Title

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Journal

Proceedings of the PowerUp Conference, 1(1)

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Publication Date

2026-09-10

DOI

None/powerup_proceedings.69148

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Suggested Citation

E. Trieschman and S. Amin, "Structural Misalignment in Financial Transmission Rights," in *Proceedings of the PowerUp Conference*, 2026.

BibTeX Entry

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@inproceedings{trieschman2026structuralmisalignme,
  author = {Trieschman, Erich and Amin, Saurabh},
  title = {Structural Misalignment in Financial Transmission Rights},
  booktitle = {Proceedings of the PowerUp Conference},
  year = {2026},
}
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Structural Misalignment in Financial Transmission Rights

Erich Trieschman[†], Saurabh Amin[†]

***Abstract*—Financial Transmission Rights (FTRs) enable electricity market participants to hedge congestion risk in Day Ahead Market (DAM) operations, but for the market to be solvent, Independent System Operators (ISOs) must ensure that FTR payouts do not exceed the collected DAM merchandising surplus that funds them. We show that FTR underfunding (or conversely, hedging efficiency) can arise structurally from misalignment between the network models used in the FTR auction and the DAM, independent of bidding behavior.**

We develop a geometric framework in which both DAM merchandising surplus and the maximum supportable FTR payout are expressed as support functions of network-feasible injection polytopes. The resulting dual representation assigns nonnegative weights to transmission element-contingency constraints, enabling constraint-level attribution of model misalignment.

Using this framework, we derive sharp implications for canonical FTR network modeling choices like uniform transmission element derates, and for structural sources of underfunding like unplanned DAM outages. We further show that multi-interval FTR products impose an intrinsic hedging inefficiency when DAM shadow prices vary over time, even under perfect model alignment.

These results provide ISOs with rigorous tools to diagnose underfunding and quantify the efficiency cost of conservative FTR network modeling choices at realized DAM outcomes. Extending these to ex ante design is future work.

***Index Terms*—financial transmission rights, electricity market design, support functions, dual degeneracy**

I. Introduction

The Financial Transmission Right (FTR) auction allocates congestion-related price risk across wholesale electricity market participants ahead of real-time operations. An FTR pays the Day Ahead Market (DAM) locational marginal price (LMP) difference between a source node and a sink node over a contract period [1]. These payments are funded by DAM merchandising surplus (realized congestion rents) collected by the Independent System Operator (ISO) when transmission elements operate at their limits (bind) [1].

FTRs provide a congestion risk hedge for physical participants (e.g., a generator hedged at a hub still faces congestion risk between its bus and the hub), transfer congestion risk to financial participants, and allocate uncertain future DAM merchandising surplus through a forward market [1], [2].

To manage revenue adequacy, ISOs impose a simultaneous feasibility test (SFT). The SFT uses a network model to link FTR positions to physical constraints and enables system-wide trading across arbitrary node pairs without separate bilateral instruments [1], [2].

If the FTR network model matched the realized DAM network model, the auction could pass through DAM merchandising surplus fully to FTR holders while remaining solvent. In practice, several factors prevent such alignment. The FTR auction clears using a single network model across the contract period, fixed months or years in advance of realized DAM operations. Heterogeneity in realized DAM network models across the contract period and unplanned DAM outages or reconfigurations can both drive misalignment. These challenges are amplified by modern grid dynamics: deep interconnection queues lead to frequent topology changes [3], while renewable integration and large data-center loads are increasing operational uncertainty [4], [5]. Misalignment creates two distinct failures. If the FTR network model is more permissive than realized DAM feasibility, awarded payouts can exceed collected DAM merchandising surplus resulting in underfunding; if the FTR network model is more restrictive, participants cannot fully hedge available DAM merchandising surplus resulting in hedging inefficiency [6], [7]. ISOs therefore face a fundamental design tradeoff between aggressive assumptions exposing underfunding risk and conservative assumptions causing hedging inefficiency.

Recent market outcomes illustrate both sides of this tradeoff. In the Southwest Power Pool (SPP), 2024 Transmission Congestion Right (TCR) funding fell below the stated 90-100% target range (83%), and the market monitor attributes much of the shortfall to network model outage gaps between the TCR auction and the DAM [6]. Prior to August 2024 when SPP tightened outage coordination requirements, transmission owners were only required to submit planned outages above a certain duration and size 14 days in advance, well after the corresponding TCR auction had taken place [6]. As a result, roughly 75% of outages were excluded from the auction network models due to lead-time alone [6]. ERCOT reports full monthly CRR funding throughout 2024 [7], supported in part by conservative transfer capability assumptions (including a uniform derating schedule) and balancing mechanisms [7]; nevertheless, its market monitor still identifies unforeseen outages and transmission reductions between the auction model and the DAM as the primary source of underfunding risk [7].

These observations motivate fundamental questions: How can the efficiency cost of conservative FTR network modeling choices be quantified? In an underfunding event, can the ISO systematically attribute the shortfall to specific constraints in the DAM and FTR network models?

We treat alignment as a geometric comparison between two network-feasible injection polytopes: one induced by the realized DAM network model and one induced by the FTR network model. Congestion rents in the DAM arise endogenously from welfare maximization under network constraints [8], [9]. We show that both DAM merchandising surplus and the maximum supportable FTR payout can be expressed as the value of the same linear congestion-rent objective evaluated over their respective feasible sets of network injections. Alignment therefore reduces to a comparison of

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these two optima: underfunding arises when the FTR-based maximum exceeds the DAM-based maximum, while hedging inefficiency arises when it falls short. This formulation isolates structural effects of network modeling from bidding behavior, but also highlights a central challenge: realistic networks yield high-dimensional polytopes with substantial dual degeneracy, requiring principled constraint-level attribution.

We develop a geometric and dual-based framework for analyzing structural alignment between the FTR auction and the DAM, with the following contributions:

Support-function formulation. We formalize the alignment problem as a support function comparison over two network-feasible polytopes, yielding a unified geometric representation of revenue adequacy and hedging inefficiency.

Dual attribution with a shared feasible set. We derive LP dual representations in which DAM and FTR model alignment problems share a common dual-feasible set and differ only through their objective cost vectors, enabling principled constraint-level comparisons of alternative network models.

Robust constraint classification. We characterize robust multiplier ranges under dual degeneracy and classify each constraint as *definitely binding*, *degenerately binding*, or *definitely slack* in a given DAM realization, enabling constraint-level diagnostics even when dual optima are non-unique.

Implications for FTR design. We show that: (i) uniform constraint limit derates produce inefficiency proportional to the derate factor; (ii) adding contingencies to the FTR network model can induce hedging inefficiency but does not permit underfunding risk; and (iii) omitting contingencies can permit underfunding but does not induce hedging inefficiency.

Inefficiency from multi-interval products. We show that temporal aggregation across DAM intervals cannot increase underfunding risk but can strictly reduce hedging efficiency when congestion patterns vary over time, even under perfect structural alignment.

These contributions enable ISOs to quantify the ex post efficiency cost of conservative FTR network modeling choices and to systematically attribute observed underfunding to specific contingencies or constraints in the DAM and FTR network models using dual information derived from realized DAM outcomes. The remainder of the paper is organized as follows: Section II formalizes the network models and market-clearing problems. Sections III–V develop the theoretical framework and main results, and Section VI discusses extensions and concludes. Proofs for all stated propositions and theorems are provided in Appendix A.

II. Network Models and Market Clearing

This section establishes notation, defines network-feasible injection sets, and characterizes market-clearing outcomes underlying both the DAM and the FTR auction. Throughout, we adopt standard DC optimal power flow (DCOPF) conventions, under which network feasibility is expressed through linear constraints on nodal injections [1], so the feasible injection sets are convex polytopes. This convexity is a design property of FTR auctions, which clear on DC network models; the AC injection sets underlying real-time operation can be non-convex, an independent source of revenue inadequacy [10] that does not arise in the DC setting we study. We focus on a single DAM interval unless otherwise noted.

A. Network Topology Representation

We fix the following sets common to both markets: $\mathcal{N}$: set of nodes (buses), with $|\mathcal{N}| = n$; $\mathcal{L}$: set of monitored transmission elements (lines and transformers), with $|\mathcal{L}| = \ell$; $\mathcal{C} = \{c_1, c_2, \ldots, c_m\}$: universal set of contingencies, with $|\mathcal{C}| = m$, indexed in a fixed order. Each $c \in \mathcal{C}$ indexes a specific network topology, such as the base case or an $N-1$ outage of a transmission element.

For each contingency $c \in \mathcal{C}$, the Power Transfer Distribution Factor (PTDF) matrix, $K_c \in \mathbb{R}^{\ell \times n}$, denotes a fixed linear mapping from nodal injections, $q \in \mathbb{R}^n$ (restricted to the balanced subspace), to line flows, $K_c q \in \mathbb{R}^\ell$; all network topology assumptions required to construct K_c are subsumed into the matrix itself [1].

For each contingency $c \in \mathcal{C}$ and transmission element $\ell \in \mathcal{L}$, let $\underline{b}_{c,\ell}$ and $\overline{b}_{c,\ell}$ denote the lower and upper flow limits, respectively. These limits may reflect thermal, voltage, or stability constraints and may vary across contingencies to capture emergency ratings [1].

