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# Accounting for Strain Rate Dependent Behavior during Consolidation of Saturated Clay

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## ABSTRACT

Two methods to account for strain rate dependence of clay behavior in computing settlement due to consolidation and secondary compression are compared. The first method explicitly accounts for strain rate effects during primary consolidation using a visco-plastic relationship and nonlinear finite difference solution. The second method indirectly accounts for strain rate effects during primary consolidation by using analytical solutions for consolidation and secondary compression in combination with a time-line based procedure for selecting the analysis parameters. The second method is computationally simpler, and potentially useful for more complicated soil layering and loading conditions. These two analysis methods are described, along with a third, traditional approach for comparison. Results are presented for one-dimensional analyses of a single clay layer for a range of stress history and loading conditions. The time-line based method is shown to provide a reasonable approximation of the more rigorous nonlinear analysis method, indicating that various traditional analysis methods can approximately account for strain rate effects (or creep) during primary consolidation through the appropriate selection of input properties.

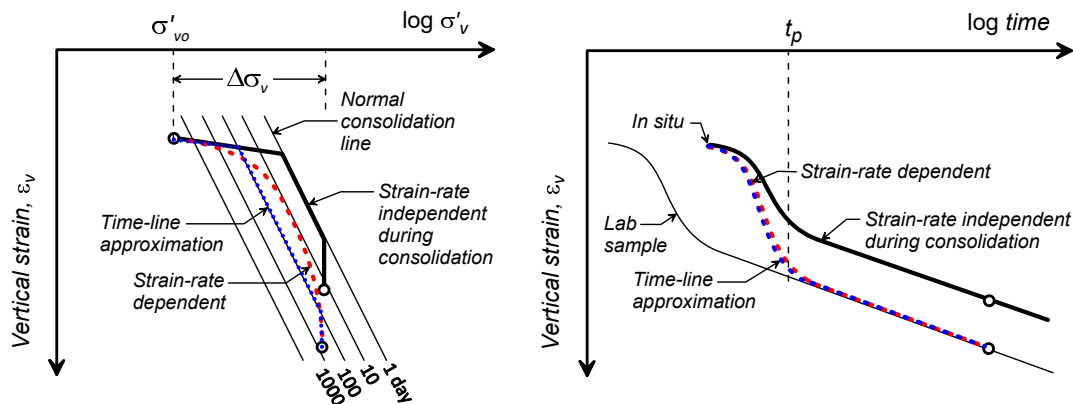
## INTRODUCTION

The viscous behaviors of saturated clay include strain-rate-dependent strength and compressibility, as well as secondary compression in which the void ratio continually decreases even when the effective stress is constant. These viscous behaviors have been examined by numerous investigators and are reasonably well understood (e.g., Bjerrum 1967, Graham et al. 1983, Leroueil 2006, Watabe et al. 2012).

A range of analysis methods, from simple to advanced, have been developed for computing settlements due to consolidation and secondary compression of saturated clays. The simpler analyses may be performed with spreadsheets, whereas specialized software is generally necessary when the analyses account for nonlinear properties, strain-rate dependent properties, large

deformations, three-dimensional geometries, complex loading histories, and other complicating features. The simpler, more traditional, approaches are sufficient for many situations, whereas a more advanced analysis can be warranted for larger or more complex projects.

