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Authors

Hahn, D.
Glendenning, N.K.

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D. Hahn and N.K. Glendenning

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Pion spectra in equilibrium models of nuclear collisions

Detlev Hahn and Norman K. Glendenning

*Nuclear Science Division
Lawrence Berkeley Laboratory
University of California
Berkeley, California 94720*

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Abstract:

We calculate pion spectra within three simple models of pion production in relativistic heavy ion collisions. All three models are based on particle freeze out from thermal and chemical equilibrium. In the first one nucleons, pions and Δ resonances freeze out at a fixed temperature and density. The spectra are a superposition of thermal pions and pions from the decay of the Δ resonance. In the second model we assume that pions are radiated from a hot, spherical source of nuclear matter, which loses energy due to this radiation. In the last calculation we employ a hydrodynamic model of the expansion process to determine the pion spectra produced by a sequential freeze out at fixed density and different times and temperatures. Only this last model gives a shape of the pion distribution similar to the experimental one. It is found that three different effects contribute to the concave shape of the spectra in this model.

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1. Introduction

Recently, exclusive pion spectra have been measured in heavy ion collisions for bombarding energies between 500 and 1800 MeV/nucleon [1,2]. The collisions had been triggered for high multiplicity or central events. The data show a concave shape of the spectra and have been fitted with a superposition of two Boltzmann distributions [1,2]; the temperatures for the two distributions differ widely by roughly a factor of two. The importance of the high temperature component was found to increase with bombarding energy [2] and with the mass [1,2]. These results have been interpreted [1,2] as showing two sources for the pions, to wit: a component originating from the decay of the Δ resonance, which is dominant at low pion energies, and a component arising from a thermal pion gas.

Here we test this hypothesis by calculating the pion spectra which are produced by a conglomerate of nucleons, pions and Δ 's in equilibrium, when the particle distributions freeze out suddenly at a fixed density and temperature.

In addition, we develop two other simple models for pion production and examine the spectra they predict. The idea behind these new approaches is that different slopes in the data are not produced by two different modes of pion production, but rather by a cooling source. First we assume that the pions are emitted from a hot sphere of nuclear matter with constant density that is in chemical and thermal equilibrium and which cools due to its pion radiation. We regard the pion velocities as high compared with the ones of the nucleons. Alternatively, our nuclear sphere may expand according to a simple hydrodynamic model [3], while the pions remain in chemical and thermal equilibrium until the density of the nuclear matter drops below a certain value. Here, the cooling is due to the expansion, since radiation effects are found to be low.

All three models rest on the same physical picture of a relativistic heavy ion collision: a hot and dense blob of nuclear matter is created by the head-on collision of two nuclei, leading to thermalization and equilibration of the matter involved during a stage of high density. This stage lasts about 5 to 10 fm/c [4,5]. Afterwards

the blob expands rapidly [4]. For a discussion of reaction rates see ref. [5] .

We should mention that pion spectra like the ones now measured have been predicted earlier as a signal for the formation of abnormal nuclear matter [6]. They have also been predicted as a manifestation of the cooling of the expanding system [7] in the context of ultrarelativistic collisions. However, we think it prudent to explore more humble explanations first. We start with the description which was suggested in ref. [1,2,5].

2. Pion and Δ freeze out from homogenous nuclear matter

In this section we examine the pion spectra which arise from the sudden break up of a blob of nuclear matter of density ρ_b and temperature T_b . That is, we assume that the expansion from the high density stage is so fast that the particle distributions freeze out simultaneously over the whole blob at some time t after the expansion has started, and that the spacial temperature variation is not important. In this paper, we do not try to correlate the thermal energy at this time t with the bombarding energy to get the compressional energy as has been done previously [5,8]. We are merely interested in the spectra, though we use the absolute number of pions to fix parameters in our models.