To unify notation across contingencies, we stack PTDF matrices and line limit vectors according to the fixed ordering of $\mathcal{C}$. Define

$$K := \begin{bmatrix} K_{c_1} \\ \vdots \\ K_{c_m} \end{bmatrix}, \quad \underline{b} := \begin{bmatrix} \underline{b}_{c_1} \\ \vdots \\ \underline{b}_{c_m} \end{bmatrix}, \quad \overline{b} := \begin{bmatrix} \overline{b}_{c_1} \\ \vdots \\ \overline{b}_{c_m} \end{bmatrix},$$

with $K \in \mathbb{R}^{m\ell \times n}$ and $\underline{b}, \overline{b} \in \mathbb{R}^{m\ell}$.

Assumption 1 (Common Network Representation). The DAM and FTR markets share a common linear network representation: identical sets $\mathcal{N}$, $\mathcal{L}$, $\mathcal{C}$, and PTDF matrices $\{K_c\}_{c \in \mathcal{C}}$. Differences between the DAM and FTR models arise exclusively through: (i) *contingency coverage* – the subset of $\mathcal{C}$ enforced in each market; (ii) *transmission element limits* – the values of $(\underline{b}_c, \overline{b}_c)$ for enforced contingencies.

B. Market-Specific Network Models

The DAM enforces a contingency set $\mathcal{C}^{\mathrm{DAM}} \subseteq \mathcal{C}$, and the FTR auction enforces a contingency set $\mathcal{C}^{\mathrm{FTR}} \subseteq \mathcal{C}$. Associated transmission element lower and upper flow limits are

$$\{(\underline{f}_c, \overline{f}_c)\}_{c \in \mathcal{C}^{\mathrm{DAM}}}, \quad \{(\underline{g}_c, \overline{g}_c)\}_{c \in \mathcal{C}^{\mathrm{FTR}}},$$

where $\underline{f}_c, \overline{f}_c, \underline{g}_c, \overline{g}_c \in \mathbb{R}^\ell$.

To compare markets with different contingency coverage, we extend each market's limits to the universal contingency set $\mathcal{C}$ by assigning infinite bounds to non-enforced contingencies. Stacking limits according to the same fixed ordering as above yields extended transmission element-contingency limits

$$\underline{f}, \overline{f}, \underline{g}, \overline{g} \in (\mathbb{R} \cup \{-\infty, +\infty\})^{m\ell}.$$

When a limit equals $\pm\infty$, the corresponding inequality constraint is interpreted as absent. Accordingly, constraints are indexed only over finite bounds via the active index sets

$$\mathcal{J}^+(\overline{b}) := \{i : \overline{b}_i < +\infty\}, \quad \mathcal{J}^-(\underline{b}) := \{i : \underline{b}_i > -\infty\}.$$

C. Network-Feasible Injection Sets

Definition 1 (Network-Feasible Injection Polytopes). For extended limit vectors $(\underline{b}, \overline{b})$ satisfying $\underline{b} \preceq \overline{b}$, define

$$\mathcal{Q}_{\text{net}}(\underline{b}, \overline{b}) := \left\{ q \in \mathbb{R}^n \,\middle|\, \begin{array}{ll} (Kq)_i \geq \underline{b}_i & \forall i \in \mathcal{J}^-(\underline{b}), \\ (Kq)_i \leq \overline{b}_i & \forall i \in \mathcal{J}^+(\overline{b}), \\ \mathbf{1}^\top q = 0 & \end{array} \right\}.$$

These constraints enforce network physics (via K), active transmission element limits under enforced contingencies, and power balance. By Assumption 1 the market-specific network-feasible sets are

$$\mathcal{Q}_{\text{net}}^{\text{DAM}} := \mathcal{Q}_{\text{net}}(\underline{f}, \overline{f}), \qquad \mathcal{Q}_{\text{net}}^{\text{FTR}} := \mathcal{Q}_{\text{net}}(\underline{g}, \overline{g}).$$

Assumption 2 (Non-emptiness). We assume $\mathbf{0} \in \mathcal{Q}_{\text{net}}^{\text{DAM}}$ and $\mathbf{0} \in \mathcal{Q}_{\text{net}}^{\text{FTR}}$. Hence both polytopes are nonempty, ensuring the maxima considered later are attained. A sufficient condition is $\underline{f}_c \preceq \mathbf{0} \preceq \overline{f}_c$ for all enforced contingencies since zero injections induce zero flows. In practice, this condition is always satisfied as transmission limits are designed to allow zero flows.

In the DAM, each unit $\omega \in \mathcal{U}$ submits quantity bounds $[\underline{q}_\omega^u, \overline{q}_\omega^u]$ and price p_ω^u, mapped to nodal injections by $M_\omega^u \in \{-1, 0, 1\}^n$ with one nonzero entry. In the FTR auction, bid $\beta \in \mathcal{B}$ specifies a source–sink pair with quantity bounds $[\underline{q}_\beta^b, \overline{q}_\beta^b]$ and price p_β^b, mapped by $M_\beta^b \in \{-1, 0, 1\}^n$ with one $+1$ and one -1 entry. DAM prices represent marginal costs (negative for demand), yielding cost minimization; FTR prices represent willingness to pay, yielding value maximization.

Definition 2 (Full Feasible Sets). Combining network feasibility with a matrix representation of bid availability, the full feasible sets are

$$\mathcal{Q}^{\text{DAM}} := \left\{ q \in \mathbb{R}^n \,\middle|\, q = M^u q^u,\ \underline{q}^u \preceq q^u \preceq \overline{q}^u,\ q \in \mathcal{Q}_{\text{net}}^{\text{DAM}} \right\},$$
$$\mathcal{Q}^{\text{FTR}} := \left\{ q \in \mathbb{R}^n \,\middle|\, q = M^b q^b,\ \underline{q}^b \preceq q^b \preceq \overline{q}^b,\ q \in \mathcal{Q}_{\text{net}}^{\text{FTR}} \right\}.$$

We abstract from market features that restrict feasible injections but do not alter network feasibility, such as unit commitment, ramping, and ancillary services in the DAM, and options in the FTR auction. Our focus is on the network constraints that govern structural revenue adequacy and hedging capacity.

D. Market Clearing and Shadow Prices

The DAM minimizes total bid-in cost subject to its full feasibility set,

$$\min_{q^u \in \mathbb{R}^{|\mathcal{U}|}} (p^u)^\top q^u \qquad \text{subject to} \quad q := M^u q^u \in \mathcal{Q}^{\text{DAM}}.$$

Let $q^{\text{DAM}*}$ denote the resulting optimal nodal injections.

Let $y^{+*}, y^{-*} \in \mathbb{R}_+^{m\ell}$ denote the dual multipliers associated with upper and lower bounds on transmission element-contingency constraints respectively, and let s^* denote the dual multiplier on power balance. Define $y^* := y^{+*} - y^{-*}$, the DAM shadow-price vector. The LMP vector is

$$\lambda := s^* \mathbf{1} - K^\top y^* \in \mathbb{R}^n,$$

which decomposes LMPs into an energy component s^* and a congestion adjustment $K^\top y^*$ [8], [9].

Definition 3 (DAM Merchandising Surplus). The congestion rents collected in the DAM by the ISO are precisely

$$\text{MS}^{\text{DAM}}(y^*) := -\lambda^\top q^{(\text{DAM})*} = y^{*\top} K q^{(\text{DAM})*},$$

where the equality follows from power balance [1].

The FTR auction maximizes total bid value subject to its full feasibility set

$$\max_{q^b \in \mathbb{R}^{|\mathcal{B}|}} (p^b)^\top q^b \qquad \text{subject to} \quad q := M^b q^b \in \mathcal{Q}^{\text{FTR}}.$$

Let $q^{\text{FTR}*}$ denote the resulting optimal nodal injections.

Definition 4 (FTR Payout). For a realized DAM shadow-price vector y^* and LMPs λ, the payout of DAM merchandising surplus to FTR positions is

$$\text{PO}^{\text{FTR}}(y^*) := -\lambda^\top q^{(\text{FTR})*} = y^{*\top} K q^{(\text{FTR})*},$$

again where the equality follows from power balance.

Definition 5 (Maximum FTR Payout). For a realized DAM shadow-price vector y^*, the maximum supportable FTR payout is

$$\text{PO}_{\max}^{\text{FTR}}(y^*) := \max_{q \in \mathcal{Q}_{\text{net}}^{\text{FTR}}} y^{*\top} K q.$$

By construction, $\text{PO}^{\text{FTR}}(y^*) \leq \text{PO}_{\max}^{\text{FTR}}(y^*)$ for any $q^{(\text{FTR})*}$.

III. Structural Alignment via Support Functions

We show that both DAM merchandising surplus and the maximum supportable FTR payout coincide with maximizations of a linear functional over the network-feasible polytopes defined in Section II (i.e., support-function evaluations [11]). This identification unifies revenue adequacy and hedging efficiency within a single geometric framework and enables the dual-based attribution developed in Section IV.