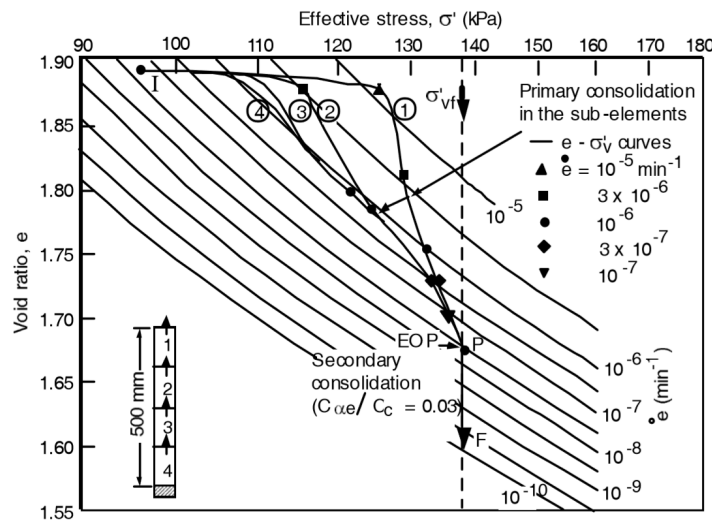
Most analysis methods do not explicitly model or simulate strain-rate effects during primary consolidation, which simplifies the analysis considerably. These analysis methods include any that assume strain-rate-independent properties, such as embodied in most closed-form analytical solutions and covered in most introductory geotechnical engineering textbooks and design manuals. This includes finite element (FE) or finite difference (FD) analysis methods, whether one-, two- or three-dimensional, that use strain-rate-independent constitutive models (e.g., elastic or elasto-plastic constitutive models). Strain-rate effects during primary consolidation may still be approximately accounted for in these types of analyses using a time-line based interpretation of the consolidation parameters (e.g., Bjerrum 1967). A time-line based approach is based on the observation that secondary compression to different ages produces normal consolidation curves that are approximately parallel as illustrated in Figure 1. A time-line based analysis is illustrated in Figure 1 (modified after Ladd et al. 1977), wherein the ultimate settlement some long time after the loading is imposed (100 years in this schematic) would be computed using the consolidation line corresponding to that age. Also shown on Figure 1 is a curve that neglects any strain-rate dependence (or creep) during primary consolidation and, for this schematic, assumes that primary consolidation in the field follows the virgin compression curve obtained from 24 hr load increment ratio (LIR) oedometer results. The assumption of rate-independence is inconsistent with experimental data demonstrating the strain-rate dependency of clay compressibility, but its assumptions are easily implemented and common in practice.



**Figure 1. Consolidation and secondary compression at a point within a clay layer assuming: rate-independent compressibility during primary consolidation, rate-dependent compressibility, and a time-line approximation (modified after Ladd et al. 1977)**

Analysis methods that explicitly simulate strain-rate effects during primary consolidation generally involve more complex soil models and often require nonlinear solution techniques. These analysis methods include isotache-based analyses (e.g., Suklje 1957, Leroueil 2006, Watabe et al. 2012) and any FE/FD analyses that use visco-plastic or other strain-rate dependent

constitutive models (e.g., Kutter and Sathialingam 1992, Yin and Graham 1994, Brandenburg 2016, Yin and Feng 2016). The concept of an isotache-based analysis is shown in Figure 2, wherein the compression curves followed by different sub-elements in a 500-mm-thick layer of St. Hilaire clay are shown for a load increment from 97 to 138 kPa (Leroueil and Marques 1996). The compression curve followed during primary consolidation is different for each sub-element because the strain rate is larger closer to the drainage boundary, and thus sub-elements closer to the drainage boundary reach isotaches corresponding to larger strain rates. These types of analysis methods produce an end-of-primary compression curve that depends on the duration of primary consolidation; this outcome is widely but not universally accepted (e.g., Mesri and Kane 2018).



**Figure 2. Consolidation of the Saint-Hilaire clay for pressure increment from 97 kPa to 138 kPa (after Mesri et al. 1995; from Leroueil and Marques 1996)**

This paper compares two approaches to accounting for strain-rate effects in clay during consolidation; one is a nonlinear method that explicitly accounts for strain-rate effects during primary consolidation, whereas the second is a time-line based method that only indirectly accounts for strain-rate effects. The nonlinear analysis uses the one-dimensional finite-difference analysis program developed by Brandenburg (2016), wherein the soil is modeled using a visco-plastic relationship that follows from the visco-plasticity constitutive model developed by Kutter and Sathialingam (1992). The time-line based analysis uses conventional one-dimensional consolidation and secondary compression theories, but with the pre-consolidation stress ( $\sigma'_{vp}$ ) and equivalent end-of-primary consolidation time ( $t_p$ ) dependent on the stress and loading history for the clay. A third analysis, presented for reference purposes, uses the same theories as the time-line based approach but with the consolidation parameters based directly on the results of 24-hr LIR tests (strain-rate independent assumption in Figure 1, referred to herein as the traditional approach). The three analysis approaches are described first, followed by a comparison of analysis results for a range of stress history and loading conditions. The time-line based approach is shown to provide a reasonable approximation of the more rigorous nonlinear analyses, indicating that

various traditional analysis methods can approximately account for strain-rate effects during primary consolidation through the appropriate selection of input properties.

