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Dirac fast scramblers

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We introduce a family of Gross-Neveu-Yukawa models with a large number of fermion and boson flavors as higher dimensional generalizations of the Sachdev-Ye-Kitaev model. The models may be derived from local lattice couplings and give rise to Lorentz invariant critical solutions in $1+1$ and $2+1$ dimensions. These solutions imply anomalous dimensions of both bosons and fermions tuned by the number ratio of boson to fermion flavors. In $1+1$ dimension the solution represents a stable critical phase, while in $2+1$ dimension it governs a quantum phase transition. We compute the out of time order correlators in the $1+1$ dimensional model, showing that it exhibits growth with the maximal Lyapunov exponent $\lambda_L = 2\pi T$ in the low temperature limit.

Introduction. Since 't Hooft's seminal work [1], large N quantum field theories—those with a large number of local degrees of freedom—have played a pivotal role in understanding strongly coupled states of matter and their holographic correspondence to gravity [2–4]. Much of the recent progress in this area was enabled by analysis of a simple large N system: the Sachdev-Ye-Kitaev (SYK) model [5–8]. In its simplest version the SYK model can be written as a quantum mechanical Hamiltonian of N Majorana fermions with all-to-all interactions:

$$H = \sum_{ijk\ell=1}^{N} J_{ijk\ell}\chi_i\chi_j\chi_k\chi_\ell. \tag{1}$$

$J_{ijk\ell}$ are random coupling constants. This model is solvable in the large N limit and has remarkable low energy properties: as a field theory, it is (nearly) conformal invariant, and related to 2d dilaton gravity [9–11]; from a condensed matter point of view, it exemplifies a non-Fermi liquid, with no well-defined quasiparticles [5,12]. Finally, from a quantum information perspective, it is a *fast scrambler*, saturating general bounds on the rate of quantum information scrambling [6,13].

The SYK model is a $0+1$ dimensional quantum field theory. A natural and much-pursued problem is to find generalizations to nonzero spatial dimensions [14–26]. One motivation for this effort is to establish concrete realizations of the AdS/CFT correspondence with richer and more realistic gravity duals. Another motivation, from the perspective of condensed matter physics, is to develop controlled theories for quantum critical points without quasiparticle excitations.

Most of the generalizations of the SYK model that have been discussed before, consist of a lattice of coupled SYK quantum dots. The problem with this approach is that the SYK interaction (1) between the internal degrees of freedom of the dots is irrelevant (at most marginal) compared to quadratic couplings (hopping) between the dots, which leads to a weakly interacting fixed point, e.g., a Fermi liquid. On the other hand, coupling the dots through four-fermion interactions leads to local quantum criticality with no higher dimensional scale invariance. In attempt to avoid this fate, some authors considered field theories with non-local couplings and thus not clearly realizable by a local microscopic Hamiltonian: for example, interactions with a "low-momentum filter" [15] or a topological kinetic term [17]. To our knowledge, the only local higher-dimensional theory exhibiting scale invariance, Lorentz symmetry, and fast scrambling is the supersymmetric $(1+1)$-d model [16,27]. Finding such a model without SUSY or beyond $(1+1)$-d remains an unrealized goal.

In this Letter we propose a family of solvable models in $(1+1)$-d and $(2+1)$-d that extend the SYK physics to higher dimensions. By doing so, we also make connection to the well-known Gross-Neveu-Yukawa (GNY) theory [28,29]. Specifically, we consider a variant of large N GNY which has a large number of real bosons ϕ_a, $a = 1,\dots,M$, and massless Dirac fermions ψ_i, $i = 1,\dots,N$. They interact via a local random Yukawa coupling. The Lagrangian in $d+1$ dimensional Euclidean space-time is

$$\mathcal{L} = \sum_{i=1}^{N} \bar{\psi}_i \partial\!\!\!/ \psi_i + \sum_{a=1}^{M} \frac{m^2}{2}\phi_a^2 + \sum_{ija} g_{ij}^a \bar{\psi}_i \phi_a \psi_j. \tag{2}$$

Here $\partial\!\!\!/ = \boldsymbol{\gamma}^\mu \partial_\mu$, $\bar{\psi} = \psi^\dagger \boldsymbol{\gamma}^0$, and $\boldsymbol{\gamma}^0,\dots,\boldsymbol{\gamma}^d$ are gamma matrices satisfying the Clifford algebra $\{\boldsymbol{\gamma}^\mu, \boldsymbol{\gamma}^\nu\} = \delta_{\mu\nu}$. The Yukawa interaction has random (but translation invariant) coefficients g_{ij}^a, which are zero-mean complex Gaussian variables satisfying

$$\overline{g_{ij}^a g_{k\ell}^b} = g^2 \delta_{i\ell}\delta_{jk}\delta_{ab}/N^2 \text{ (no sum)}. \tag{3}$$

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Here g^2 is a coupling constant, which can be positive or negative ($g \in \mathbf{i}\mathbb{R}$). We work in a large N limit where the boson/fermion number ratio tends to a constant:

$$\frac{M}{Nn_S} \longrightarrow \gamma \quad (M, N \longrightarrow \infty), \tag{4}$$

where n_S is the number of components of each spinor. This is the main difference from the usual large N limit of GNY, where the boson number remains finite. We will show that, in $d + 1 < 4$ space-time dimensions, our theory admits a family of Lorentz invariant critical solutions, whose critical exponents depend continuously on γ. Moreover, we show that the $(1 + 1)$-d critical points are fast scramblers.