Definition 6 (Network Support Function). For extended limit vectors $(\underline{b}, \overline{b})$ and arbitrary price vector y (not necessarily a DAM shadow-price vector), define

$$\phi(\underline{b}, \overline{b}; y) := \max_{q \in \mathcal{Q}_{\text{net}}(\underline{b}, \overline{b})} y^\top K q.$$

Under Assumption 2, the maximum is attained. Geometrically, $\phi(\underline{b}, \overline{b}; y)$ is the signed distance from the origin to the supporting hyperplane of $\mathcal{Q}_{\text{net}}(\underline{b}, \overline{b})$ in the direction $K^\top y$.

Definition 7 (Alignment Gap). For arbitrary price vector y, define

$$\Delta(y) := \phi(\underline{g}, \overline{g}; y) - \phi(\underline{f}, \overline{f}; y).$$

Thus $\Delta(y)$ records whether $\mathcal{Q}_{\text{net}}^{\text{FTR}}$ extends further ($\Delta(y) > 0$) or lies within ($\Delta(y) < 0$) $\mathcal{Q}_{\text{net}}^{\text{DAM}}$ in direction $K^\top y$. Figure 1 illustrates both cases.

Definition 8 (Alignment Ratio). When $\phi(\underline{f}, \overline{f}; y) > 0$, define the normalized alignment ratio

$$\eta(y) := \frac{\phi(\underline{g}, \overline{g}; y)}{\phi(\underline{f}, \overline{f}; y)} = 1 + \frac{\Delta(y)}{\phi(\underline{f}, \overline{f}; y)}.$$

If $\phi(\underline{f}, \overline{f}; y) = 0$, then $\Delta(y) = 0$ and $\eta(y)$ is undefined.

Proposition 1 (Market Interpretation). For any dual-optimal DAM shadow-price vector y^*,

$$\text{MS}^{\text{DAM}}(y^*) = \phi(\underline{f}, \overline{f}; y^*), \quad \text{PO}_{\max}^{\text{FTR}}(y^*) \ = \phi(\underline{g}, \overline{g}; y^*).$$

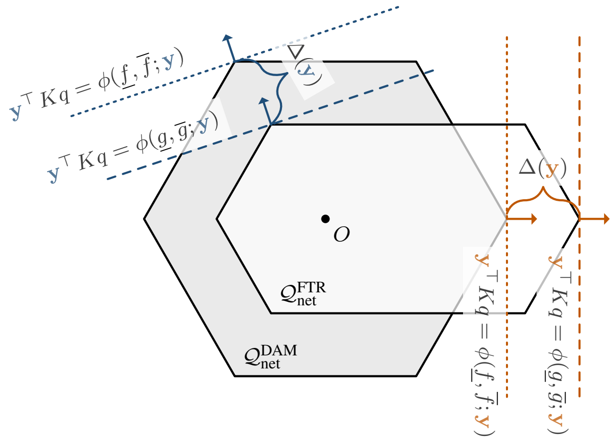


Fig. 1: Support function geometry for DAM and FTR network-feasible polytopes. For each color, the arrows indicate a support normal direction $K^\top y$ in injection space. Supporting hyperplanes (the dashed and dotted lines) are orthogonal to this direction; their points of tangency determine the support values $\phi(\underline{f}, \overline{f}; y)$ and $\phi(\underline{g}, \overline{g}; y)$; and their signed separation measured along the support normal direction is the alignment gap $\Delta(y) = \phi(\underline{g}, \overline{g}; y) - \phi(\underline{f}, \overline{f}; y)$.

Consequently,

$$\Delta(y^*) = \mathrm{PO}^{\mathrm{FTR}}_{\max}(y^*) - \mathrm{MS}^{\mathrm{DAM}}(y^*),$$

so the sign of $\Delta(y^*)$ distinguishes structural underfunding risk ($\Delta(y^*) > 0$) from structural hedging inefficiency ($\Delta(y^*) < 0$) at realized DAM shadow-price vectors.

Proposition 1 yields several immediate consequences. First, alignment is a property of the network models alone: support functions depend only on network topology (K, b) and are independent of bids and offers. Second, alignment is direction-specific: different DAM realizations may induce different DAM shadow-price vectors (y^*) and therefore different alignment outcomes under the same FTR network model. Third, each alignment evaluation reduces to a linear program, enabling tractable ex post diagnosis.

While Proposition 1 characterizes alignment at a realized DAM shadow-price vector y^*, Definitions 6-7 of $\phi(\cdot; y)$ and $\Delta(y)$ for arbitrary price vector y enable robust ex ante FTR network model analysis over sets of price vectors.

IV. Dual Attribution and Constraint-Level Decomposition

This section derives a dual representation of the support-function formulations that attributes alignment gaps to individual transmission element-contingency constraints.

The key observation is that dual feasibility depends only on network geometry K and arbitrary price vector y, and not on the limit values $(\underline{b}, \overline{b})$, so DAM and FTR instances share the same dual-feasible set. Because dual optima are generally non-unique in realistic networks, we later characterize multiplier ranges that are invariant to dual degeneracy.

A. Dual Formulation of the Support Problem

Recall the network-feasible injection polytope from Definition 1, write the two-sided limits as upper bounds, and define the stacked constraint matrix and stacked limit vector

$$A := \begin{bmatrix} K \\ -K \end{bmatrix} \in \mathbb{R}^{2m\ell \times n}, \quad b := \begin{bmatrix} \overline{b} \\ -\underline{b} \end{bmatrix} \in (\mathbb{R} \cup \{+\infty\})^{2m\ell}.$$

The network support problem from Definition 6 can then be written as

$$\begin{aligned} \phi(b; y) := \max_{q \in \mathbb{R}^n} \quad & y^\top Kq \\ \text{subject to} \quad & (Aq)_i \le b_i \quad \forall i \in \mathcal{J}(b), \\ & \mathbf{1}^\top q = 0. \end{aligned}$$

where $\mathcal{J}(b) := \{i : b_i < +\infty\}$. For brevity, we write $\phi(b; y)$ for the stacked representation.

Let $\mu \in \mathbb{R}^{2m\ell}_+$ be multipliers on $Aq \le b$ and let $s \in \mathbb{R}$ be the multiplier on $\mathbf{1}^\top q = 0$. Allowing $b_i = +\infty$ implicitly enforces $\mu_i = 0$ in the dual, so we may write $Aq \le b$ over all indices without loss of generality. The Lagrangian is

$$\mathcal{L}(q, \mu, s) = (y^\top K - \mu^\top A - s\,\mathbf{1}^\top)q + \mu^\top b,$$

and the supremum over q is finite if and only if

$$\mu^\top A + s\,\mathbf{1}^\top = y^\top K,$$

in which case $\sup_q \mathcal{L}(q, \mu, s) = \mu^\top b$.

Proposition 2 (Strong Duality). Under Assumption 2,

$$\phi(b; y) = \min_{\mu \in \mathbb{R}^{2m\ell}_+,\, s \in \mathbb{R}} \mu^\top b \quad \text{s.t.} \quad \mu^\top A + s\,\mathbf{1}^\top = y^\top K.$$

B. Shared Dual-Feasible Set

Definition 9 (Dual-Feasible Set). For any $y \in \mathbb{R}^{m\ell}$, define

$$\mathcal{M}(y) := \left\{(\mu, s) \in \mathbb{R}^{2m\ell}_+ \times \mathbb{R} \,\middle|\, \mu^\top A + s\mathbf{1}^\top = y^\top K \right\}.$$

A key structural property is that the dual feasible set depends only on the network geometry K and arbitrary price vector y, and not on the limit vector b.

Corollary 1 (Shared Feasibility System). Under the extended-limits convention, the DAM and FTR dual support problems share the same feasible set $\mathcal{M}(y^*)$. Moreover, by Proposition 2,

$$\begin{aligned} \mathrm{MS}^{\mathrm{DAM}}(y^*) &= \min_{(\mu,s) \in \mathcal{M}(y^*)} \mu^\top f, \\ \mathrm{PO}^{\mathrm{FTR}}_{\max}(y^*) &= \min_{(\mu,s) \in \mathcal{M}(y^*)} \mu^\top g, \end{aligned}$$

where

$$f := \begin{bmatrix} \overline{f} \\ -\underline{f} \end{bmatrix}, \; g := \begin{bmatrix} \overline{g} \\ -\underline{g} \end{bmatrix} \in (\mathbb{R} \cup \{+\infty\})^{2m\ell},$$

are the stacked DAM and FTR limit vectors. Thus alignment gaps arise solely from differences in the objective vectors f and g, evaluated over common $\mathcal{M}(y^*)$.

C. Constraint-Level Attribution via Dual Weights

Dual variables provide a constraint-level explanation of how the support value $\phi(b; y)$ is achieved in direction $K^\top y$: a positive μ_i indicates that constraint i restricts congestion rents in that direction, while $\mu_i = 0$ indicates an insensitivity to that limit.

