rf}}}{m\omega_x C_t}}, \end{aligned}$$

where the C_{2n} are constants that appear as the solution to $u(t)$ (see Eq. (2.62)) and which depend on Mathieu parameters a_x, q_x . Their relative value is

$$\frac{g^{(\omega_x + \Omega_{\text{rf}})}}{g^{(\omega_x)}} = \frac{C_2}{C_0} \sqrt{\frac{\omega_x + \Omega_{\text{rf}}}{\omega_x}}. \quad (6.12)$$

The formal solution to Eq. (6.11) is

$$v(t) = e^{\Gamma t} v(0) + \int_0^t e^{\Gamma(t-\tau)} \beta(\tau) d\tau. \quad (6.13)$$

The first component of $v(t)$ yields

$$\hat{a}(t) = \varepsilon_{00}(t) \hat{a}(0) + \varepsilon_{01}(t) \hat{a}_\phi(0) + \sqrt{\gamma_t} \int_0^t \varepsilon_{01}(t - \tau) \hat{b}(\tau) d\tau, \quad (6.14)$$

where the matrix elements of $e^{\Gamma t}$ are

$$\varepsilon_{00}(t) = (e^{\Gamma t})_{00} = e^{-\gamma_t t/4} \left[\cosh(\beta t) + \frac{\gamma_t}{4\beta} \sinh(\beta t) \right], \quad (6.15)$$

$$\varepsilon_{01}(t) = (e^{\Gamma t})_{01} = -e^{-\gamma_t t/4} \frac{g}{\beta} \sinh(\beta t), \quad (6.16)$$

with

$$\beta = \sqrt{\gamma_t^2/16 - g^2}. \quad (6.17)$$

In the overdamped regime, $\gamma_t/4 > g$, β is real and both modes decay without oscillation.

Assuming the electron, circuit, and bath are uncorrelated at $t = 0$, and that the circuit is continuously thermalized via its coupling to the bath, the mean phonon number evolves as

$$\bar{n}_a(t) = \langle \hat{a}^\dagger(t) \hat{a}(t) \rangle = |\varepsilon_{00}(t)|^2 n_a(0) + \gamma_t \bar{n}(\omega_t) \int_0^t |\varepsilon_{01}(s)|^2 ds, \quad (6.18)$$

where $n_a(0)$ is the initial thermal occupation $\langle \hat{a}^\dagger(0) \hat{a}(0) \rangle$. We note here that the eigenstates of the reference oscillator number operator $a^\dagger a$ are not energy eigenstates of the system, since micromotion periodically changes the total kinetic energy of the electron. However,

since micromotion is periodic, the quantum number $n = a^\dagger a$ corresponds to the energy of the electron averaged over a period $T_{\text{rf}} = 2\pi/\Omega_{\text{rf}}$ of the drive frequency [29]. At long times, the transient terms vanish and the integral saturates to $\int_0^\infty |\varepsilon_{01}(s)|^2 ds = 1/\gamma_t$, so that

$$\bar{n}_a(\infty) = \bar{n}(\omega_t). \quad (6.19)$$

Thus, the electron ultimately thermalizes to the bath temperature at the tank-circuit frequency ω_t . Because $\bar{n}(\omega_t)$ decreases with increasing ω_t at fixed T , tuning the circuit to the micromotion blue sideband ($\omega_t \approx \omega_x + \Omega_{\text{rf}}$) allows a lower final occupation than carrier cooling at $\omega_t \approx \omega_x$.

In the overdamped regime $\gamma_t \gg 4g$, the dynamics are well approximated by an exponential relaxation,

$$\bar{n}_a(t) \approx [n_a(0) - \bar{n}(\omega_t)]e^{-t/\tau_{\text{cool}}} + \bar{n}(\omega_t), \quad (6.20)$$

with a characteristic cooling time

$$\tau_{\text{cool}} \simeq \frac{\gamma_t}{4g^2}, \quad (6.21)$$

consistent with the adiabatic-elimination result where the effective damping rate of the electron mode is enhanced by $4g^2/\gamma_t$ (the Purcell-like rate).

