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CORRELATIONS AND TOTAL MUON CAPTURE RATES⁺

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ABSTRACT

The total muon capture rate for s-wave muons can be accounted for by the Primakoff expression which gives the dependence of this rate on the mass number A and the proton number Z of the absorbing nucleus. The expression is a simple three parameter phenomenological formulae which accurately describes these rates from light weight nuclei to heavy nuclei. These parameters relate to the isospin structure of the squared isovector operator which appears in a sum rule approach to such rates. A microscopic analysis of the parameters appearing in the capture rate expression is presented in the light of recent developments concerning photonuclear reactions. A shell model analysis is given and it is found that the predictions of the unperturbed shell model and also Hartree-Fock theory are in complete disagreement with the data. Considerable improvement is obtained when long range correlations are included in the ground state wave function of the absorbing nucleus.

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The capture rate of a muon in a 1S orbit is proportional to the probability of finding a muon at the nucleus and to the number of protons in the nucleus. Thus, the lowest order expression for the capture rate follows the Wheeler law $\Lambda \sim Z^4$ since the 1S probability density varies as Z^3 at the origin. Finite nuclear size corrections change Z to Z_{eff} . The effect of neutrons in the nucleus inhibit the transition $\mu + p \rightarrow n + \nu_\mu$ and reduce the capture rate. Specifically, the effect of the neutrons on the capture rate can be seen in the following semi-empirical formulae obtained by Primakoff⁽¹⁾ in the closure approximation

$$\Lambda_c(A, Z) = K Z_{\text{eff}}^3 Z \left(1 - \frac{A-Z}{2A} \delta \right) \quad (1)$$

Here, the δ is a parameter and the value $\delta = 3.15$ fits the $(A-Z)/2A$ dependence of the capture rate $\Lambda_c(A, Z)$ as shown in Fig. 1. The solid curve is the straight line result of Eq. (1).

More recently, a reinvestigation of the question of the A and Z dependence of these capture rates has been given^(2,3) and the following dependence has been proposed.

$$\frac{\Lambda_c(A, Z)}{K Z_{\text{eff}}^3 Z} = 1 + \frac{A}{2Z} (\tilde{\beta}_0 - 1) + \frac{A-2Z}{2Z} \tilde{\beta}_1 - \frac{A-Z}{2A} \tilde{\beta}_2 \quad (2)$$

The $\tilde{\beta}_0 = 0.97$, $\tilde{\beta}_1 = 0.25$ and $\tilde{\beta}_2 = 3.24$ are obtained by fitting Eq. (2) to the data. The largest term in Eq. (2) is the $(A-Z)/2A$ term since Eq. (1) accurately characterizes the data and contains only this term. Table 1 gives the values of the experimentally determined capture rate Λ_c^{exp} and the rate obtained from Eq. (2), Λ_c^{fit} , for a series of nuclei.⁽²⁾

TABLE 1

Nucleus	Λ_c^{exp} 10^6 sec^{-1}	Λ_c^{fit} 10^6 sec^{-1}	$\frac{ \Lambda_c^{\text{exp}} - \Lambda_c^{\text{fit}} }{\Lambda_c^{\text{exp}}}$ %
^8_8O	0.0974 ± 0.0031	0.1151	18.2
$^{16}_{16}\text{S}$	1.338 ± 0.007	1.244	7.0
$^{20}_{20}\text{Ca}$	2.45 ± 0.02	2.46	0.4
$^{24}_{24}\text{Cr}$	3.29 ± 0.04	3.06	7.1
$^{30}_{30}\text{Zn}$	5.74 ± 0.04	5.48	4.5
$^{42}_{42}\text{Mo}$	9.22 ± 0.06	9.55	3.6
$^{48}_{48}\text{Cd}$	10.62 ± 0.10	10.66	0.4
$^{54}_{54}\text{Ba}$	10.18 ± 0.10	10.40	2.2
$^{64}_{64}\text{Gd}$	12.09 ± 0.16	12.74	0.9
$^{82}_{82}\text{Pb}$	13.02 ± 0.11	13.05	0.3
$^{92}_{92}\text{U}$	11.0 ± 0.5	11.5	4.9

Let us now try to understand Eq. (2) in more detail and then attempt a microscopic evaluation of the parameters $\tilde{\beta}_0$, $\tilde{\beta}_1$ and $\tilde{\beta}_2$.