## **ANALYSIS APPROACHES**

### **Nonlinear consolidation analysis by iConsol**

The nonlinear finite difference program, iConsol (Brandenberg 2016) directly simulates strain-rate effects during consolidation and secondary compression. Volumetric creep behavior is modeled by relating visco-plastic volumetric strain rates to the distance in  $e$ - $\log_{10}(\sigma'_v)$  space between a current point and a corresponding point on a reference secondary compression line (RSCL). This approach follows from the generalized visco-plasticity model of Kutter and Sathialingam (1992) and reasonably approximates experimental observations regarding strain-rate-dependent compressibility and secondary compression behaviors in clays with a range of stress histories, and avoids dependency on arbitrary time clocks.

### **Traditional analysis approach**

The “traditional” analysis approach used herein uses conventional consolidation and secondary compression theories, with the consolidation parameters based directly on the results of 24-hr LIR tests and with secondary compression strains computed based on  $t_p$ . This approach implies a resetting of  $t_p$  upon application of any loading increment, or at least a subjective decision regarding how large a loading increment should be for the resetting of  $t_p$ . The arbitrary resetting of  $t_p$  regardless of loading history or loading increment can produce unrealistic responses in certain situations. This approach is included for reference purposes, as it represents the approach described in many introductory geotechnical engineering textbooks and design manuals.

### **Time-line based analysis approach**

The time-line based analysis approach used herein uses the same basic equations as the above-described traditional approach, but with  $\sigma'_{vp}$  and equivalent  $t_p$  selected following the time-line framework introduced by Bjerrum (1967). The RSCL is taken herein as the normal consolidation line corresponding to  $t_{ref} = 1$  day, consistent with the common use of 24-hour load increment testing (other testing procedures may necessitate use of a different reference time, but the following analysis approach would be the same). The virgin compression ( $C_c$ ) and recompression ( $C_r$ ) indices are set constant for simplicity, recognizing that accounting for their stress-dependency can be important for sensitive clays. The ratio of the secondary compression index ( $C_\alpha$ ) to  $C_c$  is also set constant, with typical ratios being  $0.04 \pm 0.01$  for inorganic clays and silts (Mesri 2001). The value of  $C_\alpha$  is the same for normally and over-consolidated conditions, consistent with visco-plastic modeling procedures (it is not necessary to assume  $C_\alpha$  switches to being proportional to  $C_r$  for

over-consolidated conditions because the following procedure naturally accounts for decreasing visco-plastic strain rates with increasing over-consolidation ratio). The  $\sigma'_{vp}$  for any equivalent  $t_p$  can be computed from  $\sigma'_{vp,1day}$  as:

$$\sigma'_{vp,t_p} = \sigma'_{vp,1day} \cdot 10^{-\left(\frac{C_\alpha}{C_c - C_r}\right) \log\left(\frac{t_p}{1day}\right)}$$

The over-consolidation ratio (*OCR*) at a given in-situ vertical effective stress ( $\sigma'_{vo}$ ) therefore similarly depends on the reference age:

$$OCR_{t_p} = \frac{\sigma'_{vp,t_p}}{\sigma'_{vo}} = OCR_{1day} \cdot 10^{-\left(\frac{C_\alpha}{C_c - C_r}\right) \log\left(\frac{t_p}{1day}\right)}$$

The time-line based analysis follows the conventional approach of separating the calculation of primary consolidation and secondary compression strains, but with  $\sigma'_{vp}$  and  $t_p$  chosen to provide reasonable approximations of the total long-term strains and settlements versus time. The nonlinear solutions using iConsol, as presented in the next section, indicate that the void ratios throughout a clay layer near the end of primary consolidation (assuming it is loaded into virgin compression) are close to the normal consolidation line corresponding to that particular time (similar to Figure 2). Thus,  $t_p$  is estimated herein as the time to an average degree of consolidation of 90% (i.e.,  $t_{90}$ ), unless the clay remained significantly over-consolidated during the loading.