Lattice models. Before analyzing the field theory, let us discuss its lattice realizations. In $(0 + 1)$-d, the spinor ψ can have only one component ($\boldsymbol{\gamma}^0 = 1$), and thus (2) describes a "low-rank" SYK dot [30–36] (if $\gamma = 2$, the $\mathcal{N} = 1$ supersymmetric SYK [37]). Indeed, integrating out the bosons leads to a Hamiltonian

$$H = -\frac{1}{2}\sum_{ij,k\ell}^{N} J_{ij,k\ell} c_i^\dagger c_j c_k^\dagger c_\ell, \quad J_{ij,k\ell} = \frac{1}{m^2}\sum_{a=1}^{M} g_{ij}^a g_{k\ell}^a. \tag{5}$$

As an $N^2 \times N^2$ matrix, the rank of J is at most $M = \gamma N \propto N$, instead of $\propto N^2$ in the standard (complex) SYK_4. The low-energy states of this model differ from that of SYK_4 in that the fermion scaling dimension is not fixed to $1/4$, but rather can be tuned continuously by varying γ. In this sense the model is similar to SYK_q (SYK with q fermion interaction), for $q \in (2, \infty)$, $q \neq 4$. Like the SYK models, these are fast scramblers and have a residual entropy. The higher-dimensional solutions we shall present are natural generalizations of these states.

In nonzero spatial dimensions, the field theory (2) can be obtained as the long-wavelength limit of a lattice of identical low-rank SYK dots connected by nearest-neighbor hopping:

$$H = \sum_{\text{n.n.}} c_{i,\mathbf{x}'}^\dagger c_{i,\mathbf{x}} - \frac{1}{2}\sum_{\mathbf{x}ijk\ell} J_{ij,k\ell} c_{i,\mathbf{x}}^\dagger c_{j,\mathbf{x}} c_{k,\mathbf{x}}^\dagger c_{\ell,\mathbf{x}} \overset{\text{H.S.}}{\sim} \sum_{\text{n.n.}} c_{i,\mathbf{x}'}^\dagger c_{j,\mathbf{x}} + \sum_{\mathbf{x}ija} g_{ij}^a c_{i,\mathbf{x}}^\dagger c_{j,\mathbf{x}} \varphi_{a,\mathbf{x}} + \sum_{a\mathbf{x}} \frac{\varphi_{a,\mathbf{x}}^2}{2}. \tag{6}$$

Here $c_{i,\mathbf{x}}$ are N flavors of lattice fermions, and $\varphi_{a,\mathbf{x}}$ are introduced in a Hubbard-Stratonovich transformation. We note however that our IR solution below also applies to physical bosons with an additional kinetic term $\mathcal{O}[(\partial\phi_a)^2]$.

We illustrate the connection between the microscopic Hamiltonian and the field theory degrees of freedom in two examples. In $(1 + 1)$-d, the field theory can be constructed from the slowly varying chiral fermion fields near the two Fermi points, making up the Dirac spinor $\psi_i = (\psi_i^L, \psi_i^R)^T$. The low energy bosons φ_a that couple between the chiral fermions carry momenta near $2k_F$. Finally, the gamma matrices operating in this space are

$$\boldsymbol{\gamma}^0 = \sigma^x, \quad \boldsymbol{\gamma}^0\boldsymbol{\gamma}^1 = \mathbf{i}\sigma^z,$$

where $\sigma^{x,y,z}$ are Pauli matrices.

As a $(2 + 1)$-d example, we take N flavors of fermions hopping on the honeycomb lattice. Then the two component spinor $\psi = (\psi_i^A, \psi_i^B)^T$ is constructed from fermions on the two sublattices A and B with the linearized dispersion near the Dirac point $\mathbf{K}$, $H_{\mathbf{K}+\mathbf{k}} = k_x\sigma^x + k_y\sigma^y$. The gamma matrices in this case are taken as

$$\boldsymbol{\gamma}^0 = \sigma^z, \quad \boldsymbol{\gamma}^0\boldsymbol{\gamma}^1 = \mathbf{i}\sigma^x, \quad \boldsymbol{\gamma}^0\boldsymbol{\gamma}^2 = \mathbf{i}\sigma^y.$$

A boson field that couples the two bands in the same valley $\mathbf{K}$ is constructed from the lattice bosons as $\phi_a = (\varphi_a^A - \varphi_a^B)|_{k\to 0}$, which is odd under the sublattice symmetry. Our critical theory then describes a phase transition to a phase with spontaneous breaking of the sublattice symmetry.