To begin, a detailed analysis by Primakoff⁽¹⁾ based on the impulse approximation gives the following expression for the capture rate:

$$\Lambda_c = \frac{m_\mu^2}{2\pi} |\phi_\mu|_{Av}^2 \left[G_V^2 M_V^2 + 3G_A^2 M_A^2 + (G_p^2 - 2G_p G_A) M_p^2 \right]. \quad (3)$$

The m_μ is the muon mass and the $|\phi_\mu|_{Av}^2$ is the square of the muon wave function averaged over the nucleus. The G_V , G_A and G_p are the effective vector, axial vector and induced pseudoscalar coupling constants of ref. (1). The vector matrix element M_V is

$$M_V^2 = \sum_b \left(\frac{v_{ab}}{m_\mu} \right)^2 \int \frac{d\hat{v}}{4\pi} \left| \langle b | \sum_i \tau_i^+ e^{i\vec{v}_{ab} \cdot \vec{r}_i} | a \rangle \right|^2, \quad (4)$$

the axial vector matrix element is

$$M_A^2 = \sum_b \left(\frac{v_{ab}}{m_\mu} \right)^2 \int \frac{d\hat{v}}{4\pi} \left| \langle b | \sum_i \tau_i^+ \vec{\sigma}_i e^{i\vec{v}_{ab} \cdot \vec{r}_i} | a \rangle \right|^2 \quad (5)$$

while the induced pseudoscalar matrix element is given by

$$M_p^2 = \sum_b \left(\frac{v_{ab}}{m_\mu} \right)^2 \int \frac{d\hat{v}}{4\pi} \left| \langle b | \tau_i^+ (\vec{v} \cdot \vec{\sigma}_i) e^{i\vec{v}_{ab} \cdot \vec{r}_i} | a \rangle \right|^2 \quad (6)$$

The v_{ab} is the energy of the emitted neutrino accompanying the transition of the nucleus from its initial state $|a\rangle$ to its final state $|b\rangle$ in the nucleus of one less proton and one more neutron than $|a\rangle$. Thus, $v_{ab} = m_\mu - (E_b - E_a)$, where E_a is the energy of $|a\rangle$ and E_b is the energy of $|b\rangle$. For a typical transition $E_b - E_a \approx 25$ MeV, $v_{ab} = 80$ MeV and the momentum transfer $q = 2/5 \text{ fm}^{-1}$. The τ^+ in

Eqs. (4) - (6) is defined such that $t^+|p\rangle = |n\rangle$.

We will make the usual SU4 assumption

$$M_A^2 = M_V^2 = M_P^2, \quad (7)$$

the validity of which is discussed in Ref. (4) and Ref. (5). We will therefore concentrate on M_V^2 . The methods for evaluating M_V^2 are either: a) summing up partial transitions to individual states $|b\rangle$, and b) using the closure approximation by introducing a mean neutrino energy $\bar{\nu}$. In the former case, the giant dipole resonance plays an important role for light nuclei and medium weight nuclei, while the giant quadrupole state contains most of the strength for the heavy nuclei. The contribution of the monopole state is due to retardation terms⁽⁶⁾ since all simple t_1^+ transitions are blocked by neutron orbitals. These results are illustrated in Table 2 where the results of the calculation of Joseph, Ledoyen and Goulard⁽⁵⁾ are quoted.

TABLE 2. Relative percentages of the vector matrix element $(M_V^2)_L$ in several nuclei using a shell model description.

	⁴⁰ Ca	⁵⁰ Ni	⁸⁸ Sr	¹¹⁴ Sn	¹⁴⁰ Ce	²⁰⁰ Pb
$0^+ \rightarrow 0^+$	7	6	15	13	19	28
$0^+ \rightarrow 1^-$	77	69	51	44	34	11
$0^+ \rightarrow 2^+$	14	22	28	35	39	51
$0^+ \rightarrow 3^-$	2	3	6	7	7	8
$0^+ \rightarrow 4^+$	0	0	0	1	1	2

Here, we will direct our attention to the second approach involving the closure approximation. This approximation is based on the result that if an operator $\mathcal{Q}$ acts on a state $|0\rangle$, its effect is, in general, to produce a multiple of the original state $|0\rangle$ and to admix other states $|\lambda\rangle$:

$$\mathcal{Q}|0\rangle = a_0|0\rangle + \sum_{\lambda} a_{\lambda}|\lambda\rangle \quad (8)$$

The amplitudes a_{λ} are just the matrix elements $a_{\lambda} = \langle 0|\mathcal{Q}|\lambda\rangle$. In the closure approximation, we use

$$\sum_{\lambda} |a_{\lambda}|^2 = \langle 0|\mathcal{Q}^2|0\rangle - |\langle 0|\mathcal{Q}|0\rangle|^2 \quad (9)$$

where the sum in Eq. (9) does not involve the initial state. Energy moments of the distribution of the a_{λ} 's can also be defined by

$$N_m = \sum_{\lambda} (E_{\lambda} - E_0)^m |a_{\lambda}|^2 \quad (10)$$

with E_{λ} the energy of state $|\lambda\rangle$. Our main concern will be the non-energy weighted sum rule approach for the evaluation of Eq. (4). To employ it, we have to introduce a mean neutrino energy through the defining relationship:

$$M_v^2 = \left(\frac{\bar{\nu}}{m_{\mu}}\right)^2 \sum_b \int \frac{d\hat{\nu}}{4\pi} \left| \langle b | \sum_i \tau_i^+ e^{i\vec{\nu} \cdot \vec{r}_i} | a \rangle \right|^2 \quad (11)$$