Definition 10 (Optimal Dual Sets). For limit vector $b \in \{f, g\}$ and arbitrary price vector y, define

$$\mathcal{S}(b; y) := \arg \min_{(\mu,s)\in\mathcal{M}(y)} \mu^\top b.$$

This set is a (possibly higher-dimensional) face of $\mathcal{M}(y)$.

Proposition 3 (Sensitivity Bounds). For any $(\mu^*, s^*) \in \mathcal{S}(b; y)$ and $\delta > 0$,

$$\phi(b \pm \delta e_j; y) \le \phi(b; y) \pm \delta \mu_j^*,$$

with equality when loosening $(+)$ if $\mu_j^* = 0$, and strict inequality when tightening $(-)$ if $\mu_j^* > 0$.

Thus any vector of dual weights characterizes local one-sided sensitivity of the support value to tightening or loosening individual constraints.

The dual feasibility condition, $\mu^\top A + s\,\mathbf{1}^\top = y^\top K$, requires the support normal $K^\top y$ to lie in the conic hull of the constraint normals (rows of $A = [K; -K]$), up to the power-balance direction $\mathbf{1}$. In security-constrained network models, this system imposes n equations on $2m\ell + 1$ variables; when transmission constraints greatly outnumber nodes ($m\ell \gg n$), the dual-feasible set $\mathcal{M}(y)$ is high dimensional and optima are typically attained on faces rather than at unique points. Linear dependencies among PTDF rows — arising from electrically similar contingencies — and repeated constraint limits further expand these faces by introducing directions along which the dual objective $\mu^\top b$ is invariant. Dual non-uniqueness is therefore structural, motivating a robust attribution framework.

D. Robust Attribution Under Dual Degeneracy

The dual-optimal set $\mathcal{S}(b; y)$ need not be a singleton, and individual constraint multipliers may vary across optimal dual certificates. We therefore characterize constraint importance through robust multiplier ranges, which also offer a two-sided sensitivity of the support value to tightening and loosening the limit of each constraint.

Definition 11 (Robust Multiplier Bounds). For constraint index $i \in \{1, \dots, 2m\ell\}$, limit vector b, and arbitrary price vector y, define

$$\underline{\mu}_{b,i}(y) := \min_{(\mu,s)\in\mathcal{S}(b;y)} \mu_i, \qquad \overline{\mu}_{b,i}(y) := \max_{(\mu,s)\in\mathcal{S}(b;y)} \mu_i.$$

which are computable through auxiliary linear programs.

Proposition 4 (Robust Classification). For each constraint index i,

$$\{\mu_i : (\mu, s) \in \mathcal{S}(b; y)\} = \left[\underline{\mu}_{b,i}(y),\ \overline{\mu}_{b,i}(y)\right],$$

and i falls into exactly one of the following categories:

1) *Definitely binding*: $\underline{\mu}_{b,i}(y) > 0$.
2) *Degenerately binding*: $\underline{\mu}_{b,i}(y) = 0 < \overline{\mu}_{b,i}(y)$.
3) *Definitely slack*: $\overline{\mu}_{b,i}(y) = 0$.

This classification is invariant across all dual optima. Moreover, the interval endpoints, $(\underline{\mu}_{b,i}(y),\ \overline{\mu}_{b,y}(y))$, coincide with the one-sided directional derivatives of $\phi(b; y)$ under loosening and tightening, respectively.

Given a realized DAM outcome with shadow-price vector y^*, ISOs can compute robust multiplier bounds $\underline{\mu}_i^f(y^*), \overline{\mu}_i^f(y^*)$ and $\underline{\mu}_i^g(y^*), \overline{\mu}_i^g(y^*)$ for all transmission constraints using standard LP solvers. Constraints with $\underline{\mu}_i^f(y^*) > 0$ and $\overline{\mu}_i^g(y^*) = 0$ immediately identify sources of hedging inefficiency, while constraints with $\underline{\mu}_i^g(y^*) > 0$ and $\overline{\mu}_i^f(y^*) = 0$ flag underfunding risk contributors. This attribution is robust to dual degeneracy and requires no assumptions about PTDF rank or uniqueness.

V. Classification of Network Misalignment

The dual attribution framework yields constraint-level explanations of structural misalignment between DAM and FTR network models. We first characterize misalignment induced by a single constraint difference, then apply the result to canonical DAM-FTR model differences: uniform FTR model line derates, extra FTR model contingencies, and unplanned DAM outages. Finally, we show that multi-interval FTR products induce intrinsic hedging inefficiency even under identical network models.

Definition 12 (Robust Market Dual Ranges). Fix y. Let $\underline{\mu}_j^f(y)$, $\overline{\mu}_j^f(y)$ and $\underline{\mu}_j^g(y)$, $\overline{\mu}_j^g(y)$ denote the robust dual multiplier bounds from Definition 11 for the DAM and FTR models, respectively.

Proposition 5 (Single Constraint Difference). Suppose $g = f + \delta e_j$ for some $\delta \in \mathbb{R}$. Then

$$\sup_{(\mu,s)\in\mathcal{S}(g;y)} \delta\,\mu_j \ \le\ \Delta(y)\ \le \inf_{(\mu,s)\in\mathcal{S}(f;y)} \delta\,\mu_j.$$

Corollary 2 (Robust Multiplier Characterization). If constraint j is looser in the FTR model ($\delta > 0$) and $\overline{\mu}_j^g(y) > 0$, then $\Delta(y) > 0$, creating underfunding risk precisely when constraint j is definitely or degeneratively binding in the FTR support problem.

If constraint j is tighter in the FTR model ($\delta < 0$) and $\overline{\mu}_j^f(y) > 0$, then $\Delta(y) < 0$, creating hedging inefficiency precisely when constraint j is definitely or degeneratively binding in the DAM support problem.

Otherwise, the bounds collapse and $\Delta(y) = 0$.

A. Canonical Network Model Differences

We accompany this section with a stylized 3-node network model. Details of this toy model are provided in Appendix B, along with visualizations of network feasible polytopes (Figures 3-4), and values of $MS^{\mathrm{DAM}}(y^*)$, $PO^{\mathrm{FTR}}_{\max}(y^*)$, $\mu^g(y^*)$, and $\mu^f(y^*)$ for various DAM shadow-price vectors y^* (Tables II-III).

A common ISO practice is to apply a uniform derate factor $\alpha \in (0, 1]$ to the line limits used in the FTR model [7].

Theorem 1 (Uniform Derate). If $\mathcal{C}^{\mathrm{FTR}} = \mathcal{C}^{\mathrm{DAM}}$ and $g = \alpha f$ for $\alpha \in (0, 1]$, then for any DAM shadow-price vector y^* with $\mathrm{MS}^{\mathrm{DAM}}(y^*) > 0$,

1) $\mathrm{PO}^{\mathrm{FTR}}_{\max}(y^*) = \alpha\,\mathrm{MS}^{\mathrm{DAM}}(y^*)$,

2) $\eta(y^*) = \alpha$, and
3) $\mathcal{S}(g; y^*) = \mathcal{S}(f; y^*)$.

The exact quantification $\eta(y^*) = \alpha$ across all congestion patterns enables precise efficiency-cost tradeoffs. For example, a 25% uniform derate ($\alpha = 0.75$) reduces hedging capacity to exactly 75% of available DAM merchandising surplus, providing ISOs with a quantifiable efficiency cost for this conservative decision.

In our toy model, uniformly derating all FTR line limits by $\alpha = 0.75$ contracts the feasible region (Figure 4a) and yields $\eta(y^*) = 0.75$ across all congestion patterns, with identical DAM and FTR dual multipliers in each pattern (Tables II–III).

We next analyze including an additional contingency in the FTR model that is not enforced in the DAM, a conservative modeling choice that can reduce underfunding risk [12], [13].

Theorem 2 (Extra Contingency). If $c \in \mathcal{C}^{\text{FTR}} \setminus \mathcal{C}^{\text{DAM}}$, then

$$\mathcal{Q}_{\text{net}}^{\text{FTR}} \subseteq \mathcal{Q}_{\text{net}}^{\text{DAM}}, \quad \text{and} \quad \Delta(y^*) \leq 0 \;\forall y \in \mathbb{R}^{m\ell},$$

with strict inequality ($\Delta(y^*) < 0$) if and only if some transmission element constraint c_i, under contingency c, always satisfies $\mu_{c_i}^g > 0$.

In our toy model, adding an outage contingency to the FTR model (but not the DAM) shrinks the feasible region (Figure 4b) and yields $\Delta(y^*) \leq 0$ across all congestion patterns (Table II). In pattern (a), the added contingency prices a transmission element in the FTR dual, producing strict hedging inefficiency with $\Delta(y^*) < 0$ (Table III).

Conversely, an unplanned outage in the DAM may result in a contingency being enforced operationally but omitted from the FTR model.

Theorem 3 (Missing Contingency). If $d \in \mathcal{C}^{\text{DAM}} \setminus \mathcal{C}^{\text{FTR}}$, then

$$\mathcal{Q}_{\text{net}}^{\text{FTR}} \supseteq \mathcal{Q}_{\text{net}}^{\text{DAM}}, \quad \text{and} \quad \Delta(y^*) \geq 0 \;\forall y \in \mathbb{R}^{m\ell},$$

with strict inequality ($\Delta(y^*) > 0$) if and only if some transmission element constraint, d_j, under contingency d, always satisfies $\mu_{d_j}^f > 0$.