We include, apart from the nucleons, free thermal pions and the Δ resonance in the chemical composition of our blob. For the heavy $La + La$ system considered here and the bombarding energies involved, neither the bose condensate of pions nor the heavier resonances play an important role in determining the pion spectra and are therefore left out. The energy distribution and the number of the free pions at a temperature T are given by [8]:

$$f_\pi(\epsilon) = \frac{3V}{2\pi^2\hbar^3} \frac{\epsilon p}{\exp\{\epsilon/T\} - 1} \quad (2.1)$$

$$N_\pi = \int_{m_\pi}^{\infty} f_\pi d\epsilon \quad (2.2)$$

where ϵ and p are the relativistic pion energy and momentum, and the available volume V depends on density and particle number. The velocity of light, c , has been set equal to 1.

For the nucleons and Δ 's we have:

$$f_B(\epsilon) = \frac{g_B V}{2\pi^2 \hbar^3} \frac{\epsilon p}{\exp\{(\epsilon - \mu)/T\} + 1} \quad (2.3)$$

$$N_B = \int_{m_B}^{\infty} f_B d\epsilon \quad (2.4)$$

Here, μ is the baryon chemical potential which we assume to be the same for nucleons and Δ 's and g_B is the statistical weight. B stands either for the nucleon or the Δ .

After the freeze out of the particle distributions, the Δ 's decay into nucleons and pions. The number of the pions per energy intervall thus produced is given by (see Appendix A):

$$h_\pi(\epsilon) = \frac{1}{2} \frac{m_\Delta}{m_\pi} \frac{1}{\sqrt{\gamma^2 - 1}} \cdot \int_{-G}^G \frac{f_\Delta(\epsilon' + \bar{\epsilon}) d\epsilon'}{\sqrt{(\epsilon' + \bar{\epsilon})^2 - m_\Delta^2}} \quad (2.5)$$

with

$$G = \frac{m_\Delta}{m_\pi} \sqrt{\gamma^2 - 1} p$$

$$\bar{\epsilon} = \frac{m_\Delta}{m_\pi} \gamma \epsilon$$

$$\gamma = \frac{m_\Delta^2 + m_\pi^2 - m_{nucl}^2}{2m_\Delta m_\pi} \quad (2.6)$$

Note that γ is determined by the decay process alone. It is connected to the pion energy in the system where the Δ is at rest via

$$\epsilon_\pi(\Delta_{atrest}) = \gamma m_\pi \quad (2.7)$$

The asymptotic slope S_A of the pion distribution 2.5 can easily be determined, if we assume f_Δ to be of Boltzmann type (with temperature T):

$$S_A(\epsilon) = \frac{m_\pi}{m_\Delta} \frac{T}{\gamma - \sqrt{\gamma^2 - 1} \epsilon/p} \quad (2.8)$$

The finite width of the Δ resonance has been implemented by assuming a Lorentzian shape for its energy distribution [8] and summing up the contributions from the different energy bins.

We have to determine two parameters to calculate the energy distribution of the pions, namely the temperature and density at freeze out. By comparing our spectra to the $La + La$ data of ref. [2] we vary the density and determine the temperature by fitting the absolute number of pions measured. The results, for the highest bombarding energy measured, are given in Figs. 1a and b for break up densities of $2\rho_0$ and $0.5\rho_0$, respectively.

The density dependence of these spectra is easily understood: at a high density most of the pions are produced via the Δ resonance and we clearly see the two slopes predicted in ref. [1], a high one of 80 MeV and a lower one of about 50 MeV at 300 MeV kinetic pion energy. At low density we get many more thermal pions ($N_\pi \sim V$, $N_{nucl} + N_\Delta = const$) and consequently have to lower the temperature. The Δ contribution is no longer large compared to the thermal one and the two slope picture becomes less clear.

So, to ensure two distinct slopes in the pion spectra we need a high freeze out density. But a serious discrepancy to the measured data becomes evident here, namely the Δ decay peak around $\epsilon_\pi \sim 250$ MeV in the theoretical spectra. The nonexistence of this peak in the measured pion spectra has been discussed earlier [9]. However, the problem did not appear to be serious after cascade calculations [1] produced a purely thermal pion spectrum, though the cascade creates pions *only* via the Δ resonance. The explanation of this remarkable behaviour was found to be the change in the pionic energy distribution due to the rescattering, which the pions undergo in the cascade after their creation. In other words, according to the cascade code, we do not see the original pion spectrum but a thermalized one,

whose temperature ought to be characteristic for the nucleonic energy distribution at some stage during the expansion process.