We note that the choice of lattice realization is not unique. For example, we could take the fermion fields in the $(2 + 1)$-d model to be a four-component Nambu spinor $[\psi^A, \psi^B, (\psi^B)^\dagger, -(\psi^A)^\dagger]^T$ coupling to charge-2 bosonic fields. In this case the theory (2) describes a superconducting quantum phase transition. With a different choice of four-component spinor and bosons carrying an angular momentum quantum number, the theory can describe spontaneous breaking of time reversal symmetry and establishment of a quantum Hall state.

Critical solutions. We now return to the field theory in general dimension and present its critical solutions in the IR. For generality, we shall add a boson kinetic term to the Lagrangian (2),

$$m^2\phi_a^2 \rightsquigarrow m^2\phi_a^2 + b(\partial\phi_a)^2, \quad b \geqslant 0. \tag{7}$$

In the large N limit, the fermion and boson Green functions $\mathbf{G} = \frac{1}{N}\sum_i \langle\psi_i\bar{\psi}_i\rangle$, $F = \frac{1}{M}\sum_a \langle\phi_a\phi_a\rangle$, averaged over the random couplings, are related to their respective self-energy $\boldsymbol{\Sigma}$, Π by a set of Schwinger-Dyson (SD) equations:

$$\mathbf{G}(k)^{-1} = \not{k} - \boldsymbol{\Sigma}(k), \quad F(q) = \frac{1}{m^2 + bq^2 - \Pi(q)},$$
$$\boldsymbol{\Sigma}(x) = n_S\gamma g^2\mathbf{G}(x)F(x), \quad \Pi(x) = -g^2\mathrm{tr}[\mathbf{G}(x)\mathbf{G}(-x)]. \tag{8}$$

Above, $q = (\Omega, \mathbf{q})$ and $k = (\omega, \mathbf{k})$ denote $(d + 1)$-momenta, and "tr" is over the spinor space. Equations (8) generalize the SD equations of the low-rank SYK dots, and can be derived following the same steps.

The main point of this Letter is that, in the IR limit, the SD equations admit critical solutions that have the following scale-invariant and Lorentz symmetric form [38]:

$$\mathbf{G} \sim \mathbf{i}\not{k}|k|^{2\Delta-D-1} \sim \not{x}|x|^{-2\Delta-1},$$
$$\boldsymbol{\Sigma} \sim \mathbf{i}\not{k}|k|^{-2\Delta+D-1} \sim \not{x}|x|^{2\Delta-2D-1},$$
$$F \sim |q|^{D-4\Delta} \sim |x|^{4\Delta-2D}, \quad \Pi \sim |q|^{4\Delta-D} \sim |x|^{-4\Delta}, \tag{9}$$

for a continuous range of the fermion scaling dimension $\Delta = \Delta_\psi$ (the boson scaling dimension is $\Delta_\phi = D - 2\Delta_\psi$):

$$\min\left(\frac{D}{2}, \frac{D+2}{4}\right) > \Delta > \frac{D-1}{2}, \quad D := d + 1 < 4. \tag{10}$$

For $D \geqslant 4$, no Δ satisfies these inequalities. Moreover, we have the "rank-exponent relation" (see Fig. 1):

$$\gamma = -\frac{B_{\frac{D}{2}-\Delta}C_{2\Delta}}{B_{D-\Delta}C_{2\Delta-\frac{D}{2}}}, \tag{11}$$

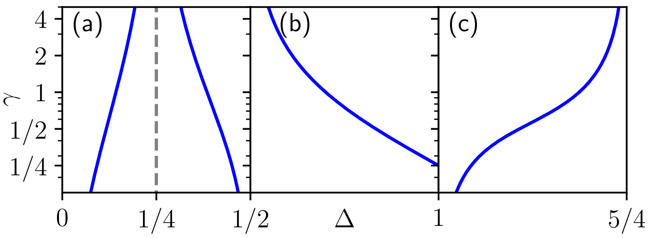

FIG. 1. The rank (γ) exponent (Δ) relation (11) in $D = 1, 2, 3$ (a)–(c) space-time dimensions.

where B and C are defined as

$$C_a = (2\pi)^{\frac{D}{2}} \frac{2^{\frac{D}{2}-a}\Gamma\left(\frac{D}{2}-a\right)}{2^a\Gamma(a)}, \quad B_a = \frac{C_{a-\frac{1}{2}}}{1-2a}. \tag{12}$$

The rank-exponent relation generalizes that in $(0+1)$-d, which was known previously [30–35,39].