Following an underfunding event, ISOs can identify the responsible outages by checking whether unplanned DAM-only contingency constraints satisfy $\underline{\mu}_i^f(y^*) > 0$. Such constraints restrict DAM merchandising surplus and permit underfunding, providing a diagnostic for improving future FTR outage coordination.

In our toy model, enforcing an additional contingency in the DAM model (but not in the FTR model) shrinks the feasible region (Figure 4c) and yields $\Delta(y^*) \geq 0$ across all congestion patterns (Table II). Underfunding occurs in patterns (a)–(b), where the missing contingency is priced in every DAM dual optimal; pattern (c), with no missing contingency constraint priced, yields exact alignment (Table III).

Missing FTR contingencies can permit underfunding, but never increase hedging inefficiency, while extra contingencies can cause hedging inefficiency, but never increase underfunding risk. The robust dual conditions above identify precisely when these effects are economically operative for a given DAM shadow-price vector y^*.

B. Multi-Interval Temporal Inconsistency

Consider a multi-interval FTR product spanning periods $\mathcal{T} = \{1, \ldots, T\}$ with realized DAM shadow-price vectors $\{y_t^*\}_{t \in \mathcal{T}}$, where the FTR auction enforces a *single* injection profile q settled against all intervals.

Theorem 4 (Multi-Interval). If $\mathcal{Q}_{\text{net}}^{\text{FTR}} = \mathcal{Q}_{\text{net}}^{\text{DAM}}$, then for any DAM shadow-price vector y^*,

$$\text{PO}_{\max}^{\text{FTR}}\left(\sum_{t \in \mathcal{T}} y_t^*\right) := \max_{q \in \mathcal{Q}_{\text{net}}^{\text{FTR}}} \sum_{t \in \mathcal{T}} y_t^{*\top} K q \leq \sum_{t \in \mathcal{T}} MS^{\text{DAM}}(y_t^*),$$

with equality if and only if all y_t^* support the same face of $\mathcal{Q}_{\text{net}}^{\text{FTR}}$.

Corollary 3 (Multi-Interval Alignment Ratio). The alignment ratio satisfies

$$\eta_{\text{multi}}\left(\{y_t^*\}_{t \in \mathcal{T}}\right) := \frac{\text{PO}_{\max}^{\text{FTR}}\left(\sum_{t \in \mathcal{T}} y_t^*\right)}{\sum_{t \in \mathcal{T}} \text{MS}^{\text{DAM}}(y_t^*)} \leq 1,$$

with strict inequality whenever congestion patterns vary temporally. This highlights the value of defining FTR contract periods to span temporally similar DAM intervals.

Multiple sources of misalignment can be combined cleanly using support-function identities. For example, under a uniform derate $g = \alpha f$ with no other network differences, Theorem 1 gives

$$\text{PO}_{\max}^{\text{FTR}}(y^*) = \alpha \,\text{MS}^{\text{DAM}}(y^*),$$

and together with Proposition 1 and Corollary 3 we have

$$\eta_{\text{multi}}\left(\{y_t^*\}_{t \in \mathcal{T}}\right) = \frac{\alpha\, \phi\left(f; \sum_{t \in \mathcal{T}} y_t^*\right)}{\sum_t \phi(f; y_t^*)} \leq \alpha.$$

Additional contingency mismatches can be incorporated via containment bounds (Theorems 2-3) in the same manner; the tight constants depend on which contingencies are priced in the relevant DAM shadow-price vectors.

VI. Conclusion

FTR-DAM misalignment remains a persistent source of hedging inefficiency and underfunding risk in wholesale electricity markets [6], [7]. We show that alignment can be characterized geometrically through support functions of network-feasible polytopes, with dual formulations yielding a constraint-level attribution that is robust to dual degeneracy and comparable across market network models. We apply this framework to canonical FTR-DAM network model differences and to temporal aggregation in FTR contract periods, diagnosing each at realized DAM outcomes.

Several extensions follow naturally: ex ante analysis over price and network-model uncertainty, broader connections to polyhedral containment and linear-programming theory [14], [15], richer network-representation choices and their effect on misalignment, and validation on ISO-scale network models.

VII. AI Usage Disclosure

All core concepts, theory, and intuition were developed by the authors, as was all final wording in this copy. AI tools were used as an interactive reviewer to clarify technical arguments and proof logic, and to provide feedback on the paper structure, clarity, and presentation, including suggested rewrites of sentences and paragraphs.

Appendix

A. Proofs

Proposition 1 (Support Function Characterization). We prove $\mathrm{MS}^{\mathrm{DAM}}(y^*) = \phi(\underline{f}, \overline{f}; y^*)$; the FTR identity follows immediately from Definitions 5 and 6.

Let $q^{\mathrm{DAM}*}$ be an optimal DAM injection and let y^{+*}, y^{-*} be the optimal multipliers on the upper and lower monitored element flow constraints in $\mathcal{Q}_{\mathrm{net}}^{\mathrm{DAM}}$, with $y^* := y^{+*} - y^{-*}$.

For any $q \in \mathcal{Q}_{\mathrm{net}}^{\mathrm{DAM}}$, feasibility gives $(Kq)_i \le \overline{f}_i$ for $i \in \mathcal{J}^+(\overline{f})$ and $(Kq)_i \ge \underline{f}_i$ for $i \in \mathcal{J}^-(\underline{f})$. Multiplying by $y^{+*} \ge 0$ and $-y^{-*} \le 0$ and summing yields

$$y^{*\top} K q \le y^{+*\top} \overline{f} - y^{-*\top} \underline{f}.$$

By complementary slackness, equality holds at the realized injection $q^{(\mathrm{DAM})*}$, hence

$$y^{*\top} K q^{(\mathrm{DAM})*} = y^{+*\top} \overline{f} - y^{-*\top} \underline{f}.$$

Substituting into the inequality, we obtain

$$y^{*\top} K q \le y^{*\top} K q^{(\mathrm{DAM})*} \quad \forall q \in \mathcal{Q}_{\mathrm{net}}^{\mathrm{DAM}}.$$

Therefore $q^{(\mathrm{DAM})*}$ attains the maximum of $y^{*\top} K q$ over $\mathcal{Q}_{\mathrm{net}}^{\mathrm{DAM}}$, so

$$\begin{aligned} \mathrm{MS}^{\mathrm{DAM}}(y^*) &= y^{*\top} K q^{(\mathrm{DAM})*} \\ &= \max_{q \in \mathcal{Q}_{\mathrm{net}}^{\mathrm{DAM}}} y^{*\top} K q = \phi(\underline{f}, \overline{f}; y^*). \end{aligned}$$

The same argument applies whenever y^* is the transmission-constraint dual of a linear pricing problem, as in security-constrained unit commitment pricing or ancillary-service co-optimization. □

Proposition 2 (Strong Duality). By Assumption 2, the primal feasible set $\mathcal{Q}_{\mathrm{net}}(\underline{b}, \overline{b})$ is nonempty and compact. The objective is linear, hence the primal optimum is finite and attained. Strong duality and dual attainment then follow from standard linear programming duality theorems for feasible and bounded LPs [16]. □

Proposition 3 (Sensitivity Bounds). Fix y and b, and let $(\mu^*, s^*) \in \mathcal{S}(b; y)$. For any perturbation b', the feasible set $\mathcal{M}(y)$ is unchanged because it depends only on (K, y) and not on b. Hence $(\mu^*, s^*) \in \mathcal{M}(y)$ is feasible for the dual problem with objective $\mu^\top b'$. By optimality,

$$\phi(b'; y) = \min_{(\mu,s) \in \mathcal{M}(y)} \mu^\top b' \;\le\; \mu^{*\top} b'.$$

For tightening with $b' = b - \delta e_j$ ($\delta > 0$),

$$\phi(b - \delta e_j; y) \le \mu^{*\top}(b - \delta e_j) = \mu^{*\top} b - \delta \mu_j^* = \phi(b; y) - \delta \mu_j^*.$$

If $\mu_j^* > 0$, the right-hand side is strictly less than $\phi(b; y)$.

For loosening with $b' = b + \delta e_j$ ($\delta > 0$),

$$\phi(b + \delta e_j; y) \le \mu^{*\top}(b + \delta e_j) = \phi(b; y) + \delta \mu_j^*.$$

If $\mu_j^* = 0$, this gives $\phi(b + \delta e_j; y) \le \phi(b; y)$. Since the primal feasible set expands when b is loosened, we also have $\phi(b + \delta e_j; y) \ge \phi(b; y)$, hence equality holds. □

Proposition 4 (Robust Classification). The set $\mathcal{S}(b; y)$ is convex, and projection onto coordinate i preserves convexity; hence the image

$$\{\mu_i : (\mu, s) \in \mathcal{S}(b; y)\},$$

is an interval. Because $\mathcal{S}(b; y)$ is a closed face of a polyhedron, the extrema $\underline{\mu}_{b,i}(y)$ and $\overline{\mu}_{b,i}(y)$ are attained, so the interval is closed. Moreover, the support function $\phi(b; y)$ is convex in b, and its subdifferential in index i satisfies

$$\partial_{b_i} \phi(b; y) = \{\mu_i : (\mu, s) \in \mathcal{S}(b; y)\},$$

so the directional derivative with respect to constraint index i is generally set-valued, allowing for multiple dual-optimal explanations of the same support value.