Whether this is correct or not, we do not see the decay peak around $\epsilon_\pi = 250$ MeV in the measured spectra. If a sizeable portion of the pions is produced via the Δ resonance, then the memory of this creation in the pionic energy distribution is lost in the later stages of the heavy ion collision. The same should happen at higher pion energies and smear out the different slopes from the thermal and the Δ pions. Therefore we think it unlikely that the different slopes measured in the experiment are caused by a superposition of thermal pions and pions from the Δ . We have to look for some other explanation.

3. Pion radiation from a uniform, cooling sphere of nuclear matter

It might be possible that the pions we see in a relativistic heavy ion collision have a history similar to the light we get from the sun; radiation from a surface with a spectrum characteristic of its temperature. This is the idea behind the model examined in this section. We assume that a blob of uniform hot nuclear matter with a spherical shape is created during the heavy ion collision. The blob is supposed to remain stationary for the life time of the high density stage, typically 10 fm/c [4]. We make this assumption to isolate the cooling effect of radiation from the one caused by expansion. The latter we investigate in the next section.

Pions are supposed to be in chemical equilibrium inside the sphere and we don't include the Δ resonance in our model. Apart from the experimental finding of the nonexistence of the Δ peak in the pion spectra, theoretical reasons can be put forward for this neglect of the Δ , namely: i.) the mass difference between the nucleon and the resonance could be higher in nuclear matter than for the free nucleon case [10]; ii.) the lifetime of the Δ in nuclear matter might be long compared to the Δ - nucleon scattering time in the high density stage and thus the Δ degree of freedom might cool together with the nucleons until it disappears; iii.) as suggested by the cascade the pion rescattering after the Δ decay might change the original pion

distribution enough that it becomes a signal for the temperature of the surrounding nuclear matter and not one of the creation process. In the latter case only the number of pions present, not their spectrum, would reveal the presence of the Δ .

Now we want to find out if pion radiation cools our sphere of nuclear matter fast enough to explain the different slopes in the experimental spectra. The flux of pions per energy intervall emitted from a sphere with temperature T at rest can be calculated (see App. B) to give

$$j_\pi(\epsilon, T) = \frac{d^2 N_\pi}{dt d\epsilon} = \pi v(\epsilon) R^2 \frac{f_\pi(\epsilon, T)}{V} \quad (3.1)$$

where R is the radius of the sphere, v the pion velocity at energy ϵ and the function f has been defined in eq. 2.1.

Then the energy loss due to this pion flux in a time dt is

$$dE_{th} = - \left[\int_{m_\pi}^{\infty} \epsilon j_\pi(\epsilon, T) d\epsilon \right] dt \quad (3.2)$$

while the total thermal energy of the sphere is given by [8]:

$$\begin{aligned} E_{th} &= \int_{m_\pi}^{\infty} \epsilon f_\pi(\epsilon, T) d\epsilon \\ &+ \int_{m_{nuc}}^{\infty} \epsilon f_{nuc}(\epsilon, T, \mu) d\epsilon - \int_{m_{nuc}}^{\infty} \epsilon f_{nuc}(\epsilon, T_0 = 0, \bar{\mu}) d\epsilon \end{aligned} \quad (3.3)$$

This thermal energy plus the compressional one add up to the available center of mass energy. The chemical potentials μ and $\bar{\mu}$ are chosen so that the nucleon density is equal to the desired break up density. Equations 3.1, 3.2 and 3.3 give us the the time dependence of the temperature, thermal energy and the pion flux. The thermal energy at time zero is used as a parameter to fit the total pion number emitted.

We test our calculations again at the highest bombarding energy of ref. [2] (1350 MeV/nucleon) and assume a density of $2.5\rho_0$, the maximal density in this collision according to a VUU calculation [11]. The high density ensures a maximal radiation. The radiation time is set to be 10 fm/c, which is roughly the life time of the high density stage [4].