For clays that remain over-consolidated at the end of primary consolidation, the elapsed time needs to be checked against a second criterion. The final vertical effective stress ( $\sigma'_{vf}$ ) and void ratio at the end of primary consolidation will fall on a normal consolidation line corresponding to some equivalent past time ( $t_{p,past}$ ). The value of  $t_{p,past}$  can be computed as,

$$t_{p,past} = t_{ref} \cdot 10^{\frac{C_c - C_r}{C_\alpha} \log\left(\frac{\sigma'_{vp,tref}}{\sigma'_{vf}}\right)}$$

The settlements due to secondary compression can then be computed as:

$$\Delta e_{sc} = C_\alpha \log\left(\frac{\max(t_{p,past}, t_{90}) + \Delta t}{\max(t_{p,past}, t_{90})}\right)$$

$$S_{sc} = \sum_{i \text{ sublayers}} \varepsilon_{v,i} H_i = \sum_{i \text{ sublayers}} \frac{\Delta e_{sc,i}}{1 + e_o} H_i$$

The ultimate consolidation settlement ( $S_{c,ult}$ ) can also now be computed using standard formulae with the age-dependent pre-consolidation stress as,

$$S_{c,ult} = \sum_{i \text{ sublayers}} \varepsilon_{v,i} H_i = \sum_{i \text{ sublayers}} \frac{\Delta e_i}{1 + e_o} H_i$$

$$\text{if } \sigma'_{vf} < \sigma'_{vp,t_p} \Rightarrow \Delta e = C_r \log\left(\frac{\sigma'_{vf}}{\sigma'_{vo}}\right)$$

$$\text{otherwise } \Rightarrow \Delta e = C_r \log\left(\frac{\sigma'_{vp,t_p}}{\min(\sigma'_{vp,t_p}, \sigma'_{vo})}\right) + C_c \log\left(\frac{\sigma'_{vf}}{\sigma'_{vp,t_p}}\right)$$

The consolidation settlement in time is computed using standard solutions for time dependent consolidation of rate-independent materials:

$$S_{c,t} = S_{c,ult} U_v$$

where  $U_v$  = average degree of consolidation, which is computed from the time factor  $T_v$ ,

$$T_v = \frac{C_v t}{(H_{dr})^2}$$

where the coefficient of consolidation  $c_v = k_v/m_v\gamma_w$ ,  $k_v$  = vertical hydraulic conductivity,  $m_v$  = coefficient of compressibility, and  $\gamma_w$  = unit weight of water. The tangent value of  $m_v$  at any value of  $\sigma'_v$  is related to the virgin compression or recompression indices as:

$$\text{if } \sigma'_v < \sigma'_{vp,t_{ref}} \Rightarrow m_v = \frac{C_r}{\sigma'_v(1+e_o)}$$

$$\text{otherwise } \Rightarrow m_v = \frac{C_c}{\sigma'_v(1+e_o)}$$

The value of  $m_v$  will vary throughout loading, but the present analyses use the value at the start of the loading increment for simplicity.

Settlement versus time is then the sum of the consolidation and secondary compression settlements, with secondary compression beginning at the applicable  $t_p$ . This calculation was performed in a spreadsheet for the following example.