With this proved, the classification follows immediately: 1) $\underline{\mu}_{b,i}(y) > 0$ iff $\mu_i > 0$ in every optimal dual certificate (definitely binding); 2) $\underline{\mu}_{b,i}(y) = 0 < \overline{\mu}_{b,i}(y)$ iff some optimal duals assign $\mu_i = 0$ and others assign $\mu_i > 0$ (degenerately binding); 3) $\overline{\mu}_{b,i}(y) = 0$ iff $\mu_i = 0$ in every optimal dual certificate (definitely slack).

Finally, since $\phi(b; y)$ is convex in b and $\mathcal{S}(b; y)$ is its subdifferential, the one-sided directional derivatives in directions $\pm e_i$ equal the extremal subgradients, i.e., $\underline{\mu}_{b,i}(y)$ (loosening) and $\overline{\mu}_{b,i}(y)$ (tightening). □

Proposition 5 (Single Constraint Difference). Fix y and suppose $g = f + \delta e_j$. Since $\mathcal{S}(f; y), \mathcal{S}(g; y) \subseteq \mathcal{M}(y)$, any $(\mu^f, s^f) \in \mathcal{S}(f; y)$ is feasible for the g-dual problem, and any $(\mu^g, s^g) \in \mathcal{S}(g; y)$ is feasible for the f-dual problem. Hence

$$\begin{aligned} \phi(g; y) &\le (\mu^f)^\top g = (\mu^f)^\top f + \delta\, \mu_j^f = \phi(f; y) + \delta\, \mu_j^f, \\ \phi(f; y) &\le (\mu^g)^\top f = (\mu^g)^\top g - \delta\, \mu_j^g = \phi(g; y) - \delta\, \mu_j^g. \end{aligned}$$

Rearranging gives $\delta\, \mu_j^g \le \Delta(y) \le \delta\, \mu_j^f$ for all choices $(\mu^f, s^f) \in \mathcal{S}(f; y)$ and $(\mu^g, s^g) \in \mathcal{S}(g; y)$. Taking sup over $\mathcal{S}(g; y)$ on the left and inf over $\mathcal{S}(f; y)$ on the right yields the claim.

In Corollary 2, the active robust multiplier is determined by the sign of δ. If $\delta > 0$, then

$$\max_{(\mu,s) \in \mathcal{S}(g;y)} \delta\, \mu_j = \delta\, \overline{\mu}_j^g(y), \qquad \min_{(\mu,s) \in \mathcal{S}(f;y)} \delta\, \mu_j = \delta\, \underline{\mu}_j^f(y),$$

while if $\delta < 0$ the extrema select the opposite endpoints. By monotonicity, $\Delta(y)$ has the same sign as δ; thus, if the corresponding upper robust bound is zero, both bounds equal zero and $\Delta(y) = 0$. □

Theorem 1 (Uniform Derate). By Corollary 1, both the DAM and FTR support problems minimize linear objectives over the same dual-feasible set $\mathcal{M}(y^*)$. Since $g = \alpha f$ with $\alpha > 0$, we have

$$\begin{aligned} \arg \min_{(\mu,s) \in \mathcal{M}(y^*)} \mu^\top g &= \arg \min_{(\mu,s) \in \mathcal{M}(y^*)} \alpha\, \mu^\top f \\ &= \arg \min_{(\mu,s) \in \mathcal{M}(y^*)} \mu^\top f, \end{aligned}$$

and therefore 3) $\mathcal{S}(g; y^*) = \mathcal{S}(f; y^*)$. Moreover, for any $(\mu, s) \in \mathcal{M}(y^*)$, $\mu^\top g = \alpha\, \mu^\top f$, so evaluating at an optimizer yields 1)

$$\mathrm{PO}_{\max}^{\mathrm{FTR}}(y^*) = \phi(g; y^*) = \alpha\, \phi(f; y^*) = \alpha\, \mathrm{MS}^{\mathrm{DAM}}(y^*).$$

The expression for 2) $\eta(y^*) = \alpha$ follows immediately. □

Theorem 2 (Extra Contingency). Enforcing an additional contingency adds transmission constraints, so $\mathcal{Q}_{\mathrm{net}}^{\mathrm{FTR}} \subseteq \mathcal{Q}_{\mathrm{net}}^{\mathrm{DAM}}$. By monotonicity of support functions under set inclusion, $\phi(g; y) \le \phi(f; y)$ for all y, hence $\Delta(y^*) \le 0$.

Suppose there exists a dual optimal $(\mu, s) \in \mathcal{S}(g; y^*)$ such that $\mu_{c_i} = 0$ for every c_i under contingency c. Proposition 3 implies that loosening all c-constraints does not change $\phi(g; y^*)$; since the DAM model omits c (equivalently loosening all c_i to infinite bounds), this yields $\phi(g; y^*) = \phi(f; y^*)$ and hence $\Delta(y^*) = 0$.

Conversely, suppose that for every dual optimal $(\mu, s) \in \mathcal{S}(g; y^*)$, there exists at least one constraint c_i under contingency c with $\mu_{c_i} > 0$. If equality $\phi(g; y^*) = \phi(f; y^*)$ were to hold, then by Proposition 3 there would exist a dual optimal with $\mu_{c_i} = 0$ for all c_i, contradicting the assumption. Therefore $\phi(g; y^*) < \phi(f; y^*)$ and $\Delta(y^*) < 0$. □

Theorem 3 (Missing Contingency). The argument mirrors the proof of Theorem 2. Omitting contingency d removes transmission constraints, so $\mathcal{Q}^{\text{FTR}}_{\text{net}} \supseteq \mathcal{Q}^{\text{DAM}}_{\text{net}}$ and $\Delta(y^*) \geq 0$.

If there exists a constraint under d that is priced in every DAM-optimal dual, this yields $\Delta(y^*) > 0$ since the FTR model omits d (equivalent to loosening all DAM d_j to infinite bounds). Conversely, if no such constraints exist, then there is a DAM-optimal dual assigning zero weight to all constraints under d, so omitting d does not change the support value and $\Delta(y^*) = 0$. □

Theorem 4 (Multi-Interval Inefficiency). By definition,

$$\text{PO}^{\text{FTR}}_{\max}\left(\sum_{t\in\mathcal{T}} y_t^*\right) = \max_{q\in\mathcal{Q}_{\text{net}}} \left(\sum_{t\in\mathcal{T}} y_t^*\right)^{\top} Kq = \phi\left(b; \sum_{t\in\mathcal{T}} y_t^*\right).$$

Since $\phi(\cdot)$ is a support function, it is subadditive, so

$$\phi\left(b; \sum_{t\in\mathcal{T}} y_t^*\right) \leq \sum_{t\in\mathcal{T}} \phi(b; y_t^*) = \sum_{t\in\mathcal{T}} \text{MS}^{\text{DAM}}(y_t^*).$$

Equality holds if and only if there exists a single injection $q^* \in \mathcal{Q}^{\text{FTR}}_{\text{net}}$ that simultaneously attains $\phi(b; y_t^*)$ for all $t \in \mathcal{T}$, i.e., all y_t^* support the same face of $\mathcal{Q}^{\text{FTR}}_{\text{net}}$. □

B. Toy Network Model

We use the stylized three-node network model in Figure 2 to illustrate the canonical sources of DAM–FTR misalignment analyzed in Section V. This appendix defines the toy model, constructs the associated network-feasible sets, and documents the congestion patterns used to evaluate alignment and dual attribution. All interpretation is deferred to the main text.

For a given load $q_L \in \mathbb{R}_+$, feasible dispatch is described by injections $q_S, q_C \in \mathbb{R}_+$ satisfying

$$\begin{aligned} q_S + q_C &= q_L, && \text{(power balance)} \\ q_S \leq \overline{q_S}, \quad q_C &\leq 300, && \text{(generator limits)} \\ |f_{SL}| = \left|\tfrac{2}{3}q_S + \tfrac{1}{3}q_C\right| &\leq 75, && \text{(line } SL \text{ limits)} \\ |f_{SC}| = \left|\tfrac{1}{3}q_S - \tfrac{1}{3}q_C\right| &\leq 25, && \text{(line } SC \text{ limits)} \end{aligned}$$

where line flows follow from DC power flow with unit reactances. Coal capacity $\overline{q}_C = 300$ is chosen sufficiently large that it does not bind in the scenarios considered, while solar capacity $\overline{q}_S$ is varied to induce congestion.