6.4.2 Lowest-order approximation

In the regime $(|a_x|, q_x^2) \ll 1$, the mode function $u(t)$ can be approximated to the lowest order as

$$u(t) \approx e^{i\nu t} \frac{1 + \frac{q_x}{2} \cos(\Omega_{\text{rf}} t)}{1 + \frac{q_x}{2}}, \quad \nu \approx \beta_x \frac{\Omega_{\text{rf}}}{2} = \omega_x, \quad \beta_x \approx \sqrt{a_x + \frac{q_x^2}{2}}. \quad (6.22)$$

Using this expression in the interaction Hamiltonian Eq. (6.3) and expanding $\hat{x}(t)$ and $\hat{Q}(t)$ in terms of $\hat{a}$ and $\hat{a}_\phi$, one recovers, under the RWA, the beam-splitter Hamiltonian

$$H_{\text{int}} = i\hbar g (\hat{a}\hat{a}_\phi^\dagger - \hat{a}^\dagger\hat{a}_\phi), \quad (6.23)$$

which coherently swaps excitations between the electron and the circuit. This is in the same form as in Eq. (2.187) where we derived the resistive cooling dynamics when the tank circuit is coupled to the axial motion of the trapped electron $\omega_t = \omega_z$. The tank circuit itself is damped at a rate γ_t and continuously rethermalized by the external resistor at temperature T_t , so it acts as an engineered reservoir with occupation $\bar{n}(\omega_t)$.

From this perspective, the cooling mechanism is a two-step process: (i) excitations are transferred from the electron mode to the circuit mode at a rate g , and (ii) the circuit then irreversibly dumps this energy into the resistive environment at a rate γ_t . The combined effect is an effective damping of the electron mode at rate $4g^2/\gamma_t$ in the overdamped limit, i.e., a Purcell-like enhancement of its coupling to the bath. The steady state is reached when $\bar{n}_a(\infty) = \bar{n}(\omega_t)$.

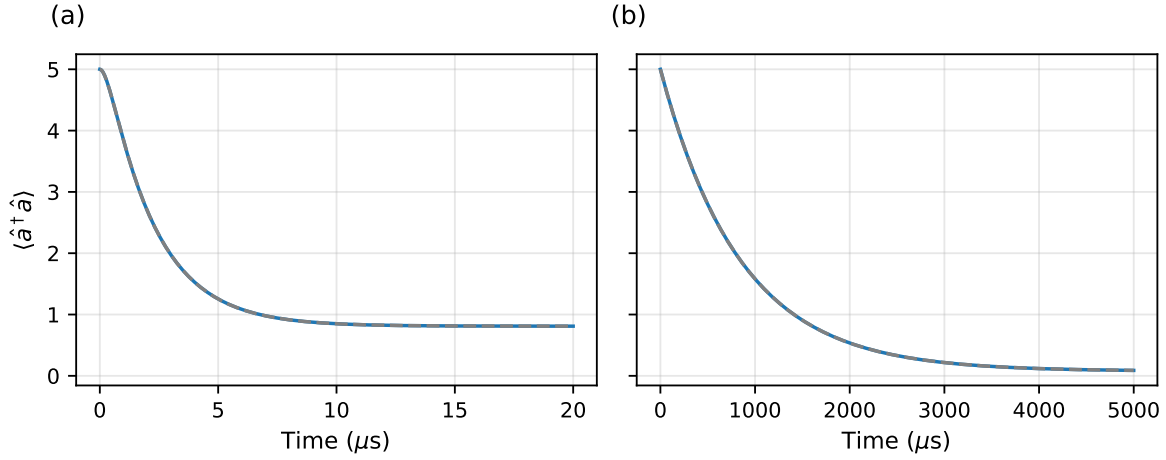


Figure 6.6: (a) Carrier (resistive) cooling dynamics and (b) micromotion sideband cooling dynamics. Solid lines show the numerical simulation results using QuTip, and dashed lines show the analytical prediction using Eq. (6.18) which overlaps with the simulation.

For the carrier case $\omega_t = \omega_x$, we have

$$g^{(\omega_x)} = \frac{e}{2(1 + \frac{q_x}{2})d_{\text{eff}}\sqrt{C_t m}}. \quad (6.24)$$

For the micromotion blue sideband, $\omega_t = \omega_x + \omega_{\text{rf}}$, we have

$$g^{(\omega_x + \Omega_{\text{rf}})} = g^{(\omega_x)} \frac{q_x}{4} \sqrt{1 + \frac{\Omega_{\text{rf}}}{\omega_x}}. \quad (6.25)$$