Then Eq. (11) reduces to

$$M_v^2 = \left(\frac{\bar{\nu}}{m_{\mu}}\right)^2 \langle a | J^{++} | a \rangle \quad , \quad (12)$$

where

$$J^{+-} \equiv \int \frac{d\vec{q}}{4\pi} \sum_{ij} t_j^- t_i^+ e^{i\vec{q} \cdot (\vec{r}_i - \vec{r}_j)} = \int \frac{d\vec{q}}{4\pi} \mathcal{Q}^- \mathcal{Q}^+ \quad (13)$$

with $\mathcal{Q}^+ = \sum e^{i\vec{q} \cdot \vec{r}_i} t_i^+$ and a similar definition for $\mathcal{Q}^-$. Now, we note that the evaluation of M_V in the closure approximation involves the ground state expectation value of a two-body operator. Its value is therefore subject to ground state correlations. Our main purpose is to show that the slope of the muon capture rate as a function of $(A-Z)/2A$ is very sensitive to such ground state correlations, changing by a factor of ten when shell model functions are improved on to include long-range correlations.

To proceed in this endeavor, we have to write Eq. (13) in the form of Eq. (2). This is accomplished by introducing the isoscalar, isovector and isotensor parts of the squared operator $\mathcal{Q}^- \mathcal{Q}^+$. In defining these isospin components we will remove that part which produces a multiple of the original state, the a_0 of Eq. (8), since we are only interested in transition matrix elements. Thus we define a two-body isoscalar part by

$$M_0 = \sum_{i,j} (e^{-i\vec{q} \cdot \vec{r}_i} - \bar{\mathcal{Q}}_Z) (e^{i\vec{q} \cdot \vec{r}_j} - \bar{\mathcal{Q}}_Z) \frac{1}{3} \vec{t}_i \cdot \vec{t}_j \quad (14)$$

a one-body isovector part by

$$M_1 = \sum_i (e^{-i\vec{q} \cdot \vec{r}_i} - \bar{\mathcal{Q}}_Z) (e^{i\vec{q} \cdot \vec{r}_i} - \bar{\mathcal{Q}}_Z) 2t_{iz} \quad (15)$$

and a two-body isotensor operator through

$$M_2 = \sum_{i,j} (e^{-i\vec{q} \cdot \vec{r}_i} - \bar{Q}_2) (e^{i\vec{q} \cdot \vec{r}_j} - \bar{Q}_2) \frac{1}{2} (\vec{t}_i \cdot \vec{t}_j - 3t_{iz}t_{jz}) \quad (16)$$

The $\bar{Q}_2$ serves the purpose of removing the multiple of the original state from the sum and it is given as

$$\bar{Q}_2 = \frac{2\langle 0 | Q_2 | 0 \rangle}{(N-Z)} \quad \text{with} \quad Q_2 = \sum e^{i\vec{q} \cdot \vec{r}_i} t_{iz}$$

Having established these definitions, it is a matter of straightforward isospin algebra to obtain

$$\langle Q^- Q^+ \rangle = Z \left[1 + \frac{A}{2Z} (\tilde{\beta}_0 - 1) + \left(\frac{A-2Z}{2Z} \right) \tilde{\beta}_1 - \left(\frac{A-Z}{2A} + \frac{|A-2Z|}{8ZA} \right) \tilde{\beta}_2 \right] \quad (17)$$

where

$$\begin{aligned} \tilde{\beta}_0 &\equiv 2 \langle \| M_0 \| \rangle + \langle \| M_2 \| \rangle \\ \tilde{\beta}_1 &\equiv \langle \| Q_2 \| \rangle \\ \tilde{\beta}_2 &\equiv \langle \| 4M_2 \| \rangle \end{aligned} \quad (18)$$

The reduced matrix elements are defined by

$$\begin{aligned} \frac{A}{2} \langle \| M_0 \| \rangle &= \langle | M_0 | \rangle \\ T_z \langle \| Q_2 \| \rangle &= \langle | Q_2 | \rangle \\ 3 \langle \| M_2 \| \rangle [3T_z^2 - T_z(T_z+1)] / 2A &= \langle | M_2 | \rangle \end{aligned} \quad (19)$$