Eliminating $q_C = q_L - q_S$ yields the equivalent constraints

$$q_S + q_L \leq 225,$$
$$-75 \leq 2q_S - q_L \leq 75,$$

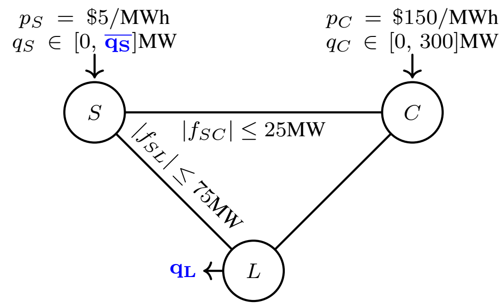


Fig. 2: Three-node toy model with nodes S (solar), C (coal), and L (load), connected in a triangle. Line reactances are normalized to one. Lines SL and SC have finite thermal limits, while line CL is unconstrained.

together with $0 \leq q_S \leq \overline{q_S}$ and $q_L \geq 0$. These inequalities define the feasible region in (q_L, q_S)-space shown in Figure 3.

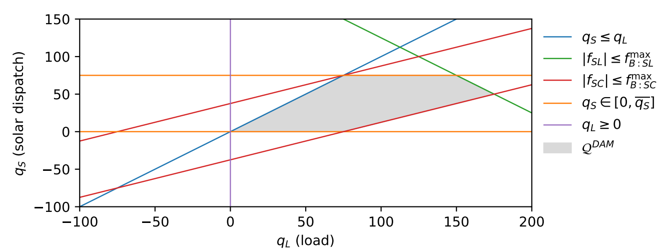


Fig. 3: Network-feasible load q_L vs solar dispatch q_S for the base network with $\overline{q_S} = 75$ MW.

We construct three network model difference variations corresponding to the misalignment mechanisms studied in Section V:

- *Uniform FTR derate:* DAM and FTR share the base network, but all FTR line limits are uniformly reduced to 75% of DAM limits.
- *Extra FTR contingency:* the FTR model includes an additional outage of line SL, while the DAM model enforces only the base network.
- *Unplanned DAM outage:* the DAM model includes an additional outage of line SC, while the FTR model enforces only the base network.

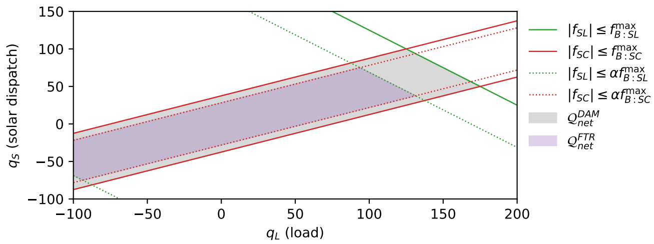
Network-feasible injection polytopes for each difference are shown in Figure 4.

Support functions are evaluated for DAM shadow-price vectors y^* corresponding to the congestion patterns in Table I.

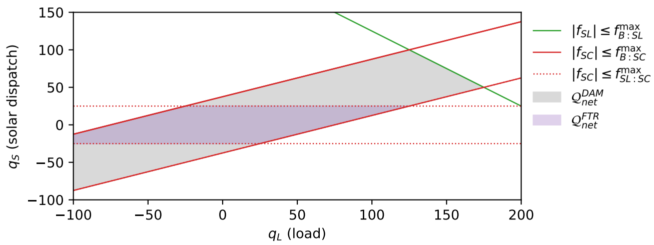
TABLE I: Congestion patterns. Entries such as $+SL$ and $-SC$ indicate binding upper or lower limits on the corresponding line under the indicated contingency.

Pattern	**Base** (B:)	**SL outage** (SL:)	**SC outage** (SC:)
(a)	$+SL$	–	–
(b)	$+SC$	–	–
(c)	$-SC$	–	–

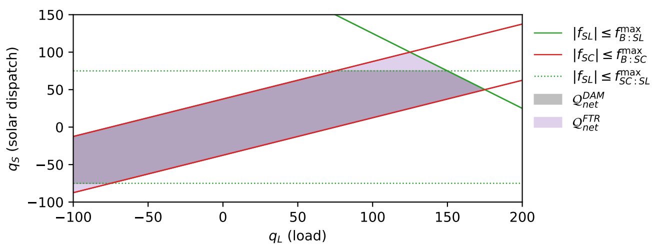
Tables II and III report alignment gaps and support-function dual values across all toy model differences and congestion patterns.



(a) Uniform FTR Derate



(b) Extra FTR Contingency



(c) Unplanned DAM Outage

Fig. 4: Network-feasible injection polytopes under alternative DAM–FTR modeling assumptions. Solid boundaries indicate constraints enforced in the base case; dotted boundaries indicate constraints enforced under an outage.

TABLE II: FTR-DAM alignment by model difference and congestion pattern.

		$MS^{DAM}(y^*)$	$\Delta(y^*)$	$\eta(y^*)$
FTR Derate	(a)	32,625	(8,156)	0.75
	(b)	5,438	(1,359)	0.75
	(c)	1,484	(371)	0.75
Extra FTR Cont.	(a)	32,625	(10,875)	0.67
	(b)	5,438	0	1.00
	(c)	1,484	0	1.00
DAM Outage	(a)	16,583	2,674	1.16
	(b)	10,875	3,625	1.33
	(c)	1,101	0	1.00

In Table III, reported dual values correspond to the difference between multipliers on the upper and lower limits of each line; line CL is omitted since it has no binding constraint. Because the feasible injection space is two-dimensional, the support problems admit a unique optimal dual for generic parameter choices, so robust and non-robust dual values coincide in this example. Although the DAM clearing problem and the DAM support problem have different objectives, the realized DAM shadow-price vector y^* satisfies the support dual optimality conditions in this toy model. In higher-dimensional networks, multiple dual certificates are typical and exact coincidence need not occur.

TABLE III: Support-function dual values by model difference and congestion pattern.

			B:SL	B:SC	SL:SL	SL:SC	SC:SL	SC:SC
FTR Derate	(a)	μ^f	435	0	-	-	-	-
		μ^g	435	0	-	-	-	-
	(b)	μ^f	0	217	-	-	-	-
		μ^g	0	217	-	-	-	-
	(c)	μ^f	0	(59)	-	-	-	-
		μ^g	0	(59)	-	-	-	-
Extra FTR Cont.	(a)	μ^f	435	0	0	0	-	-
		μ^g	0	(435)	0	435	-	-
	(b)	μ^f	0	218	0	0	-	-
		μ^g	0	217	0	0	-	-
	(c)	μ^f	0	(59)	0	0	-	-
		μ^g	0	(59)	0	0	-	-
DAM Outage	(a)	μ^f	114	0	-	-	107	0
		μ^g	221	107	-	-	0	0
	(b)	μ^f	0	0	-	-	145	0
		μ^g	145	145	-	-	0	0
	(c)	μ^f	0	(44)	-	-	0	0
		μ^g	0	(44)	-	-	0	0

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I. Review #44A

A. Paper summary

This paper studies the problem of underfunding vs. conservativeness of financial transmission rights (FTRs) based on differences in the power grid network model used in the FTR auction relative to the network model used during operations. The analysis and associated theory are based on a mathematical formulation with a geometric interpretation.

B. Comments for authors

The paper clearly formulates and systematically analyzes a mathematical formalism for FTR underfunding problems based on differences in the network topology between the FTR auction and actual operations. While somewhat straightforward, the geometric interpretation of the analysis is interesting, and the analysis of dual degeneracy is also especially valuable. The paper addresses an important issue and is clearly written.

I have a few comments regarding suggestions for improvements.

First, while the paper thoroughly addresses the case where a realized day-ahead market outcome is known, an arguably more relevant setting would be the case where the day-ahead outcomes are not known. For instance, one might seek to bound the underfunding or conservativeness of a given set of FTR allocations across a range of possible day-ahead market outcomes (e.g. the worst-case underfunding). When reading Section IV, I was hoping that this would be the direction that the paper would take next, but this is left unanswered. Given that this is a conference paper describing in progress work, I do not think this is a major flaw of the paper, and the conclusions briefly alludes to future work related to ex ante analyses. However, it would be beneficial to further discuss the assumed setting (knowledge of a realized day-ahead market outcome) and associated limitations/opportunities for future work in the introduction and conclusion.

Second, Definition 11 in Section IV-D mentions auxiliary linear programs for computing $\underline{\mu}$ and $\overline{\mu}$. If there is room available in the appendix or perhaps on a preprint version, the paper would be strengthened by including an explicit description of the mathematical formulations for these linear programs.

Third, the paper focuses on differences as network topology as the cause of FTR underfunding. While this is likely to be the dominant cause in practice, another potential source of FTR underfunding results from non-convexity of the optimal power flow feasible set as analyzed in the following reference:

B. C. Lesieutre and I. A. Hiskens, "Convexity of the set of feasible injections and revenue adequacy in FTR markets," in IEEE Transactions on Power Systems, vol. 20, no. 4, pp. 1790-1798, Nov. 2005.

The paper assumes a market structure based on the DC power flow approximation which means that non-convexities are not an issue, but this should be noted in the introduction's literature review.

C. Summarize your recommendation in one to two sentences

While I have some suggestions for some clarifying improvements, my perspective is that this paper makes solid contributions on an important problem and thus merits strong consideration for publication.