To demonstrate the above claims, *suppose*, as will be justified below, that we can ignore the bare terms $\not{k}$ and $m^2 + bq^2$ in the IR limit. Then the approximate SD equations become scale invariant. One may start from a power-law Ansatz

$$\mathbf{G}(k) = \mathbf{i}A\not{k}|k|^{2\Delta-D-1}, \tag{13}$$

and check that it is compatible with the SD equations (8) if the exponent is fixed by (11). This can be done by using the gamma-matrix identity $\not{a}\not{a} = |a|^2 I$ and the Fourier transform identities

$$|x|^{-2a} e^{\mathbf{i}q^\mu x_\mu} d^D x = C_a |q|^{2a-D}, \tag{14}$$

$$\not{x}|x|^{-2a-1} e^{\mathbf{i}q^\mu x_\mu} d^D x = -\mathbf{i}B_a \not{q}|q|^{2a-D-1}. \tag{15}$$

The value of the prefactor A depends on UV details and is unimportant; it suffices to know that $A > 0$ from the UV limit $\mathbf{G}(k) \overset{k\to\infty}{\sim} 1/(-\mathbf{i}\not{k})$.

Our neglect of the bare kinetic terms in the Green's function is justified if they are irrelevant in the IR, that is if they vanish in the low energy limit compared to the self-energy terms. The condition for the fermion kinetic term $\sim|k| \ll |\mathbf{\Sigma}(k)|$ is $\Delta > (D-1)/2$. Similarly the condition for the boson kinetic term $bq^2 \ll \Pi \sim |q|^{4\Delta-D}$ is $\Delta < (D+2)/4$. Thus, the inequalities (10) emerge as the condition for a consistent scale invariant solution [40]. Finally, the (renormalized) boson mass term is tuned to zero at criticality, or in the exceptional cases of $D = 1, 2$ is even irrelevant and flows to zero in the IR, as we discuss below.

In $(0+1)$-d, when $g^2 > 0$, the mass is known to be irrelevant, and the system always flows to a critical point with $\Delta \in (\frac{1}{4}, \frac{1}{2})$ determined uniquely by γ [see Fig. 1(a)]. This "self-tuned" criticality was first noticed in Ref. [34], see also [41] for a Monte Carlo study. The situation of $g^2 < 0$ is even more special [30]: the boson self-energy diverges $\Pi(q \to 0) \to -\infty$ and dominates the mass, and the IR fixed point has $\Delta \in (0, \frac{1}{4})$.

In $(1+1)$-d, the boson mass flows to zero in the IR provided $g^2 > 0$. To see why this must happen, let us first show that the bosons cannot remain gapped in the IR. If that were the case, the fermions would be noninteracting in the IR, $\mathbf{G} \sim (-\mathbf{i}\not{k})^{-1}$ $(\Delta = 1/2)$. However, the boson self-energy would then have a log divergence, and $\Pi(q) \sim g^2 \ln(1/|q|) > m^2$ at small enough q for any $g^2 > 0$, which makes the bosons unstable. Could the bosons then condense? In this case the condensate $F(\tau) \sim$ const. would generate a mass in the fermion dispersion leading to $G \sim \mathbf{i}\not{k}/|k|$ $(\Delta = 1)$. However, this would imply an inconsistent IR divergence $F(x) \sim \ln\ln(R/|x|)$. Since neither conventional state leads to a self-consistent solution, the IR fixed point must be critical. [A similar argument can be applied to understand the self-tuned criticality in $(0+1)$-d.] It should be noted that, since the back-scattering is relevant, our critical solutions are not Luttinger liquids.

Another peculiar aspect of $(1+1)$-d is that the critical solutions we found require a nonzero minimal rank $\gamma > \frac{1}{4}$. Indeed, the rank-exponent relation reads [Fig. 1(b)]

$$\gamma = \frac{3-2\Delta}{8\Delta-4}. \tag{16}$$

The interval $\Delta \in (\frac{1}{2}, 1)$ permitted by (10) is mapped to $\gamma \in (\frac{1}{4}, \infty)$, with the limit $\gamma \to 1/4$ corresponding to $\Delta \to 1$. The critical points for $\gamma \leqslant \frac{1}{4}$ are not included in our solutions; we speculate that the fermion Green function still has $\Delta = 1$, but with nontrivial log corrections. We remark finally that our theory at $\gamma = 1/2$ is akin to the $\mathcal{N} = 1$ SUSY theory of Ref. [16] with $q = 3$: they both have $\Delta_\phi = 1/3$.

In $(2+1)$-d, the critical points are not self-tuned, but describe a second-order transition at $g^2 = g_c^2 > 0$, between a semimetal with gapped bosons and a marginal Fermi liquid with broken symmetry. Let us also comment on the rank-exponent relation [Fig. 1(c)] which reads

$$\gamma = \frac{(2\Delta-5)(2\Delta-3)\tan\pi\Delta\tan2\pi\Delta}{8(\Delta-1)(4\Delta-3)}. \tag{17}$$

In the interval $\Delta \in (1, \frac{5}{4})$ allowed by (10), Δ is uniquely determined by γ, and increases from the noninteracting low-rank limit $\Delta_{\gamma\to0} = 1$, to the strongly coupled high-rank limit $\Delta_{\gamma\to\infty} = \frac{5}{4}$. The latter limit seems to contradict the fact that a $\bar{\psi}\psi\bar{\psi}\psi$ interaction is irrelevant in $(2+1)$-d, and that the high-rank $(\gamma \sim N \to \infty)$ limit of our model is effectively a lattice of SYK_4 dots. There is however a subtlety: the quartic couplings $J_{ij,k\ell}$ in (5) above have a variance $\sim\gamma N^{-3}$, which is much larger than N^{-3} in SYK_4 if $\gamma \sim N$. Hence, our theory in the $\gamma \to \infty$ limit is more strongly interacting than a SYK_4 lattice and only the former can approach the $\Delta = \frac{5}{4}$ fixed point (unless one goes beyond the large N limit and lets the temperature to scale with $1/N$).