The advantage of making an isospin decomposition is twofold. First,

from the above results we learn that the isotensor sum rule determines the slope of the muon capture rate. Secondly, the various components have different dependencies on the model wave functions used. In particular, as shall be shown, the tensor part is strongly model-dependent, differing by a factor of ten when simple wave functions are improved upon. The scalar part is mildly model dependent and it differs by a factor of two in such improvements, while the vector part, given by

$$\langle 0 | M_1 | 0 \rangle = \frac{1}{4}(N-Z) [1 - \langle \| Q_z \| \rangle^2]$$

is nearly model independent since it involves the expectation value of a one-body operator $\langle \| Q_z \| \rangle$ which is simply the neutron-proton form factor difference:

$$\langle \| Q_z \| \rangle = \frac{(NF_n(q) - ZF_p(q))}{(N-Z)} \quad (20)$$

Having established the connection of Eq. (2) with the isospin structure of our sum rules, let us next evaluate the coefficients $\tilde{\beta}_0$, $\tilde{\beta}_1$ and $\tilde{\beta}_2$ with a Slater determinant wave function

$$\Psi_A = \frac{1}{\sqrt{A!}} \begin{vmatrix} \phi_1(\vec{r}_1) & \dots & \phi_1(\vec{r}_A) \\ \vdots & & \vdots \\ \phi_A(\vec{r}_1) & \dots & \phi_A(\vec{r}_A) \end{vmatrix} \quad (21)$$

We will take the single particle neutron orbitals $\phi_{v_1}(n)$ to be the same as the corresponding proton orbitals $\phi_{v_1}(p)$. Now, note that the expectation value of isotensor operator M_2 vanishes in a $T=0$ state and in a

$T = \frac{1}{2}$ state. From this observation we conclude that all the $N=Z$ core orbitals do not contribute to $\langle M_2 \rangle$, nor do the neutron-excess orbitals with the $N=Z$ core. The M_2 has non-vanishing matrix elements only between two neutrons in neutron-excess orbitals. If we let v_1 and v'_1 be two such orbitals, then we have

$$\begin{aligned} \langle \psi_A | M_2 | \psi_A \rangle &= \frac{1}{4} \left\{ \sum_{v_1} \left| \langle v_1 | e^{iqz} | v_1 \rangle \right|^2 + \sum_{v_1 \neq v'_1} \left| \langle v_1 | e^{iqz} | v'_1 \rangle \right|^2 \right. \\ &\quad \left. - (N-Z) \left| \langle \parallel Q_z \parallel \rangle \right|^2 \right\} \end{aligned} \quad (22)$$

Some further observations can also be made concerning $\langle M_2 \rangle$. For one, odd multipoles vanish if all v_1 and v'_1 have the same parity. Secondly, $\langle v_1 | e^{iqz} | v'_1 \rangle = 0$ if v_1 and v'_1 belong to the same major shell since e^{iqz} excites oscillator quanta in the z -direction, leaving the x and y quanta unchanged. Thirdly, the monopole contribution vanishes through order $q^6 b^6$ (b is the oscillator size parameter) since the average monopole interaction has been subtracted out. Thus, the lowest contribution in a $q^2 b^2$ expansion is from the quadrupole term and it is of the order $q^4 b^4$ and higher. The expectation value of $Q^- Q^+$ is also easily evaluated for a Slater determinant wave function and is

$$\langle \psi_A | Q^- Q^+ | \psi_A \rangle = Z - \sum_{v_1(p) v_j(n)} \left| \langle v_1(p) | e^{iqz} | v_j(n) \rangle \right|^2 \quad (23)$$

For ^{40}Ca .

$$\langle \psi_A | Q^- Q^+ | \psi_A \rangle = 20 - 20e^{-(q^2/2v)} \left[1 + \frac{1}{2} \left(\frac{q^2}{2v} \right)^2 - \frac{1}{10} \left(\frac{q^2}{2v} \right)^3 + \frac{1}{40} \left(\frac{q^2}{2v} \right)^4 \right] \quad (24)$$

Once $\langle Q^- Q^+ \rangle$, $\langle Q_1 \rangle$ and $\langle Q_2 \rangle$ are given, the $\langle Q_0 \rangle$ follows from the isospin identity $Q^- Q^+ = 2[Q_0 - Q_1 + Q_2/3]$. Using the above results, the unperturbed shell model results can be obtained and the results are given in Table 3.

TABLE 3. Phenomenological and calculated values of the total capture rate $\langle Q^- Q^+ \rangle$ and the individual paramaters $\tilde{\beta}_0$, $\tilde{\beta}_1$, $\tilde{\beta}_2$.