— Daniel Molzahn

II. Review #44B

A. Paper summary

The paper analyzes issues stemming from the mismatch between the network topology assumed in auctions for financial transmission rights (FTRs) and the network topology that actually arises in operations. At a high level, mismatches can either lead to underfunding (if the FTR auction sells more rights than can be supported in operations) or incomplete hedging (if the FTR auction does not sell enough).

B. Comments for authors

I enjoyed the paper overall and think there is a lot of good material. The main challenge in my view is that the paper is difficult to follow at times, perhaps resulting from an effort to fit within the prescribed page limit. For example, at several points it would have been useful to include an English description of terms defined mathematically, or to remind the reader what particular notation indicated. I also think the paper would read more cleanly with Figures and Tables brought into the main text, and wonder if the decision to relegate to the Appendix was driven by the page count.

I look forward to seeing the extensions discussed in the Conclusion.

C. Summarize your recommendation in one to two sentences

This paper has a solid theoretical core, and I look forward to reading future extensions.

— Jacob Mays

III. Review #44C

A. Paper summary

This paper develops a geometric and dual-based framework for analyzing structural misalignment between FTR auctions and Day-Ahead Market (DAM) operations. The work aims to provide ISOs with rigorous tools to diagnose underfunding, attribute it to specific constraints, and quantify the efficiency cost of conservative FTR network modeling choices.

The authors formulate both DAM merchandising surplus and maximum supportable FTR payout as support functions of network-feasible injection polytopes. They derive dual LP representations where DAM and FTR alignment problems share a common dual-feasible set, differing only in objective cost vectors. To handle dual degeneracy, they characterize robust multiplier ranges and classify constraints as definitely binding, degeneratively binding, or definitely slack. The framework is applied to canonical scenarios including uniform line derates, contingency mismatches, and multi-interval FTR products.

Key theoretical results include: (1) uniform derates produce hedging inefficiency exactly proportional to the derate factor ($\eta = \alpha$); (2) extra FTR contingencies can cause hedging inefficiency but never permit underfunding, while missing contingencies can permit underfunding but never increase inefficiency; (3) multi-interval FTR products impose intrinsic hedging inefficiency when congestion patterns vary temporally, even under perfect network alignment; and (4) dual-based attribution enables constraint-level diagnosis of underfunding sources robust to dual degeneracy.

B. Comments for authors

This paper makes highly original theoretical contributions to FTR market design. The support function formulation is mathematically elegant and provides a unified geometric perspective on revenue adequacy and hedging efficiency. The recognition that DAM and FTR dual problems share a common feasible set (Corollary 1) is insightful and enables principled constraint-level comparison. The robust multiplier bounds framework (Definition 11, Proposition 4) addresses dual degeneracy in a novel and practically useful way. The exact quantification results for uniform derates (Theorem 1) and multi-interval inefficiency (Theorem 4) are new and operationally valuable. The work represents a significant theoretical advance beyond existing FTR literature.

The mathematical development is rigorous and comprehensive. All propositions and theorems are formally stated with precise assumptions (Assumptions 1-2), and complete proofs are provided in Appendix A. The support function machinery from convex analysis is correctly applied, and addressing constraint classification under degeneracy makes this more applicable to real market outcomes. The dual formulations properly handle extended limits and infinite bounds through index sets $J^+(b)$ and $J^-(b)$. The classification of network model differences (Theorems 1-3) is mathematically precise. The toy model in Appendix B provides concrete numerical validation of theoretical results. The technical quality is outstanding.

The paper is well-organized with clear progression from network models (Section II) to support functions (Section III) to dual attribution (Section IV) to applications (Section V). Definitions are precise and notation is consistent throughout. Figure 1 provides excellent geometric intuition for the alignment gap. However, the material is mathematically dense and assumes substantial background in convex analysis, linear programming duality, and power systems operations.

The target audience is clearly researchers rather than practitioners, which may limit accessibility. Some additional intuitive explanation of key theorems would improve clarity for a broader PowerUp Conference audience. Theorems 1-3 provide some brief, practical discussion. Theorem 4 characterizes inefficiency from temporal aggregation but does not provide further context.

The authors clearly recognized that outage uncertainty and mismatch are central to FTR market misalignment. Additionally, ISO practitioners would be concerned whether the results hold for ex ante FTR market design when solving security-constraint unit commitment (SCUC) of actual day-ahead markets (MIP formulation); a brief discussion in the conclusions of such fundamental limitations (which were already acknowledged in Definition 2) and practical workarounds is another natural extension of this work.

C. Summarize your recommendation in one to two sentences

The paper addresses a critical and timely problem in wholesale electricity markets with the underfunding issues. Recommendation is to accept with minor revisions for publication.

— *Anya Castillo*

IV. Brief response to reviewers (optional)

We thank the reviewers for their careful reading and constructive feedback. Below we respond to each comment.

Reviewer 44A:

- Comment 1 (Ex ante FTR design): See the universal comment on ex ante FTR design.
- Comment 2 (Auxiliary LPs): Space precluded these in the paper; we provide them here. The robust bounds $\underline{\mu}_{b,i}(y)$, $\overline{\mu}_{b,i}(y)$ (Definition 11) are computed by the linear programs

  –

$$\underline{\mu}_{b,i}(y) = \min_{\mu,s} \ \mu_i \quad \text{s.t.} \quad (\mu, s) \in \mathcal{M}(y),\ \mu^\top b = (\mu^*)^\top b,$$

–

$$\overline{\mu}_{b,i}(y) = \max_{\mu,s}\ \mu_i \quad \text{s.t.} \quad (\mu, s) \in \mathcal{M}(y),\ \mu^\top b = (\mu^*)^\top b,$$

– where $\mathcal{M}(y)$ is the dual-feasible set (Definition 9) and $(\mu^*, s^*) \in \mathcal{S}(b; y)$ is any optimal dual.

- Comment 3 (Revenue adequacy from non-convex injection sets): We thank the reviewer for pointing us to Lesieutre-Hiskens. We now cite it at the start of Section II, noting that injection-set non-convexity under AC physics is a distinct revenue-inadequacy source absent under DCOPF.

Reviewer 44B:

- Comment 1 (Clarity): See the universal comment on clarity.

Reviewer 44C:

- Comment 1 (Clarity): See the universal comment on clarity.
- Comment 2 (Interpretation of Theorem 4): We agree an interpretation was warranted. The result highlights the importance of defining FTR contract periods that span DAM intervals with similar congestion patterns; when patterns differ, an intrinsic hedging inefficiency arises even under perfect network alignment. We added a practitioner-facing line to this effect following Corollary 3.
- Comment 3 (Ex ante FTR design): See the universal comment on ex ante FTR design.
- Comment 4 (Results under SCUC pricing / MIP): We agree, and added a note to the proof of Proposition 1 (Appendix A). Unit commitment, ramping, and reserve co-optimization enter only through Q^{DAM} (Definition 2), not the network polytope $Q^{\mathrm{DAM}}_{\mathrm{net}}$. Since the identity $\mathrm{MS}^{\mathrm{DAM}}(y^*) = \phi(\underline{f}, \overline{f}; y^*)$ uses complementary slackness only on transmission-element constraints at the realized y^*, the commitment, ramping, and reserve multipliers never enter; they only affect *which* y^* can be realized. Under MIP pricing, ISOs price the fixed-commitment dispatch LP, whose transmission dual is the same y^*, so the result extends to SCUC pricing.

Universal comment on clarity

All three reviewers noted clarity could be improved. We agree; the conference page limit was the primary driver, and our journal version will prioritize interpretability for a broader practitioner/OR audience.

Universal comment on ex ante FTR design

Reviewer 44A noted that the abstract, introduction, and conclusion did not sufficiently clarify that our tools are applied to realized DAM outcomes, and Reviewer 44C raised the related limitation for ex ante FTR/SFT design. Ex ante analysis over uncertain DAM topologies $(\underline{f}, \overline{f})$ and outcomes (y^*) is a natural extension we are actively pursuing. We added clarifications to the abstract, introduction, and conclusion specifying the realized-outcome setting, and identified ex ante analysis as the prioritized next step.

V. Brief summary of changes made to the final paper

Following the reviewers' comments, we made the following changes:

- *Abstract, Introduction, and Conclusion:* Clarified that the framework is applied to realized DAM outcomes, with ex ante design identified as future work.
- *Section II:* Added a note, citing Lesieutre-Hiskens, that non-convexity of the feasible injection set under AC physics is a distinct source of revenue inadequacy; this does not arise under the convex DC approximation used in the paper.
- *Section V:* Added a practitioner-facing note following Corollary 3 (Multi-Interval Alignment Ratio) on defining FTR contract periods to span DAM intervals with similar congestion patterns, to limit intrinsic hedging inefficiency.
- *Conclusion:* Compressed the discussion of extensions and future work for space.
- *Appendix A:* Added a remark to the proof of Proposition 1 (Market Interpretation, stated in Section III) noting that the identity holds under unit commitment pricing and ancillary-service co-optimization – DAM features abstracted away in the main text. Placed in the appendix rather than the main text because of the page limit restrictions.