We now make connection to the standard approach to the GNY model, i.e., N_f fermions coupled to a single boson. Within the new large N limit with comparably many bosons, we found, at the classical saddle point, an anomalous fermion scaling dimension, which is only obtained as $1/N_f$ corrections in the standard approach. Moreover, we can compare quantitatively the rank-exponent relation (17) in $(2+1)$-d to the standard large N_f GNY exponent, by identifying $\gamma = 1/(N_f n_S)$. Solving for the scaling dimension we obtain $\Delta = 1 + 4/(3\pi^2 N_f n_S) + \mathcal{O}(1/N_f^2)$, which matches the standard $1/N_f$ result [43]. We also observe a good agreement with state-of-the-art conformal bootstrap data on the standard

TABLE I. Fermion scaling dimension Δ in the $D = 3$ GNY with N_f fermion flavors found using conformal bootstrap [42] compared to the result obtained from the rank-exponent relation (17) by setting $\gamma = 1/(N_f n_S)$.

N_f	2	3	4	8	10	20
Δ [42]	1.067	1.054	1.042	1.021	1.017	1.008
Δ (17)	1.107	1.063	1.043	1.019	1.015	1.007

GNY model [42], see Table I. These results indicate that the $1/N$ corrections of our large N theory are rather mild.

Residual entropy. A peculiar feature of SYK_q is that the entropy remains nonzero and $\propto N$ in the zero-T limit, provided the large N limit is taken first [6,12,44]. Such a residual entropy is also observed numerically in the $D = 1$ critical points [35], and we can now understand it analytically, in terms of log determinants [45]. In higher dimensions, only the zero-momentum component of the Green function is critical at zero temperature and can possibly contribute to the residual entropy. Therefore, the residual entropy is not extensive in volume, in contrast to "local critical" SYK lattices [14].

Scrambling. We turn to calculating the out of time order correlations (OTOCs) [13,46] in the low temperature limit of the $1 + 1$ dimensional field theory. The OTOCs are defined on a "double" Keldysh contour (Fig. 2):

$$\mathcal{C}(t, \mathbf{x}) = \left\langle V^\dagger(t, \mathbf{x}) V\left(t + \mathbf{i}\frac{\beta}{2}, \mathbf{x}\right) \phi^{\mathrm{q}}(0, \mathbf{0}) \phi^{\mathrm{q}}\left(\mathbf{i}\frac{\beta}{2}, \mathbf{0}\right)\right\rangle, \tag{18}$$

where $\phi^{\mathrm{q}}(t) = \phi(t + \mathbf{i}\epsilon) - \phi(t - \mathbf{i}\epsilon)$, and $(V^\dagger, V)$ can be a spinor component and its conjugate $[(\psi^A)^\dagger, \psi^A]$, or the boson (ϕ, ϕ), which is real. The OTOC measures the sensitivity of an observable V at $(t, \mathbf{x})$ to a perturbation at $(0, \mathbf{0})$ as a quantum analog of the butterfly effect; it can grow exponentially in large N systems, defining a Lyapunov exponent λ_L. The $(0 + 1)$-d version of the model are known to be fast scramblers [31,32,35], saturating the general bound on the Lyapunov exponent at low temperatures [13]:

$$\mathcal{C}(t) \sim \frac{1}{N} e^{\lambda_L t}, \quad \text{where } \lambda_L = 2\pi T. \tag{19}$$

Here we extend the result to $(1 + 1)$-d, by a method similar to that discussed in Refs. [14,26,47]. In the regime where the OTOC has a well-defined exponential growth, it is given, to leading order in $1/N$, by the sum of ladder diagrams generated by a four-point retarded kernel $\mathbb{K}(t_1, x_1, \ldots, t_4, x_4)$ that adds a rung to the ladder (see Fig. 2). The self-consistency condition (Bethe-Salpeter equation) is then equivalent to solving for the OTOC as an eigenvector of the kernel with eigenvalue 1:

$$\mathcal{F}_p(t_3, x_3, t_4, x_4) = \int \mathbb{K}\, \mathcal{F}_p(t_1, x_1, t_2, x_2). \tag{20}$$