## CONSOLIDATION EXAMPLE

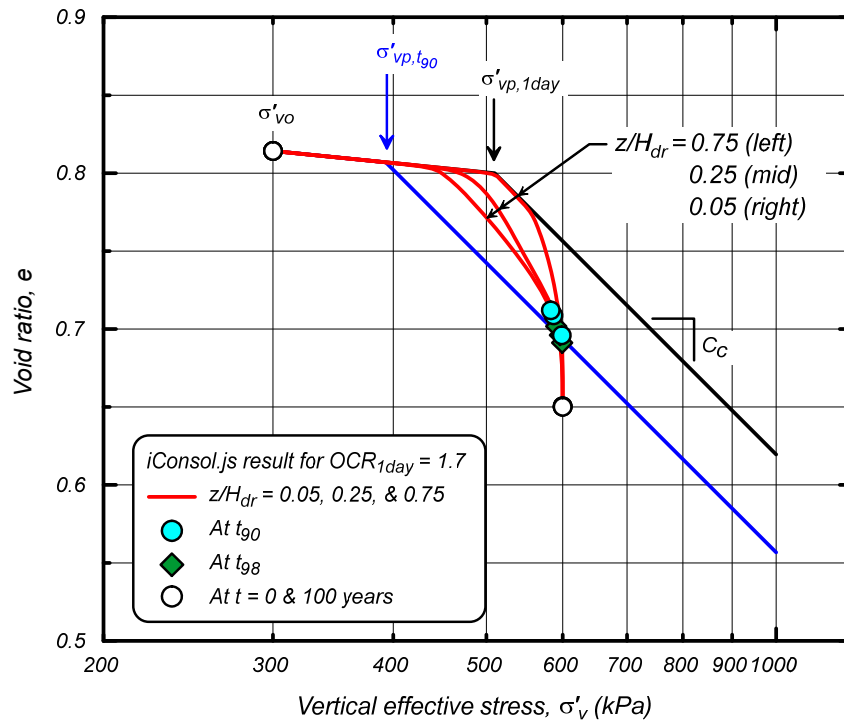
The example problem involves one-dimensional consolidation of a 12-m-thick saturated clay layer under an increment of vertical stress. Material properties are summarized in Table 1. The clay layer was modeled as neutrally buoyant ( $G_s=1$ ), to simplify comparisons by having  $\sigma'_{vo}$ ,  $\sigma'_{vf}$ , and  $\sigma'_{vp,1day}$  be constants throughout the layer. The top boundary is free draining and the bottom boundary is impermeable.

**Table 1. Soil properties for baseline analysis case.**

$C_c$	$C_r$	$\sigma'_{v,1day}$ (kPa)	$e_{ov,ref}$	$G_s$	$k_v$ (m/s)	$C_\alpha$	$t_{ref}$ (days)	$H_o$ (m)	$OCR_{1day}$	$\sigma'_{vo}$ (kPa)	$\sigma'_{vf}$ (kPa)
0.617	0.0617	510	0.8	1.0	$10^{-8}$	0.025	1	12	1.7	300	600

The results of an iConsol analysis are shown in Figure 3 in terms of void ratio ( $e$ ) versus  $\sigma'_v$  paths for three depths in the clay layer (0.6, 3.0, and 9.0 m from the top boundary). The path for a depth of 0.6 m is the highest because it experiences the highest strain rates, whereas the path for a depth of 9.0 m is the lowest because it experiences the lowest strain rates. These three paths are qualitatively consistent with those in Figure 2 for different depths in an oedometer test of a 500-mm-thick layer of St. Hilaire clay. The symbols in Figure 3 indicate when the average degree of

consolidation for the clay layer is 90% and 98%. Time-line consolidation curves are shown for both  $t_{ref} = 1$  day and  $t_{ref} = t_{90}$ . The void ratios at  $t_{90}$  to  $t_{98}$  in the iConsol analysis fall close to the consolidation curve for  $t_{ref} = t_{90}$ , which is why  $t_{ref} = t_{90}$  was selected for use in the time-line-based analysis method.



**Figure 3. Computed paths at different depths in a consolidating clay layer using iConsol, along with the time-line consolidation curves for  $t_{ref}$  equal to 1 day and  $t_{90}$ .**

Settlements versus time obtained by the three approaches are shown in Figure 4 for the case with  $OCR_{1day} = 1.7$ . The time-line-based approach closely approximates the results obtained using iConsol. The traditional approach predicts settlements that are 30-40% lower because it relies on the upper bilinear curve ( $t_{ref} = 1$  day) in Figure 3. These differences are consistent with those summarized in Brandenburg (2016).