In order to obtain the measured number of pions per nucleon we have to set the compressional energy equal to zero for these parameters, that is $E_{cm} = E_{th}$. The resulting spectrum is shown in Fig. 2a. Though the temperature has dropped from 132 to 113 MeV during the 10 fm/c radiation time, this is not enough to produce different slopes in the spectrum. Moreover, these temperatures are too high and we see a sudden drop at zero kinetic energy in the pion spectra, due to the factor v in eq. 3.1, which does not appear in the data [2].

The last two problems can be overcome by combining the models of the sections 2 and 3, i.e assuming that after the 10 fm/c radiation time a subsequent break up takes place, which sets the thermal pions inside the nuclear matter free. Results of this approach are shown in Fig. 2b. The temperatures are more realistic though still high, they fall now from 112 MeV to 99 MeV. Their difference, however, is even smaller than in the calculation shown in Fig. 2a and we see definitely only one slope.

These results show that the effect of pion radiation is to slow to cool nuclear matter by more than a few MeV during the high density stage. In addition, the trend of the pion radiation relating to the mass of the system goes in the wrong direction: a smaller mass should experience a bigger cooling effect than a heavier system, which is opposite to what the data show. We are left with the possibility of cooling by expansion.

4. Sequential freeze out from an expanding sphere

Hydrodynamic models have been successfully employed for the simulation of high energy heavy ion collisions [4]. In a hydrodynamic picture nuclear matter is always in thermal and chemical equilibrium and the density and temperature vary both in space and time. Within such a model let us assume now that pions stay in equilibrium with nuclear matter in each cell of the hydro-calculation until a certain density is reached [12]. Then pions would freeze out at different times during the expansion stage of the heavy ion collision and, supposedly, they would freeze out at different temperatures and from nuclear matter with different flow velocities. Hot

pieces of nuclear matter should freeze out first and colder ones later. In a realistic hydrodynamic model we could use the total number of pions emitted to fit the unknown freeze out density. Parts of this program have been realized in ref. [12].

Here we want to give qualitative results obtained within a very simple, analytically solvable hydrodynamic model [3], which should show us the shape of the spectra and trends like the bombarding energy dependence. The model gives us the spatial and time distributions of temperature and density in an expanding, spherical blob of nuclear matter (see Fig. 3). We handle the pion production independently from the hydrodynamic expansion. Our pions do not exchange energy or momentum with nuclear matter but are simply set free if the nuclear density falls below a certain value ρ_b . Radiation effects are neglected since they were found to be unimportant.

Apart from the independence of the pion creation from the movement of the nuclear matter, the hydrodynamic model put forward in [3] has serious flaws in itself: it is nonrelativistic, it uses the ideal gas equation of state and the relation

$$E_{th}/nucl = 1.5 T \quad (4.1)$$

to correlate temperature and thermal energy. Therefore, we expect only qualitative agreement with the data.

To obtain the pion spectra, we divide the expansion time into small steps and calculate in each timestep the volume of the shell of nuclear matter, which has passed to a density smaller than ρ_b since the previous timestep. We also determine the average temperature and radial velocity in this shell. According to eq. 2.1, the volume and the temperature give us the pionic energy distribution $f_\pi(\epsilon)$, which we boost with the flow velocity v_s of the sphere to get the distribution in the center of mass system (see App. A):

$$f_{\pi,cm}(\epsilon) = \frac{1}{2} \frac{1}{\sqrt{\gamma^2 - 1}} \int_{-G}^G \frac{f_\pi(\epsilon' + \bar{\epsilon}) d\epsilon'}{\sqrt{(\epsilon' + \bar{\epsilon})^2 - m_\pi^2}} \quad (4.2)$$

with

$$\gamma = 1 / \sqrt{1 - v_s^2}$$

$$\bar{\epsilon} = \gamma \epsilon$$

$$G = \sqrt{\gamma^2 - 1} p \quad (4.3)$$

Summing up over all the shell distributions gives us the pion spectrum.

As mentioned above the hydrodynamics we use here has some unrealistic features, which, in particular, lead to very high temperatures. To compensate for this, we assume that one third of the energy in the center of mass system is bound in compressional energy. Together with a break up density of $0.5\rho_0$ we then obtain results surprisingly close to the experimental ones both for the total number of pions and for the average slopes.