To find a Lyapunov exponent we seek exponentially growing eigenfunctions with momentum p:

$$\mathcal{F}_p(x_1, t_1, x_2, t_2) = F_p\, e^{\lambda_L(t_1+t_2)/2 + \mathbf{i}p(x_1+x_2)/2}, \tag{21}$$

where $F_p = F_p(t_1 - t_2, x_1 - x_2)$ only depends on the relative coordinate. As a result, we determine a p-dependent exponent $\lambda_L = \lambda_L(p)$. Then the OTOC in real space can be expressed as a momentum integral

$$\mathcal{C}(t, x) \sim \frac{1}{N} \int_{-\infty}^{\infty} \rho(p) F_p(0, 0) e^{\lambda_L(p)t + \mathbf{i}px} dp, \tag{22}$$

where $\rho(p)$ was shown [47] to have a pole where $\lambda_L(p = \mathbf{i}s_*) = 2\pi T$ [48]. The integral (22) is analyzed by a steepest descent method for large x, t, and has maximal growth $\propto e^{2\pi T t}$ if it is dominated by the pole in $\rho(p)$. This is the case when the fields are sufficiently separated in space, $|x|/t > v_*$, where the velocity v_* is defined as

$$v_* := [\partial_s \lambda_L(\mathbf{i}s)]_{s=s_*}. \tag{23}$$

Here $s_* > 0$ is such that $\lambda_L(\mathbf{i}s_*) = 2\pi T$.

For our $(1 + 1)$-d critical solutions, we can calculate $\lambda_L(p)$ analytically [45]. To compute the kernel we use the finite-T critical Green's functions obtained by conformal invariance from the zero temperature power laws. The calculation is simplified by factoring of correlators into functions of one chiral variable $x \pm t$. Now, solving the eigenvalue problem, we find that for any $\Delta \in (\frac{1}{2}, 1)$, $\lambda_L(p)$ satisfies the following:

$$\lambda_L(p = \pm \mathbf{i}2\pi T) = 2\pi T. \tag{24}$$

Moreover, we checked that $v_* := [\partial_s \lambda_L(\mathbf{i}s)]_{s=2\pi T} < v_B = 1$ always holds (see Fig. 1 of [45]). Therefore, there is a nonempty regime of fast scrambling near the light cone:

$$\mathcal{C}(t, x) \sim e^{2\pi T(t-|x|)}, \quad t \in \left(\frac{|x|}{v_B}, \frac{|x|}{v_*}\right). \tag{25}$$

Such a behavior is qualitatively similar to that in other higher dimensional generalizations of the SYK model [14,26,47,49,50]. However, due to lack of Lorentz symmetry, the butterfly velocity v_B in those models has a model dependent value $\mathcal{C}(t, x) \propto e^{2\pi T(t-|x|/v_B)}$, while in our case $v_B = 1$ is the speed of light.

Our result here is a concrete example of the "chiral" scrambling modes $e^{2\pi T(t\pm x)}$, which are argued to appear in generic holographic CFTs [51] as a result of broken reparametrization symmetry. It will be interesting to apply a general theory of scrambling (e.g., [52,53]) to investigate whether the $(2 + 1)$-d critical points are fast scrambling.

Discussion. We proposed a variant of the GNY theory as a generalization of the SYK physics to higher dimensions. The model is solvable in the large N limit and presents a strongly coupled Lorentz invariant critical point. Furthermore,

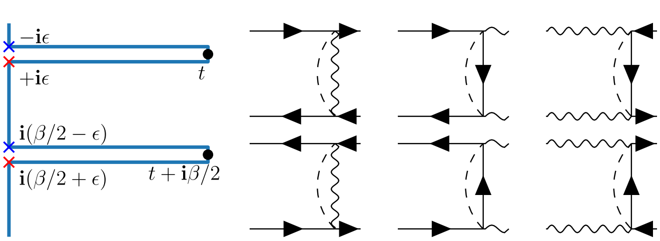

FIG. 2. Left: The Keldysh contour on which the OTOC (18) is defined, with field locations indicated. Right: The basic rungs that generate all the ladder diagrams. Straight, wavy, and dashed lines correspond to fermion, boson propagators, and average over random couplings.

direct calculation shows that in $(1+1)$-d the model exhibits maximal scrambling. A natural next step, beyond the critical saddle point solutions presented in this paper, is to derive a low-energy effective theory of the dominant fluctuations in analogy with the Schwarzian effective theory of the SYK model [54].

The model introduced here can serve as a new starting point for understanding strongly coupled quantum critical points or phases. In $(1+1)$-d we have strongly correlated gapless phases that are holographic CFTs. It will be interesting to study the effect of various perturbations, such as quenched disorder. The $(2+1)$-d solution exemplifies a strongly coupled quantum critical point with itinerant fermions. We may add a gauge field to obtain a new large N limit of QED_3. Finally, our conformal solutions also gives a new paradigm for investigating how superconductivity can emerge from critical fluctuations in the absence of quasiparticles: indeed, if the coupling constants g^a_{ij} in (2) are chosen from the GOE ensemble, with $g^a_{ij} = g^a_{ji} \in \mathbb{R}$, instead of GUE as in (3), the theory allows superconducting solutions described by Eliashberg equations [33,34].