Settlements versus time obtained using iConsol and the time-line-based approach are compared in Figure 5 for  $OCR_{1day}$  values of 1.1, 1.4, 1.7, 2.0, and 2.3. The agreement is quite reasonable given that iConsol accounts for large deformations and stress-dependent properties (i.e.,  $m_v$  varies with stress and void ratio), whereas the time-line-based approach does not.

Settlements after 10 years by the three approaches are shown in Figure 6 for  $OCR_{1day}$  values of 1.1 to 2.3. The traditional approach estimates settlements that are about 30% lower for  $OCR_{1day}$  values less than about 2.1. The settlements obtained by iConsol and the time-line-based approach are close for all cases.

Long-term settlements obtained using the time-line-based approach are relatively insensitive to the rate of consolidation, as illustrated in Figure 7 showing results for  $k_v = 0.25, 1.0, \text{ and } 4.0$



times  $10^{-8}$  m/s. This factor of 4 variation in  $k_v$  produces similar changes to  $c_v$  and causes  $t_{90}$  to range from 0.4 to 7.4 years. This changes the time for the ultimate consolidation settlement to develop, but has negligible effect on the total settlements that have developed after about 10 years.

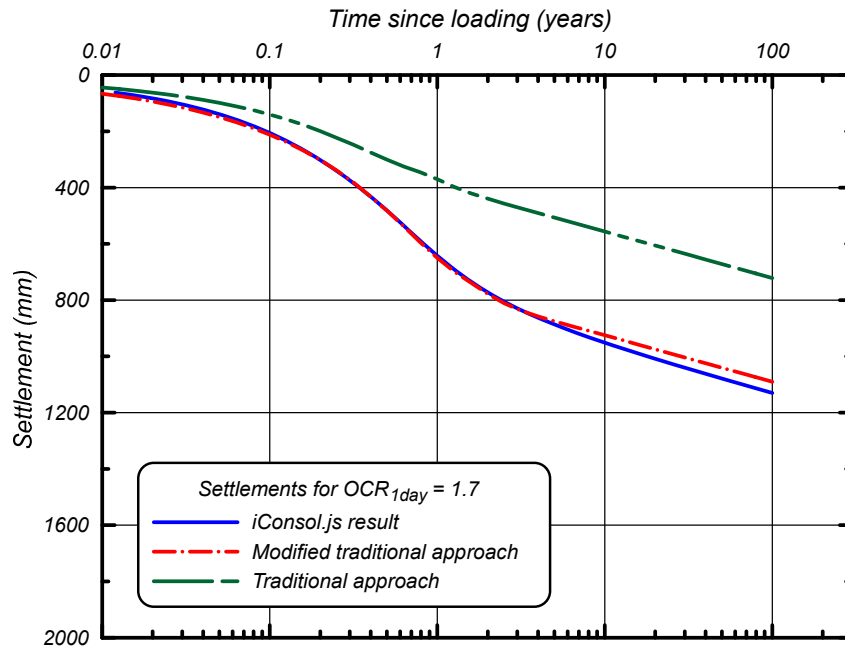


Figure 4. Consolidation settlements for the three approaches at  $OCR_{1day} = 1.7$ .