6.4.3 Numerical simulation

To validate these analytical results, we numerically integrate the full dynamics with realistic parameter choices. Eq. (6.6) and (6.7) correspond to a Hamiltonian of

$$\hat{H}_I = i\hbar\alpha[u(t)e^{-i\Omega t}\hat{a}^\dagger\hat{a}_\phi + u(t)e^{i\Omega t}\hat{a}\hat{a}_\phi^\dagger - h.c.] \quad (6.26)$$

We consider a single electron in a trap driven at $\Omega_{\text{rf}} = 2\pi \times 10$ GHz with Mathieu parameters $a_x = 0.01$ and $q_x = 0.25$, corresponding to a radial secular frequency $\omega_x \approx 2\pi \times 1$ GHz. For carrier cooling, the tank circuit is tuned to $\omega_t = \omega_x$, while for micromotion sideband cooling we choose $\omega_t = \omega_x + \Omega_{\text{rf}} = 2\pi \times 11$ GHz. We assume a cryogenic environment at $T = 40$ mK for computational efficiency, yielding thermal occupations $\bar{n}(\omega_x) \approx 0.8$ and $\bar{n}(\omega_x + \Omega_{\text{rf}}) \approx 0.08$. Assuming an effective distance $d_{\text{eff}} = 100$ μm , capacitance $C_t = 1$ pF,

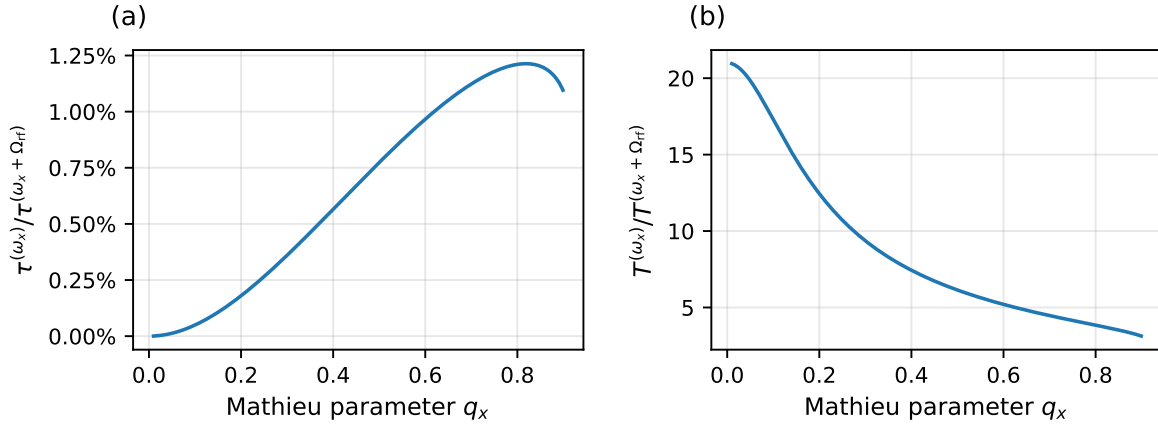


Figure 6.7: (a) Cooling time ratio of the carrier (resistive) cooling to the micromotion sideband cooling (b) Final temperature ratio of the carrier (resistive) cooling to the micromotion sideband cooling.

and quality factor of the tank circuit $Q = 1000$, we obtain

$$\gamma_t^{(\omega_x)} = \frac{\omega_x}{Q} = 2\pi \times 1 \text{ MHz}, \quad g^{(\omega_x)} \approx 2\pi \times 135 \text{ kHz}, \quad (6.27)$$

$$\gamma_t^{(\omega_x + \Omega_{rf})} = \frac{(\omega_x + \Omega_{rf})}{Q} = 2\pi \times 11 \text{ MHz}, \quad g^{(\omega_x + \Omega_{rf})} \approx 2\pi \times 23 \text{ kHz}. \quad (6.28)$$

From Eq. (6.21), we have the corresponding cooling time constant

$$\tau_{\text{cool}}^{(\omega_x)} = \frac{\gamma_t^{(\omega_x)}}{4(g^{(\omega_x)})^2} \approx 2.2 \mu\text{s}, \quad \tau_{\text{cool}}^{(\omega_x + \Omega_{rf})} = \frac{\gamma_t^{(\omega_x + \Omega_{rf})}}{4(g^{(\omega_x + \Omega_{rf})})^2} \approx 827 \mu\text{s}. \quad (6.29)$$