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[1] G. 't Hooft, A planar diagram theory for strong interactions, Nucl. Phys. B **72**, 461 (1974).

[2] J. Maldacena, The Large N limit of superconformal field theories and supergravity, Int. J. Theor. Phys. **38**, 1113 (1999).

[3] S. S. Gubser, I. R. Klebanov, and A. M. Polyakov, Gauge theory correlators from noncritical string theory, Phys. Lett. B **428**, 105 (1998).

[4] O. Aharony, S. S. Gubser, J. Maldacena, H. Ooguri, and Y. Oz, Large N field theories, string theory and gravity, Phys. Rep. **323**, 183 (2000).

[5] S. Sachdev and J. Ye, Gapless Spin Fluid Ground State in a Random, Quantum Heisenberg Magnet, Phys. Rev. Lett. **70**, 3339 (1993).

[6] A. Kitaev, A simple model of quantum holography, http://online.kitp.ucsb.edu/online/entangled15/kitaev/,http://online.kitp.ucsb.edu/online/entangled15/kitaev2/. Talks at KITP, April 7, 2015 and May 27, 2015.

[7] S. Sachdev, Bekenstein-Hawking Entropy and Strange Metals, Phys. Rev. X **5**, 041025 (2015).

[8] J. Maldacena and D. Stanford, Remarks on the Sachdev-Ye-Kitaev model, Phys. Rev. D **94**, 106002 (2016).

[9] K. Jensen, Chaos in AdS_2 Holography, Phys. Rev. Lett. **117**, 111601 (2016).

[10] J. Maldacena, D. Stanford, and Z. Yang, Conformal symmetry and its breaking in two-dimensional nearly anti-de Sitter space, Prog. Theor. Exp. Phys. **2016**, 12C104 (2016).

[11] J. Engelsöy, T. G. Mertens, and H. Verlinde, An investigation of AdS2 backreaction and holography, J. High Energy Phys. 07 (2016) 139.

[12] A. Georges, O. Parcollet, and S. Sachdev, Quantum fluctuations of a nearly critical Heisenberg spin glass, Phys. Rev. B **63**, 134406 (2001).

[13] J. Maldacena, S. H. Shenker, and D. Stanford, A bound on chaos, J. High Energy Phys. 08 (2016) 106.

[14] Y. Gu, X.-L. Qi, and D. Stanford, Local criticality, diffusion and chaos in generalized Sachdev-Ye-Kitaev models, J. High Energy Phys. 05 (2017) 125.

[15] M. Berkooz, P. Narayan, M. Rozali, and J. Simón, Higher dimensional generalizations of the SYK model, J. High Energy Phys. 01 (2017) 138.

[16] J. Murugan, D. Stanford, and E. Witten, More on supersymmetric and 2d analogs of the SYK model, J. High Energy Phys. 08 (2017) 146.

[17] G. J. Turiaci and H. Verlinde, Towards a 2d QFT analog of the SYK model, J. High Energy Phys. 10 (2017) 167.

[18] R. A. Davison, W. Fu, A. Georges, Y. Gu, K. Jensen, and S. Sachdev, Thermoelectric transport in disordered metals without quasiparticles: The Sachdev-Ye-Kitaev models and holography, Phys. Rev. B **95**, 155131 (2017).

[19] X.-Y. Song, C.-M. Jian, and L. Balents, A Strongly Correlated Metal Built from Sachdev-Ye-Kitaev Models, Phys. Rev. Lett. **119**, 216601 (2017).

[20] D. V. Khveshchenko, Thickening and sickening the SYK model, SciPost Phys. **5**, 12 (2018).

[21] A. Haldar, S. Banerjee, and V. B. Shenoy, Higher-dimensional Sachdev-Ye-Kitaev non-Fermi liquids at Lifshitz transitions, Phys. Rev. B **97**, 241106(R) (2018).

[22] D. Chowdhury, Y. Werman, E. Berg, and T. Senthil, Translationally Invariant non-Fermi Liquid Metals with Critical Fermi-Surfaces: Solvable Models, Phys. Rev. X **8**, 031024 (2018).

[23] A. A. Patel, J. McGreevy, D. P. Arovas, and S. Sachdev, Magnetotransport in a Model of a Disordered Strange Metal, Phys. Rev. X **8**, 021049 (2018).

[24] A. Altland, D. Bagrets, and A. Kamenev, Quantum Criticality of Granular Sachdev-Ye-Kitaev Matter, Phys. Rev. Lett. **123**, 106601 (2019).

[25] A. A. Patel and S. Sachdev, Theory of a Planckian Metal, Phys. Rev. Lett. **123**, 066601 (2019).

[26] B. Lian, S. L. Sondhi, and Z. Yang, The chiral SYK model, J. High Energy Phys. 09 (2019) 067.

[27] M. Mezei and G. Sárosi, Chaos in the butterfly cone, J. High Energy Phys. 01 (2020) 186.

[28] D. J. Gross and A. Neveu, Dynamical symmetry breaking in asymptotically free field theories, Phys. Rev. D **10**, 3235 (1974).