	$\tilde{\beta}_0$	$\tilde{\beta}_1$	$\tilde{\beta}_2$	$\langle Q^+ Q^- \rangle_{N=Z/Z}$
Phenomenological values	0.97	0.25	3.24	0.16
Shell model without correlations	0.34	0.30	0.27	0.27
Shell model with correlations	0.57	0.30	1.6	0.17

From the table we note that an order of magnitude discrepancy exists for the slope coefficient $\tilde{\beta}_2$, while a factor of two discrepancy exists in $\langle Q^- Q^+ \rangle/Z$. This latter disagreement between the evaluation using the shell model without correlations and the experimental result has previously been noted. (4,6)

Let us next turn to an improved wave function which includes ground state correlations and see what happens to the parameters $\tilde{\beta}_0$, $\tilde{\beta}_1$ and $\tilde{\beta}_2$. The effect of correlations on $\langle Q^- Q^+ \rangle$ using a linked cluster expansion has already been discussed and a reduction of ~30% was found for $^{16}_0$. (7) Here, we will consider the change in the ground state expectation value of a square operator Q^2 when the shell model wave function $|\phi_0\rangle$ is

perturbed by a residual interaction H_1 . The total Hamiltonian $H = H_0 + H_1$, where H_0 is the unperturbed part. For degenerate states we will take $H_0 = H$ and diagonalize H on these states. The H_1 then connects only non-degenerate states. To first order in H_1 , our new wave function is

$$|\psi_0\rangle = |\phi_0\rangle + \frac{Q}{E_0 - H_0} H_1 |\phi_0\rangle \quad (25)$$

where $Q = 1 - |\phi_0\rangle\langle\phi_0|$. Then, a first order evaluation in H_1 of the expectation value of $\mathcal{Q}^2$ with the improved $|\psi_0\rangle$ gives

$$\langle\psi_0|\mathcal{Q}^2|\psi_0\rangle = \langle\phi_0|\mathcal{Q}^2|\phi_0\rangle + 2\langle\phi_0|\mathcal{Q}^2 \frac{Q}{E_0 - H_0} H_1 |\phi_0\rangle \quad (26)$$

To evaluate Eq. (26) we will use a multipole-multipole force to give an estimate of the effect of long-ranged correlations on $\mathcal{Q}^2$. For example, a dipole-dipole interaction given by

$$H_1 = \frac{1}{2} \sum_{i,j} b_t \vec{r}_i \cdot \vec{r}_j \vec{t}_i \cdot \vec{t}_j \quad (27)$$

is used to evaluate the correlation corrections to the part of $e^{i\vec{q} \cdot \vec{r}_1}$ which excites the dipole state. The strength of the interaction is chosen to give the correct position of the dipole state. For the monopole and quadrupole parts of $e^{i\vec{q} \cdot \vec{r}_1}$, a monopole-monopole and quadrupole-quadrupole force will be used with the strength of each component, again adjusted to give the position of each isovector giant resonance, as given by the hydrodynamic model. These energies are $E_D = 2\hbar\omega$, $E_m = 4\hbar\omega$ and $E_Q = 3.2\hbar\omega$. It is straightforward to evaluate Eq. (26) for the isoscalar, isovector, and isotensor operators of Eqs. (14)-(16). Then, from these

values, the coefficients $\tilde{\beta}_0$, $\tilde{\beta}_1$ and $\tilde{\beta}_2$ can easily be obtained. The results are summarized in Table 3 under the row "shell model with correlations." Comparing these results with the phenomenological ones and the unperturbed shell model values, we see that the effect of long range correlations substantially improved on the unperturbed results. However, note that while the total transition matrix element agrees very well with the experimental one, the individual parameters still disagree by a factor of about 2 for $\tilde{\beta}_0$ and $\tilde{\beta}_2$. This discrepancy has of yet not been resolved.

Let us try to understand the large changes in the expectation value of an isotensor operator when wave functions are improved on. To do this, we will consider a problem closely related to muon capture, which is γ -absorption. Specifically, we will consider the excitation of the giant dipole resonance in γ -absorption. Recall that the integrated absorption cross section is related to the oscillator strength, f_{on} , through^(3,9)

$$\int \sigma_{\gamma\text{-abs.}}(E) dE = \frac{2\pi^2 e^2 \hbar^2}{mc} \sum_n f_{on} \quad (28)$$

with $f_{on} = |\langle n | D_z | 0 \rangle|^2 / (E_n - E_0) 2m/\hbar^2$ and where $D_z = \sum_i z_i t_{1z}$ is the dipole operator. Thus, the first moment of the dipole distribution is connected to the γ -absorption cross section. On the other hand, the zero'th moment of the dipole distribution is connected to the Bremsstrahlung cross section⁽⁸⁾

$$\int \sigma_{\gamma\text{-abs}}(E) dE/E = \frac{2\pi^2 e^2 \hbar^2}{mc} \sum_n \frac{2m}{\hbar^2} |\langle n | D_z | 0 \rangle|^2 \quad (29)$$

For $\gamma + {}^{208}\text{Pb}$ the integrated cross section is $\int \sigma_{\gamma} dE = 4.03 \text{ b} \cdot \text{MeV}$ and