Figure 6.6 compares the numerically simulated cooling dynamics for both the carrier-cooling and micromotion sideband cooling configurations. In each case, the simulations exhibit excellent agreement with the analytical prediction of Eq. (6.18), confirming that the simplified RWA model accurately captures the essential cooling dynamics. At a fixed cryostat temperature, tuning the tank circuit to the micromotion blue sideband yields a substantially lower final occupation of the electron's secular motion compared to standard carrier (resistive) cooling, albeit at the cost of a longer cooling time. This tradeoff is tunable: by adjusting the operating parameters of the Paul trap—particularly the Mathieu parameter q_x —one can vary both the relative cooling rate and the achievable steady-state temperature. Figure 6.7 illustrates how the ratios of the cooling time constants and the final temperatures between the sideband and the carrier cooling depend on q_x .

In summary, micromotion sideband cooling provides a powerful route to achieving motional occupations far below those attainable with conventional resistive cooling, effectively

coupling the electron to a colder, engineered reservoir. Although this approach generally requires longer cooling times, the balance between cooling rate and final temperature can be optimized through appropriate trap design and choice of operating parameters. One may also envision a two-stage cooling scheme in which a first tank circuit, resonant with the secular motion, rapidly pre-cools the electron via resistive (carrier) cooling, followed by a second tank circuit tuned to the micromotion blue sideband to reach significantly lower temperatures—an approach analogous to the combination of Doppler cooling and resolved-sideband cooling used for trapped ions. Together, these results establish micromotion sideband cooling as a practical and versatile pathway toward preparing ultra-cold electron motion suitable for high-fidelity quantum operations.

Chapter 7

Conclusion and Prospects

The work presented in this thesis has not yet realized a trapped-electron quantum computer. However, it establishes many of the essential ingredients: a concrete architectural proposal; a detailed theoretical understanding of the key building blocks; a demonstrated room-temperature microwave Paul trap; and a cryogenic platform and circuit design for cooling and non-destructive detection of single-electron motion. Together, these developments outline a realistic path from the present prototype experiments to a future trapped-electron quantum processor. In the following, we summarize the main achievements presented in this work and provide an outlook on future directions.

Feasibility study of trapped-electron quantum processing

Building on the well-established techniques of ion trapping in Paul traps and electron trapping in Penning traps, we have developed a theoretical framework for using electrons confined in Paul traps as spin qubits. We designed a blueprint trapped-electron quantum information processor and studied experimental proposals for trapping, cooling, electronic detection, spin readout, and single- and multi-qubit quantum logic in that context.

We analyzed the conditions under which few-electron Wigner crystals can be formed in linear Paul traps using classical trajectory simulations and used these results to determine threshold temperatures for resistive cooling. We further investigated gate schemes based on spin-dependent forces generated by oscillating magnetic field gradients. Including typical decoherence mechanisms and realistic trap parameters, we showed that two-qubit Bell-state fidelities on the order of 99.99% are in principle achievable, with a sufficiently cold motional mode and dynamical decoupling scheme.

Room-temperature trapping in a PCB Paul trap

As a first experimental step, we demonstrated electron trapping in a room-temperature, millimeter-scale PCB Paul trap driven in the GHz microwave regime. Electrons are loaded into the trap within tens of microseconds via photoionization of neutral calcium atoms.

From the detection probability and detection efficiency, we estimate the number of trapped electrons to range from single electrons to ensembles of several tens of electrons. A fraction of these electrons survive for up to at least one second, and we identify trap anharmonicities as a dominant loss mechanism.

We measured secular frequencies in the range of several tens to several hundreds of megahertz. From the width of the motional resonance, we estimate that the electron temperature is on the order of 1000 K. This experiment represents, to our knowledge, the first demonstration of electron trapping in a linear Paul trap at microwave frequencies and establishes that stable confinement is achievable with realistic RF and DC voltages, electrode geometries, and loading schemes that are compatible with later operation in a cryogenic environment.

Cryogenic experiment and micromotion sideband cooling

To progress from room-temperature trapping to a useful qubit platform, the electron's motional degrees of freedom must be cooled and detected non-destructively. As a first step in this direction, we operated the PCB Paul trap in a closed-cycle cryostat at 4 K and demonstrated trapping under cryogenic conditions. We observed more severe charging effects compared to the room-temperature setup, which informed design changes and operational strategies to mitigate charging in future iterations. We also provided an initial design of the cooling and detection circuits and discussed experimental considerations for optimizing the signal-to-noise ratio of the detection signal.