We examine the pion spectra produced by our model in Figures 4 and 5, again for the case of a $La + La$ collision at 1350 MeV/nucleon. The influence of the shape parameter α , defined in ref. [3], is shown in Fig. 4a. For all three values of α plotted here, the spectrum has the concave shape, which has been found experimentally [2], though this is more apparent for higher α 's. We think that the most realistic density distributions are obtained with $\alpha \sim 3$ [4] and have chosen this value for the remainder of this section.

Fig. 4b shows the contributions from the different time steps to the pion distribution. It reveals surprising features for us. The concave shape obtained in the summed up spectrum is not alone due to contributions from volume elements with different temperatures, but is apparent in the distributions produced in each single timestep. To display the degree of concavity we have indicated here (and in later Figures) the asymptotic slopes of the summed up pion distribution by plotting the tangents at 50 and 550 MeV (dotted lines).

A closer examination shows that three effects are equally important producing the concave shape of the spectra (see Fig. 5). The superposition of pions from volume elements with different temperatures alone produces asymptotic slopes, which are 10 MeV apart (Fig. 5), if we use Boltzmann distributions; the introduction of the Bose-Einstein distribution instead of Boltzmann doubles this difference and, finally, the boost of the pion distributions with the flow velocity v_s leads to a dif-

ference of 30 MeV in the asymptotic slopes and a definitely concave shape of the pion distribution.

Note that the flow velocities calculated with the hydrodynamic model of ref. [3] are not unrealistic high, they are of the order of one third of the velocity of light at the break up density (see Fig. 6a). The number of pions produced is dominated by contributions from medium temperatures (see Fig. 6b and c), even though the volume of the shells, which freeze out in each time step, is biggest at the later times and lower temperatures (Fig. 6d). It might be that one of the effects mentioned above, which produce the concave shape of the pion spectra, becomes more important than the others in a realistic hydrodynamic calculation, since we are here sensitive to the details of the expansion.

In Fig. 7 we display the trends of the pion spectra obtained in our model concerning the bombarding energy - and mass dependence. The concavity becomes less pronounced at lower bombarding energies (Fig. 7a), which coincides with experimental findings [2]. The nucleon number, however, seems to have no influence on the concave shape of the spectra (Fig. 7b), the difference in the asymptotic slopes is 30 MeV for both systems considered.

We have also looked for the existence of a pionic chronometer. It turns out that energetic pions (above 300 MeV) stem mostly from early times in the reaction, while pions with kinetic energies below 150 MeV are predominantly a signal from a phase of the expansion process, which is 6 fm/c older (see Fig. 8). However, this effect is not a very clear one. The whole process of pion freeze out lasts 12 fm/c in our model for the $La + La$ system at 1350 MeV/nucleon.

5. Summary

In conclusion we think it improbable that the concave shape of the pion spectra found in refs. [1,2] can be explained with the assumption of two different pion sources, the decay from the Δ resonance on the one hand and thermal pions on the other. Effects of a cooling source during the era of pion production, the Bose-Einstein distribution and the boost of the thermal pion distributions with the collec-

tive velocity work together in the model presented in section 4 to produce a concave shape of the pionic energy distribution.

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Appendix

A Energy distribution after a decay process

We want to calculate the relativistic energy distribution of particles originating from a decay process. To be more specific: let us assume that we have N particles with mass m_F and energy ϵ' distributed in a reference system S_1 according to a function $f(\epsilon')$:

$$N = \int_{m_F}^{\infty} f(\epsilon') d\epsilon' \quad (6.1)$$

Now each particle m_F shall decay into two particles with masses m_1 and m_2 and we want to know the energy distribution of one of the decay products, e.g. m_1 .

The energy of particle m_1 in the system S_2 , where m_F is at rest, can be calculated easily from energy - and momentum conservation:

$$\epsilon_0 = \gamma m_1 = \frac{1}{2} \frac{m_F^2 + m_1^2 - m_2^2}{m_F} \quad (6.2)$$

We denote the corresponding velocity of m_1 in S_2 by v_0 . Its distribution in S_2 is spherical.