$\int \sigma_Y dE/E = 260 \text{ mb.}$ (10) Using Eq. (29), and noting that

$$\sum_n |\langle n | D_z | 0 \rangle|^2 = \langle 0 | D_z^2 | 0 \rangle \quad (30)$$

the expectation value $\langle 0 | D_z^2 | 0 \rangle = 93 \text{ fm}^2$. The traditional shell model value is

$$\langle D_z^2 \rangle = \frac{Z}{8\nu_p} + \frac{N}{8\nu_n} \quad (31)$$

For ^{208}Pb , $\langle D_z^2 \rangle = 150 \text{ fm}^2$ using the oscillator result of Eq. (31); Hartree-Fock theory with Skyrme III gives for $\langle D_z^2 \rangle$ a value of about 130 fm^2 . (12)

Thus, the simplest approaches fail to account for the oscillator strength in ^{208}Pb . To see how dynamical correlations effect this evaluation, we introduce the center of mass coordinate in the z-direction, $Z_{ij} = \frac{1}{2}(z_i + z_j)$ and the corresponding relative coordinate $z_{ij} = z_i - z_j$. Then,

$$D_z^2 = \sum_{i=j} \frac{1}{4} z_i^2 + \sum_{i \neq j} Z_{ij} t_{iz} t_{jz} - \frac{1}{4} \sum z_{ij}^2 t_{iz} t_{jz} \quad (32)$$

The last term in Eq. (32) is effected by correlations and we therefore consider the function

$$R = \frac{1}{16} \left\langle \sum_{2n} z_{ij}^2 \right\rangle + \frac{1}{16} \left\langle \sum_{2p} z_{ij}^2 \right\rangle - \frac{1}{16} \left\langle \sum_{np} z_{ij}^2 \right\rangle \quad (33)$$

Now, for $\langle D_z^2 \rangle$ to decrease, R must increase. Since the np force is more attractive than the nn or pp force, the $\langle \sum_{np} z_{ij}^2 \rangle$ is reduced with respect to the $\langle \sum_{nn} z_{ij}^2 \rangle$ and $\langle \sum_{pp} z_{ij}^2 \rangle$. Consequently, we can understand the qualitative effect of dynamical correlations on $\langle D_z^2 \rangle$.

A more quantitative approach using the results of Eqs. (25) and (26) gives the following

$$\langle \Psi_0 | D_z^2 | \Psi_0 \rangle = \frac{\langle \Phi_0 | \frac{1}{2} [D_z, [H, D_z]] | \Phi_0 \rangle}{E_{T.D.}} \quad (34)$$

In obtaining Eq. (34), we have used the fact that $D_z |\Phi_0\rangle = |\Phi_D\rangle$, with $|\Phi_D\rangle$ the dipole state, and that $H_0 D_z |\Phi_0\rangle = E_{T.D.} |\Phi_0\rangle$, where $E_{T.D.}$ is the Tamm-Dancoff energy for this state. The numerator of Eq. (34) contains the exchange current enhancement κ to the integrated cross section. For ^{208}Pb the enhancement, $\kappa = \langle [D_z, [V, D_z]] \rangle / \langle [D_z, [T, D_z]] \rangle$ is 40% of the model independent kinetic energy contribution in the vicinity of the dipole state; the $\frac{1}{2} [D_z, [T, D_z]] = (\hbar^2/2M)(NZ/A)$. The contribution of very distant states to the integrated γ -absorption cross section leads to an enhancement of 100%.⁽¹²⁾ This is confirmed in the theoretical investigation of Ref. (13) and the effect is due to short-range tensor correlations. However, it should be realized that the distant state tensor correlation enhancement is essentially absent in the Bremsstrahlung weighted cross section because of the energy denominator of Eq. (29) and in Eq. (34) since we have used the dipole energy $E_{T.D.}$. Consequently we need to consider only the local enhancement. Using the Hamada-Johnston potential to evaluate $\frac{1}{2} [D_z, [V, D_z]]$ we find a κ of 0.4 with 0.2 of the enhancement coming from the one pion exchange part of this potential. Similarly, for the one boson exchange potential of Bryan and Scott,⁽¹⁴⁾ the $T=1$ σ_1 and ρ mesons double the value of the enhancement from its π -meson value of 0.2. We also note that for a zero-range-velocity independent force, the double commutator $[D_z, [V, D_z]] = 0$, so that for such a force

there is no exchange current enhancement. On the other hand, for a zero-range-velocity dependent force, as in Skyrme type interactions, this double commutator is not zero and an exchange current enhancement exists. For Skyrme III, $\kappa = 0.4$ ⁽¹¹⁾ for ^{208}Pb , while $\kappa = 0.2$ for Skyrme I ⁽¹⁵⁾ and $\kappa = 0.8$ for Skyrme II. ⁽¹⁵⁾ Here, we will use $\kappa = 0.4$.