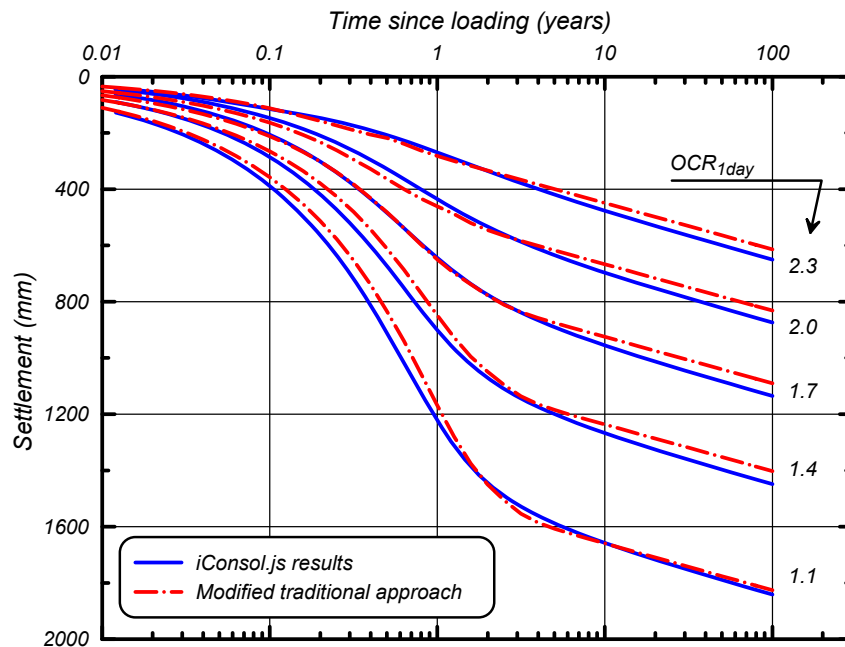


Figure 5. Consolidation settlements using the modified traditional approach and the iConsol.js program for  $OCR_{1day}$  values of 1.1 to 2.3.

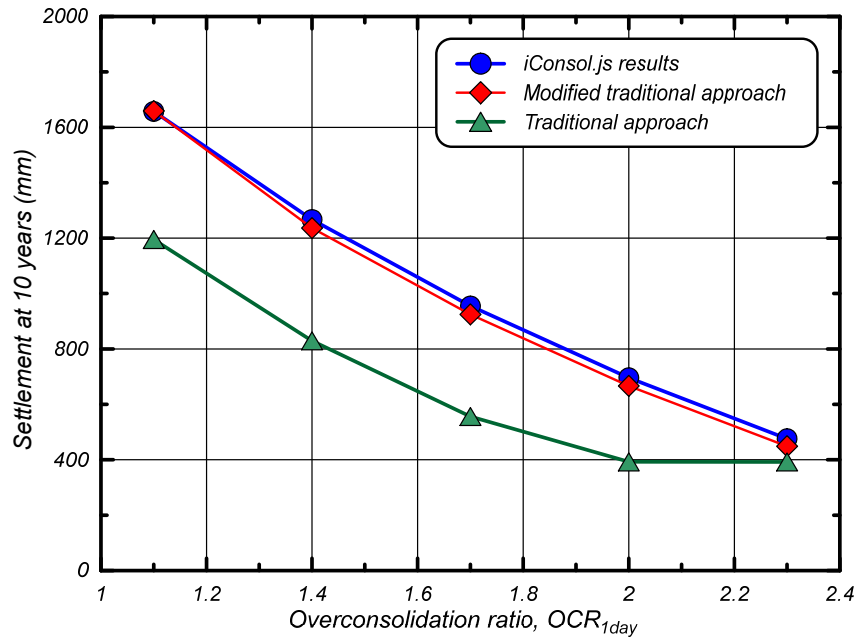


Figure 6. Total settlements at 10 years using the traditional approach, time-line based approach, and iConsol for OCR<sub>1day</sub> values of 1.1 to 2.3.