Now let dN denote the number of particles m_1 , whose parent particle m_F had a velocity between v' and $v' + dv'$ before the decay. Then

$$d^2N = \frac{1}{2v_0} dv_{0,z} dN = \frac{1}{2v_0} dv_{0,z} f(\epsilon'(v')) \frac{d\epsilon'}{dv'} dv' \quad (6.3)$$

is the fraction of dN where the component of $\vec{v}_0$ in S_2 parallel to $\vec{v}'$ is between $v_{0,z}$ and $v_{0,z} + dv_{0,z}$. This fraction turns out to be independent of $v_{0,z}$. (Of course, $|v_{0,z}| \leq v_0$)

All these d^2N particles have the same velocity v in S_1 :

$$v = \frac{\sqrt{v'^2 + 2v'v_{0,z} + v_0^2 - v'^2v_0^2 + v'^2v_{0,z}^2}}{1 + v'v_{0,z}} \quad (6.4)$$

We need the inverse relation (at fixed v')

$$v_{0,z}(v) = \frac{1}{v'} \left(\frac{\sqrt{1 - v'^2 - v_0^2 + (v'v_0)^2}}{\sqrt{1 - v^2}} - 1 \right) \quad (6.5)$$

to write d^2N in equation 6.3 as the number of particles m_1 with velocity v in S_1 , whose parent particle had the velocity v' in S_1 :

$$\begin{aligned} d^2N &= \frac{1}{2v_0} f(\epsilon'(v')) \frac{d\epsilon'}{dv'} dv' \frac{dv_{0,z}}{dv} dv \\ &= \frac{1}{2v_0} f(\epsilon'(v')) \frac{\epsilon' v'}{1 - v'^2} dv' \frac{v}{v'} \frac{\sqrt{1 - v'^2} \sqrt{1 - v_0^2}}{\sqrt{1 - v^2}^3} dv \end{aligned} \quad (6.6)$$

Integrating this expression over all contributing velocities dv' from

$$v_l = |v - v_0| / (1 - vv_0) \quad (6.7)$$

to

$$v_u = (v + v_0) / (1 + vv_0) \quad (6.8)$$

gives the total number of particles m_1 with velocities between $v, v + dv$ in S_1 :

$$g(v) = m_F \frac{v}{2v_0} \frac{\sqrt{1 - v_0^2}}{\sqrt{1 - v^2}^3} \int_{v_l}^{v_u} \frac{f(\epsilon'(v'))}{1 - v'^2} dv' \quad (6.9)$$

Rewriting this expression in terms of energies, we obtain the desired expression 2.5 for $h(\epsilon) = g(v)dv/d\epsilon$

$$h_\pi(\epsilon) = \frac{1}{2} \frac{m_F}{m_1} \frac{1}{\sqrt{\gamma^2 - 1}} \cdot \int_{-G}^G \frac{f_F(\epsilon' + \bar{\epsilon}) d\epsilon'}{\sqrt{(\epsilon' + \bar{\epsilon})^2 - m_F^2}} \quad (6.10)$$

G and $\bar{\epsilon}$ have been defined in eq. 2.6.

If we follow our calculations from eq. 6.3 on and interpret v_0 as a radial flow velocity superimposed on a distribution $f(\epsilon)$ of particles with mass m_1 , we obtain eq. 4.2. This latter equation has been found earlier [13].