The above results imply a 0.7 reduction in $\langle D_z^2 \rangle$ from correlations which give a value of $\sim 100 \text{ fm}^2$, in much better agreement with the 93 fm^2 experimental result. This reduction is obtained by noting that the shell model value of Eq. (31) follows when the numerator is evaluated with only the kinetic contribution and the denominator is set equal to $\hbar\omega$. Since the dipole state is shifted to twice the energy $\hbar\omega$, we have $E_{T,D.} = 2\hbar\omega$ and for an enhancement of 0.4 we have $[D_z, [H, D_z]] = 1.4[D_z, [T, D_z]]$. These two results give the 0.7 reduction in $\langle D_z^2 \rangle$ and we therefore conclude that the expectation value of the squared dipole operator has sizeable corrections from ground state correlations. Such effects can also be seen when the dipole-dipole force of Eq. (27) is used to evaluate Eq. (26). ⁽¹⁶⁾ When such a force is employed, the change in D_z^2 from its unperturbed value arises from $2p\text{-}2h$ states admixed into the ground state. Specifically, we obtain

$$\langle \psi_0 | D_z^2 | \psi_0 \rangle - \langle \phi_0 | D_z^2 | \phi_0 \rangle \cong - \frac{b_t}{4E_q} \left(\frac{1}{8v} \right)^2 [A^2] \quad (35)$$

where $E_q = 2(\hbar\omega + \Delta_{\text{coll}})$, with Δ_{coll} the collective shift of the dipole state. We observe that the change goes like A^2 since the square of a dipole excitation is linear in A from Eq. (31) when $v_p = v_n$ and we are considering two such independent excitations.

The above result can be simplified when use is made of the fact that in the schematic model⁽¹⁷⁾ the collective shift is related to $\langle \phi_0 | D_z^2 | \phi_0 \rangle$:

$$\Delta_{\text{coll}} = b_t \langle \phi_0 | D_z^2 | \phi_0 \rangle \quad (36)$$

The result of this equation and that of Eq. (35) gives the change in the expectation value of $\langle D_z^2 \rangle$ from 2p-2h excitations. For this change we have

$$\delta_{2p-2h} \langle D_z^2 \rangle = - \frac{\Delta_{\text{coll}}}{(\Delta_{\text{coll}} + \hbar\omega)} \langle \phi_0 | D_z^2 | \phi_0 \rangle \quad (37)$$

Next, let us look at the effect of 2p-2h excitations on the isotensor part of the squared isovector dipole operator, $D_2 = \frac{1}{2}(\sum z_i z_j \vec{t}_i \cdot \vec{t}_j - 3 \sum z_i z_j t_{iz} t_{jz})$, and on $D_- D_+ = \sum z_i z_j t_{i-} t_{j+}$ for ^{208}Pb . The effect of D_+ on the ^{208}Pb ground state wave function is essentially zero since most of the transitions are dynamically blocked by the neutron-excess orbitals. In fact, only the $1h_{11/2}$ proton transition to a $2g_{9/2}$ or $1i_{11/2}$ neutron state contributes. We will therefore consider an idealized nucleus for which $D_+ |\phi_0\rangle = 0$ or equivalently $\langle \phi_0 | D_- = 0$. Then to first order in H_1 , the expectation value of $D_- D_+$ with $|\psi_0\rangle$ given by Eq. (25) is also zero since

$$\begin{aligned} \langle \psi_0 | D_- D_+ | \psi_0 \rangle &= \langle \phi_0 | D_- D_+ | \phi_0 \rangle + \langle \phi_0 | H_1 (E_0 - H_0)^{-1} D_- D_+ | \phi_0 \rangle \\ &+ \langle \phi_0 | D_- D_+ Q (E_0 - H_0)^{-1} H_1 | \phi_0 \rangle = 0 \end{aligned}$$

Thus $\delta_{2p-2h} \langle D_- D_+ \rangle = 0$, where δ is the change in $\langle D \rangle$ when $\phi_0 \rightarrow \psi_0$.

Moreover, $\delta_{2p-2h} \langle D_1 \rangle = 0$ since the isovector operator $D_1 = \sum \frac{1}{2} z_i^2 t_{iz}$

is a one-body operator and is therefore not affected by 2p-2h states to first order in H_1 . Having established the effect of 2p-2h correlations on D_-D_+ , D_1 and D_z^2 , we can use the isospin identity $D_-D_+ = 2[D_z^2 - D_1 + D_2]$ to evaluate its effect on the isotensor part D_2 . We obtain

$$\delta_{2p-2h} \langle D_2 \rangle = -\delta_{2p-2h} \langle D_z^2 \rangle \quad (38)$$

Thus from this equation we conclude that the effects of correlations on $\langle D_2 \rangle$ are large and make $\langle D_2 \rangle$ positive since $\delta \langle D_z^2 \rangle$ is negative.