[29] J. Zinn-Justin, Four-fermion interaction near four dimensions, Nucl. Phys. B **367**, 105 (1991).

[30] Z. Bi, C.-M. Jian, Y.-Z. You, K. A. Pawlak, and C. Xu, Instability of the non-Fermi liquid state of the Sachdev-Ye-Kitaev model, Phys. Rev. B **95**, 205105 (2017).

[31] W. Fu, The Sachdev-Ye-Kitaev model and matter without quasiparticles, Ph.D. thesis, Harvard University, 2018.

[32] E. Marcus and S. Vandoren, A new class of SYK-like models with maximal chaos, J. High Energy Phys. 01 (2019) 166.

[33] I. Esterlis and J. Schmalian, Cooper pairing of incoherent electrons: An electron-phonon version of the Sachdev-Ye-Kitaev model, Phys. Rev. B **100**, 115132 (2019).
[34] Y. Wang, Solvable Strong-Coupling Quantum-Dot Model with a non-Fermi-Liquid Pairing Transition, Phys. Rev. Lett. **124**, 017002 (2020).
[35] J. Kim, X. Cao, and E. Altman, Low-rank Sachdev-Ye-Kitaev models, Phys. Rev. B **101**, 125112 (2020).
[36] J. Kim, X. Cao, and E. Altman, Scrambling versus relaxation in Fermi and non-Fermi liquids, Phys. Rev. B **102**, 085134 (2020).
[37] W. Fu, D. Gaiotto, J. Maldacena, and S. Sachdev, Supersymmetric Sachdev-Ye-Kitaev models, Phys. Rev. D **95**, 026009 (2017).
[38] The anomalous form of the fermion propagator was found in a GNY theory away from large N limit by an ϵ expansion [55].
[39] A. A. Patel and S. Sachdev, Critical strange metal from fluctuating gauge fields in a solvable random model, Phys. Rev. B **98**, 125134 (2018).
[40] Note that the condition $\Delta < D/2$ is not needed for this purpose, but is necessary in $D = 1$ so that $F(x) \to 0$ for $x \to \infty$.
[41] G. Pan, Y. Wang, and Z. Y. Meng, Self-tuned quantum criticality and non-Fermi-liquid in a Yukawa-SYK Model: A quantum Monte Carlo study, arXiv:2001.06586.
[42] L. Iliesiu, F. Kos, D. Poland, S. S. Pufu, and D. Simmons-Duffin, Bootstrapping 3d fermions with global symmetries, J. High Energy Phys. 01 (2018) 036.
[43] S. E. Derkachov, N. A. Kivel, A. S. Stepanenko, and A. N. Vasiliev, On calculation in $1/n$ expansions of critical exponents in the Gross-Neveu model with the conformal technique, arXiv:hep-th/9302034.
[44] Y. Gu, A. Kitaev, S. Sachdev, and G. Tarnopolsky, Notes on the complex Sachdev-Ye-Kitaev model, J. High Energy Phys. 02 (2020) 157.
[45] See Supplemental Material at http://link.aps.org/supplemental/10.1103/PhysRevB.103.L081113 for details the calculation of scrambling in (1+1)-d and of the residual entropy in (0+1)-d.
[46] A. I. Larkin and Yu. N. Ovchinnikov, Quasiclassical method in the theory of superconductivity, Zh. Eksp. Teor. Fiz. **55**, 2262 (1969) [Sov. J. Exp. Theor. Phys. **28**, 1200 (1969)].
[47] Y. Gu and A. Kitaev, On the relation between the magnitude and exponent of OTOCs, J. High Energy Phys. 02 (2019) 075.
[48] The argument in the aforementioned reference was formulated with SYK_q ladder kernels, but can be extended to other ones.
[49] V. Khemani, D. A. Huse, and A. Nahum, Velocity-dependent Lyapunov exponents in many-body quantum, semiclassical, and classical chaos, Phys. Rev. B **98**, 144304 (2018).
[50] H. Guo, Y. Gu, and S. Sachdev, Transport and chaos in lattice Sachdev-Ye-Kitaev models, Phys. Rev. B **100**, 045140 (2019).
[51] G. J. Turiaci and H. Verlinde, On CFT and quantum chaos, J. High Energy Phys. 12 (2016) 110.
[52] M. Blake, H. Lee, and H. Liu, A quantum hydrodynamical description for scrambling and many-body chaos, J. High Energy Phys. 10 (2018) 127.
[53] F. M. Haehl, W. Reeves, and M. Rozali, Reparametrization modes, shadow operators, and quantum chaos in higher-dimensional CFTs, J. High Energy Phys. 11 (2019) 102.
[54] J. Cotler and K. Jensen, A theory of reparameterizations for AdS3 gravity, J. High Energy Phys. 02 (2019) 079.
[55] S. Sachdev, Phase transitions of Dirac fermions, in *Quantum Phase Transitions*, 2nd ed. (Cambridge University Press, Cambridge, 2011), p. 332.