B Flux from a hot sphere

We want to calculate the pion flux from a sphere of nuclear matter (temperature T , radius R), where pions are in chemical and thermal equilibrium. A pion inside the sphere, whose distance to the surface is dr and whose velocity v forms an angle θ to the normal vector of the surface, escapes in a time dt , if

$$dr < v dt \cos \theta \quad (6.11)$$

The number of pions with an energy ϵ and this angle θ to the surface, which escape during dt , is therefore

$$dN_{\epsilon, \theta} = \frac{f_{\pi}(\epsilon)}{V} d\epsilon \frac{\sin \theta d\theta}{2} 4\pi R^2 v dt \cos \theta \quad (6.12)$$

with $\theta < \pi/2$. Integration over θ yields the flux (eq. 3.1)

$$\frac{d^2 N_{\pi}}{dt d\epsilon} = \frac{3}{2\pi(\hbar)^3} \frac{\epsilon p}{\exp\{\epsilon/T\} - 1} v R^2 \quad (6.13)$$

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Fig.1: The pionic energy distribution divided by the momentum and the energy versus the kinetic energy $\epsilon - m_\pi$ of the pions. The calculations have been done in the sudden freeze out picture for the $La + La$ system at a bombarding energy of 1350 MeV per nucleon. The dotted curve shows the contribution from the Δ decay, the dashed curve the one from the thermal pions and the solid one the sum of the two contributions. The break up density is $2 \rho_0$ in Fig. a and $0.5 \rho_0$ in Fig. b.

Fig.2: The same quantities as in Fig. 1 for the same system calculated with the radiation picture. A density of $2.5 \rho_0$ has been used to ensure a maximal influence of the radiation. Fig. a shows the spectrum obtained with the radiation alone, Fig b the one built up by a radiation period of 10 fm/c and a subsequent break up.

Fig.3: Time dependence of the temperature and density distributions obtained with the hydrodynamic model of ref.[3]. We show the results for two values of the shape parameter α . The different curves in each Figure refer to different times during the expansion. The solid curve with the highest density or temperature depicts the distribution at t_0 [3], the other curves 4, 8 and 12 fm/c later.

Fig.4: The quantities shown are the same as in Fig. 1 for the same system. In Fig. a we show the influence of the shape parameter α of the hydrodynamic model of ref. [3] on the pion spectra. In Fig. b the contributions to the final pion distribution (solid line) from each single timestep (dashed and dotted-dashed lines) are shown. The two dotted straight lines have been drawn to convince our readers that we really have different slopes in the pionic energy distribution. They are tangents on the solid line at 50 and 550 MeV kinetic pion energy, respectively.

Fig.5: The solid line in this Figure is the same as in Fig.4 b. The upper dashed line has been calculated by neglecting the collective motion in our calculation of the pion spectra (set γ equal to one in eqs. 4.2 and 4.3) The same has been done for the calculation of the lower dashed line, where we also have used the Boltzmann distribution instead of Bose-Einstein. The dotted lines are tangents on the pion spectra at 50 and 550 MeV.

Fig.6: For the $La + La$ system at 1350 MeV per nucleon bombarding energy we show the time development of some thermodynamic quantities according to ref. [3]. We look at a fixed density of $0.5\rho_0$ and subtract one third of the available energy as bound in compression. In Fig. a and b we have the flow velocity and the temperature at this density, calculated for each timestep. Fig. 6c depicts the pion production in each timestep and Fig. 6d the volume, which falls below the break up density.

Fig.7:

a.) Energy dependence of the pion spectra in the $La + La$ system. For a shape parameter $\alpha = 3$ and a break up density of $0.5\rho_0$ we show the spectra for bombarding energies of 1350 MeV/nucleon (solid line), 750 MeV/nucleon (dashed line) and 530 MeV/nucleon. The quantities depicted are the same as in Fig. 1. The tangents on the curves (dotted lines) have been described in Fig. 4.

b.) Mass dependence of the pion spectra at a bombarding energy of 1350 MeV/nucleon. The solid curve is for the $La + La$ system, the dashed one for the $Ar + KCl$ case. Parameters are the same as in Fig. a. Again the dotted lines are tangents on the pion spectra.

Fig.8:

The pionic energy distribution versus the kinetic pion energy for the $La + La$ system at 1350 MeV/nucleon (solid line). The dashed line shows the distribution of the pions, which have frozen out during the first 6 fm/c of the break up, the dotted line the contribution of the pions from the second half of the break up process, again lasting 6 fm/c.

*LAWRENCE BERKELEY LABORATORY
TECHNICAL INFORMATION DEPARTMENT
UNIVERSITY OF CALIFORNIA
BERKELEY, CALIFORNIA 94720*