Large values of $\langle D_2 \rangle$ have implications concerning the neutron radius as shall now be discussed. The total strength to T+1 is obtained from the expectation value of D_-D_+ since D_+ changes T_z by one unit and must therefore excite T+1 states when D_+ acts on a $T=T_z$ ground state. The cross section for γ -absorption to T+1 states is therefore proportional to $\langle D_-D_+ \rangle$. But $D_-D_+ = 2[D_z^2 - D_1 + D_2]$. Since $\langle D_-D_+ \rangle \geq 0$, we have $\langle D_z^2 \rangle - \langle D_1 \rangle + \langle D_2 \rangle \geq 0$. The uncorrelated value of $\langle D_2 \rangle = 0$ for ^{208}Pb , since there are no E1 matrix elements between neutron-excess orbitals for ^{208}Pb . Since $\langle D_z^2 \rangle = 93 \text{ fm}^2$, then it follows that $93 \text{ fm}^2 \geq \langle D_1 \rangle$. But $\langle D_1 \rangle = \frac{1}{12} \langle N(r_p^2)_n - Z(r_p^2)_p \rangle$. The precise value of $\langle r_p^2 \rangle_p^{1/2}$ for ^{208}Pb is determined, with some uncertainty, to be between 5.42-5.51 fm. If we use the value $\langle r_p^2 \rangle_p^{1/2} = 5.51 \text{ fm}$, then $\langle r_n^2 \rangle^{1/2} \leq 5.31 \text{ fm}$, and we conclude that neutrons are inside protons, contrary to all Hartree-Fock results. We can turn the problem around and ask what value of $\langle D_2 \rangle$ is implied by taking the Hartree-Fock value of $\langle r_n^2 \rangle^{1/2}$. For this purpose we use the values $\langle r_p^2 \rangle_p^{1/2} = 5.45 \text{ fm}$ and $\langle r_n^2 \rangle_n^{1/2} = 5.68 \text{ fm}$ obtained by Negele and Vautherin⁽¹⁸⁾ using a density dependent Skyrme interaction. Then $\langle D_1 \rangle = 135.788 \text{ fm}^2$ which implies $\langle D_2 \rangle \geq 43 \text{ fm}^2$, a large positive result;

this result is also consistent with that obtained from Eq. (38).

Having established the effect of correlations on D_z , D_1 , D_2 , we can use these results to investigate the influence of correlations on the strength to specific isospin states. The excitation strength to a specific isospin state can be obtained from

$$M(T') = \langle \phi_0(T, T_z) | D_z P_{T'} D_z | \phi_0(T, T_z) \rangle = C^2(T, T_z; 10 | T', T_z) M(\|T'\|)$$

The $P_{T'}$ is a proportion operator for $T = T'$ states. The reduced matrix elements can be shown to obey

$$\begin{aligned} M(\|T'\|) &= \langle D_z^2 \rangle - \frac{1}{2T} [T'(T' + 1) - T(T+1) - 2] \langle D_1 - D_2 \rangle \\ &+ \delta_{T', T-1}^2 \frac{2T+1}{2T-1} M_2 \end{aligned} \quad (39)$$

From this result we see that 2p-2h correlations effect $M(\|T\|)$, reducing it compared to the $M(\|T+1\|)$ and $M(\|T-1\|)$ components.

Recently improvements in the sum rule approach to muon capture have been developed by Do Dong⁽¹⁹⁾ and extended by Bernabeau⁽²⁰⁾ and Rosenfelder.⁽²¹⁾ These improvements center around removing the dependence of the capture rate on the mean neutrino energy used in Eq. (11). This removal is accomplished by introducing a combined non-energy-weighted and energy-weighted expression for the capture rate. Specifically, the neutrino energy ν_{ab} which appears in Eq. (4) - Eq. (16) is written as

$$\frac{\nu_{ab}}{m_\mu} = 1 - \frac{E_b - E_a}{m_\mu} \quad (40)$$

The second term of this equation, which involves $(E_b - E_a)/m_\mu$, then

introduces energy moments of the distribution of matrix elements into the expression for the capture rate. An evaluation of the total transition rate without including ground state correlations but using the moment expansion and a Skyrme interaction gives results which are still in disagreement with the experimental data,⁽²¹⁾ but less so than the simple shell model. Also, the combined non-energy-weighted and energy-weighted description for the parametric form of Eq. (2) as given in Ref. (2) has not been evaluated as of yet. It would be interesting to have such results.

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Fig. 1. Comparison between experimental muon (capture rate)/ $Z_{\text{eff}}^3 Z$ and Primakoff theory.