In addition to conventional resistive cooling of the secular motion, we proposed and analyzed cooling schemes that exploit the micromotion sidebands inherent to the trap RF drive. By tuning the tank circuit into resonance with the blue micromotion sideband, one can, at the same circuit temperature, achieve steady-state motional occupations far below those attainable with conventional carrier-resonant resistive cooling, at the cost of longer cooling times. In this way, the electron can be effectively coupled to a colder engineered reservoir. We derived effective cooling rates and steady-state occupation numbers for both carrier- and sideband-mediated cooling. This study informs the design of future experiments in which micromotion, typically considered a nuisance in ion trapping, is instead turned into a resource for efficient cooling of trapped electrons.

Outlook

The most immediate experimental milestone following the results in this thesis is the demonstration of resistive cooling and electronic detection of a single electron in the cryogenic Paul trap. The coupling strength between the electron motion and the tank circuit critically depends on the quality factor of the resonator, as well as the effective electron-electrode distance. The signal-to-noise ratio of the detection signal depends on the Johnson noise of the resistor, the specifications of the amplifier chain, and the stability of the electron's secular frequency. Therefore, it is crucial to reduce trap anharmonicities and trap voltage fluctuations during the experiment.

In future iterations of the experiment, several improvements are natural to approach the regime of cooling and electronically detecting a single electron: (i) *Trap design*: Shrink the trap size to reduce the effective distance d_{eff} between the electron and the detection electrodes, thereby enhancing the coupling strength. At the same time, minimize trap anharmonicity in the electrode design and use appropriate DC voltages to control higher-order anharmonic terms. (ii) *Tank and detection circuitry*: Build high- Q resonators and minimize parasitic capacitances to increase the effective impedance of the tank circuit, while implementing careful filtering to reduce pickup of the trap RF drive. (iii) *Cryogenic setup*: Commission a new cryostat with a lower base temperature around 0.4 K and higher cooling power. Mitigate charging effects by avoiding direct line-of-sight openings in the radiation shield, and explore alternative electron loading schemes and electronic detection geometries that are compatible with fully shielded operation.

Although the focus of this thesis has been on quantum information processing, the trapped-electron platform developed here has potential applications beyond quantum computing. A single electron in a Paul trap realizes an exceptionally light, high-frequency electromechanical oscillator that may serve as a precision probe of electric fields and forces at microwave frequencies [106, 107]. Small ensembles or crystals of trapped electrons can be used to explore strongly coupled plasmas [108] and electron–positron interactions [109]. Moreover, the techniques developed for trapping electrons could be adapted to positrons, opening avenues to anti-hydrogen production and precision tests of fundamental symmetries [110, 111].

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Appendix A

Spherical Harmonic Basis

j	l	m	Y_j	Names
1	0	0	1	C
2	1	-1	$-y$	E_y
3	1	0	z	E_z
4	1	1	$-x$	E_x
5	2	-2	$6xy$	U_3
6	2	-1	$-3yz$	U_4
7	2	0	$-\frac{1}{2}x^2 - \frac{1}{2}y^2 + z^2$	U_2
8	2	1	$-3xz$	U_5
9	2	2	$3x^2 - 3y^2$	U_1
10	3	-3	$-45x^2y + 15y^3$	N/A
11	3	-2	$30xyz$	N/A
12	3	-1	$\frac{3}{2}x^2y + \frac{3}{2}y^3 - 6yz^2$	N/A
13	3	0	$z^3 - \frac{3}{2}(x^2 + y^2)z$	N/A
14	3	1	$\frac{3}{2}x^3 + \frac{3}{2}xy^2 - 6xz^2$	N/A
15	3	2	$15(x^2 - y^2)z$	N/A
16	3	3	$-15x^3 + 45xy^2$	N/A