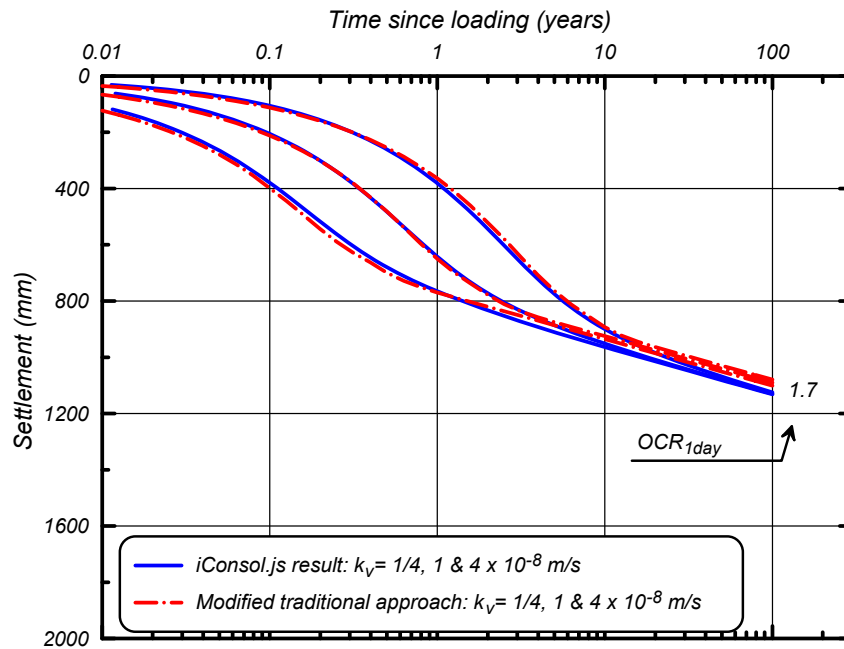


Figure 7. Effect of  $k_v$  (and hence  $c_v$ ) on the total settlement versus time

## DISCUSSION

Watabe et al. (2012) interpreted the long-term consolidation behavior of a wide range of clays using an isotache approach, and concluded there was a lower limit to the  $\sigma'_{vp}$  regardless of how small the strain rate was. This lower limit was supported by Leroueil's (2006) conclusion that  $C_d/C_c$

may be approximately constant over the range of strain rates observed in the laboratory, but likely decreases with further decreases in strain rate in the field. They subsequently developed a general isotache model, wherein the lower limit to  $\sigma'_{vp}$  is taken as 0.7 times the  $\sigma'_{vp}$  obtained from 24-hr LIR tests. For the time-line based approach used herein, it seems appropriate to similarly use a lower limit of  $\sigma'_{vp,tp}/\sigma'_{vp,1day} = 0.7$ .

The accuracy with which settlement and settlement rates can be estimated depends on numerous factors, including the availability of field observations (e.g., adjacent structures, field trials) that can be used to calibrate or validate an analysis method. The significance of accounting for strain-rate effects further depends on the potential for compensating errors or conservatism in any analysis or calibration process (e.g., incorrect models for stress distributions, not accounting for shear distortions, effects of sample disturbance). For some cases, however, accounting for strain-rate effects can be essential to understanding and correctly interpreting field observations. The isotache, time-line and nonlinear simulation methods provide alternative means for representing strain-rate effects, with each being better suited for different applications.

## CONCLUSION

Two analysis methods to account for strain-rate effects during consolidation of saturated clay were compared. One method explicitly simulated strain rate effects during primary consolidation using a visco-plastic relationship and nonlinear finite difference solution (iConsol; Brandenburg 2016). The second method indirectly accounted for strain rate effects during primary consolidation by using a time-line based approach to select the reference time and corresponding pre-consolidation stress for use with conventional analytical solutions for consolidation and secondary compression versus time. The second method is amenable to spreadsheet solutions, and for loading conditions that are more complicated than represented in readily available computational solutions. Results of one-dimensional analyses of a single clay layer for a range of stress history and loading conditions showed that the time-line based method provided a reasonable approximation of the more rigorous nonlinear analysis method. These results indicate that a time-line based approach to interpreting consolidation parameters can be used to approximate strain-rate effects in various other analysis methods (e.g., FE/FD models) that assume strain-rate-independent properties.

The accuracy of any analysis method for predicting field settlements depends on numerous factors, including the potential for compensating errors (e.g., stress distributions, shear distortions, sample disturbance) and the availability of field observations (e.g., adjacent structures, field trials) for calibration against. The relative accuracy of different analysis approaches has been evaluated by others (e.g., Watabe et al. 2012, Mesri and Kane 2018), whereas the present paper focused on comparing two alternative methods to account for strain-rate dependence of clay behavior.

## ACKNOWLEDGMENTS

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