17	4	-4	$420x^3y - 420xy^3$	N/A
18	4	-3	$-105(3x^2y - y^3)z$	N/A
19	4	-2	$-15x^3y - 15xy^3 + 90xyz^2$	N/A
20	4	-1	$-10yz^3 + \frac{15}{2}(x^2y + y^3)z$	N/A
21	4	0	$\frac{3}{8}x^4 + \frac{3}{4}x^2y^2 + \frac{3}{8}y^4 + z^4 - 3(x^2 + y^2)z^2$	N/A
22	4	1	$-10xz^3 + \frac{15}{2}(x^3 + xy^2)z$	N/A
23	4	2	$-\frac{15}{2}x^4 + \frac{15}{2}y^4 + 45(x^2 - y^2)z^2$	N/A
24	4	3	$-105(x^3 - 3xy^2)z$	N/A
25	4	4	$105x^4 - 630x^2y^2 + 105y^4$	N/A
26	5	-5	$-4725x^4y + 9450x^2y^3 - 945y^5$	N/A
27	5	-4	$3780(x^3y - xy^3)z$	N/A
28	5	-3	$\frac{315}{2}x^4y + 105x^2y^3 - \frac{105}{2}y^5 - 420(3x^2y - y^3)z^2$	N/A
29	5	-2	$210xyz^3 - 105(x^3y + xy^3)z$	N/A
30	5	-1	$-\frac{15}{8}x^4y - \frac{15}{4}x^2y^3 - \frac{15}{8}y^5 - 15yz^4 + \frac{45}{2}(x^2y + y^3)z^2$	N/A
31	5	0	$z^5 - 5(x^2 + y^2)z^3 + \frac{15}{8}(x^4 + 2x^2y^2 + y^4)z$	N/A
32	5	1	$-\frac{15}{8}x^5 - \frac{15}{4}x^3y^2 - \frac{15}{8}xy^4 - 15xz^4 + \frac{45}{2}(x^3 + xy^2)z^2$	N/A
33	5	2	$105(x^2 - y^2)z^3 - \frac{105}{2}(x^4 - y^4)z$	N/A
34	5	3	$\frac{105}{2}x^5 - 105x^3y^2 - \frac{315}{2}xy^4 - 420(x^3 - 3xy^2)z^2$	N/A
35	5	4	$945(x^4 - 6x^2y^2 + y^4)z$	N/A
36	5	5	$-945x^5 + 9450x^3y^2 - 4725xy^4$	N/A

37	6	-6	$62370x^5y - 207900x^3y^3 + 62370xy^5$	N/A
38	6	-5	$-10395(5x^4y - 10x^2y^3 + y^5)z$	N/A
39	6	-4	$-1890x^5y + 1890xy^5 + 18900(x^3y - xy^3)z^2$	N/A
40	6	-3	$-1260(3x^2y - y^3)z^3 + \frac{945}{2}(3x^4y + 2x^2y^3 - y^5)z$	N/A
41	6	-2	$\frac{105}{4}x^5y + \frac{105}{2}x^3y^3 + \frac{105}{4}xy^5 + 420xyz^4 - 420(x^3y + xy^3)z^2$	N/A
42	6	-1	$-21yz^5 + \frac{105}{2}(x^2y + y^3)z^3 - \frac{105}{8}(x^4y + 2x^2y^3 + y^5)z$	N/A
43	6	0	$-\frac{5}{16}x^6 - \frac{15}{16}x^4y^2 - \frac{15}{16}x^2y^4 - \frac{5}{16}y^6 + z^6 - \frac{15}{2}(x^2 + y^2)z^4 + \frac{45}{8}(x^4 + 2x^2y^2 + y^4)z^2$	N/A
44	6	1	$-21xz^5 + \frac{105}{2}(x^3 + xy^2)z^3 - \frac{105}{8}(x^5 + 2x^3y^2 + xy^4)z$	N/A
45	6	2	$\frac{105}{8}x^6 + \frac{105}{8}x^4y^2 - \frac{105}{8}x^2y^4 - \frac{105}{8}y^6 + 210(x^2 - y^2)z^4 - 210(x^4 - y^4)z^2$	N/A
46	6	3	$-1260(x^3 - 3xy^2)z^3 + \frac{945}{2}(x^5 - 2x^3y^2 - 3xy^4)z$	N/A
47	6	4	$-\frac{945}{2}x^6 + \frac{4725}{2}x^4y^2 + \frac{4725}{2}x^2y^4 - \frac{945}{2}y^6 + 4725(x^4 - 6x^2y^2 + y^4)z^2$	N/A
48	6	5	$-10395(x^5 - 10x^3y^2 + 5xy^4)z$	N/A
49	6	6	$10395x^6 - 155925x^4y^2 + 155925x^2y^4 - 10395y^6$	N/A

Table A.1: List of spherical harmonic basis in Cartesian coordinates up to $l = 6$.